Borrowing calculus

Syntax

$$
\begin{array}{rcll}
e & ::= & & \text{Expressions:} \\
& \mid & x & \text{– variable} \\
& \mid & {}^{\overline{x}}\{e\} & \text{– borrow} \\
& \mid & \mathbf{fn}(\overline{x :^{q} \tau})\ e_0 & \text{– function} \\
& \mid & (\overline{e}) & \text{– tuple} \\
& \mid & C^n(\overline{e}^n) & \text{– variant} \\
& \mid & [\overline{e}] & \text{– list} \\
& \mid & e_0(\overline{e}) & \text{– application} \\
& \mid & \mathbf{let}_{q_0}\ (\overline{x}) = e_0;\ e & \text{– binding} \\
& \mid & \mathbf{match}_{q_0}\ e_0\ \{\overline{C^n(\overline{x}^n) \mapsto e}\} & \text{– match} \\
& \mid & \mathbf{fold}_{q_1}\ e_1, e_2, \{x_1, x_2 \mapsto e_3\} & \text{– fold}
\end{array}
$$

$$
\begin{array}{rcll}
v & ::= & & \text{Values:} \\
& \mid & \mathbf{fn}(\overline{x :^{q} \tau})\ e_0 & \text{– function} \\
& \mid & (\overline{v}) & \text{– tuple} \\
& \mid & C^n(\overline{v}^n) & \text{– variant} \\
& \mid & [\overline{v}] & \text{– list}
\end{array}
$$

$$
\begin{array}{rcll}
\tau & ::= & & \text{Types:} \\
& \mid & (\overline{\tau^{q}}) \to \tau_0 & \text{– function} \\
& \mid & (\overline{\tau}) & \text{– tuple} \\
& \mid & \langle\overline{C^n(\overline{\tau})}\rangle & \text{– variant} \\
& \mid & [\overline{\tau}] & \text{– list}
\end{array}
$$

Typing

$$\boxed{\Gamma^{+} \vdash e^{+} :^{q^{+}} \tau^{-} \rightsquigarrow \Gamma^{-}}$$

Variables

$$\text{Var}_1\ \frac{}{\Gamma, x :^{1} \tau \vdash x :^{1} \tau \rightsquigarrow \Gamma} \qquad \text{Var}_\mu\ \frac{}{\Gamma, x :^{\mu} \tau \vdash x :^{\mu} \tau \rightsquigarrow \Gamma, x :^{\mu} \tau}\ \mu \in \{\varepsilon, \omega\}$$

$$\text{Weak}\ \frac{\Gamma \vdash x :^{\omega} \tau \rightsquigarrow \Gamma}{\Gamma \vdash x :^{1} \tau \rightsquigarrow \Gamma} \qquad \text{Borrow}\ \frac{\Gamma, \overline{x :^{\varepsilon} \tau} \vdash e :^{q} \tau \rightsquigarrow \Gamma', \overline{x :^{\varepsilon} \tau}}{\Gamma, \overline{x :^{\nu} \tau} \vdash {}^{\overline{x}}\{e\} :^{q} \tau \rightsquigarrow \Gamma', \overline{x :^{\nu} \tau}}\ \nu \in \{1, \omega\}$$

Introduction

$$\text{Fun}\ \frac{\Gamma_0, \overline{x :^{q} \tau}^{n} \vdash e_0 :^{1} \tau_0 \rightsquigarrow \Gamma_1}{\Gamma_0 \vdash \mathbf{fn}(\overline{x :^{q} \tau}^{n})\ e_0 :^{-} (\overline{\tau^{q}}^{n}) \to \tau_0 \rightsquigarrow \Gamma_1 \div \overline{x}^{n}}$$

$$\text{Tuple}\ \frac{\forall_{i \in 1..n} \quad \Gamma_i \vdash e_i :^{1} \tau_i \rightsquigarrow \Gamma_{i+1}}{\Gamma_1 \vdash (\overline{e}^{n}) :^{-} (\overline{\tau}^{n}) \rightsquigarrow \Gamma_{n+1}} \qquad \text{List}\ \frac{\forall_{i \in 1..n} \quad \Gamma_i \vdash e_i :^{1} \tau \rightsquigarrow \Gamma_{i+1}}{\Gamma_1 \vdash [\overline{e}^{n}] :^{-} [\tau] \rightsquigarrow \Gamma_{n+1}}$$

$$\text{Con}\ \frac{\Sigma \Vdash C^{n} : (\overline{\tau}^{n}) \to \tau \qquad \forall_{i \in 1..n} \quad \Gamma_i \vdash e_i :^{1} \tau_i \rightsquigarrow \Gamma_{i+1}}{\Gamma_1 \vdash C^{n}(\overline{e}^{n}) :^{-} \tau \rightsquigarrow \Gamma_{n+1}}$$

Elimination

$$\text{App}\ \frac{\Gamma \vdash e_0 :^{\varepsilon} (\overline{\tau^{q}}^{n}) \to \tau \rightsquigarrow \Gamma_1 \qquad \forall_{i \in 1..n} \quad \Gamma_i \vdash e_i :^{q_i} \tau_i \rightsquigarrow \Gamma_{i+1}}{\Gamma \vdash e_0(\overline{e}^{n}) :^{-} \tau \rightsquigarrow \Gamma_{n+1}}$$

$$\text{Let}\ \frac{\Gamma_0 \vdash e_0 :^{q_0} (\overline{\tau}^{n}) \rightsquigarrow \Gamma_1 \quad \Gamma_1, \overline{x :^{q_0} \tau}^{n} \vdash e :^{1} \tau \rightsquigarrow \Gamma_2}{\Gamma_0 \vdash \mathbf{let}_{q_0}\ (\overline{x}^{n}) = e_0;\ e :^{-} \tau \rightsquigarrow \Gamma_2}$$

$$\text{Match}\ \frac{\Gamma \vdash e_0 :^{q_0} \tau_0 \rightsquigarrow \Gamma_1 \qquad \forall_{i \in 1..m} \quad \Sigma \Vdash C_i^{n_i} : (\overline{\tau}^{n_i}) \to \tau_0 \quad \Gamma_i, \overline{x :^{q_0} \tau}^{n_i} \vdash e_i :^{1} \tau \rightsquigarrow \Gamma_{i+1}}{\Gamma \vdash \mathbf{match}_{q_0}\ e_0\ \{\overline{C^{n}(\overline{x}^{n})}^{m}\} :^{-} \tau \rightsquigarrow \Gamma_{n+1}}$$

$$\text{Fold}\ \frac{\Gamma_1 \vdash e_1 :^{q_1} [\tau_1] \rightsquigarrow \Gamma_2 \quad \Gamma_2 \vdash e_2 :^{1} \tau_2 \rightsquigarrow \Gamma_3 \quad \Gamma_3, x_1 :^{q} \tau_1, x_2 :^{1} \tau_2 \vdash e_3 :^{1} \tau \rightsquigarrow \Gamma_4}{\Gamma_1 \vdash \mathbf{fold}_{q_1}\ e_1, e_2, \{x_1, x_2 \mapsto e_3\} :^{-} \tau \rightsquigarrow \Gamma_4}$$