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THE ASTRONOMICAL JOURNAL VOLUME 78, NUMBER 3 APRIL 1973

Monochromatic phase curves and albedos for the lunar disk*

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(Received 12 December 1972)

Photoelectric observations of the entire lunar disk were made in 1964–1965 over phase angles $6° \leq i \leq 120°$ in nine narrow bands (0.35–1.0μ) and in *UBV*. Phase curves are presented as a function of wavelength; the results confirm a reddening with increasing phase angle found by previous investigators for particular areas. Observations before full Moon appear systematically brighter ($\lesssim 0.1$ mag in V) than points after at equivalent phase angles. Differences in spectral reflectivity between our data for 1964 and 1965, and also among earlier studies, are discussed, including the possibility of time-dependent emission (luminescence). Geometric albedos, phase coefficients, phase integrals, and Bond albedos are presented at all wavelengths, excluding any opposition effect. Values of the phase integral, traditionally assumed to be independent of wavelength, are found to increase with wavelength ($0.53 \leq q(\lambda) \leq 0.68$). The radiometric albedo of the Moon is estimated to be $A^* = 0.123 \pm 0.002$.

INTRODUCTION

THE integrated light of the Moon has not been studied in detail since the photoelectric observations of Rougier (1934, 1937). Rougier's phase curve, which refers to a mean wavelength of 4450 Å, is generally cited as the most accurate observational curve available for the lunar disk (Hapke 1971; Van Diggelen 1958). An accurate phase curve in the visual region (λ5500 Å) was determined earlier by Russell (1916a). Despite the small discrepancies between the two curves, the variation with wavelength of the phase curve of the lunar disk, and thus the wavelength dependence of the Moon's phase integral, has not been investigated and has been assumed to be negligible (e.g., Harris 1961; Petrova 1966).

The extensive literature on photometry of the Moon has been reviewed in several places (Minnaert 1961; Fessenkov 1962; Barabashev 1962; Kopal 1966; McCord 1968; Hapke 1971). Much of the work has been concerned with photometric properties of small areas on the Moon rather than the whole lunar disk, and, until recently, has been quite restricted in wavelength. Further studies of the lunar disk, covering a wide range of wavelength, are important since the Moon is used as a standard for interpreting observations of other planets, satellites, and asteroids. For example, the phase integral for the Moon is frequently used for estimating photometric properties of satellites for which this parameter and thus the Bond albedo cannot be determined directly (e.g., Harris 1961). The photometric properties of the Moon are also used for calibration purposes in rocket and satellite observations.

I. THE OBSERVATIONS AND DATA REDUCTION

In order to obtain basic data on the wavelength variation of the photometric properties of the entire lunar disk, the Moon was observed during 1964–1965 as part of a program of photoelectric photometry of the brighter planets directed by the Harvard College Observatory. The equipment, reduction procedures, and planetary results have been presented in a series of papers (Young and Irvine 1967; Irvine *et al.* 1968a, 1968b).

The Moon was observed over phase angles $6° \leq i \leq 120°$ in 9 narrow bands isolated by interference filters between 0.35 and 1.0μ and in *UBV* at the Le Houga Observatory in southern France. The effective wavelengths, accurate to about ±3 Å, and half-widths of the passbands, labeled with lower case letters, are shown in Table I. The light from the Moon passed

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TABLE I. Effective wavelengths of passbands.[a]

Band	u	s	p	m	l	k	h	g	e	U	B	V
λ_{eff} (Å)	3590	3926	4155	4573	5012	6264	7297	8595	10 635	3615	4400	5480
halfwidth	120	45	90	85	90	160	200	90	770	580	1020	800

[a] For details, see Young and Irvine (1967).

through an $f/15$ fused quartz "moon lens" of 20-mm diameter directly into the photometer. The resulting image in the focal plane was comparable in size to that of the planets observed as part of the same program with a 12 inch reflector and the same photometer. A diaphragm of 10-mm diameter was used in front of the moon lens to define the entrance pupil of the system and to further reduce the lunar brightness. During most of 1964, this diaphragm was placed 325 mm in front of the moon objective, while from November 1964 through 1965, it was immediately adjacent to the lens. Observations under consistent conditions have been grouped together and reduced separately.

Extensive observations of standard stars were made in order to define the color of the Sun in our observing system. We have taken $B-V$ for the Sun equal to 0.65 (Van den Bergh 1965) and the visual magnitude of the Sun as $V_\odot=-26.81$ (Harris 1961). The narrow band magnitudes of the Sun were estimated by interpolation among the magnitudes of standard stars as a function of $B-V$. The narrow band magnitude scales were then adjusted so that all colors are color excesses relative to the Sun; that is, for any narrow band x, $(x-V)_\odot=0$. Estimates of the accuracy involved in this determination are given by Irvine *et al.* (1968a).

The atmospheric extinction coefficients in each waveband were obtained from observations of standard stars several times during an observing night. The extinction program is described in detail by Young and Irvine (1967). In addition, selected standard stars and Jupiter were frequently observed with both the moon lens and the 12-inch reflector used for the planetary observations. A transformation of the form

$$m_1=m_1'+a(m_1'-m_2')+b \tag{1}$$

was then investigated in order to convert the Moon observations to the standard telescope system used for reporting the planetary results, where primed values refer to the Moon lens system and m_1 and m_2 are measured at adjacent wavelengths. A single transformation was determined for the data from November 1964 through 1965 (henceforth referred to for simplicity as the 1965 data) using comparison objects observed over the same period as the Moon measurements. For the narrow bands, one would expect the color term in Eq. (1) to be very small. In fact, the difference in transformed lunar magnitudes with and without a color term was ≤ 0.01 mag except for the bands $g(\sim 0.03)$, $e(\sim 0.02)$, $U(\sim 0.02)$, and $B(\sim 0.03)$. (The color term is somewhat uncertain for filters k and g, since its value depends heavily on a single observation of α Boo, a star with a lunar-like spectrum. The most frequent comparison object, Jupiter, and also the other comparison stars, were bluer than the Moon at these wavelengths.) The color term in the 1964 observations (i.e., observations prior to November 1964) was poorly determined because of the similarity in color of the comparison objects for that period. In consequence, we have set $a\equiv 0$ for all transformations to the standard system on which data is presented in this paper. Table II lists the standard errors in the transformation coefficient b for 1964 and 1965 on the assumption $a=0$. We shall use the 1965 data in a later section to compute the lunar albedo, in spite of some uncertainty in the red-infrared region of the spectrum, because of the greater internal consistency of the transformation as evidenced in Table II.

Observations were omitted for which gain set or voltage errors were evident or for which the extinction was obviously poorly determined. The latter included cases of large extinction residuals and cases where the residuals showed a trend during the night. Observations taken at exceptionally large air mass (≥ 3.5) were also omitted.

After correction for extinction, the observations were corrected to unit distance from the Sun and mean Earth-Moon distance (60.2665 Earth equatorial radii; *Explanatory Supplement* 1961). The lunar observations are presented in Table III. The phase angle i is defined as the angle at the center of the Moon between the

TABLE II. Transformation to the standard system.[a]

Wavelength (Å)	3590	3926	4155	4573	5012	6264	7297	8595	10 635	U	B	V
ϵ_{1964}	0.03	0.02	0.01	0.01	0.01	0.02	0.01	0.05	0.06	0.02	0.02	0.02
ϵ_{1965}	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.02	0.01	0.01

[a] ϵ is the standard error in coefficient b in Eq.(1) for $a\equiv 0$. "1964" refers to the data prior to November 1964, "1965" to subsequent dates.

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Table III. Monochromatic magnitudes of the Moon.

No.	Date	Phase	3590	3926	4155	4573	5012	6264	7297	8595	10 635	U	B	V	
1	30 July 64	+ 76.81	− 9.69	− 9.94	−10.02	−10.18	−10.31	−10.73	−10.91	−11.03	−11.14	− 9.12	− 9.49	−10.45	*
2	24 Aug 64	+ 21.53	−11.53	−11.71	−11.72	−11.85	−11.93	−12.27	−12.42	−12.50	−12.58	−10.76	−11.19	−12.07	*
3	24 Aug 64	+ 22.71	−11.62	−11.78	−11.82	−11.93	−12.01	−12.35	−12.48	−12.55	−12.64	−10.86	−11.22	−12.08	*
4	25 Aug 64	+ 33.96	−11.16	−11.36	−11.39	−11.52	−11.61	−12.01	−12.16	−12.25	−12.36	−10.41	−10.81	−11.70	
5	25 Aug 64	+ 35.02	−11.11	−11.28	−11.35	−11.49	−11.58	−11.94	−12.10	−12.18	−12.30	−10.35	−10.75	−11.66	
6	30 Aug 64	+100.53	− 8.90	− 9.10	− 9.21	− 9.37	− 9.50	− 9.85	−10.04	−10.12	−10.30	− 8.20	− 8.60	− 9.54	
7	21 Sept 64	+ 6.60	−12.01	−12.17	−12.24	−12.33	−12.43	−12.74	−12.88	−12.93	−13.06	−11.30	−11.58	−12.43	
8	22 Sept 64	+ 16.71	−11.61	−11.77	−11.84	−11.96	−12.05	−12.45	−12.60	−12.67	−12.80	−10.91	−11.22	−12.12	
9	17 Nov 64	− 24.65	−11.51	−11.65	−11.75	−11.88	−11.97	−12.43	−12.56	−12.59	−12.76	−10.86	−11.17	−12.06	
10	18 Nov 64	− 10.70	−11.98	−12.12	−12.21	−12.32	−12.41	−12.84	−12.97	−12.98	−13.14	−11.30	−11.61	−12.47	
11	09 Dec 64	−118.56	− 8.20	− 8.39	− 8.46	− 8.66	− 8.79	− 9.21	− 9.45	− 9.50	− 9.72	− 7.42	− 7.93	− 8.95	*
12	12 Dec 64	− 83.84	− 9.83	−10.00	−10.05	−10.22	−10.32	−10.75	−10.92	−11.00	−11.20	− 9.05	− 9.51	−10.45	
13	10 Jan 65	− 90.31										− 8.82	− 9.28	−10.21	
14	09 Feb 65	− 83.69	− 9.92	−10.13								− 9.17	− 9.54	−10.55	*
15	09 Mar 65	−102.17	− 9.14	− 9.33	− 9.34	− 9.52	− 9.64	−10.11	−10.29	−10.35	−10.54	− 8.42	− 8.78	− 9.77	
16	07 May 65	− 95.19	− 9.44	− 9.68	− 9.64	− 9.83		−10.40	−10.53	−10.56	−10.84	− 8.59	− 9.09	−10.01	*
17	08 May 65	− 82.11	− 9.89	−10.14	−10.11	−10.28	−10.38					− 9.08	− 9.58	−10.48	
18	10 May 65	− 55.62	−10.75	−10.99	−10.95	−11.09	−11.20	−11.68	−11.84	−11.87	−12.09	− 9.95	−10.42	−11.29	
19	11 May 65	− 42.86	−11.04	−11.28	−11.23	−11.39	−11.50	−11.99	−12.14	−12.19	−12.39	−10.26	−10.73	−11.59	
20	11 May 65	− 41.47	−10.94	−11.22	−11.16	−11.35	−11.46	−11.97	−12.15	−12.18	−12.39	−10.15	−10.67	−11.58	*
21	12 May 65	− 30.36	−11.42	−11.65	−11.60	−11.74	−11.83					−10.64	−11.07	−11.92	
22	12 May 65	− 29.23	−11.30	−11.57	−11.53	−11.71	−11.82					−10.52	−11.01	−11.91	*
23	13 May 65	− 18.17	−11.77	−12.01	−11.95	−12.08	−12.17	−12.61	−12.76	−12.75	−12.95	−11.00	−11.41	−12.23	
24	17 June 65	+ 44.24	−10.87	−11.12	−11.12	−11.27	−11.37	−11.84	−11.99	−12.03	−12.17	−10.11	−10.58	−11.47	
25	09 July 65	− 41.28	−11.00	−11.26	−11.27	−11.43	−11.54	−12.03	−12.20	−12.25	−12.36	−10.27	−10.76	−11.65	*
26	10 July 65	− 30.15	−11.30	−11.54	−11.56	−11.71	−11.81	−12.29	−12.44		−12.58	−10.60	−11.06	−11.94	*
27	11 July 65	− 19.17	−11.76	−11.96	−11.98	−12.10	−12.21	−12.62	−12.76	−12.78	−12.89	−10.95	−11.44	−12.31	*
28	08 Aug 65	− 37.51	−11.10	−11.38	−11.36	−11.52	−11.62	−12.24	−12.31	−12.38	−12.48	−10.37	−10.86	−11.70	*
29	11 Aug 65	− 6.62	−12.00	−12.23	−12.24	−12.37	−12.46	−12.98	−13.06	−13.06	−13.21	−11.21	−11.68	−12.53	
30	14 Sept 65	+ 47.74	−10.91	−11.13	−11.14	−11.27	−11.36	−11.83	−11.96	−12.01	−12.15	−10.14	−10.59	−11.46	
31	11 Oct 65	+ 17.23	−11.72	−11.94	−11.94	−12.05	−12.14	−12.61	−12.73	−12.76	−12.87	−11.02	−11.37	−12.20	
32	12 Oct 65	+ 29.27	−11.34	−11.56	−11.56	−11.68	−11.77	−12.28	−12.42	−12.44	−12.57	−10.63	−11.02	−11.86	
33	07 Nov 65	− 15.65						−12.69	−12.84	−12.87	−12.98				*

Notes (M = air mass)

(1) 30 July 64: large extinction.
(2) 24 Aug 64: $M=2.6$; large extinction.
(3) 24 Aug 64: large extinction.
(11) 09 Dec 64: $M=2.5$.
(14) 09 Feb 65: large extinction.
(16) 07 May 65: clouds; extinction poorly determined.
(20) 11 May 65: $M=3.2$.
(22) 12 May 65: $M=2.5$.
(25) 09 July 65: $M=2.9$.
(26) 10 July 65: $M=2.8$.
(27) 11 July 65: $M=2.9$.
(28) 08 Aug 65: $M=2.8$.
(33) 07 Nov 65: clouds; extinction poorly determined.

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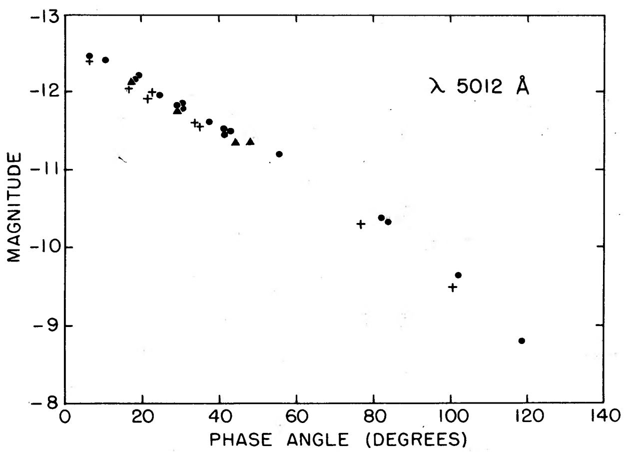

FIG. 1. Phase curve for the lunar disk at λ5012 Å. *Closed circles* (●) are 1965 negative phase angle data; *triangles* (▲) are 1965 positive phase angle data. *Crosses* (+) are 1964 data (all positive phase angle).

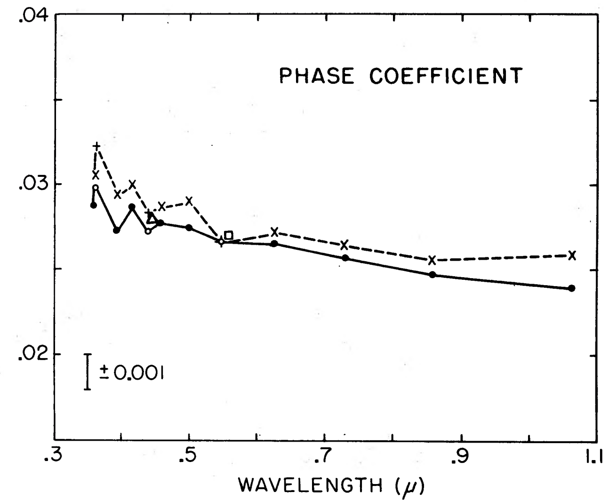

FIG. 2. Phase coefficient of the lunar disk versus wavelength (excluding the opposition effect). *Open* (○) and *closed circles* (●) (broad and narrow bands) are from our 1965 data; *crosses* and ×'s are from our 1964 data. *Triangle* from Rougier (1934); *square* (□) from Gehrels *et al.* (1964). *Error bar* is appropriate to our 1965 data. Somewhat larger errors apply to our 1964 data (see Table IV).

Sun and the observing station on Earth; negative phase angles are before full Moon while positive occur after full Moon. The starred observations in Table III are those taken under sky conditions such that an extinction problem seemed possible but was not obvious enough to cause outright rejection. In addition, all observations with air mass ≥ 2.5 are starred.

II. PHASE CURVES

A plot of the data for a typical wavelength is shown in Fig. 1. (The starred observations in Table III are included and are not distinguished as they showed no anomalies with the rest of the data). The 1964 (prior to November) data is systematically low relative to the 1965 (November 1964 and later) data. The 1964 data consists exclusively of observations obtained after full Moon (see Table III), while the 1965 observations were made primarily before full Moon. Since the fraction of the illuminated disk covered by dark maria is greater after full Moon than before, we expect the observations before full Moon to be systematically brighter than those after at equivalent phase angles. Rougier (1934) found that between quadrature and full Moon, the waxing Moon is from 0.01 to 0.09 magnitudes brighter than the waning Moon. The difference as measured by Russell (1916a) was in the same direction but more extreme (up to 0.20 mag).

It does not appear that our entire 1964–1965 difference can be explained by the before–after effect at all wavelengths, since the 1965 "after" points are not as faint relative to 1965 "before," as are the 1964 points, particularly for $\lambda \geq 6250$ Å. The difference between the 1964 and 1965 points is $\Delta m \lesssim 0.1$, being somewhat greater at longer wavelengths and at large phase angles. It is possible that systematic errors in the results, perhaps as a result of the transformation between moon lens and telescope, are present. The alternative possibility of real time differences in the lunar spectrum due to luminescence is discussed in Section III. The 1964 and 1965 data have hereafter been treated separately.

In order to obtain the phase coefficients of the lunar disk, a linear least-squares fit was made to the data with $i \leq 60°$ for each passband. The phase coefficients as a function of wavelength for 1964 and for 1965 are presented in Table IV and Fig. 2, together with the corresponding standard errors. The results agree well with earlier determinations for B and V (e.g., Rougier 1934; Gehrels *et al.* 1964).

A cubic fit to the data at all phase angles was found to almost exactly overlap the linear fit at phase angle

TABLE IV. Phase coefficient for the lunar disk ($\times 10^{-4}$ mag/deg).[a]

Wavelength (Å)	3590	3926	4155	4573	5012	6264	7297	8595	10 635	U	B	V
1964	305 ± 29	295 ± 28	301 ± 26	287 ± 24	290 ± 22	272 ± 18	265 ± 14	257 ± 14	261 ± 18	322 ± 24	283 ± 26	267 ± 19
1965	288 ± 13	273 ± 11	287 ± 10	278 ± 9	275 ± 9	266 ± 9	257 ± 7	248 ± 8	240 ± 10	298 ± 14	273 ± 9	267 ± 9

[a] "1964" refers to the data prior to November 1964, "1965" to subsequent dates.

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TABLE V. Phase curves for the lunar disk (in mag.).[a]

Wavelength (Å)	3590	3926	4155	4573	5012	6264	7297	8595	10 635	*U*	*B*	*V*
0°	0	0	0	0	0	0	0	0	0	0	0	0
10°	0.29	0.27	0.29	0.28	0.27	0.27	0.26	0.25	0.24	0.30	0.27	0.27
20°	0.58	0.55	0.57	0.56	0.55	0.53	0.52	0.50	0.48	0.60	0.55	0.53
30°	0.86	0.82	0.86	0.83	0.82	0.80	0.77	0.74	0.72	0.89	0.82	0.80
40°	1.15	1.09	1.15	1.11	1.10	1.06	1.03	0.99	0.96	1.19	1.09	1.07
50°	1.41	1.34	1.41	1.37	1.36	1.32	1.28	1.23	1.20	1.45	1.35	1.31
60°	1.66	1.60	1.67	1.62	1.62	1.58	1.54	1.49	1.44	1.72	1.61	1.57
70°	1.93	1.88	1.95	1.90	1.89	1.86	1.82	1.77	1.70	2.00	1.90	1.84
80°	2.23	2.19	2.26	2.20	2.20	2.17	2.13	2.08	1.99	2.31	2.21	2.14
90°	2.59	2.56	2.61	2.55	2.54	2.52	2.48	2.43	2.33	2.66	2.58	2.48
100°	3.01	2.99	3.02	2.95	2.94	2.93	2.87	2.82	2.72	3.08	2.99	2.87
110°	3.51	3.50	3.50	3.42	3.40	3.41	3.32	3.27	3.18	3.56	3.47	3.32
120°	4.11	4.10	4.06	3.98	3.94	3.97	3.83	3.78	3.72	4.14	4.02	3.84

[a] See Section II.

$i=40°$; therefore, we have taken as the most accurate phase curve a combination of the two curves: linear through 40° and cubic thereafter. The extrapolation to zero phase thus excludes the opposition effect. A sample curve is plotted in Fig. 3 for filter V. The standard deviation per observation from the least squares curves is typically $\sim$0.05 mag, giving an estimate of the accuracy of the curves in Table V.

The resultant phase variation of the light reflected from the Moon is presented in Table V in magnitudes for each of our pass bands. The data is from the 1965 observations only, since these covered the most complete phase angle range, and are thus weighted towards negative phase angles. The mean phase curves of Russell (1916a) and Rougier (1934) are given in Table VI for comparison, where Russell's curve has been normalized to agree with our V curve at $i=10°$.

From Table V, it can be seen that the light falls off progressively more slowly with phase angle as wavelength increases. These results constitute the first comprehensive data on the wavelength variation of the phase curve for the lunar disk over the extended visible region. The discrepancy between the curves of Rougier and Russell is confirmed to be a wavelength dependence. The present results thus confirm the reddening with phase angle found for particular areas by a number of previous investigators both in the visible (Gehrels *et al.* 1964; Mikhail 1968, 1970; Peacock 1968) and the near infrared (Hayakawa *et al.* 1968). In addition, a reddening with phase has been observed for some asteroids and Mars (Gehrels *et al.* 1964; Irvine *et al.* 1968b).

Another view of the color-phase phenomenon is shown in Fig. 4 in which the lunar spectrum (see Sec. III) is plotted over the entire wavelength range for several phase angles. Points are taken from the phase curves in Table V.

The phase-reddening is frequently expressed in terms of the reddening factor or coefficient, which is equivalent to a difference of phase coefficient between two wavelengths. It is clear from Fig. 2 and Table IV that the reddening will be largest at shorter wavelengths since the phase coefficient depends more strongly on wavelength for $\lambda<0.5\mu$. In addition (Mikhail 1968),

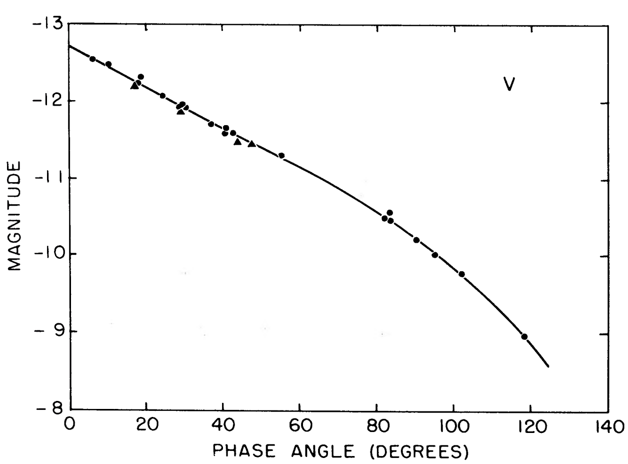

FIG. 3. Phase curve for the lunar disk for V. Data is from 1965 only; *circles* are negative phase angles, *triangles* are positive phase angles. *Line* is the phase curve described in Sec. II (see also Table V).

TABLE VI. Phase curves of Russell (1916a) and Rougier (1934) in mag.[a]

	Russell (V)	Rougier (B)
0°	0.05	0
10°	0.27	0.28
20°	0.51	0.56
30°	0.77	0.84
40°	1.01	1.13
50°	1.27	1.40
60°	1.52	1.69
70°	1.81	2.00
80°	2.12	2.34
90°	2.47	2.74
100°	2.87	3.11
110°	3.30	3.52
120°	3.80	3.98

[a] Average of before and after full Moon. Russell's curve has been normalized to agree with our V curve (see Table V) at $i=10°$.

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TABLE VII. Magnitudes at full Moon.[a]

Wavelength (Å)	3590	3926	4155	4573	5012	6264	7297	8595	10 635	U	B	V
$m(1,0)$	−12.23 ± .04	−12.41 ± .04	−12.45 ± .03	−12.56 ± .03	−12.65 ± .03	−13.11 ± .03	−13.22 ± .02	−13.22 ± .03	−13.35 ± .03	−11.51 ± .05	−11.87 ± .03	−12.72 ± .03

[a] Linear extrapolation of 1965 data (excludes opposition effect). Errors are standard errors.

the reddening depends on the specific region on the Moon being observed (probably through its albedo), on the sign of the phase angle (before–after full Moon effect), and on the range of phase angles covered. The latter effect occurs because of a sharp increase in reddening near opposition. Reddening coefficients obtained from the results in Fig. 2 are in reasonably good agreement with those of Mikhail (1970) and of Peacock (1968). The reddening coefficients obtained by Gehrels *et al.* are somewhat larger than those indicated by the present work because those authors included a number of points at rather small phase angles.

III. SPECTRAL REFLECTIVITY

We obtain magnitudes at full Moon excluding the opposition effect by a linear extrapolation of the (1965) phase curve to zero phase angle (Table VII). The absolute visual magnitude of the Moon is $V=-12.72$ and the $B-V$ color is 0.85, in close agreement with previous values (e.g., $V=-12.74$, Harris 1961; $B-V=0.84$, Gehrels *et al.* 1964; 0.85, Teifel 1959). Our value of $U-B=0.36$ is slightly bluer than that found by Gehrels *et al.* (0.40) or by Harris (0.46).

The lunar spectrum at full Moon for each of our two observing years is plotted in Fig. 5, together with a comparison spectrum from McCord and Johnson (1970) based on telescope measurements of a small area in Mare Serenitatis. As pointed out in Sec. II, our 1965 spectrum is brighter at all wavelengths than our 1964 results (at least partly due to before–after full Moon differences); we have thus normalized the three curves in Fig. 5 to agree at 0.50μ in order to facilitate comparison of the curve shapes.

In Fig. 6, we have plotted our 1965 data at $i=90°$ with comparison spectra from Harris (1961) and Mitchell and Pellicori (1970), normalized to our curve at V, and from Barabashev and Chekirda (1956), normalized at 0.50μ. Since our 1965 data is heavily weighted with before full Moon points, we have averaged only the observations at negative phase angles (all near $i=-90°$) from the data for different features given in Table II of Mitchell and Pellicori. The values from Barabashev and Chekirda are also averages over different lunar features; the phase angle for these observations is not known. The Harris

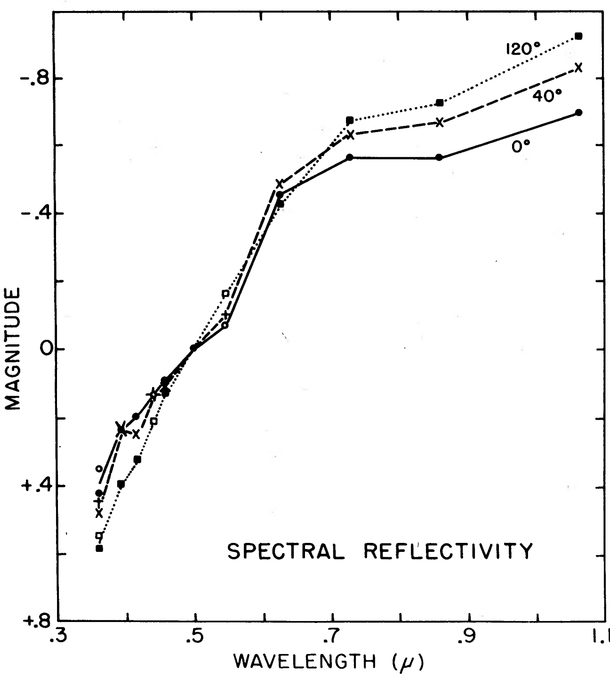

FIG. 4. Lunar spectrum relative to the Sun for three phase angles. *Open* and *closed circels* (broad and narrow bands) are for phase angle $i=0°$; *crosses* and ×'s are for $i=40°$; *open and closed squares* are for $i=120°$. The three data sets are normalized to agree at 0.50μ.

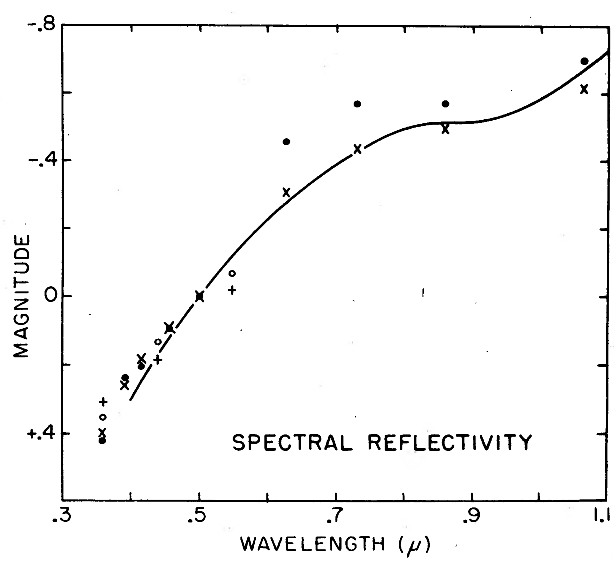

FIG. 5. Lunar spectral reflectivity. *Open and closed circles* (broad and narrow bands) are from our 1965 data; *crosses* and ×'s are from our 1964 data. *Solid line* is from McCord and Johnson (1970). The three data sets are normalized to agree at 0.50μ.

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observations, also of undetermined phase angle, refer to the whole lunar disk.

From Figs. 5 and 6, it is seen that in the wavelength region $\lambda \lesssim 0.55\mu$ our spectra agree very well with the earlier studies, particularly when one notes that the McCord and Johnson observations (Fig. 5) should be redder than ours since they do not refer to full Moon. Our V results at zero phase appear slightly faint with respect to the narrow band data, perhaps because of transformation problems associated with large filter band width (see Sec. I).

The situation for $\lambda \gtrsim 0.6\mu$ is more complex. Our 1964 spectrum (Fig. 5) agrees very well with McCord and Johnson's spectrum at the longer wavelengths, while our 1965 data, believed to be more accurate than the 1964 results, shows an excess intensity in the region $0.60 \lesssim \lambda \lesssim 0.85\mu$. The comparison results of Harris, Mitchell and Pellicori, and Barabashev and Chekirda plotted in Fig. 6 agree reasonably well with our 1965 data (here given at $i=90°$) and are thus distinctly redder than McCord and Johnson in this spectral region, even taking into account the possible phase angle difference. It should be noted that the results of Harris, Mitchell, and Pellicori, and Barabashev and Chekirda all agree very well with McCord and Johnson at $\lambda \lesssim 0.55\mu$.

Further evidence in the literature for the upturn we see near $\lambda \sim 0.6\mu$ in our 1965 lunar spectrum at zero phase is contradictory. Younkin (1970) measured only the crater Plato and found a spectrum he describes as "approximately linear" from 0.4 to 0.8μ, with several possible low-level (3%–6%) absorptions and luminescent features. Petrova (1966) finds the spectra of four different areas to vary smoothly from 0.5 to 0.7μ, with a slope close to that found by McCord. Bondarenko and Leikin (1970), in a review article of Soviet research, also report that most lunar areas have a "smooth, almost monotonic variation of intensity along the spectrum." McCord's careful photoelectric studies (e.g., 1968, 1969; McCord and Johnson 1969) of relative spectra of numerous small areas of diverse character reveal only minor variations from his standard area in Mare Serenitatis (Fig. 5). Efforts to synthesize our 1965 spectrum by various combinations of McCord's curves for different areas failed.

In contrast, Sharanov (1965) reports spectral reflectivity curves by Lebedeva for both lunar maria and continents which, although restricted in wavelength, appear to begin to turn upward at 0.6μ. Similar results indicating an upturn in the spectrum near 0.6μ were found for 14 areas on curves between 0.43 and 0.60μ by Barabashev, Ezerskii, and Fedorets (1959).

In light of these ambiguous results, the divergence of the results in Fig. 5 remains puzzling. It is possible that the differences between investigators may reflect systematic differences in calibrating with respect to the solar spectrum. Possible systematic errors in transforming our 1964 and 1965 results to a standard system have been discussed in Sec. I.

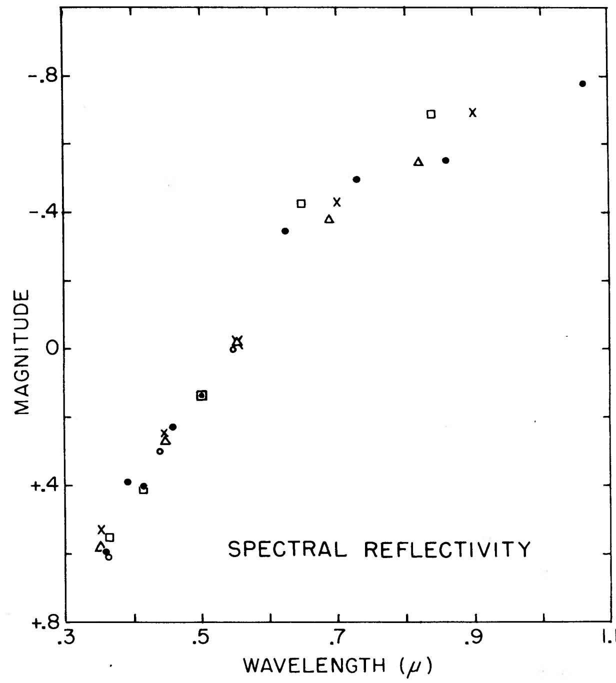

FIG. 6. Lunar spectral reflectivity. *Open and closed circles* (broad and narrow bands) are from our 1965 data at $i=90°$. *Triangles* from Harris (1961) and $\times$'s from Mitchell and Pellicori (1970; $i \approx -90°$) are normalized to our curve at V. *Squares* from Barabashev and Chekirda (1956) are normalized to our curve at 0.50μ.

An alternate possibility which must be examined is that real temporal changes (luminescence) occur in the lunar spectrum. Evidence for both localized and widespread brightness and color changes has been presented by a variety of studies (cf. the review by Kopal 1966; Foukal 1968; W. Cameron 1972; Sekiguchi 1971; Rackham 1967). Furthermore, enstatite achondrites have been found to possess a strong luminescent feature between 0.6 and 0.8μ (Derham and Geake 1964). The prominence of the "hump" in our 1965 data is greatest near full Moon (although the steepening of the spectrum at larger phase angles will tend to obscure the effect), which is also suggestive of time variability (see Fig. 4).

On the other hand, no evidence of *localized* time variations in spectral reflectivity was found by McCord (1968) in his study of differential (area–area) colors. An earlier study (McCord 1967) did appear to verify luminescence, but at a level of only a few percent. Likewise, the measured luminescent efficiency of returned lunar samples is orders of magnitude too low to provide observable effects if the source of energy is the solar wind, solar UV radiation, or thermoluminescence (e.g., Blair and Edgington 1970; Greenman and Gross 1970; Geake *et al.* 1970; Nash and Greer 1970). Whether it is possible to accelerate solar

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TABLE VIII. Geometric albedo[a] p, phase integral[b] q, and Bond albedo A for the lunar disk.

Wavelength (Å)	3590	3926	4155	4573	5012	6264	7297	8595	10 635	U	B	V
p	0.072	0.085	0.088	0.098	0.106	0.162	0.179	0.179	0.202	0.077	0.094	0.113
q	0.544	0.578	0.551	0.575	0.585	0.603	0.633	0.661	0.676	0.532	0.587	0.611
A	0.039	0.049	0.048	0.056	0.062	0.098	0.113	0.118	0.136	0.041	0.055	0.069

[a] Uses linear fit to 1965 data (excludes opposition effect). Cf. Table VII for error estimate.
[b] Computed from phase curves in Table V.

wind particles to requisite energies through interaction with the Earth's magnetosphere, as suggested by A. Cameron (1964) and Speiser (1967), appears to be as yet unresolved. It does seem that localized lunar transient phenomena correlate rather highly with both sunrise and the Earth's magnetopause (W. Cameron, 1972).

In spite of the uncertainty in the spectrum near zero phase for the 1965 data, values of the phase integral $q(\lambda)$ deduced in the next section vary smoothly with wavelength and seem not to be affected.

IV. GEOMETRIC ALBEDOS, PHASE INTEGRALS, AND BOND ALBEDOS

The geometric albedo p of the full Moon is defined by

$$\log_{10} p = 0.4[m_{\odot} - m_{☾}(1,0)] - 2\log_{10}\sin\sigma, \quad (2)$$

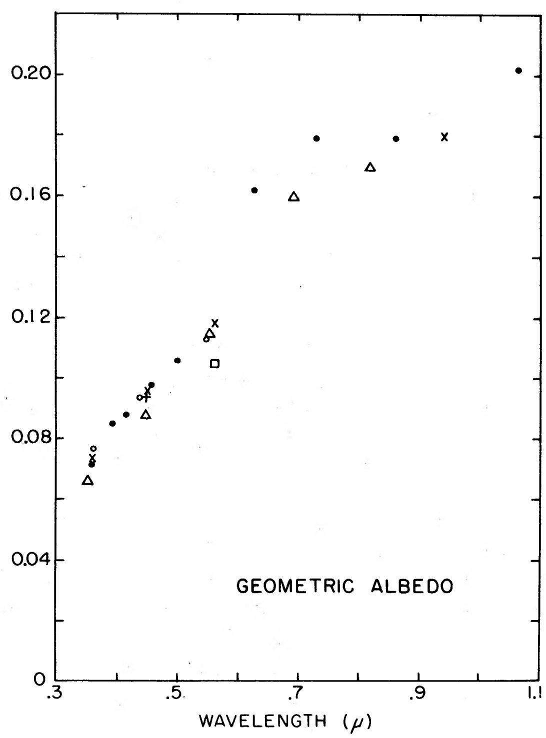

FIG. 7. Geometric albedo versus wavelength (excluding the opposition effect). *Open and closed circles* (broad and narrow bands) are from our 1965 data. *Triangles* from Harris (1961); square from Russell (1916b); *cross* from Rougier (1937). X's are from Gehrels *et al.* (1964), normalized to our curve at 0.56μ.

where $\sigma = 0.0045216$ is the angular radius of the Moon at mean Earth–Moon distance. The geometric albedo of the Moon as a function of wavelength is given in Table VIII and plotted in Fig. 7, where $m_{☾}(1,0)$ is taken from the linear extrapolation of the 1965 data and thus excludes the opposition effect. Results of Harris (1961), Russell (1916b), Rougier (1937), and Gehrels *et al.* (1964) are shown in Fig. 7 for comparison. Gehrels' values have been normalized to agree with our curve at $\lambda 0.56\mu$ since his data were computed including the opposition effect. The 1964 data have not been used because of the larger standard errors in $m_{☾}(1,0)$ for 1964 as compared to 1965 (a factor of 1.5 to 2), and our greater confidence in the transformation to standard conditions for 1965 (see Sec. I). Geometric albedos deduced from our 1964 measurements are about five percent lower than the values presented in Table VIII for $\lambda \leq 0.5\mu$, and ten to twenty percent lower at longer wavelengths (cf. Fig. 5).

Inclusion of the opposition effect will significantly increase values of $m_{☾}(1,0)$ and hence, values of p. There is considerable uncertainty in the magnitude of this effect, however, with estimates of the brightness increase in the visual ranging from 44% (Pohn *et al.* 1969) to 100% (Gehrels *et al.*) as the phase angle decreases from 5° to 0°. Moreover, both theoretical studies of the effect of multiple scattering (e.g., Irvine 1966) and observations of the Moon (Mikhail 1970) and Mars (O'Leary and Rea 1968) indicate that the opposition effect will be a function of wavelength, being inversely correlated with surface albedo. Curiously, experiments on lunar samples (O'Leary and Briggs, 1970) indicate the opposite wavelength dependence. Further study is clearly required before reliable estimates of the opposition effect can be made.

It can be seen from Fig. 7 that our geometric albedos agree very well in the region $0.35 \leq \lambda \leq 0.55\mu$ with the determinations by Gehrels *et al.* and by Rougier. The steeper slope of the points from Harris probably results from his assumption that the colors determined at large phase angles apply also at $i = 0°$ (i.e., his neglect of the wavelength dependence of the phase curves). Between 0.60 and 0.85μ, our results show an excess reflectivity (cf. Sec. III) relative to Harris, particularly when the phase angle dependence of the spectral reflectivity would imply a redder spectrum for Harris.

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The spherical (Bond) albedo of a planet or satellite is customarily defined as the fraction of incident solar flux reflected in all directions and is expressed as

$$A = pq, \tag{3}$$

where p is the geometric albedo at zero phase, q the phase integral

$$q = 2\int_0^{\pi} di \sin i\, \Phi(i), \tag{4}$$

and $\Phi(i)$ is the phase function of the planet or satellite. In computing the phase integrals for the Moon, we have used the combination phase curves described in Sec. II; i.e., a cubic least-squares fit to the 1965 data at all phase angles with a linear fit substituted for the $i < 40°$ portion of the cubic curve. The phase integrals and Bond albedos for the Moon are listed in Table VIII and are plotted in Figs. 8 and 9, respectively. There was insufficient phase angle coverage in our 1964 data for a computation of q.

These results provide the first direct evidence for a wavelength dependence of the phase integral for the entire Moon. They are significant also for the study of other solar system objects for which $q(\lambda)$ can not be directly determined and for which the lunar value is frequently used in estimating Bond albedos; e.g., for Saturn's ring particles (Franklin and Cook 1965), asteroids (Matson 1972) and satellites of the outer planets (Johnson 1971; Harris 1961; Moroz 1966).

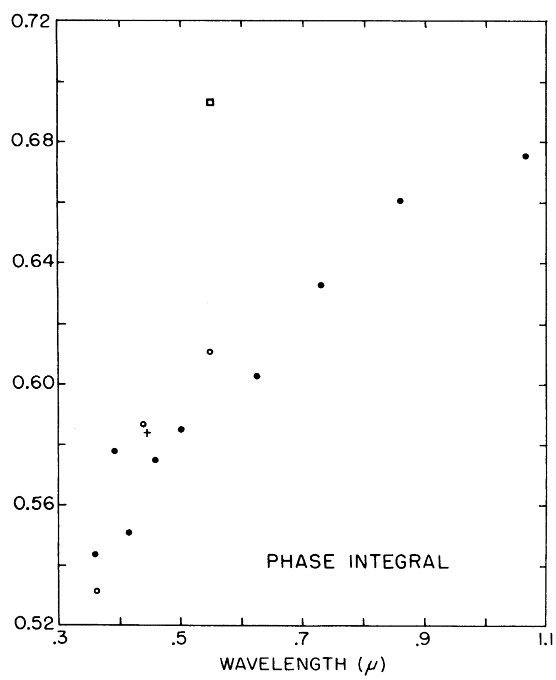

FIG. 8. Phase integral versus wavelength (excluding the opposition effect). *Open and closed circles* (broad and narrow bands) are computed from the phase curves in Table V. *Square* from Russell (1916b); *cross* from Rougier (1937).

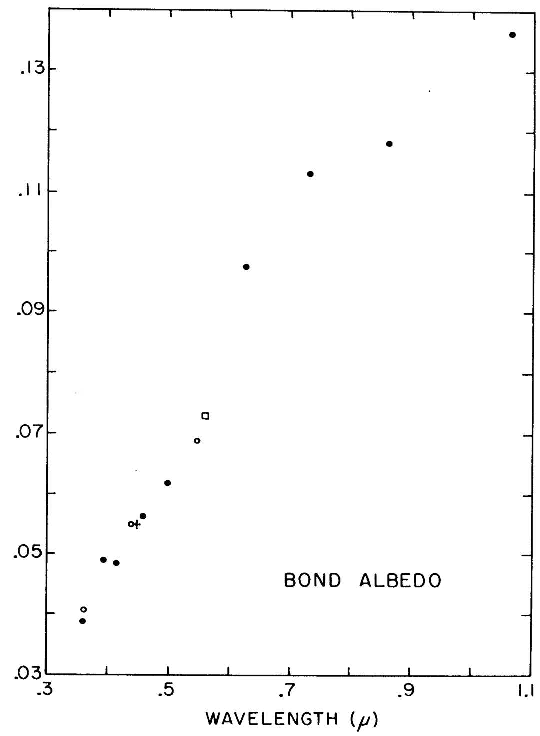

FIG. 9. Bond albedo versus wavelength. *Open and closed circles* (broad and narrow bands) are from our 1965 data. *Square* from Russell (1916b); *cross* from Rougier (1937).

While the agreement of our determination at B with that of Rougier (0.584) is very good, Russell's value (at V) lies far off our curve. This difference can be attributed to inaccuracies in his phase curve for small phase angles (cf. Table VI). Values of q are strongly dependent upon the phase coefficients listed in Table IV, and are probably accurate to $\sim 5\%$ apart from uncertainties introduced by the opposition effect (see below).

The phase integral, because $\Phi(i)$ is normalized to unity at $i=0$, will significantly decrease in value if the opposition effect is included in the phase curve. If the opposition effect, as expected, correlates inversely with wavelength (since the lunar albedo increases with wavelength), then the increase in $q(\lambda)$ with wavelength will be steeper than that shown in Fig. 8.

Fortunately, the Bond albedos A are independent of the opposition effect to roughly the extent that $(\Delta i)^2 \ll 1$, where Δi is the angular width of the opposition effect in radians [since $A \propto \int di \sin i\, j(i)$, where j is the reflected flux for the Moon]. The values of A given in Table VIII and Fig. 9 will thus be uncertain because of the above-mentioned observational uncertainties in p and q and possible systematic or luminescent effects, particularly in filters k and h (see Sec. III). The agreement between our Bond albedos, which increase approximately linearly with wavelength for $\lambda \leq 0.6\mu$

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(see Fig. 9), and those determined by Russell (1916b) and Rougier (1937) is excellent.

V. RADIOMETRIC ALBEDO

The radiometric albedo A^* describes the reflecting properties of the Moon with respect to the total incident electromagnetic energy

$$A^* = \int_0^\infty d\lambda E_\odot(\lambda) A(\lambda) \bigg/ \int_0^\infty d\lambda E_\odot(\lambda), \qquad (5)$$

where $E_\odot(\lambda)$ is the solar flux. We take ultraviolet values of $p(\lambda)$ from Lebedinsky *et al.* (1967) for $0.28 \leq \lambda \leq 0.36\mu$ and from Lebedinsky *et al.* (1968a) for $0.21 \leq \lambda \leq 0.27\mu$, normalizing to our value at 0.36μ and assuming $p(\lambda)$ is approximately constant from 0.27 to 0.29μ (Carver and Horton 1967). The phase integral $q(\lambda)$ in the ultraviolet was extrapolated linearly from Table VIII on the basis of Ahmad and Deutschman (1972). In the infrared we have determined $p(\lambda)$ for $1 \leq \lambda \leq 2.5\mu$ from McCord and Johnson (1970) by normalizing to our value at 1μ. We then interpolated smoothly to the values for $3.4 \leq \lambda \leq 3.9\mu$ from Lebedinsky *et al.* (1968b). We have somewhat arbitrarily extrapolated our $q(\lambda)$ monotonically to a value $q(2.5\mu) = 0.737$, and then utilized the value determined by Lebedinsky *et al.* (1968b) for the 3.5μ region. The balloon spectrum of Wattson and Danielson (1965) agrees well with McCord and Johnson except in showing a sharp upturn near 2.5μ which may be instrumental.

Having thus obtained $A = pq$ and taking $E_\odot$ from Allen (1963), we find

$$A^* = 0.123 \pm 0.002, \qquad (6)$$

where the uncertainty includes only that intrinsic to our determinations of $A(\lambda)$ in the visible (including the 1964–1965 difference). A considerably larger uncertainty arises from differences in the infrared spectrum obtained by various investigators (Moroz 1966; Cruikshank 1969; Binder 1965; Hayakawa *et al.* 1968). Moroz's results are the most discordant with the spectrum adopted above, and would lead to a value $A^* = 0.11$.

The value of A^* in Eq. (6) agrees very well with that (0.122) obtained by Saari and Shorthill (1972) by an entirely different procedure based on measurement of the infrared brightness temperature.

ACKNOWLEDGMENTS

This research was supported by the National Aeronautics and Space Administration under grant NGL 22-010-023. Numerical calculations were performed at the University of Massachusetts Research Computing Center. We are grateful to Dr. D. P. Cruikshank, Dr. Z. Kopal, and Dr. A. T. Young for their comments on a draft of this manuscript, and to Dr. D. H. Menzel for providing the initial impetus for this project.

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