Since the eigenvalues of T are those values of λ which make det[T - λ I] = 0, it follows that $\lambda_i = d - 1$ for i = 1, 2, …, n-1, $\lambda_n = d + n - 1$.

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TEST MATRIX FOR INVERSION

In reference to M. L. Pei's letter regarding a test matrix for inversion *[Comm. ACM 5, 10 (Oct.1962)]*, some additional features may be found useful. The letter gives the matrix:

$$T = t_{ij} = \begin{cases} d, & i = j \\ 1, & i \neq j \end{cases}$$

and its inverse:

$$V = v_{ij} = \begin{cases} \dfrac{d+n-2}{d(d+n-2)-(n-1)}, & i = j \\ \dfrac{-1}{d(d+n-2)-(n-1)}, & i \neq j \end{cases}$$

where n is the order of the matrix.

Recalling the method for computing an inverse, it will seen that:

$$v_{11} = \frac{T_{11}}{\Delta(n)}$$

where T_{11} is the cofactor of t_{11} and Δ(n) is the determinant of T. However, T_{11} is simply Δ(n-1) and we have the relation:

$$\frac{\Delta(n-1)}{\Delta(n)} = \frac{d+n-2}{d(d+n-2)-(n-1)} = \frac{d+(n-2)}{(d-1)[d+(n-1)]}$$

Noting that $\Delta(2) = d^2-1=(d+1)(d-1)$, we have by induction:

$$\Delta(n) = (d-1)^{n-1}(d+n-1)$$

Since the eigenvalues of T are those values of λ which make det[T - λ I] = 0, it follows that $\lambda_i = d-1$ for i = 1, 2, …, n-1, $\lambda_n = d+n-1$. Therefore , in addition to having a known explicit inverse, the matrix T has a known determinant and known eigenvalues.

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