

TEST MATRIX FOR INVERSION

In reference to M. L. Pei's letter regarding a test matrix for inversion [*Comm. ACM* 5, 10 (Oct.1962)], some additional features may be found useful. The letter gives the matrix:

$$T = t_{ij} = \begin{cases} d, & i = j \\ 1, & i \neq j \end{cases}$$

and its inverse:

$$V = v_{ij} = \begin{cases} \frac{d + n - 2}{d(d + n - 2) - (n - 1)}, & i = j \\ \frac{-1}{d(d + n - 2) - (n - 1)}, & i \neq j \end{cases}$$

where n is the order of the matrix.

Recalling the method for computing an inverse, it will seen that:

$$v_{11} = \frac{T_{11}}{\Delta(n)}$$

where T_{11} is the cofactor of t_{11} and $\Delta(n)$ is the determinant of T. However, T_{11} is simply $\Delta(n-1)$ and we have the relation:

$$\frac{\Delta(n-1)}{\Delta(n)} = \frac{d + n - 2}{d(d + n - 2) - (n - 1)} = \frac{d + (n - 2)}{(d - 1)[d + (n - 1)]}$$

Noting that $\Delta(2) = d^2 - 1 = (d+1)(d-1)$, we have by induction:

$$\Delta(n) = (d - 1)^{n-1}(d + n - 1)$$

Since the eigenvalues of T are those values of λ which make $\det[T - \lambda I] = 0$, it follows that $\lambda_i = d-1$ for $i = 1, 2, \dots, n-1$, $\lambda_n = d+n-1$. Therefore, in addition to having a known explicit inverse, the matrix T has a known determinant and known eigenvalues.

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<https://developer.apple.com/library/archive/documentation/UserExperience/Conceptual/SpeechSynthesisProgrammingGuide/FineTuning/FineTuning.html>