1. (30 points) Maximize $f(x, y) = x + y$ where x, y must lie on the unit circle.

   > **Solution:** See `https://en.wikipedia.org/wiki/Lagrange_multiplier#Example_1`
   > $f(1/\sqrt{2}, 1/\sqrt{2}) = \sqrt{2}$

2. (30 points) Maximize $f(p_1, p_2) = -(p_1 \log p_1 + p_2 \log p_2)$ subject to $p_1 + p_2 = 1$.
   The function is called information entropy and the p_i are supposed to be probabilities. Therefore, you are determining the probability distribution that gives maximal information.

   > **Solution:** $p_1 = 1/2 = p_2$, see below.

3. (5 points (bonus)) Given an arbitrary but still finite collection of probabilities, $\{p_1, \ldots, p_n\}$, which means $\sum_{i=1}^{n} p_i = 1$, maximize

   $$f(p_1, \ldots, p_n) = -\sum_{i=1}^{n} p_i \log p_i$$

   What is the name of this probability distribution? Can you interpret this result in terms of entropy?

   > **Solution:** `https://en.wikipedia.org/wiki/Lagrange_multiplier#Example_3:_Entropy`