Directions: Questions 2 and 3 are short free-response questions that require about 13 minutes each to answer and are worth 7 points each. Questions 1 is a long free-response question that require about 25 minutes each to answer and are worth 12 points. Show your work for each part in the space provided after that part.
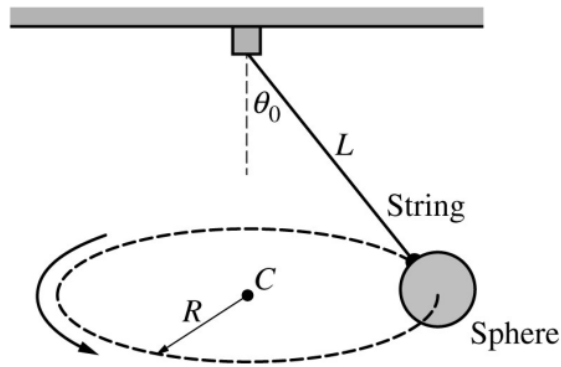

1. A small sphere of mass M is suspended by a string of length L. The sphere is made to move in a horizontal circle of radius R at a constant speed, as shown above. The center of the circle is labeled point C, and the string makes an θ_0 with the vertical.

   (a) Two students are discussing the motion of the sphere and make the following statements.

   Student 1: None of the forces exerted on the sphere are in the direction of point C, the center of the circular path. Therefore, I don't see how there can be a centripetal force exerted on the sphere to make it move in a circle.

   Student 2: I see another problem. The tension force exerted by the string is at an angle from the vertical. Therefore, its vertical component must be less than the weight Mg of the sphere. That means the net force on the sphere has a downward vertical component, and the sphere should move downward as well as moving around in a circle.

      i. What is one aspect of Student 1's reasoning that is incorrect?
      ii. What is one aspect of Student 2's reasoning that is incorrect?

   (b)   i. Derive an equation for the magnitude of the net force exerted on the sphere. Express your answer in terms of M, θ, and physical constants, as appropriate.
      ii. Describe one aspect or step in your derivation of part (b)(i) that can be correctly linked to your answer to either part (a)(i) or part (a)(ii).
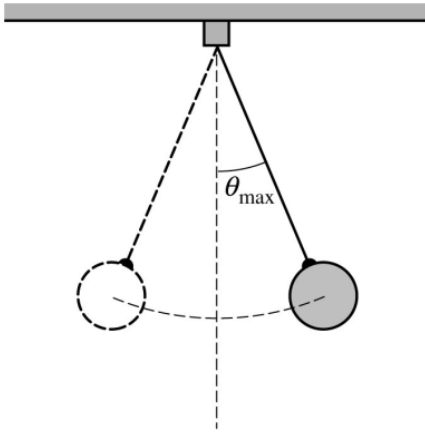
Instead of moving in a horizontal circle, the sphere now moves in a vertical plane so that it is a simple pendulum, as shown above. The maximum angle θ_{max} that the string makes from the vertical can be assumed to be small. The graph below shows data for the square of the pendulum period T as a function of string length L.
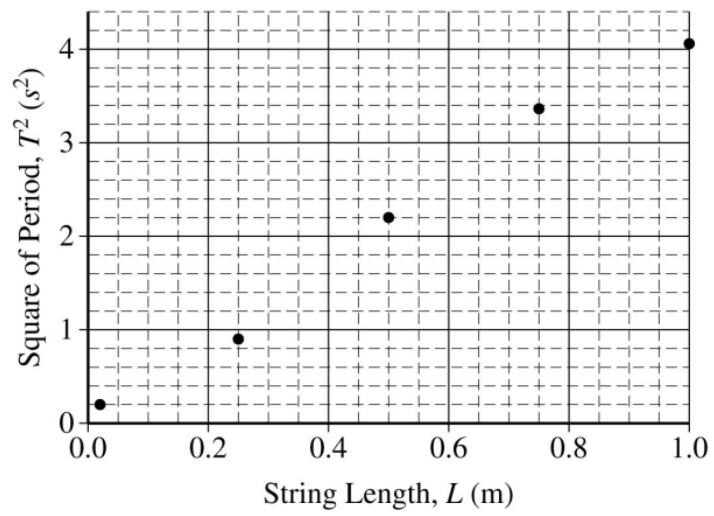

(c) On the graph above, draw a best-fit line for the data. Then use the line to calculate a numerical value for the gravitational acceleration g.
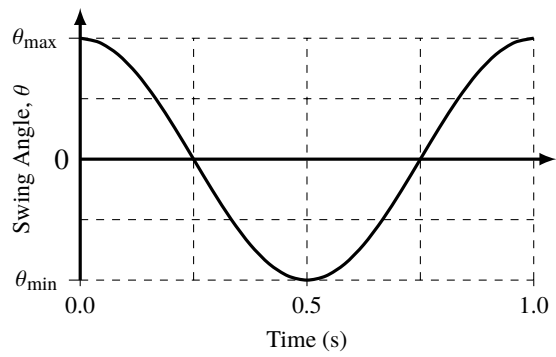
(d) The graph above shows the angle θ from the vertical as a function of time for the pendulum. On the axes below, sketch a graph of the gravitational potential energy of the sphere-Earth system for the same time interval. Take the zero of potential energy to be when the potential energy has its a minimum value.
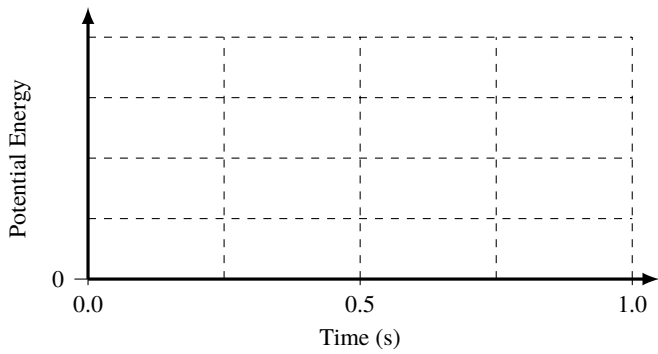
(e) As the sphere swings back and forth, it must also rotate a small amount during each swing. The figures below indicate the direction that the sphere rotates as it is swinging in each direction.
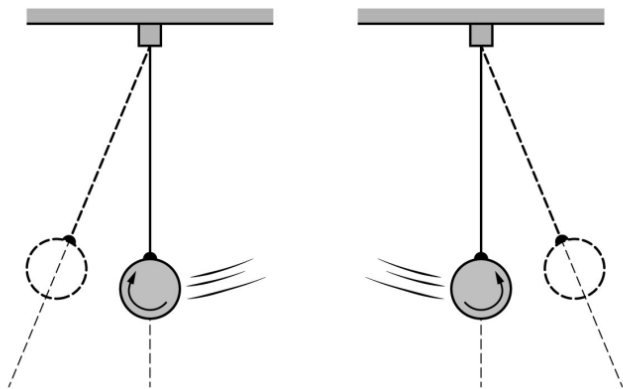
In order for the sphere's rotation to change direction, a torque must be exerted on the sphere. When the sphere is at its maximum rightward displacement, what is the direction of the torque exerted on the sphere with respect to the point of attachment between the sphere and string?

_____ Clockwise   _____ Counterclockwise

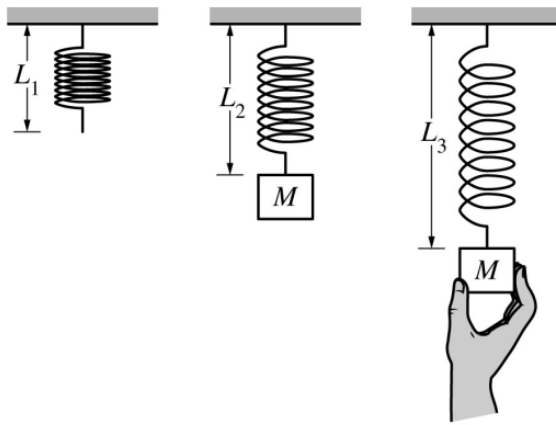Briefly state why the torque is in the direction you indicated.

Figure 1   Figure 2   Figure 3

2. A spring with unstretched length L_1 is hung vertically, with the top end fixed in place, as shown in Figure 1 above. A block of mass M is attached to the bottom of the spring, as shown in Figure 2, and the spring has length $L_2 > L_1$ when the block hangs at rest. The block is then pulled downward and held in place so that the spring is stretched to a length $L_3 > L_2$, as shown in Figure 3.

(a) On the dot below, which represents the block in Figure 3, draw and label the forces (not components) exerted on the block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

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(b) The student releases the block. Consider the time during which the block is moving upward toward its equilibrium position and the spring length is still longer than L_2. In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, indicate why the total mechanical energy is increasing, decreasing, or constant for each of the systems listed below.

- System 1: The block
- System 2: The block and the spring
- System 3: The block, the spring, and Earth

Use E_1, E_2, and E_3 to denote the total mechanical energy of systems 1, 2, and 3, respectively.
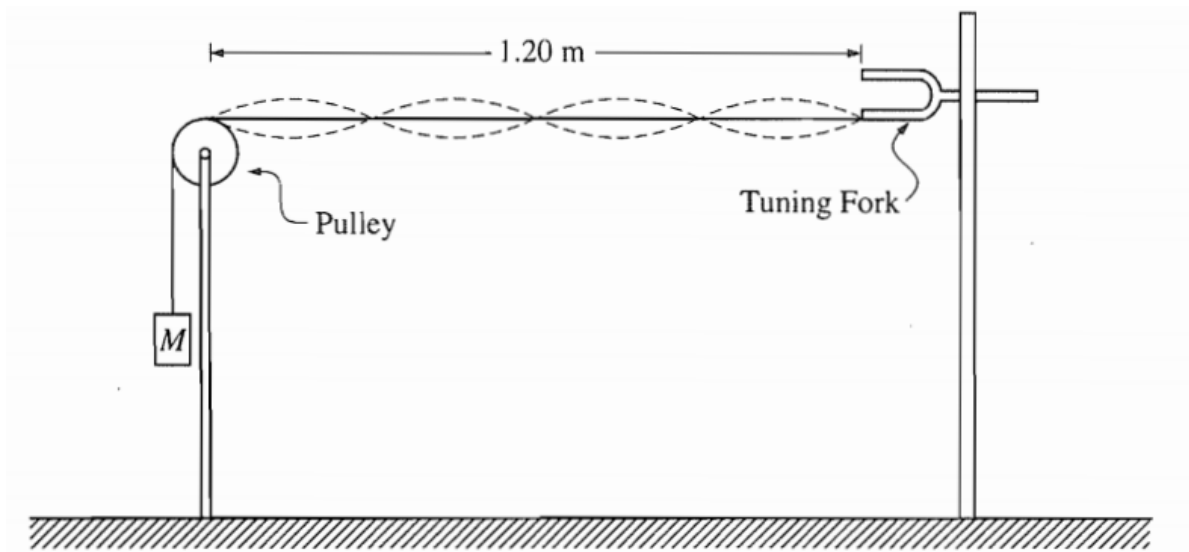
3. To demonstrate standing waves, one end of a string is attached to a tuning fork with frequency 120 Hz. The other end of the string passes over a pulley and is connected to a suspended mass M as shown in the figure above. The value of M is such that we standing wave pattern has four "loops". The length of the string from the tuning fork to the point where the string touches the top of the pulley is 1.20 m. The linear mass density of the string is 1.0×10^{-4} kg/m, and remains constant throughout the experiment.

   (a) Determine the wavelength of the standing wave.

   (b) Determine the speed of the transverse waves along the string.

   (c) The speed of waves along the string increases with increase tension in the string. Indicate whether the value of M should be increases or decreased in order to double the number of loops in the standing wave pattern. Justify your answer.

   (d) If a point on the string at an antinode moves a total vertical distance of 4 cm during one complete cycle, what is the amplitude of the standing wave?