Student #: ______________________ Student Name: ______________________________

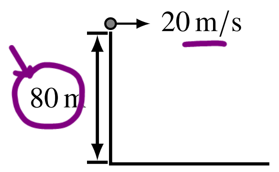
AP PHYSICS C: KINEMATICS

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

Questions 1–2

A ball of mass 0.5 kg is launched horizontally from the top of a cliff 80 m high with a speed of 20 m/s at time $t = 0$.



$y = \frac{1}{2} g t^2$

$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(80)}{10}} = 4$

1. The horizontal distance x traveled by the ball before striking the ground is

   (A) 20 m
   (B) 40 m
   (C) 80 m
   (D) 160 m
   (E) 320 m

   C

2. The speed of the ball just before striking the ground is

   (A) 4 m/s
   (B) 14 m/s
   (C) 20 m/s
   (D) 44 m/s
   (E) 64 m/s

   D

   20 m/s, $v_y = gt = 40\,\mathrm{m/s}$, ?

   $\sqrt{20^2 + 40^2}$ < 40 ~~20~~

3. A space explorer throws a tool downward on a planet with an initial velocity of 2.0 m/s from a height of 6 m above the surface. The tool strikes the surface in a time of 2 s. The acceleration due to gravity on the planet is

   (A) $1\,\mathrm{m/s^2}$
   (B) $2\,\mathrm{m/s^2}$
   (C) $3\,\mathrm{m/s^2}$
   (D) $4\,\mathrm{m/s^2}$
   (E) $10\,\mathrm{m/s^2}$

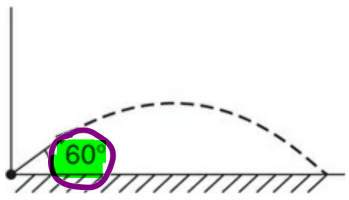
   A

   $\Delta y = v_0 t + \frac{1}{2} g t^2$

   $\cancel{6} = (2)(2) + \frac{1}{2} g (2^2)$

   $1 = \frac{2}{4} = \frac{1}{2} g (4)$

4. A golf ball is hit from level ground and has a horizontal range of 100 m. The ball leaves the golf club at an angle of 60° to the level ground. At what other angle(s) can the ball be struck at the same initial velocity and still have a range of 100 m?

   (A) 30°
   (B) 20° and 80°
   (C) 10° and 120°
   (D) 45° and 135°
   (E) There is no other angle other than 60° in which the ball will have a range of 100 m.

   A

5. A stack of coffee filters falls from rest through the air. Due to air resistance, the filters fall with an acceleration proportional to the velocity of fall, that is, $a = -kv$, where k is a positive constant. The velocity of the falling filters as a function of time of fall is

   (A) $-kv^2$
   (B) $-12kv^2$
   (C) $-k$
   (D) $\ln(kt)$
   (E) $v_0 e^{-kt}$

   E

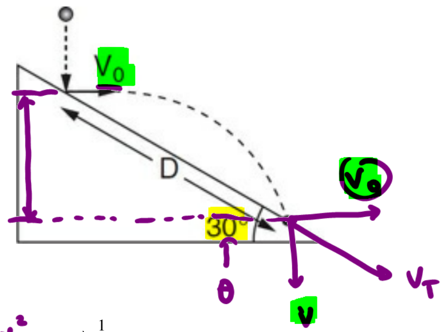
   $\frac{dv}{dt} = -kv$

   $\int \frac{dv}{v} = -\int k\,t$

   $\ln v = -kt + C$

   $v(t) = e^{-kt + c} = e^{-kt} \underbrace{e^{c}}_{v_0}$

6. A rubber ball is dropped from rest onto a plane angled at $\theta = 30°$ to the horizontal floor and bounces off the plane with a horizontal speed v_0. The ball lands on the plane a distance D along the plane, as shown below. In terms of v_0, D, and g, the speed of the ball just before striking the plane is

   $D\sin\theta$, v_0, v_T, θ, v

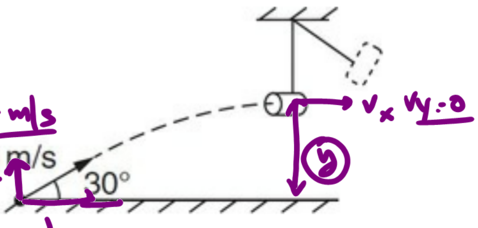
   B

   (A) v_0
   (B) $\left(v_0^2 + 2D\sin\theta g\right)^{\frac{1}{2}}$
   (C) $\left(v_0 + \frac{D\sin\theta}{g}\right)^{\frac{1}{2}}$
   (D) $\left(v_0^2 + \frac{D\sin\theta}{g}\right)^{\frac{1}{2}}$
   (E) $(2D\sin\theta g)^{\frac{1}{2}}$

   $v^2 = 2gD\sin\theta$

   $v_T = \sqrt{v^2 + v_0^2}$

7. A small ball is launched with a speed of 8 m/s at an angle of 30° from the horizontal. A cup is hung so that it is in position to catch the ball when it reaches its maximum height. How far above the floor should the cup be hung to catch the ball?

   4 m/s, $8\sin 30°$, $8\cos 30°$, v_x, $v_y = 0$, y

   D

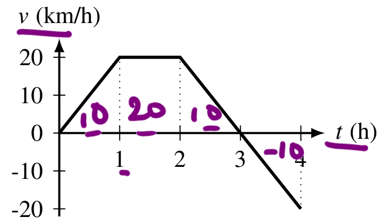
   (A) 2.4 m
   (B) 1.6 m
   (C) 1.0 m
   (D) 0.8 m
   (E) 0.4 m

   $\cancel{v^2} = v_0^2 + 2gy$ (−10)

   $0 = 4^2 - 20y$

   $20y = 16 \rightarrow y = \frac{16}{20}$

8. The velocity vs. time graph below represents the motion of a bicycle rider. The displacement of the rider between 0 and 4 h is

   10, 20, 10, −10

   (A) +10 km
   (B) +20 km
   (C) +30 km
   (D) +40 km
   (E) −10 km

   C

9. An object starts from rest at $t = 0$ and position $x = 0$, then moves in a straight line with an acceleration described by the equation $a = 4t^2$ in $\mathrm{m/s^2}$. What is the position of the object at $t = 3$ s?

   (A) 6 m
   (B) 1 m
   (C) 27 m
   (D) 54 m
   (E) 108 m

   C

   $v = \int a\,dt = 4\int t^2\,dt$

   $v = \frac{4}{3} t^3$

   $x = \int v\,dt = \frac{4}{3}\int t^3\,dt = \frac{1}{3} t^4 = \frac{1}{3} 3^4$

   $\frac{1}{4} t^4$

Questions 10–11

A car of mass m travels along a straight horizontal road. The car begins with a speed v_0, but accelerates according to the velocity function $v = \left(v_0^2 + \frac{Ct^2}{m}\right)$, where t is time.

10. The speed of the car is zero at a time t of

E

(A) zero
(B) $2t$
(C) $4t$
(D) $\sqrt{8t}$
(E) The speed of the car is never zero.

11. The acceleration of the car as a function of time is

D

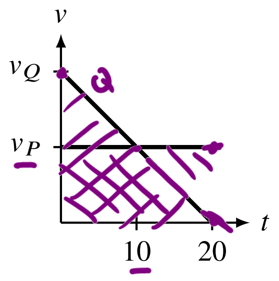
$a = \frac{dv}{dt} = \frac{2Ct}{m}$

(A) $\left(v_0^2 + \frac{Ct^2}{m}\right)$
(B) $\left(v_0^2 + \frac{2Ct^2}{m}\right)$
(C) $\left(v_0 + \frac{Ct}{m}\right)$
(D) $\left(\frac{2Ct}{m}\right)$
(E) $\left(\frac{2Ct^2}{m}\right)$

Questions 12–13

The graph shown below represents the velocity vs. time graphs for two cars, P and Q. Car P begins with a speed v_P, and Car Q begins with a speed v_Q which is twice the velocity of Car P, that is, $v_Q = 2v_P$.

Both car start at the same position at $t = 0$.



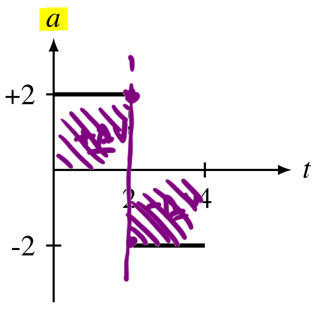
12. Which of the following is true at a time of 10 s?

(A) The cars occupy the same position.
(B) Car P is at rest.
(C) $v_Q > v_P$
(D) $v_P > v_Q$
(E) Car Q is ahead of Car P.

13. Which of the following is true at a time of 20 s?

(A) The cars occupy the same position.
(B) Car P is at rest.
(C) $v_Q > v_P$
(D) $a_P = a_Q$
(E) Car P is ahead of Car Q.

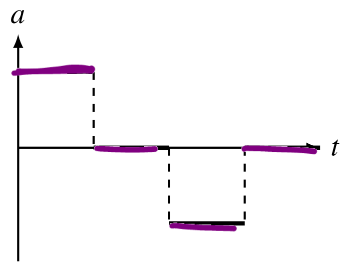
14. The motion of an object is represented by the acceleration vs. time graph below. The object is intially at rest. Which of the following statements is true about the motion of the object?

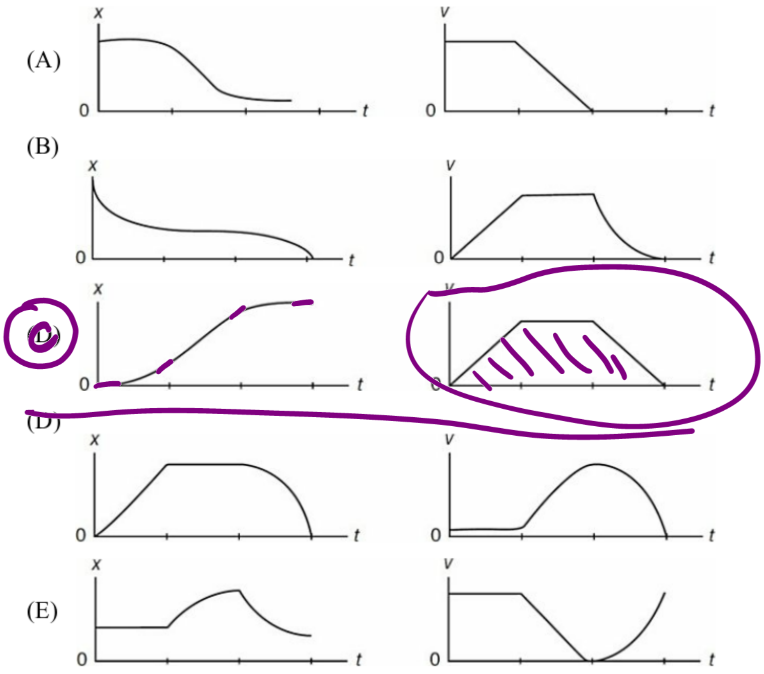


C

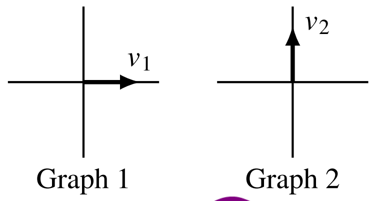
(A) The object returns to its original position. ??
(B) The velocity of the object is zero at a time of 2 s.
(C) The velocity of the object is zero at a time of 4 s.
(D) The displacement of the object is zero at a time of 4 s. ??
(E) The acceleration of the object is zero at a time of 2 s.

15. Which of the following pairs of graphs could show the position vs. time and velocity vs. time graphs for the acceleration vs. time graph shown ~~above~~ below? Assume $v = 0$ and $x = 0$ at $t = 0$.

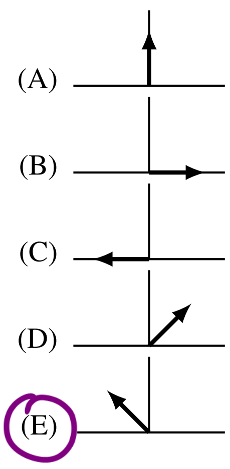




16. Two velocity vectors v_1 and v_2 each have a magnitude of 10 m/s. Graph 1 shows the velocity v_1 at $t = 0$ s, and then the same object has a velocity v_2 at $t = 2$ s, shown in Graph 2. Which of the following vectors best represents the average acceleration vector that causes the object's velocity to change from v_1 to v_2 ?



$\vec{a} = \frac{\Delta \vec{v}}{t}$



$\vec{v}_2$ $\Delta\vec{v}$ $\vec{v}_1$

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$\underline{\vec{v}_1 + \Delta\vec{v}} = \vec{v}_2$

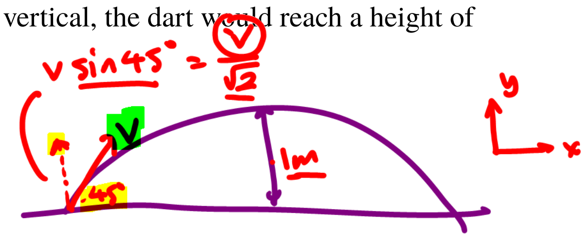
E

17. A toy dart gun fires a dart at an angle of 45° to the horizontal and the dart reaches a maximum height of 1 meter. If the dart were fired straight up into the air along the vertical, the dart would reach a height of

B

(A) 1 m
(B) 2 m
(C) 3 m
(D) 4 m
(E) 5 m

$v \sin 45° = \frac{v}{\sqrt{2}}$



final velocity squared

y-dir

$0 = \left(\frac{v}{\sqrt{2}}\right)^2 - 2gy \longrightarrow v^2 = 2gy$

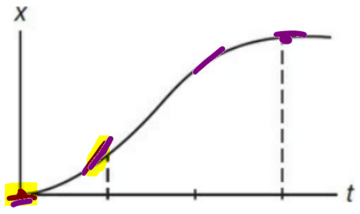
$\frac{v^2}{2} = 2g(1)$

$v^2 = 4g$

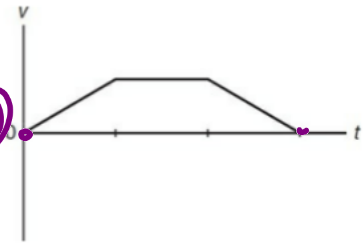
$\frac{4g}{2g} = \frac{2g}{2g}y$

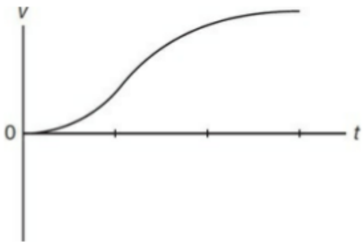
$y = 2$

18. The graph below shows the displacement as a function of time for a car moving in a straight line. Which of the following graphs shows the velocity vs. time graph for the same time intervals?

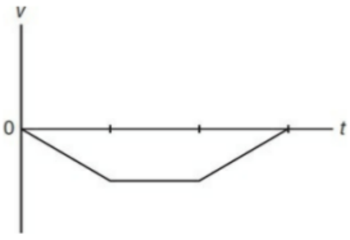


A

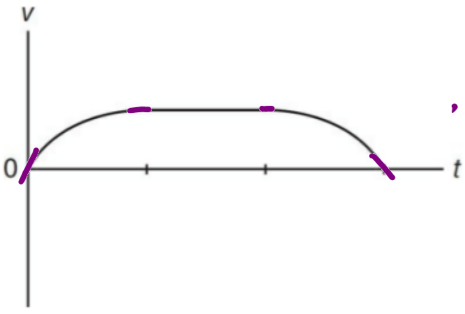
(A) 

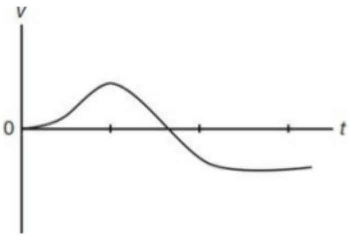
(B) 

x

(C) 

x

(D) 

(E) 

x

Questions 19–20

An object is released from rest and falls through a resistive medium. The resistance causes the velocity of the object to change according to the equation $v = 16t - \frac{1}{2}t^4$, where v is in m/s and time is in s.

19. Which of the following is a possible equation for the acceleration of the object as a function of time?

B

$a = \frac{dv}{dt} = 16 - 2t^3$

(A) $16 - 2t^2$
(B) $16 - 2t^3$
(C) $16 - 2t$
(D) $8t^3 - 2t^2$
(E) $32t^3 - 2t^5$

20. What is the terminal velocity of the object as it falls?

$a = 0$

E

$a = 0$ $16 - 2t^3 = 0$
$2t^3 = 16$
$t^3 = 8$
$t = 2$

(A) 5 m/s
(B) 10 m/s
(C) 24 m/s
(D) 32 m/s
(E) The object never reaches a terminal velocity.

AP PHYSICS C: KINEMATICS
SECTION II
4 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. The position x of an object is described with respect to time t by the following equation: $x = 2t^3 - 15t^2 + 36t - 8$, where x is in meters and t in seconds. Answer the following questions.

   (a) Find its displacement between $t = 3$ and 5 s.

   (b) Write out an expression for the velocity of the object with respect to time.

   (c) Write out an expression for the acceleration of the object with respect to time.

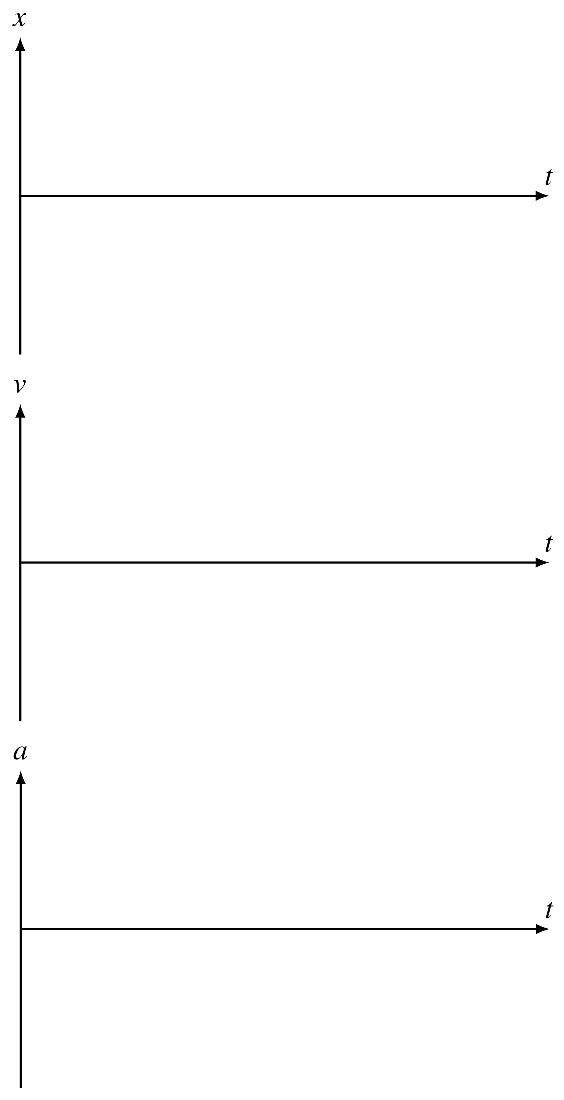
   (d) At what point(s) in time is the velocity of the object zero?

   (e) At each of those points (from d above), is the acceleration positive, negative, or zero?

   (f) During what intervals of time is the velocity of the object positive?

   (g) During what intervals of time is the acceleration of the object positive?

   (h) On the graph (on the next page), sketch position x, velocity v and acceleration a as functions of time.

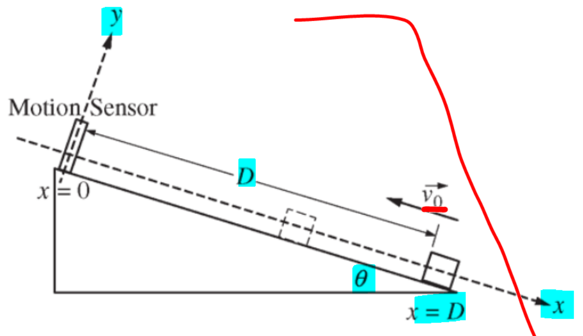


2. A steel ball is dropped from a point with (x, y) coordinate of $(8\,\text{m}, 16\,\text{m})$. At the same time, another ball is launched from the origin with a speed of $20\,\text{m/s}$ at an angle of $30°$.

   (a) Find the minimum distance of separation occur of the two balls.

   (b) At what time does this separation occur?

   (c) Give the coordinates of the two balls for the minimum separation.



3. A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0. The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ and physical constants, as appropriate.

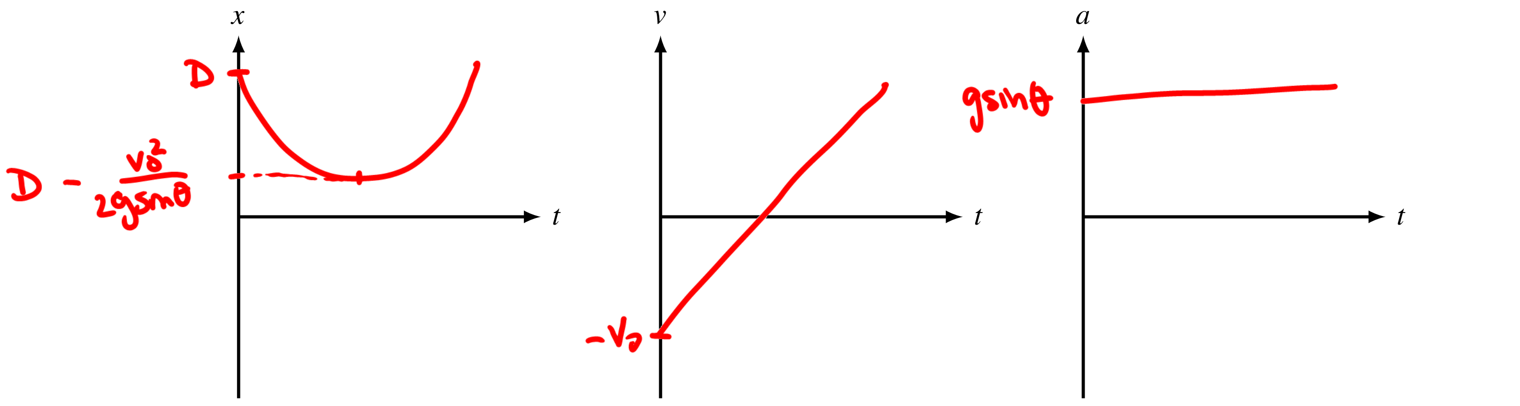
i. Determine the acceleration a of the block.

ii. Determine an expression for the velocity v of the block.

iii. Determine an expression for the position x of the block.

(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

(c) On the axes provided below, sketch graphs of position x, velocity v, and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k. Derive an expression for the distance the block slides before stopping. Express your answer in terms of m, D, v_0, θ, μ_k, and physical constants, as appropriate.

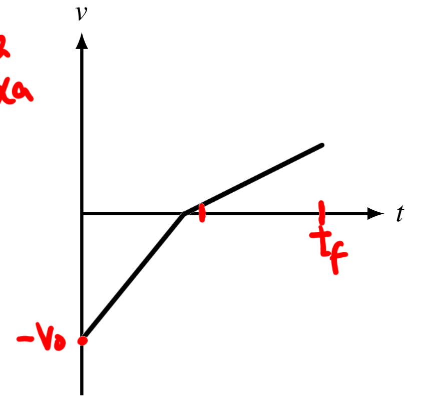
(e) Suppose the ramp now has friction. The same block is projected up with the same initial speed v_0 and comes back down the ramp. On the axes provided below, sketch a graph of the velocity v as a function of time t for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time t_f. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

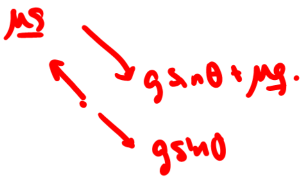
$F_k = \mu_k N = \mu_k mg$

$-F_k = ma$

$-\mu_k \cancel{m} g = \cancel{m} a$

$a = -\mu g$





$\cancel{v_f^2} = v_i^2 + 2ad$

$v_0^2 - 2\mu_k g d$

$\boxed{d = \frac{v_0^2}{2\mu_k g}}$

(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ and physical constants, as appropriate.

i. Determine the acceleration a of the block.

ii. Determine an expression for the velocity v of the block.

iii. Determine an expression for the position x of the block.

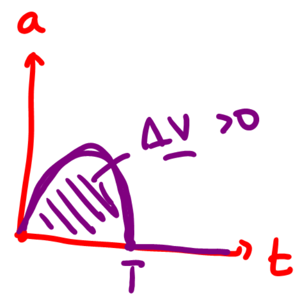
i) $\cancel{m}g\sin\theta = \cancel{m}a$ $\quad \boxed{a = g\sin\theta}$

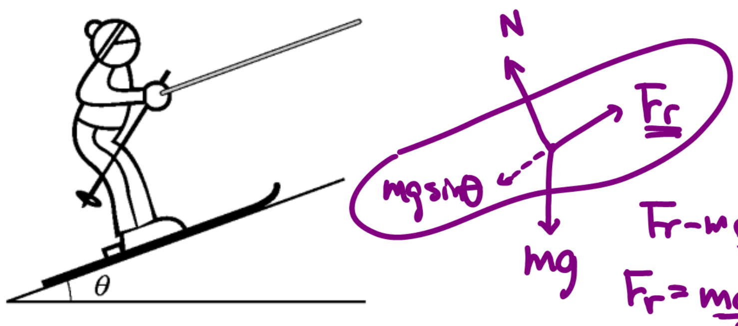
ii) $v = v_i + at$ $\quad \boxed{v = -v_0 + gt\sin\theta}$

iii) $x = x_i + v_it + \frac{1}{2}at^2$ $\quad \boxed{x = D - v_0t + \frac{1}{2}gt^2\sin\theta}$

b) $\cancel{v_f^2} = v_i^2 + 2a(x - x_i)$ $\quad v = 0$

$0 = v_0^2 + (2g\sin\theta)(x - D) \longrightarrow - \; x - D = -\frac{v_0^2}{2g\sin\theta} \rightarrow \boxed{x = D - \frac{v_0^2}{2g\sin\theta}}$





$F_r - mg\sin\theta = ma$

$F_r = ma + mg\sin\theta$ ← $a(t)$

4. A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a = a_{\max}\sin\left(\frac{\pi t}{T}\right) \qquad (0 < t < T)$$
$$= 0 \qquad (t \geq T)$$

where $a_{\max}$ and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

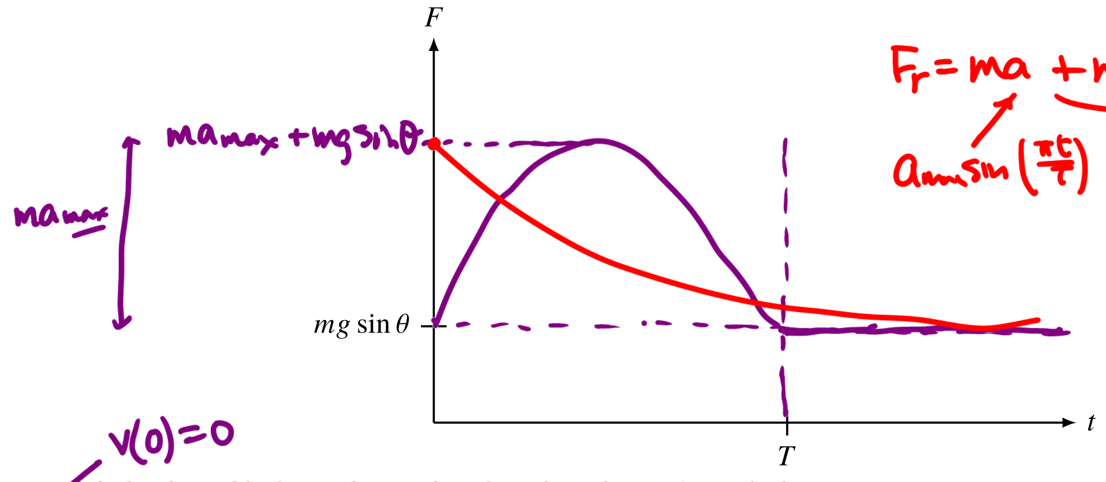
(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

(b) Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

(c) Determine the magnitude of the force exerted by the rope on the skier at terminal speed. after $t > T$ $a = 0$

(d) Derive an expression for the total impulse imparted to the skier during the acceleration.

(e) Suppose that the magnitude of the acceleration is instead modeled as $a = a_{\max}e^{-\pi t/2T}$ for all $t > 0$, where $a_{\max}$ and T are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model F_1 and the new model F_2.



$F_r = ma + mg\sin\theta$

$a_{max}\sin\left(\frac{\pi t}{T}\right)$

F_2: $a_{max}e^{-\frac{\pi t}{2T}}$

$v(0) = 0$

(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

$v(t) = \int a(t)\,dt = \int a_{max}\sin\left(\frac{\pi t}{T}\right)dt$

$v(t) = -a_{max}\frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right) + C$

$v(0) = -a_{max}\left(\frac{T}{\pi}\right)\underbrace{\cos(0)}_{=1} + C$

$C = a_{max}\left(\frac{T}{\pi}\right)$

$\boxed{v(t) = \frac{a_{max}T}{\pi}\left(1 - \cos\left(\frac{\pi t}{T}\right)\right) \quad 0 \leq t < T}$

at T, $v(T) = \frac{a_{max}T}{\pi}\underbrace{\left(1 - \underset{(-1)}{\cos\pi}\right)}_{2} = \frac{2a_{max}T}{\pi}$

b) $W_{net} = \Delta K = \frac{1}{2}mv_2^2 - \cancel{\frac{1}{2}mv_1^2}$

$W_{net} = \frac{1}{2}m\left(\frac{2a_{max}T}{\pi}\right)^2$

$W_{net} = \frac{1}{2}m\left(\frac{4a_{max}^2T^2}{\pi^2}\right)$

$\boxed{W_{net} = \frac{2ma_{max}^2T^2}{\pi^2}}$

c) $a = 0$ for $t > T$

$\boxed{F_r = mg\sin\theta}$

d) $J_{total} = \Delta p = mv_2 - \cancel{mv_1}$

$= m\left(\frac{2a_{max}T}{\pi}\right)$

$\boxed{J_{net} = \frac{2ma_{max}T}{\pi}}$