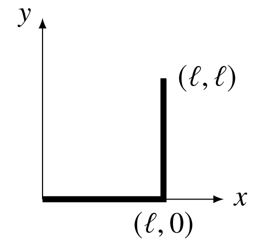
Student #: ____________________ Student Name: ____________________

AP PHYSICS C CLASS C: MOMENTUM AND CENTER OF MASS

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

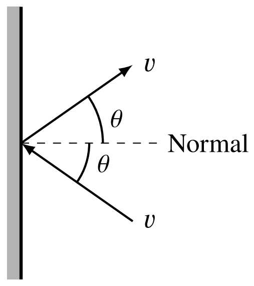
1. A uniform rod of length 2ℓ is bent, as shown in the figure below. What are the coordinates of the center of mass of the rod?



   (A) $(\ell/4, 3\ell/4)$
   (B) $(3\ell/4, \ell/4)$
   (C) $(2\ell/3, \ell/3)$
   (D) $(2\ell/3, 2\ell/3)$
   (E) $(\ell/2, \ell/3)$

2. Two uniform spheres of mass M and $4M$ are connected by a rod whose mass is negligible, and the distance between the centers of the spheres is d. The $4M$ sphere is then moved a distance of $d/3$ toward the smaller sphere. How far has the center of mass of the entire object moved?
   (A) The center of mass has not moved, because both spheres still have their original masses.
   (B) $d/15$
   (C) $d/5$
   (D) $4d/15$
   (E) $8d/15$

3. A rubber ball of mass m strikes a wall with a speed v at an angle θ below the normal line and rebounds from the wall at the same speed and angle above the normal line as shown. The magnitude of the change in momentum of the ball is



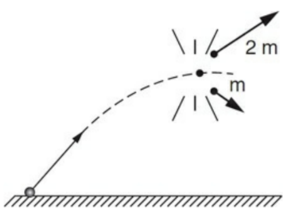
   (A) mv
   (B) $2mv$
   (C) $mv\cos\theta$
   (D) $2mv\cos\theta$
   (E) zero

4. Two blocks are connected by a compressed spring and rest on a frictionless surface. The blocks are released from rest and pushed apart by the compressed spring. If one mass is twice the mass of the other, which of the following is the same for both blocks?
   (A) magnitude of momentum
   (B) acceleration
   (C) speed
   (D) kinetic energy
   (E) potential energy

5. A known net force F acts on an unknown mass for a known time Δt. From this information, you could determine the
   (A) change in kinetic energy of the object
   (B) change in velocity of the object
   (C) acceleration of the object
   (D) mass of the object
   (E) change in momentum of the object

6. A small mass m is moving with a speed v toward a stationary mass M. The speed of the center of mass of the system is
   (A) $\left(\dfrac{m}{m+M}\right)v$
   (B) $\left(\dfrac{m+M}{m}\right)v$
   (C) $\left(\dfrac{m}{M}\right)v$
   (D) $\left(1+\dfrac{m}{M}\right)v$
   (E) $\left(1+\dfrac{M}{3m}\right)v$

Questions 7–8
A projectile is launched at an angle to the level ground as shown. At the top of the trajectory at point P, the projectile explodes into two pieces of mass $2m$ and m.

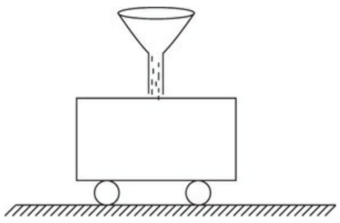


7. Which of the following arrows best represents the direction of the velocity of the center of mass of the projectile at point P after the explosion?
   (A) ←
   (B) ↙
   (C) ↘
   (D) →
   (E) ↗

8. Which of the following statements is true of the center of mass of the projectile after the explosion?
   (A) The center of mass will continue on a parabolic path and land on the ground at the place where it would have landed had it not exploded.
   (B) The center of mass will alter its parabolic path and land on the ground farther from where it would have landed had it not exploded.
   (C) The center of mass will alter its parabolic path and land on the ground at a shorter distance than it would have landed had it not exploded.
   (D) The center of mass will fall straight downward from the point of explosion.
   (E) The center of mass will travel straight upward from the point of explosion.

9. A mass traveling in the $+x$ direction collides with a mass at rest. Which of the following statements is true?
   (A) After the collision, the two masses will move with parallel velocities
   (B) After the collision, the masses will move with anti-parallel velocities
   (C) After the collision, the masses will both move along the x-axis
   (D) After the collision, the y-components of the velocities of the two particles will sum to zero.
   (E) None of the above

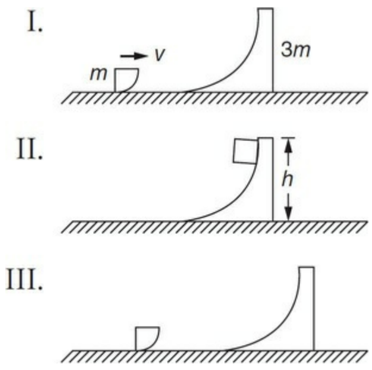
10. A 1000 kg (empty mass) railroad car is rolling without friction on a horizontal track at a speed of 2 m/s. Sand is poured into the open top of the car for the time interval from $t = 0$ to $t = 4$ s. The mass of the sand poured into the car as a function of time is $m(t) = 60t^2$. The velocity of the car at a time of 4 s is most nearly



(A) 1 m/s
(B) 2 m/s
(C) 3 m/s
(D) 4 m/s
(E) 5 m/s

Questions 11–12

A small block of mass m slides on a horizontal frictionless surface toward a ramp of mass $3m$ which is also free to move on the surface. The small block slides up to a height h on the ramp with no friction (Figure I), then they move together (Figure II), and the small block slides back down the ramp to the horizontal surface (Figure III). Both the block and the ramp continue to slide on the horizontal surface after they separate.



11. Which of the following is true regarding the conservation laws throughout this process?

(A) Kinetic energy is conserved from I to II.
(B) Momentum is conserved from I to III.
(C) Kinetic energy is conserved from II to III.
(D) Potential energy is conserved from I to II.
(E) Potential energy is conserved from II to III.

12. Which of the following is a true statement regarding Figure III?

(A) The small block is moving to the left and the ramp is moving to the right.
(B) The small block is moving to the right and the ramp is moving to the left.
(C) The small block is moving to the right and the ramp is moving to the right.
(D) The small block is moving to the left and the ramp is moving to the left.
(E) The small block and the large block are moving with the same velocity.

13. A moving object is changing its momentum during a time interval. If a graph of momentum vs. time is plotted, the net force acting on the mass at any time can be determined by finding the

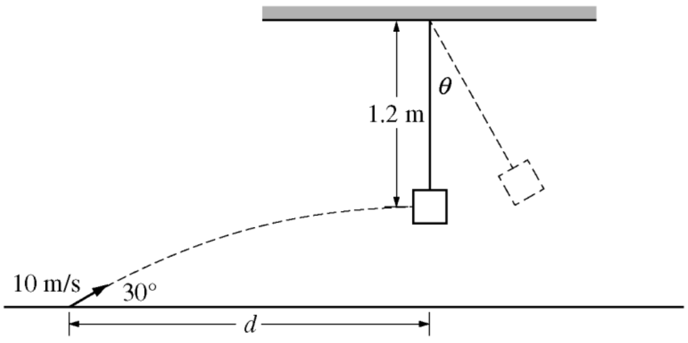
(A) slope of line tangent to the graph at that time
(B) area under the graph
(C) y-intercept of the graph
(D) x-intercept of the graph
(E) change in slope of the graph from beginning to end

14. A mass m_1 initially moving at speed v_0 collides with and sticks to a spring attached to a second, initially stationary mass m_2. The two masses continue to move to the right on a frictionless surface as the length of the spring oscillates. At the instant that the spring is maximally extended, the velocity of the first mass is

(A) v_0
(B) $m_1^2 v_0/(m_1+m_2)^2$
(C) $m_2 v_0/m_1$
(D) $m_1 v_0/m_2$
(E) $m_1 v_0/(m_1+m_2)$

AP PHYSICS C CLASS 5: MOMENTUM, IMPULSE, COLLISIONS, AND CENTER OF MASS
SECTION II
3 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.



1. A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.
   (a) Determine the speed of the dart just before it strikes the block.
   (b) Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.
   (c) Calculate the speed of the block just after the dart strikes.
   (d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.
   (e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.
   (f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.
      i. Would the angle θ that the dart and block swing to increase, decrease, or stay the same? Justify your answer.

      ____ Increase ____ Decrease ____ Stay the same

      ii. Would the period of oscillation after the collision increase, decrease, or stay the same? Justify your answer.

      ____ Increase ____ Decrease ____ Stay the same

2. A crash test car of mass 1000 kg moving at constant speed of 12 m/s collides completely inelastically with an object of mass M at time $t = 0$. The object was initially at rest. The speed v in m/s of the car-object system after the collision is given as a function of time t in seconds by the expression

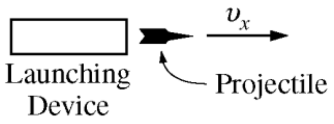
$$v = \frac{8}{1 + 5t}$$

(a) Calculate the mass M of the object.

(b) Assuming an initial position of $x = 0$, determine an expression for the position of the car-object system after the collision as a function of time t.

(c) Determine an expression for the resisting force on the car-object system after the collision as a function of time t.

(d) Determine the impulse delivered to the car-object system from $t = 0$ to $t = 2$ s.



3. A projectile is fired horizontally from a launching device, exiting with a speed v_x. While the projectile is in the launching device, the impulse imparted to it is J, and the average force on it is F_{avg}. Assume the force becomes zero just as the projectile reaches the end of the launching device. Express your answers to parts (a) and (b) in terms of v_x, J, F_{avg}, and fundamental constants, as appropriate.

   (a) Determine an expression for the time required for the projectile to travel the length of the launching device.

   (b) Determine an expression for the mass of the projectile.

   The projectile is fired horizontally into a block of wood that is clamped to a tabletop so that it cannot move. The projectile travels a distance d into the block before it stops. Express all algebraic answers to the following in terms of d and the given quantities previously indicated, as appropriate.

   (c) Derive an expression for the work done in stopping the projectile.

   (d) Derive an expression for the average force F_b exerted on the projectile as it comes to rest in the block.

   Now a new projectile and block are used, identical to the first ones, but the block is not clamped to the table. The projectile is again fired into the block of wood and travels a new distance d_n into the block while the block slides across the table a short distance D. Assume the following: the projectile enters the block with speed v_x, the average force F_b between the projectile and the block has the same value as determined in part (d), the average force of friction between the table and the block is f_T, and the collision is instantaneous so the frictional force is negligible during the collision.

   (e) Derive an expression for d_n in terms of d, D, f_T, and F_b, as appropriate.

   (f) Derive an expression for d_n in terms of d, the mass m of the projectile, and the mass M of the block.