Topic 11: Electrostatics

Advanced Placement Physics C

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Intro

Electrostatic Force

The Charges Are

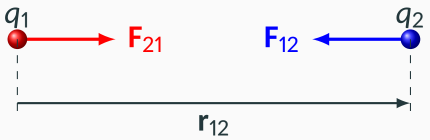
We should already know a bit about charge particles:

- A **proton** carries a **positive** charge
- An **electron** carries a **negative** charge
- A *net charge* of an object means an excess of protons or electrons
- Similar charges are repel; opposite charges attract

We start with electrostatics:

- Charges that are not moving relative to one another

Coulomb's Law for Electrostatic Force



The **electrostatic force** (or **coulomb force**) is a mutually repulsive/attractive force between all charged objects. The force that charge q_1 exerts on q_2 is given by **Coulomb's law**:

$$\boxed{\mathbf{F}_{12} = \frac{kq_1q_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12}}$$

Coulomb's Law for Electrostatic Force

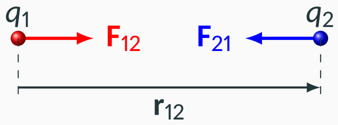
$$\boxed{\mathbf{F}_{12} = \frac{kq_1q_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12}}$$

Quantity	Symbol	SI Unit
Electrostatic force	$\mathbf{F}_{12}$	N
Coulomb's constant (electrostatic constant)	k	$\mathrm{N\,m^2/C^2}$
Point charges 1 and 2	q_1, q_2	C
Distance between point charges	$\lvert\mathbf{r}_{12}\rvert$	m
Unit vector of direction between point charges	$\hat{\mathbf{r}}_{12}$	

Coulomb's constant $k = \dfrac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\,\mathrm{N\,m^2/C^2}$ where

$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N\,m^2}$ is called the "permittivity of free space"

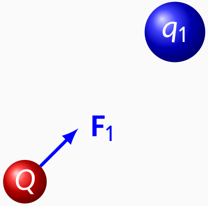
Coulomb's Law for Electrostatic Force



- If q_1 exerts an electrostatic force $\mathbf{F}_{12}$ on q_2, then q_2 likewise exerts a force of $\mathbf{F}_{21} = -\mathbf{F}_{12}$ on q_1. The two forces are equal in magnitude and opposite in direction (3rd law of motion).
- q_1 and q_2 are assumed to be *point charges* that do not occupy any space
- The (more familiar) scalar form is often used as well:

$$\boxed{F_q = \frac{kq_1q_2}{r^2}}$$

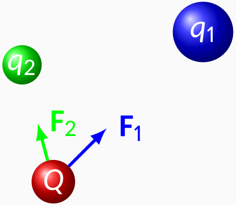
More Than One Charge



For a charge Q that is subjected to the influence of multiple discrete point charges q_i, the total electrostatic force that Q experiences is the vector sum of all the forces $\mathbf{F}_i$:

$$\boxed{\mathbf{F} = \sum_i \mathbf{F}_i = kQ\left(\sum_{i=1}^{N} \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i\right)}$$

More Than One Charge



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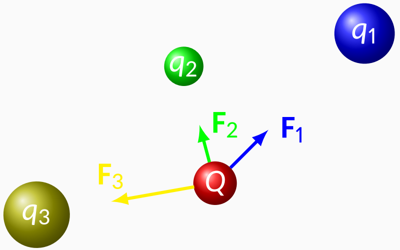

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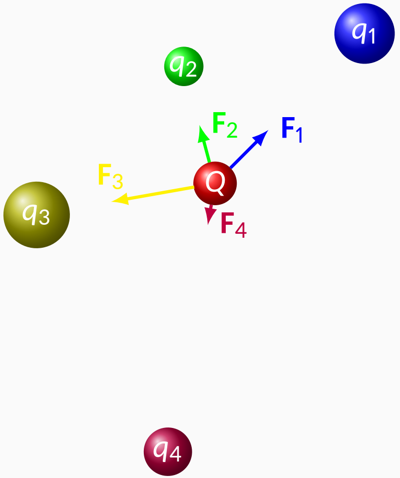

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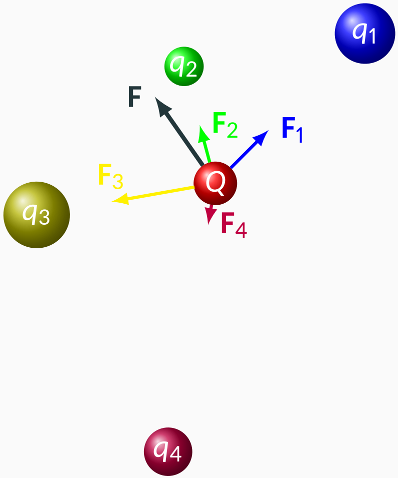

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Electric Field

Electric Field

The expression for **electric field** is obtained by repeating the same procedure as with gravitational field, by groupping the variables in Coulomb's law:

$$F_q = \underbrace{\left[\frac{kq_1}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}\right]}_{\mathbf{E}} q_2$$

The electric field **E** created by q_1 is a vector function (called a **vector field**) that shows how it influences other charged particles around it.

Electric Field Near a Point Charge

The electric field a distance r away from a point charge q is given by:

$$\boxed{\mathbf{E}(q, \mathbf{r}) = \frac{kq}{|\mathbf{r}|^2}\hat{\mathbf{r}}}$$

Quantity	Symbol	SI Unit
Electric field intensity	$\mathbf{E}$	N / C
Coulomb's constant	k	$\mathrm{N\,m^2/C^2}$
Source charge	q	C
Distance from source charge	$\lvert\mathbf{r}\rvert$	m
Outward unit vector from point source	$\hat{\mathbf{r}}$	

The direction of **E** is radially outward from a positive point charge and radially inward towards a negative charge.

More Than One Charge

When multiple point charges are present, the total electric field at any position $\mathbf{r}$ is the vector sum of all the fields $\mathbf{E}_i$:

$$\boxed{\mathbf{E} = \sum_i \mathbf{E}_i = k\left(\sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i\right)}$$

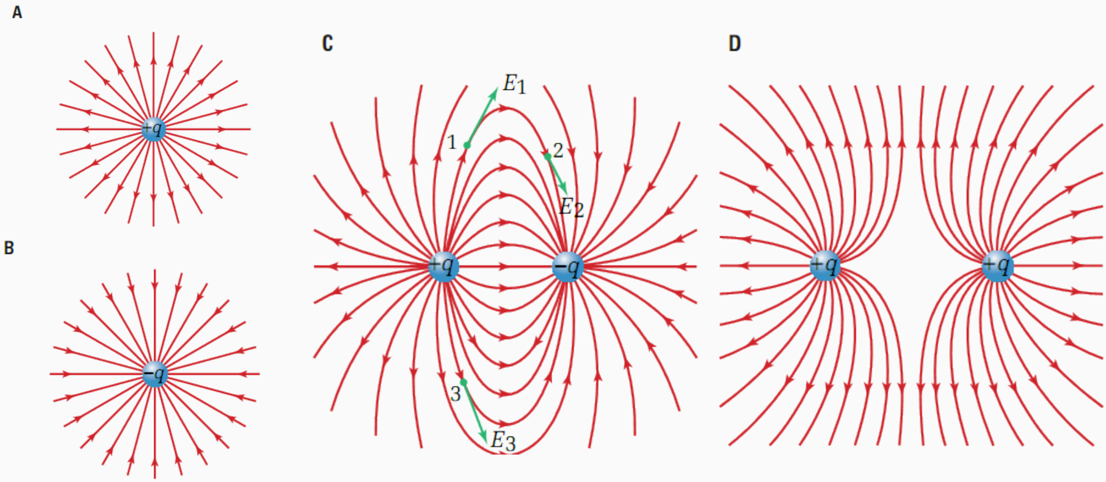
Think Electric Field

$\mathbf{E}$ iself *doesn't do anything* until another charge interacts with it. And when there is a charge q, the electrostatic force $\mathbf{F}_q$ that the charge experiences is proportional to q and $\mathbf{E}$, regardless of how the electric field is generated:

$$\boxed{\mathbf{F}_q = \mathbf{E}q}$$

A positive charge in the electric field experiences a electrostatic force $\mathbf{F}$ in the same direction as $\mathbf{E}$.

Electric Field Lines



Electric Potential & Potential Energy

Electrical Potential Energy

The work done by the electrostatic force is given by:

$$W = \int \mathbf{F}_q \cdot d\mathbf{r} = kq_1q_2 \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{kq_1q_2}{r}\Big|_{r_1}^{r_2} = -\Delta U_q$$

where Q_q is defined as the **electric potential energy**:

$$\boxed{U_q = \frac{kq_1q_2}{r}}$$

U_q can be $(+)$ or $(-)$, because charges can be either $(+)$ or $(-)$.

How it Differs from Gravitational Potential Energy

Two positive charges:

$$U_q > 0$$

Two negative charges:

$$U_q > 0$$

One positive and one negative charge:

$$U_q < 0$$

- $U_q > 0$ means positive work is done to bring two charges together from $r = \infty$ to r (both charges of the same sign)
- $U_q < 0$ means negative work (the charges are opposite signs)
- For gravitational potential U_g is always < 0

Electric Potential

When I move an object of mass m against a gravitational force from one point to another, the work that I do is directly proportional to m, i.e. there is a "constant" in that scales with *any* mass, as long as they move between those same two points:

$$W = \Delta U_g = Km$$

In the trivial case (small changes in height, no change in g), this constant is just

$$\frac{\Delta U_g}{m} = g\Delta h$$

Electric Potential

This is also true for moving a charged particle q against an electric electric field created by q_s, and the "constant" is called the **electric potential**. For a point charge, it is defined as:

$$\boxed{V = \frac{U_q}{q} = \frac{kq_s}{r}}$$

The unit for electric potential is a *volt* which is *one joule per coulomb*:

$$1\,\mathrm{V} = 1\,\mathrm{J}/\mathrm{C}$$

We can easily the relationship between V and **E**:

$$\boxed{\Delta V = \int \mathbf{E} \cdot d\mathbf{r}}$$

Potential Difference (Voltage)

The change in electric potential is called the **electric potential difference** or **voltage**:

$$\boxed{\Delta V = \frac{\Delta U_q}{q}} \quad \text{and} \quad \boxed{dV = \frac{dU_q}{q}}$$

Here, we can relate ΔV to an equation that we knew from Grade 11 Physics, which related to the energy dissipated in a resistor in a circuit ΔU to the voltage drop ΔV:

$$\boxed{\Delta U_q = q\Delta V}$$

Electric potential difference also has the unit *volts* (V)

Getting Those Names Right

Remember that these three scalar quantities, as opposed to electrostatic force $\mathbf{F}_q$ and electric field $\mathbf{E}$ which are vectors

- Electric potential energy:

$$U = \frac{kq_1q_2}{r}$$

- Electric potential:

$$V = \frac{kq}{r}$$

- Electric potential difference (voltage):

$$\Delta V = \frac{\Delta U_q}{q}$$

Relating U_q, $\mathbf{F}_q$ and $\mathbf{E}$

From the fundamental theorem calculus, we can relate electrostatic force ($\mathbf{F}_q$) to electric potential energy (U_q) by the gradient operator, and electric field ($\mathbf{E}$) to the electric potential (V) the same way:

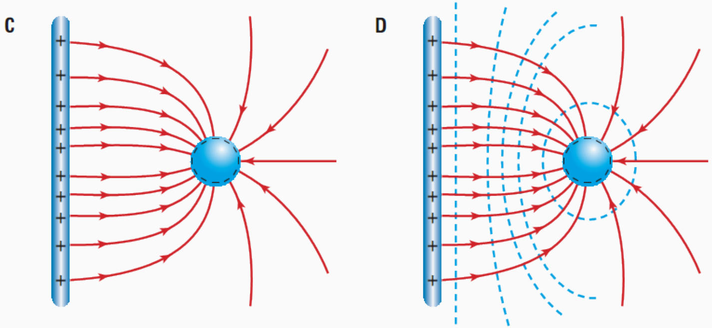
$$\mathbf{F}_q = -\nabla U_q = -\frac{\partial U_q}{\partial r}\hat{\mathbf{r}} \qquad \mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{\mathbf{r}}$$

- Electrostatic force $\mathbf{F}_q$ always points from high to low potential energy (steepest descent direction)
- Electric field can also be expressed as the change of electric potential per unit distance, which has the unit

$$1\,\mathrm{N/C} = 1\,\mathrm{V/m}$$

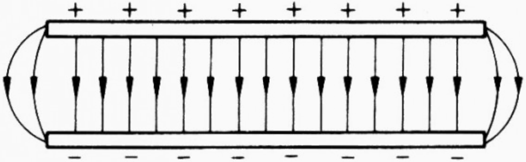
- Electric field is also called "potential gradient"

Equipotential Lines



The dotted blue lines are called **equipotential lines.** They are always *perpendicular* to the electric field lines. Charges moving in the direction of the equipotential lines have constant electric potential

Electric Field Between Parallel Charged Plates

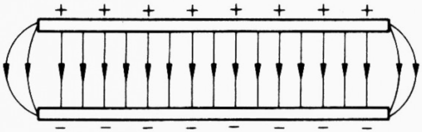


- Two plates, each producing an electric field pointing in the same direction
- The total electric field is twice the value of *one* infinite plane, pointing from the positively charged plate towards the negatively charged plate

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

- **E** outside the plates is very low (close to zero), except for fringe effects at the edges of the plates

Electric Field and Electric Potential Difference



In the case of two parallel plates, the electric field is uniform, and the relationship simplifies to:

$$\boxed{E = \frac{\Delta V}{d}}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C
Electric potential difference between plates	ΔV	V
Distance between plates	d	m