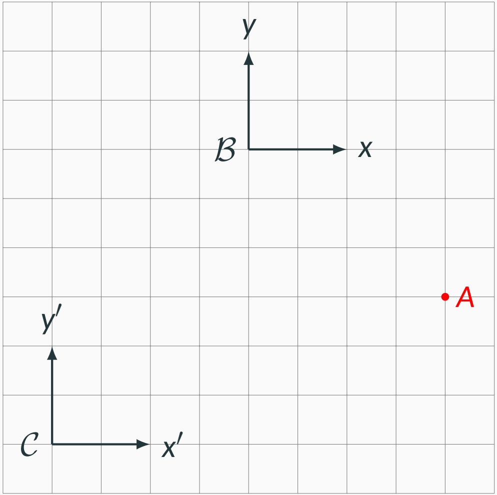
Relative Motion

All motion quantities must be measured relative to a *frame of reference*

- **Frame of reference:** the *coordinate system* from which all physical measurements are made.
- In *classical* mechanics, the coordinate system is the Cartesian system
- There is no absolute motion/rest: all motions are relative
- **Principle of Relativity:** All laws of physics are equal in all inertial (non-accelerating) frames of reference

Relative Motion

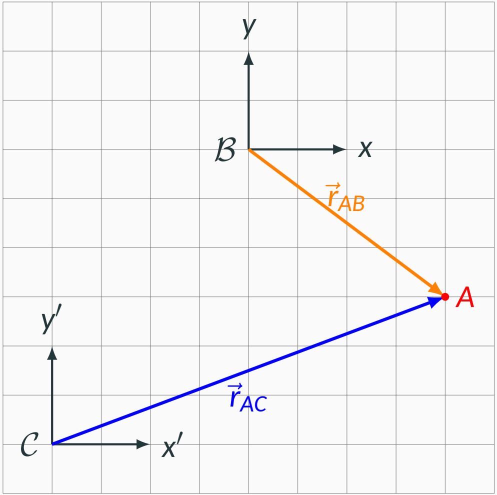


Two frames of reference

- $\mathcal{B}$ with axes x, y
- $\mathcal{C}$ with axes x', y'

The two reference frames may (or may not) be moving relative to each other. The motion of the two reference frames affect how motion of A is calculated.

Relative Motion

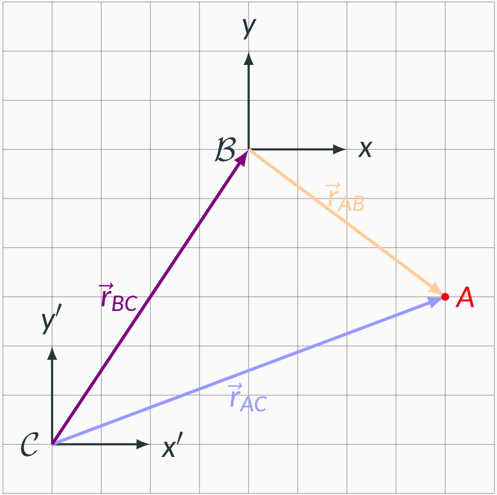


The position of A can be described by

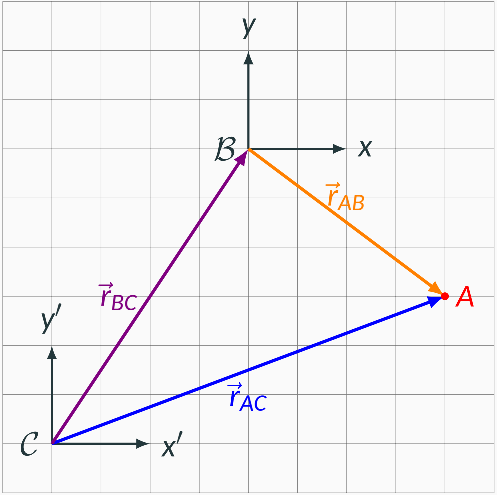
- $\vec{r}_{AB}(t)$ (relative to frame $\mathcal{B}$)
- $\vec{r}_{AC}(t)$ (relative to frame $\mathcal{C}$)

It is obvious that $\vec{r}_{AB}(t)$ and $\vec{r}_{AC}(t)$ are different vectors

Relative Motion



Relative Motion



Starting from the definition of **relative position**:

$$\boxed{\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC}}$$

Differentiating all terms with respect to time, we get the equation for **relative velocity**:

$$\boxed{\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}}$$

Differentiating with respect to time again, and we obtain the expression for **relative acceleration**:

$$\boxed{\vec{a}_{AC} = \vec{a}_{AB} + \vec{a}_{BC}}$$

Relative Velocity

In classical mechanics, the equation for relative velocities follows the **Galilean velocity addition rule**, which applies to speeds much less than the speed of light:

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

The velocity of A relative to reference frame $\mathcal{C}$ is the velocity of A relative to reference frame $\mathcal{B}$, plus the velocity of $\mathcal{B}$ relative to $\mathcal{C}$. If we add another reference frame $\mathcal{D}$, the equation becomes:

$$\vec{v}_{AD} = \vec{v}_{AB} + \vec{v}_{BC} + \vec{v}_{CD}$$