

# **WELCOME TO AP PHYSICS 1 & 2**

# Pre-requisites

- **Physics 11 and 12:** You will need to be comfortable with the topics covered in high-school level physics courses.
- **Vectors:** You need to be comfortable with vector operations, including addition and subtraction, multiplication/division by constants, as well as dot products and cross products.

If you already have a background in both differential and integral calculus, you may consider taking the AP Physics C exams instead.

# Classroom Rules

- Treat each other with respect
- Raise your hands if you have a question. Don't wait too long
- E-mail me at [tleung@olympiadsmail.ca](mailto:tleung@olympiadsmail.ca) for any questions related to physics and math and engineering
- Do **not** try to find me on social media

# Topic 1: Kinematics

## Advanced Placement Physics 1

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Dr. Timothy Leung

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Olympiads School

## Files for You to Download

- `PhysAP1-courseOutline.pdf`—The course outline
- `PhysAP1-01-kinematics.pdf`—This set of slides
- `PhysAP1-01-Homework.pdf`—Homework problems for kinematics

# Kinematics

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# Kinematics

**Kinematics** is a discipline within mechanics concerning the mathematical description of the motion of bodies. It describes the relationships between

- Position
- Displacement
- Distance
- Velocity
- Speed
- Acceleration

Kinematics does not deal with the causes of motion. In the AP Physics 1 exam, kinematics account for approximately 10 % to 16 % of the marks.

# Position

**Position** ( $\mathbf{x}$ ) describes the location of an object within a coordinate system. The SI unit for position is **meter** (m).

$$\mathbf{x}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$$

Vectors in 2D/3D Cartesian space are often expressed using the **IJK notation**

- $\hat{\mathbf{i}}$ ,  $\hat{\mathbf{j}}$  and  $\hat{\mathbf{k}}$  are **basis vectors** representing the directions of the  $x$ ,  $y$  and  $z$  axes. Basis vectors are **unit vectors** (i.e. length 1)
- The IJK notation does not explicitly give the magnitude or the direction of the vector (needs to be calculated using the Pythagorean theorem)



# Displacement

**Displacement** ( $\Delta \mathbf{x}$ ) is the vector change in position from the initial position  $\mathbf{x}_0$  within the same coordinate system. The unit for displacement is also meter.

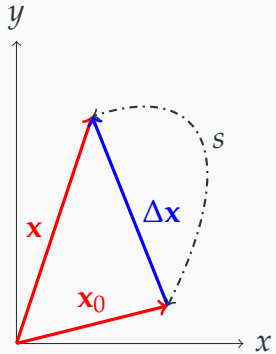
$$\Delta \mathbf{x}(t) = \mathbf{x}(t) - \mathbf{x}_0 = (x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k}$$

IJK notation makes vector addition and subtraction less prone to errors

# Distance

**Distance**  $s$  is a quantity that is *related* to displacement. It is:

- The length of the path taken by an object when it from  $\mathbf{x}_0$  to  $\mathbf{x}$
- A scalar quantity
- Always positive, i.e.  $s \geq 0$
- Although the magnitude of the displacement vector is also a scalar, it is not necessarily the same as distance
- $s \geq |\Delta \mathbf{x}|$



Pay close attention to the difference between distance and displacement.

# Average Velocity

**Average velocity**  $\bar{\mathbf{v}}$  of an object is its displacement  $\Delta \mathbf{x}$  over a *finite* time interval  $\Delta t$ . The unit for velocity is **meters per second** (m/s):

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t}$$

Since the  $\hat{\mathbf{i}}$ ,  $\hat{\mathbf{j}}$  and  $\hat{\mathbf{k}}$  directions ( $x$ ,  $y$ ,  $z$  axes) are *linearly independent*<sup>1</sup>, each component of average velocity can be calculated by separating each direction:

$$\bar{\mathbf{v}} = \frac{\Delta x}{\Delta t} \hat{\mathbf{i}} + \frac{\Delta y}{\Delta t} \hat{\mathbf{j}} + \frac{\Delta z}{\Delta t} \hat{\mathbf{k}}$$

(Note: A bar is drawn over the symbol if it is averaged over time.)

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<sup>1</sup>mathematical way of saying that what happens in one axis does not affect another

# Instantaneous Velocity

If displacement  $d\mathbf{x}$  is calculated a very small<sup>2</sup> time interval  $dt$ , then velocity is called the **instantaneous velocity**:

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t} \quad \rightarrow \quad \boxed{\mathbf{v} = \frac{d\mathbf{x}}{dt}}$$

The instantaneous velocity is the slope of the tangent on the position-time graph.

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<sup>2</sup>In calculus, a very small change is called *infinitesimally small*

# Instantaneous & Average Speed

**Average speed** is similar to average velocity: it is the distance  $s$  traveled over a finite time interval  $\Delta t$ . Since distance is always positive, so too is the average speed

$$\bar{v} = \frac{s}{\Delta t}$$

Likewise, when the time interval is made infinitesimally small, then the speed is called the **instantaneous speed**  $v$ . Instantaneous speed  $v$  is the magnitude of the instantaneous velocity vector.

# Instantaneous & Average Acceleration

In the same way that velocity describes how quickly position changes with time, **average acceleration**  $\bar{\mathbf{a}}$  is the change in velocity  $\Delta \mathbf{v}$  over a finite time interval  $\Delta t$ . The unit for acceleration is **meters per second squared** ( $\text{m/s}^2$ ).

$$\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}(t) - \mathbf{v}(t_0)}{t - t_0}$$

Making the time interval  $\Delta t = t - t_0$  infinitesimally small gives the **instantaneous acceleration**  $\mathbf{a}(t)$ .

## If You Are Curious (Not Part of AP Physics)

For the curious minds, the time rate of change of acceleration is called **jerk**, with a unit of  $\text{m/s}^3$ :

$$\bar{\mathbf{j}} = \frac{\Delta \mathbf{a}}{\Delta t}$$

The time rate of change in jerk is called **jounce** or **snap**, with a unit of  $\text{m/s}^4$ :

$$\bar{\mathbf{s}} = \frac{\Delta \mathbf{j}}{\Delta t}$$

The next motion quantities are are called **crackle** and **pop**, but these quantities are almost never used.

# Acceleration as Functions of Velocity and Position

Sometimes, acceleration are expressed as a function of velocity or position rather than of time, depending on the forces acting on them. For example:

- Gravitational or electrostatic forces:  $a(x) = Ax^{-2}$
- Spring force:  $a(x) = -Bx$
- Damping force (e.g. shock absorbers):  $a(v) = Cv$
- Aerodynamic drag:  $a(v) = Dv^2$

In these cases, solving for the motion quantities  $x(t)$ ,  $v(t)$  and  $a(t)$  may require calculus, numerical integration methods, or the conservation of energy.



# Kinematic Equations

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# Kinematic Equations

Without calculus, kinematic problems in AP Physics 1 only deal with constant acceleration. The 1D kinematic equations that will be used in Physics 1 are:

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- Initial position:  $x_0$
- Position at time  $t$ :  $x$
- Initial velocity:  $v_0$
- Velocity at time  $t$ :  $v$
- Acceleration (constant):  $a$

These equations are sometimes called the “Big-five” or “Big-four” in Grade 11/12 Physics. In AP, you are given only 3 equations in your equation sheet.

# Motion Graphs

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# Motion Graphs

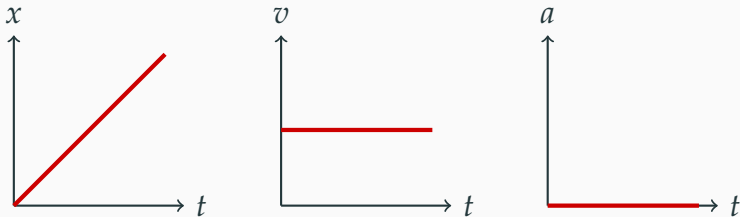
You should already be familiar with the *basic* 1D motion graphs. These are still used in AP Physics.

- Position vs. time ( $x-t$ ) graph
- Velocity vs. time ( $v-t$ ) graph
- Acceleration vs. time ( $a-t$ ) graph

They are the graphical representation of the kinematic equations from the previous slide.

# Uniform Motion

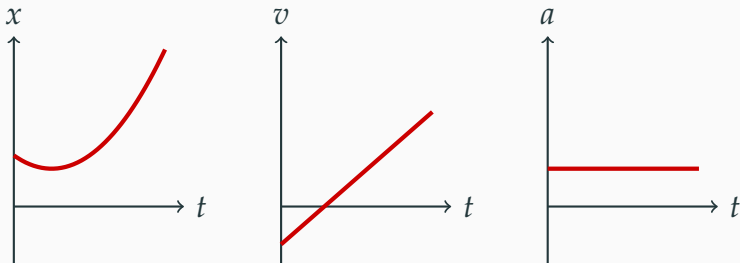
An object moves with constant velocity (neither magnitude nor direction changes) and therefore no acceleration.



- Constant velocity has a straight line in the  $x$ - $t$  graph
- The slope of the  $x$ - $t$  graph is the velocity  $v$ , which is constant
- The slope of the  $v$ - $t$  graph is the acceleration  $a$ , which is zero by definition

# Uniform Acceleration

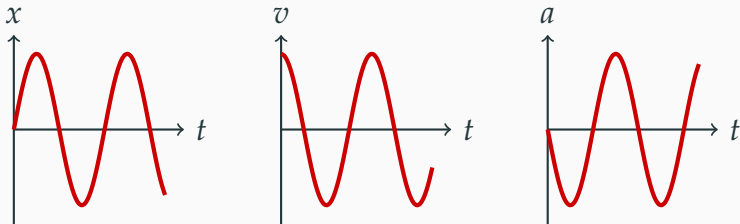
An object moves with a constant non-zero acceleration:



- The  $x$ - $t$  graph is part of a *parabola*
  - If the parabola is *convex* (opens up), acceleration is (+)
  - If the parabola is *concave* (opens down), acceleration is (—)
- The  $v$ - $t$  graph is a straight line; its slope (a constant) is the acceleration
- The  $a$ - $t$  graph is a horizontal straight line

# Simple Harmonic Motion

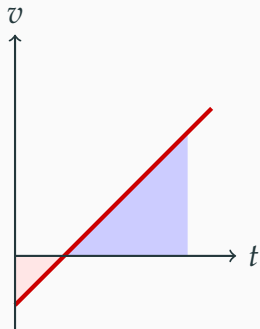
For **harmonic motions** (vibrations, oscillations),  $x$ ,  $v$  and  $a$  are all non-constant, and they all change with time as sinusoidal functions.



**Bottom line:** regardless of the type motion,

- The  $v$ - $t$  graph is the slope of the  $x$ - $t$  graph
- The  $a$ - $t$  graph is the slope of the  $v$ - $t$  graph

# Area Under Motion Graphs



The area under the  $v-t$  graph is the displacement  $\Delta x$ :

- Area *above* the time axis: + displacement
- Area *below* the time axis: – displacement

The area under the  $a-t$  graph is the change in velocity  $\Delta v$ :

- Area *above* the time axis: + change in velocity
- Area *below* the time axis: – change in velocity

The area under the  $x-t$  graph has no physical meaning.



# Velocity Squared vs. Displacement

If velocity information is given as a function of position<sup>3</sup> then a motion graph can be plotted using this kinematic equation:

$$\underbrace{v^2}_y = \underbrace{v_0^2}_b + \underbrace{2a}_m \underbrace{(x - x_0)}_x$$

by plotting  $v^2$  on the  $y$ -axis and displacement  $\Delta x = x - x_0$  on the  $x$ -axis. The slope of the graph is  $m = 2a$ . The square of the initial velocity ( $v_0^2$ ) is the  $y$ -intercept.

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<sup>3</sup>Depends on experimental set up

# Graphing “Linear” Functions

This concept extends to graphing other physical quantities not relating to motion:

- To find the index of refraction of a material using Snell’s law, plot  $\sin \theta_i$  vs.  $\sin \theta_2$  (rather than  $\theta_1$  vs.  $\theta_2$ ). The slope is the index  $n$ :

$$\underbrace{\sin \theta_1}_y = \underbrace{n}_m \underbrace{\sin \theta_2}_x$$

- To relate the period of oscillation of a simple pendulum to the length of the pendulum, plot  $T$  vs.  $\sqrt{L}$ :

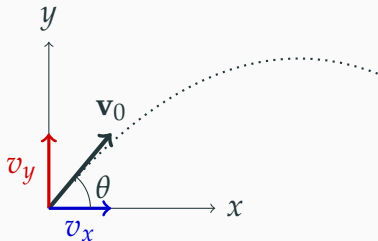
$$\underbrace{T}_y = \frac{2\pi}{\underbrace{\sqrt{g}}_m} \underbrace{\sqrt{L}}_x$$

# Projectile Motion

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# Projectile Motion

A **projectile** is an object that is launched with an initial velocity of  $\mathbf{v}_0$  along a parabolic trajectory and accelerates only due to gravity.



- $x$ -axis: *horizontal*, pointing *forward*
- $y$ -axis: *vertical*, pointing *up*
- Angle  $\theta$  measured *above* the horizontal
- The origin is usually where the projectile is launched

## Horizontal Direction

The initial velocity  $\mathbf{v}_0$  can be decomposed into its  $x$  and  $y$  components using the launch angle  $\theta$ :

$$\mathbf{v}_0 = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}} = [v_0 \cos \theta] \hat{\mathbf{i}} + [v_0 \sin \theta] \hat{\mathbf{j}}$$

There is no horizontal acceleration (i.e.  $a_x = 0$ ), therefore  $v_x$  is constant. The kinematic equations reduce to a single equation:

$$x = v_x t = [v_0 \cos \theta] t$$

where  $x$  is the horizontal position at time  $t$

## Vertical Direction

There is constant vertical acceleration due to gravity alone, i.e.  $a_y = -g$ . ( $a_y$  is *negative* due to the way we defined the coordinate system.) The important equation is this one:

$$y = [v_0 \sin \theta] t - \frac{1}{2}gt^2$$

These two kinematic equations may also be useful:

$$\begin{aligned}v_y &= [v_0 \sin \theta] - gt \\v_y^2 &= [v_0^2 \sin^2 \theta] - 2gy\end{aligned}$$

# Solving Projectile Motion Problems

Horizontal and vertical motions are linearly independent, but variables are shared in both directions:

- Time  $t$
- Launch angle  $\theta$  (above the horizontal)
- Initial speed  $v_0$

When solving any projectile motion problems

- *Two* equations with *two* unknowns
- If an object lands on an incline, there will be a third equation relating  $x$  and  $y$

# Symmetric Trajectory

A projectile's trajectory is *symmetric* if the object lands at the same height as when it launched. The angle  $\theta$  is measured above the the horizontal.

- Time of flight

$$t_{\max} = \frac{2v_0 \sin \theta}{g}$$

- Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum height

$$y_{\max} = \frac{v_0^2 \sin^2 \theta}{2g}$$



## Maximum Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum range occurs at  $\theta = 45^\circ$
- For a given initial speed  $v_0$  and range  $R$ , launch angle  $\theta$  is given by:

$$\theta_1 = \frac{1}{2} \sin^{-1} \left( \frac{Rg}{v_0^2} \right)$$

But there is another angle that *gives the same range*!

$$\theta_2 = 90^\circ - \theta_1$$

# Relative Motion

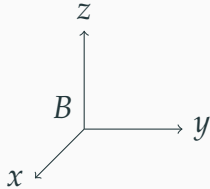
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# Relative Motion

## All motion quantities must be measured relative to a frame of reference

- A frame of reference is the coordinate system from which all physical measurements are made.
- In *classical* mechanics, the coordinate system is the Cartesian system
- There is no absolute motion/rest: all motions are relative
- All laws of physics are equal in all inertial (non-accelerating) frames of reference (principle of relativity)

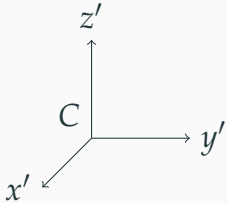
# Relative Motion



Two frames of reference

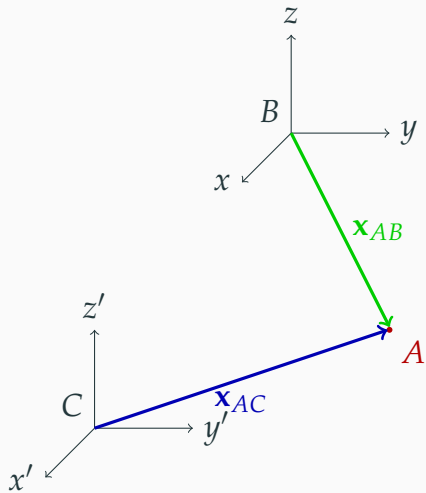
- B with axes  $x, y, z$
- C with axes  $x', y', z'$

The two reference frames may (or may not) be moving relative to each other. The motion of the two reference frames affect how motion of **A** is calculated.



• **A**

# Relative Motion

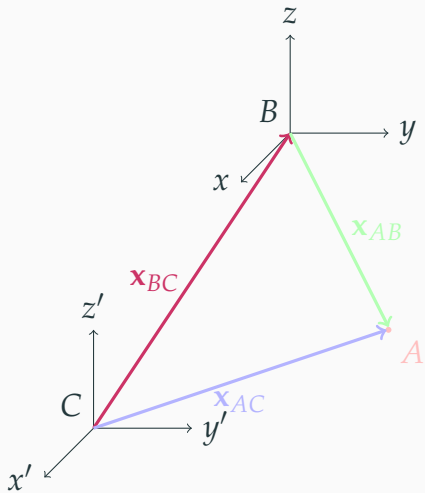


The position of  $A(t)$  (as a function of time) can be described by

- $\mathbf{x}_{AB}(t)$  (relative to frame B)
- $\mathbf{x}_{AC}(t)$  (relative to frame C)

Without needing mathematically rigorous vector notation, it is obvious that  $\mathbf{x}_{AB}(t)$  and  $\mathbf{x}_{AC}(t)$  are different vectors

# Relative Motion



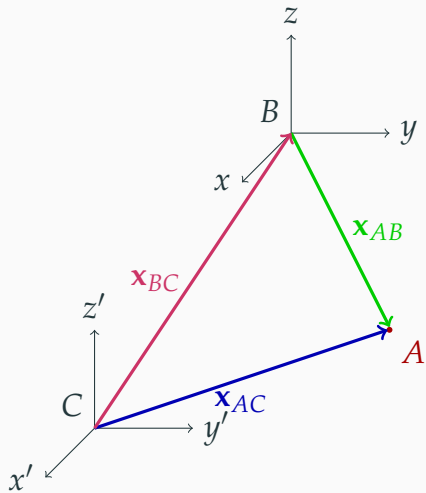
The position vector of the origins of the two reference frames is given by  $\mathbf{x}_{BC}$

- The vector pointing from the origin of frame C to the origin of frame B
- If the two frames are moving relative to each other, then  $\mathbf{x}_{BC}$  is also a function of time

Without using vector notations, the relationship between the vectors is obvious:

$$\mathbf{x}_{AC} = \mathbf{x}_{AB} + \mathbf{x}_{BC}$$

# Relative Motion



Starting from the definition of **relative position**:

$$\mathbf{x}_{AC} = \mathbf{x}_{AB} + \mathbf{x}_{BC}$$

Using the definitions for velocity to get a similar equation for **relative velocity**:

$$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$$

and **relative acceleration**:

$$\mathbf{a}_{AC} = \mathbf{a}_{AB} + \mathbf{a}_{BC}$$

# Relative Velocity

In classical mechanics, the equation for relative velocities follows the **Galilean velocity addition rule**, which applies to speeds much less than the speed of light:

$$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$$

The velocity of  $A$  relative to reference frame  $C$  is the velocity of  $A$  relative to reference frame  $B$ , plus the velocity of  $B$  relative to  $C$ . If we add another reference frame  $D$ , the equation becomes:

$$\mathbf{v}_{AD} = \mathbf{v}_{AB} + \mathbf{v}_{BC} + \mathbf{v}_{CD}$$



# Typical Problems

In the AP Physics 1 exam, kinematics questions appear in both multiple-choice and free-response sections. The problems themselves are not necessarily very different from Grade 11/12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use  $g = 10 \text{ m/s}^2$  in your calculations to make your lives simpler
- Many problems are *symbolic*, which means that they deal with the algebraic expressions, not actual numbers
- Often coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet