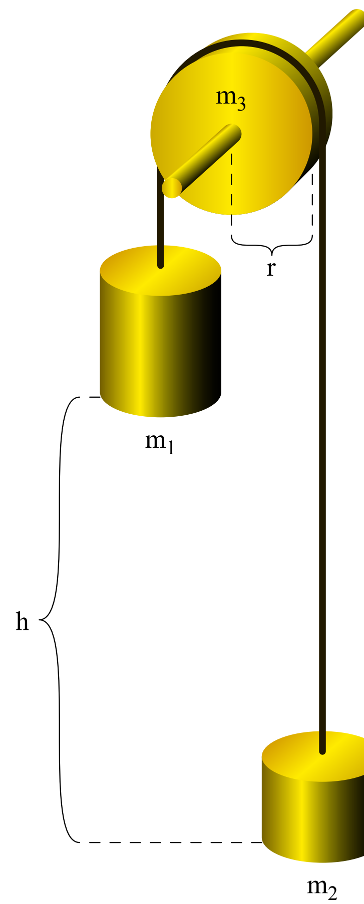
Rotational Dynamics

Practice

practice problem 1



A kind of Atwood's machine is built from two cylinders of mass m_1 and m_2; a cylindrical pulley of mass m_3 and radius r; a light, frictionless axle; and a piece of light, unstretchable string. The heavier mass m_1 is held above the ground a height h and then relased from rest.

a. Draw a free body diagram showing all the forces acting on…
   i. the heavier mass
   ii. the lighter mass
   iii. the pulley
b. Write the equation stating Newton's second law of translational motion for…
   i. the heavier mass
   ii. the lighter mass

   and rotational motion for…
   iii. the pulley
c. Determine the translational acceleration of…
   i. the heavier mass
   ii. the lighter mass

   and the rotational acceleration of…
   iii. the pulley
d. Determine the tension in the side of the string connected to
   i. the heavier mass
   ii. the lighter mass

   and the upward force of the axle on…
   iii. the pulley
e. Lastly, determine…
   i. the time it takes for the heavier mass to reach the ground
   ii. its speed on impact
   iii. the rotational speed of the pulley at this time

solution

This might seem like a big problem, but it's actually just a bunch of small ones. Since problems in rotational dynamics tend to get complicated very quickly, it seems like a good way to introduce this topic.

a. Answer it.
   i. Answer it.

ii. Answer it.

iii. Answer it.

b. Answer it.

i. Answer it.

ii. Answer it.

iii. Answer it.

c. Answer it.

i. Answer it.

ii. Answer it.

iii. Answer it.

d. Answer it.

i. Answer it.

ii. Answer it.

iii. Answer it.

e. Answer it.

i. Answer it.

ii. Answer it.

iii. Answer it.

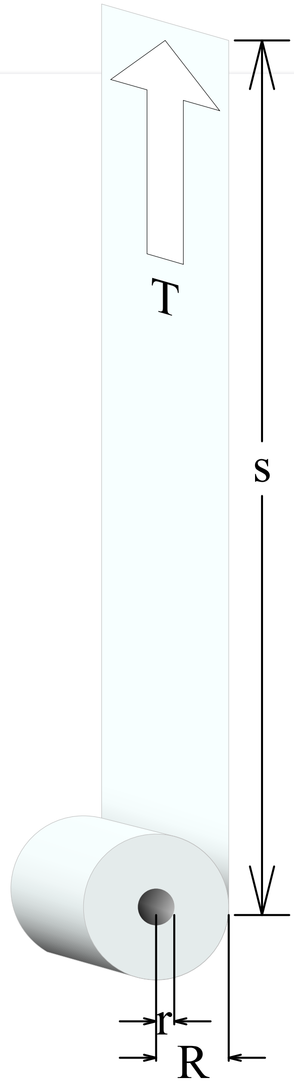
practice problem 2

A roll of toilet paper is held by the first piece and allowed to unfurl as shown in the diagram to the right. The roll has an outer radius R = 6.0 cm, an inner radius r = 1.8 cm, a mass m = 200 g, and falls a distance s = 3.0 m. Assuming the outer diameter of the roll does not change significantly during the fall, determine…

a. the tension in the sheets
b. the translational acceleration of the roll
c. the angular acceleration of the roll
d. the final translational speed of roll
e. the final angular speed of the roll

solution

a. Apply Newton's second law of motion in both its translational and rotational forms. (Let down be the positive direction.)



translational	rotational
$\sum \mathbf{F} = m\mathbf{a}$	$\sum \boldsymbol{\tau} = I\boldsymbol{\alpha}$
$mg - T = ma$	$RT = ½m(R^2 + r^2)\,\dfrac{a}{R}$

Now work the magic of algebra. Begin by rewriting the rotational equation a bit then substitute from the translational side and solve for tension. This is not as easy to *do* as it is to *say*, however. Those of you not comfortable with the algebra of pure symbols may want to substitute numbers in now, simplify a bit, and then solve. Whatever gets you the right answer is probably a right method.

$$2R^2T = (R^2 + r^2)ma$$

$$2R^2T = (R^2 + r^2)(mg - T)$$

$$2R^2T + (R^2 + r^2)T = (R^2 + r^2)mg$$

$$T = \frac{(R^2 + r^2)mg}{3R^2 + r^2}$$

$$T = \frac{[(0.060\text{ m})^2 + (0.018\text{ m})^2](0.200\text{ kg})(9.8\text{ m/s}^2)}{3(0.060\text{ m})^2 + (0.018\text{ m})^2}$$

$$T = 0.691\text{ N}$$

b. Jump back to the translational statement of Newton's second law then jump through the usual hoops: algebra, numbers, answer. The answer should be less than the acceleration due to gravity (since tension is dragging upward against weight).

$$mg - T = ma$$

$$a = g - \frac{T}{m}$$

$$a = 9.8 \text{ m/s}^2 - \frac{0.691 \text{ N}}{0.200 \text{ kg}}$$

$$a = 6.34 \text{ m/s}^2$$

c. At last, an easy problem. Just be sure to use the outer radius of the roll, not the inner.

$$\alpha = \frac{a}{R}$$

$$\alpha = \frac{6.34 \text{ m/s}^2}{0.060 \text{ m}}$$

$$\alpha = 106 \text{ rad/s}^2$$

$$\alpha = 16.8 \text{ rev/s}^2$$

d. Another easy problem. At least, you should find it easy if you've gotten this far through this book.

$$v^2 = v_0{}^2 + 2a\Delta s$$
$$v = \sqrt{(2 \times 6.34 \text{ m/s}^2 \times 3.0 \text{ m})}$$
$$v = 6.17 \text{ m/s}$$

e. One more easy problem. There are several ways to solve this part. Whatever method you chose, use the correct radius.

$$\omega = \frac{v}{R}$$

$$\omega = \frac{6.17 \text{ m/s}}{0.060 \text{ m}}$$

$$\omega = 103 \text{ rad/s}$$

$$\omega = 16.4 \text{ rev/s}$$

practice problem 3

The top shown below consists of a cylindrical spindle of negligible mass attached to a conical base of mass m = 0.50 kg. The radius of the spindle is r = 1.2 cm and the radius of the cone is R = 10 cm. A string is wound around the spindle. The top is thrown forward with an initial speed of v_0 = 10 m/s while at the same time the string is yanked backward. The top moves forward a distance s = 2.5 m, then stops and spins in place.

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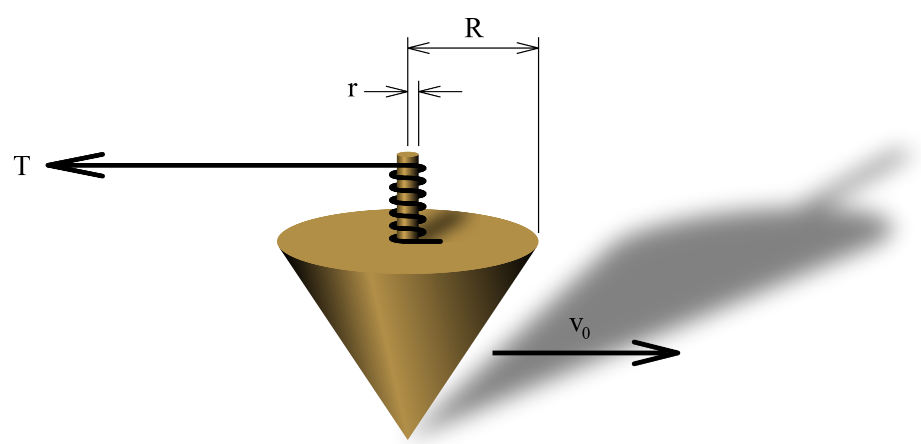

Using rotational dynamics (and kinematics) determine…

a. the moment of inertia I of the top (essentially, the moment of inertia of a cone)
b. the tension T in the string
c. the final angular velocity ω of the top
d. the length ℓ of string wound around the spindle

solution

a. We already have a nice formula for the moment of inertia of a cone. It would be a shame to ruin it with messy numbers, but compute we must.

$$I = \frac{3}{10} MR^2$$

$$I = \frac{3}{10} (0.50 \text{ kg})(0.10 \text{ m})^2$$

$$I = 0.0015 \text{ kg m}^2$$

b. Use the second equation of translational motion. The net force on the top comes from the tension of the string (since weight and normal force cancel and friction is assumed negligible). Replace acceleration with a suitably rearranged version of one of the translational equations of motion. Substitute and solve.

$$\sum F = T = ma \qquad \& \qquad v^2 = v_0^{\,2} + 2a\Delta s$$

$$T = \frac{mv^2}{2\Delta s}$$

$$T = \frac{(0.50\text{ kg})(10\text{ m/s})^2}{2(2.5\text{ m})}$$

$$T = 10\text{ N}$$

c. Not only does the tension supply the net external force needed to stop the forward motion of the top, it also applies the net external torque needed to get the top spinning. We need to explore both realms of Newton's second law to solve this part of the problem.

<u>translational</u>	<u>rotational</u>
$\sum F = ma$	$\sum \tau = I\alpha$
$T = ma$	$rT = I\alpha$

On the translational side, replace acceleration with an equation of motion that can be used to find time. On the rotational side, replace angular acceleration with an equation of motion that uses time.

<u>translational</u>	<u>rotational</u>
$v = v_0 + a\Delta t$	$\omega = \omega_0 + \alpha\Delta t$
$a = \dfrac{v_0}{\Delta t}$	$\alpha = \dfrac{\omega}{\Delta t}$
$T = m\dfrac{v_0}{\Delta t}$	$rT = \left(\dfrac{3}{10}mR^2\right)\left(\dfrac{\omega}{\Delta t}\right)$

Now, combine the two formulas by substituting T from the translational equation into T in the rotational equation, then watch stuff drop out. Do not cancel the radii, however. One is the radius of the spindle (r) and the other is the radius of the base (R).

$$r\left(m\frac{v_0}{\Delta t}\right) = \left(\frac{3}{10}mR^2\right)\left(\frac{\omega}{\Delta t}\right)$$

$$rv_0 = \frac{3}{10}R^2\omega$$

Solve for ω, input numbers, and compute the answer. (Pay attention to the units.)

$$\omega = \frac{10rv_0}{3R^2}$$

$$\omega = \frac{10(0.012\text{ m})(10\text{ m/s})}{3(0.10\text{ m})^2}$$

$$\omega = 40\text{ rad/s}$$

$$\omega = 6.37\text{ rotations per second}$$

d. Find the time first. How long was the string in contact with the top? Use translational equations for this.

$$s = \frac{v + v_0}{2}\Delta t$$

$$\Delta t = \frac{2s}{v + v_0}$$

$$\Delta t = \frac{2(2.5\text{ m})}{0\text{ m/s} + 10\text{ m/s}}$$

$$\Delta t = 0.50\text{ s}$$

Pop this number into the rotational equivalent of the previous equation.

$$\theta = \frac{\omega + \omega_0}{2}\Delta t$$

$$\theta = \frac{0.50\text{ rad/s}}{2}0.50\text{ s}$$

$$\theta = 10\text{ rad}$$

Multiply by the radius of the spindle to determine the length of the string.

$$\ell = r\theta$$

$$\ell = (0.012\text{ m})(10\text{ rad})$$

$$\ell = 0.12\text{ m} = 12\text{ cm}$$

practice problem 4

Write something different.

solution

Answer it.

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