Topic 18: Special Relativity

AP and IBHL Physics

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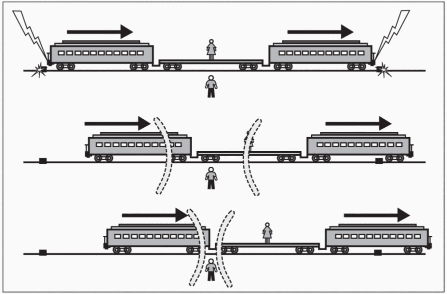
Olympiads School

Simultaneity

The Relativity of Simultaneity

This *thought experiment* is similar to the one that Einstein presented. Suppose lightning bolt strikes the two ends of a high-speed moving train. Does it happen simultaneously?



- Two *independent* events: lightning striking the front, and lightning striking the back of the train
- The man on the ground sees the lightning bolt striking at the same time
- The woman on the moving train sees the lightning bolt on the front first

Relativity of Simultaneity

From the man's perspective:

- He is stationary, but the train is moving
- When the lightnings strike, he is at an equal distance from the front and the back of the train
- Flashes from the two lightning bolts arrive at his eyes at the same time
- Since the speed of light is a constant regardless of motion

Therefore, his conclusions are:

- The two lightnings must have happened at the same time
- The woman in the train made the wrong observation: she only *thinks* that the lightning struck the front first because she is moving toward the light from the front

Relativity of Simultaneity

From the woman's perspective:

- *She* is stationary, but the man and the rest of the world are moving
- When the lightnings strike, she is at an equal distance from the two ends of the train
- The flash from the front arrive first, then the back
- Since the speed of light is a constant regardless of motion

Therefore, her conclusions are:

- Lightnings must have struck the front first
- The man on the road made the wrong observation: he only *thinks* that the lightning struck at the same time because he's moving toward the light from the back

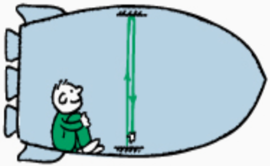
Relativity of Simultaneity

- The two observers disagree on the result, but
  - Neither person is wrong
  - Neither person is misinformed
- Both observers are valid *inertial* frames of reference, and therefore both can consider themselves at rest
- This means that *simultaneity depends on your motion*

Relativity of Simultaneity: Events that are simultaneous in one inertial frame of reference are not simultaneous in another.

Time Dilation

Relativity of Time: Time Dilation

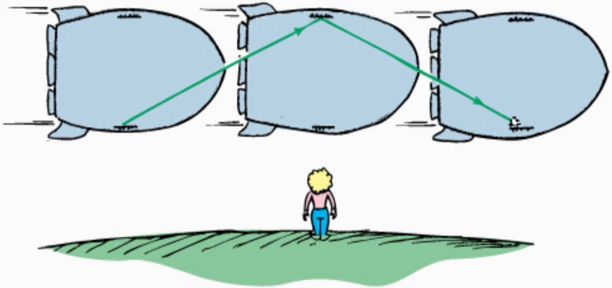


I'm on a spaceship travelling in deep space, and I shine a light from A to B. The distance between A and B is:

$$|AB| = c\Delta t_0$$

I know the speed of light c, and I know how long it took for the light pulse to reach B. (The reason I used Δt_0 will be obvious later.)

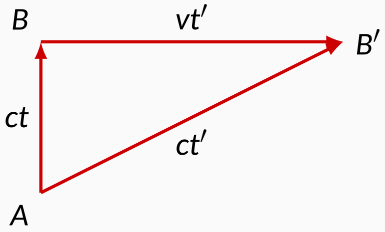
Relativity of Time: Time Dilation



You are on a small planet watching my spaceship go past you at speed v. You would see that same beam of light travel from A to B' instead.

Relativity of Time: Time Dilation

We can relate the time interval observed by me on the spaceship (t) and your time interval on the small planet (t') using Pythagorean theorem:



$$(ct')^2 = (vt')^2 + (ct)^2$$

$$\left(c^2 - v^2\right) t'^2 = c^2 t^2$$

$$\left(1 - \frac{v^2}{c^2}\right) t'^2 = t^2$$

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Relativity of time: the passage of time as measured by two observers in two different inertial references are different

Relativity of Time: Time Dilation

The passage of time as measured by two observers in two different inertial frames of reference are related by:

$$\boxed{t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

Variable	Symbol	SI Unit
Proper time (ordinary time)	t	s
Dilated time (expanded time)	t'	s
Speed	v	m/s
Speed of light	c	m/s

- **Proper time** is measured by an observer *at rest* relative to the events
- **Dilated time** is measured by a *moving* observer in another inertial frame

Example Problem

Example 1a: Kim is riding a rocket that speeds past an asteroid at $v = 0.600c$. If Kim sees 10.0 spass on her watch, how long would that time interval be as seen by Jim, an observer on the asteroid?

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$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{10.0}{\sqrt{1 - 0.600^2}} = 16.7\,\text{s}$$

- Jim observes that in the time it took Kim's clock to run 10.0 s, his watch has already gone 16.7 s, therefore
- Jim concludes that Kim's watch must be running slow

Relativity of Time: A moving clock appears to run slow.

Example Problem

Example 1b: Kim is riding a rocket that speeds past an asteroid at 0.600*c*. If Jim, an observer in the *asteroid*, sees 10.0 spass on his watch, how long would that time interval be as seen by Kim?

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$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{10.}{\sqrt{1 - 0.600^2}} = 16.7\,\mathrm{s}$$

- This problem is exactly the same as the last one!
- Kim observes that in the time it took Jim's clock to run 10.0 s, her watch has already gone 16.7 s, therefore
- Kim concludes that Jim's watch must be running slow

How can that be?

How can the observer in the asteroid sees time in the rocket runs slowly, while the observer in the rocket *also* sees time in the asteroid runs slowly?

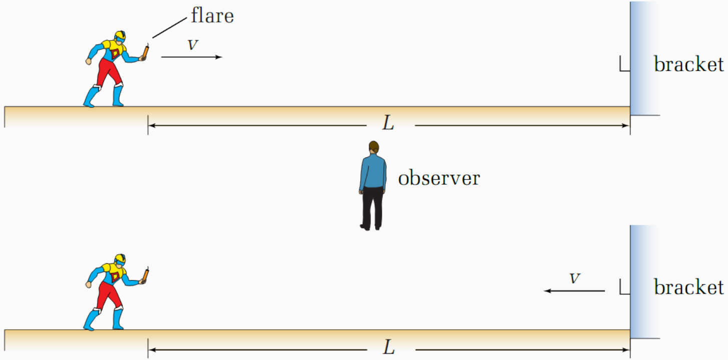
Answer: the relativty of simultaneity. The clocks on the asteroid and on the rocket are *not* synchronized.

- In example 1a, when Kim (on the rocket) starts measuring a 10.0 s time interval, in order for Jim to compare that interval to *his* watch, he has to start and end at the same time (simultaeously!) as Kim.
- But simultaneity is only relative. In Kim's reference frame, Jim never got the timing right!
- This problem reverses itself when Kim tries to synchronize her watch to Jim's 10.0 s interval.

Length Contraction

Relativity of Space

Captain Quick is a comic book hero who can run at nearly the speed of light. In his hand, he is carrying a bomb set to explode in 1.5 μs. The bomb must be placed into its bracket before this happens. The distance (L) between the flare and the bracket is 402 m.



Relativity of Space

Suppose Captain Quick runs at $2.00 \times 10^{8}\,\mathrm{m/s}$, according to classical mechanics, he will not make it in time:

$$t = \frac{L}{v} = \frac{402\,\mathrm{m}}{2.00 \times 10^{8}\,\mathrm{m/s}} = 2.01 \times 10^{-6}\,\mathrm{s} = 2.01\,\mu\mathrm{s}$$

But according to relativistic mechanics, he makes it just in time...

Relativity of Space

To a stationary observer, the time on the flare is slowed:

$$t' = \frac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1.5\times 10^{-6}}{\sqrt{1-\left(\frac{2.00}{3.00}\right)^2}} = 2.01\times 10^{-6}\,\mathrm{s}$$

The stationary observer sees a passage of time of $t' = 2.01\,\mu\mathrm{s}$, but Captain Quick, who is in the same reference frame as the flare, experiences a passage of time of $t = 1.50\,\mu\mathrm{s}$, precisely the time for the flare to explode.

Relativity of Space

If Captain Quick sees only $t = 1.50\,\mu\text{s}$, then how far did he travel?

- Both Captain Quick and the observer on the side of the road agree that he is traveling at $v = 2.00 \times 10^8\ \text{m/s}$
- The only possibility is that *the distance actually got shorter* in Captain Quick's frame of reference, by this amount:

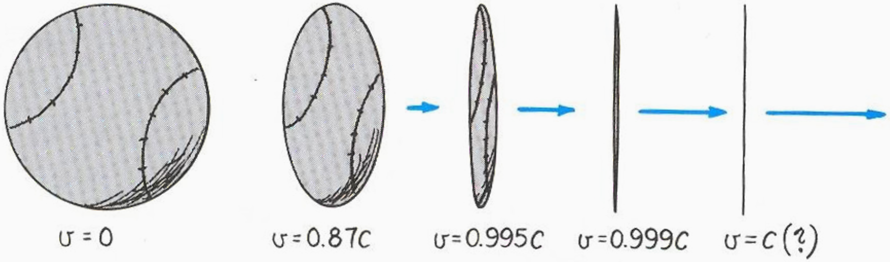
$$\boxed{L' = L\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

For this example:

$$L' = L\sqrt{1 - \left(\frac{v}{c}\right)^2} = 402\sqrt{1 - \left(\frac{2.00}{3.00}\right)^2} = 300\,\text{m}$$

Length Contraction

Length contraction only occurs in the direction of motion



Example Problem

Example 2: A spacecraft passes Earth at a speed of $2.00 \times 10^8\ \mathrm{m/s}$. If observers on Earth measure the length of the spacecraft to be $554\ \mathrm{m}$, how long would it be according to its passengers?

Lorentz Factor

Lorentz Factor

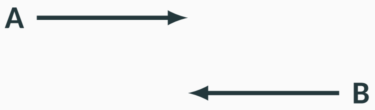
The **Lorentz factor** γ is a short-hand for writing length contraction, time dilation and relativistic mass:

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

Then time dilation and length contraction can be written simply as:

$$\boxed{t' = \gamma t} \quad \boxed{L' = \frac{L}{\gamma}}$$

Summary



If observers A and B are moving at constant velocity relative to one another (doesn't matter if they're moving toward, or away from each other)

- They cannot agree whether any events happens at the same time or not
- Each sees the other's clock running slow
- Each sees the other "contracted" in length along the direction of motion

Lorentz Transformation

Time dilation and length contraction only tell part of the story. To account for the loss of simultaneity from one inertial frame to another, we need to use the **Lorentz transformation:**

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

The Lorentz transformation “solves” many paradoxes (e.g. the twin paradox) from the time-dilation and length-contraction equations, but aren’t really there.

Lorentz Transformation

For slow speeds $v \ll c$, Lorentz transformation reduces to the Galilean transformation from classical mechanics, from which the velocity addition rule is formulated:

$$
\begin{aligned}
x' &= x - vt \\
y' &= y \\
z' &= z \\
t' &= t'
\end{aligned}
$$

Relative Velocity

Relative Velocity

Unlike in classical mechanics, velocities (speeds) do not simply add. We have to account for time dilation and length contraction, which are included in the Lorentz transformation

Einstein velocity addition rule:

$$\boxed{\boldsymbol{v}_{AC} = \frac{\boldsymbol{v}_{AB} + \boldsymbol{v}_{BC}}{1 + \dfrac{\boldsymbol{v}_{AB} \cdot \boldsymbol{v}_{BC}}{c^2}}}$$

If $v_{AB} \ll c$ and $v_{BC} \ll c$, we recover Galilean velocity addition rule