Class 18: Faraday's Law and Magnetic Induction

Advanced Placement Physics C

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Olympiads School

Faraday’s Law

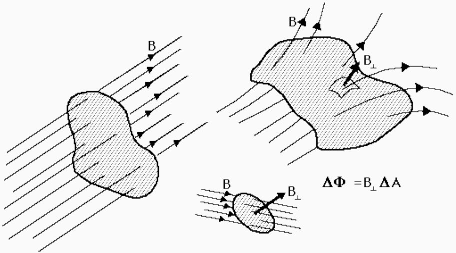
Magnetic Flux

Question: If a current-carrying wire can generate a magnetic field, can a magnetic field affect the current in a wire?

Answer: Yes, sort of...

To understand how to *induce* a current by a magnetic field, we need to look at fluxes again.

Magnetic Flux



Magnetic flux is defined as:

$$\boxed{\Phi_m = \int \vec{B} \cdot \mathrm{d}\vec{A}}$$

where $\vec{B}$ is the magnetic field, and $\mathrm{d}\vec{A}$ is the infinitesimal area pointing **outwards**. Note that magnetic flux can also be expressed as:

$$\boxed{\Phi_m = \int (\vec{B} \cdot \hat{n})\mathrm{d}A}$$

where $\hat{n}$ is the outward normal direction

Magnetic Flux Over a Closed Surface

The SI unit for magnetic flux is a "weber" (Wb), in honor of German physicist Wilhelm Weber, who invented the electromagnetic telegraph with Carl Gauss. The unit is defined as:

$$1\,\mathrm{Wb} = 1\,\mathrm{T} \cdot \mathrm{m}^2$$

The magnetic flux over a closed surface is always zero (**Gauss's law for magnetism**):

$$\boxed{\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0}$$

Since magnetic field lines only exist as a loop, that means there should be equal amount of "flux" flowing out of a closed surface as entering the surface.

Changing Magnetic Flux

Changes to magnetic flux can be due to a number of reasons:

1. **Changing magnetic field** $\vec{B}$: the magnetic field is created by a time-dependent source (e.g. alternating current)
2. **Changing area** A: the surface area from which the flux is calculated is changing
3. **Changing orientation of magnetic field** $\vec{B} \cdot \mathrm{d}\vec{A}$: the surface area is moving relative to the magnetic field

Faraday's Law

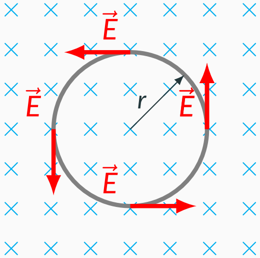
Faraday's law states that the rate of change of magnetic flux produces an electric field $\vec{E}$ in a circuit, and therefore an electromotive force $\mathcal{E}$ (voltage):

$$\boxed{\mathcal{E} = \oint \vec{E} \cdot \mathrm{d}\vec{\ell} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}}$$

The negative sign highlighted in red is the result of **Lenz's law**, which is related to the conservation energy

When Magnetic Flux is Changing

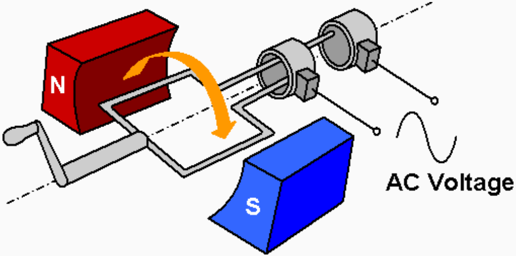
- When the magnetic flux Φ_m is changing, an electromotive force (*emf*, $\mathcal{E}$) is created in the wire.
- Unlike in a circuit, where the *emf* is concentrated at the terminals of the battery, the induced *emf* is spread across the entire wire.



- Since *emf* is work per unit charge, that means that there is an electric field inside the wire to move the charges.
- In this example:
  - Magnetic field $\vec{B}$ into the page
  - The direction of the electric field $\vec{E}$ corresponds to an *increase* in magnetic flux

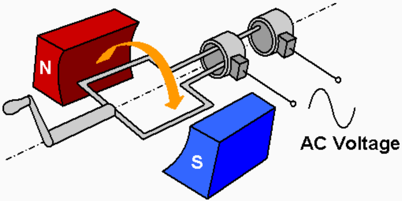
AC Generators

A simple AC (alternating current) generator makes use of the fact that a coil rotating against a fixed magnetic field has a changing flux.



Let's say the permanent magnets produce a uniform magnetic field B, and the coil between them has N turns, and an area A. Now let's say that the coil is rotating with an angular frequency ω.

AC Generators



When the coil is turning, the angle between the coil and the magnetic field is:

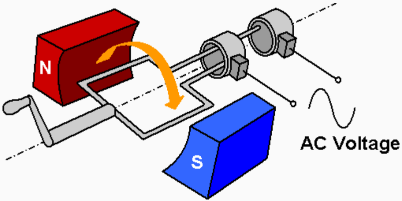
$$\theta = \omega t + \delta$$

where δ is the initial angle. The magnetic flux through the coil is given by:

$$\Phi_m = NBA\cos\theta = NBA\cos(\omega t + \delta)$$

as the coil turns

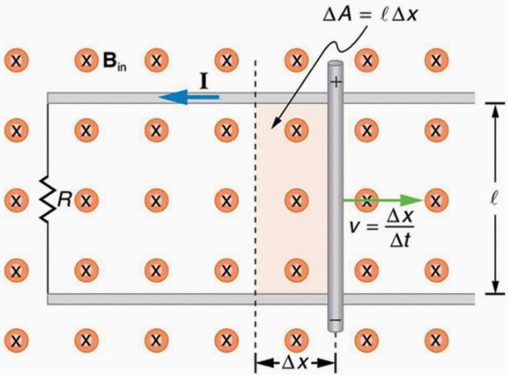
AC Generators



The electromotive force *emf* produced is therefore the rate of change of the magnetic flux:

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = \underbrace{NBA\omega}_{\mathcal{E}_{\mathrm{max}}} \sin(\omega t + \delta)$$

Motional EMF



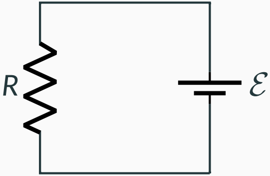
When sliding the rod to the right with speed v, the magnetic flux through the loop (and its rate of change) is:

$$\Phi_m = BA = B\ell x$$

$$\frac{d\Phi_m}{dr} = B\ell \frac{dx}{dt} = \boxed{B\ell v = \mathcal{E}}$$

We can use the Lorentz force law on the charges on the rod to find the direction of the current I.

Motional EMF

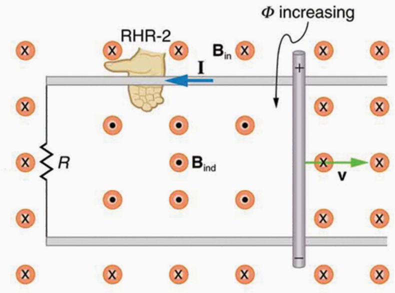


- An equivalent circuit is shown on the left
- The amount of current can be found using Ohm's law: $V = IR$
- Note that the "motional emf" produced is spread over the entire circuit
    - In constrast, in a voltaic cell (or battery), the *emf* is concentrated between the two terminals.

Lenz’s Law

Lenz's Law

Something very interesting happens when the current starts running on the wire.



It produces an "induced magnetic field" out of the page, in the opposite direction as the field that generated the current in the first place.

Lenz’s Law

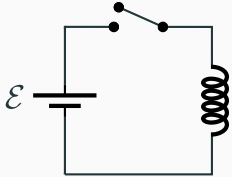
LENZ’S LAW
The induced *emf* and induced current are in such are direction as to oppose the change that produces them

So basically, the conservation of energy

Inductance

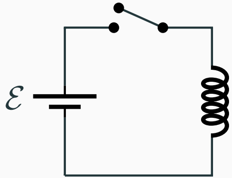
Back *emf*

Consider a very simple circuit consisting of a voltage source and a coil



- When the switch is closed and current begins to flow, the coil begins to generate a magnetic flux inside
- As the current changes (initially increasing with time), it self-induces a “back *emf*” that opposes the change in current
- A current can’t jump from zero to some value (or from some value to zero) instantaneously

Back *emf*



- Breaking the circuit causes the magnetic flux to change very rapidly
- The rapid change of Φ_m creates a large induced back *emf* that is proportional to the time rate of change of magnetic flux $\Phi_m'(t)$
- The back *emf* creates a large voltage drop across the switch
- Large voltage across two metal contact produces a very strong electric field–strong enough to tear electrons away from air molecules ("dielectric breakdown")
- Air conducts electricity in the form of a "spark"

Self Inductance

A solenoid carrying a current generates a magnetic field B; its magnitude at the core is proportional to the current I:

$$B = \left[\frac{\mu_0 N}{\ell}\right] I$$

Since $\vec{B} \propto I$, the magnetic flux through the core of the solenoid (really $\Phi_m = NBA$, where A is the cross-sectional area of the solenoid and N is the number of coils) is therefore also proportional to I, i.e.

$$\boxed{\Phi_m = LI}$$

where L is the called the **self inductance** of the coil.

Self Inductance

For a solenoid, we can see that the self inductance is given by:

$$\boxed{L = \frac{\Phi_m}{I} = \mu_0 n^2 A\ell}$$

Quantity	**Symbol**	**SI Unit**
Self inductance	L	H
Vacuum permeability	μ_0	$\mathrm{T \cdot m / A}$
Number of coils per unit length	n	A
Cross-section area of the solenoid	A	m^2
Length of the solenoid	ℓ	m

Note that $A\ell$ is the *enclosed volume* of the solenoid.

Self Inductance and Induced EMF

If the current changes, the magnetic flux changes as well, therefore inducing an electromotive force in the circuit According Faraday's law:

$$\boxed{\mathcal{E} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = -L\frac{\mathrm{d}I}{\mathrm{d}t}}$$

The self-induced *emf* is proportional to the rate of change of current

Magnetic Energy

At any instant, the magnitude of the induced emf is

$$\mathcal{E} = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

so the power absorbed by the inductor is

$$P(t) = \mathcal{E}I = LI\frac{\mathrm{d}I}{\mathrm{d}t}$$

Integrating in time gives the magnetic (potential) energy that is stored in the magnetic field:

$$U_m = \int_0^t P(t)\mathrm{d}t = L\int I\frac{\mathrm{d}I}{\mathrm{d}t}\mathrm{d}t = L\int_0^I I\mathrm{d}I = \frac{1}{2}LI^2$$

Magnetic Energy

Just as a capacitor stores energy in its electric field, an inductor coil carrying a current I stores energy in its magnetic field, given by:

$$\boxed{U_m = \frac{1}{2}LI^2}$$

We can also define a **magnetic energy density**:

$$\boxed{\eta_m = \frac{B^2}{2\mu_0}}$$