Student #: ______________________________ Student Name: ______________________________________

AP PHYSICS C: ROTATIONAL MOTION

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\text{m/s}^2$ in all problems.

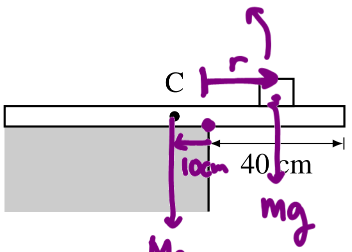
1. A meter stick of mass 0.1 kg rests on a table as shown. A length of 40 cm extends over the edge of the table. How far from the edge of the table could a 0.05 kg mass be placed on the meter stick so that the stick just begins to tip?

↙M

M

$I_{net} = 0$

$+\circlearrowright$



- (A) 5 cm
- (B) 10 cm
- (C) 15 cm
- [✓ (D)] 20 cm
- (E) 30 cm

D

$Mg(10) = mg(r)$

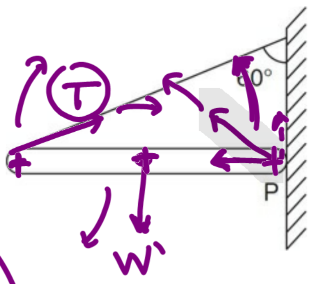
$(0.1)(10) = (0.05)(r)$

$r = 20\text{cm}$

2. A metal bar of constant density and weight W is attached to a pivot on the wall at point P and supported by a rope that makes an angle of 60° with the vertical wall. The reaction force exerted by the pivot on the bar at point P is best represented by which arrow?

$\Sigma F = 0$

$\Sigma \tau = 0$



- (A) ↗
- (B) ↑
- (C) ↓
- [✓ (D)] ↖
- (E) ↘

D

If there is no rotational motion, that means that the total torque calculated at ANY location must be zero.

3. A ballet dancer is spinning around a vertical axis with her arms fully extended. How are her angular momentum and kinetic energy affected as she pulls her arms in toward her body as she spins?

decrease her moment of inertia

$L = I\omega$

↑ ↓

constant (no external force)

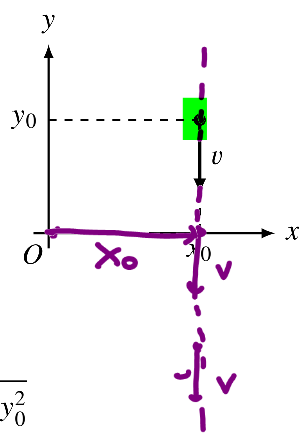
- (A) Her angular momentum remains constant, but her kinetic energy increases. ← no external work
- (B) Her angular momentum increases, but her kinetic energy remains constant. x
- (C) Her angular momentum decreases, but her kinetic energy remains constant. x
- (D) Her angular momentum increases, but her kinetic energy decreases. x
- [✓ (E)] Both her angular momentum and kinetic energy remain constant.

E

4. A particle of mass m moves with a constant speed v at a distance x_0 parallel to the y-axis as shown. When the particle is in the position shown below, the magnitude of its angular momentum relative to the origin is

$L = \text{constant}$

$L = rp$
$= x_0(mv)$

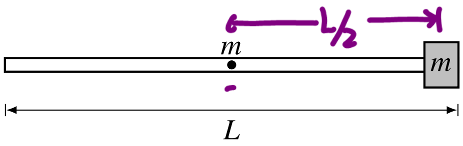


- [✓ (A)] mvx_0
- (B) mvy_0
- (C) $mv\sqrt{x_0^2 + y_0^2}$
- (D) $\dfrac{mv}{\sqrt{x_0^2 + y_0^2}}$
- (E) zero

A

5. A uniform rod of length L and mass m has a rotational inertia of $\frac{1}{12}mL^2$ about its center. A particle, also of mass m, is attached to one end of the stick. The combined rotational inertia of the stick and particle about the center of the rod is

moment of inertia



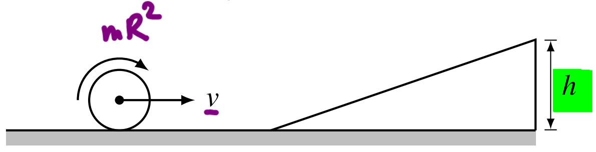
A

- [✓ (A)] $\dfrac{mL^2}{3}$
- (B) $\dfrac{12mL^2}{13}$
- (C) $\dfrac{13mL^2}{12}$
- (D) $\dfrac{mL^2}{156}$
- (E) $\dfrac{13mL^2}{156}$

particle $I = mr^2$
$= m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4}$

$I_T = \frac{1}{12}mL^2 + \frac{1}{4}mL^2$

6. A hoop of radius R and mass m has a rotational inertia of mR^2. The hoop rolls without slipping along a horizontal floor with a constant speed v and then rolls up a long incline. The hoop can roll up the incline to a maximum vertical height of



A

- [✓ (A)] $\dfrac{v^2}{g}$
- (B) $\dfrac{2v^2}{g}$
- (C) $\dfrac{v^2}{2g}$
- (D) $\dfrac{4v^2}{g}$
- (E) $\dfrac{v^2}{4g}$

$K_T + K_r = U_g$

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

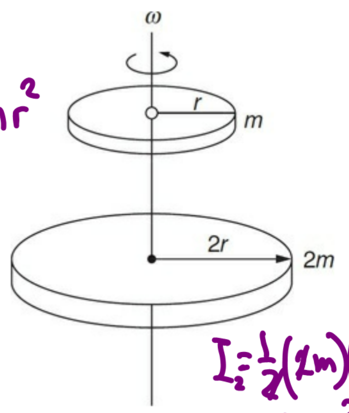
$\frac{1}{2}mv^2 + \frac{1}{2}(m\cancel{R^2})\left(\frac{v^2}{\cancel{R^2}}\right) = mgh$

$\cancel{m}v^2 = \cancel{m}gh$

$h = \frac{v^2}{g}$

7. Two disks are fixed to a vertical axle that is rotating with a constant angular speed ω. The smaller disk has a mass m and a radius r, and the larger disk has a mass $2m$ and radius $2r$. The general equation for the rotational inertia of a disk of mass M and radius R is $\frac{1}{2}MR^2$. The ratio of the angular momentum of the larger disk to the smaller disk is

$I_1 = \frac{1}{2}mr^2$



$I_2 = \frac{1}{2}(2m)(2r)^2$
$= 4mr^2$

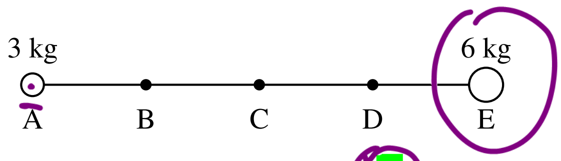
E

- (A) 1 : 4
- (B) 4 : 1
- (C) 1 : 2
- (D) 2 : 1
- [✓ (E)] 8 : 1

$I_1 : I_2 \rightarrow 1 : 8$

$I_2 : I_1 \rightarrow 8 : 1$

8. A light rod has a mass attached at each end. At one end is a 6 kg mass, and at the other end is a 3 kg mass. An axis can be placed at any of the points shown. Through which point should an axis be placed so that the rotational inertia is the greatest about that axis?



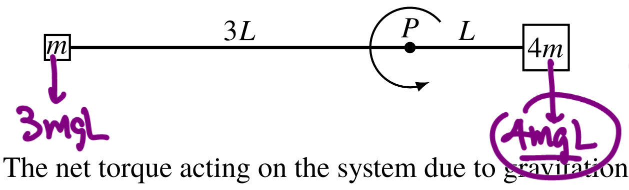
A

$I = MR^2$ maximize

- [✓ (A)] A
- (B) B
- (C) C
- (D) D
- (E) E

Question 9–10

A light rod of negligible mass is pivoted at point P a distance L from one end as shown. A mass m is attached to the left end of the rod at a distance of $3L$ from the pivot, and another mass $4m$ is attached to the other end a distance L from the pivot. The system begins from rest in the horizontal position.



9. The net torque acting on the system due to gravitational forces is

E

- (A) $4mgL$ clockwise
- (B) $3mgL$ clockwise
- (C) $3mgL$ counterclockwise
- (D) mgL counterclockwise
- [✓ (E)] mgL clockwise

$I = 4mL^2 + m(3L)^2$

$= 13mL^2$

10. The angular acceleration of the system when it is released from rest is

D

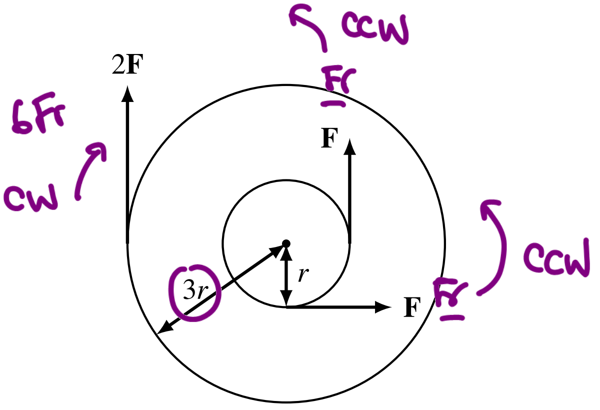
$I = I\alpha$

$\cancel{m}gL = (13\cancel{m}L^{\cancel{2}})\,\alpha$

$\alpha = \frac{g}{13L}$

- (A) zero
- (B) $\frac{g}{5L}$
- (C) $\frac{g}{4L}$
- [✓ (D)] $\frac{g}{13L}$
- (E) $\frac{g}{L}$

11. Two wheels are attached to each other and fixed so that they can only turn together. The smaller wheel has a radius of r and the larger wheel has a radius of $3r$. The two wheels can rotate together on a frictionless axle. Three forces act tangentially on the edge of the wheels as shown. The magnitude of the net torque acting on the system of wheels is



D

- (A) Fr
- (B) $2Fr$
- (C) $3Fr$
- [✓ (D)] $4Fr$
- (E) $6Fr$

12. A disk is mounted on a fixed axle. The rotational inertia of the disk is I. The angular velocity of the disk is decreased from ω_i to ω_f during a time Δt due to friction in the axle. The magnitude of the average net torque acting on the wheel is

$I = I\alpha = I\left(\frac{\Delta\omega}{\Delta t}\right)$

$= \frac{I(\omega_f - \omega_i)}{\Delta t}$

C

- (A) $\frac{\omega_f - \omega_i}{\Delta t}$
- (B) $\frac{(\omega_f - \omega_i)^2}{\Delta t}$
- [✓ (C)] $\frac{I(\omega_f - \omega_i)}{\Delta t}$
- (D) $\frac{I(\omega_f - \omega_i)^2}{\Delta t}$
- (E) $\frac{I(\omega_f - \omega_i)}{\Delta t^2}$

13. The average power developed by the friction in the axle of the disk from the previous question to bring it to a complete stop is

$P = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t} = \frac{\frac{1}{2}I\omega_i^2}{\Delta t}$

$= \frac{I\omega_i^2}{2\Delta t}$

D

- (A) $\frac{\omega_i}{\Delta t}$
- (B) $\frac{\omega_i^2}{\Delta t}$
- (C) $\frac{I\omega_i}{2\Delta t}$
- [✓ (D)] $\frac{I\omega_i^2}{2\Delta t}$
- (E) $\frac{I\omega_i}{\Delta t^2}$

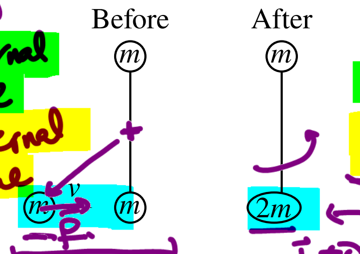
14. Astronauts are conducting an experiment in a negligible gravity environment. Two spheres of mass m are attached to either end of a light rod. As the rod and spheres float motionless in space, an astronaut launches a piece of sticky clay, also of mass m, toward one of the spheres so that the clay strikes and sticks to the sphere perpendicular to the rod. Which of the following statements is true of the motion of the rod, clay, and spheres after the collision?

∫ collision

→ no external force

↳ no external torque

$\vec{L} \neq 0$



$\vec{L} \neq 0$

$p = \text{constant}$

$L = \text{constant}$

- (A) Linear momentum is not conserved, but angular momentum is conserved.
- (B) Angular momentum is not conserved, but linear momentum is conserved.
- (C) Kinetic energy is conserved, but angular momentum is not conserved.
- (D) Kinetic energy is conserved, but linear momentum is not conserved.
- [✓ (E)] Both linear momentum and angular momentum are conserved, but kinetic energy is not conserved.

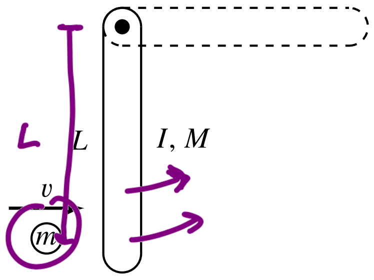
E

Completely inelastic

∴ K not conserved

15. A rod of mass M, length L, and rotational inertia I hangs at rest from a frictionless axle as shown. A ball of mass m with a speed v strikes the rod perpendicularly at the end of the rod. As a result of the collision, the ball stops. The angular speed of the rod immediately after the collision is

$L = mvL$



$mvL = I\omega$

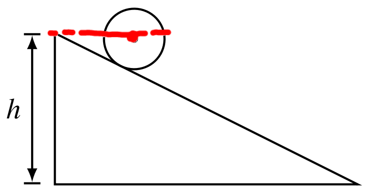
$\omega = \frac{mvL}{I}$

D

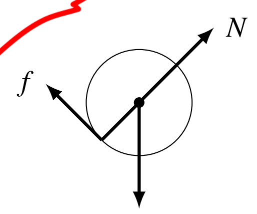
- (A) vL
- (B) $\frac{v}{L}$
- (C) $\frac{mv}{I}$
- [✓ (D)] $\frac{mvL}{I}$
- (E) $\frac{mv}{IL}$

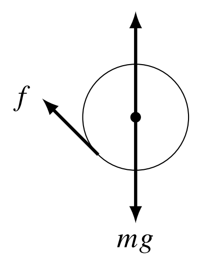
Questions 16–17

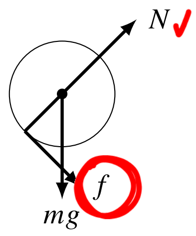
A hollow sphere of mass m and radius R begins from rest at a height h and rolls down a rough inclined plane. The rotational inertia of the hollow sphere is $\frac{2}{3}mR^2$.

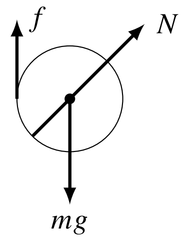


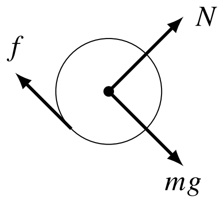
16. Which of the following diagrams best represents the forces acting on the sphere as it rolls down the plane?

(A)


(B)


(C)


(D)


(E)


17. The speed of the sphere when it reaches the bottom of the plane is

(A) $\sqrt{\frac{8gh}{5}}$
[✓ (B) $\sqrt{\frac{6gh}{5}}$]
(C) $\sqrt{\frac{5gh}{6}}$
(D) $\sqrt{\frac{7gh}{10}}$
(E) $\sqrt{\frac{gh}{2}}$

B

$$\underline{mgh} = \underline{\frac{1}{2}mv^2} + \underline{\frac{1}{2}I\omega^2}$$

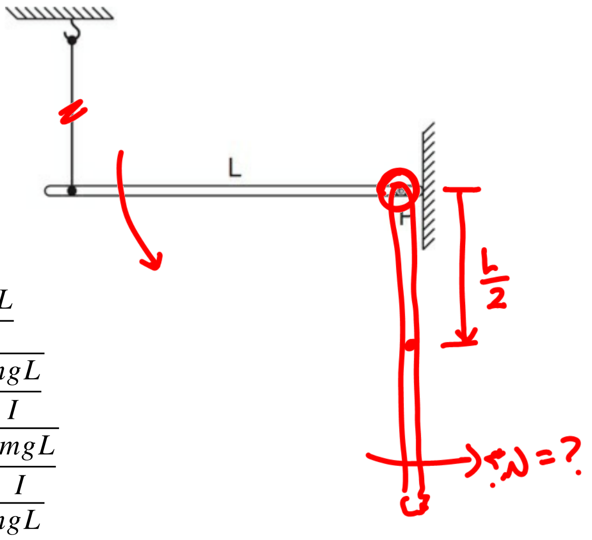
$$= \frac{1}{2}mv^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}m\cancel{R^2}\right)\left(\frac{v^2}{\cancel{R^2}}\right)$$

$$\cancel{m}gh = \frac{1}{2}mv^2 + \frac{1}{3}mv^2 = \frac{5}{6}\cancel{m}v^2$$

$$v^2 = \frac{6}{5}gh \rightarrow v = \sqrt{\frac{6}{5}gh}$$

Calculated at the end (pivot)

18. One end of a stick of length L, rotational inertia I, and mass m is pivoted on an axle with negligible friction at point P. The other end is tied to a string and held in a horizontal position. When the string is cut, the stick rotates counterclockwise. The angular speed ω of the stick when it reaches the bottom of its swing is



(A) $\frac{mgL}{I}$
[✓ (B) $\sqrt{\frac{mgL}{I}}$]
(C) $\sqrt{\frac{2mgL}{I}}$
(D) $\sqrt{\frac{mgL}{2I}}$
(E) $\sqrt{\frac{4mgL}{I}}$

B

$$\underbrace{mg\frac{L}{\cancel{2}}}_{-\Delta U_g} = \frac{1}{\cancel{2}}I_P\omega^2$$

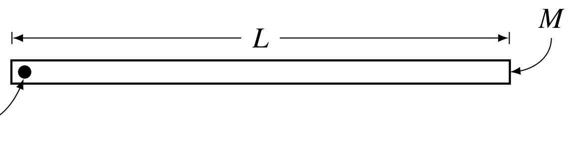
$$\omega = \sqrt{\frac{mgL}{I}}$$

AP PHYSICS C: ROTATIONAL MOTION
SECTION II
6 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

$$\frac{1}{2}I_p\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$I_p \neq I_{cm}$



1. A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

   (a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.

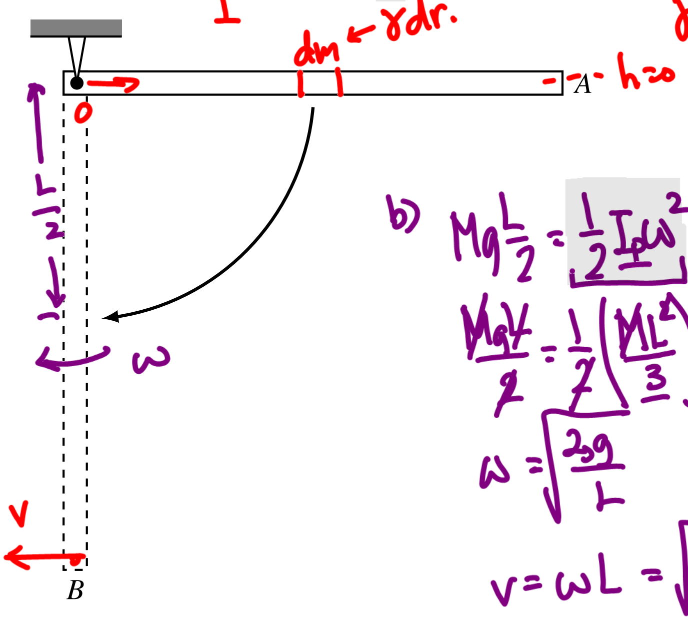
I

$\gamma = \frac{M}{L}$ $dm = \gamma dr$

$dm = \frac{M}{L}dr$

$$I = \int r^2 dm = \int r^2(\gamma dr) = \gamma\int_0^L r^2 dr$$

$$I = \gamma\left(\frac{r^3}{3}\right)\Big|_0^L = \frac{\gamma L^3}{3} = \frac{(\gamma L)L^2}{3} = \boxed{\frac{ML^2}{3}}$$



b) $Mg\frac{L}{2} = \underline{\frac{1}{2}I_p\omega^2}$ ← no need to use $\underline{\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2}$ same

$$\cancel{M}g\cancel{L}/\cancel{2} = \frac{1}{\cancel{2}}\left(\frac{\cancel{M}L^{\cancel{2}}}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

$$v = \omega L = \sqrt{\frac{3g}{\cancel{L}}}(\cancel{L}) = \boxed{\sqrt{3gL}}$$

The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B.

   (b) Derive an expression for the velocity of the free end of the rod at position B. Express your answer in terms of M, L, and physical constants, as appropriate.

An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position A and uses photogates to measure the velocity of the free end at position B. The data are recorded below.

$V_B = \sqrt{3g}\sqrt{L}$ — y, m, x

L Length (m)	0.25	0.50	0.75	1.00	1.25	1.50
v Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3
V_B^2 (m^2/s^2)	7.3	144	21.2	27.0	33.6	39.7

$V_B^2 = 3gL$ — y, m x

   (c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g.

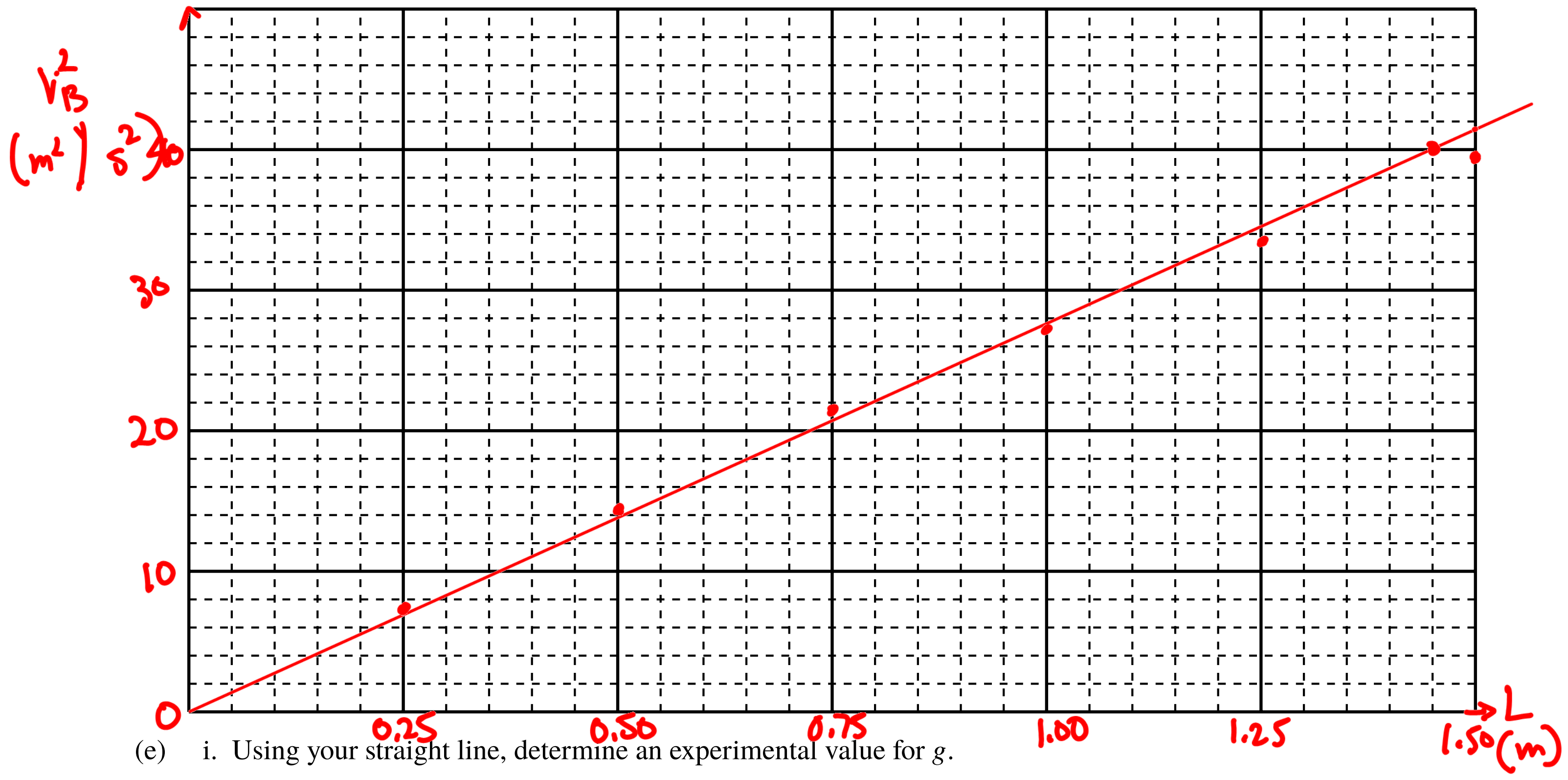
slope $= \sqrt{3g}$

Horizontal axis: $\sqrt{L}$ or L → slope = 3g

Vertical axis: V_B V_B^2

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

   (d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



   (e) i. Using your straight line, determine an experimental value for g.

   ii. Describe two ways in which the effects of air resistance could be reduced.

$$\text{slope} = 3g \rightarrow g = \frac{\text{slope}}{3} = \left(\frac{41}{1.50}\right)\frac{1}{3} = \boxed{9.1\ m/s^2} < 9.8$$

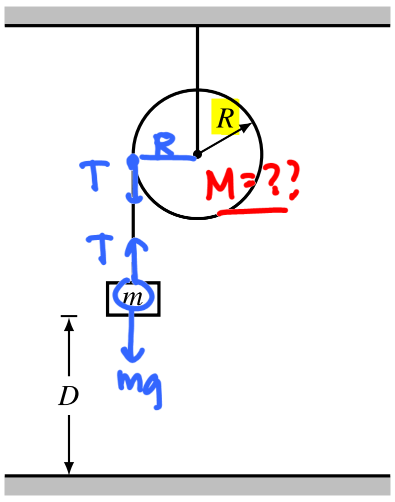
ii) 1. Perform the experiment in a vacuum. 2. Shorter rod, 3. More massive rod, 4. More aerodynamic rod. 5. Thinner rod

c) $TR = I\alpha$] ← disk

$mg - T = ma$
$T = m(g-a)$] mass

$I = \frac{TR}{\alpha} = \frac{(TR^2)}{a} = \boxed{\frac{m(g-a)R^2}{a}}$

↖ a/R



$\boxed{mR^2\left(\frac{g}{a} - 1\right)}$

2. A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor.

   (a) Calculate the linear acceleration a of the falling block in terms of the given quantities.

   (b) The time t is measured for various heights D and the data are recorded in the following table.

a) $D = \frac{1}{2}at^2$ →

$\boxed{a = \frac{2D}{t^2}}$

D (m)	t (s)	t^2
0.5	0.68	0.4624
1	1.02	1.0404
1.5	1.19	1.4161
2	1.38	1.9044

   i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.

   ii. On the grid below, plot the quantities determined in (b)i., label the axes, and draw the best-fit line to the data.

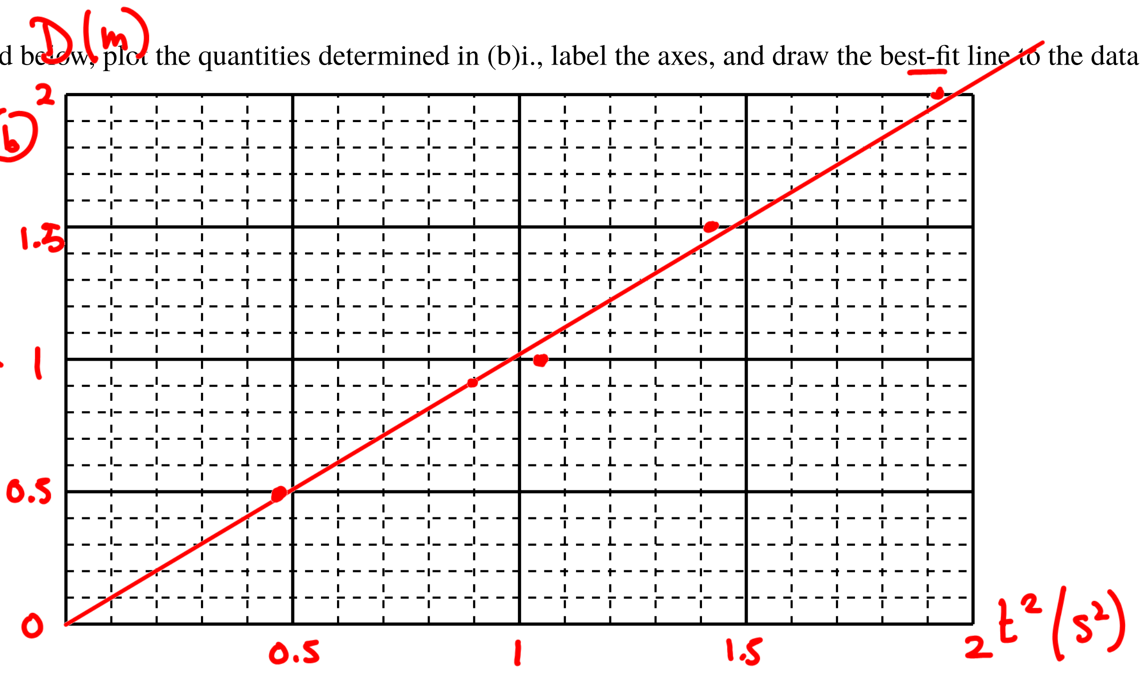
b) i) $\underbrace{D}_{y} = \underbrace{\frac{1}{2}a}_{m}\underbrace{t^2}_{x} + (b)$

plot D vs. t^2, the line is linear, and the slope is $\frac{1}{2}a$.

slope $m = \frac{\Delta D}{\Delta (t)^2} = \frac{1}{1}$

$m = 1.0$

$a = 2m = \boxed{2.0 \text{ m/s}^2}$



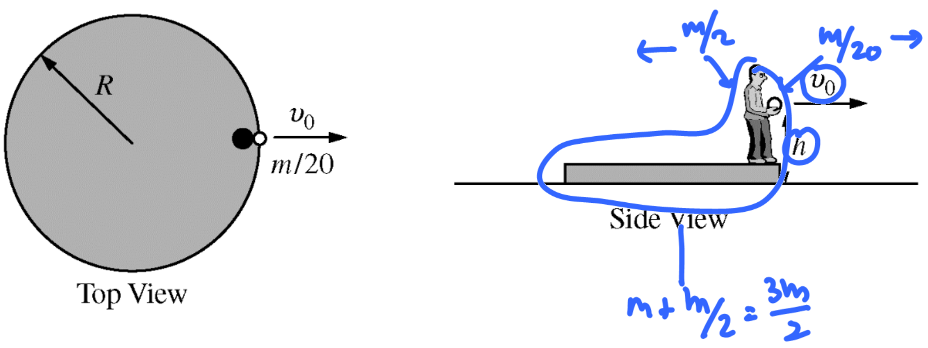
   iii. Use your graph to calculate the magnitude of the acceleration.

   (c) Calculate the rotational inertia of the pulley in terms of m, R, a, and fundamental constants.

I

   (d) The value of acceleration found in (b)iii, along with numerical values for the given quantities and your answer to (c), can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

1. There may be errors in the acceleration from experimental results
2. Since the The string was wrapped around the disk several times, the string may have increased The effective radius of The disk.
3. The string slips → m will have a higher acceleration.



3. A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, m, and fundamental constants, as appropriate.

   (a) Derive an expression for the length of time it will take the stone to strike the ice.

   (b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

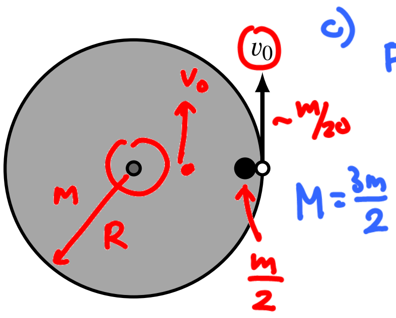
   (c) Derive an expression for the time it will take the disk to stop sliding.

a) $h = \frac{1}{2}gt^2$

$\boxed{t = \sqrt{\frac{2h}{g}}}$

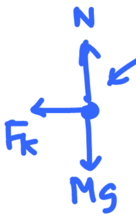
b) $\frac{\cancel{m}}{20}v_0 = \left(\frac{3\cancel{m}}{2}\right)v$

$\boxed{v = \frac{v_0}{30}}$



Top View

c)



$F_k = -\mu N = Ma$

$= -\mu \cancel{M}g = \cancel{M}a$

$a = -\mu g$

$t = \frac{\Delta v}{a} = \left(\frac{-v_0}{30}\right)\left(\frac{1}{-\mu g}\right) = \boxed{\frac{v_0}{30\mu g}}$

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

   (d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

   (e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d)? Justify your answer.

____ Greater than [✓ ____ Less than] ____ Equal to

$L = I\omega$ $\frac{v_0}{R}$

d) When the stone is thrown, there is no external torque ∴ no change in angular momentum

$L_i = L_f = 0$

$L_{stone} = L_{disk} + L_{person}$

$\underbrace{\left(\frac{m}{20}\right)R^2}_{I}\underbrace{\left(\frac{v_0}{R}\right)}_{\omega} = \underbrace{\frac{mR^2}{2}}\omega + \underbrace{\frac{m}{2}R^2}_{I}\omega$

$\frac{\cancel{m}Rv_0}{20} = \cancel{m}R^2\omega$

$\boxed{\omega = \frac{v_0}{20R}}$

(e). L_{stone} after being thrown is only ½ of the previous value,

· I_{disk} does not change

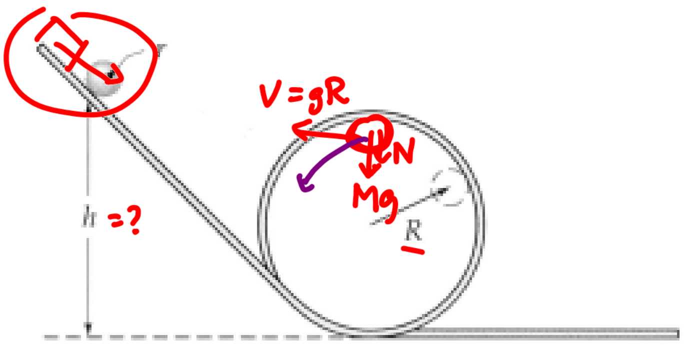
$I_{pers.}$ decrease to ¼ of original value.

→ total I decreases $\frac{3}{4}$ original value.

∴ ω must decrease to decrease L

4. A uniform ball of radius r rolls without slipping along the loop-the-loop track in the figure below. The ball starts at rest at a height of h above the bottom of the loop.

Mass M



(a) If it is not to leave the track at the top of the loop, what is the least value h can have (in terms of radius R of the loop)?

(b) What would h have to be if, instead of rolling, the ball slides without friction?

a) $F_c = ma_c$

$Mg + N = M\frac{v^2}{R}$

at minimum v → N=0

$\cancel{M}g = \frac{\cancel{M}v^2}{R}$

$v^2 = gR$

$$\underbrace{Mgh}_{\text{at the top}} = \underbrace{Mg(2R) + \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2}_{\text{top of loop}}$$

$$Mgh = 2MgR + \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}Mr^2\right)\left(\frac{v^2}{r^2}\right)$$

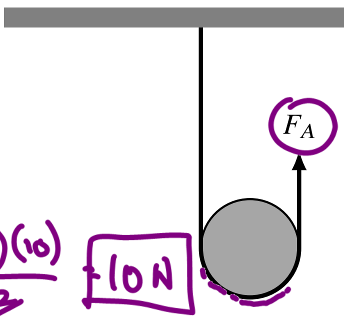
$$\cancel{M}gh = 2\cancel{M}gR + \frac{1}{2}\cancel{M}v^2 + \frac{1}{5}\cancel{M}v^2$$

$$gh = 2gR + \frac{7}{10}v^2 = 2gR + \frac{7}{10}gR = \frac{27}{10}gR$$

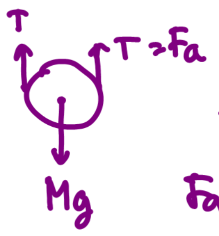
$$\boxed{h = \frac{27}{10}R}$$

b) $\cancel{M}gh = \cancel{M}g(2R) + \frac{1}{2}\cancel{M}v^2 = 2gR + \frac{1}{2}gR = \frac{5}{2}gR \rightarrow \boxed{h = \frac{5}{2}R}$

Start at a lower height



a)



$2T = Mg$

$F_a = T = \frac{Mg}{2} = \frac{(\cancel{2.0})(10)}{\cancel{2}}$ $\boxed{10\,N}$

5. A disk of mass $M = 2.0\,\text{kg}$ and radius $R = 0.10\,\text{m}$ is supported by a rope of negligible mass, as shown above. The rope is attached to the ceiling at one end and passes under the disk. The other end of the rope is pulled upward with a force F_A. The rotational inertia of the disk around its center is $MR^2/2$.

   (a) Calculate the magnitude of the force F_A necessary to hold the disk at rest.

   At time $t = 0$, the force F_A is increased to 12 N, causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.

   (b) Calculate the linear acceleration of the disk.

   (c) Calculate the angular speed of the disk at $t = 3.0\,\text{s}$.

   (d) Calculate the increase in total mechanical energy of the disk from $t = 0$ to $t = 3.0\,\text{s}$.

   (e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk. Justify your answer.

   ____ Greater than [✓ Less than] ____ The same as

b)

T ≠ Fa
if T=Fa
there is
No torque

$\sum F = F_a + T - Mg = Ma$

$\sum \tau = F_a \cdot R - T \cdot R = I\alpha$

$F_a R - T \cdot R = \left(\frac{1}{2}MR^{\cancel{2}}\right)\left(\frac{a}{\cancel{R}}\right)$

$F_a \cancel{R} - T \cdot \cancel{R} = \frac{1}{2}M\cancel{R}a$

$F_a - T = \frac{1}{2}Ma$

$T = F_a - \frac{1}{2}Ma$

$F_a + \left(F_a - \frac{1}{2}Ma\right) - Mg = Ma$

$\frac{2F_a}{M} - \cancel{M}g = \frac{3}{2}\cancel{M}a$

$a = \frac{2}{3}\left(\frac{2F_a}{M} - g\right)$

$a = \frac{2}{3}\left(\frac{\cancel{2}(12)}{\cancel{2}} - 10\right)$

$\boxed{a = 1.33\,m/s^2}$

↑
constant

c) $\omega = \cancel{\omega_0} + \overset{\overset{\frac{a}{R}}{\downarrow}}{\alpha} t$

$\omega = \frac{at}{R} = \frac{\left(\frac{4}{\cancel{3}}\right)(\cancel{3})}{0.10} = \boxed{40\,rad/s}$

d) $\Delta E = \Delta U_g + \Delta K = \underbrace{Mg\Delta h}_{\Delta U_g} + \overbrace{\left[\frac{1}{2}M\underset{\overset{\uparrow}{at\,,\ R\omega}}{v^2} + \frac{1}{2}I\omega^2\right]}^{\Delta K} = Mg\left(\frac{1}{2}at^2\right) + \frac{1}{2}M(R\omega)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$

$= \underbrace{\frac{1}{2}Mgat^2} + \frac{3}{4}MR^2\omega^2 = \cdots\cdots = \boxed{144\,J}$

e) hoop $\underbrace{I = MR^2}$ ← The moment of inertia for a hoop of the same mass is HIGHER than the disk (i.e. the mass near the center of the disk is pushed towards the outside of the disk)

This means that more of the work done by the applied force will be converted into the rotational kinetic energy and not translational kinetic energy, therefore linear acceleration will be less.