Student #: ___________________________ Student Name: ___________________________________

TOPIC 3: WORK AND ENERGY

$g = 10\ m/s^2$

1. A 1 kg ball is thrown vertically downward from a 50-meter-high tower with an initial speed of 4 m/s. Just before striking the ground, the speed of the ball is 20 m/s. The energy lost to air friction is most nearly

   (A) 101 J
   (B) 210 J
   (C) 308 J
   (D) 406 J
   (E) 508 J

$E_i = U_{gi} + K_i = mgh + \frac{1}{2}mv^2 = (10)(50) + \frac{1}{2}(16) = 508\ J$

$E_f = K_f = \frac{1}{2}mv^2 = \frac{1}{2}(20)^2 = 200\ J$

308 J

$E_i + W_{nc} = E_f \rightarrow W_{nc} = -308\ J$

2. An archer pulls a bowstring back a distance of 20 cm with an average force of 75 N. The arrow has a mass of 20.0 g. When he releases the string, what is the velocity of the arrow when it leaves the bow?

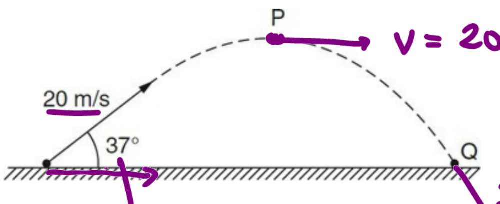
   (A) 1.2 m/s
   (B) 22 m/s
   (C) 32 m/s
   (D) 39 m/s
   (E) 42 m/s

$W = F \cdot \Delta x = K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2F\Delta x}{m}}$

$= 38.7$

Questions 3–4

A 2 kg projectile is launched with a speed of 20 m/s from horizontal ground at an angle of 37° to the horizontal as shown. Point P is at the top of the path, and point Q is at the end of the path, just before the projectile again reaches the ground.



$v = 20\cos 37° = 16\ m/s$

$\frac{4}{5}$

$\frac{1}{2}mv^2$

3. The kinetic energy of the projectile at point P is

   (A) 108 J
   (B) 225 J
   (C) 256 J
   (D) 400 J
   (E) 525 J

5, 3, 4

$\frac{1}{2}mv^2$

C

4. The kinetic energy of the projectile at point Q is

   (A) 108 J
   (B) 225 J
   (C) 256 J
   (D) 400 J
   (E) 525 J

D

5. If a projectile thrown directly upward reaches a maximum height h and spends a total time in the air of T before returning to its original position, the average power of the gravitational force during the trajectory is

   (A) $P = 2mgh/T$
   (B) $P = -2mgh/T$
   (C) 0
   (D) $P = mgh/T$
   (E) $P = -mgh/T$

Constant

$W = F \cdot \Delta x = 0$

6. Given that the constant net force on an object and the object's displacement, which of the following quantities can be calculated?

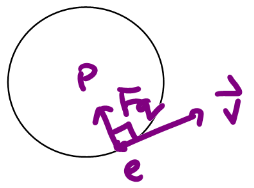
   (A) ~~the net change in the object's velocity~~
   (B) ~~the net change in the object's mechanical energy~~
   (C) ~~the average acceleration~~
   (D) the net change in the object's kinetic energy
   (E) ~~the net change in the object's potential energy~~

$W_{net} = \Delta K$

m?

7. An electron travels in a circle around a hydrogen nucleus at a very high speed. The work done by the electrostatic force acting on the electron after one complete revolution is

   (A) zero
   (B) positive
   (C) negative
   (D) equal to the kinetic energy of the electron
   (E) equal to the potential energy of the electron



$F_q \perp$ motion

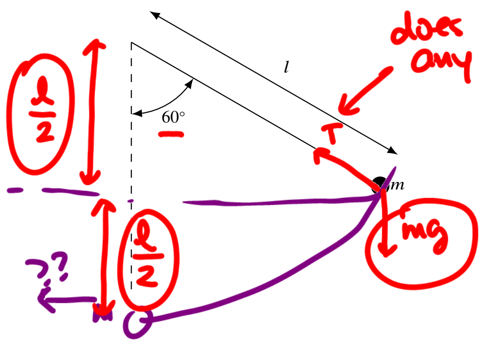
8. An object is moved from rest at point P to rest at point Q in a gravitational field. The net work against the gravitational field depends on the

   (A) mass of the object and the positions of P and Q
   (B) ~~mass of the object only~~ ×
   (C) ~~positions of P and Q only~~
   (D) ~~length moved between points P and Q~~
   (E) ~~coefficient of friction~~

conservative

$W_g = \Delta U_g \leftarrow mgh$

9. A pendulum bob of mass m is released from rest as shown in the figure below. What is the tension in the string as the pendulum swings through the lowest point of its motion?



   (A) $T = \frac{1}{2}mg$
   (B) $T = mg$
   (C) $T = \frac{3}{2}mg$
   (D) $T = 2mg$
   (E) None of the above

$mg\frac{l}{2} \rightarrow \frac{1}{2}mv^2 \rightarrow v = \sqrt{gl}$

$v^2 = gl$

$F_c = T - mg$

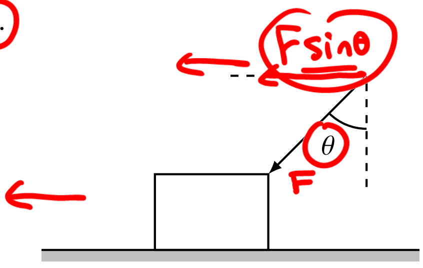
$T = F_c + mg = \frac{mv^2}{l} + mg$

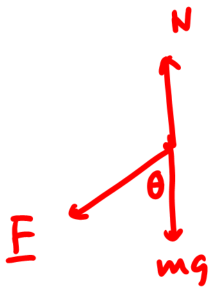
$= 2mg$

mg

Questions 10–11

A force is applied to a block of mass m at a downward angle of θ to the vertical as shown. The block moves with a constant speed across a rough floor for a distance x.





$N = mg + F\cos\theta$

$\mu = \frac{F_f}{N} = \frac{F\sin\theta}{mg + F\cos\theta}$

10. The work done by the applied force on the block is

A

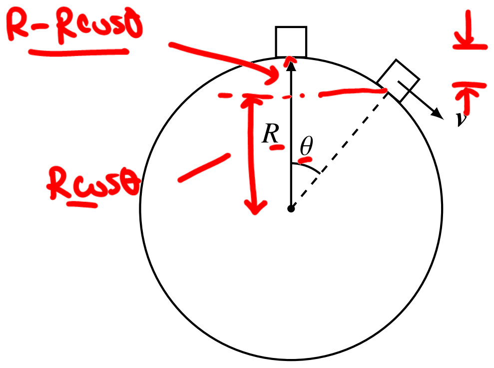
(A) $Fx \sin\theta$
(B) $Fx \cos\theta$
(C) $Fmx \sin\theta$
(D) $Fmx \cos\theta$
(E) zero

11. The coefficient of friction between the block and the floor is

(A) $\dfrac{F}{mg}$
(B) $\dfrac{F\cos\theta}{mg}$
(C) $\dfrac{F\cos\theta}{F\sin\theta + mg}$
(D) $\dfrac{F\sin\theta}{F\cos\theta + mg}$
(E) $\dfrac{F\cos\theta}{F\sin\theta}$

Questions 12–13

A small block rests on the top of a smooth sphere of radius R when it is given a light tap so that it just begins sliding on the sphere. When the block reaches the angle θ, it loses contact with the surface of the sphere.



$K = mg\Delta h =$
$= mg(R - R\cos\theta)$

12. The kinetic energy of the block as it leaves the surface of the sphere is

(A) mgR
(B) $mgR\cos\theta$
(C) $mgR\sin\theta$
(D) $mg(R - R\cos\theta)$
(E) $mg(R - R\sin\theta)$

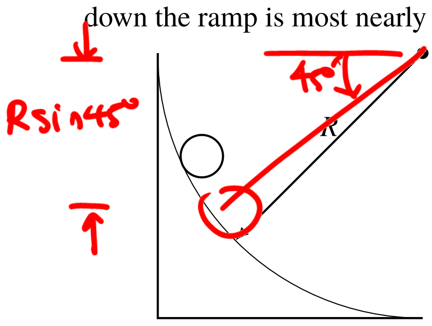
13. The speed of the block as it leaves the surface of the sphere is

$\sqrt{2\cancel{m}g(R - R\cos\theta)} = \left(\frac{1}{2}\right)\cancel{m}v^2$

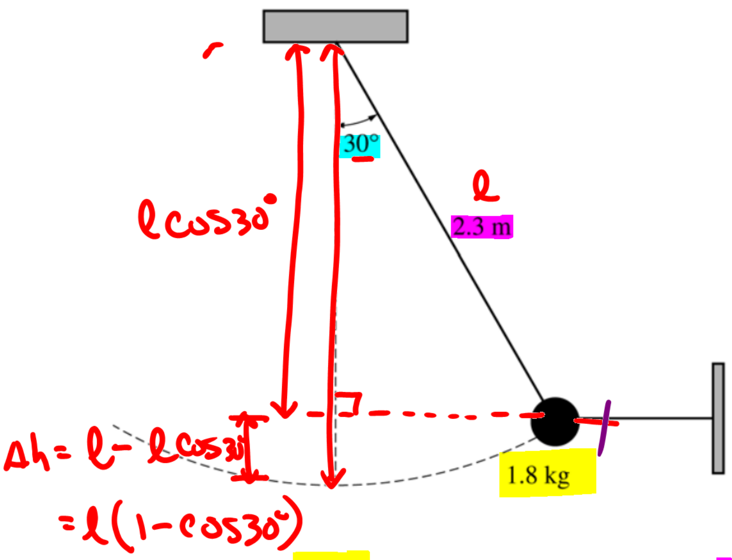
(A) $\sqrt{2gm}$
(B) $\sqrt{2gRm}$
(C) $\sqrt{2gR\cos\theta}$
(D) $\sqrt{2g(R - R\cos\theta)}$
(E) $\sqrt{2g(R - R\sin\theta)}$

14. A small ball starts from rest and rolls down a quarter-circle ramp of radius R. The speed of the ball at the point halfway down the ramp is most nearly

$\sqrt{2\cancel{m}gR\sin 45°} = \left(\frac{1}{2}\right)\cancel{m}v^2$

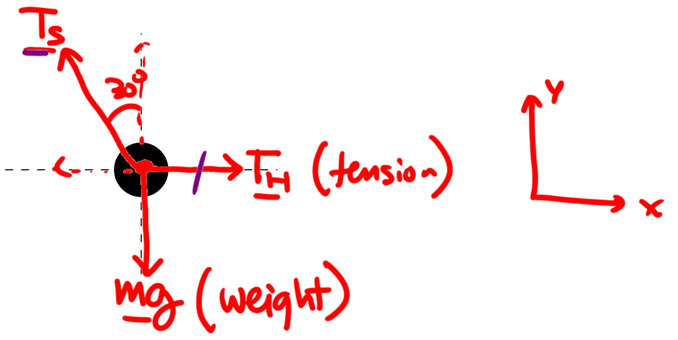


(A) gR
(B) $2gR$
(C) $\sqrt{gR\sin 45°}$
(D) $\sqrt{2gR\sin 45°}$
(E) The speed cannot be determined without knowing the mass of the ball.



15. A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.

(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



(b) Calculate the tension in the horizontal string.

(c) The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

b) $\hat{j}$-direction

$T_s \cos 30° = mg$

$T_s = \frac{mg}{\cos 30°}$

$\hat{i}$-dir

$T_s \sin 30° = T_H$

$T_H = \frac{mg}{\cos 30°} \sin 30°$

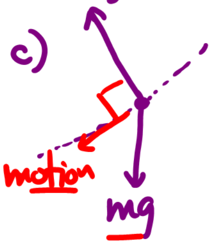
$= mg \tan 30°$

$= (1.8)(10) \tan 30°$

$= 10.4$

$\boxed{T_H = 10\,N}$

c)



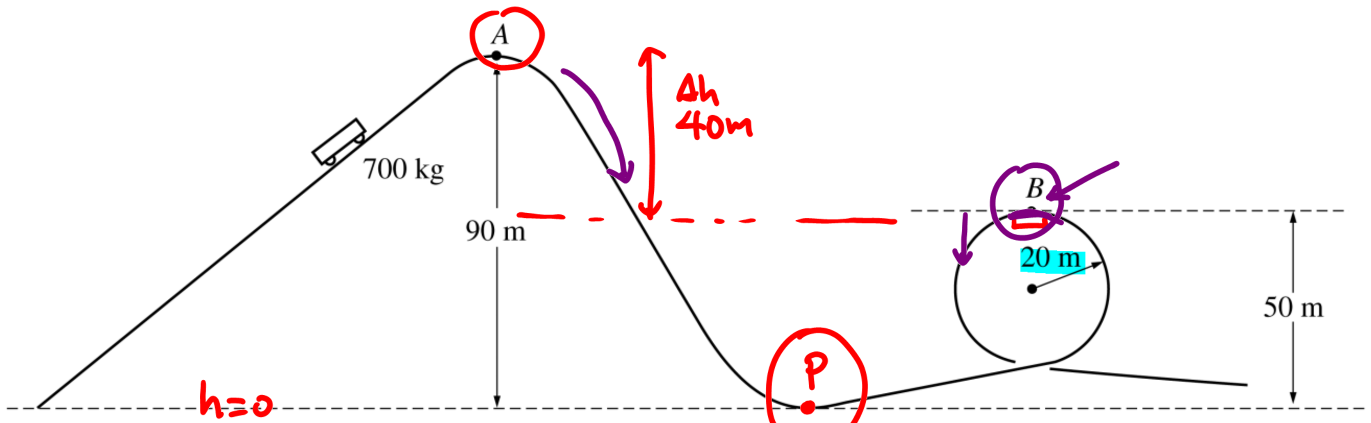
T_s does not do any work

$mg\Delta h = \frac{1}{2} m v^2$

$v = \sqrt{2g\Delta h}$

$= \sqrt{2g\ell(1-\cos 30°)}$

$=$ ______ m/s



16. A roller coaster ride at an amusement park lifts a car of mass 700 kg to point A at a height of 90 m above the lowest point on the track, as shown above. The car starts from rest at point A, rolls with negligible friction down the incline and follows the track around a loop of radius 20 m. Point B, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

 (a) i. Indicate on the figure the point P at which the maximum speed of the car is attained.

 ii. Calculate the value v_{max} of this maximum speed.

 (b) Calculate the speed v_B of the car at point B.

 (c) i. On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point B.

$\underbrace{mgh}_{U_g} = \underbrace{\frac{1}{2}mv^2}_{K}$

$V = \sqrt{2gh}$

$V = 42.42 = \boxed{\text{a,ii) } 42\,m/s}$

b) $mg\Delta h = \frac{1}{2}mv^2$

$V = \sqrt{2g\Delta h}$

$= 28.28 = \boxed{28\,m/s}$

at b $v^2 = 800$

$F_c = mg + N = \frac{mv^2}{r}$

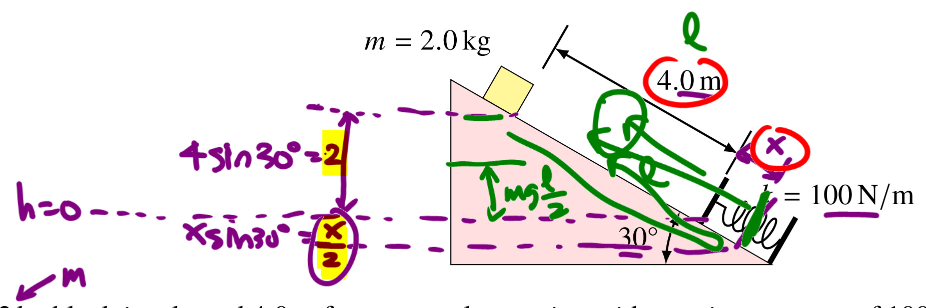
$N = \frac{mv^2}{r} - mg = \frac{(700)(800)}{20} - 7000$

$\boxed{N = 21000N}$

$\boxed{mg = 7000N}$

 ii. Calculate the magnitude of all the forces identified in (c)i.

 (d) Now suppose that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)? Justify your answer.



17. A 2 kg block is released 4.0 m from a massless spring with a spring constant of 100 N/m that is fixed along a frictionless plane inclined at 30°, as shown in the figure ~~below~~ above. Plese give answer to 3 significant figures.

(a) Find the maximum compression of the spring.

(b) If the plane is not frictionless, and the coefficient of kinetic friction between it and the block is 0.20, find the maximum compression.

(c) For the rough incline ($\mu = 0.20$), how far up the incline will the block travel after leaving the spring?

a) N, motion, mg

$$\underbrace{U_g + \cancel{K} + \cancel{U_e}}_{\text{release}} = \underbrace{U_g' + \cancel{K'} + U_e'}_{\text{max compression.}}$$

$$mgh = -mg\frac{x}{2} + \frac{1}{2}kx^2$$

$$40 = -10x + 50x^2$$

$$50x^2 - 10x - 40 = 0$$

$$5x^2 - x - 4 = 0$$

$$\boxed{x_1 = 1.00} \quad x_2 = -\cancel{0.800}$$

$$\boxed{x = 1.00\,\text{m}}$$

b) $f_k = \mu mg\cos\theta$, $N = mg\cos\theta$, mg, $\theta = 30°$

$$\underbrace{U_g + \cancel{K} + \cancel{U_e}}_{E_i} + W_f = U_g' + \cancel{K'} + U_e'$$

$$mgh - \underbrace{\mu mg\cos\theta}_{3.646}(4+x) = -mg\frac{x}{2} + \frac{1}{2}kx^2$$

$$40 - 13.86 - 3.646x = -10x + 50x^2$$

$$50x^2 - 6.536x - 26.14 = 0$$

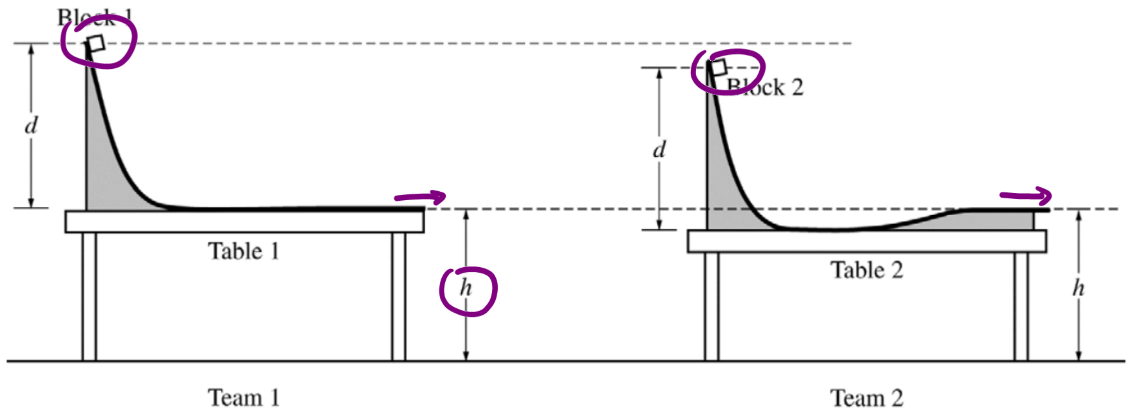
$$\boxed{x_1 = 0.79\,\text{m}}, \quad \cancel{x_2 = -0.66\,\text{m}}$$

c) $N = mg\cos\theta$, $f_k = \mu mg\cos\theta$, mg, $\theta = 30°$

$$U_g + \cancel{K} + U_e + W_f = U_g' + \cancel{K'} + \cancel{U_e'}$$

$$\underbrace{-mg\frac{x}{2}}_{-7.9} + \frac{1}{2}kx^2 - \underbrace{\mu mg\cos\theta}_{3.646}(\ell + x) = \underbrace{mg\frac{\ell}{2}}_{10}$$

$$\boxed{\ell = \_\_\_\_\,\text{m}}$$

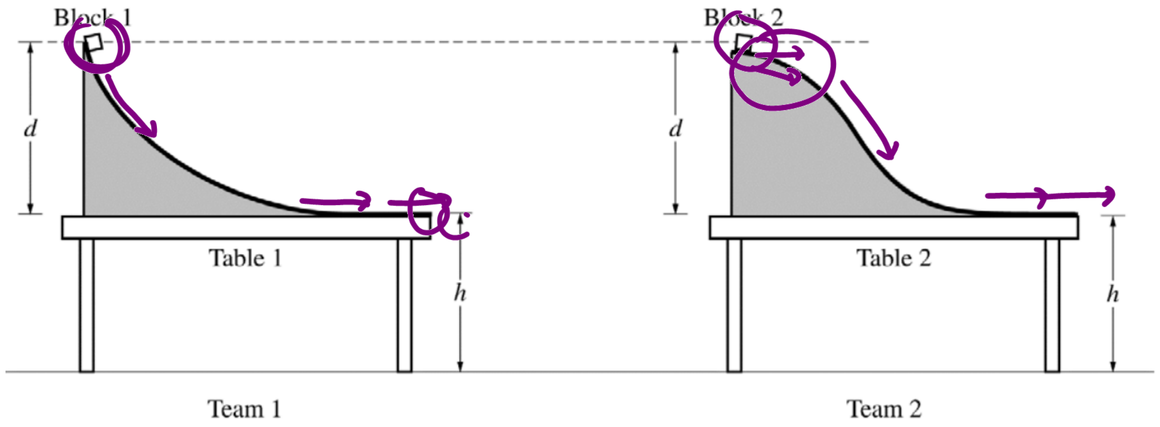


18. A physics class is asked to design a low-friction slide that will launch a block horizontally from the top of a lab table. Teams 1 and 2 assemble the slides shown above and use identical blocks 1 and 2, respectively. Both slides start at the same height d above the tabletop. However, team 2's table is lower than team 1's table. To compensate for the lower table, team 2 constructs the right end of the slide to rise above the tabletop so that the block leaves the slide horizontally at the same height h above the floor as does team 1's block (see figure above).

    (a) Both blocks are released from rest at the top of their respective slides. Do block 1 and block 2 land the same distance from their respective tables? Justify your answer.

    ____ Yes ✓ No

    In another experiment, teams 1 and 2 use tables and low-friction slides with the same height. However, the two slides have different shapes, as shown below.

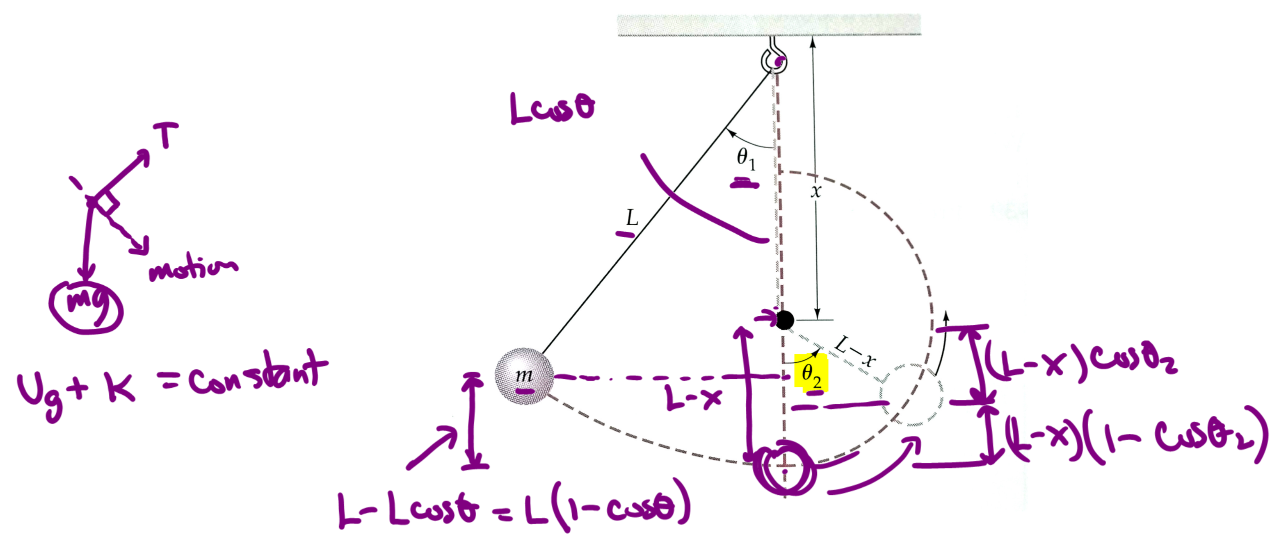


    (b) Both blocks are released from rest at the top of their respective slides at the same time.

    i. Which block, if either, lands farther from its respective table? Briefly explain your reasoning without manipulating equations.

    ____ Block 1 ____ Block 2 ✓ The two blocks land the same distance from their respective tables.

    ii. Which block, if either, hits the floor first? Briefly explain your reasoning without manipulating equations. ✓ Block 1 ____ Block 2 ____ The two blocks hit the floor at the same time.



19. A pendulum of length L has a bob of mass m. It is released from some angle θ_1. The string hits a peg at a distance x directly below the pivot, as shown in the figure below, effectively shortening the length of the pendulum. Find the maximum angle θ_2 between the string and the vertical when the bob is to the right of the peg.

at ~~bottom~~ $\theta_1 \rightarrow$ bottom   bottom $\rightarrow \theta_2$

$$\cancel{mg}L(1-\cos\theta_1) = \frac{1}{2}mv^2 = \cancel{mg}(L-x)(1-\cos\theta_2)$$

$$\cancel{mgL} - mgL\cos\theta_1 = (mgL - mgx)(1-\cos\theta_2)$$

$$= \cancel{mgL} - mgL\cos\theta_2 - mgx + mgx\cos\theta_2$$

$$\cancel{mg}x - \cancel{mg}L\cos\theta_1 = (\cancel{mg}x - \cancel{mg}L)\cos\theta_2$$

$$x - L\cos\theta_1 = (x-L)\cos\theta_2 \longrightarrow \cos\theta_2 = \frac{x - L\cos\theta_1}{x-L}$$

$$\boxed{\theta_2 = \cos^{-1}\left(\frac{x - L\cos\theta_1}{x-L}\right)}$$