Student #: ___________________________ Student Name: __________________________________

AP PHYSICS C CLASS 15: GAUSS'S LAW

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

1. A non-conducting sphere does not have a uniform charge density, but the density ρ varies with the distance r from the center of the sphere according to the equation $\rho = \beta r$ where β is a positive constant. The electric field inside the sphere ($r < R$) at a distance r from the center of the sphere is

   (A) $\dfrac{\beta r^2}{12\epsilon_0}$
   (B) $\dfrac{\beta r^3}{3\epsilon_0}$
   (C) $\dfrac{\beta r}{2\epsilon_0}$
   (D) $\dfrac{\beta r^2}{2\epsilon_0}$
   (E) $\dfrac{\beta r^2}{4\epsilon_0}$

2. The electric potential at the surface of the sphere from the last question is

   (A) $\dfrac{\beta R^3}{12\epsilon_0}$
   (B) $\dfrac{\beta R}{2\epsilon_0}$
   (C) $\dfrac{\beta R^3}{3\epsilon_0}$
   (D) $\dfrac{\beta R^2}{2\epsilon_0}$
   (E) $\dfrac{\beta R^2}{4\epsilon_0}$

3. According to Gauss's law, the net electric flux passing through a closed surface is

   (A) positive if the flux is entering the surface
   (B) negative if the flux is exiting the surface
   (C) positive if the net charge inside the surface is zero
   (D) negative if the net charge inside the surface is zero
   (E) zero if the net charge inside the surface is zero

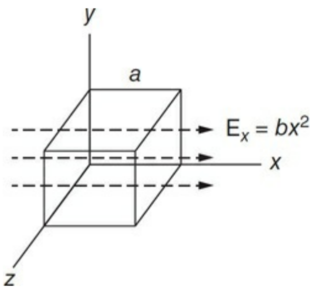
4. According to Gauss's law, which of the following statements is true?

   (A) It is possible to have a nonzero electric field, but zero electric flux.
   (B) It is possible to have a nonzero electric flux, but zero electric field.
   (C) It is possible to have a nonzero electric flux through a closed surface even if the enclosed charge in a surface is zero.
   (D) If a surface is not closed (such as a sheet of paper), the flux through it must be zero.
   (E) It is possible for charges located outside a closed surface to produce a net positive flux through the surface.

5. Gauss's law is most convenient to use when calculating an electric field due to

   (A) charges outside a closed surface
   (B) charges inside a closed surface that has high symmetry
   (C) charges inside a closed surface that has low symmetry
   (D) a potential difference that is negative
   (E) a potential difference that is positive

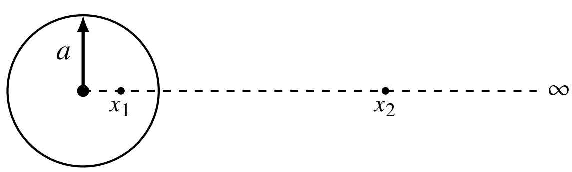
Question ??-??



6. A cube has sides of length a. The cube rests so that one side rests on the x-axis as shown. An electric field is established in the x-direction according to the function $E_x = bx^2$, where b is a positive constant. Which of the following statements is true?
   (A) There is a net charge inside the cube.
   (B) There is no net charge inside the cube.
   (C) The flux passing through the cube is negative.
   (D) The flux passing through the cube is zero.
   (E) The flux diminishes while passing through the cube.

7. The charge inside the cube can be expressed by the equation
   (A) $\epsilon_0 ba$
   (B) $\epsilon_0 ba^2$
   (C) $\epsilon_0 ba^3$
   (D) $\epsilon_0 ba^4$
   (E) $\epsilon_0 b^2 2a^2$

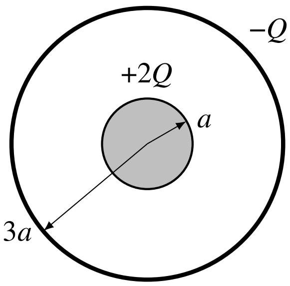
Questions ??–??: A nonconducting spherical charge distribution has a nonuniform positive charge density ρ. The center of the sphere is point O, the radius of the sphere is a. The sphere is centered on the x-axis. A point inside the sphere lies on the x-axis at a distance x_1 from the center of the sphere. Another point, x_2, is outside the sphere on the x-axis.



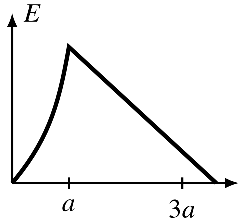
8. The electric field at point x_2 can be determined by
   (A) using Gauss's law to determine the electric field from O to a, then using Gauss's law to determine the electric field from a to x_2, then finding the difference between the two electric fields.
   (B) using Gauss's law to determine the electric field from O to a, then using Gauss's law to determine the electric field from a to x_2, then finding the sum of the two electric fields.
   (C) integrating the electric potential outside the sphere from infinity to a, then integrating the electric potential inside the sphere from a to x_1, then finding the difference between the two potential integrals.
   (D) integrating the electric potential outside the sphere from infinity to a, then integrating the electric potential inside the sphere from a to x_1, then finding the sum of the two potential integrals.
   (E) determining the derivative of the potential function inside and outside the sphere, then finding the difference between the two derivatives.

9. The electric potential at point x_1 can be determined by
   (A) determining the derivative of the electric field function inside and outside the sphere, then finding the difference between the two derivatives.
   (B) determining the derivative of the electric field function inside and outside the sphere, then finding the sum of the two derivatives.
   (C) integrating the derivative of the product of the electric field and potential functions, then finding their sum.
   (D) integrating the electric field outside the sphere from infinity to a, then integrating the electric field inside the sphere from a to x_1, then finding the sum of the two potentials.
   (E) integrating the electric field outside the sphere from infinity to a, then integrating the electric field inside the sphere from a to x_1, then finding the difference between the two potentials.

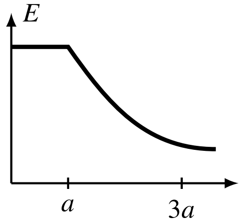
Questions ??–??: A solid conducting sphere of radius a is placed inside a conducting spherical shell of radius $3a$, as shown. A charge $+2Q$ is placed on the inner sphere, and a charge $-Q$ is placed on the outer sphere.



10. Which of the following graphs best represents the electric field $\vec{E}$ as a function of the distance r from the center of the spheres?



(A)



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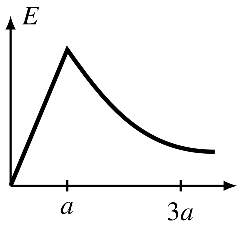
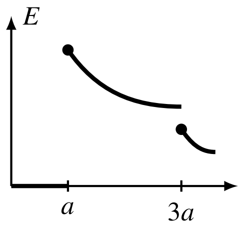
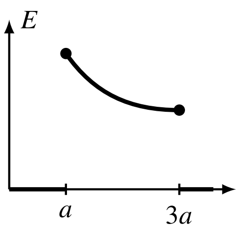


(C)

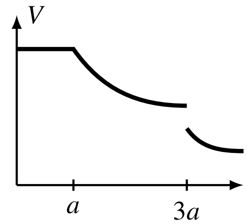


(D)

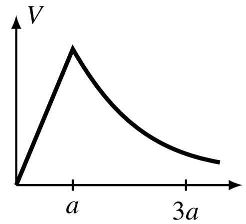


(E)

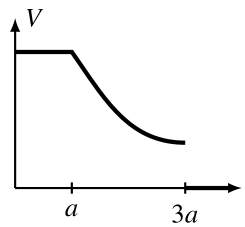
11. Which of the following graphs best represents the electric potential V as a function of the distance r from the center of the spheres?



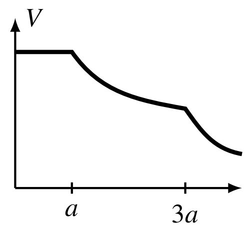
(A)



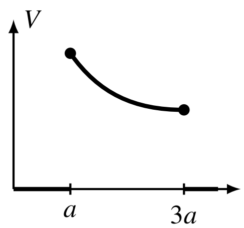
(B)



(C)



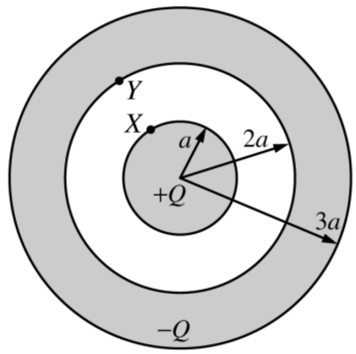
(D)



(E)

AP PHYSICS C CLASS 15: GAUSS'S LAW
SECTION II
3 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.



1. In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.
   (a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.
      i. Within the solid sphere ($r < a$)
      ii. Between the solid sphere and the spherical shell ($a < r < 2a$)
      iii. Within the spherical shell ($2a < r < 3a$)
      iv. Outside the spherical shell ($r > 3a$)
   (b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.
   (c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.