

Class 12: Electrostatics Part 1 (Point Charges)

Advanced Placement Physics C

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Fall 2021

Olympiads School

Electrostatic Force

Review: The Charges Are

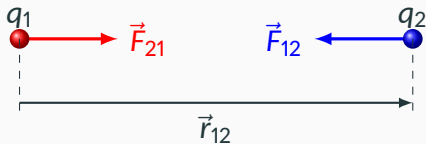
We should already know a bit about charge particles:

- A **proton** carries a **positive** charge
- An **electron** carries a **negative** charge
- A *net charge* of an object means an excess of protons or electrons
- Similar charges are repel; opposite charges attract

We start with electrostatics:

- Charges that are not moving relative to one another

Coulomb's Law for Electrostatic Force



The **electrostatic force** (or **coulomb force**) is a mutually repulsive/attractive force between all charged objects. The force that charge q_1 exerts on q_2 is given by **Coulomb's law**:

$$\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_{12}|^2}\hat{r}_{12}$$

Coulomb's Law for Electrostatic Force

$$\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_{12}|^2}\hat{r}_{12}$$

Quantity	Symbol	SI Unit
Electrostatic force	\vec{F}_{12}	N
Coulomb's constant	k	$\text{N} \cdot \text{m}^2 / \text{C}^2$
Point charges 1 and 2	q_1, q_2	C
Distance between point charges	$ \vec{r}_{12} $	m
Unit vector of direction between point charges	\hat{r}_{12}	

Coulomb's Constant

$$\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_{12}|^2} \hat{r}_{12}$$

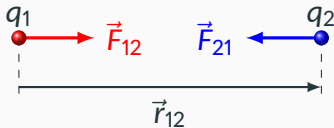
The constant k in the Coulomb's law is called the **coulomb's constant**, defined as:

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

where ϵ_0 is a fundamental constant called the **permittivity of free space**, or **vacuum permittivity**. It measures a vacuum's ability to resist the formation of an electric field:

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

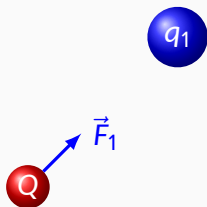
Coulomb's Law for Electrostatic Force



- If q_1 exerts an electrostatic force \vec{F}_{12} on q_2 , then q_2 likewise exerts a force of $\vec{F}_{21} = -\vec{F}_{12}$ on q_1 . The two forces are equal in magnitude and opposite in direction. (Third law of motion)
- q_1 and q_2 are assumed to be *point charges* that do not occupy any space
- The scalar form is often used as well, since the direction of F_q can easily be found:

$$F_q = \frac{kq_1q_2}{r^2}$$

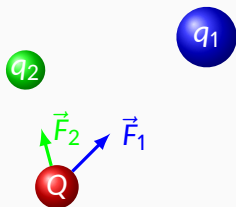
More Than One Charge



For a charge Q that is subjected to the influence of multiple discrete point charges q_i , the total electrostatic force that Q experiences is the vector sum of all the forces \vec{F}_i :

$$\vec{F} = \sum_i \vec{F}_i = kQ \left(\sum_{i=1}^N \frac{q_i}{r_i^2} \hat{r}_i \right)$$

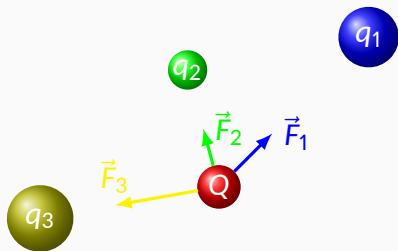
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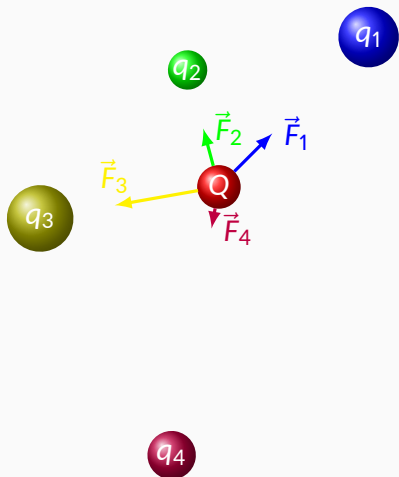
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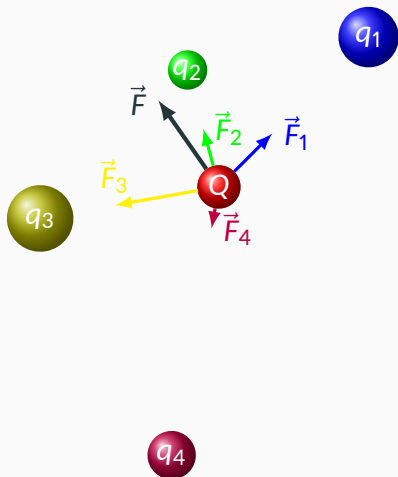
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Continuous Distribution of Charges

As $N \rightarrow \infty$, the summation becomes an integral, and can now be used to describe the force from charges with *spatial extend* i.e. charges that take up physical space (e.g. a continuous distribution of charges):

$$\vec{F} = \int d\vec{F} = kQ \int \frac{dq}{r^2} \hat{r}$$

Electric Field

Electric Field

The expression for **electric field** is obtained by repeating the same procedure as with gravitational field, by grouping the variables in Coulomb's law:

$$F_q = \underbrace{\left[\frac{kq_1}{|\vec{r}_{12}|^2} \hat{r} \right]}_{\vec{E}} q_2$$

The electric field \vec{E} created by q_1 is a vector function (called a **vector field**) that shows how it influences other charged particles around it.

Electric Field Near a Point Charge

The electric field a distance r away from a point charge q is given by:

$$\vec{E}(q, \vec{r}) = \frac{kq}{|\vec{r}|^2} \hat{r}$$

Quantity	Symbol	SI Unit
Electric field intensity	\vec{E}	N/C
Coulomb's constant	k	$\text{N} \cdot \text{m}^2/\text{C}^2$
Source charge	q	C
Distance from source charge	$ \vec{r} $	m
Outward unit vector from point source	\hat{r}	

The direction of \vec{E} is radially outward from a positive point charge and radially inward toward a negative charge.

More Than One Charge

When multiple point charges are present, the total electric field at any position \vec{r} is the vector sum of all the fields \vec{E}_i :

$$\vec{E} = \sum_i \vec{E}_i = k \left(\sum_{i=1}^N \frac{q_i}{r_i^2} \hat{r}_i \right)$$

More Than One Charge

As $N \rightarrow \infty$, the summation becomes an integral, and can now be used to describe the electric field generated by charges with *spatial extend*:

$$\vec{E} = \int d\vec{E} = k \int \frac{dq}{r^2} \hat{r}$$

This integral may be difficult to compute if the geometry of is complicated, but in general, there are usually symmetry that can be exploited.

Think Electric Field

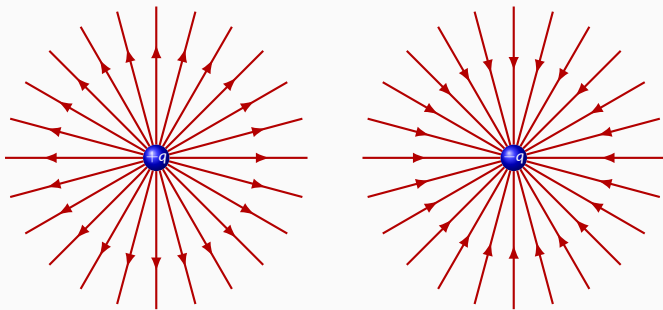
\vec{E} itself *doesn't do anything* until another charge interacts with it. And when there is a charge q , the electrostatic force \vec{F}_q that the charge experiences is proportional to q and \vec{E} , regardless of how the electric field is generated:

$$\boxed{\vec{F}_q = q\vec{E}}$$

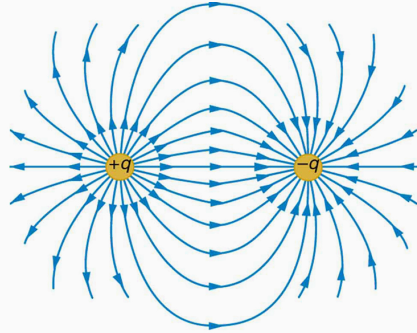
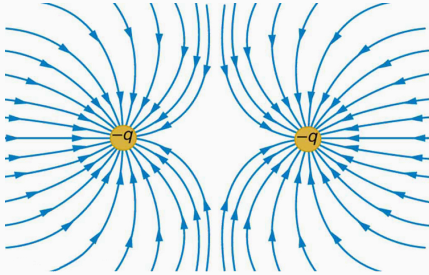
A positive charge in the electric field experiences an electrostatic force \vec{F} in the same direction as \vec{E} .

Electric Field Lines

Electric field lines can be used to visualize the direction of the electric field.



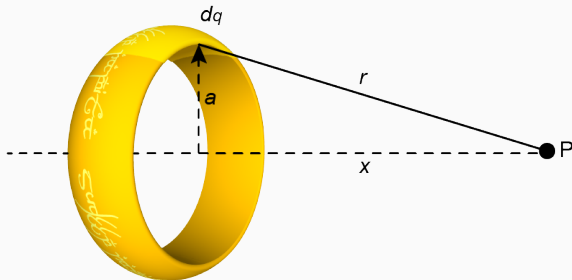
Electric Field from Multiple Charges



- Electric field lines must begin and/or end at a charge
- Field lines do not cross
- Direction of the electric field is tangent to the field lines

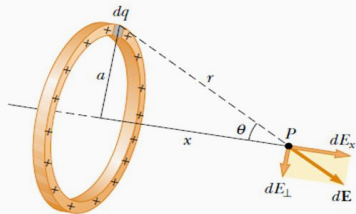
Lord of the Ring Charge

Suppose you have been given *The One Ring To Rule Them All*, and you found out that it is charged! What is its electric field at point P along its axis?



Note that calculating the electric field away from the axis is very difficult.

Electric Field Along Axis of a Ring Charge



- We can separate the electric field $d\vec{E}$ (generated by charge dq) into axial (dE_x) and radial (dE_{\perp}) components
- Based on symmetry, dE_{\perp} doesn't contribute to anything; but dE_x is pretty easy to find:

$$dE_x = \frac{k dq}{r^2} \cos \theta = \frac{k dq}{r^2} \frac{x}{r} = \frac{k x dq}{(x^2 + a^2)^{3/2}}$$

Integrating this over all charges dq , we have:

$$E_x = \frac{kx}{(x^2 + a^2)^{3/2}} \int dq = \boxed{\frac{kQx}{(x^2 + a^2)^{3/2}}}$$

Electric Field Along Axis of a Uniformly Charged Disk

Let's extend what we know to a disk of radius a and charge density σ

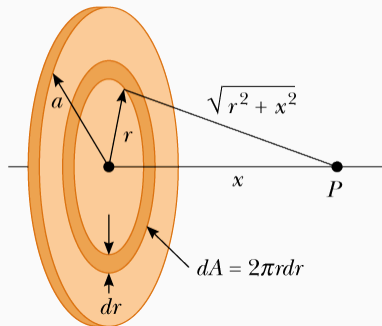
We start with the solution from the ring problem, and replace Q with $dq = 2\pi\sigma r dr$:

$$dE_x = \frac{2\pi k r \sigma x}{(x^2 + r^2)^{3/2}} dr$$

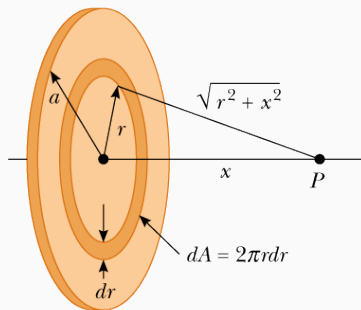
Integrating over the entire disk:

$$E_x = \pi k x \sigma \int_0^a \frac{2r}{(x^2 + r^2)^{3/2}} dr$$

This is not an easy integral!



Eclectic Field Along Axis of a Uniformly Charged Disk



Luckily for us, the integral is in the form of $\int u^n du$, with $u = x^2 + r^2$ and $n = \frac{-3}{2}$. You can find the integral in any math textbook:

$$E_x = 2\pi k\sigma \left(1 - \frac{x}{\sqrt{x^2 + a^2}} \right)$$

Electric Potential Energy

Electric Potential Energy

The electrostatic force is a conservative force, therefore the work done by F_q is related to the **electric potential energy** U_q :

$$W = \int \vec{F}_q \cdot d\vec{r} = kq_1q_2 \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{kq_1q_2}{r} \Big|_{r_1}^{r_2} = -\Delta U_q$$

where

$$U_q = \frac{kq_1q_2}{r}$$

- U_q can be (+) or (-), because charges can be either (+) or (-)
- Positive work done by F_q decreases U_q , while
- Negative work done by F_q increases U_q
- W depends on r_1 and r_2 but not *how* the charge moves from $r_1 \rightarrow r_2$

How it Differs from Gravitational Potential Energy

Two positive charges:

$$U_q > 0$$

Two negative charges:

$$U_q > 0$$

One positive and one
negative charge:

$$U_q < 0$$

- $U_q > 0$ means positive work is done to bring two charges together from $r = \infty$ to r (both charges of the same sign)
- $U_q < 0$ means negative work (the charges are opposite signs)
- For gravitational potential U_g is always < 0

Relating U_q to \vec{F}_q

From the fundamental theorem calculus, we can relate electrostatic force (\vec{F}_q) to electric potential energy (U_q) by the gradient operator:

$$\Delta U_q = - \int \vec{F}_q \cdot d\vec{r} \quad \rightarrow \quad \vec{F}_q(r) = -\nabla U_q = -\frac{\partial U_q}{\partial r} \hat{r}$$

Electrostatic force \vec{F}_q always points from high to low potential energy (steepest descent direction)

Electric Potential

Electric Potential: Using Gravity as Example

An object at a specific location inside a gravitational field has a gravitational potential energy proportional to its mass, i.e.

$$U_g = V_g m$$

This “constant” V_g is called the **gravitational potential**, which is the *gravitational potential energy per unit mass*. In the trivial case with a uniform gravitational field:

$$V_g = \frac{U_g}{m} = gh$$

This also applies to the general case of the gravitational potential energy:

$$V_g = \frac{U_g}{m} = -\frac{Gm}{r}$$

Electric Potential

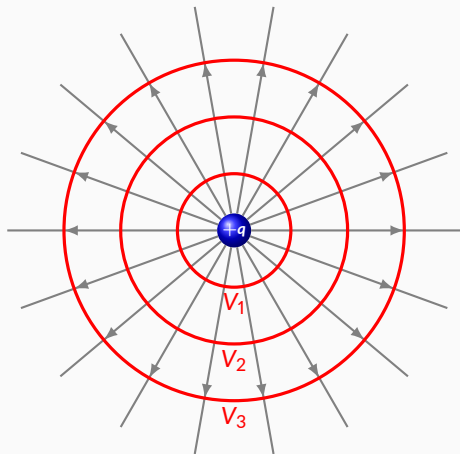
This is also true for moving a charged particle q against an electric electric field created by q_s , and the “constant” is called the **electric potential**. The unit for electric potential is a *volt* which is *one joule per coulomb*, i.e. $1\text{V} = 1\text{J/C}$

$$V = \frac{U_q}{q}$$

The electric potential from a source point charge q_s is therefore:

$$V = \frac{kq_s}{r}$$

Electric Potential

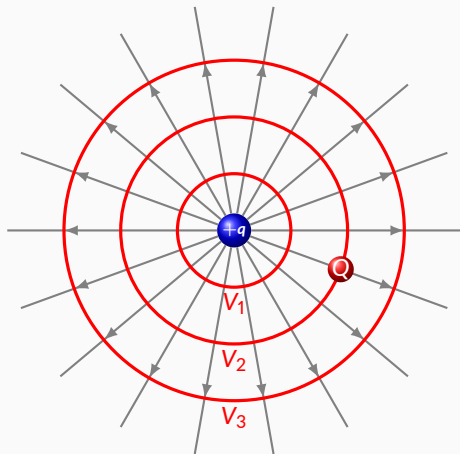


For a point charge q , every point at a distance r will have the same electric potential $V(r)$.

- The red lines have the same electric potential; they are called **equipotential lines**, or **equipotential contours**
- Equipotential lines are perpendicular to the electric field lines
- Electric field lines always points from higher V toward lower V , i.e.

$$V_1 > V_2 > V_3$$

Electric Potential

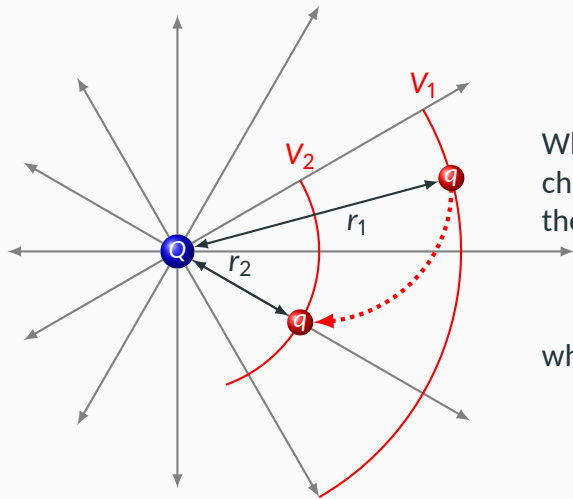


A charge Q that is placed inside this electric field will now have an electric potential energy of:

$$U_q = QV = Q \left[\frac{kq}{r} \right]$$

in agreement with equation for electric potential energy

Potential Difference



When a charge is moved from r_1 to r_2 , the change in electric potential energy is related to the change in electric potential by:

$$\Delta U_q = U_2 - U_1 = q\Delta V$$

where ΔV is called the **potential difference**

Potential Difference (Voltage)

The change in electric potential is called the **electric potential difference** or **voltage**:

$$\Delta V = \frac{\Delta U_q}{q} \quad \text{and} \quad dV = \frac{dU_q}{q}$$

Here, we can relate ΔV to an equation that we knew from Grade 11 Physics, which related to the energy dissipated in a resistor in a circuit ΔU to the voltage drop ΔV :

$$\Delta U_q = q\Delta V$$

Electric potential difference also has the unit *volts* (V)

Relating V to \vec{E}

In the same way that the fundamental theorem of calculus relates the electrostatic force (\vec{F}_q) and electric potential energy (U_q) by the gradient operator, electric field (\vec{E}) and electric potential (V) are also related the same way:

$$\Delta V_q = - \int \vec{E}_q \cdot d\vec{r} \quad \rightarrow \quad \vec{E}(r) = -\nabla V_q = -\frac{\partial V}{\partial r} \hat{r}$$

- Electrostatic field \vec{E} always points from high to low electric potential
- Electric field is also called “potential gradient”

Getting Those Names Right

Remember that these three scalar quantities, as opposed to electrostatic force \vec{F}_q and electric field \vec{E} which are vectors

- Electric potential energy:

$$U_q = \frac{kq_1q_2}{r}$$

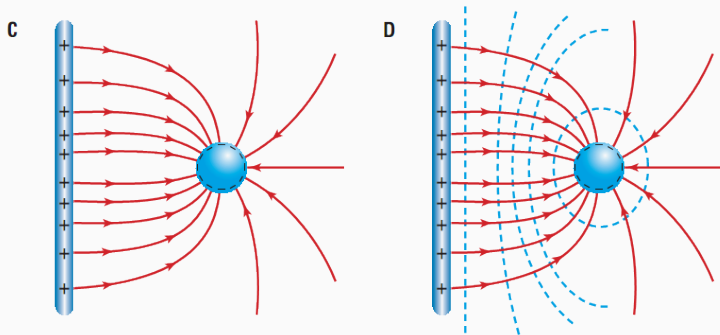
- Electric potential:

$$V = \frac{U_q}{q}$$

- Electric potential difference (voltage):

$$\Delta V = \frac{\Delta U_q}{q}$$

Equipotential Lines



The dotted blue lines are called **equipotential lines**. They are always *perpendicular* to the electric field lines. Charges moving in the direction of the equipotential lines have constant electric potential