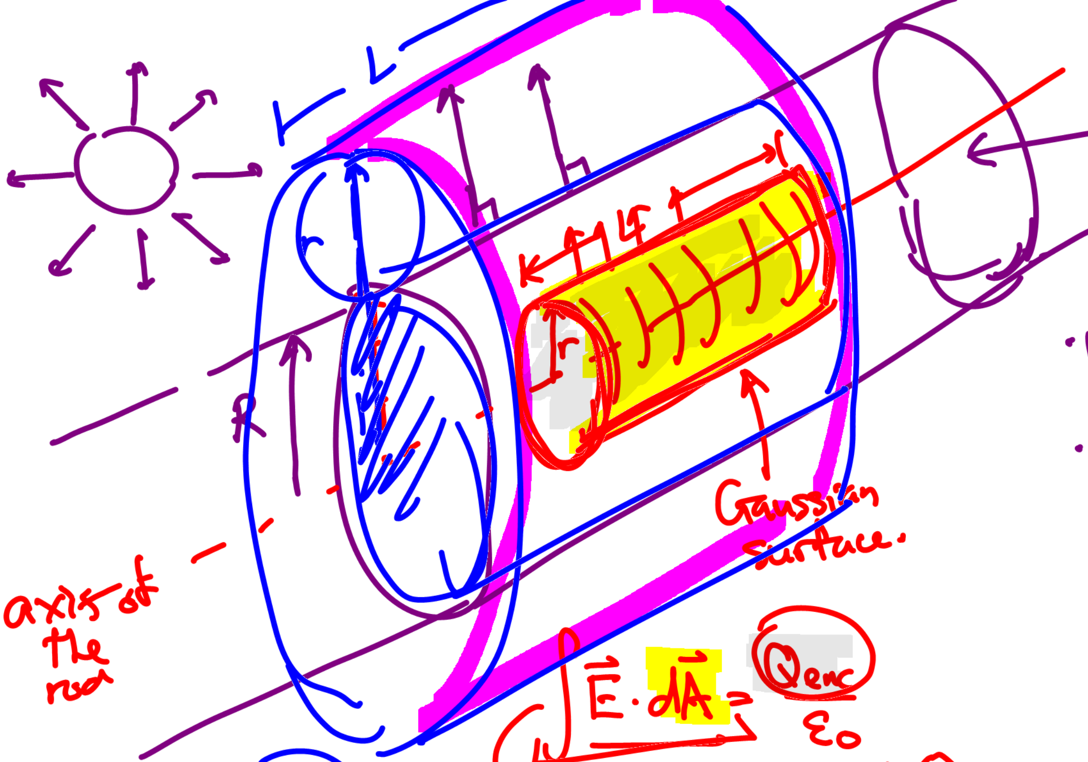


infinitely long charged rod with a constant charge density of ρ.

- What is electric field inside the rod?
- what is the electric field outside of the rod??

$$\int \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2\pi \cancel{r} \cancel{L}) = \frac{\pi r^{\cancel{2}} \cancel{L} \rho}{\varepsilon_0}$$

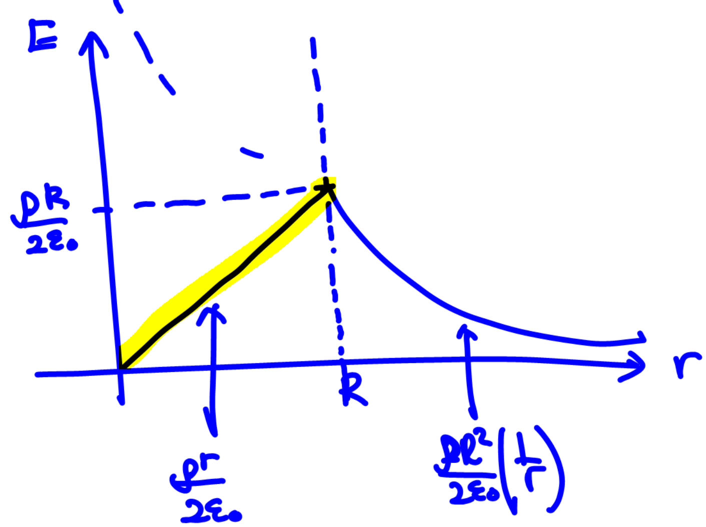
$$2E = \frac{\rho r}{\varepsilon_0} \rightarrow E = \frac{\rho r}{2\varepsilon_0} \propto r$$

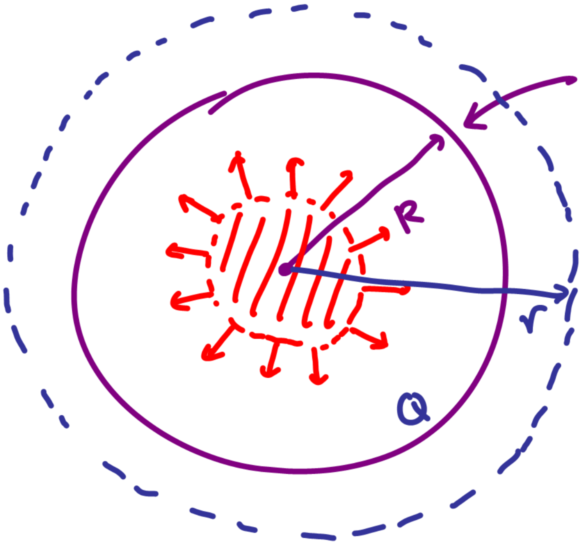
$r > R$

$$\int E\cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

constant

$$E\cdot(2\pi r \Delta) = \frac{\pi R^2 \Delta \rho}{\varepsilon_0} \rightarrow E = \left[\frac{\rho R^2}{2\varepsilon_0}\right]\frac{1}{r} \rightarrow E \propto \frac{1}{r}$$





sphere with uniform charge density ρ

total charge

$$Q = \rho V = \tfrac{4}{3}\pi R^3 \rho \rightarrow \rho = \frac{3Q}{4\pi R^3} \Big\} \text{ constant.}$$

positive.

1. What is the electric field inside the sphere?
2. What is the electric field outside of the sphere?

1. Gaussian surface inside the sphere at radius $r < R$

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(A) = \frac{Q_{enc}}{\varepsilon_0}$$

$$\rho V = \left(\frac{3Q}{4\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{Q}{R^3} r^3$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0 R^3} r^3$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 R^3} r} \rightarrow E = \frac{kQ}{R^3} r$$

$E \propto r$

2. Gaussian surface outside of the sphere at radius $r > R$

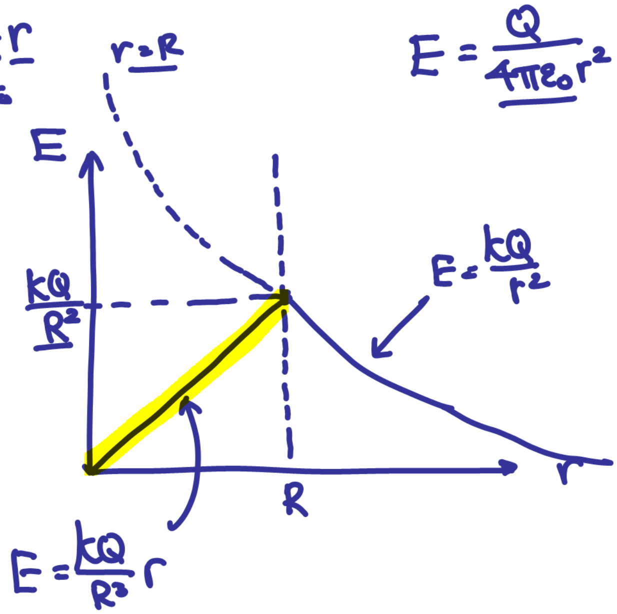
$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

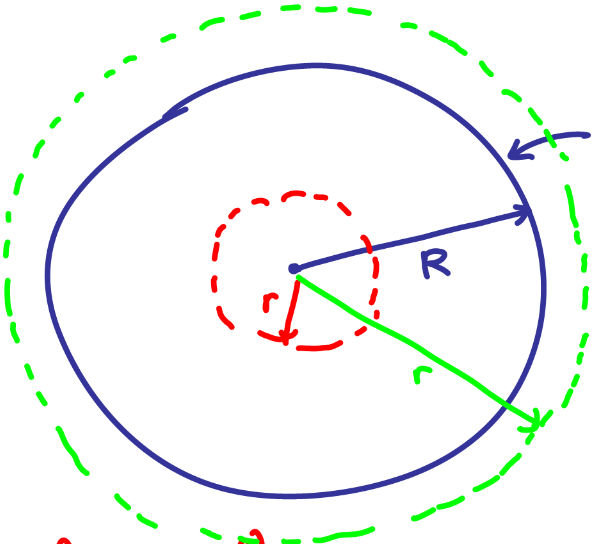
$$E \cdot A = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \rightarrow \frac{kQ}{r^2}$$

looks a point charge.





charged spherical shell
total charge +Q.

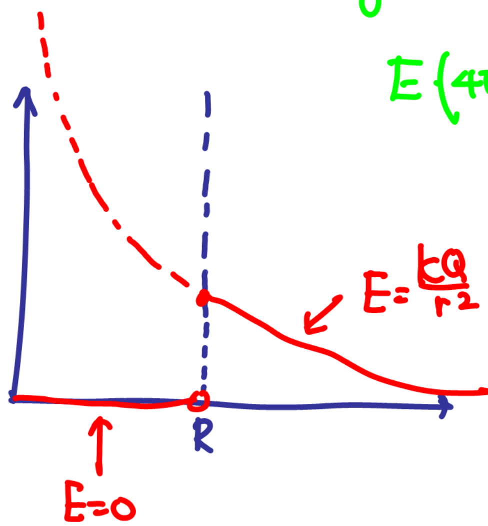
1. What is the electric field inside the sphere?
2. What is the electric field outside the sphere?

1. $\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0} = 0$

$\vec{E} = 0.$

2. $\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$

$E(4\pi r^2) = \frac{Q}{\varepsilon_0} \longrightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$



DOES GAUSS' LAW HAVE AN EQUIVALENT FOR GRAVITY?

Answer: YES!!!!

no one ever writes it this way!
Just for illustration!

electricity

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = 4\pi k Q_{enc} \longrightarrow$$

$k = \frac{1}{4\pi\varepsilon_0}$

$\frac{1}{\varepsilon_0} = 4\pi k$

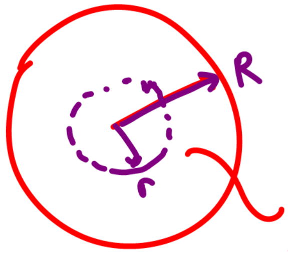
Gravity

inward.

$$\boxed{\int \vec{g} \cdot d\vec{A} = -4\pi G M_{enc}}$$

Gauss's law for gravity.

A solid sphere (radius R) with charge density that varies with distance r to the center



$\rho(r) = br^2$

(+) constant

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0} \rightarrow E \cdot (\cancel{4\pi} \cancel{r^2}) = \frac{\cancel{4\pi} b \cancel{r^5}^{\,r^3}}{5\varepsilon_0}$$

???

$$E \cdot A = \frac{Q_{enc}}{\varepsilon_0}$$

$$\boxed{E = \frac{br^3}{5\varepsilon_0}}$$

What is the electric field inside the sphere?

aside:

Volume of a sphere

$$V = \int_0^R dV = \int_0^R (4\pi r^2 dr) = \frac{4}{3}\pi R^3$$

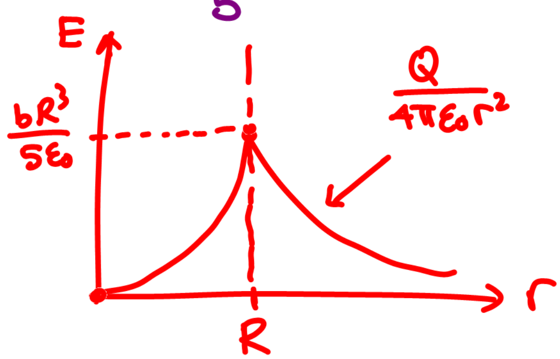
charge:

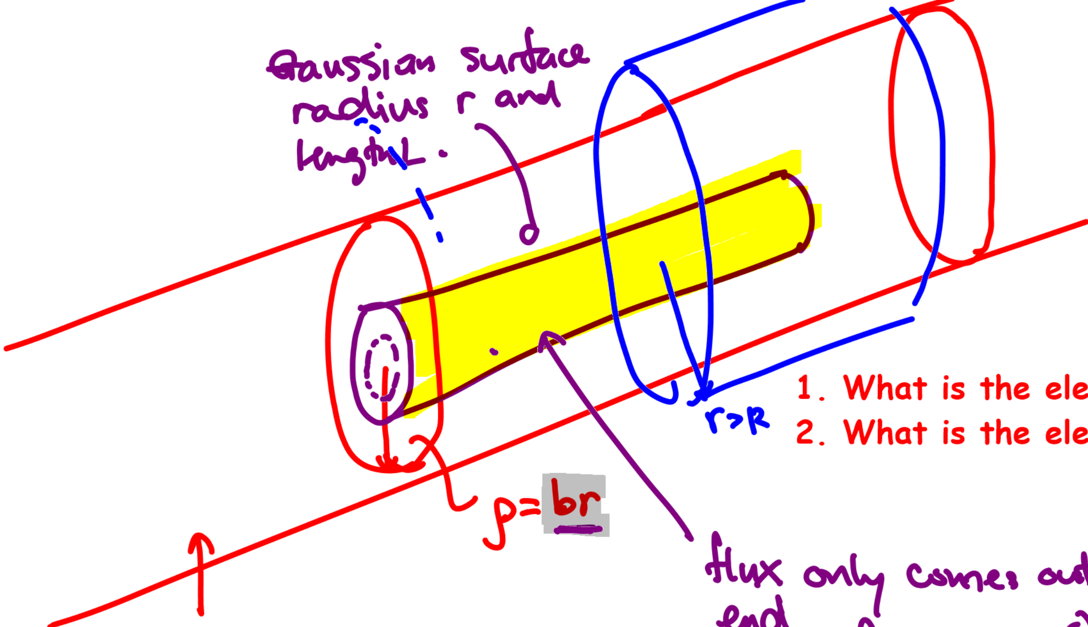
$$\boxed{Q = \int dQ = \int \rho \, dV = \int \rho (4\pi r^2 dr)}$$

$$Q_{enc} = \int_0^r dQ = \int \rho(r) 4\pi r^2 dr$$

$$= \int (br^2) 4\pi r^2 dr = 4\pi b \int_0^r r^4 dr$$

$$Q_{enc} = \frac{4\pi b}{5} r^5$$





1. What is the electric field inside the rod?
2. What is the electric field outside the rod?

Infinitely long rod with radius R

flux only comes out of the side, not the end.

$$\boxed{Q_{enc} = \int_0^r dQ = \int_0^r \rho\, dV = \int_0^r \rho\,(2\pi r L\, dr)}$$

$$= \int_0^r (br)(2\pi r L\, dr) = 2\pi b L \int_0^r r^2 dr$$

$$= \frac{2\pi b L}{3} r^3$$

$$Q_{enc} = \int_0^R dQ = \frac{2\pi b L}{3} R^3$$

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E \cdot (\cancel{2\pi r L}) = \frac{\cancel{2\pi b L}}{3\varepsilon_0} r^{\cancel{3}\,2}$$

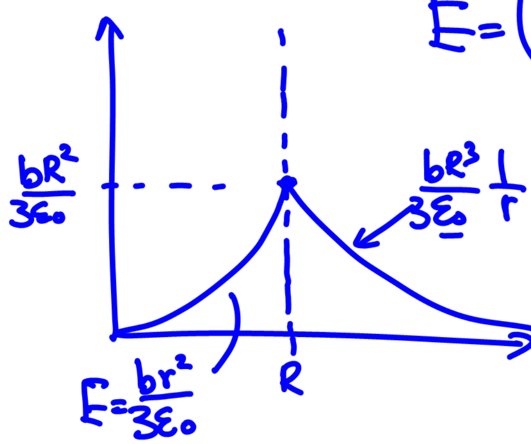
$$\boxed{E = \frac{br^2}{3\varepsilon_0}}$$

outside

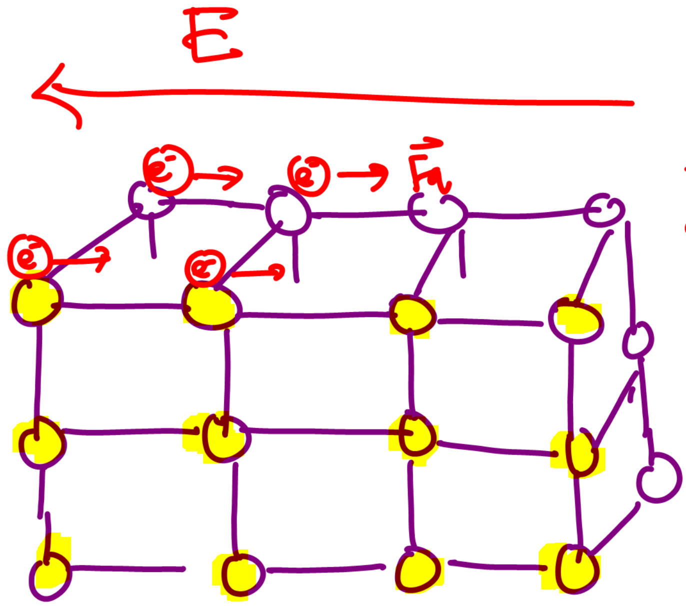
$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(\cancel{2\pi}\, r \cancel{L}) = \frac{\cancel{2\pi} b \cancel{L} R^3}{3\varepsilon_0}$$

$$E = \left(\frac{bR^3}{3\varepsilon_0}\right)\frac{1}{r}$$



In a conductor, each atom has 1 or 2 "free electrons" that can "hop" from atom to atom



The motion of the electrons create electric current

Atoms are organized in a "lattice" structure

+ block of metal

(−) charged block of metal

more or less electrically neutral.

electrons are pushed away from the (+) charge, ∴ (+) charge on the edge

excess electrons on this side

~ electrically neutral.

between $R \to R_2$

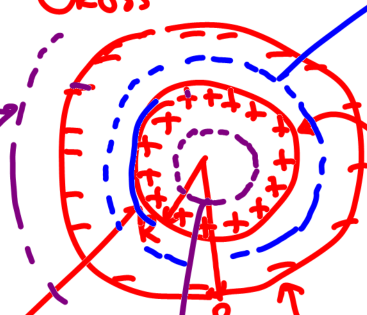
$$E(2\pi r \cancel{\ell}) = \frac{\gamma \cancel{\ell}}{\varepsilon_0} \to E = \left(\frac{\gamma}{2\pi\varepsilon_0}\right)\frac{1}{r}$$

CROSS SECTION

for $r > R_2$

$Q_{enc} = 0$

$\underline{\vec{E} = 0}$



Infinitely long rod made of a conducting material, radius R, charge density γ

Infinitely long & thin cylindrical shell with charge -Q

Charge on the rod is at the surface, but NOT distributed uniformly inside.

inside the rod. $Q_{enc} = 0 \to \underline{E = 0}$