WELCOME TO AP PHYSICS 1 & 2

Pre-requisites

- **Physics 11 and 12:** You will need to be comfortable with the topics covered in high-school level physics courses.
- **Vectors:** You need to be comfortable with vector operations, including addition and subtraction, multiplication/division by constants, as well as dot products and cross products.

If you already have a background in both differential and integral calculus, you may consider taking the AP Physics C exams instead.

Classroom Rules

- Treat each other with respect
- Raise your hands if you have a question. Don't wait too long
- E-mail me at `tleung@olympiadsmail.ca` for any questions related to physics and math and engineering
- Do ***not*** try to find me on social media

Topic 1: Kinematics

Advanced Placement Physics 1

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June 29, 2020

Olympiads School

Files for You to Download

- `PhysAP1-courseOutline.pdf`—The course outline
- `PhysAP1-01-kinematics.pdf` - Slides on *kinematics* (this set of slides).
- `PhysAP1-02-dynamics.pdf` - Slides for *dynamics*, the next topic
- `PhysAP1-01-Homework.pdf`—Homework problems for kinematics
- `PhysAP1-02-Homework.pdf`—Homework problems for dynmaics

Please download/print the PDF file for the class slides before each class. There is no point copying notes that are already on the slides. Instead, focus on things that aren't necessarily on the slides. If you wish to print the slides, we recommend printing *four* slides per page.

Vectors

Please refer to the handout to make sure that you are familiar with basic vector operations. We will be using a slightly more advanced notation method for this course.

Kinematics

Kinematics

Kinematics is a discipline within mechanics concerning the mathematical description of the motion of bodies. It describes the relationships between

- Position
- Displacement
- Distance
- Velocity
- Speed
- Acceleration

Kinematics does not deal with the causes of motion. In the AP Physics 1 exam, kinematics account for approximately $10\,\%$ to $16\,\%$ of the marks.

Position

Position $\mathbf{x}$ describes the location of an object in a coordinate system. The origin of the coordinate system is called the "reference point". The SI unit for position is **meter**, m.

$$\mathbf{x}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}$$

Vectors in 2D/3D Cartesian space are often expressed using the "IJK notation"

- $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ are **basis vectors** representing the directions of the x, y and z axes. Basis vectors are **unit vectors** (i.e. length 1)
- The IJK notation does not explicitly give the magnitude or the direction of the vector (needs to be calculated using the Pythagorean theorem)

Displacement

Displacement $\Delta\mathbf{x}(t)$ is the vector change in position from the initial position $\mathbf{x}_0$ within the same coordinate system. The unit for displacement is also meter.

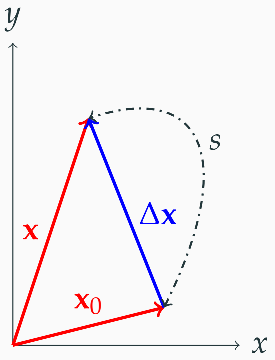
$$\Delta\mathbf{x}(t) = \mathbf{x} - \mathbf{x}_0 = (x - x_0)\hat{\boldsymbol{\imath}} + (y - y_0)\hat{\boldsymbol{\jmath}} + (z - z_0)\hat{\boldsymbol{k}}$$

- IJK notation makes vector addition and subtraction less prone to errors
- Since the reference point $\mathbf{x}_{\text{ref}} = \mathbf{0}$, the position vector $\mathbf{x}$ is also its displacement from the reference point

Distance

Distance $s(t)$ is a quantity that is *related* to displacement. It is:

- The length of the path taken by an object when it from $\mathbf{x}_0$ to $\mathbf{x}$
- A scalar quantity
- Always positive, i.e. $s \geq 0$
- Although the magnitude of the displacement vector is also a scalar, it is not necessarily the same as distance
- $s \geq |\Delta \mathbf{x}|$



Pay close attention to the difference between distance and displacement.

Average Velocity

Average velocity $\overline{\mathbf{v}}$ of an object is its displacement $\Delta\mathbf{x}$ over a *finite* time interval Δt. The unit for velocity is **meters per second** ($\mathrm{m/s}$):

$$\boxed{\overline{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta t}}$$

Since the $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ directions (x, y, z axes) are *linearly independent*[1], each component of average velocity can be calculated by separating each direction:

$$\boxed{\overline{\mathbf{v}} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} + \frac{\Delta z}{\Delta t}\hat{k}}$$

(Note: A bar is drawn over the symbol if it is averaged over time.)

[1]mathematical way of saying that what happens in one axis does not affect another

Instantaneous Velocity

If displacement $d\mathbf{x}$ is calculated a very small[2] time interval dt, then velocity is called the **instantaneous velocity**:

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t} \quad \rightarrow \quad \boxed{\mathbf{v} = \frac{d\mathbf{x}}{dt}}$$

The instantaneous velocity is the slope of the tangent on the position-time graph.

[2] In calculus, a very small change is called *infinitesimally small*

Instantaneous & Average Speed

Average speed is similar to average velocity: it is the distance s traveled over a finite time interval Δt. Since distance is always positive, so too is the average speed

$$\boxed{\overline{v} = \frac{s}{\Delta t}}$$

Likewise, when the time interval is made infinitesimally small, then the speed is called the **instantaneous speed** v. Instantaneous speed v is the magnitude of the instantaneous velocity vector.

Instantaneous & Average Acceleration

In the same way that velocity descrbes how quickly position changes with time, **average acceleration** $\bar{\mathbf{a}}$ is the change in velocity $\Delta\mathbf{v}$ over a finite time interval Δt.The unit for acceleration is $\mathrm{m/s^2}$.

$$\boxed{\bar{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{\mathbf{v} - \mathbf{v}_0}{\Delta t}}$$

Making the time interval Δt infinitesimally small gives the **instantaneous acceleration** $\mathbf{a}(t)$.

If You Are Curious

For the curious minds (i.e. we don't need it for AP Physics), the change in acceleration in time is called **jerk**, with a unit of $\mathrm{m/s^3}$:

$$\bar{\mathbf{j}} = \frac{\Delta \mathbf{a}}{\Delta t}$$

The change in jerk in time is called **jounce** or **snap**, with a unit of $\mathrm{m/s^4}$:

$$\bar{\mathbf{s}} = \frac{\Delta \mathbf{j}}{\Delta t}$$

The next motion quantities are are called **crackle** and **pop**, but these quantities are almost never used.

Acceleration as Functions of Velocity and Position

Sometimes, acceleration are expressed as a function of velocity or position rather than of time, depending on the forces acting on them. For example:

- Gravitational or electrostatic forces: $a(x) = Ax^{-2}$
- Spring force: $a(x) = -Bx$
- Damping force (e.g. shock absorbers): $a(v) = Cv$
- Aerodynamic drag: $a(v) = Dv^2$

In these cases, solving for the motion quantities $x(t)$, $v(t)$ and $a(t)$ requires calculus or numerical iterative methods.

Kinematic Equations

Kinematic Equations

Without calculus, kinematic problems in AP Physics 1 only deal with constant acceleration. Non-constant acceleration problems are studied in depth in Physics C. The 1D kinematic equations that will be used in Physics 1 are:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

- Initial position: x_0
- Position at time t: x
- Initial velocity: v_0
- Velocity at time t: v
- Acceleration (constant): a

These equations are sometimes called the “Big-five” or “Big-four” in Grade 11/12 Physics. In AP, you are given only 3 equations in your equation sheet.

Non-Constant Acceleration

What happens when acceleration is not constant? Our options are:

- Give up (bad idea!)
- Use calculus
- Use a numerical iterative method
- Use conservation of energy

Motion Graphs

Motion Graphs

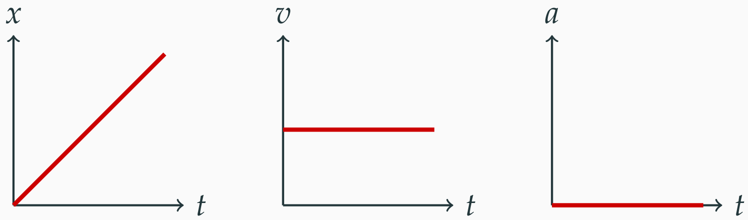
You should already be familiar with the *basic* motion graphs from Grade 11/12 Physics:

- Position vs. time (x-t) graph
- Velocity vs. time (v-t) graph
- Acceleration vs. time (a-t) graph

Note that these graphs are only valid for 1D motion. Think of them as the graphical representation of the kinematic equations from the previous slide

Uniform Motion

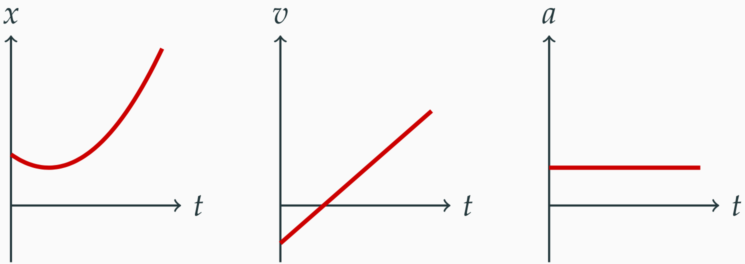
Uniform motion is when an object moves with constant velocity (neither magnitude nor direction changes) and therefore no acceleration. In 1D, the motion graphs are:



- Constant velocity has a straight line in the x-t graph
- The slope of the x-t graph is the velocity v, which is constant
- The slope of the v-t graph is the acceleration a, which is zero by definition

Uniform Acceleration

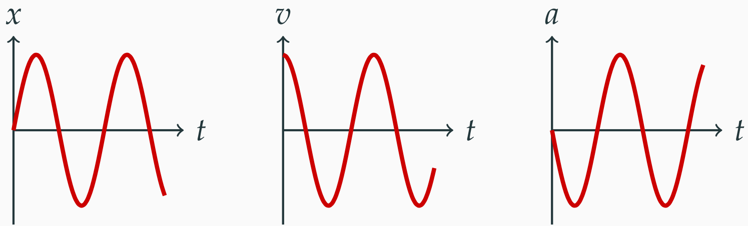
Uniform acceleration means that an object moves with a constant non-zero acceleration.



- The x-t graph is part of a *parabola*
  - If the parabola is *convex* (opens up), acceleration is $(+)$
  - If the parabola is *concave* (opens down), acceleration is $(-)$
- The v-t graph is a straight line; its slope (a constant) is the acceleration
- The a-t graph is a horizontal straight line

Simple Harmonic Motion

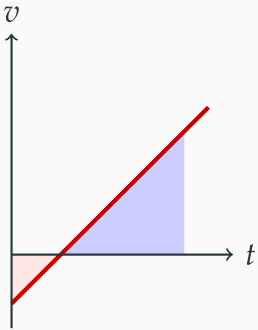
For **harmonic motions** (vibrations, oscillations), x, v and a are all non-constant, and they all change with time as sinusoidal functions.



Bottom line: regardless of the type motion,

- The v-t graph is the slope of the x-t graph
- The a-t graph is the slope of the v-t graph

Area Under Motion Graphs



The area under the v-t graph is the displacement Δx:

- Area *above* the time axis: $+$ displacement
- Area *below* the time axis: $-$ displacement

The area under the a-t graph is the change in velocity Δv:

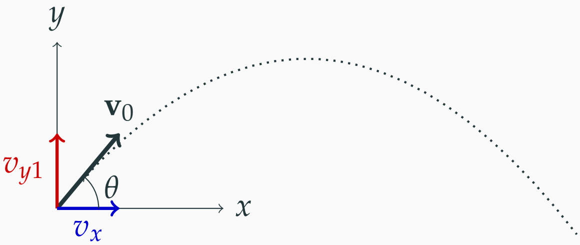
- Area *above* the time axis: $+$ change in velocity
- Area *below* the time axis: $-$ change in velocity

The area under the x-t graph has no physical meaning.

Projectile Motion

Projectile Motion

A **projectile** is an object that is launched with an initial velocity of $\mathbf{v}_0$ along a parabolic trajectory and accelerates only due to gravity.



- x-axis usually is the *horizontal* direction, with $+$ direction pointing *forward*
- y-axis usually is the *vertical* direction, with $+$ direction pointing *up*
- The reference point is usually where the projectile is launched
- Consistent with the right-handed Cartesian coordinate system

Horizontal (x) Direction

The initial velocity $\mathbf{v}_0$ can be decomposed into its x and y components using the launch angle θ:

$$\mathbf{v}_0 = v_x\hat{\boldsymbol{\imath}} + v_{y0}\hat{\boldsymbol{\jmath}} = \left[v_0 \cos\theta\right]\hat{\boldsymbol{\imath}} + \left[v_0 \sin\theta\right]\hat{\boldsymbol{\jmath}}$$

There is no horizontal acceleration (i.e. $a_x = 0$), therefore v_x is constant. The kinematic equations reduce to a single equation:

$$x = v_x t = \left[v_0 \cos\theta\right] t$$

where x is the horizontal position at time t

Vertical (y) Direction

There is constant vertical acceleration due to gravity alone, i.e. $a_y = -g$. (a_y is *negative* due to the way we defined the coordinate system.) The important equation is this one:

$$y = [v_0 \sin\theta]\, t - \frac{1}{2} g t^2$$

These two kinematic equations may also be useful:

$$\begin{aligned} v_y &= [v_0 \sin\theta] - gt \\ v_y^2 &= \left[v_0^2 \sin^2\theta\right] - 2gy \end{aligned}$$

Solving Projectile Motion Problems

Horizontal and vertical motions are independent of each other, but there are variables that are shared in both directions, namely:

- Time t
- Launch angle θ (above the horizontal)
- Initial speed v_0

When solving any projectile motion problems

- *Two* equations with *two* unknowns
- If an object lands on an incline, there will be a third equation relating x and y

Symmetric Trajectory

A projectile's trajectory is symmetric if the object lands at the same height as when it launched. The angle θ is measured above the the horizontal.

- Time of flight
$$t_{\max} = \frac{2v_0 \sin\theta}{g}$$
- Range
$$R = \frac{v_0^2 \sin(2\theta)}{g}$$
- Maximum height
$$y_{\max} = \frac{v_0^2 \sin^2\theta}{2g}$$

Maximum Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum range occurs at $\theta = 45^\circ$
- For a given initial speed v_0 and range R, launch angle θ is given by:

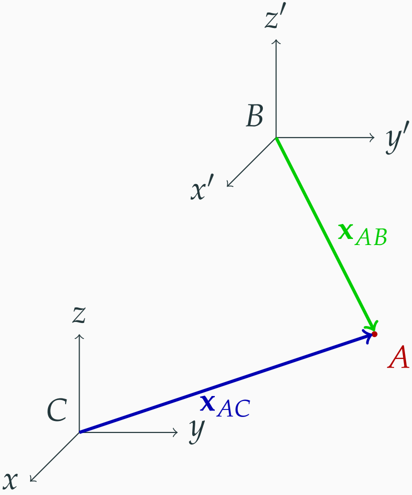
$$\theta_1 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

But there is another angle that *gives the same range*!

$$\theta_2 = 90^\circ - \theta_1$$

Relative Motion

Relative Motion



All motion quantities must be measured relative to a frame of reference

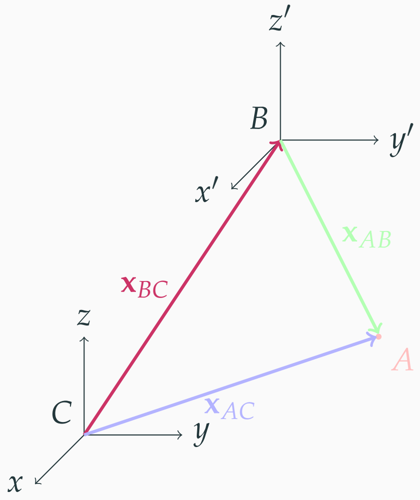
Two **frames of reference** (i.e. **coordinate systems**)

- C with axes x, y, z
- B with axes x', y', z'

The position of A can be described by $\mathbf{x}_{AC}$ (A relative to frame C) or $\mathbf{x}_{AB}$ (A relative to B)

- Obviously $\mathbf{x}_{AB}$ and $\mathbf{x}_{AC}$ are different
- If the object moves, then $\mathbf{x}_{AB}$ and $\mathbf{x}_{AC}$ are functions of time

Relative Motion



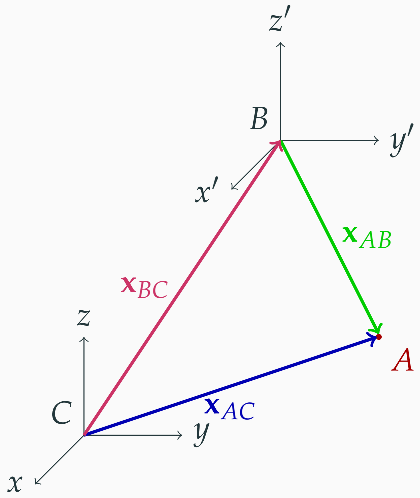
We can relate the positions of the origins of the two frames of reference by $\mathbf{x}_{BC}$ (origin of frame B relative to frame C)

- The vector pointing from the origin of frame C to the origin of frame B
- If the two frames are moving relative to each other, then $\mathbf{x}_{BC}$ is a function of time

Without needing elaborate specific vector notations, it should be obvious that

$$\mathbf{x}_{AC} = \mathbf{x}_{AB} + \mathbf{x}_{BC}$$

Relative Motion



Starting from the definition of **relative position**:

$$\mathbf{x}_{AC} = \mathbf{x}_{AB} + \mathbf{x}_{BC}$$

Using the definitions for velocity to get a similar equation for **relative velocity**:

$$\boxed{\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}}$$

Likewise, **relative acceleration** has a similar expression:

$$\mathbf{a}_{AC} = \mathbf{a}_{AB} + \mathbf{a}_{BC}$$

Relative Velocity Example

If an airplane (P) flies in windy air (A) we must consider the velocity of the airplane relative to air, i.e. $\mathbf{v}_{PA}$ and the velocity of the air relative to Earth E, i.e. $\mathbf{v}_{AE}$. The velocity of the airplane relative to Earth is therefore

$$\mathbf{v}_{PE} = \mathbf{v}_{PA} + \mathbf{v}_{AE}$$

Simple example: If an airplane is flying at a constant velocity of $253\,\mathrm{km/h}$ south relative to the air and the air velocity is $24\,\mathrm{km/h}$ east, what is the velocity of the airplane relative to Earth?

Relative Velocity

In classical mechanics, the equation for relative motion follows the **Galilean velocity addition rule**:

$$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$$

The velocity of A relative to reference frame C is the velocity of A relative to reference frame B, plus the velocity of B relative to C. If we add another reference frame D, the equation becomes:

$$\mathbf{v}_{AD} = \mathbf{v}_{AB} + \mathbf{v}_{BC} + \mathbf{v}_{CD}$$

Typical Problems

In the AP Physics 1 exam, questions involving kinematics usually appear in the multiple-choice section. The problems themselves are not necessarily very different from Grade 11/12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use $g = 10\,\mathrm{m/s^2}$ in your calculations to make your lives simpler
- A lot of problems are *symbolic*, which means that they deal with the algebraic expressions, not actual numbers
- Will be coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet