Student #: ___________________________ Student Name: _________________________________

AP PHYSICS C: HARMONIC MOTION

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

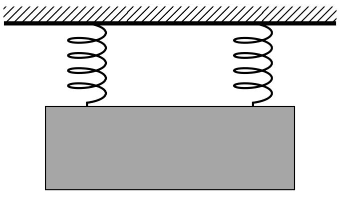
Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

1. A mass oscillates on the end of a spring that obeys Hooke's law. Which of the following statements is true?
   (A) The amplitude of oscillation is equal to the potential energy of the spring.
   (B) The kinetic energy of the oscillating mass is constant.
   (C) Maximum potential energy occurs when the mass reaches the equilibrium position.
   (D) The potential energy of the spring at the amplitude is equal to the kinetic energy at the equilibrium position.
   (E) The kinetic energy of the spring at the amplitude is equal to the potential energy at the equilibrium position.

2. A superball is dropped from a height of 5.0 m above a floor. The ball bounces off the floor in a perfectly elastic collision so that it rises to the same height with each bounce. The motion of the ball can be described as
   (A) harmonic motion with a period of 2 s
   (B) harmonic motion with a period of 1 s
   (C) harmonic motion with a period of 1/2 s
   (D) motion with a constant velocity
   (E) motion with a constant momentum

3. An object oscillates in simple harmonic motion along the x-axis according to the equation $x = 6\cos(4t)$. The period of oscillation of the object is
   (A) 1/4 s
   (B) 4 s
   (C) $\pi/4$ s
   (D) $\pi/2$ s
   (E) 4π s

4. A mass m oscillates on the end of a string of length L. The frequency of the pendulum is f. How would you increase the frequency of the pendulum to $2f$?



   (A) Increase the length of the pendulum to $4L$
   (B) Decrease the length of the pendulum to $L/4$
   (C) Increase the length of the pendulum to $2L$
   (D) Decrease the length of the pendulum to $L/2$
   (E) Decrease the mass of the pendulum to $m/2$

5. A mass hangs from two parallel springs, each with the same spring constant k. Compared to the period T of the same mass oscillating on one of the springs, the period of oscillation of the mass with both springs connected to it is
   (A) $T/4$
   (B) $T/\sqrt{2}$
   (C) T (unchanged)
   (D) $2T$
   (E) $4T$

6. Which of the following is generally true for an object in simple harmonic motion on a spring of constant k?
   (A) The greater the spring constant k, the greater the amplitude of the motion.
   (B) The greater the spring constant k, the greater the period of the motion.
   (C) The greater the spring constant k, the greater the frequency of the motion.
   (D) The lower the spring constant k, the greater the frequency of the motion.
   (E) The lower the spring constant k, the greater the kinetic energy of the motion.

Questions 7–9: A harmonic oscillator follows the equation $\frac{d^2x}{dt^2} = -4x$. The spring constant k is $4\,\mathrm{N/m}$.

7. The angular frequency ω of the harmonic motion is
   (A) zero
   (B) 2 rad/s
   (C) 4 rad/s
   (D) 8 rad/s
   (E) 16 rad/s

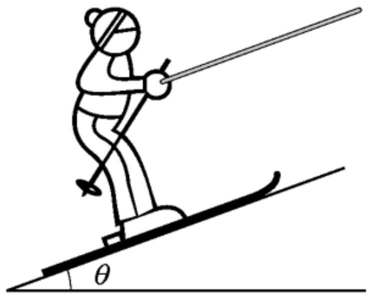
8. The mass m oscillating on the spring is
   (A) 1 kg
   (B) 2 kg
   (C) 4 kg
   (D) 8 kg
   (E) 16 kg

9. The period T of oscillation is
   (A) zero
   (B) $\pi/4$s
   (C) $\pi/2$s
   (D) π s
   (E) 2π s

10. A pendulum of length L has a period of 2 s on Earth. A planetary explorer takes the same pendulum of length L to another planet where its period is 1 s. The gravitational acceleration on the surface of this planet is most nearly
   (A) $8g$
   (B) $4g$
   (C) $2g$
   (D) $g/2$
   (E) $g/4$

AP PHYSICS C: SIMPLE HARMONIC MOTION
SECTION II
6 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.



1. A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a = a_{\max} \sin\left(\frac{\pi t}{T}\right) \qquad (0 < t < T)$$
$$= 0 \qquad (t \geq T)$$

where $a_{\max}$ and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

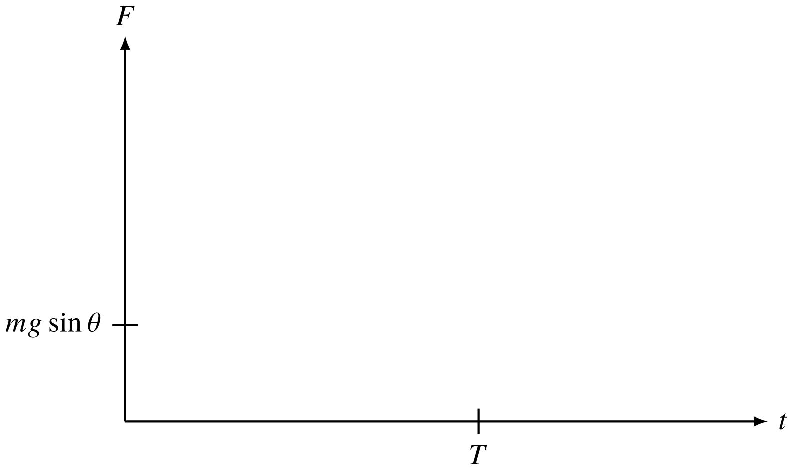
(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

(b) Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

(c) Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

(d) Derive an expression for the total impulse imparted to the skier during the acceleration.

(e) Suppose that the magnitude of the acceleration is instead modeled as $a = a_{\max} e^{-\pi t/2T}$ for all $t > 0$, where $a_{\max}$ and T are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model F_1 and the new model F_2.



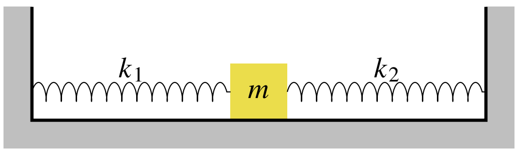
2. In heavy seas, the bow of a battle ship undergoes a simple harmonic vertical pitching motion with a period of 8.0 s and an amplitude of 2.0 m.

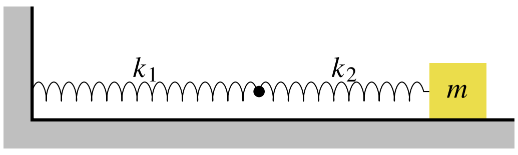
   (a) What is the maximum vertical velocity of the battle ship's bow?

   (b) What is its maximum acceleration?

   (c) An 80 kg sailor is standing on the scale in the bunk room in the bow. What are the maximum and minimum reading on the scale in newtons?

3. Show that for the situations in the figures below, the object of mass m oscillates with a frequency of $f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}}$ where k_{eff} is given by (a) $k_{\text{eff}} = k_1 + k_2$ and (b) $\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2}$. Hint: find the net force on the mass and write $F = -k_{\text{eff}}x$. Note that in (b), the springs stretch by different amounts, the sum of which is x.

(a) 

(b) 

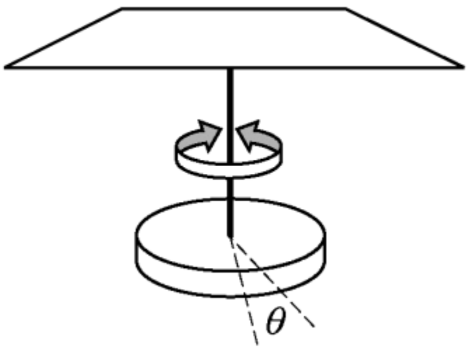
4. A simple pendulum of length L is released from rest from an angle of θ_0.

   (a) Assuming the motion of the pendulum to be simple harmonic motion, find its speed as it passes through $\theta = 0$.

   (b) Using the conservation of energy, find this speed exactly.

   (c) Show that your results for (a) and (b) are the same when θ_0 is small.

   (d) Find the difference in your results for $\theta_0 = 0.20\,\mathrm{rad}$ and $L = 1\,\mathrm{m}$.



5. The torsion pendulum shown above consists of a disk of rotational inertia I suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle θ, the twisted rod exerts a restoring torque t that is proportional to the angular displacement: $t = -\beta\theta$, where β is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.

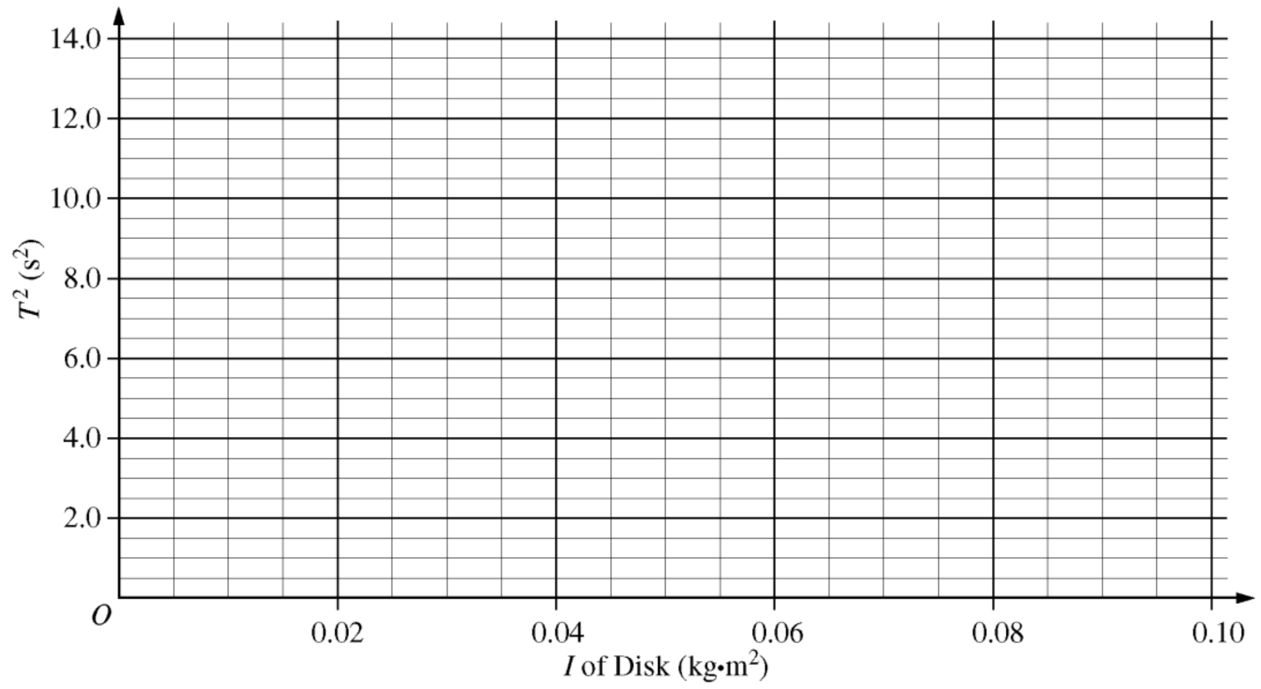
   (a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement θ of the torsion pendulum as a function of time t.

   (b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.

   To determine the torsion constant β of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

Rotational Inertia I of Disk ($kg \cdot m^2$)	Average Time for Ten Oscillations (s)	Period T (s)	T^2 (s^2)
0.025	22.4	2.24	5.0
0.036	26.8	2.68	7.2
0.049	29.5	2.95	8.7
0.064	33.3	3.33	11.1
0.081	35.9	3.59	12.9

(c) On the graph below, plot the data points. Draw a straight line that best represents the data.



(d) Determine the equation for your line.

(e) Calculate the torsion constant β of the rod from your line.

(f) What is the physical significance of the intercept of your line with the vertical axis?