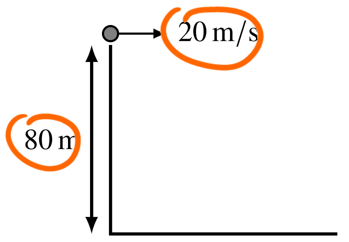
Student #: ____________________   Student Name: ____________________

TOPIC 1: KINEMATICS

Questions 1–2

A ball of mass 0.5 kg is launched horizontally from the top of a cliff 80 m high with a speed of 20 m/s at time $t = 0$.

$g = 10\,m/s^2$



$y = \frac{1}{2} g t^2$

$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(80)}{10}} = \sqrt{16} = 4s$

1. The horizontal distance x traveled by the ball before striking the ground is

   (A) 20 m (B) 40 m (C) 80 m (D) 160 m (E) 320 m

$x = vt = (20)(4) = 80m$

2. The speed of the ball just before striking the ground is

   ~~(A)~~ 4 m/s ~~(B)~~ 14 m/s ~~(C)~~ 20 m/s (D) 44 m/s ~~(E)~~ 64 m/s

20m/s

$v = \sqrt{20^2 + 40^2} =$

$v_y = gt = (10)(4) = 40\,m/s$

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3. A space explorer throws a tool downward on a planet with an initial velocity of 2 m/s from a height of 6 m above the surface. The tool strikes the surface in a time of 2 s. The acceleration due to gravity on the planet is

   (A) $1\,m/s^2$ (B) $2\,m/s^2$ (C) $3\,m/s^2$ (D) $4\,m/s^2$ (E) $10\,m/s^2$

(+)

$\Delta y = v_0 t \pm \frac{1}{2} g t^2$ $6 = (2)(2) + \frac{1}{2} g (2^2)$ $6 = 4 + 2g$ $g = 1\,m/s^2$

Questions 4–5

A sprinter starting from rest runs a 100 m race on a straight track. The sprinter covers the first 10 m with a constant acceleration in 2 seconds. The sprinter runs the remaining 90 m with the same velocity he had at the end of 2 s.

4. The sprinter's velocity at the end of the first 2 s is

   (A) 5 m/s (B) 10 m/s (C) 20 m/s (D) 40 m/s (E) 60 m/s

$d = \frac{v_0 + v}{2}(t)$

$d = \frac{v}{2} t$

$v = \frac{2d}{t} = \frac{(2)(10)}{2}$

$= 10\,m/s$

5. The total time it takes for the sprinter to run the full 100 m is

   (A) 2 s (B) 9 s (C) 10 s (D) 11 s (E) 12 s

$t_2 = \frac{90}{10} = 9\,s$

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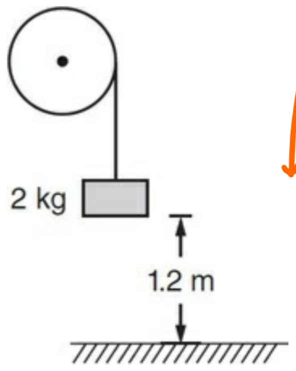
6. A block of mass 2 kg is attached to a string that is wrapped around a pulley of negligible mass and allowed to descend from rest a vertical distance of 1.2 m in a time of 1.5 s. The acceleration of the block is most nearly



(+)

(A) $0.2\,m/s^2$
(B) $0.6\,m/s^2$
(C) $1.1\,m/s^2$
(D) $1.4\,m/s^2$
(E) $1.5\,m/s^2$

$d = \frac{1}{2} a t^2$

$a = \frac{2d}{t^2} = \frac{(2)(1.2)}{1.5^2} = \frac{2.4}{2.25}$

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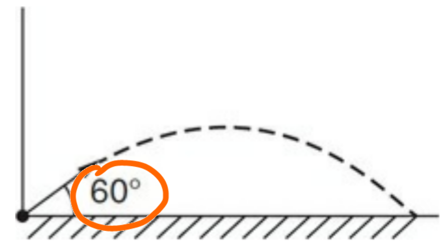
7. A golf ball is hit from level ground and has a horizontal range of 100 m. The ball leaves the golf club at an angle of 60° to the level ground. At what other angle(s) can the ball be struck at the same initial velocity and still have a range of 100 m?



(A) 30°
(B) 20° and 80°
(C) 10° and 120°
(D) 45° and 135°
(E) There is no other angle other than 60° in which the ball will have a range of 100 m.

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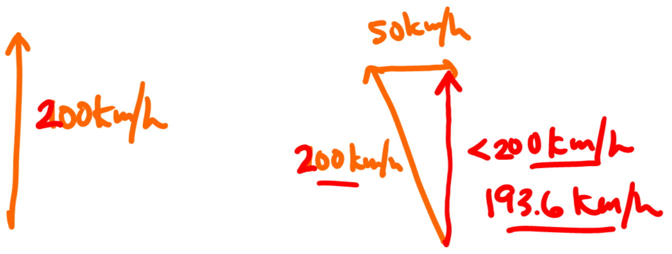
8. A small airplane can fly at 200 km/h with no wind. The pilot of the plane would like to fly to a destination 100 km due north of his present position, but there is a crosswind of 50 km/h east. How much time is required for the plane to fly north to its destination?

   ~~(A)~~ less than 1/2 h
   ~~(B)~~ 1/2 h
   (C) more than 1/2 h
   ~~(D)~~ 1 h
   ~~(E)~~ more than 1 h



Questions 9–10

A particle moves on a horizontal surface with a constant acceleration of 6 m/s^2 in the x-direction and 4 m/s^2 in the y-direction. The initial velocity of the particle is 3 m/s in the x-direction.

9. The speed of the particle after 4 s is

   (A) ~~16 m/s~~ (B) ~~27 m/s~~ (C) 31 m/s (D) ~~44 m/s~~ (E) ~~985 m/s~~

x-dir
$V_x = V_{xo} + at$
$= (3) + (6)(4)$
$= 27$ m/s

y-dir
$V_y = V_{yo} + a_y t$
$= 0 + (4)(4) = 16$ m/s

10. The displacement of the particle from its initial position is

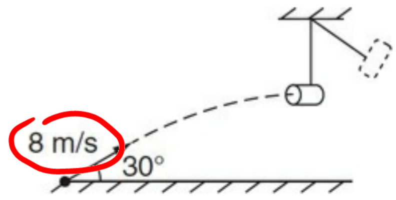
   (A) ~~16 m~~ (B) ~~32 m~~ (C) ~~60 m~~ (D) 68 m (E) ~~92 m~~

x-dir $x = \cancel{x_0} + V_0 t + \frac{1}{2}at^2 = (3)(4) + \frac{1}{2}(6)(4^2) = 12 + 48 = 60$ m

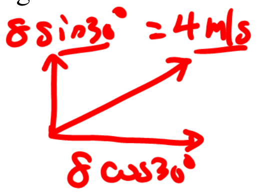
y-dir $y = \cancel{V_0 t} + \frac{1}{2}a_y t^2 = \frac{1}{2}(4)(4^2) = 32$

$\sqrt{60^2 + 32^2} = 68$ m

11. A small ball is launched with a speed of 8 m/s at an angle of 30° from the horizontal. A cup is hung so that it is in position to catch the ball when it reaches its maximum height. How far above the floor should the cup be hung to catch the ball?



(A) 2.4 m
(B) 1.6 m
(C) 1.0 m
(D) 0.8 m
(E) 0.4 m



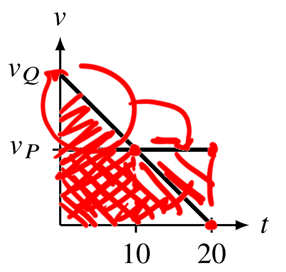
$\cancel{v^2} = v_0^2 + 2ay$

$y = \frac{-v_0^2}{2a} = \frac{-(4^2)}{(2)(10)} = \frac{-16}{20}$

$= 0.8$ m.

Questions 12–13

The graph shown below represents the velocity vs. time graphs for two cars, P and Q. Car P begins with a speed v_P, and Car Q begins with a speed v_Q which is twice the velocity of Car P, that is, $v_Q = 2v_P$.



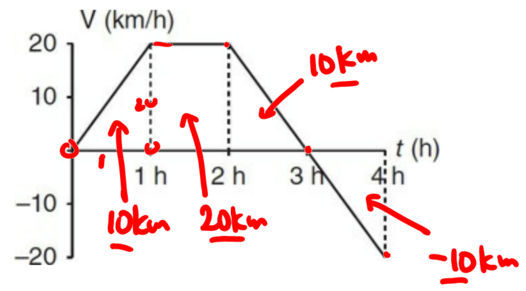
12. Which of the following is true at a time of 10 s?

   (A) The cars occupy the same position.
   (B) Car P is at rest.
   (C) $v_Q > v_P$
   (D) $v_P > v_Q$
   (E) Car Q is ahead of Car P.

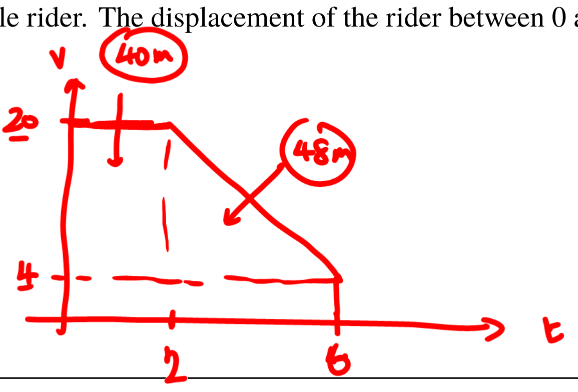
13. Which of the following is true at a time of 20 s?

   (A) The cars occupy the same position.
   (B) Car P is at rest.
   (C) $v_Q > v_P$
   (D) $a_P = a_Q$
   (E) Car P is ahead of Car Q.

14. The velocity vs. time graph below represents the motion of a bicycle rider. The displacement of the rider between 0 and 4 h is



(A) +10 km
(B) +20 km
(C) +30 km
(D) +40 km
(E) −10 km



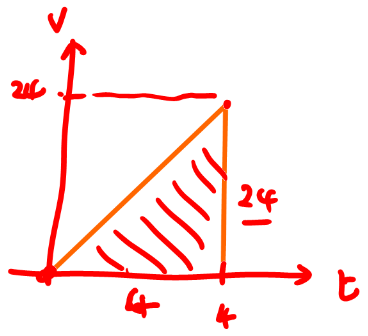
15. A car is initially moving with a positive velocity of 20 m/s when it passes the origin at time $t = 0$. The car continues to move at 20 m/s between $t = 0$ and $t = 2$ s. At $t = 2$ s, the driver presses the brake, giving the car an acceleration of -4 m/s^2. The displacement of the car at $t = 6$ s is

   (A) 40 m (B) 32 m (C) 48 m (D) 64 m (E) 88 m

16. A 600 kg car accelerates uniformly from rest. After 4 s, it reaches a speed of 24 m/s. During the 4 s, the car has traveled a distance of

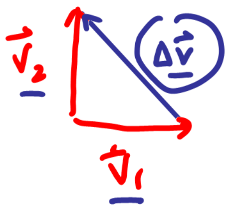
   (A) 12 m (B) 24 m (C) 36 m (D) 48 m (E) 96 m

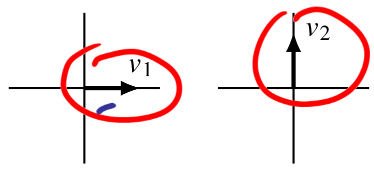
$d = \frac{V_0 + V}{2} t = \left(\frac{24}{2}\right)(4) = 48$ m



$\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$

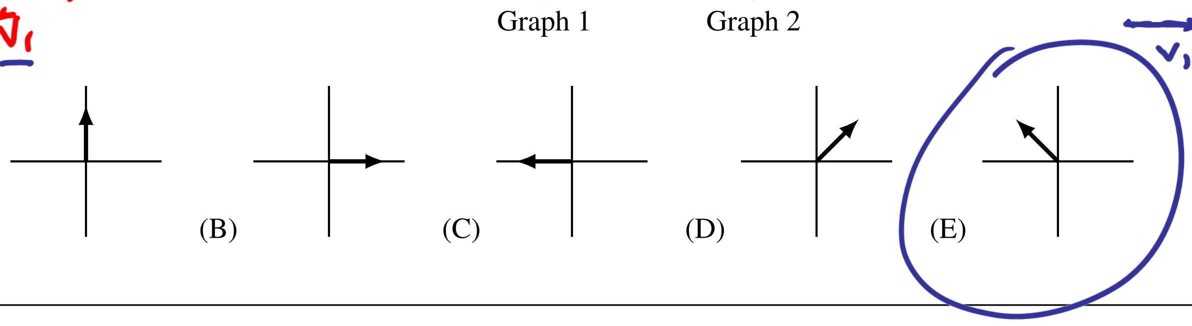
17. Two velocity vectors v_1 and v_2 each have a magnitude of 10 m/s. Graph 1 shows the velocity v_1 at $t = 0$ s, and then the same object has a velocity v_2 at $t = 2$ s, shown in Graph 2. Which of the following vectors represents the average acceleration vector that causes the object's velocity to change from v_1 to v_2 ?





Graph 1 Graph 2

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$



(A) (B) (C) (D) (E)

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18. A toy dart gun fires a dart at an angle of 45° to the horizontal and the dart reaches a maximum height of 1 meter. If the dart were fired straight up into the air along the vertical, the dart would reach a height of

(A) 1 m (B) 2 m (C) 3 m (D) 4 m (E) 5 m

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19. A ball is hit straight up into the air with an upward positive velocity. Wich of the following describes the velocity and acceleration of the ball at the instant it reaches the top of its flight?

	Velocity	Acceleration
(A)	0	0
(B)	0	g
(C)	$2v_0$	g
(D)	$\frac{1}{2}v_0$	0
(E)	0	$\frac{1}{2}g$

$v_i = \frac{v_0}{\sqrt{2}}$ v_0 45° $\frac{v_0}{\sqrt{2}}$

$v^2 = v_i^2 - 2gy$

$0 = \frac{v_0^2}{2} - 2gy$

$y = \frac{v_0^2}{4g}$ = v_0^2 $y = \frac{v_0^2}{2g}$

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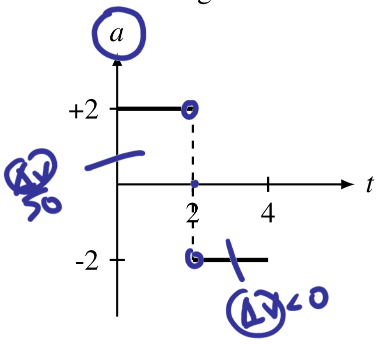
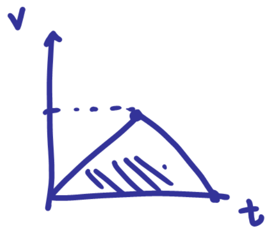
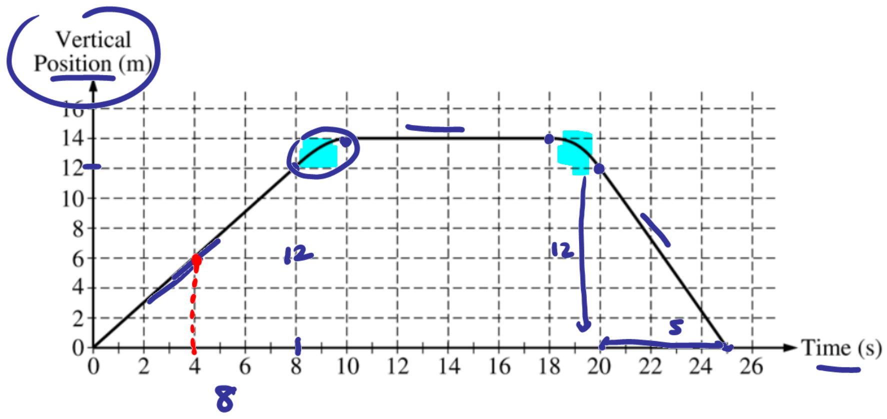
20. The motion of an object is represented by the acceleration vs. time graph below. The object begins from rest. Which of the following statements is true about the motion of the object?



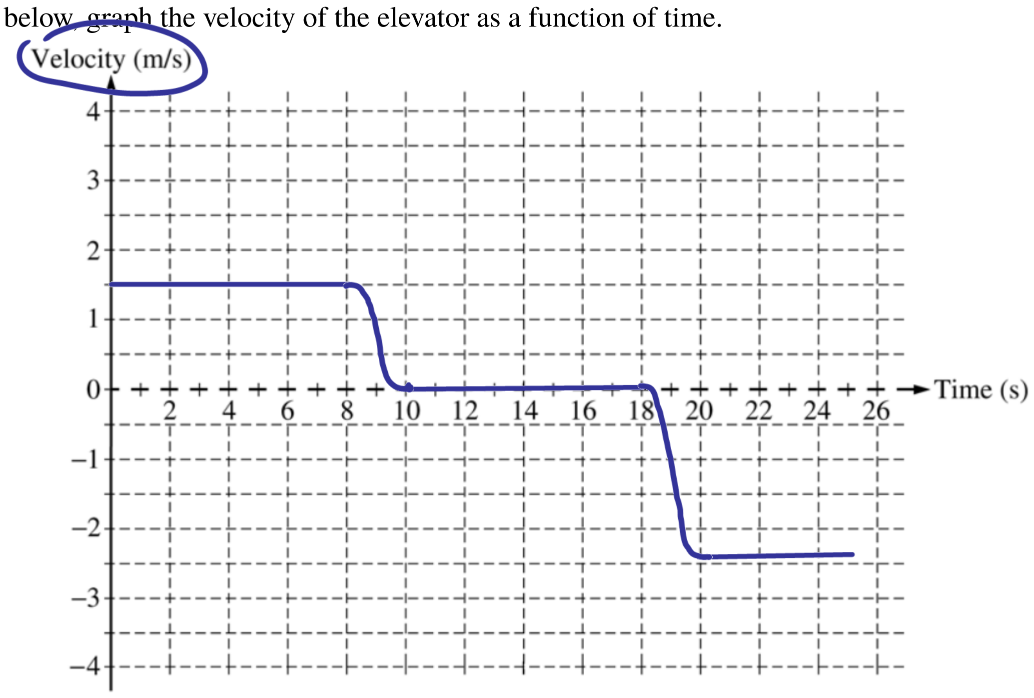
(A) The object returns to its original position.
(B) The velocity of the object is zero at a time of 2 s.
(C) The velocity of the object is zero at a time of 4 s.
(D) The displacement of the object is zero at a time of 4 s.
(E) The acceleration of the object is zero at a time of 2 s.





21. The vertical position of an elevator as a function of time is shown above.

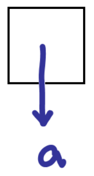
(a) On the grid below, graph the velocity of the elevator as a function of time.



(b) i. Calculate the average acceleration for the time period $t = 8$ s to $t = 10$ s.

ii. On the box below that represents the elevator, draw a vector to represent the direction of this average acceleration.

$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{\Delta t} = \frac{-1.5}{2} = \boxed{-0.75 \, m/s^2}$

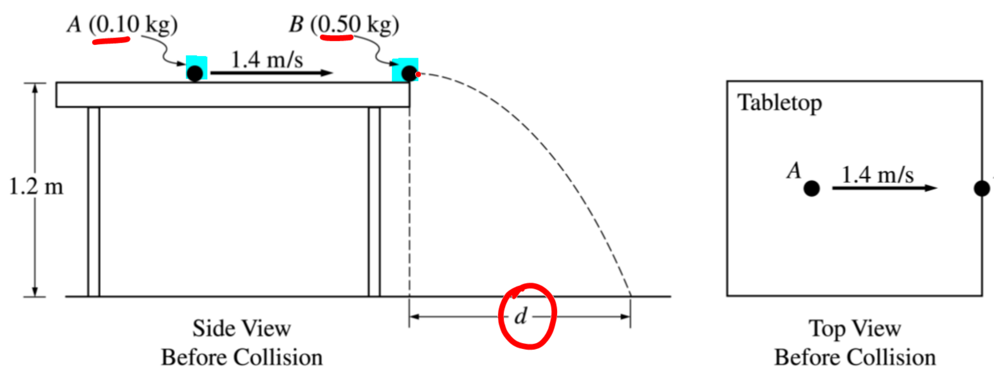


(c) Suppose that there is a passenger of mass 70 kg in the elevator. Calculate the apparent weight of the passenger at time $t = 4$ s.

at $t = 4$ s $a = 0$ ∴ apparent weight = actual weight

$F_g = mg = (70)(10) = \boxed{700 \, N}$

700 N [down]



Note: Figures not drawn to scale.

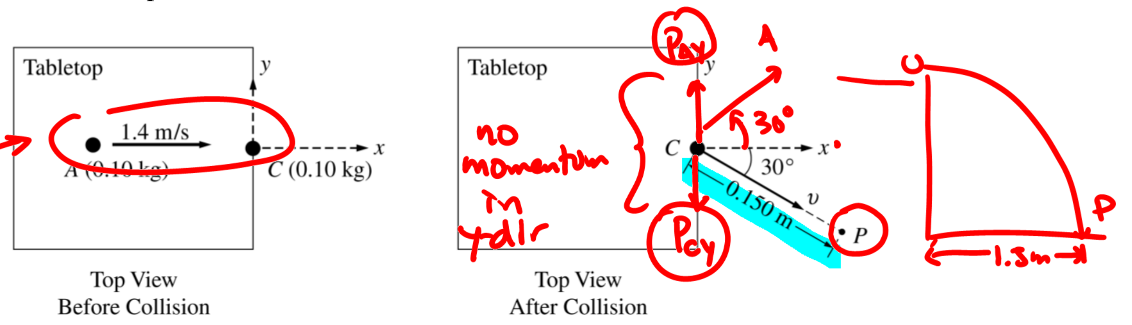
22. An incident ball A of mass 0.10 kg is sliding at 1.4 m/s on the horizontal tabletop of negligible friction shown above. It makes a head-on collision with a target ball B of mass 0.50 kg at rest at the edge of the table. As a result of the collision, the incident ball rebounds, sliding backwards at 0.70 m/s immediately after the collision.

(a) Calculate the speed of the 0.50 kg target ball immediately after the collision.

The tabletop is 1.20 m above a level, horizontal floor. The target ball is projected horizontally and initially strikes the floor at a horizontal displacement d from the point of collision.

(b) Calculate the horizontal displacement d.

no momentum in y dir



In another experiment on the same table, the target ball B is replaced by target ball C of mass 0.10 kg. The incident ball A again slides at 1.4 m/s, as shown above left, but this time makes a glancing collision with the target ball C that is at rest at the edge of the table. The target ball C strikes the floor at point P, which is at a horizontal displacement of 0.15 m from the point of the collision, and at a horizontal angle of 30° from the $+x$-axis, as shown above right.

(c) Calculate the speed v of the target ball C immediately after the collision.

(d) Calculate the y-component of incident ball A's momentum immediately after the collision.

(a) Calculate the speed of the 0.50 kg target ball immediately after the collision.

$P_A + P_B = P_A' + P_B'$

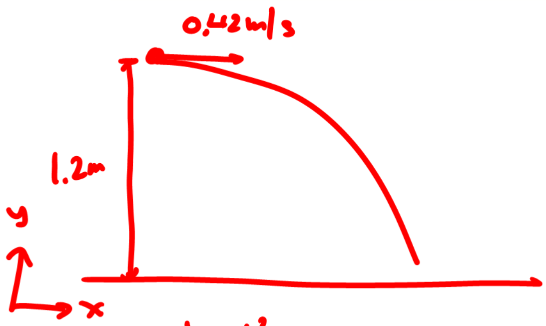
$m_A v_A = m_A v_A' + m_B v_B'$

$v_B' = \frac{m_A (v_A - v_A')}{m_B}$

$= \frac{0.10}{0.50}(1.4 - (-0.7))$

$v_B' = \frac{0.1}{0.5}(2.1) = \boxed{0.42\ m/s}$

(b) Calculate the horizontal displacement d.



y-dir $y = -\frac{1}{2} g t^2$

$+1.2 = \frac{+1}{2}(10) t^2$

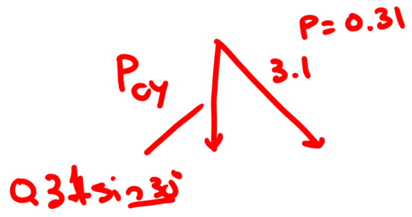
$t = \sqrt{\frac{(2)(1.2)}{10}} = 0.49 s$

$d = v_B' t = 2.057 = \boxed{2.1 m}$

(c) Calculate the speed v of the target ball C immediately after the collision.

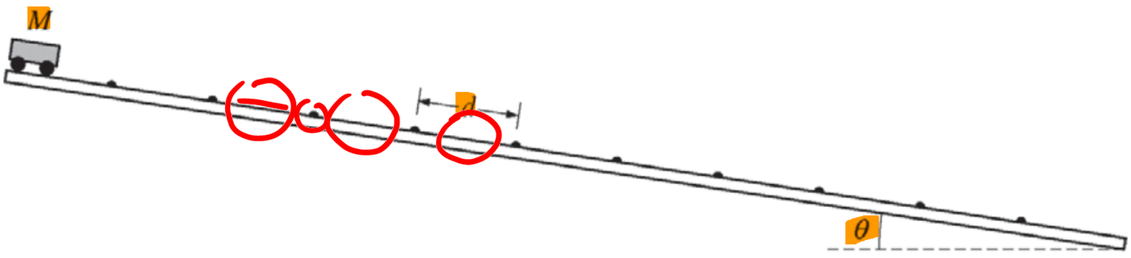
$t = 0.49 s$ ↑ to hit the ground

$v_c' = \frac{d}{t} = \frac{1.5}{0.49} = 3.062 = \boxed{3.1 m/s}$



$P_{cy} = -0.153$ kg m/s

$\boxed{P_{Ay} = 0.15\ kg\ m/s}$



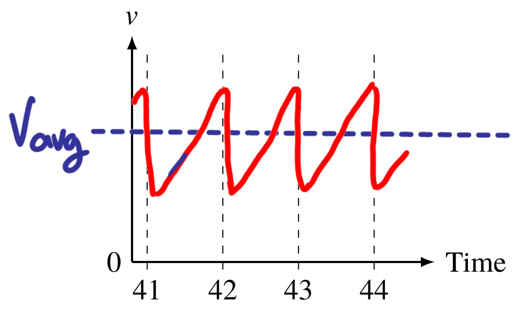
Note: Figure not drawn to scale.

23. (Suggested time 25 minutes) A long track, inclined at an angle θ to the horizontal, has small speed bumps on it. The bumps are evenly spaced a distance d apart, as shown in the figure above. The track is actually much longer than shown, with over 100 bumps. A cart of mass M is released from rest at the top of the track. A student notices that after reaching the 40th bump the cart's average speed between successive bumps no longer increases, reaching a maximum value v_{avg}. This means the time interval taken to move from one bump to the next bump becomes constant.

(a) Consider the cart's motion between bump 41 and bump 44.

i. In the figure below, sketch a graph of the cart's velocity v as a function of time from the moment it reaches bump 41 until the moment it reaches bump 44.

ii. Over the same time interval, draw a dashed horizontal line at $v = v_{avg}$. Label this line "v_{avg}".



(b) Suppose the distance between the bumps is increased but everything else stays the same. Is the maximum speed of the cart now greater than, less than, or the same as it was with the bumps closer together? Briefly explain your reasoning.

- more time to accelerate between bumps

_✓_ Greater than ___ Less than ___ Same as

- the bumps exert less impulse

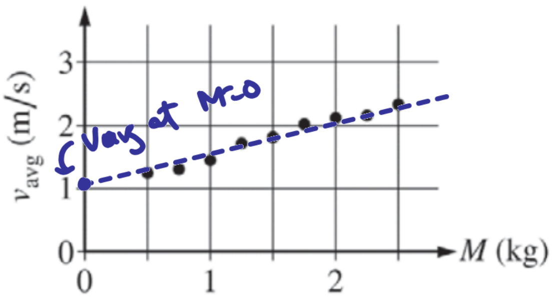
(c) With the bumps returned to the original spacing, the track is tilted to a greater ramp angle θ. Is the maximum speed of the cart greater than, less than, or the same as it was when the ramp angle was smaller? Briefly explain your reasoning.

higher accelration at higher angle

_✓_ Greater than ___ Less than ___ Same as

(d) Before deriving an equation for a quantity such as v_{avg}, it can be useful to come up with an equation that is intuitively expected to be true. That way, the derivation can be checked later to see if it makes sense physically. A student comes up with the following equation for the cart's maximum average speed: $v_{avg} = C\dfrac{Mg\sin\theta}{d}$, where C is a positive constant.

i. To test the equation, the student rolls a cart down the long track with speed bumps many times in front of a motion detector. The student varies the mass M of the cart with each trial but keeps everything else the same. The graph shown below is the student's plot of the data for v_{avg} as a function of M.



- equation predicts that at M=0, Vavg = 0.
- graph predicts a non-zero Vavg when M=0.

Are these data consistent with the student's equation? Briefly explain your reasoning.

___ Yes _✓_ No

ii. Another student suggests that whether or not the data above are consistent with the equation, the equation could be incorrect for other reasons. Does the equation make physical sense? Briefly explain your reasoning.

___ Yes _✓_ No

- Equation has d in denominator increasing d decreases Vavg
- however, we know that increasing d increases Vavg ∴
- ∴ equation is incorrect.

$\theta = 22 + 10 = 32^\circ$

24. A projectile is launched from point O at an angle of 22° with an initial velocity of 15 m/s up an incline plane that makes an angle of 10° with the horizontal. The projectile hits the incline plane at point M.

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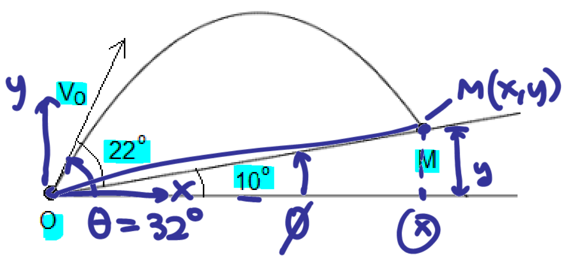


(a) Find the time it takes for the projectile to hit the incline plane.

(b) Find the distance OM.

$x = V_0 \cos\theta\, t \longrightarrow t = \dfrac{x}{V_0 \cos\theta}$

$y = V_0 \sin\theta\, t - \frac{1}{2} g t^2 \longrightarrow x \tan\phi = \cancel{V_0} \sin\theta \left(\dfrac{x}{\cancel{V_0}\cos\theta}\right) - \frac{1}{2} g \dfrac{x^2}{V_0^2 \cos^2\theta}$

$\tan\phi = \dfrac{y}{x} \longrightarrow y = x \tan\phi$

$\cancel{x} \tan\phi = \cancel{x} \tan\theta - \frac{1}{2} g \dfrac{x^{\cancel{2}}}{V_0^2 \cos^2\theta}$

$\frac{1}{2} g \dfrac{x}{V_0^2 \cos^2\theta} = \tan\theta - \tan\phi$

$x = \dfrac{2 V_0^2 \cos^2\theta (\tan\theta - \tan\phi)}{g}$

$x = 14.79748$

a) $t = \dfrac{x}{V_0 \cos\theta} = \dfrac{14.798}{(15)(\cos 32^\circ)} = 1.16$

$\boxed{t = 1.2\ \text{s}}$

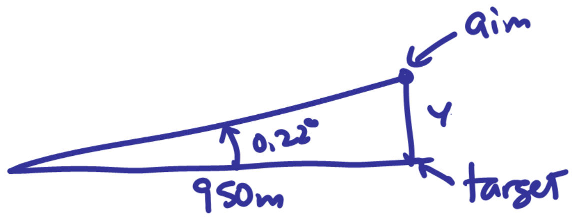
b) $OM = \dfrac{x}{\cos\phi} = \dfrac{14.798}{\cos 10} = \boxed{15\ \text{m}}$

25. A high-powered rifle shoots bullets that leave the muzzle at 1.1×10^3 m/s. If a bullet is to hit a target 950 m away at the level, the gun must be aimed at a point above the target. Neglecting air resistance, how far above the target is this point?

same height

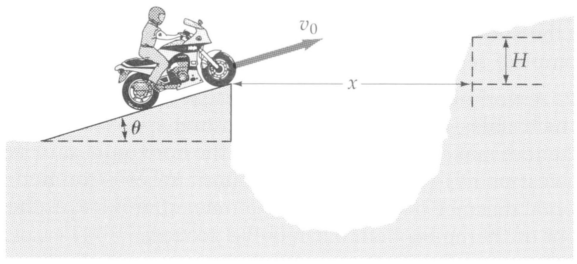
$$R = \frac{V_0^2 \sin(2\theta)}{g} \longrightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{V_0^2}\right)$$

$$\theta = \frac{1}{2}\sin^{-1}\left[\frac{(9.81)(950)}{(1.1\times10^3)^2}\right] = \underline{0.22^\circ}$$



$$\tan 0.22 = \frac{y}{950}$$

$$y = 950 \tan 0.22^\circ = \boxed{3.7\,m}$$



26. A trail bike take off from a ramp with velocity $\boldsymbol{v}_0$ at angle θ to clear a ditch of width x and land on the other side, which is elevated at a height H.

    (a) For a given angle θ and distance x, what is the upper limit for H such that the bike has an chance of making the jump?

    (b) For H less than this upper limit, what is the minimum take-off speed v_0 necessary for a successful jump?

    Neglect the size of the trail bike, and assume that covering a horizontal distance x and a vertical distance H is sufficient to clear the ditch.