Topic 18: Special Relativity, Part 1

AP and IBHL Physics

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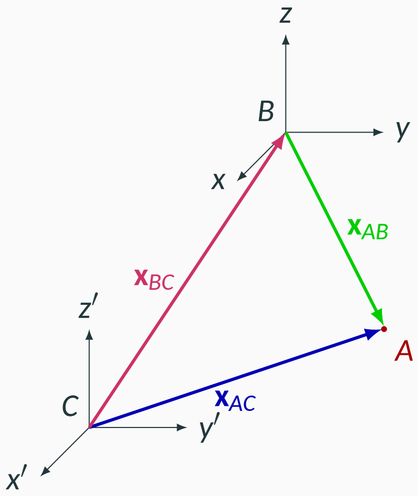
Olympiads School

Reference Frame

Relative Motion in Classical Physics



In the first topic (kinematics) we studied the definitions **relative position**:

$$\boxed{\mathbf{x}_{AC} = \mathbf{x}_{AB} + \mathbf{x}_{BC}}$$

relative velocity:

$$\boxed{\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}}$$

and **relative acceleration**:

$$\boxed{\mathbf{a}_{AC} = \mathbf{a}_{AB} + \mathbf{a}_{BC}}$$

Relative Motion

All motion quantities must be measured relative to a frame of reference

- A frame of reference is the coordinate system from which all physical measurements are made.
- Because all motions are relative, there is no absolute motion/rest

Frame of Reference

Think of a **frame of reference** (or just “frame”) as a hypothetical mobile “laboratory” an observer uses to make measurements (e.g. mass, lengths, time). At a minimum, it includes:

- Some rulers (i.e. coordinate system) to measure positions and lengths
- A clock to measure the passage of time
- A scale to compare forces
- A balance to measure masses

Frame of Reference

- We assume that the hypothetical laboratory is *perfect*—all the hypothetical "instruments" have zero errors
- What matters is the *motion* (at rest, uniform motion, acceleration etc) of your laboratory, and how it affects the measurement that you make
- "From the point of view of..."

Inertial Frame of Reference

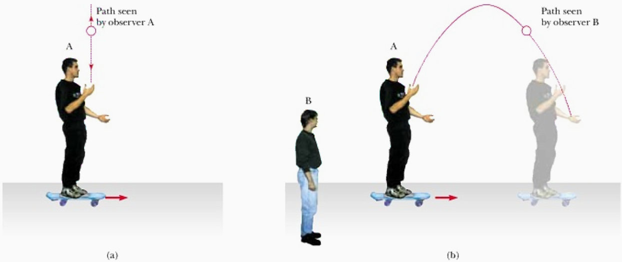
An **inertial frame of reference** (or a **rest frame**) is one that is moving in uniform motion

- In an inertial frame, Newton's first and second laws of motion are valid
- Since all uniform motion are treated the same way, you may consider any inertial frames of reference to be *at rest*

> **The Principle of Relativity:** All laws of motion must apply equally in all inertial frames of reference.

Inertial Frame of Reference

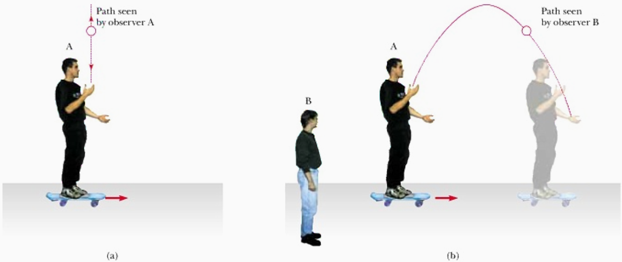
Observer A moves uniformly with the skateboard, while Observer B stands on the side of the road. So, when A tosses a ball upward:



- A sees only vertical motion, while
- B sees the ball traveling in a parabolic curve, although
- A & B observe different motion, but they agree on the *equations* that govern the motion

Inertial Frame of Reference

Observer A sees the same motion (only vertical motion) regardless of whether he is moving uniformly w.r.t. B or not (as long as *neither* are accelerating)



- Valid for A to conclude that he is at rest, but that B and the rest of the world are moving
- Likewise, it is also valid for B to think that he is at rest, but it is only A and his skateboard that are moving

Newtonian (Classical) Relativity

When studying kinematics and dynamics, we made some untested assumptions that seemed obvious: space and time are *absolute*

- 1 m is 1 m no matter where you are, or how you are moving
- 1 s is 1 s no matter where you are, or how you are moving
- Measurements of space and time do not depend on motion

If space and time are absolute, then *all* velocities are relative to the observer

- Measured velocities depend on the motion of the observer
- Galilean velocity addition rule:

$$\boxed{\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}}$$

Maxwell's Equations

Maxwell's equations on electrodynamics in a vacuum:

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Disturbances in **E** and **B** travel as an *electromagnetic wave* with a speed c:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299\,792\,458\,\mathrm{m/s}$$

Peculiar features of Maxwell’s equation

- Does not mention the *medium* in which EM waves travels
- When applying *Galilean transformation* (from which we obtain the velocity addition rule) to Maxwell’s equations, asymmetry is introduced
- Gauss’s law for magnetism break down: magnetic field lines appear to have beginnings/ends
- In *some* inertial frames of reference, Maxwell’s equations are simple and elegant, but transform the equation into another inertial frame, the equations are ugly and complex!
- Physicists at the time began to theorize that (perhaps) there is an actual *preferred* inertial frame of references
- This seems to violate the *principle of relativity*

The Illusive Ether

Maxwell's hypothesis: the speed of light c_0 is relative to a hypothetical subtance called **luminiferous aether** (or just **ether**) that permeates the universe. Ether must have some fantastic properties:

- All space is filled with ether
- Massless
- Zero viscosity
- Non-dispersive
- Incompressible
- Continuous at a very small (sub-atomic) scale

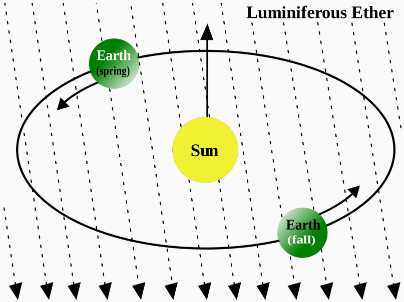
It was thought that the preferred inertial reference frame is that of the ether

Spoiler Alert

Spoiler alert: Ether doesn’t exist.

The Michelson-Morley Experiment

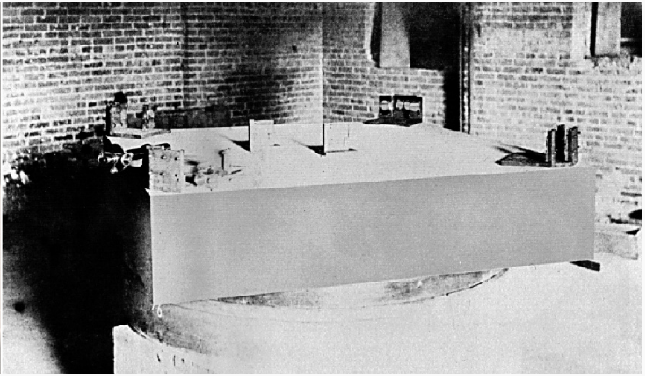
If ether exists, then at different times of the year, the Earth will have a different relative velocity with respect to it:



And it causes light to speed up or slow down. By measuring and comparing the speed of light at various times of the year, we should be able to determine to flow of ether relative to Earth.

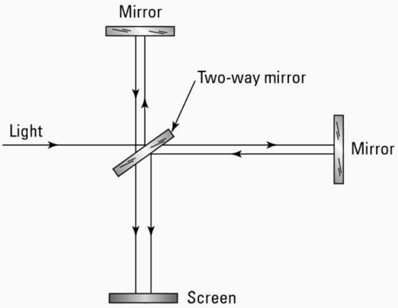
Michelson-Morley Experiment

American phycisists Albert Michelson[1] and Edward Morley designed an ingenious but very difficult experiment to detect ether using an **interferometer** designed by Michelson



[1] Nobel Prize in Physics, 1907

The Michelson Interferometer



- A beam of light is split into two using a two-way (half-silvered) mirror
- The two beams are reflected off mirrors and finally arriving at the screen where interference patterns are observed
- The two paths are the same length, so if the *speed* of the light changes, we should see an interference pattern
- **Except none were ever found!** The interference patterns that could be observed were well within experimental errors, and far below expected values

What To Do with “Null Result”

The Michelson-Morley experiment failed to detect the flow of ether, even after many refinements. What does this mean?

- Majority view
    - **The experiment was flawed!** It is actually a reasonable guess, since the experiment is known to be a difficult one, errors can be introduced
    - Keep improving the experiment (or design a better experiment) and Earth’s motion relative to ether will eventually be found
- Minority view:
    - **The ether hypothesis is wrong!**
    - The experiment showed it for what it is: ether either cannot be detected or it doesn’t exist
- A few physicists: The must be **another explanation** that saves both experiment and theory

Hendrik Lorentz

Dutch physicist Hendrik Antoon Lorentz[2] was one of the first to consider the findings of Michelson-Morley experiment to be significant

- Lorentz's hypothesis: objects traveling in the direction of ether must contract in length, nullifying the experimental results:

$$\boxed{\beta = \sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

- But *No known physical phenomenon* causes an object to contract

[2] 1853–1928, Nobel Prize in Physics, 1902

Strange Behavior in Absolute Space Time

French mathematician Henri Poincaré also hypothesized that ether affects the flow of time the direction of motion. His equation is similar to the hypothesis by Lorentz and contains the same factor:

$$\boxed{t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

But *no known physical phenomenon* can alter the flow of time either!

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Both Poincaré and Lorentz depended their hypothesis on

- Absolute time and space
- Existence of ether

Making Maxwell's Equations Work

Albert Einstein
in 1905

- In 1905, at the age of 26, Albert Einstein was working as a patent clerk in Bern, Switzerland while completing his Ph.D.
  - Believed in the principle of relativity, and therefore
  - Rejected the concept of a "preferred" frame of reference
- The failure of the Michelson-Morley experiment to find the flow of ether proves that it does not exist
- In order to make Maxwell's equations to work again, Einstein revisited two most fundamental concepts in physics: *space* and *time*

Special Relativity

Published in *Annalen der Physik* on September 26, 1905 in the article *On the Electrodynamics of Moving Bodies*

- Submitted on June 30, 1905 and passed for publication by a referee
- Einstein's third paper (of four) that year
- Mentions only five other scientists by name: Issac Newton, James Clerk Maxwell, Heinrich Hertz, Christian Doppler and Hendrik Lorentz, but does not contain references to any publications
- Ignored by most physicists at first, until Max Planck took interests
- Called "special relativity" because it describes a "special case" without effects of gravity

Postulates of Special Relativity

The Principle of Relativity: *All* laws of physics must apply equally in *all* inertial frames of reference.

- Reaffirms the principle in which physics is based on
- Extends the principle to include electrodynamics

The Principle of Invariant Light Speed: As measured in any inertial frame of reference, light always propagates in a vaccum with a definite velocity c, independent of the state of motion of the emitting body.

- Reaffirms the results from Michelson-Morley experiment
- Disproves the existence of ether

The two postulates are unremarkable by themselves, but when combined, the consequences are profound

What's so Special About Special Relativity?

Classical (Newtonian) relativity:

- Space and time are absolute (invariant), therefore
- The speed of light must be relative to the observer

Einstein's special relativity:

- The speed of light is absolute (invariant), therefore
- Space and time must be relative to the observer

We must modify our traditional concepts:

- Measurement of *space*
- Measurement of *time*
- Concept of *simultaneity*