Student #: ______________________________ Student Name: ______________________________

AP PHYSICS C: CIRCULAR MOTION

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\text{m/s}^2$ in all problems.

1. A girl stands on a rotating merry-go-round without holding on to a rail. The force that keeps her moving in a circle is the

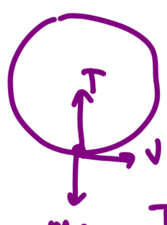
   (A) frictional force on the girl directed away from the center of the merry-go-round
   (B) frictional force on the girl directed toward the center of the merry-go-round
   (C) normal force on the girl directed away from the center of the merry-go-round
   (D) normal force on the girl directed toward the center of the merry-go-round
   (E) weight of the girl

B

2. A ball of mass m is swung in a vertical circle of radius R. The speed of the ball at the bottom of the circle is v. The tension in the string at the bottom of the circle is

   (A) mg
   (B) $mg + \frac{mv^2}{R}$
   (C) $mg - \frac{mv^2}{R}$
   (D) $\frac{mv^2}{R}$
   (E) 0

B

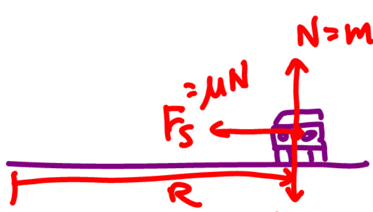


$T - mg = \frac{mv^2}{R}$

3. A car of mass m drives on a flat circular track of radius R. To maintain a constant speed v on the track, the coefficient of friction μ between the tires and the road must be

   (A) mg
   (B) $mg + \frac{mv^2}{R}$
   (C) $mg - \frac{mv^2}{R}$
   (D) $\frac{v^2}{gR}$
   (E) $\sqrt{\frac{v^2}{gR}}$

D



$F_s = \mu mg \rightarrow F_c = ma_c$

$\mu \cancel{m} g = \cancel{m}\frac{v^2}{R}$

$\mu = \frac{v^2}{gR}$

Questions 4–5

4. A ball on the end of a string is swung in a circle of radius 2 m according to the equation $\theta = 4t^2 + 3t$, where θ is in radians and t is in seconds. The angular acceleration of the ball is

   (A) $6\,\text{rad/s}^2$
   (B) $4t^2 + 3t\,\text{rad/s}^2$
   (C) $8t + 3\,\text{rad/s}^2$
   (D) $\frac{3}{4}t^3 + 3t^2\,\text{rad/s}^2$
   (E) $8\,\text{rad/s}^2$

E

$\omega = \dot{\theta} = 8t + 3$

$\alpha = \dot{\omega} = 8$

5. The linear speed v of the ball at $t = 3$ s is

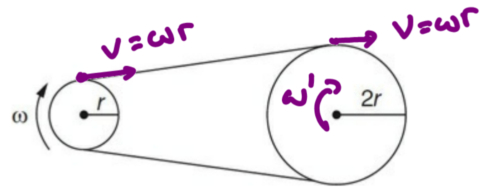
   (A) 27 m/s
   (B) 54 m/s
   (C) 108 m/s
   (D) 135 m/s
   (E) 210 m/s

B

$\omega(3) = 8(3) + 3 = 27$

$v(3) = r\omega = (2)(27) = 54$

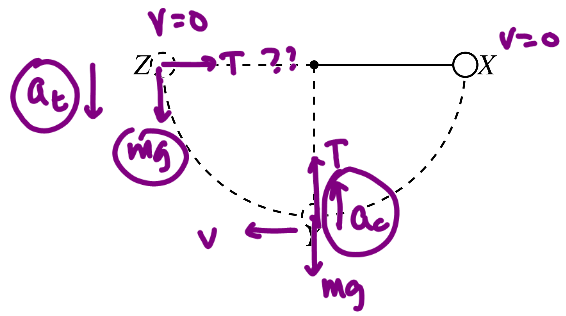
6. A belt is wrapped around two wheels as shown. The smaller wheel has a radius r, and the larger wheel has a radius $2r$. When the wheels turn, the belt does not slip on the wheels, and gives the smaller wheel an angular speed ω. The angular speed of the larger wheel is



$\omega' = \frac{v}{2r} = \frac{\omega \cancel{r}}{2\cancel{r}} = \frac{\omega}{2}$

   (A) $\frac{1}{4}\omega$
   (B) $\frac{1}{2}\omega$
   (C) ω
   (D) 2ω
   (E) 4ω

B



7. A ball on the end of a string is released from rest at point X as shown. The ball swings under the influence of gravity from point X through points Y and Z. What are the directions of the acceleration vectors at points Y and Z, respectively?

B

(A) Point Y: $\uparrow$ Point Z: $\leftarrow$
(B) Point Y: $\uparrow$ Point Z: $\downarrow$
(C) Point Y: $\nwarrow$ Point Z: $\searrow$
(D) Point Y: $\searrow$ Point Z: $\leftarrow$
(E) Point Y: $\leftarrow$ Point Z: $\rightarrow$

8. A merry-go-round is initially at rest, and begins to rotate with a constant angular acceleration α. The angular speed ω of the merry-go-round after making two complete revolutions is

$\Delta\theta = 4\pi$

E

(A) 2α
(B) 4α
(C) $\sqrt{2\pi\alpha}$
(D) $4\pi\alpha$
(E) $\sqrt{8\pi\alpha}$

$\omega^2 = \cancel{\omega_0^2} + 2\alpha\,\Delta\theta$

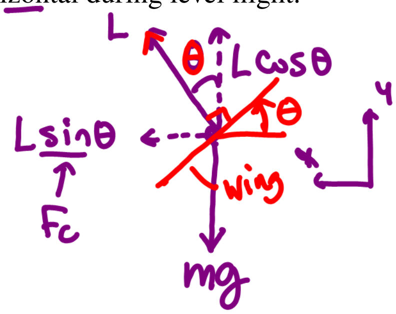
$\omega = \sqrt{2\alpha(4\pi)} = \sqrt{8\pi\alpha}$

9. An airplane travelling at 300 m/s banks its wings to enter into a horizontal circular turn. The circular path has a radius of 2.7 km. Which of these values *best* represents the angle of the wings relative to the horizontal if the airplane experiences no change in altitude during the turn? Assume that the wings are completely horizontal during level flight.

r
v

E

(A) 8°
(B) 16°
(C) 20°
(D) 25°
(E) 74°



no change in altitude $= \Sigma F_y = 0$

$F_c = ma_c$

$L\sin\theta = m\frac{v^2}{r}$

$L\cos\theta = mg$

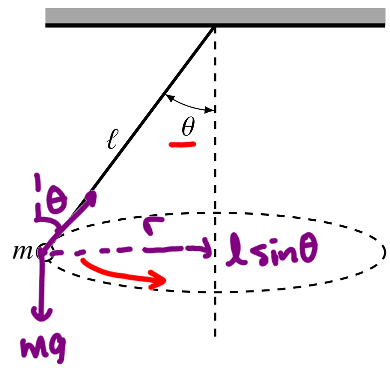
$L = \frac{mg}{\cos\theta}$

$\cancel{m}g\tan\theta = \frac{\cancel{m}v^2}{r}$

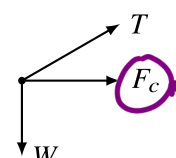
$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right) = 73.6°$

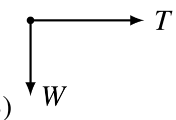
Questions 10–12

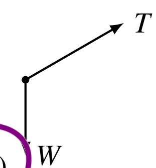
A conical pendulum consists of a ball attached to a string that moves in a horizontal circle, as shown below.

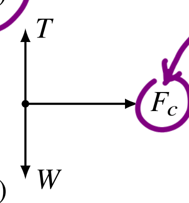


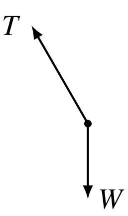
10. Which of the following diagrams best shows the forces acting on the ball as it moves in a circle?

(A) 

(B) 

(C) 

(D) 

(E) 

Centripetal force (Fc) should never appear on a FBD because it is a net force.

11. In terms of m, ℓ and θ and universal constants, the magnitude of the tension T is

D

(A) mg
(B) $mg\cos\theta$
(C) $mg\sin\theta$
(D) $\frac{mg}{\cos\theta}$
(E) $\frac{mg}{\sin\theta}$

no vertical motion

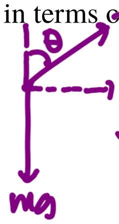
$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$

12. Which of the following expressions correct gives the speed at which the ball is travelling in terms of θ, ℓ and universal constants?

E

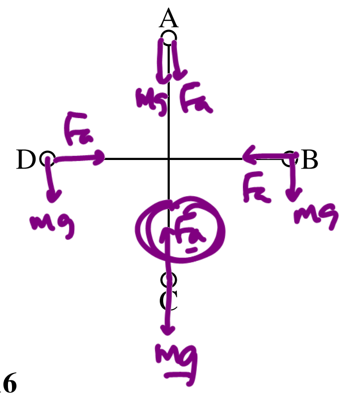
(A) $\sqrt{\ell g}$
(B) $\sqrt{\ell g \sin\theta}$
(C) $\sqrt{\ell g \cos\theta}$
(D) $\sqrt{\ell g \tan\theta}$
(E) $\sqrt{\ell g \sin\theta \tan\theta}$



$T\sin\theta = \frac{mv^2}{r}$

$mg\tan\theta = \frac{mv^2}{\ell\sin\theta}$

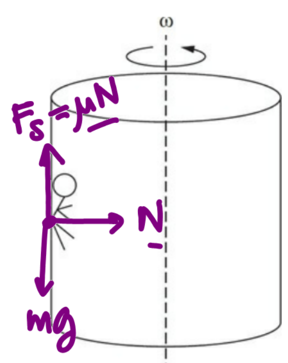
$v = \sqrt{g\ell\sin\theta\tan\theta}$



13. A typical curve on a high-speed racetrack has a radius of 200 m and is banked at 31° above the horizontal. Assuming that a race car has a mass of 1500 kg and that the coefficient of friction between the car's tires and the track is 0.90 what is the maximum speed at which a race car can travel through the curve and not slide toward the top of the track?

r
μ
θ

(A) 80 m/s
(B) 87 m/s
(C) 100 m/s
(D) 120 m/s
(E) 210 m/s



14. An amusement park ride consists of a cylindrical room that spins so that people leaning up against the wall can stick to the wall even if the floor is lowered out from under them. The rotating room reaches a maximum angular speed ω, the floor is lowered, and the riders stick to the wall. Which of the following statements is true?

(A) The weight of each rider provides the centripetal force keeping the rider moving in a circle.
(B) The normal force applied by the wall must equal the weight of the rider.
(C) The difference between the normal force applied by the wall and the weight of the rider is equal to the centripetal force acting on the rider.
(D) The frictional force between the wall and the rider must equal the weight of the rider.
(E) The frictional force between the wall and the rider provides the centripetal force acting on the rider.

Questions 15–16

Two balls of equal mass are attached to each end of a rod that is spinning about its center in the vertical plane with a constant angular speed ω. Each ball is a radius r from the center of the rod. A bug holds on to one of the balls as the system rotates. Four points, A, B, C, and D, are marked at the quarter circle points on the circle.

15. At which point would the bug need to apply the most adhesive force to remain on the ball?

C

(A) A
(B) B
(C) C
(D) D
(E) The bug would apply the same force at all points to remain on the ball.

16. The minimum force necessary the bug would have to apply to remain on the ball at point C is

E

(A) $m\omega r$
(B) $m\omega^2 r$
(C) mg
(D) $m\omega^2 r - mg$
(E) $m\omega^2 r + mg$

$F_c = ma_c$

$F_a - mg = m\omega^2 r$

$F_a = m\omega^2 r + mg$

$\Sigma F_y = 0$ (no vertical motion)

$N\cos\theta - \underbrace{\mu N}_{f_s}\sin\theta - mg = 0$

$N(\cos\theta - \mu\sin\theta) = mg$

$\Sigma F_x = F_c = N\sin\theta + \mu N\cos\theta = \frac{mv^2}{r}$

$N(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r}$

$\frac{N(\sin\theta + \mu\cos\theta)}{N(\cos\theta - \mu\sin\theta)} = \frac{\frac{mv^2}{r}}{mg}$

$v = \sqrt{rg\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}} = \_\_\_\_\_\_ \text{ m/s}$

AP PHYSICS C: CIRCULAR MOTION
SECTION II
3 Questions

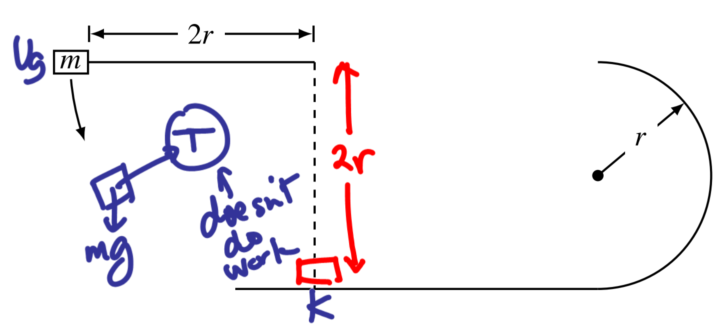
Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. A mass m attached to a string of length $2r$ swings, starting at rest when the string is horizontal, until the string is vertical. At the instant the string is vertical, the mass makes contact with the horizontal surface, the string is cut, and the mass continues along a frictionless track as shown below.

a) $\underset{U_g}{\cancel{m}g(2r)} = \underset{K}{\frac{1}{2}\cancel{m}v^2}$

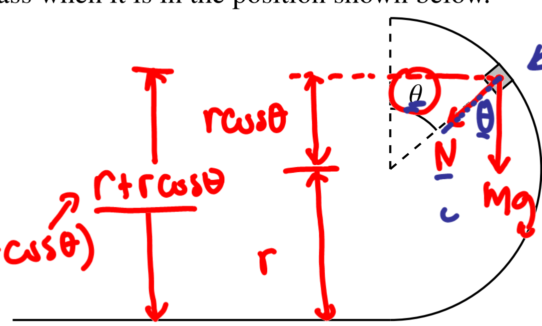
$v^2 = 4gr$

$\boxed{v = 2\sqrt{gr} \text{ or } \sqrt{4gr}}$



(a) What is the speed of the mass attached to the string the instant the string is cut?

(b) Sketch the forces acting on the mass when it is in the position shown below.



$h = r(1+\cos\theta)$

N does not do any work as the block goes up (only force doing work is gravity)

When the mass is in the position shown above,

(c) Find the object's speed as a function of θ

(d) Find the object's centripetal acceleration as a function of θ

(e) Determine at what angle θ the mass will fall of the track

N=0 just before it falls off.

c) $\underbrace{\cancel{m}gh + \frac{1}{2}\cancel{m}v^2}_{\text{at } \theta} = \underbrace{\frac{1}{2}\cancel{m}v_b^2}_{\text{at bottom}}$

$2gh + v^2 = v_b^2$

$2gr(1+\cos\theta) + v^2 = 4gr$ ← from part (a).

$2gr + 2gr\cos\theta + v^2 = \overset{2}{\cancel{4}}gr$

$v^2 = 2gr - 2gr\cos\theta$

$v^2 = 2gr(1-\cos\theta)$

$\boxed{v = \sqrt{2gr(1-\cos\theta)}}$

d) $a_c = \frac{v^2}{r} = \frac{2g\cancel{r}(1-\cos\theta)}{\cancel{r}}$

$a_c = 2g(1-\cos\theta)$

e) $F_c = ma_c$

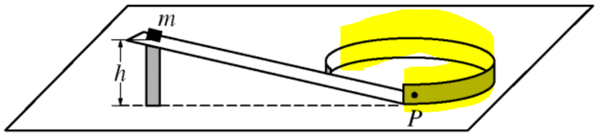
just before it falls off

$\cancel{N} + mg\cos\theta = m(2g(1-\cos\theta))$

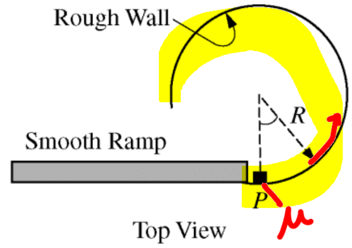
$mg\cos\theta = 2mg(1-\cos\theta)$

$mg\cos\theta = 2mg - 2mg\cos\theta$

$3\cancel{mg}\cos\theta = 2\cancel{mg} \rightarrow \cos\theta = \frac{2}{3} \rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right) = \boxed{48.2°}$



Side View



Top View

2. A small block of mass m starts from rest at the top of a frictionless ramp, which is at a height h above a horizontal tabletop, as shown in the side view above. The block slides down the smooth ramp and reaches point P with a speed v_0. After the block reaches point P at the bottom of the ramp, it slides on the tabletop guided by a circular vertical wall with radius R, as shown in the top view. The tabletop has negligible friction, and the coefficient of kinetic friction between the block and the circular wall is μ.

   (a) Derive an expression for the height of the ramp h. Express your answer in terms of v_0, m, and fundamental constants, as appropriate.

   $U_g = K \rightarrow \cancel{m}gh = \frac{1}{2}\cancel{m}v_0^2 \rightarrow \boxed{h = \frac{v_0^2}{2g}}$

A short time after passing point P, the block is in contact with the wall and moves with a speed of v.

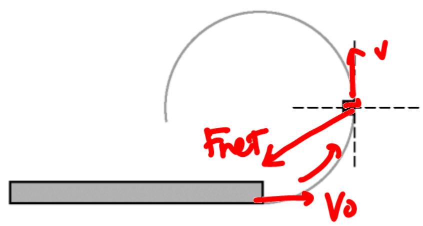
   (b) i. Is the vertical component of the net force on the block upward, downward, or zero?

   ____ Upward ____ Downward _✓_ Zero

   Justify your answer.

   There is no vertical motion after reaching P.

   ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.



Top View

$\frac{-1}{v} = \frac{\mu}{R}t - \frac{1}{v_0} \rightarrow \underline{v(t) = ?}$

Express your answers to the following in terms of v_0, v, m, R, μ, and fundamental constants, as appropriate.

   (c) Determine an expression for the magnitude of the normal force N exerted on the block by the circular wall as a function of v.

   (d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of v.

   (e) Derive an expression for $v(t)$, the speed of the block as a function of time t after passing point P on the track.

(c) normal force provides centripetal force

$\boxed{N = \frac{mv^2}{R}}$

as the block slows down, N decreases

d) tangential acceleration provided by kinetic friction

$F_k = ma_t$

$\mu N = ma_t$

$\frac{\mu \cancel{m} v^2}{R} = \cancel{m}a_t$

$\boxed{a_t = \frac{\mu v^2}{R}}$

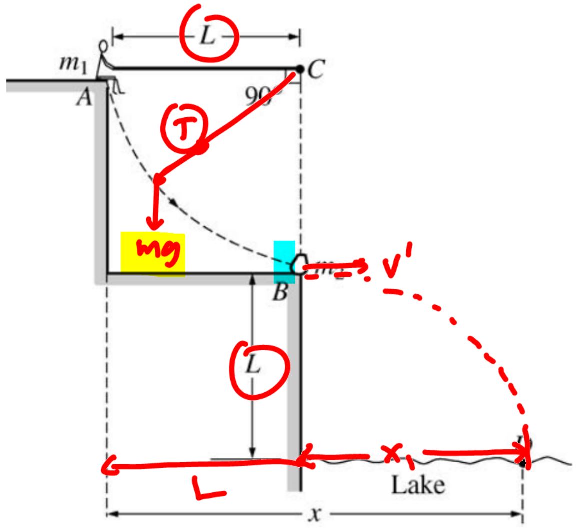
$t = 0$
$v = v_0$

e) $\overbrace{\frac{dv}{dt}}^{a_t} = \frac{\mu v^2}{R}$

$\int \frac{dv}{v^2} = \frac{\mu}{R}\int dt$

$\frac{-1}{v} = \frac{\mu}{R}t + C$

$\frac{-1}{v_0} = C$



b) T ; $m_1 g$; $\rightarrow v^2 = 2gL$

a) $m_1 g L = \frac{1}{2} m_1 v^2$

$v^2 = 2gL$

$v = \sqrt{2gL}$

$F_c = m_1 a_c$

$\rightarrow T - m_1 g = \frac{m_1 v^2}{L}$

$= \frac{m_1}{\cancel{L}} 2g\cancel{L}$

$T - m_1 g = 2 m_1 g$

$\boxed{T = 3 m_1 g}$

3. A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown above. The person then drops off the ledge and swings down on the rope toward position B on a lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and fundamental constants. ← g

(a) The speed of the person just before the collision with the object ← at B

(b) The tension in the rope just before the collision with the object

(c) The speed of the person and object just after the collision ← conservation of momentum

(d) The ratio of the kinetic energy of the person-object system before the collision to the kinetic energy after the collision

(e) The total horizontal displacement x of the person from position A until the person and object land in the water at point D.

c) $m_1 \rightarrow v$ $\quad m_2$ $\qquad m_1 m_2 \rightarrow v' = ??$

$m_1 v = (m_1 + m_2) v'$ →

↑ from (a)

$v' = \left(\frac{m_1}{m_1+m_2}\right) v = \boxed{\left[\frac{m_1}{m_1+m_2}\right]\sqrt{2gL}}$

$v'^2 = \left(\frac{m_1}{m_1+m_2}\right)^2 2gL$

d) $\frac{K_i}{K_f} = \frac{m_1 g L}{\frac{1}{2}(m_1+m_2) v'^2}$ ← U_g

$\frac{K_i}{K_f} = \frac{\cancel{m_1 g L}}{\cancel{\frac{1}{2}}\cancel{(m_1+m_2)}\left(\frac{m_1^{\cancel{2}}}{(m_1+m_2)^{\cancel{2}}}\right)\cancel{2gL}} = \boxed{\frac{m_1+m_2}{m_1}}$

e) projectile motion

vertical direction

$y = \frac{1}{2} g t^2 \rightarrow L = \frac{1}{2} g t^2$

$t = \sqrt{\frac{2L}{g}}$ ← time to fall into water.

$x_1 = v' t = \left[\frac{m_1}{m_1+m_2}\right]\sqrt{\cancel{2}\cancel{g}L}\sqrt{\frac{\cancel{2}L}{\cancel{g}}}$

$x_1 = \frac{2m_1}{m_1+m_2} L$

$x = x_1 + L = \boxed{\left(\frac{2m_1}{m_1+m_2} + 1\right) L}$