WELCOME TO AP PHYSICS

Pre-requisites

- **Physics 11 and 12** You will need to be comfortable with the topics covered in high-school level physics courses.
- **Calculus** The AP Physics C exams are calculus based, and you will be required to perform basic differentiation and integration. You don't need to be an expert, but basic knowledge is required. Differentiation and integration in the course are generally not difficult, but there are occasional challenges.
- **Vectors** You need to be comfortable with vector operations, including addition and subtraction, multiplication and division by constants, as well as dot products and cross products.

The AP Physics Exams

There are 4 AP Physics exams:

- Physics 1
- Physics 2
- Physics C–Mechanics
- Physics C–Electricity and Magnetism

Offered in first or second week of May of each year. The Physics C exams are calculus based; Physics 1 and 2 exams are algebra based.

Classroom Rules

Same as in Physics 11 and 12

- Treat each other with respect
- Raise your hands if you have a question. Don't wait too long
- E-mail me at `tleung@olympiadsmail.ca` for any questions related to physics and math and engineering
- Do ***not*** try to find me on social media

Topic 1: Introduction & Kinematics

Advanced Placement Physics C

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Olympiads School

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Files for You to Download

- `PhysAP-courseOutline.pdf`—The course outline
- `PhysAP-equationSheet.pdf`—An equation sheet that you use during the exams
- `PhysAP-01-kinematics.pdf`—These presentation slides
- `PhysAP-01-Homework.pdf`—Homework problems for Topic 1.

Please download/print the PDF file for the class slides before each class. There is no point copying notes that are already on the slides. Instead, focus on things that aren't necessarily on the slides. If you wish to print the slides, we recommend printing 4 slides per page.

Kinematics

Kinematics is a discipline with in mechanics for describing the motion of points, bodies (objects), and systems of bodies (groups of objects).

- Relationship between
  - Position
  - Displacement
  - Distance
  - Velocity
  - Speed
  - Acceleration
- Kinematics does not deal with what causes motion

Position

Position is a vector describing the location of an object in a coordinate system (usually *Cartesian*; can also be *polar*, *cylindrical* or *spherical*). The origin of the coordinate system is called “reference point”.

$$\mathbf{x}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}$$

- The SI unit for position is a **meter**, m
- The components x, y and z are the coordinates along those axes
- The vector is a function of time t

Displacement

Displacement is the change in position from 1 to 2 within the same coordinate system:

$$\Delta\mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1 = (x_2 - x_1)\hat{\boldsymbol{\imath}} + (y_2 - y_1)\hat{\boldsymbol{\jmath}} + (z_2 - z_1)\hat{\boldsymbol{k}}$$

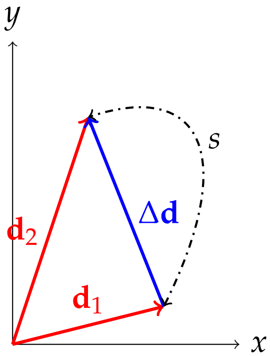
- IJK notation makes vector addition and subtraction less prone to errors
- Since reference point $\mathbf{x}_{\mathrm{ref}} = \mathbf{0}$, the position $\mathbf{x}$ is also its displacement from the reference point

Distance

Similar to Displacement

Distance s is a quantity that is *related* to displacement.

- The length of the path taken when an object moves from $\mathbf{d}_1$ to $\mathbf{d}_2$
- A scalar quantity
- Always positive, i.e. $s \geq 0$
- Although the magnitude of the displacement vector is also a scalar, it is not necessarily the same as distance
- In general, $s \geq |\Delta \mathbf{d}|$



Instantaneous Velocity

Time Derivative of Position

If position $\mathbf{x}$ is a differentiable function in time t, then velocity $\mathbf{v}$ can be found at any time t. The **instantaneous velocity** of an object is the rate of change of its position vector w.r.t. time:

$$\boxed{\mathbf{v}(t) = \frac{d\mathbf{x}(t)}{dt}}$$

Since $\mathbf{x}$ has x, y and z components in the $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ directions that are linearly independent, we can take the derivative w.r.t. time in every component:

$$\mathbf{v}(t) = \frac{d\mathbf{x}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{\imath}} + \frac{dy}{dt}\hat{\boldsymbol{\jmath}} + \frac{dz}{dt}\hat{\boldsymbol{k}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\hat{\boldsymbol{k}}$$

Integrating Velocity to Get Position/Displacement

If instantaneous velocity $\mathbf{v}$ is the rate of change of position $\mathbf{x}$ w.r.t. time t, then $\mathbf{x}$ is the time integral of $\mathbf{v}$:

$$\mathbf{x}(t) = \int \mathbf{v}(t)dt + \mathbf{x}_0$$

The constant of integration $\mathbf{x}_0 = \mathbf{x}(0)$ is the *initial position* at $t = 0$. As both $\mathbf{x}$ and $\mathbf{v}$ are vectors, we integrate each component to get $\mathbf{x}$:

$$\mathbf{x}(t) = \left(\int v_x \hat{\boldsymbol{\imath}} + \int v_y \hat{\boldsymbol{\jmath}} + \int v_z \hat{\boldsymbol{k}} \right) dt + \mathbf{x}_0$$

Average Velocity

The **average velocity** of an object is the change in position $\Delta\mathbf{x}$ over a finite time interval Δt:

$$\boxed{\bar{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta t}}$$

Like instantaneous velocity, we can find the x, y and z components of average velocity by separating components in each direction:

$$\bar{\mathbf{v}} = \frac{\Delta x}{\Delta t}\hat{\boldsymbol{\imath}} + \frac{\Delta y}{\Delta t}\hat{\boldsymbol{\jmath}} + \frac{\Delta z}{\Delta t}\hat{\boldsymbol{k}}$$

Instantaneous Speed

Intantaneous speed the rate of change of *distance* w.r.t. time:

$$\boxed{v = \frac{ds}{dt}}$$

- Since distance of any path must always be positive $s > 0$, instantaneous speed must also be positive
- Instantaneous speed v is the magnitude of the instantaneous velocity vector $\mathbf{v}$

Average Speed

Likewise, **average speed** is similar to average velocity: it is the distance travelled over a finite time interval.

$$\boxed{\overline{v} = \frac{s}{\Delta t}}$$

Path

Sometimes instead of explicitly describing the position $x = x(t)$ and $y = y(t)$, the path of an object can be given in terms of x coordinate $y = y(x)$, while giving the x (or y) coordinate as a function of time.

- In this case, substitute the expression for $x(t)$ into $y = y(x)$ to get an expression of $y = y(t)$
- Take derivative using chain rule to get $v_y = v_y(t)$

Instantaneous Acceleration

In the same way that velocity is the rate of change in position w.r.t. time, **acceleration** is the rate of change in velocity w.r.t. time:

$$\boxed{\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = \frac{d^2\mathbf{x}(t)}{dt^2}}$$

Acceleration is the second derivative of position, i.e.

1. Take derivative of $\mathbf{x}(t)$ to get $\mathbf{v}(t) = \mathbf{x}'(t)$
2. Take derivative again of $\mathbf{v}(t)$ to get $\mathbf{a}(t) = \mathbf{v}'(t)$

Special Notation When Differentiating With Time

Physicists and engineers use a special notation when the derivative is taken w.r.t. *time*, by writing a dot above the variable:

- Velocity:

$$\boxed{\mathbf{v}(t) = \dot{\mathbf{x}}}$$

- Acceleration:

$$\boxed{\mathbf{a}(t) = \dot{\mathbf{v}} = \ddot{\mathbf{x}}}$$

We will use this notation whenever it is convenient

Integrating Acceleration to Get Velocity

Velocity $\mathbf{v}(t)$ is the time integral of acceleration $\mathbf{a}(t)$:

$$\mathbf{v}(t) = \int \mathbf{a}(t)dt + \mathbf{v}_0$$

Again, since both $\mathbf{v}$ and $\mathbf{a}$ are vectors, we need to integrate in each direction:

$$\mathbf{v}(t) = \left(\int a_x \hat{\boldsymbol{\imath}} + \int a_y \hat{\boldsymbol{\jmath}} + \int a_z \hat{\mathbf{k}} \right) dt + \mathbf{v}_0$$

For Those Who Are Curious

The time derivative of acceleration is called **jerk**, with a unit of $\mathrm{m/s^3}$:

$$\mathbf{j} = \frac{d\mathbf{a}}{dt} = \frac{d^2\mathbf{v}}{dt^2} = \frac{d^3\mathbf{x}}{dt^3}$$

The time derivative of jerk is **jounce**, or **snap**, with a unit of $\mathrm{m/s^4}$:

$$\mathbf{s} = \frac{d\mathbf{j}}{dt} = \frac{d^2\mathbf{a}}{dt^2} = \frac{d^3\mathbf{v}}{dt^3} = \frac{d^4\mathbf{x}}{dt^4}$$

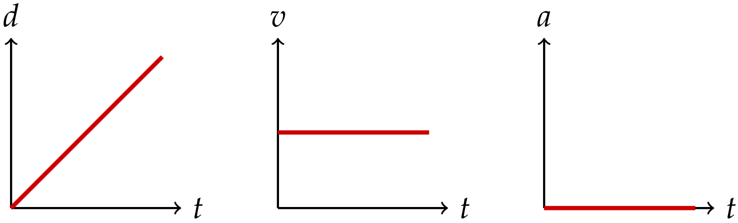
The next two derivatives of snap is facetiously called **crackle** and **pop**, but these higher derivatives of position vector are rarely seen. We will not be using these in AP Physics.

Motion Graphs

For 1D motion, we can describe motion graphically using motion graphs, by plotting

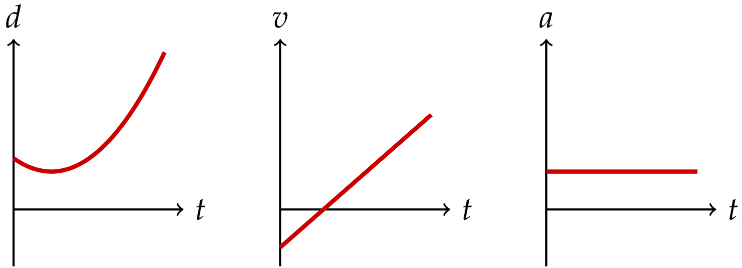
- Position vs. time $(x - t)$ graph
- Velocity vs. time $(v - t)$ graph
- Acceleration vs. time $(a - t)$ graph

Uniform Motion



- Constant velocity has a straight line in the $d - t$ graph
- The slope of the $d - t$ graph is the velocity v
- The slope of the $v - t$ graph is the acceleration a, which is zero in this case

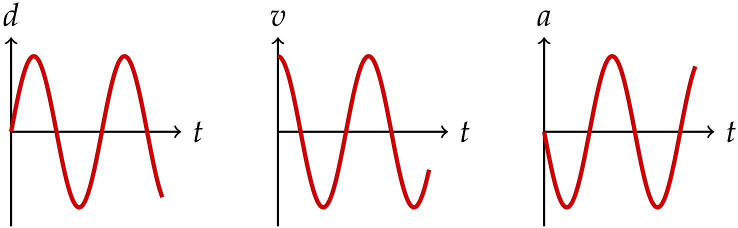
Uniform Acceleration



- The $d - t$ graph for motion with constant acceleration is part of a *parabola*
  - If the parabola is *convex*, then acceleration is positive
  - If the parabola is *concave*, then acceleration is negative
- The $v - t$ graph is a straight line; its slope (a constant) is the acceleration

Simple Harmonic Motion

For oscillatory motion, or **harmonic motion**, neither position, velocity nor acceleration are constant:

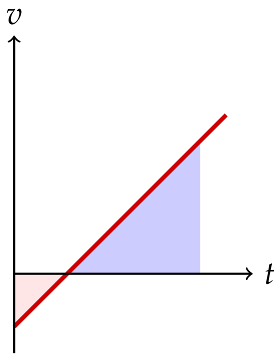


Bottom line: regardless of the type motion,

- The $v - t$ graph is the slope of the $d - t$ graph
- The $a - t$ graph is the slope of the $v - t$ graph

Area Under $v - t$ Graph

The area under the $v - t$ graph is the displacement $x - x_0$. (This should be obvious, since x is the time integral of v.)



- If the area is *below* the x (time) axis, then the displacement is negative;
- If the area is *above* the time axis, then displacement is positive

Kinematic Equations For Constant Acceleration

Although kinematic problems in AP Physics often require calculus, these basic kinematic equations[1] are still a very powerful tool.

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- The variables of interests are:

$$\mathbf{x}_0 \quad \mathbf{x} \quad \mathbf{v}_0 \quad \mathbf{v} \quad t \quad \mathbf{a}$$

- Only applicable for constant acceleration

You will still encounter situations where integration is necessary.

[1] In Physics 11 and 12, depending on *where* you learned these equation, they may be called the "Big-five" or "Big-four" equations. In AP, you will only be given 3 equations in your equation sheet.

Projectile Motion

- For 2D problems, resolve the problem into its horizontal (x) and vertical (y) directions, and apply kinematic equations independently
- For projectile motion, there is no acceleration in the x direction, i.e. $a_x = 0$, therefore the kinematic equations reduce to just

$$x = v_x t\hat{\boldsymbol{\imath}}$$

- The only acceleration is in the $\hat{\jmath}$ direction. In the standard Cartesian coordinate system, this usually means that $\hat{\jmath}$ direction is *up*:

$$a_y = -g\hat{\boldsymbol{\jmath}}$$

- The variable that connects the two directions is time t

Symmetric Trajectory

Trajectory is symmetric if the object lands at the same height as when it started. The angle θ is measured *above the the horizontal*.

- Time of flight

$$t_{\text{max}} = \frac{2v_i \sin\theta}{g}$$

- Range

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

- Maximum height

$$h_{\text{max}} = \frac{v_i^2 \sin^2\theta}{2g}$$

Maximum Range

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

- For a given initial speed v_i, maximum range occurs at $\theta = 45^\circ$
- For a given initial speed v_i and range R, I can find a launch angle θ that gives the required range:

$$\theta_1 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_i^2}\right)$$

- But there is another angle that *gives the same range*!

$$\theta_2 = 90^\circ - \theta_1$$

Relative Motion

Notation

When expressing relative motion, the first subscript (A) represents the moving object, and the second subscript (B) represents the frame of reference:

$$\mathbf{v}_{AB}$$

If an airplane ("P") is traveling at $251\,\mathrm{km/h}$ [N] relative to Earth ("E"), its velocity is expressed as:

$$\mathbf{v}_{PE} = 251\,\mathrm{km/h}\ [\mathrm{N}]$$

Relative Motion

If the airplane flies in windy air ("A") we must consider the velocity of the airplane relative to air $\mathbf{v}_{PA}$ and the velocity of the air relative to Earth $\mathbf{v}_{AE}$. The velocity of the airplane relative to Earth is therefore

$$\mathbf{v}_{PE} = \mathbf{v}_{PA} + \mathbf{v}_{AE}$$

If an airplane is flying at a constant velocity of $253\,\mathrm{km/h}$ [S] relative to the air and the air velocity is $24\,\mathrm{km/h}$ [N], what is the velocity of the airplane relative to Earth?

Relative Motion

In classical mechanics, the equation for relative motion follows the **Galilean velocity addition rule**[2]:

$$\boxed{\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}}$$

The velocity of A relative to reference frame C is the velocity of A relative to reference frame B, plus the velocity of B relative to C.

If we add another frame of reference ("D"), the equation becomes:

$$\mathbf{v}_{AD} = \mathbf{v}_{AB} + \mathbf{v}_{BC} + \mathbf{v}_{CD}$$

[2]This equation was thought to be so obvious that no one bother to give it a name until Einstein proved that it was incorrect for speeds close to the speed of light

Typical Problems

For both AP Physics 1 and AP Physics C exams, questions involving kinematics usually appear in the multiple-choice section. The problems themselves are not very different compared to the Grade 12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use $g = 10\,\mathrm{m/s^2}$ to make your lives simpler
- A lot of problems are *symbolic*, which means that they deal with the equations, not actual numbers
- Will be coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet