WELCOME TO AP AND IBHL PHYSICS

Prerequisites

- **Physics 11 and 12:** You will need to be comfortable with the topics covered in high-school level physics courses.
- **Vectors:** You need to be comfortable with vector operations, including addition and subtraction, multiplication/division by constants, as well as dot products and cross products.

If you already have a background in both differential and integral calculus, you may consider taking the AP Physics C exams instead.

Topic 1: Kinematics

AP and IBHL Physics

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Summer 2021

Olympiads School

Kinematics

Kinematics

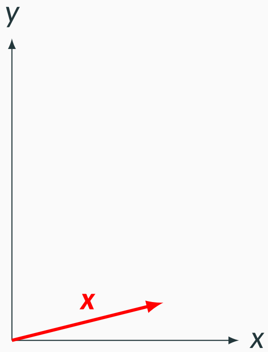
Kinematics is a discipline within mechanics concerning the mathematical description of the motion of bodies. It describes the relationships between

- Position
- Displacement
- Distance
- Velocity
- Speed
- Acceleration

Position

Position ($\boldsymbol{x}$) describes the location of an object within a coordinate system. The SI unit for position is **meter** (m).

$$\boldsymbol{x}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}$$



Vectors in 2D/3D Cartesian space are often expressed using the **IJK notation**

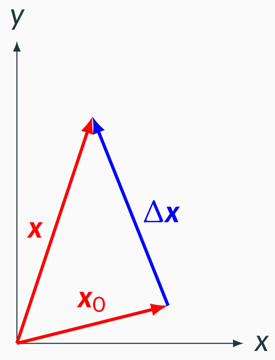
- $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ are unit vectors representing the directions of the positive x, y and z axes.
- This notation does not explicitly give the magnitude and direction of the vector (needs to be calculated using the Pythagorean theorem)

Displacement

Displacement ($\Delta \boldsymbol{x}$) is the vector change in position from the initial position $\boldsymbol{x}_0$ within the same coordinate system.

$$\begin{aligned}\Delta \boldsymbol{x} &= \boldsymbol{x} - \boldsymbol{x}_0 \\ &= (x - x_0)\hat{\boldsymbol{\imath}} + (y - y_0)\hat{\boldsymbol{\jmath}} + (z - z_0)\hat{\boldsymbol{k}}\end{aligned}$$

IJK notation makes vector addition and subtraction less prone to errors

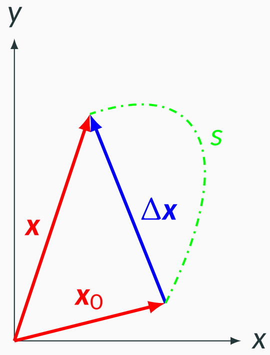


Distance

Distance s is a quantity that is *related* to displacement. It is:

- Length of the path taken by an object when it moves from $\boldsymbol{x}_0$ to $\boldsymbol{x}$
- A scalar quantity
- Always positive, i.e. $s \geq 0$
- Although the magnitude of the displacement vector is also a scalar, it is not necessarily the same as distance
- $s \geq |\Delta \boldsymbol{x}|$

Pay close attention to the difference between distance and displacement.



Average Velocity

Average velocity $\bar{v}$ of an object is its displacement $\Delta \boldsymbol{x}$ over a *finite* time interval Δt. The unit for velocity is **meters per second** (m/s):

$$\boxed{\bar{\boldsymbol{v}} = \frac{\Delta \boldsymbol{x}}{\Delta t}}$$

Since the $\hat{\imath}$, $\hat{\jmath}$ and $\hat{\boldsymbol{k}}$ directions (x, y, z axes) are *linearly independent*[1], each component of average velocity can be calculated by separating each direction:

$$\boxed{\bar{\boldsymbol{v}} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} + \frac{\Delta z}{\Delta t}\hat{\boldsymbol{k}}}$$

(Note: A bar is drawn over the symbol if it is averaged over time.)

[1] mathematical way of saying that what happens in one axis does not affect another

Instantaneous Velocity

If displacement dx is calculated a very small[2] time interval dt, then velocity is called the **instantaneous velocity**:

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \boxed{v = \frac{dx}{dt}}$$

The instantaneous velocity is the slope of the tangent on the position-time graph.

[2] In calculus, a very small change is called *infinitesimally small*

Instantaneous & Average Speed

Average speed is similar to average velocity: it is the distance s traveled over a finite time interval Δt. Since distance is always positive, so too is the average speed

$$\boxed{\bar{v} = \frac{s}{\Delta t}}$$

Likewise, when the time interval is made infinitesimally small, then the speed is called the **instantaneous speed** v. Instantaneous speed v is the magnitude of the instantaneous velocity vector.

Instantaneous & Average Acceleration

In the same way that velocity describes how quickly position changes with time, **average acceleration** $\bar{\boldsymbol{a}}$ is the change in velocity $\Delta \boldsymbol{v}$ over a finite time interval Δt. The unit for acceleration is **meters per second squared** ($\mathrm{m/s^2}$).

$$\boxed{\bar{\boldsymbol{a}} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{\boldsymbol{v}(t) - \boldsymbol{v}(t_0)}{t - t_0}}$$

Making the time interval $\Delta t = t - t_0$ infinitesimally small gives the **instantaneous acceleration** $\boldsymbol{a}(t)$.

If You Are Curious (Not Part of AP Physics)

For the curious minds, the time rate of change of acceleration is called **jerk**, with a unit of $\mathrm{m/s^3}$:

$$\bar{j} = \frac{\Delta a}{\Delta t}$$

The time rate of change in jerk is called **jounce** or **snap**, with a unit of $\mathrm{m/s^4}$:

$$\bar{s} = \frac{\Delta j}{\Delta t}$$

The next motion quantities are are called **crackle** and **pop**, but these quantities are almost never used.

Acceleration as Functions of Velocity and Position

Sometimes, acceleration are expressed as a function of velocity or position rather than of time, depending on the forces acting on them. For example:

- Gravitational or electrostatic forces: $a(x) = Ax^{-2}$
- Spring force: $a(x) = -Bx$
- Damping force (e.g. shock absorbers): $a(v) = Cv$
- Aerodynamic drag: $a(v) = Dv^2$

In these cases, solving for the motion quantities $x(t)$, $v(t)$ and $a(t)$ may require calculus, numerical integration methods, or the conservation of energy.

Kinematic Equations

Kinematic Equations

Without calculus, kinematic problems in AP Physics 1 only deal with constant acceleration. The 1D kinematic equations that will be used in Physics 1 are:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- Initial position: x_0
- Position at time t: x
- Initial velocity: v_0
- Velocity at time t: v
- Acceleration (constant): a

These equations are sometimes called the "Big-five" or "Big-four" in Grade 11/12 Physics. In AP, you are given only 3 equations in your equation sheet.

Motion Graphs

Motion Graphs

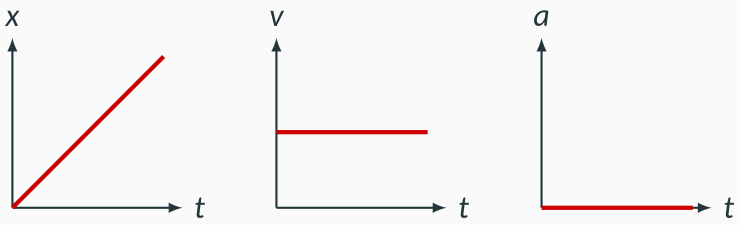
You should already be familiar with the *basic* 1D motion graphs. These are still used in AP Physics.

- Position vs. time (x-t) graph
- Velocity vs. time (v-t) graph
- Acceleration vs. time (a-t) graph

They are the graphical representation of the kinematic equations from the previous slide.

Uniform Motion

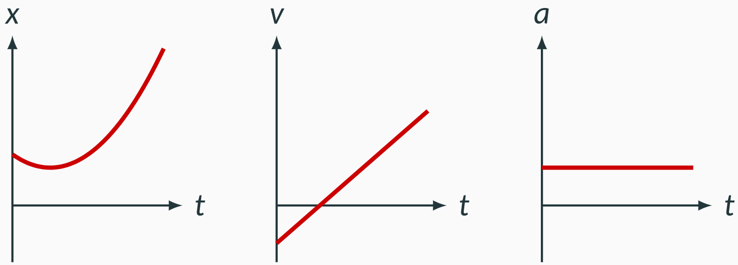
An object moves with constant velocity (neither magnitude nor direction changes) and therefore no acceleration.



- Constant velocity has a straight line in the x-t graph
- The slope of the x-t graph is the velocity v, which is constant
- The slope of the v-t graph is the acceleration a, which is zero by definition

Uniform Acceleration

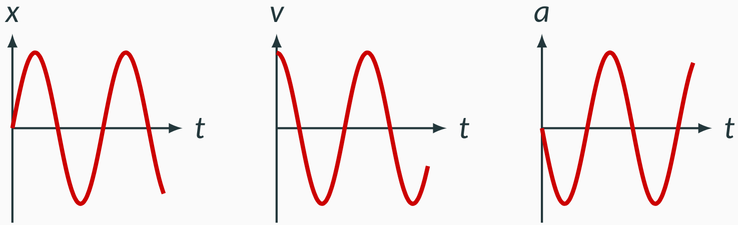
An object moves with a constant non-zero acceleration:



- The x-t graph is part of a *parabola*
  - If the parabola is *convex* (opens up), acceleration is $(+)$
  - If the parabola is *concave* (opens down), acceleration is $(-)$
- The v-t graph is a straight line; its slope (a constant) is the acceleration
- The a-t graph is a horizontal straight line

Simple Harmonic Motion

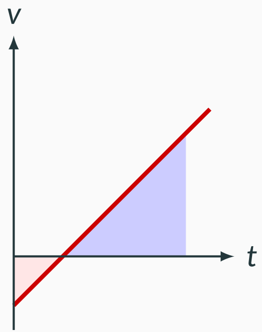
For **harmonic motions** (vibrations, oscillations), x, v and a are all non-constant, and they all change with time as sinusoidal functions.



Bottom line: regardless of the type motion,

- The v-t graph is the slope of the x-t graph
- The a-t graph is the slope of the v-t graph

Area Under Motion Graphs



The area under the v-t graph is the displacement Δx:

- Area *above* the time axis: $+$ displacement
- Area *below* the time axis: $-$ displacement

The area under the a-t graph is the change in velocity Δv:

- Area *above* the time axis: $+$ change in velocity
- Area *below* the time axis: $-$ change in velocity

The area under the x-t graph has no physical meaning.

Velocity Squared vs. Displacement

If velocity information is given as a function of position[3] then a motion graph can be plotted using this kinematic equation:

$$\underbrace{v^2}_{y} = \underbrace{v_0^2}_{b} + \underbrace{2a}_{m} \underbrace{(x - x_0)}_{x}$$

by plotting v^2 on the y-axis and displacement $\Delta x = x - x_0$ on the x-axis. The slope of the graph is $m = 2a$. The square of the initial velocity (v_0^2) is the y-intercept.

[3]Depends on experimental set up

Graphing "Linear" Functions

This concept extends to graphing other physical quantities not relating to motion:

- To find the index of refraction of a material using Snell's law, plot $\sin\theta_i$ vs. $\sin\theta_2$ (rather than θ_1 vs. θ_2). The slope is the index n:

$$\underbrace{\sin\theta_1}_{y} = \underbrace{n}_{m}\underbrace{\sin\theta_2}_{x}$$

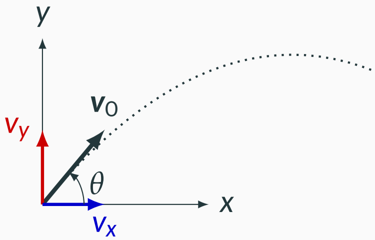
- To relate the period of oscillation of a simple pendulum to the length of the pendulum, plot T vs. $\sqrt{L}$:

$$\underbrace{T}_{y} = \underbrace{\frac{2\pi}{\sqrt{g}}}_{m}\underbrace{\sqrt{L}}_{x}$$

Projectile Motion

Projectile Motion

A **projectile** is an object that is launched with an initial velocity of $\boldsymbol{v}_0$ along a parabolic trajectory and accelerates only due to gravity.



- x-axis: *horizontal*, pointing *forward*
- y-axis: *vertical*, pointing *up*
- Angle θ measured *above* the horizontal
- The origin is usually where the projectile is launched

Horizontal Direction

The initial velocity $\boldsymbol{v}_0$ can be decomposed into its x and y components using the launch angle θ:

$$\boldsymbol{v}_0 = v_x \hat{\boldsymbol{\imath}} + v_y \hat{\boldsymbol{\jmath}} = \left[v_0 \cos\theta\right] \hat{\boldsymbol{\imath}} + \left[v_0 \sin\theta\right] \hat{\boldsymbol{\jmath}}$$

There is no horizontal acceleration (i.e. $a_x = 0$), therefore v_x is constant. The kinematic equations reduce to a single equation:

$$x = v_x t = \left[v_0 \cos\theta\right] t$$

where x is the horizontal position at time t

Vertical Direction

There is constant vertical acceleration due to gravity alone, i.e. $a_y = -g$. (a_y is *negative* due to the way we defined the coordinate system.) The important equation is this one:

$$y = \left[v_0 \sin\theta\right] t - \frac{1}{2} g t^2$$

These two kinematic equations may also be useful:

$$\begin{aligned} v_y &= \left[v_0 \sin\theta\right] - gt \\ v_y^2 &= \left[v_0^2 \sin^2\theta\right] - 2gy \end{aligned}$$

Solving Projectile Motion Problems

Horizontal and vertical motions are linearly independent, but variables are shared in both directions:

- Time t
- Launch angle θ (above the horizontal)
- Initial speed v_0

When solving any projectile motion problems

- *Two* equations with *two* unknowns
- If an object lands on an incline, there will be a third equation relating x and y

Symmetric Trajectory

A projectile's trajectory is *symmetric* if the object lands at the same height as when it launched. The angle θ is measured above the horizontal.

- Time of flight

$$t_{\text{max}} = \frac{2v_0 \sin\theta}{g}$$

- Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum height

$$y_{\text{max}} = \frac{v_0^2 \sin^2\theta}{2g}$$

Maximum Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum range occurs at $\theta = 45°$
- For a given initial speed v_0 and range R, launch angle θ is given by:

$$\theta_1 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

But there is another angle that *gives the same range*!

$$\theta_2 = 90° - \theta_1$$

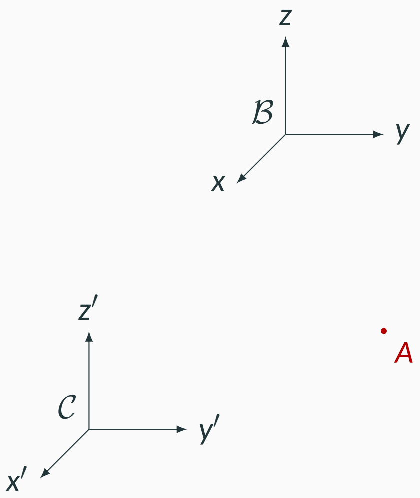
Relative Motion

Relative Motion

All motion quantities must be measured relative to a frame of reference

- Frame of reference: a coordinate system from which physical measurements are made.
- In *classical* mechanics, the coordinate system is the Cartesian coordinate system
- There is no absolute motion/rest: all motions are relative
- Principle of relativity: All laws of physics are equal in all inertial (non-accelerating) frames

Relative Motion

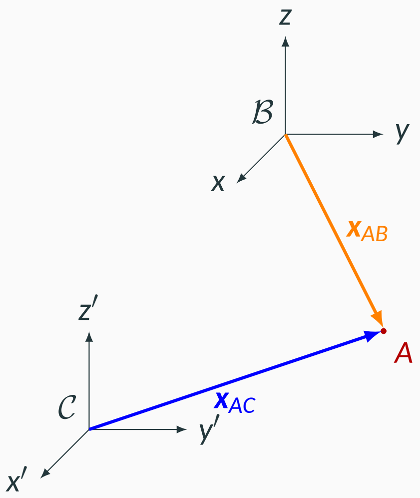


The position and motion of A can be calculatde from two frames of reference

- $\mathcal{B}$ with axes x, y, z
- $\mathcal{C}$ with axes x', y', z'

The two reference frames may (or may not) be moving relative to each other. The motion of the two reference frames affect how motion of A is calculated.

Relative Motion

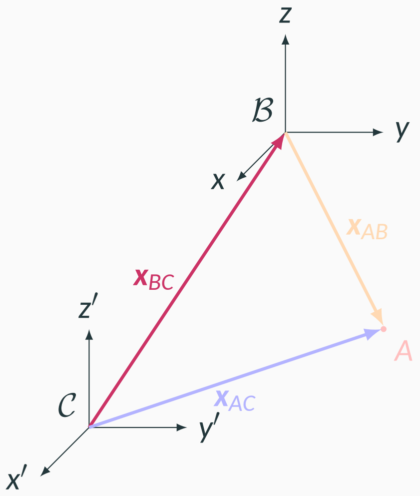


The position of $A(t)$ (as a function of time) can be described by

- $\boldsymbol{x}_{AB}(t)$ (relative to frame $\mathcal{B}$)
- $\boldsymbol{x}_{AC}(t)$ (relative to frame $\mathcal{C}$)

Without needing mathematically rigorous vector notation, it is obvious that $\boldsymbol{x}_{AB}(t)$ and $\boldsymbol{x}_{AC}(t)$ are different vectors

Relative Motion



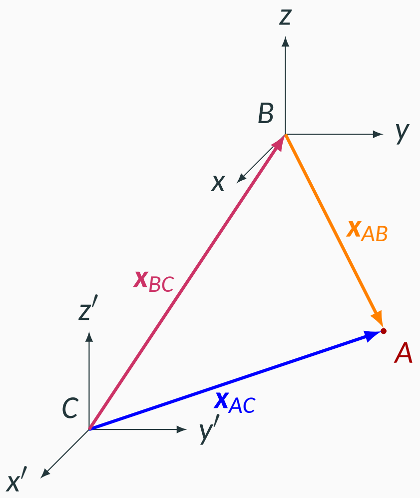
The position vector of the origins of the two reference frames is given by $\mathbf{x}_{BC}$

- The vector pointing from the origin of frame $\mathcal{C}$ to the origin of frame $\mathcal{B}$
- If the two frames are moving relative to each other, then $\mathbf{x}_{BC}$ is also a function of time

Without using vector notations, the relationship between the vectors is obvious:

$$\boxed{\mathbf{x}_{AC} = \mathbf{x}_{AB} + \mathbf{x}_{BC}}$$

Relative Motion



Starting from the definition of **relative position**:

$$\boxed{\boldsymbol{x}_{AC} = \boldsymbol{x}_{AB} + \boldsymbol{x}_{BC}}$$

Using the definitions for velocity to get a similar equation for **relative velocity**:

$$\boxed{\boldsymbol{v}_{AC} = \boldsymbol{v}_{AB} + \boldsymbol{v}_{BC}}$$

and **relative acceleration**:

$$\boxed{\boldsymbol{a}_{AC} = \boldsymbol{a}_{AB} + \boldsymbol{a}_{BC}}$$

Relative Velocity

In classical mechanics, the equation for relative velocities follows the **Galilean velocity addition rule**, which applies to speeds much less than the speed of light:

$$\boldsymbol{v}_{AC} = \boldsymbol{v}_{AB} + \boldsymbol{v}_{BC}$$

The velocity of A relative to reference frame C is the velocity of A relative to reference frame B, plus the velocity of B relative to C. If we add another reference frame D, the equation becomes:

$$\boldsymbol{v}_{AD} = \boldsymbol{v}_{AB} + \boldsymbol{v}_{BC} + \boldsymbol{v}_{CD}$$

Typical Problems

In the AP Physics 1 exam, kinematics questions appear in both multiple-choice and free-response sections. The problems themselves are not necessarily very different from Grade 11/12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use $g = 10\,\mathrm{m/s^2}$ in your calculations to make your lives simpler
- Many problems are *symbolic*, which means that they deal with the algebraic expressions, not actual numbers
- Often coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet