

# Topic 11: Electrostatics

## Advanced Placement Physics 1 & 2

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# Electrostatic Force

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# The Charges Are

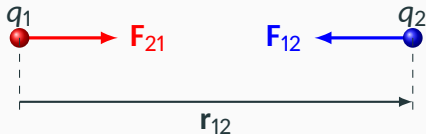
We should already know a bit about charge particles:

- A **proton** carries a **positive** charge
- An **electron** carries a **negative** charge
- A *net charge* of an object means an excess of protons or electrons
- Similar charges are repel; opposite charges attract

We start with electrostatics:

- Charges that are not moving relative to one another

# Coulomb's Law for Electrostatic Force



The **electrostatic force** (or **coulomb force**) is a mutually repulsive/attractive force between all charged objects. The force that charge  $q_1$  exerts on  $q_2$  is given by **Coulomb's law**:

$$\mathbf{F}_{12} = \frac{kq_1q_2}{|\mathbf{r}_{12}|^2} \hat{\mathbf{r}}_{12}$$

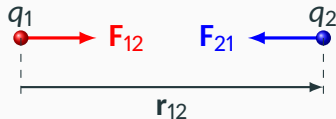
# Coulomb's Law for Electrostatic Force

$$\mathbf{F}_{12} = \frac{kq_1q_2}{|\mathbf{r}_{12}|^2} \hat{\mathbf{r}}_{12}$$

Quantity	Symbol	SI Unit
Electrostatic force	$\mathbf{F}_{12}$	N
Coulomb's constant (electrostatic constant)	$k$	$\text{N m}^2/\text{C}^2$
Point charges 1 and 2	$q_1, q_2$	C
Distance between point charges	$ \mathbf{r}_{12} $	m
Unit vector of direction between point charges	$\hat{\mathbf{r}}_{12}$	

**Coulomb's constant**  $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$  where  
 $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2$  is called the “permittivity of free space”

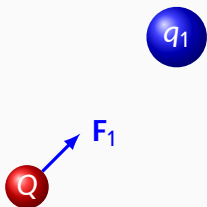
# Coulomb's Law for Electrostatic Force



- If  $q_1$  exerts an electrostatic force  $\mathbf{F}_{12}$  on  $q_2$ , then  $q_2$  likewise exerts a force of  $\mathbf{F}_{21} = -\mathbf{F}_{12}$  on  $q_1$ . The two forces are equal in magnitude and opposite in direction (3rd law of motion).
- $q_1$  and  $q_2$  are assumed to be *point charges* that do not occupy any space
- The (more familiar) scalar form is often used as well:

$$F_q = \frac{kq_1q_2}{r^2}$$

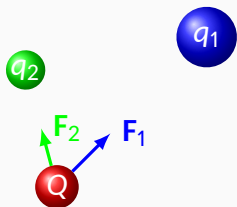
## More Than One Charge



For a charge  $Q$  that is subjected to the influence of multiple discrete point charges  $q_i$ , the total electrostatic force that  $Q$  experiences is the vector sum of all the forces  $\mathbf{F}_i$ :

$$\mathbf{F} = \sum_i \mathbf{F}_i = kQ \left( \sum_{i=1}^N \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \right)$$

## More Than One Charge

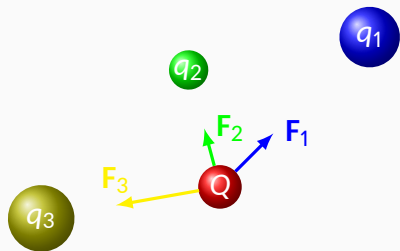


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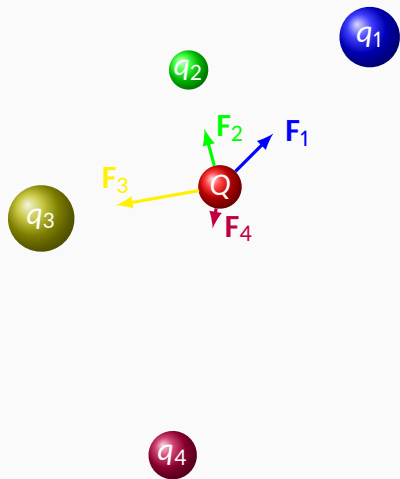
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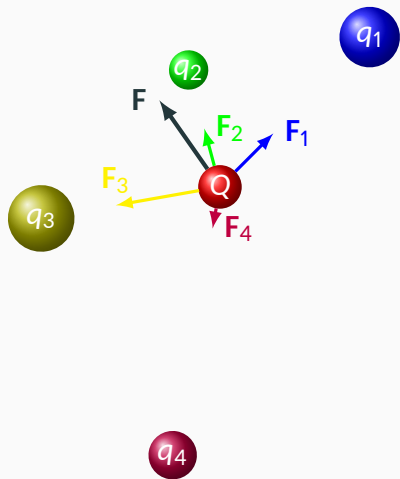
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# Electric Field

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# Electric Field

The expression for **electric field** is obtained by repeating the same procedure as with gravitational field, by grouping the variables in Coulomb's law:

$$F_q = \underbrace{\left[ \frac{kq_1}{|\mathbf{r}_{12}|^2} \hat{\mathbf{r}} \right]}_{\mathbf{E}} q_2$$

The electric field **E** created by  $q_1$  is a vector function (called a **vector field**) that shows how it influences other charged particles around it.

# Electric Field Near a Point Charge

The electric field a distance  $r$  away from a point charge  $q$  is given by:

$$\mathbf{E}(q, \mathbf{r}) = \frac{kq}{|\mathbf{r}|^2} \hat{\mathbf{r}}$$

Quantity	Symbol	SI Unit
Electric field intensity	$\mathbf{E}$	N/C
Coulomb's constant	$k$	$\text{N m}^2/\text{C}^2$
Source charge	$q$	C
Distance from source charge	$ \mathbf{r} $	m
Outward unit vector from point source	$\hat{\mathbf{r}}$	

The direction of  $\mathbf{E}$  is radially outward from a positive point charge and radially inward towards a negative charge.

## More Than One Charge

When multiple point charges are present, the total electric field at any position  $\mathbf{r}$  is the vector sum of all the fields  $\mathbf{E}_i$ :

$$\mathbf{E} = \sum_i \mathbf{E}_i = k \left( \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \right)$$

# Think Electric Field

**E** itself *doesn't do anything* until another charge interacts with it. And when there is a charge  $q$ , the electrostatic force  $\mathbf{F}_q$  that the charge experiences is proportional to  $q$  and **E**, regardless of how the electric field is generated:

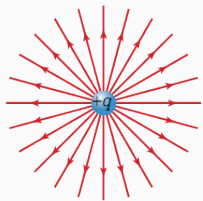
$$\mathbf{F}_q = q\mathbf{E}$$

A positive charge in the electric field experiences a electrostatic force **F** in the same direction as **E**.

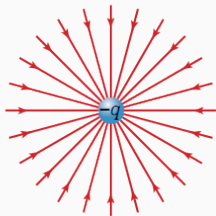


# Electric Field Lines

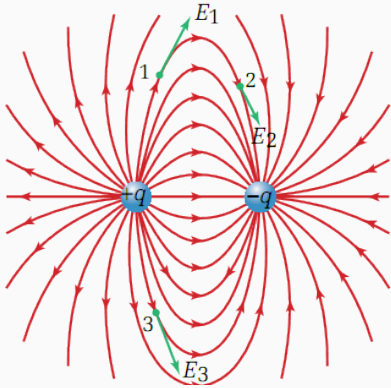
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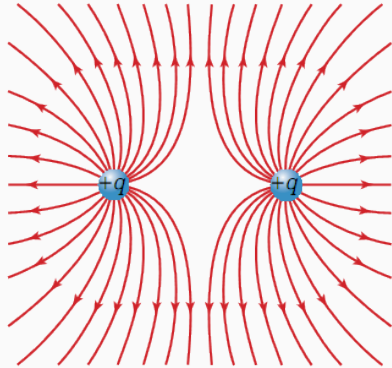
B



C



D



# Electric Potential & Potential Energy

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# Electrical Potential Energy

If you know calculus, you can easily find that the work done by the electrostatic force is given by this integral:

$$W = \int \mathbf{F}_q \cdot d\mathbf{r} = kq_1q_2 \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{kq_1q_2}{r} \Big|_{r_1}^{r_2} = -\Delta U_q$$

And if you don't know calculus, all you need to know is the result:  $U_q$  is defined as the **electric potential energy**:

$$U_q = \frac{kq_1q_2}{r}$$

$U_q$  can be (+) or (−), because charges can be either (+) or (−).

## How it Differs from Gravitational Potential Energy

Two positive charges:

$$U_q > 0$$

Two negative charges:

$$U_q > 0$$

One positive and one  
negative charge:

$$U_q < 0$$

- $U_q > 0$ : positive work is done to bring two charges together from  $r = \infty$  to  $r$  (both charges of the same sign)
- $U_q < 0$ : work is done to pull the objects from  $r$  to  $r = \infty$
- In comparison, gravitational potential  $U_g$  is always  $< 0$

# Electric Potential

**Using gravity as an example:** An object at a specific location inside a gravitational field has a gravitational potential energy that is proportional to its mass, i.e.

$$U_g = V_g m$$

This “constant”  $V_g$  is called the **gravitational potential**, which is the *gravitational potential energy per unit mass*. In the trivial case with a uniform  $\mathbf{g}$ :

$$V_g = \frac{U_g}{m} = g\Delta h$$

This also applies to the general case of the gravitational potential energy:

$$V_g = \frac{U_g}{m} = -\frac{GM}{r}$$

# Electric Potential

This is also true for charged particle  $q$  in an electric field created by  $q_s$ , and the “constant” is called the **electric potential**. For a point charge, it is defined as:

$$V = \frac{U_q}{q} = \frac{kq_s}{r}$$

The unit for electric potential is a *volt*<sup>1</sup> which is *one joule per coulomb*:

$$1\text{V} = 1\text{J/C}$$

The relationship between  $V$  and  $\mathbf{E}$  is given by:

$$|\mathbf{E}| \approx \frac{\Delta V}{\Delta r}$$

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<sup>1</sup>Named after Italian physicist Alessandro Volta

## Potential Difference (Voltage)

The change in electric potential is called the **electric potential difference** or **voltage**:

$$\Delta V = \frac{\Delta U_q}{q}$$

Here, we can relate  $\Delta V$  to an equation that we knew from Grade 11 Physics, which related to the energy dissipated in a resistor in a circuit  $\Delta U$  to the voltage drop  $\Delta V$ :

$$\Delta U_q = q\Delta V$$

Electric potential difference also has the unit *volts* (V)

## Getting Those Names Right

Remember that these three scalar quantities, as opposed to electrostatic force  $\mathbf{F}_q$  and electric field  $\mathbf{E}$  which are vectors

- Electric potential energy:

$$U_q = \frac{kq_1q_2}{r}$$

- Electric potential:

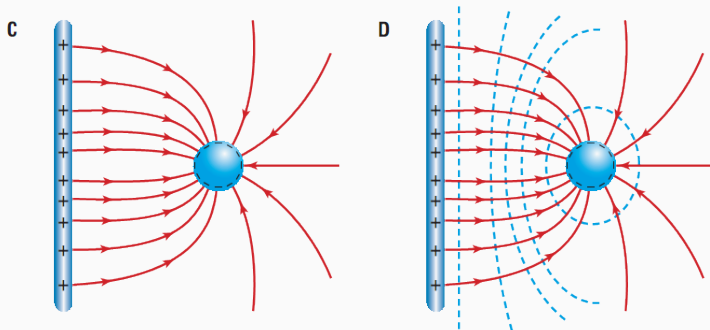
$$V = \frac{kq}{r}$$

- Electric potential difference (voltage):

$$\Delta V = \frac{\Delta U_q}{q}$$

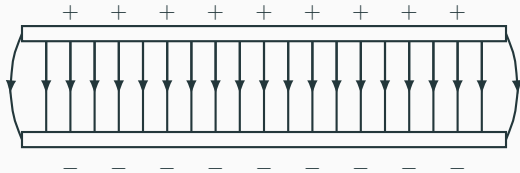


# Equipotential Lines



The dotted blue lines are called **equipotential lines**. They are always *perpendicular* to the electric field lines. Charges moving in the direction of the equipotential lines have constant electric potential

# Electric Field Between Parallel Charged Plates

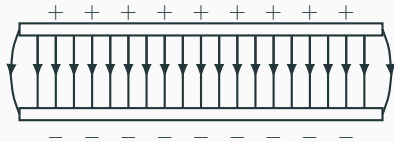


- Two plates, each producing an electric field pointing in the same direction
- The total electric field is twice the value of *one* infinite plane, pointing from the positively charged plate towards the negatively charged plate

$$E = \frac{\sigma}{\epsilon_0}$$

- **E** outside the plates is very low (close to zero), except for fringe effects at the edges of the plates

# Electric Field and Electric Potential Difference



In the case of two parallel plates, the electric field is uniform, and the relationship simplifies to:

$$E = \frac{\Delta V}{d}$$

Quantity	Symbol	SI Unit
Electric field intensity	$E$	N/C
Electric potential difference between plates	$\Delta V$	V
Distance between plates	$d$	m