CHAPTER 1: Introduction, Measurement, Estimating

Answers to Questions

1. (*a*) Fundamental standards should be accessible, invariable, indestructible, and reproducible. A particular person's foot would not be very accessible, since the person could not be at more than one place at a time. The standard would be somewhat invariable if the person were an adult, but even then, due to swelling or injury, the length of the standard foot could change. The standard would not be indestructible – the foot would not last forever. The standard could be reproducible – tracings or plaster casts could be made as secondary standards.

   (*b*) If any person's foot were to be used as a standard, "standard" would vary significantly depending on the person whose foot happened to be used most recently for a measurement. The standard would be very accessible, because wherever a measurement was needed, it would be very easy to find someone with feet. The standard would be extremely variable – perhaps by a factor of 2. That also renders the standard as not reproducible, because there could be many reproductions that were quite different from each other. The standard would be almost indestructible in that there is essentially a limitless supply of feet to be used.

2. There are various ways to alter the signs. The number of meters could be expressed in one significant figure, as "900 m (3000 ft)". Or, the number of feet could be expressed with the same precision as the number of meters, as "914 m (2999 ft)". The signs could also be moved to different locations, where the number of meters was more exact. For example, if a sign was placed where the elevation was really 1000 m to the nearest meter, then the sign could read "1000 m (3280 ft)".

3. Including more digits in an answer does not necessarily increase its accuracy. The accuracy of an answer is determined by the accuracy of the physical measurement on which the answer is based. If you draw a circle, measure its diameter to be 168 mm and its circumference to be 527 mm, their quotient, representing π, is 3.136904762. The last seven digits are meaningless – they imply a greater accuracy than is possible with the measurements.

4. The problem is that the precision of the two measurements are quite different. It would be more appropriate to give the metric distance as 11 km, so that the numbers are given to about the same precision (nearest mile or nearest km).

5. A measurement must be measured against a scale, and the units provide that scale. Units must be specified or the answer is meaningless – the answer could mean a variety of quantities, and could be interpreted in a variety of ways. Some units are understood, such as when you ask someone how old they are. You assume their answer is in years. But if you ask someone how long it will be until they are done with their task, and they answer "five", does that mean five minutes or five hours or five days? If you are in an international airport, and you ask the price of some object, what does the answer "ten" mean? Ten dollars, or ten pounds, or ten marks, or ten euros?

6. If the jar is rectangular, for example, you could count the number of marbles along each dimension, and then multiply those three numbers together for an estimate of the total number of marbles. If the jar is cylindrical, you could count the marbles in one cross section, and then multiply by the number of layers of marbles. Another approach would be to estimate the volume of one marble. If we assume that the marbles are stacked such that their centers are all on vertical and horizontal lines, then each marble would require a cube of edge 2R, or a volume of $8R^3$, where R is the radius of a marble. The number of marbles would then be the volume of the container divided by $8R^3$.

7. The result should be written as 8.32 cm. The factor of 2 used to convert radius to diameter is exact – it has no uncertainty, and so does not change the number of significant figures.

8. $\sin 30.0^\circ = 0.500$

9. Since the size of large eggs can vary by 10%, the random large egg used in a recipe has a size with an uncertainty of about $\pm 5\%$. Thus the amount of the other ingredients can also vary by about $\pm 5\%$ and not adversely affect the recipe.

10. In estimating the number of car mechanics, the assumptions and estimates needed are:
    the population of the city
    the number of cars per person in the city
    the number of cars that a mechanic can repair in a day
    the number of days that a mechanic works in a year
    the number of times that a car is taken to a mechanic, per year
    We estimate that there is 1 car for every 2 people, that a mechanic can repair 3 cars per day, that a mechanic works 250 days a year, and that a car needs to be repaired twice per year.

    (*a*) For San Francisco, we estimate the population at one million people. The number of mechanics is found by the following calculation.

$$\left(1\times10^{6}\text{ people}\right)\left(\frac{1\text{ car}}{2\text{ people}}\right)\left(\frac{2\,\frac{\text{repairs}}{\text{year}}}{1\text{ car}}\right)\left(\frac{1\text{ yr}}{250\text{ workdays}}\right)\left(\frac{1\text{ mechanic}}{3\,\frac{\text{repairs}}{\text{workday}}}\right)=\boxed{1300\text{ mechanics}}$$

    (*b*) For Upland, Indiana, the population is about 4000. The number of mechanics is found by a similar calculation, and would be $\boxed{5\text{ mechanics}}$. There are actually two repair shops in Upland, employing a total of 6 mechanics.

Solutions to Problems

$\boxed{1.}$ (*a*) 14 billion years = $\boxed{1.4\times10^{10}\text{ years}}$

(*b*) $\left(1.4\times10^{10}\text{ y}\right)\left(3.156\times10^{7}\text{ s}/1\text{ y}\right)=\boxed{4.4\times10^{17}\text{ s}}$

2. (*a*) 214 $\boxed{3\text{ significant figures}}$
(*b*) 81.60 $\boxed{4\text{ significant figures}}$
(*c*) 7.03 $\boxed{3\text{ significant figures}}$
(*d*) 0.03 $\boxed{1\text{ significant figure}}$
(*e*) 0.0086 $\boxed{2\text{ significant figures}}$
(*f*) 3236 $\boxed{4\text{ significant figures}}$
(*g*) 8700 $\boxed{2\text{ significant figures}}$

3. (*a*) $1.156 = \boxed{1.156\times10^{0}}$
   (*b*) $21.8 = \boxed{2.18\times10^{1}}$
   (*c*) $0.0068 = \boxed{6.8\times10^{-3}}$
   (*d*) $27.635 = \boxed{2.7635\times10^{1}}$
   (*e*) $0.219 = \boxed{2.19\times10^{-1}}$
   (*f*) $444 = \boxed{4.44\times10^{2}}$

4. (*a*) $8.69\times10^{4} = \boxed{86{,}900}$
   (*b*) $9.1\times10^{3} = \boxed{9{,}100}$
   (*c*) $8.8\times10^{-1} = \boxed{0.88}$
   (*d*) $4.76\times10^{2} = \boxed{476}$
   (*e*) $3.62\times10^{-5} = \boxed{0.0000362}$

5. The uncertainty is taken to be 0.01 m.

$$\%\text{ uncertainty} = \frac{0.01\text{ m}}{1.57\text{ m}}\times100\% = \boxed{1\%}$$

$\boxed{6.}$ $$\%\text{ uncertainty} = \frac{0.25\text{ m}}{3.76\text{ m}}\times100\% = \boxed{6.6\%}$$

7. (*a*) $\%\text{ uncertainty} = \dfrac{0.2\text{ s}}{5\text{ s}}\times100\% = \boxed{4\%}$
   (*b*) $\%\text{ uncertainty} = \dfrac{0.2\text{ s}}{50\text{ s}}\times100\% = \boxed{0.4\%}$
   (*c*) $\%\text{ uncertainty} = \dfrac{0.2\text{ s}}{300\text{ s}}\times100\% = \boxed{0.07\%}$

8. To add values with significant figures, adjust all values to be added so that their exponents are all the same.

$$9.2\times10^{3}\text{s} + 8.3\times10^{4}\text{s} + 0.008\times10^{6}\text{s} = 9.2\times10^{3}\text{s} + 83\times10^{3}\text{s} + 8\times10^{3}\text{s} = \left(9.2+83+8\right)\times10^{3}\text{s}$$

$$= 100\times10^{3}\text{s} = \boxed{1.00\times10^{5}\text{s}}$$

When adding, keep the least accurate value, and so keep to the "ones" place in the parentheses.

9. $\left(2.079\times10^{2}\text{m}\right)\left(0.082\times10^{-1}\right) = \boxed{1.7\text{ m}}$. When multiplying, the result should have as many digits as the number with the least number of significant digits used in the calculation.

10. To find the approximate uncertainty in the area, calculate the area for the specified radius, the minimum radius, and the maximum radius. Subtract the extreme areas. The uncertainty in the area is then half this variation in area. The uncertainty in the radius is assumed to be $0.1\times10^{4}\text{cm}$.

$$A_{\text{specified}} = \pi r^2_{\text{specified}} = \pi\left(3.8\times10^4\,\text{cm}\right)^2 = 4.5\times10^9\,\text{cm}^2$$

$$A_{\text{min}} = \pi r^2_{\text{min}} = \pi\left(3.7\times10^4\,\text{cm}\right)^2 = 4.30\times10^9\,\text{cm}^2$$

$$A_{\text{max}} = \pi r^2_{\text{max}} = \pi\left(3.9\times10^4\,\text{cm}\right)^2 = 4.78\times10^9\,\text{cm}^2$$

$$\Delta A = \tfrac{1}{2}\left(A_{\text{max}} - A_{\text{min}}\right) = \tfrac{1}{2}\left(4.78\times10^9\,\text{cm}^2 - 4.30\times10^9\,\text{cm}^2\right) = 0.24\times10^9\,\text{cm}^2$$

Thus the area should be quoted as $\boxed{A = \left(4.5\pm0.2\right)\times10^9\,\text{cm}^2}$

11. To find the approximate uncertainty in the volume, calculate the volume for the specified radius, the minimum radius, and the maximum radius. Subtract the extreme volumes. The uncertainty in the volume is then half this variation in volume.

$$V_{\text{specified}} = \tfrac{4}{3}\pi r^3_{\text{specified}} = \tfrac{4}{3}\pi\left(2.86\text{ m}\right)^3 = 9.80\times10^1\,\text{m}^3$$

$$V_{\text{min}} = \tfrac{4}{3}\pi r^3_{\text{min}} = \tfrac{4}{3}\pi\left(2.77\text{ m}\right)^3 = 8.903\times10^1\,\text{m}^3$$

$$V_{\text{max}} = \tfrac{4}{3}\pi r^3_{\text{max}} = \tfrac{4}{3}\pi\left(2.95\text{ m}\right)^3 = 10.754\times10^1\,\text{m}^3$$

$$\Delta V = \tfrac{1}{2}\left(V_{\text{max}} - V_{\text{min}}\right) = \tfrac{1}{2}\left(10.754\times10^1\,\text{m}^3 - 8.903\times10^1\,\text{m}^3\right) = 0.926\times10^1\,\text{m}^3$$

The percent uncertainty is $\dfrac{\Delta V}{V_{\text{specified}}} = \dfrac{0.923\times10^1\,\text{m}^3}{9.80\times10^1\,\text{m}^3}\times100 = 0.09444 = \boxed{9\,\%}$

12.

(*a*)	286.6 mm	$286.6\times10^{-3}\,\text{m}$	$\boxed{0.286\ 6\text{ m}}$	
(*b*)	$85\,\mu\text{V}$	$85\times10^{-6}\,\text{V}$	$\boxed{0.000\ 085\text{ V}}$	
(*c*)	760 mg	$760\times10^{-6}\,\text{kg}$	$\boxed{0.000\ 760\text{ kg}}$	(if last zero is significant)
(*d*)	60.0 ps	$60.0\times10^{-12}\,\text{s}$	$\boxed{0.000\ 000\ 000\ 0600\text{ s}}$	
(*e*)	22.5 fm	$22.5\times10^{-15}\,\text{m}$	$\boxed{0.000\ 000\ 000\ 000\ 022\ 5\text{ m}}$	
(*f*)	2.50 gigavolts	$2.5\times10^9\,\text{volts}$	$\boxed{2,500,000,000\text{ volts}}$	

13.

(*a*)	$1\times10^6\,\text{volts}$	$\boxed{\text{1 megavolt}} = 1\text{ Mvolt}$
(*b*)	$2\times10^{-6}\,\text{meters}$	$\boxed{\text{2 micrometers}} = 2\,\mu\text{m}$
(*c*)	$6\times10^3\,\text{days}$	$\boxed{\text{6 kilodays}} = 6\text{ kdays}$
(*d*)	$18\times10^2\,\text{bucks}$	$\boxed{\text{18 hectobucks}} = 18\text{ hbucks}$
(*e*)	$8\times10^{-9}\,\text{pieces}$	$\boxed{\text{8 nanopieces}} = 8\text{ npieces}$

14. (*a*) Assuming a height of 5 feet 10 inches, then $5'10'' = \left(70\text{ in}\right)\left(1\text{ m}/39.37\text{ in}\right) = \boxed{1.8\text{ m}}$

(*b*) Assuming a weight of 165 lbs, then $\left(165\text{ lbs}\right)\left(0.456\text{ kg}/1\text{ lb}\right) = \boxed{75.2\text{ kg}}$

Technically, pounds and mass measure two separate properties. To make this conversion, we have to assume that we are at a location where the acceleration due to gravity is 9.8 m/s^2.

$\boxed{15.}$ (*a*) $93\text{ million miles} = \left(93\times10^{6}\text{ miles}\right)\left(1610\text{ m}/1\text{ mile}\right) = \boxed{1.5\times10^{11}\text{ m}}$

(*b*) $1.5\times10^{11}\text{ m} = 150\times10^{9}\text{ m} = \boxed{150\text{ gigameters}}$ or $1.5\times10^{11}\text{ m} = 0.15\times10^{12}\text{ m} = \boxed{0.15\text{ terameters}}$

16. (*a*) $1\text{ ft}^2 = \left(1\text{ ft}^2\right)\left(1\text{ yd}/3\text{ ft}\right)^2 = \boxed{0.111\text{ yd}^2}$

(*b*) $1\text{ m}^2 = \left(1\text{ m}^2\right)\left(3.28\text{ ft}/1\text{ m}\right)^2 = \boxed{10.8\text{ ft}^2}$

17. Use the speed of the airplane to convert the travel distance into a time.

$$1.00\text{ km}\left(\frac{1\text{ h}}{950\text{ km}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{3.8\text{ s}}$$

18. (*a*) $1.0\times10^{-10}\text{ m} = \left(1.0\times10^{-10}\text{ m}\right)\left(39.37\text{ in}/1\text{ m}\right) = \boxed{3.9\times10^{-9}\text{ in}}$

(*b*) $\left(1.0\text{ cm}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)\left(\frac{1\text{ atom}}{1.0\times10^{-10}\text{ m}}\right) = \boxed{1.0\times10^{8}\text{ atoms}}$

19. To add values with significant figures, adjust all values to be added so that their units are all the same.

$$1.80\text{ m} + 142.5\text{ cm} + 5.34\times10^{5}\ \mu\text{m} = 1.80\text{ m} + 1.425\text{ m} + 0.534\text{ m} = 3.759\text{ m} = \boxed{3.76\text{ m}}$$

When adding, the final result is to be no more accurate than the least accurate number used. In this case, that is the first measurement, which is accurate to the hundredths place.

20. (*a*) $\left(1\text{k}/\text{h}\right)\left(\frac{0.621\text{ mi}}{1\text{ km}}\right) = \boxed{0.621\text{mi}/\text{h}}$

(*b*) $\left(1\text{m}/\text{s}\right)\left(\frac{3.28\text{ ft}}{1\text{ m}}\right) = \boxed{3.28\text{ft}/\text{s}}$

(*c*) $\left(1\text{km}/\text{h}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \boxed{0.278\text{m}/\text{s}}$

21. One mile is $1.61\times10^{3}\text{ m}$. It is 110 m longer than a 1500-m race. The percentage difference is

$$\frac{110\text{ m}}{1500\text{ m}}\times100\% = \boxed{7.3\%}$$

22. (*a*) $1.00\text{ ly} = \left(2.998\times10^{8}\text{ m}/\text{s}\right)\left(3.156\times10^{7}\text{ s}\right) = \boxed{9.46\times10^{15}\text{ m}}$

(*b*) $\left(1.00\text{ ly}\right)\left(\frac{9.462\times10^{15}\text{ m}}{1.00\text{ ly}}\right)\left(\frac{1\text{ AU}}{1.50\times10^{11}\text{m}}\right) = \boxed{6.31\times10^{4}\text{ AU}}$

(*c*) $\left(2.998\times10^{8}\text{ m}/\text{s}\right)\left(\frac{1\text{ AU}}{1.50\times10^{11}\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ hr}}\right) = \boxed{7.20\text{AU}/\text{h}}$

$\boxed{23.}$ The surface area of a sphere is found by $A = 4\pi r^2 = 4\pi\left(d/2\right)^2 = \pi d^2$.

(*a*) $A_{\text{Moon}} = \pi D_{\text{Moon}}^2 = \pi\left(3.48\times10^6\,\text{m}\right)^2 = \boxed{3.80\times10^{13}\,\text{m}^2}$

(*b*) $\dfrac{A_{\text{Earth}}}{A_{\text{Moon}}} = \dfrac{\pi D_{\text{Earth}}^2}{\pi D_{\text{Moon}}^2} = \left(\dfrac{D_{\text{Earth}}}{D_{\text{Moon}}}\right)^2 = \left(\dfrac{R_{\text{Earth}}}{R_{\text{Moon}}}\right)^2 = \left(\dfrac{6.38\times10^6\,\text{m}}{1.74\times10^6\,\text{m}}\right)^2 = \boxed{13.4}$

24. (*a*) $2800 = 2.8\times10^3 \approx 1\times10^3 = \boxed{10^3}$

(*b*) $86.30\times10^2 = 8.630\times10^3 \approx 10\times10^3 = \boxed{10^4}$

(*c*) $0.0076 = 7.6\times10^{-3} \approx 10\times10^{-3} = \boxed{10^{-2}}$

(*d*) $15.0\times10^8 = 1.5\times10^9 \approx 1\times10^9 = \boxed{10^9}$

25. The textbook is approximately 20 cm deep and 4 cm wide. With books on both sides of a shelf, with a little extra space, the shelf would need to be about 50 cm deep. If the aisle is 1.5 meter wide, then about 1/4 of the floor space is covered by shelving. The number of books on a single shelf level is then $\frac{1}{4}\left(3500\,\text{m}^2\right)\left(\dfrac{1\text{ book}}{\left(0.25\text{ m}\right)\left(0.04\text{ m}\right)}\right) = 8.75\times10^4\,\text{books}$. With 8 shelves of books, the total number of books stored is as follows.

$$\left(8.75\times10^4\,\frac{\text{books}}{\text{shelf level}}\right)\left(8\text{ shelves}\right) \approx \boxed{7\times10^5\text{ books}}.$$

26. The distance across the United States is about 3000 miles.

$$\left(3000\text{ mi}\right)\left(1\text{ km}/0.621\text{ mi}\right)\left(1\text{ hr}/10\text{ km}\right) \approx \boxed{500\text{ hr}}$$

Of course, it would take more time on the clock for the runner to run across the U.S. The runner could obviously not run for 500 hours non-stop. If they could run for 5 hours a day, then it would take about 100 days for them to cross the country.

$\boxed{27.}$ An NCAA-regulation football field is 360 feet long (including the end zones) and 160 feet wide, which is about 110 meters by 50 meters, or 5,500 m^2. The mower has a cutting width of 0.5 meters. Thus the distance to be walked is

$$d = \frac{\text{Area}}{\text{width}} = \frac{5500\,\text{m}^2}{0.5\text{ m}} = 11000\text{ m} = 11\text{ km}$$

At a speed of 1 km/hr, then it will take about $\boxed{11\text{ h}}$ to mow the field.

28. A commonly accepted measure is that a person should drink eight 8-oz. glasses of water each day. That is about 2 quarts, or 2 liters of water per day. Then approximate the lifetime as 70 years.

$$\left(70\text{ y}\right)\left(365\text{ d}/1\text{ y}\right)\left(2\text{ L}/1\text{ d}\right) \approx \boxed{5\times10^4\,\text{L}}$$

29. Consider the body to be a cylinder, about 170 cm tall, and about 12 cm in cross-sectional radius (a 30-inch waist). The volume of a cylinder is given by the area of the cross section times the height.

$$V = \pi r^2 h = \pi\left(12\text{ cm}\right)^2\left(170\text{ cm}\right) = 9\times10^4\,\text{cm}^3 \approx \boxed{8\times10^4\,\text{cm}^3}$$

30. Estimate one side of a house to be about 40 feet long, and about 10 feet high. Then the wall area of that particular wall is 400 ft^2. There would perhaps be 4 windows in that wall, each about 3 ft wide and 4 feet tall, so 12 ft^2 per window, or about 50 ft^2 of window per wall. Thus the percentage of wall area that is window area is $\frac{50\text{ ft}^2}{400\text{ ft}^2}\times 100 = 12.5\%$. Thus a rough estimate would be $\boxed{10\% - 15\%}$ of the house's outside wall area.

31. Assume that the tires last for 5 years, and so there is a tread wearing of 0.2 cm/year. Assume the average tire has a radius of 40 cm, and a width of 10 cm. Thus the volume of rubber that is becoming pollution each year from one tire is the surface area of the tire, times the thickness per year that is wearing. Also assume that there are 150,000,000 automobiles in the country – approximately one automobile for every two people. So the mass wear per year is given by

$$\left(\frac{\text{Mass}}{\text{year}}\right) = \left(\frac{\text{Surface area}}{\text{tire}}\right)\left(\frac{\text{Thickness wear}}{\text{year}}\right)(\text{density of rubber})(\#\text{ of tires})$$

$$= \left[2\pi(0.4\text{ m})(0.1\text{ m})\right](0.002\ \text{m/y})\left(1200\,\text{kg/m}^3\right)(600,000,000\text{ tires})$$

$$= \boxed{4\times 10^8\text{ kg/y}}$$

32. For the equation $v = At^3 - Bt$, the units of At^3 must be the same as the units of v. So the units of A must be the same as the units of v/t^3, which would be $\boxed{\text{distance/time}^4}$. Also, the units of Bt must be the same as the units of v. So the units of B must be the same as the units of v/t, which would be $\boxed{\text{distance/time}^2}$.

33. (*a*) The quantity vt^2 has units of $(\text{m/s})(\text{s}^2) = \text{m}\bullet\text{s}$, which do not match with the units of meters for *x*. The quantity $2at$ has units $(\text{m/s}^2)(\text{s}) = \text{m/s}$, which also do not match with the units of meters for *x*. Thus this equation $\boxed{\text{cannot be correct}}$.

    (*b*) The quantity $v_0 t$ has units of $(\text{m/s})(\text{s}) = \text{m}$, and $\frac{1}{2}at^2$ has units of $(\text{m/s}^2)(\text{s}^2) = \text{m}$. Thus, since each term has units of meters, this equation $\boxed{\text{can be correct}}$.

    (*c*) The quantity $v_0 t$ has units of $(\text{m/s})(\text{s}) = \text{m}$, and $2at^2$ has units of $(\text{m/s}^2)(\text{s}^2) = \text{m}$. Thus, since each term has units of meters, this equation $\boxed{\text{can be correct}}$.

34. The percentage accuracy is $\frac{2\text{ m}}{2\times 10^7\text{ m}}\times 100\% = \boxed{1\times 10^{-5}\%}$. The distance of 20,000,000 m needs to be distinguishable from 20,000,002 m, which means that $\boxed{\text{8 significant figures}}$ are needed in the distance measurements.

35. Multiply the number of chips per wafer times the number of wafers that can be made fro a cylinder.

$$\left(100\frac{\text{chips}}{\text{wafer}}\right)\left(\frac{1\text{ wafer}}{0.60\text{ mm}}\right)\left(\frac{300\text{ mm}}{1\text{ cylinder}}\right) = \boxed{50,000\frac{\text{chips}}{\text{cylinder}}}$$

36. (*a*) # of seconds in 1.00 y: $1.00\text{ y} = \left(1.00\text{ y}\right)\left(\dfrac{3.156\times10^{7}\text{s}}{1\text{ y}}\right) = \boxed{3.16\times10^{7}\text{s}}$

(*b*) # of nanoseconds in 1.00 y: $1.00\text{ y} = \left(1.00\text{ y}\right)\left(\dfrac{3.156\times10^{7}\text{s}}{1\text{ y}}\right)\left(\dfrac{1\times10^{9}\text{ns}}{1\text{ s}}\right) = \boxed{3.16\times10^{16}\text{ns}}$

(*c*) # of years in 1.00 s: $1.00\text{ s} = \left(1.00\text{ s}\right)\left(\dfrac{1\text{ y}}{3.156\times10^{7}\text{s}}\right) = \boxed{3.17\times10^{-8}\text{y}}$

37. Assume that the alveoli are spherical, and that the volume of a typical human lung is about 2 liters, which is .002 m^3. The diameter can be found from the volume of a sphere, $\frac{4}{3}\pi r^3$.

$$\tfrac{4}{3}\pi r^{3} = \tfrac{4}{3}\pi\left(d/2\right)^{3} = \frac{\pi d^{3}}{6}$$

$$\left(3\times10^{8}\right)\pi\frac{d^{3}}{6} = 2\times10^{-3}\text{m}^{3} \quad\rightarrow\quad d = \left[\frac{6\left(2\times10^{-3}\right)}{3\times10^{8}\pi}\text{m}^{3}\right]^{1/3} = \boxed{2\times10^{-4}\text{m}}.$$

38. $1\text{ hectare} = \left(1\text{ hectare}\right)\left(\dfrac{10^{4}\text{ m}^{2}}{1\text{ hectare}}\right)\left(\dfrac{3.28\text{ ft}}{1\text{ m}}\right)^{2}\left(\dfrac{1\text{ acre}}{4\times10^{4}\text{ft}^{2}}\right) = \boxed{2.69\text{ acres}}$

39. (*a*) $\left(\dfrac{10^{-15}\text{kg}}{1\text{ bacterium}}\right)\left(\dfrac{1\text{ proton or neutron}}{10^{-27}\text{kg}}\right) = \boxed{10^{12}\text{ protons or neutrons}}$

(*b*) $\left(\dfrac{10^{-17}\text{kg}}{1\text{ DNA molecule}}\right)\left(\dfrac{1\text{ proton or neutron}}{10^{-27}\text{kg}}\right) = \boxed{10^{10}\text{ protons or neutrons}}$

(*c*) $\left(\dfrac{10^{2}\text{kg}}{1\text{ human}}\right)\left(\dfrac{1\text{ proton or neutron}}{10^{-27}\text{kg}}\right) = \boxed{10^{29}\text{ protons or neutrons}}$

(*d*) $\left(\dfrac{10^{41}\text{kg}}{1\text{ galaxy}}\right)\left(\dfrac{1\text{ proton or neutron}}{10^{-27}\text{kg}}\right) = \boxed{10^{68}\text{ protons or neutrons}}$

40. There are about 300,000,000 people in the United States. Assume that half of them have cars, that they each drive 12,000 miles per year, and their cars get 20 miles per gallon of gasoline.

$$\left(3\times10^{8}\text{people}\right)\left(\frac{1\text{ automobile}}{2\text{ people}}\right)\left(\frac{12{,}000\text{ mi}}{1\text{ y}}\right)\left(\frac{1\text{ gallon}}{20\text{ mi}}\right) \approx \boxed{1\times10^{11}\text{ gallons}/\text{y}}$$

41. Approximate the gumball machine as a rectangular box with a square cross-sectional area. In counting gumballs across the bottom, there are about 10 in a row. Thus we estimate that one layer contains about 100 gumballs. In counting vertically, we see that there are bout 15 rows. Thus we estimate that there are about $\boxed{\text{1500 gumballs}}$ in the machine.

42. The volume of water used by the people can be calculated as follows:

$$\left(4\times10^{4}\text{people}\right)\left(\frac{1200\text{ L/day}}{4\text{ people}}\right)\left(\frac{365\text{ day}}{1\text{ y}}\right)\left(\frac{1000\text{ cm}^{3}}{1\text{ L}}\right)\left(\frac{1\text{ km}}{10^{5}\text{ cm}}\right)^{3} = 4.4\times10^{-3}\text{ km}^{3}/\text{y}$$

The depth of water is found by dividing the volume by the area.

$$d = \frac{V}{A} = \frac{4.4\times10^{-3}\ \text{km}^3/\text{y}}{50\ \text{km}^2} = \left(8.76\times10^{-5}\ \frac{\text{km}}{\text{y}}\right)\left(\frac{10^5\,\text{cm}}{1\ \text{km}}\right) = 8.76\,\text{cm}/\text{y} \approx \boxed{9\,\text{cm}/\text{y}}$$

43. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. For our 1-ton rock, we can calculate the volume to be

$$V = (1\ \text{T})\left(\frac{2000\ \text{lb}}{1\ \text{T}}\right)\left(\frac{1\,\text{ft}^3}{186\ \text{lb}}\right) = 10.8\ \text{ft}^3 .$$

Then the radius is found by

$$d = 2r = 2\left(\frac{3V}{4\pi}\right)^{1/3} = 2\left[\frac{3\left(10.8\ \text{ft}^3\right)}{4\pi}\right]^{1/3} = 2.74\ \text{ft} \approx \boxed{3\ \text{ft}}$$

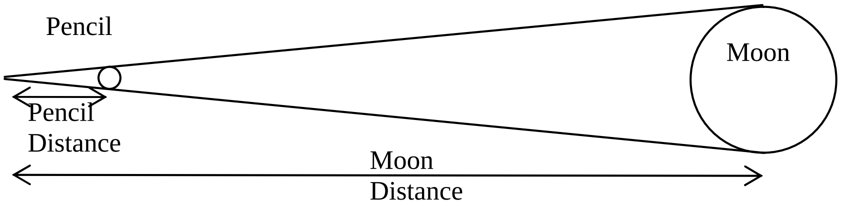
44. To calculate the mass of water, we need to find the volume of water, and then convert the volume to mass.

$$\left[\left(4\times10^1\ \text{km}^2\right)\left(\frac{10^5\text{cm}}{1\ \text{km}}\right)^2\right](1.0\ \text{cm})\left(\frac{10^{-3}\,\text{kg}}{1\ \text{cm}^3}\right)\left(\frac{1\ \text{ton}}{10^3\,\text{kg}}\right) = \boxed{4\times10^5\,\text{ton}}$$

To find the number of gallons, convert the volume to gallons.

$$\left[\left(4\times10^1\ \text{km}^2\right)\left(\frac{10^5\text{cm}}{1\ \text{km}}\right)^2\right](1.0\ \text{cm})\left(\frac{1\ \text{L}}{1\times10^3\,\text{cm}^3}\right)\left(\frac{1\ \text{gal}}{3.78\ \text{L}}\right) = \boxed{1\times10^8\,\text{gal}}$$

45. A pencil has a diameter of about 0.7 cm. If held about 0.75 m from the eye, it can just block out the Moon. The ratio of pencil diameter to arm length is the same as the ratio of Moon diameter to Moon distance. From the diagram, we have the following ratios.



$$\frac{\text{Pencil diameter}}{\text{Pencil distance}} = \frac{\text{Moon diameter}}{\text{Moon distance}} \quad \rightarrow$$

$$\text{Moon diameter} = \frac{\text{pencil diameter}}{\text{pencil distance}}(\text{Moon distance}) = \frac{7\times10^{-3}\,\text{m}}{0.75\ \text{m}}\left(3.8\times10^5\,\text{km}\right) \approx \boxed{3500\ \text{km}}$$

46. The person walks $4\,\text{km}/\text{h}$, 10 hours each day. The radius of the Earth is about 6380 km, and the distance around the world at the equator is the circumference, $2\pi R_{\text{Earth}}$. We assume that the person can "walk on water", and so ignore the existence of the oceans.

$$2\pi(6380\ \text{km})\left(\frac{1\ \text{h}}{4\ \text{km}}\right)\left(\frac{1\ \text{d}}{10\ \text{h}}\right) = \boxed{1\times10^3\ \text{d}}$$

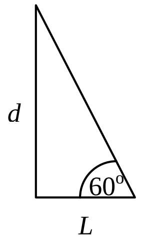
47. A cubit is about a half of a meter, by measuring several people's forearms. Thus the dimensions of Noah's ark would be $\boxed{\text{150 m long , 25 m wide, 15 m high}}$. The volume of the ark is found by multiplying the three dimensions.

$$V=(150\text{ m})(25\text{ m})(15\text{ m})=5.625\times10^{4}\,\text{m}^{3}\approx\boxed{6\times10^{4}\,\text{m}^{3}}$$

48. The volume of the oil will be the area times the thickness. The area is $\pi r^{2}=\pi\left(d/2\right)^{2}$, and so

$$V=\pi\left(d/2\right)^{2}t\ \rightarrow\ d=2\sqrt{\frac{V}{\pi t}}=2\sqrt{\frac{1000\text{cm}^{3}\left(\frac{1\text{ m}}{100\text{ cm}}\right)^{3}}{\pi\left(2\times10^{-10}\text{m}\right)}}=\boxed{3\times10^{3}\text{ m}}.$$

49. Consider the diagram shown. L is the distance she walks upstream, which is about 120 yards. Find the distance across the river from the diagram.



$$\tan60^{\circ}=\frac{d}{L}\ \rightarrow\ d=L\tan60^{\circ}=(120\text{ yd})\tan60^{\circ}=\boxed{210\text{ yd}}$$

$$(210\text{ yd})\left(\frac{3\text{ ft}}{1\text{ yd}}\right)\left(\frac{0.305\text{ m}}{1\text{ ft}}\right)=\boxed{190\text{ m}}$$

50. $\left(\frac{8\text{ s}}{1\text{ y}}\right)\left(\frac{1\text{ y}}{3.156\times10^{7}\text{s}}\right)\times100\%=\boxed{3\times10^{-5}\%}$

$\boxed{51.}$ The volume of a sphere is found by $V=\frac{4}{3}\pi r^{3}$.

$$V_{\text{Moon}}=\tfrac{4}{3}\pi R_{\text{Moon}}^{3}=\tfrac{4}{3}\pi\left(1.74\times10^{6}\text{m}\right)^{3}=\boxed{2.21\times10^{19}\text{m}^{3}}$$

$$\frac{V_{\text{Earth}}}{V_{\text{Moon}}}=\frac{\frac{4}{3}\pi R_{\text{Earth}}^{3}}{\frac{4}{3}\pi R_{\text{Moon}}^{3}}=\left(\frac{R_{\text{Earth}}}{R_{\text{Moon}}}\right)^{3}=\left(\frac{6.38\times10^{6}\text{m}}{1.74\times10^{6}\text{m}}\right)^{3}=49.3.$$

Thus it would take about $\boxed{49.3}$ Moons to create a volume equal to that of the Earth.

52. (*a*) $1.0\,\overset{\text{o}}{\text{A}}=\left(1.0\,\overset{\text{o}}{\text{A}}\right)\left(\frac{10^{-10}\text{m}}{1\overset{\text{o}}{\text{A}}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{m}}\right)=\boxed{0.10\text{ nm}}$

(*b*) $1.0\,\overset{\text{o}}{\text{A}}=\left(1.0\,\overset{\text{o}}{\text{A}}\right)\left(\frac{10^{-10}\text{m}}{1\overset{\text{o}}{\text{A}}}\right)\left(\frac{1\text{ fm}}{10^{-15}\text{m}}\right)=\boxed{1.0\times10^{5}\text{fm}}$

(*c*) $1.0\text{ m}=(1.0\text{ m})\left(\frac{1\overset{\text{o}}{\text{A}}}{10^{-10}\text{m}}\right)=\boxed{1.0\times10^{10}\,\overset{\text{o}}{\text{A}}}$

(*d*) $1.0\text{ ly}=(1.0\text{ ly})\left(\frac{9.46\times10^{15}\text{m}}{1\text{ ly}}\right)\left(\frac{1\overset{\text{o}}{\text{A}}}{10^{-10}\text{m}}\right)=\boxed{9.5\times10^{25}\,\overset{\text{o}}{\text{A}}}$

53. (*a*) Note that $\sin 15.0^{\circ} = 0.259$ and $\sin 15.5^{\circ} = 0.267$.

$$\left(\frac{\Delta\theta}{\theta}\right)100 = \left(\frac{0.5^{\circ}}{15.0^{\circ}}\right)100 = \boxed{3\%} \qquad \left(\frac{\Delta\sin\theta}{\sin\theta}\right)100 = \left(\frac{8\times10^{-3}}{0.259}\right)100 = \boxed{3\%}$$

(*b*) Note that $\sin 75.0^{\circ} = 0.966$ and $\sin 75.5^{\circ} = 0.968$.

$$\left(\frac{\Delta\theta}{\theta}\right)100 = \left(\frac{0.5^{\circ}}{75.0^{\circ}}\right)100 = \boxed{0.7\%} \qquad \left(\frac{\Delta\sin\theta}{\sin\theta}\right)100 = \left(\frac{2\times10^{-3}}{0.966}\right)100 = \boxed{0.2\%}$$

A consequence of this result is that when using a protractor, and you have a fixed uncertainty in the angle ($\pm 0.5^{\circ}$ in this case), you should measure the angles from a reference line that gives a large angle measurement rather than a small one. Note above that the angles around 75° had only a 0.2% error in sin θ, while the angles around 15° had a 3% error in sin θ.

54. Utilize the fact that walking totally around the Earth along the meridian would trace out a circle whose full 360° would equal the circumference of the Earth.

$$(1\text{ minute})\left(\frac{1^{\circ}}{60\text{ minute}}\right)\left(\frac{2\pi\left(6.38\times10^{3}\text{km}\right)}{360^{\circ}}\right)\left(\frac{0.621\text{ m}}{1\text{ km}}\right) = \boxed{1.15\text{ mi}}$$

CHAPTER 2: Describing Motion: Kinematics in One Dimension

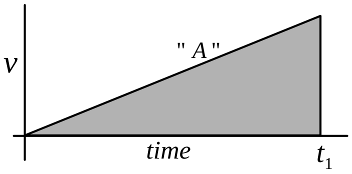
Answers to Questions

1. A car speedometer measures only speed. It does not give any information about the direction, and so does not measure velocity.

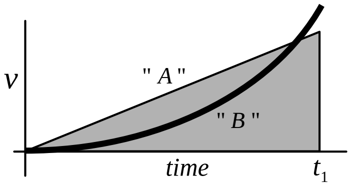
2. By definition, if an object has a constant velocity, then both the object's speed and its direction of motion are constant. Therefore the object CANNOT have a varying speed if its velocity is constant.

3. When an object moves with constant velocity, its average velocity over any time interval is exactly equal to its instantaneous velocity at all times

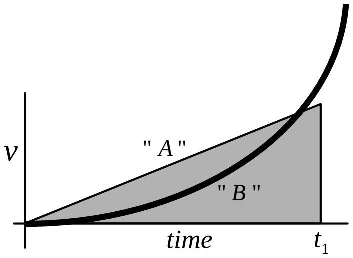
4. For both cars, the time elapsed is the distance traveled divided by the average velocity. Since both cars travel the same distance, the car with the larger average velocity will have the smaller elapsed time. Consider this scenario. Assume that one car has a constant acceleration down the track. Then a graph of its speed versus time would look like line "*A*" on the first graph. The shaded area of the graph represents the distance traveled, and the graph is plotted to such a time that the shaded area represents the length of the track. The time for this car to finish the race is labeled "t_1".



   Now let the second car have a much smaller acceleration initially, but with an increasing acceleration. A graph of its velocity, superimposed on the above graph and labeled "*B*", might look like the second diagram.



   It is seen that at the time t_1 when the first car finished the race, the second car is going faster than the first car, because the heavy line is "higher" on the graph than the line representing the first car. However, the area under the "B" line (the distance that the second car has traveled) is smaller than the shaded area, and so is less than the full track length. For the area under the "B" line to be the same as the area under the "A" line, the graph would need to look like the third diagram, indicating a longer time for the second car to finish the race.



5. There is no general relationship between the magnitude of speed and the magnitude of acceleration. For example, one object may have a large but constant speed. The acceleration of that object is then 0. Another object may have a small speed but be gaining speed, and therefore have a positive acceleration. So in this case the object with the greater speed has the lesser acceleration.

   Or consider two objects that are dropped from rest at different times. If we ignore air resistance, then the object dropped first will always have a greater speed than the object dropped second, but both will have the same acceleration of 9.80 m/s^2.

6. The acceleration of both the motorcycle and the bicycle are the same, since the same change in velocity occurred during the same time interval.

   If you do a further calculation, you will find that the distance traveled by the motorcycle during the acceleration is 17 times the distance traveled by the bicycle.

7. If an object is traveling to the north but slowing down, it has a northward velocity and a southward acceleration.

8. The velocity of an object can be negative when its acceleration is positive. If we define the positive direction to be to the right, then an object traveling to the left that is having a reduction in speed will have a negative velocity with a positive acceleration.

   If again we define the positive direction to be to the right, then an object traveling to the right that is having a reduction in speed will have a positive velocity and a negative acceleration.

9. If north is defined as the positive direction, then an object traveling to the south and increasing in speed has both a negative velocity and a negative acceleration. Or, if up is defined as the positive direction, then an object falling due to gravity has both a negative velocity and a negative acceleration.

10. If the two cars emerge side by side, then the one moving faster is passing the other one. Thus car A is passing car B. With the acceleration data given for the problem, the ensuing motion would be that car A would pull away from car B for a time, but eventually car B would catch up to and pass car A.

11. Assume that north is the positive direction. If a car is moving south and gaining speed at an increasing rate, then the acceleration will be getting larger in magnitude. However, since the acceleration is directed southwards, the acceleration is negative, and is getting more negative. That is a decreasing acceleration as the speed increases.

    Another example would be an object falling WITH air resistance. As the object falls, it gains speed, the air resistance increases. As the air resistance increases, the acceleration of the falling object decreases, and it gains speed less quickly the longer it falls.

12. Assuming that the catcher catches the ball at the same height at which it left the bat, then the ball will be traveling with a speed of 120 km/h when caught. This is proven in problem 41.

13. As a freely falling object speeds up, its acceleration due to gravity stays the same. If air resistance is considered, then the acceleration of the object is due to both gravity and air resistance. The total acceleration gets smaller as the object speeds up, until the object reaches a terminal velocity, at which time its total acceleration is zero. Thereafter its speed remains constant.

14. To estimate the height, throw the ball upward and time the flight from throwing to catching. Then, ignoring air resistance, the time of rising would be half of the time of flight. With that "half" time, assuming that the origin is at the top of the path and that downward is positive, knowing that the ball started from the top of the path with a speed of 0, use the equation $y = \frac{1}{2}gt^2$ with that time and the acceleration due to gravity to find the distance that the ball fell. With the same "half" time, we know that at the top of the path, the speed is 0. Taking the upward direction as positive, use the equation $v = v_0 + at \quad \rightarrow \quad 0 = v_0 - gt \quad \rightarrow \quad v_0 = gt$ to find the throwing speed.

15. The average speed is NOT 80 km/h. Since the two distances traveled were the same, the times of travel were unequal. The time to travel from A to B at 70 km/h is longer than the time to travel from B to C at 90 km/h. Thus we cannot simply average the speed numbers. To find the average speed, we need to calculate (total distance) / (total time). We assume the distance from A to B and the distance from B to C are both d km. The time to travel a distance d with a speed v is $t = d / v$.

$\overline{v} = \frac{d_{AB} + d_{BC}}{t_{AB} + t_{BC}} = \frac{(d \text{ km}) + (d \text{ km})}{\frac{d \text{ km}}{70 \text{ km/h}} + \frac{d \text{ km}}{90 \text{ km/h}}} = 78.75 \text{ km/h}$. The average speed is 78.75 km/h.

16. The sounds will not occur at equal time intervals because the longer any particular nut falls, the faster it will be going. With equal distances between nuts, each successive nut, having fallen a longer time when its predecessor reaches the plate, will have a higher average velocity and thus travel the inter-nut distance in shorter periods of time. Thus the sounds will occur with smaller and smaller intervals between sounds.

    To hear the sounds at equal intervals, the nuts would have to be tied at distances corresponding to equal time intervals. Since for each nut the distance of fall and time of fall are related by $d_i = \frac{1}{2} g t_i^2$, assume that $d_1 = \frac{1}{2} g t_1^2$. If we want $t_2 = 2t_1$, $t_3 = 3t_1$, $t_4 = 4t_1$, $\cdots$, then $d_2 = \frac{1}{2} g (2t_1)^2 = 4d_1$, $d_3 = \frac{1}{2} g (3t_1)^2 = 9d_1$, $d_4 = \frac{1}{2} g (4t_1)^2 = 16d_1$, etc.

17. The elevator moving from the second floor to the fifth floor is NOT an example of constant acceleration. The elevator accelerates upward each time it starts to move, and it accelerates downward each time it stops.

    Ignoring air resistance, a rock falling from a cliff would have a constant acceleration. (If air resistance is included, then the acceleration will be decreasing as the rock falls.) A dish resting on a table has an acceleration of 0, so the acceleration is constant.

18. As an object rises WITH air resistance, the acceleration is larger in magnitude than *g*, because both gravity and air resistance will be causing a downward acceleration. As the object FALLS with air resistance, the acceleration will be smaller in magnitude than *g*, because gravity and resistance will be opposing each other. Because of the smaller acceleration being applied over the same distance, the return speed will be slower than the launch speed.

19. If an object is at the instant of reversing direction (like an object thrown upward, at the top of its path), it instantaneously has a zero velocity and a non-zero acceleration at the same time. A person at the exact bottom of a "bungee" cord plunge also has an instantaneous velocity of zero but a non-zero (upward) acceleration at the same time.

20. An object moving with a constant velocity has a non-zero velocity and a zero acceleration at the same time. So a car driving at constant speed on a straight, level roadway would meet this condition.

21. The object starts with a constant velocity in the positive direction. At about $t = 17$ s, when the object is at the 5 meter position, it begins to gain speed – it has a positive acceleration. At about $t = 27$ s, when the object is at about the 12 m position, it begins to slow down – it has a negative acceleration. The object instantaneously stops at about $t = 37$ s, reaching its maximum distance from the origin of 20 m. The object then reverses direction, gaining speed while moving backwards. At about $t = 47$ s, when the object is again at about the 12 m position, the object starts to slow down, and appears to stop at $t = 50$ s, 10 m from the starting point.

22. Initially, the object moves in the positive direction with a constant acceleration, until about $t = 45$ s, when it has a velocity of about 37 m/s in the positive direction. The acceleration then decreases, reaching an instantaneous acceleration of 0 at about $t = 50$ s, when the object has its maximum speed of about 38 m/s. The object then begins to slow down, but continues to move in the positive

direction. The object stops moving at t = 90 s and stays at rest until about t = 108 s. Then the object begins to move to the right again, at first with a large acceleration, and then a lesser acceleration. At the end of the recorded motion, the object is still moving to the right and gaining speed.

Solutions to Problems

1. The average speed is given by:

$$\bar{v} = d/\Delta t = 235\text{ km}/3.25\text{ h} = \boxed{72.3\text{ km/h}}.$$

2. The time of travel can be found by rearranging the average speed equation.

$$\bar{v} = d/\Delta t \rightarrow \Delta t = d/\bar{v} = (15\text{ km})/(25\text{ km/h}) = \boxed{0.60\text{ h}} = 36\text{ min}$$

3. The distance of travel (displacement) can be found by rearranging the average speed equation. Also note that the units of the velocity and the time are not the same, so the speed units will be converted.

$$\bar{v} = \frac{d}{\Delta t} \rightarrow d = \bar{v}\Delta t = (110\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right)(2.0\text{ s}) = 0.061\text{ km} = \boxed{61\text{ m}}$$

4. (*a*) $35\text{ mi/h} = (35\text{ mi/h})(1.61\text{ km/mi}) = \boxed{56\text{ km/h}}$

(*b*) $35\text{ mi/h} = (35\text{ mi/h})(1610\text{ m/mi})(1\text{ h}/3600\text{ s}) = \boxed{16\text{ m/s}}$

(*c*) $35\text{ mi/h} = (35\text{ mi/h})(5280\text{ ft/mi})(1\text{ h}/3600\text{ s}) = \boxed{51\text{ ft/s}}$

5. The average velocity is given by $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{-4.2\text{ cm} - 3.4\text{ cm}}{6.1\text{ s} - 3.0\text{ s}} = \dfrac{-7.6\text{ cm}}{3.1\text{ s}} = \boxed{-2.5\text{ cm/s}}$.

6. The average velocity is given by $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{8.5\text{ cm} - 3.4\text{ cm}}{4.5\text{ s} - (-2.0\text{ s})} = \dfrac{5.1\text{ cm}}{6.5\text{ s}} = \boxed{0.78\text{ cm/s}}$.

$\boxed{\text{The average speed cannot be calculated.}}$ To calculate the average speed, we would need to know the actual distance traveled, and it is not given.

7. The time for the first part of the trip is calculated from the initial speed and the first distance.

$$\text{ave speed}_1 = v_1 = \frac{d_1}{\Delta t_1} \rightarrow \Delta t_1 = \frac{d_1}{v_1} = \frac{130\text{ km}}{95\text{ km/h}} = 1.37\text{ h} = 82\text{ min}$$

The time for the second part of the trip is therefore

$$\Delta t_2 = \Delta t_{tot} - \Delta t_1 = 3.33\text{ h} - 1.37\text{ h} = 1.96\text{ h} = 118\text{ min}$$

The distance for the second part of the trip is calculated from the average speed for that part of the trip and the time for that part of the trip.

$$\text{ave speed}_2 = v_2 = \frac{d_2}{\Delta t_2} \rightarrow d_2 = v_2\Delta t_2 = (65\text{ km/h})(1.96\text{ h}) = 127.5\text{ km} = 1.3\times10^2\text{ km}$$

(*a*) The total distance is then $d_{total} = d_1 + d_2 = 130\text{ km} + 127.5\text{ km} = 257.5\text{ km} \approx \boxed{2.6\times10^2\text{ km}}$

(*b*) The average speed is NOT the average of the two speeds. Use the definition of average speed.

$$\text{ave speed} = \frac{d_{total}}{\Delta t_{total}} = \frac{257.5\text{ km}}{3.33\text{ h}} = \boxed{77\text{ km/h}}$$

8. The speed of sound is intimated in the problem as 1 mile per 5 seconds. The speed is calculated by:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \left(\frac{1\text{ mi}}{5\text{ s}}\right)\left(\frac{1610\text{ m}}{1\text{ mi}}\right) = \boxed{300\text{ m/s}}.$$

Note that only 1 significant figure is given, (5 sec), and so only 1 significant figure is justified in the result.

$\boxed{9.}$ The distance traveled is 2.0 miles $(8\text{ laps} \times 0.25\text{ mi/lap})$. The displacement is 0 because the ending point is the same as the starting point.

(*a*) Average speed $= \dfrac{d}{\Delta t} = \dfrac{2.0\text{ mi}}{12.5\text{ min}} = \left(\dfrac{2\text{ mi}}{12.5\text{ min}}\right)\left(\dfrac{1610\text{ m}}{1\text{ mi}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = \boxed{4.3\text{ m/s}}$

(*b*) Average velocity $= \overline{v} = \Delta x/\Delta t = \boxed{0\text{ m/s}}$

10. The distance traveled is $116\text{ km} + \frac{1}{2}(116\text{ km}) = 174\text{ km}$, and the displacement is $116\text{ km} - \frac{1}{2}(116\text{ km}) = 58\text{ km}$. The total time is 14.0 s + 4.8 s = 18.8 s.

(*a*) Average speed $= \dfrac{d}{\Delta t} = \dfrac{174\text{ m}}{18.8\text{ s}} = \boxed{9.26\text{ m/s}}$

(*b*) Average velocity $= \overline{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{58\text{ m}}{18.8\text{ s}} = \boxed{3.1\text{ m/s}}$

11. Since the locomotives have the same speed, they each travel half the distance, 4.25 km. Find the time of travel from the average speed.

$$\text{ave speed} = v = \frac{d}{\Delta t} \rightarrow \Delta t = \frac{d}{v} = \frac{4.25\text{ km}}{95\text{ km/h}} = 0.0447\text{ h}\left(\frac{60\text{ min}}{1\text{ h}}\right) = 2.68\text{ min} \cong \boxed{2.7\text{ min}}$$

12. Both objects will have the same time of travel. If the truck travels a distance d_{truck}, then the distance the car travels will be $d_{\text{car}} = d_{\text{truck}} + 110\text{ m}$. Using the equation for average speed, $\overline{v} = d/\Delta t$, solve for time, and equate the two times.

$$\Delta t = \frac{d_{\text{truck}}}{\overline{v}_{\text{truck}}} = \frac{d_{\text{car}}}{\overline{v}_{\text{car}}} \qquad \frac{d_{\text{truck}}}{75\text{ km/h}} = \frac{d_{\text{truck}} + 110\text{ m}}{88\text{ km/h}}$$

Solving for d_{truck} gives $d_{\text{truck}} = (110\text{ m})\dfrac{(75\text{ km/h})}{(88\text{ km/h} - 75\text{ km/h})} = 634.6\text{ m}$.

The time of travel is

$$\Delta t = \frac{d_{\text{truck}}}{\overline{v}_{\text{truck}}} = \left(\frac{634.6\text{ m}}{75000\text{ m/h}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = 0.5077\text{ min} = 30.46\text{ s} = \boxed{3.0 \times 10^1\text{ s}}.$$

Also note that $\Delta t = \dfrac{d_{\text{car}}}{\overline{v}_{\text{car}}} = \left(\dfrac{634.6\text{ m} + 110\text{ m}}{88000\text{ m/h}}\right)\left(\dfrac{60\text{ min}}{1\text{ h}}\right) = 0.5077\text{ min} = 30.46\text{ s}$.

ALTERNATE SOLUTION:
The speed of the car relative to the truck is $88\ \text{km/h} - 75\ \text{km/h} = 13\ \text{km/h}$. In the reference frame of the truck, the car must travel 110 m to catch it.

$$\Delta t = \frac{0.11\ \text{km}}{13\,\text{km/h}}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 30.46\ \text{s}$$

13. The average speed for each segment of the trip is given by $\overline{v} = \frac{d}{\Delta t}$, so $\Delta t = \frac{d}{\overline{v}}$ for each segment.

For the first segment, $\Delta t_1 = \frac{d_1}{\overline{v}_1} = \frac{3100\ \text{km}}{790\ \text{km/h}} = 3.924\ \text{h}$.

For the second segment, $\Delta t_2 = \frac{d_2}{\overline{v}_2} = \frac{2800\ \text{km}}{990\ \text{km/h}} = 2.828\ \text{h}$.

Thus the total time is $\Delta t_{\text{tot}} = \Delta t_1 + \Delta t_2 = 3.924\ \text{h} + 2.828\ \text{h} = 6.752\ \text{h} \approx \boxed{6.8\ \text{h}}$.

The average speed of the plane for the entire trip is

$$\overline{v} = \frac{d_{\text{tot}}}{\Delta t_{\text{tot}}} = \frac{3100\ \text{km} + 2800\ \text{km}}{6.752\ \text{h}} = 873.8 \approx \boxed{8.7\times10^2\ \text{km/h}}.$$

14. The distance traveled is 500 km (250 km outgoing, 250 km return, keep 2 significant figures). The displacement (Δx) is 0 because the ending point is the same as the starting point.

(*a*) To find the average speed, we need the distance traveled (500 km) and the total time elapsed. During the outgoing portion, $\overline{v}_1 = \frac{d_1}{\Delta t_1}$ and so $\Delta t_1 = \frac{d_1}{\overline{v}_1} = \frac{250\ \text{km}}{95\,\text{km/h}} = 2.632\ \text{h}$. During the return portion, $\overline{v}_2 = \frac{d_2}{\Delta t_2}$, and so $\Delta t_2 = \frac{d_2}{\overline{v}_2} = \frac{250\ \text{km}}{55\,\text{km/h}} = 4.545\ \text{h}$. Thus the total time, including lunch, is $\Delta t_{\text{total}} = \Delta t_1 + \Delta t_{\text{lunch}} + \Delta t_2 = 8.177\ \text{h}$. Average speed $= \frac{d_{total}}{\Delta t_{total}} = \frac{500\ \text{km}}{8.177\ \text{h}} = \boxed{61\,\text{km/h}}$.

(*b*) Average velocity = $\boxed{\overline{v} = \Delta x/\Delta t = 0}$

15. The average speed of sound is given by $\overline{v} = d/\Delta t$, and so the time for the sound to travel from the end of the lane back to the bowler is $\Delta t_{\text{sound}} = \frac{d}{\overline{v}_{\text{sound}}} = \frac{16.5\ \text{m}}{340\ \text{m/s}} = 4.85\times10^{-2}\,\text{s}$. Thus the time for the ball to travel from the bowler to the end of the lane is given by $\Delta t_{\text{ball}} = \Delta t_{\text{total}} - \Delta t_{\text{sound}} = 2.50\ \text{s} - 4.85\times10^{-2}\,\text{s} = 2.4515\ \text{s}$. And so the speed of the ball is:

$$\overline{v}_{\text{ball}} = \frac{d}{\Delta t_{\text{ball}}} = \frac{16.5\ \text{m}}{2.4515\ \text{s}} = \boxed{6.73\,\text{m/s}}.$$

16. The average acceleration is given by

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{95\,\text{km/h} - 0\,\text{km/h}}{6.2\ \text{s}} = \frac{(95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)}{6.2\ \text{s}} = \boxed{4.3\,\text{m/s}^2}.$$

17. (*a*) The average acceleration of the sprinter is $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{10.0\,\mathrm{m/s} - 0.0\,\mathrm{m/s}}{1.35\ \mathrm{s}} = \boxed{7.41\,\mathrm{m/s^2}}$.

(*b*) $\bar{a} = \left(7.41\,\mathrm{m/s^2}\right)\left(\dfrac{1\ \mathrm{km}}{1000\ \mathrm{m}}\right)\left(\dfrac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right)^2 = \boxed{9.60\times 10^4\ \mathrm{km/h^2}}$

$\boxed{18.}$ The time can be found from the average acceleration, $\bar{a} = \dfrac{\Delta v}{\Delta t}$.

$$\Delta t = \frac{\Delta v}{\bar{a}} = \frac{110\,\mathrm{km/h} - 80\,\mathrm{km/h}}{1.6\,\mathrm{m/s^2}} = \frac{\left(30\,\mathrm{km/h}\right)\left(\dfrac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)}{1.6\,\mathrm{m/s^2}} = \boxed{5.2\ \mathrm{s}}$$

19. The initial velocity of the car is the average speed of the car before it accelerates.

$$\bar{v} = \frac{d}{\Delta t} = \frac{110\ \mathrm{m}}{5.0\ \mathrm{s}} = 22\ \mathrm{m/s} = v_0$$

The final speed is $v = 0$, and the time to stop is 4.0 s. Use Eq. 2-11a to find the acceleration.

$$v = v_0 + at \quad \rightarrow$$

$$a = \frac{v - v_0}{t} = \frac{0 - 22\,\mathrm{m/s}}{4.0\ \mathrm{s}} = \boxed{-5.5\,\mathrm{m/s^2}} = \left(-5.5\,\mathrm{m/s^2}\right)\left(\frac{1\,g}{9.80\,\mathrm{m/s^2}}\right) = \boxed{-0.56\ g\text{'s}}$$

20. To estimate the velocity, find the average velocity over each time interval, and assume that the car had that velocity at the midpoint of the time interval. To estimate the acceleration, find the average acceleration over each time interval, and assume that the car had that acceleration at the midpoint of the time interval. A sample of each calculation is shown.

From 2.00 s to 2.50 s, for average velocity:

$$t_{\mathrm{mid}} = \frac{2.50\ \mathrm{s} + 2.00\ \mathrm{s}}{2} = 2.25\ \mathrm{s}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{13.79\ \mathrm{m} - 8.55\ \mathrm{m}}{2.50\ \mathrm{s} - 2.00\ \mathrm{s}} = \frac{5.24\ \mathrm{m}}{0.50\ \mathrm{s}} = 10.48\,\mathrm{m/s}$$

From 2.25 s to 2.75 s, for average acceleration:

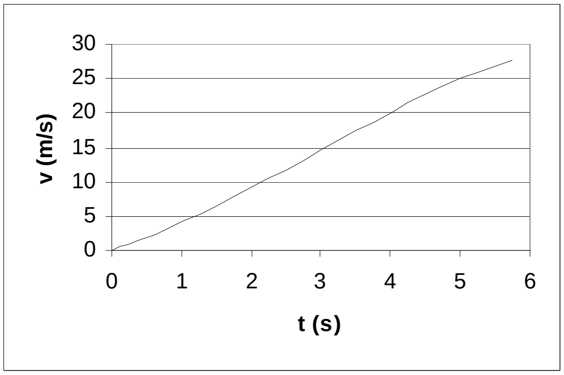
$$t_{\mathrm{mid}} = \frac{2.25\ \mathrm{s} + 2.75\ \mathrm{s}}{2} = 2.50\ \mathrm{s}$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{13.14\,\mathrm{m/s} - 10.48\,\mathrm{m/s}}{2.75\ \mathrm{s} - 2.25\ \mathrm{s}} = \frac{2.66\,\mathrm{m/s}}{0.50\ \mathrm{s}} = 5.32\,\mathrm{m/s^2}$$

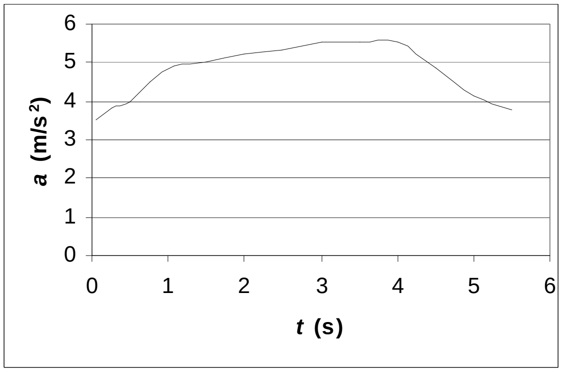
Table of Calculations

t (s)	x (m)	t (s)	v (m/s)	t (s)	a (m/s^2)
0.00	0.00	0.00	0.00		
				0.063	3.52
		0.125	0.44		
0.25	0.11			0.25	3.84
		0.375	1.40		
0.50	0.46			0.50	4.00
		0.625	2.40		
0.75	1.06			0.75	4.48
		0.875	3.52		
1.00	1.94			1.06	4.91
		1.25	5.36		
1.50	4.62			1.50	5.00
		1.75	7.86		
2.00	8.55			2.00	5.24
		2.25	10.48		
2.50	13.79			2.50	5.32
		2.75	13.14		
3.00	20.36			3.00	5.52
		3.25	15.90		
3.50	28.31			3.50	5.56
		3.75	18.68		
4.00	37.65			4.00	5.52
		4.25	21.44		
4.50	48.37			4.50	4.84
		4.75	23.86		
5.00	60.30			5.00	4.12
		5.25	25.92		
5.50	73.26			5.50	3.76
		5.75	27.80		
6.00	87.16				

Graph of the velocity



Graph of the acceleration



21. By definition, the acceleration is $a=\dfrac{v-v_0}{t}=\dfrac{25\,\text{m/s}-13\,\text{m/s}}{6.0\text{ s}}=\boxed{2.0\,\text{m/s}^2}$.
The distance of travel can be found from Eq. 2-11b.

$$x-x_0=v_0t+\tfrac{1}{2}at^2=(13\,\text{m/s})(6.0\text{ s})+\tfrac{1}{2}(2.0\,\text{m/s}^2)(6.0\text{ s})^2=\boxed{114\text{ m}}$$

22. The acceleration can be found from Eq. (2-11c).

$$v^2=v_0^2+2a(x-x_0)\ \rightarrow\ a=\frac{v^2-v_0^2}{2(x-x_0)}=\frac{0-(23\,\text{m/s})^2}{2(85\text{ m})}=\boxed{-3.1\,\text{m/s}^2}.$$

23. Assume that the plane starts from rest. The runway distance is found by solving Eq. 2-11c for $x-x_0$.

$$v^2=v_0^2+2a(x-x_0)\ \rightarrow\ x-x_0=\frac{v^2-v_0^2}{2a}=\frac{(33\,\text{m/s})^2-0}{2(3.0\,\text{m/s}^2)}=\boxed{1.8\times10^2\,\text{m}}$$

24. The sprinter starts from rest. The average acceleration is found from Eq. 2-11c.

$$v^2=v_0^2+2a(x-x_0)\ \rightarrow\ a=\frac{v^2-v_0^2}{2(x-x_0)}=\frac{(11.5\,\text{m/s})^2-0}{2(15.0\text{ m})}=4.408\,\text{m/s}^2\approx\boxed{4.41\,\text{m/s}^2}.$$

The elapsed time is found by solving Eq. 2-11a for time.

$$v=v_0+at\ \rightarrow\ t=\frac{v-v_0}{a}=\frac{11.5\,\text{m/s}-0}{4.408\,\text{m/s}^2}=\boxed{2.61\text{ s}}$$

25. The words “slowing down uniformly” implies that the car has a constant acceleration. The distance of travel is found form combining Eqs. 2-7 and 2-8.

$$x-x_0=\frac{v_0+v}{2}t=\left(\frac{21.0\,\text{m/s}+0\,\text{m/s}}{2}\right)(6.00\text{ sec})=\boxed{63.0\text{ m}}.$$

$\boxed{26.}$ The final velocity of the car is zero. The initial velocity is found from Eq. 2-11c with $v=0$ and solving for v_0.

$$v^2=v_0^2+2a(x-x_0)\ \rightarrow\ v_0=\sqrt{v^2-2a(x-x_0)}=\sqrt{0-2(-7.00\,\text{m/s}^2)(92\text{ m})}=\boxed{36\,\text{m/s}}$$

27. The final velocity of the driver is zero. The acceleration is found from Eq. 2-11c with $v = 0$ and solving for a.

$$a = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{0 - \left[\left(85\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{2\left(0.80\text{ m}\right)} = -348.4\,\text{m/s}^2 \approx \boxed{-3.5\times 10^2\text{ m/s}^2}$$

Converting to "g's": $a = \dfrac{-3.484\times 10^2\text{ m/s}^2}{\left(9.8\,\text{m/s}^2\right)/g} = \boxed{-36\,g\text{'s}}$.

28. The origin is the location of the car at the beginning of the reaction time. The initial speed of the car is $\left(95\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s}$. The location where the brakes are applied is found from the equation for motion at constant velocity: $x_0 = v_0 t_R = \left(26.39\,\text{m/s}\right)\left(1.0\text{ s}\right) = 26.39\text{ m}$. This is now the starting location for the application of the brakes. In each case, the final speed is 0.

(*a*) Solve Eq. 2-11c for the final location.

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \quad \rightarrow \quad x = x_0 + \frac{v^2 - v_0^2}{2a} = 26.39\text{ m} + \frac{0 - \left(26.39\,\text{m/s}\right)^2}{2\left(-4.0\,\text{m/s}^2\right)} = \boxed{113\text{ m}}$$

(*b*) Solve Eq. 2-11c for the final location with the second acceleration.

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 26.39\text{ m} + \frac{0 - \left(26.39\,\text{m/s}\right)^2}{2\left(-8.0\,\text{m/s}^2\right)} = \boxed{70\text{ m}}$$

29. The origin is the location of the car at the beginning of the reaction time. The location where the brakes are applied is found from the equation for motion at constant velocity: $x_0 = v_0 t_R$
This is the starting location for the application of the brakes. Solve Eq. 2-11c for the final location of the car, with $v = 0$.

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = v_0 t_R - \frac{v_0^2}{2a}$$

30. The critical condition is that the total distance covered by the passing car and the approaching car must be less than 400 m so that they do not collide. The passing car has a total displacement composed of several individual parts. These are: i) the 10 m of clear room at the rear of the truck, ii) the 20 m length of the truck, iii) the 10 m of clear room at the front of the truck, and iv) the distance the truck travels. Since the truck travels at a speed of $\overline{v} = 25\,\text{m/s}$, the truck will have a displacement of $\Delta x_{\text{truck}} = \left(25\,\text{m/s}\right)t$. Thus the total displacement of the car during passing is $\Delta x_{\substack{\text{passing}\\\text{car}}} = 40\,\text{m} + \left(25\,\text{m/s}\right)t$.

To express the motion of the car, we choose the origin to be at the location of the passing car when the decision to pass is made. For the passing car, we have an initial velocity of $v_0 = 25\,\text{m/s}$ and an acceleration of $a = 1.0\,\text{m/s}^2$. Find $\Delta x_{\substack{\text{passing}\\\text{car}}}$ from Eq. 2-11b.

$$\Delta x_{\substack{\text{passing}\\\text{car}}} = x_c - x_0 = v_0 t + \tfrac{1}{2}at = \left(25\,\text{m/s}\right)t + \tfrac{1}{2}\left(1.0\,\text{m/s}^2\right)t^2$$

Set the two expressions for $\Delta x_{\substack{\text{passing}\\\text{car}}}$ equal to each other in order to find the time required to pass.

$$40\text{ m}+(25\text{ m/s})t_{\text{pass}}=(25\text{ m/s})t_{\text{pass}}+\tfrac{1}{2}(1.0\text{ m/s}^2)t_{\text{pass}}^2 \rightarrow 40\text{ m}=\tfrac{1}{2}(1.0\text{ m/s}^2)t_{\text{pass}}^2 \rightarrow$$

$$t_{\text{pass}}=\sqrt{80\text{s}^2}=8.94\text{ s}$$

Calculate the displacements of the two cars during this time.

$$\Delta x_{\substack{\text{passing}\\\text{car}}}=40\text{ m}+(25\text{ m/s})(8.94\text{ s})=264\text{ m}$$

$$\Delta x_{\substack{\text{approaching}\\\text{car}}}=v_{\substack{\text{approaching}\\\text{car}}}t=(25\text{ m/s})(8.94\text{ s})=224\text{ m}$$

Thus the two cars together have covered a total distance of 488 m, which is more than allowed.

$\boxed{\text{The car should not pass.}}$

31. During the final part of the race, the runner must have a displacement of 1100 m in a time of 180 s (3 min). Assume that the starting speed for the final part is the same as the average speed thus far.

$$\text{Average speed}=\frac{d}{\Delta t}=\frac{8900\text{ m}}{(27\times 60)\text{ s}}=5.494\text{ m/s}=v_0$$

The runner will accomplish this by accelerating from speed v_0 to speed v for t seconds, covering a distance d_1, and then running at a constant speed of v for $(180-t)$ seconds, covering a distance d_2. We have these relationships:

$$v=v_o+at \qquad d_1=v_ot+\tfrac{1}{2}at^2 \qquad d_2=v(180-t)=(v_0+at)(180-t)$$

$$1100\text{ m}=d_1+d_2=v_ot+\tfrac{1}{2}at^2+(v_0+at)(180-t) \rightarrow 1100\text{ m}=180v_0+180at-\tfrac{1}{2}at^2 \rightarrow$$

$$1100\text{ m}=(180\text{ s})(5.494\text{ m/s})+(180\text{ s})(0.2\text{ m/s}^2)t-\tfrac{1}{2}(0.2\text{ m/s}^2)t^2$$

$$0.1t^2-36t+111=0 \qquad t=357\text{ s},\ 3.11\text{ s}$$

Since we must have $t<180\text{ s}$, the solution is $t=3.1\text{ s}$.

$\boxed{32.}$ The car's initial speed is $v_o=(45\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=12.5\text{ m/s}$.

Case I: trying to stop. The constraint is, with the braking deceleration of the car $\left(a=-5.8\text{ m/s}^2\right)$, can the car stop in a 28 m displacement? The 2.0 seconds has no relation to this part of the problem. Using equation (2-11c), the distance traveled during braking is

$$(x-x_0)=\frac{v^2-v_0^2}{2a}=\frac{0-(12.5\text{ m/s})^2}{2(-5.8\text{ m/s}^2)}=13.5\text{ m} \quad \boxed{\text{She can stop the car in time.}}$$

Case II: crossing the intersection. The constraint is, with the acceleration of the car $\left[a=\left(\frac{65\text{ km/h}-45\text{ km/h}}{6.0\text{ s}}\right)\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=0.9259\text{ m/s}^2\right]$, can she get through the intersection (travel 43 meters) in the 2.0 seconds before the light turns red? Using equation (2.11b), the distance traveled during the 2.0 sec is

$$(x-x_0)=v_0t+\tfrac{1}{2}at^2=(12.5\text{ m/s})(2.0\text{ s})+\tfrac{1}{2}(0.927\text{ m/s}^2)(2.0\text{ s})^2=26.9\text{ m}.$$

$\boxed{\text{She should stop.}}$

33. Choose downward to be the positive direction, and take $y_0 = 0$ at the top of the cliff. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80\,\mathrm{m/s^2}$. The displacement is found from equation (2-11b), with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad y - 0 = 0 + \tfrac{1}{2}\left(9.80\,\mathrm{m/s^2}\right)\left(3.25\ \mathrm{s}\right)^2 \quad \rightarrow \quad y = \boxed{51.8\ \mathrm{m}}$$

34. Choose downward to be the positive direction. The initial velocity is $v_0 = 0$, the final velocity is $v = \left(85\,\mathrm{km/h}\right)\left(\dfrac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 23.61\,\mathrm{m/s}$, and the acceleration is $a = 9.80\,\mathrm{m/s^2}$. The time can be found by solving Eq. 2-11a for the time.

$$v = v_0 + at \quad \rightarrow \quad t = \frac{v - v_0}{a} = \frac{23.61\,\mathrm{m/s} - 0}{9.80\,\mathrm{m/s^2}} = \boxed{2.4\ \mathrm{s}}$$

35. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the top of the Empire State Building. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80\,\mathrm{m/s^2}$.

(*a*) The elapsed time can be found from Eq. 2-11b, with *x* replaced by *y*.

$$y - y_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2\left(380\ \mathrm{m}\right)}{9.80\,\mathrm{m/s^2}}} = 8.806\,\mathrm{s} \approx \boxed{8.8\ \mathrm{s}}\,.$$

(*b*) The final velocity can be found from equation (2-11a).

$$v = v_0 + at = 0 + \left(9.80\,\mathrm{m/s^2}\right)\left(8.806\ \mathrm{s}\right) = \boxed{86\,\mathrm{m/s}}$$

$\boxed{36.}$ Choose upward to be the positive direction, and take $y_0 = 0$ to be at the height where the ball was hit. For the upward path, $v_0 = 22\,\mathrm{m/s}$, $v = 0$ at the top of the path, and $a = -9.80\,\mathrm{m/s^2}$.

(*a*) The displacement can be found from Eq. 2-11c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - \left(22\,\mathrm{m/s}\right)^2}{2\left(-9.80\,\mathrm{m/s^2}\right)} = \boxed{25\ \mathrm{m}}$$

(*b*) The time of flight can be found from Eq. 2-11b, with *x* replaced by *y*, using a displacement of 0 for the displacement of the ball returning to the height from which it was hit.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow \quad t\left(v_0 + \tfrac{1}{2}at\right) = 0 \quad \rightarrow \quad t = 0\ ,\ t = \frac{2v_0}{-a} = \frac{2\left(22\,\mathrm{m/s}\right)}{-9.80\,\mathrm{m/s^2}} = \boxed{4.5\ \mathrm{s}}$$

The result of *t* = 0 s is the time for the original displacement of zero (when the ball was hit), and the result of *t* = 4.5 s is the time to return to the original displacement. Thus the answer is *t* = 4.5 seconds.

37. Choose upward to be the positive direction, and take $y_0 = 0$ to be the height from which the ball was thrown. The acceleration is $a = -9.80\,\mathrm{m/s^2}$. The displacement upon catching the ball is 0, assuming it was caught at the same height from which it was thrown. The starting speed can be found from Eq. 2-11b, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow$$

$$v_0 = \frac{y - y_0 - \tfrac{1}{2}at^2}{t} = -\tfrac{1}{2}at = -\tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)\left(3.0\ \mathrm{s}\right) = 14.7\,\mathrm{m/s} \approx \boxed{15\,\mathrm{m/s}}$$

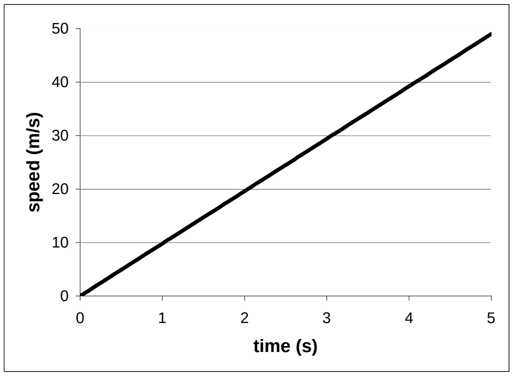
The height can be calculated from Eq. 2-11c, with a final velocity of $v = 0$ at the top of the path.

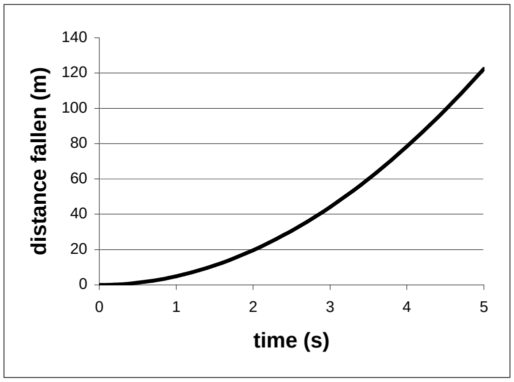
$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - \left(14.7\,\mathrm{m/s}\right)^2}{2\left(-9.8\,\mathrm{m/s^2}\right)} = \boxed{11\ \mathrm{m}}$$

38. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the height where the object was released. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80\,\mathrm{m/s^2}$.

(*a*) The speed of the object will be given by Eq. 2-11a with $v_0 = 0$, and so $v = at = \left(9.80\,\mathrm{m/s^2}\right)t$. This is the equation of a straight line passing through the origin with a slope of $9.80\,\mathrm{m/s^2}$.

(*b*) The distance fallen will be given by equation (2-11b) with $v_0 = 0$, and so $y = y_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \left(4.90\,\mathrm{m/s^2}\right)t^2$. This is the equation of a parabola, centered on the t-axis, opening upward.





39. Choose downward to be the positive direction, and take $y_0 = 0$ to be the height where the object was released. The initial velocity is $v_0 = -5.20\,\mathrm{m/s^2}$, the acceleration is $a = 9.80\,\mathrm{m/s^2}$, and the displacement of the package will be $y = 125\ \mathrm{m}$. The time to reach the ground can be found from Eq. 2-11b, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t^2 + \frac{2v_0}{a}t - \frac{2y}{a} = 0 \quad \rightarrow \quad t^2 + \frac{2\left(-5.2\,\mathrm{m/s}\right)}{9.80\,\mathrm{m/s^2}}t - \frac{2\left(125\ \mathrm{m}\right)}{9.80\,\mathrm{m/s^2}} = 0 \quad \rightarrow$$

$$t = 5.61\ \mathrm{s}\ ,\ -4.55\ \mathrm{s}$$

The correct time is the positive answer, $\boxed{t = 5.61\ \mathrm{s}}$.

40. Choose downward to be the positive direction, and take $y_0 = 0$ to be the height from which the object is released. The initial velocity is $v_0 = 0$, the acceleration is $a = g$. Then we can calculate the position as a function of time from Eq. 2-11b, with *x* replaced by *y*, as $y\left(t\right) = \frac{1}{2}gt^2$. At the end of each second, the position would be as follows:

$$y(0) = 0\ ; \quad y(1) = \tfrac{1}{2}g\ ; \quad y(2) = \tfrac{1}{2}g\left(2\right)^2 = 4y(1)\ ; \quad y(3) = \tfrac{1}{2}g\left(3\right)^2 = 9y(1)$$

The distance traveled during each second can be found by subtracting two adjacent position values from the above list.

$$d(1) = y(1) - y(0) = y(1)\ ; \quad d(2) = y(2) - y(1) = 3y(1)\ ; \quad d(3) = y(3) - y(2) = 5y(1)$$

We could do this in general.

$$y(n)=\tfrac{1}{2}gn^2 \qquad y(n+1)=\tfrac{1}{2}g(n+1)^2$$

$$d(n+1)=y(n+1)-y(n)=\tfrac{1}{2}g(n+1)^2-\tfrac{1}{2}gn^2=\tfrac{1}{2}g\left((n+1)^2-n^2\right)$$

$$=\tfrac{1}{2}g\left(n^2+2n+1-n^2\right)=\tfrac{1}{2}g(2n+1)$$

The value of $(2n+1)$ is always odd, in the sequence $\boxed{1, 3, 5, 7, \ldots}$

41. Choose upward to be the positive direction, and take $y_0 = 0$ to be the height from which the ball is thrown. The initial velocity is v_0, the acceleration is $a=-g$, and the final location for the round trip is $y=0$. The velocity under those conditions can be found from Eq. 2-11c, with *x* replaced by *y*.

$$v^2-v_0^2=2ay=0 \quad\rightarrow\quad v^2=v_0^2 \quad\rightarrow\quad \boxed{v=\pm v_0}$$

The two results represent two different velocities for the same displacement of 0. The positive sign $(v=v_0)$ is the initial velocity, when the ball is moving upwards, and the negative sign $(v=-v_0)$ is the final velocity, when the ball is moving downwards. Both of these velocities have the same magnitude, and so the ball has the same speed at the end of its flight as at the beginning.

42. Choose upward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is thrown. We have $v_0 = 18.0\,\mathrm{m/s}$, $a=-9.80\,\mathrm{m/s^2}$, and $y-y_0 = 11.0\ \mathrm{m}$.

(*a*) The velocity can be found from Eq, 2-11c, with *x* replaced by *y*.

$$v^2=v_0^2+2a(y-y_0)=0 \quad\rightarrow$$

$$v=\pm\sqrt{v_0^2+2ay}=\pm\sqrt{(18.0\,\mathrm{m/s})^2+2(-9.80\,\mathrm{m/s^2})(11.0\ \mathrm{m})}=\pm 10.4\,\mathrm{m/s}$$

Thus the speed is $\boxed{|v|=10.4\,\mathrm{m/s}}$

(*b*) The time to reach that height can be found from equation (2-11b).

$$y=y_0+v_0t+\tfrac{1}{2}at^2 \quad\rightarrow\quad t^2+\frac{2(18.0\,\mathrm{m/s})}{-9.80\,\mathrm{m/s^2}}t+\frac{2(-11.0\ \mathrm{m})}{-9.80\,\mathrm{m/s^2}}=0 \quad\rightarrow$$

$$t^2-3.673t+2.245=0 \quad\rightarrow\quad \boxed{t=2.90\ \mathrm{s}\ ,\ 0.775\ \mathrm{s}}$$

(*c*) $\boxed{\text{There are two times at which the object reaches that height}}$ – once on the way up $(t=0.775\ \mathrm{s})$, and once on the way down $(t=2.90\ \mathrm{s})$.

43. The 10-cm (100 mm) apple has a diameter of about 6 mm as measured in the photograph. Thus any distances measured from the picture need to be multiplied by 100 / 6. Choose the downward direction to be positive. Choose $y_0 = 0$ to be stem of the apple on the THIRD image from the top of the picture. It is the first picture in which the stem of the apple is visible. The velocity of the apple at that position is not 0, but it is not known either. Call it v_0. We will choose that the time at that point is $t=0$, and we call the time interval from one picture to the next to be T. The acceleration of the apple is $a=g=9.8\,\mathrm{m/s^2}$.

The 3rd picture after the $t = 0$ picture (the first one that is not overlapping with another image) has the stem 16.5 mm from the origin of coordinates, at a time of $t = 3T$. The actual position would be found by

$$y_1 = (16.5\text{ mm})(100/6) = 275\text{ mm} = 0.275\text{ m}.$$

The 6th picture after the $t = 0$ picture (the next to last one in the picture) has the stem 42 mm from the origin of coordinates, at a time of $t = 6T$. The actual position would be found by

$$y_2 = (42\text{ mm})(100/6) = 700\text{ mm} = 0.70\text{ m}.$$

Now we have two sets of position-time data, relative to the origin. Both of those sets of position-time data must satisfy equation Eq. 2-11b.

$$y_1 = y_0 + v_0 t_1 + \tfrac{1}{2}at_1^2 \quad \rightarrow \quad 0.275 = 3v_0 T + \tfrac{1}{2}g(3T)^2$$

$$y_2 = y_0 + v_0 t_2 + \tfrac{1}{2}at_2^2 \quad \rightarrow \quad 0.70 = 6v_0 T + \tfrac{1}{2}g(6T)^2$$

Multiply the first equation by 2, and then subtract it from the second equation to eliminate the dependence on v_0. The resulting equation can be solved for T.

$$\left.\begin{array}{r} 0.55\,\text{m} = 6v_0 T + 9gT^2 \\ 0.70\,\text{m} = 6v_0 T + 18gT^2 \end{array}\right\} \rightarrow \quad 0.15\,\text{m} = 9gT^2 \quad \rightarrow \quad T = \sqrt{\frac{0.15\,\text{m}}{9(9.8\,\text{m/s}^2)}} = \boxed{4.1\times10^{-2}\,\text{s}}$$

This is equivalent to $\dfrac{1\text{ flash}}{T} = \dfrac{1\text{ flash}}{4.1\times10^{-2}\,\text{s}} = \boxed{24\text{ flashes per second}}$.

44. Choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is dropped. Call the location of the top of the window y_w, and the time for the stone to fall from release to the top of the window is t_w. Since the stone is dropped from rest, using Eq. 2-11b with *y* substituting for *x*, we have $y_w = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}gt_w^2$. The location of the bottom of the window is $y_w + 2.2\text{ m}$, and the time for the stone to fall from release to the bottom of the window is $t_w + 0.28\text{ s}$. Since the stone is dropped from rest, using Eq. 2-11b, we have $y_w + 2.2\text{ m} = y_0 + v_0 + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}g(t_w + 0.28\text{ s})^2$. Substituting the first expression for y_w into the second one.

$$\tfrac{1}{2}gt_w^2 + 2.2\text{ m} = \tfrac{1}{2}g(t_w + 0.28\text{ s})^2 \quad \rightarrow \quad t_w = 0.662\text{ s}$$

Use this time in the first equation.

$$y_w = \tfrac{1}{2}gt_w^2 = \tfrac{1}{2}(9.8\,\text{m/s}^2)(0.662\text{ s})^2 = \boxed{2.1\text{ m}}.$$

45. For the falling rock, choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is dropped. The initial velocity is $v_0 = 0\,\text{m/s}$, the acceleration is $a = g$, the displacement is $y = H$, and the time of fall is t_1. Using Eq. 2-11b with *y* substituting for *x*, we have $H = y_0 + v_0 t + \tfrac{1}{2}t^2 = 0 + 0 + \tfrac{1}{2}gt_1^2$.

For the sound wave, use the constant speed equation that $v_s = \dfrac{d}{\Delta t} = \dfrac{H}{T - t_1}$, which can be rearranged to give $t_1 = T - \dfrac{H}{v_s}$, where $T = 3.2\ \text{s}$ is the total time elapsed from dropping the rock to hearing the sound. Insert this expression for t_1 into the equation for *H*, and solve for *H*.

$$H = \tfrac{1}{2}g\left(T - \frac{H}{v_s}\right)^2 \rightarrow \frac{g}{2v_s^2}H^2 - \left(\frac{gT}{v_s}+1\right)H + \tfrac{1}{2}gT^2 = 0 \rightarrow$$

$$4.239\times10^{-5}H^2 - 1.092H + 50.18 = 0 \rightarrow H = 46.0\ \text{m},\ 2.57\times10^4\ \text{m}$$

If the larger answer is used in $t_1 = T - \dfrac{H}{v_s}$, a negative time of fall results, and so the physically correct answer is $\boxed{H = 46\ \text{m}}$.

46. Choose upward to be the positive direction, and $y_0 = 0$ to be the location of the nozzle. The initial velocity is v_0, the acceleration is $a = -9.8\ \text{m/s}^2$, the final location is $y = -1.5\ \text{m}$, and the time of flight is $t = 2.0\ \text{s}$. Using Eq. 2-11b and substituting *y* for *x* gives the following.

$$y = y_0 + v_0t + \tfrac{1}{2}at^2 \rightarrow v_0 = \frac{y - \frac{1}{2}at^2}{t} = \frac{-1.5\ \text{m} - \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)\left(2.0\ \text{s}\right)^2}{2.0\ \text{s}} = \boxed{9.1\ \text{m/s}}$$

$\boxed{47.}$ Choose downward to be the positive direction, and $y_0 = 0$ to be at the top of the cliff. The initial velocity is $v_0 = -12.0\ \text{m/s}$, the acceleration is $a = 9.80\ \text{m/s}^2$, and the final location is $y = 70.0\ \text{m}$.

(*a*) Using Eq. 2-11b and substituting *y* for *x*, we have

$$y = y_0 + v_0t + \tfrac{1}{2}at^2 \rightarrow \left(4.9\ \text{m/s}^2\right)t^2 - \left(12.0\ \text{m/s}\right)t - 70\ \text{m} = 0 \rightarrow t = -2.749\ \text{s},\ 5.198\ \text{s}.$$

The positive answer is the physical answer: $\boxed{t = 5.20\ \text{s}}$.

(*b*) Using Eq. 2-11a, we have $v = v_0 + at = -12.0\ \text{m/s} + \left(9.80\ \text{m/s}^2\right)\left(5.198\ \text{s}\right) = \boxed{38.9\ \text{m/s}}$.

(*c*) The total distance traveled will be the distance up plus the distance down. The distance down will be 70 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0. Then using Eq. 2-11c:

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \rightarrow y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - \left(-12.0\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = -7.35\ \text{m}.$$

Thus the distance up is 7.35 m, the distance down is 77.35 m, and the total distance traveled is $\boxed{84.7\ \text{m}}$.

48. Choose upward to be the positive direction, and $y_0 = 0$ to be the level from which the ball was thrown. The initial velocity is v_0, the instantaneous velocity is $v = 13\ \text{m/s}$, the acceleration is $a = -9.80\ \text{m/s}^2$, and the location of the window is $y = 28\ \text{m}$.

(*a*) Using Eq. 2-11c and substituting *y* for *x*, we have

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \rightarrow$$

$$v_0 = \pm\sqrt{v^2 - 2a\left(y - y_0\right)} = \pm\sqrt{\left(13\ \text{m/s}\right)^2 - 2\left(-9.8\ \text{m/s}^2\right)\left(28\ \text{m}\right)} = \boxed{27\ \text{m/s}}$$

Choose the positive value because the initial direction is upward.

(*b*) At the top of its path, the velocity will be 0, and so we can use the initial velocity as found above, along with Eq. 2-11c.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (27\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = \boxed{37\ \mathrm{m}}$$

(*c*) We want the time elapsed from throwing (speed $v_0 = 27\,\mathrm{m/s}$) to reaching the window (speed $v = 13\,\mathrm{m/s}$). Using Eq. 2-11a, we have:

$$v = v_0 + at \quad \rightarrow \quad t = \frac{v - v_0}{a} = \frac{13\,\mathrm{m/s} - 27\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = \boxed{1.4\ \mathrm{s}}.$$

(*d*) We want the time elapsed from the window (speed $v_0 = 13\,\mathrm{m/s}$) to reaching the street (speed $v = -27\,\mathrm{m/s}$). Using Eq. 2-11a, we have:

$$v = v_0 + at \quad \rightarrow \quad t = \frac{v - v_0}{a} = \frac{-27\,\mathrm{m/s} - 13\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = \boxed{4.1\ \mathrm{s}}.$$

49. Slightly different answers may be obtained since the data comes from reading the graph.

(*a*) The greatest velocity is found at the highest point on the graph, which is at $\boxed{t \approx 48\ \mathrm{s}}$.

(*b*) The indication of a constant velocity on a velocity-time graph is a slope of 0, which occurs from $\boxed{t = 90\ \mathrm{s}\ \text{ to }\ t \approx 108\ \mathrm{s}}$.

(*c*) The indication of a constant acceleration on a velocity-time graph is a constant slope, which occurs from $\boxed{t = 0\ \mathrm{s}\ \text{ to }\ t \approx 38\ \mathrm{s}}$, again from $\boxed{t \approx 65\ \mathrm{s}\ \text{ to }\ t \approx 83\ \mathrm{s}}$, and again from $\boxed{t = 90\ \mathrm{s}\ \text{ to }\ t \approx 108\ \mathrm{s}}$.

(*d*) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from $\boxed{t \approx 65\ \mathrm{s}\ \text{ to }\ t \approx 83\ \mathrm{s}}$.

50. Slightly different answers may be obtained since the data comes from reading the graph.

(*a*) The instantaneous velocity is given by the slope of the tangent line to the curve. At $t = 10.0\ \mathrm{s}$, the slope is approximately $v(10) \approx \dfrac{3\,\mathrm{m} - 0}{10.0\ \mathrm{s} - 0} = \boxed{0.3\,\mathrm{m/s}}$.

(*b*) At $t = 30.0\ \mathrm{s}$, the slope of the tangent line to the curve, and thus the instantaneous velocity, is approximately $v(30) \approx \dfrac{22\ \mathrm{m} - 8\ \mathrm{m}}{35\ \mathrm{s} - 25\ \mathrm{s}} = \boxed{1.4\,\mathrm{m/s}}$.

(*c*) The average velocity is given by $\bar{v} = \dfrac{x(5)\,\mathrm{m} - x(0)\,\mathrm{m}}{5.0\ \mathrm{s} - 0\ \mathrm{s}} = \dfrac{1.5\ \mathrm{m} - 0}{5.0\ \mathrm{s}} = \boxed{.30\,\mathrm{m/s}}$.

(*d*) The average velocity is given by $\bar{v} = \dfrac{x(30)\,\mathrm{m} - x(25)\,\mathrm{m}}{30.0\ \mathrm{s} - 25.0\ \mathrm{s}} = \dfrac{16\ \mathrm{m} - 9\ \mathrm{m}}{5.0\ \mathrm{s}} = \boxed{1.4\,\mathrm{m/s}}$.

(*e*) The average velocity is given by $\bar{v} = \dfrac{x(50)\,\mathrm{m} - x(40)\,\mathrm{m}}{50.0\ \mathrm{s} - 40.0\ \mathrm{s}} = \dfrac{10\ \mathrm{m} - 19.5\ \mathrm{m}}{10.0\ \mathrm{s}} = \boxed{-0.95\,\mathrm{m/s}}$.

51. Slightly different answers may be obtained since the data comes from reading the graph.

(*a*) The indication of a constant velocity on a position-time graph is a constant slope, which occurs from $\boxed{t = 0\ \mathrm{s}\ \text{ to }\ t \approx 18\ \mathrm{s}}$.

(*b*) The greatest velocity will occur when the slope is the highest positive value, which occurs at about $\boxed{t = 27\ \text{s}}$.

(*c*) The indication of a 0 velocity on a position-time graph is a slope of 0, which occurs at about from $\boxed{t = 38\ \text{s}}$.

(*d*) $\boxed{\text{The object moves in both directions.}}$ When the slope is positive, from $t = 0\ \text{s}$ to $t = 38\ \text{s}$, the object is moving in the positive direction. When the slope is negative, from $t = 38\ \text{s}$ to $t = 50\ \text{s}$, the object is moving in the negative direction.

52. Slightly different answers may be obtained since the data comes from reading the graph. We assume that the short, nearly horizontal portions of the graph are the times that shifting is occurring, and those times are not counted as being "in" a certain gear.

(*a*) The average acceleration in 2nd gear is given by $\bar{a}_2 = \dfrac{\Delta v_2}{\Delta t_2} = \dfrac{24\ \text{m/s} - 14\ \text{m/s}}{8\ \text{s} - 4\ \text{s}} = \boxed{2.5\ \text{m/s}^2}$.

The average acceleration in 4th gear is given by $\bar{a}_4 = \dfrac{\Delta v_4}{\Delta t_4} = \dfrac{44\ \text{m/s} - 37\ \text{m/s}}{27\ \text{s} - 16\ \text{s}} = \boxed{0.64\ \text{m/s}^2}$.

(*b*) The distance traveled can be determined from a velocity-time graph by calculating the area between the graph and the $v = 0$ axis, bounded by the times under consideration. For this case, we will approximate the area as a rectangle.

$$\text{height} = \bar{v} = \frac{v_f + v_0}{2} = \frac{44\ \text{m/s} + 37\ \text{m/s}}{2} = 40.5\ \text{m/s} \quad \text{width} = \Delta t = 27\ \text{s} - 16\ \text{s} = 11\ \text{s}$$

Thus the distance traveled is $d = \bar{v}\Delta t = (40.5\ \text{m/s})(11\ \text{s}) = \boxed{450\ \text{m}}$.

53. Slightly different answers may be obtained since the data comes from reading the graph. We assume that the short, nearly horizontal portions of the graph are the times that shifting is occurring, and those times are not counted as being "in" a certain gear.

(*a*) The average acceleration in first gear is given by $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{14\ \text{m/s} - 0\ \text{m/s}}{4\ \text{s} - 0\ \text{s}} = \boxed{4\ \text{m/s}^2}$.

(*b*) The average acceleration in third gear is given by $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{37\ \text{m/s} - 24\ \text{m/s}}{14\ \text{s} - 9\ \text{s}} = \boxed{3\ \text{m/s}^2}$.

(*c*) The average acceleration in fifth gear is given by $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{52\ \text{m/s} - 44\ \text{m/s}}{50\ \text{s} - 27\ \text{s}} = \boxed{0.35\ \text{m/s}^2}$.

(*d*) The average acceleration through the first four gears is given by

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{44\ \text{m/s} - 0\ \text{m/s}}{27\ \text{s} - 0\ \text{s}} = \boxed{1.6\ \text{m/s}^2}.$$

54. Slightly different answers may be obtained since the data comes from reading the graph.

(*a*) To estimate the distance the object traveled during the first minute, we need to find the area under the graph, from $t = 0$ s to $t = 60$ s. Each "block" of the graph represents an "area" of $\Delta x = (10\ \text{m/s})(10\ \text{s}) = 100\ \text{m}$. By counting and estimating, there are about 17.5 blocks under the 1st minute of the graph, and so the distance traveled during the 1st minute is about $\boxed{1750\ \text{m}}$.

(*b*) For the second minute, there are about 5 blocks under the graph, and so the distance traveled during the second minute is about $\boxed{500\ \text{m}}$.

Alternatively, average accelerations can be estimated for various portions of the graph, and then the uniform acceleration equations may be applied. For instance, for part (*a*), break the motion up into two segments, from 0 to 50 seconds and then from 50 to 60 seconds.

(*a*) t = 0 to 50: $\overline{a}_1 = \dfrac{\Delta v}{\Delta t} = \dfrac{38\,\text{m/s} - 14\,\text{m/s}}{50\text{ s} - 0\text{ s}} = 0.48\,\text{m/s}^2$

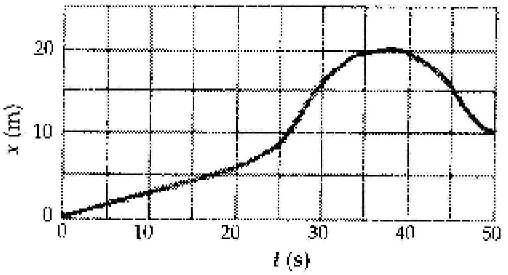
$$d_1 = v_{o1}t_1 + \tfrac{1}{2}\overline{a}_1 t_1^2 = (14\,\text{m/s})(50\text{ s}) + \tfrac{1}{2}(0.48\,\text{m/s}^2)(50\text{ s})^2 = 1300\text{ m}$$

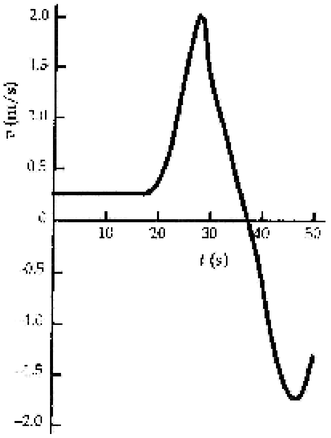
$$\overline{a}_2 = \frac{\Delta v}{\Delta t} = \frac{31\,\text{m/s} - 38\,\text{m/s}}{60\text{ s} - 50\text{ s}} = -0.70\,\text{m/s}^2$$

$$d_2 = v_{o2}t_2 + \tfrac{1}{2}\overline{a}_2 t_2^2 = (38\,\text{m/s})(10\text{ s}) + \tfrac{1}{2}(-0.70\,\text{m/s}^2)(10\text{ s})^2 = 345\text{ m}$$

$$d_1 + d_2 = 1645\text{ m}$$

55. The *v* vs. *t* graph is found by taking the slope of the *x* vs. *t* graph. Both graphs are shown here.





56. (*a*) During the interval from A to B, it is $\boxed{\text{moving in the negative direction}}$, because its displacement is negative.

(*b*) During the interval from A to B, it is $\boxed{\text{speeding up}}$, because the magnitude of its slope is increasing (changing from less steep to more steep).

(*c*) During the interval from A to B, $\boxed{\text{the acceleration is negative}}$, because the graph is concave downward, indicating that the slope is getting more negative, and thus the acceleration is negative.

(*d*) During the interval from D to E, it is $\boxed{\text{moving in the positive direction}}$, because the displacement is positive.

(*e*) During the interval from D to E, it is $\boxed{\text{speeding up}}$, because the magnitude of its slope is increasing (changing from less steep to more steep).

(*f*) During the interval from D to E, $\boxed{\text{the acceleration is positive}}$, because the graph is concave upward, indicating the slope is getting more positive, and thus the acceleration is positive.

(*g*) During the interval from C to D, $\boxed{\text{the object is not moving in either direction}}$.

$\boxed{\text{The velocity and acceleration are both 0.}}$

57. (*a*) For the free-falling part of the motion, choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the person jumped. The initial velocity is $v_0 = 0$, acceleration is $a = 9.80\,\text{m/s}^2$, and the location of the net is $y = 15.0\ \text{m}$. Find the speed upon reaching the net from Eq. (2-11c) with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a\left(y - y_{(\ )}\right) \quad \rightarrow \quad v = \pm\sqrt{0 + 2a\left(y - 0\right)} = \pm\sqrt{2\left(9.80\,\text{m/s}^2\right)\left(15.0\ \text{m}\right)} = 17.1\,\text{m/s}$$

The positive root is selected since the person is moving downward.
For the net-stretching part of the motion, choose downward to be the positive direction, and $y_0 = 15.0\ \text{m}$ to be the height at which the person first contacts the net. The initial velocity is $v_0 = 17.1\,\text{m/s}$, the final velocity is $v = 0$, and the location at the stretched position is $y = 16.0\ \text{m}$. Find the acceleration from Eq. (2-11c) with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2\left(y - y_0\right)} = \frac{0^2 - \left(17.1\,\text{m/s}\right)^2}{2\left(1.0\ \text{m}\right)} = \boxed{-150\,\text{m/s}^2}$$

(*b*) For the acceleration to be smaller, in the above equation we see that the displacement would have to be larger. This means that the net should be $\boxed{\text{"loosened"}}$.

58. Choose the upward direction to be positive, and $y_0 = 0$ to be the level from which the object was thrown. The initial velocity is v_0 and the velocity at the top of the path is $v = 0\,\text{m/s}$. The height at the top of the path can be found from Eq. (2-11c) with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow \quad y - y_0 = \frac{-v_0^2}{2a}.$$

From this we see that the displacement is inversely proportional to the acceleration, and so if the acceleration is reduced by a factor of 6 by going to the Moon, and the initial velocity is unchanged, the $\boxed{\text{displacement increases by a factor of 6}}$.

59. The initial velocity of the car is $v_0 = \left(100\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 27.8\,\text{m/s}$. Choose $x_0 = 0$ to be location at which the deceleration begins. We have $v = 0\,\text{m/s}$ and $a = -30g = -294\,\text{m/s}^2$. Find the displacement from Eq. (2-11c).

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \quad \rightarrow \quad x = x_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - \left(27.8\,\text{m/s}\right)^2}{2\left(-2.94\times10^2\ \text{m/s}^2\right)} = 1.31\,\text{m} \approx \boxed{1.3\ \text{m}}$$

60. Choose downward to be the positive direction, and $y_0 = 0$ to be at the height of the bridge. Agent Bond has an initial velocity of $v_0 = 0$, an acceleration of $a = g$, and will have a displacement of $y = 12\ \text{m} - 1.5\ \text{m} = 11.5\ \text{m}$. Find the time of fall from Eq. 2-11b with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2\left(11.5\ \text{m}\right)}{9.80\,\text{m/s}^2}} = 1.532\ \text{s}$$

If the truck is approaching with $v = 25\,\text{m/s}$, then he needs to jump when the truck is a distance away given by $d = vt = \left(25\,\text{m/s}\right)\left(1.532\ \text{s}\right) = 38.30\ \text{m}$. Convert this distance into "poles".

$$d = \left(38.30\ \text{m}\right)\left(1\ \text{pole}/25\ \text{m}\right) = 1.53\ \text{poles}$$

So he should jump when the truck is about $\boxed{\text{1.5 poles}}$ away from the bridge.

61. (*a*) Choose downward to be the positive direction, and $y_0 = 0$ to be the level from which the car was dropped. The initial velocity is $v_0 = 0$, the final location is $y = H$, and the acceleration is $a = g$. Find the final velocity from Eq. 2-11c, replacing *x* with *y*.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \rightarrow v = \pm\sqrt{v_0^2 + 2a\left(y - y_0\right)} = \pm\sqrt{2gH}\ .$$

The speed is the magnitude of the velocity, $\boxed{v = \sqrt{2gH}}$.

(*b*) Solving the above equation for the height, we have that $H = \dfrac{v^2}{2g}$. Thus for a collision of

$$v = \left(60\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 16.67\,\text{m/s}$$, the corresponding height is:

$$H = \frac{v^2}{2g} = \frac{\left(16.67\,\text{m/s}\right)^2}{2\left(9.80\,\text{m/s}^2\right)} = 14.17\,\text{m} \approx \boxed{14\text{ m}}.$$

(*c*) For a collision of $v = \left(100\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 27.78\,\text{m/s}$, the corresponding height is:

$$H = \frac{v^2}{2g} = \frac{\left(27.78\,\text{m/s}\right)^2}{2\left(9.80\,\text{m/s}^2\right)} = 39.37\,\text{m} \approx \boxed{39\text{ m}}.$$

62. The average speed is the distance divided by the time.

$$\overline{v} = \frac{d}{t} = \left(\frac{1\times10^9\text{ km}}{1\text{ y}}\right)\left(\frac{1\text{ y}}{365\text{ d}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right) = 1.142\times10^5\text{ km/h} \approx \boxed{1\times10^5\text{ km/h}}$$

63. Use the information for the first 180 m to find the acceleration, and the information for the full motion to find the final velocity. For the first segment, the train has $v_0 = 0\,\text{m/s}$, $v_1 = 25\,\text{m/s}$, and a displacement of $x_1 - x_0 = 180\text{ m}$. Find the acceleration from Eq. 2-11c.

$$v_1^2 = v_0^2 + 2a\left(x_1 - x_0\right) \rightarrow a = \frac{v_1^2 - v_0^2}{2\left(x_1 - x_0\right)} = \frac{\left(25\,\text{m/s}\right)^2 - 0}{2\left(180\text{ m}\right)} = 1.736\,\text{m/s}^2$$

Find the speed of the train after it has traveled the total distance (total displacement of $x_2 - x_0 = 275\text{ m}$) using Eq. 2-11c.

$$v_2^2 = v_0^2 + 2a\left(x_2 - x_0\right) \rightarrow v_2 = \sqrt{v_0^2 + 2a\left(x_2 - x_0\right)} = \sqrt{2\left(1.736\,\text{m/s}^2\right)\left(275\text{ m}\right)} = \boxed{31\,\text{m/s}}.$$

64. For the motion in the air, choose downward to be the positive direction, and $y_0 = 0$ to be at the height of the diving board. Then diver has $v_0 = 0$, (assuming the diver does not jump upward or downward), $a = g = 9.8\,\text{m/s}^2$, and $y = 4.0\text{ m}$ when reaching the surface of the water. Find the diver's speed at the water's surface from Eq. 2-11c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a\left(y - y_0\right)x \rightarrow v = \pm\sqrt{v_0^2 + 2a\left(y - y_0\right)} = \sqrt{0 + 2\left(9.8\,\text{m/s}^2\right)\left(4.0\text{ m}\right)} = 8.85\,\text{m/s}$$

For the motion in the water, again choose down to be positive, but redefine $y_0 = 0$ to be at the surface of the water. For this motion, $v_0 = 8.85\,\text{m/s}$, $v = 0$, and $y - y_0 = 2.0\text{ m}$. Find the acceleration from Eq. 2-11c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(y - y_0)x} = \frac{0 - (8.85\,\text{m/s})^2}{2(2.0\text{ m})} = -19.6\,\text{m/s}^2 \approx \boxed{-20\,\text{m/s}^2}$$

The negative sign indicates that the acceleration is directed upwards.

65. This problem can be analyzed as a series of three one-dimensional motions: the acceleration phase, the constant speed phase, and the deceleration phase. The maximum speed of the train is:

$$(90\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 25\,\text{m/s}\,.$$

In the acceleration phase, the initial velocity is $v_0 = 0\,\text{m/s}$, the acceleration is $a = 1.1\,\text{m/s}^2$, and the final velocity is $v = 25\,\text{m/s}$. Find the elapsed time for the acceleration phase from Eq. 2-11a.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{acc}} = \frac{v - v_0}{a} = \frac{25\,\text{m/s} - 0}{1.1\,\text{m/s}^2} = 22.73\text{ s}\,.$$

Find the displacement during the acceleration phase from Eq. 2-11b.

$$(x - x_0)_{\text{acc}} = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(1.1\,\text{m/s}^2)(22.73\text{ s})^2 = 284\text{ m}\,.$$

In the deceleration phase, the initial velocity is $v_0 = 25\,\text{m/s}$, the acceleration is $a = -2.0\,\text{m/s}^2$, and the final velocity is $v = 0\,\text{m/s}$. Find the elapsed time for the deceleration phase from equation Eq. 2-11a.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{dec}} = \frac{v - v_0}{a} = \frac{0 - 25\,\text{m/s}}{-2.0\,\text{m/s}^2} = 12.5\text{ s}\,.$$

Find the distance traveled during the deceleration phase from Eq. 2-11b.

$$(x - x_0)_{\text{dec}} = v_0 t + \tfrac{1}{2}at^2 = (25\,\text{m/s})(12.5\text{ s}) + \tfrac{1}{2}(-2.0\,\text{m/s}^2)(12.5\text{ s})^2 = 156\text{ m}\,.$$

The total elapsed time and distance traveled for the acceleration / deceleration phases are:

$$\begin{aligned} t_{\text{acc}} + t_{\text{dec}} &= 22.7\text{ s} + 12.5\text{ s} = 35.2\text{ s} \\ (x - x_0)_{\text{acc}} + (x - x_0)_{\text{dec}} &= 284\text{ m} + 156\text{ m} = 440\text{ m} \end{aligned}\,.$$

(*a*) If the stations are spaced 1.80 km = 1800 m apart, then there is a total of $\frac{9000\text{ m}}{1800\text{ m}} = 5$ inter-station segments. A train making the entire trip would thus have a total of 5 inter-station segments and 4 stops of 20 s each at the intermediate stations. Since 440 m is traveled during acceleration and deceleration, 1360 m of each segment is traveled at an average speed of $\bar{v} = 25\,\text{m/s}$. The time for that 1360 m is given by

$$d = \bar{v}t \quad \rightarrow \quad t_{\substack{\text{constant}\\\text{speed}}} = \frac{d}{\bar{v}} = \frac{1360\text{ m}}{25\,\text{m/s}} = 54.4\text{ s}\,.$$

Thus a total inter-station segment will take 35.2 s + 54.4 s = 89.6 s. With 5 inter-station segments of 89.6 s each, and 4 stops of 20 s each, the total time is given by:

$$t_{0.8\text{ km}} = 5(89.6\,\text{s}) + 4(20\,\text{s}) = 528\text{ s} = \boxed{8.8\text{ min}}\,.$$

(*b*) If the stations are spaced 3.0 km =3000 m apart, then there is a total of $\frac{9000\text{ m}}{3000\text{ m}} = 3$ inter-station segments. A train making the entire trip would thus have a total of 3 inter-station segments and 2 stops of 20 s each at the intermediate stations. Since 440 m is traveled during acceleration and deceleration, 2560 m of each segment is traveled at an average speed of $\bar{v} = 25\,\text{m/s}$. The time for that 2560 m is given by $d = \bar{v}t \rightarrow t = \frac{d}{\bar{v}} = \frac{2560\text{ m}}{25\,\text{m/s}} = 102.4\text{ s}$.

Thus a total inter-station segment will take 35.2 s + 102.4 s = 137.6 s. With 3 inter-station segments of 137.6 s each, and 2 stops of 20 s each, the total time is

$$t_{3.0\text{ km}} = 3(137.6\,\text{s}) + 2(20\,\text{s}) = 453\text{ s} = \boxed{7.5\text{ min}}.$$

66. Choose downward to be the positive direction, and $y_0 = 0$ to be at the start of the pelican's dive. The pelican has an initial velocity is $v_0 = 0$ and an acceleration of $a = g$, and a final location of $y = 16.0\text{ m}$. Find the total time of the pelican's dive from Eq. 2-11b, with *x* replaced by *y*.

$$y = y_0 + v_0t + \tfrac{1}{2}at^2 \rightarrow y = 0 + 0 + \tfrac{1}{2}at^2 \rightarrow t_{\text{dive}} = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(16.0\text{ m})}{9.80\,\text{m/s}^2}} = 1.81\text{ s}.$$

The fish can take evasive action if he sees the pelican at a time of 1.81 s – 0.20 s = 1.61 s into the dive. Find the location of the pelican at that time from Eq. 2-11b.

$$y = y_0 + v_0t + \tfrac{1}{2}at = 0 + 0 + \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.61\text{ s})^2 = 12.7\text{ m}$$

Thus the fish must spot the pelican at a minimum height from the surface of the water of $16.0\text{ m} - 12.7\text{ m} = \boxed{3.3\text{ m}}$.

67. First consider the "uphill lie", in which the ball is being putted down the hill. Choose $x_0 = 0$ to be the ball's original location, and the direction of the ball's travel as the positive direction. The final velocity of the ball is $v = 0\,\text{m/s}$, the acceleration of the ball is $a = -2.0\,\text{m/s}^2$, and the displacement of the ball will be $x - x_0 = 6.0\text{ m}$ for the first case, and $x - x_0 = 8.0\text{ m}$ for the second case. Find the initial velocity of the ball from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v_0 = \sqrt{v^2 - 2a(x - x_0)} = \begin{cases} \sqrt{0 - 2(-2.0\,\text{m/s}^2)(6.0\text{ m})} = 4.9\,\text{m/s} \\ \sqrt{0 - 2(-2.0\,\text{m/s}^2)(8.0\text{ m})} = 5.7\,\text{m/s} \end{cases}$$

The range of acceptable velocities for the uphill lie is $\boxed{4.9\,\text{m/s to }5.7\,\text{m/s}}$, with a spread of 0.8 m/s.

Now consider the "downhill lie", in which the ball is being putted up the hill. Use a very similar set-up for the problem, with the basic difference being that the acceleration of the ball is now $a = -3.0\,\text{m/s}^2$. Find the initial velocity of the ball from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v_0 = \sqrt{v^2 - 2a(x - x_0)} = \begin{cases} \sqrt{0 - 2(-3.0\,\text{m/s}^2)(6.0\text{ m})} = 6.0\,\text{m/s} \\ \sqrt{0 - 2(-3.0\,\text{m/s}^2)(8.0\text{ m})} = 6.9\,\text{m/s} \end{cases}$$

The range of acceptable velocities for the downhill lie is $\boxed{6.0\,\text{m/s to }6.9\,\text{m/s}}$, with a spread of 0.9 m/s.

Because the range of acceptable velocities is smaller for putting down the hill, more control in putting is necessary, and so the downhill putt is more difficult.

68. (*a*) The train's constant speed is $v_{\text{train}} = 6.0\,\text{m/s}$, and the location of the empty box car as a function of time is given by $x_{\text{train}} = v_{\text{train}}t = (6.0\,\text{m/s})t$. The fugitive has $v_0 = 0\,\text{m/s}$ and $a = 4.0\,\text{m/s}^2$ until his final speed is $8.0\,\text{m/s}$. The elapsed time during acceleration is $t_{\text{acc}} = \dfrac{v - v_0}{a} = \dfrac{8.0\,\text{m/s}}{4.0\,\text{m/s}^2} = 2.0\text{ s}$. Let the origin be the location of the fugitive when he starts to run. The first possibility to consider is, "Can the fugitive catch the train before he reaches his maximum speed?" During the fugitive's acceleration, his location as a function of time is given by $x_{\text{fugitive}} = x_0 + v_0t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(4.0\,\text{m/s}^2)t^2$. For him to catch the train, we must have $x_{\text{train}} = x_{\text{fugitive}} \rightarrow (6.0\,\text{m/s})t = \frac{1}{2}(4.0\,\text{m/s}^2)t^2$. The solutions of this are $t = 0\text{ s}, 3\text{ s}$. Thus the fugitive cannot catch the car during his 2.0 s of acceleration.

Now the equation of motion of the fugitive changes. After the 2.0 s acceleration, he runs with a constant speed of $8.0\,\text{m/s}$. Thus his location is now given (for times $t > 2\text{ s}$) by the following.

$$x_{\text{fugitive}} = \tfrac{1}{2}(4.0\,\text{m/s}^2)(2.0\text{ s})^2 + (8.0\,\text{m/s})(t - 2.0\text{ s}) = (8.0\,\text{m/s})t - 8.0\text{ m}.$$

So now, for the fugitive to catch the train, we again set the locations equal.

$$x_{\text{train}} = x_{\text{fugitive}} \rightarrow (6.0\,\text{m/s})t = (8.0\,\text{m/s})t - 8.0\text{ m} \rightarrow t = \boxed{4.0\text{ s}}$$

(*b*) The distance traveled to reach the box car is given by

$$x_{\text{fugitive}}(t = 4.0\text{ s}) = (8.0\,\text{m/s})(4.0\text{ s}) - 8.0\text{ m} = \boxed{24\text{ m}}.$$

69. Choose downward to be the positive direction, and $y_0 = 0$ to be at the roof from which the stones are dropped. The first stone has an initial velocity of $v_0 = 0$ and an acceleration of $a = g$. Eqs. 2-11a and 2-11b (with *x* replaced by *y*) give the velocity and location, respectively, of the first stone as a function of time.

$$v = v_0 + at \rightarrow v_1 = gt_1 \qquad y = y_0 + v_0t + \tfrac{1}{2}at^2 \rightarrow y_1 = \tfrac{1}{2}gt_1^2.$$

The second stone has the same initial conditions, but its elapsed time $t - 1.50\text{ s}$, and so has velocity and location equations as follows.

$$v_2 = g(t_1 - 1.50\text{ s}) \qquad y_2 = \tfrac{1}{2}g(t_1 - 1.50\text{ s})^2$$

The second stone reaches a speed of $v_2 = 12.0\,\text{m/s}$ at a time given by

$$t_1 = 1.50\text{ s} + \frac{v_2}{g} = 1.50\text{ s} + \frac{12.0\,\text{m/s}}{9.80\,\text{m/s}^2} = 2.72\text{ s}.$$

The location of the first stone at that time is

$$y_1 = \tfrac{1}{2}gt_1^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(2.72\text{ s})^2 = 36.4\text{ m}.$$

The location of the second stone at that time is

$$y_2 = \tfrac{1}{2}g(t_1 - 1.50\text{ s})^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(2.72 - 1.50\text{ s})^2 = 7.35\text{ m}.$$

Thus the distance between the two stones is $y_1 - y_2 = 36.4\text{ m} - 7.35\text{ m} = \boxed{29.0\text{ m}}$.

70. To find the average speed for the entire race, we must take the total distance divided by the total time. If one lap is a distance of L, then the total distance will be $10L$. The time elapsed at a given constant speed is given by $t = d/v$, so the time for the first 9 laps would be $t_1 = \dfrac{9L}{198.0\text{ km/h}}$, and the time for the last lap would be $t_2 = L/v_2$, where v_2 is the average speed for the last lap. Write an expression for the average speed for the entire race, and then solve for v_2.

$$\overline{v} = \frac{d_{\text{total}}}{t_1 + t_2} = \frac{10L}{\dfrac{9L}{198.0\text{ km/h}} + \dfrac{L}{v_2}} = 200.0\text{ km/h} \quad \rightarrow$$

$$v_2 = \frac{1}{\dfrac{10}{200.0\text{ km/h}} - \dfrac{9}{198.0\text{ km/h}}} = \boxed{220.0\text{ km/h}}$$

71. The initial velocity is $v_0 = (18\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 5.0\text{ m/s}$. The final velocity is $v_0 = (75\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 20.83\text{ m/s}$. The displacement is $x - x_0 = 4.0\text{ km} = 4000\text{ m}$. Find the average acceleration from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(20.83\text{ m/s})^2 - (5.0\text{ m/s})^2}{2(4000\text{ m})} = \boxed{5.1\times10^{-2}\text{ m/s}^2}$$

72. Assume that $y_0 = 0$ for each child is the level at which the child loses contact with the trampoline surface. Choose upward to be the positive direction.

(*a*) The second child has $v_{02} = 5.0\text{ m/s}$, $a = -g = -9.8\text{ m/s}^2$, and $v = 0\text{ m/s}$ at the maximum height position. Find the child's maximum height from Eq. 2-11c, with *x* replaced by *y*.

$$v^2 = v_{02}^2 + 2a(y_2 - y_0) \quad \rightarrow \quad y_2 = y_0 + \frac{v^2 - v_{02}^2}{2a} = 0 + \frac{0 - (5.0\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 1.276\text{ m} \approx \boxed{1.3\text{ m}}$$

(*b*) Since the first child can bounce up to one-and-a-half times higher than the second child, the first child can bounce up to a height of $1.5(1.276\text{ m}) = 1.913\text{ m} = y_1 - y_0$. Eq. 2-11c is again used to find the initial speed of the first child.

$$v^2 = v_{01}^2 + 2a(y_1 - y_0) \quad \rightarrow$$

$$v_{01} = \pm\sqrt{v^2 - 2a(y_1 - y_0)} = \sqrt{0 - 2(-9.8\text{ m/s}^2)(1.913\text{ m})} = 6.124\text{ m/s} \approx \boxed{6.1\text{ m/s}}$$

The positive root was chosen since the child was initially moving upward.

(*c*) To find the time that the first child was in the air, use Eq. 2-11b with a total displacement of 0, since the child returns to the original position.

$$y = y_0 + v_{01}t_1 + \tfrac{1}{2}at_1^2 \quad \rightarrow \quad 0 = (6.124\text{ m/s})t_1 + \tfrac{1}{2}(-9.8\text{ m/s}^2)t_1^2 \quad \rightarrow \quad t_1 = 0\text{ s},\ 1.2497\text{ s}$$

The time of 0 s corresponds to the time the child started the jump, so the correct answer is $\boxed{1.2\text{ s}}$.

73. For the car to pass the train, the car must travel the length of the train AND the distance the train travels. The distance the car travels can thus be written as either $d_{\text{car}} = v_{\text{car}}t = (95\,\text{km/h})t$ or $d_{\text{car}} = L_{\text{train}} + v_{\text{train}}t = 1.10\text{ km} + (75\,\text{km/h})t$. To solve for the time, equate these two expressions for the distance the car travels.

$$(95\,\text{km/h})t = 1.10\text{ km} + (75\,\text{km/h})t \quad\rightarrow\quad t = \frac{1.10\text{ km}}{20\,\text{km/h}} = 0.055\text{ h} = \boxed{3.3\text{ min}}$$

The distance the car travels during this time is $d = (95\,\text{km/h})(0.055\text{ h}) = 5.225\,\text{km} \approx \boxed{5.2\text{ km}}$.

If the train is traveling the opposite direction from the car, then the car must travel the length of the train MINUS the distance the train travels. Thus the distance the car travels can be written as either $d_{\text{car}} = (95\,\text{km/h})t$ or $d_{\text{car}} = 1.10\text{ km} - (75\,\text{km/h})t$. To solve for the time, equate these two expressions for the distance the car travels.

$$(95\,\text{km/h})t = 1.10\text{ km} - (75\,\text{km/h})t \quad\rightarrow\quad t = \frac{1.10\text{ km}}{170\,\text{km/h}} = 6.47\times10^{-3}\text{ h} = \boxed{23.3\text{ s}}$$

The distance the car travels during this time is $d = (95\,\text{km/h})(6.47\times10^{-3}\text{ h}) = \boxed{0.61\text{ km}}$.

74. For the baseball, $v_0 = 0$, $x - x_0 = 3.5\text{ m}$, and the final speed of the baseball (during the throwing motion) is $v = 44\,\text{m/s}$. The acceleration is found from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \quad\rightarrow\quad a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(44\,\text{m/s})^2 - 0}{2(3.5\text{ m})} = \boxed{280\,\text{m/s}^2}$$

75. (*a*) Choose upward to be the positive direction, and $y_0 = 0$ at the ground. The rocket has $v_0 = 0$, $a = 3.2\,\text{m/s}^2$, and $y = 1200\text{ m}$ when it runs out of fuel. Find the velocity of the rocket when it runs out of fuel from Eq 2-11c, with *x* replaced by *y*.

$$v_{1200\text{ m}}^2 = v_0^2 + 2a(y - y_0) \quad\rightarrow$$

$$v_{1200\text{ m}} = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \pm\sqrt{0 + 2(3.2\,\text{m/s}^2)(1200\text{ m})} = 87.64\,\text{m/s} \approx \boxed{88\,\text{m/s}}$$

The positive root is chosen since the rocket is moving upwards when it runs out of fuel.

(*b*) The time to reach the 1200 m location can be found from equation (2-11a).

$$v_{1200\text{ m}} = v_0 + at_{1200\text{ m}} \quad\rightarrow\quad t_{1200\text{ m}} = \frac{v_{1200\text{ m}} - v_0}{a} = \frac{87.64\,\text{m/s} - 0}{3.2\,\text{m/s}^2} = 27.39\text{ s} \approx \boxed{27\text{ s}}$$

(*c*) For this part of the problem, the rocket will have an initial velocity $v_0 = 87.64\,\text{m/s}$, an acceleration of $a = -9.8\,\text{m/s}^2$, and a final velocity of $v = 0$ at its maximum altitude. The altitude reached from the out-of-fuel point can be found from equation (2-11c).

$$v^2 = v_{1200\text{ m}}^2 + 2a(y - 1200\text{ m}) \quad\rightarrow$$

$$y_{\max} = 1200\text{ m} + \frac{0 - v_{1200\text{ m}}^2}{2a} = 1200\text{ m} + \frac{-(87.64\,\text{m/s})^2}{2(-9.8\,\text{m/s}^2)} = 1200\text{ m} + 390\text{ m} = \boxed{1590\text{ m}}$$

(*d*) The time for the "coasting" portion of the flight can be found from Eq. 2-11a.

$$v = v_{1200\text{ m}} + at_{\text{coast}} \quad\rightarrow\quad t_{\text{coast}} = \frac{v - v_0}{a} = \frac{0 - 87.64\,\text{m/s}}{-9.8\,\text{m/s}^2} = 8.94\text{ s}$$

Thus the total time to reach the maximum altitude is $t = 27\ \text{s} + 8.94\ \text{s} \approx \boxed{36\ \text{s}}$.

(*e*) For this part of the problem, the rocket has $v_0 = 0\,\text{m/s}$, $a = -9.8\,\text{m/s}^2$, and a displacement of $-1600\ \text{m}$ (it falls from a height of 1600 m to the ground). Find the velocity upon reaching the Earth from Eq. 2-11c.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2a\left(y - y_0\right)} = \pm\sqrt{0 + 2\left(-9.80\,\text{m/s}^2\right)\left(-1600\ \text{m}\right)} = \boxed{-177\,\text{m/s}}$$

The negative root was chosen because the rocket is moving downward, which is the negative direction.

(*f*) The time for the rocket to fall back to the Earth is found from Eq. 2-11a.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{fall}} = \frac{v - v_0}{a} = \frac{-177\,\text{m/s} - 0}{-9.80\,\text{m/s}^2} = 18.1\ \text{s}$$

Thus the total time for the entire flight is $t = 36\ \text{s} + 18.1\ \text{s} = \boxed{54\ \text{s}}$.

76. The speed limit is $50\,\text{km/h}\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 13.89\,\text{m/s}$.

(*a*) For your motion, you would need to travel $\left(10 + 15 + 50 + 15 + 70\right)\text{m} = 160\ \text{m}$ to get through the third light. The time to travel the 160 m is found using the distance and the constant speed.

$$d = \bar{v}\,t \quad \rightarrow \quad t = \frac{d}{\bar{v}} = \frac{160\ \text{m}}{13.89\,\text{m/s}} = 11.52\ \text{s}$$

$\boxed{\text{Yes}}$, you can make it through all three lights without stopping.

(*b*) The second car needs to travel 150 m before the third light turns red. This car accelerates from $v_0 = 0\,\text{m/s}$ to a maximum of $v = 13.89\,\text{m/s}$ with $a = 2.0\,\text{m/s}^2$. Use Eq. 2-11a to determine the duration of that acceleration.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{acc}} = \frac{v - v_0}{a} = \frac{13.89\,\text{m/s} - 0\,\text{m/s}}{2.0\,\text{m/s}^2} = 6.94\ \text{s}$$

The distance traveled during that time is found from Eq. 2-11b.

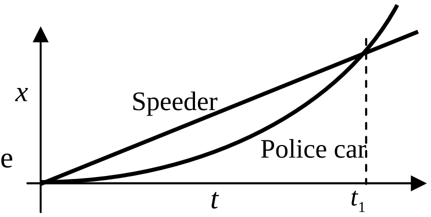
$$\left(x - x_0\right)_{\text{acc}} = v_0 t_{\text{acc}} + \tfrac{1}{2}a t_{\text{acc}}^2 = 0 + \tfrac{1}{2}\left(2.0\,\text{m/s}^2\right)\left(6.94\ \text{s}\right)^2 = 48.2\ \text{m}\,.$$

Since 6.94 sec have elapsed, there are 13 – 6.94 = 6.06 sec remaining to clear the intersection. The car travels another 6 seconds at a speed of 13.89 m/s, covering a distance of $d_{\substack{\text{constant}\\ \text{speed}}} = \bar{v}\,t = \left(13.89\,\text{m/s}\right)\left(6.06\ \text{s}\right) = 84.2\ \text{m}$. Thus the total distance is 48.2 m + 84.2 m = 132.4 m. $\boxed{\text{No}}$, the car cannot make it through all three lights without stopping.

77. Take the origin to be the location where the speeder passes the police car. The speeder's constant speed is $v_{\text{speeder}} = \left(120\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\ \text{km/h}}\right) = 33.3\,\text{m/s}$, and the location of the speeder as a function of time is given by $x_{\text{speeder}} = v_{\text{speeder}} t_{\text{speeder}} = \left(33.3\,\text{m/s}\right) t_{\text{speeder}}$. The police car has an initial velocity of $v_0 = 0\,\text{m/s}$ and a constant acceleration of a_{police}. The location of the police car as a function of time is given by Eq. 2-11b.

$$x_{\text{police}} = v_0 t + \tfrac{1}{2}at^2 = \tfrac{1}{2}a_{\text{police}} t_{\text{police}}^2\,.$$

(*a*) The position vs. time graphs would qualitatively look like the graph shown here.



(*b*) The time to overtake the speeder occurs when the speeder has gone a distance of 750 m. The time is found using the speeder's equation from above.

$$750\text{ m} = (33.3\,\text{m/s})t_{\text{speeder}} \quad\rightarrow\quad t_{\text{speeder}} = \frac{750\text{ m}}{33.3\,\text{m/s}} = 22.5\text{ s} \approx \boxed{23\text{ s}}$$

(*c*) The police car's acceleration can be calculated knowing that the police car also had gone a distance of 750 m in a time of 22.5 s.

$$750\text{ m} = \tfrac{1}{2}a_p(22.5\text{ s})^2 \quad\rightarrow\quad a_p = \frac{2(750\text{ m})}{(22.5\text{ s})^2} = 2.96\,\text{m/s}^2 \approx \boxed{3.0\,\text{m/s}^2}$$

(*d*) The speed of the police car at the overtaking point can be found from Eq. 2-11a.

$$v = v_0 + at = 0 + (2.96\,\text{m/s}^2)(22.5\text{ s}) = 66.67\,\text{m/s} \approx \boxed{67\,\text{m/s}}$$

Note that this is exactly twice the speed of the speeder.

78. Choose downward to be the positive direction, and the origin to be at the roof of the building from which the stones were dropped. The first stone has $y_0 = 0$, $v_0 = 0$, a final location of $y = H$ (as yet unknown), and $a = g$. If the time for the first stone to reach the ground is t_1, then Eq. 2-11c gives the following, replacing *x* with *y*:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad\rightarrow\quad H = \tfrac{1}{2}(9.80\,\text{m/s}^2)t_1^2 .$$

The second stone has $v_0 = 25.0\,\text{m/s}$, $y_0 = 0$, a final location of $y = H$, and $a = g$. The time for the second stone to reach the ground is $t_1 - 2.00\text{ s}$, and so Eq. 2-11c for the second stone is

$$H = (25.0\,\text{m/s})(t_1 - 2.00) + \tfrac{1}{2}(9.80\,\text{m/s}^2)(t_1 - 2.00)^2 .$$

(*a*) Set the two expressions for H equal to each other, and solve for t_1.

$$\tfrac{1}{2}(9.80\,\text{m/s}^2)t_1^2 = (25.0\,\text{m/s})(t_1 - 2) + \tfrac{1}{2}(9.80\,\text{m/s}^2)(t_1 - 2)^2 \quad\rightarrow\quad t_1 = \boxed{5.63\text{ s}}$$

(*b*) The building height is given by $H = \tfrac{1}{2}gt_1^2 = \tfrac{1}{2}(9.80\,\text{m/s}^2)(5.63\text{ s})^2 = \boxed{155\text{ m}}$.

(*c*) The speed of the stones is found using Eq. 2-11a.

$$\#1{:}\quad v = v_0 + at = gt_1 = (9.80\,\text{m/s}^2)(5.63\text{ s}) = \boxed{55.2\,\text{m/s}}$$

$$\#2{:}\quad v = v_0 + at = v_0 + g(t_1 - 2) = 25.0\,\text{m/s} + (9.80\,\text{m/s}^2)(3.63\text{ s}) = \boxed{60.6\,\text{m/s}}$$

79. Choose upward to be the positive direction, and the origin to be at ground level. The initial velocity of the first stone is $v_{0A} = 11.0\,\text{m/s}$, and the acceleration of both stones is $a = -9.80\,\text{m/s}^2$. The starting location is $y_{0A} = H_A$, and it takes 4.5 s for the stone to reach the final location $y = 0$. Use Eq. 2-11b (with *x* replaced by *y*) to find a value for H_A.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad\rightarrow\quad 0 = H_A + (11.0\,\text{m/s})(4.5\text{ s}) - \tfrac{1}{2}(9.80\,\text{m/s}^2)(4.5\text{ s})^2 \quad\rightarrow$$

$$H_A = 49.7\text{ m}$$

Assume that the 12th floor balcony is three times higher above the ground than the 4th floor balcony. Thus the height of 4th floor balcony is $\tfrac{1}{3}(49.7\text{ m}) = 16.6\text{ m}$. So for the second stone, $y_{0B} = 16.6\text{ m}$, and it takes 4.5 s for the stone to reach the final location $y = 0$. Use Eq. 2-11b to find the starting velocity, v_{0B}.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad 0 = 16.6\text{ m} + v_{0B}(4.5\text{ s}) - \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)(4.5\text{ s})^2 \quad \rightarrow$$

$$v_{0B} = \boxed{18\,\text{m/s}}$$

80. Choose downward to be the positive direction, and the origin to be at the location of the plane. The parachutist has $v_0 = 0$, $a = g = 9.8\,\text{m/s}^2$, and will have $y - y_0 = 2850$ m when she pulls the ripcord. Eq. 2-11b, with *x* replaced by *y*, is used to find the time when she pulls the ripcord.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{2(y - y_0)/a} = \sqrt{2(2850\text{ m})/\left(9.80\,\text{m/s}^2\right)} = \boxed{24.1\text{ s}}$$

The speed is found from Eq. 2-11a.

$$v = v_0 + at = 0 + \left(9.80\,\text{m/s}^2\right)(24.1\text{ s}) = 236\,\text{m/s} \cong \boxed{2.3\times10^2\text{ m/s}} = 850\,\text{km/h}$$

81. The speed of the conveyor belt is given by $d = \bar{v}\Delta t \;\rightarrow\; \bar{v} = \dfrac{d}{\Delta t} = \dfrac{1.1\text{ m}}{2.5\text{ min}} = \boxed{0.44\,\text{m/min}}$. The rate of burger production, assuming the spacing given is center to center, can be found as

$$\left(\frac{1\text{ burger}}{0.15\text{ m}}\right)\left(\frac{0.44\text{ m}}{1\text{ min}}\right) = \boxed{2.9\,\frac{\text{burgers}}{\text{min}}}.$$

82. Choose upward to be the positive direction, and the origin to be at the level where the ball was thrown. The velocity at the top of the ball's path will be $v = 0$, and the ball will have an acceleration of $a = -g$. If the maximum height that the ball reaches is $y = H$, then the relationship between the initial velocity and the maximum height can be found from Eq. 2-11c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad 0 = v_0^2 + 2(-g)H \quad \rightarrow \quad H = v_0^2/2g.$$

We are told that $v_{0\text{ Bill}} = 1.5 v_{0\text{ Joe}}$, so $\dfrac{H_{\text{Bill}}}{H_{\text{Joe}}} = \dfrac{\left(v_{0\text{ Bill}}\right)^2/2g}{\left(v_{0\text{ Joe}}\right)^2/2g} = \dfrac{\left(v_{0\text{ Bill}}\right)^2}{\left(v_{0\text{ Joe}}\right)^2} = 1.5^2 = 2.25 \approx \boxed{2.3}$.

83. As shown in problem 41, the speed with which the ball was thrown upward is the same as its speed on returning to the ground. From the symmetry of the two motions (both motions have speed = 0 at top, have same distance traveled and have same acceleration), the time for the ball to rise is 1.2 s. Choose upward to be the positive direction, and the origin to be at the level where the ball was thrown. For the ball, $v = 0$ at the top of the motion, and $a = -g$. Find the initial velocity from Eq. 2-11a.

$$v = v_0 + at \quad \rightarrow \quad v_0 = v - at = 0 - \left(-9.80\,\text{m/s}^2\right)(1.2\text{ s}) = \boxed{12\,\text{m/s}}$$

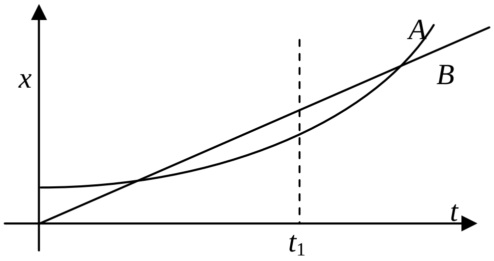
84. Choose downward to be the positive direction, and the origin to be at the top of the building. The barometer has $y_0 = 0$, $v_0 = 0$, and $a = g = 9.8\,\text{m/s}^2$. Use Eq. 2-11b to find the height of the building, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)t^2$$

$$y_{t=2.0} = \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)(2.0\text{ s})^2 = 20\text{ m} \qquad y_{t=2.3} = \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)(2.3\text{ s})^2 = 26\text{ m}$$

The difference in the estimates is $\boxed{6\text{ m}}$.

The intent of the method was probably to use the change in air pressure between the ground level and the top of the building to find the height of the building. The very small difference in time measurements, which could be due to human reaction time, makes a 6 m difference in the height. This could be as much as 2 floors in error.



85. (*a*) The two bicycles will have the same velocity at any time when the instantaneous slopes of their x vs. t graphs are the same. That occurs near the time t_1 as marked on the graph.
    (*b*) Bicycle A has the larger acceleration, because its graph is concave upward, indicating a positive acceleration. Bicycle B has no acceleration because its graph has a constant slope.
    (*c*) The bicycles are passing each other at the times when the two graphs cross, because they both have the same position at that time. The graph with the steepest slope is the faster bicycle, and so is the one that is passing at that instant. So at the first crossing, bicycle B is passing bicycle A. At the second crossing, bicycle A is passing bicycle B.
    (*d*) Bicycle B has the highest instantaneous velocity at all times until the time $t_{1.}$, where both graphs have the same slope. For all times after t_1, bicycle A has the highest instantaneous velocity. The largest instantaneous velocity is for bicycle A at the latest time shown on the graph.
    (*e*) The bicycles appear to have the same average velocity. If the starting point of the graph for a particular bicycle is connected to the ending point with a straight line, the slope of that line is the average velocity. Both appear to have the same slope for that "average" line.

CHAPTER 3: Kinematics in Two Dimensions; Vectors

Answers to Questions

1. Their velocities are NOT equal, because the two velocities have different directions.

2. (*a*) During one year, the Earth travels a distance equal to the circumference of its orbit, but has a displacement of 0 relative to the Sun.
   (*b*) The space shuttle travels a large distance during any flight, but the displacement from one launch to the next is 0.
   (*c*) Any kind of cross country "round trip" air travel would result in a large distance traveled, but a displacement of 0.
   (*d*) The displacement for a race car from the start to the finish of the Indy 500 auto race is 0.

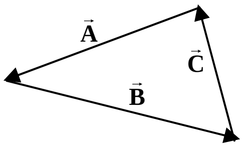
3. The displacement can be thought of as the "straight line" path from the initial location to the final location. The length of path will always be greater than or equal to the displacement, because the displacement is the shortest distance between the two locations. Thus the displacement can never be longer than the length of path, but it can be less. For any path that is not a single straight line segment, the length of path will be longer than the displacement.

4. Since both the batter and the ball started their motion at the same location (where the ball was hit) and ended their motion at the same location (where the ball was caught), the displacement of both was the same.

5. The magnitude of the vector sum need not be larger than the magnitude of either contributing vector. For example, if the two vectors being added are the exact opposite of each other, the vector sum will have a magnitude of 0. The magnitude of the sum is determined by the angle between the two contributing vectors.

6. If the two vectors are in the same direction, the magnitude of their sum will be a maximum, and will be 7.5 km. If the two vectors are in the opposite direction, the magnitude of their sum will be a minimum, and will be 0.5 km. If the two vectors are oriented in any other configuration, the magnitude of their sum will be between 0.5 km and 7.5 km.

7. Two vectors of unequal magnitude can never add to give the zero vector. However, three vectors of unequal magnitude can add to give the zero vector. If their geometric sum using the tail-to-tip method gives a closed triangle, then the vector sum will be zero. See the diagram, in which $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}=0$



8. (*a*) The magnitude of a vector can equal the length of one of its components if the other components of the vector are all 0; i.e. if the vector lies along one of the coordinate axes.
   (*b*) The magnitude of a vector can never be less than one of its components, because each component contributes a positive amount to the overall magnitude, through the Pythagorean relationship. The square root of a sum of squares is never less than the absolute value of any individual term.

9. A particle with constant speed can be accelerating, if its direction is changing. Driving on a curved roadway at constant speed would be an example. However, a particle with constant velocity cannot be accelerating – its acceleration must be zero. It has both constant speed and constant direction.

10. To find the initial speed, use the slingshot to shoot the rock directly horizontally (no initial vertical speed) from a height of 1 meter. The vertical displacement of the rock can be related to the time of flight by Eq. 2-11b. Take downward to be positive.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}at^2 \quad \rightarrow \quad 1\text{ m} = \tfrac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{2\left(1\text{ m}\right)\Big/\left(9.8\,\text{m/s}^2\right)} = 0.45\text{ s} .$$

Measure the horizontal range R of the rock with the meter stick. Then, if we measure the horizontal range R, we know that $R = v_x t = v_x\left(0.45\text{ s}\right)$, and so $v_x = R\big/0.45\text{ s}$. The only measurements are the height of fall and the range, both of which can be measured by a meter stick.

11. Assume that the bullet was fired from behind and below the airplane. As the bullet rose in the air, its vertical speed would be slowed by both gravity and air resistance, and its horizontal speed would be slowed by air resistance. If the altitude of the airplane was slightly below the maximum height of the bullet, then at the altitude of the airplane, the bullet would be moving quite slowly in the vertical direction. If the bullet's horizontal speed had also slowed enough to approximately match the speed of the airplane, then the bullet's velocity relative to the airplane would be small. With the bullet moving slowly, it could safely be caught by hand.

12. The moving walkway will be moving at the same speed as the "car". Thus, if you are on the walkway, you are moving the same speed as the car. Your velocity relative to the car is 0, and it is easy to get into the car. But it is very difficult to keep your balance while trying to sit down into a moving car from a stationary platform. It is easier to keep your balance by stepping on to the moving platform while walking, and then getting into the car with a velocity of 0 relative to the car.

13. Your reference frame is that of the train you are riding. If you are traveling with a relatively constant velocity (not over a hill or around a curve or drastically changing speed), then you will interpret your reference frame as being at rest. Since you are moving forward faster than the other train, the other train is moving backwards relative to you. Seeing the other train go past your window from front to rear makes it look like the other train is going backwards. This is similar to passing a semi truck on the interstate – out of a passenger window, it looks like the truck is going backwards.

14. When you stand still under the umbrella in a vertical rain, you are in a cylinder-shaped volume in which there is no rain. The rain has no horizontal component of velocity, and so the rain cannot move from outside that cylinder into it. You stay dry. But as you run, you have a forward horizontal velocity relative to the rain, and so the rain has a backwards horizontal velocity relative to you. It is the same as if you were standing still under the umbrella but the rain had some horizontal component of velocity towards you. The perfectly vertical umbrella would not completely shield you.

15. (*a*) The ball lands at the same point from which it was thrown inside the train car – back in the thrower's hand.
    (*b*) If the car accelerates, the ball will land behind the point from which it was thrown.
    (*c*) If the car decelerates, the ball will land in front of the point from which it was thrown.
    (*d*) If the car rounds a curve (assume it curves to the right), then the ball will land to the left of the point from which it was thrown.
    (*e*) The ball will be slowed by air resistance, and so will land behind the point from which it was thrown.

16. Both rowers need to cover the same "cross river" distance. The rower with the greatest speed in the "cross river" direction will be the one that reaches the other side first. The current has no bearing on the problem because the current doesn't help either of the boats move across the river. Thus the rower heading straight across will reach the other side first. All of his rowing effort has gone into

crossing the river. For the upstream rower, some of his rowing effort goes into battling the current, and so his "cross river" speed will be only a fraction of his rowing speed.

17. The baseball is hit and caught at approximately the same height, and so the range formula of $R = v_0^2 \sin 2\theta_0 / g$ is particularly applicable. Thus the baseball player is judging the initial speed of the ball and the initial angle at which the ball was hit.

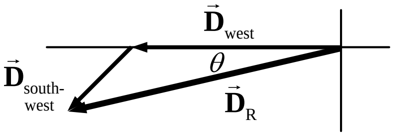
18. The arrow should be aimed above the target, because gravity will deflect the arrow downward from a horizontal flight path. The angle of aim (above the horizontal) should increase as the distance from the target increases, because gravity will have more time to act in deflecting the arrow from a straight-line path. If we assume that the arrow when shot is at the same height as the target, then the range formula is applicable: $R = v_0^2 \sin 2\theta_0 / g \quad \rightarrow \quad \theta = \frac{1}{2}\sin^{-1}\left(Rg/v_0^2\right)$. As the range and hence the argument of the inverse sine function increases, the angle increases.

$\boxed{19.}$ The horizontal component of the velocity stays constant in projectile motion, assuming that air resistance is negligible. Thus the horizontal component of velocity 1.0 seconds after launch will be the same as the horizontal component of velocity 2.0 seconds after launch. In both cases the horizontal velocity will be given by $v_x = v_0 \cos\theta = \left(30\,\text{m/s}\right)\left(\cos 30^\circ\right) = 26\,\text{m/s}$.

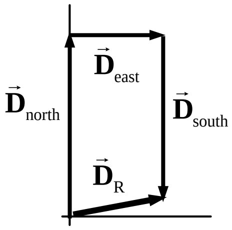
20. (*a*) Cannonball A, with the larger angle, will reach a higher elevation. It has a larger initial vertical velocity, and so by Eq. 2-11c, will rise higher before the vertical component of velocity is 0.
    (*b*) Cannonball A, with the larger angle, will stay in the air longer. It has a larger initial vertical velocity, and so takes more time to decelerate to 0 and start to fall.
    (*c*) The cannonball with a launch angle closest to 45° will travel the farthest. The range is a maximum for a launch angle of 45°, and decreases for angles either larger or smaller than 45°.

Solutions to Problems

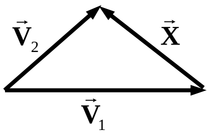
1. The resultant vector displacement of the car is given by $\vec{\mathbf{D}}_{\text{R}} = \vec{\mathbf{D}}_{\text{west}} + \vec{\mathbf{D}}_{\text{south-west}}$. The westward displacement is $215 + 85\cos 45^\circ = 275.1\text{ km}$ and the south displacement is $85\sin 45^\circ = 60.1\text{ km}$. The resultant displacement has a magnitude of $\sqrt{275.1^2 + 60.1^2} = 281.6\text{ km}$ $\approx \boxed{282\text{ km}}$. The direction is $\theta = \tan^{-1} 60.1/275.1 = 12.3^\circ \approx \boxed{12^\circ \text{ south of west}}$.

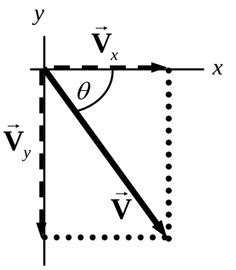

$\vec{\mathbf{D}}_{\text{west}}$
θ
$\vec{\mathbf{D}}_{\text{south-west}}$
$\vec{\mathbf{D}}_{\text{R}}$

2. The truck has a displacement of $18 + (-16) = 2$ blocks north and 10 blocks east. The resultant has a magnitude of $\sqrt{2^2 + 10^2} = \boxed{10\text{ blocks}}$ and a direction of $\tan^{-1} 2/10 = \boxed{11^\circ \text{ north of east}}$.


$\vec{\mathbf{D}}_{\text{east}}$
$\vec{\mathbf{D}}_{\text{north}}$
$\vec{\mathbf{D}}_{\text{south}}$
$\vec{\mathbf{D}}_{\text{R}}$

3. Label the "INCORRECT" vector as vector $\vec{\mathbf{X}}$. Then Fig. 3-6 (c) illustrates the relationship $\vec{\mathbf{V}}_1 + \vec{\mathbf{X}} = \vec{\mathbf{V}}_2$ via the tail-to-tip method. Thus $\vec{\mathbf{X}} = \boxed{\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1}$.

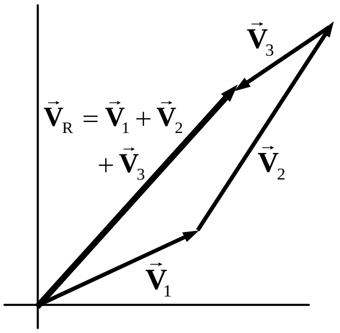




y
$\vec{\mathbf{V}}_x$
x
θ
$\vec{\mathbf{V}}_y$
$\vec{\mathbf{V}}$

4. Given that $V_x = 6.80$ units and $V_y = -7.40$ units, the magnitude of $\vec{\mathbf{V}}$ is given by $V = \sqrt{V_x^2 + V_y^2} = \sqrt{6.80^2 + (-7.40)^2} = \boxed{10.0\text{ units}}$. The direction is given by an angle of $\theta = \tan^{-1}\dfrac{-7.40}{6.80} = \boxed{-47^\circ}$, or 47° below the positive x-axis.

5. The vectors for the problem are drawn approximately to scale. The resultant has a length of $\boxed{58\text{ m}}$ and a direction $\boxed{48^\circ}$ north of east. If calculations are done, the actual resultant should be 57.4 m at 47.5° north of east.



$\vec{\mathbf{V}}_3$
$\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$
$+\vec{\mathbf{V}}_3$
$\vec{\mathbf{V}}_2$
$\vec{\mathbf{V}}_1$

6. The sum is found by adding the components of vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$

$$\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = (8.0, -3.7, 0.0) + (3.9, -8.1, -4.4) = \boxed{(11.9, -11.8, -4.4)}$$

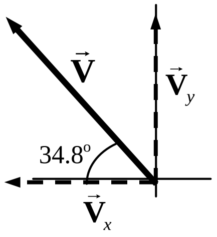
$$V = \left|\vec{\mathbf{V}}\right| = \sqrt{(11.9)^2 + (-11.8)^2 + (-4.4)^2} = \boxed{17.3}$$

7. (*a*) See the accompanying diagram

(*b*) $V_x = -14.3\cos 34.8^\circ = \boxed{-11.7\text{ units}}$ $\quad V_y = 14.3\sin 34.8^\circ = \boxed{8.16\text{ units}}$

(*c*) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-11.7)^2 + (8.16)^2} = \boxed{14.3\text{ units}}$

$$\theta = \tan^{-1}\frac{8.16}{11.7} = \boxed{34.8^\circ \text{ above the } -x \text{ axis}}$$



$\vec{\mathbf{V}}$
$\vec{\mathbf{V}}_y$
34.8°
$\vec{\mathbf{V}}_x$

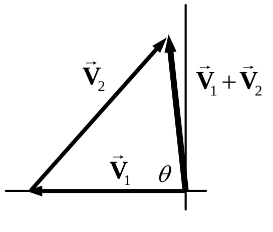
8. (*a*) $V_{1x} = \boxed{-6.6\text{ units}}$ $\quad V_{1y} = \boxed{0\text{ units}}$

$V_{2x} = 8.5\cos 45^\circ = \boxed{6.0\text{ units}}$ $\quad V_{2y} = 8.5\sin 45^\circ = \boxed{6.0\text{ units}}$

(*b*) $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = (V_{1x} + V_{2x}, V_{1y} + V_{2y}) = (-0.6, 6.0)$

$\left|\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2\right| = \sqrt{(-0.6)^2 + (6.0)^2} = 6.0\text{ units}$ $\quad \theta = \tan^{-1}\dfrac{6.0}{0.6} = 84^\circ$

The sum has a magnitude of $\boxed{6.0\text{ units}}$, and is $\boxed{84^\circ\text{ clockwise from the – negative x-axis}}$, or 96° counterclockwise from the positive x-axis.



$\vec{\mathbf{V}}_2$
$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$
$\vec{\mathbf{V}}_1$
θ

9. (*a*) $v_{\text{north}} = (735\text{ km/h})(\cos 41.5^\circ) = \boxed{550\text{ km/h}}$ $\quad v_{\text{west}} = (735\text{ km/h})(\sin 41.5^\circ) = \boxed{487\text{ km/h}}$

(*b*) $\Delta d_{\text{north}} = v_{\text{north}}t = (550\text{ km/h})(3.00\text{ h}) = \boxed{1650\text{ km}}$

$\Delta d_{\text{west}} = v_{\text{west}}t = (487\text{ km/h})(3.00\text{ h}) = \boxed{1460\text{ km}}$

$\boxed{10.}$ $A_x = 44.0\cos 28.0^\circ = 38.85$ $\quad A_y = 44.0\sin 28.0^\circ = 20.66$

$B_x = -26.5\cos 56.0^\circ = -14.82$ $\quad B_y = 26.5\sin 56.0^\circ = 21.97$

$C_x = 31.0\cos 270^\circ = 0.0$ $\quad C_y = 31.0\sin 270^\circ = -31.0$

(*a*) $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_x = 38.85+\left(-14.82\right)+0.0 = 24.03 = \boxed{24.0}$

$\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_y = 20.66+21.97+\left(-31.0\right) = 11.63 = \boxed{11.6}$

(*b*) $\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right| = \sqrt{\left(24.03\right)^2+\left(11.63\right)^2} = \boxed{26.7}$ $\qquad \theta = \tan^{-1}\dfrac{11.63}{24.03} = \boxed{25.8^\circ}$

11. $A_x = 44.0\cos 28.0^\circ = 38.85$ $\quad A_y = 44.0\sin 28.0^\circ = 20.66$

$C_x = 31.0\cos 270^\circ = 0.0$ $\quad C_y = 31.0\sin 270^\circ = -31.0$

$\left(\vec{\mathbf{A}}-\vec{\mathbf{C}}\right)_x = 38.85-0.0 = 38.85$ $\qquad \left(\vec{\mathbf{A}}-\vec{\mathbf{C}}\right)_y = 20.66-\left(-31.0\right) = 51.66$

$\left|\vec{\mathbf{A}}-\vec{\mathbf{C}}\right| = \sqrt{\left(38.85\right)^2+\left(51.66\right)^2} = \boxed{64.6}$ $\qquad \theta = \tan^{-1}\dfrac{51.66}{38.85} = \boxed{53.1^\circ}$

12. $A_x = 44.0\cos 28.0^\circ = 38.85$ $\quad A_y = 44.0\sin 28.0^\circ = 20.66$

$B_x = -26.5\cos 56.0^\circ = -14.82$ $\quad B_y = 26.5\sin 56.0^\circ = 21.97$

(*a*) $\left(\vec{\mathbf{B}}-\vec{\mathbf{A}}\right)_x = \left(-14.82\right)-38.85 = -53.67$ $\qquad \left(\vec{\mathbf{B}}-\vec{\mathbf{A}}\right)_y = 21.97-20.66 = 1.31$

Note that since the *x* component is negative and the *y* component is positive, the vector is in the 2nd quadrant.

$\left|\vec{\mathbf{B}}-\vec{\mathbf{A}}\right| = \sqrt{\left(-53.67\right)^2+\left(1.31\right)^2} = \boxed{53.7}$ $\qquad \theta_{B-A} = \tan^{-1}\dfrac{1.31}{-53.67} = \boxed{1.4^\circ \text{ above } -x \text{ axis}}$

(*b*) $\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_x = 38.85-\left(-14.82\right) = 53.67$ $\qquad \left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_y = 20.66-21.97 = -1.31$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{\left(53.67\right)^2+\left(-1.31\right)^2} = \boxed{53.7}$ $\qquad \theta = \tan^{-1}\dfrac{-1.31}{53.7} = \boxed{1.4^\circ \text{ below } +x \text{ axis}}$

Comparing the results shows that $\vec{\mathbf{B}}-\vec{\mathbf{A}}$ is the opposite of $\vec{\mathbf{A}}-\vec{\mathbf{B}}$.

13. $A_x = 44.0\cos 28.0^\circ = 38.85$ $\quad A_y = 44.0\sin 28.0^\circ = 20.66$

$B_x = -26.5\cos 56.0^\circ = -14.82$ $\quad B_y = 26.5\sin 56.0^\circ = 21.97$

$C_x = 31.0\cos 270^\circ = 0.0$ $\quad C_y = 31.0\sin 270^\circ = -31.0$

(*a*) $\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_x = 38.85-\left(-14.82\right)+0.0 = 53.67$

$\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_y = 20.66-21.97+\left(-31.0\right) = -32.31$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}}\right| = \sqrt{\left(53.67\right)^2+\left(-32.31\right)^2} = \boxed{62.6}$ $\qquad \theta = \tan^{-1}\dfrac{-32.31}{53.67} = \boxed{31.0^\circ \text{below } +x \text{ axis}}$

(*b*) $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right)_x = 38.85+\left(-14.82\right)-0.0 = 24.03$

$\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right)_y = 20.66+21.97-\left(-31.0\right) = 73.63$

$\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right| = \sqrt{\left(24.03\right)^2+\left(73.63\right)^2} = \boxed{77.5} \qquad \theta = \tan^{-1}\frac{73.63}{24.03} = \boxed{71.9^\circ}$

(*c*) $\left(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_x = 0.0-38.85-\left(-14.82\right) = -24.03$

$\left(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_y = -31.0-20.66-21.97 = -73.63$

Note that since both components are negative, the vector is in the 3rd quadrant.

$\left|\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{\left(-24.03\right)^2+\left(-73.63\right)^2} = \boxed{77.5} \qquad \theta = \tan^{-1}\frac{-73.63}{-24.03} = \boxed{71.9^\circ \text{ below } -x \text{ axis}}$

Note that the answer to (c) is the exact opposite of the answer to (b).

14. $A_x = 44.0\cos 28.0^\circ = 38.85 \qquad A_y = 44.0\sin 28.0^\circ = 20.66$

$B_x = -26.5\cos 56.0^\circ = -14.82 \qquad B_y = 26.5\sin 56.0^\circ = 21.97$

$C_x = 31.0\cos 270^\circ = 0.0 \qquad C_y = 31.0\sin 270^\circ = -31.0$

(*a*) $\left(\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right)_x = -14.82-2\left(38.85\right) = -92.52 \qquad \left(\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right)_y = 21.97-2\left(20.66\right) = -19.35$

Note that since both components are negative, the vector is in the 3rd quadrant.

$\left|\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right| = \sqrt{\left(-92.52\right)^2+\left(-19.35\right)^2} = \boxed{94.5} \qquad \theta = \tan^{-1}\frac{-19.35}{-92.52} = \boxed{11.8^\circ \text{ below } -x \text{ axis}}$

(*b*) $\left(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right)_x = 2\left(38.85\right)-3\left(-14.82\right)+2\left(0.0\right) = 122.16$

$\left(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right)_y = 2\left(20.66\right)-3\left(21.97\right)+2\left(-31.0\right) = -86.59$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$\left|2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right| = \sqrt{\left(122.16\right)^2+\left(-86.59\right)^2} = \boxed{149.7} \qquad \theta = \tan^{-1}\frac{-86.59}{122.16} = \boxed{35.3^\circ \text{ below } +x \text{ axis}}$

15. The *x* component is negative and the *y* component is positive, since the summit is to the west of north. The angle measured counterclockwise from the positive *x* axis would be 122.4°. Thus the components are found to be

$x = -4580\sin 32.4^\circ = -2454 \text{ m} \qquad y = 4580\cos 32.4^\circ = 3867 \text{ m} \qquad z = 2450 \text{ m}$

$\boxed{\vec{\mathbf{r}} = \left(-2450 \text{ m}, 3870 \text{ m}, 2450 \text{ m}\right)} \qquad \left|\vec{\mathbf{r}}\right| = \sqrt{\left(-2454\right)^2+\left(4580\right)^2+\left(2450\right)^2} = \boxed{5190 \text{ m}}$

16. $70.0 = \sqrt{x^2+\left(-55.0\right)^2} \rightarrow 4900 = x^2+3025 \rightarrow x^2 = 1875 \rightarrow x = \boxed{\pm 43.3 \text{ units}}$

17. Choose downward to be the positive *y* direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $v_{x0} = 3.5 \text{ m/s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80 \text{ m/s}^2$, $y_0 = 0$, and the final location $y = 6.5 \text{ m}$. The time for the tiger to reach the ground is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 6.5\text{m} = 0 + 0 + \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2\left(6.5\text{m}\right)}{9.8\,\text{m/s}^2}} = 1.15\text{ sec}$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$\Delta x = v_x t = \left(3.5\,\text{m/s}\right)\left(1.15\text{ sec}\right) = \boxed{4.0\text{ m}}$$

18. Choose downward to be the positive *y* direction. The origin will be at the point where the diver dives from the cliff. In the horizontal direction, $v_{x0} = 1.8\,\text{m/s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m/s}^2$, $y_0 = 0$, and the time of flight is $t = 3.0\text{ s}$. The height of the cliff is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(3.0\,\text{s}\right)^2 = \boxed{44\text{ m}}$$

The distance from the base of the cliff to where the diver hits the water is found from the horizontal motion at constant velocity:

$$\Delta x = v_x t = \left(1.8\,\text{m/s}\right)\left(3\text{ s}\right) = \boxed{5.4\text{ m}}$$

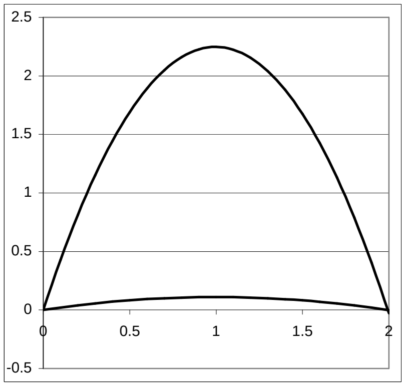
19. Apply the range formula from Example 3-8.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow$$

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{\left(2.0\text{ m}\right)\left(9.8\,\text{m/s}^2\right)}{\left(6.8\,\text{m/s}\right)^2} = 0.4239$$

$$2\theta_0 = \sin^{-1} 0.4239 \quad \rightarrow \quad \theta_0 = \boxed{13^\circ, 77^\circ}$$

There are two angles because each angle gives the same range. If one angle is $\theta = 45^\circ + \delta$, then $\theta = 45^\circ - \delta$ is also a solution. The two paths are shown in the graph.



20. Choose upward to be the positive *y* direction. The origin is the point from which the pebbles are released. In the vertical direction, $a_y = -9.80\,\text{m/s}^2$, the velocity at the window is $v_y = 0$, and the vertical displacement is 4.5 m. The initial *y* velocity is found from Eq. 2-11c.

$$v_y^2 = v_{y0}^2 + 2a_y\left(y - y_0\right) \quad \rightarrow$$

$$v_{y0} = \sqrt{v_y^2 - 2a_y\left(y - y_0\right)} = \sqrt{0 - 2\left(-9.80\,\text{m/s}^2\right)\left(4.5\text{ m}\right)} = 9.39\,\text{m/s}$$

Find the time for the pebbles to travel to the window from Eq. 2-11a.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{0 - 9.4\,\text{m/s}}{-9.8\,\text{m/s}^2} = 0.958\text{ s}$$

Find the horizontal speed from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x/t = 5.0\text{ m}/0.958\text{ s} = \boxed{5.2\,\text{m/s}}$$

This is the speed of the pebbles when they hit the window.

$\boxed{21.}$ Choose downward to be the positive *y* direction. The origin will be at the point where the ball is thrown from the roof of the building. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m/s}^2$, $y_0 = 0$, and the displacement is 45.0 m. The time of flight is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 45.0\text{ m} = \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2\left(45.0\text{ m}\right)}{9.80\,\text{m/s}^2}} = 3.03\text{ sec}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity:

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x/t = 24.0\text{ m}/3.03\text{ s} = \boxed{7.92\,\text{m/s}}\ .$$

22. Choose the point at which the football is kicked the origin, and choose upward to be the positive *y* direction. When the football reaches the ground again, the *y* displacement is 0. For the football, $v_{y0} = \left(18.0\sin 35.0^\circ\right)\text{m/s}$, $a_y = -9.80\,\text{m/s}^2$ and the final *y* velocity will be the opposite of the starting *y* velocity (reference problem 3-28). Use Eq. 2-11a to find the time of flight.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{\left(-18.0\sin 35.0^\circ\right)\text{m/s} - \left(18.0\sin 35.0^\circ\right)\text{m/s}}{-9.80\,\text{m/s}^2} = \boxed{2.11\text{ s}}$$

23. Choose downward to be the positive *y* direction. The origin is the point where the ball is thrown from the roof of the building. In the vertical direction, $v_{y0} = 0$, $y_0 = 0$, and $a_y = 9.80\,\text{m/s}^2$. The initial horizontal velocity is 22.2 m/s and the horizontal range is 36.0 m. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x/v_x = 36.0\text{ m}/22.2\,\text{m/s} = 1.62\text{ s}$$

The vertical displacement, which is the height of the building, is found by applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(1.62\text{ s}\right)^2 = \boxed{12.9\text{ m}}$$

$\boxed{24.}$ (*a*) Use the “Level horizontal range” formula from Example 3-8.

$$R = \frac{v_0^2\sin 2\theta_0}{g} \quad \rightarrow \quad v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{\left(7.80\text{ m}\right)\left(9.80\,\text{m/s}^2\right)}{\sin 2\left(28.0^\circ\right)}} = \boxed{9.60\,\text{m/s}}$$

(*b*) Now increase the speed by 5.0% and calculate the new range. The new speed would be $9.60\,\text{m/s}\left(1.05\right) = 10.1\,\text{m/s}$ and the new range would be

$$R = \frac{v_0^2\sin 2\theta_0}{g} = \frac{\left(10.1\,\text{m/s}\right)^2\sin 2\left(28.0^\circ\right)}{9.80\,\text{m/s}^2} = 8.60\text{ m}\ ,$$

an increase of $\boxed{0.80\text{ m }\left(10\%\text{ increase}\right)}$.

25. Calculate the range as derived in Example 3-8: $R = \dfrac{v_0^2\sin 2\theta_0}{g}$. If the launching speed and angle are held constant, the range is inversely proportional to the value of g. The acceleration due to gravity on the Moon is 1/6th that on Earth.

$$R_{\text{Earth}} = \frac{v_0^2\sin 2\theta_0}{g_{\text{Earth}}} \qquad R_{\text{Moon}} = \frac{v_0^2\sin 2\theta_0}{g_{\text{Moon}}} \quad \rightarrow \quad R_{\text{Earth}}\,g_{\text{Earth}} = R_{\text{Moon}}\,g_{\text{Moon}}$$

$$R_{\text{Moon}} = R_{\text{Earth}}\frac{g_{\text{Earth}}}{g_{\text{Moon}}} = 6R_{\text{Earth}}$$

Thus on the Moon, the person can jump $\boxed{\text{6 times farther}}$.

26. (*a*) Choose downward to be the positive *y* direction. The origin is the point where the bullet leaves the gun. In the vertical direction, $v_{y0} = 0$, $y_0 = 0$, and $a_y = 9.80\,\text{m/s}^2$. In the horizontal direction, $\Delta x = 75.0\text{ m}$ and $v_x = 180\,\text{m/s}$. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x/v_x = 75.0\text{ m}/180\,\text{m/s} = 0.4167\text{ s}$$

This time can now be used in Eq. 2-11b to find the vertical drop of the bullet.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(0.4167\text{ s}\right)^2 = \boxed{0.851\text{ m}}$$

(*b*) For the bullet to hit the target at the same level, the level horizontal range formula of Example 3-8 applies. The range is 75.0 m, and the initial velocity is 180 m/s. Solving for the angle of launch results in the following.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad \sin 2\theta_0 = \frac{Rg}{v_0^2} \quad \rightarrow \quad \theta_0 = \frac{1}{2}\sin^{-1}\frac{\left(75.0\text{ m}\right)\left(9.80\,\text{m/s}^2\right)}{\left(180\,\text{m/s}\right)^2} = \boxed{0.650^\circ}$$

Because of the symmetry of the range formula, there is also an answer of the complement of the above answer, which would be 89.35°. That is an unreasonable answer from a practical physical viewpoint – it is pointing the gun almost straight up.

27. Choose downward to be the positive *y* direction. The origin is the point where the supplies are dropped. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m/s}^2$, $y_0 = 0$, and the final position is $y = 160\,\text{m}$. The time of flight is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 160\text{ m} = 0 + 0 + \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)t^2 \quad \rightarrow$$

$$t = \sqrt{\frac{2\left(160\text{ m}\right)}{9.80\,\text{m/s}^2}} = \boxed{5.71\text{ s}}$$

Note that the speed of the airplane does not enter into this calculation.

28. The horizontal component of the speed does not change during the course of the motion, and so $v_{xf} = v_{x0}$. The net vertical displacement is 0 if the firing level equals the landing level. Eq. 2-11c then gives $v_{yf}^2 = v_{y0}^2 + 2a_y\Delta y = v_{y0}^2$. Thus $v_{yf}^2 = v_{y0}^2$, and from the horizontal $v_{xf}^2 = v_{x0}^2$. The initial speed is $v_0 = \sqrt{v_{y0}^2 + v_{x0}^2}$. The final speed is $v_f = \sqrt{v_{yf}^2 + v_{xf}^2} = \sqrt{v_{y0}^2 + v_{x0}^2} = v_0$. Thus $\boxed{v_f = v_0}$.

29. Choose upward to be the positive *y* direction. The origin is point from which the football is kicked. The initial speed of the football is $v_0 = 20.0\,\text{m/s}$. We have $v_{y0} = v_0 \sin 37.0^\circ = 12.04\,\text{m/s}$, $y_0 = 0$, and $a_y = -9.80\,\text{m/s}^2$. In the horizontal direction, $v_x = v_0 \cos 37.0^\circ = 15.97\,\text{m/s}$, and $\Delta x = 36.0\text{ m}$. The time of flight to reach the goalposts is found from the horizontal motion at constant speed:

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x/v_x = 36.0\text{ m}/15.97\,\text{m/s} = 2.254\text{ s}.$$

Now use this time with the vertical motion data and Eq. 2-11b to find the height of the football when it reaches the horizontal location of the goalposts.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = 0 + \left(12.04\,\text{m/s}\right)\left(2.254\text{ s}\right) + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)\left(2.254\,\text{s}\right)^2 = 2.24\text{ m}$$

Since the ball's height is less than 3.00 m, $\boxed{\text{the football does not clear the bar}}$. It is 0.76 m too low when it reaches the horizontal location of the goalposts.

30. Choose the origin to be where the projectile is launched, and upwards to be the positive y direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, and $v_{y0} = v_0 \sin\theta_0$.

(*a*) The maximum height is found from Eq. 2-11c, $v_y^2 = v_{y0}^2 + 2a_y\left(y - y_0\right)$, with $v_y = 0$ at the maximum height.

$$y_{\max} = 0 + \frac{v_y^2 - v_{y0}^2}{2a_y} = \frac{-v_0^2\sin^2\theta_0}{-2g} = \frac{v_0^2\sin^2\theta_0}{2g} = \frac{\left(65.2\,\text{m/s}\right)^2\sin^2 34.5^\circ}{2\left(9.80\,\text{m/s}^2\right)} = \boxed{69.6\text{ m}}$$

(*b*) The total time in the air is found from Eq. 2-11b, with a total vertical displacement of 0 for the ball to reach the ground.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0 = v_0\sin\theta_0 t - \tfrac{1}{2}gt^2 \quad\rightarrow$$

$$t = \frac{2v_0\sin\theta_0}{g} = \frac{2\left(65.2\,\text{m/s}\right)\sin 34.5^\circ}{\left(9.80\,\text{m/s}^2\right)} = \boxed{7.54\text{ s}} \text{ and } t = 0$$

The time of 0 represents the launching of the ball.

(*c*) The total horizontal distance covered is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = \left(v_0\cos\theta_0\right)t = \left(65.2\,\text{m/s}\right)\left(\cos 34.5^\circ\right)\left(7.54\text{ s}\right) = \boxed{405\text{ m}}$$

(*d*) The velocity of the projectile 1.50 s after firing is found as the vector sum of the horizontal and vertical velocities at that time. The horizontal velocity is a constant $v_0\cos\theta_0 = \left(65.2\,\text{m/s}\right)\left(\cos 34.5^\circ\right) = 53.7\,\text{m/s}$. The vertical velocity is found from Eq. 2-11a.

$$v_y = v_{y0} + at = v_0\sin\theta_0 - gt = \left(65.2\,\text{m/s}\right)\sin 34.5^\circ - \left(9.80\,\text{m/s}^2\right)\left(1.50\,\text{s}\right) = 22.2\,\text{m/s}$$

Thus the speed of the projectile is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{53.7^2 + 22.2^2} = \boxed{58.1\,\text{m/s}}$.

The direction above the horizontal is given by $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{22.2}{53.7} = \boxed{22.5^\circ}$.

31. Choose the origin to be at ground level, under the place where the projectile is launched, and upwards to be the positive y direction. For the projectile, $v_0 = 65.0\,\text{m/s}$, $\theta_0 = 37.0^\circ$, $a_y = -g$, $y_0 = 125$, and $v_{y0} = v_0\sin\theta_0$

(*a*) The time taken to reach the ground is found from Eq. 2-11b, with a final height of 0.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0 = 125 + v_0\sin\theta_0 t - \tfrac{1}{2}gt^2 \quad\rightarrow$$

$$t = \frac{-v_0\sin\theta_0 \pm \sqrt{v_0^2\sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)\left(125\right)}}{2\left(-\tfrac{1}{2}g\right)} = \frac{-39.1 \pm 63.1}{-9.8} = 10.4\text{ s},\ -2.45\text{ s} = \boxed{10.4\text{ s}}$$

Choose the positive sign since the projectile was launched at time $t = 0$.

(*b*) The horizontal range is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = \left(v_0\cos\theta_0\right)t = \left(65.0\,\text{m/s}\right)\cos 37.0^\circ\left(10.4\text{ s}\right) = \boxed{541\text{ m}}$$

(*c*) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_x = v_0\cos\theta_0 = \left(65.0\,\text{m/s}\right)\cos 37.0^\circ = \boxed{51.9\,\text{m/s}}$. The vertical component is found from Eq. 2-11a.

$$v_y = v_{y0} + at = v_0 \sin\theta_0 - gt = (65.0\,\mathrm{m/s})\sin 37.0^\circ - (9.80\,\mathrm{m/s^2})(10.4\ \mathrm{s})$$
$$= \boxed{-63.1\,\mathrm{m/s}}$$

(*d*) The magnitude of the velocity is found from the *x* and *y* components calculated in part c) above.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(51.9\,\mathrm{m/s})^2 + (-63.1\,\mathrm{m/s})^2} = \boxed{81.7\,\mathrm{m/s}}$$

(*e*) The direction of the velocity is $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-63.1}{51.9} = -50.6^\circ$, and so the object is moving $\boxed{50.6^\circ \text{ below the horizon}}$.

(*f*) The maximum height above the cliff top reached by the projectile will occur when the y-velocity is 0, and is found from Eq. 2-11c.

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \quad \rightarrow \quad 0 = v_0^2 \sin^2\theta_0 - 2gy_{\max}$$

$$y_{\max} = \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{(65.0\,\mathrm{m/s})^2 \sin^2 37.0^\circ}{2(9.80\,\mathrm{m/s^2})} = \boxed{78.1\ \mathrm{m}}$$

32. Choose the origin to be the point of release of the shot put. Choose upward to be the positive *y* direction. Then $y_0 = 0$, $v_{y0} = (15.5 \sin 34.0^\circ)\,\mathrm{m/s} = 8.67\,\mathrm{m/s}$, $a_y = -9.80\,\mathrm{m/s^2}$, and $y = -2.20$ m at the end of the motion. Use Eq. 2-11b to find the time of flight.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad \tfrac{1}{2}a_y t^2 + v_{y0}t - y = 0 \quad \rightarrow$$

$$t = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 - 4\left(\tfrac{1}{2}a_y\right)(-y)}}{2\tfrac{1}{2}a_y} = \frac{-8.67 \pm \sqrt{(8.67)^2 - 2(-9.80)(2.20)}}{-9.80} = 1.99\ \mathrm{s}, -0.225\ \mathrm{s}$$

Choose the positive result since the time must be greater than 0. Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$\Delta x = v_x t = (15.5\cos 34^\circ)\,\mathrm{m/s}\,(1.99\ \mathrm{s}) = \boxed{25.6\ \mathrm{m}}$$

33. Choose the origin to be where the projectile is launched, and upwards to be the positive *y* direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, and $v_{y0} = v_0 \sin\theta_0$. The range of the projectile is given by the range formula from Example 3-8, $R = \frac{v_0^2 \sin 2\theta_0}{g}$. The maximum height of the projectile will occur when its vertical speed is 0. Apply Eq. 2-11c.

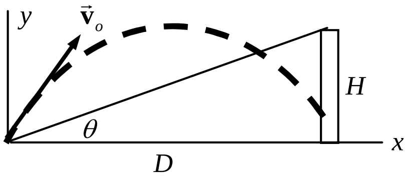
$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \quad \rightarrow \quad 0 = v_0^2 \sin^2\theta_0 - 2gy_{\max} \quad \rightarrow \quad y_{\max} = \frac{v_0^2 \sin^2\theta_0}{2g}$$

Now find the angle for which $R = y_{\max}$.

$$R = y_{\max} \quad \rightarrow \quad \frac{v_0^2 \sin 2\theta_0}{g} = \frac{v_0^2 \sin^2\theta_0}{2g} \quad \rightarrow \quad \sin 2\theta_0 = \frac{\sin^2\theta_0}{2} \quad \rightarrow$$

$$2\sin\theta_0 \cos\theta_0 = \frac{\sin^2\theta_0}{2} \quad \rightarrow \quad 4\cos\theta_0 = \sin\theta_0 \quad \rightarrow \quad \tan\theta_0 = 4 \quad \rightarrow \quad \theta_0 = \tan^{-1} 4 = \boxed{76^\circ}$$

34. Choose the origin to be the location from which the balloon is fired, and choose upward as the positive *y* direction. Assume the boy in the tree is a distance *H* up from the point at which the balloon is fired, and that the tree is a distance *D* horizontally from the point at which the balloon is fired. The equations of motion for the balloon and boy are as follows, using constant acceleration relationships.



$$x_{\text{Balloon}} = v_0 \cos\theta_0 t \qquad y_{\text{Balloon}} = 0 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \qquad y_{\text{Boy}} = H - \tfrac{1}{2}gt^2$$

Use the horizontal motion at constant velocity to find the elapsed time after the balloon has traveled *D* to the right.

$$D = v_0 \cos\theta_0 t_D \quad \rightarrow \quad t_D = \frac{D}{v_0 \cos\theta_0}$$

Where is the balloon vertically at that time?

$$y_{\text{Balloon}} = v_0 \sin\theta_0 t_D - \tfrac{1}{2}gt_D^2 = v_0 \sin\theta_0 \frac{D}{v_0\cos\theta_0} - \tfrac{1}{2}g\left(\frac{D}{v_0\cos\theta_0}\right)^2 = D\tan\theta_0 - \tfrac{1}{2}g\left(\frac{D}{v_0\cos\theta_0}\right)^2$$

Where is the boy vertically at that time? Note that $H = D\tan\theta_o$.

$$y_{\text{Boy}} = H - \tfrac{1}{2}gt_D^2 = H - \tfrac{1}{2}g\left(\frac{D}{v_0\cos\theta_0}\right)^2 = D\tan\theta_0 - \tfrac{1}{2}g\left(\frac{D}{v_0\cos\theta_0}\right)^2$$

The boy and the balloon are at the same height and the same horizontal location at the same time. Thus they collide!

35. Choose the origin to be the location on the ground directly below the airplane at the time the supplies are dropped, and choose upward as the positive *y* direction. For the supplies, $y_0 = 235\text{ m}$, $v_{y0} = 0$, $a_y = -g$, and the final *y* location is $y = 0\text{ m}$. The initial (and constant) *x* velocity of the supplies is $v_x = 69.4\,\text{m/s}$.

(*a*) The time for the supplies to reach the ground is found from Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = y_0 + 0 + \tfrac{1}{2}at^2 \quad \rightarrow$$

$$t = \sqrt{\frac{-2y_0}{a}} = \sqrt{\frac{-2(235\text{ m})}{\left(-9.80\,\text{m/s}^2\right)}} = 6.93\text{ s}$$

Then the horizontal distance of travel for the package is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (69.4\,\text{m/s})(6.93\text{ s}) = \boxed{481\text{ m}}$$

(*b*) Now the supplies have to travel a horizontal distance of only 425 m. Thus the time of flight will be less, and is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x / v_x = 425\text{ m}/69.4\,\text{m/s} = 6.124\text{ s}.$$

The *y* motion must satisfy Eq. 2-11b for this new time, but the same vertical displacement and acceleration.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow$$

$$v_{y0} = \frac{y - y_0 - \tfrac{1}{2}a_y t^2}{t} = \frac{0 - 235\text{ m} - \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)(6.124\text{ s})^2}{6.124\text{ s}} = \boxed{-8.37\,\text{m/s}}$$

Notice that since this is a negative velocity, the object must be projected DOWN.

(*c*) The horizontal component of the speed of the supplies upon landing is the constant horizontal speed of 69.4 m/s. The vertical speed is found from Eq. 2-11a.

$$v_y = v_{y0} + a_y t = -8.37\,\text{m/s} + \left(-9.80\,\text{m/s}^2\right)\left(6.124\text{ s}\right) = 68.4\,\text{m/s}$$

Thus the speed is given by

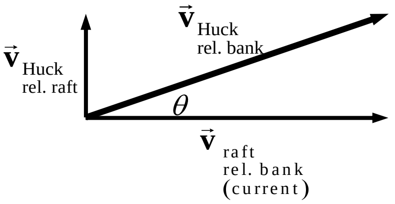
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(69.4\,\text{m/s}\right)^2 + \left(68.4\,\text{m/s}\right)^2} = \boxed{97.4\,\text{m/s}}$$

36. Call the direction of the boat relative to the water the positive direction.

(*a*) $$\vec{\mathbf{v}}_{\substack{\text{jogger}\\\text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger}\\\text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\\text{water}}} = 2.2\ \text{m/s} + 7.5\ \text{m/s}$$
$$= \boxed{9.7\ \text{m/s in the direction the boat is moving}}$$

(*b*) $$\vec{\mathbf{v}}_{\substack{\text{jogger}\\\text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger}\\\text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\\text{water}}} = -2.2\ \text{m/s} + 7.5\ \text{m/s}$$
$$= \boxed{5.3\ \text{m/s in the direction the boat is moving}}$$

37. Call the direction of the flow of the river the *x* direction, and the direction of Huck walking relative to the raft the *y* direction.

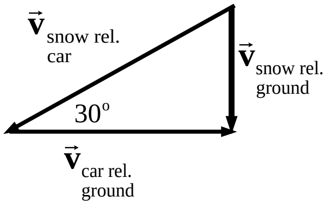

$\vec{\mathbf{v}}$ Huck rel. bank
$\vec{\mathbf{v}}$ Huck rel. raft
θ
$\vec{\mathbf{v}}$ raft rel. bank (current)

$$\vec{\mathbf{v}}_{\substack{\text{Huck}\\\text{rel. bank}}} = \vec{\mathbf{v}}_{\substack{\text{Huck}\\\text{rel. raft}}} + \vec{\mathbf{v}}_{\substack{\text{raft rel.}\\\text{bank}}} = \left(0, 0.6\right)\text{m/s} + \left(1.7, 0\right)\text{m/s}$$
$$= \left(1.7, 0.6\right)\text{m/s}$$

Magnitude: $v_{\substack{\text{Huck}\\\text{rel. bank}}} = \sqrt{1.7^2 + 0.6^2} = \boxed{1.8\,\text{m/s}}$

Direction: $\theta = \tan^{-1}\dfrac{0.6}{1.7} = \boxed{19^\circ\text{ relative to river}}$

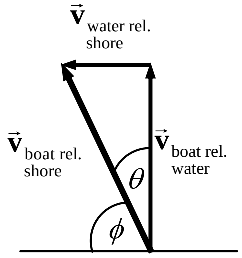
38. We have $v_{\substack{\text{car rel.}\\\text{ground}}} = 25\,\text{m/s}$. Use the diagram, illustrating $\vec{\mathbf{v}}_{\substack{\text{snow rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{snow rel.}\\\text{car}}} + \vec{\mathbf{v}}_{\substack{\text{car rel.}\\\text{ground}}}$, to calculate the other speeds.


$\vec{\mathbf{v}}$ snow rel. car
$\vec{\mathbf{v}}$ snow rel. ground
30°
$\vec{\mathbf{v}}$ car rel. ground

$$\cos 30^\circ = \frac{v_{\substack{\text{car rel.}\\\text{ground}}}}{v_{\substack{\text{snow rel.}\\\text{car}}}} \rightarrow v_{\substack{\text{snow rel.}\\\text{car}}} = 25\,\text{m/s}/\cos 30^\circ = \boxed{29\,\text{m/s}}$$

$$\tan 30^\circ = \frac{v_{\substack{\text{snow rel.}\\\text{ground}}}}{v_{\substack{\text{car rel.}\\\text{ground}}}} \rightarrow v_{\substack{\text{snow rel.}\\\text{ground}}} = \left(25\,\text{m/s}\right)\tan 30^\circ = \boxed{14\,\text{m/s}}$$

39. Call the direction of the flow of the river the *x* direction, and the direction the boat is headed the *y* direction.


$\vec{\mathbf{v}}$ water rel. shore
$\vec{\mathbf{v}}$ boat rel. shore
$\vec{\mathbf{v}}$ boat rel. water
θ
ϕ

(*a*) $$v_{\substack{\text{boat rel.}\\\text{shore}}} = \sqrt{v^2_{\substack{\text{water rel.}\\\text{shore}}} + v^2_{\substack{\text{boat rel.}\\\text{water}}}} = \sqrt{1.20^2 + 2.30^2} = \boxed{2.59\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{1.20}{2.30} = 27.6^\circ\ ,\ \phi = 90^\circ - \theta = \boxed{62.4^\circ\text{ relative to shore}}$$

(*b*) The position of the boat after 3.00 seconds is given by

$$\Delta d = v_{\substack{\text{boat rel.}\\\text{shore}}} t = \left[(1.20, 2.30)\,\text{m/s}\right](3.00\,\text{sec})$$

$$= \boxed{(3.60\text{ m downstream}, 6.90\text{ m across the river})}$$

As a magnitude and direction, it would be 7.8 m away from the starting point, at an angle of 62.4° relative to the shore.

40. If each plane has a speed of 785 km/hr, then their relative speed of approach is 1570 km/hr. If the planes are 11 km apart, then the time for evasive action is found from

$$\Delta d = vt \quad \rightarrow \quad t = \frac{\Delta d}{v} = \left(\frac{11.0\text{ km}}{1570\,\text{km/hr}}\right)\left(\frac{3600\text{ sec}}{1\text{ hr}}\right) = \boxed{25.2\text{ s}}$$

41. Call east the positive *x* direction and north the positive *y* direction. Then the following vector velocity relationship exists.

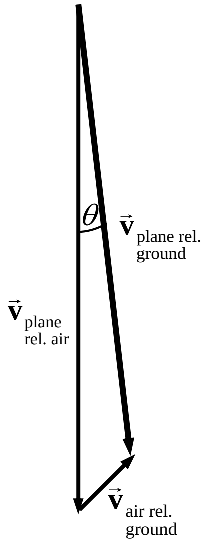
(*a*) $$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$$

$$= (0, -600)\,\text{km/h} + \left(100\cos 45.0^\circ, 100\sin 45.0^\circ\right)\text{km/h}$$

$$= (70.7, -529)\,\text{km/h}$$

$$v_{\substack{\text{plane rel.}\\\text{ground}}} = \sqrt{(70.7\,\text{km/h})^2 + (-529\,\text{km/h})^2} = \boxed{540\,\text{km/h}}$$

$$\theta = \tan^{-1}\frac{70.7}{529} = \boxed{7.6^\circ\text{ east of south}}$$



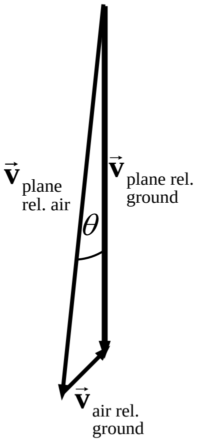
(*b*) The plane is away from its intended position by the distance the air has caused it to move. The wind speed is 100 km/h, so after 10 min (1/6 h), the plane is off course by $\Delta x = v_x t = (100\,\text{km/h})\left(\tfrac{1}{6}\,\text{h}\right) = \boxed{17\text{ km}}$.

42. Call east the positive *x* direction and north the positive *y* direction. Then the following vector velocity relationship exists.

$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}} \quad \rightarrow$$

$$\left(0, -v_{\substack{\text{plane rel.}\\\text{ground}}}\right) = (-600\sin\theta, 600\cos\theta)\,\text{km/h}$$

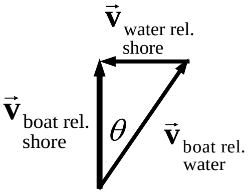
$$+ \left(100\cos 45.0^\circ, 100\sin 45.0^\circ\right)\text{km/h}$$



Equate *x* components in the above equation.

$$0 = -600\sin\theta + 100\cos 45.0^\circ \quad \rightarrow$$

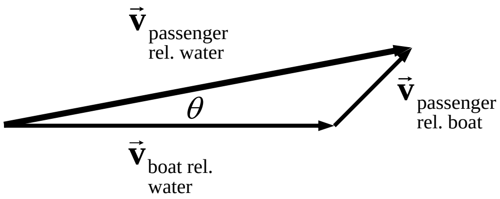
$$\theta = \sin^{-1}\frac{100\cos 45.0^\circ}{600} = \boxed{6.77^\circ\text{, west of south}}$$



43. From the diagram in figure 3-29, it is seen that

$$v_{\substack{\text{boat rel.}\\\text{shore}}} = v_{\substack{\text{boat rel.}\\\text{water}}} \cos\theta = (1.85\,\text{m/s})\cos 40.4^\circ = \boxed{1.41\,\text{m/s}}.$$

44. Call the direction of the boat relative to the water the *x* direction, and upward the *y* direction. Also see the diagram.


$\vec{\mathbf{v}}$ passenger rel. water
θ
$\vec{\mathbf{v}}$ passenger rel. boat
$\vec{\mathbf{v}}$ boat rel. water

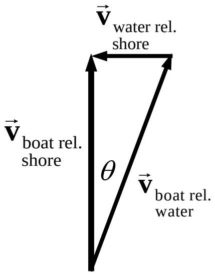
$$\vec{\mathbf{v}}_{\substack{\text{passenger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{passenger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}}$$

$$= \left(0.50\cos 45^\circ, 0.50\sin 45^\circ\right)\text{m/s}$$

$$+ \left(1.50, 0\right)\text{m/s} = \left(1.854, 0.354\right)\text{m/s}$$

$$v_{\substack{\text{passenger}\\ \text{rel. water}}} = \sqrt{1.854^2 + 0.354^2} = \boxed{1.89\ \text{m/s}}$$

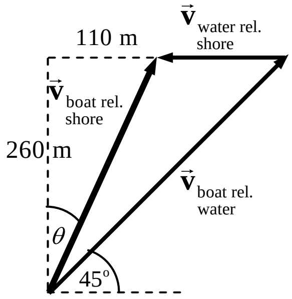
45. Call the direction of the flow of the river the *x* direction, and the direction straight across the river the *y* direction. The boat is traveling straight across the river. The boat is headed at $\theta = 28.5^\circ$ upstream, at a speed of $v_{\substack{\text{boat rel.}\\ \text{water}}} = 2.60\ \text{m/s}$.


$\vec{\mathbf{v}}$ water rel. shore
$\vec{\mathbf{v}}$ boat rel. shore
θ
$\vec{\mathbf{v}}$ boat rel. water

(*a*) $\sin\theta = v_{\substack{\text{water rel.}\\ \text{shore}}} \Big/ v_{\substack{\text{boat rel.}\\ \text{water}}} \rightarrow v_{\substack{\text{water rel.}\\ \text{shore}}} = \left(2.60\ \text{m/s}\right)\sin 28.5^\circ = \boxed{1.24\ \text{m/s}}$

(*b*) $\cos\theta = v_{\substack{\text{boat rel.}\\ \text{shore}}} \Big/ v_{\substack{\text{boat rel.}\\ \text{water}}} \rightarrow v_{\substack{\text{boat rel.}\\ \text{shore}}} = \left(2.60\ \text{m/s}\right)\cos 28.5^\circ = \boxed{2.28\ \text{m/s}}$

46. Call the direction of the flow of the river the *x* direction, and the direction straight across the river the *y* direction. From the diagram, $\theta = \tan^{-1} 110\ \text{m} \big/ 260\ \text{m} = 22.9^\circ$. Equate the vertical components of the velocities to find the speed of the boat relative to the shore.


110 m
$\vec{\mathbf{v}}$ water rel. shore
$\vec{\mathbf{v}}$ boat rel. shore
260 m
$\vec{\mathbf{v}}$ boat rel. water
θ
45°

$$v_{\substack{\text{boat rel.}\\ \text{shore}}} \cos\theta = v_{\substack{\text{boat rel.}\\ \text{water}}} \sin 45^\circ \quad \rightarrow$$

$$v_{\substack{\text{boat rel.}\\ \text{shore}}} = \left(1.70\ \text{m/s}\right)\frac{\sin 45^\circ}{\cos 22.9^\circ} = 1.305\ \text{m/s}$$

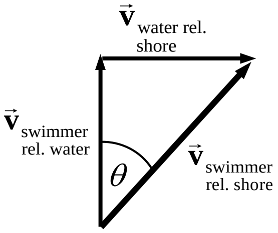
Equate the horizontal components of the velocities.

$$v_{\substack{\text{boat rel.}\\ \text{shore}}} \sin\theta = v_{\substack{\text{boat rel.}\\ \text{water}}} \cos 45^\circ - v_{\substack{\text{water}\\ \text{rel. shore}}} \quad \rightarrow$$

$$v_{\substack{\text{water}\\ \text{rel. shore}}} = v_{\substack{\text{boat rel.}\\ \text{water}}} \cos 45^\circ - v_{\substack{\text{boat rel.}\\ \text{shore}}} \sin\theta$$

$$= \left(1.70\ \text{m/s}\right)\cos 45^\circ - \left(1.305\ \text{m/s}\right)\sin 22.9^\circ = \boxed{0.69\ \text{m/s}}$$

$\boxed{47.}$ Call the direction of the flow of the river the *x* direction, and the direction straight across the river the *y* direction. Call the location of the swimmer's starting point the origin.


$\vec{\mathbf{v}}$ water rel. shore
$\vec{\mathbf{v}}$ swimmer rel. water
θ
$\vec{\mathbf{v}}$ swimmer rel. shore

$$\vec{\mathbf{v}}_{\substack{\text{swimmer}\\ \text{rel. shore}}} = \vec{\mathbf{v}}_{\substack{\text{swimmer}\\ \text{rel. water}}} + \vec{\mathbf{v}}_{\substack{\text{water rel.}\\ \text{shore}}} = \left(0, 0.45\ \text{m/s}\right) + \left(0.40\ \text{m/s}, 0\right)$$

$$= \left(0.40, 0.45\right)\text{m/s}$$

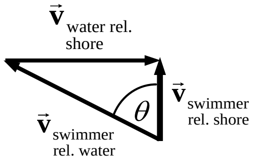
(*a*) Since the swimmer starts from the origin, the distances covered in the *x* and *y* directions will be exactly proportional to the speeds in those directions.

$$\frac{\Delta x}{\Delta y} = \frac{v_x t}{v_y t} = \frac{v_x}{v_y} \quad \rightarrow \quad \frac{\Delta x}{75\ \text{m}} = \frac{0.40\ \text{m/s}}{0.45\ \text{m/s}} \quad \rightarrow \quad \Delta x = \boxed{67\ \text{m}}$$

(*b*) The time is found from the constant velocity relationship for either the *x* or *y* directions.

$$\Delta y = v_y t \quad \rightarrow \quad t = \frac{\Delta y}{v_y} = \frac{75\text{ m}}{0.45\,\text{m/s}} = \boxed{170\text{ s}}$$

48. (*a*) Call the direction of the flow of the river the *x* direction, and the direction straight across the river the *y* direction.


$\vec{\mathbf{v}}_{\text{water rel. shore}}$
$\vec{\mathbf{v}}_{\text{swimmer rel. shore}}$
θ
$\vec{\mathbf{v}}_{\text{swimmer rel. water}}$

$$\sin\theta = \frac{v_{\text{water rel. shore}}}{v_{\text{swimmer rel. water}}} = \frac{0.40\,\text{m/s}}{0.45\,\text{m/s}} \rightarrow \theta = \sin^{-1}\frac{0.40}{0.45} = 62.73^\circ = \boxed{62^\circ}$$

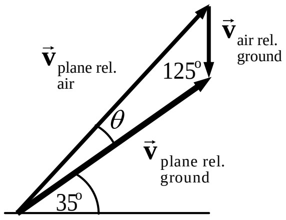
(*b*) From the diagram her speed with respect to the shore is

$$v_{\text{swimmer rel. shore}} = v_{\text{swimmer rel. water}}\cos\theta = \left(0.45\,\text{m/s}\right)\cos 62.73^\circ = 0.206\,\text{m/s}$$

The time to cross the river can be found from the constant velocity relationship.

$$\Delta x = vt \quad \rightarrow \quad t = \frac{\Delta x}{v} = \frac{75\text{ m}}{0.206\,\text{m/s}} = 364\text{ s} = \boxed{3.6\times10^2\text{ s} = 6.1\text{ min}}$$

49. Call east the positive *x* direction and north the positive *y* direction. The following is seen from the diagram. Apply the law of sines to the triangle formed by the three vectors.


$\vec{\mathbf{v}}_{\text{air rel. ground}}$
$\vec{\mathbf{v}}_{\text{plane rel. air}}$
125°
θ
$\vec{\mathbf{v}}_{\text{plane rel. ground}}$
35°

$$\frac{v_{\text{plane rel. air}}}{\sin 125^\circ} = \frac{v_{\text{air rel. ground}}}{\sin\theta} \quad \rightarrow \quad \sin\theta = \frac{v_{\text{air rel. ground}}}{v_{\text{plane rel. air}}}\sin 125^\circ \quad \rightarrow$$

$$\theta = \sin^{-1}\left(\frac{95}{620}\sin 125^\circ\right) = 7.211^\circ$$

So the plane should head in a direction of $35.0^\circ + 7.2^\circ = \boxed{42.2^\circ\text{ north of east}}$.

50. Take the origin to be the location at which the speeder passes the police car, in the reference frame of the unaccelerated police car. The speeder is traveling at $145\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 40.28\,\text{m/s}$ relative to the ground, and the policeman is traveling at $95\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s}$ relative to the ground. Relative to the unaccelerated police car, the speeder is traveling at $13.89\,\text{m/s} = v_s$, and the police car is not moving. Do all of the calculations in the frame of reference of the unaccelerated police car.

The position of the speeder in the chosen reference frame is given by $\Delta x_s = v_s t$. The position of the policeman in the chosen reference frame is given by $\Delta x_p = \frac{1}{2}a_p\left(t-1\right)^2, t > 1$. The police car overtakes the speeder when these two distances are the same.; i.e., $\Delta x_s = \Delta x_p$.

$$\Delta x_s = \Delta x_p \quad \rightarrow \quad v_s t = \tfrac{1}{2}a_p\left(t-1\right)^2 \quad \rightarrow \quad \left(13.89\,\text{m/s}\right)t = \tfrac{1}{2}\left(2\,\text{m/s}^2\right)\left(t^2 - 2t + 1\right) = t^2 - 2t + 1$$

$$t^2 - 15.89t + 1 = 0 \quad \rightarrow \quad t = \frac{15.89 \pm \sqrt{15.89^2 - 4}}{2} = 0.0632\text{ s}, 15.83\text{ s}$$

Since the police car doesn't accelerate until $t = 1.00\text{ s}$, the correct answer is $\boxed{t = 15.8\text{ s}}$.

51. Take the origin to be the location at which the speeder passes the police car. The speed of the speeder is v_s. The position of the speeder after the 7.00 seconds is $\Delta x_s = v_s t = v_s\left(7.00\text{ s}\right)$. The position of the police car is calculated based on the fact that the car traveled 1 second at the original velocity, and then 6 seconds under acceleration. Note that the police car's velocity must have the units changed.

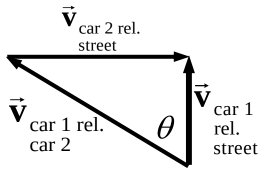
$$v_p = \left(95\text{ km/h}\right)\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 26.39\text{ m/s} \qquad a_p = 2.00\text{ m/s}^2$$

$$\Delta x_p = v_p\left(1.00\text{ s}\right) + v_p\left(6.00\text{ s}\right) + \tfrac{1}{2}a_p\left(6.00\text{ s}\right)^2 = 220.7\text{ m}$$

The police car overtakes the speeder when these two distances are the same; i.e., $\Delta x_s = \Delta x_p$.

$$v_s\left(7\text{ s}\right) = 220.7\text{ m} \quad\rightarrow\quad v_s = \frac{220.7\text{ m}}{7\text{ s}}\left(\frac{3.6\text{ km/h}}{1\text{ m/s}}\right) = \boxed{114\text{ km/h}}$$

52. Call east the positive *x* direction and north the positive *y* direction. From the first diagram, this relative velocity relationship is seen.

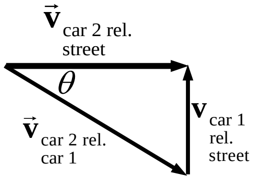


$\vec{\mathbf{v}}_{\text{car 2 rel. street}}$
$\vec{\mathbf{v}}_{\text{car 1 rel. car 2}}$
θ
$\vec{\mathbf{v}}_{\text{car 1 rel. street}}$

$$\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}} + \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} \quad\rightarrow\quad v_{\substack{\text{car 1 rel.}\\\text{car 2}}} = \sqrt{\left(-55\right)^2 + \left(35\right)^2} = \boxed{65\text{ km/h}}$$

$$\theta = \tan^{-1} 55/35 = \boxed{58^\circ\text{ West of North}}$$

For the other relative velocity relationship:



$\vec{\mathbf{v}}_{\text{car 2 rel. street}}$
θ
$\vec{\mathbf{v}}_{\text{car 2 rel. car 1}}$
$\vec{\mathbf{v}}_{\text{car 1 rel. street}}$

$$\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} + \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} \quad\rightarrow\quad v_{\substack{\text{car 2 rel.}\\\text{car 1}}} = \sqrt{\left(55\right)^2 + \left(-35\right)^2} = \boxed{65\text{ km/h}}$$

$$\theta = \tan^{-1} 35/55 = \boxed{32^\circ\text{ South of East}}$$

Notice that the two relative velocities are opposites of each other: $\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} = -\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}}$

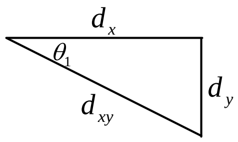
53. Since the arrow will start and end at the same height, use the range formula derived in Example 3-8. The range is 27 m, and the initial speed of the arrow is 35 m/s.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad\rightarrow\quad \sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{\left(27\text{ m}\right)\left(9.80\text{ m/s}^2\right)}{\left(35\text{ m/s}\right)^2} = 0.216$$

$$\theta_0 = \tfrac{1}{2}\sin^{-1} 0.216 = 6.2^\circ, 83.8^\circ$$

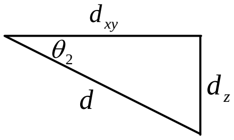
Only the first answer is practical, so the result is $\boxed{\theta_0 = 6.2^\circ}$.

54. The plumber's displacement in component notation is $\boxed{\vec{\mathbf{d}} = \left(50\text{ m}, -25\text{ m}, -10\text{ m}\right)}$. Since this is a 3-dimensional problem, it requires 2 angles to determine his location (similar to latitude and longitude on the surface of the Earth). In the *x*-*y* plane, this follows.



$$\theta_1 = \tan^{-1}\frac{d_y}{d_x} = \tan^{-1}\frac{25}{50} = 27^\circ\text{ South of East}$$

$$d_{xy} = \sqrt{d_x^2 + d_y^2} = \sqrt{(50)^2 + (-25)^2} = 55.9\text{ m}$$


d_{xy}
θ_2
d
d_z

For the vertical motion, consider another right triangle, made up of d_{xy} as one leg, and the vertical displacement d_z as the other leg. See the second figure, and the following calculations.

$$\theta_2 = \tan^{-1}\frac{d_z}{d_{xy}} = \tan^{-1}\frac{10\text{ m}}{55.9\text{ m}} = 10^\circ \text{ Below the Horizontal}$$

$$d = \sqrt{d_{xy}^2 + d_z^2} = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{(50)^2 + (-25)^2 + (-10)^2} = 57\text{ m}$$

The result is that the displacement is $\boxed{57\text{ m}}$, at an angle of $\boxed{27^\circ \text{ South of East}}$, and $\boxed{10^\circ \text{ Below the Horizontal}}$.

55. Assume a constant upward slope, and so the deceleration is along a straight line. The starting velocity along that line is $120\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 33.3\,\text{m/s}$. The ending velocity is 0 m/s. The acceleration is found from Eq. 2-11a.

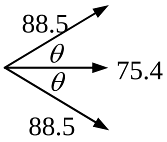
$$v = v_0 + at \quad \rightarrow \quad 0 = 33.3\,\text{m/s} + a(6.0\text{ s}) \quad \rightarrow \quad a = -\frac{33.3\,\text{m/s}}{6.0\text{ s}} = -5.56\,\text{m/s}^2$$

The horizontal acceleration is $a_{\text{horiz}} = a\cos\theta = -5.56\,\text{m/s}^2\left(\cos 32^\circ\right) = \boxed{-4.7\,\text{m/s}^2}$.

The vertical acceleration is $a_{\text{vert}} = a\sin\theta = -5.56\,\text{m/s}^2\left(\sin 30^\circ\right) = \boxed{-2.8\,\text{m/s}^2}$

The horizontal acceleration is to the left in the textbook diagram, and the vertical acceleration is down.

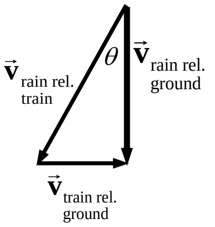
56. Magnitude $= \sqrt{75.4^2 + y^2} = 88.5 \quad \rightarrow \quad y = \pm\sqrt{88.5^2 - 75.4^2} = \pm 46.34 = \boxed{\pm 46.3}$


88.5
θ
75.4
θ
88.5

Direction $= \tan^{-1}\frac{46.34}{75.4} = \boxed{31.6^\circ \text{ relative to } x \text{ axis}}$

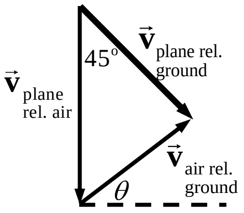
See the diagram for the two possible answers.

57. Choose the *x* direction to be the direction of train travel (the direction the passenger is facing) and choose the *y* direction to be up. This relationship exists among the velocities: $\vec{\mathbf{v}}_{\substack{\text{rain rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{rain rel.}\\\text{train}}} + \vec{\mathbf{v}}_{\substack{\text{train rel.}\\\text{ground}}}$. From the diagram, find the expression for the speed of the raindrops.


$\vec{\mathbf{v}}_{\substack{\text{rain rel.}\\\text{train}}}$
θ
$\vec{\mathbf{v}}_{\substack{\text{rain rel.}\\\text{ground}}}$
$\vec{\mathbf{v}}_{\substack{\text{train rel.}\\\text{ground}}}$

$$\tan\theta = \frac{v_{\substack{\text{train rel.}\\\text{ground}}}}{v_{\substack{\text{rain rel.}\\\text{ground}}}} = \frac{v_{\text{T}}}{v_{\substack{\text{rain rel.}\\\text{ground}}}} \quad \rightarrow \quad \boxed{v_{\substack{\text{rain rel.}\\\text{ground}}} = \frac{v_{\text{T}}}{\tan\theta}}.$$

58. Call east the positive *x* direction and north the positive *y* direction. Then this relative velocity relationship follows (see the accompanying diagram).


45°
$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}}$
$\vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}}$
$\vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$
θ

$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$$

Equate the *x* components of the velocity vectors.

$$(125\,\text{km/h})\cos 45^\circ = 0 + v_{\text{wind }x} \quad\rightarrow\quad v_{\text{wind }x} = 88.4\,\text{km/h}\ .$$

From the *y* components of the above equation:

$$-125\sin 45^\circ = -155 + v_{\text{wind-y}} \quad\rightarrow\quad v_{\text{wind-y}} = 155 - 125\sin 45^\circ = 66.6\,\text{km/h}$$

The magnitude of the wind velocity is

$$v_{\text{wind}} = \sqrt{v^2_{\text{wind-x}} + v^2_{\text{wind-y}}} = \sqrt{(88.4\,\text{km/h})^2 + (66.6\,\text{km/h})^2} = \boxed{111\,\text{km/h}}\,.$$

The direction of the wind is $\theta = \tan^{-1}\dfrac{v_{\text{wind-y}}}{v_{\text{wind-x}}} = \tan^{-1}\dfrac{66.6}{88.4} = \boxed{37.0^\circ \text{ north of east}}$.

59. Work in the frame of reference in which the train is at rest. Then, relative to the train, the car is moving at 20 km/h. The car has to travel 1 km in that frame of reference to pass the train, and so the time to pass can be found from the constant horizontal velocity relationship.

$$\Delta x = v_x t \quad\rightarrow\quad t_{\substack{\text{same}\\\text{direction}}} = \frac{\Delta x}{(v_x)_{\substack{\text{same}\\\text{direction}}}} = \frac{1\text{ km}}{20\,\text{km/h}} = 0.05\text{ h}\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{180\text{ s}}$$

The car travels 1 km in the frame of reference of the stationary train, but relative to the ground, the car is traveling at 95 km/hr and so relative to the ground the car travels this distance:

$$\Delta x = v_x t_{\substack{\text{same}\\\text{direction}}} = (95\,\text{km/h})(0.05\text{ h}) = \boxed{4.8\text{ km}}$$

If the car and train are traveling in opposite directions, then the velocity of the car relative to the train will be 170 km/h. Thus the time to pass will be

$$t_{\substack{\text{opposite}\\\text{direction}}} = \frac{\Delta x}{(v_x)_{\substack{\text{opposite}\\\text{direction}}}} = \frac{1\text{ km}}{170\,\text{km/h}} = \left(\frac{1}{170}\text{ h}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{21.2\text{ s}}\,.$$

The distance traveled by the car relative to the ground will be

$$\Delta x = v_x t_{\substack{\text{opposite}\\\text{direction}}} = (95\,\text{km/h})\left(\frac{1}{170}\text{ h}\right) = \boxed{0.56\text{ km}}\,.$$

60. The time of flight is found from the constant velocity relationship for horizontal motion.

$$\Delta x = v_x t = \quad\rightarrow\quad t = \Delta x/v_x = 8.0\text{ m}/9.1\,\text{m/s} = \boxed{0.88\text{ s}}$$

The *y* motion is symmetric in time – it takes half the time of flight to rise, and half to fall. Thus the time for the jumper to fall from his highest point to the ground is 0.44 sec. His vertical speed is zero at the highest point. From this time, starting vertical speed, and the acceleration of gravity, the maximum height can be found. Call upward the positive *y* direction. The point of maximum height is the starting position y_0, the ending position is $y = 0$, the starting vertical speed is 0, and $a = -g$. Use Eq. 2-11b to find the height.

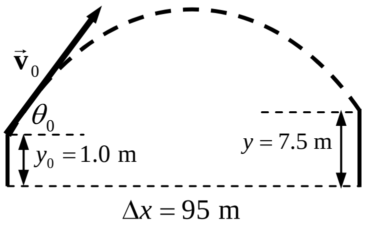
$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0 = y_0 + 0 - \tfrac{1}{2}(9.8\,\text{m/s}^2)(0.44\text{ s})^2 \quad\rightarrow\quad y_0 = \boxed{0.95\text{ m}}\,.$$

61. Assume that the golf ball takes off and lands at the same height, so that the range formula derived in Example 3-8 can be applied. The only variable is to be the acceleration due to gravity.

$$R_{\text{Earth}} = v_0^2\sin 2\theta_0/g_{\text{Earth}} \qquad R_{\text{Moon}} = v_0^2\sin 2\theta_0/g_{\text{Moon}}$$

$$\frac{R_{\text{Earth}}}{R_{\text{Moon}}} = \frac{v_0^2\sin 2\theta_0/g_{\text{Earth}}}{v_0^2\sin 2\theta_0/g_{\text{Moon}}} = \frac{1/g_{\text{Earth}}}{1/g_{\text{Moon}}} = \frac{g_{\text{Moon}}}{g_{\text{Earth}}} = \frac{35\text{ m}}{180\text{ m}} = 0.19 \quad\rightarrow$$

$$g_{\text{Moon}} = 0.19\,g_{\text{Earth}} \approx \boxed{1.9\,\text{m/s}^2}$$



62. The minimum speed will be that for which the ball just clears the fence; i.e., the ball has a height of 7.5 m when it is 95 m horizontally from home plate. The origin is at home plate, with upward as the positive *y* direction. For the ball, $y_0 = 1.0\text{ m}$, $y = 7.5\text{ m}$, $a_y = -g$, $v_{y0} = v_0 \sin\theta_0$, $v_x = v_0 \cos\theta_0$, and $\theta_0 = 38^\circ$. See the diagram (not to scale). For the horizontal motion at constant velocity, $\Delta x = v_x t = v_0 \cos\theta_0 t$, and so $t = \dfrac{\Delta x}{v_0 \cos\theta_0}$. For the vertical motion, apply Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = y_0 + v_0(\sin\theta_0)t - \tfrac{1}{2}gt^2$$

Substitute the value of the time of flight for the first occurrence only in the above equation, and then solve for the time.

$$y = y_0 + v_0 t \sin\theta_0 - \tfrac{1}{2}gt^2 \rightarrow y = y_0 + v_0 \sin\theta_0 \frac{\Delta x}{v_0 \cos\theta_0} - \tfrac{1}{2}gt^2 \rightarrow$$

$$t = \sqrt{2\left(\frac{y_0 - y + \Delta x \tan\theta_0}{g}\right)} = \sqrt{2\left(\frac{1.0\text{ m} - 7.5\text{ m} + (95\text{ m})\tan 38^\circ}{9.80\,\text{m/s}^2}\right)} = 3.718\text{ s}$$

Finally, use the time with the horizontal range to find the initial speed.

$$\Delta x = v_0 \cos\theta_0 t \rightarrow v_0 = \frac{\Delta x}{t\cos\theta_0} = \frac{95\text{ m}}{(3.718\text{ s})\cos 38^\circ} = \boxed{32\,\text{m/s}}$$

63. Choose downward to be the positive *y* direction. The origin is at the point from which the divers push off the cliff. In the vertical direction, the initial velocity is $v_{y0} = 0$, the acceleration is $a_y = 9.80\,\text{m/s}^2$, and the displacement is 35 m. The time of flight is found from Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \rightarrow 35\text{ m} = 0 + 0 + \tfrac{1}{2}(9.8\,\text{m/s}^2)t^2 \rightarrow t = \sqrt{\frac{2(35\text{ m})}{9.8\,\text{m/s}^2}} = \boxed{2.7\text{ s}}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \rightarrow v_x = \Delta x/t = 5.0\text{ m}/2.7\text{ s} = \boxed{1.9\,\text{m/s}}$$

64. Choose the origin to be the location on the ground directly underneath the ball when served, and choose upward as the positive *y* direction. Then for the ball, $y_0 = 2.50\text{ m}$, $v_{y0} = 0$, $a_y = -g$, and the *y* location when the ball just clears the net is $y = 0.90\text{ m}$. The time for the ball to reach the net is calculated from Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \rightarrow 0.90\text{ m} = 2.50\text{ m} + 0 + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2 \rightarrow$$

$$t_{\substack{\text{to}\\\text{net}}} = \sqrt{\frac{2(-1.60\text{ m})}{-9.80\,\text{m/s}^2}} = 0.57143\text{ s}$$

The *x* velocity is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \rightarrow v_x = \frac{\Delta x}{t} = \frac{15.0\text{ m}}{0.57143\text{ s}} = 26.25 \approx \boxed{26.3\,\text{m/s}}.$$

This is the minimum speed required to clear the net.

To find the full time of flight of the ball, set the final *y* location to be *y* = 0, and again use Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 0.0\text{ m} = 2.50\text{ m} + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2 \;\rightarrow$$

$$t_{\text{total}} = \sqrt{\frac{2\left(-2.50\text{ m}\right)}{-9.80\,\text{m/s}^2}} = 0.7143 \approx \boxed{0.714\,\text{s}}$$

The horizontal position where the ball lands is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = \left(26.25\,\text{m/s}\right)\left(0.7143\text{ s}\right) = 18.75 \approx \boxed{18.8\text{ m}}.$$

Since this is between 15.0 and 22.0 m, $\boxed{\text{the ball lands in the "good" region}}$.

65. Work in the frame of reference in which the car is at rest at ground level. In this reference frame, the helicopter is moving horizontally with a speed of

$$215\,\text{km/h} - 155\,\text{km/h} = 60\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 16.67\,\text{m/s}.$$

For the vertical motion, choose the level of the helicopter to be the origin, and downward to be positive. Then the package's *y* displacement is $y = 78.0\text{ m}$, $v_{y0} = 0$, and $a_y = g$. The time for the package to fall is calculated from Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 78.0\text{ m} = \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)t^2 \quad\rightarrow\quad t = \sqrt{\frac{2\left(78.0\text{ m}\right)}{9.80\,\text{m/s}^2}} = 3.99\text{ sec}$$

The horizontal distance that the package must move, relative to the "stationary" car, is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = \left(16.67\,\text{m/s}\right)\left(3.99\text{ s}\right) = 66.5\text{ m}$$

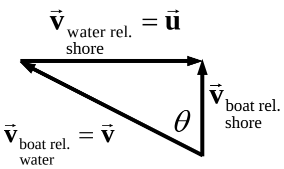
Thus the angle under the horizontal for the package release will be

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{78.0\text{ m}}{66.5\text{ m}}\right) = 49.55^\circ \approx \boxed{50^\circ} \text{ (to 2 significant figures).}$$

66. (*a*) For the upstream trip, the boat will cover a distance of $D/2$ with a net speed of $v-u$, so the time is $t_1 = \dfrac{D/2}{v-u} = \dfrac{D}{2\left(v-u\right)}$. For the downstream trip, the boat will cover a distance of $D/2$ with a net speed of $v+u$, so the time is $t_2 = \dfrac{D/2}{v+u} = \dfrac{D}{2\left(v+u\right)}$. Thus the total time for the round trip will be $t = t_1 + t_2 = \dfrac{D}{2\left(v-u\right)} + \dfrac{D}{2\left(v+u\right)} = \boxed{\dfrac{Dv}{\left(v^2-u^2\right)}}$.

(*b*) For the boat to go directly across the river, it must be angled against the current in such a way that the net velocity is straight across the river, as in the picture. This equation must be satisfied:

$$\vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{shore}}} = \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} + \vec{\mathbf{v}}_{\substack{\text{water rel.}\\ \text{shore}}} = \vec{\mathbf{v}} + \vec{\mathbf{u}}.$$


$\vec{\mathbf{v}}_{\substack{\text{water rel.}\\ \text{shore}}} = \vec{\mathbf{u}}$
$\vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{shore}}}$
θ
$\vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} = \vec{\mathbf{v}}$

Thus $v_{\substack{\text{boat rel.}\\ \text{shore}}} = \sqrt{v^2 - u^2}$, and the time to go a distance $D/2$ across the river is $t_1 = \frac{D/2}{\sqrt{v^2-u^2}} = \frac{D}{2\sqrt{v^2-u^2}}$. The same relationship would be in effect for crossing back, so the time to come back is given by $t_2 = t_1$ and the total time is $t = t_1 + t_2 = \boxed{\frac{D}{\sqrt{v^2-u^2}}}$.

The speed v must be greater than the speed u. The velocity of the boat relative to the shore when going upstream is $v-u$. If $v<u$, the boat will not move upstream at all, and so the first part of the trip would be impossible. Also, in part b, we see that *v* is longer than *u* in the triangle, since *v* is the hypotenuse.

67. Choose the origin to be the point from which the projectile is launched, and choose upward as the positive *y* direction. The *y* displacement of the projectile is 155 m, and the horizontal range of the projectile is 195 m. The acceleration in the *y* direction is $a_y = -g$, and the time of flight is 7.6 s. The horizontal velocity is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \frac{\Delta x}{t} = \frac{195\text{ m}}{7.6\text{ s}} = 25.7\,\text{m/s}$$

Calculate the initial *y* velocity from the given data and Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 155\text{ m} = v_{y0}(7.6\text{ s}) + \tfrac{1}{2}(-9.80\,\text{m/s}^2)(7.6\text{ s})^2 \;\rightarrow\; v_{y0} = 57.6\,\text{m/s}$$

Thus the initial velocity and direction of the projectile are:

$$v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{(25.7\,\text{m/s})^2 + (57.6\,\text{m/s})^2} = \boxed{63\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{57.6\,\text{m/s}}{25.7\,\text{m/s}} = \boxed{66^\circ}$$

68. Choose downward to be the positive *y* direction for this problem.

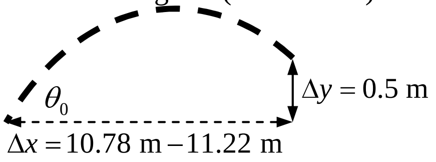
(*a*) The vertical component of her acceleration is directed downward, and its magnitude will be given by $a_y = a\sin\theta = (1.80\,\text{m/s}^2)\sin 30.0^\circ = \boxed{0.900\,\text{m/s}^2}$.

(*b*) The time to reach the bottom of the hill is calculated from Eq. 2-11b, with a *y* displacement of 335 m, $v_{y0} = 0$, and $a_y = 0.900\,\text{m/s}^2$.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 335\text{ m} = 0 + 0 + \tfrac{1}{2}(0.900\,\text{m/s}^2)(t)^2 \quad\rightarrow$$

$$t = \sqrt{\frac{2(335\text{ m})}{(0.900\,\text{m/s}^2)}} = \boxed{27.3\text{ s}}$$

69. The proper initial speeds will be those for which the ball has traveled a horizontal distance somewhere between 10.78 m and 11.22 m while it changes height from 2.10 m to 2.60 m with a shooting angle of 38.0°. Choose the origin to be at the shooting location of the basketball, with upward as the positive *y* direction. Then the vertical displacement is $y = 0.5\text{ m}$, $a_y = -9.80\,\text{m/s}^2$, $v_{y0} = v_0\sin\theta_0$, and the (constant) *x* velocity is $v_x = v_0\cos\theta_0$. See the diagram (not to scale).



For the horizontal motion at constant velocity,

$$\Delta x = v_x t = v_0\cos\theta_0 t \quad\text{and so}\quad t = \frac{\Delta x}{v_0\cos\theta_0}.$$

For the vertical motion, applying Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = v_0 \sin\theta t - \tfrac{1}{2}gt^2$$

Substitute the expression for the time of flight and solve for the initial velocity.

$$y = v_0 \sin\theta t - \tfrac{1}{2}gt^2 = v_0 \sin\theta \frac{\Delta x}{v_0 \cos\theta_0} - \tfrac{1}{2}g\left(\frac{\Delta x}{v_0 \cos\theta_0}\right)^2 = \Delta x \tan\theta - \frac{g(\Delta x)^2}{2v_0^2 \cos^2\theta_0}$$

$$v_0 = \sqrt{\frac{g(\Delta x)^2}{2\cos^2\theta_0(-y + \Delta x \tan\theta)}}$$

For $\Delta x = 10.78\ \text{m}$, the shortest shot:

$$v_0 = \sqrt{\frac{(9.80\,\text{m/s}^2)(10.78\ \text{m})^2}{2\cos^2 38.0^\circ\left[\left(-0.5\ \text{m} + (10.78\ \text{m})\tan 38.0^\circ\right)\right]}} = \boxed{10.8\,\text{m/s}}.$$

For $\Delta x = 11.22\ \text{m}$, the longest shot:

$$v_0 = \sqrt{\frac{(9.80\,\text{m/s}^2)(11.22\ \text{m})^2}{2\cos^2 38.0^\circ\left[\left(-0.5\ \text{m} + (11.22\ \text{m})\tan 38.0^\circ\right)\right]}} = \boxed{11.0\,\text{m/s}}.$$

70. Choose the origin to be the location at water level directly underneath the diver when she left the board. Choose upward as the positive *y* direction. For the diver, $y_0 = 5.0\ \text{m}$, the final y position is $y = 0.0\ \text{m}$ (water level), $a_y = -g$, the time of flight is $t = 1.3\ \text{s}$, and the horizontal displacement is $\Delta x = 3.0\ \text{m}$.

(*a*) The horizontal velocity is determined from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \frac{\Delta x}{t} = \frac{3.0\ \text{m}}{1.3\ \text{s}} = 2.31\,\text{m/s}$$

The initial *y* velocity is found using Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \ \rightarrow\ 0\ \text{m} = 5.0\ \text{m} + v_{y0}(1.3\ \text{s}) + \tfrac{1}{2}(-9.80\,\text{m/s}^2)(1.3\ \text{s})^2 \ \rightarrow$$

$$v_{y0} = 2.52\,\text{m/s}$$

The magnitude and direction of the initial velocity is

$$v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{(2.31\,\text{m/s})^2 + (2.52\,\text{m/s})^2} = \boxed{3.4\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{2.52\,\text{m/s}}{2.31\,\text{m/s}} = \boxed{48^\circ\ \text{above the horizontal}}$$

(*b*) The maximum height will be reached when the *y* velocity is zero. Use Eq. 2-11c.

$$v_y^2 = v_{y0}^2 + 2a\Delta y \quad \rightarrow \quad 0 = (2.52\,\text{m/s})^2 + 2(-9.80\,\text{m/s}^2)(y_{\text{max}} - 5.0\ \text{m}) \quad \rightarrow$$

$$y_{\text{max}} = \boxed{5.3\ \text{m}}$$

(*c*) To find the velocity when she enters the water, the horizontal velocity is the (constant) value of $v_x = 2.31\,\text{m/s}$. The vertical velocity is found from Eq. 2-11a.

$$v_y = v_{y0} + at = 2.52\,\text{m/s} + (-9.80\,\text{m/s}^2)(1.3\ \text{s}) = -10.2\,\text{m/s}.$$

The magnitude and direction of this velocity is given by

$$v=\sqrt{v_x^2+v_y^2}=\sqrt{\left(2.31\,\mathrm{m/s}\right)^2+\left(-10.2\,\mathrm{m/s}\right)^2}=10.458\,\mathrm{m/s}\approx\boxed{10\,\mathrm{m/s}}$$

$$\theta=\tan^{-1}\frac{v_y}{v_x}=\tan^{-1}\frac{-10.2\,\mathrm{m/s}}{2.31\,\mathrm{m/s}}=\boxed{-77^\circ\ \left(\text{below the horizontal}\right)}$$

71. (*a*) Choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive *y* direction. At the end of its flight over the 8 cars, the car must be at $y=-1.5\ \mathrm{m}$. Also for the car, $v_{y0}=0$, $a_y=-g$, $v_x=v_0$, and $\Delta x=20\ \mathrm{m}$. The time of flight is found from the horizontal motion at constant velocity: $\Delta x=v_x t \rightarrow t=\Delta x/v_0$. That expression for the time is used in Eq. 2-11b for the vertical motion.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_y t^2 \quad\rightarrow\quad y=0+0+\tfrac{1}{2}(-g)\left(\frac{\Delta x}{v_0}\right)^2 \quad\rightarrow$$

$$v_0=\sqrt{\frac{-g\left(\Delta x\right)^2}{2\left(y\right)}}=\sqrt{\frac{-\left(9.80\,\mathrm{m/s^2}\right)\left(20\ \mathrm{m}\right)^2}{2\left(-1.5\ \mathrm{m}\right)}}=\boxed{36\,\mathrm{m/s}}$$

(*b*) Again choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive *y* direction. The *y* displacement of the car at the end of its flight over the 8 cars must again be $y=-1.5\ \mathrm{m}$. For the car, $v_{y0}=v_0\sin\theta_0$, $a_y=-g$, $v_x=v_0\cos\theta_0$, and $\Delta x=20\ \mathrm{m}$. The launch angle is $\theta_0=10^\circ$. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x=v_x t \quad\rightarrow\quad t=\frac{\Delta x}{v_0\cos\theta_0}$$

That expression for the time is used in Eq. 2-11b for the vertical motion.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_y t^2 \quad\rightarrow\quad y=v_0\sin\theta_0\frac{\Delta x}{v_0\cos\theta_0}+\tfrac{1}{2}(-g)\left(\frac{\Delta x}{v_0\cos\theta_0}\right)^2 \quad\rightarrow$$

$$v_0=\sqrt{\frac{g\left(\Delta x\right)^2}{2\left(\Delta x\tan\theta_0-y\right)\cos^2\theta_0}}=\sqrt{\frac{\left(9.80\,\mathrm{m/s^2}\right)\left(20\ \mathrm{m}\right)^2}{2\left(\left(20\ \mathrm{m}\right)\tan10^\circ+1.5\ \mathrm{m}\right)\cos^2 10^\circ}}=\boxed{20\,\mathrm{m/s}}$$

72. Choose the origin to be the point at ground level directly below where the ball was hit. Call upwards the positive *y* direction. For the ball, we have $v_0=28\,\mathrm{m/s}$, $\theta_0=61^\circ$, $a_y=-g$, $y_0=0.9\ \mathrm{m}$, and $y=0.0\ \mathrm{m}$.

(*a*) To find the horizontal displacement of the ball, the horizontal velocity and the time of flight are needed. The (constant) horizontal velocity is given by $v_x=v_0\cos\theta_0$. The time of flight is found from Eq. 2-11b.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0=y_0+v_0\sin\theta_0 t-\tfrac{1}{2}gt^2 \quad\rightarrow$$

$$t=\frac{-v_0\sin\theta_0\pm\sqrt{v_0^2\sin^2\theta_0-4\left(-\tfrac{1}{2}g\right)y_0}}{2\left(-\tfrac{1}{2}g\right)}$$

$$= \frac{-(28\,\mathrm{m/s})\sin 61^\circ \pm \sqrt{(28\,\mathrm{m/s})^2 \sin^2 61^\circ - 4\left(-\tfrac{1}{2}\right)\left(9.80\,\mathrm{m/s^2}\right)(0.9\,\mathrm{m})}}{2\left(-\tfrac{1}{2}\right)\left(9.80\,\mathrm{m/s^2}\right)}$$

$$= 5.034\ \mathrm{s}, -0.0365\ \mathrm{s}$$

Choose the positive time, since the ball was hit at $t = 0$. The horizontal displacement of the ball will be found by the constant velocity relationship for horizontal motion.

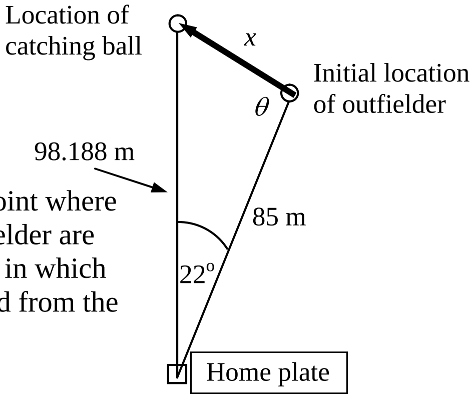
$$\Delta x = v_x t = v_0 \cos\theta_0 t = (28\,\mathrm{m/s})\left(\cos 61^\circ\right)(5.034\mathrm{s}) = 68.34\ \mathrm{m} \approx \boxed{68\ \mathrm{m}}$$

(*b*) The center fielder catches the ball right at ground level. He ran 105 m – 68.34 m = 36.66 m to catch the ball, so his average running speed would be

$$v_{avg} = \frac{\Delta d}{t} = \frac{36.66\ \mathrm{m}}{5.034\,\mathrm{s}} = 7.282\,\mathrm{m/s} \approx \boxed{7.3\,\mathrm{m/s}}$$

73. Since the ball is being caught at the same height from which it was struck, use the range formula to find the horizontal distance the ball travels.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(32\,\mathrm{m/s})^2 \sin\left(2\times 55^\circ\right)}{9.8\,\mathrm{m/s^2}} = 98.188\ \mathrm{m}$$



Then as seen from above, the location of home plate, the point where the ball must be caught, and the initial location of the outfielder are shown in the diagram. The dark arrow shows the direction in which the outfielder must run. The length of that distance is found from the law of cosines as applied to the triangle.

$$x = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

$$= \sqrt{98.188^2 + 85^2 - 2(98.188)(85)\cos 22^\circ} = 37.27\ \mathrm{m}$$

The angle θ at which the outfielder should run is found from the law of sines.

$$\frac{\sin 22^\circ}{x} = \frac{\sin\theta}{98\ \mathrm{m}} \quad \rightarrow \quad \theta = \sin^{-1}\left(\frac{98.188}{37.27}\sin 22^\circ\right) = 81^\circ \text{ or } 99^\circ$$

Since $98.188^2 > 85^2 + 37.27^2$, the angle must be obtuse, so we choose $\theta = 97^\circ$.
Assume that the outfielder's time for running is the same as the time of flight of the ball. The time of flight of the ball is found from the horizontal motion of the ball at constant velocity.

$$R = v_x t = v_0 \cos\theta_0 t \quad \rightarrow \quad t = \frac{R}{v_0 \cos\theta_0} = \frac{98.188\ \mathrm{m}}{(32\,\mathrm{m/s})\cos 55^\circ} = 5.35\ \mathrm{s}$$

Thus the average velocity of the outfielder must be $v_{avg} = \dfrac{\Delta d}{t} = \dfrac{37.27\ \mathrm{m}}{5.35\ \mathrm{s}} = \boxed{7.0\,\mathrm{m/s}}$ at an angle of $\boxed{97^\circ}$ relative to the outfielder's line of sight to home plate.

74. Choose the origin to be the point at the top of the building from which the ball is shot, and call upwards the positive *y* direction. The initial velocity is $v_0 = 18\,\mathrm{m/s}$ at an angle of $\theta_0 = 42^\circ$. The acceleration due to gravity is $a_y = -g$.

(*a*) $v_x = v_0 \cos\theta_0 = (18\,\mathrm{m/s})\cos 42^\circ = 13.38 \approx \boxed{13\,\mathrm{m/s}}$

$v_{y0} = v_0 \sin\theta_0 = (18\,\mathrm{m/s})\sin 42^\circ = 12.04 \approx \boxed{12\,\mathrm{m/s}}$

(*b*) Since the horizontal velocity is known and the horizontal distance is known, the time of flight can be found from the constant velocity equation for horizontal motion.

$$\Delta x = v_x t \quad \rightarrow \quad t = \frac{\Delta x}{v_x} = \frac{55\text{ m}}{13.38\,\text{m/s}} = 4.111\text{ s}$$

With that time of flight, calculate the vertical position of the ball using Eq. 2-11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = \left(12.04\,\text{m/s}\right)\left(4.111\text{ s}\right) + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)\left(4.111\text{ s}\right)^2$$

$$= -33.3 = \boxed{-33\text{ m}}$$

So the ball will strike 33 m below the top of the building.

75. When shooting the gun vertically, half the time of flight is spent moving upwards. Thus the upwards flight takes two seconds. Choose upward as the positive *y* direction. Since at the top of the flight, the vertical velocity is zero, find the launching velocity from Eq. 2-11a.

$$v_y = v_{y0} + at \quad \rightarrow \quad v_{y0} = v_y - at = 0 = \left(9.8\,\text{m/s}^2\right)\left(2.0\text{ s}\right) = 19.6\,\text{m/s}$$

Using this initial velocity and an angle of 45° in the range formula will give the maximum range for the gun.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{\left(19.6\,\text{m/s}\right)^2 \sin\left(2 \times 45^\circ\right)}{9.80\,\text{m/s}^2} = \boxed{39\text{ m}}$$

$$\left(W/t\right)_{\text{savings}}=\left(W/t\right)_{5300}-\left(W/t\right)_{500}=\left(5300\text{ W}-500\text{ W}\right)\left(\frac{\left(273+32\right)\text{K}}{\left(273+21\right)\text{K}}-1\right)=\boxed{180\text{ W}}$$

68. To find the mass of water removed, find the energy that is removed from the low temperature reservoir from the work input and the Carnot efficiency. Then use the latent heat of vaporization to determine the mass of water from the energy required for the condensation. Note that the heat of vaporization used is that given in chapter 14 for evaporation at 20$^\text{o}$C.

$$e=1-\frac{T_\text{L}}{T_\text{H}}=\frac{W}{Q_\text{H}}=\frac{W}{W+Q_\text{L}}\quad\rightarrow\quad Q_\text{L}=W\frac{T_\text{L}}{\left(T_\text{H}-T_\text{L}\right)}=mL_{\text{vapor}}$$

$$m=\frac{W}{L_{\text{vapor}}}\frac{T_\text{L}}{\left(T_\text{H}-T_\text{L}\right)}=\frac{\left(600\text{ W}\right)\left(3600\text{ s}\right)}{\left(2.45\times10^{6}\text{ J/kg}\right)}\frac{\left(273+8\right)\text{K}}{17\text{ K}}=\boxed{14.6\text{ kg}}$$

CHAPTER 4: Dynamics: Newton's Laws of Motion

Answers to Questions

1. The child tends to remain at rest (Newton's 1st Law), unless a force acts on her. The force is applied to the wagon, not the child, and so the wagon accelerates out from under the child, making it look like the child falls backwards relative to the wagon. If the child is standing in the wagon, the force of friction between the child and the bottom of the wagon will produce an acceleration of the feet, pulling the feet out from under the child, also making the child fall backwards.

2. (*a*) Mary sees the box stay stationary with respect to the ground. There is no horizontal force on the box since the truck bed is smooth, and so the box cannot accelerate. Thus Mary would describe the motion of the box in terms of Newton's 1st law – there is no force on the box, so it does not accelerate.
   (*b*) Chris, from his non-inertial reference frame, would say something about the box being "thrown" backwards in the truck, and perhaps use Newton's 2nd law to describe the effects of that force. But the source of that force would be impossible to specify.

3. If the acceleration of an object is zero, then by Newton's second law, the *net* force must be zero. There can be forces acting on the object as long as the vector sum of the forces is zero.

4. If only once force acts on the object, then the net force cannot be zero. Thus the object cannot have zero acceleration, by Newton's second law. The object can have zero velocity for an instant. For example, an object thrown straight up under the influence of gravity has a velocity of zero at the top of its path, but has a non-zero net force and non-zero acceleration throughout the entire flight.

5. (*a*) A force is needed to bounce the ball back up, because the ball changes direction, and so accelerates. If the ball accelerates, there must be a force.
   (*b*) The pavement exerts the force on the golf ball.

6. When you try to walk east, you push on the ground (or on the log in this case) with a westward force. When you push westward on the massive Earth, the Earth moves imperceptibly, but by Newton's 3rd law there is an eastward force on you, which propels you forward. When walking on the log, the relatively light and unrestricted log is free to move, and so when you push it westward, it moves westward as you move eastward.

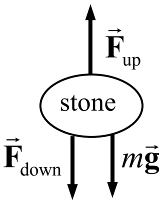
7. By Newton's 3rd law, the desk or wall exerts a force on your foot equal in magnitude to the force with which you hit the desk or wall. If you hit the desk or wall with a large force, then there will be a large force on your foot, causing pain. Only a force on your foot causes pain.

8. (*a*) When you are running, the stopping force is a force of friction between your feet and the ground. You push forward with your feet on the ground, and thus the ground pushes backwards on you, slowing your speed.
   (*b*) A fast person can run about 10 meters per second, perhaps takes a distance of 5 meters over which to stop. Those 5 meters would be about 5 strides, of 1 meter each. The acceleration can be found from Eq. 2-11c.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{0 - \left(10\,\mathrm{m/s}\right)^2}{10\ \mathrm{m}} = -10\,\mathrm{m/s^2}$$

$\boxed{9.}$ When giving a sharp pull, the key is the suddenness of the application of the force. When a large, sudden force is applied to the bottom string, the bottom string will have a large tension in it. Because of the stone's inertia, the upper string does not immediately experience the large force. The bottom string must have more tension in it, and will break first.

If a slow and steady pull is applied, the tension in the bottom string increases. We approximate that condition as considering the stone to be in equilibrium until the string breaks. The free-body diagram for the stone would look like this diagram. While the stone is in equilibrium, Newton's 2nd law states that $F_{up} = F_{down} + mg$. Thus the tension in the upper string is going to be larger than the tension in the lower string because of the weight of the stone, and so the upper string will break first.


$\vec{\mathbf{F}}_{\text{up}}$
stone
$\vec{\mathbf{F}}_{\text{down}}$
$m\vec{\mathbf{g}}$

$\boxed{10.}$ The acceleration of both rocks is found by dividing their weight (the force of gravity on them) by their mass. The 2-kg rock has a force of gravity on it that is twice as great as the force of gravity on the 1-kg rock, but also twice as great a mass as the 1-kg rock, so the acceleration is the same for both.

11. Only the pounds reading would be correct. The spring scale works on the fact that a certain **force** (the weight of the object being weighed) will stretch the spring a certain distance. That distance is proportional to the product of the mass and the acceleration due to gravity. Since the acceleration due to gravity is smaller by a factor of 6 on the moon, the weight of the object is smaller by a factor of 6, and the spring will be pulled to only one-sixth of the distance that it was pulled on the Earth. The mass itself doesn't change when moving to the Moon, and so a mass reading on the Moon would be incorrect.

12. When you pull the rope at an angle, only the horizontal component of the pulling force will be accelerating the box across the table. This is a smaller horizontal force than originally used, and so the horizontal acceleration of the box will decrease.

13. Let us find the acceleration of the Earth, assuming the mass of the freely falling object is $m = 1$ kg. If the mass of the Earth is M, then the acceleration of the Earth would be found using Newton's 3rd law and Newton's 2nd law.

$$F_{\text{Earth}} = F_{\text{object}} \quad \rightarrow \quad Ma_{\text{Earth}} = mg \quad \rightarrow \quad a_{\text{Earth}} = g\,m/M$$

Since the Earth has a mass that is on the order of 10^{25} kg, then the acceleration of the Earth is on the order of $10^{-25}\,g$, or about $10^{-24}\ \text{m/s}^2$. This tiny acceleration is undetectable.

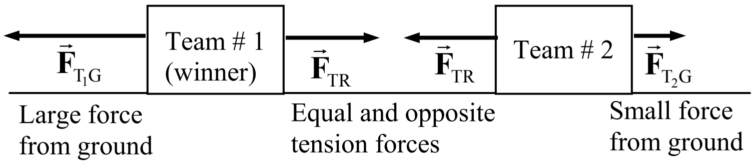
14. (*a*) To lift the object on the Earth requires a force the same size as its weight on Earth, $F_{\text{Earth}} = mg_{\text{Earth}} = 98\ \text{N}$. To lift the object on the Moon requires a force the same size as its weight on the Moon, $F_{\text{Moon}} = mg_{\text{Moon}} = mg_{\text{Moon}}/6 = 16\ \text{N}$.

    (*b*) The horizontal accelerating force would be the same in each case, because the mass of the object is the same on both the Earth and the Moon, and both objects would have the same acceleration to throw them with the same speed. So by Newton's second law, the forces would have to be the same.

15. In a tug of war, the team that pushes hardest against the ground wins. It is true that both teams have the same force on them due to the tension in the rope. But the winning team pushes harder against the ground and thus the ground pushes harder on the winning team, making a net unbalanced force.

The free body diagram below illustrates this. The forces are $\vec{\mathbf{F}}_{\mathrm{T_1G}}$, the force on team 1 from the ground, $\vec{\mathbf{F}}_{\mathrm{T_2G}}$, the force on team 2 from the ground, and $\vec{\mathbf{F}}_{\mathrm{TR}}$, the force on each team from the rope.

Thus the net force on the winning team $\left(\vec{\mathbf{F}}_{\mathrm{T_1G}} - \vec{\mathbf{F}}_{\mathrm{TR}}\right)$ is in the winning direction.


$\vec{\mathbf{F}}_{\mathrm{T_1G}}$
Team # 1 (winner)
$\vec{\mathbf{F}}_{\mathrm{TR}}$
$\vec{\mathbf{F}}_{\mathrm{TR}}$
Team # 2
$\vec{\mathbf{F}}_{\mathrm{T_2G}}$
Large force from ground
Equal and opposite tension forces
Small force from ground

16. (*a*) The magnitude is 40 N.
    (*b*) The direction is downward.
    (*c*) It is exerted on the person.
    (*d*) It is exerted by the bag of groceries.

17. If you are at rest, the net force on you is zero. Hence the ground exerts a force on you exactly equal to your weight. The two forces acting on you sum to zero, and so you don't accelerate. If you squat down and then push with a larger force against the ground, the ground then pushes back on you with a larger force by Newton's third law, and you can then rise into the air.

18. In a whiplash situation, the car is violently pushed forward. Since the victim's back is against the seat of the car, the back moves forward with the car. But the head has no direct horizontal force to push it, and so it "lags behind". The victim's body is literally pushed forward, out from under their head – the head is not thrown backwards. The neck muscles must eventually pull the head forward, and that causes the whiplash. To avoid this, use the car's headrests.

19. The truck bed exerts a force of static friction on the crate, causing the crate to accelerate.

20. On the way up, there are two forces on the block that are parallel to each other causing the deceleration – the component of weight parallel to the plane, and the force of friction on the block. Since the forces are parallel to each other, both pointing down the plane, they add, causing a larger magnitude force and a larger acceleration. On the way down, those same two forces are opposite of each other, because the force of friction is now directed up the plane. With these two forces being opposite of each other, their net force is smaller, and so the acceleration is smaller.

21. Assume your weight is W. If you weighed yourself on an inclined plane that is inclined at angle θ, the bathroom scale would read the magnitude of the normal force between you and the plane, which would be $W\cos\theta$.

Solutions to Problems

1. Use Newton's second law to calculate the force.

$$\sum F = ma = (60.0\ \mathrm{kg})\left(1.25\ \mathrm{m/s^2}\right) = \boxed{75.0\ \mathrm{N}}$$

2. Use Newton's second law to calculate the mass.

$$\sum F = ma \rightarrow m = \frac{\sum F}{a} = \frac{265\ \mathrm{N}}{2.30\,\mathrm{m/s^2}} = \boxed{115\ \mathrm{kg}}$$

3. Use Newton's second law to calculate the tension.

$$\sum F = F_{\mathrm{T}} = ma = (960\ \mathrm{kg})(1.20\ \mathrm{m/s^2}) = \boxed{1.15\times10^3\,\mathrm{N}}$$

4. In all cases, $W = mg$, where g changes with location.

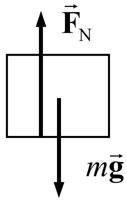
(*a*) $W_{\text{Earth}} = mg_{\text{Earth}} = (76\ \mathrm{kg})(9.8\,\mathrm{m/s^2}) = \boxed{7.4\times10^2\,\mathrm{N}}$

(*b*) $W_{\text{Moon}} = mg_{\text{Moon}} = (76\ \mathrm{kg})(1.7\,\mathrm{m/s^2}) = \boxed{1.3\times10^2\,\mathrm{N}}$

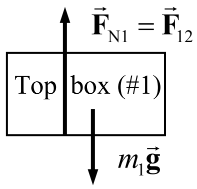
(*c*) $W_{\text{Mars}} = mg_{\text{Mars}} = (76\ \mathrm{kg})(3.7\,\mathrm{m/s^2}) = \boxed{2.8\times10^2\,\mathrm{N}}$

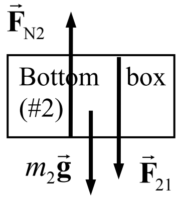
(*d*) $W_{\text{Space}} = mg_{\text{Space}} = (76\ \mathrm{kg})(0\,\mathrm{m/s^2}) = \boxed{0\ \mathrm{N}}$

5. (*a*) The 20.0 kg box resting on the table has the free-body diagram shown. Its weight is $mg = (20.0\ \mathrm{kg})(9.80\,\mathrm{m/s^2}) = \boxed{196\ \mathrm{N}}$. Since the box is at rest, the net force on the box must be 0, and so the normal force must also be $\boxed{196\ \mathrm{N}}$.



(*b*) Free-body diagrams are shown for both boxes. $\vec{\mathbf{F}}_{12}$ is the force on box 1 (the top box) due to box 2 (the bottom box), and is the normal force on box 1. $\vec{\mathbf{F}}_{21}$ is the force on box 2 due to box 1, and has the same magnitude as $\vec{\mathbf{F}}_{12}$ by Newton's 3rd law. $\vec{\mathbf{F}}_{\mathrm{N2}}$ is the force of the table on box 2. That is the normal force on box 2. Since both boxes are at rest, the net force on each box must be 0. Write Newton's 2nd law in the vertical direction for each box, taking the upward direction to be positive.





$$\sum F_1 = F_{\mathrm{N1}} - m_1 g = 0$$

$$F_{\mathrm{N1}} = m_1 g = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = \boxed{98.0\ \mathrm{N}} = F_{12} = F_{21}$$

$$\sum F_2 = F_{\mathrm{N2}} - F_{21} - m_2 g = 0$$

$$F_{\mathrm{N2}} = F_{21} + m_2 g = 98.0\ \mathrm{N} + (20.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = \boxed{294\ \mathrm{N}}$$

6. Find the average acceleration from Eq. 2-2. The average force on the car is found from Newton's second law.

$$v = 0 \quad v_0 = (95\,\mathrm{km/h})\left(\frac{0.278\,\mathrm{m/s}}{1\,\mathrm{km/h}}\right) = 26.4\,\mathrm{m/s} \quad a_{avg} = \frac{v - v_0}{t} = \frac{0 - 26.4\,\mathrm{m/s}}{8.0\ \mathrm{s}} = -3.30\ \mathrm{m/s^2}$$

$$F_{avg} = ma_{avg} = (1100\ \mathrm{kg})(-3.3\ \mathrm{m/s^2}) = \boxed{-3.6\times10^3\,\mathrm{N}}$$

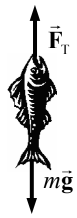
The negative sign indicates the direction of the force, in the opposite direction to the initial velocity.

7. The average force on the pellet is its mass times its average acceleration. The average acceleration is found from Eq. 2-11c. For the pellet, $v_0 = 0$, $v = 125\,\mathrm{m/s}$, and $x - x_0 = 0.800\ \mathrm{m}$.

$$a_{avg} = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(125\,\mathrm{m/s})^2 - 0}{2(0.800\ \mathrm{m})} = 9770\ \mathrm{m/s^2}$$

$$F_{avg} = ma_{avg} = (7.00\times10^{-3}\ \mathrm{kg})(9770\ \mathrm{m/s^2}) = \boxed{68.4\ \mathrm{N}}$$

8. We assume that the fishline is pulling vertically on the fish, and that the fish is not jerking the line. A free-body diagram for the fish is shown. Write Newton's 2nd law for the fish in the vertical direction, assuming that up is positive. The tension is at its maximum.



$$\sum F = F_T - mg = ma \quad \rightarrow \quad F_T = m(g+a) \quad \rightarrow$$

$$m = \frac{F_T}{g+a} = \frac{22\text{ N}}{9.8\,\text{m/s}^2 + 2.5\,\text{m/s}^2} = \boxed{1.8\text{ kg}}$$

Thus a mass of 1.8 kg is the maximum that the fishline will support with the given acceleration. Since the line broke, the fish's mass must be greater than 1.8 kg (about 4 lbs).

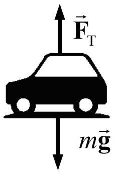
9. The problem asks for the average force on the glove, which in a direct calculation would require knowledge about the mass of the glove and the acceleration of the glove. But no information about the glove is given. By Newton's 3rd law, the force exerted by the ball on the glove is equal and opposite to the force exerted by the glove on the ball. So calculate the average force on the ball, and then take the opposite of that result to find the average force on the glove. The average force on the ball is its mass times its average acceleration. Use Eq. 2-11c to find the acceleration of the ball, with $v = 0$, $v_0 = 35.0\,\text{m/s}$, and $x - x_0 = 0.110$ m. The initial direction of the ball is the positive direction.

$$a_{avg} = \frac{v^2 - v_0^2}{2(x-x_0)} = \frac{0 - (35.0\,\text{m/s})^2}{2(0.110\text{ m})} = -5568\ \text{m/s}^2$$

$$F_{avg} = ma_{avg} = (0.140\text{ kg})(-5568\ \text{m/s}^2) = \boxed{-7.80\times 10^2\,\text{N}}$$

Thus the average force on the glove was 780 N, in the direction of the initial velocity of the ball.

$\boxed{10.}$ Choose up to be the positive direction. Write Newton's 2nd law for the vertical direction, and solve for the tension force.



$$\sum F = F_T - mg = ma \quad \rightarrow F_T = m(g+a)$$

$$F_T = (1200\text{ kg})(9.80\,\text{m/s}^2 + 0.80\,\text{m/s}^2) = \boxed{1.3\times 10^4\,\text{N}}$$

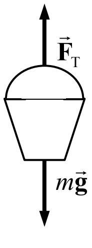
11. Use Eq. 2-11b with $v_0 = 0$ to find the acceleration.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad a = \frac{2(x-x_0)}{t^2} = \frac{2(402\text{ m})}{(6.40\text{ s})^2} = 19.6\,\text{m/s}^2\left(\frac{1\text{ "}g\text{"}}{9.80\,\text{m/s}^2}\right) = \boxed{2.00\ g\text{'s}}$$

The accelerating force is found by Newton's 2nd law.

$$F = ma = (485\text{ kg})(19.6\,\text{m/s}^2) = \boxed{9.51\times 10^3\,\text{N}}$$

12. Choose up to be the positive direction. Write Newton's 2nd law for the vertical direction, and solve for the acceleration.

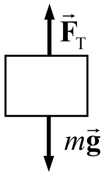


$$\sum F = F_T - mg = ma$$

$$a = \frac{F_T - mg}{m} = \frac{163\text{ N} - (12.0\text{ kg})(9.80\,\text{m/s}^2)}{12.0\text{ kg}} = \boxed{3.8\,\text{m/s}^2}$$

Since the acceleration is positive, the bucket has an $\boxed{\text{upward}}$ acceleration.

13. In both cases, a free-body diagram for the elevator would look like the adjacent diagram. Choose up to be the positive direction. To find the MAXIMUM tension, assume that the acceleration is up. Write Newton's 2nd law for the elevator.



$$\sum F = ma = F_T - mg \quad \rightarrow$$

$$F_T = ma + mg = m(a+g) = m(0.0680g + g) = (4850\text{ kg})(1.0680)(9.80\,\text{m/s}^2)$$

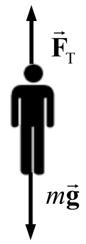
$$= \boxed{5.08\times10^4\,\text{N}}$$

To find the MINIMUM tension, assume that the acceleration is down. Then Newton's 2nd law for the elevator becomes

$$\sum F = ma = F_T - mg \quad \rightarrow \quad F_T = ma + mg = m(a+g) = m(-0.0680g + g)$$

$$= (4850\text{ kg})(0.9320)(9.80\,\text{m/s}^2) = \boxed{4.43\times10^4\,\text{N}}$$

14. If the thief were to hang motionless on the sheets, or descend at a constant speed, the sheets would not support him, because they would have to support the full 75 kg. But if he descends with an acceleration, the sheets will not have to support the total mass. A free-body diagram of the thief in descent is shown. If the sheets can support a mass of 58 kg, then the tension force that the sheets can exert is $F_T = (58\text{ kg})(9.8\,\text{m/s}^2) = 570\text{ N}$.



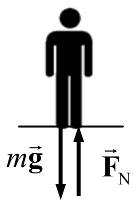
Assume that is the tension in the sheets. Then write Newton's 2nd law for the thief, taking the upward direction to be positive.

$$\sum F = F_T - mg = ma \quad \rightarrow \quad a = \frac{F_T - mg}{m} = \frac{570\text{ N} - (75\text{ kg})(9.80\,\text{m/s}^2)}{75\text{ kg}} = -2.2\,\text{m/s}^2$$

The negative sign shows that the acceleration is downward.

If the thief descends with an acceleration of 2.2 m/s^2 or greater, the sheets will support his descent.

15. There will be two forces on the person – their weight, and the normal force of the scales pushing up on the person. A free-body diagram for the person is shown. Choose up to be the positive direction, and use Newton's 2nd law to find the acceleration.

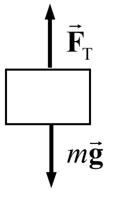


$$\sum F = F_N - mg = ma \quad \rightarrow \quad 0.75mg - mg = ma \quad \rightarrow$$

$$a = -0.25g = -0.25(9.8\,\text{m/s}^2) = \boxed{-2.5\,\text{m/s}^2}$$

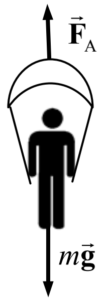
Due to the sign of the result, the direction of the acceleration is $\boxed{\text{down}}$. Thus the elevator must have started to move down since it had been motionless.

16. Choose UP to be the positive direction. Write Newton's 2nd law for the elevator.



$$\sum F = F_T - mg = ma \quad \rightarrow$$

$$a = \frac{F_T - mg}{m} = \frac{21{,}750\text{ N} - (2125\text{ kg})(9.80\,\text{m/s}^2)}{2125\text{ kg}} = 0.4353\,\text{m/s}^2 \approx \boxed{0.44\,\text{m/s}^2}$$



17. (*a*) There will be two forces on the skydivers – their combined weight, and the upward force of air resistance, $\vec{\mathbf{F}}_A$. Choose up to be the positive direction. Write Newton's 2nd law for the skydivers.

$$\sum F = F_A - mg = ma \rightarrow 0.25mg - mg = ma \rightarrow$$

$$a = -0.75g = -0.75\left(9.8\,\text{m/s}^2\right) = \boxed{-7.4\,\text{m/s}^2}$$

Due to the sign of the result, the direction of the acceleration is down.

(*b*) If they are descending at constant speed, then the net force on them must be zero, and so the force of air resistance must be equal to their weight.

$$F_A = mg = (132\text{ kg})\left(9.80\,\text{m/s}^2\right) = \boxed{1.29\times10^3\,\text{N}}$$

18. (*a*) Use Eq. 2-11c to find the speed of the person just before striking the ground. Take down to be the positive direction. For the person, $v_0 = 0$, $y - y_0 = 3.9$ m, and $a = 9.8\,\text{m/s}^2$.

$$v^2 - v_0^2 = 2a(y - y_0) \rightarrow v = \sqrt{2a(y-y_0)} = \sqrt{2\left(9.8\,\text{m/s}^2\right)(3.9\text{ m})} = 8.743 = \boxed{8.7\,\text{m/s}}$$

(*b*) For the deceleration, use Eq. 2-11c to find the average deceleration, choosing down to be positive.

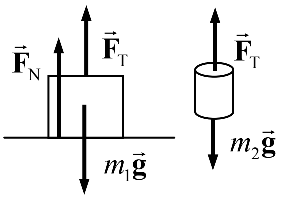
$$v_0 = 8.743\,\text{m/s} \qquad v = 0 \qquad y - y_0 = 0.70\text{ m} \qquad v^2 - v_0^2 = 2a(y-y_0) \rightarrow$$

$$a = \frac{-v_0^2}{2\Delta y} = \frac{-(8.743\,\text{m/s})^2}{2(0.70\text{ m})} = -54.6\,\text{m/s}^2$$

The average force is found from Newton's 2nd law.

$$F = ma = (42\text{ kg})\left(-54.6\,\text{m/s}^2\right) = \boxed{-2.3\times10^3\,\text{N}}.$$

The negative sign shows that the force is in the negative direction, which is upward.



19. Free body diagrams for the box and the weight are shown below. The tension exerts the same magnitude of force on both objects.

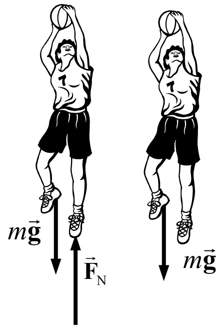
(*a*) If the weight of the hanging weight is less than the weight of the box, the objects will not move, and the tension will be the same as the weight of the hanging weight. The acceleration of the box will also be zero, and so the sum of the forces on it will be zero. For the box,

$$F_N + F_T - m_1g = 0 \rightarrow F_N = m_1g - F_T = m_1g - m_2g = 77.0\text{N} - 30.0\text{ N} = \boxed{47.0\text{ N}}$$

(*b*) The same analysis as for part (*a*) applies here.

$$F_N = m_1g - m_2g = 77.0\text{ N} - 60.0\text{ N} = \boxed{17.0\text{ N}}$$

(*c*) Since the hanging weight has more weight than the box on the table, the box on the table will be lifted up off the table, and normal force of the table on the box will be $\boxed{0\text{ N}}$.

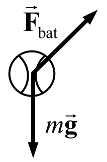


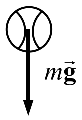
20. (*a*) Just before the player leaves the ground on a jump, the forces on the player would be his weight and the force of the floor pushing up the player. If the player is jumping straight up, then the force of the floor pushing on the player will be straight up – a normal force. See the first diagram. In this case, while they are touching the floor, $F_N > mg$.

(*b*) While the player is in the air, the only force on the player is their weight. See the second diagram.

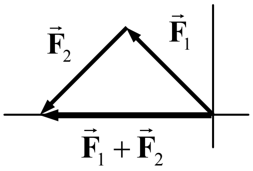
21. (*a*) Just as the ball is being hit, if we ignore air resistance, there are two main forces on the ball – the weight of the ball, and the force of the bat on the ball.

    (*b*) As the ball flies toward the outfield, the only force on it is its weight, if air resistance is ignored.

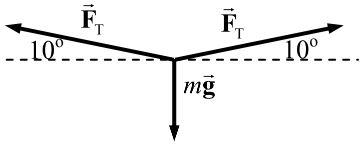

$\vec{\mathbf{F}}_{\text{bat}}$
$m\vec{\mathbf{g}}$


$m\vec{\mathbf{g}}$

22. The two forces must be oriented so that the northerly component of the first force is exactly equal to the southerly component of the second force. Thus the second force must act $\boxed{\text{southwesterly}}$. See the diagram.


$\vec{\mathbf{F}}_2$
$\vec{\mathbf{F}}_1$
$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$

23. Consider the point in the rope directly below Arlene. That point can be analyzed as having three forces on it – Arlene's weight, the tension in the rope towards the right point of connection, and the tension in the rope towards the left point of connection. Assuming the rope is massless, those two tensions will be of the same magnitude. Since the point is not accelerating the sum of the forces must be zero. In particular, consider the sum of the vertical forces on that point, with UP as the positive direction.

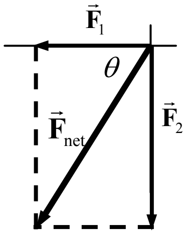

$\vec{\mathbf{F}}_{\text{T}}$
$\vec{\mathbf{F}}_{\text{T}}$
10°
10°
$m\vec{\mathbf{g}}$

$$\sum F = F_{\text{T}}\sin 10.0^{\circ} + F_{\text{T}}\sin 10.0^{\circ} - mg = 0 \quad \rightarrow$$

$$F_{\text{T}} = \frac{mg}{2\sin 10.0^{\circ}} = \frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{2\sin 10.0^{\circ}} = \boxed{1.41\times 10^3\text{ N}}$$

24. The net force in each case is found by vector addition with components.

    (*a*) $F_{\text{Net x}} = -F_1 = -10.2\text{ N} \quad F_{\text{Net y}} = -F_2 = -16.0\text{ N}$

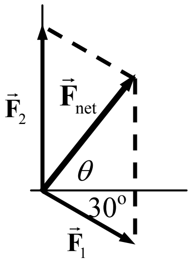

$\vec{\mathbf{F}}_1$
θ
$\vec{\mathbf{F}}_{\text{net}}$
$\vec{\mathbf{F}}_2$

$$F_{\text{Net}} = \sqrt{(-10.2)^2 + (-16.0)^2} = \boxed{19.0\text{ N}} \qquad \theta = \tan^{-1}\frac{-16.0}{-10.2} = 57.5^{\circ}$$

The actual angle from the *x*-axis is then 237.5°.

$$a = \frac{F_{\text{Net}}}{m} = \frac{19.0\text{ N}}{27.0\text{ kg}} = \boxed{0.703\,\text{m/s}^2} \text{ at } \boxed{237^{\circ}}$$

(*b*) $F_{\text{Net x}} = F_1\cos 30^{\circ} = 8.83\text{ N} \quad F_{\text{Net y}} = F_2 - F_1\sin 30^{\circ} = 10.9\text{ N}$

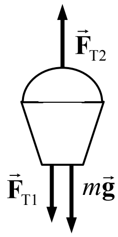

$\vec{\mathbf{F}}_2$
$\vec{\mathbf{F}}_{\text{net}}$
θ
30°
$\vec{\mathbf{F}}_1$

$$F_{\text{Net}} = \sqrt{(8.83\text{ N})^2 + (10.9\text{ N})^2} = \boxed{14.0\text{ N}} \qquad \theta = \tan^{-1}\frac{10.9}{8.83} = \boxed{51.0^{\circ}}$$

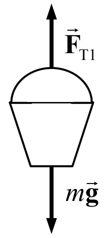
$$a = \frac{F_{\text{Net}}}{m} = \frac{14.0\text{ N}}{27.0\text{ kg}} = \boxed{0.520\,\text{m/s}^2} \text{ at } \boxed{51.0^{\circ}}$$

25. We draw free-body diagrams for each bucket.

    (*a*) Since the buckets are at rest, their acceleration is 0. Write Newton's 2nd law for each bucket, calling UP the positive direction.


$\vec{\mathbf{F}}_{\text{T2}}$
$\vec{\mathbf{F}}_{\text{T1}}$
$m\vec{\mathbf{g}}$

Top (# 2)


$\vec{\mathbf{F}}_{\text{T1}}$
$m\vec{\mathbf{g}}$

Bottom (# 1)

$$\sum F_1 = F_{\text{T1}} - mg = 0 \quad \rightarrow$$

$$F_{\text{T1}} = mg = (3.2\text{ kg})(9.8\,\text{m/s}^2) = \boxed{31\text{ N}}$$

$$\sum F_2 = F_{\text{T2}} - F_{\text{T1}} - mg = 0 \quad \rightarrow$$

$$F_{\text{T2}} = F_{\text{T1}} + mg = 2mg = 2(3.2\text{ kg})(9.8\,\text{m/s}^2) = \boxed{63\text{ N}}$$

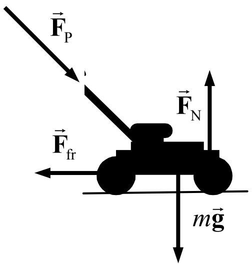
(*b*) Now repeat the analysis, but with a non-zero acceleration. The free-body diagrams are unchanged.

$$\sum F_1 = F_{T1} - mg = ma \quad \to$$

$$F_{T1} = mg + ma = (3.2\text{ kg})\left(9.80\,\text{m/s}^2 + 1.60\,\text{m/s}^2\right) = \boxed{36\text{ N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = ma \quad \to \quad F_{T2} = F_{T1} + mg + ma = 2F_{T1} = \boxed{73\text{ N}}$$

26. (*a*) We assume that the mower is being pushed to the right. $\vec{\mathbf{F}}_{fr}$ is the friction force, and $\vec{\mathbf{F}}_P$ is the pushing force along the handle.



(*b*) Write Newton's 2nd law for the horizontal direction. The forces must sum to 0 since the mower is not accelerating.

$$\sum F_x = F_P \cos 45.0^\circ - F_{fr} = 0 \quad \to$$

$$F_{fr} = F_P \cos 45.0^\circ = (88.0\text{ N})\cos 45.0^\circ = \boxed{62.2\text{ N}}$$

(*c*) Write Newton's 2nd law for the vertical direction. The forces must sum to 0 since the mower is not accelerating in the vertical direction.

$$\sum F_y = F_N - mg - F_P \sin 45.0^\circ = 0 \quad \to$$

$$F_N = mg + F_P \sin 45^\circ = (14.0\text{ kg})\left(9.80\,\text{m/s}^2\right) + (88.0\text{ N})\sin 45.0^\circ = \boxed{199\text{ N}}$$

(*d*) First use Eq. 2-11a to find the acceleration.

$$v - v_0 = at \quad \to \quad a = \frac{v - v_0}{t} = \frac{1.5\,\text{m/s} - 0}{2.5\text{ s}} = 0.60\,\text{m/s}^2$$

Now use Newton's 2nd law for the *x* direction to find the necessary pushing force.

$$\sum F_x = F_P \cos 45.0^\circ - F_f = ma \quad \to$$

$$F_P = \frac{F_f + ma}{\cos 45.0^\circ} = \frac{62.2\text{ N} + (14.0\text{ kg})\left(0.60\,\text{m/s}^2\right)}{\cos 45.0^\circ} = \boxed{99.9\text{ N}}$$

27. Choose the *y* direction to be the "forward" direction for the motion of the snowcats, and the *x* direction to be to the right on the diagram in the textbook. Since the housing unit moves in the forward direction on a straight line, there is no acceleration in the *x* direction, and so the net force in the *x* direction must be 0. Write Newton's 2nd law for the *x* direction.

$$\sum F_x = F_{Ax} + F_{Bx} = 0 \quad \to \quad -F_A \sin 50^\circ + F_B \sin 30^\circ = 0 \quad \to$$

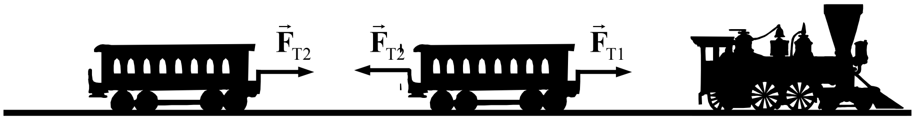
$$F_B = \frac{F_A \sin 50^\circ}{\sin 30^\circ} = \frac{(4500\text{ N})\sin 50^\circ}{\sin 30^\circ} = \boxed{6.9\times 10^3\text{ N}}$$

Since the *x* components add to 0, the magnitude of the vector sum of the two forces will just be the sum of their *y* components.

$$\sum F_y = F_{Ay} + F_{By} = F_A \cos 50^\circ + F_B \cos 30^\circ = (4500\text{ N})\cos 50^\circ + (6900\text{ N})\cos 30^\circ = \boxed{8.9\times 10^3\text{ N}}$$

28. Since all forces of interest in this problem are horizontal, draw the free-body diagram showing only the horizontal forces. $\vec{\mathbf{F}}_{T1}$ is the tension in the coupling between the locomotive and the first car, and it pulls to the right on the first car. $\vec{\mathbf{F}}_{T2}$ is the tension in the coupling between the first car an the second car. It pulls to the right on car 2, labeled $\vec{\mathbf{F}}_{T2R}$ and to the left on car 1, labeled $\vec{\mathbf{F}}_{T2L}$. Both cars

have the same mass m and the same acceleration a. Note that $\left|\vec{\mathbf{F}}_{\mathrm{T2R}}\right| = \left|\vec{\mathbf{F}}_{\mathrm{T2L}}\right| = F_{T2}$ by Newton's 3rd law.


$\vec{\mathbf{F}}_{\mathrm{T2}}$
$\vec{\mathbf{F}}_{\mathrm{T2}}$
$\vec{\mathbf{F}}_{\mathrm{T1}}$

Write a Newton's 2nd law expression for each car.

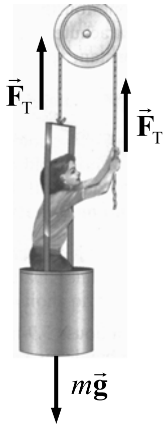
$$\sum F_1 = F_{T1} - F_{T2} = ma \qquad \sum F_2 = F_{T2} = ma$$

Substitute the expression for ma from the second expression into the first one.

$$F_{T1} - F_{T2} = ma = F_{T2} \;\rightarrow\; F_{\mathrm{T1}} = 2F_{\mathrm{T2}} \;\rightarrow\; \boxed{F_{\mathrm{T1}}/F_{\mathrm{T2}} = 2}$$

This can also be discussed in the sense that the tension between the locomotive and the first car is pulling 2 cars, while the tension between the cars is only pulling one car.

29. The window washer pulls down on the rope with her hands with a tension force F_{T}, so the rope pulls up on her hands with a tension force F_{T}. The tension in the rope is also applied at the other end of the rope, where it attaches to the bucket. Thus there is another force F_{T} pulling up on the bucket. The bucket-washer combination thus has a net force of $2F_{\mathrm{T}}$ upwards. See the adjacent free-body diagram, showing only forces on the bucket-washer combination, not forces exerted by the combination (the pull down on the rope by the person) or internal forces (normal force of bucket on person).


$\vec{\mathbf{F}}_{\mathrm{T}}$
$\vec{\mathbf{F}}_{\mathrm{T}}$
$m\vec{\mathbf{g}}$

(*a*) Write Newton's 2nd law in the vertical direction, with up as positive. The net force must be zero if the bucket and washer have a constant speed.

$$\sum F = F_{\mathrm{T}} + F_{\mathrm{T}} - mg = 0 \;\rightarrow\; 2F_{\mathrm{T}} = mg \;\rightarrow$$

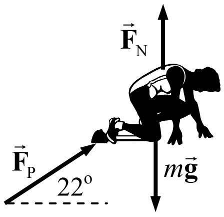
$$F_{\mathrm{T}} = \frac{mg}{2} = \frac{(65\text{ kg})(9.8\,\mathrm{m/s^2})}{2} = \boxed{320\text{ N}}$$

(*b*) Now the force is increased by 15%, so $F_{\mathrm{T}} = 320\text{ N}(1.15) = 368\text{ N}$. Again write Newton's 2nd law, but with a non-zero acceleration.

$$\sum F = F_{\mathrm{T}} + F_{\mathrm{T}} - mg = ma \;\rightarrow$$

$$a = \frac{2F_{\mathrm{T}} - mg}{m} = \frac{2(368\text{ N}) - (65\text{ kg})(9.80\,\mathrm{m/s^2})}{65\text{ kg}} = \boxed{1.5\,\mathrm{m/s^2}}$$

30. Since the sprinter exerts a force of 720 N on the ground at an angle of 22° below the horizontal, by Newton's 3rd law the ground will exert a force of 720 N on the sprinter at an angle of 22° above the horizontal. A free-body diagram for the sprinter is shown.


$\vec{\mathbf{F}}_{\mathrm{N}}$
$\vec{\mathbf{F}}_{\mathrm{P}}$
22°
$m\vec{\mathbf{g}}$

(*a*) The horizontal acceleration will be found from the net horizontal force. Using Newton's 2nd law, we have the following.

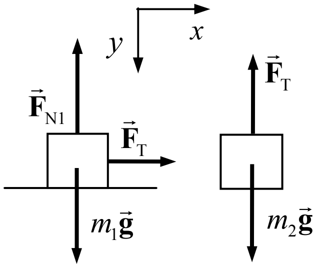
$$\sum F_x = F_{\mathrm{P}}\cos 22^\circ = ma_x \;\rightarrow\; a_x = \frac{F_{\mathrm{P}}\cos 22^\circ}{m} = \frac{(720\text{ N})\cos 22^\circ}{65\text{ kg}}$$

$$= 10.27\,\mathrm{m/s^2} \approx \boxed{1.0\times 10^1\,\mathrm{m/s^2}}$$

(*b*) Eq. 2-11a is used to find the final speed. The starting speed is 0.

$$v = v_0 + at \quad \rightarrow \quad v = 0 + at = \left(10.27\,\mathrm{m/s^2}\right)\left(0.32\ \mathrm{s}\right) = 3.286\,\mathrm{m/s} \approx \boxed{3.3\,\mathrm{m/s}}$$

31. (*a*) See the free-body diagrams included.



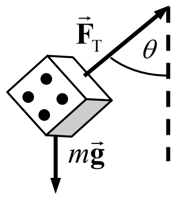
y
x
$\vec{\mathbf{F}}_{\mathrm{N1}}$
$\vec{\mathbf{F}}_{\mathrm{T}}$
$m_1\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\mathrm{T}}$
$m_2\vec{\mathbf{g}}$

(*b*) For block 1, since there is no motion in the vertical direction, we have $F_{\mathrm{N1}} = m_1 g$. We write Newton's 2nd law for the x direction: $\sum F_{1x} = F_{\mathrm{T}} = m_1 a_{1x}$. For block 2, we only need to consider vertical forces: $\sum F_{2y} = m_2 g - F_{\mathrm{T}} = m_2 a_{2y}$. Since the two blocks are connected, the magnitudes of their accelerations will be the same, and so let $a_{1x} = a_{2y} = a$. Combine the two force equations from above, and solve for a by substitution.

$$F_{\mathrm{T}} = m_1 a \qquad m_2 g - F_{\mathrm{T}} = m_2 a \quad \rightarrow \quad m_2 g - m_1 a = m_2 a \quad \rightarrow$$

$$m_1 a + m_2 a = m_2 g \quad \rightarrow \quad \boxed{a = g\frac{m_2}{m_1 + m_2} \qquad F_{\mathrm{T}} = m_1 a = g\frac{m_1 m_2}{m_1 + m_2}}$$

32. Consider a free-body diagram of the dice. The car is moving to the right. The acceleration of the dice is found from Eq. 2-11a.



$\vec{\mathbf{F}}_{\mathrm{T}}$
θ
$m\vec{\mathbf{g}}$

$$v = v_0 + = a_x t \quad \rightarrow \quad a_x = \frac{v - v_0}{t} = \frac{28\,\mathrm{m/s} - 0}{6.0\ \mathrm{s}} = 4.67\,\mathrm{m/s^2}$$

Now write Newton's 2nd law for both the vertical (y) and horizontal (x) directions.

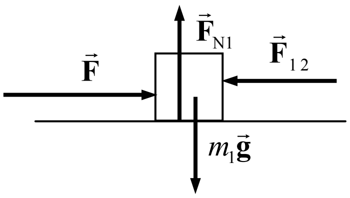
$$\sum F_y = F_{\mathrm{T}}\cos\theta - mg = 0 \quad \rightarrow \quad F_{\mathrm{T}} = \frac{mg}{\cos\theta} \qquad \sum F_x = F_{\mathrm{T}}\sin\theta = ma_x$$

Substitute the expression for the tension from the y equation into the x equation.

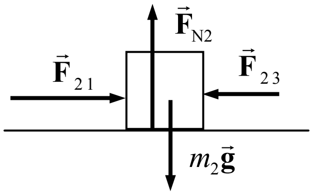
$$ma_x = F_{\mathrm{T}}\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta \quad \rightarrow \quad a_x = g\tan\theta$$

$$\theta = \tan^{-1}\frac{a_x}{g} = \tan^{-1}\frac{4.67\,\mathrm{m/s^2}}{9.8\,\mathrm{m/s^2}} = \boxed{25^{\circ}}$$

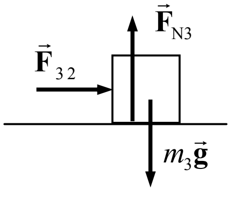
33. (*a*) In the free-body diagrams below, $\vec{\mathbf{F}}_{12}$ = force on block 1 exerted by block 2, $\vec{\mathbf{F}}_{21}$ = force on block 2 exerted by block 1, $\vec{\mathbf{F}}_{23}$ = force on block 2 exerted by block 3, and $\vec{\mathbf{F}}_{32}$ = force on block 3 exerted by block 2. The magnitudes of $\vec{\mathbf{F}}_{21}$ and $\vec{\mathbf{F}}_{12}$ are equal, and the magnitudes of $\vec{\mathbf{F}}_{23}$ and $\vec{\mathbf{F}}_{32}$ are equal, by Newton's 3rd law.



$\vec{\mathbf{F}}$
$\vec{\mathbf{F}}_{\mathrm{N1}}$
$\vec{\mathbf{F}}_{12}$
$m_1\vec{\mathbf{g}}$



$\vec{\mathbf{F}}_{21}$
$\vec{\mathbf{F}}_{\mathrm{N2}}$
$\vec{\mathbf{F}}_{23}$
$m_2\vec{\mathbf{g}}$



$\vec{\mathbf{F}}_{32}$
$\vec{\mathbf{F}}_{\mathrm{N3}}$
$m_3\vec{\mathbf{g}}$

(*b*) All of the vertical forces on each block add up to zero, since there is no acceleration in the vertical direction. Thus for each block, $F_N = mg$. For the horizontal direction, we have

$$\sum F = F - F_{12} + F_{21} - F_{23} + F_{32} = F = (m_1 + m_2 + m_3)a \rightarrow \boxed{a = \frac{F}{m_1 + m_2 + m_3}}$$

(*c*) For each block, the net force must be *ma* by Newton's 2nd law. Each block has the same acceleration since they are in contact with each other.

$$\boxed{F_{1net} = \frac{m_1 F}{m_1 + m_2 + m_3}} \quad \boxed{F_{2net} = \frac{m_2 F}{m_1 + m_2 + m_3}} \quad \boxed{F_{3net} = \frac{m_3 F}{m_1 + m_2 + m_3}}$$

(*d*) From the free-body diagram, we see that for m_3, $F_{32} = F_{3net} = \boxed{\frac{m_3 F}{m_1 + m_2 + m_3}}$. And by Newton's 3rd law, $F_{32} = F_{23} = F_{3net} = \boxed{\frac{m_3 F}{m_1 + m_2 + m_3}}$. Of course, $\vec{\mathbf{F}}_{23}$ and $\vec{\mathbf{F}}_{32}$ are in opposite directions.

Also from the free-body diagram, we see that for m_1,

$F - F_{12} = F_{1net} = \frac{m_1 F}{m_1 + m_2 + m_3} \rightarrow F_{12} = F - \frac{m_1 F}{m_1 + m_2 + m_3} \rightarrow \boxed{F_{12} = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}}$. By Newton's 3rd law, $F_{12} = F_{21} = \boxed{\frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}}$.

(*e*) Using the given values, $a = \frac{F}{m_1 + m_2 + m_3} = \frac{96.0\text{ N}}{36.0\text{ kg}} = \boxed{2.67\text{ m/s}^2}$. Since all three masses are the same value, the net force on each mass is $F_{net} = ma = (12.0\text{ kg})(2.67\text{ m/s}^2) = 32.0\text{ N}$.

This is also the value of F_{32} and F_{23}. The value of F_{12} and F_{21} is

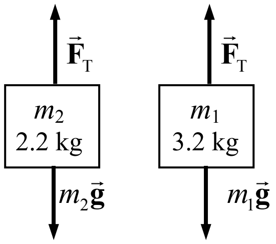
$$F_{12} = F_{21} = (m_2 + m_3)a = (24\text{ kg})(2.67\text{ m/s}^2) = 64.0\text{ N}.$$

To summarize:

$$F_{\text{net }1} = F_{\text{net }2} = F_{\text{net }3} = \boxed{32.0\text{ N}} \qquad F_{12} = F_{21} = \boxed{64.0\text{ N}} \qquad F_{23} = F_{32} = \boxed{32.0\text{ N}}$$

The values make sense in that in order of magnitude, we should have $F > F_{21} > F_{32}$, since *F* is the net force pushing the entire set of blocks, F_{12} is the net force pushing the right two blocks, and F_{23} is the net force pushing the right block only.

34. First, draw a free-body diagram for each mass. Notice that the same tension force is applied to each mass. Choose UP to be the positive direction. Write Newton's 2nd law for each of the masses.



$$F_T - m_2 g = m_2 a_2 \qquad F_T - m_1 g = m_1 a_1$$

Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_1 = -a_2$. Substitute this into the force expressions and solve for the acceleration by subtracting the second equation from the first.

$$F_T - m_1 g = -m_1 a_2 \rightarrow F_T = m_1 g - m_1 a_2$$

$$F_T - m_2 g = m_2 a_2 \rightarrow m_1 g - m_1 a_2 - m_2 g = m_2 a_2 \rightarrow m_1 g - m_2 g = m_1 a_2 + m_2 a_2$$

$$a_2 = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{3.2\text{ kg} - 2.2\text{ kg}}{3.2\text{ kg} + 2.2\text{ kg}}(9.8\text{ m/s}^2) = 1.815\text{ m/s}^2$$

The lighter block starts with a speed of 0, and moves a distance of 1.80 meters with the acceleration found above. Using Eq. 2-11c, the velocity of the lighter block at the end of this accelerated motion can be found.

$$v^2 - v_0^2 = 2a(y - y_0) \rightarrow v = \sqrt{v_0^2 + 2a(y - y_0)} = \sqrt{0 + 2(1.815\,\text{m/s}^2)(1.80\text{ m})} = 2.556\,\text{m/s}$$

Now the lighter block has different conditions of motion. Once the heavier block hits the ground, the tension force disappears, and the lighter block is in free fall. It has an initial speed of 2.556 m/s upward as found above, with an acceleration of –9.8 m/s^2 due to gravity. At its highest point, its speed will be 0. Eq. 2-11c can again be used to find the height to which it rises.

$$v^2 - v_0^2 = 2a(y - y_0) \rightarrow (y - y_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - (2.556\,\text{m/s})^2}{2(-9.8\,\text{m/s}^2)} = 0.33\text{ m}$$

Thus the total height above the ground is 1.80 m + 1.80 m + 0.33 m = $\boxed{3.93\text{ m}}$.

35. Please refer to the free-body diagrams given in the textbook for this problem. Initially, treat the two boxes and the rope as a single system. Then the only accelerating force on the system is $\vec{\mathbf{F}}_\text{P}$. The mass of the system is 23.0 kg, and so using Newton's 2nd law, the acceleration of the system is $a = \frac{F_\text{P}}{m} = \frac{40.0\text{ N}}{23.0\text{ kg}} = \boxed{1.74\,\text{m/s}^2}$. This is the acceleration of each piece of the system.

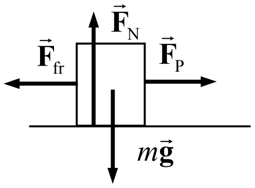
    Now consider the left box alone. The only force on it is $\vec{\mathbf{F}}_\text{BT}$, and it has the acceleration found above. Thus F_BT can be found from Newton's 2nd law.

    $$F_\text{BT} = m_\text{B}a = (12.0\text{ kg})(1.74\,\text{m/s}^2) = \boxed{20.9\text{ N}}.$$

    Now consider the rope alone. The net force on it is $\vec{\mathbf{F}}_\text{TA} - \vec{\mathbf{F}}_\text{TB}$, and it also has the acceleration found above. Thus F_TA can be found from Newton's 2nd law.

    $$F_\text{TA} - F_\text{TB} = m_C a \rightarrow F_\text{TA} = F_\text{TB} + m_\text{C}a = 20.9\text{ N} + (1.0\text{ kg})(1.74\,\text{m/s}^2) = \boxed{22.6\text{ N}}$$

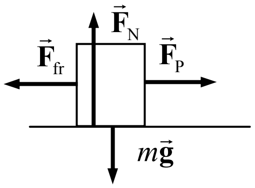
36. A free-body diagram for the crate is shown. The crate does not accelerate vertically, and so $F_\text{N} = mg$. The crate does not accelerate horizontally, and so $F_\text{P} = F_\text{fr}$. Putting this together, we have


$\vec{\mathbf{F}}_\text{N}$
$\vec{\mathbf{F}}_\text{fr}$
$\vec{\mathbf{F}}_\text{P}$
$m\vec{\mathbf{g}}$

    $$F_\text{P} = F_\text{fr} = \mu_k F_\text{N} = \mu_k mg = (0.30)(35\text{ kg})(9.8\,\text{m/s}^2) = 103 = \boxed{1.0 \times 10^2\text{ N}}$$

    If the coefficient of kinetic friction is zero, then the horizontal force required is $\boxed{0\text{ N}}$, since there is no friction to counteract. Of course, it would take a force to START the crate moving, but once it was moving, no further horizontal force would be necessary to maintain the motion.

$\boxed{37.}$ A free-body diagram for the box is shown. Since the box does not accelerate vertically, $F_\text{N} = mg$


$\vec{\mathbf{F}}_\text{N}$
$\vec{\mathbf{F}}_\text{fr}$
$\vec{\mathbf{F}}_\text{P}$
$m\vec{\mathbf{g}}$

(*a*) To start the box moving, the pulling force must just overcome the force of static friction, and that means the force of static friction will reach its maximum value of $F_\text{fr} = \mu_s F_\text{N}$. Thus we have for the starting motion,

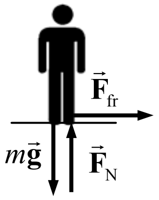
$$F_{\mathrm{P}} = F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}} = \mu_s mg \quad \rightarrow \quad \mu_s = \frac{F_{\mathrm{P}}}{mg} = \frac{48.0\ \mathrm{N}}{(5.0\ \mathrm{kg})(9.8\,\mathrm{m/s^2})} = \boxed{0.98}$$

(*b*) The same force diagram applies, but now the friction is kinetic friction, and the pulling force is NOT equal to the frictional force, since the box is accelerating to the right.

$$\sum F = F_{\mathrm{P}} - F_{\mathrm{fr}} = ma \quad \rightarrow \quad F_{\mathrm{P}} - \mu_k F_{\mathrm{N}} = ma \quad \rightarrow \quad F_{\mathrm{P}} - \mu_k mg = ma \quad \rightarrow$$

$$\mu_k = \frac{F_{\mathrm{P}} - ma}{mg} = \frac{48.0\ \mathrm{N} - (5.0\ \mathrm{kg})(0.70\,\mathrm{m/s^2})}{(5.0\ \mathrm{kg})(9.8\,\mathrm{m/s^2})} = \boxed{0.91}$$

38. A free-body diagram for you as you stand on the train is shown. You do not accelerate vertically, and so $F_{\mathrm{N}} = mg$. The maximum static frictional force is $\mu_s F_N$, and that must be greater than or equal to the force needed to accelerate you.

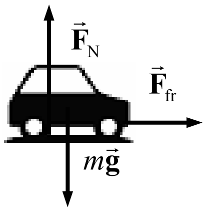


$\vec{\mathbf{F}}_{\mathrm{fr}}$
$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\mathrm{N}}$

$$F_{\mathrm{fr}} \geq ma \quad \rightarrow \quad \mu_s F_{\mathrm{N}} \geq ma \quad \rightarrow \quad \mu_s mg \geq ma \quad \rightarrow \quad \mu_s \geq a/g = 0.20g/g = \boxed{0.20}$$

The static coefficient of friction must be at least 0.20 for you to not slide.

39. A free-body diagram for the accelerating car is shown. The car does not accelerate vertically, and so $F_{\mathrm{N}} = mg$. The static frictional force is the accelerating force, and so $F_{\mathrm{fr}} = ma$. If we assume the maximum acceleration, then we need the maximum force, and so the static frictional force would be its maximum value of $\mu_s F_{\mathrm{N}}$. Thus we have

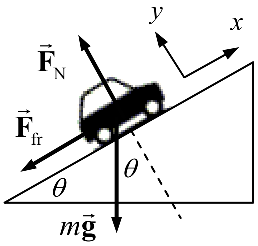


$\vec{\mathbf{F}}_{\mathrm{N}}$
$\vec{\mathbf{F}}_{\mathrm{fr}}$
$m\vec{\mathbf{g}}$

$$F_{\mathrm{fr}} = ma \quad \rightarrow \quad \mu_s F_{\mathrm{N}} = ma \quad \rightarrow \quad \mu_s mg = ma \quad \rightarrow$$

$$a = \mu_s g = 0.80(9.8\,\mathrm{m/s^2}) = \boxed{7.8\,\mathrm{m/s^2}}$$

40. See the included free-body diagram. To find the maximum angle, assume that the car is just ready to slide, so that the force of static friction is a maximum. Write Newton's 2nd law for both directions. Note that for both directions, the net force must be zero since the car is not accelerating.



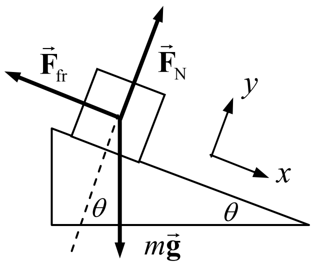
y
x
$\vec{\mathbf{F}}_{\mathrm{N}}$
$\vec{\mathbf{F}}_{\mathrm{fr}}$
θ
θ
$m\vec{\mathbf{g}}$

$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = 0 \quad \rightarrow \quad mg\sin\theta = F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}} = \mu_s mg\cos\theta$$

$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = 0.8 \quad \rightarrow \quad \theta = \tan^{-1}0.8 = 39^\circ = \boxed{40^\circ} \quad (1\ \text{sig fig})$$

41. Start with a free-body diagram. Write Newton's 2nd law for each direction.



$\vec{\mathbf{F}}_{\mathrm{fr}}$
$\vec{\mathbf{F}}_{\mathrm{N}}$
y
x
θ
θ
$m\vec{\mathbf{g}}$

$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = ma_x$$

$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = ma_y = 0$$

Notice that the sum in the *y* direction is 0, since there is no motion (and hence no acceleration) in the *y* direction. Solve for the force of friction.

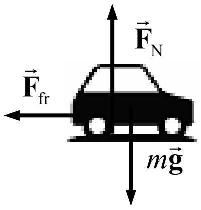
$$mg\sin\theta - F_{\mathrm{fr}} = ma_x \quad \rightarrow$$

$$F_{\mathrm{fr}} = mg\sin\theta - ma_x = (15.0\ \mathrm{kg})\left[(9.80\,\mathrm{m/s^2})(\sin 32^\circ) - 0.30\,\mathrm{m/s^2}\right] = 73.40\ \mathrm{N} \approx \boxed{73\ \mathrm{N}}$$

Now solve for the coefficient of kinetic friction. Note that the expression for the normal force comes from the y direction force equation above.

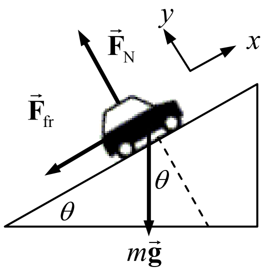
$$F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta \;\rightarrow\; \mu_k = \frac{F_{\text{fr}}}{mg\cos\theta} = \frac{73.40\text{ N}}{(15.0\text{ kg})(9.80\,\text{m/s}^2)(\cos 32^{\circ})} = \boxed{0.59}$$

42. The direction of travel for the car is to the right, and that is also the positive horizontal direction. Using the free-body diagram, write Newton's 2nd law in the x direction for the car on the level road. We assume that the car is just on the verge of skidding, so that the magnitude of the friction force is $F_{\text{fr}} = \mu_s F_{\text{N}}$.


$\vec{\mathbf{F}}_{\text{N}}$
$\vec{\mathbf{F}}_{\text{fr}}$
$m\vec{\mathbf{g}}$

$$\sum F_x = -F_{\text{fr}} = ma \qquad F_{\text{fr}} = -ma = -\mu_s mg \;\rightarrow\; \mu_s = \frac{a}{g} = \frac{4.80\,\text{m/s}^2}{9.80\,\text{m/s}^2} = 0.4898$$

Now put the car on an inclined plane. Newton's 2nd law in the x-direction for the car on the plane is used to find the acceleration. We again assume the car is on the verge of slipping, so the static frictional force is at its maximum.

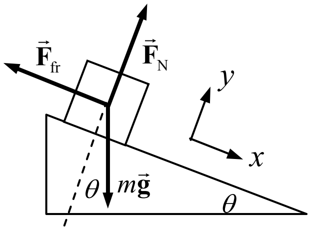

y
x
$\vec{\mathbf{F}}_{\text{N}}$
$\vec{\mathbf{F}}_{\text{fr}}$
θ
θ
$m\vec{\mathbf{g}}$

$$\sum F_x = -F_{\text{fr}} - mg\sin\theta = ma \;\rightarrow$$

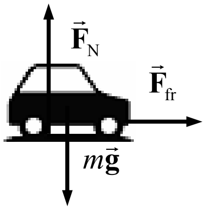
$$a = \frac{-F_{\text{fr}} - mg\sin\theta}{m} = \frac{-\mu_s mg\cos\theta - mg\sin\theta}{m} = -g\left(\mu_s\cos\theta + \sin\theta\right)$$

$$= -\left(9.80\,\text{m/s}^2\right)\left(0.4898\cos 13^{\circ} + \sin 13^{\circ}\right) = \boxed{-6.9\,\text{m/s}^2}$$

43. (*a*) Here is a free-body diagram for the box at rest on the plane. The force of friction is a STATIC frictional force, since the box is at rest.
(*b*) If the box were sliding down the plane, the only change is that the force of friction would be a KINETIC frictional force.
(*c*) If the box were sliding up the plane, the force of friction would be a KINETIC frictional force, and it would point down the plane, in the opposite direction to that shown in the diagram.


$\vec{\mathbf{F}}_{\text{fr}}$
$\vec{\mathbf{F}}_{\text{N}}$
y
x
θ
$m\vec{\mathbf{g}}$
θ

44. Assume that the static frictional force is the only force accelerating the racer. Then consider the free-body diagram for the racer as shown. It is apparent that the normal is equal to the weight, since there is no vertical acceleration. It is also assumed that the static frictional force is at its maximum. Thus


$\vec{\mathbf{F}}_{\text{N}}$
$\vec{\mathbf{F}}_{\text{fr}}$
$m\vec{\mathbf{g}}$

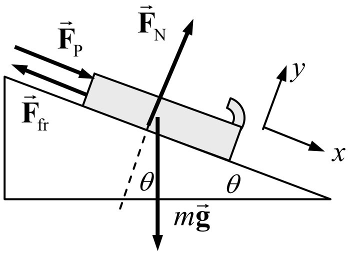
$$F_f = ma \;\rightarrow\; \mu_s mg = ma \;\rightarrow\; \mu_s = a/g$$

The acceleration of the racer can be calculated from Eq. 2-11b, with an initial speed of 0.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \;\rightarrow\; a = 2\left(x - x_0\right)/t^2$$

$$\mu_s = \frac{a}{g} = \frac{2\left(x - x_0\right)}{gt^2} = \frac{2(1000\text{ m})}{\left(9.8\,\text{m/s}^2\right)(12\text{ sec})^2} = \boxed{1.4}$$

45. A free-body diagram for the bobsled is shown. The acceleration of the sled is found from Eq. 2-11c. The final velocity also needs to be converted to m/s.


$\vec{\mathbf{F}}_{\text{P}}$
$\vec{\mathbf{F}}_{\text{N}}$
y
$\vec{\mathbf{F}}_{\text{fr}}$
x
θ
θ
$m\vec{\mathbf{g}}$

$$v = \left(60\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 16.667\,\text{m/s}$$

$$v^2 - v_0^2 = 2a_x\left(x - x_0\right) \quad \rightarrow$$

$$a_x = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{\left(16.667\,\text{m/s}\right)^2 - 0}{2\left(75\text{ m}\right)} = 1.852\,\text{m/s}^2$$

Now write Newton's 2nd law for both directions. Since the sled does not accelerate in the y direction, the net force on the y direction must be 0. Then solve for the pushing force.

$$\sum F_y = F_\text{N} - mg\cos\theta = 0 \quad \rightarrow \quad F_\text{N} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta + F_\text{P} - F_\text{fr} = ma_x$$

$$F_\text{P} = ma_x - mg\sin\theta + F_\text{fr} = ma_x - mg\sin\theta + \mu_k F_\text{N}$$

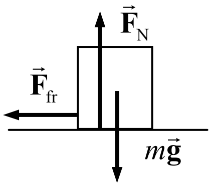
$$= ma_x - mg\sin\theta + \mu_k mg\cos\theta = m\left[a_x + g\left(\mu_k\cos\theta - \sin\theta\right)\right]$$

$$= \left(22\text{ kg}\right)\left[1.852\,\text{m/s}^2 + \left(9.8\,\text{m/s}^2\right)\left(0.10\cos 6.0^\circ - \sin 6.0^\circ\right)\right] = 39.6\text{ N} \approx \boxed{40\text{ N}}$$

46. The analysis of the blocks at rest can be done exactly the same as that presented in Example 4-20, up to the equation for the acceleration, $a = \dfrac{m_{II}g - F_\text{fr}}{m_I + m_{II}}$. Now, for the stationary case, the force of friction is static friction. To find the minimum value of m_I, we assume the maximum static frictional force. Thus $a = \dfrac{m_{II}g - \mu_s m_I g}{m_I + m_{II}}$. Finally, for the system to stay at rest, the acceleration must be zero. Thus

$$m_{II}g - \mu_s m_I g = 0 \quad \rightarrow \quad m_I = m_{II}/\mu_s = 2.0\text{ kg}/0.30 = \boxed{6.7\text{ kg}}$$

47. A free-body diagram for the box is shown, assuming that it is moving to the right. The "push" is not shown on the free-body diagram because as soon as the box moves away from the source of the pushing force, the push is no longer applied to the box. It is apparent from the diagram that $F_\text{N} = mg$ for the vertical direction. We write Newton's 2nd law for the horizontal direction, with positive to the right, to find the acceleration of the box.



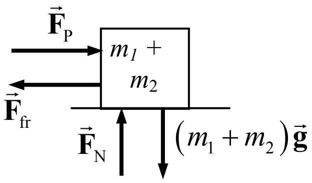
$$\sum F_x = -F_\text{fr} = ma \quad \rightarrow \quad ma = -\mu_k F_\text{N} = -\mu_k mg \quad \rightarrow$$

$$a = -\mu_k g = -0.2\left(9.8\,\text{m/s}^2\right) = -1.96\,\text{m/s}^2$$

Eq. 2-11c can be used to find the distance that the box moves before stopping. The initial speed is 4.0 m/s, and the final speed will be 0.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(4.0\,\text{m/s}\right)^2}{2\left(-1.96\,\text{m/s}^2\right)} = \boxed{4.1\text{ m}}$$

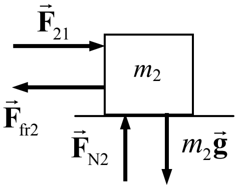
48. (*a*) Since the two blocks are in contact, they can be treated as a single object as long as no information is needed about internal forces (like the force of one block pushing on the other block). Since there is no motion in the vertical direction, it is apparent that $F_\text{N} = \left(m_1 + m_2\right)g$, and so $F_\text{fr} = \mu_k F_\text{N} = \mu_k\left(m_1 + m_2\right)g$. Write Newton's 2nd law for the horizontal direction.


$\vec{\mathbf{F}}_\text{P}$
$m_I +$
m_2
$\vec{\mathbf{F}}_\text{fr}$
$\vec{\mathbf{F}}_\text{N}$
$\left(m_1 + m_2\right)\vec{\mathbf{g}}$

$$\sum F_x = F_\text{P} - F_\text{fr} = \left(m_1 + m_2\right)a \quad \rightarrow$$

$$a=\frac{F_P-F_{fr}}{m_1+m_2}=\frac{F_P-\mu_k\left(m_1+m_2\right)g}{m_1+m_2}=\frac{620\text{ N}-(0.15)(185\text{ kg})\left(9.8\text{ m/s}^2\right)}{185\text{ kg}}=\boxed{1.9\text{ m/s}^2}$$

(*b*) To solve for the contact forces between the blocks, an individual block must be analyzed. Look at the free-body diagram for the second block. $\vec{\mathbf{F}}_{21}$ is the force of the first block pushing on the second block. Again, it is apparent that $F_{N2}=m_2g$ and so $F_{fr2}=\mu_kF_{N2}=\mu_km_2g$. Write Newton's 2nd law for the horizontal direction.

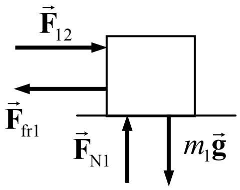


$$\sum F_x=F_{21}-F_{fr2}=m_2a\quad\rightarrow$$

$$F_{21}=\mu_km_2g+m_2a=(0.15)(110\text{ kg})\left(9.8\text{ m/s}^2\right)+(110\text{ kg})\left(1.9\text{ m/s}^2\right)=\boxed{3.7\times10^2\text{ N}}$$

By Netwon's 3rd law, there will also be a 370 N force to the left on block # 1 due to block # 2.

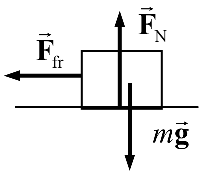
(*c*) If the crates are reversed, the acceleration of the system will remain the same – the analysis from part (a) still applies. We can also repeat the analysis from part (b) to find the force of one block on the other, if we simply change m_1 to m_2 in the free-body diagram and the resulting equations. The result would be



$$\sum F_x=F_{12}-F_{fr1}=m_1a\quad\rightarrow$$

$$F_{12}=\mu_km_1g+m_1a=(0.15)(75\text{ kg})\left(9.8\text{ m/s}^2\right)+(75\text{ kg})\left(1.9\text{ m/s}^2\right)=\boxed{2.5\times10^2\text{ N}}$$

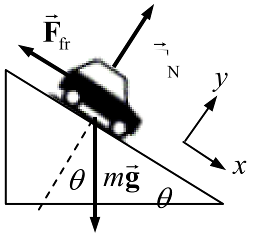
49. The force of static friction is what decelerates the crate if it is not sliding on the truck bed. If the crate is not to slide, but the maximum deceleration is desired, then the maximum static frictional force must be exerted, and so $F_{fr}=\mu_sF_N$. The direction of travel is to the right. It is apparent that $F_N=mg$ since there is no acceleration in the y direction. Write Newton's 2nd law for the truck in the horizontal direction.



$$\sum F_x=-F_{fr}=ma\quad\rightarrow\quad-\mu_smg=ma\quad\rightarrow\quad a=-\mu_sg=-(0.75)\left(9.8\text{ m/s}^2\right)=\boxed{-7.4\text{ m/s}^2}$$

The negative sign indicates the direction of the acceleration – opposite to the direction of motion.

50. Consider a free-body diagram of the car on the icy inclined driveway. Assume that the car is not moving, but just ready to slip, so that the static frictional force has its maximum value of $F_{fr}=\mu_sF_N$. Write Newton's 2nd law in each direction for the car, with a net force of 0 in each case.



$$\sum F_y=F_N-mg\cos\theta=0\quad\rightarrow\quad F_N=mg\cos\theta$$

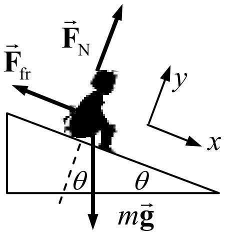
$$\sum F_x=mg\sin\theta-F_{fr}=0\quad\rightarrow\quad mg\sin\theta=\mu_smg\cos\theta$$

$$\mu_s=\sin\theta/\cos\theta=\tan\theta\quad\rightarrow\quad\theta=\tan^{-1}\mu_s=\tan^{-1}0.15=8.5^{\circ}$$

The car will not be able to stay at rest on any slope steeper than 8.5°.

$\boxed{\text{Only the driveway across the street is safe for parking.}}$

51. We assume that the child starts from rest at the top of the slide, and then slides a distance $x - x_0$ along the slide. A force diagram is shown for the child on the slide. First, ignore the frictional force and so consider the no-friction case. All of the motion is in the x direction, so we will only consider Newton's 2nd law for the x direction.



$$\sum F_x = mg\sin\theta = ma \quad \rightarrow \quad a = g\sin\theta$$

Use Eq. 2-11c to calculate the speed at the bottom of the slide.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad v_{(\text{No friction})} = \sqrt{v_0^2 + 2a\left(x - x_0\right)} = \sqrt{2g\sin\theta\left(x - x_0\right)}$$

Now include kinetic friction. We must consider Newton's 2nd law in both the x and y directions now. The net force in the y direction must be 0 since there is no acceleration in the y direction.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = ma = mg\sin\theta - F_{fr} = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g\left(\sin\theta - \mu_k\cos\theta\right)$$

With this acceleration, we can again use Eq. 2-11c to find the speed after sliding a certain distance.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad v_{(\text{friction})} = \sqrt{v_0^2 + 2a\left(x - x_0\right)} = \sqrt{2g\left(\sin\theta - \mu_k\cos\theta\right)\left(x - x_0\right)}$$

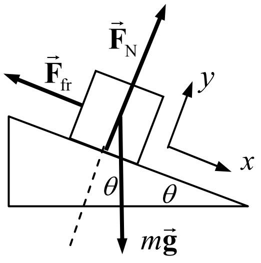
Now let the speed with friction be half the speed without friction, and solve for the coefficient of friction. Square the resulting equation and divide by $g\cos\theta$ to get the result.

$$v_{(\text{friction})} = \tfrac{1}{2}v_{(\text{No friction})} \quad \rightarrow \quad \sqrt{2g\left(\sin\theta - \mu_k\cos\theta\right)\left(x - x_0\right)} = \tfrac{1}{2}\sqrt{2g\left(\sin\theta\right)\left(x - x_0\right)}$$

$$2g\left(\sin\theta - \mu_k\cos\theta\right)\left(x - x_0\right) = \tfrac{1}{4}2g\left(\sin\theta\right)\left(x - x_0\right)$$

$$\mu_k = \tfrac{3}{4}\tan\theta = \tfrac{3}{4}\tan 28^\circ = \boxed{0.40}$$

$\boxed{52.}$ (*a*) Consider the free-body diagram for the carton on the surface. There is no motion in the y direction and thus no acceleration in the y direction. Write Newton's 2nd law for both directions.



$$\sum F_x = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

$$ma = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

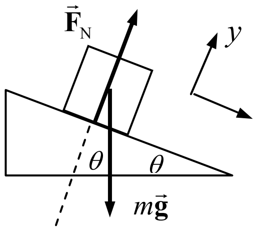
$$a = g\left(\sin\theta - \mu_k\cos\theta\right)$$

$$= \left(9.80\,\text{m/s}^2\right)\left(\sin 22.0^\circ - 0.12\cos 22.0^\circ\right) = 2.58 = \boxed{2.6\,\text{m/s}^2}$$

(*b*) Now use Eq. 2-11c, with an initial velocity of 0, to find the final velocity.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad v = \sqrt{2a\left(x - x_0\right)} = \sqrt{2\left(2.58\,\text{m/s}^2\right)\left(9.30\text{ m}\right)} = \boxed{6.9\,\text{m/s}}$$

53. (*a*) Consider the free-body diagram for the carton on the frictionless surface. There is no acceleration in the y direction. Write Newton's 2nd law for the x direction.



$$\sum F_x = mg\sin\theta = ma \quad \rightarrow \quad a = g\sin\theta$$

Use Eq. 2-11c with $v_0 = -3.0\,\text{m/s}$ and $v = 0\,\text{m/s}$ to find the distance that it slides before stopping.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow$$

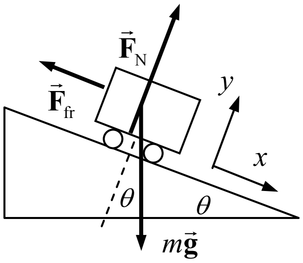
$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - (-3.0\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)\sin 22.0^\circ} = \boxed{-1.2\ \text{m}}$$

The negative sign means that the block is displaced up the plane, which is the negative direction.

(*b*) The time for a round trip can be found from Eq. 2-11a. The free-body diagram (and thus the acceleration) is the same whether the block is rising or falling. For the entire trip, $v_0 = -3.0\,\text{m/s}$ and $v = +3.0\,\text{m/s}$.

$$v = v_0 + at \quad \rightarrow \quad t = \frac{v - v_0}{a} = \frac{(3.0\,\text{m/s}) - (-3.0\,\text{m/s})}{(9.8\,\text{m/s}^2)\sin 22^\circ} = \boxed{1.6\ \text{s}}$$

54. See the free-body diagram for the descending roller coaster. It starts its descent with $v_0 = (6.0\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 1.667\,\text{m/s}$. The total displacement in the *x* direction is $x - x_0 = 45.0\ \text{m}$. Write Newton's second law for both the *x* and *y* directions.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = ma = mg\sin\theta - F_{fr} = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g(\sin\theta - \mu_k\cos\theta)$$

Now use Eq. 2-11c to solve for the final velocity.

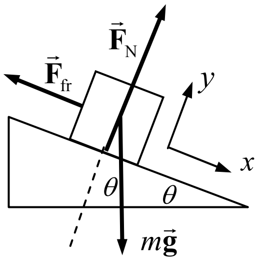
$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow$$

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{v_0^2 + 2g(\sin\theta - \mu_k\cos\theta)(x - x_0)}$$

$$= \sqrt{(1.667\,\text{m/s})^2 + 2(9.8\,\text{m/s}^2)\left[\sin 45^\circ - (0.18)\cos 45^\circ\right](45.0\ \text{m})}$$

$$= 22.68\,\text{m/s} \approx \boxed{23\,\text{m/s}} \approx 82\,\text{km/h}$$

55. Consider a free-body diagram of the box. Write Newton's 2nd law for both directions. The net force in the *y* direction is 0 because there is no acceleration in the *y* direction.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

Now solve for the force of friction and the coefficient of friction.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

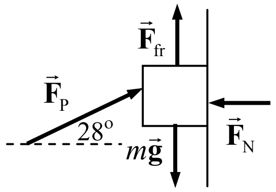
$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

$$F_{fr} = mg\sin\theta - ma = m(g\sin\theta - a) = (18.0\ \text{kg})\left[(9.80\,\text{m/s}^2)(\sin 37.0^\circ) - 0.270\,\text{m/s}^2\right]$$

$$= 101.3\,\text{N} \approx \boxed{101\,\text{N}}$$

$$F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta \;\rightarrow\; \mu_k = \frac{F_{\text{fr}}}{mg\cos\theta} = \frac{101.3\text{ N}}{(18.0\text{ kg})(9.80\,\text{m/s}^2)\cos 37.0^\circ} = \boxed{0.719}$$

56. Consider a free-body diagram for the box, showing force on the box. When $F_{\text{P}} = 13\text{ N}$, the block does not move. Thus in that case, the force of friction is static friction, and must be at its maximum value, given by $F_{\text{fr}} = \mu_s F_{\text{N}}$. Write Newton's 2nd law in both the *x* and *y* directions. The net force in each case must be 0, since the block is at rest.


$\vec{\mathbf{F}}_{\text{fr}}$
$\vec{\mathbf{F}}_{\text{P}}$
28°
$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{N}}$

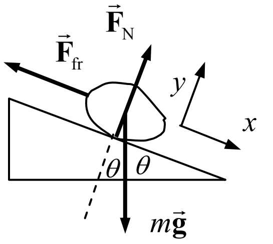
$$\sum F_x = F_{\text{P}}\cos\theta - F_{\text{N}} = 0 \;\rightarrow\; F_{\text{N}} = F_{\text{P}}\cos\theta$$

$$\sum F_y = F_{\text{fr}} + F_{\text{P}}\sin\theta - mg = 0 \;\rightarrow\; F_{\text{fr}} + F_{\text{P}}\sin\theta = mg$$

$$\mu_s F_{\text{N}} + F_{\text{P}}\sin\theta = mg \;\rightarrow\; \mu_s F_{\text{P}}\cos\theta + F_{\text{P}}\sin\theta = mg$$

$$m = \frac{F_{\text{P}}}{g}(\mu_s\cos\theta + \sin\theta) = \frac{13\text{ N}}{9.80\,\text{m/s}^2}\left(0.40\cos 28^\circ + \sin 28^\circ\right) = \boxed{1.1\text{ kg}}$$

57. (*a*) Consider the free-body diagram for the snow on the roof. If the snow is just ready to slip, then the static frictional force is at its maximum value, $F_{\text{fr}} = \mu_s F_{\text{N}}$. Write Newton's 2nd law in both directions, with the net force equal to zero since the snow is not accelerating.



$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \;\rightarrow\; F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = 0 \;\rightarrow$$

$$mg\sin\theta = F_{\text{fr}} = \mu_s F_{\text{N}} = \mu_s mg\cos\theta \;\rightarrow\; \mu_s = \tan\theta = \tan 30^\circ = \boxed{0.58}$$

If $\mu_s > 0.58$, then the snow would not be on the verge of slipping.

(*b*) The same free-body diagram applies for the sliding snow. But now the force of friction is kinetic, so $F_{\text{fr}} = \mu_k F_{\text{N}}$, and the net force in the *x* direction is not zero. Write Newton's 2nd law for the *x* direction again, and solve for the acceleration.

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = ma$$

$$a = \frac{mg\sin\theta - F_{\text{fr}}}{m} = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g(\sin\theta - \mu_k\cos\theta)$$

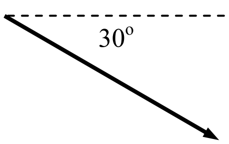
Use Eq. 2-11c with $v_i = 0$ to find the speed at the end of the roof.

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$v = \sqrt{v_0 + 2a(x - x_0)} = \sqrt{2g(\sin\theta - \mu_k\cos\theta)(x - x_0)}$$

$$= \sqrt{2(9.8\,\text{m/s}^2)(\sin 30^\circ - (0.20)\cos 30^\circ)(5.0\text{ m})} = 5.66 = \boxed{5.7\,\text{m/s}}$$

(*c*) Now the problem becomes a projectile motion problem. The projectile has an initial speed of 5.7 m/s, directed at an angle of 30° below the horizontal. The horizontal component of the speed, (5.66 m/s) cos 30° = 4.90 m/s, will stay constant. The vertical component will change due to gravity. Define the positive direction to be downward. Then the starting vertical velocity is (5.66 m/s) sin 30° = 2.83 m/s, the vertical acceleration is 9.8 m/s^2, and the vertical displacement is 10.0 m. Use Eq. 2-11c to find the final vertical speed.



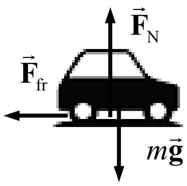
$$v_y^2 - v_{y0y}^2 = 2a(y - y_0)$$

$$v_y = \sqrt{v_{y0}^2 + 2a(y - y_0)} = \sqrt{(2.83\,\text{m/s})^2 + 2(9.8\,\text{m/s}^2)(10.0\text{ m})} = 14.3\text{ m/s}$$

To find the speed when it hits the ground, the horizontal and vertical components of velocity must again be combined, according to the Pythagorean theorem.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(4.90\,\text{m/s})^2 + (14.3\text{ m/s})^2} = \boxed{15\,\text{m/s}}$$

58. (*a*) A free-body diagram for the car is shown, assuming that it is moving to the right. It is apparent from the diagram that $F_N = mg$ for the vertical direction. Write Newton's 2nd law for the horizontal direction, with positive to the right, to find the acceleration of the car. Since the car is assumed to NOT be sliding, use the maximum force of static friction.



$$\sum F_x = -F_{fr} = ma \rightarrow ma = -\mu_s F_N = -\mu_s mg \rightarrow a = -\mu_s g$$

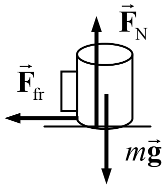
Eq. 2-11c can be used to find the distance that the car moves before stopping. The initial speed is given as v, and the final speed will be 0.

$$v^2 - v_0^2 = 2a(x - x_0) \rightarrow (x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - v^2}{2(-\mu_s g)} = \boxed{\frac{v^2}{2\mu_s g}}$$

(*b*) Using the given values:

$$v = (95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.38\,\text{m/s} \qquad (x - x_0) = \frac{v^2}{2\mu_s g} = \frac{(26.38\,\text{m/s})^2}{2(0.75)(9.8\,\text{m/s}^2)} = \boxed{47\text{ m}}$$

59. A free-body diagram for the coffee cup is shown. Assume that the car is moving to the right, and so the acceleration of the car (and cup) will be to the left. The deceleration of the cup is caused by friction between the cup and the dashboard. For the cup to not slide on the dash, and to have the minimum deceleration time means the largest possible static frictional force is acting, so $F_{fr} = \mu_s F_N$. The normal force on the cup is equal to its weight, since there is no vertical acceleration. The horizontal acceleration of the cup is found from Eq. 2-11a, with a final velocity of zero.



$$v_0 = (45\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 12.5\,\text{m/s}$$

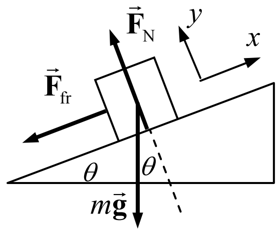
$$v - v_0 = at \rightarrow a = \frac{v - v_0}{t} = \frac{0 - 12.5\,\text{m/s}}{3.5\text{ s}} = -3.57\,\text{m/s}^2$$

Write Newton's 2nd law for the horizontal forces, considering to the right to be positive.

$$\sum F_x = -F_{fr} = ma \rightarrow ma = -\mu_s F_N = -\mu_s mg \rightarrow \mu_s = -\frac{a}{g} = -\frac{(-3.57\,\text{m/s}^2)}{9.80\,\text{m/s}^2} = \boxed{0.36}$$

60. We derive two expressions for acceleration – one from the kinematics, and one from the dynamics. From Eq. 2-11c with a starting speed of v_0 up the plane and a final speed of zero, we have

$$v^2 - v_o^2 = 2a(x - x_0) \rightarrow a = \frac{-v_o^2}{2(x - x_0)} = \frac{-v_o^2}{2d}$$



Write Newton's 2nd law for both the x and y directions. Note that the net force in the y direction is zero, since the block does not accelerate in the y direction.

$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = 0 \quad\rightarrow\quad F_{\mathrm{N}} = mg\cos\theta$$

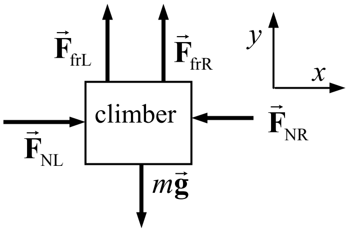
$$\sum F_x = -mg\sin\theta - F_{\mathrm{fr}} = ma \quad\rightarrow\quad a = \frac{-mg\sin\theta - F_{\mathrm{fr}}}{m}$$

Now equate the two expressions for the acceleration, substitute in the relationship between the frictional force and the normal force, and solve for the coefficient of friction.

$$a = \frac{-mg\sin\theta - F_{\mathrm{fr}}}{m} = \frac{-v_o^2}{2d} \quad\rightarrow\quad \frac{mg\sin\theta + \mu_k mg\cos\theta}{m} = \frac{v_o^2}{2d} \quad\rightarrow$$

$$\boxed{\mu_k = \frac{v_o^2}{2gd\cos\theta} - \tan\theta}$$

61. Since the walls are vertical, the normal forces are horizontal, away from the wall faces. We assume that the frictional forces are at their maximum values, so $F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}}$ applies at each wall. We assume that the rope in the diagram is not under any tension and so does not exert any forces. Consider the free-body diagram for the climber. F_{NR} is the normal force on the climber from the right wall, and F_{NL} is the normal force on the climber from the left wall. The static frictional forces are $F_{\mathrm{frL}} = \mu_{s\mathrm{L}} F_{\mathrm{NL}}$ and $F_{\mathrm{frR}} = \mu_{s\mathrm{R}} F_{\mathrm{NR}}$. Write Newton's 2nd law for both the x and y directions. The net force in each direction must be zero if the climber is stationary.



$$\sum F_x = F_{\mathrm{NL}} - F_{\mathrm{NR}} = 0 \quad\rightarrow\quad F_{\mathrm{NL}} = F_{\mathrm{NR}} \qquad \sum F_y = F_{\mathrm{frL}} + F_{\mathrm{frR}} - mg = 0$$

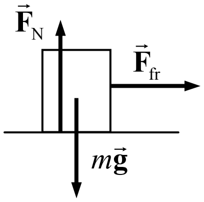
Substitute the information from the x equation into the y equation.

$$F_{\mathrm{frL}} + F_{\mathrm{frR}} = mg \quad\rightarrow\quad \mu_{s\mathrm{L}} F_{\mathrm{NL}} + \mu_{s\mathrm{R}} F_{\mathrm{NR}} = mg \quad\rightarrow\quad \left(\mu_{s\mathrm{L}} + \mu_{s\mathrm{R}}\right) F_{\mathrm{NL}} = mg$$

$$F_{\mathrm{NL}} = \frac{mg}{\left(\mu_{s\mathrm{L}} + \mu_{s\mathrm{R}}\right)} = \frac{(75\text{ kg})\left(9.80\text{ m/s}^2\right)}{1.4} = 525\text{ N}$$

And so $F_{\mathrm{NL}} = F_{\mathrm{NR}} = 525\text{ N}$. These normal forces arise as Newton's 3rd law reaction forces to the climber pushing on the walls. Thus the climber must exert a force of at least $5.3\times10^2\text{ N}$ against each wall.

62. Notice the symmetry of this problem – in the first half of the motion, the object accelerates with a constant acceleration to a certain speed, and then in the second half of the motion, the object decelerates with the same magnitude of acceleration back to a speed of 0. Half the time elapses during the first segment of the motion, and half the distance is traveled during the first segment of the motion. Thus we analyze half of the motion, and then double the time found to get the total time.



Friction is the accelerating and decelerating force. We assume that the boxes do not slip on the belt since slippage would increase the travel time. To have the largest possible acceleration, and hence the largest possible force, so that the travel time can be a minimum, the box must be moved by the maximum value of the static frictional force, and so $F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}}$. See the free-body diagram for the box on the first half of the trip, assuming that the conveyor belt is level. Since there is no vertical acceleration of the box, it is apparent that $F_\text{N} = mg$, and so $F_\text{fr} = \mu_s mg$. Use Newton's 2nd law in the horizontal direction to find the acceleration.

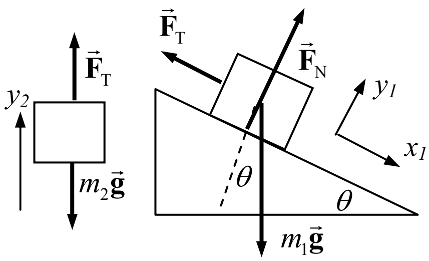
$$\sum F = F_\text{fr} = \mu_s mg = ma \quad \rightarrow \quad a = \mu_s g = (0.60)\left(9.8\,\text{m/s}^2\right) = 5.88\,\text{m/s}^2$$

Now use Eq. 2-11b to determine the time taken to move half the distance with the calculated acceleration, starting from rest.

$$d/2 = x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{d/a}\ .$$

Thus the total time for the trip will be $t_\text{total} = 2\sqrt{d/a} = 2\sqrt{(11.0\text{ m})/\left(5.88\,\text{m/s}^2\right)} = \boxed{2.7\text{ s}}$

63. (*a*) Draw a free-body diagram for each block. Write Newton's 2nd law for each block. Notice that the acceleration of block # 1 in the y_1 direction will be zero, since it has no motion in the y_1 direction.


$\vec{\mathbf{F}}_\text{T}$
$\vec{\mathbf{F}}_\text{T}$
$\vec{\mathbf{F}}_\text{N}$
y_1
y_2
x_1
$m_2\vec{\mathbf{g}}$
θ
θ
$m_1\vec{\mathbf{g}}$

$$\sum F_{y1} = F_\text{N} - m_1 g\cos\theta = 0 \quad \rightarrow \quad F_\text{N} = m_1 g\cos\theta$$

$$\sum F_{x1} = m_1 g\sin\theta - F_\text{T} = m_1 a_{x1}$$

$$\sum F_{y2} = F_\text{T} - m_2 g = m_2 a_{y2} \quad \rightarrow \quad F_\text{T} = m_2\left(g + a_{y2}\right)$$

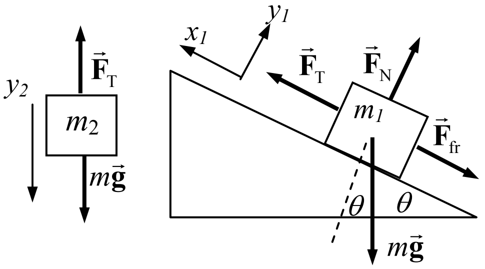
Since the blocks are connected by the cord, $a_{y2} = a_{x1} = a$. Substitute the expression for the tension force from the last equation into the *x* direction equation for block 1, and solve for the acceleration.

$$m_1 g\sin\theta - m_2\left(g + a\right) = m_1 a \quad \rightarrow \quad m_1 g\sin\theta - m_2 g = m_1 a + m_2 a$$

$$a = \boxed{g\frac{\left(m_1\sin\theta - m_2\right)}{\left(m_1 + m_2\right)}}$$

(*b*) If the acceleration is to be down the plane, it must be positive. That will happen if $\boxed{m_1\sin\theta > m_2\ \left(\text{down the plane}\right)}$. The acceleration will be up the plane (negative) if $\boxed{m_1\sin\theta < m_2\ \left(\text{up the plane}\right)}$. If $m_1\sin\theta = m_2$, then the system will not accelerate. It will move with a constant speed if set in motion by a push.

$\boxed{64.}$ (*a*) Given that m_2 is moving down, m_1 must be moving up the incline, and so the force of kinetic friction on m_1 will be directed down the incline. Since the blocks are tied together, they will both have the same acceleration, and so $a_{y2} = a_{x1} = a$. Write Newton's 2nd law for each mass.


x_1
y_1
y_2
$\vec{\mathbf{F}}_\text{T}$
m_2
$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_\text{T}$
$\vec{\mathbf{F}}_\text{N}$
m_1
$\vec{\mathbf{F}}_\text{fr}$
θ
θ
$m\vec{\mathbf{g}}$

$$\sum F_{y2} = mg - F_\text{T} = ma \quad \rightarrow \quad F_\text{T} = mg - ma$$

$$\sum F_{x1} = F_\text{T} - mg\sin\theta - F_\text{fr} = ma$$

$$\sum F_{y1} = F_\text{N} - mg\cos\theta = 0 \quad \rightarrow \quad F_\text{N} = mg\cos\theta$$

Take the information from the two *y* equations and substitute into the *x* equation to solve for the acceleration.

$$mg - ma - mg\sin\theta - \mu_k mg\cos\theta = ma$$

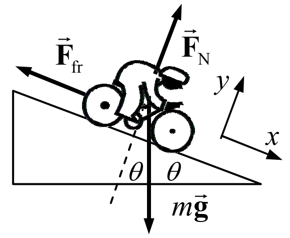
$$a = g\frac{(1-\sin\theta-\mu_k\cos\theta)}{2} = (9.8\,\mathrm{m/s^2})\frac{(1-\sin 25^\circ - 0.15\cos 25^\circ)}{2} = \boxed{2.2\,\mathrm{m/s^2}}$$

(*b*) To have an acceleration of zero, the expression for the acceleration must be zero.

$$a = g\frac{(1-\sin\theta-\mu_k\cos\theta)}{2} = 0 \;\rightarrow\; 1-\sin\theta-\mu_k\cos\theta = 0 \;\rightarrow$$

$$\mu_k = \frac{1-\sin\theta}{\cos\theta} = \frac{1-\sin 25^\circ}{\cos 25^\circ} = \boxed{0.64}$$

65. Consider a free-body diagram for the cyclist coasting downhill at a constant speed. Since there is no acceleration, the net force in each direction must be zero. Write Newton's 2nd law for the x direction.

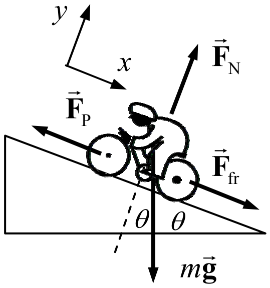


$\vec{\mathbf{F}}_{\mathrm{fr}}$
$\vec{\mathbf{F}}_{\mathrm{N}}$
y
x
θ
θ
$m\vec{\mathbf{g}}$

$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = 0 \;\rightarrow\; F_{\mathrm{fr}} = mg\sin\theta$$

This establishes the size of the air friction force at 6.0 km/h, and so can be used in the next part.

Now consider a free-body diagram for the cyclist climbing the hill. F_{P} is the force pushing the cyclist uphill. Again, write Newton's 2nd law for the x direction, with a net force of 0.



y
x
$\vec{\mathbf{F}}_{\mathrm{N}}$
$\vec{\mathbf{F}}_{\mathrm{P}}$
$\vec{\mathbf{F}}_{\mathrm{fr}}$
θ
θ
$m\vec{\mathbf{g}}$

$$\sum F_x = F_{\mathrm{fr}} + mg\sin\theta - F_{\mathrm{P}} = 0 \;\rightarrow$$

$$F_{\mathrm{P}} = F_{\mathrm{fr}} + mg\sin\theta = 2mg\sin\theta$$

$$= 2(65\text{ kg})(9.8\,\mathrm{m/s^2})(\sin 6.0^\circ) = \boxed{1.3\times10^2\,\mathrm{N}}$$

66. The average acceleration of the blood is given by $a = \frac{v-v_0}{t} = \frac{0.35\,\mathrm{m/s} - 0.25\,\mathrm{m/s}}{0.10\text{ s}} = 1.0\,\mathrm{m/s^2}$.

Thus the net force on the blood, exerted by the heart, would be

$$F = ma = (20\times10^{-3}\,\mathrm{kg})(1.0\,\mathrm{m/s^2}) = \boxed{0.020\text{ N}}.$$

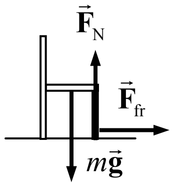
67. The acceleration of a person having a 30 "*g*" deceleration is $a = (30\,"g")\left(\frac{9.8\,\mathrm{m/s^2}}{"g"}\right) = 290\,\mathrm{m/s^2}$.

The average force causing that acceleration is $F = ma = (70\text{ kg})(290\,\mathrm{m/s^2}) = \boxed{2.1\times10^4\,\mathrm{N}}$. Since the person is undergoing a deceleration, the acceleration and force would both be directed opposite to the direction of motion. Use Eq. 2-11c to find the distance traveled during the deceleration. Take the initial velocity to be in the positive direction, so that the acceleration will have a negative value, and the final velocity will be 0.

$$v_0 = (100\,\mathrm{km/h})\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 27.78\,\mathrm{m/s}$$

$$v^2 - v_0^2 = 2a(x-x_0) \;\rightarrow\; (x-x_0) = \frac{v^2-v_0^2}{2a} = \frac{0-(27.78\,\mathrm{m/s})^2}{2(-290\,\mathrm{m/s^2})} = \boxed{1.3\text{ m}}$$

68. (*a*) Assume that the earthquake is moving the Earth to the right. If an object is to "hold its place", then the object must also be accelerating to the right with the Earth. The force that will accelerate that object will be the static frictional force, which would also have to be to the right. If the force were not large enough, the Earth would move out from under the chair somewhat, giving the appearance that the chair were being "thrown" to the left. Consider the free-body diagram shown for a chair on the floor. It is apparent that the normal force is equal to the weight since there is no motion in the vertical direction. Newton's 2nd law says that $F_{\text{fr}} = ma$. We also assume that the chair is just on the verge of slipping, which means that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}} = \mu_s mg$. Equate the two expressions for the frictional force to find the coefficient of friction.

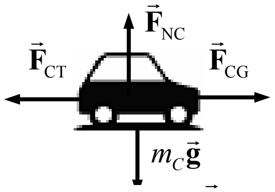

$\vec{\mathbf{F}}_{\text{N}}$
$\vec{\mathbf{F}}_{\text{fr}}$
$m\vec{\mathbf{g}}$

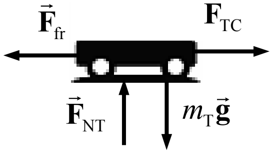
$$ma = \mu_s mg \quad \rightarrow \quad \boxed{\mu_s = a/g}$$

If the static coefficient is larger than this, then there will be a larger maximum frictional force, and the static frictional force will be more than sufficient to hold the chair in place on the floor.

(*b*) For the 1989 quake, $\dfrac{a}{g} = \dfrac{4.0\,\text{m/s}^2}{9.8\,\text{m/s}^2} = 0.41$. Since $\mu_s = 0.25$, $\boxed{\text{the chair would slide}}$.

$\boxed{69.}$ We draw three free-body diagrams – one for the car, one for the trailer, and then "add" them for the combination of car and trailer. Note that since the car pushes against the ground, the ground will push against the car with an equal but oppositely directed force. $\vec{\mathbf{F}}_{\text{CG}}$ is the force on the car due to the ground, $\vec{\mathbf{F}}_{\text{TC}}$ is the force on the trailer due to the car, and $\vec{\mathbf{F}}_{\text{CT}}$ is the force on the car due to the trailer. Note that by Newton's 3rd law, $\left|\vec{\mathbf{F}}_{\text{CT}}\right| = \left|\vec{\mathbf{F}}_{\text{TC}}\right|$.

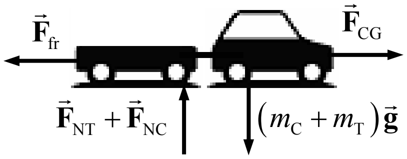

$\vec{\mathbf{F}}_{\text{NC}}$
$\vec{\mathbf{F}}_{\text{CT}}$
$\vec{\mathbf{F}}_{\text{CG}}$
$m_C\vec{\mathbf{g}}$


$\vec{\mathbf{F}}_{\text{fr}}$
$\vec{\mathbf{F}}_{\text{TC}}$
$\vec{\mathbf{F}}_{\text{NT}}$
$m_{\text{T}}\vec{\mathbf{g}}$

From consideration of the vertical forces in the individual free-body diagrams, it is apparent that the normal force on each object is equal to its weight. This leads to the conclusion that

$$F_{\text{fr}} = \mu_k F_{\text{NT}} = \mu_k m_{\text{T}} g = (0.15)(450\text{ kg})(9.8\,\text{m/s}^2) = 660\text{ N}.$$

Now consider the combined free-body diagram. Write Newton's 2nd law for the horizontal direction, This allows the calculation of the acceleration of the system.


$\vec{\mathbf{F}}_{\text{fr}}$
$\vec{\mathbf{F}}_{\text{CG}}$
$\vec{\mathbf{F}}_{\text{NT}} + \vec{\mathbf{F}}_{\text{NC}}$
$(m_{\text{C}} + m_{\text{T}})\vec{\mathbf{g}}$

$$\sum F = F_{\text{CG}} - F_{\text{fr}} = (m_{\text{C}} + m_{\text{T}})a \quad \rightarrow$$

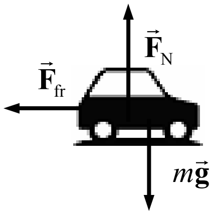
$$a = \frac{F_{\text{CG}} - F_{\text{fr}}}{m_{\text{C}} + m_{\text{T}}} = \frac{3800\text{ N} - 660\text{ N}}{1600\text{ kg}} = 1.9625\,\text{m/s}^2$$

Finally, consider the free-body diagram for the trailer alone. Again write Newton's 2nd law for the horizontal direction, and solve for F_{TC}.

$$\sum F = F_{\text{TC}} - F_{\text{fr}} = m_{\text{T}} a \quad \rightarrow$$

$$F_{\text{TC}} = F_{\text{fr}} + m_{\text{T}} a = 660\text{ N} + (450\text{ kg})(1.9625\,\text{m/s}^2) = \boxed{1.54 \times 10^3\,\text{N}}$$

70. Assume that kinetic friction is the net force causing the deceleration. See the free-body diagram for the car, assuming that the right is the positive direction, and the direction of motion of the skidding car. Since there is no acceleration in the vertical direction, and so $F_N = mg$. Applying Newton's 2nd law to the x direction gives



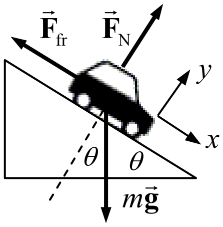
$$\sum F = -F_f = ma \quad \rightarrow \quad -\mu_k F_N = -\mu_k mg = ma \quad \rightarrow \quad a = -\mu_k g .$$

Use Eq. 2-11c to determine the initial speed of the car, with the final speed of the car being zero.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow$$

$$v_0 = \sqrt{v^2 - 2a\left(x-x_0\right)} = \sqrt{0 - 2\left(-\mu_k g\right)\left(x - x_0\right)} = \sqrt{2\left(0.8\right)\left(9.8\,\mathrm{m/s^2}\right)\left(72\ \mathrm{m}\right)} = \boxed{34\,\mathrm{m/s}}$$

71. We include friction from the start, and then for the no-friction result, set the coefficient of friction equal to 0. Consider a free-body diagram for the car on the hill. Write Newton's 2nd law for both directions. Note that the net force on the y direction will be zero, since there is no acceleration in the y direction.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma \quad \rightarrow$$

$$a = g\sin\theta - \frac{F_{fr}}{m} = g\sin\theta - \frac{\mu_k mg\cos\theta}{m} = g\left(\sin\theta - \mu_k\cos\theta\right)$$

Use Eq. 2-11c to determine the final velocity, assuming that the car starts from rest.

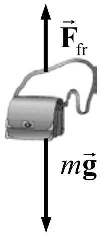
$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad v = \sqrt{0 + 2a\left(x-x_0\right)} = \sqrt{2g\left(x-x_0\right)\left(\sin\theta - \mu_k\cos\theta\right)}$$

The angle is given by $\sin\theta = 1/4 \quad \rightarrow \quad \theta = \sin^{-1} 0.25 = 14.5^\circ$

$$(a)\quad \mu_k = 0 \quad \rightarrow \quad v = \sqrt{2g\left(x-x_0\right)x\sin\theta} = \sqrt{2\left(9.8\,\mathrm{m/s^2}\right)\left(55\ \mathrm{m}\right)\sin 14.5^\circ} = \boxed{16\,\mathrm{m/s}}$$

$$(b)\quad \mu_k = 0.10 \quad \rightarrow \quad v = \sqrt{2\left(9.8\,\mathrm{m/s^2}\right)\left(55\ \mathrm{m}\right)\left(\sin 14.5^\circ - 0.10\cos 14.5^\circ\right)} = \boxed{13\,\mathrm{m/s}}$$

72. See the free-body diagram for the falling purse. Assume that down is the positive direction, and that the air resistance force $\vec{\mathbf{F}}_{fr}$ is constant. Write Newton's 2nd law for the vertical direction.



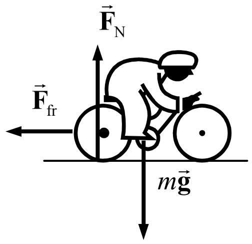
$$\sum F = mg - F_{fr} = ma \quad \rightarrow \quad F_{fr} = m\left(g - a\right)$$

Now obtain an expression for the acceleration from Eq. 2-11c with $v_0 = 0$, and substitute back into the friction force.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad a = \frac{v^2}{2\left(x - x_0\right)}$$

$$F_f = m\left(g - \frac{v^2}{2\left(x-x_0\right)}\right) = \left(2.0\ \mathrm{kg}\right)\left(9.8\,\mathrm{m/s^2} - \frac{\left(29\,\mathrm{m/s}\right)^2}{2\left(55\ \mathrm{m}\right)}\right) = \boxed{4.3\ \mathrm{N}}$$

73. Consider the free-body diagram for the cyclist in the mud, assuming that the cyclist is traveling to the right. It is apparent that $F_N = mg$ since there is no vertical acceleration. Write Newton's 2nd law for the horizontal direction, positive to the right.



$$\sum F_x = -F_{fr} = ma \;\rightarrow\; -\mu_k mg = ma \;\rightarrow\; a = -\mu_k g$$

Use Eq. 2-11c to determine the distance the cyclist could travel in the mud before coming to rest.

$$v^2 - v_0^2 = 2a(x - x_0) \;\rightarrow\; (x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{-v_0^2}{-2\mu_k g} = \frac{(12\,\text{m/s})^2}{2(0.60)(9.8\,\text{m/s}^2)} = 12\text{ m}$$

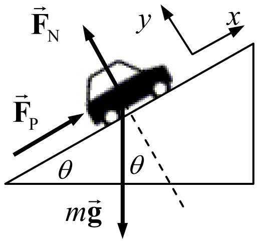
Since there is only 11 m of mud, $\boxed{\text{the cyclist will emerge from the mud.}}$ The speed upon emerging is found from Eq. 2-11c.

$$v^2 - v_0^2 = 2a(x - x_0) \;\rightarrow$$

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{v_i^2 - 2\mu_k g(x - x_0)} = \sqrt{(12\,\text{m/s})^2 - 2(0.60)(9.8\,\text{m/s}^2)(11\text{ m})}$$

$$= \boxed{3.8\,\text{m/s}}$$

74. The given data can be used to calculate the force with which the road pushes against the car, which in turn is equal in magnitude to the force the car pushes against the road. The acceleration of the car on level ground is found from Eq. 2-11a.



$$v - v_0 = at \;\rightarrow\; a = \frac{v - v_0}{t} = \frac{21\,\text{m/s} - 0}{14.0\text{ s}} = 1.50\,\text{m/s}^2$$

The force pushing the car in order to have this acceleration is found from Newton's 2nd law.

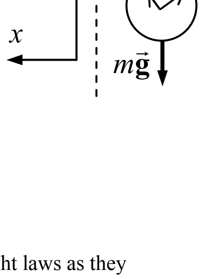
$$F_P = ma = (1100\text{ kg})(1.50\,\text{m/s}^2) = 1650\text{ N}$$

We assume that this is the force pushing the car on the incline as well. Consider a free-body diagram for the car climbing the hill. We assume that the car will have a constant speed on the maximum incline. Write Newton's 2nd law for the x direction, with a net force of zero since the car is not accelerating.

$$\sum F_x = F_P - mg\sin\theta = 0 \;\rightarrow\; \sin\theta = \frac{F_P}{mg}$$

$$\theta = \sin^{-1}\frac{F_P}{mg} = \sin^{-1}\frac{1650\text{ N}}{(1100\text{ kg})(9.8\,\text{m/s}^2)} = \boxed{8.8^\circ}$$

75. Consider the free-body diagram for the watch. Write Newton's 2nd law for both the x and y directions. Note that the net force in the y direction is 0 because there is no acceleration in the y direction.



$$\sum F_y = F_T\cos\theta - mg = 0 \;\rightarrow\; F_T = \frac{mg}{\cos\theta}$$

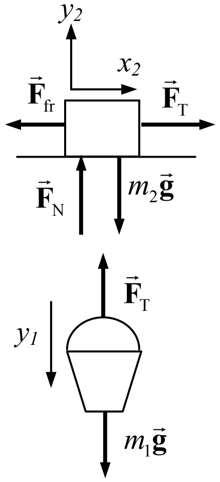
$$\sum F_x = F_T\sin\theta = ma \;\rightarrow\; \frac{mg}{\cos\theta}\sin\theta = ma$$

$$a = g\tan\theta = (9.8\,\text{m/s}^2)\tan 25^\circ = 4.6\,\text{m/s}^2$$

Use Eq. 2-11a with $v_0 = 0$ to find the final velocity (takeoff speed).

$$v - v_0 = at \;\rightarrow\; v = v_0 + at = 0 + \left(4.6\,\mathrm{m/s^2}\right)\left(18\ \mathrm{s}\right) = \boxed{82\,\mathrm{m/s}}$$

$\boxed{76.}$ (*a*) Consider the free-body diagrams for both objects, initially stationary. As sand is added, the tension will increase, and the force of static friction on the block will increase until it reaches its maximum of $F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}}$. Then the system will start to move. Write Newton's 2nd law for each object, when the static frictional force is at its maximum, but the objects are still stationary.


y_2
x_2
$\vec{\mathbf{F}}_{\mathrm{fr}}$
$\vec{\mathbf{F}}_{\mathrm{T}}$
$\vec{\mathbf{F}}_{\mathrm{N}}$
$m_2\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\mathrm{T}}$
y_1
$m_1\vec{\mathbf{g}}$

$$\sum F_{y\ \text{bucket}} = m_1 g - F_{\mathrm{T}} = 0 \;\rightarrow\; F_{\mathrm{T}} = m_1 g$$

$$\sum F_{y\ \text{block}} = F_{\mathrm{N}} - m_2 g = 0 \;\rightarrow\; F_{\mathrm{N}} = m_2 g$$

$$\sum F_{x\ \text{block}} = F_{\mathrm{T}} - F_{\mathrm{fr}} = 0 \;\rightarrow\; F_{\mathrm{T}} = F_{\mathrm{fr}}$$

Equate the two expressions for tension, and substitute in the expression for the normal force to find the masses.

$$m_1 g = F_{\mathrm{fr}} \;\rightarrow\; m_1 g = \mu_s F_{\mathrm{N}} = \mu_s m_2 g \;\rightarrow$$

$$m_1 = \mu_s m_2 = (0.450)(28.0\ \mathrm{kg}) = 12.6\ \mathrm{kg}$$

Thus $12.6\ \mathrm{kg} - 1.35\ \mathrm{kg} = 11.25 = \boxed{11.3\ \mathrm{kg}}$ of sand was added.

(*b*) The same free-body diagrams can be used, but now the objects will accelerate. Since they are tied together, $a_{y1} = a_{x2} = a$. The frictional force is now kinetic friction, given by $F_{\mathrm{fr}} = \mu_k F_{\mathrm{N}} = \mu_k m_2 g$. Write Newton's 2nd laws for the objects in the direction of their acceleration.

$$\sum F_{y\ \text{bucket}} = m_1 g - F_{\mathrm{T}} = m_1 a \;\rightarrow\; F_{\mathrm{T}} = m_1 g - m_1 a$$

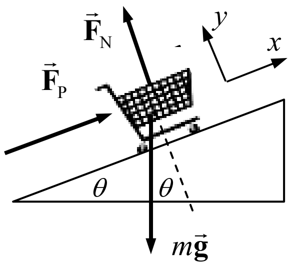
$$\sum F_{x\ \text{block}} = F_{\mathrm{T}} - F_{\mathrm{fr}} = m_2 a \;\rightarrow\; F_{\mathrm{T}} = F_{\mathrm{fr}} + m_2 a$$

Equate the two expressions for tension, and solve for the acceleration.

$$m_1 g - m_1 a = \mu_k m_2 g + m_2 a$$

$$a = g\frac{(m_1 - \mu_k m_2)}{(m_1 + m_2)} = \left(9.80\,\mathrm{m/s^2}\right)\frac{\left(12.6\mathrm{kg} - (0.320)(28.0\mathrm{kg})\right)}{(12.6\mathrm{kg} + 28.0\mathrm{kg})} = \boxed{0.88\,\mathrm{m/s^2}}$$

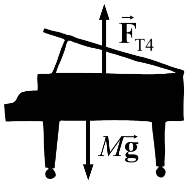
77. Consider a free-body diagram for a grocery cart being pushed up an incline. Assuming that the cart is not accelerating, we write Newton's 2nd law for the *x* direction.


$\vec{\mathbf{F}}_{\mathrm{N}}$
y
x
$\vec{\mathbf{F}}_{\mathrm{P}}$
θ
θ
$m\vec{\mathbf{g}}$

$$\sum F_x = F_{\mathrm{P}} - mg\sin\theta = 0 \;\rightarrow\; \sin\theta = \frac{F_{\mathrm{P}}}{mg}$$

$$\theta = \sin^{-1}\frac{F_{\mathrm{P}}}{mg} = \sin^{-1}\frac{20\ \mathrm{N}}{(20\ \mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)} = \boxed{5.9^{\circ}}$$

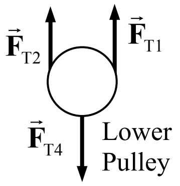
78. (*a*) To find the minimum force, assume that the piano is moving with a constant velocity. Since the piano is not accelerating, $F_{\mathrm{T4}} = Mg$. For the lower pulley, since the tension in a rope is the same throughout, and since the pulley is not accelerating, it is seen that

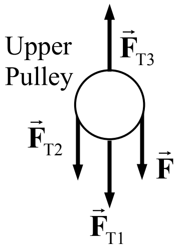

$\vec{\mathbf{F}}_{\mathrm{T4}}$
$M\vec{\mathbf{g}}$

$$F_{\mathrm{T1}} + F_{\mathrm{T2}} = 2F_{\mathrm{T1}} = Mg \quad \rightarrow \quad F_{\mathrm{T1}} = F_{\mathrm{T2}} = Mg/2 .$$

It also can be seen that since $F = F_{\mathrm{T2}}$, that $\boxed{F = Mg/2}$.

(*b*) Draw a free-body diagram for the upper pulley. From that diagram, we see that


$\vec{\mathbf{F}}_{\mathrm{T2}}$
$\vec{\mathbf{F}}_{\mathrm{T1}}$
$\vec{\mathbf{F}}_{\mathrm{T4}}$
Lower
Pulley

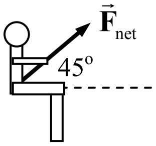

Upper
Pulley
$\vec{\mathbf{F}}_{\mathrm{T3}}$
$\vec{\mathbf{F}}_{\mathrm{T2}}$
$\vec{\mathbf{F}}$
$\vec{\mathbf{F}}_{\mathrm{T1}}$

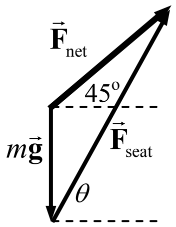
$$F_{\mathrm{T3}} = F_{\mathrm{T1}} + F_{\mathrm{T2}} + F = \frac{3Mg}{2}$$

To summarize:

$$\boxed{F_{\mathrm{T1}} = F_{\mathrm{T2}} = Mg/2 \qquad F_{\mathrm{T3}} = 3Mg/2 \qquad F_{\mathrm{T4}} = Mg}$$

79. The acceleration of the pilot will be the same as that of the plane, since the pilot is at rest with respect to the plane. Consider first a free-body diagram of the pilot, showing only the net force. By Newton's 2nd law, the net force MUST point in the direction of the acceleration, and its magnitude is ma. That net force is the sum of ALL forces on the pilot. If we assume that the force of gravity and the force of the cockpit seat on the pilot are the only forces on the pilot, then in terms of vectors, $\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{g}} + \vec{\mathbf{F}}_{\text{seat}} = m\vec{\mathbf{a}}$. Solve this equation for the force of the seat to find $\vec{\mathbf{F}}_{\text{seat}} = \vec{\mathbf{F}}_{\text{net}} - m\vec{\mathbf{g}} = m\vec{\mathbf{a}} - m\vec{\mathbf{g}}$. A vector diagram of that equation is as shown. Solve for the force of the seat on the pilot using components.


$\vec{\mathbf{F}}_{\text{net}}$
45°


$\vec{\mathbf{F}}_{\text{net}}$
45°
$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{seat}}$
θ

$$F_{x\text{ seat}} = F_{x\text{ net}} = ma\cos 45^\circ = (75\text{ kg})(3.5\,\text{m/s}^2)\cos 45^\circ = 180\text{ N}$$

$$F_{y\text{ seat}} = mg + F_{y\text{ net}} = mg + ma\sin 45^\circ$$
$$= (75\text{ kg})(9.8\,\text{m/s}^2) + (75\text{ kg})(3.5\,\text{m/s}^2)\cos 45^\circ = 920\text{ N}$$

The magnitude of the cockpit seat force is

$$F = \sqrt{F_{x\text{ seat}}^2 + F_{y\text{ seat}}^2} = \sqrt{(180\text{ N})^2 + (920\text{ N})^2} = \boxed{940\text{ N}}$$

The angle of the cockpit seat force is

$$\theta = \tan^{-1}\frac{F_{y\text{ seat}}}{F_{x\text{ seat}}} = \tan^{-1}\frac{920\text{ N}}{180\text{ N}} = \boxed{79^\circ} \text{ above the horizontal.}$$

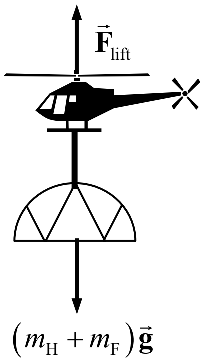
80. The initial speed is $v_i = (45\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 12.5\,\text{m/s}$. Use Eq. 2-11a to find the deceleration of the child.

$$v - v_0 = at \quad \rightarrow \quad a = \frac{v - v_0}{t} = \frac{0 - 12.5\,\text{m/s}}{0.20\text{ s}} = -62.5\,\text{m/s}^2 .$$

The net force on the child is given by Newton's 2nd law.

$$F_{\text{net}} = ma = (12\text{ kg})(-62.5\,\text{m/s}^2) = \boxed{-7.5\times 10^2\,\text{N}}, \text{ opposite to the velocity}$$

We also assumed that friction between the seat and child is zero, and we assumed that the bottom of the seat is horizontal. If friction existed or if the seat was tilted back, then the force that the straps would have to apply would be less.

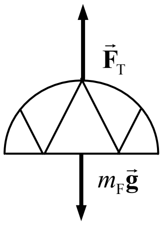

$\vec{\mathbf{F}}_{\text{lift}}$
$(m_{\text{H}}+m_{\text{F}})\vec{\mathbf{g}}$

81 (*a*) The helicopter and frame will both have the same acceleration, and so can be treated as one object if no information about internal forces (like the cable tension) is needed. A free-body diagram for the helicopter-frame combination is shown. Write Newton's 2nd law for the combination, calling UP the positive direction.

$$\sum F = F_{\text{lift}} - (m_{\text{H}}+m_{\text{F}})g = (m_{\text{H}}+m_{\text{F}})a \quad \rightarrow$$

$$F_{\text{lift}} = (m_{\text{H}}+m_{\text{F}})(g+a) = (7650\text{ kg}+1250\text{ kg})\left(9.80\,\text{m/s}^2+0.80\,\text{m/s}^2\right)$$

$$= \boxed{9.43\times10^4\,\text{N}}$$



(*b*) Now draw a free-body diagram for the frame alone, in order to find the tension in the cable. Again use Newton's 2nd law.

$$\sum F = F_{\text{T}} - m_{\text{F}}g = m_{\text{F}}a \quad \rightarrow$$

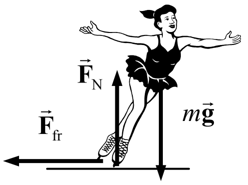
$$F_{\text{T}} = m_{\text{F}}(g+a) = (1250\text{ kg})\left(9.80\,\text{m/s}^2+0.80\,\text{m/s}^2\right) = \boxed{1.33\times10^4\,\text{N}}$$

(*c*) The tension in the cable is the same at both ends, and so the cable exerts a force of $\boxed{1.33\times10^4\,\text{N}}$ downward on the helicopter.

82. (*a*) We assume that the maximum horizontal force occurs when the train is moving very slowly, and so the air resistance is negligible. Thus the maximum acceleration is given by

$$a_{\text{max}} = \frac{F_{\text{max}}}{m} = \frac{4.0\times10^5\,\text{N}}{6.6\times10^5\,\text{kg}} = \boxed{0.61\,\text{m/s}^2}.$$

(*b*) At top speed, we assume that the train is moving at constant velocity. Therefore the net force on the train is 0, and so the air resistance must be of the same magnitude as the horizontal pushing force, which is $\boxed{1.5\times10^5\,\text{N}}$.



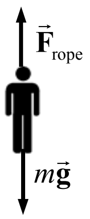
83. Consider the free-body diagram for the decelerating skater, moving to the right. It is apparent that $F_{\text{N}} = mg$ since there is no acceleration in the vertical direction. From Newton's 2nd law in the horizontal direction, we have

$$\sum F = F_{\text{fr}} = ma \quad \rightarrow \quad -\mu_k mg = ma \quad \rightarrow \quad a = -\mu_k g$$

Now use Eq. 2-11c to find the starting speed.

$$v^2 - v_0^2 = 2a(x-x_0) \quad \rightarrow$$

$$v_0 = \sqrt{v^2 - 2a(x-x_0)} = \sqrt{0 + 2\mu_k g(x-x_0)} = \sqrt{2(0.10)\left(9.8\,\text{m/s}^2\right)(75\text{ m})} = \boxed{12\,\text{m/s}}$$



84. First calculate Karen's speed from falling. Let the downward direction be positive, and use Eq. 2-11c with $v_0 = 0$.

$$v^2 - v_0^2 = 2a(y-y_0) \quad \rightarrow \quad v = \sqrt{0 + 2a(y-y_0)} = \sqrt{2\left(9.8\,\text{m/s}^2\right)(2.0\text{ m})} = 6.26\,\text{m/s}$$

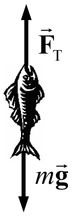
Now calculate the average acceleration as the rope stops Karen, again using Eq. 2-11c, with down as positive.

$$v^2 - v_0^2 = 2a(y-y_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(y-y_0)} = \frac{0-(6.26\,\text{m/s})^2}{2(1.0\text{ m})} = -19.6\,\text{m/s}^2$$

The negative sign indicates that the acceleration is upward. Since this is her acceleration, the net force on Karen is given by Newton's 2nd law, $F_{\text{net}} = ma$. That net force will also be upward. Now consider the free-body diagram shown of Karen as she decelerates. Call DOWN the positive direction, and Newton's 2nd law says that $F_{\text{net}} = ma = mg - F_{\text{rope}} \rightarrow F_{\text{rope}} = mg - ma$. The ratio of this force to Karen's weight would be $\dfrac{F_{\text{rope}}}{mg} = \dfrac{mg - ma}{g} = 1.0 - \dfrac{a}{g} = 1.0 - \dfrac{-19.6\,\text{m/s}^2}{9.8\,\text{m/s}^2} = 3.0$. Thus the rope pulls upward on Karen with an average force of $\boxed{\text{3.0 times her weight}}$.

A completely analogous calculation for Bill gives the same speed after the 2.0 m fall, but since he stops over a distance of 0.30 m, his acceleration is –65 m/s^2, and the rope pulls upward on Bill with an average force of $\boxed{\text{7.7 times his weight}}$. Thus $\boxed{\text{Bill is more likely to get hurt}}$ in the fall.

85. See the free-body diagram for the fish being pulled upward vertically. From Newton's 2nd law, calling the upward direction positive, we have
$\sum F_y = F_{\text{T}} - mg = ma \quad \rightarrow \quad F_{\text{T}} = m(g + a)$



(*a*) If the fish has a constant speed, then its acceleration is zero, and so $F_{\text{T}} = mg$. Thus the heaviest fish that could be pulled from the water in this case is $\boxed{45\text{ N }(10\text{ lb})}$.

(*b*) If the fish has an acceleration of 2.0 m/s^2, and F_{T} is at its maximum of 45 N, then solve the equation for the mass of the fish.

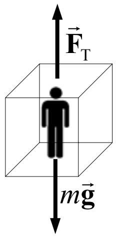
$$m = \frac{F_{\text{T}}}{g + a} = \frac{45\text{ N}}{9.8\,\text{m/s}^2 + 2.0\,\text{m/s}^2} = 3.8\text{ kg} \quad \rightarrow$$

$$mg = (3.8\text{ kg})(9.8\,\text{m/s}^2) = \boxed{37\text{ N }(\approx 8.4\text{ lb})}$$

(*c*) It is not possible to land a 15-lb fish using 10-lb line, if you have to lift the fish vertically. If the fish were reeled in while still in the water, and then a net used to remove the fish from the water, it might still be caught with the 10-lb line.

86. Choose downward to be positive. The elevator's acceleration is calculated by Eq. 2-11c.

$$v^2 - v_0^2 = 2a(y - y_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(y - y_0)} = \frac{0 - (3.5\,\text{m/s})^2}{2(2.6\text{ m})} = -2.356\,\text{m/s}^2$$

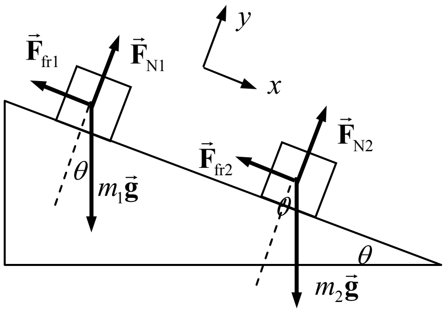


See the free-body diagram of the elevator. Write Newton's 2nd law for the elevator.

$$\sum F_y = mg - F_{\text{T}} = ma$$

$$F_{\text{T}} = m(g - a) = (1300\text{ kg})(9.80\,\text{m/s}^2 - -2.356\,\text{m/s}^2) = \boxed{1.58 \times 10^4\,\text{N}}$$

87. (*a*) Draw a free-body diagram for each block, with no connecting tension. Because of the similarity of the free-body diagrams, we shall just analyze block 1. Write Newton's 2nd law in both the *x* and *y* directions. The net force in the *y* direction is zero, because there is no acceleration in the *y* direction.


y
x
$\vec{\mathbf{F}}_{\text{fr1}}$
$\vec{\mathbf{F}}_{\text{N1}}$
θ
$m_1\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{fr2}}$
$\vec{\mathbf{F}}_{\text{N2}}$
θ
θ
$m_2\vec{\mathbf{g}}$

$$\sum F_{y1} = F_{\text{N1}} - m_1 g\cos\theta = 0 \quad \rightarrow \quad F_{\text{N1}} = m_1 g\cos\theta$$

$$\sum F_{x1} = m_1 g\sin\theta - F_{\text{fr1}} = m_1 a_1 \quad \rightarrow$$

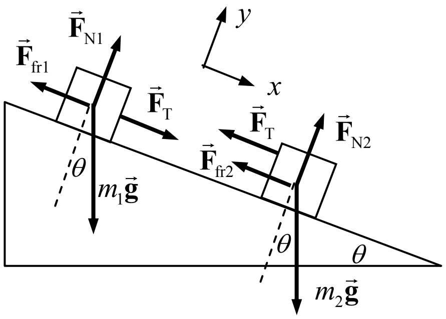
$$a_1 = \frac{m_1 g \sin\theta - F_{\text{fr1}}}{m_1} = \frac{m_1 g \sin\theta - \mu_k m_1 g \cos\theta}{m_1} = g\left(\sin\theta - \mu_1 \cos\theta\right)$$
$$= \left(9.8\,\text{m/s}^2\right)\left(\sin 30^\circ - 0.10\cos 30^\circ\right) = \boxed{4.1\,\text{m/s}^2}$$

The same analysis for block 2 would give

$$a_2 = g\left(\sin\theta - \mu_2 \cos\theta\right) = \left(9.8\,\text{m/s}^2\right)\left(\sin 30^\circ - 0.20\cos 30^\circ\right) = \boxed{3.2\,\text{m/s}^2}$$

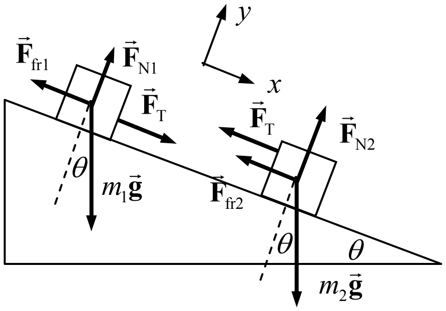
Since $a_2 < a_1$, if both blocks were released from rest, block # 1 would "gain" on block 2.

(*b*) Now let the rope tension be present. Before writing equations, consider that without the tension, $a_1 > a_2$. In the free-body diagram shown, m_1 now has even more force accelerating down the plane because of the addition of the tension force, which means m_1 has an even larger acceleration than before. And m_2 has less force accelerating it down the plane because of the addition of the tension force. Thus m_2 has a smaller acceleration than before. And so in any amount of time considered, m_1 will move more distance down the plane upon release than block m_2. The cord will go slack almost immediately after the blocks are released, and the blocks revert to the original free-body diagram. The conclusion is that the accelerations in this part of the problem are the same as they would be in part (*a*):



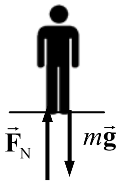
$$\boxed{a_1 = 4.1\,\text{m/s}^2 \;;\; a_2 = 3.2\,\text{m/s}^2}\,.$$

(*c*) Now reverse the position of the masses. Write Newton's 2nd law for each block, and assume that they have the same acceleration. The *y* equations and frictional forces are unchanged from the analysis in part (*a*), so we only write the *x* equations. After writing them, add them together (to eliminate the tension) and solve for the acceleration.



$$\sum F_{x1} = m_1 g \sin\theta - F_{\text{fr1}} - F_{\text{T}} = m_1 a$$
$$\sum F_{x2} = m_2 g \sin\theta - F_{\text{fr2}} + F_{\text{T}} = m_2 a$$
$$\left(m_1 + m_2\right) g \sin\theta - F_{\text{fr1}} - F_{\text{fr2}} = \left(m_1 + m_2\right) a$$
$$a = \frac{\left(m_1 + m_2\right) g \sin\theta - \mu_1 m_1 g \cos\theta - \mu_2 m_2 g \cos\theta}{\left(m_1 + m_2\right)} = g\left[\sin\theta - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}\cos\theta\right]$$
$$= \left(9.8\,\text{m/s}^2\right)\left[\sin 30^\circ - \frac{(0.10)(1.0\text{ kg}) + (0.20)(2.0\text{ kg})}{3.0\text{ kg}}\cos 30^\circ\right] = \boxed{3.5\,\text{m/s}^2}$$

88. See the free-body diagram of the person in the elevator. The scale will read the normal force. Choose upward to be positive. From Newton's 2nd law,



$$\sum F = F_{\text{N}} - mg = ma \quad\rightarrow\quad F_{\text{N}} = m\left(g + a\right)$$

(*a*, *b*, *c*) If the elevator is either at rest or moving with a constant vertical speed, either up or down, the acceleration is zero, and so

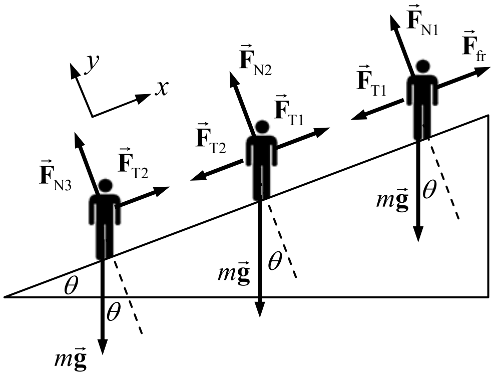
$$F_N = mg = (75.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{7.35\times10^2\text{ N}} \quad m = \frac{F_N}{g} = \boxed{75.0\text{ kg}}$$

(*d*) When accelerating upward, the acceleration is $+3.0\text{ m/s}^2$, and so

$$F_N = m(g+a) = (75.0\text{ kg})(12.8\text{ m/s}^2) = \boxed{9.60\times10^2\text{ N}} \quad m = \frac{F_N}{g} = \boxed{98.0\text{ kg}}$$

(*e*) When accelerating downward, the acceleration is -3.0 m/s^2, and so

$$F_N = m(g+a) = (75.0\text{ kg})(6.80\text{ m/s}^2) = \boxed{5.10\times10^2\text{ N}} \quad m = \frac{F_N}{g} = \boxed{52.0\text{ kg}}$$



89. Since the climbers are on ice, the frictional force for the lower two climbers is negligible. Consider the free-body diagram as shown. Note that all the masses are the same. Write Newton's 2nd law in the *x* direction for the lowest climber, assuming he is at rest.

$$\sum F_x = F_{T2} - mg\sin\theta = 0$$

$$F_{T2} = mg\sin\theta = (75\text{ kg})(9.8\text{ m/s}^2)\sin 21.0^\circ$$

$$= \boxed{2.6\times10^2\text{ N}}$$

Write Newton's 2nd law in the *x* direction for the middle climber, assuming he is at rest.

$$\sum F_x = F_{T1} - F_{T2} - mg\sin\theta = 0 \quad\rightarrow\quad F_{T1} = F_{T2} + mg\sin\theta = 2F_{T2} = \boxed{5.3\times10^2\text{ N}}$$

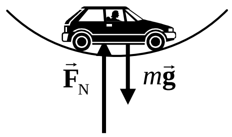
CHAPTER 5: Circular Motion; Gravitation

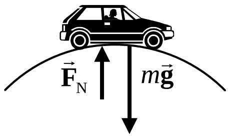
Answers to Questions

1. The problem with the statement is that there is nothing to cause an outward force, and so the water removed from the clothes is not thrown outward. Rather, the spinning drum pushes INWARD on the clothes and water. But where there are holes in the drum, the drum can't push on the water, and so the water is not pushed in. Instead, the water moves tangentially to the rotation, out the holes, in a straight line, and so the water is separated from the clothes.

2. The centripetal acceleration for an object moving in circular motion is inversely proportional to the radius of the curve, given a constant speed $\left(a = v^2/r\right)$. So for a gentle curve (which means a large radius), the acceleration is smaller, while for a sharp curve (which means a small radius), the acceleration is larger.

3. The force that the car exerts on the road is the Newton's 3rd law reaction to the normal force of the road on the car, and so we can answer this question in terms of the normal force. The car exerts the greatest force on the road at the dip between two hills. There the normal force from the road has to both support the weight AND provide a centripetal upward force to make the car move in an upward curved path. The car exerts the least force on the road at the top of a hill. We have all felt the "floating upward" sensation as we have driven over the crest of a hill. In that case, there must be a net downward centripetal force to cause the circular motion, and so the normal force from the road does not completely support the weight.





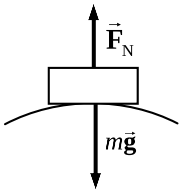
4. There are at least three distinct major forces on the child. The force of gravity is acting downward on the child. There is a normal force from the seat of the horse acting upward on the child. There must be friction between the seat of the horse and the child as well, or the child could not be accelerated by the horse. It is that friction that provides the centripetal acceleration. There may be smaller forces as well, such as a reaction force on the child's hands if the child is holding on to part of the horse. Any force that has a radially inward component will contribute to the centripetal acceleration.

5. For the water to remain in the bucket, there must be a centripetal force forcing the water to move in a circle along with the bucket. That centripetal force gets larger with the tangential velocity of the water, since $F_R = mv^2/r$. The centripetal force at the top of the motion comes from a combination of the downward force of gravity and the downward normal force of the bucket on the water. If the bucket is moving faster than some minimum speed, the water will stay in the bucket. If the bucket is moving too slow, there is insufficient force to keep the water moving in the circular path, and it spills out.

6. The three major "accelerators" are the accelerator pedal, the brake pedal, and the steering wheel. The accelerator pedal (or gas pedal) can be used to increase speed (by depressing the pedal) or to decrease speed in combination with friction (by releasing the pedal). The brake pedal can be used to decrease speed by depressing it. The steering wheel is used to change direction, which also is an acceleration. There are some other controls which could also be considered accelerators. The parking brake can be used to decrease speed by depressing it. The gear shift lever can be used to decrease speed by downshifting. If the car has a manual transmission, then the clutch can be used to

decrease speed by depressing it (friction will slow the car). Finally, shutting the car off can be used to decrease its speed. Any change in speed or direction means that an object is accelerating.

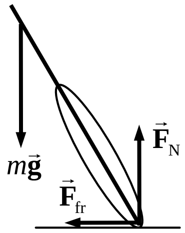
7. When the child is on a level surface, the normal force between his chest and the sled is equal to the child's weight, and thus he has no vertical acceleration. When he goes over the hill, the normal force on him will be reduced. Since the child is moving on a curved path, there must be a net centripetal force towards the center of the path, and so the normal force does not completely support the weight. Write Newton's 2nd law for the radial direction, with inward as positive.



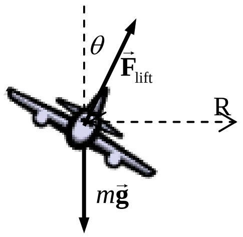
$$\sum F_{\text{R}} = mg - F_{\text{N}} = mv^2/r \quad \rightarrow \quad F_{\text{N}} = mg - mv^2/r$$

We see that the normal force is reduced from *mg* by the centripetal force.

8. When a bicycle rider leans inward, the bike tire pushes down on the ground at an angle. The road surface then pushes back on the tire both vertically (to provide the normal force which counteracts gravity) and horizontally toward the center of the curve (to provide the centripetal frictional force, enabling them to turn).

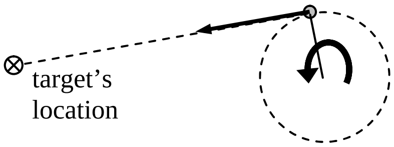


9. Airplanes bank when they turn because in order to turn, there must be a force that will be exerted towards the center of a circle. By tilting the wings, the lift force on the wings has a non-vertical component which points toward the center of the curve, providing the centripetal force. The banking angle can be computed from the free-body diagram. The sum of vertical forces must be zero for the plane to execute a level turn, and so $F_{\text{lift}}\cos\theta = mg$. The horizontal component of the lifting force must provide the centripetal force to move the airplane in a circle.



$$F_{\text{lift}}\sin\theta = mv^2/r \quad \rightarrow \quad \frac{mg}{\cos\theta}\sin\theta = mv^2/r \quad \rightarrow \quad \tan\theta = \frac{v^2}{Rg}$$

10. She should let go of the string when the ball is at a position where the tangent line to the circle at the ball's location, when extended, passes through the target's position. That tangent line indicates the direction of the velocity at that instant, and if the centripetal force is removed, then the ball will follow that line horizontally. See the top-view diagram.



$\boxed{11.}$ The apple does exert a gravitational force on the Earth. By Newton's 3rd law, the force on the Earth due to the apple is the same magnitude as the force on the apple due to the Earth – the weight of the apple. The force is also independent of the state of motion of the apple. So for both a hanging apple and a falling apple, the force on the Earth due to the apple is equal to the weight of the apple.

12. The gravitational force on the Moon is given by $G\frac{M_{\text{Earth}}M_{\text{Moon}}}{R^2}$, where *R* is the radius of the Moon's orbit. This is a radial force, and so can be expressed as $M_{\text{Moon}}\,v_{\text{Moon}}^2/R$. This can be changed using the relationship $v_{\text{Moon}} = 2\pi R/T$, where *T* is the orbital period of the Moon, to $4\pi^2 M_{\text{Moon}}\,R/T^2$. If we equate these two expressions for the force, we get the following:

$G\frac{M_{\text{Earth}}M_{\text{Moon}}}{R^2} = 4\pi^2 M_{\text{Moon}} R/T^2 \quad \rightarrow \quad \frac{R^3}{T^2} = \frac{GM_{\text{Earth}}}{4\pi^2}$. Thus the mass of the Earth determines the ratio R^3/T^2. If the mass of the Earth were doubled, then the ratio R^3/T^2 would double, and so $R'^3/T'^2 = R^3/T^2$, where the primes indicated the "after doubling" conditions. For example, the radius might stay the same, and the period decrease by a factor of $\sqrt{2}$, which means the speed increased by a factor of $\sqrt{2}$. Or the period might stay the same, and the radius increase by a factor of $2^{1/3}$, which means the speed increased by the same factor of $2^{1/3}$. Or if both R and T were to double, keeping the speed constant, then R^3/T^2 would double. There are an infinite number of other combinations that would also satisfy the doubling of R^3/T^2.

13. The gravitational pull is the same in each case, by Newton's 3rd law. The magnitude of that pull is given by $F = G\frac{M_{\text{Earth}}M_{\text{Moon}}}{r^2_{\text{Earth-Moon}}}$. To find the acceleration of each body, the gravitational pulling force is divided by the mass of the body. Since the Moon has the smaller mass, it will have the larger acceleration.

14. The difference in force on the two sides of the Earth from the gravitational pull of either the Sun or the Moon is the primary cause of the tides. That difference in force comes about from the fact that the two sides of the Earth are a different distance away from the pulling body. Relative to the Sun, the difference in distance (Earth diameter) of the two sides from the Sun, relative to the average distance to the Sun, is given by $2R_{\text{Earth}}\Big/R_{\substack{\text{Earth}\\\text{to Sun}}} = 8.5\times10^{-5}$. The corresponding relationship between the Earth and the Moon is $2R_{\text{Earth}}\Big/R_{\substack{\text{Earth}\\\text{to Moon}}} = 3.3\times10^{-2}$. Since the relative change in distance is much greater for the Earth-Moon combination, we see that the Moon is the primary cause of the Earth's tides.

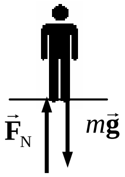
15. An object weighs more at the poles, due to two effects which complement (not oppose) each other. First of all, the Earth is slightly flattened at the poles and expanded at the equator, relative to a perfect sphere. Thus the mass at the poles is slightly closer to the center, and so experiences a slightly larger gravitational force. Secondly, objects at the equator have a centripetal acceleration due to the rotation of the Earth that objects at the poles do not have. To provide that centripetal acceleration, the apparent weight (the radially outward normal force of the Earth on an object) is slightly less than the gravitational pull inward. So the two effects both make the weight of an object at the equator less than that at the poles.

16. The Moon is not pulled away from the Earth because both the Moon and the Earth are experiencing the same radial acceleration due to the Sun. They both have the same period around the Sun because they are both, on average, the same distance from the Sun, and so they travel around the Sun together.

17. The centripetal acceleration of Mars is smaller than that of Earth. The acceleration of each planet can be found by dividing the gravitational force on each planet by the planet's mass. The resulting acceleration is inversely proportional to the square of the distance of the planet from the Sun. Since Mars is further from the Sun than the Earth is, the acceleration of Mars will be smaller. Also see the equation below.

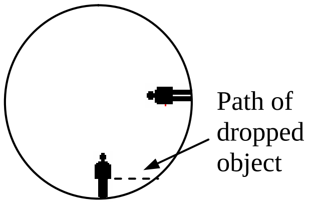
$$F_{\text{on planet}} = G\frac{M_{\text{sun}}M_{\text{planet}}}{r^2_{\text{Sun to planet}}} \qquad a_{\text{planet}} = \frac{F_{\text{on planet}}}{M_{\text{planet}}} = G\frac{M_{\text{sun}}}{r^2_{\text{Sun to planet}}}$$

18. In order to orbit, a satellite must reach an orbital speed relative to the center of the Earth. Since the satellite is already moving eastward when launched (due to the rotation speed at the surface of the Earth), it requires less additional speed to launch it east to obtain the final orbital speed.



19. The apparent weight (the normal force) would be largest when the elevator is accelerating upward. From the free-body diagram, with up as positive, we have $F_N - mg = ma \;\rightarrow\; F_N = m(g+a)$. With a positive acceleration, the normal force is greater than your weight. The apparent weight would be the least when in free fall, because there the apparent weight is zero, since $a = -g$. When the elevator is moving with constant speed, your apparent weight would be the same as it is on the ground, since $a = 0$ and so $F_N = mg$.

20. A satellite remains in orbit due to the combination of gravitational force on the satellite directed towards the center of the orbit and the tangential speed of the satellite. First, the proper tangential speed had to be established by some other force than the gravitational force. Then, if the satellite has the proper combination of speed and radius such that the force required for circular motion is equal to the force of gravity on the satellite, then the satellite will maintain circular motion.



21. The passengers, as seen in the diagram, are standing on the floor.
   (*a*) If a passenger held an object beside their waist and then released it, the object would move in a straight line, tangential to the circle in which the passenger's waist was moving when the object was released. In the figure, we see that the released object would hit the rotating shell, and so fall to the floor, but behind the person. The passenger might try to explain such motion by inventing some kind of "retarding" force on dropped objects, when really there is no such force.
   (*b*) The floor exerts a centripetal force on the feet, pushing them towards the center. This force has the same direction ("upwards", away from the floor) that a passenger would experience on Earth, and so it seems to the passenger that gravity must be pulling them "down". Actually, the passengers are pushing down on the floor, because the floor is pushing up on them.
   (*c*) The "normal" way of playing catch, for example, would have to change. Since the artificial gravity is not uniform, passengers would have to re-learn how to throw something across the room to each other. There would not be projectile motion as we experience it on Earth. Also, if the cylinder were small, there might be a noticeable difference in the acceleration of our head vs. our feet. Since the head is closer to the center of the circle than the feet, and both the head and the feet have the same period of rotation, the centripetal acceleration $\left(a_R = 4\pi^2 r/T^2\right)$ is smaller for the head. This might cause dizziness or a light-headed feeling.

22. When the runner has both feet off the ground, the only force on the runner is gravity – there is no normal force from the ground on the runner. This lack of normal force is interpreted as "free fall" and "apparent weightlessness".

23. By Kepler's 2nd law, the Earth moves faster around the Sun when it is nearest the Sun. Kepler's 2nd law says that an imaginary line drawn from the Sun to the Earth sweeps out equal areas in equal

times. So when the Earth is close to the Sun, it must move faster to sweep out a given area than when the Earth is far from the Sun. Thus the Earth is closer to the Sun in January.

24. Let the mass of Pluto be M, the mass of the moon be m, the radius of the moon's orbit be R, and the period of the moon's orbit be T. Then Newton's second law for the moon orbiting Pluto will be $F = \frac{GmM}{R^2}$. If that moon's orbit is a circle, then the form of the force must be centripetal, and so $F = mv^2/R$. Equate these two expressions for the force on the moon, and substitute the relationship for a circular orbit that $v = 2\pi R/T$.

$$\frac{GmM}{R^2} = \frac{mv^2}{R} = \frac{4\pi^2 mR}{T^2} \quad \rightarrow \quad M = \frac{4\pi^2 R^3}{GT^2}.$$

Thus a value for the mass of Pluto can be calculated knowing the period and radius of the moon's orbit.

Solutions to Problems

1. (*a*) Find the centripetal acceleration from Eq. 5-1.

$$a_R = v^2/r = (1.25\,\mathrm{m/s})^2/1.10\ \mathrm{m} = \boxed{1.42\,\mathrm{m/s^2}}$$

(*b*) The net horizontal force is causing the centripetal motion, and so will be the centripetal force.

$$F_R = ma_R = (25.0\ \mathrm{kg})(1.42\,\mathrm{m/s^2}) = \boxed{35.5\ \mathrm{N}}$$

2. Find the centripetal acceleration from Eq. 5-1.

$$a_R = v^2/r = \frac{(525\,\mathrm{m/s})^2}{6.00\times10^3\,\mathrm{m}} = (45.94\,\mathrm{m/s^2})\left(\frac{1\,g}{9.80\,\mathrm{m/s^2}}\right) = \boxed{4.69\,g\text{'s}}$$

3. The centripetal acceleration is $a_R = v^2/R_{\substack{\text{Earth}\\\text{orbit}}} = \frac{\left(2\pi R_{\substack{\text{Earth}\\\text{orbit}}}/T\right)^2}{R_{\substack{\text{Earth}\\\text{orbit}}}} = \frac{4\pi^2 R_{\substack{\text{Earth}\\\text{orbit}}}}{T^2}$. The force (from Newton's 2nd law) is $F_R = m_{\text{Earth}} a_R$. The period is one year, converted into seconds.

$$a_R = \frac{4\pi^2 R_{\substack{\text{Earth}\\\text{orbit}}}}{T^2} = \frac{4\pi^2(1.50\times10^{11}\,\mathrm{m})}{(3.15\times10^7\ \mathrm{sec})^2} = \boxed{5.97\times10^{-3}\ \mathrm{m/s^2}}$$

$$F_R = ma = (5.97\times10^{24}\,\mathrm{kg})(5.97\times10^{-3}\ \mathrm{m/s^2}) = \boxed{3.56\times10^{22}\ \mathrm{N}}$$

$\boxed{\text{The Sun}}$ exerts this force on the Earth. It is a gravitational force.

4. The speed can be found from the centripetal force and centripetal acceleration.

$$F_R = ma_R = mv^2/r \quad \rightarrow \quad v = \sqrt{\frac{F_R r}{m}} = \sqrt{\frac{(210\ \mathrm{N})(0.90\ \mathrm{m})}{2.0\ \mathrm{kg}}} = \boxed{9.7\,\mathrm{m/s}}$$

5. The orbit radius will be the sum of the Earth's radius plus the 400 km orbit height. The orbital period is about 90 minutes. Find the centripetal acceleration from these data.

$$r = 6380\,\text{km} + 400\,\text{km} = 6780\,\text{km} = 6.78\times10^{6}\,\text{m} \qquad T = 90\ \text{min}\left(\frac{60\ \text{sec}}{1\ \text{min}}\right) = 5400\ \text{sec}$$

$$a_{\text{R}} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(6.78\times10^{6}\,\text{m}\right)}{\left(5400\ \text{sec}\right)^2} = \left(9.18\,\text{m/s}^2\right)\left(\frac{1\,g}{9.80\,\text{m/s}^2}\right) = 0.937 \approx \boxed{0.9\,g\text{'s}}$$

Notice how close this is to g, because the shuttle is not very far above the surface of the Earth, relative to the radius of the Earth.

6. To find the period, the rotational speed (in rev/min) is reciprocated to have min/rev, and then converted to sec/rev. Use the period to find the speed, and then the centripetal acceleration.

$$T = \left(\frac{1\ \text{min}}{45\ \text{rev}}\right)\left(\frac{60\ \text{sec}}{1\ \text{min}}\right) = 1.333\frac{\text{sec}}{\text{rev}} \qquad r = 0.16\ \text{m} \qquad v = \frac{2\pi r}{T} = \frac{2\pi\left(0.16\ \text{m}\right)}{1.333\ \text{sec}} = 0.754\,\text{m/s}$$

$$a_{\text{R}} = v^2/r = \frac{\left(0.754\,\text{m/s}\right)^2}{0.16\ \text{m}} = \boxed{3.6\,\text{m/s}^2}$$

$\boxed{7.}$ See the free-body diagram in the textbook. Since the object is moving in a circle with a constant speed, the net force on the object at any point must point to the center of the circle.

(*a*) Take positive to be downward. Write Newton's 2nd law in the downward direction.

$$\sum F_{\text{R}} = mg + F_{\text{T1}} = ma_{\text{R}} = mv^2/r \quad \rightarrow$$

$$F_{\text{T1}} = m\left(v^2/r - g\right) = \left(0.300\ \text{kg}\right)\left(\frac{\left(4.00\,\text{m/s}\right)^2}{0.720\ \text{m}} - 9.80\,\text{m/s}^2\right) = \boxed{3.73\ \text{N}}$$

This is a downward force, as expected.

(*b*) Take positive to be upward. Write Newton's 2nd law in the upward direction.

$$\sum F_{R} = F_{\text{T2}} - mg = ma = mv^2/r \quad \rightarrow$$

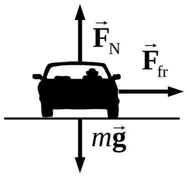
$$F_{\text{T1}} = m\left(v^2/r + g\right) = \left(0.300\ \text{kg}\right)\left(\frac{\left(4.00\,\text{m/s}\right)^2}{0.720\ \text{m}} + 9.80\,\text{m/s}^2\right) = \boxed{9.61\ \text{N}}$$

This is an upward force, as expected.

8. The centripetal force that the tension provides is given by $F_{\text{R}} = mv^2/r$. Solve that for the speed.

$$v = \sqrt{\frac{F_{\text{R}}r}{\text{m}}} = \sqrt{\frac{\left(75\ \text{N}\right)\left(1.3\ \text{m}\right)}{0.45\,\text{kg}}} = \boxed{15\,\text{m/s}}$$

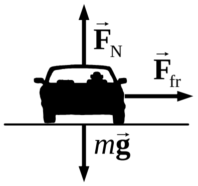
9. A free-body diagram for the car at one instant of time is shown. In the diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion.



$$F_{\text{R}} = F_{\text{fr}} \quad \rightarrow \quad mv^2/r = \mu_s F_{\text{N}} = \mu_s mg \quad \rightarrow \quad v = \sqrt{\mu_s rg} = \sqrt{\left(0.80\right)\left(77\ \text{m}\right)\left(9.8\,\text{m/s}^2\right)} = \boxed{25\,\text{m/s}}$$

Notice that the result is $\boxed{\text{independent of the car's mass}}$.

10. In the free-body diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value.



$$F_R = F_{\text{fr}} \rightarrow mv^2/r = \mu_s F_{\text{N}} = \mu_s mg \rightarrow \mu_s = \frac{v^2}{rg} = \frac{\left[(95\,\text{km/hr})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/hr}}\right)\right]^2}{(85\text{ m})(9.8\,\text{m/s}^2)} = \boxed{0.84}$$

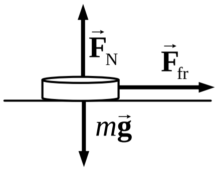
Notice that the result is independent of the car's mass.

11. Since the motion is all in a horizontal circle, gravity has no influence on the analysis. Set the general expression for centripetal force equal to the stated force in the problem.

$$F_{\text{R}} = mv^2/r = 7.85W = 7.85mg \rightarrow v = \sqrt{7.85rg} = \sqrt{7.85(12.0\text{m})(9.8\,\text{m/s}^2)} = \boxed{30.4\,\text{m/s}}$$

$$(30.4\,\text{m/s})\left(\frac{1\text{ rev}}{2\pi(12.0\text{ m})}\right) = \boxed{0.403\,\text{rev/s}}$$

12. The force of static friction is causing the circular motion – it is the centripetal force. The coin slides off when the static frictional force is not large enough to move the coin in a circle. The maximum static frictional force is the coefficient of static friction times the normal force, and the normal force is equal to the weight of the coin as seen in the free-body diagram, since there is no vertical acceleration. In the free-body diagram, the coin is coming out of the paper and the center of the circle is to the right of the coin, in the plane of the paper.

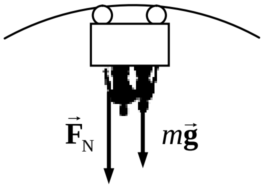


The rotational speed must be changed into a linear speed.

$$v = \left(36\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi(0.11\text{m})}{1\text{ rev}}\right) = 0.4147\,\text{m/s}$$

$$F_{\text{R}} = F_{\text{fr}} \rightarrow mv^2/r = \mu_s F_{\text{N}} = \mu_s mg \rightarrow \mu_s = \frac{v^2}{rg} = \frac{(0.4147\,\text{m/s})^2}{(0.11\text{ m})(9.8\,\text{m/s}^2)} = \boxed{0.16}$$

13. At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's 2nd law for the passengers.

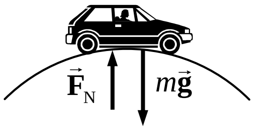


$$\sum F = F_{\text{N}} + mg = ma = mv^2/r \rightarrow F_{\text{N}} = m\left(v^2/r - g\right)$$

We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car – they are in free fall. The limiting condition is

$$v_{\text{min}}^2/r - g = 0 \rightarrow v_{\text{min}} = \sqrt{rg} = \sqrt{(9.8\,\text{m/s}^2)(7.4\text{ m})} = \boxed{8.5\,\text{m/s}}$$

14. (*a*) A free-body diagram of the car at the instant it is on the top of the hill is shown. Since the car is moving in a circular path, there must be a net centripetal force downward. Write Newton's 2nd law for the car, with down as the positive direction.



$$\sum F_R = mg - F_N = ma = mv^2/r \quad \rightarrow$$

$$F_N = m\left(g - v^2/r\right) = (950\text{ kg})\left(9.8\text{ m/s}^2 - \frac{(22\text{ m/s})^2}{95\text{ m}}\right) = \boxed{4.5\times10^3\text{ N}}$$

(*b*) The free-body diagram for the passengers would be the same as the one for the car, leading to the same equation for the normal force on the passengers.

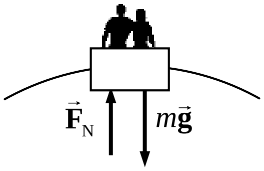
$$F_N = m\left(g - v^2/r\right) = (72\text{ kg})\left(9.8\text{ m/s}^2 - \frac{(22\text{ m/s})^2}{95\text{ m}}\right) = \boxed{3.4\times10^2\text{ N}}$$

Notice that this is significantly less than the 700-N weight of the passenger. Thus the passenger will feel "light" as they drive over the hill.

(*c*) For the normal force to be zero, we see that we must have

$$F_N = m\left(g - v^2/r\right) = 0 \quad \rightarrow \quad g = v^2/r \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{\left(9.8\text{ m/s}^2\right)(95\text{ m})} = \boxed{31\text{ m/s}}.$$

$\boxed{15.}$ The free-body diagram for passengers at the top of a Ferris wheel is as shown. F_N is the normal force of the seat pushing up on the passenger. The sum of the forces on the passenger is producing the centripetal motion, and so must be a centripetal force. Call the downward direction positive. Newton's 2nd law for the passenger is:



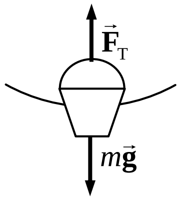
$$\sum F_R = mg - F_N = ma = mv^2/r$$

Since the passenger is to feel "weightless", they must lose contact with their seat, and so the normal force will be 0.

$$mg = mv^2/r \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{\left(9.8\text{ m/s}^2\right)(7.5\text{ m})} = 8.6\text{ m/s}$$

$$\left(8.6\frac{\text{m}}{\text{s}}\right)\left(\frac{1\text{ rev}}{2\pi(7.5\text{ m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{11\text{ rpm}}$$

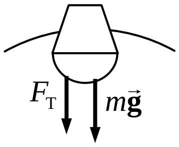
16. (*a*) At the bottom of the motion, a free-body diagram of the bucket would be as shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's 2nd law for the bucket, with up as the positive direction.



$$\sum F_R = F_T - mg = ma = mv^2/r \quad \rightarrow$$

$$v = \sqrt{\frac{r\left(F_T - mg\right)}{m}} = \sqrt{\frac{(1.10\text{ m})\left[25.0\text{ N} - (2.00\text{ kg})\left(9.80\text{ m/s}^2\right)\right]}{2.00\text{ kg}}} = 1.723 \approx \boxed{1.7\text{ m/s}}$$

(*b*) A free-body diagram of the bucket at the top of the motion is shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's 2nd law for the bucket, with down as the positive direction.



$$\sum F_R = F_T + mg = ma = mv^2/r \quad \rightarrow \quad v = \sqrt{\frac{r\left(F_T + mg\right)}{m}}$$

If the tension is to be zero, then

$$v=\sqrt{\frac{r\left(0+mg\right)}{m}}=\sqrt{rg}=\sqrt{\left(1.1\text{ m}\right)\left(9.8\,\text{m}/\text{s}^2\right)}=\boxed{3.3\,\text{m}/\text{s}}$$

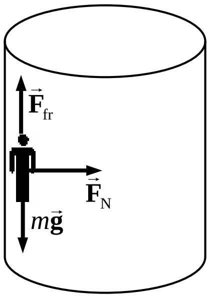
The bucket must move faster than 3.3 m/s in order for the rope not to go slack.

17. The centripetal acceleration of a rotating object is given by $a_\text{R}=v^2/r$. Thus

$$v=\sqrt{a_\text{R}r}=\sqrt{\left(1.15\times10^5g\right)r}=\sqrt{\left(1.15\times10^5\right)\left(9.80\,\text{m}/\text{s}^2\right)\left(9.00\times10^{-2}\,\text{m}\right)}=3.18\times10^2\text{ m}/\text{s}.$$

$$\left(3.18\times10^2\text{ m}/\text{s}\right)\left(\frac{1\text{ rev}}{2\pi\left(9.00\times10^{-2}\text{m}\right)}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)=\boxed{3.38\times10^4\,\text{rpm}}.$$

18. Consider the free-body diagram for a person in the "Rotor-ride". $\vec{\mathbf{F}}_\text{N}$ is the normal force of contact between the rider and the wall, and $\vec{\mathbf{F}}_\text{fr}$ is the static frictional force between the back of the rider and the wall. Write Newton's 2nd law for the vertical forces, noting that there is no vertical acceleration.



$$\sum F_y=F_\text{fr}-mg=0\quad\rightarrow\quad F_\text{fr}=mg$$

If we assume that the static friction force is a maximum, then

$$F_\text{fr}=\mu_sF_\text{N}=mg\quad\rightarrow\quad F_\text{N}=mg/\mu_s.$$

But the normal force must be the force causing the centripetal motion – it is the only force pointing to the center of rotation. Thus $F_\text{R}=F_\text{N}=mv^2/r$. Using $v=2\pi r/T$, we have $F_N=\dfrac{4\pi^2mr}{T^2}$. Equate the two expressions for the normal force and solve for the coefficient of friction. Note that since there are 0.5 rev per sec, the period is 2.0 sec.

$$F_N=\frac{4\pi^2mr}{T^2}=\frac{mg}{\mu_s}\quad\rightarrow\quad\mu_s=\frac{gT^2}{4\pi^2r}=\frac{\left(9.8\,\text{m}/\text{s}^2\right)\left(2\text{ s}\right)^2}{4\pi^2\left(4.6\text{ m}\right)}=\boxed{0.22}.$$

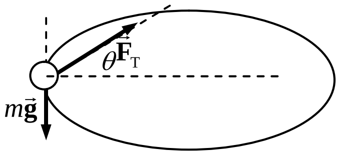
Any larger value of the coefficient of friction would mean that the normal force could be smaller to achieve the same frictional force, and so the period could be longer or the cylinder smaller.

There is no force pushing outward on the riders. Rather, the wall pushes against the riders, so by Newton's 3rd law the riders push against the wall. This gives the sensation of being pressed into the wall.

19. Since mass *m* is dangling, the tension in the cord must be equal to the weight of mass *m*, and so $F_\text{T}=mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass *M*, and so $F_\text{T}=F_\text{R}=Ma_\text{R}=M\,v^2/r$. Equate the two expressions for the tension and solve for the velocity.

$$M\,v^2/r=mg\quad\rightarrow\quad v=\boxed{\sqrt{mgR/M}}.$$

20. A free-body diagram for the ball is shown. The tension in the suspending cord must not only hold the ball up, but also provide the centripetal force needed to make the ball move in a circle. Write Newton's 2nd law for the vertical direction, noting that the ball is not accelerating vertically.



$$\sum F_y = F_\text{T} \sin\theta - mg = 0 \quad \rightarrow \quad F_\text{T} = \frac{mg}{\sin\theta}$$

The force moving the ball in a circle is the horizontal portion of the tension. Write Newton's 2nd law for that radial motion.

$$\sum F_\text{R} = F_\text{T} \cos\theta = ma_\text{R} = m v^2/r$$

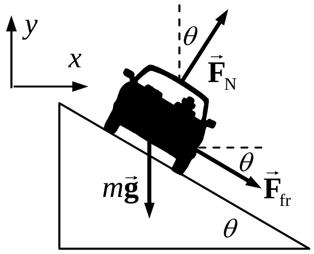
Substitute the expression for the tension from the first equation into the second equation, and solve for the angle. Also substitute in the fact that for a rotating object, $v = 2\pi r/T$. Finally we recognize that if the string is of length L , then the radius of the circle is $r = L\cos\theta$.

$$F_\text{T} \cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2} = \frac{4\pi^2 mL\cos\theta}{T^2} \quad \rightarrow$$

$$\sin\theta = \frac{gT^2}{4\pi^2 L} \quad \rightarrow \quad \theta = \sin^{-1}\frac{gT^2}{4\pi^2 L} = \sin^{-1}\frac{\left(9.80\,\text{m/s}^2\right)\left(0.500\text{ s}\right)^2}{4\pi^2\left(0.600\text{ m}\right)} = \boxed{5.94^\circ}$$

The tension is then given by $F_\text{T} = \dfrac{mg}{\sin\theta} = \dfrac{\left(0.150\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)}{\sin 5.94^\circ} = \boxed{14.2\text{ N}}$

21. Since the curve is designed for 75 km/h, traveling at a higher speed with the same radius means that more centripetal force will be required. That extra centripetal force will be supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. Note that from Example 5-7 in the textbook, the no-friction banking angle is given by



$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[\left(75\,\text{km/h}\right)\left(\dfrac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{\left(88\text{ m}\right)\left(9.8\,\text{m/s}^2\right)} = 26.7^\circ$$

Write Newton's 2nd law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_\text{fr} = \mu_s F_\text{N}$. Solve each equation for the normal force.

$$\sum F_y = F_\text{N}\cos\theta - mg - F_\text{fr}\sin\theta = 0 \quad \rightarrow \quad F_\text{N}\cos\theta - \mu_s F_\text{N}\sin\theta = mg \quad \rightarrow$$

$$F_\text{N} = \frac{mg}{\left(\cos\theta - \mu_s\sin\theta\right)}$$

$$\sum F_x = F_\text{N}\sin\theta + F_\text{fr}\cos\theta = F_\text{R} = mv^2/r \quad \rightarrow \quad F_\text{N}\sin\theta + \mu_s F_\text{N}\cos\theta = mv^2/r \quad \rightarrow$$

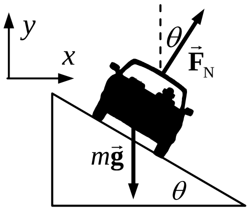
$$F_\text{N} = \frac{mv^2/r}{\left(\sin\theta + \mu_s\cos\theta\right)}$$

Equate the two expressions for F_N , and solve for the coefficient of friction. The speed of rounding the curve is given by $v = \left(95\,\text{km/h}\right)\left(\dfrac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s}$.

$$\frac{mg}{\left(\cos\theta - \mu_s\sin\theta\right)} = \frac{mv^2/r}{\left(\sin\theta + \mu_s\cos\theta\right)} \quad \rightarrow$$

$$\mu_s = \frac{\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)}{\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)} = \frac{\left(\frac{v^2}{r} - g\tan\theta\right)}{\left(g + \frac{v^2}{r}\tan\theta\right)} = \frac{\left(\frac{(26.39\,\text{m/s})^2}{88\text{ m}} - \left(9.8\,\text{m/s}^2\right)\tan 26.7^\circ\right)}{\left(9.8\,\text{m/s}^2 + \frac{(26.39\,\text{m/s})^2}{88\text{ m}}\tan 26.7^\circ\right)} = \boxed{0.22}$$

22. The car moves in a horizontal circle, and so there must be a net horizontal centripetal force. The car is not accelerating vertically. Write Newton's 2nd law for both the *x* and *y* directions.



$$\sum F_y = F_N\cos\theta - mg = 0 \quad\rightarrow\quad F_N = \frac{mg}{\cos\theta}$$

$$\sum F_x = \sum F_R = F_N\sin\theta = ma_x$$

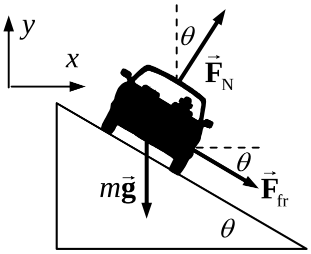
The amount of centripetal force needed for the car to round the curve is

$$F_R = mv^2/r = (1200\text{ kg})\frac{\left[(95\,\text{km/h})\left(\frac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{67\text{ m}} = 1.247\times10^4\,\text{N}.$$

The actual horizontal force available from the normal force is

$$F_N\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (1200\text{ kg})\left(9.80\,\text{m/s}^2\right)\tan 12^\circ = 2.500\times10^3\,\text{N}.$$

Thus more force is necessary for the car to round the curve than can be supplied by the normal force. That extra force will have to have a horizontal component to the right in order to provide the extra centripetal force. Accordingly, we add a frictional force pointed down the plane. That corresponds to the car not being able to make the curve without friction.



Again write Newton's 2nd law for both directions, and again the *y* acceleration is zero.

$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \quad\rightarrow\quad F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = mv^2/r$$

Substitute the expression for the normal force from the *y* equation into the *x* equation, and solve for the friction force.

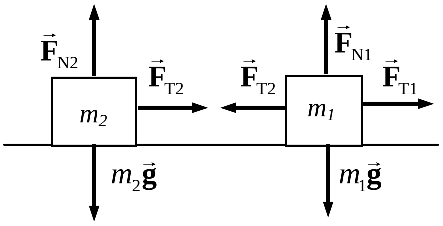
$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = mv^2/r \quad\rightarrow\quad (mg + F_{fr}\sin\theta)\sin\theta + F_{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta$$

$$F_{fr} = m\frac{v^2}{r}\cos\theta - mg\sin\theta = \left(1.247\times10^4\,\text{N}\right)\cos 12^\circ - (1200\text{ kg})\left(9.80\,\text{m/s}^2\right)\sin 12^\circ$$

$$= 9.752\times10^3\,\text{N}$$

So a frictional force of $\boxed{9.8\times10^3\,\text{N down the plane}}$ is needed to provide the necessary centripetal force to round the curve at the specified speed.

23. If the masses are in line and both have the same frequency, then they will always stay in line. Consider a free-body diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1, by Newton's 3rd law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal to its weight. Write Newton's 2nd law for the horizontal direction for both masses, noting that they are in uniform circular motion.



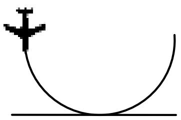
$$\sum F_{1R} = F_{T1} - F_{T2} = m_1 a_1 = m_1 v_1^2/r_1 \qquad \sum F_{2R} = F_{T2} = m_2 a_2 = m_2 v_2^2/r_2$$

The speeds can be expressed in terms of the frequency as follows: $v = \left(f \frac{\text{rev}}{\text{sec}}\right)\left(\frac{2\pi r}{1 \text{ rev}}\right) = 2\pi r f$.

$$F_{T2} = m_2 v_2^2/r_2 = m_2\left(2\pi r_2 f\right)^2/r_2 = \boxed{4\pi^2 m_2 r_2 f^2}$$

$$F_{T1} = F_{T2} + m_1 v_1^2/r_1 = 4\pi m_2 r_2 f^2 + m_1\left(2\pi r_1 f\right)^2/r_1 = \boxed{4\pi^2 f^2\left(m_1 r_1 + m_2 r_2\right)}$$

24. The fact that the pilot can withstand 9.0 *g*'s without blacking out, along with the speed of the aircraft, will determine the radius of the circle that he must fly as he pulls out of the dive. To just avoid crashing into the sea, he must begin to form that circle (pull out of the dive) at a height equal to the radius of that circle.



$$a_R = v^2/r = 9.0g \quad \rightarrow \quad r = \frac{v^2}{9.0g} = \frac{\left(310\,\text{m/s}\right)^2}{9.0\left(9.80\,\text{m/s}^2\right)} = \boxed{1.1\times10^3\,\text{m}}$$

25. From example 5.8, we are given that the track radius is 500 m, and the tangential acceleration is 3.2 m/s^2. Thus the tangential force is

$$F_{\tan} = ma_{\tan} = \left(1100\text{ kg}\right)\left(3.2\,\text{m/s}^2\right) = \boxed{3.5\times10^3\,\text{N}}.$$

The centripetal force is given by

$$F_R = mv^2/r = \left(1100\text{ kg}\right)\left(15\,\text{m/s}\right)^2/\left(500\text{ m}\right) = \boxed{5.0\times10^2\,\text{N}}.$$

26. The car has constant tangential acceleration, which is the acceleration that causes the speed to change. Thus use constant acceleration equations to calculate the tangential acceleration. The initial speed is 0, the final speed is $320\,\text{km/h}\left(\frac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right) = 88.89\,\text{m/s}$, and the distance traveled is one half of a circular arc of radius 220 m, so $\Delta x_{\tan} = 220\pi\text{ m}$. Find the tangential acceleration using Eq. 2-11c.

$$v_{\tan}^2 - v_{0\tan}^2 = 2a_{\tan}\Delta x_{\tan} \quad \rightarrow \quad a_{\tan} = \frac{v_{\tan}^2 - v_{0\tan}^2}{2\Delta x_{\tan}} = \frac{\left(88.89\,\text{m/s}\right)^2}{2\left(220\pi\text{ m}\right)} = 5.72\,\text{m/s}^2$$

With this tangential acceleration, we can find the speed that the car has halfway through the turn, using Eq. 2-11c, and then calculate the radial acceleration.

$$v_{\tan}^2 - v_{0\tan}^2 = 2a_{\tan}\Delta x_{\tan} \quad \rightarrow \quad v_{\tan} = \sqrt{v_{0\tan}^2 + 2a_{\tan}\Delta x_{\tan}} = \sqrt{2\left(5.72\,\text{m/s}^2\right)\left(110\pi\text{ m}\right)} = 62.9\,\text{m/s}$$

$$a_R = v^2/r = \frac{\left(62.9\,\text{m/s}\right)^2}{220\text{ m}} = 18.0\,\text{m/s}^2$$

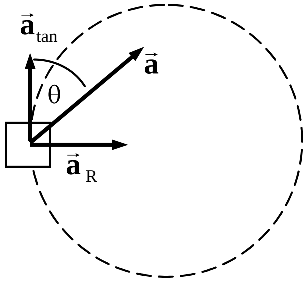
The total acceleration is given by the Pythagorean combination of the tangential and centripetal accelerations. $a_{\text{total}} = \sqrt{a_{\text{R}}^2 + a_{\text{tan}}^2}$. If static friction is to provide the total acceleration, then $F_{\text{fr}} = ma_{\text{total}} = m\sqrt{a_{\text{R}}^2 + a_{\text{tan}}^2}$. We assume that the car is on the verge of slipping, and is on a level surface, and so the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}} = \mu_s mg$. If we equate these two expressions for the frictional force, we can solve for the coefficient of static friction.

$$F_{\text{fr}} = ma_{\text{total}} = m\sqrt{a_{\text{R}}^2 + a_{\text{tan}}^2} = \mu_s mg \quad \rightarrow$$

$$\mu_s = \frac{\sqrt{a_{\text{R}}^2 + a_{\text{tan}}^2}}{g} = \frac{\sqrt{\left(18.0\,\text{m/s}^2\right)^2 + \left(5.72\,\text{m/s}^2\right)^2}}{9.80\,\text{m/s}^2} = 1.92 \approx \boxed{1.9}$$

This is an exceptionally large coefficient of friction, and so the curve had better be banked.

27. We show a top view of the particle in circular motion, traveling clockwise. Because the particle is in circular motion, there must be a radially-inward component of the acceleration.


$\vec{\mathbf{a}}_{\text{tan}}$
$\vec{\mathbf{a}}$
θ
$\vec{\mathbf{a}}_{\text{R}}$

(*a*) $a_{\text{R}} = a\sin\theta = v^2/r \quad \rightarrow$

$$v = \sqrt{ar\sin\theta} = \sqrt{\left(1.05\,\text{m/s}^2\right)\left(2.90\ \text{m}\right)\sin 32.0^\circ} = \boxed{1.27\,\text{m/s}}$$

(*b*) The particle's speed change comes from the tangential acceleration, which is given by $a_{\text{tan}} = a\cos\theta$. If the tangential acceleration is constant, then using Eq. 2-11a,

$$v_{\text{tan}} - v_{0\ \text{tan}} = a_{\text{tan}}t \quad \rightarrow$$

$$v_{\text{tan}} = v_{0\ \text{tan}} + a_{\text{tan}}t = 1.27\,\text{m/s} + \left(1.05\,\text{m/s}^2\right)\left(\cos 32.0^\circ\right)\left(2.00\ \text{s}\right) = \boxed{3.05\,\text{m/s}}$$

28. The spacecraft is three times as far from the Earth's center as when at the surface of the Earth. Therefore, since the force as gravity decreases as the square of the distance, the force of gravity on the spacecraft will be one-ninth of its weight at the Earth's surface.

$$F_G = \tfrac{1}{9}mg_{\substack{\text{Earth's}\\\text{surface}}} = \frac{\left(1350\ \text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{9} = \boxed{1.47\times10^3\,\text{N}}$$

This could also have been found using Newton's law of Universal Gravitation.

29. (*a*) Mass is independent of location and so the mass of the ball is $\boxed{21.0\ \text{kg}}$ on both the Earth and the planet.

(*b*) The weight is found by $W = mg$.

$$W_{\text{Earth}} = mg_{\text{Earth}} = \left(21.0\ \text{kg}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{206\ \text{N}}$$

$$W_{\text{Planet}} = mg_{\text{Planet}} = \left(21.0\ \text{kg}\right)\left(12.0\,\text{m/s}^2\right) = \boxed{252\ \text{N}}.$$

30. The force of gravity on an object at the surface of a planet is given by Newton's law of Universal Gravitation, using the mass and radius of the planet. If that is the only force on an object, then the acceleration of a freely-falling object is acceleration due to gravity.

$$F_G = G\frac{M_{\text{Moon}}m}{r_{\text{Moon}}^2} = mg_{\text{Moon}} \quad \rightarrow$$

$$g_{\text{Moon}} = G\frac{M_{\text{Moon}}}{r^2_{\text{Moon}}} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(7.35\times10^{22}\,\text{kg}\right)}{\left(1.74\times10^{6}\,\text{m}\right)^2} = \boxed{1.62\,\text{m/s}^2}$$

31. The acceleration due to gravity at any location on or above the surface of a planet is given by $g_{\text{planet}} = G\,M_{\text{Planet}}/r^2$, where *r* is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{M_{\text{Earth}}}{\left(1.5R_{\text{Earth}}\right)^2} = \frac{1}{1.5^2}G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}} = \frac{1}{1.5^2}g_{\text{Earth}} = \frac{9.8\,\text{m/s}^2}{1.5^2} = \boxed{4.4\,\text{m/s}^2}$$

32. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text{planet}} = G\,M_{\text{Planet}}/r^2$, where *r* is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{1.66M_{\text{Earth}}}{R^2_{\text{Earth}}} = 1.66\left(G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}}\right) = 1.66g_{\text{Earth}} = 1.66\left(9.80\,\text{m/s}^2\right) = \boxed{16.3\,\text{m/s}^2}$$

33. Assume that the two objects can be treated as point masses, with $m_1 = m$ and $m_2 = 4\ \text{kg} - m$. The gravitational force between the two masses is given by

$$F = G\frac{m_1m_2}{r^2} = G\frac{m\left(4-m\right)}{r^2} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{4m-m^2}{\left(0.25\ \text{m}\right)^2} = 2.5\times10^{-10}\,\text{N}.$$

This can be rearranged into a quadratic form of $m^2 - 4m + 0.234 = 0$. Use the quadratic formula to solve for *m*, resulting in two values which are the two masses.

$$\boxed{m_1 = 3.9\ \text{kg}\ ,\ m_2 = 0.1\ \text{kg}}.$$

34. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text{planet}} = G\,M_{\text{Planet}}/r^2$, where *r* is the distance from the center of the planet to the location in question.

For this problem, $M_{\text{Planet}} = M_{\text{Earth}} = 5.97\times10^{24}\,\text{kg}$

(*a*) $r = R_{\text{Earth}} + 3200\ \text{m} = 6.38\times10^{6}\,\text{m} + 3200\ \text{m}$

$$g = G\frac{M_{\text{Earth}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)}{\left(6.38\times10^{6}\,\text{m} + 3200\ \text{m}\right)^2} = \boxed{9.77\,\text{m/s}^2}$$

(*b*) $r = R_{\text{Earth}} + 3200\ \text{km} = 6.38\times10^{6}\,\text{m} + 3.20\times10^{6}\ \text{m} = 9.58\times10^{6}\,\text{m}$

$$g = G\frac{M_{\text{Earth}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)}{\left(9.58\times10^{6}\,\text{m}\right)^2} = \boxed{4.34\,\text{m/s}^2}$$

35. In general, the acceleration due to gravity of the Earth is given by $g = G\,M_{\text{Earth}}/r^2$, where *r* is the distance from the center of the Earth to the location in question. So for the location in question,

$$g = \tfrac{1}{10}g_{\text{surface}} \quad\rightarrow\quad G\frac{M_{\text{Earth}}}{r^2} = \tfrac{1}{10}G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}} \quad\rightarrow\quad r^2 = 10\,R^2_{\text{Earth}}$$

$$r = \sqrt{10}R_{\text{Earth}} = \sqrt{10}\left(6.38\times10^{6}\,\text{m}\right) = \boxed{2.02\times10^{7}\,\text{m}}$$

36. The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}} = G\,M_{\text{star}}/r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}} = G\frac{M_{\text{star}}}{r^2} = G\frac{5M_{\text{Sun}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{5\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1\times10^{4}\,\text{m}\right)^2} = \boxed{7\times10^{12}\ \text{m/s}^2}$$

37. The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}} = G\,M_{\text{star}}/r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}} = G\frac{M_{\text{sun}}}{R^2_{\text{Moon}}} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.74\times10^{6}\ \text{m}\right)^2} = \boxed{4.38\times10^{7}\ \text{m/s}^2}$$

38. The distance from the Earth’s center is

$$r = R_{\text{Earth}} + 250\ \text{km} = 6.38\times10^{6}\,\text{m} + 2.5\times10^{5}\,\text{m} = 6.63\times10^{6}\,\text{m}\,.$$

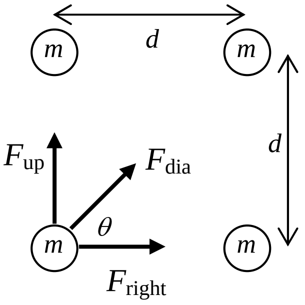
Calculate the acceleration due to gravity at that location.

$$g = G\frac{M_{\text{Earth}}}{r^2} = G\frac{M_{\text{Earth}}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{5.97\times10^{24}\,\text{kg}}{\left(6.63\times10^{6}\,\text{m}\right)^2} = 9.059\,\text{m/s}^2$$

$$= 9.059\,\text{m/s}^2\left(\frac{1\text{"}g\text{"}}{9.80\,\text{m/s}^2}\right) = \boxed{0.924g\text{'s}}$$

This is only about a 7.5% reduction from the value of g at the surface of the Earth.

39. Calculate the force on the sphere in the lower left corner, using the free-body diagram shown. From the symmetry of the problem, the net forces in the x and y directions will be the same. Note $\theta = 45^\circ$



$$F_x = F_{\text{right}} + F_{\text{dia}}\cos\theta = G\frac{m^2}{d^2} + G\frac{m^2}{\left(\sqrt{2}d\right)^2}\frac{1}{\sqrt{2}} = G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)$$

and so $F_y = F_x = G\dfrac{m^2}{d^2}\left(1+\dfrac{1}{2\sqrt{2}}\right)$. The net force can be found by the Pythagorean combination of the two component forces. Due to the symmetry of the arrangement, the net force will be along the diagonal of the square.

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{2F_x^2} = F_x\sqrt{2} = G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)\sqrt{2} = G\frac{m^2}{d^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$= \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(9.5\ \text{kg}\right)^2}{\left(0.60\ \text{m}\right)^2}\left(\sqrt{2}+\frac{1}{2}\right) = \boxed{3.2\times10^{-8}\,\text{N at }45^\circ}$$

The force points towards the center of the square.

40. We are to calculate the force on Earth, so we need the distance of each planet from Earth.

$$r_{\substack{\text{Earth}\\ \text{Venus}}} = \left(150-108\right)\times10^{6}\ \text{km} = 4.2\times10^{10}\,\text{m} \qquad r_{\substack{\text{Earth}\\ \text{Jupiter}}} = \left(778-150\right)\times10^{6}\ \text{km} = 6.28\times10^{11}\,\text{m}$$

$$r_{\substack{\text{Earth}\\ \text{Saturn}}} = \left(1430-150\right)\times10^{6}\ \text{km} = 1.28\times10^{12}\,\text{m}$$

Jupiter and Saturn will exert a rightward force, while Venus will exert a leftward force. Take the right direction as positive.

$$F_{\substack{\text{Earth-}\\\text{planets}}} = G\frac{M_{\text{Earth}}M_{\text{Jupiter}}}{r^2_{\substack{\text{Earth}\\\text{Jupiter}}}} + G\frac{M_{\text{Earth}}M_{\text{Saturn}}}{r^2_{\substack{\text{Earth}\\\text{Saturn}}}} - G\frac{M_{\text{Earth}}M_{\text{Venus}}}{r^2_{\substack{\text{Earth}\\\text{Venus}}}}$$

$$= GM^2_{\text{Earth}}\left(\frac{318}{\left(6.28\times10^{11}\,\text{m}\right)^2} + \frac{95.1}{\left(1.28\times10^{12}\,\text{m}\right)^2} - \frac{0.815}{\left(4.2\times10^{10}\,\text{m}\right)^2}\right)$$

$$= \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\,\text{kg}\right)^2\left(4.02\times10^{-22}\,\text{m}^{-2}\right) = \boxed{9.56\times10^{17}\,\text{N}}$$

The force of the Sun on the Earth is as follows.

$$F_{\substack{\text{Earth-}\\\text{Sun}}} = G\frac{M_{\text{Earth}}M_{\text{Sun}}}{r^2_{\substack{\text{Earth}\\\text{Sun}}}} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.50\times10^{11}\,\text{m}\right)^2} = 3.52\times10^{22}\,\text{N}$$

And so the ratio is $F_{\substack{\text{Earth-}\\\text{planets}}}\Big/F_{\substack{\text{Earth-}\\\text{Sun}}} = 9.56\times10^{17}\,\text{N}\big/3.52\times10^{22}\,\text{N} = \boxed{2.71\times10^{-5}}$, which is 27 millionths.

41. The expression for the acceleration due to gravity at the surface of a body is $g_{\text{body}} = G\dfrac{M_{\text{body}}}{R^2_{\text{body}}}$, where R_{body} is the radius of the body. For Mars, $g_{\text{Mars}} = 0.38g_{\text{Earth}}$. Thus

$$G\frac{M_{\text{Mars}}}{R^2_{\text{Mars}}} = 0.38\,G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}} \quad \rightarrow$$

$$M_{\text{Mars}} = 0.38M_{\text{Earth}}\left(\frac{R_{\text{Mars}}}{\text{R}_{\text{Earth}}}\right)^2 = 0.38\left(5.97\times10^{24}\,\text{kg}\right)\left(\frac{3400\text{ km}}{6380\text{ km}}\right)^2 = \boxed{6.4\times10^{23}\,\text{kg}}$$

42. The speed of an object in an orbit of radius *r* around the Sun is given by $v = \sqrt{G\,M_{\text{Sun}}/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for M_{Sun} , using data for the Earth.

$$\sqrt{G\frac{M_{\text{Sun}}}{r}} = \frac{2\pi r}{T} \quad \rightarrow \quad M_{\text{Sun}} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2\left(1.50\times10^{11}\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(3.15\times10^{7}\,\text{sec}\right)^2} = \boxed{2.01\times10^{30}\text{ kg}}$$

This is the same result obtained in Example 5-16 using Kepler’s third law.

43. The speed of a satellite in a circular orbit around a body is given by $v = \sqrt{G\,M_{\text{body}}/r}$, where *r* is the distance from the satellite to the center of the body. So for this satellite,

$$v = \sqrt{G\frac{M_{\text{body}}}{r}} = \sqrt{G\frac{M_{\text{Earth}}}{R_{\text{Earth}} + 3.6\times10^{6}\,\text{m}}} = \sqrt{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)}{\left(9.98\times10^{6}\,\text{m}\right)}}$$

$$= \boxed{6.32\times10^{3}\,\text{m/s}}$$

44. The shuttle must be moving at "orbit speed" in order for the satellite to remain in the orbit when released. The speed of a satellite in circular orbit around the Earth is given by

$$v=\sqrt{G\frac{M_{\text{Earth}}}{r}}=\sqrt{G\frac{M_{\text{Earth}}}{\left(R_{\text{Earth}}+650\text{ km}\right)}}=\sqrt{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\text{ kg}\right)}{\left(6.38\times10^{6}\text{ m}+6.5\times10^{5}\text{ m}\right)}}$$

$$=\boxed{7.53\times10^{3}\text{ m/s}}$$

45. The centripetal acceleration will simulate gravity. Thus $v^2/r=0.60g \rightarrow v=\sqrt{0.60gr}$. Also for a rotating object, the speed is given by $v=2\pi r/T$. Equate the two expressions for the speed and solve for the period.

$$v=\sqrt{0.60gr}=\frac{2\pi r}{T}\rightarrow T=\frac{2\pi r}{\sqrt{0.60gr}}=\frac{2\pi\left(16\text{ m}\right)}{\sqrt{\left(0.60\right)\left(9.8\text{ m/s}^2\right)\left(16\text{ m}\right)}}=\boxed{10\text{ sec}}$$

46. The speed of an object in an orbit of radius *r* around the Earth is given by $v=\sqrt{GM_{\text{Earth}}/r}$, and is also given by $v=2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*. Also, for a "near-Earth" orbit, $r=R_{\text{Earth}}$.

$$\sqrt{G\frac{M_{\text{Earth}}}{r}}=\frac{2\pi r}{T}\rightarrow T=2\pi\sqrt{\frac{r^3}{GM_{\text{Earth}}}}$$

$$T=2\pi\sqrt{\frac{R_{\text{Earth}}^3}{GM_{\text{Earth}}}}=2\pi\sqrt{\frac{\left(6.38\times10^{6}\text{ m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ m}\right)}}=\boxed{5070\text{ s}\sim84\text{ min}}$$

$\boxed{\text{No}}$, the result does not depend on the mass of the satellite.

47. At the top of Mt. Everest (elevation 8848 meters), the distance of the orbit from the center of the Earth would be $r=R_{\text{Earth}}+8848\text{ m}=6.38\times10^{6}\text{ m}+8848\text{ m}$. The orbit speed is given by

$$v=\sqrt{G\frac{M_{\text{Earth}}}{r}}=\sqrt{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\text{ m}\right)}{\left(6.38\times10^{6}\text{ m}+8848\text{ m}\right)}}=\boxed{7.90\times10^{3}\text{ m/s}}.$$

A comment – a launch would have some initial orbit speed from the fact that the Earth is rotating to the east. That is why most space launches are to the east.

48. The speed of an object in an orbit of radius *r* around the Moon is given by $v=\sqrt{GM_{\text{Moon}}/r}$, and is also given by $v=2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{GM_{\text{Moon}}/r}=2\pi r/T\rightarrow$$

$$T=2\pi\sqrt{\frac{r^3}{GM_{\text{Moon}}}}=2\pi\sqrt{\frac{\left(R_{\text{Moon}}+100\text{ km}\right)^3}{GM_{\text{Moon}}}}=2\pi\sqrt{\frac{\left(1.74\times10^{6}\text{ m}+1.0\times10^{5}\text{ m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\text{ kg}\right)}}$$

$$=\boxed{7.08\times10^{3}\text{ s}\left(\sim2\text{ h}\right)}$$

49. The speed of an object in an orbit of radius *r* around a planet is given by $v = \sqrt{G M_{\text{planet}}/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{G\frac{M_{\text{Planet}}}{r}} = \frac{2\pi r}{T} \quad \rightarrow \quad T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Planet}}}}$$

For this problem, the inner orbit is at $r_{\text{inner}} = 7.3\times10^7\,\text{m}$, and the outer orbit is at $r_{\text{inner}} = 1.7\times10^8\,\text{m}$. Use these values to calculate the periods.

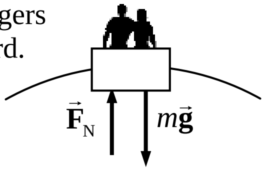
$$T_{\text{inner}} = 2\pi\sqrt{\frac{\left(7.3\times10^7\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.7\times10^{26}\,\text{kg}\right)}} = \boxed{2.0\times10^4\,\text{s}}$$

$$T_{\text{outer}} = 2\pi\sqrt{\frac{\left(1.7\times10^8\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.7\times10^{26}\,\text{kg}\right)}} = \boxed{7.1\times10^4\,\text{s}}$$

Saturn's rotation period (day) is 10 hr 39 min which is about $3.8\times10^4\,\text{sec}$. Thus the inner ring will appear to move across the sky "faster" than the Sun (about twice per Saturn day), while the outer ring will appear to move across the sky "slower" than the Sun (about once every two Saturn days).

50. The apparent weight is the normal force on the passenger. For a person at rest, the normal force is equal to the actual weight. If there is acceleration in the vertical direction, either up or down, then the normal force (and hence the apparent weight) will be different than the actual weight. The speed of the Ferris wheel is $v = 2\pi r/T = 2\pi\left(12.0\text{ m}\right)/15.5\text{ s} = 4.86\,\text{m/s}$.

(*a*) At the top, consider the free-body diagram shown. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's 2nd law with downward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.

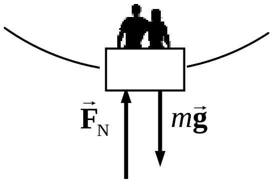


$$\sum F = F_{\text{R}} = mg - F_{\text{N}} = ma = mv^2/r \quad \rightarrow \quad F_{\text{N}} = mg - mv^2/r$$

The ratio of apparent weight to real weight is given by

$$\frac{mg - mv^2/r}{mg} = \frac{g - v^2/r}{g} = 1 - \frac{v^2}{rg} = 1 - \frac{\left(4.86\,\text{m/s}\right)^2}{\left(12.0\text{ m}\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{0.799}$$

(*b*) At the bottom, consider the free-body diagram shown. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's 2nd law with upward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.

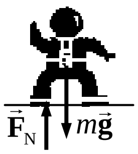


$$\sum F = F_{\text{R}} = F_{\text{N}} - mg = ma = mv^2/r \quad \rightarrow \quad F_{\text{N}} = mg + mv^2/r$$

The ratio of apparent weight to real weight is given by

$$\frac{mg + mv^2/r}{mg} = 1 + \frac{v^2}{rg} = \boxed{1.201}$$

51. Consider the free-body diagram for the astronaut in the space vehicle. The Moon is below the astronaut in the figure. We assume that the astronaut is touching the inside of the space vehicle, or in a seat, or strapped in somehow, and so a force will be exerted on the astronaut by the spacecraft. That force has been labeled $\vec{\mathbf{F}}_\text{N}$. The magnitude of that force is the apparent weight of the astronaut. Take down as the positive direction.



(*a*) If the spacecraft is moving with a constant velocity, then the acceleration of the astronaut must be 0, and so the net force on the astronaut is 0.

$$\sum F = mg - F_N = 0 \quad \to$$

$$F_N = mg = G\frac{mM_\text{Moon}}{r^2} = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{(75\ \text{kg})\left(7.4\times10^{22}\ \text{kg}\right)}{\left(4.2\times10^{6}\ \text{m}\right)^2} = 21\ \text{N}$$

Since the value here is positive, the normal force points in the original direction as shown on the free-body diagram. The astronaut will be pushed "upward" by the floor or the seat. Thus the astronaut will perceive that he has a "weight" of $\boxed{\text{21 N, towards the Moon}}$.

(*b*) Now the astronaut has an acceleration towards the Moon. Write Newton's 2nd law for the astronaut, with down as the positive direction.

$$\sum F = mg - F_N = ma \quad \to \quad F_N = mg - ma = 21\ \text{N} - (75\ \text{kg})\left(2.9\ \text{m}/\text{s}^2\right) = -2.0\times10^2\ \text{N}$$

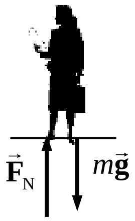
Because of the negative value, the normal force points in the opposite direction from what is shown on the free-body diagram – it is pointing towards the Moon. So perhaps the astronaut is pinned against the "ceiling" of the spacecraft, or safety belts are pulling down on the astronaut. The astronaut will perceive being "pushed downwards", and so has an upward apparent weight of $\boxed{2.0\times10^2\ \text{N, away from the Moon}}$.

52. Consider the motion of one of the stars. The gravitational force on the star is given by $F = G\dfrac{mm}{d^2}$, where *d* is the distance separating the two stars. But since the star is moving in a circle of radius *d*/2, the force on the star can be expressed as $F_\text{R} = mv^2/r = m\dfrac{v^2}{d/2}$. Equate these two force expressions, and use $v = 2\pi r/T = d\pi/T$.

$$G\frac{mm}{d^2} = m\frac{v^2}{d/2} = m\frac{(d\pi/T)^2}{d/2} \quad \to$$

$$m = \frac{2\pi^2 d^3}{GT^2} = \frac{2\pi^2\left(3.6\times10^{11}\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left[(5.7\ \text{y})\left(3.15\times10^{7}\ \text{sec}/\text{y}\right)\right]^2} = \boxed{4.3\times10^{29}\ \text{kg}}$$

53. Consider a free-body diagram for the woman in the elevator. $\vec{\mathbf{F}}_\text{N}$ is the force the spring scale exerts. Write Newton's 2nd law for the vertical direction, with up as positive.



$$\sum F = F_\text{N} - mg = ma \quad \to \quad F_\text{N} = m(g+a)$$

(*a*, *b*) For constant speed motion in a straight line, the acceleration is 0, and so

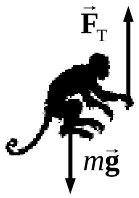
$$F_\text{N} = mg = (55\ \text{kg})\left(9.8\ \text{m}/\text{s}^2\right) = \boxed{5.4\times10^2\ \text{N}}$$

(*c*) Here $a = +0.33g$ and so $F_\text{N} = 1.33mg = 1.33(55\ \text{kg})\left(9.8\ \text{m}/\text{s}^2\right) = \boxed{7.2\times10^2\ \text{N}}$

(*d*) Here $a = -0.33g$ and so $F_N = 0.67\,mg = 0.67(55\text{ kg})(9.8\,\text{m/s}^2) = \boxed{3.6\times10^2\text{ N}}$

(*e*) Here $a = -g$ and so $F_N = \boxed{0\text{ N}}$

54. Draw a free-body diagram of the monkey. Then write Newton's 2nd law for the vertical direction, with up as positive.



$$\sum F = F_T - mg = ma \quad\rightarrow\quad a = \frac{F_T - mg}{m}$$

For the maximum tension of 220 N,

$$a = \frac{220\text{ N} - (17.0\text{ kg})(9.80\,\text{m/s}^2)}{(17.0\text{ kg})} = 3.1\,\text{m/s}^2$$

Thus the elevator must have an $\boxed{\text{upward acceleration greater than } a = 3.1\,\text{m/s}^2}$ for the cord to break. Any downward acceleration would result in a tension less than the monkey's weight.

55. (*a*) The speed of an object in near-surface orbit around a planet is given by $v = \sqrt{GM/R}$, where M is the planet mass and R is the planet radius. The speed is also given by $v = 2\pi R/T$, where T is the period of the object in orbit. Equate the two expressions for the speed.

$$\sqrt{G\frac{M}{R}} = \frac{2\pi R}{T} \quad\rightarrow\quad G\frac{M}{R} = \frac{4\pi^2 R^2}{T^2} \quad\rightarrow\quad \frac{M}{R^3} = \frac{4\pi^2}{GT^2}$$

The density of a uniform spherical planet is given by $\rho = \dfrac{M}{\text{Volume}} = \dfrac{M}{\frac{4}{3}\pi R^3}$. Thus

$$\rho = \frac{3M}{4\pi R^3} = \frac{3}{4\pi}\frac{4\pi^2}{GT^2} = \frac{3\pi}{GT^2}$$

(*b*) For Earth,

$$\rho = \frac{3\pi}{GT^2} = \frac{3\pi}{(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2)[(85\text{ min})(60\text{ s/min})]^2} = \boxed{5.4\times10^3\text{ kg/m}^3}$$

56. Use Kepler's 3rd law for objects orbiting the Earth. The following are given.

$$T_2 = \text{period of Moon} = (27.4\text{ day})\left(\frac{86,400\text{ s}}{1\text{ day}}\right) = 2.367\times10^6\text{ sec}$$

$$r_2 = \text{radius of Moon's orbit} = 3.84\times10^8\text{ m}$$

$$r_1 = \text{radius of near-Earth orbit} = R_{\text{Earth}} = 6.38\times10^6\text{ m}$$

$$(T_1/T_2)^2 = (r_1/r_2)^3 \quad\rightarrow$$

$$T_1 = T_2(r_1/r_2)^{3/2} = (2.367\times10^6\text{ sec})\left(\frac{6.38\times10^6\text{ m}}{3.84\times10^8\text{ m}}\right)^{3/2} = \boxed{5.07\times10^3\text{sec}}\,(\sim 84.5\text{ min})$$

57. Use Kepler's 3rd law for objects orbiting the Sun.

$$\left(\frac{r_{\text{Icarus}}}{r_{\text{Earth}}}\right)^3 = \left(\frac{T_{\text{Icarus}}}{T_{\text{Earth}}}\right)^2 \quad\rightarrow\quad r_{\text{Icarus}} = r_{\text{Earth}}\left(\frac{T_{\text{Icarus}}}{T_{\text{Earth}}}\right)^{2/3} = (1.50\times10^{11}\text{ m})\left(\frac{410\text{ d}}{365\text{ d}}\right)^{2/3} = \boxed{1.62\times10^{11}\text{ m}}$$

58. Use Kepler's 3rd law for objects orbiting the Sun.

$$\left(T_{\text{Neptune}}/T_{\text{Earth}}\right)^2 = \left(r_{\text{Neptune}}/r_{\text{Earth}}\right)^3 \quad \rightarrow$$

$$T_{\text{Neptune}} = T_{\text{Earth}}\left(\frac{r_{\text{Neptune}}}{r_{\text{Earth}}}\right)^{3/2} = (1\text{ year})\left(\frac{4.5\times10^{9}\text{ km}}{1.5\times10^{8}\text{ km}}\right)^{3/2} = \boxed{1.6\times10^{2}\text{ years}}$$

59. Use Kepler's 3rd law to relate the orbits of Earth and Halley's comet around the Sun.

$$\left(r_{\text{Halley}}/r_{\text{Earth}}\right)^3 = \left(T_{\text{Halley}}/T_{\text{Earth}}\right)^2 \quad \rightarrow$$

$$r_{\text{Halley}} = r_{\text{Earth}}\left(T_{\text{Halley}}/T_{\text{Earth}}\right)^{2/3} = \left(150\times10^{6}\text{ km}\right)\left(76\text{ y}/1\text{ y}\right)^{2/3} = \boxed{2690\times10^{6}\text{ km}}$$

This value is half the sum of the nearest and farthest distances of Halley's comet from the Sun. Since the nearest distance is very close to the Sun, we will approximate that nearest distance as 0. Then the farthest distance is twice the value above, or $5380\times10^{6}\text{ km}$. This distance approaches the mean orbit distance of Pluto, which is $5900\times10^{6}\text{ km}$. $\boxed{\text{It is still in the Solar System, nearest to Pluto's orbit.}}$

60. There are two expressions for the velocity of an object in circular motion around a mass *M*: $v = \sqrt{GM/r}$ and $v = 2\pi r/T$. Equate the two expressions and solve for *T*.

$$\sqrt{GM/r} = 2\pi r/T \quad \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{\left(\left(3\times10^{4}\text{ly}\right)\frac{\left(3\times10^{8}\text{ m/s}\right)\left(3.16\times10^{7}\text{sec}\right)}{1\text{ ly}}\right)^{3}}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(4\times10^{41}\text{kg}\right)}} = 5.8\times10^{15}\text{s} = 1.8\times10^{8}\text{ y}$$

$$\approx \boxed{2\times10^{8}\text{ y}}$$

61. (*a*) The relationship between satellite period *T*, mean satellite distance *r*, and planet mass *M* can be derived from the two expressions for satellite speed: $v = \sqrt{GM/r}$ and $v = 2\pi r/T$. Equate the two expressions and solve for *M*.

$$\sqrt{GM/r} = 2\pi r/T \quad \rightarrow \quad M = \frac{4\pi^2 r^3}{GT^2}$$

Substitute the values for Io to get the mass of Jupiter.

$$M_{\substack{\text{Jupiter-}\\ \text{Io}}} = \frac{4\pi^2\left(4.22\times10^{8}\text{m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.77\text{d}\times\frac{24\text{ h}}{1\text{ d}}\times\frac{3600\text{ s}}{1\text{ h}}\right)^2} = \boxed{1.90\times10^{27}\text{kg}}.$$

(*b*) For the other moons:

$$M_{\substack{\text{Jupiter-}\\ \text{Europa}}} = \frac{4\pi^2\left(6.71\times10^{8}\text{m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(3.55\times24\times3600\text{ s}\right)^2} = \boxed{1.90\times10^{27}\text{kg}}$$

$$M_{\substack{\text{Jupiter-}\\ \text{Ganymede}}} = \frac{4\pi^2\left(1.07\times10^{9}\text{m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.16\times24\times3600\text{ s}\right)^2} = \boxed{1.89\times10^{27}\text{kg}}$$

$$M_{\substack{\text{Jupiter-}\\\text{Callisto}}} = \frac{4\pi^2\left(1.883\times10^9\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(16.7\times24\times3600\text{ s}\right)^2} = \boxed{1.90\times10^{27}\,\text{kg}}$$

$\boxed{\text{Yes}}$, the results are consistent – only about 0.5% difference between them.

62. Knowing the period of the Moon and the distance to the Moon, we can calculate the speed of the Moon by $v = 2\pi r/T$. But the speed can also be calculated for any Earth satellite by $v = \sqrt{G M_{\text{Earth}}/r}$. Equate the two expressions for the speed, and solve for the mass of the Earth.

$$\sqrt{G M_{\text{Earth}}/r} = 2\pi r/T \quad \rightarrow$$

$$M_{\text{Earth}} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2\left(3.84\times10^8\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left[\left(27.4\text{ d}\right)\left(86,400\,\text{s/d}\right)\right]^2} = \boxed{5.98\times10^{24}\,\text{kg}}$$

63. Use Kepler's 3rd law to find the radius of each moon of Jupiter, using Io's data for r_2 and T_2.

$$\left(r_1/r_2\right)^3 = \left(T_1/T_2\right)^2 \quad \rightarrow \quad r_1 = r_2\left(T_1/T_2\right)^{2/3}$$

$$r_{\text{Europa}} = r_{\text{Io}}\left(T_{\text{Europa}}/T_{\text{Io}}\right)^{2/3} = \left(422\times10^3\,\text{km}\right)\left(3.55\text{ d}/1.77\text{ d}\right)^{2/3} = \boxed{671\times10^3\,\text{km}}$$

$$r_{\text{Ganymede}} = \left(422\times10^3\,\text{km}\right)\left(7.16\text{ d}/1.77\text{ d}\right)^{2/3} = \boxed{1070\times10^3\,\text{km}}$$

$$r_{\text{Callisto}} = \left(422\times10^3\,\text{km}\right)\left(16.7\text{ d}/1.77\text{ d}\right)^{2/3} = \boxed{1880\times10^3\,\text{km}}$$

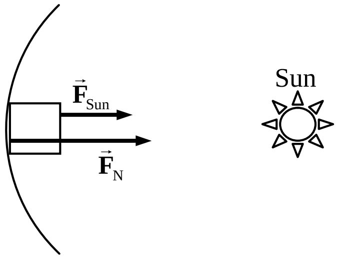
The agreement with the data in the table is excellent.

64. (*a*) Use Kepler's 3rd law to relate the Earth and the hypothetical planet in their orbits around the Sun.

$$\left(T_{\text{planet}}/T_{\text{Earth}}\right)^2 = \left(r_{\text{planet}}/r_{\text{Earth}}\right)^3 \quad \rightarrow \quad T_{\text{planet}} = T_{\text{Earth}}\left(r_{\text{planet}}/r_{\text{Earth}}\right)^{3/2} = \left(1\text{ y}\right)\left(3/1\right)^{3/2} \approx \boxed{5\text{ y}}$$

(*b*) No mass data can be calculated from this relationship, because the relationship is mass-independent. Any object at the orbit radius of 3 times the Earth's orbit radius would have a period of 5.2 years, regardless of its mass.

65. If the ring is to produce an apparent gravity equivalent to that of Earth, then the normal force of the ring on objects must be given by $F_{\text{N}} = mg$. The Sun will also exert a force on objects on the ring. See the free-body diagram.



$\vec{\mathbf{F}}_{\text{Sun}}$
$\vec{\mathbf{F}}_{\text{N}}$
Sun

Write Newton's 2nd law for the object, with the fact that the acceleration is centripetal.

$$\sum F = F_{\text{R}} = F_{\text{Sun}} + F_{\text{N}} = mv^2/r$$

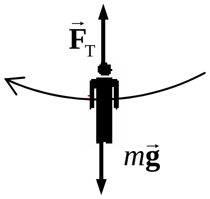
Substitute in the relationships that $v = 2\pi r/T$, $F_{\text{N}} = mg$, and $F_{\text{Sun}} = G\dfrac{M_{\text{Sun}}m}{r^2}$, and solve for the period of the rotation.

$$F_{Sun} + F_N = mv^2/r \quad \rightarrow \quad G\frac{M_{\text{Sun}}m}{r^2} + mg = \frac{4\pi^2 mr}{T^2} \quad \rightarrow \quad G\frac{M_{\text{Sun}}}{r^2} + g = \frac{4\pi^2 r}{T^2}$$

$$T=\sqrt{\frac{4\pi^2 r}{G\frac{M_{\text{Sun}}}{r^2}+g}}=\sqrt{\frac{4\pi^2\left(1.50\times10^{11}\,\text{m}\right)}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.50\times10^{11}\,\text{m}\right)^2}+9.8\,\text{m/s}^2}}=7.8\times10^5\,\text{s}=\boxed{9.0\text{ d}}$$

The force of the Sun is only about 1/1600 the size of the normal force. The force of the Sun could have been ignored in the calculation with no effect in the result as given above.

66. A free-body diagram of Tarzan at the bottom of his swing is shown. The upward tension force is created by his pulling down on the vine. Write Newton's 2nd law in the vertical direction. Since he is moving in a circle, his acceleration will be centripetal, and points upward when he is at the bottom.



$$\sum F=F_{\text{T}}-mg=ma=mv^2/r\quad\rightarrow\quad v=\sqrt{\frac{\left(F_{\text{T}}-mg\right)r}{m}}$$

The maximum speed will be obtained with the maximum tension.

$$v_{\max}=\sqrt{\frac{\left(\vec{\mathbf{F}}_{\text{T max}}-mg\right)r}{m}}=\sqrt{\frac{\left(1400\text{ N}-(80\text{ kg})\left(9.8\,\text{m/s}^2\right)\right)5.5\text{ m}}{80\text{ kg}}}=\boxed{6.5\,\text{m/s}}$$

67. The acceleration due to the Earth's gravity at a location at or above the surface is given by $g=GM_{\text{Earth}}/r^2$, where r is the distance from the center of the Earth to the location in question. Find the location where $g=\frac{1}{2}g_{\text{surface}}$.

$$\frac{GM_{\text{Earth}}}{r^2}=\frac{1}{2}\frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2}\quad\rightarrow\quad r^2=2R_{\text{Earth}}^2\quad\rightarrow\quad r=\sqrt{2}R_{\text{Earth}}$$

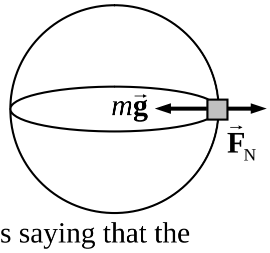
The distance above the Earth's surface is

$$r-R_{\text{Earth}}=\left(\sqrt{2}-1\right)R_{\text{Earth}}=\left(\sqrt{2}-1\right)\left(6.38\times10^6\,\text{m}\right)=\boxed{2.64\times10^6\,\text{m}}.$$

68. The radius of either skater's motion is 0.80 m, and the period is 2.5 sec. Thus their speed is given by $v=2\pi r/T=\frac{2\pi(0.80\text{ m})}{2.5\text{ s}}=2.0\,\text{m/s}$. Since each skater is moving in a circle, the net radial force on each one is given by Eq. 5-3.

$$F_{\text{R}}=mv^2/r=\frac{(60.0\text{ kg})(2.0\,\text{m/s})^2}{0.80\text{ m}}=\boxed{3.0\times10^2\,\text{N}}.$$

69. Consider this free-body diagram for an object at the equator. Since the object is moving in a circular path, there must be a net force on the object, pointing towards the center of the Earth, producing a centripetal acceleration. Write Newton's 2nd law, with the inward direction positive.



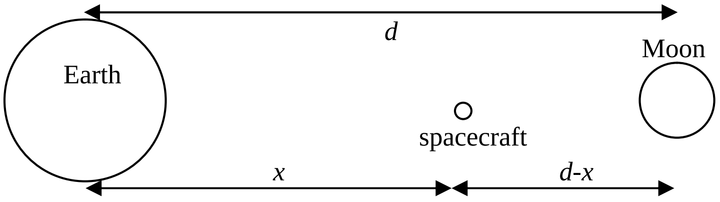
$$\sum F_{\text{R}}=mg-F_{\text{N}}=mv^2/R_{\text{Earth}}\quad\rightarrow\quad F_N=m\left(g-v^2/R_{\text{Earth}}\right)$$

We see that the normal force, which without the rotation would be the "expected" value of *mg*, has now been reduced. This effect can be described as saying that the acceleration due to gravity has been reduced, by an amount equal to v^2/R_{Earth}. To calculate this "change" in *g*, use $v=2\pi R_{\text{Earth}}/T$ to get the following.

$$\Delta g = -v^2/R_{\text{Earth}} = -\frac{4\pi^2 R_{\text{Earth}}}{T^2} = -\frac{4\pi^2\left(6.38\times10^6\,\text{m}\right)}{\left[(1\text{ d})(86,400\text{ s}/1\text{ d})\right]^2} = -0.03374\,\text{m/s}^2$$

This is a reduction of $-0.03374\,\text{m/s}^2 \times \dfrac{1\,g}{9.80\,\text{m/s}^2} = \boxed{-3.44\times10^{-3}\,g}$.

70. For the forces to balance means that the gravitational force on the spacecraft due to the Earth must be the same as that due to the Moon. Write the gravitational forces on the spacecraft, equate them, and solve for the distance *x*.



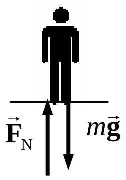
$$F_{\substack{\text{Earth-}\\\text{spacecraft}}} = G\frac{M_{\text{Earth}}m_{\text{spacecraft}}}{x^2} \quad ; \quad F_{\substack{\text{Moon}\\\text{spacecraft}}} = G\frac{M_{\text{Moon}}m_{\text{spacecraft}}}{(d-x)^2}$$

$$G\frac{M_{\text{Earth}}m_{\text{spacecraft}}}{x^2} = G\frac{M_{\text{Moon}}m_{\text{spacecraft}}}{(d-x)^2} \rightarrow \frac{x^2}{M_{\text{Earth}}} = \frac{(d-x)^2}{M_{\text{Moon}}} \rightarrow \frac{x}{\sqrt{M_{\text{Earth}}}} = \frac{d-x}{\sqrt{M_{\text{Moon}}}}$$

$$x = d\frac{\sqrt{M_{\text{Earth}}}}{\left(\sqrt{M_{\text{Moon}}}+\sqrt{M_{\text{Earth}}}\right)} = \left(3.84\times10^8\,\text{m}\right)\frac{\sqrt{5.97\times10^{24}\,\text{kg}}}{\left(\sqrt{7.35\times10^{22}\,\text{kg}}+\sqrt{5.97\times10^{24}\,\text{kg}}\right)} = \boxed{3.46\times10^8\,\text{m}}$$

This is only about 22 Moon radii away from the Moon.

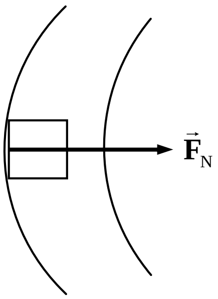
71. Consider a free-body diagram of yourself in the elevator. $\vec{\mathbf{F}}_{\text{N}}$ is the force of the scale pushing up on you, and reads the normal force. Since the scale reads 82 kg, if it were calibrated in Newtons, the normal force would be $F_{\text{N}} = (82\text{ kg})\left(9.8\,\text{m/s}^2\right) = 804\text{ N}$. Write Newton's 2nd law in the vertical direction, with upward as positive.



$$\sum F = F_{\text{N}} - mg = ma \rightarrow a = \frac{F_{\text{N}}-mg}{m} = \frac{804\text{ N} - (65\text{ kg})\left(9.8\,\text{m/s}^2\right)}{65\text{ kg}} = \boxed{2.6\ \text{m/s}^2\text{ upward}}$$

Since the acceleration is positive, the acceleration is upward.

72. To experience a gravity-type force, objects must be on the inside of the outer wall of the tube, so that there can be a centripetal force to move the objects in a circle. See the free-body diagram for an object on the inside of the outer wall, and a portion of the tube. The normal force of contact between the object and the wall must be maintaining the circular motion. Write Newton's 2nd law for the radial direction.



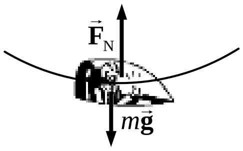
$$\sum F_{\text{R}} = F_{\text{N}} = ma = mv^2/r$$

If this is to have the same effect as Earth gravity, then we must also have that $F_{\text{N}} = mg$. Equate the two expressions for normal force and solve for the speed.

$$F_{\text{N}} = mv^2/r = mg \rightarrow v = \sqrt{gr} = \sqrt{\left(9.8\,\text{m/s}^2\right)(550\,\text{m})} = 73\,\text{m/s}$$

$$(73\,\text{m/s})\left(\frac{1\text{ rev}}{2\pi(550\text{ m})}\right)\left(\frac{86,400\text{ s}}{1\text{ d}}\right) = 1825\,\text{rev/d} \approx \boxed{1.8\times10^3\text{ rev/d}}$$

73. (*a*) See the free-body diagram for the pilot in the jet at the bottom of the loop. For $a_R = v^2/r = 6g$,



$$v^2/r = 6.0g \quad \rightarrow \quad r = \frac{v^2}{6.0g} = \frac{\left[(1300\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{6.0\left(9.8\,\text{m/s}^2\right)} = \boxed{2.2\times10^3\,\text{m}}$$

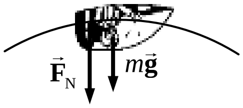
(*b*) The net force must be centripetal, to make the pilot go in a circle. Write Newton's 2nd law for the vertical direction, with up as positive. The normal force is the apparent weight.

$$\sum F_R = F_N - mg = mv^2/r$$

The centripetal acceleration is to be $v^2/r = 6.0g$.

$$F_N = mg + mv^2/r = 7mg = 7(78\text{ kg})\left(9.80\,\text{m/s}^2\right) = 5350\text{ N} = \boxed{5.4\times10^3\,\text{N}}$$

(*c*) See the free-body diagram for the pilot at the top of the loop. Notice that the normal force is down, because the pilot is upside down. Write Newton's 2nd law in the vertical direction, with down as positive.

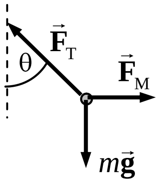


$$\sum F_R = F_N + mg = mv^2/r = 6mg \quad \rightarrow \quad F_N = 5mg = \boxed{3.8\times10^3\,\text{N}}$$

74. The force of gravity on an object at the surface is given by $F_{\text{grav}} = mg_P$. But by Newton's law of Universal Gravitation, the force of gravity on an object at the surface is given by $F_{\text{grav}} = G\frac{mM_{\text{planet}}}{r^2}$. Equate the expressions for the force of gravity and solve for the mass of the planet.

$$G\frac{mM_{\text{planet}}}{r^2} = mg_P \quad \rightarrow \quad \boxed{M_{\text{planet}} = \frac{r^2 g_P}{G}}.$$

75. (*a*) See the free-body diagram for the plumb bob. The attractive gravitational force on the plumb bob is $F_M = G\frac{mm_M}{D_M^2}$. Since the bob is not accelerating, the net force in any direction will be zero. Write the net force for both vertical and horizontal directions. Use $g = G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}$



$$\sum F_{\text{vertical}} = F_T\cos\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{\cos\theta}$$

$$\sum F_{\text{horizontal}} = F_M - F_T\sin\theta = 0 \quad \rightarrow \quad F_M = F_T\sin\theta = mg\tan\theta$$

$$G\frac{mm_M}{D_M^2} = mg\tan\theta \quad \rightarrow \quad \theta = \tan^{-1} G\frac{m_M}{gD_M^2} = \boxed{\tan^{-1}\frac{m_M R_{\text{Earth}}^2}{M_{\text{Earth}} D_M^2}}$$

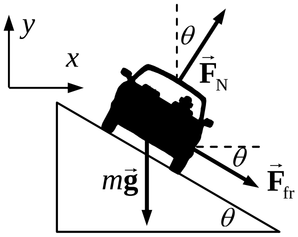
(*b*) We estimate the mass of Mt. Everest by taking its volume times its mass density. If we approximate Mt. Everest as a cone with the same size diameter as height, then its volume is $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2000\text{ m})^2(4000\text{ m}) = 1.7\times10^{10}\,\text{m}^3$. The density is $\rho = 3\times10^3\text{ kg/m}^3$. Find the mass by multiplying the volume times the density.

$$M = \rho V = \left(3\times10^3\text{ kg/m}^3\right)\left(1.7\times10^{10}\,\text{m}^3\right) = \boxed{5\times10^{13}\,\text{kg}}$$

(*c*) With $D = 5000$ m, use the relationship derived in part (a).

$$\theta = \tan^{-1}\frac{M_{\mathrm{M}}R_{Earth}^{2}}{M_{Earth}D_{M}^{2}} = \tan^{-1}\frac{\left(5\times10^{13}\,\mathrm{kg}\right)\left(6.38\times10^{6}\,\mathrm{m}\right)^{2}}{\left(5.97\times10^{24}\,\mathrm{kg}\right)\left(5000\,\mathrm{m}\right)^{2}} = \boxed{8\times10^{-4}\,\mathrm{degrees}}$$

76. Since the curve is designed for a speed of 95 km/h, traveling at that speed would mean no friction is needed to round the curve. From Example 5-7 in the textbook, the no-friction banking angle is given by



y
x
θ
$\vec{\mathbf{F}}_{\mathrm{N}}$
θ
$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\mathrm{fr}}$
θ

$$\theta = \tan^{-1}\frac{v^{2}}{rg} = \tan^{-1}\frac{\left[\left(95\,\mathrm{km/h}\right)\left(\dfrac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^{2}}{\left(67\ \mathrm{m}\right)\left(9.8\,\mathrm{m/s^{2}}\right)} = 46.68^{\circ}$$

Driving at a higher speed with the same radius means that more centripetal force will be required than is present by the normal force alone. That extra centripetal force will be supplied by a force of static friction, downward along the incline, as shown in the first free-body diagram for the car on the incline. Write Newton's 2nd law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}}$.

$$\sum F_y = F_{\mathrm{N}}\cos\theta - mg - F_{\mathrm{fr}}\sin\theta = 0 \ \rightarrow\ F_{\mathrm{N}}\cos\theta - \mu_s F_{\mathrm{N}}\sin\theta = mg \ \rightarrow$$

$$F_{\mathrm{N}} = \frac{mg}{\left(\cos\theta - \mu_s\sin\theta\right)}$$

$$\sum F_x = F_{\mathrm{N}}\sin\theta + F_{\mathrm{fr}}\cos\theta = mv^{2}/r \ \rightarrow\ F_{\mathrm{N}}\sin\theta + \mu_s F_{\mathrm{N}}\cos\theta = mv^{2}/r \ \rightarrow$$

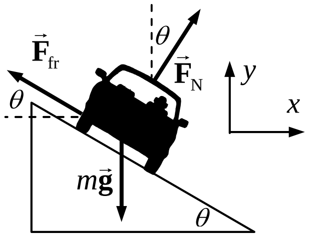
$$F_{\mathrm{N}} = \frac{mv^{2}/r}{\left(\sin\theta + \mu_s\cos\theta\right)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^{2}/r}{\left(\sin\theta + \mu_s\cos\theta\right)} = \frac{mg}{\left(\cos\theta - \mu_s\sin\theta\right)} \ \rightarrow$$

$$v = \sqrt{rg\frac{\left(\sin\theta + \mu_s\cos\theta\right)}{\left(\cos\theta - \mu_s\sin\theta\right)}} = \sqrt{\left(67\ \mathrm{m}\right)\left(9.8\,\mathrm{m/s^{2}}\right)\frac{\left(\sin 46.68^{\circ} + 0.30\cos 46.68^{\circ}\right)}{\left(\cos 46.68^{\circ} - 0.30\sin 46.68^{\circ}\right)}} = 36\,\mathrm{m/s}$$

Now for the slowest possible speed. Driving at a slower speed with the same radius means that less centripetal force will be required than that supplied by the normal force. That decline in centripetal force will be supplied by a force of static friction, upward along the incline, as shown in the second free-body diagram for the car on the incline. Write Newton's 2nd law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}}$.



θ
$\vec{\mathbf{F}}_{\mathrm{fr}}$
$\vec{\mathbf{F}}_{\mathrm{N}}$
y
x
θ
$m\vec{\mathbf{g}}$
θ

$$\sum F_y = F_{\mathrm{N}}\cos\theta - mg + F_{\mathrm{fr}}\sin\theta = 0 \ \rightarrow$$

$$F_{\mathrm{N}}\cos\theta + \mu_s F_{\mathrm{N}}\sin\theta = mg \ \rightarrow\ F_{\mathrm{N}} = \frac{mg}{\left(\cos\theta + \mu_s\sin\theta\right)}$$

$$\sum F_x = F_{\mathrm{N}} \sin\theta - F_{\mathrm{fr}} \cos\theta = mv^2/r \quad \rightarrow \quad F_{\mathrm{N}} \sin\theta - \mu_s F_{\mathrm{N}} \cos\theta = mv^2/r \quad \rightarrow$$

$$F_{\mathrm{N}} = \frac{mv^2/r}{(\sin\theta - \mu_s \cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/r}{(\sin\theta - \mu_s\cos\theta)} = \frac{mg}{(\cos\theta + \mu_s \sin\theta)} \quad \rightarrow$$

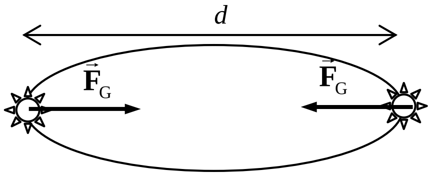
$$v = \sqrt{rg\frac{(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}} = \sqrt{(67\ \mathrm{m})(9.8\ \mathrm{m/s^2})\frac{(\sin 46.67^\circ - 0.30\cos 46.67^\circ)}{(\cos 46.67^\circ + 0.30 \sin 46.67^\circ)}} = 19\ \mathrm{m/s}$$

Thus the range is $\boxed{19\ \mathrm{m/s} \le v \le 36\ \mathrm{m/s}}$, which is $\boxed{68\ \mathrm{km/h} \le v \le 130\ \mathrm{km/h}}$.

77. For an object to be apparently weightless would mean that the object would have a centripetal acceleration equal to *g*. This is really the same as asking what the orbital period would be for an object orbiting the Earth with an orbital radius equal to the Earth's radius. To calculate, use $g = a_C = v^2/R_{\mathrm{Earth}}$, along with $v = 2\pi R_{\mathrm{Earth}}/T$, and solve for *T*.

$$g = \frac{v^2}{R_{\mathrm{Earth}}} = \frac{4\pi^2 R_{\mathrm{Earth}}}{T^2} \quad \rightarrow \quad T = 2\pi\sqrt{\frac{R_{\mathrm{Earth}}}{g}} = 2\pi\sqrt{\frac{6.38\times10^6\ \mathrm{m}}{9.80\ \mathrm{m/s^2}}} = \boxed{5.07\times10^3\ \mathrm{s}}\left(\sim 84.5\ \mathrm{min}\right)$$

78. See the diagram for the two stars.



d
$\vec{\mathbf{F}}_{\mathrm{G}}$
$\vec{\mathbf{F}}_{\mathrm{G}}$

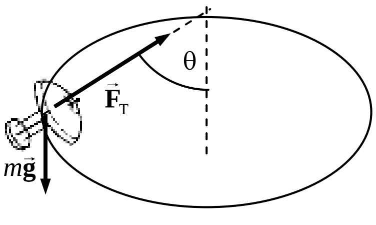
(*a*) The two stars don't crash into each other because of their circular motion. The force on them is centripetal, and maintains their circular motion. Another way to consider it is that the stars have a velocity, and the gravity force causes CHANGE in velocity, not actual velocity. If the stars were somehow brought to rest and then released under the influence of their mutual gravity, they would crash into each other.

(*b*) Set the gravity force on one of the stars equal to the centripetal force, using the relationship that $v = 2\pi r/T = \pi d/T$, and solve for the mass.

$$F_G = G\frac{M^2}{d^2} = F_{\mathrm{R}} = M\frac{v^2}{d/2} = M\frac{2(\pi d/T)^2}{d} = \frac{2\pi^2 Md}{T^2} \quad \rightarrow \quad G\frac{M^2}{d^2} = \frac{2\pi^2 Md}{T^2} \quad \rightarrow$$

$$M = \frac{2\pi^2 d^3}{GT^2} = \frac{2\pi^2\left(8.0\times10^{10}\ \mathrm{m}\right)^3}{\left(6.67\times10^{-11}\ \mathrm{N{\bullet}m^2/kg^2}\right)\left(12.6\ \mathrm{y}\times\frac{3.15\times10^7\ \mathrm{s}}{1\ \mathrm{y}}\right)^2} = \boxed{9.6\times10^{26}\ \mathrm{kg}}$$

79. The lamp must have the same speed and acceleration as the train. The forces on the lamp as the train rounds the corner are shown in the free-body diagram included. The tension in the suspending cord must not only hold the lamp up, but also provide the centripetal force needed to make the lamp move in a circle. Write Newton's 2nd law for the vertical direction, noting that the lamp is not accelerating vertically.



$$\sum F_y = F_T \cos\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{\cos\theta}$$

The force moving the lamp in a circle is the horizontal portion of the tension. Write Newton's 2nd law for that radial motion.

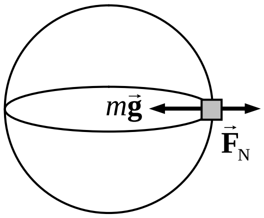
$$\sum F_R = F_T \sin\theta = ma_R = mv^2/r$$

Substitute the expression for the tension from the first equation into the second equation, and solve for the speed.

$$F_T \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = mv^2/r \quad \rightarrow$$

$$v = \sqrt{rg\tan\theta} = \sqrt{(235\text{ m})(9.80\text{ m/s}^2)\tan 17.5^\circ} = \boxed{26.9\text{ m/s}}$$

80. For a body on the equator, the net motion is circular. Consider the free-body diagram as shown. F_N is the normal force, which is the apparent weight. The net force must point to the center of the circle for the object to be moving in a circular path at constant speed. Write Newton's 2nd law with the inward direction as positive.



$$\sum F_R = mg_{\text{Jupiter}} - F_N = mv^2/R_{\text{Jupiter}} \quad \rightarrow$$

$$F_N = m\left(g_{\text{Jupiter}} - v^2/R_{\text{Jupiter}}\right) = m\left(G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{v^2}{R_{\text{Jupiter}}}\right)$$

Use the fact that for a rotating object, $v = 2\pi r/T$.

$$F_N = m\left(G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{4\pi^2 R_{\text{Jupiter}}}{T^2_{\text{Jupiter}}}\right)$$

Thus the perceived acceleration of the object on the surface of Jupiter is

$$G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{4\pi^2 R_{\text{Jupiter}}}{T^2_{\text{Jupiter}}} = \left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.9\times10^{27}\text{ kg}\right)}{\left(7.1\times10^7\text{ m}\right)^2} - \frac{4\pi^2\left(7.1\times10^7\text{ m}\right)}{\left[(595\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right)\right]^2}$$

$$= 22.94\text{ m/s}^2\left(\frac{1\,g}{9.8\text{ m/s}^2}\right) = \boxed{2.3\,g\text{'s}}$$

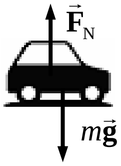
Thus you would not be crushed at all. You would certainly feel "heavy", but not at all crushed.

81. The speed of an orbiting object is given by $v = \sqrt{GM/r}$, where *r* is the radius of the orbit, and *M* is the mass around which the object is orbiting. Solve the equation for *M*.

$$v = \sqrt{GM/r} \quad \rightarrow \quad M = \frac{rv^2}{G} = \frac{\left(5.7\times10^{17}\text{ m}\right)\left(7.8\times10^5\text{ m/s}\right)^2}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)} = \boxed{5.2\times10^{39}\text{ kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\#\text{ solar masses} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{5.2\times10^{39}\text{ kg}}{2\times10^{30}\text{ kg}} = \boxed{2.6\times10^9\text{ solar masses}}$$



82. A generic free-body diagram for the car at any of the three locations is shown. Write Newton's 2nd law for the vertical direction, with downward positive.

$$\sum F = mg - F_N = ma$$

(*a*) At point B, the net force is 0, so the acceleration is 0, and so $F_N = mg$. At point A, the net force is positive, so $mg > F_N$, which is interpreted as a relatively small normal force. At point B, the net force is negative, so $mg < F_N$, which is interpreted as a relatively large normal force. And so $\boxed{F_{NC} > F_{NB} > F_{NA}}$.

(*b*) The driver "feels heavy" when the normal force is larger than his weight, so the driver will feel heavy at point C. The driver "feels light" when the normal force is smaller than his weight, so the driver will feel light at point A. $\boxed{\text{From feels heaviest to feels lightest, C, B, A.}}$

(*c*) In general, $mg - F_N = mv^2/r$ if the car is executing a part of the road that is curved up or down with a radius of *R*. If the normal force goes to 0, then the car loses contact with the road. This happens if $mg = mv^2/r \rightarrow \boxed{v_{max} = \sqrt{gR}}$. Any faster and the car will lose contact with the road.

83. (*a*) The speed of a satellite orbiting the Earth is given by $v = \sqrt{GM_{Earth}/r}$. For the GPS satellites, $r = R_{Earth} + (11,000)(1.852\text{ km}) = 2.68\times10^7\text{ m}$.

$$v = \sqrt{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\text{kg}\right)}{2.68\times10^7\text{ m}}} = \boxed{3.86\times10^3\text{ m/s}}$$

(*b*) The period can be found from the speed and the radius.

$$v = 2\pi r/T \rightarrow T = \frac{2\pi r}{v} = \frac{2\pi\left(2.68\times10^7\text{ m}\right)}{3.86\times10^3\text{ m/s}} = \boxed{4.36\times10^4\text{sec}} = 12.1\text{ hours}$$

84. (*a*) The speed of an object orbiting a mass *M* is given by $v = \sqrt{GM/r}$. The mass of the asteroid is found by multiplying the density times the volume. The period of an object moving in a circular path is given by $T = 2\pi r/v$. Combine these relationships to find the period.

$$M = \rho V = \left(2.3\times10^3\frac{\text{kg}}{\text{m}^3}\right)\left(40000\times6000\times6000\text{ m}^3\right) = 3.312\times10^{15}\text{kg}$$

$$T = \frac{2\pi r}{\sqrt{G\frac{M}{r}}} = \frac{2\pi\left(1.5\times10^4\text{ m}\right)}{\sqrt{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(3.312\times10^{15}\text{kg}\right)}{\left(1.5\times10^4\text{ m}\right)}}} = 2.456\times10^4\text{sec}$$

$$\approx \boxed{2\times10^4\text{sec}} \approx 7\text{ h}$$

(*b*) If the asteroid were a sphere, then the mass would be given by $M = \rho V = \frac{4}{3}\pi\rho r^3$. Solve this for the radius.

$$r = \left(\frac{3M}{4\pi\rho}\right)^{1/3} = \left(\frac{3\left(3.312\times10^{15}\text{kg}\right)}{4\pi\left(2.3\times10^3\frac{\text{kg}}{\text{m}^3}\right)}\right)^{1/3} = 7005\text{ m} \approx \boxed{7\times10^3\text{m}}$$

(*c*) The acceleration due to gravity is found from the mass and the radius.

$$g = GM/r^2 = \left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(3.312\times10^{15}\,\text{kg}\right)}{\left(7005\ \text{m}\right)^2} = 4.502\times10^{-3}\ \text{m/s}^2$$

$$\approx \boxed{5\times10^{-3}\ \text{m/s}^2}$$

85. The relationship between orbital speed and orbital radius for objects in orbit around the Earth is given by $v=\sqrt{GM_{\text{Earth}}/r}$. There are two orbital speeds involved – the one at the original radius, $v_0=\sqrt{GM_{\text{Earth}}/r_0}$, and the faster speed at the reduced radius, $v=\sqrt{GM_{\text{Earth}}/\left(r_0-\Delta r\right)}$.

(*a*) At the faster speed, 25,000 more meters will be traveled during the "catch-up" time, *t*. Note that $r_0 = 6.38\times10^6\,\text{m} + 4\times10^5\,\text{m} = 6.78\times10^6\,\text{m}$.

$$vt = v_0t + 2.5\times10^4\,\text{m} \rightarrow \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0-\Delta r}}\right)t = \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0}}\right)t + 2.5\times10^4\,\text{m} \rightarrow$$

$$t = \frac{2.5\times10^4\,\text{m}}{\sqrt{GM_{\text{Earth}}}}\left(\frac{1}{\sqrt{r_0-\Delta r}}-\frac{1}{\sqrt{r_0}}\right)$$

$$= \frac{2.5\times10^4\,\text{m}}{\sqrt{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\,\text{kg}\right)}}\left(\frac{1}{\sqrt{6.78\times10^6\,\text{m}-1\times10^3\,\text{m}}}-\frac{1}{\sqrt{6.78\times10^6\,\text{m}}}\right)$$

$$= 4.42\times10^4\,\text{s} \approx \boxed{12\,\text{h}}$$

(*b*) Again, 25,000 more meters must be traveled at the faster speed in order to catch up to the satellite.

$$vt = v_0t + 2.5\times10^4\,\text{m} \rightarrow \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0-\Delta r}}\right)t = \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0}}\right)t + 2.5\times10^4\,\text{m} \rightarrow$$

$$\sqrt{\frac{1}{r_0-\Delta r}} = \frac{1}{\sqrt{r_0}} + \frac{2.5\times10^4\,\text{m}}{t\sqrt{GM_{\text{Earth}}}} \rightarrow \sqrt{r_0-\Delta r} = \left[\frac{1}{\sqrt{r_0}} + \frac{2.5\times10^4\,\text{m}}{t\sqrt{GM_{\text{Earth}}}}\right]^{-1} \rightarrow$$

$$\Delta r = r_0 + \left[\frac{1}{\sqrt{r_0}} + \frac{2.5\times10^4\,\text{m}}{t\sqrt{GM_{\text{Earth}}}}\right]^{-2} = 1755\ \text{m} \approx \boxed{1.8\times10^3\,\text{m}}$$

86. (*a*) Use Kepler's 3rd law to relate the orbits of the Earth and the comet around the Sun.

$$\left(\frac{r_{\text{comet}}}{r_{\text{Earth}}}\right)^3 = \left(\frac{T_{\text{comet}}}{T_{\text{Earth}}}\right)^2 \rightarrow r_{\text{comet}} = r_{\text{Earth}}\left(\frac{T_{\text{comet}}}{T_{\text{Earth}}}\right)^{2/3} = \left(1\ \text{AU}\right)\left(\frac{3000\ \text{y}}{1\ \text{y}}\right)^{2/3} = \boxed{208\ \text{AU}}$$

(*b*) The mean distance is the numeric average of the closest and farthest distances.

$$208\ \text{AU} = \frac{1\ \text{AU} + r_{\max}}{2} \rightarrow r_{\max} = \boxed{415\ \text{AU}}\,.$$

(*c*) Refer to figure 5-29, which illustrates Kepler's second law. If the time for each shaded region is made much shorter, then the area of each region can be approximated as a triangle. The area of each triangle is half the "base" (speed of comet multiplied by the amount of time) times the "height" (distance from Sun). So we have the following.

$$\text{Area}_{\min} = \text{Area}_{\max} \quad \rightarrow \quad \tfrac{1}{2}\left(v_{\min} t\right) r_{\min} = \tfrac{1}{2}\left(v_{\max} t\right) r_{\max} \quad \rightarrow$$
$$v_{\min} / v_{\max} = r_{\max} / r_{\min} = \boxed{415/1}\ .$$

87. Let us assume that each person has a mass of 70 kg (a weight of ~ 150 lb). We shall assume that the people can be treated as point masses, and that their centers of mass are about 0.5 m apart. Finally, we assume that we can feel a gravitational force of about 1 N. The expression for the gravitational force becomes

$$F = G\frac{m_1 m_2}{r^2} \quad \rightarrow \quad G = \frac{Fr^2}{m_1 m_2} = \frac{(1\text{ N})(0.5\text{ m})^2}{(70\text{ kg})^2} = \boxed{5\times10^{-5}\text{ N}\bullet\text{m}^2/\text{kg}^2}\ .$$

This is roughly one million times larger than G actually is.

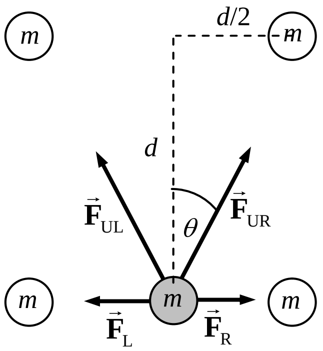
88. The speed of rotation of the Sun about the galactic center, under the assumptions made, is given by $v = \sqrt{G\frac{M_{\text{galaxy}}}{r_{\text{Sun orbit}}}}$ and so $M_{\text{galaxy}} = \frac{r_{\text{Sun orbit}} v^2}{G}$. Substitute in the relationship that $v = 2\pi r_{\text{Sun orbit}}/T$.

$$M_{\text{galaxy}} = \frac{4\pi^2\left(r_{\text{Sun orbit}}\right)^3}{GT^2} = \frac{4\pi^2\left[(30,000)\left(9.5\times10^{15}\text{ m}\right)\right]^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left[\left(200\times10^6\text{ y}\right)\left(\frac{3.15\times10^7\text{ s}}{1\text{ y}}\right)\right]^2}$$
$$= 3.452\times10^{41}\text{ kg} \approx \boxed{3\times10^{41}\text{ kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\#\text{ stars} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{3.452\times10^{41}\text{ kg}}{2.0\times10^{30}\text{ kg}} = 1.726\times10^{11} \approx \boxed{2\times10^{11}\text{ stars}}$$

89. See the free-body diagram. $\theta = \tan^{-1} 0.25/0.50 = 27^\circ$. Because of the symmetry of the problem, the forces $\vec{\mathbf{F}}_{\text{L}}$ and $\vec{\mathbf{F}}_{\text{R}}$ cancel each other. Likewise, the horizontal components of $\vec{\mathbf{F}}_{\text{UR}}$ and $\vec{\mathbf{F}}_{\text{UL}}$ cancel each other. Thus the only forces on the fifth mass will be the vertical components of $\vec{\mathbf{F}}_{\text{UR}}$ and $\vec{\mathbf{F}}_{\text{UL}}$. These components are also equal, and so only one needs to be calculated, and then doubled.


m
d/2
m
d
$\vec{\mathbf{F}}_{\text{UL}}$
θ
$\vec{\mathbf{F}}_{\text{UR}}$
m
m
m
$\vec{\mathbf{F}}_{\text{L}}$
$\vec{\mathbf{F}}_{\text{R}}$

$$F_{\text{net}} = 2F_{UR}\cos\theta = 2G\frac{m^2}{d^2 + (d/2)^2}\cos\theta$$
$$= 2\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{(1.0\text{ kg})^2}{\frac{5}{4}(0.50\text{ m})^2}\cos 27^\circ = \boxed{3.8\times10^{-10}\text{ N , upward}}$$

90. The gravitational force on the satellite is given by $F_{\text{grav}} = G\frac{M_{\text{Earth}} m}{r^2}$, where r is the distance of the satellite from the center of the Earth. Since the satellite is moving in circular motion, then the net force on the satellite can be written as $F_{\text{net}} = mv^2/r$. By substituting $v = 2\pi r/T$ for a circular orbit,

we have $F_{\text{net}} = \frac{4\pi^2 mr}{T^2}$. Then, since gravity is the only force on the satellite, the two expressions for force can be equated, and solved for the orbit radius.

$$G\frac{M_{\text{Earth}}m}{r^2} = \frac{4\pi^2 mr}{T^2} \quad \rightarrow$$

$$r = \left(\frac{GM_{\text{Earth}}T^2}{4\pi^2}\right)^{1/3} = \left[\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(6.0\times10^{24}\,\text{kg}\right)\left(6200\text{ s}\right)^2}{4\pi^2}\right]^{1/3} = 7.304\times10^6\,\text{m}.$$

(*a*) From this value the gravitational force on the satellite can be calculated.

$$F_{\text{grav}} = G\frac{M_{\text{Earth}}m}{r^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(6.0\times10^{24}\,\text{kg}\right)\left(5500\text{ kg}\right)}{\left(7.304\times10^6\,\text{m}\right)^2} = 4.126\times10^4\,\text{N}$$

$$\approx \boxed{4.1\times10^4\,\text{N}}$$

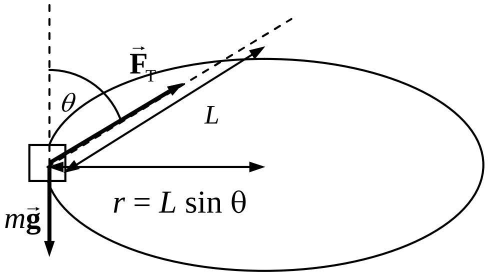
(*b*) The altitude of the satellite above the Earth's surface is given by

$$r - R_{\text{Earth}} = 7.304\times10^6\,\text{m} - 6.38\times10^6\,\text{m} = \boxed{9.2\times10^5\,\text{m}}.$$

91. The radial acceleration is given by $a_R = v^2/r$. Substitute in the speed of the tip of the sweep hand, given by $v = 2\pi r/T$, to get $a_R = \frac{4\pi^2 r}{T^2}$. For the tip of the sweep hand, $r = 0.015$ m, and $T = 60$ sec.

$$a_R = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(0.015\text{ m}\right)}{\left(60\text{ s}\right)^2} = \boxed{1.6\times10^{-4}\,\text{m/s}^2}.$$

92. A free-body diagram for the sinker weight is shown. L is the length of the string actually swinging the sinker. The radius of the circle of motion is moving is $r = L\sin\theta$. Write Newton's 2nd law for the vertical direction, noting that the sinker is not accelerating vertically. Take up to be positive.



$$\sum F_y = F_{\text{T}}\cos\theta - mg = 0 \quad \rightarrow \quad F_{\text{T}} = \frac{mg}{\cos\theta}$$

The radial force is the horizontal portion of the tension. Write Newton's 2nd law for the radial motion.

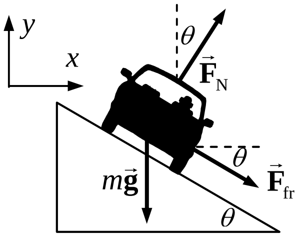
$$\sum F_{\text{R}} = F_{\text{T}}\sin\theta = ma_{\text{R}} = mv^2/r$$

Substitute the tension from the vertical equation, and the relationships $r = L\sin\theta$ and $v = 2\pi r/T$.

$$F_{\text{T}}\sin\theta = mv^2/r \quad \rightarrow \quad \frac{mg}{\cos\theta}\sin\theta = \frac{4\pi^2 mL\sin\theta}{T^2} \quad \rightarrow \quad \cos\theta = \frac{gT^2}{4\pi^2 L}$$

$$\theta = \cos^{-1}\frac{gT^2}{4\pi^2 L} = \cos^{-1}\frac{\left(9.8\,\text{m/s}^2\right)\left(0.50\text{ s}\right)^2}{4\pi^2\left(0.25\text{ m}\right)} = \boxed{76^\circ}$$

93. From Example 5-7 in the textbook, the no-friction banking angle is given by $\theta = \tan^{-1}\frac{v_0^2}{Rg}$. The centripetal force in this case is provided by a component of the normal force. Driving at a higher speed with the same radius requires more centripetal force than that provided by the normal force alone. The additional centripetal force is supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. The center of the circle of the car's motion is to the right of the car in the diagram. Write Newton's 2nd law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. Assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}}$.



$$\sum F_y = F_{\text{N}}\cos\theta - mg - F_{\text{fr}}\sin\theta = 0 \;\rightarrow\; F_{\text{N}}\cos\theta - \mu_s F_{\text{N}}\sin\theta = mg \;\rightarrow$$

$$F_{\text{N}} = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

$$\sum F_x = F_{\text{R}} = F_{\text{N}}\sin\theta + F_{\text{fr}}\cos\theta = mv^2/R \;\rightarrow\; F_{\text{N}}\sin\theta + \mu_s F_{\text{N}}\cos\theta = mv^2/R \;\rightarrow$$

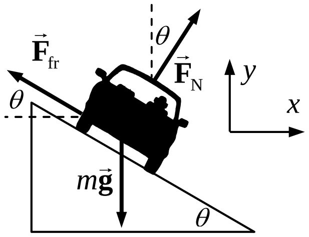
$$F_{\text{N}} = \frac{mv^2/R}{(\sin\theta + \mu_s\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed, which is the maximum speed that the car can have.

$$\frac{mv^2/R}{(\sin\theta + \mu_s\cos\theta)} = \frac{mg}{(\cos\theta - \mu_s\sin\theta)} \;\rightarrow$$

$$v_{\text{max}} = \sqrt{Rg\frac{\sin\theta}{\cos\theta}\frac{(1+\mu_s/\tan\theta)}{(1-\mu_s\tan\theta)}} = v_0\sqrt{\frac{(1+Rg\mu_s/v_0^2)}{(1-\mu_s v_0^2/Rg)}}$$

Driving at a slower speed with the same radius requires less centripetal force than that provided by the normal force alone. The decrease in centripetal force is supplied by a force of static friction, upward along the incline. See the free-body diagram for the car on the incline. Write Newton's 2nd law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. Assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}}$.



$$\sum F_y = F_{\text{N}}\cos\theta - mg + F_{\text{fr}}\sin\theta = 0 \;\rightarrow$$

$$F_{\text{N}}\cos\theta + \mu_s F_{\text{N}}\sin\theta = mg \;\rightarrow\; F_{\text{N}} = \frac{mg}{(\cos\theta + \mu_s\sin\theta)}$$

$$\sum F_x = F_{\text{R}} = F_{\text{N}}\sin\theta - F_{\text{fr}}\cos\theta = mv^2/R \;\rightarrow\; F_{\text{N}}\sin\theta - \mu_s F_{\text{N}}\cos\theta = mv^2/R \;\rightarrow$$

$$F_{\text{N}} = \frac{mv^2/R}{(\sin\theta - \mu_s\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/R}{(\sin\theta-\mu_s\cos\theta)}=\frac{mg}{(\cos\theta+\mu_s\sin\theta)} \quad\rightarrow$$

$$v_{\min}=\sqrt{Rg\frac{\sin\theta}{\cos\theta}\frac{(1-\mu_s/\tan\theta)}{(1+\mu_s\tan\theta)}}=v_0\sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}$$

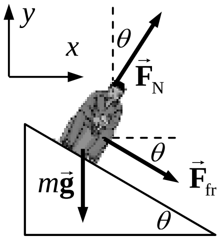
Thus $\boxed{v_{\min}=v_0\sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}}$ and $\boxed{v_{\max}=v_0\sqrt{\frac{(1+Rg\mu_s/v_0^2)}{(1-\mu_s v_0^2/Rg)}}}$.

94. The speed of the train is $(160\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)=44.44\,\text{m/s}$

(*a*) If there is no tilt, then the friction force must supply the entire centripetal force on the passenger.

$$F_\text{R}=mv^2/R=\frac{(75\text{ kg})(44.44\,\text{m/s})^2}{(620\text{ m})}=238.9\,\text{N}\approx\boxed{2.4\times10^2\,\text{N}}$$

(*b*) For the banked case, the normal force will contribute to the radial force needed. Write Newton's 2nd law for both the *x* and *y* directions. The *y* acceleration is zero, and the *x* acceleration is radial.



$$\sum F_y=F_\text{N}\cos\theta-mg-F_\text{fr}\sin\theta=0 \quad\rightarrow\quad F_\text{N}=\frac{mg+F_\text{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x=F_\text{N}\sin\theta+F_\text{fr}\cos\theta=mv^2/r$$

Substitute the expression for the normal force from the *y* equation into the *x* equation, and solve for the friction force.

$$\frac{mg+F_\text{fr}\sin\theta}{\cos\theta}\sin\theta+F_\text{fr}\cos\theta=mv^2/r \quad\rightarrow$$

$$(mg+F_\text{fr}\sin\theta)\sin\theta+F_\text{fr}\cos^2\theta=m\frac{v^2}{r}\cos\theta \quad\rightarrow$$

$$F_\text{fr}=m\left(\frac{v^2}{r}\cos\theta-g\sin\theta\right)$$

$$=(75\text{ kg})\left[\frac{(44.44\,\text{m/s})^2}{620\text{ m}}\cos8.0^\circ-(9.80\,\text{m/s}^2)\sin8.0^\circ\right]=134\,\text{N}\approx\boxed{1.3\times10^2\,\text{N}}$$

CHAPTER 6: Work and Energy

Answers to Questions

1. Some types of physical labor, particularly if it involves lifting objects, such as shoveling dirt or carrying shingles up to a roof, are “work” in the physics sense of the word. Or, pushing a lawn mower would be work corresponding to the physics definition. When we use the word “work” for employment, such as “go to work” or “school work”, there is often no sense of physical labor or of moving something through a distance by a force.

2. Since “centripetal” means “pointing to the center of curvature”, then a centripetal force will not do work on an object, because if an object is moving in a curved path, by definition the direction towards the center of curvature is always perpendicular to the direction of motion. For a force to do work, the force must have a component in the direction of displacement. So the centripetal force does no work.

3. The normal force can do work on an object if the normal force has a component in the direction of displacement of an object. If someone were to jump up in the air, then the floor pushing upward on the person (the normal force) would do positive work and increase the person’s kinetic energy. Likewise when they hit the floor coming back down, the force of the floor pushing upwards (the normal force) would do negative work and decrease the person’s kinetic energy.

4. The woman does work by moving the water with her hands and feet, because she must exert a force to move the water some distance. As she stops swimming and begins to float in the current, the current does work on her because she gains kinetic energy. Once she is floating the same speed as the water, her kinetic energy does not change, and so no net work is being done on her.

5. The kinetic force of friction opposes the relative motion between two objects. As in the example suggested, as the tablecloth is pulled from under the dishes, the relative motion is for the dishes to be left behind as the tablecloth is pulled, and so the kinetic friction opposes that and moves the dishes in the same direction as the tablecloth. This is a force that is in the direction of displacement, and so positive work is done. Also note that the cloth is moving faster than the dishes in this case, so that the friction is kinetic, not static.

6. While it is true that no work is being done on the wall by you, there is work being done inside your arm muscles. Exerting a force via a muscle causes small continual motions in your muscles, which is work, and which causes you to tire. An example of this is holding a heavy load at arm’s length. While at first you may hold the load steady, after a time your arm will begin to shake, which indicates the motion of muscles in your arm.

7. (*a*) In this case, the same force is applied to both springs. Spring 1 will stretch less, and so more work is done on spring 2.
   (*b*) In this case, both springs are stretched the same distance. It takes more force to stretch spring 1, and so more work is done on spring 1.

8. At point C the block’s speed will be less than $2v_B$. The same amount of work was done on the block in going from A to B as from B to C since the force and the displacement are the same for each segment. Thus the change in kinetic energy will be the same for each segment. From A to B, the

block gained $\frac{1}{2}mv_{\mathrm{B}}^2$ of kinetic energy. If the same amount is gained from B to C, then the total kinetic energy at C is $\frac{1}{2}mv_{\mathrm{C}}^2 = 2\left(\frac{1}{2}mv_{\mathrm{B}}^2\right)$ which results in $v_{\mathrm{C}} = \sqrt{2}\, v_{\mathrm{B}}$, or $v_{\mathrm{C}} \approx 1.4 v_{\mathrm{B}}$

9. Your gravitational PE will change according to $\Delta PE = mg\Delta y$. If we choose some typical values of $m = 80\text{ kg}$ and $\Delta y = 0.75\text{ m}$, then $\Delta PE = (80\text{ kg})(9.8\,\text{m/s}^2)(0.75\text{ m}) = 590\text{ J}$

$\boxed{10.}$ Since each balloon has the same initial kinetic energy, and each balloon undergoes the same overall change in gravitational PE, each balloon will have the same kinetic energy at the ground, and so each one has the same speed at impact.

11. The two launches will result in the same largest angle. Applying conservation of energy between the launching point and the highest point, we have $E_1 = E_2 \quad \rightarrow \quad \frac{1}{2}mv^2 + mgh = mgh_{\max}$. The direction of the launching velocity does not matter, and so the same maximum height (and hence maximum angle) will results from both launches. Also, for the first launch, the ball will rise to some maximum height and then come back to the launch point with the same speed as when launched. That then exactly duplicates the second launch.

12. The spring can leave the table if it is compressed enough. If the spring is compressed an amount x_0, then the gain in elastic PE is $\frac{1}{2}kx_0^2$. As the spring is compressed, its center of mass is lowered by some amount. If the spring is uniform, then the center of mass is lowered by $x_0/2$, and the amount of decrease in gravitational PE is $\frac{1}{2}mgx_0$. If the gain in elastic PE is more than the loss in gravitational PE, so that $\frac{1}{2}kx_0^2 > \frac{1}{2}mgx_0$ or $x_0 > mg/k$, then the released spring should rise up off of the table, because there is more than enough elastic PE to restore the spring to its original position. That extra elastic energy will enable the spring to "jump" off the table – it can raise its center of mass to a higher point and thus rise up off the table. Where does that "extra" energy come from? From the work you did in compressing the spring.

$\boxed{13.}$ If the instructor releases the ball without pushing it, the ball should return to exactly the same height (barring any dissipative forces) and just touch the instructor's nose as it stops. But if the instructor pushes the ball, giving it extra kinetic energy and hence a larger total energy, the ball will then swing to a higher point before stopping, and hit the instructor in the face when it returns.

$\boxed{14.}$ When water at the top of a waterfall falls to the pool below, initially the water's gravitational PE is turned into kinetic energy. That kinetic energy then can do work on the pool water when it hits it, and so some of the pool water is given energy, which makes it splash upwards and outwards and creates outgoing water waves, which carry energy. Some of the energy will become heat, due to viscous friction between the falling water and the pool water. Some of the energy will become kinetic energy of air molecules, making sound waves that give the waterfall its "roar".

15. Start the description with the child suspended in mid-air, at the top of a hop. All of the energy is gravitational PE at that point. Then, the child falls, and gains kinetic energy. When the child reaches the ground, most of the energy is kinetic. As the spring begins to compress, the kinetic energy is changed into elastic PE. The child also goes down a little bit further as the spring compresses, and so more gravitational PE is also changed into elastic PE. At the very bottom of a hop, the energy is all elastic PE. Then as the child rebounds, the elastic PE is turned into kinetic energy and gravitational PE. When the child reaches the top of the bounce, all of the elastic PE has

been changed into gravitational PE, because the child has a speed of 0 at the top. Then the cycle starts over again. Due to friction, the child must also add energy to the system by pushing down on the pogo stick while it is on the ground, getting a more forceful reaction from the ground.

16. As the skier goes down the hill, the gravitational PE is transformed mostly into kinetic energy, and small amount is transformed into heat energy due to the friction between the skis and the snow and air friction. As the skier strikes the snowdrift, the kinetic energy of the skier turns into kinetic energy of the snow (by making the snow move), and also into some heat from the friction in moving through the snowdrift.

17. (*a*) If there is no friction to dissipate any of the energy, then the gravitational PE that the child has at the top of the hill all turns into kinetic energy at the bottom of the hill. The same kinetic energy will be present regardless of the slope – the final speed is completely determined by the height. The time it takes to reach the bottom of the hill will be longer for a smaller slope.

    (*b*) If there is friction, then the longer the path is, the more work that friction will do, and so the slower the speed will be at the bottom. So for a steep hill, the sled will have a greater speed at the bottom than for a shallow hill.

18. Stepping on the log requires that the entire body mass be raised up the height of the log, requiring work (that is not recoverable) proportional to the entire body mass. Stepping over the log only requires the raising of the legs, making for a small mass being raised and thus less work. Also, when jumping down, energy is expended to stop the "fall" from the log. The potential energy that you had at the top of the log is lost when coming down from the log.

19. If we assume that all of the arrow's kinetic energy is converted into work done against friction, then the following relationship exists:

$$W = \Delta KE = KE_f - KE_i \quad \rightarrow \quad F_{\mathrm{fr}} d \cos 180^{\circ} = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_0^2 \quad \rightarrow \quad -F_{\mathrm{fr}} d = -\tfrac{1}{2} m v_0^2 \quad \rightarrow$$

$$d = \frac{m v_0^2}{2 F_{\mathrm{fr}}}$$

Thus the distance is proportional to the square of the initial velocity. So if the initial velocity is doubled, the distance will be multiplied by a factor of 4. Thus the faster arrow penetrates 4 times further than the slower arrow.

20. (*a*) Consider that there is no friction to dissipate any energy. Start the pendulum at the top of a swing, and define the lowest point of the swing as the zero location for gravitational PE. The pendulum has maximum gravitational PE at the top of a swing. Then as it falls, the gravitational PE is changed to kinetic energy. At the bottom of the swing, the energy is all kinetic energy. Then the pendulum starts to rise, and kinetic energy is changed to gravitational PE. Since there is no dissipation, all of the original gravitational PE is converted to kinetic energy, and all of the kinetic energy is converted to gravitational PE. The pendulum rises to the same height on both sides of every swing, and reaches the same maximum speed at the bottom on every swing.

    (*b*) If there is friction to dissipate the energy, then on each downward swing, the pendulum will have less kinetic energy at the bottom than it had gravitational PE at the top. And then on each swing up, the pendulum will not rise as high as the previous swing, because energy is being lost to frictional dissipation any time the pendulum is moving. So each time it swings, it has a smaller maximum displacement. When a grandfather clock is wound up, a weight is elevated so that it has some PE. That weight then falls at the proper rate to put energy back in to the pendulum to replace the energy that was lost to dissipation.

21. The superball cannot rebound to a height greater than its original height when dropped. If it did, it would violate conservation of energy. When a ball collides with the floor, the KE of the ball is converted into elastic PE by deforming the ball, much like compressing a spring. Then as the ball springs back to its original shape, that elastic PE is converted to back to KE. But that process is "lossy" – not all of the elastic PE gets converted back to KE. Some of the PE is lost, primarily to friction. The superball rebounds higher than many other balls because it is less "lossy" in its rebound than many other materials.

22. The work done to lift the suitcase is equal to the change in PE of the suitcase, which is the weight of the suitcase times the change in height (the height of the table).
    (*a*) Work does NOT depend on the path, as long as there are no non-conservative forces doing work.
    (*b*) Work does NOT depend on the time taken.
    (*c*) Work DOES depend on the height of the table – the higher the table, the more work it takes to lift the suitcase.
    (*d*) Work DOES depend on the weight of the suitcase – the more the suitcase weighs, the more work it takes to lift the suitcase.

23. The power needed to lift the suitcase is the work required to lift the suitcase, divided by the time that it takes.
    (*a*) Since work does NOT depend on the path, the power will not depend on the path either, assuming the time is the same for all paths.
    (*b*) The power DOES depend on the time taken. The more time taken, the lower the power needed.
    (*c*) The power needed DOES depend on the height of the table. A higher table requires more work to lift the suitcase. If we assume that the time to lift the suitcase is the same in both cases, then to lift to the higher table takes more power.
    (*d*) The power DOES depend on the weight of the suitcase. A heavier suitcase requires more force to lift, and so requires more work. Thus the heavier the suitcase, the more power is needed to lift it (in the same amount of time).

24. The climber does the same amount of work whether climbing straight up or via a zig-zag path, ignoring dissipative forces. But if a longer zig-zag path is taken, it takes more time to do the work, and so the power output needed from the climber is less. That will make the climb easier. It is easier for the human body to generate a small amount of power for long periods of time rather than to generate a large power for a small period of time.

25. Assuming that there are no dissipative forces to consider, for every meter that the load is raised, two meters of rope must be pulled up. This is due to the rope passing over the bottom pulley. The work done by the person pulling must be equal to the work done on the piano. Since the force on the piano is twice that exerted by the person pulling, and since work is force times distance, the person must exert their smaller force over twice the distance that the larger pulley force moves the piano.

Solutions to Problems

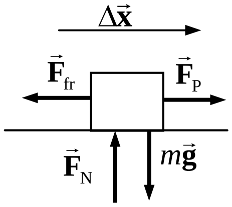
1. The force and the displacement are both downwards, so the angle between them is 0°.

$$W_G = mgd\cos\theta = (265\text{ kg})(9.80\text{ m/s}^2)(2.80\text{ m})\cos 0^\circ = \boxed{7.27\times 10^3\text{ J}}$$

2. The minimum force required to lift the firefighter is equal to his weight. The force and the displacement are both upwards, so the angle between them is 0°.

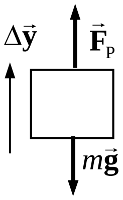
$$W_{\text{climb}} = F_{\text{climb}} d \cos\theta = mgd \cos\theta = (65.0 \text{ kg})(9.80 \text{ m/s}^2)(20.0 \text{m}) \cos 0^\circ = \boxed{1.27 \times 10^4 \text{ J}}$$

3. (*a*) See the free-body diagram for the crate as it is being pulled. Since the crate is not accelerating horizontally, $F_P = F_{fr} = 230 \text{ N}$. The work done to move it across the floor is the work done by the pulling force. The angle between the pulling force and the direction of motion is 0°.


$\Delta\vec{\mathbf{x}}$
$\vec{\mathbf{F}}_{\text{fr}}$
$\vec{\mathbf{F}}_{\text{P}}$
$\vec{\mathbf{F}}_{\text{N}}$
$m\vec{\mathbf{g}}$

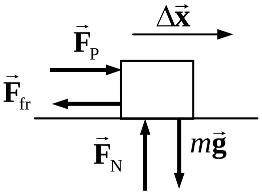
$$W_P = F_P d \cos 0^\circ = (230 \text{ N})(4.0 \text{ m})(1) = \boxed{9.2 \times 10^2 \text{ J}}$$

(*b*) See the free-body diagram for the crate as it is being lifted. Since the crate is not accelerating vertically, the pulling force is the same magnitude as the weight. The angle between the pulling force and the direction of motion is 0°.



$$W_P = F_P d \cos 0^\circ = mgd = (1300 \text{ N})(4.0 \text{ m}) = \boxed{5.2 \times 10^3 \text{ J}}$$

4. Draw a free-body diagram for the crate as it is being pushed across the floor. Since it is not accelerating vertically, $F_N = mg$. Since it is not accelerating horizontally, $F_P = F_{fr} = \mu_k F_N = \mu_k mg$. The work done to move it across the floor is the work done by the pushing force. The angle between the pushing force and the direction of motion is 0°.


$\Delta\vec{\mathbf{x}}$
$\vec{\mathbf{F}}_{\text{P}}$
$\vec{\mathbf{F}}_{\text{fr}}$
$\vec{\mathbf{F}}_{\text{N}}$
$m\vec{\mathbf{g}}$

$$W_{\text{push}} = F_{\text{push}} d \cos 0^\circ = \mu_k mgd\,(1) = (0.50)(160 \text{ kg})(9.80 \text{ m/s}^2)(10.3 \text{ m})$$
$$= \boxed{8.1 \times 10^3 \text{ J}}$$

5. Since the acceleration of the box is constant, use Eq. 2-11b to find the distance moved. Assume that the box starts from rest.

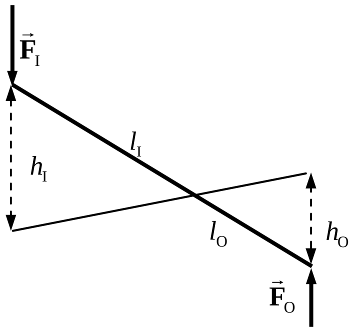
$$\Delta x = x - x_0 = v_0 t + \tfrac{1}{2} a t^2 = 0 + \tfrac{1}{2}(2.0 \text{ m/s}^2)(7 \text{ s})^2 = 49 \text{ m}$$

Then the work done in moving the crate is

$$W = F \Delta x \cos 0^\circ = ma\Delta x = (5 \text{ kg})(2.0 \text{ m/s}^2)(49 \text{ m}) = \boxed{4.9 \times 10^2 \text{ J}}$$

6. The first book is already in position, so no work is required to position it. The second book must be moved upwards by a distance *d*, by a force equal to its weight, *mg*. The force and the displacement are in the same direction, so the work is *mgd*. The third book will need to be moved a distance of 2*d* by the same size force, so the work is 2*mgd*, This continues through all seven books, with each needing to be raised by an additional amount of *d* by a force of *mg*. The total work done is

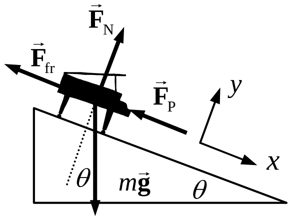
$$W = mgd + 2mgd + 3mgd + 4mgd + 5mgd + 6mgd + 7mgd$$
$$= 28mgd = 28(1.7 \text{ kg})(9.8 \text{ m/s}^2)(0.043 \text{ m}) = \boxed{2.0 \times 10^1 \text{ J}}.$$



7. Consider the diagram shown. If we assume that the man pushes straight down on the end of the lever, then the work done by the man (the “input” work) is given by $W_\mathrm{I} = F_\mathrm{I} h_\mathrm{I}$. The object moves a shorter distance, as seen from the diagram, and so $W_\mathrm{O} = F_\mathrm{O} h_\mathrm{O}$. Equate the two amounts of work.

$$F_\mathrm{O} h_\mathrm{O} = F_\mathrm{I} h_\mathrm{I} \rightarrow \frac{F_\mathrm{O}}{F_\mathrm{I}} = \frac{h_\mathrm{I}}{h_\mathrm{O}}$$

But by similar triangles, we see that $\frac{h_\mathrm{I}}{h_\mathrm{O}} = \frac{l_\mathrm{I}}{l_\mathrm{O}}$, and so $\boxed{\frac{F_\mathrm{O}}{F_\mathrm{I}} = \frac{l_\mathrm{I}}{l_\mathrm{O}}}$.


$\vec{\mathbf{F}}_\mathrm{N}$
$\vec{\mathbf{F}}_\mathrm{fr}$
$\vec{\mathbf{F}}_\mathrm{P}$
y
x
θ
$m\vec{\mathbf{g}}$
θ

8. The piano is moving with a constant velocity down the plane. $\vec{\mathbf{F}}_\mathrm{P}$ is the force of the man pushing on the piano.

(*a*) Write Newton’s 2nd law on each direction for the piano, with an acceleration of 0.

$$\sum F_y = F_\mathrm{N} - mg\cos\theta = 0 \rightarrow F_\mathrm{N} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_\mathrm{P} - F_\mathrm{fr} = 0 \rightarrow$$

$$F_\mathrm{P} = mg\sin\theta - F_\mathrm{fr} = mg\left(\sin\theta - \mu_k\cos\theta\right)$$

$$= \left(330\text{ kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\left(\sin 28^\circ - 0.40\cos 28^\circ\right) = \boxed{3.8\times10^2\text{ N}}$$

(*b*) The work done by the man is the work done by $\vec{\mathbf{F}}_\mathrm{P}$. The angle between $\vec{\mathbf{F}}_\mathrm{P}$ and the direction of motion is 180°.

$$W_\mathrm{P} = F_\mathrm{P} d\cos 180^\circ = -\left(380\text{ N}\right)\left(3.6\text{ m}\right) = \boxed{-1.4\times10^3\text{ J}}.$$

(*c*) The angle between $\vec{\mathbf{F}}_\mathrm{fr}$ and the direction of motion is 180°.

$$W_\mathrm{fr} = F_\mathrm{fr} d\cos 180^\circ = -\mu_k mgd\cos\theta = -\left(0.40\right)\left(330\text{ kg}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(3.6\text{ m}\right)\cos 28^\circ$$

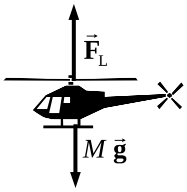
$$= \boxed{-4.1\times10^3\text{ J}}$$

(*d*) The angle between the force of gravity and the direction of motion is 62°. So the work done by gravity is

$$W_G = F_G d\cos 62^\circ = mgd\cos 62^\circ = \left(330\text{ kg}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(3.6\text{ m}\right)\cos 62^\circ = \boxed{5.5\times10^3\text{ J}}.$$

(*e*) Since the piano is unaccelerated, the net force on the piano is 0, and so the net work done on the piano is also 0. This can also be seen by adding the three work amounts calculated.

$$W_\mathrm{Net} = W_\mathrm{P} + W_\mathrm{fr} + W_\mathrm{G} = -1400\text{ J} - 4100\text{ J} + 5500\text{ J} = \boxed{0\text{ J}}$$

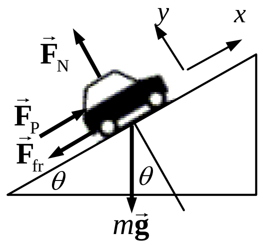


9. (*a*) Write Newton’s 2nd law for the vertical direction, with up as positive.

$$\sum F_y = F_\mathrm{L} - Mg = Ma = M\left(0.10g\right) \rightarrow F_\mathrm{L} = \boxed{1.10\,Mg}$$

(*b*) The lifting force and the displacement are in the same direction, so the work done by the lifting force in lifting the helicopter a vertical distance *h* is

$$W_\mathrm{L} = F_\mathrm{L} h\cos 0^\circ = \boxed{1.10\,Mgh}.$$



y
x
$\vec{\mathbf{F}}_\mathrm{N}$
$\vec{\mathbf{F}}_\mathrm{P}$
$\vec{\mathbf{F}}_\mathrm{fr}$
θ
θ
$m\vec{\mathbf{g}}$

10. Draw a free-body diagram of the car on the incline. Include a frictional force, but ignore it in part (*a*) of the problem. The minimum work will occur when the car is moved at a constant velocity.

(*a*) Write Newton's 2nd law in both the *x* and *y* directions, noting that the car is unaccelerated.

$$\sum F_y = F_\mathrm{N} - mg\cos\theta = 0 \quad\rightarrow\quad F_\mathrm{N} = mg\cos\theta$$

$$\sum F_x = F_\mathrm{P} - mg\sin\theta = 0 \quad\rightarrow\quad F_\mathrm{P} = mg\sin\theta$$

The work done by $\vec{\mathbf{F}}_\mathrm{P}$ in moving the car a distance *d* along the plane (parallel to $\vec{\mathbf{F}}_\mathrm{P}$) is given by

$$W_\mathrm{P} = F_\mathrm{P} d\cos 0^\circ = mgd\sin\theta = (950\ \mathrm{kg})(9.80\,\mathrm{m/s^2})(810\ \mathrm{m})\sin 9.0^\circ = \boxed{1.2\times10^6\,\mathrm{J}}$$

(*b*) Now include the frictional force, given by $F_\mathrm{fr} = \mu_k F_\mathrm{N}$. We still assume that the car is not accelerated. We again write Newton's 2nd law for each direction. The *y*-forces are unchanged by the addition of friction, and so we still have $F_\mathrm{N} = mg\cos\theta$.

$$\sum F_x = F_\mathrm{P} - F_\mathrm{fr} - mg\sin\theta = 0 \quad\rightarrow\quad F_\mathrm{P} = F_\mathrm{fr} + mg\sin\theta = \mu_k mg\cos\theta + mg\sin\theta\,.$$

The work done by $\vec{\mathbf{F}}_\mathrm{P}$ in moving the car a distance *d* along the plane (parallel to $\vec{\mathbf{F}}_\mathrm{P}$) is given by

$$W_\mathrm{P} = F_\mathrm{P} d\cos 0^\circ = mgd(\sin\theta + \mu_k\cos\theta)$$

$$= (950\ \mathrm{kg})(9.80\,\mathrm{m/s^2})(810\ \mathrm{m})(\sin 9.0^\circ + 0.25\cos 9.0^\circ) = \boxed{3.0\times10^6\,\mathrm{J}}$$

11. The work done is equal to the area under the graph. The area is roughly trapezoidal, and so the area of the region is found as follows.

$$W = \tfrac{1}{2}(F_{\|\max} + F_{\|\min})(d_B - d_A) = \tfrac{1}{2}(250\ \mathrm{N} + 150\ \mathrm{N})(35.0\ \mathrm{m} - 10.0\ \mathrm{m}) = \boxed{5.0\times10^3\,\mathrm{J}}$$

12. The work done will be the area under the F_x vs. *x* graph.

(*a*) From $x = 0.0$ to $x = 10.0\ \mathrm{m}$, the shape under the graph is trapezoidal. The area is

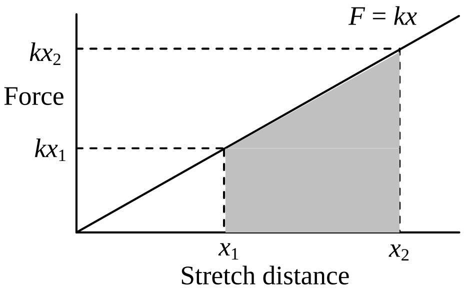
$$W_a = (400\ \mathrm{N})\frac{10\ \mathrm{m} + 4\ \mathrm{m}}{2} = \boxed{2.8\times10^3\,\mathrm{J}}$$

(*b*) From $x = 10.0\ \mathrm{m}$ to $x = 15.0\ \mathrm{m}$, the force is in the opposite direction from the direction of motion, and so the work will be negative. Again, since the shape is trapezoidal, we find

$$W_a = (-200\ \mathrm{N})\frac{5\ \mathrm{m} + 2\ \mathrm{m}}{2} = -700\ \mathrm{J}\,.$$

Thus the total work from $x = 0.0$ to $x = 15.0\ \mathrm{m}$ is $2800\ \mathrm{J} - 700\ \mathrm{J} = \boxed{2.1\times10^3\,\mathrm{J}}$

13. The force exerted to stretch a spring is given by $F_\mathrm{stretch} = kx$ (the opposite of the force exerted by the spring, which is given by $F = -kx$. A graph of F_stretch vs. *x* will be a straight line of slope *k* thorough the origin. The stretch from x_1 to x_2, as shown on the graph, outlines a trapezoidal area. This area represents the work, and is calculated by

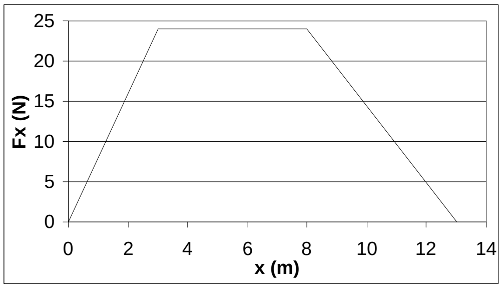


$$W = \tfrac{1}{2}(kx_1 + kx_2)(x_2 - x_1) = \tfrac{1}{2}k(x_1 + x_2)(x_2 - x_1)$$

$$= \tfrac{1}{2}(88\,\mathrm{N/m})(0.096\ \mathrm{m})(0.020\ \mathrm{m}) = \boxed{8.4\times10^{-2}\,\mathrm{J}}.$$

14. See the graph of force vs. distance. The work done is the area under the graph. It can be found from the formula for a trapezoid.

$$W = \tfrac{1}{2}(13.0\text{ m} + 5.0\text{ m})(24.0\text{ N}) = \boxed{216\text{ J}}$$



15. Find the velocity from the kinetic energy, using Eq. 6-3.

$$KE = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2(KE)}{m}} = \sqrt{\frac{2\left(6.21\times10^{-21}\text{ J}\right)}{5.31\times10^{-26}}} = \boxed{484\text{ m/s}}$$

16. (*a*) Since $KE = \tfrac{1}{2}mv^2$, then $v = \sqrt{2(KE)/m}$ and so $v \propto \sqrt{KE}$. Thus if the kinetic energy is doubled, the speed will be multiplied by a factor of $\boxed{\sqrt{2}}$.

(*b*) Since $KE = \tfrac{1}{2}mv^2$, then $KE \propto v^2$. Thus if the speed is doubled, the kinetic energy will be multiplied by a factor of $\boxed{4}$.

17. The work done on the electron is equal to the change in its kinetic energy.

$$W = \Delta KE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(1.90\times10^{6}\text{ m/s}\right)^2 = \boxed{-1.64\times10^{-18}\text{ J}}$$

18. The work done on the car is equal to the change in its kinetic energy, and so

$$W = KE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(1250\text{ kg})\left[(105\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2 = \boxed{-5.32\times10^{5}\text{ J}}$$

19. The force exerted by the bow on the arrow is in the same direction as the displacement of the arrow. Thus $W = Fd\cos 0^\circ = Fd = (110\text{ N})(0.78\text{ m}) = 85.8\text{ J}$. But that work changes the KE of the arrow, by the work-energy theorem. Thus

$$Fd = W = KE_2 - KE_1 = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad \rightarrow \quad v_2 = \sqrt{\frac{2Fd}{m} + v_1^2} = \sqrt{\frac{2(85.8\text{ J})}{0.088\text{ kg}} + 0} = \boxed{44\text{ m/s}}$$

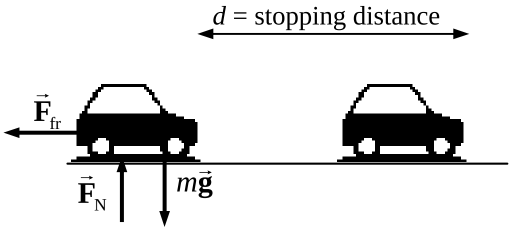
$\boxed{20.}$ The work done by the ball on the glove will be the opposite of the work done by the glove on the ball. The work done on the ball is equal to the change in the kinetic energy of the ball.

$$W_{\text{on ball}} = \left(KE_2 - KE_1\right)_{\text{ball}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(0.140\text{ kg})(32\text{ m/s})^2 = -72\text{ J}$$

So $W_{\text{on glove}} = 72\text{ J}$. But $W_{\text{on glove}} = F_{\text{on glove}}\,d\cos 0^\circ$, because the force on the glove is in the same direction as the motion of the glove.

$$72\text{ J} = F_{\text{on glove}}(0.25\text{ m}) \quad \rightarrow \quad F_{\text{on glove}} = \frac{72\text{ J}}{0.25\text{ m}} = \boxed{2.9\times10^{2}\text{ N}}.$$

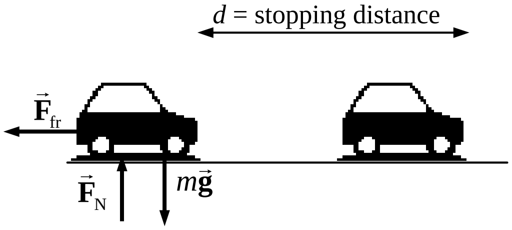
21. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car. Assume the maximum possible frictional force, which results in the minimum braking distance. Thus $F_{\text{fr}} = \mu_s F_{\text{N}}$. The normal force is equal to the car's weight if it is on a level surface, and so $F_{\text{fr}} = \mu_s mg$. In the diagram, the car is traveling to the right.



$$W = \Delta KE \quad \rightarrow \quad F_{\text{fr}} d \cos 180^{\circ} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad \rightarrow \quad -\mu_s mgd = -\tfrac{1}{2}mv_1^2 \quad \rightarrow \quad d = \frac{v_1^2}{2g\mu_s}$$

Since $d \propto v_1^2$, if v_1 increases by 50%, or is multiplied by 1.5, then d will be multiplied by a factor of $(1.5)^2$, or $\boxed{2.25}$.

$\boxed{22.}$ The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car, which is assumed to be static friction since the driver locked the brakes. Thus $F_{\text{fr}} = \mu_k F_{\text{N}}$. Since the car is on a level surface, the normal force is equal to the car's weight, and so $F_{\text{fr}} = \mu_k mg$ if it is on a level surface. See the diagram for the car. The car is traveling to the right.



$$W = \Delta KE \quad \rightarrow \quad F_{\text{fr}} d \cos 180^{\circ} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad \rightarrow \quad -\mu_k mgd = 0 - \tfrac{1}{2}mv_1^2 \quad \rightarrow$$

$$v_1 = \sqrt{2\mu_k gd} = \sqrt{2(0.42)(9.8\,\text{m/s}^2)(88\text{ m})} = \boxed{27\,\text{m/s}}$$

The mass does not affect the problem, since both the change in kinetic energy and the work done by friction are proportional to the mass. The mass cancels out of the equation.

23. The original speed of the softball is $(95\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s}$. The final speed is 90% of this, or 23.75 m/s. The work done by air friction causes a change in the kinetic energy of the ball, and thus the speed change. In calculating the work, notice that the force of friction is directed oppositely to the direction of motion of the ball.

$$W_{\text{fr}} = F_{\text{fr}} d \cos 180^{\circ} = KE_2 - KE_1 = \tfrac{1}{2}m\left(v_2^2 - v_1^2\right) \quad \rightarrow$$

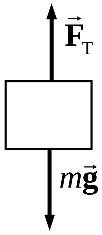
$$F_{\text{fr}} = \frac{m\left(v_2^2 - v_1^2\right)}{-2d} = \frac{mv_1^2\left(0.9^2 - 1\right)}{-2d} = \frac{(0.25\text{ kg})(26.39\,\text{m/s})^2\left(0.9^2 - 1\right)}{-2(15\text{ m})} = \boxed{1.1\text{ N}}$$

24. If the rock has 80.0 J of work done to it, and it loses all 80.0 J by stopping, then the force of gravity must have done –80.0 J of work on the rock. The force is straight down, and the displacement is straight up, so the angle between the force and the displacement is 180°. The work done by the gravity force can be used to find the distance the rock rises.

$$W_{\text{G}} = F_{\text{G}} d \cos\theta = mgd \cos 180^{\circ} = -80.0\text{ J}$$

$$d=\frac{W_{\mathrm{G}}}{-mg}=\frac{-80.0\ \mathrm{J}}{-(1.85\ \mathrm{kg})(9.80\,\mathrm{m/s^2})}=\boxed{4.41\ \mathrm{m}}$$

25. (*a*) From the free-body diagram for the load being lifted, write Newton's 2nd law for the vertical direction, with up being positive.



$$\sum F=F_{\mathrm{T}}-mg=ma=0.160mg \quad\rightarrow$$

$$F_{\mathrm{T}}=1.16mg=1.16(285\ \mathrm{kg})(9.80\,\mathrm{m/s^2})=\boxed{3.24\times10^3\,\mathrm{N}}$$

(*b*) The net work done on the load is found from the net force.

$$W_{\mathrm{net}}=F_{\mathrm{net}}d\cos 0^{\circ}=(0.160mg)d=0.160(285\ \mathrm{kg})(9.80\,\mathrm{m/s^2})(22.0\ \mathrm{m})$$

$$=\boxed{9.83\times10^3\,\mathrm{J}}$$

(*c*) The work done by the cable on the load is

$$W_{\mathrm{cable}}=F_{\mathrm{T}}d\cos 0^{\circ}=(1.160mg)d=1.16(285\ \mathrm{kg})(9.80\,\mathrm{m/s^2})(22.0\ \mathrm{m})=\boxed{7.13\times10^4\,\mathrm{J}}$$

(*d*) The work done by gravity on the load is

$$W_{\mathrm{G}}=mgd\cos180^{\circ}=-mgd=-(285\ \mathrm{kg})(9.80\,\mathrm{m/s^2})(22.0\ \mathrm{m})=\boxed{-6.14\times10^4\,\mathrm{J}}$$

(*e*) Use the work-energy theory to find the final speed, with an initial speed of 0.

$$W_{\mathrm{net}}=KE_2-KE_1=\tfrac{1}{2}mv_2^2-\tfrac{1}{2}mv_1^2 \quad\rightarrow$$

$$v_2=\sqrt{\frac{2W_{net}}{m}+v_1^2}=\sqrt{\frac{2(9.83\times10^3\,\mathrm{J})}{285\ \mathrm{kg}}+0}=\boxed{8.31\,\mathrm{m/s}}$$

26. The elastic PE is given by $PE_{\mathrm{elastic}}=\frac{1}{2}kx^2$ where *x* is the distance of stretching or compressing of the spring from its natural length.

$$x=\sqrt{\frac{2PE_{\mathrm{elastic}}}{k}}=\sqrt{\frac{2(25\ \mathrm{J})}{440\,\mathrm{N/m}}}=\boxed{0.34\ \mathrm{m}}$$

27. Subtract the initial gravitational PE from the final gravitational PE.

$$\Delta PE_{\mathrm{G}}=mgy_2-mgy_1=mg(y_2-y_1)=(7.0\ \mathrm{kg})(9.8\,\mathrm{m/s^2})(1.2\ \mathrm{m})=\boxed{82\ \mathrm{J}}$$

28. Subtract the initial gravitational PE from the final gravitational PE.

$$\Delta PE_{\mathrm{grav}}=mgy_2-mgy_1=mg(y_2-y_1)=(64\ \mathrm{kg})(9.8\,\mathrm{m/s^2})(4.0\ \mathrm{m})=\boxed{2.5\times10^3\,\mathrm{J}}$$

29. Assume that all of the kinetic energy of the car becomes PE of the compressed spring.

$$\tfrac{1}{2}mv^2=\tfrac{1}{2}kx^2 \quad\rightarrow\quad k=\frac{mv^2}{x^2}=\frac{(1200\ \mathrm{kg})\left[(65\,\mathrm{km/h})\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^2}{(2.2\ \mathrm{m})^2}=\boxed{8.1\times10^4\ \mathrm{N/m}}$$

30. (*a*) Relative to the ground, the PE is given by

$$PE_{\mathrm{G}}=mg(y_{\mathrm{book}}-y_{\mathrm{ground}})=(2.10\ \mathrm{kg})(9.80\,\mathrm{m/s^2})(2.20\ \mathrm{m})=\boxed{45.3\ \mathrm{J}}$$

(*b*) Relative to the top of the person's head, the PE is given by

$$PE_G = mg\left(y_{\text{book}} - y_{\text{head}}\right)h = (2.10\text{ kg})\left(9.80\,\text{m/s}^2\right)(0.60\text{ m}) = \boxed{12\text{ J}}$$

(*c*) The work done by the person in lifting the book from the ground to the final height is the same as the answer to part (a), $\boxed{45.3\text{ J}}$. In part (*a*), the PE is calculated relative to the starting location of the application of the force on the book. The work done by the person is not related to the answer to part (*b*).

31. (*a*) The change in PE is given by

$$\Delta PE_G = mg\left(y_2 - y_1\right) = (55\text{ kg})\left(9.80\,\text{m/s}^2\right)(3300\text{ m} - 1600\text{ m}) = \boxed{9.2\times 10^5\text{ J}}$$

(*b*) The minimum work required by the hiker would equal the change in PE, which is $\boxed{9.2\times 10^5\text{ J}}$.

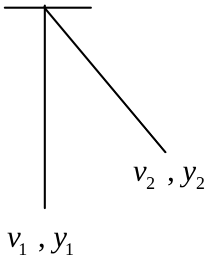
(*c*) $\boxed{\text{Yes}}$. The actual work may be more than this, because the climber almost certainly had to overcome some dissipative forces such as air friction. Also, as the person steps up and down, they do not get the full amount of work back from each up-down event. For example, there will be friction in their joints and muscles.

32. The spring will stretch enough to hold up the mass. The force exerted by the spring will be equal to the weight of the mass.

$$mg = k(\Delta x) \quad\to\quad \Delta x = \frac{mg}{k} = \frac{(2.5\text{ kg})\left(9.80\,\text{m/s}^2\right)}{53\,\text{N/m}} = 0.46\text{ m}$$

Thus the ruler reading will be $\boxed{46\,\text{cm} + 15\,\text{cm} = 61\,\text{cm}}$.

33. The only forces acting on Jane are gravity and the vine tension. The tension pulls in a centripetal direction, and so can do no work – the tension force is perpendicular at all times to her motion. So Jane's mechanical energy is conserved. Subscript 1 represents Jane at the point where she grabs the vine, and subscript 2 represents Jane at the highest point of her swing. The ground is the zero location for PE $(y = 0)$. We have $v_1 = 5.3\,\text{m/s}$, $y_1 = 0$, and $v_2 = 0$ (top of swing). Solve for y_2, the height of her swing.

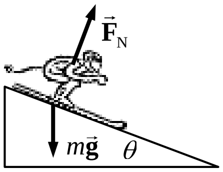


$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\to\quad \tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \quad\to$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(5.3\,\text{m/s})^2}{2\left(9.8\,\text{m/s}^2\right)} = \boxed{1.4\text{ m}}$$

$\boxed{\text{No}}$, the length of the vine does not enter into the calculation, unless the vine is less than 0.7 m long. If that were the case, she could not rise 1.4 m high. Instead she would wrap the vine around the tree branch.

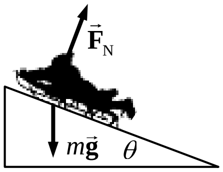
34. The forces on the skier are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the skier's mechanical energy is conserved. Subscript 1 represents the skier at the top of the hill, and subscript 2 represents the skier at the bottom of the hill. The ground is the zero location for PE $(y = 0)$. We have $v_1 = 0$, $y_1 = 185\text{ m}$, and $y_2 = 0$ (bottom of the hill). Solve for v_2, the speed at the bottom.



$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\to\quad 0 + mgy_1 = \tfrac{1}{2}mv_2^2 + 0 \quad\to$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(185\ \mathrm{m}\right)} = \boxed{60.2\,\mathrm{m/s}}\left(\approx 135\,\mathrm{mi/h}\right)$$

35. The forces on the sled are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the sled's mechanical energy is conserved. Subscript 1 represents the sled at the bottom of the hill, and subscript 2 represents the sled at the top of the hill. The ground is the zero location for PE $\left(y=0\right)$. We have $y_1 = 0$, $v_2 = 0$, and $y_2 = 1.35\ \mathrm{m}$. Solve for v_1, the speed at the bottom.



$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \quad\rightarrow$$

$$v_1 = \sqrt{2gy_2} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(1.35\ \mathrm{m}\right)} = \boxed{5.14\,\mathrm{m/s}}$$

Notice that the angle is not used in the calculation.

36. We assume that all the forces on the jumper are conservative, so that the mechanical energy of the jumper is conserved. Subscript 1 represents the jumper at the bottom of the jump, and subscript 2 represents the jumper at the top of the jump. Call the ground the zero location for PE $\left(y=0\right)$. We have $y_1 = 0$, $v_2 = 0.70\,\mathrm{m/s}$, and $y_2 = 2.10\ \mathrm{m}$. Solve for v_1, the speed at the bottom.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + 0 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow$$

$$v_1 = \sqrt{v_2^2 + 2gy_2} = \sqrt{\left(0.70\,\mathrm{m/s}\right)^2 + 2\left(9.80\,\mathrm{m/s^2}\right)\left(2.10\ \mathrm{m}\right)} = \boxed{6.45\,\mathrm{m/s}}$$

$\boxed{37.}$ (*a*) Since there are no dissipative forces present, the mechanical energy of the person – trampoline – Earth combination will be conserved. The level of the unstretched trampoline is the zero level for both the elastic and gravitational PE. Call up the positive direction. Subscript 1 represents the jumper at the top of the jump, and subscript 2 represents the jumper upon arriving at the trampoline. There is no elastic PE involved in this part of the problem. We have $v_1 = 5.0\,\mathrm{m/s}$, $y_1 = 3.0\ \mathrm{m}$, and $y_2 = 0$. Solve for v_2, the speed upon arriving at the trampoline.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + 0 \quad\rightarrow$$

$$v_2 = \pm\sqrt{v_1^2 + 2gy_1} = \pm\sqrt{\left(5.0\,\mathrm{m/s}\right)^2 + 2\left(9.8\,\mathrm{m/s^2}\right)\left(3.0\ \mathrm{m}\right)} = \pm 9.154\,\mathrm{m/s} \approx \boxed{9.2\,\mathrm{m/s}}$$

The speed is the absolute value of v_2.

(*b*) Now let subscript 3 represent the jumper at the maximum stretch of the trampoline. We have $v_2 = 9.154\,\mathrm{m/s}$, $y_2 = 0$, $x_2 = 0$, $v_3 = 0$, and $x_3 = y_3$. There is no elastic energy at position 2, but there is elastic energy at position 3. Also, the gravitational PE at position 3 is negative, and so $y_3 < 0$. A quadratic relationship results from the conservation of energy condition.

$$E_2 = E_3 \quad\rightarrow\quad \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}kx_3^2 \quad\rightarrow$$

$$\tfrac{1}{2}mv_2^2 + 0 + 0 = 0 + mgy_3 + \tfrac{1}{2}ky_3^2 \quad\rightarrow\quad \tfrac{1}{2}ky_3^2 + mgy_3 - \tfrac{1}{2}mv_2^2 = 0 \quad\rightarrow$$

$$y_3 = \frac{-mg \pm \sqrt{m^2g^2 - 4\left(\tfrac{1}{2}k\right)\left(-\tfrac{1}{2}mv_2^2\right)}}{2\left(\tfrac{1}{2}k\right)} = \frac{-mg \pm \sqrt{m^2g^2 + kmv_2^2}}{k}$$

$$= \frac{-\left(65\ \mathrm{kg}\right)\left(9.8\,\mathrm{m/s^2}\right) \pm \sqrt{\left(65\ \mathrm{kg}\right)^2\left(9.8\,\mathrm{m/s^2}\right)^2 + \left(6.2\times10^4\ \mathrm{N/m}\right)\left(65\ \mathrm{kg}\right)\left(9.154\,\mathrm{m/s}\right)^2}}{\left(6.2\times10^4\ \mathrm{N/m}\right)}$$

$$= -0.307 \text{ m}, 0.286 \text{ m}$$

Since $y_3 < 0$, $y_3 = \boxed{-0.31 \text{ m}}$.

The first term under the quadratic is about 1000 times smaller than the second term, indicating that the problem could have been approximated by not even including gravitational PE for the final position. If that approximation would have been made, the result would have been found by taking the negative result from the following solution.

$$E_2 = E_3 \quad \rightarrow \quad \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}ky_3^2 \quad \rightarrow \quad y_3 = v_2\sqrt{\frac{m}{k}} = \left(9.2 \text{ m/s}\right)\sqrt{\frac{65 \text{ kg}}{6.2\times10^4 \text{ N/m}}} = \pm0.30 \text{ m}$$

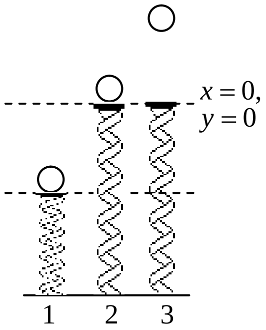
38. Use conservation of energy. Subscript 1 represents the projectile at the launch point, and subscript 2 represents the projectile as it reaches the ground. The ground is the zero location for PE $(y = 0)$. We have $v_1 = 185 \text{ m/s}$, $y_1 = 265 \text{ m}$, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + 0 \quad \rightarrow$$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{\left(185 \text{ m/s}\right)^2 + 2\left(9.80 \text{ m/s}^2\right)\left(265 \text{ m}\right)} = \boxed{199 \text{ m/s}}$$

Note that the angle of launch does not enter into the problem.

39. Use conservation of energy. The level of the ball on the uncompressed spring taken as the zero location for both gravitational PE $(y = 0)$ and elastic PE $(x = 0)$. Take up to be positive for both.



(*a*) Subscript 1 represents the ball at the launch point, and subscript 2 represents the ball at the location where it just leaves the spring, at the uncompressed length. We have $v_1 = 0$, $x_1 = y_1 = -0.150 \text{ m}$, and $x_2 = y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + 0 + 0 \quad \rightarrow \quad v_2 = \sqrt{\frac{kx_1^2 + 2mgy_1}{m}}$$

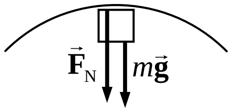
$$v_2 = \sqrt{\frac{\left(950 \text{ N/m}\right)\left(0.150 \text{ m}\right)^2 + 2\left(0.30 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(-0.150 \text{ m}\right)}{\left(0.30 \text{ kg}\right)}} = \boxed{8.3 \text{ m/s}}$$

(*b*) Subscript 3 represents the ball at its highest point. We have $v_1 = 0$, $x_1 = y_1 = -0.150 \text{ m}$, $v_3 = 0$, and $x_3 = 0$. Solve for y_3.

$$E_1 = E_3 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}kx_3^2 \quad \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = 0 + mgy_2 + 0 \quad \rightarrow \quad y_2 - y_1 = \frac{kx_1^2}{2mg} = \frac{\left(950 \text{ N/m}\right)\left(0.150 \text{ m}\right)^2}{2\left(0.30 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)} = \boxed{3.6 \text{ m}}$$

40. Draw a free-body diagram for the block at the top of the curve. Since the block is moving in a circle, the net force is centripetal. Write Newton's 2nd law for the block, with down as positive. If the block is to be on the verge of falling off the track, then $F_N = 0$.



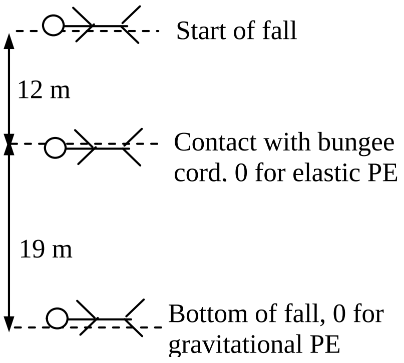
$$\sum F_R = F_N + mg = mv^2/r \quad \rightarrow \quad mg = mv_{top}^2/r \quad \rightarrow \quad v_{top} = \sqrt{gr}$$

Now use conservation of energy for the block. Since the track is frictionless, there are no non-conservative forces, and mechanical energy will be conserved. Subscript 1 represents the block at the release point, and subscript 2 represents the block at the top of the loop. The ground is the zero location for PE $(y = 0)$. We have $v_1 = 0$, $y_1 = h$, $v_2 = \sqrt{gr}$, and $y_2 = 2r$. Solve for *h*.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad 0 + mgh = \tfrac{1}{2}mgr + 2mgr \quad \rightarrow \quad h = \boxed{2.5\,r}$$

41. The block-spring combination is assumed to initially be at equilibrium, so the spring is neither stretched nor unstretched. At the release point, the speed of the mass is 0, and so the initial energy is all PE, given by $\tfrac{1}{2}kx_0^2$. That is the total energy of the system. Thus the energy of the system when the block is at a general location with some non-zero speed will still have this same total energy value. This is expressed by $E_{total} = \boxed{\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_0^2}$.

42. Consider this diagram for the jumper's fall.

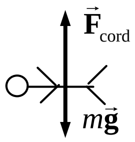

Start of fall
12 m
Contact with bungee cord. 0 for elastic PE
19 m
Bottom of fall, 0 for gravitational PE

(*a*) The mechanical energy of the jumper is conserved. Use *y* for the distance from the 0 of gravitational PE and *x* for the amount of bungee cord "stretch" from its unstretched length. Subscript 1 represents the jumper at the start of the fall, and subscript 2 represents the jumper at the lowest point of the fall. The bottom of the fall is the zero location for gravitational PE $(y = 0)$, and the location where the bungee cord just starts to be stretched is the zero location for elastic PE $(x = 0)$. We have $v_1 = 0$, $y_1 = 31\text{ m}$, $x_1 = 0$, $v_2 = 0$, $y_2 = 0$, and $x_2 = 19\text{ m}$. Apply conservation of energy.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$k = \frac{2mgy_1}{x_2^2} = \frac{2(62\text{ kg})(9.8\text{ m/s}^2)(31\text{ m})}{(19\text{ m})^2} = 104.4\text{ N/m} \approx \boxed{1.0\times10^2\text{ N/m}}$$

(*b*) The maximum acceleration occurs at the location of the maximum force, which occurs when the bungee cord has its maximum stretch, at the bottom of the fall. Write Newton's 2nd law for the force on the jumper, with upward as positive.



$$F_{net} = F_{cord} - mg = kx_2 - mg = ma \quad \rightarrow$$

$$a = \frac{kx_2}{m} - g = \frac{(104.4\text{ N/m})(19\text{ m})}{(62\text{ kg})} - 9.8\text{ m/s}^2 = 22.2\text{ m/s}^2 \approx \boxed{22\text{ m/s}^2}$$

43. Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved. Subscript 1 represents the coaster at point 1, etc. The height of point 2 is the zero location for gravitational PE. We have $v_1 = 0$ and $y_1 = 35\text{ m}$.

Point 2: $\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad ; \; y_2 = 0 \rightarrow \; mgy_1 = \tfrac{1}{2}mv_2^2 \quad \rightarrow$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(35\text{ m})} = \boxed{26\text{ m/s}}$$

Point 3: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_3^2 + mgy_3$; $y_3 = 28 \text{ m} \rightarrow mgy_1 = \frac{1}{2}mv_3^2 + mgy_3 \rightarrow$

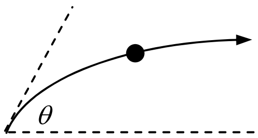
$$v_3 = \sqrt{2g\left(y_1 - y_3\right)} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(7 \text{ m}\right)} = \boxed{12 \text{ m/s}}$$

Point 4: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_4^2 + mgy_4$; $y_4 = 15 \text{ m} \rightarrow mgy_1 = \frac{1}{2}mv_4^2 + mgy_1 \rightarrow$

$$v_4 = \sqrt{2g\left(y_1 - y_4\right)} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(20 \text{ m}\right)} = \boxed{20 \text{ m/s}}$$

44. (*a*) See the diagram for the thrown ball. The speed at the top of the path will be the horizontal component of the original velocity.

$$v_{\text{top}} = v_0 \cos\theta = \left(12 \text{ m/s}\right)\cos 33^\circ = \boxed{10 \text{ m/s}}$$

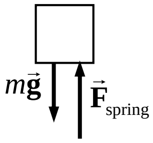


(*b*) Since there are no dissipative forces in the problem, the mechanical energy of the ball is conserved. Subscript 1 represents the ball at the release point, and subscript 2 represents the ball at the top of the path. The ground is the zero location for PE $\left(y = 0\right)$. We have $v_1 = 12 \text{ m/s}$, $y_1 = 0$, and $v_2 = v_1 \cos\theta$. Solve for y_2.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow \tfrac{1}{2}mv_1^2 + 0 = \tfrac{1}{2}mv_1^2\cos^2\theta + mgy_2 \rightarrow$$

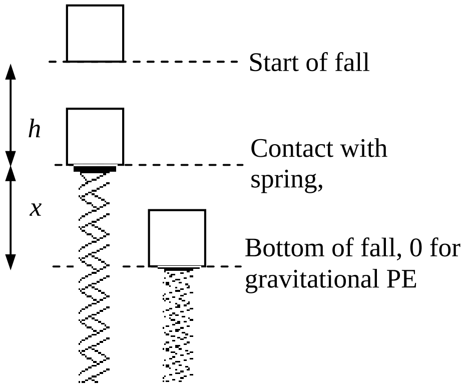
$$y_2 = \frac{v_1^2\left(1 - \cos^2\theta\right)}{2g} = \frac{\left(12 \text{ m/s}\right)^2\left(1 - \cos^2 33^\circ\right)}{2\left(9.8 \text{ m/s}^2\right)} = \boxed{2.2 \text{ m}}$$

45. The maximum acceleration of 5.0 *g* occurs where the force is at a maximum. The maximum force occurs at the bottom of the motion, where the spring is at its maximum compression. Write Newton's 2nd law for the elevator at the bottom of the motion, with up as the positive direction.



$$F_{\text{net}} = F_{\text{spring}} - Mg = Ma = 5.0Mg \rightarrow F_{\text{spring}} = 6.0Mg$$

Now consider the diagram for the elevator at various points in its motion. If there are no non-conservative forces, then mechanical energy is conserved. Subscript 1 represents the elevator at the start of its fall, and subscript 2 represents the elevator at the bottom of its fall. The bottom of the fall is the zero location for gravitational PE $\left(y = 0\right)$. There is also a point at the top of the spring that we will define as the zero location for elastic PE ($x = 0$). We have $v_1 = 0$, $y_1 = x + h$, $x_1 = 0$, $v_2 = 0$, $y_2 = 0$, and $x_2 = x$. Apply conservation of energy.



$$E_1 = E_2 \rightarrow \tfrac{1}{2}Mv_1^2 + Mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}Mv_2^2 + Mgy_2 + \tfrac{1}{2}kx_2^2 \rightarrow$$

$$0 + Mg\left(x + h\right) + 0 = 0 + 0 + \tfrac{1}{2}kx_2^2 \rightarrow Mg\left(x + h\right) = \tfrac{1}{2}kx_2^2$$

$$F_{spring} = 6.0Mg = kx \rightarrow x = \frac{6.0Mg}{k} \rightarrow Mg\left(\frac{6Mg}{k} + h\right) = \tfrac{1}{2}k\left(\frac{6Mg}{k}\right)^2 \rightarrow \boxed{k = \frac{12Mg}{h}}$$

46. (*a*) The work done against gravity is the change in PE.

$$W_{\substack{\text{against}\\\text{gravity}}} = \Delta PE = mg\left(y_2 - y_1\right) = \left(75 \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)\left(150 \text{ m}\right) = \boxed{1.1 \times 10^5 \text{ J}}$$

(*b*) The work done by the force on the pedals in one revolution is equal to the tangential force times the circumference of the circular path of the pedals. That work is also equal to the energy change of the bicycle during that revolution. Note that a vertical rise on the incline is related to the distance along the incline by $\text{rise} = \text{distance}\left(\sin\theta\right)$.

$$W_{\substack{\text{pedal}\\ \text{force}}} = F_{\text{tan}} 2\pi r = \Delta PE_{1\text{ rev}} = mg\left(\Delta y\right)_{1\text{ rev}} = mgd_{1\text{ rev}}\sin\theta \quad\rightarrow$$

$$F_{\text{tan}} = \frac{mgd_{1\text{ rev}}\sin\theta}{2\pi r} = \frac{\left(75\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)\left(5.1\text{ m}\right)\sin 7.8^{\circ}}{2\pi\left(0.18\text{ m}\right)} = \boxed{4.5\times10^2\,\text{N}}$$

47. Use conservation of energy, where all of the kinetic energy is transformed to thermal energy.

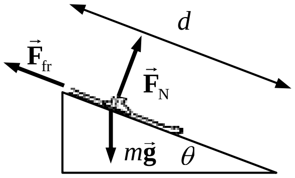
$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad \tfrac{1}{2}mv^2 = E_{\text{thermal}} = \tfrac{1}{2}\left(2\right)\left(7650\text{ kg}\right)\left[\left(95\,\text{km/h}\right)\left(\frac{0.238\,\text{m/s}}{1\,\text{km/h}}\right)\right]^2 = \boxed{5.3\times10^6\,\text{J}}$$

48. Apply the conservation of energy to the child, considering work done by gravity and work changed into thermal energy. Subscript 1 represents the child at the top of the slide, and subscript 2 represents the child at the bottom of the slide. The ground is the zero location for PE $\left(y=0\right)$. We have $v_1 = 0$, $y_1 = 3.5\text{ m}$, $v_2 = 2.2\,\text{m/s}$, and $y_2 = 0$. Solve for the work changed into thermal energy.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + W_{\text{thermal}} \quad\rightarrow$$

$$W_{\text{thermal}} = mgy_1 - \tfrac{1}{2}mv_2^2 = \left(21.7\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)\left(3.5\text{ m}\right) - \tfrac{1}{2}\left(21.7\text{ kg}\right)\left(2.2\,\text{m/s}\right)^2 = \boxed{6.9\times10^2\,\text{J}}$$

49. (*a*) See the free-body diagram for the ski. Write Newton's 2nd law for forces perpendicular to the direction of motion, noting that there is no acceleration perpendicular to the plane.



$$\sum F_{\perp} = F_{\text{N}} - mg\cos\theta \quad\rightarrow\quad F_{\text{N}} = mg\cos\theta \quad\rightarrow$$

$$F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta$$

Now use conservation of energy, including the non-conservative friction force. Subscript 1 represents the ski at the top of the slope, and subscript 2 represents the ski at the bottom of the slope. The location of the ski at the bottom of the incline is the zero location for gravitational PE $\left(y=0\right)$. We have $v_1 = 0$, $y_1 = d\sin\theta$, and $y_2 = 0$. Write the conservation of energy condition, and solve for the final speed. Note that $F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta$

$$W_{\text{NC}} = \Delta KE + \Delta PE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgy_2 - mgy_1 \quad\rightarrow\quad W_{\text{NC}} + E_1 = E_2$$

$$F_{\text{fr}}d\cos180^{\circ} + \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad -\mu_k mgd\cos\theta + mgd\sin\theta = \tfrac{1}{2}mv_2^2 \quad\rightarrow$$

$$v_2 = \sqrt{2gd\left(\sin\theta - \mu_k\cos\theta\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(75\text{ m}\right)\left(\sin 22^{\circ} - 0.090\cos 22^{\circ}\right)}$$

$$= 20.69\,\text{m/s} \approx \boxed{21\,\text{m/s}}$$

(*b*) Now, on the level ground, $F_f = \mu_k mg$, and there is no change in PE. Let us again use conservation of energy, including the non-conservative friction force, to relate position 2 with position 3. Subscript 3 represents the ski at the end of the travel on the level, having traveled a distance d_3 on the level. We have $v_2 = 20.69\,\text{m/s}$, $y_2 = 0$, $v_3 = 0$, and $y_3 = 0$.

$$W_{\text{NC}} + E_2 = E_3 \quad \rightarrow \quad F_f d_3 \cos 180^\circ + \tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 \quad \rightarrow$$

$$-\mu_k mgd_3 + \tfrac{1}{2}mv_2^2 = 0 \;\rightarrow\; d_3 = \frac{v_2^2}{2g\mu_k} = \frac{(20.69\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)(0.090)} = 242.7\text{ m} \approx \boxed{2.4\times 10^2\,\text{m}}$$

50. (*a*) Apply energy conservation with no non-conservative work. Subscript 1 represents the ball as it is dropped, and subscript 2 represents the ball as it reaches the ground. The ground is the zero location for gravitational PE. We have $v_1 = 0$, $y_1 = 13.0\text{ m}$, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}mv_2^2 \quad \rightarrow$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\,\text{m/s}^2)(13.0\text{ m})} = \boxed{16.0\,\text{m/s}}$$

(*b*) Apply energy conservation, but with non-conservative work due to friction included. The work done by friction will be given by $W_{\text{NC}} = F_{\text{fr}} d \cos 180^\circ$, since the force of friction is in the opposite direction as the motion. The distance d over which the frictional force acts will be the 13.0 m distance of fall. With the same parameters as above, and $v_2 = 8.00\,\text{m/s}$, solve for the force of friction.

$$W_{nc} + E_1 = E_2 \quad \rightarrow \quad -F_{\text{fr}}d + \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad -F_{\text{fr}}d + mgy_1 = \tfrac{1}{2}mv_2^2 \quad \rightarrow$$

$$F_{\text{fr}} = m\left(g\frac{y_1}{d} - \frac{v_2^2}{2d}\right) = (0.145\text{ kg})\left(9.80\,\text{m/s}^2 - \frac{(8.00\,\text{m/s})^2}{2(13.0\text{ m})}\right) = \boxed{1.06\text{ N}}$$

51. (*a*) Calculate the energy of the ball at the two maximum heights, and subtract to find the amount of energy "lost". The energy at the two heights is all gravitational PE, since the ball has no KE at those maximum heights.

$$E_{\text{lost}} = E_{\text{initial}} - E_{\text{final}} = mgy_{\text{initial}} - mgy_{\text{final}}$$

$$\frac{E_{\text{lost}}}{E_{\text{initial}}} = \frac{mgy_{\text{initial}} - mgy_{\text{final}}}{mgy_{\text{initial}}} = \frac{y_{\text{initial}} - y_{\text{final}}}{y_{\text{initial}}} = \frac{2.0\text{ m} - 1.5\text{ m}}{2.0\text{ m}} = 0.25 = \boxed{25\%}$$

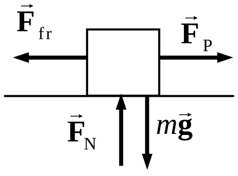
(*b*) Due to energy conservation, the KE of the ball just as it leaves the ground is equal to its final PE.

$$PE_{\text{final}} = KE_{\text{ground}} \quad \rightarrow \quad mgy_{\text{final}} = \tfrac{1}{2}mv_{\text{ground}}^2 \quad \rightarrow$$

$$v_{\text{ground}} = \sqrt{2gy_{\text{final}}} = \sqrt{2(9.8\,\text{m/s}^2)(1.5\text{ m})} = \boxed{5.4\,\text{m/s}}$$

(*c*) The energy "lost" was changed primarily into heat energy – the temperature of the ball and the ground would have increased slightly after the bounce. Some of the energy may have been changed into acoustic energy (sound waves). Some may have been lost due to non-elastic deformation of the ball or ground.

52. Since the crate moves along the floor, there is no change in gravitational PE, so use the work-energy theorem: $W_{\text{net}} = KE_2 - KE_1$. There are two forces doing work: F_{P}, the pulling force, and $F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg$, the frictional force. $KE_1 = 0$ since the crate starts from rest. Note that the two forces doing work do work over different distances.


$\vec{\mathbf{F}}_{\text{fr}}$
$\vec{\mathbf{F}}_{\text{P}}$
$\vec{\mathbf{F}}_{\text{N}}$
$m\vec{\mathbf{g}}$

$$W_{\text{P}} = F_{\text{P}}d_{\text{P}}\cos 0^\circ \qquad W_{\text{fr}} = F_{\text{fr}}d_{\text{fr}}\cos 180^\circ = -\mu_k mgd_{\text{fr}}$$

$$W_{\text{net}} = W_{\text{P}} + W_{\text{fr}} = KE_2 - KE_1 = \tfrac{1}{2}mv_2^2 - 0 \quad \rightarrow$$

$$v_2 = \sqrt{\frac{2}{m}\left(W_{\text{P}} + W_{\text{fr}}\right)} = \sqrt{\frac{2}{m}\left(F_{\text{P}}d_{\text{P}} - \mu_k mgd_{\text{fr}}\right)}$$

$$= \sqrt{\frac{2}{(110\text{ kg})}\left[(350\text{ N})(30\text{ m}) - (0.30)(110\text{ kg})\left(9.8\,\text{m/s}^2\right)(15\text{ m})\right]} = \boxed{10\,\text{m/s}}$$

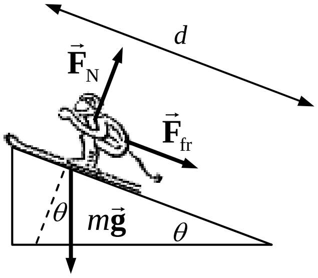
53. Since there is a non-conservative force, consider energy conservation with non-conservative work included. Subscript 1 represents the roller coaster at point 1, and subscript 2 represents the roller coaster at point 2. Point 2 is taken as the zero location for gravitational PE. We have $v_1 = 1.70\,\text{m/s}$, $y_1 = 35\text{ m}$, and $y_2 = 0$. Solve for v_2. The work done by the non-conservative friction force is given by $W_{\text{NC}} = F_{\text{fr}}d\cos 180^\circ = -0.20mgd$, since the force is one-fifth of *mg*, and the force is directed exactly opposite to the direction of motion.

$$W_{\text{NC}} + E_1 = E_2 \quad \rightarrow \quad -0.2mgd + \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

$$v_2 = \sqrt{-0.4gd + v_1^2 + 2gy_1} = \sqrt{-0.4\left(9.80\,\text{m/s}^2\right)(45.0\text{ m}) + (1.70\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)(35\text{ m})}$$

$$= 22.64\,\text{m/s} \approx \boxed{23\,\text{m/s}}$$

54. Consider the free-body diagram for the skier in the midst of the motion. Write Newton's 2nd law for the direction perpendicular to the plane, with an acceleration of 0.



$$\sum F_\perp = F_{\text{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\text{N}} = mg\cos\theta \quad \rightarrow$$

$$F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta$$

Apply conservation of energy to the skier, including the non-conservative friction force. Subscript 1 represents the skier at the bottom of the slope, and subscript 2 represents the skier at the point furthest up the slope. The location of the skier at the bottom of the incline is the zero location for gravitational PE $(y = 0)$. We have $v_1 = 12.0\,\text{m/s}$, $y_1 = 0$, $v_2 = 0$, and $y_2 = d\sin\theta$.

$$W_{\text{NC}} + E_1 = E_2 \quad \rightarrow \quad F_{\text{fr}}d\cos 180^\circ + \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

$$-\mu_k mgd\cos\theta + \tfrac{1}{2}mv_1^2 + 0 = 0 + mgd\sin\theta \quad \rightarrow$$

$$\mu_k = \frac{\tfrac{1}{2}v_1^2 - gd\sin\theta}{gd\cos\theta} = \frac{v_1^2}{2gd\cos\theta} - \tan\theta = \frac{(12.0\,\text{m/s})^2}{2\left(9.80\,\text{m/s}^2\right)(12.2\text{ m})\cos 18.0^\circ} - \tan 18.0^\circ$$

$$= \boxed{0.308}$$

55. Use conservation of energy, including the non-conservative frictional force. The block is on a level surface, so there is no gravitational PE change to consider. The frictional force is given by $F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg$, since the normal force is equal to the weight. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic PE (*x* = 0). Take right to be the positive direction. We have $v_1 = 0$, $x_1 = -0.050\text{ m}$, $v_2 = 0$, and $x_2 = 0.023\text{ m}$.

$$W_{\text{NC}} + E_1 = E_2 \;\rightarrow\; F_{\text{fr}}\Delta x\cos 180^\circ + \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \;\rightarrow$$

$$-\mu_k mg\Delta x + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}kx_2^2 \;\rightarrow$$

$$\mu_k = \frac{k\left(x_1^2 - x_2^2\right)}{2mg\Delta x} = \frac{\left(180\,\text{N/m}\right)\left[\left(0.050\text{m}\right)^2 - \left(0.023\text{m}\right)^2\right]}{2\left(0.620\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)\left(0.073\text{ m}\right)} = \boxed{0.40}$$

56. Use conservation of energy, including the non-conservative frictional force. The block is on a level surface, so there is no gravitational PE change to consider. Since the normal force is equal to the weight, the frictional force is $F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg$. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic PE ($x = 0$). Take right to be the positive direction. We have $v_1 = 0$, $x_1 = -0.18\text{ m}$, and $v_2 = 0$. The value of the spring constant is found from the fact that a 20-N force compresses the spring 18 cm, and so $k = F/x = 22\text{ N}/0.18\text{ m} = 122.2\,\text{N/m}$. The value of x_2 must be positive.

$$W_{\text{NC}} + E_1 = E_2 \;\rightarrow\; F_{\text{fr}}\Delta x\cos 180^\circ + \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \;\rightarrow$$

$$-\mu_k mg\left(x_2 - x_1\right) + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}kx_2^2 \;\rightarrow\; x_2^2 + \frac{2\mu_k mg}{k}x_2 - \left(\frac{2\mu_k mg}{k}x_1 + x_1^2\right) = 0 \;\rightarrow$$

$$x_2^2 + \frac{2\left(0.30\right)\left(0.28\right)\left(9.80\right)}{122.2}x_2 - \left(\frac{2\left(0.30\right)\left(0.28\right)\left(9.80\right)}{122.2}\left(-0.18\right) + \left(-0.18\right)^2\right) = 0 \;\rightarrow$$

$$x_2^2 + 0.01347x_2 - 0.02997 = 0 \;\rightarrow\; x_2 = 0.1665\,\text{m}, -0.1800\text{m} \;\rightarrow\; x_2 = \boxed{0.17\text{ m}}$$

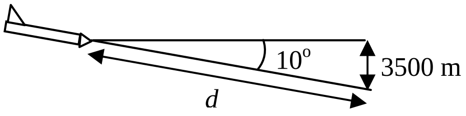
57. (*a*) If there is no air resistance, then conservation of mechanical energy can be used. Subscript 1 represents the glider when at launch, at subscript 2 represents the glider at landing. The landing location is the zero location for elastic PE ($x = 0$). We have $y_1 = 500\text{ m}$, $y_2 = 0$, and $v_1 = 500\,\text{km/h}\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 138.9\,\text{m/s}$. Solve for v_2

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow$$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{\left(138.9\,\text{m/s}\right)^2 + 2\left(9.80\,\text{m/s}^2\right)\left(3500\text{ m}\right)} = 296\,\text{m/s}\left(\frac{3.6\,\text{km/h}}{1\,\text{m/s}}\right)$$

$$= 1067\,\text{km/h} \approx \boxed{1.1\times10^3\text{ km/h}}$$

(*b*) Now include the work done by the non-conservative frictional force. Consider the diagram of the glider. Calculate the work done by friction.



$$W_{\text{NC}} = F_{\text{fr}}d\cos 180^\circ = -F_{\text{fr}}d = -F_{\text{fr}}\frac{3500\text{ m}}{\sin 10^\circ}$$

Use the same subscript representations as above, with y_1, v_1, and y_2 as before, and $v_2 = 200\,\text{km/h}\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 55.56\,\text{m/s}$. Write the energy conservation equation and solve for the frictional force.

$$W_{\mathrm{NC}} + E_1 = E_2 \quad \rightarrow \quad -F_{\mathrm{fr}}d + \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad F_f = \frac{m\left(v_1^2 - v_2^2 + 2gy_1\right)}{2d}$$

$$= \frac{(980\text{ kg})\left[(138.9\,\text{m/s})^2 - (55.56\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)(3500\text{ m})\right]}{2\left(\dfrac{3500\text{ m}}{\sin 10^\circ}\right)}$$

$$= 2062\text{ N} \approx \boxed{2\times10^3\,\text{N}}$$

58. The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W = Fd\cos 0^\circ = mgh$. The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$P = \frac{W}{t} = \frac{mgh}{t} \quad \rightarrow \quad t = \frac{mgh}{P} = \frac{(315\text{ kg})\left(9.80\,\text{m/s}^2\right)(16.0\text{ m})}{1750\text{ W}} = \boxed{28.2\text{ s}}$$

59. The 18 hp is the power generated by the engine in creating a force on the ground to propel the car forward. The relationship between the power and the force is given by $P = \dfrac{W}{t} = \dfrac{Fd}{t} = F\dfrac{d}{t} = Fv$. Thus the force to propel the car forward is found by $F = P/v$. If the car has a constant velocity, then the total resistive force must be of the same magnitude as the engine force, so that the net force is zero. Thus the total resistive force is also found by $F = P/v$.

$$F = \frac{P}{v} = \frac{(18\text{ hp})(746\text{ W/1 hp})}{(88\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)} = \boxed{5.5\times10^2\,\text{N}}$$

60. The power is given by Eq. 6-16. The energy transformed is the change in kinetic energy of the car.

$$P = \frac{\text{energy transformed}}{\text{time}} = \frac{\Delta KE}{t} = \frac{\tfrac{1}{2}m\left(v_2^2 - v_1^2\right)}{t} = \frac{(1400\text{ kg})\left[(95\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{2(7.4\text{ s})}$$

$$= \boxed{6.6\times10^4\,\text{W}} \approx 88\text{ hp}$$

61. (*a*) $1\text{ hp} = (1\text{ hp})\left(\dfrac{550\,\text{ft}\bullet\text{lb/s}}{1\text{ hp}}\right)\left(\dfrac{4.45\text{ N}}{1\text{ lb}}\right)\left(\dfrac{1\text{ m}}{3.28\text{ ft}}\right) = 746\,\text{N}\bullet\text{m/s} = \boxed{746\text{ W}}$

(*b*) $75\,\text{W} = (75\text{ W})\left(\dfrac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.10\text{ hp}}$

62. (*a*) $1\text{ kW}\bullet\text{h} = 1\text{ kW}\bullet\text{h}\left(\dfrac{1000\text{ W}}{1\text{ kW}}\right)\left(\dfrac{3600\text{ s}}{1\text{ h}}\right)\left(\dfrac{1\text{ J/s}}{1\text{ W}}\right) = \boxed{3.6\times10^6\,\text{J}}$

(*b*) $(520\text{ W})(1\text{ month}) = (520\text{ W})(1\text{ month})\left(\dfrac{1\text{ kW}}{1000\text{ W}}\right)\left(\dfrac{30\text{ d}}{1\text{ month}}\right)\left(\dfrac{24\text{ h}}{1\text{ d}}\right) = 374\,\text{kW}\bullet\text{h}$

$$\approx \boxed{370\,\mathrm{kW{\bullet}h}}$$

(*c*) $374\,\mathrm{kW{\bullet}h} = 374\,\mathrm{kW{\bullet}h}\left(\dfrac{3.6\times10^{6}\,\mathrm{J}}{1\,\mathrm{kW{\bullet}h}}\right) = \boxed{1.3\times10^{9}\,\mathrm{J}}$

(*d*) $\left(374\,\mathrm{kW{\bullet}h}\right)\left(\dfrac{\$0.12}{1\,\mathrm{kW{\bullet}h}}\right) = \$44.88 \approx \boxed{\$45}$

Kilowatt-hours is a measure of energy, not power, and so $\boxed{\text{no}}$, the actual rate at which the energy is used does not figure into the bill. They could use the energy at a constant rate, or at a widely varying rate, and as long as the total used is 370 kilowatt-hours, the price would be $45.

63. The energy transfer from the engine must replace the lost kinetic energy. From the two speeds, calculate the average rate of loss in kinetic energy while in neutral.

$$v_1 = 85\,\mathrm{km/h}\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 23.61\,\mathrm{m/s} \qquad v_2 = 65\,\mathrm{km/h}\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 18.06\,\mathrm{m/s}$$

$$\Delta KE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(1150\text{ kg})\left[(18.06\,\mathrm{m/s})^2 - (23.61\,\mathrm{m/s})^2\right] = -1.330\times10^{5}\,\mathrm{J}$$

$$P = \frac{W}{t} = \frac{1.330\times10^{5}\,\mathrm{J}}{6.0\text{ s}} = 2.216\times10^{4}\,\mathrm{W}\text{ , or } \left(2.216\times10^{4}\,\mathrm{W}\right)\frac{1\text{ hp}}{746\text{ W}} = 29.71\text{ hp}$$

So $\boxed{2.2\times10^{4}\,\mathrm{W}}$ or $\boxed{3.0\times10^{1}\,\mathrm{hp}}$ is needed from the engine.

64. Since $P = \dfrac{W}{t}$, we have $W = Pt = 3.0\text{ hp}\left(\dfrac{746\text{ W}}{1\text{ hp}}\right)(1\text{ hr})\left(\dfrac{3600\text{ s}}{1\text{ h}}\right) = \boxed{8.1\times10^{6}\,\mathrm{J}}$

$\boxed{65.}$ The work done in accelerating the shot put is given by its change in kinetic energy: The power is the energy change per unit time.

$$P = \frac{W}{t} = \frac{KE_2 - KE_1}{t} = \frac{\frac{1}{2}m\left(v_2^2 - v_1^2\right)}{t} = \frac{\frac{1}{2}(7.3\text{ kg})\left[(14\,\mathrm{m/s})^2 - 0\right]}{1.5\text{ s}} = 476.9\text{ W} \approx \boxed{4.8\times10^{2}\,\mathrm{W}}$$

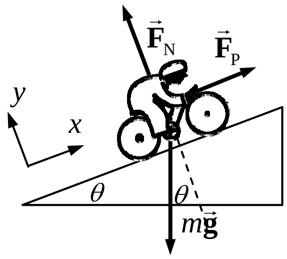
66. The force to lift the water is equal to its weight, and so the work to lift the water is equal to the weight times the distance. The power is the work done per unit time.

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{(18.0\text{ kg})\left(9.80\,\mathrm{m/s^2}\right)(3.60\text{ m})}{60\text{ sec}} = \boxed{10.6\text{ W}}$$

67. The minimum force needed to lift the football player vertically is equal to his weight, *mg*. The distance over which that force would do work would be the change in height, $\Delta y = (140\text{ m})\sin 32^{\circ} = 74.2\text{ m}$. So the work done in raising the player is $W = mg\Delta y$ and the power output required is the work done per unit time.

$$P = \frac{W}{t} = \frac{mg\Delta y}{t} = \frac{(95\text{ kg})\left(9.80\,\mathrm{m/s^2}\right)(74.2\text{ m})}{66\text{ sec}} = 1047\text{ W} \approx \boxed{1.0\times10^{3}\,\mathrm{W}}.$$

68. See the free-body diagram for the bicycle on the hill. Write Newton's 2nd law for the *x* direction, noting that the acceleration is 0. Solve for the magnitude of $\vec{\mathbf{F}}_\text{P}$. The power output related to that force is given by Eq. 6-17, $P = F_\text{P}v$. Use that relationship to find the velocity.

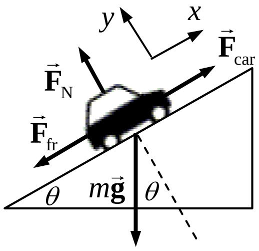


$$\sum F_x = F_\text{P} - mg\sin\theta = 0 \;\rightarrow\; F_\text{P} = mg\sin\theta$$

$$P = vF_\text{P} \;\rightarrow\; v = \frac{P}{F_\text{P}} = \frac{P}{mg\sin\theta} = \frac{(0.25\text{ hp})(746\text{ W/hp})}{(68\text{ kg})(9.8\text{ m/s}^2)\sin 6.0^\circ}$$

$$= \boxed{2.7\text{ m/s}}$$

69. Consider the free-body diagram for the car. The car has a constant velocity, so the net force on the car is zero. F_fr is the friction force, and F_car is the force of the road pushing on the car. It is equal in magnitude to the force of the car pushing on the road, and so we can think of F_car as the force the car is able to generate by the engine. Write Newton's 2nd law in the *x* direction.



y
x
$\vec{\mathbf{F}}_\text{car}$
$\vec{\mathbf{F}}_\text{N}$
$\vec{\mathbf{F}}_\text{fr}$
θ
$m\vec{\mathbf{g}}$
θ

$$\sum F_x = F_\text{car} - F_\text{fr} - mg\sin\theta \;\rightarrow\; F_\text{car} = F_\text{fr} + mg\sin\theta$$

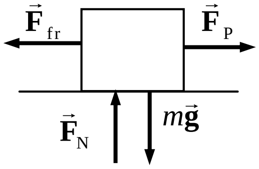
Use Eq. 6-17 to express the power output of the car, and then calculate the angle from that expression.

$$P = \left(F_\text{fr} + mg\sin\theta\right)v \;\rightarrow$$

$$\theta = \sin^{-1}\left[\frac{1}{mg}\left(\frac{P}{v} - F_\text{fr}\right)\right] = \sin^{-1}\left[\frac{1}{(1200\text{ kg})(9.80\text{ m/s}^2)}\left(\frac{120\text{hp}(746\text{ W/1 hp})}{(75\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)} - 650\text{ N}\right)\right]$$

$$= \boxed{18^\circ}$$

70. Draw a free-body diagram for the box being dragged along the floor. The box has a constant speed, so the acceleration is 0 in all directions. Write Newton's 2nd law for both the *x* (horizontal) and *y* (vertical) directions.



$\vec{\mathbf{F}}_\text{fr}$
$\vec{\mathbf{F}}_\text{P}$
$\vec{\mathbf{F}}_\text{N}$
$m\vec{\mathbf{g}}$

$$\sum F_y = F_\text{N} - mg = 0 \;\rightarrow\; F_\text{N} = mg$$

$$\sum F_x = F_\text{P} - F_\text{fr} = 0 \;\rightarrow\; F_\text{P} = F_\text{fr} = \mu_k F_\text{N} = \mu_k mg$$

The work done by F_P in moving the crate a distance Δx is given by $W = F_\text{P}\Delta x\cos 0^\circ = \mu_k mg\Delta x$. The power required is the work done per unit time.

$$P = \frac{W}{t} = \frac{\mu_k mg\Delta x}{t} = \mu_k mg\frac{\Delta x}{t} = \mu_k mgv_x = (0.45)(310\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m/s}) = 1641\text{ W}$$

$$1641\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{2.2\text{ hp}}$$

71. First, consider a free-body diagram for the cyclist going down hill. Write Newton's 2nd law for the *x* direction, with an acceleration of 0 since the cyclist has a constant speed.

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = 0 \quad \rightarrow \quad F_{\text{fr}} = mg\sin\theta$$

Now consider the diagram for the cyclist going up the hill. Again, write Newton's 2nd law for the x direction, with an acceleration of 0.

$$\sum F_x = F_{\text{fr}} - F_{\text{P}} + mg\sin\theta = 0 \quad \rightarrow \quad F_{\text{P}} = F_{\text{fr}} + mg\sin\theta$$

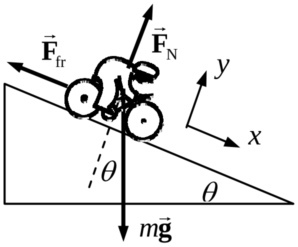
Assume that the friction force is the same when the speed is the same, so the friction force when going uphill is the same magnitude as when going downhill.

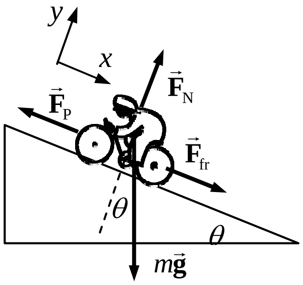
$$F_{\text{P}} = F_{\text{fr}} + mg\sin\theta = 2mg\sin\theta$$

The power output due to this force is given by Eq. 6-17.

$$P = F_{\text{P}}v = 2mgv\sin\theta = 2(75\text{ kg})(9.8\,\text{m/s}^2)(5.0\,\text{m/s})\sin 7.0^\circ$$

$$= \boxed{9.0\times 10^2\,\text{W}}$$

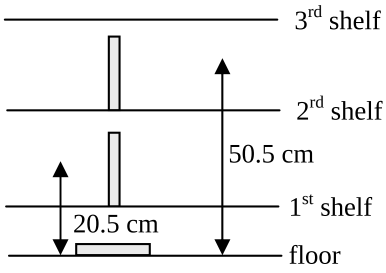



y
x
$\vec{\mathbf{F}}_{\text{N}}$
$\vec{\mathbf{F}}_{\text{P}}$
$\vec{\mathbf{F}}_{\text{fr}}$
θ
θ
$m\vec{\mathbf{g}}$

72. The kinetic energy of the moving car is changed into the elastic PE of the bumper, before it deforms.

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx^2 \quad \rightarrow \quad k = \frac{mv^2}{x^2} = \frac{(1300\text{ kg})\left[(8\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(0.015\text{ m})^2} = \boxed{2.9\times 10^7\text{ N/m}}$$

73. The minimum work required to shelve a book is equal to the weight of the book times the vertical distance the book is moved – its increase in PE. See the diagram. Each book that is placed on the lowest shelf has its center of mass moved upwards by 20.5 cm. So the work done to move 25 books to the lowest shelf is $W_1 = 25mg(0.205\text{ m})$. Each book that is placed on the second shelf has its center of mass moved upwards by 50.5 cm, so the work done to move 25 books to the second shelf is $W_2 = 25mg(0.505\text{ m})$. Similarly, $W_3 = 25mg(0.805\text{ m})$, $W_4 = 25mg(1.105\text{ m})$, and $W_5 = 25mg(1.405\text{ m})$. The total work done is the sum of the five work expressions.



$$W = 25mg(0.205\,\text{m} + .505\,\text{m} + .805\,\text{m} + 1.105\,\text{m} + 1.405\,\text{m})$$

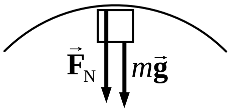
$$= 25(1.5\text{ kg})(9.80\,\text{m/s}^2)(4.025\text{ m}) = 1479\,\text{J} \approx \boxed{1.5\times 10^3\,\text{J}}$$

74. Assume that there are no non-conservative forces doing work, so the mechanical energy of the jumper will be conserved. Subscript 1 represents the jumper at the launch point of the jump, and subscript 2 represents the jumper at the highest point. The starting height of the jump is the zero location for PE $(y = 0)$. We have $y_1 = 0$, $y_2 = 1.1\text{ m}$, and $v_2 = 6.5\,\text{m/s}$. Solve for v_1.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

$$v_1 = \sqrt{v_2^2 + 2gy_2} = \sqrt{(6.5\,\text{m/s})^2 + 2(9.8\,\text{m/s}^2)(1.1\text{ m})} = \boxed{8.0\,\text{m/s}}$$

75. (*a*) Consider a free-body diagram for the block at the top of the curve. Since the block is moving in a circle, the net force is centripetal. Write Newton's 2nd law for the block, with down as vertical. If the block is to be on the verge of falling off the track, then $F_N = 0$.



$$\sum F_R = F_N + mg = mv_{top}^2/r \quad \rightarrow \quad mg = mv_{top}^2/r \quad \rightarrow \quad v_{top} = \sqrt{gr}$$

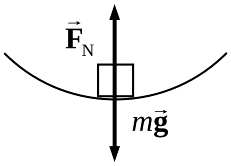
Now use conservation of energy for the block. Since the track is frictionless, there are no non-conservative forces, and mechanical energy will be conserved. Subscript 1 represents the block at the release point, and subscript 2 represents the block at the top of the loop. The ground is the zero location for PE $(y=0)$. We have $v_1 = 0$, $y_1 = h$, $v_2 = v_{top} = \sqrt{gr}$, and $y_2 = 2r$. Solve for *h*.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mgh = \tfrac{1}{2}mgr + 2mgr \quad \rightarrow \quad h = \boxed{2.5r}$$

(*b*) Now the release height is $2h = 5r$. Use conservation of energy again. Subscript 1 represents the block at the (new) release point, and subscript 2 represents the block at the bottom of the loop. We have $v_1 = 0$, $y_1 = 5r$, and $y_2 = 0$. Solve for v_2^2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad v_2^2 = 10rg.$$

Now consider the free-body diagram for the block at the bottom of the loop. The net force must be upward and radial. Write Newton's 2nd law for the vertical direction, with up as positive.



$$\sum F_R = F_N - mg = mv^2/r \quad \rightarrow$$

$$F_N = mg + mv^2/r = mg + \frac{m10rg}{r} = \boxed{11mg}$$

(*c*) Use conservation of energy again. Subscript 2 is as in part (*b*) above, and subscript 3 represents the block at the top of the loop. We have $y_2 = 0$, $v_2 = \sqrt{10rg}$, and $y_3 = 2r$. Solve for v_3^2.

$$E_2 = E_3 \quad \rightarrow \quad \tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 \quad \rightarrow \quad 5mrg + 0 = \tfrac{1}{2}mv_3^2 + 2rmg \quad \rightarrow$$

$$v_3^2 = 6rg$$

Refer to the free-body diagram and analysis of part (a) to find the normal force.

$$\sum F_R = F_N + mg = mv_{top}^2/r \quad \rightarrow \quad F_N = mv^2/r - mg = \frac{6rmg}{r} - mg = \boxed{5mg}$$

(*d*) When moving on the level, the normal force is the same as the weight, $F_N = \boxed{mg}$.

76. (*a*) Use conservation of energy, including the work done by the non-conservative force of the snow on the pilot. Subscript 1 represents the pilot at the top of the snowbank, and subscript 2 represents the pilot at the bottom of the crater. The bottom of the crater is the zero location for PE $(y=0)$. We have v_1 = 35 m/s , y_1 = 1.1 m, v_2 = 0, and y_2 = 0. Solve for the non-conservative work.

$$W_{NC} + E_1 = E_2 \quad \rightarrow \quad W_{NC} + \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

$$W_{NC} = -\tfrac{1}{2}mv_1^2 - mgy_1 = -\tfrac{1}{2}(78\text{ kg})(35\,\text{m/s})^2 - (78\text{ kg})(9.8\,\text{m/s}^2)(1.1\text{ m})$$

$$= -4.862\times10^4\,\text{J} \approx \boxed{-4.9\times10^4\text{ J}}$$

(*b*) The work done by the snowbank is done by an upward force, while the pilot moves down.

$$W_{NC} = F_{snow}d\cos180^\circ = -F_{snow}d \quad \rightarrow$$

$$F_{\text{snow}} = -\frac{W_{\text{NC}}}{d} = -\frac{-4.862\times10^4 \text{ J}}{1.1 \text{ m}} = 4.420\times10^4 \text{ N} \approx \boxed{4.4\times10^4 \text{ N}}$$

(*c*) To find the work done by air friction, another non-conservative force, use energy conservation including the work done by the non-conservative force of air friction. Subscript 1 represents the pilot at the start of the descent, and subscript 3 represents the pilot at the top of the snowbank. The top of the snowbank is the zero location for PE $(y=0)$. We have $v_1 = 0$ m/s , $y_1 = 370$ m, $v_2 = 35$ m/s, and $y_2 = 0$. Solve for the non-conservative work.

$$W_{\text{NC}} + E_1 = E_2 \quad\rightarrow\quad W_{\text{NC}} + \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow$$

$$W_{\text{NC}} = \tfrac{1}{2}mv_2^2 - mgy_1 = \tfrac{1}{2}(78 \text{ kg})(35 \text{ m/s})^2 - (78 \text{ kg})(9.8 \text{ m/s}^2)(370 \text{ m})$$

$$= -2.351\times10^5 \text{ J} \approx \boxed{-2.4\times10^5 \text{ J}}$$

$\boxed{77.}$ (*a*) The tension in the cord is perpendicular to the path at all times, and so the tension in the cord does not do any work on the ball. Thus the mechanical energy of the ball is conserved. Subscript 1 represents the ball when it is horizontal, and subscript 2 represents the ball at the lowest point on its path. The lowest point on the path is the zero location for PE $(y=0)$. We have $v_1 = 0$, $y_1 = L$, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad mgL = \tfrac{1}{2}mv_2^2 \quad\rightarrow\quad v_2 = \boxed{\sqrt{2gL}}$$

(*b*) Use conservation of energy, to relate points 2 and 3. Point 2 is as described above. Subscript 3 represents the ball at the top of its circular path around the peg. The lowest point on the path is the zero location for PE $(y=0)$. We have $v_1 = \sqrt{2gL}$, $y_1 = 0$, and $y_2 = 2(L-h) = 2(L-0.80L) = 0.40L$. Solve for v_2.

$$E_2 = E_3 \quad\rightarrow\quad \tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 \quad\rightarrow\quad \tfrac{1}{2}m(2gL) = \tfrac{1}{2}mv_2^2 + mg(0.40L) \quad\rightarrow$$

$$\boxed{v_2 = \sqrt{1.2gL}}$$

78. (*a*) The work done by the hiker against gravity is the change in gravitational PE.

$$W_{\text{G}} = mg\Delta y = (65 \text{ kg})(9.8 \text{ m/s}^2)(3700 \text{ m} - 2300 \text{ m}) = 8.918\times10^5 \text{ J} \cong \boxed{8.9\times10^5 \text{ J}}$$

(*b*) The average power output is found by dividing the work by the time taken.

$$P_{output} = \frac{W_{grav}}{t} = \frac{8.918\times10^5 \text{ J}}{(5 \text{ h})(3600 \text{ s/1 h})} = 49.54 \text{ W} \approx \boxed{50 \text{ W}}$$

$$49.54 \text{ W}\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = \boxed{6.6\times10^{-2} \text{ hp}}$$

(*c*) The output power is the efficiency times the input power.

$$P_{\text{output}} = 0.15P_{\text{input}} \quad\rightarrow\quad P_{\text{input}} = \frac{P_{\text{output}}}{0.15} = \frac{49.54 \text{ W}}{0.15} = \boxed{3.3\times10^2 \text{ W}} = \boxed{0.44 \text{ hp}}$$

79. (*a*) The work done by gravity as the elevator falls is the opposite of the change in gravitational PE.

$$W_{\text{G}} = -\Delta PE = PE_1 - PE_2 = mg(y_1 - y_2) = (920 \text{ kg})(9.8 \text{ m/s}^2)(28 \text{ m})$$

$$= 2.524\times10^5 \text{ J} \approx \boxed{2.5\times10^5 \text{ J}}$$

Gravity is the only force doing work on the elevator as it falls (ignoring friction), so this result is also the net work done on the elevator as it falls.

(*b*) The net work done on the elevator is equal to its change in kinetic energy. The net work done just before striking the spring is the work done by gravity found above.

$$W_G = KE_2 - KE_1 \quad \rightarrow \quad mg\left(y_1 - y_2\right) = \tfrac{1}{2}mv_2 - 0 \quad \rightarrow$$

$$v_2 = \sqrt{2g\left(y_1 - y_2\right)} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(28\text{ m}\right)} = 23.43\,\text{m/s} \approx \boxed{23\,\text{m/s}}$$

(*c*) Use conservation of energy. Subscript 1 represents the elevator just before striking the spring, and subscript 2 represents the elevator at the bottom of its motion. The level of the elevator just before striking the spring is the zero location for both gravitational PE and elastic PE. We have $v_1 = 23.43\,\text{m/s}$, $y_1 = 0$, and $v_2 = 0$. We assume that $y_2 < 0$.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}ky_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 \quad \rightarrow$$

$$y_2^2 + 2\frac{mg}{k}y_2 - \frac{m}{k}v_1^2 = 0 \quad \rightarrow \quad y_2 = \frac{-\dfrac{2mg}{k} \pm \sqrt{\dfrac{4m^2g^2}{k^2} + 4\dfrac{mv_1^2}{k}}}{2} = \frac{-mg \pm \sqrt{m^2g^2 + mkv_1^2}}{k}$$

We must choose the negative root so that y_2 is negative. Thus

$$y_2 = \frac{-\left(920\text{ kg}\right)\left(9.8\,\text{m/s}^2\right) - \sqrt{\left(920\text{ kg}\right)^2\left(9.8\,\text{m/s}^2\right) + \left(920\text{ kg}\right)\left(2.2\times10^5\text{ N/m}\right)\left(23.43\,\text{m/s}\right)^2}}{2.2\times10^5\text{ N/m}}$$

$$= \boxed{-1.56\text{ m}}$$

80. The force to lift a person is equal to the person's weight, so the work to lift a person up a vertical distance *h* is equal to *mgh*. The work needed to lift *N* people is *Nmgh*, and so the power needed is the total work divided by the total time. We assume the mass of the average person to be 70 kg,

$$P = \frac{W}{t} = \frac{Nmgh}{t} = \frac{47000\left(70\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)\left(200\text{ m}\right)}{3600\text{ s}} = 1.79\times10^6\,\text{W} \approx \boxed{2\times10^6\,\text{W}}.$$

81. (*a*) Use conservation of mechanical energy, assuming there are no non-conservative forces. Subscript 1 represents the water at the top of the dam, and subscript 2 represents the water as it strikes the turbine blades. The level of the turbine blades is the zero location for PE $\left(y = 0\right)$.

We have $v_1 = 0$, $y_1 = 80\text{ m}$, and $y_2 = 0$. Solve for v_2.

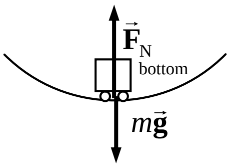
$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}mv_2^2 \quad \rightarrow$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(81\text{ m}\right)} = 39.84\,\text{m/s} \approx \boxed{4.0\times10^1\text{ m/s}}$$

(*b*) The energy of the water at the level of the turbine blades is all kinetic energy, and so is given by $\tfrac{1}{2}mv_2^2$. 58% of that energy gets transferred to the turbine blades. The rate of energy transfer to the turbine blades is the power developed by the water.

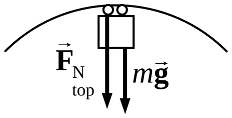
$$P = 0.58\left(\tfrac{1}{2}\frac{m}{t}v_2^2\right) = \frac{\left(0.58\right)\left(650\text{ kg/s}\right)\left(39.84\,\text{m/s}\right)^2}{2} = \boxed{3.0\times10^5\,\text{W}}$$

82. Consider the free-body diagram for the coaster at the bottom of the loop. The net force must be an upward centripetal force.



$$\sum F_{\text{bottom}} = F_{\underset{\text{bottom}}{\text{N}}} - mg = m v_{\text{bottom}}^2 \big/ R \quad \to \quad F_{\underset{\text{bottom}}{\text{N}}} = mg + m v_{\text{bottom}}^2 \big/ R$$

Now consider the force diagram at the top of the loop. Again, the net force must be centripetal, and so must be downward.



$$\sum F_{\text{top}} = F_{\underset{\text{top}}{\text{N}}} + mg = m v_{\text{top}}^2 \big/ R \quad \to \quad F_{\underset{\text{top}}{N}} = m v_{\text{top}}^2 \big/ R - mg \,.$$

Assume that the speed at the top is large enough that $F_{\underset{\text{top}}{N}} > 0$, and so $v_{\text{top}} > \sqrt{Rg}$. Now apply the conservation of mechanical energy. Subscript 1 represents the coaster at the bottom of the loop, and subscript 2 represents the coaster at the top of the loop. The level of the bottom of the loop is the zero location for PE $(y = 0)$. We have $y_1 = 0$ and $y_2 = 2R$.

$$E_1 = E_2 \quad \to \quad \tfrac{1}{2} m v_1^2 + mgy_1 = \tfrac{1}{2} m v_2^2 + mgy_2 \quad \to \quad v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gR \,.$$

The difference in apparent weights is the difference in the normal forces.

$$F_{\underset{\text{bottom}}{\text{N}}} - F_{\underset{\text{top}}{\text{N}}} = \left(mg + m v_{\text{bottom}}^2 \big/ R \right) - \left(m v_{\text{top}}^2 \big/ R - mg \right) = 2mg + m\left(v_{\text{bottom}}^2 - v_{\text{top}}^2 \right) \big/ R$$

$$= 2mg + m\left(4gR\right) \big/ R = \boxed{6mg}$$

Notice that the result does not depend on either v or R.

83. (*a*) Assume that the energy of the candy bar is completely converted into a change of PE:

$$E_{\underset{\text{bar}}{\text{candy}}} = \Delta PE = mg\Delta y \quad \to \quad \Delta y = \frac{E_{\underset{\text{bar}}{\text{candy}}}}{mg} = \frac{1.1 \times 10^6\,\text{J}}{(82\text{ kg})\left(9.8\,\text{m/s}^2\right)} = \boxed{1.4 \times 10^3\,\text{m}} \,.$$

(*b*) If the person jumped to the ground, the same energy is all converted into kinetic energy.

$$E_{\underset{\text{bar}}{\text{candy}}} = \tfrac{1}{2} m v^2 \quad \to \quad v = \sqrt{\frac{2E_{\underset{\text{bar}}{\text{candy}}}}{m}} = \sqrt{\frac{2\left(1.1 \times 10^6\,\text{J}\right)}{(82\text{ kg})}} = \boxed{1.6 \times 10^2\text{ m/s}}$$

84. Since there are no non-conservative forces, the mechanical energy of the projectile will be conserved. Subscript 1 represents the projectile at launch and subscript 2 represents the projectile as it strikes the ground. The ground is the zero location for PE $(y = 0)$. We have $v_1 = 175\,\text{m/s}$, $y_1 = 165\,\text{m}$, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad \to \quad \tfrac{1}{2} m v_1^2 + mgy_1 = \tfrac{1}{2} m v_2^2 + mgy_2 \quad \to \quad \tfrac{1}{2} m v_1^2 + mgy_1 = \tfrac{1}{2} m v_2^2 \quad \to$$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{\left(175\,\text{m/s}\right)^2 + 2\left(9.8\,\text{m/s}^2\right)\left(165\text{ m}\right)} = \boxed{184\,\text{m/s}}$$

Notice that the launch angle does not enter the problem, and so does not influence the final speed.

85. The spring constant for the scale can be found from the 0.6 mm compression due to the 710 N force.

$k = \dfrac{F}{x} = \dfrac{710\text{ N}}{6.0 \times 10^{-4}\,\text{m}} = 1.183 \times 10^6\text{ N/m}$. Use conservation of energy for the jump. Subscript 1 represents the initial location, and subscript 2 represents the location at maximum compression of the scale spring. Assume that the location of the uncompressed scale spring is the 0 location for gravitational PE. We have $v_1 = v_2 = 0$ and $y_1 = 1.0\,\text{m}$. Solve for y_2, which must be negative.

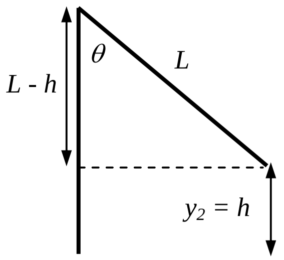
$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 \quad \rightarrow$$

$$mgy_1 = mgy_2 + \tfrac{1}{2}ky_2^2 \quad \rightarrow \quad y_2^2 + 2\frac{mg}{k}y_2 - 2\frac{mg}{k}y_1 = y_2^2 + 1.200\times10^{-3}y_2 - 1.200\times10^{-3} = 0$$

$$y_2 = -3.52\times10^{-2}\,\mathrm{m}, 3.40\times10^{-2}\,\mathrm{m}$$

$$F_{\mathrm{scale}} = k\left|x\right| = \left(1.183\times10^{6}\ \mathrm{N/m}\right)\left(3.52\times10^{-2}\,\mathrm{m}\right) = \boxed{4.2\times10^{4}\,\mathrm{N}}$$

86. (*a*) Use conservation of energy for the swinging motion. Subscript 1 represents the student initially grabbing the rope, and subscript 2 represents the student at the top of the swing. The location where the student initially grabs the rope is the zero location for PE $(y = 0)$.



We have $v_1 = 5.0\,\mathrm{m/s}$, $y_1 = 0$, and $v_2 = 0$. Solve for y_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

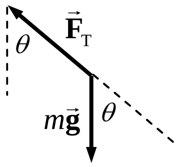
$$\tfrac{1}{2}mv_1^2 = mgy_2 \quad \rightarrow \quad y_2 = \frac{v_1^2}{2g} = h$$

Calculate the angle from the relationship in the diagram.

$$\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - \frac{v_1^2}{2gL} \quad \rightarrow$$

$$\theta = \cos^{-1}\left(1 - \frac{v_1^2}{2gL}\right) = \cos^{-1}\left(1 - \frac{\left(5.0\,\mathrm{m/s}\right)^2}{2\left(9.8\,\mathrm{m/s^2}\right)\left(10\ \mathrm{m}\right)}\right) = \boxed{29^\circ}$$

(*b*) At the release point, the speed is 0, and so there is no radial acceleration, since $a_{\mathrm{R}} = v^2/r$. Thus the centripetal force must be 0. Use the free-body diagram to write Newton's 2nd law for the radial direction.



$$\sum F_{\mathrm{R}} = F_{\mathrm{T}} - mg\cos\theta = 0 \quad \rightarrow$$

$$F_{\mathrm{T}} = mg\cos\theta = \left(65\ \mathrm{kg}\right)\left(9.8\,\mathrm{m/s^2}\right)\cos 29^\circ = \boxed{5.6\times10^{2}\,\mathrm{N}}$$

(*c*) Write Newton's 2nd law for the radial direction for any angle, and solve for the tension.

$$\sum F_{\mathrm{R}} = F_{\mathrm{T}} - mg\cos\theta = mv^2/r \quad \rightarrow \quad F_{\mathrm{T}} = mg\cos\theta + mv^2/r$$

As the angle decreases, the tension increases, and as the speed increases, the tension increases. Both effects are greatest at the bottom of the swing, and so that is where the tension will be at its maximum.

$$F_{\underset{\mathrm{max}}{\mathrm{T}}} = mg\cos 0 + mv_1^2/r = \left(65\ \mathrm{kg}\right)\left(9.8\,\mathrm{m/s^2}\right) + \frac{\left(65\ \mathrm{kg}\right)\left(5.0\,\mathrm{m/s}\right)^2}{10\ \mathrm{m}} = \boxed{8.0\times10^{2}\,\mathrm{N}}$$

87. The minimum vertical force needed to raise the athlete is equal to the athlete's weight. If the athlete moves upward a distance Δy, then the work done by the lifting force is $W = Fd\cos 0^\circ = mg\Delta y$, the change in PE. The power output needed to accomplish this work in a certain time *t* is the work divided by the time.

$$P = \frac{W}{t} = \frac{mg\Delta y}{t} = \frac{\left(72\ \mathrm{kg}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(5.0\ \mathrm{m}\right)}{9.0\ \mathrm{s}} = \boxed{3.9\times10^{2}\,\mathrm{W}}$$

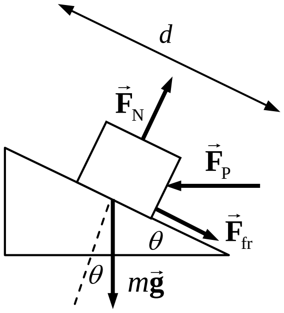
88. The energy to be stored is the power multiplied by the time: $E = Pt$. The energy will be stored as the gravitational PE increase in the water: $E = \Delta PE = mg\Delta y = \rho V g \Delta y$, where ρ is the density of the water, and V is the volume of the water.

$$Pt = \rho V g \Delta y \quad \rightarrow \quad V = \frac{Pt}{\rho g \Delta y} = \frac{\left(120\times10^6\,\text{W}\right)\left(3600\text{ s}\right)}{\left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(520\text{ m}\right)} = \boxed{8.5\times10^4\,\text{m}^3}$$

89. If the original spring is stretched a distance *x* from equilibrium, then the potential energy stored is $PE_{\text{full}} = \frac{1}{2}kx^2$. Alternatively, think of the original spring as being made up of the two halves of the spring, connected from end to end. Each half of the spring has a spring constant k', to be determined. As the spring is stretched a distance *x*, each half-spring is stretched a distance *x*/2. Each half-spring will have an amount of potential energy stored of $PE_{\text{half}} = \frac{1}{2}k'\left(x/2\right)^2$. The amount of energy in the two half-springs must equal the amount of energy in the full spring.

$$PE_{\text{full}} = 2PE_{\text{half}} \quad \rightarrow \quad \tfrac{1}{2}kx^2 = 2\left[\tfrac{1}{2}k'\left(x/2\right)^2\right] \quad \rightarrow \quad k' = \boxed{2k}$$

90. Consider the free-body diagram for the block. The block is moving up the plane.



d
$\vec{\mathbf{F}}_{\text{N}}$
$\vec{\mathbf{F}}_{\text{P}}$
$\vec{\mathbf{F}}_{\text{fr}}$
θ
θ
$m\vec{\mathbf{g}}$

(*a*) $KE_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}\left(6.0\text{ kg}\right)\left(2.2\,\text{m/s}\right)^2 = 14.52\text{ J} \approx \boxed{15\text{ J}}$

(*b*) $W_{\text{P}} = F_{\text{P}}d\cos 37^{\circ} = \left(75\text{ N}\right)\left(8.0\text{ m}\right)\cos 37^{\circ} = 479.2\text{ J} \approx \boxed{4.8\times10^2\text{ J}}$

(*c*) $W_{\text{fr}} = F_{\text{fr}}d\cos 180^{\circ} = -\left(25\text{ N}\right)\left(8.0\text{ m}\right) = \boxed{-2.0\times10^2\text{ J}}$

(*d*) $W_G = mgd\cos 127^{\circ} = \left(6.0\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)\left(8.0\text{ m}\right)\cos 127^{\circ}$

$$= -283.1\text{ J} \approx \boxed{-2.8\times10^2\text{ J}}$$

(*e*) $W_{\text{N}} = F_{\text{N}}d\cos 90^{\circ} = \boxed{0\text{ J}}$

(*f*) By the work-energy theorem,

$$W_{\text{total}} = KE_2 - KE_1 \quad \rightarrow$$

$$KE_2 = W_{\text{total}} + KE_1 = W_{\text{P}} + W_{\text{fr}} + W_{\text{G}} + W_{\text{N}} + KE_1 = 10.62\text{ J} \approx \boxed{11\text{ J}}$$

91. The power output for either scenario is given by the change in kinetic energy, divided by the time required to change the kinetic energy. Subscripts of *f* and *i* are used for final and initial values of speed and kinetic energy. Subscript 1 represents the acceleration from 35 km/h to 55 km/h, and subscript 2 represents the acceleration from 55 km/h to 75 km/h.

$$P_1 = \frac{KE_{1f} - KE_{1i}}{t_1} = \frac{\frac{1}{2}m\left(v_{1f}^2 - v_{1i}^2\right)}{t_1} \qquad P_2 = \frac{KE_{2f} - KE_{2i}}{t_2} = \frac{\frac{1}{2}m\left(v_{2f}^2 - v_{2i}^2\right)}{t_2}$$

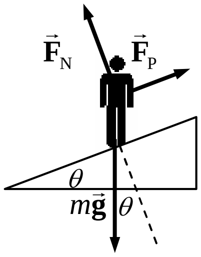
Equate the two expressions for power, and solve for t_2.

$$\frac{\frac{1}{2}m\left(v_{1f}^2 - v_{1i}^2\right)}{t_1} = \frac{\frac{1}{2}m\left(v_{2f}^2 - v_{2i}^2\right)}{t_2} \quad \rightarrow \quad t_2 = t_1\frac{\left(v_{2f}^2 - v_{2i}^2\right)}{\left(v_{1f}^2 - v_{1i}^2\right)}$$

Since the velocities are included as a ratio, any consistent set of units may be used for the velocities. Thus no conversion from km/h to some other units is needed.

$$t_2 = t_1\frac{\left(v_{2f}^2 - v_{2i}^2\right)}{\left(v_{1f}^2 - v_{1i}^2\right)} = \left(3.2\text{ s}\right)\frac{\left(75\,\text{km/h}\right)^2 - \left(55\,\text{km/h}\right)^2}{\left(55\,\text{km/h}\right)^2 - \left(35\,\text{km/h}\right)^2} = \boxed{4.6\text{ s}}$$

92. See the free-body diagram for the patient on the treadmill. We assume that there are no dissipative forces. Since the patient has a constant velocity, the net force parallel to the plane must be 0. Write Newton's 2nd law for forces parallel to the plane, and then calculate the power output of force $\vec{\mathbf{F}}_{\text{P}}$.



$$\sum F_{\text{parallel}} = F_{\text{P}} - mg\sin\theta = 0 \quad\rightarrow\quad F_{\text{P}} = mg\sin\theta$$

$$P = F_P v = mgv\sin\theta = (75\text{ kg})(9.8\,\text{m/s}^2)(3.3\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\sin 15^{\circ}$$

$$= 174.4\,\text{W} \approx \boxed{170\,\text{W}}$$

This is about 2 to 3 times the wattage of typical household light bulbs (60–100 W).

93. (*a*) Assume that there are no non-conservative forces on the rock, and so its mechanical energy is conserved. Subscript 1 represents the rock as it leaves the volcano, and subscript 2 represents the rock at its highest point. The location as the rock leaves the volcano is the zero location for PE $(y=0)$. We have $y_1 = 0$, $y_2 = 500\text{ m}$, and $v_2 = 0$. Solve for v_1.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 = mgy_2 \quad\rightarrow$$

$$v_1 = \sqrt{2gy_2} = \sqrt{2(9.80\,\text{m/s}^2)(500\text{ m})} = 98.99\,\text{m/s} \approx \boxed{1\times10^2\text{ m/s}}.$$

(*b*) The power output is the energy transferred to the launched rocks per unit time. The launching energy of a single rock is $\tfrac{1}{2}mv_1^2$, and so the energy of 1000 rocks is $1000\left(\tfrac{1}{2}mv_1^2\right)$. Divide this energy by the time it takes to launch 1000 rocks to find the power output needed to launch the rocks.

$$P = \frac{1000\left(\tfrac{1}{2}mv_1^2\right)}{t} = \frac{500(500\text{ kg})(98.99\,\text{m/s})^2}{60\text{ sec}} = \boxed{4\times10^7\,\text{W}}$$

94. (*a*) The maximum power output from the falling water would occur if all of the potential energy available were converted into work to turn the wheel. The rate of potential energy delivery to the wheel from the falling water is the power available.

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{(95\text{ kg})(9.8\,\text{m/s}^2)(2.0\,\text{m})}{1\text{ sec}} = \boxed{1.9\times10^3\,\text{W}}$$

(*b*) To find the speed of the water as it hits the wheel, use energy conservation with no non-conservative forces. Subscript 1 represents the water at the start of the descent, and subscript 2 represents the water as it hits the wheel at the bottom of the descent. The bottom of the descent is the zero location for PE $(y=0)$. We have $v_1 = 0$, $y_1 = 2.0\text{ m}$, and $y_2 = 0$. Solve for v_2.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad mgy_1 = \tfrac{1}{2}mv_2^2 \quad\rightarrow$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.8\,\text{m/s}^2)(2.0\text{ m})} = \boxed{6.3\,\text{m/s}}$$

CHAPTER 7: Linear Momentum

Answers to Questions

1. For momentum to be conserved, the system under analysis must be "closed" – not have any forces on it from outside the system. A coasting car has air friction and road friction on it, for example, which are "outside" forces and thus reduce the momentum of the car. If the ground and the air were considered part of the system, and their velocities analyzed, then the momentum of the entire system would be conserved, but not necessarily the momentum of any single component, like the car.

2. Consider this problem as a very light object hitting and sticking to a very heavy object. The large object – small object combination (Earth + jumper) would have some momentum after the collision, but due to the very large mass of the Earth, the velocity of the combination is so small that it is not measurable. Thus the jumper lands on the Earth, and nothing more happens.

3. When you release an inflated but untied balloon at rest, the gas inside the balloon (at high pressure) rushes out the open end of the balloon. That escaping gas and the balloon form a closed system, and so the momentum of the system is conserved. The balloon and remaining gas acquires a momentum equal and opposite to the momentum of the escaping gas, and so move in the opposite direction to the escaping gas.

4. If the rich man would have faced away from the shore and thrown the bag of coins directly away from the shore, he would have acquired a velocity towards the shore by conservation of momentum. Since the ice is frictionless, he would slide all the way to the shore.

5. When a rocket expels gas in a given direction, it puts a force on that gas. The momentum of the gas-rocket system stays constant, and so if the gas is pushed to the left, the rocket will be pushed to the right due to Newton's 3rd law. So the rocket must carry some kind of material to be ejected (usually exhaust from some kind of engine) to change direction.

6. The air bag greatly increases the amount of time over which the stopping force acts on the driver. If a hard object like a steering wheel or windshield is what stops the driver, then a large force is exerted over a very short time. If a soft object like an air bag stops the driver, then a much smaller force is exerted over a much longer time. For instance, if the air bag is able to increase the time of stopping by a factor of 10, then the average force on the person will be decreased by a factor of 10. This greatly reduces the possibility of serious injury or death.

7. "Crumple zones" are similar to air bags in that they increase the time of interaction during a collision, and therefore lower the average force required for the change in momentum that the car undergoes in the collision.

8. From Eq. 7-7 for a 1-D elastic collision, $v_A - v_B = v'_B - v'_A$. Let "A" represent the bat, and let "B" represent the ball. The positive direction will be the (assumed horizontal) direction that the bat is moving when the ball is hit. We assume the batter can swing the bat with equal strength in either case, so that v_A is the same in both pitching situations. Because the bat is so much heavier than the ball, we assume that $v'_A \approx v_A$ – the speed of the bat doesn't change significantly during the collision. Then the velocity of the baseball after being hit is $v'_B = v'_A + v_A - v_B \approx 2v_A - v_B$. If $v_B = 0$, the ball tossed up into the air by the batter, then $v'_B \approx 2v_A$ – the ball moves away with twice the speed of the

bat. But if $v_B < 0$, the pitched ball situation, we see that the magnitude of $v'_B > 2v_A$, and so the ball moves away with greater speed. If, for example, the pitching speed of the ball was about twice the speed at which the batter could swing the bat, then we would have $v'_B \approx 4v_A$. Thus the ball has greater speed after being struck, and thus it is easier to hit a home run. This is similar to the "gravitational slingshot" effect discussed in problem 85.

9. The impulse is the product of the force and the time duration that the force is applied. So the impulse from a small force applied over a long time can be larger than the impulse applied by a large force over a small time.

$\boxed{10.}$ The momentum of an object can be expressed in terms of its kinetic energy, as follows.

$$p = mv = \sqrt{m^2v^2} = \sqrt{m\left(mv^2\right)} = \sqrt{2m\left(\tfrac{1}{2}mv^2\right)} = \sqrt{2mKE} .$$

Thus if two objects have the same kinetic energy, then the one with more mass has the greater momentum.

11. Consider two objects, each with the same magnitude of momentum, moving in opposite directions. They have a total momentum of 0. If they collide and have a totally inelastic collision, in which they stick together, then their final common speed must be 0 so that momentum is conserved. But since they are not moving after the collision, they have no kinetic energy, and so all of their kinetic energy has been lost.

12. The turbine blades should be designed so that the water rebounds. If the water rebounds, that means that a larger momentum change for the water has occurred than if it just came to a stop. And if there is a larger momentum change for the water, there will also be a larger momentum change for the blades, making them spin faster.

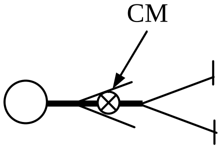
13. (*a*) The downward component of the momentum is unchanged. The horizontal component of momentum changes from rightward to leftward. Thus the change in momentum is to the left in the picture.
    (*b*) Since the force on the wall is opposite that on the ball, the force on the wall is to the right.

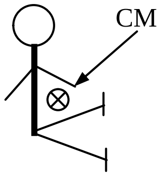
14. (*a*) The momentum of the ball is not conserved during any part of the process, because there is an external force acting on the ball at all times – the force of gravity. And there is an upward force on the ball during the collision. So considering the ball as the system, there are always external forces on it, and so its momentum is not conserved.
    (*b*) With this definition of the system, all of the forces are internal, and so the momentum of the Earth-ball system is conserved during the entire process.
    (*c*) The answer here is the same as for part (*b*).

15. In order to maintain balance, your CM must be located directly above your feet. If you have a heavy load in your arms, your CM will be out in front of your body and not above your feet. So you lean backwards to get your CM directly above your feet. Otherwise, you would fall over forwards.

16. The 1-m length of pipe is uniform – it has the same density throughout, and so its CM is at its geometric center, which is its midpoint. The arm and leg are not uniform – they are more dense where there is muscle, primarily in the parts that are closest to the body. Thus the CM of the arm or leg is closer the body than the geometric center. The CM is located closer to the more massive part of the arm or leg.

17. When you are lying flat on the floor, your CM is inside of the volume of your body. When you sit up on the floor with your legs extended, your CM is outside of the volume of your body.





18. The engine does not directly accelerate the car. The engine puts a force on the driving wheels, making them rotate. The wheels then push backwards on the roadway as they spin. The Newton's 3rd law reaction to this force is the forward-pushing of the roadway on the wheels, which accelerates the car. So it is the (external) road surface that accelerates the car.

19. The motion of the center of mass of the rocket will follow the original parabolic path, both before and after explosion. Each individual piece of the rocket will follow a separate path after the explosion, but since the explosion was internal to the system (consisting of the rocket), the center of mass of all the exploded pieces will follow the original path.

Solutions to Problems

1. $p = mv = (0.028\text{ kg})(8.4\text{ m/s}) = \boxed{0.24\text{ kg}\bullet\text{m/s}}$

2. From Newton's second law, $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t$. For a constant mass object, $\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}$. Equate the two expressions for $\Delta\vec{\mathbf{p}}$.

$$\vec{\mathbf{F}}\Delta t = m\Delta\vec{\mathbf{v}} \quad\rightarrow\quad \Delta\vec{\mathbf{v}} = \frac{\vec{\mathbf{F}}\Delta t}{m}.$$

If the skier moves to the right, then the speed will decrease, because the friction force is to the left.

$$\Delta v = -\frac{F\Delta t}{m} = -\frac{(25\text{ N})(20\text{ s})}{65\text{ kg}} = \boxed{-7.7\text{ m/s}}$$

The skier loses 7.7 m/s of speed.

3. Choose the direction from the batter to the pitcher to be the positive direction. Calculate the average force from the change in momentum of the ball.

$$\Delta p = F\Delta t = m\Delta v \quad\rightarrow$$

$$F = m\frac{\Delta v}{\Delta t} = (0.145\text{ kg})\left(\frac{52.0\text{ m/s} - -39.0\text{ m/s}}{3.00\times10^{-3}\text{s}}\right) = \boxed{4.40\times10^{3}\text{ N, towards the pitcher}}$$

$\boxed{4.}$ The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let "A" represent the boat and child together, and let "B" represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad (m_A + m_B)v = m_A v'_A + m_B v'_B \quad\rightarrow$$

$$v'_A = -\frac{m_B v'_B}{m_A} = -\frac{(6.40\text{ kg})(10.0\text{ m/s})}{(26.0\text{ kg} + 45.0\text{ kg})} = \boxed{-0.901\text{ m/s}}$$

The boat and child move in the opposite direction as the thrown package.

5. The force on the gas can be found from its change in momentum.

$$F=\frac{\Delta p}{\Delta t}=\frac{v\Delta m}{\Delta t}=v\frac{\Delta m}{\Delta t}=\left(4.0\times10^{4}\ \mathrm{m/s}\right)\left(1500\ \mathrm{kg/s}\right)=6.0\times10^{7}\ \mathrm{N\ downward}$$

The force on the rocket is the Newton's 3rd law pair (equal and opposite) to the force on the gas, and so is $\boxed{6.0\times10^{7}\ \mathrm{N\ upward}}$.

6. The tackle will be analyzed as a one-dimensional momentum conserving situation. Let "A" represent the halfback, and "B" represent the tackling cornerback.

$$p_{\text{initial}}=p_{\text{final}}\quad\rightarrow\quad m_Av_A+m_Bv_B=\left(m_A+m_B\right)v'\quad\rightarrow$$

$$v'=\frac{m_Av_A+m_Bv_B}{m_A+m_B}=\frac{\left(95\ \mathrm{kg}\right)\left(4.1\ \mathrm{m/s}\right)+\left(85\ \mathrm{kg}\right)\left(5.5\ \mathrm{m/s}\right)}{\left(95\ \mathrm{kg}\right)+\left(85\ \mathrm{kg}\right)}=\boxed{4.8\ \mathrm{m/s}}$$

7. Consider the horizontal motion of the objects. The momentum in the horizontal direction will be conserved. Let "A" represent the car, and "B" represent the load. The positive direction is the direction of the original motion of the car.

$$p_{\text{initial}}=p_{\text{final}}\quad\rightarrow\quad m_Av_A+m_Bv_B=\left(m_A+m_B\right)v'\quad\rightarrow$$

$$v'=\frac{m_Av_A+m_Bv_B}{m_A+m_B}=\frac{\left(12{,}600\ \mathrm{kg}\right)\left(18.0\ \mathrm{m/s}\right)+0}{\left(12{,}600\ \mathrm{kg}\right)+\left(5350\ \mathrm{kg}\right)}=\boxed{12.6\ \mathrm{m/s}}$$

8. Consider the motion in one dimension, with the positive direction being the direction of motion of the first car. Let "A" represent the first car, and "B" represent the second car. Momentum will be conserved in the collision. Note that $v_B=0$.

$$p_{\text{initial}}=p_{\text{final}}\quad\rightarrow\quad m_Av_A+m_Bv_B=\left(m_A+m_B\right)v'\quad\rightarrow$$

$$m_B=\frac{m_A\left(v_A-v'\right)}{v'}=\frac{\left(9300\ \mathrm{kg}\right)\left(15.0\ \mathrm{m/s}-6.0\ \mathrm{m/s}\right)}{6.0\ \mathrm{m/s}}=\boxed{1.4\times10^{4}\ \mathrm{kg}}$$

9. The force stopping the wind is exerted by the person, so the force on the person would be equal in magnitude and opposite in direction to the force stopping the wind. Calculate the force from Eq. 7-2, in magnitude only.

$$\frac{m_{\text{wind}}}{\Delta t}=\frac{40\ \mathrm{kg/s}}{\mathrm{m}^2}\left(1.50\ \mathrm{m}\right)\left(0.50\ \mathrm{m}\right)=30\ \mathrm{kg/s}\qquad\Delta v_{\text{wind}}=100\ \mathrm{km/h}\left(\frac{1\ \mathrm{m/s}}{3.6\ \mathrm{km/h}}\right)=27.8\ \mathrm{m/s}$$

$$F_{\substack{\text{on}\\\text{person}}}=F_{\substack{\text{on}\\\text{wind}}}=\frac{\Delta p_{\text{wind}}}{\Delta t}=\frac{m_{\text{wind}}\Delta v_{\text{wind}}}{\Delta t}=\frac{m_{\text{wind}}}{\Delta t}\Delta v_{\text{wind}}=\left(30\ \mathrm{kg/s}\right)\left(27.8\ \mathrm{m/s}\right)$$

$$=833\ \mathrm{N}\approx\boxed{8\times10^{2}\ \mathrm{N}}$$

The typical maximum frictional force is $F_{\text{fr}}=\mu_s mg=\left(1.0\right)\left(70\ \mathrm{kg}\right)\left(9.8\ \mathrm{m/s^2}\right)=690\ \mathrm{N}$, and so we see that $\boxed{F_{\substack{\text{on}\\\text{person}}}>F_{\text{fr}}}$ – the wind is literally strong enough to blow a person off his feet.

10. Momentum will be conserved in the horizontal direction. Let "A" represent the car, and "B" represent the snow. For the horizontal motion, $v_B=0$ and $v_B'=v_A'$. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A = (m_A + m_B) v'_A$$

$$v'_A = \frac{m_A v_A}{m_A + m_B} = \frac{(3800\text{ kg})(8.60\text{ m/s})}{3800\text{ kg} + \left(\frac{3.50\text{ kg}}{\text{min}}\right)(90.0\text{ min})} = 7.94\text{ m/s} \approx \boxed{7.9\text{ m/s}}$$

11. Consider the motion in one dimension, with the positive direction being the direction of motion of the original nucleus. Let "A" represent the alpha particle, with a mass of 4 u, and "B" represent the new nucleus, with a mass of 218 u. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_A + m_B) v = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_A = \frac{(m_A + m_B) v - m_B v'_B}{m_A} = \frac{(222\text{ u})(420\text{ m/s}) - (218\text{ u})(350\text{ m/s})}{4.0\text{ u}} = \boxed{4.2 \times 10^3\text{ m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

$\boxed{12.}$ Consider the motion in one dimension with the positive direction being the direction of motion of the bullet. Let "A" represent the bullet, and "B" represent the block. Since there is no net force outside of the block-bullet system (like frictions with the table), the momentum of the block and bullet combination is conserved. Note that $v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_B = \frac{m_A (v_A - v'_A)}{m_B} = \frac{(0.023\text{ kg})(230\text{ m/s} - 170\text{ m/s})}{2.0\text{ kg}} = \boxed{0.69\text{ m/s}}$$

13. (*a*) Consider the motion in one dimension with the positive direction being the direction of motion before the separation. Let "A" represent the upper stage (that moves away faster) and "B" represent the lower stage. It is given that $m_A = m_B$, $v_A = v_B = v$, and $v'_B = v'_A - v_{rel}$. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_A + m_B) v = m_A v'_A + m_B v'_B = m_A v'_A + m_B (v'_A - v_{rel}) \quad \rightarrow$$

$$v'_A = \frac{(m_A + m_B) v + m_B v_{rel}}{(m_A + m_B)} = \frac{(975\text{ kg})(5800\text{ m/s}) + \frac{1}{2}(975\text{ kg})(2200\text{ m/s})}{(975\text{ kg})}$$

$$= \boxed{6.9 \times 10^3\text{ m/s , away from Earth}}$$

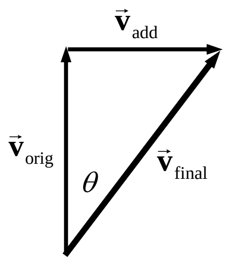
$$v'_B = v'_A - v_{rel} = 6.9 \times 10^3\text{ m/s} - 2.20 \times 10^3\text{ m/s} = \boxed{4.7 \times 10^3\text{ m/s , away from Earth}}$$

(*b*) The change in KE had to be supplied by the explosion.

$$\Delta KE = KE_f - KE_i = \left(\tfrac{1}{2} m_A v'^2_A + \tfrac{1}{2} m_B v'^2_B\right) - \tfrac{1}{2}(m_A + m_B) v^2$$

$$= \tfrac{1}{2}(487.5\text{ kg})\left[(6900\text{ m/s})^2 + (4700\text{ m/s})^2\right] - \tfrac{1}{2}(975\text{ kg})(5800\text{ m/s})^2$$

$$= \boxed{5.9 \times 10^8\text{ J}}$$


$\vec{\mathbf{v}}_{\text{add}}$
$\vec{\mathbf{v}}_{\text{orig}}$
θ
$\vec{\mathbf{v}}_{\text{final}}$

14. To alter the course by 35.0°, a velocity perpendicular to the original velocity must be added. Call the direction of the added velocity, $\vec{\mathbf{v}}_{\text{add}}$, the positive direction. From the diagram, we see that $v_{\text{add}} = v_{\text{orig}} \tan\theta$. The momentum in the perpendicular direction will be conserved, considering that the gases are given perpendicular momentum in the opposite direction of $\vec{\mathbf{v}}_{\text{add}}$. The gas is expelled in the opposite direction to $\vec{\mathbf{v}}_{\text{add}}$, and so a negative value is used for $v_{\perp\text{ gas}}$.

$$\underset{\text{before}}{p_\perp} = \underset{\text{after}}{p_\perp} \quad \to \quad 0 = m_{\text{gas}} v_{\perp\text{ gas}} + \left(m_{\text{rocket}} - m_{\text{gas}}\right) v_{\text{add}} \quad \to$$

$$m_{\text{gas}} = \frac{m_{\text{rocket}} v_{\text{add}}}{\left(v_{\text{add}} - v_{\perp\text{ gas}}\right)} = \frac{(3180\text{ kg})(115\text{ m/s})\tan 35.0^\circ}{\left[(115\text{ m/s})\tan 35^\circ - -1750\text{ m/s}\right]} = \boxed{1.40\times10^1\text{kg}}$$

$\boxed{15.}$ (*a*) The impulse is the change in momentum. The direction of travel of the struck ball is the positive direction.

$$\Delta p = m\Delta v = \left(4.5\times10^{-2}\text{kg}\right)\left(45\text{ m/s} - 0\right) = \boxed{2.0\text{ kg}\bullet\text{m/s}}$$

(*b*) The average force is the impulse divided by the interaction time.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{2.0\text{ kg}\bullet\text{m/s}}{3.5\times10^{-3}\text{s}} = \boxed{5.8\times10^2\text{ N}}$$

16. (*a*) The impulse given to the nail is the opposite of the impulse given to the hammer. This is the change in momentum. Call the direction of the initial velocity of the hammer the positive direction.

$$\Delta p_{\text{nail}} = -\Delta p_{\text{hammer}} = mv_i - mv_f = (12\text{ kg})(8.5\text{ m/s}) - 0 = \boxed{1.0\times10^2\text{ kg}\bullet\text{m/s}}$$

(*b*) The average force is the impulse divided by the time of contact.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{1.0\times10^2\text{ kg}\bullet\text{m/s}}{8.0\times10^{-3}\text{s}} = \boxed{1.3\times10^4\text{N}}$$

17. The impulse given the ball is the change in the ball's momentum. From the symmetry of the problem, the vertical momentum of the ball does not change, and so there is no vertical impulse. Call the direction AWAY from the wall the positive direction for momentum perpendicular to the wall.

$$\Delta p_\perp = m\underset{\text{final}}{v_\perp} - m\underset{\text{initial}}{v_\perp} = m\left(v\sin 45^\circ - -v\sin 45^\circ\right) = 2mv\sin 45^\circ$$

$$= 2\left(6.0\times10^{-2}\text{km}\right)\left(25\text{ m/s}\right)\sin 45^\circ = \boxed{2.1\text{kg}\bullet\text{m/s} \text{ , to the left}}$$

18. (*a*) The average force on the car is the impulse (change in momentum) divided by the time of interaction. The positive direction is the direction of the car's initial velocity.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = (1500\text{ kg})\left(\frac{0 - 50\text{ km/h}\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)}{0.15\text{ s}}\right) = -1.389\times10^5\text{ N} \approx \boxed{-1.4\times10^5\text{N}}$$

(*b*) The deceleration is found from Newton's 2nd law.

$$\bar{F}=m\bar{a} \rightarrow \bar{a}=\frac{\bar{F}}{m}=\frac{\left(-1.389\times10^{5}\,\text{N}\right)}{1500\,\text{kg}}=\boxed{-93\,\text{m/s}^{2}}$$

19. Call east the positive direction.

(*a*) $p_{\substack{\text{original}\\ \text{fullback}}}=mv_{\substack{\text{original}\\ \text{fullback}}}=(95\text{ kg})(4.0\,\text{m/s})=\boxed{3.8\times10^{2}\text{ kg}\bullet\text{m/s}}$

(*b*) The impulse on the fullback is the change in the fullback's momentum.

$$\Delta p_{\text{fullback}}=m\left(v_{\substack{\text{final}\\ \text{fullback}}}-v_{\substack{\text{final}\\ \text{fullback}}}\right)=(95\text{ kg})(0-4.0\,\text{m/s})=\boxed{-3.8\times10^{2}\text{ kg}\bullet\text{m/s}}$$

(*c*) The impulse on the tackler is the opposite of the impulse on the fullback, so $\boxed{3.8\times10^{2}\text{ kg}\bullet\text{m/s}}$

(*d*) The average force on the tackler is the impulse on the tackler divided by the time of interaction.

$$\bar{F}=\frac{\Delta p}{\Delta t}=\frac{3.8\times10^{2}\text{ kg}\bullet\text{m/s}}{0.75\text{ s}}=\boxed{5.1\times10^{2}\,\text{N}}$$

20. (*a*) The impulse given the ball is the area under the *F* vs. *t* graph. Approximate the area as a triangle of "height" 250 N, and "width" 0.01 sec.

$$\Delta p=\tfrac{1}{2}(250\text{ N})(0.01\text{ s})=\boxed{1.25\text{N}\bullet\text{s}}$$

(*b*) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball's travel after being served.

$$\Delta p=m\Delta v=m\left(v_{f}-v_{\text{i}}\right) \rightarrow v_{f}=v_{i}+\frac{\Delta p}{m}=0+\frac{1.25\text{N}\bullet\text{s}}{6.0\times10^{-2}\,\text{kg}}=\boxed{21\text{m/s}}$$

21. Find the velocity upon reaching the ground from energy conservation. Assume that all of the initial potential energy at the maximum height h_{max} is converted into kinetic energy. Take down to be the positive direction, so the velocity at the ground is positive.

$$mgh_{\text{max}}=\tfrac{1}{2}mv_{\text{ground}}^{2} \rightarrow v_{\text{ground}}=\sqrt{2gh_{\text{max}}}$$

When contacting the ground, the impulse on the person causes a change in momentum. That relationship is used to find the time of the stopping interaction. The force of the ground acting on the person is negative since it acts in the upward direction.

$$\bar{F}\Delta t=m\left(0-v_{\text{ground}}\right) \rightarrow \Delta t=-\frac{mv_{\text{ground}}}{\bar{F}}$$

We assume that the stopping force is so large that we call it the total force on the person – we ignore gravity for the stopping motion. The average acceleration of the person during stopping $\left(\bar{a}=\bar{F}/m\right)$ is used with Eq. 2-11b to find the displacement during stopping, h_{stop}.

$$y-y_{0}=v_{0}t+\tfrac{1}{2}at^{2} \rightarrow h_{\text{stop}}=v_{\text{ground}}\left(-\frac{mv_{\text{ground}}}{\bar{F}}\right)+\tfrac{1}{2}\left(\frac{\bar{F}}{m}\right)\left(-\frac{mv_{\text{ground}}}{\bar{F}}\right)^{2} \rightarrow$$

$$h_{\text{stop}}=-\frac{mv_{\text{ground}}^{2}}{\bar{F}}+\frac{1}{2}\frac{mv_{\text{ground}}^{2}}{\bar{F}}=-\frac{1}{2}\frac{mv_{\text{ground}}^{2}}{\bar{F}}=-\frac{gh_{\text{max}}m}{\bar{F}} \rightarrow h_{\text{max}}=-\frac{\bar{F}h_{\text{stop}}}{mg}$$

We assume that the person lands with both feet striking the ground simultaneously, so the stopping force is divided between both legs. Thus the critical average stopping force is twice the breaking strength of a single leg.

$$h_{\text{max}} = -\frac{\bar{F} h_{\text{stop}}}{mg} = \frac{2F_{\text{break}} h_{\text{stop}}}{mg} = \frac{2\left(170\times10^{6}\ \text{N/m}^2\right)\left(2.5\times10^{-4}\ \text{m}^2\right)\left(0.60\ \text{m}\right)}{\left(75\ \text{kg}\right)\left(9.8\,\text{m/s}^2\right)} = \boxed{69\ \text{m}}$$

22. Let A represent the 0.440-kg ball, and B represent the 0.220-kg ball. We have $v_{\text{A}} = 3.30\,\text{m/s}$ and $v_{\text{B}} = 0$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_{\text{A}} - v_{\text{B}} = -\left(v'_{\text{A}} - v'_{\text{B}}\right) \ \rightarrow\ v'_{\text{B}} = v_{\text{A}} + v'_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_{\text{A}} v_{\text{A}} + m_{\text{B}} v_{\text{B}} = m_{\text{A}} v'_{\text{A}} + m_{\text{B}} v'_{\text{B}} \ \rightarrow\ m_{\text{A}} v_{\text{A}} = m_{\text{A}} v'_{\text{A}} + m_{\text{B}}\left(v_{\text{A}} + v'_{\text{A}}\right) \ \rightarrow$$

$$v'_{\text{A}} = \frac{\left(m_{\text{A}} - m_{\text{B}}\right)}{\left(m_{\text{A}} + m_{\text{B}}\right)} v_{\text{A}} = \frac{0.220\ \text{kg}}{0.660\ \text{kg}}\left(3.30\,\text{m/s}\right) = \boxed{1.10\,\text{m/s}\left(\text{east}\right)}$$

$$v'_{\text{B}} = v_{\text{A}} + v'_{\text{A}} = 3.30\,\text{m/s} + 1.10\,\text{m/s} = \boxed{4.40\,\text{m/s}\left(\text{east}\right)}$$

$\boxed{23.}$ Let A represent the 0.450-kg puck, and let B represent the 0.900-kg puck. The initial direction of puck A is the positive direction. We have $v_{\text{A}} = 3.00\,\text{m/s}$ and $v_{\text{B}} = 0$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_{\text{A}} - v_{\text{B}} = -\left(v'_{\text{A}} - v'_{\text{B}}\right) \ \rightarrow\ v'_{\text{B}} = v_{\text{A}} + v'_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_{\text{A}} v_{\text{A}} + m_{\text{B}} v_{\text{B}} = m_{\text{A}} v'_{\text{A}} + m_{\text{B}} v'_{\text{B}} \ \rightarrow\ m_{\text{A}} v_{\text{A}} = m_{\text{A}} v'_{\text{A}} + m_{\text{B}}\left(v_{\text{A}} + v'_{\text{A}}\right) \ \rightarrow$$

$$v'_{\text{A}} = \frac{\left(m_{\text{A}} - m_{\text{B}}\right)}{\left(m_{\text{A}} + m_{\text{B}}\right)} v_{\text{A}} = \frac{-0.450\ \text{kg}}{1.350\ \text{kg}}\left(3.00\,\text{m/s}\right) = -1.00\,\text{m/s} = \boxed{1.00\,\text{m/s}\left(\text{west}\right)}$$

$$v'_{\text{B}} = v_{\text{A}} + v'_{\text{A}} = 3.00\,\text{m/s} - 1.00\,\text{m/s} = \boxed{2.00\,\text{m/s}\left(\text{east}\right)}$$

24. Let A represent the ball moving at 2.00 m/s, and call that direction the positive direction. Let B represent the ball moving at 3.00 m/s in the opposite direction. So $v_{\text{A}} = 2.00\,\text{m/s}$ and $v_{\text{B}} = -3.00\,\text{m/s}$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_{\text{A}} - v_{\text{B}} = -\left(v'_{\text{A}} - v'_{\text{B}}\right) \ \rightarrow\ v'_{\text{B}} = 5.00\,\text{m/s} + v'_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision, noting that $m_{\text{A}} = m_{\text{B}}$.

$$m_{\text{A}} v_{\text{A}} + m_{\text{B}} v_{\text{B}} = m_{\text{A}} v'_{\text{A}} + m_{\text{B}} v'_{\text{B}} \ \rightarrow\ v_{\text{A}} + v_{\text{B}} = v'_{\text{A}} + v'_{\text{B}} \ \rightarrow$$

$$-1.00\,\text{m/s} = v'_{\text{A}} + \left(v'_{\text{A}} + 5.00\,\text{m/s}\right) \ \rightarrow\ 2v'_{\text{A}} = -6.00\,\text{m/s} \ \rightarrow\ v'_{\text{A}} = \boxed{-3.00\,\text{m/s}}$$

$$v'_{\text{B}} = 5.00\,\text{m/s} + v'_{\text{A}} = \boxed{2.00\,\text{m/s}}$$

The two balls have exchanged velocities. This will always be true for 1-D elastic collisions of objects of equal mass.

25. Let A represent the 0.060-kg tennis ball, and let B represent the 0.090-kg ball. The initial direction of the balls is the positive direction. We have $v_A = 2.50\,\text{m/s}$ and $v_B = 1.15\,\text{m/s}$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = 1.35\,\text{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A + m_B v_B = m_A v'_A + m_B(1.35\,\text{m/s} + v'_A) \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B(v_B - 1.35\,\text{m/s})}{m_A + m_B} = \frac{(0.060\text{ kg})(2.50\,\text{m/s}) + (0.090\text{ kg})(1.15\,\text{m/s} - 1.35\,\text{m/s})}{0.150\text{ kg}}$$

$$= \boxed{0.88\,\text{m/s}}$$

$$v'_B = 1.35\,\text{m/s} + v'_A = \boxed{2.23\,\text{m/s}}$$

Both balls move in the direction of the tennis ball's initial motion.

26. Let A represent the moving softball, and let B represent the ball initially at rest. The initial direction of the softball is the positive direction. We have $v_A = 8.5\,\text{m/s}$, $v_B = 0$, and $v'_A = -3.7\,\text{m/s}$.

(*a*) Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A - v_B + v'_A = 8.5\,\text{m/s} - 0 - 3.7\,\text{m/s} = \boxed{4.8\,\text{m/s}}$$

(*b*) Use momentum conservation to solve for the mass of the target ball.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow$$

$$m_B = m_A \frac{(v_A - v'_A)}{(v'_B - v_B)} = (0.220\text{ kg})\frac{(8.5\,\text{m/s} - -3.7\,\text{m/s})}{4.8\,\text{m/s}} = \boxed{0.56\text{ kg}}$$

27. Let the original direction of the cars be the positive direction. We have $v_A = 4.50\,\text{m/s}$ and $v_B = 3.70\,\text{m/s}$

(*a*) Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A - v_B + v'_A = 0.80\,\text{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A + m_B v_B = m_A v'_A + m_B(0.80\,\text{m/s} + v'_A) \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B(v_B - 0.80\,\text{m/s})}{m_A + m_B} = \frac{(450\text{ kg})(4.50\,\text{m/s}) + (550\text{ kg})(2.90\,\text{m/s})}{1000\text{ kg}} = \boxed{3.62\,\text{m/s}}$$

$$v'_B = 0.80\,\text{m/s} + v'_A = \boxed{4.42\,\text{m/s}}$$

(*b*) Calculate $\Delta p = p' - p$ for each car.

$$\Delta p_A = m_A v'_A - m_A v_A = (450\text{ kg})(3.62\,\text{m/s} - 4.50\,\text{m/s}) = -3.96\times10^2\text{ kg}\bullet\text{m/s}$$

$$\approx \boxed{-4.0\times10^2\text{ kg}\bullet\text{m/s}}$$

$$\Delta p_B = m_B v'_B - m_B v_B = (550\text{ kg})(4.42\,\text{m/s} - 3.70\,\text{m/s}) = 3.96\times10^2\text{ kg}\bullet\text{m/s}$$

$$\approx \boxed{4.0\times10^2\text{ kg}\bullet\text{m/s}}$$

The two changes are equal and opposite because momentum was conserved.

28. (*a*) Momentum will be conserved in one dimension. Call the direction of the first ball the positive direction. Let A represent the first ball, and B represent the second ball. We have $v_B = 0$ and $v_B' = \frac{1}{2}v_A$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v_A' - v_B'\right) \quad \rightarrow \quad v_A' = -\tfrac{1}{2}v_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow \quad m_A v_A = -\tfrac{1}{2}m_A v_A + m_B \tfrac{1}{2}v_A \quad \rightarrow$$

$$m_B = 3m_A = 3\left(0.280\text{ kg}\right) = \boxed{0.840\text{ kg}}$$

(*b*) The fraction of the kinetic energy given to the second ball is as follows.

$$\frac{KE_B'}{KE_A} = \frac{\frac{1}{2}m_B v_B'^2}{\frac{1}{2}m_A v_A^2} = \frac{3m_A\left(\frac{1}{2}v_A\right)^2}{m_A v_A^2} = \boxed{0.75}$$

29. Let A represent the cube of mass *M*, and B represent the cube of mass *m*. Find the speed of A immediately before the collision, v_A, by using energy conservation.

$$Mgh = \tfrac{1}{2}Mv_A^2 \quad \rightarrow \quad v_A = \sqrt{2gh} = \sqrt{2\left(9.8\text{ m/s}^2\right)\left(0.30\text{ m}\right)} = 2.42\text{ m/s}$$

Use Eq. 7-7 for elastic collisions to obtain a relationship between the velocities in the collision. We have $v_B = 0$ and $M = 2m$.

$$v_A - v_B = -\left(v_A' - v_B'\right) \quad \rightarrow \quad v_B' = v_A + v_A'$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow \quad m_A v_A = m_A v_A' + m_B\left(v_A + v_A'\right) \quad \rightarrow$$

$$2mv_A = 2mv_A' + m\left(v_A + v_A'\right) \quad \rightarrow \quad v_A' = \frac{v_A}{3} = \frac{\sqrt{2gh}}{3} = \frac{\sqrt{2\left(9.8\text{ m/s}^2\right)\left(0.30\text{ m}\right)}}{3} = 0.808\text{ m/s}$$

$$v_B' = v_A + v_A' = \tfrac{4}{3}v_A = 3.23\text{ m/s}$$

Each mass is moving horizontally initially after the collision, and so each has a vertical velocity of 0 as they start to fall. Use constant acceleration Eq. 2-11b with down as positive and the table top as the vertical origin to find the time of fall.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad H = 0 + 0 + \tfrac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{2H/g}$$

Each cube then travels a horiztonal distance found by $\Delta x = v_x \Delta t$.

$$\Delta x_m = v_A' \Delta t = \frac{\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{2}{3}\sqrt{hH} = \tfrac{2}{3}\sqrt{\left(0.30\text{ m}\right)\left(0.90\text{ m}\right)} = 0.3464\text{ m} \approx \boxed{0.35\text{ m}}$$

$$\Delta x_M = v_B' \Delta t = \frac{4\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{8}{3}\sqrt{hH} = \tfrac{8}{3}\sqrt{\left(0.30\text{ m}\right)\left(0.90\text{ m}\right)} = 1.386\text{ m} \approx \boxed{1.4\text{ m}}$$

30. (*a*) Use Eq. 7-7, along with $v_B = 0$, to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v_A' - v_B'\right) \quad \rightarrow \quad v_B' = v_A + v_A'$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' = m_A v_A' + m_B\left(v_A + v_A'\right) = m_A v_A' + m_B v_A + m_B v_A' \quad \rightarrow$$

$$m_A v_A - m_B v_A = m_A v_A' + m_B v_A' \quad \rightarrow \quad \left(m_A - m_B\right)v_A = \left(m_A + m_B\right)v_A' \quad \rightarrow \quad v_A' = \frac{\left(m_A - m_B\right)}{\left(m_A + m_B\right)}v_A$$

Substitute this result into the result of Eq. 7-7.

$$v'_{\mathrm{B}}=v_{\mathrm{A}}+v'_{\mathrm{A}}=v_{\mathrm{A}}+\frac{(m_{\mathrm{A}}-m_{\mathrm{B}})}{(m_{\mathrm{A}}+m_{\mathrm{B}})}v_{\mathrm{A}}=v_{\mathrm{A}}\frac{(m_{\mathrm{A}}+m_{\mathrm{B}})}{(m_{\mathrm{A}}+m_{\mathrm{B}})}+\frac{(m_{\mathrm{A}}-m_{\mathrm{B}})}{(m_{\mathrm{A}}+m_{\mathrm{B}})}v_{\mathrm{A}}=v_{\mathrm{A}}\frac{2m_{\mathrm{A}}}{(m_{\mathrm{A}}+m_{\mathrm{B}})}$$

(*b*) If $m_A \ll m_B$, then approximate $m_A = 0$

$$v'_A=\frac{(m_A-m_B)}{(m_A+m_B)}v_A=\frac{(-m_B)}{(+m_B)}v_A=-v_A \qquad v'_B=\frac{2m_Av_A}{(m_A+m_B)}=0$$

The result is $\boxed{v'_A=-v_A\ ;\ v'_B=0}$. An example of this is a ball bouncing off of the floor. The massive floor has no speed after the collision, and the velocity of the ball is reversed (if dissipative forces are not present).

(*c*) If $m_A \gg m_B$, then approximate $m_B = 0$.

$$v'_A=\frac{(m_A-m_B)}{(m_A+m_B)}v_A=\frac{(m_A)}{(m_A)}v_A=v_A \qquad v'_B=\frac{2m_Av_A}{(m_A+m_B)}=\frac{2m_Av_A}{(m_A)}=2v_A$$

The result is $\boxed{v'_A=v_A\ ;\ v'_B=2v_{\mathrm{A}}}$. An example of this would be a golf club hitting a golf ball. The speed of the club immediately after the collision is essentially the same as its speed before the collision, and the golf ball takes off with twice the speed of the club.

(*d*) If $m_A = m_B$, then set $m_A = m_B = m$.

$$v'_A=\frac{(m-m)}{(m+m)}v_A=0 \qquad v'_B=\frac{2mv_A}{(m+m)}=\frac{2mv_A}{2m}=v_A$$

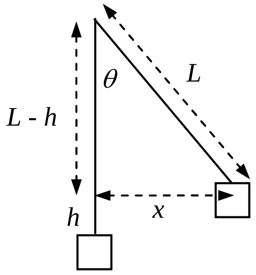
The result is $\boxed{v'_A=0\ ;\ v'_B=v_{\mathrm{A}}}$. An example of this is one billiard ball making a head-on collision with another. The first ball stops, and the second ball takes off with the same speed that the first one had.

31. From the analysis in Example 7-10, the initial projectile speed is given by $v=\frac{m+M}{m}\sqrt{2gh}$.

Compare the two speeds with the same masses.

$$\frac{v_2}{v_1}=\frac{\frac{m+M}{m}\sqrt{2gh_2}}{\frac{m+M}{m}\sqrt{2gh_1}}=\frac{\sqrt{h_2}}{\sqrt{h_1}}=\sqrt{\frac{h_2}{h_1}}=\sqrt{\frac{5.2}{2.6}}=\sqrt{2}\ \rightarrow\ \boxed{v_2=\sqrt{2}v_1}$$

32. From the analysis in the Example 7-10, we know that



$$v=\frac{m+M}{m}\sqrt{2gh}\ \rightarrow$$

$$h=\frac{1}{2g}\left(\frac{mv}{m+M}\right)^2=\frac{1}{2(9.8\,\mathrm{m/s^2})}\left(\frac{(0.028\text{ kg})(230\,\mathrm{m/s})}{0.028\text{ kg}+3.6\text{kg}}\right)^2$$

$$=0.1607\,\mathrm{m}\approx\boxed{0.16\text{ m}}$$

From the diagram we see that

$$L^2=(L-h)^2+x^2$$

$$x=\sqrt{L^2-(L-h)^2}=\sqrt{(2.8\text{ m})^2-(2.8\text{ m}-0.1607\text{ m})^2}=\boxed{0.94\text{ m}}$$

33. (*a*) In example 7-10, $KE_i = \frac{1}{2}mv^2$ and $KE_f = \frac{1}{2}(m+M)v'^2$. The speeds are related by

$$v' = \frac{m}{m+M}v .$$

$$\frac{\Delta KE}{KE_i} = \frac{KE_f - KE_i}{KE_i} = \frac{\frac{1}{2}(m+M)v'^2 - \frac{1}{2}mv^2}{\frac{1}{2}mv^2} = \frac{(m+M)\left(\frac{m}{m+M}v\right)^2 - mv^2}{mv^2}$$

$$= \frac{\frac{m^2v^2}{m+M} - mv^2}{mv^2} = \frac{m}{m+M} - 1 = \boxed{\frac{-M}{m+M}}$$

(*b*) For the given values, $\frac{-M}{m+M} = \frac{-380\text{ g}}{394\text{ g}} = -0.96$. Thus 96% of the energy is lost.

34. Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Call the direction of the heavier particle's motion the positive direction. Let A represent the heavier particle, and B represent the lighter particle. We have $m_A = 1.5m_B$, and $v_A = v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A v'_A + m_B v'_B \rightarrow v'_A = -\frac{m_B v'_B}{m_A} = -\frac{2}{3}v'_B$$

The negative sign indicates direction.

Since there was no mechanical energy before the explosion, the kinetic energy of the particles after the explosion must equal the energy added.

$$E_{\text{added}} = KE'_A + KE'_B = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B = \frac{1}{2}(1.5m_B)\left(\frac{2}{3}v'_B\right)^2 + \frac{1}{2}m_B v'^2_B = \frac{5}{3}\left(\frac{1}{2}m_B v'^2_B\right) = \frac{5}{3}KE'_B$$

$$KE'_B = \frac{3}{5}E_{\text{added}} = \frac{3}{5}(7500\text{ J}) = 4500\text{ J} \qquad KE'_A = E_{\text{added}} - KE'_B = 7500\text{ J} - 4500\text{ J} = 3000\text{ J}$$

Thus $\boxed{KE'_A = 3.0\times10^3\text{ J} \quad KE'_B = 4.5\times10^3\text{ J}}$

35 Use conservation of momentum in one dimension. Call the direction of the sports car's velocity the positive *x* direction. Let A represent the sports car, and B represent the SUV. We have $v_B = 0$ and $v'_A = v'_B$. Solve for v_A.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + 0 = (m_A + m_B)v'_A \rightarrow v_A = \frac{m_A + m_B}{m_A}v'_A$$

The kinetic energy that the cars have immediately after the collision is lost due to negative work done by friction. The work done by friction can also be calculated using the definition of work. We assume the cars are on a level surface, so that the normal force is equal to the weight. The distance the cars slide forward is Δx. Equate the two expressions for the work done by friction, solve for v'_A, and use that to find v_A.

$$W_{\text{fr}} = (KE_{\text{final}} - KE_{\text{initial}})_{\substack{\text{after}\\\text{collision}}} = 0 - \frac{1}{2}(m_A + m_B)v'^2_A$$

$$W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180^\circ = -\mu_k(m_A + m_B)g\Delta x$$

$$-\frac{1}{2}(m_A + m_B)v'^2_A = -\mu_k(m_A + m_B)g\Delta x \rightarrow v'_A = \sqrt{2\mu_k g\Delta x}$$

$$v_A = \frac{m_A + m_B}{m_A} v_A' = \frac{m_A + m_B}{m_A}\sqrt{2\mu_k g\Delta x} = \frac{920\,\text{kg} + 2300\,\text{kg}}{920\,\text{kg}}\sqrt{2(0.80)(9.8\,\text{m/s}^2)(2.8\text{ m})}$$
$$= 23.191\,\text{m/s} \approx \boxed{23\,\text{m/s}}$$

36. Consider conservation of energy during the rising and falling of the ball, between contacts with the floor. The gravitational potential energy at the top of a path will be equal to the kinetic energy at the start and the end of each rising-falling cycle. Thus $mgh = \frac{1}{2}mv^2$ for any particular bounce cycle. Thus for an interaction with the floor, the ratio of the energies before and after the bounce is $\frac{KE_{\text{after}}}{KE_{\text{before}}} = \frac{mgh'}{mgh} = \frac{1.20\text{ m}}{1.50\text{ m}} = 0.80$. We assume that each bounce will further reduce the energy to 80% of its pre-bounce amount. The number of bounces to lose 90% of the energy can be expressed as follows.

$$(0.8)^n = 0.1 \quad \rightarrow \quad n = \frac{\log 0.1}{\log 0.8} = 10.3$$

Thus after $\boxed{\text{11 bounces}}$, more than 90% of the energy is lost.

As an alternate method, after each bounce, 80% of the available energy is left. So after 1 bounce, 80% of the original energy is left. After the second bounce, only 80% of 80%, or 64% of the available energy is left. After the third bounce, 51 %. After the fourth bounce, 41%. After the fifth bounce, 33 %. After the sixth bounce, 26%. After the seventh bounce, 21%. After the eight bounce, 17%. After the ninth bounce, 13%. After the tenth bounce, 11%. After the eleventh bounce, 9% is left. So again, it takes 11 bounces.

37. (*a*) For a perfectly elastic collision, Eq. 7-7 says $v_A - v_B = -(v_A' - v_B')$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v_A' - v_B'}{v_B - v_A} = -\frac{(v_A - v_B)}{v_B - v_A} = 1.$$

For a completely inelastic collision, $v_A' = v_B'$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v_A' - v_B'}{v_B - v_A} = 0.$$

(*b*) Let A represent the falling object, and B represent the heavy steel plate. The speeds of the steel plate are $v_B = 0$ and $v_B' = 0$. Thus $e = -v_A'/v_A$. Consider energy conservation during the falling or rising path. The potential energy of body A at height *h* is transformed into kinetic energy just before it collides with the plate. Choose down to be the positive direction.

$$mgh = \tfrac{1}{2}mv_A^2 \quad \rightarrow \quad v_A = \sqrt{2gh}$$

The kinetic energy of body A immediately after the collision is transformed into potential energy as it rises. Also, since it is moving upwards, it has a negative velocity.

$$mgh' = \tfrac{1}{2}mv_A'^2 \quad \rightarrow \quad v_A' = -\sqrt{2gh'}$$

Substitute the expressions for the velocities into the definition of the coefficient of restitution.

$$e = -v_A'/v_A = -\frac{-\sqrt{2gh'}}{\sqrt{2gh}} \quad \rightarrow \quad \boxed{e = \sqrt{h'/h}}$$

38. Let A represent the more massive piece, and B the less massive piece. Thus $m_A = 3m_B$. In the explosion, momentum is conserved. We have $v_A = v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad 0 = m_A v'_A + m_B v'_B = 3m_B v'_A + m_B v'_B \quad \rightarrow \quad v'_A = -\tfrac{1}{3} v'_B$$

For each block, the kinetic energy gained during the explosion is lost to negative work done by friction on the block.

$$W_{\text{fr}} = KE_f - KE_i = -\tfrac{1}{2} m v^2$$

But work is also calculated in terms of the force doing the work and the distance traveled.

$$W_{\text{fr}} = F_{\text{fr}} \Delta x \cos 180^\circ = -\mu_k F_N \Delta x = -\mu_k m g \Delta x$$

Equate the two work expressions, solve for the distance traveled, and find the ratio of distances.

$$-\tfrac{1}{2} m v^2 = -\mu_k m g \Delta x \quad \rightarrow \quad \Delta x = \frac{v^2}{g \mu_k} \qquad \frac{(\Delta x)_A}{(\Delta x)_B} = \frac{\dfrac{v'^2_A}{g\mu_k}}{\dfrac{v'^2_B}{g\mu_k}} = \frac{v'^2_A}{v'^2_B} = \frac{\left(-\tfrac{1}{3} v'_B\right)^2}{v'^2_B} = \frac{1}{9}$$

And so $\boxed{(\Delta x)_{\text{heavy}} / (\Delta x)_{\text{light}} = 1/9}$

39. In each case, use momentum conservation. Let A represent the 15.0-kg object, and let B represent the 10.0-kg object. We have $v_A = 5.5\,\text{m/s}$ and $v_B = -4.0\,\text{m/s}$.

(*a*) In this case, $v'_A = v'_B$.

$$m_A v_A + m_B v_B = (m_A + m_B) v'_A \quad \rightarrow$$

$$v'_B = v'_A = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(15.0\text{ kg})(5.5\,\text{m/s}) + (10.0\text{ kg})(-4.0\,\text{m/s})}{25.0\text{ kg}} = \boxed{1.7\,\text{m/s}}$$

(*b*) In this case, use Eq. 7-7 to find a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = v_A - v_B + v'_A$$

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B = m_A v'_A + m_B (v_A + v_B + v'_A) \quad \rightarrow$$

$$v'_A = \frac{(m_A - m_B) v_A + 2 m_B v_B}{m_A + m_B} = \frac{(5.0\text{ kg})(5.5\,\text{m/s}) + 2(10.0\text{ kg})(-4.0\,\text{m/s})}{25.0\text{ kg}} = \boxed{-2.1\,\text{m/s}}$$

$$v'_B = v_A - v_B + v'_A = 5.5\,\text{m/s} - (-4.0\,\text{m/s}) - 2.1\,\text{m/s} = \boxed{7.4\,\text{m/s}}$$

(*c*) In this case, $v'_A = 0$.

$$m_A v_A + m_B v_B = m_B v'_B \quad \rightarrow$$

$$v'_B = \frac{m_A v_A + m_B v_B}{m_B} = \frac{(15.0\text{ kg})(5.5\,\text{m/s}) + (10.0\text{ kg})(-4.0\,\text{m/s})}{10.0\text{ kg}} = \boxed{4.3\,\text{m/s}}$$

To check for "reasonableness", first note the final directions of motion. A has stopped, and B has gone in the opposite direction. This is reasonable. Secondly, calculate the change in kinetic energy.

$$\Delta KE = \tfrac{1}{2} m_B v'^2_B - \left(\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2\right)$$

$$= \tfrac{1}{2}(10.0\text{ kg}) - \left[\tfrac{1}{2}(15.0\text{ kg})(5.5\,\text{m/s})^2 + \tfrac{1}{2}(10.0\text{ kg})(-4.0\,\text{m/s})^2\right] = -220\text{ J}$$

Since the system has lost kinetic energy and the directions are possible, this interaction is $\boxed{\text{“reasonable”}}$.

(*d*) In this case, $v_B' = 0$.

$$m_A v_A + m_B v_B = m_A v_A' \quad \rightarrow$$

$$v_A' = \frac{m_A v_A + m_B v_B}{m_A} = \frac{(15.0\ \text{kg})(5.5\,\text{m/s}) + (10.0\ \text{kg})(-4.0\,\text{m/s})}{15.0\ \text{kg}} = \boxed{2.8\,\text{m/s}}$$

This answer is $\boxed{\text{not reasonable}}$ because it has A moving in its original direction while B has stopped. Thus A has somehow passed through B. If B has stopped, A should have rebounded in the negative direction.

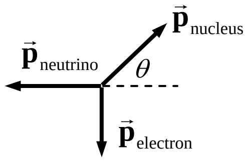
(*e*) In this case, $v_A' = -4.0\,\text{m/s}$.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow$$

$$v_B' = \frac{(15.0\ \text{kg})(5.5\,\text{m/s} - -4.0\,\text{m/s}) + (10.0\ \text{kg})(-4.0\,\text{m/s})}{10.0\ \text{kg}} = \boxed{10.3\,\text{m/s}}$$

The directions are reasonable, in that each object rebounds. However, the speed of both objects is larger than its speed in the perfectly elastic case (b). Thus the system has gained kinetic energy, and unless there is some other source adding energy, this is $\boxed{\text{not reasonable}}$.

40. Use this diagram for the momenta after the decay. Since there was no momentum before the decay, the three momenta shown must add to 0 in both the *x* and *y* directions.



$\vec{\mathbf{p}}_{\text{nucleus}}$
$\vec{\mathbf{p}}_{\text{neutrino}}$
θ
$\vec{\mathbf{p}}_{\text{electron}}$

$$(p_{\text{nucleus}})_x = p_{\text{neutrino}} \qquad (p_{\text{nucleus}})_y = p_{\text{electron}}$$

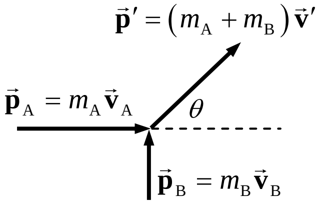
$$p_{\text{nucleus}} = \sqrt{(p_{\text{nucleus}})_x^2 + (p_{\text{nucleus}})_y^2} = \sqrt{(p_{\text{neutrino}})^2 + (p_{\text{electron}})^2}$$

$$= \sqrt{(5.40\times10^{-23}\ \text{kg}\bullet\text{m/s})^2 + (9.30\times10^{-23}\ \text{kg}\bullet\text{m/s})^2} = \boxed{1.08\times10^{-22}\ \text{kg}\bullet\text{m/s}}$$

$$\theta = \tan^{-1}\frac{(p_{\text{nucleus}})_y}{(p_{\text{nucleus}})_x} = \tan^{-1}\frac{(p_{\text{electron}})}{(p_{\text{neutrino}})} = \tan^{-1}\frac{(9.30\times10^{-23}\ \text{kg}\bullet\text{m/s})}{(5.40\times10^{-23}\ \text{kg}\bullet\text{m/s})} = 59.9^{\circ}$$

The second nucleus' momentum is $\boxed{150^{\circ}\ \text{from the momentum of the electron}}$.

41. Consider the diagram for the momenta of the eagles. Momentum will be conserved in both the *x* and *y* directions.



$\vec{\mathbf{p}}' = (m_A + m_B)\vec{\mathbf{v}}'$
$\vec{\mathbf{p}}_A = m_A\vec{\mathbf{v}}_A$
θ
$\vec{\mathbf{p}}_B = m_B\vec{\mathbf{v}}_B$

$$p_x' = (m_A + m_B)v_x' = m_A v_A \quad \rightarrow \quad v_x' = \frac{m_A v_A}{m_A + m_B}$$

$$p_y' = (m_A + m_B)v_y' = m_B v_B \quad \rightarrow \quad v_y' = \frac{m_B v_B}{m_A + m_B}$$

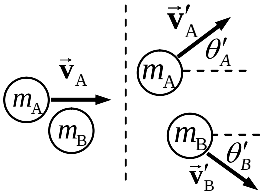
$$v' = \sqrt{v_x'^2 + v_y'^2} = \sqrt{\left(\frac{m_A v_A}{m_A + m_B}\right)^2 + \left(\frac{m_B v_B}{m_A + m_B}\right)^2} = \frac{\sqrt{(m_A v_A)^2 + (m_B v_B)^2}}{m_A + m_B}$$

$$= \frac{\sqrt{(4.3\ \text{kg})^2(7.8\,\text{m/s})^2 + (5.6\ \text{kg})^2(10.2\,\text{m/s})^2}}{4.3\ \text{kg} + 5.6\ \text{kg}} = \boxed{6.7\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{v'_y}{v'_x} = \tan^{-1}\frac{\dfrac{m_B v_B}{m_A + m_B}}{\dfrac{m_A v_A}{m_A + m_B}} = \tan^{-1}\frac{m_B v_B}{m_A v_A} = \tan^{-1}\frac{(5.6\text{ kg})(10.2\text{ m/s})}{(4.3\text{ kg})(7.8\text{ m/s})} = \boxed{60^\circ\text{ rel. to eagle A}}$$

42. (*a*) p_x : $m_A v_A = m_A v'_A \cos\theta'_A + m_B v'_B \cos\theta'_B$

p_y : $0 = m_A v'_A \sin\theta'_A - m_B v'_B \sin\theta'_B$



(*b*) Solve the *x* equation for $\cos\theta'_B$ and the *y* equation for $\sin\theta'_B$, and then find the angle from the tangent function.

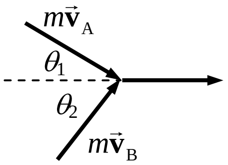
$$\tan\theta'_B = \frac{\sin\theta'_B}{\cos\theta'_B} = \frac{\dfrac{m_A v'_A \sin\theta'_A}{m_B v'_B}}{\dfrac{m_A\left(v_A - v'_A\cos\theta'_A\right)}{m_B v'_B}} = \frac{v'_A \sin\theta'_A}{\left(v_A - v'_A\cos\theta'_A\right)}$$

$$\theta'_B = \tan^{-1}\frac{v'_A \sin\theta'_A}{v_A - v'_A\cos\theta'_A} = \tan^{-1}\frac{(1.10\text{ m/s})\sin 30.0^\circ}{1.80\text{ m/s} - (1.10\text{ m/s})\cos 30.0^\circ} = \boxed{33.0^\circ}$$

With the value of the angle, solve the *y* equation for the velocity.

$$v'_B = \frac{m_A v'_A \sin\theta'_A}{m_B \sin\theta'_B} = \frac{(0.400\text{ kg})(1.10\text{ m/s})\sin 30.0^\circ}{(0.500\text{ kg})\sin 33.0^\circ} = \boxed{0.808\text{ m/s}}$$

43. Call the final direction of the joined objects the positive *x* axis. A diagram of the collision is shown. Momentum will be conserved in both the *x* and *y* directions. Note that $v_A = v_B = v$ and $v' = v/3$.



p_y : $-mv\sin\theta_1 + mv\sin\theta_2 = 0 \;\rightarrow\; \sin\theta_1 = \sin\theta_2 \;\rightarrow\; \theta_1 = \theta_2$

p_x : $mv\cos\theta_1 + mv\cos\theta_2 = (2m)(v/3) \;\rightarrow\; \cos\theta_1 + \cos\theta_2 = \frac{2}{3}$

$\cos\theta_1 + \cos\theta_2 = 2\cos\theta_1 = \frac{2}{3} \;\rightarrow\; \theta_1 = \cos^{-1}\frac{1}{3} = 70.5^\circ = \theta_2$

$\theta_1 + \theta_2 = \boxed{141^\circ}$

44. Write momentum conservation in the *x* and *y* directions, and KE conservation. Note that both masses are the same. We allow $\vec{v}_A$ to have both *x* and *y* components.

p_x : $mv_B = mv'_{Ax} \;\rightarrow\; v_B = v'_{Ax}$

p_y : $mv_A = mv'_{Ay} + mv'_B \;\rightarrow\; v_A = v'_{Ay} + v'_B$

KE : $\frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2}mv_A'^2 + \frac{1}{2}mv_B'^2 \;\rightarrow\; v_A^2 + v_B^2 = v_A'^2 + v_B'^2$

Substitute the results from the momentum equations into the KE equation.

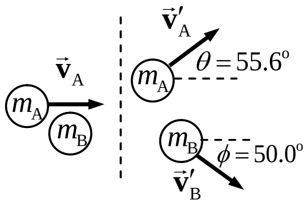
$$\left(v'_{Ay} + v'_B\right)^2 + \left(v'_{Ax}\right)^2 = v_A'^2 + v_B'^2 \;\rightarrow\; v_{Ay}'^2 + 2v_{Ay}'^2 v'_B + v_B'^2 + v_{Ay}'^2 = v_A'^2 + v_B'^2 \;\rightarrow$$

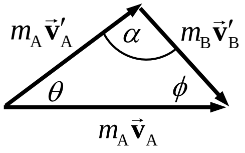
$$v_A'^2 + 2v_{Ay}'^2 v'_B + v_B'^2 = v_A'^2 + v_B'^2 \;\rightarrow\; 2v_{Ay}'^2 v'_B = 0 \;\rightarrow\; v'_{Ay} = 0 \text{ or } v'_B = 0$$

Since we are given that $v'_B \neq 0$, we must have $v'_{Ay} = 0$. This means that the final direction of A is the $\boxed{x\text{ direction}}$. Put this result into the momentum equations to find the final speeds.

$v'_A = v'_{Ax} = v_B = \boxed{3.7\text{ m/s}}$ $\qquad v'_B = v_A = \boxed{2.0\text{ m/s}}$

45. Let A represent the incoming neon atom, and B represent the target atom. A momentum diagram of the collision looks like the first figure. The figure can be re-drawn as a triangle, the second figure, since $m_A\vec{\mathbf{v}}_A = m_A\vec{\mathbf{v}}'_A + m_B\vec{\mathbf{v}}'_B$. Write the law of sines for this triangle, relating each final momentum magnitude to the initial momentum magnitude.





$$\frac{m_A v'_A}{m_A v_A} = \frac{\sin\phi}{\sin\alpha} \quad\rightarrow\quad v'_A = v_A\frac{\sin\phi}{\sin\alpha}$$

$$\frac{m_B v'_B}{m_A v_A} = \frac{\sin\theta}{\sin\alpha} \quad\rightarrow\quad v'_B = v_A\frac{m_A}{m_B}\frac{\sin\theta}{\sin\alpha}$$

The collision is elastic, so write the KE conservation equation, and substitute the results from above. Also note that $\alpha = 180.0 - 55.6^\circ - 50.0^\circ = 74.4^\circ$

$$\tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2 \quad\rightarrow\quad m_A v_A^2 = m_A\left(v_A\frac{\sin\phi}{\sin\alpha}\right)^2 + m_B\left(v_A\frac{m_A}{m_B}\frac{\sin\theta}{\sin\alpha}\right)^2 \quad\rightarrow$$

$$m_B = \frac{m_A\sin^2\theta}{\sin^2\alpha - \sin^2\phi} = \frac{(20.0\text{ u})\sin^2 55.6^\circ}{\sin^2 74.4 - \sin^2 50.0^\circ} = \boxed{39.9\text{ u}}$$

46. Use Eq. 7-9a, extended to three particles.

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(1.00\text{ kg})(0) + (1.50\text{ kg})(0.50\text{ m}) + (1.10\text{ kg})(0.75\text{ m})}{1.00\text{ kg} + 1.50\text{ kg} + 1.10\text{ kg}}$$
$$= \boxed{0.44\text{ m}}$$

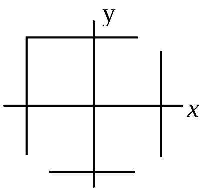
47. Choose the carbon atom as the origin of coordinates.

$$x_{CM} = \frac{m_C x_C + m_O x_O}{m_C + m_O} = \frac{(12\text{ u})(0) + (16\text{ u})(1.13\times10^{-10}\text{ m})}{12\text{ u} + 16\text{ u}} = \boxed{6.5\times10^{-11}\text{ m}} \text{ from the C atom.}$$

$\boxed{48.}$ Find the CM relative to the front of the car.

$$x_{CM} = \frac{m_{car}x_{car} + m_{front}x_{front} + m_{back}x_{back}}{m_{car} + m_{front} + m_{back}}$$
$$= \frac{(1050\text{ kg})(2.50\text{ m}) + 2(70.0\text{ kg})(2.80\text{ m}) + 3(70.0\text{ kg})(3.90\text{ m})}{1050\text{ kg} + 2(70.0\text{ kg}) + 3(70.0\text{ kg})} = \boxed{2.74\text{ m}}$$

49. Consider this diagram of the cars on the raft. Notice that the origin of coordinates is located at the CM of the raft. Reference all distances to that location.



$$x_{CM} = \frac{(1200\text{ kg})(9\text{ m}) + (1200\text{ kg})(9\text{ m}) + (1200\text{ kg})(-9\text{ m})}{3(1200\text{ kg}) + 6800\text{ kg}} = \boxed{1.04\text{ m}}$$

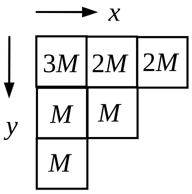
$$y_{CM} = \frac{(1200\text{ kg})(9\text{ m}) + (1200\text{ kg})(-9\text{ m}) + (1200\text{ kg})(-9\text{ m})}{3(1200\text{ kg}) + 6800\text{ kg}} = \boxed{-1.04\text{ m}}$$

50. By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0, and the depth CM coordinate will be 0. The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the volume times the density, and so $m_1 = \rho\left(l_0\right)^3$, $m_2 = \rho\left(2l_0\right)^3$, $m_3 = \rho\left(3l_0\right)^3$. Measuring from the left edge of the smallest block, the locations of the CM's of the individual cubes are $x_1 = \frac{1}{2}l_0$, $x_2 = 2l_0$, $x_3 = 4.5l_0$. Use Eq. 7-9a to calculate the CM of the system.

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{\rho l_0^3\left(\frac{1}{2}l_0\right) + 8\rho l_0^3\left(2l_0\right) + 27\rho l_0^3\left(4.5l_0\right)}{\rho l_0^3 + 8\rho l_0^3 + 27\rho l_0^3}$$

$$= \boxed{3.8\,l_0 \text{ from the left edge of the smallest cube}}$$

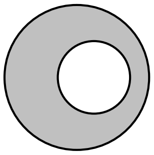
51. Let each crate have a mass M. A top view of the pallet is shown, with the total mass of each stack listed. Take the origin to be the back left corner of the pallet.



$$x_{CM} = \frac{(5M)(l/2) + (3M)(3l/2) + (2M)(5l/2)}{10M} = \boxed{1.2l}$$

$$y_{CM} = \frac{(7M)(l/2) + (2M)(3l/2) + (1M)(5l/2)}{10M} = \boxed{0.9l}$$

52. Consider the following. We start with a full circle of radius 2*R*, with its CM at the origin. Then we draw a circle of radius *R*, with its CM at the coordinates $(0.80R, 0)$. The full circle can now be labeled as a "gray" part and a "white" part. The *y* coordinate of the CM of the entire circle, the CM of the gray part, and the CM of the white part are all at $y = 0$ by the symmetry of the system. The *x* coordinate of the entire circle is at $x_{\text{CM}} = 0$, and can be calculated by $x_{\text{CM}} = \frac{m_{\text{gray}}x_{\text{gray}} + m_{\text{white}}x_{\text{white}}}{m_{\text{total}}}$. Rearrange this equation.



$$x_{\text{CM}} = \frac{m_{\text{gray}}x_{\text{gray}} + m_{\text{white}}x_{\text{white}}}{m_{\text{total}}} \quad \rightarrow$$

$$x_{\text{gray}} = \frac{m_{\text{total}}x_{\text{CM}} - m_{\text{white}}x_{\text{white}}}{m_{\text{gray}}} = \frac{m_{\text{total}}x_{\text{CM}} - m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}} = \frac{-m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}}$$

This is functionally the same as treating the white part of the figure as a hole of negative mass. The mass of each part can be found by multiplying the area of the part times the uniform density of the plate.

$$x_{\text{gray}} = \frac{-m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}} = \frac{-\rho\pi R^2\left(0.80R\right)}{\rho\pi\left(2R\right)^2 - \rho\pi R^2} = \frac{-0.80R}{3} = \boxed{-0.27R}$$

53. Take the upper leg, lower leg, and foot all together. Note that Table 7-1 gives the relative mass of BOTH legs and feet, so a factor of 1/2 is needed. Assume a person of mass 70 kg.

$$\left(70\text{ kg}\right)\frac{\left(21.5 + 9.6 + 3.4\right)}{100}\frac{1}{2} = \boxed{12\text{ kg}}.$$

54. With the shoulder as the origin of coordinates for measuring the center of mass, we have the following relative locations from Table 7-1 for the arm components, as percentages of the height. Down is positive.

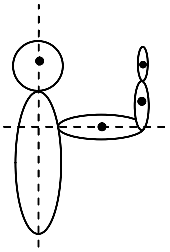
$$x_{\substack{\text{upper}\\\text{arm}}} = 81.2 - 7.7 = 9.5 \qquad x_{\substack{\text{lower}\\\text{arm}}} = 81.2 - 55.3 = 25.9 \qquad x_{\text{hand}} = 81.2 - 43.1 = 38.1$$

To find the CM, we can also use relative mass percentages. Since the expression includes the total mass in the denominator, there is no need to divide all masses by 2 to find single component masses. Simply use the relative mass percentages given in the table.

$$x_{\text{CM}} = \frac{x_{\substack{\text{upper}\\\text{arm}}} m_{\substack{\text{upper}\\\text{arm}}} + x_{\substack{\text{lower}\\\text{arm}}} m_{\substack{\text{lower}\\\text{arm}}} + x_{\text{hand}} m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}} = \frac{(9.5)(6.6) + (25.9)(4.2) + (38.1)(1.7)}{6.6 + 4.2 + 1.7}$$

$$= \boxed{\text{19\% of the person's height along the line from the shoulder to the hand}}$$

55. Take the shoulder to be the origin of coordinates. We assume that the arm is held with the upper arm parallel to the floor and the lower arm and hand extended upward. Measure *x* horizontally from the shoulder, and *y* vertically. Since the expression includes the total mass in the denominator, there is no need to divide all masses by 2 to find single component masses. Simply use the relative mass percentages given in the table.



$$x_{\text{CM}} = \frac{x_{\substack{\text{upper}\\\text{arm}}} m_{\substack{\text{upper}\\\text{arm}}} + x_{\substack{\text{lower}\\\text{arm}}} m_{\substack{\text{lower}\\\text{arm}}} + x_{\text{hand}} m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}}$$

$$= \frac{(81.2 - 71.7)(6.6) + (81.2 - 62.2)(4.2 + 1.7)}{6.6 + 4.2 + 1.7} = 14.0$$

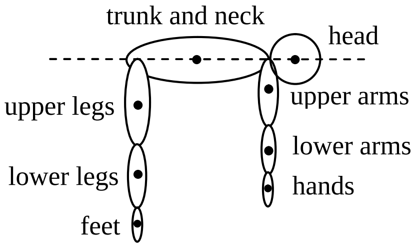
$$y_{\text{CM}} = \frac{y_{\substack{\text{upper}\\\text{arm}}} m_{\substack{\text{upper}\\\text{arm}}} + y_{\substack{\text{lower}\\\text{arm}}} m_{\substack{\text{lower}\\\text{arm}}} + y_{\text{hand}} m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}}$$

$$= \frac{(0)(6.6) + (62.2 - 55.3)(4.2) + (62.2 - 43.1)(1.7)}{6.6 + 4.2 + 1.7} = 4.92$$

Convert the distance percentages to actual distance using the person's height.

$$x_{\text{CM}} = (14.0\%)(155\text{ cm}) = \boxed{21.7\text{ cm}} \qquad y_{\text{CM}} = (4.92\%)(155\text{ cm}) = \boxed{7.6\text{ cm}}$$

56. See the diagram of the person. The head, trunk, and neck are all lined up so that their CM's are on the torso's median line. Call down the positive *y* direction. The *y* distances of the CM of each body part from the median line, in terms of percentage of full height, are shown below, followed by the percentage each body part is of the full body mass.



On median line:	head (h):	0	;	6.9% body mass
	Trunk & neck (t n):	0	;	46.1% body mass
From shoulder hinge point:	upper arms (u a):	81.2 – 71.7 = 9.5	;	6.6% body mass
	lower arms (l a):	81.2 – 55.3 = 25.9	;	4.2% body mass
	hands (ha):	81.2 – 43.1 = 38.1	;	1.7% body mass

From hip hinge point:	upper legs (u l):	52.1 – 42.5 = 9.6	;	21.5% body mass
	lower legs (l l):	52.1 – 18.2 = 33.9	;	9.6% body mass
	feet (f):	52.1 – 1.8 = 50.3	;	3.4% body mass

Using this data, calculate the vertical location of the CM.

$$y_{CM} = \frac{y_{\text{h}} m_{\text{h}} + y_{\text{t n}} m_{\text{t n}} + y_{\text{u a}} m_{\text{u a}} + y_{\text{l a}} m_{\text{l a}} + y_{\text{ha}} m_{\text{ha}} + y_{\text{u l}} m_{\text{u l}} + y_{\text{l l}} m_{\text{l l}} + y_{\text{f}} m_{\text{f}}}{m_{\substack{\text{full}\\\text{body}}}}$$

$$= \frac{0 + 0 + (9.5)(6.6) + (25.9)(4.2) + (38.1)(1.7) + (9.6)(21.5) + (33.9)(9.6) + (50.3)(3.4)}{100}$$

$$= 9.4$$

Thus the center of mass is $\boxed{\text{9.4\% of the full body height below the torso's median line}}$. For a person of height 1.7 m, this is about 16 cm. That is most likely slightly $\boxed{\text{outside the body}}$.

57. (*a*) Find the CM relative to the center of the Earth.

$$x_{CM} = \frac{m_E x_E + m_M x_M}{m_E + m_M} = \frac{\left(5.98\times10^{24}\,\text{kg}\right)(0) + \left(7.35\times10^{22}\,\text{kg}\right)\left(3.84\times10^{8}\,\text{m}\right)}{5.98\times10^{24}\,\text{kg} + 7.35\times10^{22}\,\text{kg}}$$

$$= \boxed{4.66\times10^{6}\,\text{m from the center of the Earth}}$$

This is actually inside the volume of the Earth, since $R_E = 6.38\times10^{6}\,\text{m}$

(*b*) It is this Earth – Moon CM location that actually traces out the orbit as discussed in chapter 5. The Earth and Moon will orbit about this location in (approximately) circular orbits. The motion of the Moon, for example, around the Sun would then be a sum of two motions: i) the motion of the Moon about the Earth – Moon CM; and ii) the motion of the Earth – Moon CM about the Sun. To an external observer, the Moon's motion would appear to be a small radius, higher frequency circular motion (motion about the Earth – Moon CM) combined with a large radius, lower frequency circular motion (motion about the Sun).

58. (*a*) Measure all distances from the original position of the woman.

$$x_{\text{CM}} = \frac{m_{\text{W}} x_{\text{W}} + m_{\text{M}} x_{\text{M}}}{m_{\text{W}} + m_{\text{M}}} = \frac{(55\text{ kg})(0) + (80\text{ kg})(10.0\text{ m})}{135\text{ kg}} = \boxed{5.9\text{ m from the woman}}$$

(*b*) Since there is no force external to the man-woman system, the CM will not move, relative to the original position of the woman. The woman's distance will no longer be 0, and the man's distance has changed to 7.5 m.

$$x_{\text{CM}} = \frac{m_{\text{W}} x_{\text{W}} + m_{\text{M}} x_{\text{M}}}{m_{\text{W}} + m_{\text{M}}} = \frac{(55\text{ kg})x_{\text{W}} + (80\text{ kg})(7.5\text{ m})}{135\text{ kg}} = 5.9\text{ m} \rightarrow$$

$$x_{\text{W}} = \frac{(5.9\text{ m})(135\text{ kg}) - (80\text{ kg})(7.5\text{ m})}{55\text{ kg}} = 3.6\text{ m}$$

$$x_{\text{M}} - x_{\text{W}} = 7.5\text{ m} - 3.6\text{ m} = \boxed{3.9\text{ m}}$$

(*c*) When the man collides with the woman, he will be at the original location of the center of mass.

$$x_{\substack{\text{M}\\\text{final}}} - x_{\substack{\text{M}\\\text{initial}}} = 5.9\,\text{m} - 10.0\text{ m} = -4.1\text{ m}$$

He has moved $\boxed{4.1\text{ m}}$ from his original position.

59. The point that will follow a parabolic trajectory is the center of mass. Find the CM relative to the bottom of the mallet. Each part of the hammer (handle and head) can be treated as a point mass located at the CM of the respective piece. So the CM of the handle is 12.0 cm from the bottom of the handle, and the CM of the head is 28.0 cm from the bottom of the handle.

$$x_{\text{CM}} = \frac{m_{\text{handle}}x_{\text{handle}} + m_{\text{head}}x_{\text{head}}}{m_{\text{handle}} + m_{\text{head}}} = \frac{(0.500\text{ kg})(24.0\text{ cm}) + (2.00\text{ kg})(28.0\text{ cm})}{2.50\text{ kg}} = \boxed{24.8\text{ cm}}$$

Note that this is inside the head of the mallet.

60. The CM of the system will follow the same path regardless of the way the mass splits, and so will still be $2d$ from the launch point when the parts land. Assume that the explosion is designed so that m_{I} still is stopped in midair and falls straight down.

(*a*) $$x_{\text{CM}} = \frac{m_{\text{I}}x_{\text{I}} + m_{\text{II}}x_{\text{II}}}{m_{\text{I}} + m_{\text{II}}} \rightarrow 2d = \frac{m_{\text{I}}d + 3m_{\text{I}}x_{\text{II}}}{4m_{\text{I}}} = \frac{d + 3x_{\text{II}}}{4} \rightarrow x_{\text{II}} = \boxed{\tfrac{7}{3}d}$$

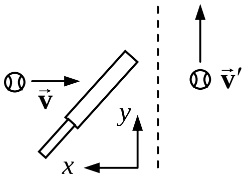
(*b*) $$x_{\text{CM}} = \frac{m_{\text{I}}x_{\text{I}} + m_{\text{II}}x_{\text{II}}}{m_{\text{I}} + m_{\text{II}}} \rightarrow 2d = \frac{3m_{\text{II}}d + m_{\text{II}}x_{\text{II}}}{4m_{\text{II}}} = \frac{3d + x_{\text{II}}}{4} \rightarrow x_{\text{II}} = \boxed{5d}$$

61. Call the origin of coordinates the CM of the balloon, gondola, and person at rest. Since the CM is at rest, the total momentum of the system relative to the ground is 0. The man climbing the rope cannot change the total momentum of the system, and so the CM must stay at rest. Call the upward direction positive. Then the velocity of the man with respect to the balloon is $-v$. Call the velocity of the balloon with respect to the ground v_{BG}. Then the velocity of the man with respect to the ground is $v_{\text{MG}} = -v + v_{\text{BG}}$. Apply Eq. 7-10.

$$0 = mv_{\text{MG}} + Mv_{\text{BG}} = m(-v + v_{\text{BG}}) + Mv_{\text{BG}} \rightarrow \boxed{v_{\text{BG}} = v\frac{m}{m+M}\text{ , upward}}$$

If the passenger stops, $\boxed{\text{the balloon also stops}}$, and the CM of the system remains at rest.

62. To find the average force, divide the change in momentum by the time over which the momentum changes. Choose the *x* direction to be the opposite of the baseball's incoming direction. The velocity with which the ball is moving after hitting the bat can be found from conservation of energy, and knowing the height the ball rises.



$$\left(KE_{\text{initial}} = PE_{\text{final}}\right)_{\substack{\text{after}\\\text{collision}}} \rightarrow \tfrac{1}{2}mv'^2 = mg\Delta y \rightarrow$$

$$v' = \sqrt{2g\Delta y} = \sqrt{2\left(9.80\text{ m/s}^2\right)(55.6\text{ m})} = 33.0\text{ m/s}$$

The average force can be calculated from the change in momentum and the time of contact.

$$\bar{F}_x = \frac{\Delta p_x}{\Delta t} = \frac{m(v'_x - v_x)}{\Delta t} = \frac{(0.145\text{ kg})(0 - -35.0\text{ m/s})}{1.4\times10^{-3}\text{s}} = 3.6\times10^3\text{ N}$$

$$\bar{F}_y = \frac{\Delta p_y}{\Delta t} = \frac{m(v'_y - v_y)}{\Delta t} = \frac{(0.145\text{ kg})(33.0\text{ m/s} - 0)}{1.4\times10^{-3}\text{s}} = 3.4\times10^3\text{ N}$$

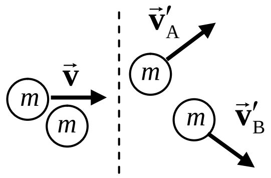
$$\bar{F} = \sqrt{\bar{F}_x^2 + \bar{F}_y^2} = \boxed{5.0\times10^3\text{ N}} \qquad \theta = \tan^{-1}\frac{\bar{F}_y}{\bar{F}_x} = \boxed{43^\circ}$$

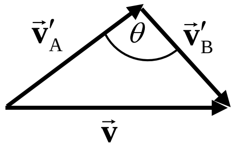
63. Momentum will be conserved in two dimensions. The fuel was ejected in the *y* direction as seen from the ground, and so the fuel had no *x*-component of velocity.

$$p_x: \quad m_{\text{rocket}}v_0 = \left(m_{\text{rocket}} - m_{\text{fuel}}\right)v'_x + m_{\text{fuel}}0 = \tfrac{2}{3}m_{\text{rocket}}v'_x \quad \rightarrow \quad \boxed{v'_x = \tfrac{3}{2}v_0}$$

$$p_y: \quad 0 = m_{\text{fuel}}v_{\text{fuel}} + \left(m_{\text{rocket}} - m_{\text{fuel}}\right)v'_y = \tfrac{1}{3}m_{\text{rocket}}\left(2v_0\right) + \tfrac{2}{3}m_{\text{rocket}}v'_y \quad \rightarrow \quad \boxed{v'_y = -v_0}$$

64. In an elastic collision between two objects of equal mass, with the target object initially stationary, the angle between the final velocities of the objects is 90°. Here is a proof of that fact. Momentum conservation as a vector relationship says $m\vec{v} = m\vec{v}'_{\text{A}} + m\vec{v}'_{\text{B}} \rightarrow \vec{v} = \vec{v}'_{\text{A}} + \vec{v}'_{\text{B}}$. Kinetic energy conservation says $\frac{1}{2}mv^2 = \frac{1}{2}mv'^2_{\text{A}} + \frac{1}{2}mv'^2_{\text{B}} \rightarrow v^2 = v'^2_{\text{A}} + v'^2_{\text{B}}$. The vector equation resulting from momentum conservation can be illustrated by the second diagram. Apply the law of cosines to that triangle of vectors, and then equate the two expressions for v^2.

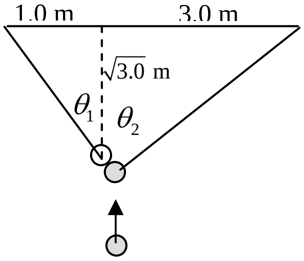

$\vec{\mathbf{v}}'_{\text{A}}$
$\vec{\mathbf{v}}$
m
m
m
m
$\vec{\mathbf{v}}'_{\text{B}}$


$\vec{\mathbf{v}}'_{\text{A}}$
θ
$\vec{\mathbf{v}}'_{\text{B}}$
$\vec{\mathbf{v}}$

$$v^2 = v'^2_{\text{A}} + v'^2_{\text{B}} - 2v'_{\text{A}}v'_{\text{B}}\cos\theta$$

Equating the two expressions for v^2 gives

$$v'^2_{\text{A}} + v'^2_{\text{B}} - 2v'_{\text{A}}v'_{\text{B}}\cos\theta = v'^2_{\text{A}} + v'^2_{\text{B}} \quad \rightarrow \quad \cos\theta = 0 \quad \rightarrow \quad \theta = 90^\circ$$



For this specific circumstance, see the third diagram. We assume that the target ball is hit "correctly" so that it goes in the pocket. Find θ_1 from the geometry of the "left' triangle: $\theta_1 = \tan^{-1}\dfrac{1.0}{\sqrt{3.0}} = 30^\circ$. Find θ_2 from the geometry of the "right" triangle: $\theta_2 = \tan^{-1}\dfrac{3.0}{\sqrt{3.0}} = 60^\circ$. Since the balls will separate at a 90° angle, if the target ball goes in the pocket, this does appear to be a $\boxed{\text{good possibility of a scratch shot}}$.

65. (*a*) The momentum of the astronaut – space capsule combination will be conserved since the only forces are "internal" to that system. Let A represent the astronaut and B represent the space capsule, and let the direction the astronaut moves be the positive direction. Due to the choice of reference frame, $v_{\text{A}} = v_{\text{B}} = 0$. We also have $v'_{\text{A}} = 2.50\,\text{m/s}$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{B} = 0 = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}} \quad \rightarrow$$

$$v'_{\text{B}} = -v'_{\text{A}}\frac{m_{\text{A}}}{m_{\text{B}}} = -\left(2.50\,\text{m/s}\right)\frac{140\text{ kg}}{1800\text{ kg}} = \boxed{-0.194\,\text{m/s}}$$

The negative sign indicates that the space capsule is moving in the opposite direction to the astronaut.

(*b*) The average force on the astronaut is the astronaut's change in momentum, divided by the time of interaction.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{m\left(v'_{\text{A}} - v_{\text{A}}\right)}{\Delta t} = \frac{\left(140\text{ kg}\right)\left(2.50\,\text{m/s} - 0\right)}{0.40\text{ s}} = \boxed{8.8 \times 10^2\,\text{N}}$$

66. Since the only forces on the astronauts are internal to the 2-astronaut system, their CM will not change. Call the CM location the origin of coordinates. That is also the original location of the two astronauts.

$$x_{\text{CM}} = \frac{m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} \quad \rightarrow \quad 0 = \frac{(60\text{ kg})(12\text{ m}) + (80\text{ kg})x_{\text{B}}}{140\text{ kg}} \quad \rightarrow \quad x = -9\text{ m}$$

Their distance apart is $x_{\text{A}} - x_{\text{B}} = 12\text{ m} - (-9\text{m}) = \boxed{21\text{ m}}$.

67. Let A represent the incoming ball, and B represent the target ball. We have $v_{\text{B}} = 0$ and $v'_{\text{A}} = -\tfrac{1}{4}v_{\text{A}}$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_{\text{A}} - v_{\text{B}} = -(v'_{\text{A}} - v'_{\text{B}}) \quad \rightarrow \quad v'_{\text{B}} = v_{\text{A}} + v'_{\text{A}} = \tfrac{3}{4}v_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_{\text{A}}v_{\text{A}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}} = m_{\text{A}}\left(-\tfrac{1}{4}v_{\text{A}}\right) + m_{\text{B}}\left(\tfrac{3}{4}v_{\text{A}}\right) \quad \rightarrow \quad \boxed{m_{\text{B}} = \tfrac{5}{3}m_{\text{A}}}$$

68. We assume that all motion is along a single direction. The distance of sliding can be related to the change in the kinetic energy of a car, as follows.

$$W_{\text{fr}} = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180^{\text{o}}\theta = -\mu_k F_{\text{N}}\Delta x = -\mu_k mg\Delta x \quad \rightarrow$$

$$-\mu_k g\Delta x = \tfrac{1}{2}\left(v_f^2 - v_i^2\right)$$

For post-collision sliding, $v_f = 0$ and v_i is the speed immediately after the collision, v'. Use this relationship to find the speed of each car immediately after the collision.

$$\text{Car A:} \quad -\mu_k g\Delta x'_{\text{A}} = -\tfrac{1}{2}v'^2_{\text{A}} \quad \rightarrow \quad v'_{\text{A}} = \sqrt{2\mu_k g\Delta x'_{\text{A}}} = \sqrt{2(0.60)(9.8\text{ m/s}^2)(18\text{ m})} = 14.55\text{ m/s}$$

$$\text{Car B:} \quad -\mu_k g\Delta x'_{\text{B}} = -\tfrac{1}{2}v'^2_{\text{B}} \quad \rightarrow \quad v'_{\text{B}} = \sqrt{2\mu_k g\Delta x'_{\text{B}}} = \sqrt{2(0.60)(9.8\text{ m/s}^2)(30\text{ m})} = 18.78\text{ m/s}$$

During the collision, momentum is conserved in one dimension. Note that $v_{\text{B}} = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_{\text{A}}v_{\text{A}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}}$$

$$v_A = \frac{m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}}}{m_{\text{A}}} = \frac{(1900\text{ kg})(14.55\text{ m/s}) + (1100\text{ kg})(18.78\text{ m/s})}{1900\text{ kg}} = 25.42\text{ m/s}$$

For pre-collision sliding, again apply the friction – energy relationship, with $v_f = v_{\text{A}}$ and v_i is the speed when the brakes were first applied.

$$-\mu_k g\Delta x_{\text{A}} = \tfrac{1}{2}\left(v_{\text{A}}^2 - v_i^2\right) \quad \rightarrow \quad v_i = \sqrt{v_A^2 + 2\mu_k g\Delta x_A} = \sqrt{(25.42\text{ m/s})^2 + 2(0.60)(9.8\text{ m/s}^2)(15\text{ m})}$$

$$= 28.68\text{ m/s}\left(\frac{1\text{ mi/h}}{0.447\text{ m/s}}\right) = \boxed{64\text{ mi/h}}$$

69. Because all of the collisions are perfectly elastic, no energy is lost in the collisions. With each collision, the horizontal velocity is constant, and the vertical velocity reverses direction. So, after each collision, the ball rises again to the same height from which it dropped. Thus, after five bounces, the bounce height will be $\boxed{4.00\text{ m}}$, the same as the starting height.

70. This is a ballistic "pendulum" of sorts, similar to Example 7-10 in the textbook. There is no difference in the fact that the block and bullet are moving vertically instead of horizontally. The collision is still totally inelastic and conserves momentum, and the energy is still conserved in the

rising of the block and embedded bullet after the collision. So we simply quote the equation from that example.

$$v = \frac{m+M}{m}\sqrt{2gh} \quad \rightarrow$$

$$h = \frac{1}{2g}\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2\left(9.80\,\text{m/s}^2\right)}\left(\frac{(0.0290\,\text{kg})(510\,\text{m/s})}{0.0290\,\text{kg} + 1.40\ \text{kg}}\right)^2 = \boxed{5.47\ \text{m}}$$

71. This is a ballistic "pendulum" of sorts, similar to Example 7-10 in the textbook. Momentum is conserved in the totally inelastic collision, and so $mv = (m+M)v'$. The kinetic energy present immediately after the collision is lost due to negative work being done by friction.

$$W_{\text{fr}} = \Delta KE = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right)_{\substack{\text{after}\\\text{collision}}} \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x \cos 180^\circ\theta = -\mu_k F_{\text{N}}\Delta x = -\mu_k mg\Delta x \quad \rightarrow$$

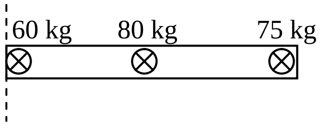
$$-\mu_k g\Delta x = \tfrac{1}{2}\left(v_f^2 - v_i^2\right) = -\tfrac{1}{2}v'^2 \quad \rightarrow \quad v' = \sqrt{2\mu_k g\Delta x}$$

Use this expression for v' in the momentum equation in order to solve for v.

$$mv = (m+M)v' = (m+M)\sqrt{2\mu_k g\Delta x} \quad \rightarrow$$

$$v = \left(\frac{m+M}{m}\right)\sqrt{2\mu_k g\Delta x} = \left(\frac{0.025\ \text{kg} + 1.35\ \text{kg}}{0.025\ \text{kg}}\right)\sqrt{2(0.25)\left(9.8\,\text{m/s}^2\right)(9.5\ \text{m})} = \boxed{3.8\times 10^2\ \text{m/s}}$$

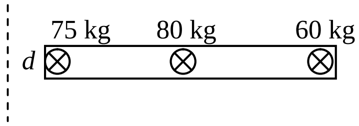
72. Calculate the CM relative to the 60-kg person's seat, at one end of the boat. See the first diagram. Don't forget to include the boat's mass.



$$x_{\text{CM}} = \frac{m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}} + m_{\text{C}}x_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}$$

$$= \frac{(60\ \text{kg})(0) + (80\ \text{kg})(1.6\ \text{m}) + (75\ \text{kg})(3.2\ \text{m})}{215\ \text{kg}} = 1.712\ \text{m}$$

Now, when the passengers exchange positions, the boat will move some distance "*d*" as shown, but the CM will not move. We measure the location of the CM from the same place as before, but now the boat has moved relative to that origin.



$$x_{\text{CM}} = \frac{m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}} + m_{\text{C}}x_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}$$

$$1.712\ \text{m} = \frac{(75\ \text{kg})(d) + (80\ \text{kg})(1.6\ \text{m} + d) + (60\ \text{kg})(3.2\ \text{m} + d)}{215\ \text{kg}} = \frac{215d\ \text{kg}\bullet\text{m} + 320\ \text{kg}\bullet\text{m}}{215\ \text{kg}}$$

$$d = 0.224\ \text{m}$$

Thus the boat will move $\boxed{\text{0.22 m towards the initial position of the 75 kg person}}$.

73. (*a*) The meteor striking and coming to rest in the Earth is a totally inelastic collision. Let A represent the Earth and B represent the meteor. Use the frame of reference in which the Earth is at rest before the collision, and so $v_{\text{A}} = 0$. Write momentum conservation for the collision.

$$m_{\text{B}}v_{\text{B}} = \left(m_{\text{A}} + m_{\text{B}}\right)v' \quad \rightarrow$$

$$v' = v_{\mathrm{B}} \frac{m_{\mathrm{B}}}{m_{\mathrm{A}} + m_{\mathrm{B}}} = \left(1.5\times10^{4}\ \mathrm{m/s}\right)\frac{1.0\times10^{8}\,\mathrm{kg}}{6.0\times10^{24}\,\mathrm{kg}+1.0\times10^{8}\,\mathrm{kg}} = \boxed{2.5\times10^{-13}\ \mathrm{m/s}}$$

(*b*) The fraction of the meteor's KE transferred to the Earth is the final KE of the Earth divided by the initial KE of the meteor.

$$\frac{KE_{\substack{\text{final}\\ \text{Earth}}}}{KE_{\substack{\text{initial}\\ \text{meteor}}}} = \frac{\frac{1}{2}m_{\mathrm{A}}v'^{2}}{\frac{1}{2}m_{\mathrm{B}}v_{\mathrm{B}}^{2}} = \frac{\frac{1}{2}\left(6.0\times10^{24}\,\mathrm{kg}\right)\left(2.5\times10^{-13}\ \mathrm{m/s}\right)^{2}}{\frac{1}{2}\left(1.0\times10^{8}\,\mathrm{kg}\right)\left(1.5\times10^{4}\ \mathrm{m/s}\right)^{2}} = \boxed{1.7\times10^{-17}}$$

(*c*) The Earth's change in KE can be calculated directly.

$$\Delta KE_{\text{Earth}} = KE_{\substack{\text{final}\\ \text{Earth}}} - KE_{\substack{\text{initial}\\ \text{Earth}}} = \tfrac{1}{2}m_{\mathrm{A}}v'^{2} - 0 = \tfrac{1}{2}\left(6.0\times10^{24}\,\mathrm{kg}\right)\left(2.5\times10^{-13}\ \mathrm{m/s}\right) = \boxed{0.19\ \mathrm{J}}$$

74. Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger KE.

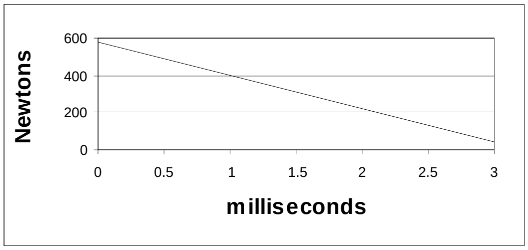
$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad 0 = m_{\mathrm{A}}v'_{\mathrm{A}} + m_{\mathrm{B}}v'_{\mathrm{B}} \quad\rightarrow\quad v'_{\mathrm{B}} = -\frac{m_{\mathrm{A}}v'_{\mathrm{A}}}{m_{\mathrm{B}}}$$

$$KE_{A} = 2KE_{B} \quad\rightarrow\quad \tfrac{1}{2}m_{\mathrm{A}}v'^{2}_{\mathrm{A}} = 2\left(\tfrac{1}{2}m_{\mathrm{B}}v'^{2}_{\mathrm{B}}\right) = m_{\mathrm{B}}\left(-\frac{m_{\mathrm{A}}v'_{\mathrm{A}}}{m_{\mathrm{B}}}\right)^{2} \quad\rightarrow\quad \frac{m_{\mathrm{A}}}{m_{\mathrm{B}}} = \boxed{\frac{1}{2}}.$$

The fragment with the larger KE energy has half the mass of the other fragment.

75. (*a*) The force is linear, with a maximum force of 580 N at 0 seconds, and a minimum force of 40 N at 3 milliseconds.

(*b*) The impulse given the bullet is the "area" under the *F* vs. *t* graph. The area is trapezoidal.



$$\text{Impulse} = \left(\frac{580\ \mathrm{N} + 40\ \mathrm{N}}{2}\right)\left(3.0\times10^{-3}\,\mathrm{s}\right)$$

$$= \boxed{0.93\,\mathrm{N{\bullet}s}}$$

(*c*) The impulse given the bullet is the change in momentum of the bullet. The starting speed of the bullet is 0.

$$\text{Impulse} = \Delta p = m\left(v - v_{0}\right) \quad\rightarrow\quad m = \frac{\text{Impulse}}{v} = \frac{0.93\,\mathrm{N{\bullet}s}}{220\,\mathrm{m/s}} = \boxed{4.2\times10^{-3}\,\mathrm{kg}}$$

76. For the swinging balls, their velocity at the bottom of the swing and the height to which they rise are related by conservation of energy. If the zero of gravitational potential energy is taken to be the lowest point of the swing, then the kinetic energy at the low point is equal to the potential energy at the highest point of the swing, where the speed is zero. Thus we have $\frac{1}{2}mv^{2}_{\text{bottom}} = mgh$ for any swinging ball, and so the relationship between speed and height is $v^{2}_{\text{bottom}} = 2gh$.

(*a*) Calculate the speed of the lighter ball at the bottom of its swing.

$$v_{\mathrm{A}} = \sqrt{2gh_{\mathrm{A}}} = \sqrt{2\left(9.8\,\mathrm{m/s^{2}}\right)\left(0.30\ \mathrm{m} - 0.30\ \mathrm{m}\cos 60^{\circ}\right)} = 1.715\,\mathrm{m/s} \approx \boxed{1.7\,\mathrm{m/s}}$$

(*b*) Assume that the collision is elastic, and use the results of problem 30. Take the direction that ball A is moving just before the collision as the positive direction.

$$v'_{\mathrm{A}}=\frac{(m_{\mathrm{A}}-m_{\mathrm{B}})}{(m_{\mathrm{A}}+m_{\mathrm{B}})}v_{\mathrm{A}}=\frac{(0.040\text{ kg}-0.060\text{ kg})}{(0.040\text{ kg}+0.060\text{ kg})}(1.715\,\mathrm{m/s})=-0.343\,\mathrm{m/s}\approx\boxed{-0.34\,\mathrm{m/s}}$$

$$v'_{\mathrm{B}}=\frac{2m_{\mathrm{A}}}{(m_{\mathrm{A}}+m_{\mathrm{B}})}v_{A}=\frac{2(0.040\text{ kg})}{(0.040\text{ kg}+0.060\text{ kg})}(1.715\,\mathrm{m/s})=1.372\,\mathrm{m/s}\approx\boxed{1.4\,\mathrm{m/s}}$$

Notice that ball A has rebounded backwards.

(*c*) After each collision, use the conservation of energy relationship again.

$$h'_{\mathrm{A}}=\frac{v'^2_{\mathrm{A}}}{2g}=\frac{(-0.343\,\mathrm{m/s})^2}{2(9.8\,\mathrm{m/s^2})}=\boxed{6.0\times10^{-3}\,\mathrm{m}}\qquad h'_{\mathrm{B}}=\frac{v'^2_{\mathrm{B}}}{2g}=\frac{(1.372\,\mathrm{m/s})^2}{2(9.8\,\mathrm{m/s^2})}=\boxed{9.6\times10^{-2}\,\mathrm{m}}$$

77. Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Let A represent the alpha particle, and B represent the smaller nucleus. Call the direction of the alpha particle's motion the positive direction. We have $m_{\mathrm{B}}=57m_{\mathrm{A}}$, $v_{\mathrm{A}}=v_{\mathrm{B}}=0$, and $v'_{\mathrm{A}}=3.8\times10^5\,\mathrm{m/s}$.

$$p_{\text{initial}}=p_{\text{final}}\quad\rightarrow\quad 0=m_Av'_A+m_Bv'_B\quad\rightarrow$$

$$v'_B=-\frac{m_Av'_A}{m_B}=-\frac{v'_A}{57}=-\frac{3.8\times10^5\text{ m/s}}{57}=-6.7\times10^3\text{ m/s}\quad,\quad |v'_B|=\boxed{6.7\times10^3\text{ m/s}}$$

The negative sign indicates that the nucleus is moving in the opposite direction of the alpha particle.

78. The original horizontal distance can be found from the range formula from Example 3-8.

$$R=v_0^2\sin2\theta_0/g=(25\,\mathrm{m/s})^2(\sin60^\circ)/(9.8\,\mathrm{m/s^2})=55.2\text{ m}$$

The height at which the objects collide can be found from Eq. 2-11c for the vertical motion, with $v_y=0$ at the top of the path. Take up to be positive.

$$v_y^2=v_{y0}^2+2a(y-y_0)\quad\rightarrow\quad(y-y_0)=\frac{v_y^2-v_{y0}^2}{2a}=\frac{0-[(25\,\mathrm{m/s})\sin30^\circ]^2}{2(-9.8\,\mathrm{m/s^2})}=7.97\text{ m}$$

Let *m* represent the bullet and *M* the skeet. When the objects collide, the skeet is moving horizontally at $v_0\cos\theta=(25\,\mathrm{m/s})\cos30^\circ=21.65\,\mathrm{m/s}=v_x$, and the bullet is moving vertically at $v_y=200\,\mathrm{m/s}$. Write momentum conservation in both directions to find the velocities after the totally inelastic collision.

$$p_x:\quad Mv_x=(M+m)v'_x\quad\rightarrow\quad v'_x=\frac{Mv_x}{M+m}=\frac{(0.25\text{ kg})(21.65\,\mathrm{m/s})}{(0.25+0.015)\,\mathrm{kg}}=20.42\,\mathrm{m/s}$$

$$p_y:\quad mv_y=(M+m)v'_y\quad\rightarrow\quad v'_y=\frac{mv_y}{M+m}=\frac{(0.015\text{ kg})(200\,\mathrm{m/s})}{(0.25+0.015)\,\mathrm{kg}}=11.32\,\mathrm{m/s}$$

(*a*) The speed v'_y can be used as the starting vertical speed in Eq. 2-11c to find the height that the skeet-bullet combination rises above the point of collision.

$$v_y^2=v_{y0}^2+2a(y-y_0)_{\text{extra}}\quad\rightarrow\quad(y-y_0)_{\text{extra}}=\frac{v_y^2-v_{y0}^2}{2a}=\frac{0-(11.32\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})}=\boxed{6.5\text{ m}}$$

(*b*) From Eq. 2-11b applied to the vertical motion after the collision, we can find the time for the skeet-bullet combination to reach the ground.

$$y = y_0 + v'_y t + \tfrac{1}{2}at^2 \;\rightarrow\; 0 = 7.97\text{ m} + (11.32\,\text{m/s})t + \tfrac{1}{2}(-9.8\,\text{m/s}^2)t^2 \;\rightarrow$$

$$4.9t^2 - 11.32t - 7.97 = 0 \;\rightarrow\; t = 2.88\text{ s}\,,\; -0.565\text{ s}$$

The positive time root is used to find the horizontal distance traveled by the combination after the collision.

$$x_{\text{after}} = v'_x t = (20.42\,\text{m/s})(2.88\text{ s}) = 58.7\text{ m}$$

If the collision would not have happened, the skeet would have gone $\tfrac{1}{2}R$ horizontally.

$$\Delta x = x_{\text{after}} - \tfrac{1}{2}R = 58.7\text{ m} - \tfrac{1}{2}(55.2\text{ m}) = 31.1\text{ m} \approx \boxed{31\text{ m}}$$

79. (*a*) Use conservation of energy to find the speed of mass *m* before the collision. The potential energy at the starting point is all transformed into kinetic energy just before the collision.

$$mgh_{\text{A}} = \tfrac{1}{2}mv_{\text{A}}^2 \;\rightarrow\; v_{\text{A}} = \sqrt{2gh_{\text{A}}} = \sqrt{2(9.80\,\text{m/s}^2)(3.60\text{ m})} = 8.40\,\text{m/s}$$

Use Eq. 7-7 to obtain a relationship between the velocities, noting that $v_B = 0$.

$$v_{\text{A}} - v_{\text{B}} = v'_{\text{B}} - v'_{\text{A}} \;\rightarrow\; v'_{\text{B}} = v'_{\text{A}} + v_{\text{A}}$$

Apply momentum conservation for the collision, and substitute the result from Eq. 7-7.

$$mv_{\text{A}} = mv'_{\text{A}} + Mv'_{\text{B}} = mv'_{\text{A}} + M\left(v_{\text{A}} + v'_{\text{A}}\right) \;\rightarrow$$

$$v'_{\text{A}} = \frac{m-M}{m+M}v_{\text{A}} = \left(\frac{2.20\text{ kg} - 7.00\text{ kg}}{9.20\text{ kg}}\right)(8.4\,\text{m/s}) = -4.38\,\text{m/s} \approx \boxed{-4.4\,\text{m/s}}$$

$$v'_{\text{B}} = v'_{\text{A}} + v_{\text{A}} = -4.4\,\text{m/s} + 8.4\,\text{m/s} = \boxed{4.0\,\text{m/s}}$$

(*b*) Again use energy conservation to find the height to which mass *m* rises after the collision. The kinetic energy of *m* immediately after the collision is all transformed into potential energy. Use the angle of the plane to change the final height into a distance along the incline.

$$\tfrac{1}{2}mv'^2_{\text{A}} = mgh'_{\text{A}} \;\rightarrow\; h'_{\text{A}} = \frac{v'^2_{\text{A}}}{2g}$$

$$d'_{\text{A}} = \frac{h'_{\text{A}}}{\sin 30} = \frac{v'^2_{\text{A}}}{2g\sin 30} = \frac{(-4.38\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)g\sin 30} = 1.96\text{ m} \approx \boxed{2.0\text{ m}}$$

80. Let A represent mass *m*, and B represent mass *M*. Use Eq. 7-7 to obtain a relationship between the velocities, noting that $v_B = 0$.

$$v_{\text{A}} - v_{\text{B}} = v'_{\text{B}} - v'_{\text{A}} \;\rightarrow\; v'_{\text{A}} = v'_{\text{B}} - v_{\text{A}}.$$

After the collision, v'_{A} will be negative since *m* is moving in the negative direction. For there to be a second collision, then after *m* moves up the ramp and comes back down, with a positive velocity at the bottom of the incline of $-v'_{\text{A}}$, the speed of *m* must be greater than the speed of *M* so that *m* can catch *M*. Thus $-v'_{\text{A}} > v'_{\text{B}}$, or $v'_{\text{A}} < -v'_{\text{B}}$. Substitute the result from Eq. 7-7 into the inequality.

$$v'_{\text{B}} - v_{\text{A}} < -v'_{\text{B}} \;\rightarrow\; v'_{\text{B}} < \tfrac{1}{2}v_{\text{A}}.$$

Now write momentum conservation for the original collision, and substitute the result from Eq. 7-7.

$$mv_{\text{A}} = mv'_{\text{A}} + Mv'_{\text{B}} = m\left(v'_{\text{B}} - v_{\text{A}}\right) + Mv'_{\text{B}} \;\rightarrow\; v'_{\text{B}} = \frac{2m}{m+M}v_{\text{A}}$$

Finally, combine the above result with the inequality from above.

$$\frac{2m}{m+M}v_{\mathrm{A}} < \tfrac{1}{2}v_{\mathrm{A}} \quad \rightarrow \quad 4m < m+M \quad \rightarrow \quad \boxed{m < \tfrac{1}{3}M = 2.33\,\mathrm{kg}}$$

81. The interaction between the planet and the spacecraft is elastic, because the force of gravity is conservative. Thus kinetic energy is conserved in the interaction. Consider the problem a 1-dimensional collision, with A representing the spacecraft and B representing Saturn. Because the mass of Saturn is so much bigger than the mass of the spacecraft, Saturn's speed is not changed appreciably during the interaction. Use Eq. 7-7, with $v_{\mathrm{A}} = 10.4\,\mathrm{km/s}$ and $v_{\mathrm{B}} = v'_{\mathrm{B}} = -9.6\,\mathrm{km/s}$.

$$v_{\mathrm{A}} - v_{\mathrm{B}} = -v'_{\mathrm{A}} + v'_{\mathrm{B}} \quad \rightarrow \quad v'_{\mathrm{A}} = 2v_{\mathrm{B}} - v_{\mathrm{A}} = 2\left(-9.6\,\mathrm{km/s}\right) - 10.4\,\mathrm{km/s} = \boxed{-29.6\,\mathrm{km/s}}$$

Thus there is almost a threefold increase in the spacecraft's speed.

CHAPTER 8: Rotational Motion

Answers to Questions

1. The odometer designed for 27-inch wheels increases its reading by the circumference of a 27-inch wheel $\left(27\pi"\right)$ for every revolution of the wheel. If a 24-inch wheel is used, the odometer will still register $\left(27\pi"\right)$ for every revolution, but only $24\pi"$ of linear distance will have been traveled. Thus the odometer will read a distance that is further than you actually traveled, by a factor of $27/24 = 1.125$. The odometer will read 12.5% too high.

2. If a disk rotates at constant angular velocity, a point on the rim has radial acceleration only – no tangential acceleration. If the disk's angular velocity increases uniformly, the point will have both radial and tangential acceleration. If the disk rotates at constant angular velocity, neither component of linear acceleration is changing – both radial and tangential acceleration are constant. If the disk rotates with a uniformly increasing angular velocity, then the radial acceleration is changing, but the tangential acceleration is a constant non-zero value.

3. A non-rigid body cannot be described by a single value of angular velocity. Since the body is non-rigid, the angular position of one part of the body changes with respect to other parts of the body. Consider the solar system as an example of a non-rigid body or system. Each planet orbits in basically the same direction around the Sun, but each planet has its own angular velocity which is different than that of the other planets.

4. Since the torque involves the product of force times lever arm, a small force can exert a greater torque than a larger force if the small force has a large enough lever arm.

5. If the lever arm is zero, then the force does not exert any torque and so cannot produce an angular acceleration. There will be no change in the angular state of motion. However, the force will add to the net force on the body and so will change the linear acceleration of the body. The body's linear state of motion will change.

6. When you do a sit-up, torque from your abdomen muscles must rotate the upper half of the body from a laying-down position to a sitting-up position. The larger the moment of inertia of the upper half of the body, the more torque is needed, and thus the harder the sit-up is to do. With the hands behind the head, the moment of inertia of the upper half of the body is larger than with the hands outstretched in front.

7. The tension force in the bicycle chain can be assumed to be the same at both the front and rear sprockets. The force is related to the torque at each sprocket by $F = \tau/R$, and so $\tau_R/R_R = \tau_F/R_F$. The torque at the rear sprocket is what actually accelerates the bicycle, and so $\tau_R = \tau_F\, R_R/R_F$.

   We see that, to achieve a given torque at the back sprocket, a larger front torque (due to pedaling) must be present when the rear sprocket is small. Thus it is harder to pedal with a small rear sprocket.

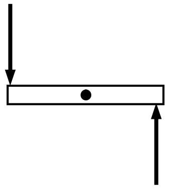
   Likewise, to achieve a given torque at the back sprocket, a larger front torque (due to pedaling) must be present when the front sprocket is larger. Thus it is harder to pedal with a larger front sprocket.

8. The legs have a lower moment of inertia when the leg mass is concentrated next to the body. That means the legs will require less torque to have a given angular acceleration, or, alternatively, a higher angular acceleration can be developed. Thus the animal can run fast.

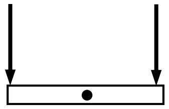
9. The long beam increases the rotational inertia of the walker. If the walker gets off-center from the tightrope, gravity will exert a torque on the walker causing the walker to rotate with their feet as a pivot point. With a larger rotational inertia, the angular acceleration caused by that gravitational torque will be smaller, and the walker will therefore have more time to compensate.

   The long size of the beam allows the walker to make relatively small shifts in their center of mass to bring them back to being centered on the tightrope. It is much easier for the walker to move a long, narrow object with the precision needed for small adjustments than a short, heavy object like a barbell.

10. Just because the net force on a system is zero, the net torque need not be zero. Consider a uniform object with two equal forces on it, and shown in the first diagram. The net force on the object is zero (it would not start to translate under the action of these forces), but there is a net counterclockwise torque about the center of the rod (it would start to rotate under the action of these forces).



   Just because the net torque on a system is zero, the net force need not be zero. Consider an object with two equal forces on it, as shown in the second diagram. The net torque on the object is zero (it would not start to rotate under the action of these forces), but there is a net downward force on the rod (it would start to translate under the action of these forces).



11. Applying conservation of energy at the top and bottom of the incline, assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \rightarrow Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. For a solid ball, $I = \frac{2}{5}MR^2$. If the ball rolls without slipping (no work done by friction) then $\omega = v/R$, and so

$$Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\tfrac{2}{5}MR^2\,v^2/R^2 \quad \rightarrow \quad v = \sqrt{10gh/7} \;.$$

   This speed is independent of the angle of the incline, and so both balls will have the same speed at the bottom. The ball on the incline with the smaller angle will take more time to reach the bottom than the ball on the incline with the larger angle.

12. Applying conservation of energy at the top and bottom of the incline, and assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \rightarrow Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. For a solid ball, $I = \frac{2}{5}MR^2$. If the ball rolls without slipping (no work done by friction) then $\omega = v/R$, and so

$$Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\tfrac{2}{5}MR^2\,v^2/R^2 \quad \rightarrow \quad v = \sqrt{10gh/7}$$

   This speed is independent of the mass and radius of the ball, and so both balls will have the same speed at the bottom. In fact, this is true for ANY height of fall, so the two balls will have identical instantaneous speeds all along their descent, and so both balls will take the same time to reach the bottom. The total kinetic energy is $KE = KE_{\text{trans}} + KE_{\text{rot}} = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{2}{5}MR^2\,v^2/R^2 = \frac{7}{10}Mv^2$, and so the ball with the larger mass has the greater total kinetic energy. Another way to consider this is that the initial potential energy of Mgh is all converted to kinetic energy. The larger mass has more potential energy to begin with (due to the larger mass), and so has more kinetic energy at the bottom.

13. Applying conservation of energy at the top and bottom of the incline, assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \rightarrow Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. If the objects roll without slipping, then $\omega = v/R$, and so $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(v/R\right)^2 \rightarrow v = \sqrt{\frac{2Mgh}{M + I/R^2}}$. For a solid ball, $I = \frac{2}{5}MR^2$, and for a cylinder, $I = \frac{1}{2}MR^2$. Thus $v_{\text{sphere}} = \sqrt{10gh/7}$ and $v_{\text{cyl}} = \sqrt{4gh/3}$. Since $v_{\text{sphere}} > v_{\text{cyl}}$, the sphere has the greater speed at the bottom. That is true for any amount of height change, and so the sphere is always moving faster than the cylinder after they start to move. Thus the sphere will reach the bottom first. Since both objects started with the same potential energy, both have the same total kinetic energy at the bottom. But since both objects have the same mass and the cylinder is moving slower, the cylinder has the smaller translational KE and thus the greater rotational KE.

14. Momentum and angular momentum are conserved for closed systems – systems in which there are no external forces or torques applied to the system. Probably no macroscopic systems on Earth are truly closed, and so external forces and torques (like those applied by air friction, for example) affect the systems over time.

15. If a large number of people went to the equator, the rotational inertia of the Earth would increase, since the people would be further from the axis of rotation. Angular momentum would be conserved in such an interaction, and so since the rotational inertia increased, the angular velocity would decrease – the Earth would “slow down” a small amount. The length of a day would therefore increase.

16. In order to do a somersault, the diver needs some initial angular momentum when she leaves the diving board, because angular momentum will be conserved during the free-fall motion of the dive. She cannot exert a torque on herself in isolation, and so if there is no angular momentum initially, there will be no rotation during the rest of the dive.

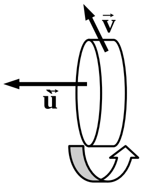
17. The moment of inertia will increase, because most the mass of the disk will be further from the axis of rotation than it was with the original axis position.

18. Your angular velocity will not change. Before you let go of the masses, your body has a certain angular momentum, which is the product of your moment of inertia and your angular velocity. No torques are put upon you by the act of dropping the masses, and so your angular momentum does not change. If you don’t change your moment of inertia by changing the position of your body, then your angular velocity will not change. The masses, when dropped, will have a horizontal motion that is tangential to the circle in which they were moving before they were dropped. An object traveling horizontally at some distance from a vertical line (like your axis of rotation) has angular momentum relative to that vertical line. The masses keep the angular momentum that they had before being dropped.

19. The two spheres would have different rotational inertias. The sphere that is hollow will have a larger rotational inertia than the solid sphere. If the two spheres are allowed to roll down an incline without slipping, the sphere with the smaller moment of inertia (the solid one) will reach the bottom of the ramp first. See question number 13 for an explanation of why this happens.

20. Using the right hand rule, point the fingers in the direction of the Earth's rotation, from west to east. Then the thumb points north. Thus the Earth's angular velocity points along its axis of rotation, towards the North Star.

21. See the diagram. To the left is west, the direction of the angular velocity. The direction of the linear velocity of a point on the top of the wheel would be into the paper, which is north. If the angular acceleration is east, which is opposite the angular velocity, the wheel is slowing down – its angular speed is decreasing. The tangential linear acceleration of the point on top will be in the opposite direction to its linear velocity – it will point south.



22. The angular momentum of the turntable – person system will be conserved, since no external torques are being applied as the person walks to the center. As the person walks to the center, the overall moment of inertia of the system gets smaller, since the person is closer to the axis of rotation. Since the angular momentum is constant, the angular velocity must increase. So the turntable will begin to rotate faster as you walk to the center. This is similar to the spinning ice skater who pulls her arms in to increase her angular speed.

23. The shortstop, while in mid-air, cannot exert a torque on himself, and so his angular momentum will be conserved while in the air. If the upper half of his body rotates in a certain direction during the throwing motion, then to conserve angular momentum, the lower half of his body will rotate in the opposite direction.

24. Consider a helicopter in the air with the rotor spinning. To change the rotor's angular speed, a torque must be applied to the rotor. That torque has to come from the helicopter, and so by Newton's 3rd law, and equal and opposite torque will be applied by the rotor to the helicopter. Any change in rotor speed would therefore cause the body of the helicopter to spin in a direction opposite to the change in the rotor's angular velocity.

    Some large helicopters have two rotor systems, spinning in opposite directions. That makes any change in the speed of the rotor pair require a net torque of 0, and so the helicopter body would not tend to spin. Smaller helicopters have a tail rotor which rotates in a vertical plane, causing a force on the tail of the helicopter in the opposite direction of the tendency of the tail to spin.

Solutions to Problems

1. (*a*) $\left(30^\circ\right)\left(2\pi\ \text{rad}/360^\circ\right)=\boxed{\pi/6\,\text{rad}}=\boxed{0.52\ \text{rad}}$

    (*b*) $\left(57^\circ\right)\left(2\pi\ \text{rad}/360^\circ\right)=\boxed{19\pi/60\,\text{rad}}=\boxed{0.99\ \text{rad}}$

    (*c*) $\left(90^\circ\right)\left(2\pi\ \text{rad}/360^\circ\right)=\boxed{\pi/2\,\text{rad}}=\boxed{1.57\ \text{rad}}$

    (*d*) $\left(360^\circ\right)\left(2\pi\ \text{rad}/360^\circ\right)=\boxed{2\pi\ \text{rad}}=\boxed{6.28\ \text{rad}}$

    (*e*) $\left(420^\circ\right)\left(2\pi\ \text{rad}/360^\circ\right)=\boxed{7\pi/3\,\text{rad}}=\boxed{7.33\ \text{rad}}$

2. The angle in radians is the diameter of the object divided by the distance to the object.

$$\Delta\theta_{\text{Sun}}=\frac{2R_{\text{Sun}}}{r_{\text{Earth}-\text{Sun}}}=\frac{2\left(6.96\times10^{5}\,\text{km}\right)}{149.6\times10^{6}\,\text{km}}=\boxed{9.30\times10^{-3}\,\text{rad}}$$

$$\Delta\theta_{\text{Moon}} = \frac{2R_{\text{Moon}}}{r_{\text{Earth-Moon}}} = \frac{2\left(1.74\times10^{3}\,\text{km}\right)}{384\times10^{3}\,\text{km}} = \boxed{9.06\times10^{-3}\,\text{rad}}$$

Since these angles are practically the same, solar eclipses occur.

3. We find the diameter of the spot from

$$\theta = \frac{\text{diameter}}{r_{\text{Earth-Moon}}} \rightarrow \text{diameter} = \theta\, r_{\text{Earth-Moon}} = \left(1.4\times10^{-5}\,\text{rad}\right)\left(3.8\times10^{8}\,\text{m}\right) = \boxed{5.3\times10^{3}\,\text{m}}$$

4. The initial angular velocity is $\omega_o = \left(6500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right) = 681\,\text{rad/s}$. Use the definition of angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 681\,\text{rad/s}}{3.0\ \text{s}} = -227\ \text{rad/s}^2 \approx \boxed{-2.3\times10^{2}\ \text{rad/s}^2}$$

5. The ball rolls $2\pi r = \pi d$ of linear distance with each revolution.

$$15.0\ \text{rev}\left(\frac{\pi d\ \text{m}}{1\ \text{rev}}\right) = 3.5\ \text{m} \rightarrow d = \frac{3.5\ \text{m}}{15.0\ \pi} = \boxed{7.4\times10^{-2}\,\text{m}}$$

6. In each revolution, the wheel moves forward a distance equal to its circumference, πd .

$$\Delta x = N_{\text{rev}}\left(\pi d\right) \rightarrow N = \frac{\Delta x}{\pi d} = \frac{8000\ \text{m}}{\pi\left(0.68\ \text{m}\right)} = \boxed{3.7\times10^{3}\,\text{rev}}$$

7. (*a*) $\omega = \left(\frac{2500\ \text{rev}}{1\ \text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 261.8\,\text{rad/sec} \sim \boxed{2.6\times10^{2}\ \text{rad/sec}}$

(*b*) $v = \omega r = \left(261.8\,\text{rad/sec}\right)\left(0.175\ \text{m}\right) = \boxed{46\,\text{m/s}}$

$$a_{\text{R}} = \omega^2 r = \left(261.8\,\text{rad/sec}\right)^2\left(0.175\ \text{m}\right) = \boxed{1.2\times10^{4}\ \text{m/s}^2}$$

8. The angular speed of the merry-go-round is $2\pi\ \text{rad}/4.0\ \text{s} = 1.57\,\text{rad/s}$

(*a*) $v = \omega r = \left(1.57\,\text{rad/sec}\right)\left(1.2\ \text{m}\right) = \boxed{1.9\,\text{m/s}}$

(*b*) The acceleration is radial. There is no tangential acceleration.

$$a_{\text{R}} = \omega^2 r = \left(1.57\,\text{rad/sec}\right)^2\left(1.2\ \text{m}\right) = \boxed{3.0\,\text{m/s}^2\ \text{towards the center}}$$

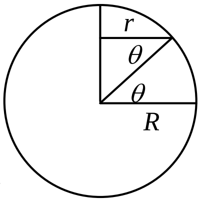
9. (*a*) The Earth makes one orbit around the Sun in one year.

$$\omega_{\text{orbit}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\ \text{rad}}{1\ \text{yr}}\right)\left(\frac{1\ \text{yr}}{3.16\times10^{7}\,\text{s}}\right) = \boxed{1.99\times10^{-7}\ \text{rad/s}}$$

(*b*) The Earth makes one revolution about its axis in one day.

$$\omega_{\text{rotation}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\ \text{rad}}{1\ \text{d}}\right)\left(\frac{1\ \text{d}}{86{,}400\ \text{s}}\right) = \boxed{7.27\times10^{-5}\ \text{rad/s}}$$

10. Each location will have the same angular velocity (1 revolution per day), but the radius of the circular path varies with the location. From the diagram, we see $r = R\cos\theta$, where R is the radius of the Earth, and r is the radius at latitude θ.



(*a*) $v=\omega r=\frac{2\pi}{T}r=\left(\frac{2\pi\text{ rad}}{1\text{ d}}\right)\left(\frac{1\text{ d}}{86400\text{ s}}\right)\left(6.38\times10^{6}\text{ m}\right)=\boxed{4.64\times10^{2}\text{ m/s}}$

(*b*) $v=\omega r=\frac{2\pi}{T}r=\left(\frac{2\pi\text{ rad}}{1\text{ d}}\right)\left(\frac{1\text{ d}}{86400\text{ s}}\right)\left(6.38\times10^{6}\text{ m}\right)\cos66.5^{\circ}=\boxed{1.85\times10^{2}\text{ m/s}}$

(*c*) $v=\omega r=\frac{2\pi}{T}r=\left(\frac{2\pi\text{ rad}}{1\text{ d}}\right)\left(\frac{1\text{ d}}{86400\text{ s}}\right)\left(6.38\times10^{6}\text{ m}\right)\cos45.0^{\text{O}}=\boxed{3.28\times10^{2}\text{ m/s}}$

11. The centripetal acceleration is given by $a=\omega^2 r$. Solve for the angular velocity.

$$\omega=\sqrt{\frac{a}{r}}=\sqrt{\frac{(100,000)\left(9.8\text{ m/s}^2\right)}{0.070\text{ m}}}=3741\frac{\text{rad}}{\text{s}}\left(\frac{1\text{ rev}}{2\pi\text{rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)=\boxed{3.6\times10^{4}\text{ rpm}}$$

12. Convert the rpm values to angular velocities.

$$\omega_0=\left(130\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right)=13.6\text{ rad/s}$$

$$\omega=\left(280\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right)=29.3\text{ rad/s}$$

(*a*) The angular acceleration is found from Eq. 8-9a.

$$\omega=\omega_0+\alpha t\quad\rightarrow\quad\alpha=\frac{\omega-\omega_0}{t}=\frac{29.3\text{ rad/s}-13.6\text{ rad/s}}{4.0\text{ s}}=3.93\text{ rad/s}^2\approx\boxed{3.9\text{ rad/s}^2}$$

(*b*) To find the components of the acceleration, the instantaneous angular velocity is needed.

$$\omega=\omega_0+\alpha t=13.6\text{ rad/s}+\left(3.93\text{ rad/s}^2\right)(2.0\text{ s})=21.5\text{ rad/s}$$

The instantaneous radial acceleration is given by $a_{\text{R}}=\omega^2 r$.

$$a_{\text{R}}=\omega^2 r=\left(21.5\text{ rad/s}\right)^2(0.35\text{ m})=\boxed{1.6\times10^{2}\text{ m/s}^2}$$

The tangential acceleration is given by $a_{\text{tan}}=\alpha r$.

$$a_{\text{tan}}=\alpha r=\left(3.93\text{ rad/s}^2\right)(0.35\text{ m})=\boxed{1.4\text{ m/s}^2}$$

13. The tangential speed of the turntable must be equal to the tangential speed of the roller, if there is no slippage.

$$v_1=v_2\quad\rightarrow\quad\omega_1R_1=\omega_2R_2\quad\rightarrow\quad\boxed{\omega_1/\omega_2=R_2/R_1}$$

14. (*a*) The angular rotation can be found from Eq. 8-3a. The initial angular frequency is 0 and the final angular frequency is 1 rpm.

$$\alpha=\frac{\omega-\omega_0}{t}=\frac{\left(1\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1.0\text{ min}}{60\text{ s}}\right)-0}{720\text{ s}}=1.454\times10^{-4}\text{ rad/s}^2\approx\boxed{1.5\times10^{-4}\text{ rad/s}^2}$$

(*b*) After 5.0 min (300 s), the angular speed is as follows.

$$\omega=\omega_0+\alpha t=0+\left(1.454\times10^{-4}\text{ rad/s}^2\right)(300\text{ s})=4.363\times10^{-2}\text{ rad/s}$$

Find the components of the acceleration of a point on the outer skin from the angular speed and the radius.

$$a_{\text{tan}} = \alpha R = \left(1.454\times10^{-4}\ \text{rad/s}^2\right)\left(4.25\ \text{m}\right) = \boxed{6.2\times10^{-4}\ \text{m/s}^2}$$

$$a_{\text{rad}} = \omega^2 R = \left(4.363\times10^{-2}\ \text{rad/s}\right)^2\left(4.25\ \text{m}\right) = \boxed{8.1\times10^{-3}\ \text{m/s}^2}$$

15. The angular displacement can be found from the following uniform angular acceleration relationship.

$$\theta = \tfrac{1}{2}\left(\omega_o + \omega\right)t = \tfrac{1}{2}\left(0 + 15000\,\text{rev/min}\right)\left(220\ \text{s}\right)\left(1\,\text{min}/60\,\text{s}\right) = \boxed{2.8\times10^4\,\text{rev}}$$

16. (*a*) For constant angular acceleration:

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{1200\,\text{rev/min} - 4500\,\text{rev/min}}{2.5\ \text{s}} = \frac{-3300\,\text{rev/min}}{2.5\,\text{s}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= \boxed{-1.4\times10^2\ \text{rad/s}^2}$$

(*b*) For the angular displacement, given constant angular acceleration:

$$\theta = \tfrac{1}{2}\left(\omega_o + \omega\right)t = \tfrac{1}{2}\left(4500\,\text{rev/min} + 1200\,\text{rev/min}\right)\left(2.5\ \text{s}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{1.2\times10^2\text{rev}}$$

17. (*a*) The angular acceleration can be found from $\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$ with $\omega_o = 0$.

$$\alpha = \frac{2\theta}{t^2} = \frac{2\left(20\ \text{rev}\right)}{\left(1.0\ \text{min}\right)^2} = \boxed{4.0\times10^1\ \text{rev/min}^2}$$

(*b*) The final angular speed can be found from $\theta = \tfrac{1}{2}\left(\omega_o + \omega\right)t$, with $\omega_o = 0$.

$$\omega = \frac{2\theta}{t} - \omega_o = \frac{2\left(20\ \text{rev}\right)}{1.0\ \text{min}} = \boxed{4.0\times10^1\ \text{rpm}}$$

18. Use Eq. 8-9d combined with Eq. 8-2a.

$$\overline{\omega} = \frac{\omega + \omega_0}{2} = \frac{240\,\text{rpm} + 360\,\text{rpm}}{2} = 300\,\text{rpm}$$

$$\theta = \overline{\omega} t = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\left(6.5\ \text{s}\right) = 32.5\ \text{rev}$$

Each revolution corresponds to a circumference of travel distance.

$$32.5\ \text{rev}\left[\frac{\pi\left(0.33\ \text{m}\right)}{1\ \text{rev}}\right] = \boxed{34\ \text{m}}$$

19. (*a*) The angular acceleration can be found from $\omega^2 = \omega_o^2 + 2\alpha\theta$.

$$\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{0 - \left(850\,\text{rev/min}\right)^2}{2\left(1500\ \text{rev}\right)} = \left(-241\frac{\text{rev}}{\text{min}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 = \boxed{-0.42\frac{\text{rad}}{\text{s}^2}}$$

(*b*) The time to come to a stop can be found from $\theta = \tfrac{1}{2}\left(\omega_o + \omega\right)t$.

$$t = \frac{2\theta}{\omega_o + \omega} = \frac{2\left(1500\ \text{rev}\right)}{850\,\text{rev/min}}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{210\ \text{s}}$$

20. Since there is no slipping between the wheels, the tangential component of the linear acceleration of each wheel must be the same.

(*a*) $a_{\substack{\text{tan}\\\text{small}}} = a_{\substack{\text{tan}\\\text{argel}}} \quad \rightarrow \quad \alpha_{\text{small}} r_{\text{small}} = \alpha_{\text{large}} r_{\text{large}} \quad \rightarrow$

$$\alpha_{\text{large}} = \alpha_{\text{small}} \frac{r_{\text{small}}}{r_{\text{large}}} = \left(7.2\,\text{rad/s}^2\right)\left(\frac{2.0\text{ cm}}{25.0\text{ cm}}\right) = 0.576\,\text{rad/s}^2 \approx \boxed{0.58\,\text{rad/s}^2}$$

(*b*) Assume the pottery wheel starts from rest. Convert the speed to an angular speed, and then use Eq. 8-9a.

$$\omega = \left(65\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 6.81\,\text{rad/s}$$

$$\omega = \omega_0 + \alpha t \quad \rightarrow \quad t = \frac{\omega - \omega_0}{\alpha} = \frac{6.81\,\text{rad/s}}{0.576\,\text{rad/s}^2} = \boxed{12\text{ s}}$$

$\boxed{21.}$ (*a*) The angular acceleration can be found from $\omega^2 = \omega_o^2 + 2\alpha\theta$, with the angular velocities being found from $\omega = v/r$.

$$\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{\left(v^2 - v_o^2\right)}{2r^2\theta} = \frac{\left[\left(45\,\text{km/h}\right)^2 - \left(95\,\text{km/h}\right)^2\right]\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)^2}{2\left(0.40\text{ m}\right)^2\left(65\text{ rev}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)}$$

$$= -4.133\,\text{rad/s}^2 \approx \boxed{-4.1\,\text{rad/s}^2}$$

(*b*) The time to stop can be found from $\omega = \omega_o + \alpha t$, with a final angular velocity of 0.

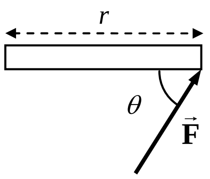
$$t = \frac{\omega - \omega_o}{\alpha} = \frac{v - v_o}{r\alpha} = \frac{-\left(45\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)}{\left(0.40\text{ m}\right)\left(-4.133\,\text{rad/s}^2\right)} = \boxed{7.6\text{ s}}$$

22. (*a*) The maximum torque will be exerted by the force of her weight, pushing tangential to the circle in which the pedal moves.

$$\tau = r_\perp F = r_\perp mg = \left(0.17\text{ m}\right)\left(55\text{ kg}\right)\left(9.8\,\text{m/s}^2\right) = \boxed{92\,\text{m}\bullet\text{N}}$$

(*b*) She could exert more torque by pushing down harder with her legs, raising her center of mass. She could also pull upwards on the handle bars as she pedals, which will increase the downward force of her legs.

23. The torque is calculated by $\tau = rF\sin\theta$. See the diagram, from the top view.



(*a*) For the first case, $\theta = 90^\circ$.

$$\tau = rF\sin\theta = \left(0.74\text{ m}\right)\left(55\text{ N}\right)\sin 90^\circ = \boxed{41\text{ m}\bullet\text{N}}$$

(*b*) For the second case, $\theta = 45^\circ$.

$$\tau = rF\sin\theta = \left(0.74\text{ m}\right)\left(55\text{ N}\right)\sin 45^\circ = \boxed{29\text{ m}\bullet\text{N}}$$

24. Each force is oriented so that it is perpendicular to its lever arm. Call counterclockwise torques positive. The torque due to the three applied forces is given by

$$\tau_{\substack{\text{applied}\\\text{forces}}} = (28\text{ N})(0.24\text{ m}) - (18\text{ N})(0.24\text{ m}) - (35\text{ N})(0.12\text{ m}) = -1.8\text{ m}\bullet\text{N}.$$

Since this torque is clockwise, we assume the wheel is rotating clockwise, and so the frictional torque is counterclockwise. Thus the net torque is

$$\tau_{\text{net}} = (28\text{ N})(0.24\text{ m}) - (18\text{ N})(0.24\text{ m}) - (35\text{ N})(0.12\text{ m}) + 0.40\text{ m}\bullet\text{N} = -1.4\text{ m}\bullet\text{N}$$

$$= \boxed{1.4\text{ m}\bullet\text{N},\text{ clockwise}}$$

25. There is a counterclockwise torque due to the force of gravity on the left block, and a clockwise torque due to the force of gravity on the right block. Call clockwise the positive direction.

$$\sum \tau = mgL_2 - mgL_1 = \boxed{mg(L_2 - L_1)\text{ , clockwise}}$$

26. (*a*) The force required to produce the torque can be found from $\tau = rF\sin\theta$. The force is applied perpendicularly to the wrench, so $\theta = 90^\circ$. Thus

$$F = \frac{\tau}{r} = \frac{88\text{ m}\bullet\text{N}}{0.28\text{ m}} = \boxed{3.1\times10^{2}\text{ N}}$$

(*b*) The net torque still must be $88\text{ m}\bullet\text{N}$. This is produced by 6 forces, one at each of the 6 points. Those forces are also perpendicular to the lever arm, and so

$$\tau_{\text{net}} = (6F_{\text{point}})r_{\text{point}} \rightarrow F_{\text{point}} = \frac{\tau}{6r} = \frac{88\text{ m}\bullet\text{N}}{6(0.0075\text{ m})} = \boxed{2.0\times10^{3}\text{ N}}$$

27. For a sphere rotating about an axis through its center, the moment of inertia is given by

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(10.8\text{ kg})(0.648\text{ m})^2 = \boxed{1.81\text{ kg}\bullet\text{m}^2}\quad.$$

28. Since all of the significant mass is located at the same distance from the axis of rotation, the moment of inertia is given by

$$I = MR^2 = (1.25\text{ kg})\left(\frac{0.667}{2}\text{m}\right)^2 = \boxed{0.139\text{ kg}\bullet\text{m}^2}.$$

The hub mass can be ignored because its distance from the axis of rotation is very small, and so it has a very small rotational inertia.

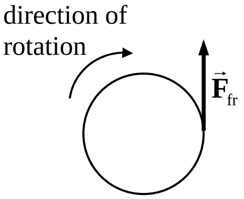
29. (*a*) The small ball can be treated as a particle for calculating its moment of inertia.

$$I = MR^2 = (0.650\text{ kg})(1.2\text{ m})^2 = \boxed{0.94\text{ kg}\bullet\text{m}^2}$$

(*b*) To keep a constant angular velocity, the net torque must be zero, and so the torque needed is the same magnitude as the torque caused by friction.

$$\sum \tau = \tau_{\text{applied}} - \tau_{\text{fr}} = 0 \rightarrow \tau_{\text{applied}} = \tau_{\text{fr}} = F_{\text{fr}}r = (0.020\text{ N})(1.2\text{ m}) = \boxed{2.4\times10^{-2}\text{ m}\bullet\text{N}}$$

30. (*a*) The torque exerted by the frictional force is $\tau = rF_{\text{fr}}\sin\theta$. The frictional force is assumed to be tangential to the clay, and so the angle is $\theta = 90^\circ$.



$$\tau_{\text{total}} = rF_{\text{fr}}\sin\theta = (6.0\times10^{-2}\text{ m})(1.5\text{ N})\sin 90^\circ = \boxed{9.0\times10^{-2}\text{ m}\bullet\text{N}}$$

(*b*) The time to stop is found from $\omega = \omega_o + \alpha t$, with a final angular velocity of 0. The angular acceleration can be found from $\tau_{\text{total}} = I\alpha$. The net torque (and angular acceleration) is negative since the object is slowing.

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{\omega - \omega_o}{\tau / I} = \frac{0 - (1.6\,\text{rev/s})(2\pi\ \text{rad/rev})}{(-9.0\times 10^{-2}\,\text{m}\bullet\text{N})/(0.11\,\text{kg}\bullet\text{m}^2)} = \boxed{12\ \text{s}}$$

31. (*a*) To calculate the moment of inertia about the *y*-axis (vertical), use

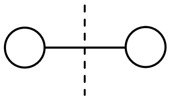
$$I = \sum M_i R_{ix}^2 = m(0.50\ \text{m})^2 + M(0.50\ \text{m})^2 + m(1.00\ \text{m})^2 + M(1.00\ \text{m})^2$$
$$= (m+M)\left[(0.50\ \text{m})^2 + (1.00\ \text{m})^2\right] = (4.9\ \text{kg})\left[(0.50\ \text{m})^2 + (1.00\ \text{m})^2\right] = \boxed{6.1\ \text{kg}\bullet\text{m}^2}$$

(*b*) To calculate the moment of inertia about the *x*-axis (horizontal), use

$$I = \sum M_i R_{iy}^2 = (2m + 2M)(0.25\ \text{m})^2 = \boxed{0.61\ \text{kg}\bullet\text{m}^2}.$$

(*c*) Because of the larger *I* value, it is harder to accelerate the array about the $\boxed{\text{vertical axis}}$.

32 The oxygen molecule has a "dumbbell" geometry, rotating about the dashed line, as shown in the diagram. If the total mass is *M*, then each atom has a mass of *M*/2. If the distance between them is *d*, then the distance from the axis of rotation to each atom is *d*/2. Treat each atom as a particle for calculating the moment of inertia.



$$I = (M/2)(d/2)^2 + (M/2)(d/2)^2 = 2(M/2)(d/2)^2 = \tfrac{1}{4}Md^2 \quad \rightarrow$$
$$d = \sqrt{4I/M} = \sqrt{4(1.9\times 10^{-46}\,\text{kg}\bullet\text{m}^2)/(5.3\times 10^{-26}\,\text{kg})} = \boxed{1.2\times 10^{-10}\,\text{m}}$$

33. The firing force of the rockets will create a net torque, but no net force. Since each rocket fires tangentially, each force has a lever arm equal to the radius of the satellite, and each force is perpendicular to the lever arm. Thus $\tau_{\text{net}} = 4FR$. This torque will cause an angular acceleration according to $\tau = I\alpha$, where $I = \tfrac{1}{2}MR^2$ for a cylinder. The angular acceleration can be found from the kinematics by $\alpha = \dfrac{\Delta\omega}{\Delta t}$. Equating the two expressions for the torque and substituting enables us to solve for the force.

$$4FR = I\alpha = \tfrac{1}{2}MR^2 \frac{\Delta\omega}{\Delta\tau}$$
$$F = \frac{MR\Delta\omega}{8\Delta t} = \frac{(3600\ \text{kg})(4.0\ \text{m})(32\,\text{rev/min})(2\pi\ \text{rad/rev})(1\ \text{min}/60\ \text{s})}{8(5.0\ \text{min})(60\ \text{s/min})}$$
$$= 20.11\ \text{N} \approx \boxed{2.0\times 10^{1}\,\text{N}}$$

34. (*a*) The moment of inertia of a cylinder is found in Figure 8-21.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(0.580\ \text{kg})(8.50\times 10^{-2}\,\text{m})^2 = 2.0953\times 10^{-3}\,\text{kg}\bullet\text{m}^2 \approx \boxed{2.10\times 10^{-3}\,\text{kg}\bullet\text{m}^2}$$

(*b*) The wheel slows down "on its own" from 1500 rpm to rest in 55.0s. This is used to calculate the frictional torque.

$$\tau_{\text{fr}} = I\alpha_{\text{fr}} = I\frac{\Delta\omega}{\Delta t} = (2.0953\times 10^{-3}\,\text{kg}\bullet\text{m}^2)\frac{(0 - 1500\,\text{rev/min})(2\pi\ \text{rad/rev})(1\ \text{min}/60\ \text{s})}{55.0\ \text{s}}$$
$$= -5.984\times 10^{-3}\,\text{m}\bullet\text{N}$$

The net torque causing the angular acceleration is the applied torque plus the (negative) frictional torque.

$$\sum\tau = \tau_{\text{applied}} + \tau_{\text{fr}} = I\alpha \quad \rightarrow$$

$$\tau_{\text{applied}} = I\alpha - \tau_{\text{fr}} = I\frac{\Delta\omega}{\Delta t} - \tau_{\text{fr}}$$

$$= \left(2.0953\times10^{-3}\,\text{kg}\bullet\text{m}^2\right)\frac{\left(1500\,\text{rev/min}\right)\left(2\pi\ \text{rad/rev}\right)\left(1\ \text{min/60 s}\right)}{5.00\ \text{s}} + \left(5.984\times10^{-3}\,\text{m}\bullet\text{N}\right)$$

$$= \boxed{7.2\times10^{-2}\,\text{m}\bullet\text{N}}$$

35. The torque can be calculated from $\tau = I\alpha$. The rotational inertia of a rod about its end is given by $I = \frac{1}{3}ML^2$.

$$\tau = I\alpha = \tfrac{1}{3}ML^2\frac{\Delta\omega}{\Delta t} = \tfrac{1}{3}\left(2.2\ \text{kg}\right)\left(0.95\ \text{m}\right)^2\frac{\left(3.0\,\text{rev/s}\right)\left(2\pi\ \text{rad/rev}\right)}{0.20\ \text{s}} = \boxed{62\ \text{m}\bullet\text{N}}$$

$\boxed{36.}$ The torque needed is the moment of inertia of the system (merry-go-round and children) times the angular acceleration of the system. Let the subscript "mgr" represent the merry-go-round.

$$\tau = I\alpha = \left(I_{\text{mgr}} + I_{\text{children}}\right)\frac{\Delta\omega}{\Delta t} = \left(\tfrac{1}{2}M_{\text{mgr}}R^2 + 2m_{\text{child}}R^2\right)\frac{\omega-\omega_0}{t}$$

$$= \left[\tfrac{1}{2}\left(760\ \text{kg}\right) + 2\left(25\ \text{kg}\right)\right]\left(2.5\ \text{m}\right)^2\frac{\left(15\,\text{rev/min}\right)\left(2\pi\ \text{rad/rev}\right)\left(1\ \text{min/60 s}\right)}{10.0\ \text{s}}$$

$$= 422.15\,\text{m}\bullet\text{N} \approx \boxed{4.2\times10^2\,\text{m}\bullet\text{N}}$$

The force needed is calculated from the torque and the radius. Assume that the force is all directed perpendicularly to the radius.

$$\tau = F_\perp R\sin\theta \quad \rightarrow \quad F_\perp = \tau/R = 4.2215\times10^2\,\text{m}\bullet\text{N}/2.5\ \text{m} = \boxed{1.7\times10^2\,\text{N}}$$

37. The torque on the rotor will cause an angular acceleration given by $\alpha = \tau/I$. The torque and angular acceleration will have the opposite sign of the initial angular velocity because the rotor is being brought to rest. The rotational inertia is that of a solid cylinder. Substitute the expressions for angular acceleration and rotational inertia into the equation $\omega^2 = \omega_o^2 + 2\alpha\theta$, and solve for the angular displacement.

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$\theta = \frac{\omega^2-\omega_o^2}{2\alpha} = \frac{0-\omega_o^2}{2\left(\tau/I\right)} = \frac{-\omega_o^2}{2\left(\tau/\tfrac{1}{2}MR^2\right)} = \frac{-MR^2\omega_o^2}{4\tau}$$

$$= \frac{-\left(4.80\ \text{kg}\right)\left(0.0710\ \text{m}\right)^2\left[\left(10,300\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\right]^2}{4\left(-1.20\ \text{N}\bullet\text{m}\right)} = 5865\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)$$

$$= \boxed{993\ \text{rev}}$$

The time can be found from $\theta = \frac{1}{2}\left(\omega_o + \omega\right)t$.

$$t = \frac{2\theta}{\omega_o + \omega} = \frac{2(993\text{ rev})}{10{,}300\text{ rev/min}}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{10.9\text{ s}}$$

38. (*a*) The torque gives angular acceleration to the ball only, since the arm is considered massless. The angular acceleration of the ball is found from the given tangential acceleration.

$$\tau = I\alpha = MR^2\alpha = MR^2\frac{a_{\text{tan}}}{R} = MRa_{\text{tan}} = (3.6\text{ kg})(0.31\text{ m})\left(7.0\,\text{m/s}^2\right)$$
$$= 7.812\,\text{m}\bullet\text{N} \approx \boxed{7.8\,\text{m}\bullet\text{N}}$$

(*b*) The triceps muscle must produce the torque required, but with a lever arm of only 2.5 cm, perpendicular to the triceps muscle force.

$$\tau = Fr_\perp \;\rightarrow\; F = \tau/r_\perp = 7.812\,\text{m}\bullet\text{N}\big/\left(2.5\times10^{-2}\text{ m}\right) = \boxed{3.1\times10^{2}\,\text{N}}$$

39. (*a*) The angular acceleration can be found from

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega}{t} = \frac{v/r}{t} = \frac{(10.0\,\text{m/s})/(0.31\,\text{m})}{0.350\text{ s}} = 92.17\,\text{rad/s}^2 \approx \boxed{92\,\text{rad/s}^2}$$

(*b*) The force required can be found from the torque, since $\tau = Fr\sin\theta$. In this situation the force is perpendicular to the lever arm, and so $\theta = 90^\circ$. The torque is also given by $\tau = I\alpha$, where I is the moment of inertia of the arm-ball combination. Equate the two expressions for the torque, and solve for the force.

$$Fr\sin\theta = I\alpha$$

$$F = \frac{I\alpha}{r\sin\theta} = \frac{m_{\text{ball}}d_{\text{ball}}^2 + \frac{1}{3}m_{\text{arm}}L_{\text{arm}}^2}{r\sin 90^\circ}\alpha$$
$$= \frac{(1.00\text{ kg})(0.31\text{ m})^2 + \frac{1}{3}(3.70\text{ kg})(0.31\text{ m})^2}{(0.025\text{ m})}\left(92.17\,\text{rad/s}^2\right) = \boxed{7.9\times10^{2}\,\text{N}}$$

40. (*a*) The moment of inertia of a thin rod, rotating about its end, is given in Figure 8-21(g). There are three blades to add.

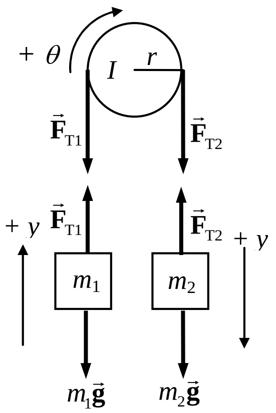
$$I_{\text{total}} = 3\left(\tfrac{1}{3}ML^2\right) = ML^2 = (160\text{ kg})(3.75\text{ m})^2 = 2250\,\text{kg}\bullet\text{m}^2 \approx \boxed{2.3\times10^{2}\,\text{kg}\bullet\text{m}^2}$$

(*b*) The torque required is the rotational inertia times the angular acceleration, assumed constant.

$$\tau = I_{\text{total}}\alpha = I_{\text{total}}\frac{\omega - \omega_0}{t} = \left(2250\,\text{kg}\bullet\text{m}^2\right)\frac{(5.0\,\text{rev/sec})(2\pi\,\text{rad/rev})}{8.0\text{ s}} = \boxed{8.8\times10^{3}\,\text{m}\bullet\text{N}}$$

41. We assume that $m_2 > m_1$, and so m_2 will accelerate down, m_1 will accelerate up, and the pulley will accelerate clockwise. Call the direction of acceleration the positive direction for each object. The masses will have the same acceleration since they are connected by a cord. The rim of the pulley will have that same acceleration since the cord is making it rotate, and so $\alpha_{\text{pulley}} = a/r$. From the free-body diagrams for each object, we have the following.

$$\sum F_{y\,1} = F_{\text{T1}} - m_1g = m_1a \;\rightarrow\; F_{\text{T1}} = m_1g + m_1a$$
$$\sum F_{y\,2} = m_2g - F_{\text{T2}} = m_2a \;\rightarrow\; F_{\text{T2}} = m_2g - m_2a$$


$+\ \theta$
I
r
$\vec{\mathbf{F}}_{\text{T1}}$
$\vec{\mathbf{F}}_{\text{T2}}$
$+\ y$
$\vec{\mathbf{F}}_{\text{T1}}$
$\vec{\mathbf{F}}_{\text{T2}}$
$+\ y$
m_1
m_2
$m_1\vec{\mathbf{g}}$
$m_2\vec{\mathbf{g}}$

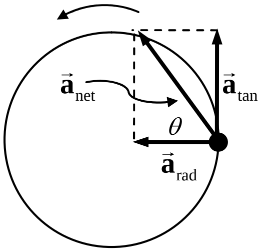
$$\sum \tau = F_{T2}r - F_{T1}r = I\alpha = I\frac{a}{r}$$

Substitute the expressions for the tensions into the torque equation, and solve for the acceleration.

$$F_{T2}r - F_{T1}r = I\frac{a}{r} \quad \rightarrow \quad (m_2g - m_2a)r - (m_1g + m_1a)r = I\frac{a}{r} \quad \rightarrow \quad \boxed{a = \frac{(m_2 - m_1)}{(m_1 + m_2 + I/r^2)}g}$$

If the moment of inertia is ignored, then from the torque equation we see that $F_{T2} = F_{T1}$, and the acceleration will be $\boxed{a_{I=0} = \frac{(m_2 - m_1)}{(m_1 + m_2)}g}$. We see that the acceleration with the moment of inertia included will be smaller than if the moment of inertia is ignored.

42. A top view diagram of the hammer is shown, just at the instant of release, along with the acceleration vectors.



(*a*) The angular acceleration is found from Eq. 8-9c.

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta \quad \rightarrow$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta} = \frac{(v/r)^2 - 0}{2\Delta\theta} = \frac{[(28.0\,\text{m/s})/(1.20\text{ m})]^2}{2(8\pi\text{ rad})} = \boxed{10.8\,\text{rad/s}^2}$$

(*b*) The tangential acceleration is found from the angular acceleration and the radius.

$$a_{tan} = \alpha r = (10.8\,\text{rad/s}^2)(1.20\text{ m}) = \boxed{13.0\,\text{m/s}^2}$$

(*c*) The centripetal acceleration is found from the speed and the radius.

$$a_{rad} = v^2/r = (28.0\,\text{m/s})^2/(1.20\text{ m}) = \boxed{653\,\text{m/s}^2}$$

(*d*) The net force is the mass times the net acceleration. It is in the same direction as the net acceleration, also.

$$F_{net} = ma_{net} = m\sqrt{a_{tan}^2 + a_{rad}^2} = (7.30\text{ kg})\sqrt{(13.0\,\text{m/s}^2)^2 + (653\,\text{m/s}^2)^2} = \boxed{4.77\times10^3\,\text{N}}$$

(*e*) Find the angle from the two acceleration vectors.

$$\theta = \tan^{-1}\frac{a_{tan}}{a_{rad}} = \tan^{-1}\frac{13.0\,\text{m/s}^2}{653\,\text{m/s}^2} = \boxed{1.14^\circ}$$

43. The energy required to bring the rotor up to speed from rest is equal to the final rotational KE of the rotor.

$$KE_{rot} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(3.75\times10^{-2}\,\text{kg}\bullet\text{m}^2)\left[8250\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 = \boxed{1.40\times10^4\,\text{J}}$$

44. Work can be expressed in rotational quantities as $W = \tau\,\Delta\theta$, and so power can be expressed in rotational quantities as $P = \frac{W}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t} = \tau\omega$.

$$P = \tau\omega = (280\,\text{m}\bullet\text{N})\left(3800\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 1.114\times10^5\,\text{W} \approx \boxed{1.1\times10^5\,\text{W}}$$

$$1.114\times10^5\,\text{W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{1.5\times10^2\,\text{hp}}$$

45. The total kinetic energy is the sum of the translational and rotational kinetic energies. Since the ball is rolling without slipping, the angular velocity is given by $\omega = v/R$. The rotational inertia of a sphere about an axis through its center is $I = \frac{2}{5}mR^2$.

$$KE_{\text{total}} = KE_{\text{trans}} + KE_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\tfrac{2}{5}mR^2\frac{v^2}{R^2} = \tfrac{7}{10}mv^2$$

$$= 0.7\left(7.3\text{ kg}\right)\left(3.3\,\text{m/s}\right)^2 = \boxed{56\text{ J}}$$

46. (*a*) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$KE_{\text{daily}} = \tfrac{1}{2}I\omega_{\text{daily}}^2 = \tfrac{1}{2}\left(\tfrac{2}{5}MR_{\text{Earth}}^2\right)\omega_{\text{daily}}^2$$

$$= \tfrac{1}{5}\left(6.0\times10^{24}\,\text{kg}\right)\left(6.4\times10^{6}\,\text{m}\right)^2\left[\left(\frac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right]^2 = \boxed{2.6\times10^{29}\,\text{J}}$$

(*b*) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$KE_{\text{yearly}} = \tfrac{1}{2}I\omega_{\text{yearly}}^2 = \tfrac{1}{2}\left(MR_{\substack{\text{Sun-}\\\text{Earth}}}^2\right)\omega_{\text{yearly}}^2$$

$$= \tfrac{1}{2}\left(6.0\times10^{24}\,\text{kg}\right)\left(1.5\times10^{11}\,\text{m}\right)^2\left[\left(\frac{2\pi\text{ rad}}{365\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right]^2 = \boxed{2.7\times10^{33}\,\text{J}}$$

Thus the total KE is $KE_{\text{daily}} + KE_{\text{yearly}} = 2.6\times10^{29}\,\text{J} + 2.6\times10^{33}\,\text{J} = \boxed{2.6\times10^{33}\,\text{J}}$. The KE due to the daily motion is about 10,000 smaller than that due to the yearly motion.

47. The work required is the change in rotational kinetic energy. The initial angular velocity is 0.

$$W = \Delta KE_{\text{rot}} = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = \tfrac{1}{2}\tfrac{1}{2}MR^2\omega_f^2 = \tfrac{1}{4}\left(1640\text{ kg}\right)\left(7.50\text{ m}\right)^2\left(\frac{2\pi\text{ rad}}{8.00\text{ s}}\right)^2 = \boxed{1.42\times10^{4}\,\text{J}}$$

48. Apply conservation of energy to the sphere, as done in Example 8-13.

(*a*) The work of Example 8-13 is exactly applicable here. The symbol *d* is to represent the distance the sphere rolls along the plane. The sphere is rolling without slipping, so $v_{\text{CM}} = \omega R$.

$$v_{\text{CM}} = \sqrt{\tfrac{10}{7}gh} = \sqrt{\tfrac{10}{7}gd\sin\theta} = \sqrt{\tfrac{10}{7}\left(9.80\,\text{m/s}^2\right)\left(10.0\text{ m}\right)\left(\sin 30.0^\circ\right)} = 8.367$$

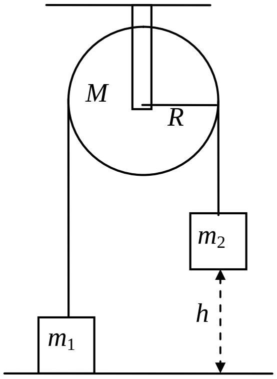
$$= \boxed{8.37\,\text{m/s}}$$

$$\omega = v_{\text{CM}}/R = 8.367\,\text{m/s}/\left(2.00\times10^{-1}\,\text{m}\right) = \boxed{41.8\,\text{rad/s}}$$

(*b*) $$\frac{KE_{\text{trans}}}{KE_{\text{rot}}} = \frac{\tfrac{1}{2}Mv_{\text{CM}}^2}{\tfrac{1}{2}I_{\text{CM}}\omega^2} = \frac{\tfrac{1}{2}Mv_{\text{CM}}^2}{\tfrac{1}{2}\left(\tfrac{2}{5}MR^2\right)\dfrac{v_{\text{CM}}^2}{R^2}} = \boxed{2.5}$$

(*c*) Only the angular speed depends on the radius. $\boxed{\text{None of the results depend on the mass.}}$

49. The only force doing work in this system is gravity, so mechanical energy will be conserved. The initial state of the system is the configuration with m_1 on the ground and all objects at rest. The final state of the system has m_2 just reaching the ground, and all objects in motion. Call the zero level of gravitational potential energy to be the ground level. Both masses will have the same speed since they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega = v/R$. All objects have an initial speed of 0.



$$E_i = E_f$$

$$\tfrac{1}{2}m_1v_i^2 + \tfrac{1}{2}m_2v_i^2 + \tfrac{1}{2}I\omega_i^2 + m_1gy_{1i} + m_2gy_{2i} = \tfrac{1}{2}m_1v_f^2 + \tfrac{1}{2}m_2v_f^2 + \tfrac{1}{2}I\omega_f^2 + m_1gy_{1f} + m_2gy_{2f}$$

$$m_2gh = \tfrac{1}{2}m_1v_f^2 + \tfrac{1}{2}m_2v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_f^2}{R^2}\right) + m_1gh$$

$$v_f = \sqrt{\frac{2(m_2 - m_1)gh}{\left(m_1 + m_2 + \tfrac{1}{2}M\right)}} = \sqrt{\frac{2(26.5\text{ kg} - 18.0\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m})}{\left(26.5\text{ kg} + 18.0\text{ kg} + \left(\tfrac{1}{2}\right)7.50\text{ kg}\right)}} = \boxed{3.22\text{ m/s}}$$

50. Since the lower end of the pole does not slip on the ground, the friction does no work, and so mechanical energy is conserved. The initial energy is the potential energy, treating all the mass as if it were at the CM. The final energy is rotational KE, for rotation about the point of contact with the ground. The linear velocity of the falling tip of the rod is its angular velocity divided by the length.

$$PE = KE \quad\rightarrow\quad mgh = \tfrac{1}{2}I\omega^2 \quad\rightarrow\quad mg\,L/2 = \tfrac{1}{2}\left(\tfrac{1}{3}mL^2\right)\left(v_{\text{end}}/L\right)^2 \quad\rightarrow$$

$$v_{\text{end}} = \sqrt{3gL} = \sqrt{3(9.80\text{ m/s}^2)(2.30\text{ m})} = \boxed{8.22\text{ m/s}}$$

51. The angular momentum is given by Eq. 8-18.

$$L = I\omega = MR^2\omega = (0.210\text{ kg})(1.10\text{ m})^2(10.4\text{ rad/s}) = \boxed{2.64\text{ kg}\bullet\text{m}^2/\text{s}}$$

52. (*a*) The angular momentum is given by Eq. 8-18.

$$L = I\omega = \tfrac{1}{2}MR^2\omega = \tfrac{1}{2}(2.8\text{ kg})(0.18\text{ m})^2\left[\left(\frac{1500\text{ rev}}{1\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]$$

$$= \boxed{7.1\text{ kg}\bullet\text{m}^2/\text{s}}$$

(*b*) The torque required is the change in angular momentum per unit time. The final angular momentum is zero.

$$\tau = \frac{L - L_0}{\Delta t} = \frac{0 - 7.1\text{ kg}\bullet\text{m}^2/\text{s}}{6.0\text{ s}} = \boxed{-1.2\text{ m}\bullet\text{N}}$$

The negative sign indicates that the torque is used to oppose the initial angular momentum.

53. (*a*) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms is internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.

(*b*) $L_i = L_f \rightarrow I_i\omega_i = I_f\omega_f \rightarrow I_f = I_i \dfrac{\omega_i}{\omega_f} = I_i \dfrac{1.30\,\text{rev/s}}{0.80\,\text{rev/s}} = 1.625\,I_i \approx 1.6\,I_i$

The rotational inertia has increased by a factor of $\boxed{1.6}$.

54. There is no net torque on the diver because the only external force (gravity) passes through the center of mass of the diver. Thus the angular momentum of the diver is conserved. Subscript 1 refers to the tuck position, and subscript 2 refers to the straight position.

$$L_1 = L_2 \rightarrow I_1\omega_1 = I_2\omega_2 \rightarrow \omega_2 = \omega_1 \frac{I_1}{I_2} = \left(\frac{2\text{ rev}}{1.5\text{ sec}}\right)\left(\frac{1}{3.5}\right) = \boxed{0.38\text{ rev/s}}$$

55. The skater's angular momentum is constant, since no external torques are applied to her.

$$L_i = L_f \rightarrow I_i\omega_i = I_f\omega_f \rightarrow I_f = I_i \frac{\omega_i}{\omega_f} = \left(4.6\text{ kg}\bullet\text{m}^2\right)\frac{0.50\,\text{rev/s}}{3.0\,\text{rev/s}} = \boxed{0.77\text{ kg}\bullet\text{m}^2}$$

She accomplishes this by starting with her arms extended (initial angular velocity) and then $\boxed{\text{pulling her arms in to the center of her body}}$ (final angular velocity).

56. Because there is no external torque applied to the wheel-clay system, the angular momentum will be conserved. We assume that the clay is thrown with no angular momentum so that its initial angular momentum is 0. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the clay and the wheel. Subscript 1 represents before the clay is thrown, and subscript 2 represents after the clay is thrown.

$$L_1 = L_2 \rightarrow I_1\omega_1 = I_2\omega_2 \rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \frac{I_\text{wheel}}{I_\text{wheel} + I_\text{clay}} = \omega_1\left(\frac{\frac{1}{2}M_\text{wheel}R^2_\text{wheel}}{\frac{1}{2}M_\text{wheel}R^2_\text{wheel} + \frac{1}{2}M_\text{clay}R^2_\text{clay}}\right)$$

$$= \left(1.5\,\text{rev/s}\right)\left[\frac{(5.0\text{ kg})(0.20\text{ m})^2}{(5.0\text{ kg})(0.20\text{ m})^2 + (3.1\text{ kg})\left(8.0\times10^{-2}\,\text{m}\right)^2}\right] = 1.36\,\text{rev/s} \approx \boxed{1.4\,\text{rev/s}}$$

$\boxed{57.}$ (*a*) $L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}(55\text{ kg})(0.15\text{ m})^2\left(3.5\dfrac{\text{rev}}{\text{s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{14\,\text{kg}\bullet\text{m}^2/\text{s}}$

(*b*) If the rotational inertia does not change, then the change in angular momentum is strictly due to a change in angular velocity.

$$\tau = \frac{\Delta L}{\Delta t} = \frac{0 - 14\,\text{kg}\bullet\text{m}^2/\text{s}}{5.0\text{ s}} = \boxed{-2.7\text{m}\bullet\text{N}}$$

The negative sign indicates that the torque is in the opposite direction as the initial angular momentum.

58. (*a*) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$L_\text{daily} = I\omega_\text{daily} = \left(\tfrac{2}{5}MR^2_\text{Earth}\right)\omega_\text{daily}$$

$$= \tfrac{2}{5}\left(6.0\times10^{24}\,\text{kg}\right)\left(6.4\times10^6\,\text{m}\right)^2\left[\left(\frac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right] = \boxed{7.1\times10^{33}\text{ kg}\bullet\text{m}^2/\text{s}}$$

(*b*) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(MR^2_{\substack{\text{Sun-}\\\text{Earth}}}\right)\omega_{\text{daily}}$$

$$= \left(6.0\times10^{24}\,\text{kg}\right)\left(1.5\times10^{11}\,\text{m}\right)^2\left[\left(\frac{2\pi\ \text{rad}}{365\ \text{day}}\right)\left(\frac{1\ \text{day}}{86{,}400\ \text{s}}\right)\right] = \boxed{2.7\times10^{40}\ \text{kg}\bullet\text{m}^2/\text{s}}$$

59. Since there are no external torques on the system, the angular momentum of the 2-disk system is conserved. The two disks have the same final angular velocity.

$$L_i = L_f \quad\to\quad I\omega + I(0) = 2I\omega_f \quad\to\quad \boxed{\omega_f = \tfrac{1}{2}\omega}$$

60. The angular momentum of the disk – rod combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the disk and the rod. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The rod has no initial angular momentum.

$$L_1 = L_2 \quad\to\quad I_1\omega_1 = I_2\omega_2 \quad\to$$

$$\omega_2 = \omega_1\frac{I_1}{I_2} = \omega_1\frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{rod}}} = \omega_1\left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + \frac{1}{12}M\left(2R\right)^2}\right] = \left(2.4\,\text{rev}/\text{s}\right)\left(\frac{3}{5}\right) = \boxed{1.4\,\text{rev}/\text{s}}$$

61. Since the person is walking radially, no torques will be exerted on the person-platform system, and so angular momentum will be conserved. The person will be treated as a point mass. Since the person is initially at the center, they have no initial rotational inertia.

(*a*) $L_i = L_f \quad\to\quad I_{\text{platform}}\omega_i = \left(I_{\text{platform}} + I_{\text{person}}\right)\omega_f$

$$\omega_f = \frac{I_{\text{platform}}}{I_{\text{platform}} + mR^2}\omega_i = \frac{920\ \text{kg}\bullet\text{m}^2}{920\ \text{kg}\bullet\text{m}^2 + \left(75\ \text{kg}\right)\left(3.0\ \text{m}\right)^2}\left(2.0\,\text{rad}/\text{s}\right) = 1.154\,\text{rad}/\text{s} \approx \boxed{1.2\,\text{rad}/\text{s}}$$

(*b*) $KE_i = \frac{1}{2}I_{\text{platform}}\omega_i^2 = \frac{1}{2}\left(920\,\text{kg}\bullet\text{m}^2\right)\left(2.0\,\text{rad}/\text{s}\right)^2 = \boxed{1.8\times10^3\,\text{J}}$

$$KE_f = \tfrac{1}{2}\left(I_{\text{platform}} + I_{\text{person}}\right)\omega_f^2 = \tfrac{1}{2}\left(I_{\text{platform}} + m_{\text{person}}r^2_{\text{person}}\right)\omega_f^2$$

$$= \tfrac{1}{2}\left[920\,\text{kg}\bullet\text{m}^2 + \left(75\ \text{kg}\right)\left(3.0\ \text{m}\right)^2\right]\left(1.154\,\text{rad}/\text{s}\right)^2 = 1062\ \text{J} \approx \boxed{1.1\times10^3\,\text{J}}$$

62. The angular momentum of the merry-go-round and people combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the merry-go-round and the people. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The people have no initial angular momentum.

$$L_1 = L_2 \quad\to\quad I_1\omega_1 = I_2\omega_2 \quad\to$$

$$\omega_2 = \omega_1\frac{I_1}{I_2} = \omega_1\frac{I_{\text{m-g-r}}}{I_{\text{m-g-r}} + I_{\text{people}}} = \omega_1\left[\frac{I_{\text{m-g-r}}}{I_{\text{m-g-r}} + 4M_{\text{person}}R^2}\right]$$

$$= \left(0.80\,\text{rad}/\text{s}\right)\left[\frac{1760\,\text{kg}\bullet\text{m}^2}{1760\,\text{kg}\bullet\text{m}^2 + 4\left(65\ \text{kg}\right)\left(2.1\ \text{m}\right)^2}\right] = \boxed{0.48\,\text{rad}/\text{s}}$$

If the people jump off the merry-go-round radially, then they exert no torque on the merry-go-round, and thus cannot change the angular momentum of the merry-go-round. The merry-go-round would continue to rotate at $\boxed{0.80\,\text{rad/s}}$.

63. Since the lost mass carries away no angular momentum, the angular momentum of the remaining mass will be the same as the initial angular momentum.

$$L_i = L_f \quad \to \quad I_i\omega_i = I_f\omega_f \quad \to \quad \frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{\frac{2}{5}M_iR_i^2}{\frac{2}{5}M_fR_f^2} = \frac{M_iR_i^2}{(0.5M_i)(0.01R_f)^2} = 2.0\times10^4$$

$$\omega_f = 2.0\times10^4\omega_i = 2.0\times10^4\left(\frac{2\pi\text{ rad}}{30\text{ day}}\right)\left(\frac{1\text{ d}}{86400\text{ s}}\right) = 4.848\times10^{-2}\text{ rad/s} \approx \boxed{5\times10^{-2}\text{ rad/s}}$$

The period would be a factor of 20,000 smaller, which would make it about 130 seconds.
The ratio of angular kinetic energies of the spinning mass would be

$$\frac{KE_f}{KE_i} = \frac{\frac{1}{2}I_f\omega_f^2}{\frac{1}{2}I_i\omega_i^2} = \frac{\frac{1}{2}\left[\frac{2}{5}(0.5M_i)(0.01R_i)^2\right](2.0\times10^4\omega_i)^2}{\frac{1}{2}\left(\frac{2}{5}M_iR_i^2\right)\omega_i^2} = 2.0\times10^4 \quad \to \quad \boxed{KE_f = 2\times10^4 KE_i}$$

64. For our crude estimate, we model the hurricane as a rigid cylinder of air. Since the "cylinder" is rigid, each part of it has the same angular velocity. The mass of the air is the product of the density of air times the volume of the air cylinder.

$$M = \rho V = \rho\pi R^2 h = (1.3\,\text{kg/m}^3)\pi(1.00\times10^5\,\text{m})^2(4.0\times10^3\,\text{m}) = 1.634\times10^{14}\,\text{kg}$$

(*a*) $KE = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(v_{\text{edge}}/R\right)^2 = \frac{1}{4}Mv_{\text{edge}}^2$

$$= \tfrac{1}{4}(1.634\times10^{14}\,\text{kg})\left[(120\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = 4.539\times10^{16}\,\text{J} \approx \boxed{5\times10^{16}\,\text{J}}$$

(*b*) $L = I\omega = \left(\frac{1}{2}MR^2\right)\left(v_{\text{edge}}/R\right) = \frac{1}{2}MRv_{\text{edge}}$

$$= \tfrac{1}{2}(1.634\times10^{14}\,\text{kg})(1.00\times10^5\,\text{m})\left[(120\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right] = 2.723\times10^{20}\,\text{kg}\bullet\text{m}^2/\text{s}$$

$$\approx \boxed{3\times10^{20}\,\text{kg}\bullet\text{m}^2/\text{s}}$$

65. Angular momentum will be conserved in the Earth – asteroid system, since all forces and torques are internal to the system. The initial angular velocity of the satellite, just before collision, can be found from $\omega_{\text{asteroid}} = v_{\text{asteroid}}/R_{\text{Earth}}$. Assuming the asteroid becomes imbedded in the Earth at the surface, the Earth and the asteroid will have the same angular velocity after the collision. We model the Earth as a uniform sphere, and the asteroid as a point mass.

$$L_i = L_f \quad \to \quad I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = (I_{\text{Earth}} + I_{\text{asteroid}})\omega_f$$

The moment of inertia of the satellite can be ignored relative to that of the Earth on the right side of the above equation, and so the percent change in Earth's angular velocity is found as follows.

$$I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = I_{\text{Earth}}\omega_f \quad \to \quad \frac{(\omega_f - \omega_{\text{Earth}})}{\omega_{\text{Earth}}} = \frac{I_{\text{asteroid}}}{I_{\text{Earth}}}\frac{\omega_{\text{asteroid}}}{\omega_{\text{Earth}}}$$

$$\%\text{ change} = \frac{\left(\omega_f - \omega_{\text{Earth}}\right)}{\omega_{\text{Earth}}}\left(100\right) = \frac{m_{\text{asteroid}}R^2_{\text{Earth}}}{\frac{2}{5}M_{\text{Earth}}R^2_{\text{Earth}}}\,\frac{\dfrac{v_{\text{asteroid}}}{R_{\text{Earth}}}}{\omega_{\text{Earth}}} = \frac{m_{\text{asteroid}}}{\frac{2}{5}M_{\text{Earth}}}\,\frac{v_{\text{asteroid}}}{\omega_{\text{Earth}}R_{\text{Earth}}}\left(100\right)$$

$$= \frac{\left(1.0\times10^{5}\,\text{kg}\right)\left(3.0\times10^{4}\text{ m/s}\right)}{\left(0.4\right)\left(5.97\times10^{24}\,\text{kg}\right)\left(\dfrac{2\pi\text{ rad}}{86400\,\text{s}}\right)\left(6.38\times10^{6}\,\text{m}\right)}\left(100\right) = \boxed{2.7\times10^{-16}\,\%}$$

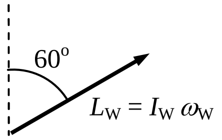
66. When the person and the platform rotate, they do so about the vertical axis. Initially there is no angular momentum pointing along the vertical axis, and so any change that the person – wheel – platform undergoes must result in no net angular momentum along the vertical axis.

(*a*) If the wheel is moved so that its angular momentum points upwards, then the person and platform must get an equal but opposite angular momentum, which will point downwards. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$L_i = L_f \quad\rightarrow\quad 0 = I_{\text{W}}\omega_{\text{W}} + I_{\text{P}}\omega_{\text{P}} \quad\rightarrow\quad \boxed{\omega_{\text{P}} = -\frac{I_{\text{W}}}{I_{\text{P}}}\omega_{\text{W}}}$$

The negative sign means that the platform is rotating in the opposite direction of the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is spinning clockwise.

(*b*) If the wheel is pointing at a 60^{o} angle to the vertical, then the component of its angular momentum that is along the vertical direction is $I_{\text{W}}\omega_{\text{W}}\cos 60^{\text{o}}$. See the diagram. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.



$$L_i = L_f \quad\rightarrow\quad 0 = I_{\text{W}}\omega_{\text{W}}\cos 60^{\text{o}} + I_{\text{P}}\omega_{\text{P}} \quad\rightarrow\quad \boxed{\omega_{\text{P}} = -\frac{I_{\text{W}}}{2I_{\text{P}}}\omega_{\text{W}}}$$

Again, the negative sign means that the platform is rotating in the opposite direction of the wheel.

(*c*) If the wheel is moved so that its angular momentum points downwards, then the person and platform must get an equal but opposite angular momentum, which will point upwards. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$L_i = L_f \quad\rightarrow\quad 0 = -I_{\text{W}}\omega_{\text{W}} + I_{\text{P}}\omega_{\text{P}} \quad\rightarrow\quad \boxed{\omega_{\text{P}} = \omega_{\text{W}}\, I_{\text{W}}/I_{\text{P}}}$$

The platform is rotating in the same direction as the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is also spinning counterclockwise.

(*d*) Since the total angular momentum is 0, if the wheel is stopped from rotating, the platform will also stop. Thus $\boxed{\omega_{\text{P}} = 0}$.

67. The angular momentum of the person – turntable system will be conserved. Call the direction of the person's motion the positive rotation direction. Relative to the ground, the person's speed will be $v + v_{\text{T}}$, where v is the person's speed relative to the turntable, and v_{T} is the speed of the rim of the turntable with respect to the ground. The turntable's angular speed is $\omega_{\text{T}} = v_{\text{T}}/R$, and the person's

angular speed relative to the ground is $\omega_{\text{P}} = \frac{v + v_{\text{T}}}{R} = \frac{v}{R} + \omega_{\text{T}}$. The person is treated as a point particle for calculation of the moment of inertia.

$$L_i = L_f \quad \rightarrow \quad 0 = I_{\text{T}}\omega_{\text{T}} + I_{\text{P}}\omega_{\text{P}} = I_{\text{T}}\omega_{\text{T}} + mR^2\left(\omega_{\text{T}} + \frac{v}{R}\right) \quad \rightarrow$$

$$\omega_{\text{T}} = -\frac{mRv}{I_{\text{T}} + mR^2} = -\frac{(55\text{ kg})(3.25\text{ m})(3.8\text{ m/s})}{1700\text{ kg}\bullet\text{m}^2 + (55\text{ kg})(3.25\text{ m})^2} = \boxed{-0.30\text{ rad/s}}$$

68. Since the spool rolls without slipping, each point on the edge of the spool moves with a speed of $v = r\omega = v_{\text{CM}}$ relative to the center of the spool, where v_{CM} is the speed of the center of the spool relative to the ground. Since the spool is moving to the right relative to the ground, and the top of the spool is moving to the right relative to the center of the spool, the top of the spool is moving with a speed of $2v_{\text{CM}}$ relative to the ground. This is the speed of the rope, assuming it is unrolling without slipping and is at the outer edge of the spool. The speed of the rope is the same as the speed of the person, since the person is holding the rope. So the person is walking with a speed of twice that of the center of the spool. Thus if the person moves forward a distance L, in the same time the center of the spool, traveling with half the speed, moves forward a distance $\boxed{L/2}$. The rope, to stay connected both to the person and to the spool, must therefore unwind by an amount $\boxed{L/2}$ also.

69. The spin angular momentum of the Moon can be calculated by $L_{\text{spin}} = I_{\text{spin}}\omega_{\text{spin}} = \frac{2}{5}MR^2_{\text{Moon}}\omega_{\text{spin}}$. The orbital angular momentum can be calculated by $L_{\text{orbit}} = I_{\text{orbit}}\omega_{\text{orbit}} = MR^2_{\text{orbit}}\omega_{\text{orbit}}$. Because the same side of the Moon always faces the Earth, $\omega_{\text{spin}} = \omega_{\text{orbit}}$.

$$\frac{L_{\text{spin}}}{L_{\text{orbit}}} = \frac{\frac{2}{5}MR^2_{\text{Moon}}\omega_{\text{spin}}}{MR^2_{\text{orbit}}\omega_{\text{orbit}}} = \frac{2}{5}\left(\frac{R_{\text{Moon}}}{R_{\text{orbit}}}\right)^2 = 0.4\left(\frac{1.74\times10^6\text{ m}}{3.84\times10^8\text{ m}}\right)^2 = \boxed{8.21\times10^{-6}}$$

70. As discussed in section 8-3, from the reference frame of the axle of the wheel, the points on the wheel are all moving with the same speed of $v = r\omega$, where v is the speed of the axle of the wheel relative to the ground. The top of the tire has a velocity of v to the right relative to the axle, so it has a velocity of $2v$ to the right relative to the ground.

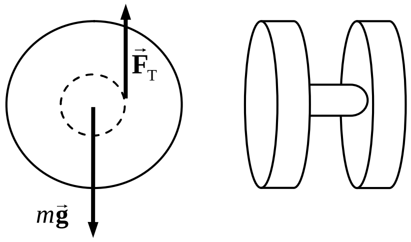
$$\vec{\mathbf{v}}_{\substack{\text{top rel}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{top rel}\\\text{center}}} + \vec{\mathbf{v}}_{\substack{\text{center rel}\\\text{ground}}} = (v\text{ to the right}) + (v\text{ to the right}) = 2v\text{ to the right}$$

$$v_{\substack{\text{top rel}\\\text{ground}}} = 2v = 2(v_0 + at) = 2at = 2\left(1.00\text{ m/s}^2\right)(3.0\text{ s}) = \boxed{6.0\text{ m/s}}$$

71. The torque is found from $\tau = I\alpha$. The angular acceleration can be found from $\omega = \omega_o + \alpha t$, with an initial angular velocity of 0. The rotational inertia is that of a cylinder.

$$\tau = I\alpha = \tfrac{1}{2}MR^2\left(\frac{\omega - \omega_o}{t}\right) = 0.5(1.4\text{ kg})(0.20\text{ m})^2\frac{(1800\text{ rev/s})(2\pi\text{ rad/rev})}{6.0\text{ s}} = \boxed{53\text{ m}\bullet\text{N}}$$

72. (*a*) There are two forces on the yo-yo: gravity and the string tension. If we assume that the top of the string is held in a fixed position, then the tension does no work, and so mechanical energy is conserved. The initial gravitational PE is converted into rotational and translational KE. Since the yo-yo rolls without slipping at the point of contact of the string, the velocity of the CM is simply related to the angular velocity of the yo-yo: $v_{\text{CM}} = r\omega$, where *r* is the radius of the inner hub. Let *m* be the mass of the inner hub, and *M* and *R* be the mass and radius of each outer disk. Calculate the rotational inertia of the yo-yo about its CM, and then use conservation of energy to find the linear speed of the CM. We take the 0 of gravitational PE to be at the bottom of its fall.



$$I_{\text{CM}} = \tfrac{1}{2}mr^2 + 2\left(\tfrac{1}{2}MR^2\right) = \tfrac{1}{2}mr^2 + MR^2$$

$$= \tfrac{1}{2}\left(5.0\times10^{-3}\,\text{kg}\right)\left(5.0\times10^{-3}\,\text{m}\right)^2 + \left(5.0\times10^{-2}\,\text{kg}\right)\left(3.75\times10^{-2}\,\text{m}\right)^2 = 7.038\times10^{-5}\,\text{kg}\bullet\text{m}^2$$

$$m_{\text{total}} = m + 2M = 5.0\times10^{-3}\,\text{kg} + 2\left(5.0\times10^{-2}\,\text{kg}\right) = 0.105\text{ kg}$$

$$PE_i = KE_f \quad \rightarrow$$

$$m_{\text{total}}gh = \tfrac{1}{2}m_{\text{total}}v_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 = \tfrac{1}{2}m_{\text{total}}v_{\text{CM}}^2 + \tfrac{1}{2}\frac{I_{\text{CM}}}{r^2}v_{\text{CM}}^2 = \left(\tfrac{1}{2}m_{\text{total}} + \tfrac{1}{2}\frac{I_{\text{CM}}}{r^2}\right)v_{\text{CM}}^2 \quad \rightarrow$$

$$v_{\text{CM}} = \sqrt{\frac{m_{\text{total}}gh}{\tfrac{1}{2}\left(m_{\text{total}} + \dfrac{I_{\text{CM}}}{r^2}\right)}} = \sqrt{\frac{\left(0.105\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)\left(1.0\text{ m}\right)}{\tfrac{1}{2}\left[\left(0.105\text{ kg}\right) + \dfrac{\left(7.038\times10^{-5}\,\text{kg}\bullet\text{m}^2\right)}{\left(5.0\times10^{-3}\,\text{m}\right)^2}\right]}} = 0.8395 = \boxed{0.84\,\text{m/s}}$$

(*b*) Calculate the ratio $KE_{\text{rot}}/KE_{\text{tot}}$.

$$\frac{KE_{\text{rot}}}{KE_{\text{tot}}} = \frac{KE_{\text{rot}}}{PE_{\text{tot}}} = \frac{\tfrac{1}{2}I_{\text{CM}}\omega^2}{m_{\text{total}}gh} = \frac{\tfrac{1}{2}\dfrac{I_{\text{CM}}}{r^2}v_{\text{CM}}^2}{m_{\text{total}}gh} = \frac{I_{\text{CM}}v_{\text{CM}}^2}{2r^2m_{\text{total}}gh}$$

$$= \frac{\left(7.038\times10^{-5}\,\text{kg}\bullet\text{m}^2\right)\left(0.8395\,\text{m/s}\right)^2}{2\left(5.0\times10^{-3}\,\text{m}\right)^2\left(0.105\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)\left(1.0\text{ m}\right)} = 0.96 = \boxed{96\%}$$

73. (*a*) The linear speed of the chain must be the same as it passes over both sprockets. The linear speed is related to the angular speed by $v = \omega R$, and so

$$\omega_R R_R = \omega_F R_F \text{ .}$$

If the spacing of the teeth on the sprockets is a distance *d*, then the number of teeth on a sprocket times the spacing distance must give the circumference of the sprocket.

$$Nd = 2\pi R \text{ and so } R = \frac{Nd}{2\pi}\text{. Thus } \omega_R\frac{N_R d}{2\pi} = \omega_F\frac{N_F d}{2\pi} \quad \rightarrow \quad \boxed{\frac{\omega_R}{\omega_F} = \frac{N_F}{N_R}}$$

(*b*) $\boxed{\omega_R/\omega_F = 52/13 = 4.0}$

(*c*) $\boxed{\omega_R/\omega_F = 42/28 = 1.5}$

74. Since the lost mass carries away no angular momentum, the angular momentum of the remaining mass will be the same as the initial angular momentum.

$$L_i = L_f \quad \rightarrow \quad I_i\omega_i = I_f\omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{\frac{2}{5}M_iR_i^2}{\frac{2}{5}M_fR_f^2} = \frac{\left(8.0M_{\text{Sun}}\right)\left(6.96\times10^8\,\text{m}\right)^2}{\left(0.25\right)\left(8.0M_{\text{Sun}}\right)\left(1.1\times10^4\,\text{m}\right)^2} = 1.601\times10^{10}$$

$$\omega_f = 1.601\times10^{10}\,\omega_i = 1.601\times10^{10}\left(\frac{1\,\text{rev}}{12\text{ day}}\right) = 1.334\times10^9\text{ rev/day}$$

$$\approx \boxed{1.3\times10^9\text{ rev/day} = 1.5\times10^4\text{ rev/s}}$$

75. (*a*) The initial energy of the flywheel is used for two purposes – to give the car translational kinetic energy 20 times, and to replace the energy lost due to friction, from air resistance and from braking. The statement of the problem leads us to ignore any gravitational potential energy changes.

$$W_{\text{fr}} = KE_{\text{final}} - KE_{\text{initial}} \quad \rightarrow \quad F_{\text{fr}}\Delta x\cos180^\circ = \tfrac{1}{2}M_{\text{car}}v_{\text{car}}^2 - KE_{\text{flywheel}}$$

$$KE_{\text{flywheel}} = F_{\text{fr}}\Delta x + \tfrac{1}{2}M_{\text{car}}v_{\text{car}}^2$$

$$= \left(450\text{ N}\right)\left(3.5\times10^5\,\text{m}\right) + \left(20\right)\tfrac{1}{2}\left(1400\text{ kg}\right)\left[\left(95\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2$$

$$= 1.672\times10^8\,\text{J} \approx \boxed{1.7\times10^8\,\text{J}}$$

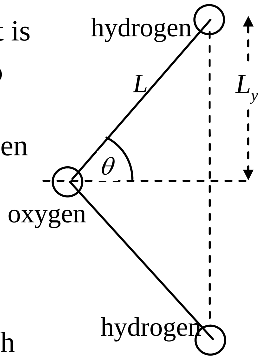
(*b*) $KE_{\text{flywheel}} = \tfrac{1}{2}I\omega^2$

$$\omega = \sqrt{\frac{2\,KE}{I}} = \sqrt{\frac{2\,KE}{\frac{1}{2}M_{\text{flywheel}}R_{\text{flywheel}}^2}} = \sqrt{\frac{2\left(1.672\times10^8\,\text{J}\right)}{\frac{1}{2}\left(240\text{ kg}\right)\left(0.75\text{ m}\right)^2}} = \boxed{2.2\times10^3\text{ rad/s}}$$

(*c*) To find the time, use the relationship that $\text{Power} = \frac{\text{Work}}{t}$, where the work done by the motor will be equal to the kinetic energy of the flywheel.

$$P = \frac{W}{t} \quad \rightarrow \quad t = \frac{W}{P} = \frac{\left(1.672\times10^8\,\text{J}\right)}{\left(150\text{ hp}\right)\left(746\text{ W/hp}\right)} = 1.494\times10^3\,\text{s} \approx \boxed{25\text{ min}}$$

76. The mass of a hydrogen atom is 1.01 atomic mass units. The atomic mass unit is 1.66×10^{-27} kg. Since the axis passes through the oxygen atom, it will have no rotational inertia.



(*a*) If the axis is perpendicular to the plane of the molecule, then each hydrogen atom is a distance *L* from the axis of rotation.

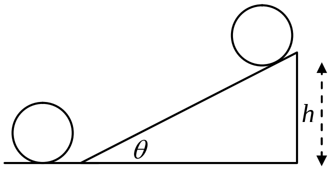
$$I_{\text{perp}} = 2m_HL^2 = 2\left(1.01\right)\left(1.66\times10^{-27}\,\text{kg}\right)\left(0.96\times10^{-9}\,\text{m}\right)^2$$

$$= \boxed{3.1\times10^{-45}\,\text{kg}\bullet\text{m}^2}$$

(*b*) If the axis is in the plane of the molecule, bisecting the H-O-H bonds, each hydrogen atom is a distance of $L_y = L\sin\theta = \left(9.6\times10^{-10}\,\text{m}\right)\sin52^\circ$ $= 7.564\times10^{-10}\,\text{m}$. Thus the moment of inertia is

$$I_{\text{plane}} = 2m_H L_y^2 = 2(1.01)\left(1.66\times10^{-27}\,\text{kg}\right)\left(7.564\times10^{-10}\,\text{m}\right)^2 = \boxed{1.9\times10^{-45}\,\text{kg}\bullet\text{m}^2}$$

77. (*a*) Assuming that there are no dissipative forces doing work, conservation of energy may be used to find the final height *h* of the hoop. Take the bottom of the incline to be the zero level of gravitational potential energy. We assume that the hoop is rolling without sliding, so that $\omega = v/R$. Relate the conditions at the bottom of the incline to the conditions at the top by conservation of energy. The hoop has both translational and rotational kinetic energy at the bottom, and the rotational inertia of the hoop is given by $I = mR^2$.



$$E_{\text{bottom}} = E_{\text{top}} \rightarrow \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgh \rightarrow \tfrac{1}{2}mv^2 + \tfrac{1}{2}mR^2\frac{v^2}{R^2} = mgh \rightarrow$$

$$h = \frac{v^2}{g} = \frac{(3.3\,\text{m/s})^2}{9.8\,\text{m/s}^2} = 1.111\text{ m}$$

The distance along the plane is given by $d = \dfrac{h}{\sin\theta} = \dfrac{1.111\text{ m}}{\sin 15^\circ} = 4.293\text{ m} \approx \boxed{4.3\text{ m}}$

(*b*) The time can be found from the constant accelerated linear motion. Use the relationship

$$\Delta x = \tfrac{1}{2}(v + v_o)t \rightarrow t = \frac{2\Delta x}{v + v_o} = \frac{2(4.293\text{ m})}{0 + 3.3\,\text{m/s}} = 2.602\text{ s}.$$

This is the time to go up the plane. The time to come back down the plane is the same, and so the total time is $\boxed{5.2\text{ s}}$.

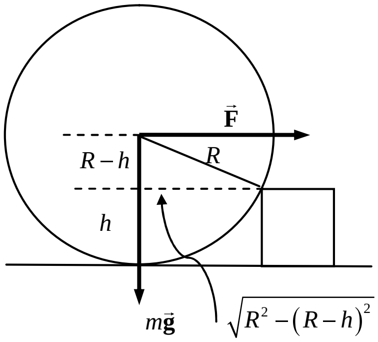
78. (*a*) The force of gravity acting through the CM will cause a clockwise torque which produces an angular acceleration. At the moment of release, the force of gravity is perpendicular to the lever arm from the hinge to the CM.

$$\tau = I\alpha \rightarrow \alpha = \frac{\tau_{\text{gravity}}}{I_{\text{rod about end}}} = \frac{Mg\,L/2}{\tfrac{1}{3}ML^2} = \boxed{\frac{3g}{2L}}$$

(*b*) At the end of the rod, there is a tangential acceleration equal to the angular acceleration times the distance from the hinge. There is no radial acceleration since at the moment of release, the speed of the end of the rod is 0. Thus the tangential acceleration is the entire linear acceleration.

$$a_{\text{linear}} = a_{\tan} = \alpha L = \boxed{\tfrac{3}{2}g}$$

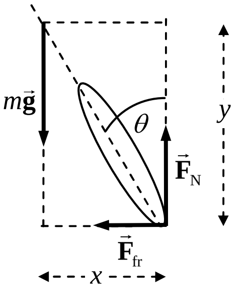
79. The wheel is rolling about the point of contact with the step, and so all torques are to be taken about that point. As soon as the wheel is off the floor, there will be only two forces that can exert torques on the wheel – the pulling force and the force of gravity. There will not be a normal force of contact between the wheel and the floor once the wheel is off the floor, and any force on the wheel from the point of the step cannot exert a torque about that very point. Calculate the net torque on the wheel, with clockwise torques positive. The minimum force occurs when the net torque is 0.



$$\sum\tau = F\left(R-h\right) - mg\sqrt{R^2-\left(R-h\right)^2} = 0$$

$$F = \frac{Mg\sqrt{R^2-\left(R-h\right)^2}}{R-h} = \boxed{\frac{Mg\sqrt{2Rh-h^2}}{R-h}}$$

80. (*a*) In order not to fall over, the net torque on the cyclist about an axis through the CM and parallel to the ground must be zero. Consider the free-body diagram shown. Sum torques about the CM, with counterclockwise as positive, and set the sum equal to zero.



$$\sum\tau = F_{\text{N}}x - F_{\text{fr}}y = 0 \quad\to\quad \frac{F_{\text{fr}}}{F_{\text{N}}} = \frac{x}{y} = \tan\theta$$

(*b*) The cyclist is not accelerating vertically, so $F_{\text{N}} = mg$. The cyclist is accelerating horizontally, because he is traveling in a circle. Thus the frictional force must be supplying the centripetal force, so $F_{\text{fr}} = mv^2/r$.

$$\tan\theta = \frac{F_{\text{fr}}}{F_{\text{N}}} = \frac{mv^2/r}{mg} = \frac{v^2}{rg} \quad\to\quad \theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left(4.2\,\text{m/s}\right)^2}{\left(6.4\text{ m}\right)\left(9.8\,\text{m/s}^2\right)} = 15.71^{\circ} \approx \boxed{16^{\circ}}$$

(*c*) From $F_{\text{fr}} = mv^2/r$, the smallest turning radius results in the maximum force. The maximum static frictional force is $F_{\text{fr}} = \mu F_{\text{N}}$. Use this to calculate the radius.

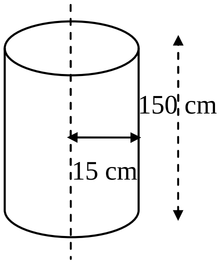
$$mv^2/r_{\min} = \mu F_{\text{N}} = \mu mg \quad\to\quad r_{\min} = \frac{v^2}{\mu g} = \frac{\left(4.2\,\text{m/s}\right)^2}{\left(0.70\right)\left(9.8\,\text{m/s}^2\right)} = \boxed{2.6\text{ m}}$$

81. Assume that the angular acceleration is uniform. Then the torque required to whirl the rock is the moment of inertia of the rock (treated as a particle) times the angular acceleration.

$$\tau = I\alpha = \left(mr^2\right)\left(\frac{\omega-\omega_0}{t}\right) = \frac{\left(0.50\text{ kg}\right)\left(1.5\text{ m}\right)^2}{5.0\text{ s}}\left[\left(120\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right] = \boxed{2.8\,\text{m}\bullet\text{N}}$$

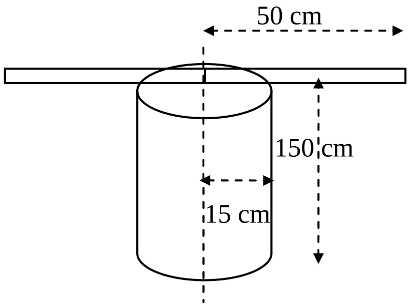
That torque comes from the arm swinging the sling, and so comes from the arm muscles.

82. Assume a mass of 50 kg, corresponding to a weight of about 110 lb. From Table 7-1, we find that the total arm and hand mass is about 12.5% of the total mass, and so the rest of the body is about 87.5% of the total mass. Model the skater as a cylinder of mass 44 kg, and model each arm as a thin rod of mass 3 kg. Estimate the body as 150 cm tall with a radius of 15 cm. Estimate the arm dimension as 50 cm long.



With the arms held tightly, we approximate that the arms are part of the body cylinder. A sketch of the skater in this configuration is then as shown in the first diagram. In this configuration, the rotational inertia is

$$I_{\text{in}} = I_{\text{cylinder}} = \tfrac{1}{2}M_{\substack{\text{total}\\\text{body}}}R_{\text{body}}^2 .$$



With the arms extended, the configuration changes to the second diagram. In this configuration, the rotational inertia is

$$I_{\text{out}} = I_{\text{body}} + I_{\text{arms}} = \tfrac{1}{2}M_{\text{body}}R_{\text{body}}^2 + 2\tfrac{1}{3}M_{\text{arm}}L_{\text{arm}}^2$$

The forces and torques involved in changing the configuration of the skater are internal to the skater, and so the skater's angular momentum is conserved during a configuration change. Thus

$$L_{\text{in}} = L_{\text{out}} \quad \rightarrow \quad I_{\text{in}}\omega_{\text{in}} = I_{\text{out}}\omega_{\text{out}} \quad \rightarrow$$

$$\frac{\omega_{\text{out}}}{\omega_{\text{in}}} = \frac{I_{\text{in}}}{I_{\text{out}}} = \frac{\tfrac{1}{2}M_{\substack{\text{total}\\\text{body}}}R_{\text{body}}^2}{\tfrac{1}{2}M_{\text{body}}R_{\text{body}}^2 + 2\tfrac{1}{3}M_{\text{arm}}L_{\text{arm}}^2} = \frac{\tfrac{1}{2}(50\text{ kg})(0.15\text{ m})^2}{\tfrac{1}{2}(44\text{ kg})(0.15\text{ m})^2 + 2\tfrac{1}{3}(3\text{ kg})(0.50\text{ m})^2}$$

$$= 0.575 \approx \boxed{0.6}$$

83. (*a*) The angular momentum of M_{A} will be

$$L_{\text{A}} = I_A\omega_1 = \tfrac{1}{2}M_AR^2\omega_1 = \tfrac{1}{2}(6.0\text{ kg})(0.60\text{ m})^2(7.2\,\text{rad/s}) = \boxed{7.8\,\text{kg}\bullet\text{m}^2/\text{s}}\,.$$

(*b*) The torque required to accelerate M_{A} will be

$$\tau = \frac{\Delta L}{\Delta t} = \frac{7.8\,\text{kg}\bullet\text{m}^2/\text{s} - 0}{2.0\text{ s}} = \boxed{3.9\text{ m}\bullet\text{N}}$$

(*c*) Since there are no torques external to the two plates, the angular momentum of the two plates will be conserved. Since the two plates stick together, they will have a common final angular velocity. This is a totally inelastic collision.

$$L_i = L_f \quad \rightarrow \quad I_{\text{A}}\omega_1 = (I_{\text{A}} + I_{\text{B}})\omega_2 \quad \rightarrow$$

$$\omega_2 = \frac{I_{\text{A}}}{I_{\text{A}} + I_{\text{B}}}\omega_1 = \frac{\tfrac{1}{2}M_{\text{A}}R^2}{\tfrac{1}{2}M_{\text{A}}R^2 + \tfrac{1}{2}M_{\text{B}}R^2}\omega_1 = \frac{M_{\text{A}}}{M_{\text{A}} + M_{\text{B}}}\omega_1 = \left(\frac{6.0\text{ kg}}{15.0\text{ kg}}\right)(7.2\,\text{rad/s})$$

$$= \boxed{2.9\ \text{rad/s}}$$

$\boxed{84.}$ Since frictional losses can be ignored, energy will be conserved for the marble. Define the 0 position of PE to be the bottom of the track, so that the bottom of the ball is initially a height *h* above the 0 position of PE. Since $r \ll R$, the marble's CM is very close to the surface of the track. While the marble is on the loop, we then approximate that it will be moving in a circle of radius *R*. When the marble is at the top of the loop, we approximate that its CM is a distance of 2*R* above the 0 position of PE. For the marble to just be on the verge of leaving the track means the normal force between the marble and the track is zero, and so the centripetal force at the top must be equal to the gravitational force on the marble.

$$\frac{mv_{\substack{\text{top of}\\\text{loop}}}^2}{R} = mg \quad \rightarrow \quad v_{\substack{\text{top of}\\\text{loop}}}^2 = gR$$

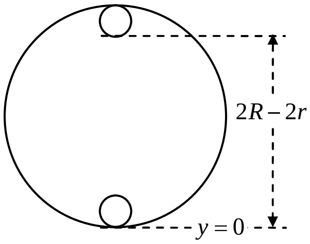
We assume that the marble is rolling without slipping, and so $\omega = v/r$, and that the marble is released from rest. Use energy conservation to relate the release point to the point at the top of the loop. Note that the marble has both translational and rotational kinetic energy.

$$E_{\text{release}} = E_{\substack{\text{top of}\\\text{loop}}} \rightarrow KE_{\text{release}} + PE_{\text{release}} = KE_{\substack{\text{top of}\\\text{loop}}} + PE_{\substack{\text{top of}\\\text{loop}}}$$

$$0 + mgh = \tfrac{1}{2}mv^2_{\substack{\text{top of}\\\text{loop}}} + \tfrac{1}{2}I\omega^2_{\substack{\text{top of}\\\text{loop}}} + mg2R = \tfrac{1}{2}mv^2_{\substack{\text{top of}\\\text{loop}}} + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v^2_{\substack{\text{top of}\\\text{loop}}}}{r^2} + 2mgR$$

$$mgh = \tfrac{7}{10}mv^2_{\substack{\text{top of}\\\text{loop}}} + 2mgR = \tfrac{7}{10}mgR + 2mgR = 2.7mgR \rightarrow \boxed{h = 2.7R}$$

85. Since frictional losses can be ignored, energy will be conserved for the marble. Define the 0 position of PE to be bottom of the track, so that the bottom of the ball is initially a height *h* above the 0 position of PE. Since we are not to assume that $r \ll R$, then while the marble is on the loop portion of the track, it is moving in a circle of radius $R - r$, and when at the top of the loop, the bottom of the marble is a height of $2(R - r)$ above the 0 position of PE (see the diagram). For the marble to just be on the verge of leaving the track means the normal force between the marble and the track is zero, and so the centripetal force at the top must be equal to the gravitational force on the marble.



$$\frac{mv^2_{\substack{\text{top of}\\\text{loop}}}}{R - r} = mg \rightarrow v^2_{\substack{\text{top of}\\\text{loop}}} = g(R - r)$$

We assume that the marble is rolling without slipping and so $\omega = v/r$, and that the marble is released from rest. Use energy conservation to relate the release point to the point at the top of the loop. Note that the marble has both translational and rotational kinetic energy.

$$E_{\text{release}} = E_{\substack{\text{top of}\\\text{loop}}} \rightarrow KE_{\text{release}} + PE_{\text{release}} = KE_{\substack{\text{top of}\\\text{loop}}} + PE_{\substack{\text{top of}\\\text{loop}}}$$

$$0 + mgh = \tfrac{1}{2}mv^2_{\substack{\text{top of}\\\text{loop}}} + \tfrac{1}{2}I\omega^2_{\substack{\text{top of}\\\text{loop}}} + mg2(R - r) = \tfrac{1}{2}mv^2_{\substack{\text{top of}\\\text{loop}}} + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v^2_{\substack{\text{top of}\\\text{loop}}}}{r^2} + 2mg(R - r)$$

$$mgh = \tfrac{7}{10}mv^2_{\substack{\text{top of}\\\text{loop}}} + 2mg(R - r) = \tfrac{7}{10}mg(R - r) + 2mg(R - r) = 2.7mg(R - r)$$

$$\boxed{h = 2.7(R - r)}$$

86. (*a*) The angular acceleration can be found from $\omega^2 = \omega_o^2 + 2\alpha\theta$, with the angular velocities being given by $\omega = v/r$.

$$\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{\left(v^2 - v_o^2\right)}{2r^2\theta} = \frac{\left[(60.0\,\text{km/h})^2 - (90.0\,\text{km/h})^2\right]\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)^2}{2(0.45\text{ m})^2(85\text{ rev})(2\pi\text{ rad/rev})}$$

$$= -1.6053\,\text{rad/s}^2 \approx \boxed{-1.61\,\text{rad/s}^2}$$

(*b*) The time to stop can be found from $\omega = \omega_o + \alpha t$, with a final angular velocity of 0.

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{v - v_o}{r\alpha} = \frac{0 - (60.0\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)}{(0.45\text{ m})\left(-1.6053\,\text{rad/s}^2\right)} = \boxed{23\text{ s}}$$

CHAPTER 9: Static Equilibrium; Elasticity and Fracture

Answers to Questions

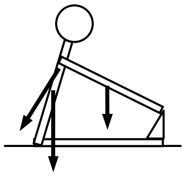
1. If the object has a net force on it of zero, then its center of mass does not accelerate. But since it is not in equilibrium, it must have a net torque, and therefore have an angular acceleration. Some examples are:
    a) A compact disk in a player as it comes up to speed, after just being put in the player.
    b) A hard drive on a computer when the computer is first turned on.
    c) A window fan immediately after the power to it has been shut off.

2. The bungee jumper is not in equilibrium, because the net force on the jumper is not zero. If the jumper were at rest and the net force were zero, then the jumper would stay at rest by Newton's 1st law. The jumper has a net upward force when at the bottom of the dive, and that is why the jumper is then pulled back upwards.

3. If the fingers are not the same distance from the CG, the finger closer to the CG will support a larger fraction of the weight of the meter stick so that the net torque on the stick is zero. That larger vertical force means there will be more friction between the stick and that closer finger, and thus the finger further from the CG will be easier to move. The more distant finger will slide easier, and therefore move in closer to the CG. That finger, when it becomes the one closest to the CG, will then have more friction and will "stick". The other finger will then slide. You then repeat the process. Whichever finger is farther from the CG will slide closer to it, until the two fingers eventually meet at the CG.

4. Like almost any beam balance, the movable weights are connected to the fulcrum point by relatively long lever arms, while the platform on which you stand is connected to the fulcrum point by a very short lever arm. The scale "balances" when the torque provided by your weight (large mass, small lever arm) is equal to that provided by the sliding weights (small mass, large lever arm).

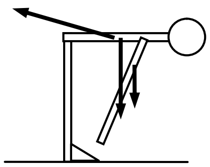
5. (*a*) If we assume that the pivot point of rotation is the lower left corner of the wall in the picture, then the gravity force acting through the CM provides the torque to keep the wall upright. Note that the gravity force would have a relatively small lever arm (about half the width of the wall) and so the sideways force would not have to be particularly large to start to move the wall.
    (*b*) With the horizontal extension, there are factors that make the wall less likely to overturn.
    - The mass of the second wall is larger, and so the torque caused by gravity (helping to keep the wall upright) will be larger for the second wall.
    - The center of gravity of the second wall is further to the right of the pivot point and so gravity exerts a larger torque to counteract the torque due to $\vec{\mathbf{F}}$.
    - The weight of the ground above the new part of the wall provides a large clockwise torque that helps to counteract the torque due to $\vec{\mathbf{F}}$.



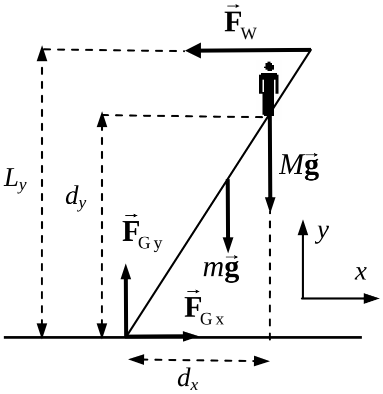
6. For rotating the upper half body, the pivot point is near the waist and hips. In that position, the arms have a relatively small torque, even when extended, due to their smaller mass, and the more massive trunk–head combination has a very short lever arm, and so also has a relatively small torque. Thus the force of gravity on the upper body causes relatively little torque about the hips tending to rotate you forward, and so the back muscles need to produce little torque to keep you from rotating forward. The force on the upper half body due to the back muscles is small, and so the

(partially rightward) force at the base of the spinal column, to keep the spine in equilibrium, will be small.

When standing and bending over, the lever arm for the upper body is much larger than while sitting, and so causes a much larger torque. The CM of the arms is also further from the support point, and so causes more torque. The back muscles, assumed to act at the center of the back, do not have a very long lever arm. Thus the back muscles will have to exert a large force to cause a counter-torque that keeps you from falling over. And accordingly, there will have to be a large force (mostly to the right in the picture) at the base of the spine to keep the spine in equilibrium.



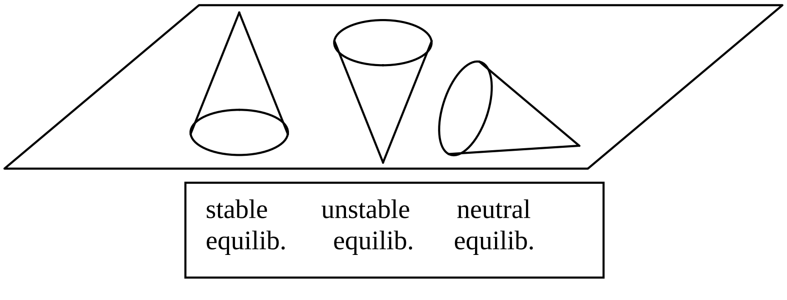
7. When the person stands near the top, the ladder is more likely to slip. In the accompanying diagram, the force of the person pushing down on the ladder $(M\vec{\mathbf{g}})$ causes a clockwise torque about the contact point with the ground, with lever arm d_x. The only force causing a counterclockwise torque about that same point is the reaction force of the wall on the ladder, $\vec{\mathbf{F}}_{\mathrm{W}}$. While the ladder is in equilibrium, $\vec{\mathbf{F}}_{\mathrm{W}}$ will be the same magnitude as the frictional force at the ground, $\vec{\mathbf{F}}_{\mathrm{G}x}$. Since $\vec{\mathbf{F}}_{\mathrm{G}x}$ has a maximum value, $\vec{\mathbf{F}}_{\mathrm{W}}$ will have the same maximum value, and so $\vec{\mathbf{F}}_{\mathrm{W}}$ will have a maximum counterclockwise torque that it can exert. As the person climbs the ladder, their lever arm gets longer and so the torque due to their weight gets larger. Eventually, if the torque caused by the person is larger than the maximum torque caused by $\vec{\mathbf{F}}_{\mathrm{W}}$, the ladder will start to slip – it will not stay in equilibrium.


$\vec{\mathbf{F}}_{\mathrm{W}}$
$M\vec{\mathbf{g}}$
L_y
d_y
$\vec{\mathbf{F}}_{\mathrm{G}y}$
y
$m\vec{\mathbf{g}}$
x
$\vec{\mathbf{F}}_{\mathrm{G}x}$
d_x

8. The mass of the meter stick is equal to that of the rock. For purposes of calculating torques, the meter stick can be treated as if all of its mass were at the 50 cm mark. Thus the CM of the meter stick is the same distance from the pivot point as the rock, and so their masses must be the same in order to exert the same torque.

9. If the sum of the forces on an object are not zero, then the CM of the object will accelerate in the direction of the net force. If the sum of the torques on the object are zero, then the object has no angular acceleration. Some examples are:
    a) A satellite in a circular orbit around the Earth.
    b) A block sliding down an inclined plane.
    c) An object that is in projectile motion but not rotating
    d) The startup motion of an elevator, changing from rest to having a non-zero velocity.

10.



11. Configuration (b) is more likely to be stable. In configuration (a), the CG of the bottom brick is at the edge of the table, and the CG of the top brick is to the right of the edge of the table. Thus the CG of the two-brick system is not above the base of support, and so gravity will exert a torque to roll the bricks clockwise off the table. Another way to see this is that more than 50% of the brick mass is not above the base of support – 50% of the bottom brick and 75% of the top brick are to the right of the edge of the table. It is not in stable, neutral, or unstable equilibrium.

    In configuration (b), exactly half of the mass (75% of the top brick and 25% of the bottom brick) is over the edge of the table. Thus the CG of the pair is at the edge of the table – it is in unstable equilibrium.

$\boxed{12.}$ When walking, you must keep your CG over your feet. If you have a heavy load in your arms, your CG is shifted forward, and so you must lean backwards to realign your CG over your feet.

$\boxed{13.}$ When you rise on your tiptoes, your CM shifts forward. Since you are already standing with your nose and abdomen against the door, your CM cannot shift forward. Thus gravity exerts a torque on you and you are unable to stay on your tiptoes – you will return to being flat-footed on the floor.

14. When you start to stand up from a normal sitting position, your CM is not over your point of support (your feet), and so gravity will exert a torque about your feet that rotates you back down into the chair. You must lean forward in order that your CM be over your feet so that you can stand up.

15. In the midst of doing a sit-up, the abdomen muscles provide a torque to rotate you up away from the floor, while the force of gravity on your upper half-body is tending to pull you back down to the floor, providing the difficulty for doing sit-ups. The force of gravity on your lower half-body provides a torque that opposes the torque caused by the force of gravity on your upper half-body, making the sit-up a little easier. With the legs bent, the lever arm for the lower half-body is shorter, and so less counter-torque is available.

16. Position "A" is unstable equilibrium, position "B" is stable equilibrium, and position "C" is neutral equilibrium.

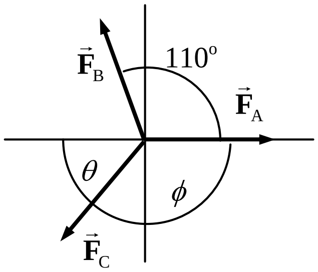
17. The Young's modulus for a bungee cord is much smaller than that for ordinary rope. From its behavior, we know that the bungee cord stretches relatively easily, compared to ordinary rope. From Eq. 9-4, we have $E = \dfrac{F/A}{\Delta L/L_o}$. The value of Young's modulus is inversely proportional to the change in length of a material under a tension. Since the change in length of a bungee cord is much larger than that of an ordinary rope if other conditions are identical (stressing force, unstretched length, cross-sectional area of rope or cord), it must have a smaller Young's modulus.

18. An object under shear stress has equal and opposite forces applied across it. One blade of the scissors pushes down on the cardboard while the other arm pushes up. These two forces cause the cardboard to shear between the two blades. Thus the name "shears" is justified.

19. The left support is under tension, since the force from the support pulls on the beam. Thus it would not be wise to use concrete or stone for that support. The right support is under compression, since it pushes on the beam. Concrete or stone would be acceptable for that support.

Solutions to Problems

1. If the tree is not accelerating, then the net force in all directions is 0.



$$\sum F_x = F_A + F_B \cos 110 + F_{Cx} = 0 \quad \rightarrow$$

$$F_{Cx} = -F_A - F_B \cos 110 = -310 \text{ N} - (425 \text{ N})\cos 110 = -164.6 \text{ N}$$

$$\sum F_y = F_B \sin + F_{Cy} = 0 \quad \rightarrow$$

$$F_{Cy} = -F_B \sin 110 = -(425 \text{ N})\sin 110 = -399.4 \text{ N}$$

$$F_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = \sqrt{(-164.6 \text{ N})^2 + (-399.4 \text{ N})^2} = 432.0 \text{ N} \approx \boxed{4.3\times10^2 \text{ N}}$$

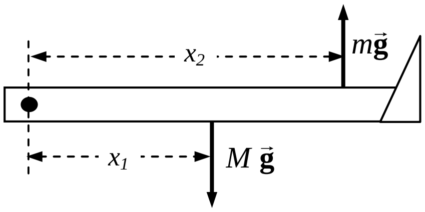
$$\theta = \tan^{-1}\frac{F_{Cy}}{F_{Cx}} = \tan^{-1}\frac{-399.4 \text{ N}}{-164.6 \text{ N}} = 67.6^\circ \ , \ \phi = 180^\circ - 67.6^\circ = 112.4^\circ \approx \boxed{112^\circ}$$

And so $\vec{\mathbf{F}}_C$ is 430 N, at an angle of 112° clockwise from $\vec{\mathbf{F}}_A$.

2. The torque is the force times the lever arm.

$$\tau = Fr = (58 \text{ kg})(9.8 \text{ m/s}^2)(3.0 \text{ m}) = \boxed{1.7\times10^3 \text{ m}\bullet\text{N} \text{, clockwise}}$$

3. Because the mass m is stationary, the tension in the rope pulling up on the sling must be mg, and so the force of the sling on the leg must be mg, upward. Calculate torques about the hip joint, with counterclockwise torque taken as positive. See the free-body diagram for the leg. Note that the forces on the leg exerted by the hip joint are not drawn, because they do not exert a torque about the hip joint.

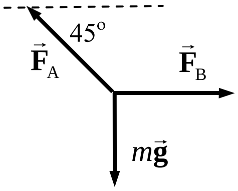


$$\sum \tau = mgx_2 - Mgx_1 = 0 \quad \rightarrow \quad m = M\frac{x_1}{x_2} = (15.0 \text{ kg})\frac{(35.0 \text{ cm})}{(80.5 \text{ cm})} = \boxed{6.52 \text{ kg}}$$

4. The torque is the force times the lever arm.

$$\tau = Fr \quad \rightarrow \quad r = \frac{\tau}{F} = \frac{1100 \text{ m}\bullet\text{N}}{(58 \text{ kg})(9.8 \text{ m/s}^2)} = \boxed{1.9 \text{ m}}$$

5. Write Newton's 2nd law for the junction, in both the x and y directions.

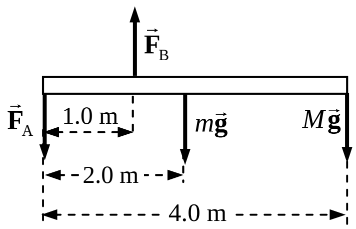


$$\sum F_x = F_B - F_A \cos 45^\circ = 0$$

From this, we see that $F_A > F_B$. Thus set $F_A = 1550 \text{ N}$.

$$\sum F_y = F_A \sin 45^\circ - mg = 0$$

$$mg = F_A \sin 45^\circ = (1550 \text{ N})\sin 45^\circ = \boxed{1.1\times10^3 \text{ N}}$$



6. (*a*) Let $m = 0$. Calculate the net torque about the left end of the diving board, with counterclockwise torques positive. Since the board is in equilibrium, the net torque is zero.

$$\sum \tau = F_B(1.0\text{ m}) - Mg(4.0\text{ m}) = 0 \quad \rightarrow$$

$$F_B = 4Mg = 4(58\text{ kg})(9.80\text{ m/s}^2) = 2274\text{ N} \approx \boxed{2.3\times10^3\text{ N}}$$

Use Newton's 2nd law in the vertical direction to find F_A.

$$\sum F_y = F_B - Mg - F_A \quad \rightarrow$$

$$F_A = F_B - Mg = 4Mg - Mg = 3Mg = 3(58\text{ kg})(9.80\text{ m/s}^2) = 1705\text{ N} \approx \boxed{1.7\times10^3\text{ N}}$$

(*b*) Repeat the basic process, but with $m = 35$ kg. The weight of the board will add more clockwise torque.

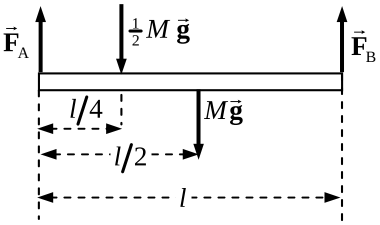
$$\sum \tau = F_B(1.0\text{ m}) - mg(2.0\text{ m}) - Mg(4.0\text{ m}) = 0 \quad \rightarrow$$

$$F_B = 4Mg + 2mg = [4(58\text{ kg}) + 2(35\text{ kg})](9.80\text{ m/s}^2) = 2960\text{ N} \approx \boxed{3.0\times10^3\text{ N}}$$

$$\sum F_y = F_B - Mg - mg - F_A \quad \rightarrow$$

$$F_A = F_B - Mg - mg = 4Mg + 2mg - Mg - mg = 3Mg + mg$$

$$= [3(58\text{ kg}) + 35\text{ kg}](9.80\text{ m/s}^2) = 2048\text{ N} \approx \boxed{2.0\times10^3\text{ N}}$$



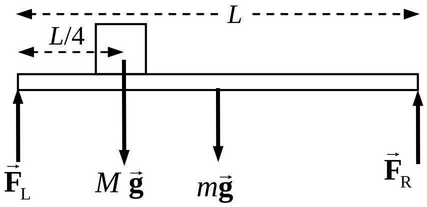
7. The CG of each beam is at its center. Calculate torques about the left end of the beam, and take counterclockwise torques to be positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.

$$\sum \tau = F_B l - Mg(l/2) - \tfrac{1}{2}Mg(l/4) = 0 \quad \rightarrow$$

$$F_B = \tfrac{5}{8}Mg = \tfrac{5}{8}(940\text{ kg})(9.80\text{ m/s}^2) = 5758\text{ N} \approx \boxed{5.8\times10^3\text{ N}}$$

$$\sum F_y = F_A + F_B - Mg - \tfrac{1}{2}Mg = 0 \quad \rightarrow$$

$$F_A = \tfrac{3}{2}Mg - F_B = \tfrac{7}{8}Mg = \tfrac{7}{8}(940\text{ kg})(9.80\text{ m/s}^2) = 8061\text{ N} \approx \boxed{8.1\times10^3\text{ N}}$$



8. Let m be the mass of the beam, and M be the mass of the piano. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.

$$\sum \tau = F_R L - mg(\tfrac{1}{2}L) - Mg(\tfrac{1}{4}L) = 0$$

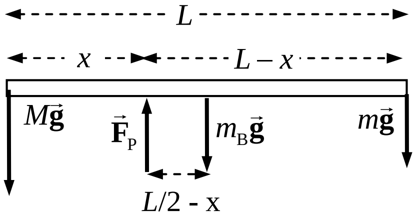
$$F_R = (\tfrac{1}{2}m + \tfrac{1}{4}M)g = [\tfrac{1}{2}(140\text{ kg}) + \tfrac{1}{4}(320\text{ kg})](9.80\text{ m/s}^2) = 1.47\times10^3\text{ N}$$

$$\sum F_y = F_L + F_R - mg - Mg = 0$$

$$F_L = (m+M)g - F_R = (460\text{ kg})(9.80\text{ m/s}^2) - 1.47\times10^3\text{ N} = 3.04\times10^3\text{ N}$$

The forces on the supports are equal in magnitude and opposite in direction to the above two results.

$$\boxed{F_R = 1.5\times10^3\text{ N down}} \qquad \boxed{F_L = 3.0\times10^3\text{ N down}}$$



9. The pivot should be placed so that the net torque on the board is zero. We calculate torques about the pivot point, with counterclockwise torques as positive. The upward force $\vec{\mathbf{F}}_\text{P}$ at the pivot point is shown, but it exerts no torque about the pivot point. The mass of the board is m_B, and the CG is at the middle of the board.

(*a*) Ignore the force $m_\text{B}g$.

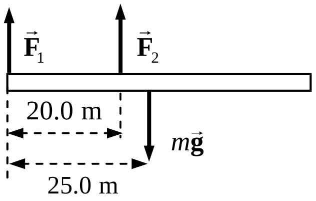
$$\sum \tau = Mgx - mg\left(L-x\right) = 0 \quad \rightarrow$$

$$x = \frac{m}{m+M}L = \frac{\left(25\text{ kg}\right)}{\left(25\text{ kg}+75\text{ kg}\right)}\left(9.0\text{ m}\right) = 2.25\,\text{m} \approx \boxed{2.3\text{ m from adult}}$$

(*b*) Include the force $m_\text{B}g$.

$$\sum \tau = Mgx - mg\left(L-x\right) - m_\text{B}g\left(L/2-x\right) = 0$$

$$x = \frac{\left(m+m_\text{B}/2\right)}{\left(M+m+m_\text{B}\right)}L = \frac{\left(25\,\text{kg}+7.5\,\text{kg}\right)}{\left(75\,\text{kg}+25\,\text{kg}+15\,\text{kg}\right)}\left(9.0\text{ m}\right) = 2.54\,\text{m} \approx \boxed{2.5\text{ m from adult}}$$



10. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.

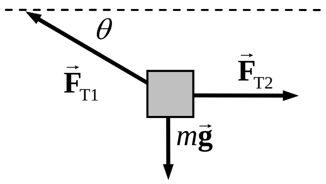
$$\sum \tau = F_2\left(20.0\text{ m}\right) - mg\left(25.0\text{ m}\right) = 0 \quad \rightarrow$$

$$F_2 = \frac{25.0}{20.0}mg = \left(1.25\right)\left(1250\text{ kg}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{1.53\times10^4\,\text{N}}$$

$$\sum F_y = F_1 + F_2 - mg = 0$$

$$F_1 = mg - F_2 = mg - 1.25mg = -0.25mg = -\left(0.25\right)\left(1250\text{ kg}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{-3.06\times10^3\,\text{N}}$$

Notice that $\vec{\mathbf{F}}_1$ points down.



11. Using the free-body diagram, write Newton's second law for both the horizontal and vertical directions, with net forces of zero.

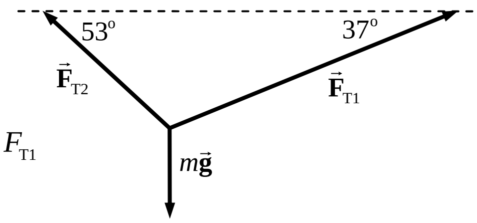
$$\sum F_x = F_\text{T2} - F_\text{T1}\cos\theta = 0 \quad \rightarrow \quad F_\text{T2} = F_\text{T1}\cos\theta$$

$$\sum F_y = F_\text{T1}\sin\theta - mg = 0 \quad \rightarrow \quad F_\text{T1} = \frac{mg}{\sin\theta}$$

$$F_\text{T2} = F_\text{T1}\cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mg}{\tan\theta} = \frac{\left(170\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)}{\tan 33^\circ} = \boxed{2.6\times10^3\,\text{N}}$$

$$F_\text{T1} = \frac{mg}{\sin\theta} = \frac{\left(170\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)}{\sin 33^\circ} = \boxed{3.1\times10^3\,\text{N}}$$

12. Draw a free-body diagram of the junction of the three wires. The tensions can be found from the conditions for force equilibrium.


53°
37°
$\vec{\mathbf{F}}_{\mathrm{T2}}$
$\vec{\mathbf{F}}_{\mathrm{T1}}$
$m\vec{\mathbf{g}}$

$$\sum F_x = F_{\mathrm{T1}}\cos 37^\circ - F_{\mathrm{T2}}\cos 53^\circ = 0 \quad\rightarrow\quad F_{\mathrm{T2}} = \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}}$$

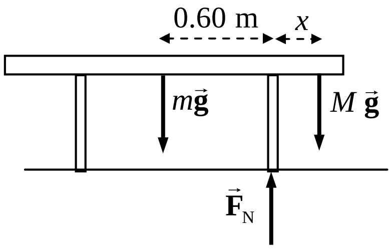
$$\sum F_y = F_{\mathrm{T1}}\sin 37^\circ + F_{\mathrm{T2}}\sin 53^\circ - mg = 0$$

$$F_{\mathrm{T1}}\sin 37^\circ + \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}}\sin 53^\circ - mg = 0 \quad\rightarrow$$

$$F_{\mathrm{T1}} = \frac{(33\text{ kg})(9.8\text{ m/s}^2)}{\sin 37^\circ + \dfrac{\cos 37^\circ}{\cos 53^\circ}\sin 53^\circ} = 1.946\times 10^2\text{ N} \approx \boxed{1.9\times 10^2\text{ N}}$$

$$F_{\mathrm{T2}} = \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}} = \frac{\cos 37^\circ}{\cos 53^\circ}\left(1.946\times 10^2\text{ N}\right) = 2.583\times 10^2\text{ N} \approx \boxed{2.6\times 10^2\text{ N}}$$

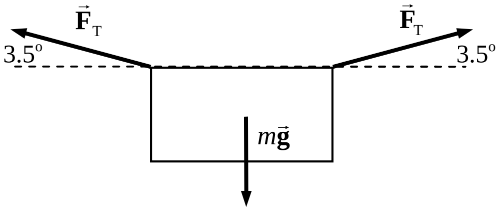
13. The table is symmetric, so the person can sit near either edge and the same distance will result. We assume that the person (mass *M*) is on the right side of the table, and that the table (mass *m*) is on the verge of tipping, so that the left leg is on the verge of lifting off the floor. There will then be no normal force between the left leg of the table and the floor. Calculate torques about the right leg of the table, so that the normal force between the table and the floor causes no torque. Counterclockwise torques are taken to be positive. The conditions of equilibrium for the table are used to find the person's location.


0.60 m
x
$m\vec{\mathbf{g}}$
$M\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\mathrm{N}}$

$$\sum \tau = mg(0.60\text{ m}) - Mgx = 0 \quad\rightarrow\quad x = (0.60\text{ m})\frac{m}{M} = (0.60\text{ m})\frac{20.0\text{ kg}}{66.0\text{ kg}} = 0.182\text{ m}$$

Thus the distance from the edge of the table is $0.50\text{ m} - 0.182\text{ m} = \boxed{0.32\text{ m}}$

14. Draw a force diagram for the sheet, and write Newton's second law for the vertical direction. Note that the tension is the same in both parts of the clothesline.

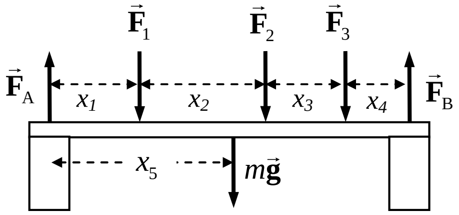

$\vec{\mathbf{F}}_{\mathrm{T}}$
$\vec{\mathbf{F}}_{\mathrm{T}}$
3.5°
3.5°
$m\vec{\mathbf{g}}$

$$\sum F_y = F_{\mathrm{T}}\sin 3.5^\circ + F_{\mathrm{T}}\sin 3.5^\circ - mg = 0 \quad\rightarrow$$

$$F_{\mathrm{T}} = \frac{mg}{2\left(\sin 3.5^\circ\right)} = \frac{(0.60\text{ kg})(9.80\text{ m/s}^2)}{2\left(\sin 3.5^\circ\right)} = \boxed{48\text{ N}}$$

The 48-N tension is much higher than the ~ 6-N weight of the sheet because of the angle. Only the vertical components of the tension are supporting the sheet, and since the angle is small, the tension has to be large to have a large enough vertical component.

15. The beam is in equilibrium, and so both the net torque and net force on it must be zero. From the free-body diagram, calculate the net torque about the center of the left support, with counterclockwise torques as positive. Calculate the net force, with upward as positive. Use those two equations to find F_{A} and F_{B}.


$\vec{\mathbf{F}}_1$
$\vec{\mathbf{F}}_2$
$\vec{\mathbf{F}}_3$
$\vec{\mathbf{F}}_{\mathrm{A}}$
$\vec{\mathbf{F}}_{\mathrm{B}}$
x_1
x_2
x_3
x_4
x_5
$m\vec{\mathbf{g}}$

$$\sum \tau = F_B\left(x_1+x_2+x_3+x_4\right)-F_1x_1-F_2\left(x_1+x_2\right)-F_3\left(x_1+x_2+x_3\right)-mgx_5$$

$$F_B = \frac{F_1x_1+F_2\left(x_1+x_2\right)+F_3\left(x_1+x_2+x_3\right)+mgx_5}{\left(x_1+x_2+x_3+x_4\right)}$$

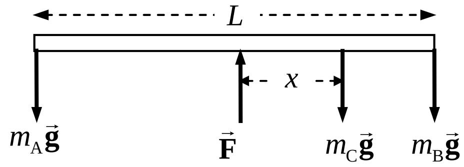
$$= \frac{(4300\text{ N})(2.0\text{ m})+(3100\text{ N})(6.0\text{ m})+(2200\text{ N})(9.0\text{ m})+(250\text{ kg})\left(9.8\,\text{m/s}^2\right)(5.0\text{ m})}{10.0\text{ m}}$$

$$= 5925\text{ N} \approx \boxed{5.9\times10^3\text{ N}}$$

$$\sum F = F_A + F_B - F_1 - F_2 - F_3 - mg = 0$$

$$F_A = F_1+F_2+F_3+mg-F_B = 9600\text{ N} + (250\text{ kg})\left(9.8\,\text{m/s}^2\right) - 5925\text{ N} = 6125\text{ N} \approx \boxed{6.1\times10^3\text{ N}}$$

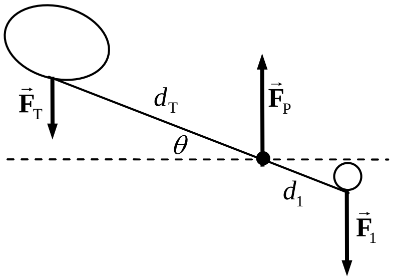
16. From the free-body diagram, the conditions of equilibrium are used to find the location of the girl (mass m_C). The 50-kg boy is represented by m_A, and the 35-kg girl by m_B. Calculate torques about the center of the see-saw, and take counterclockwise torques to be positive. The upward force of the fulcrum on the see-saw $\left(\vec{\mathbf{F}}\right)$ causes no torque about the center.



$$\sum \tau = m_A g\left(\tfrac{1}{2}L\right) - m_C g x - m_B g\left(\tfrac{1}{2}L\right) = 0$$

$$x = \frac{\left(m_A - m_B\right)}{m_C}\left(\tfrac{1}{2}L\right) = \frac{(50\text{ kg} - 35\text{ kg})}{25\text{ kg}}\tfrac{1}{2}(3.6\text{ m}) = \boxed{1.1\text{ m}}$$

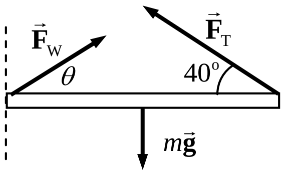
17. Since each half of the forceps is in equilibrium, the net torque on each half of the forceps is zero. Calculate torques with respect to an axis perpendicular to the plane of the forceps, through point P, counterclockwise being positive. Consider a force diagram for one half of the forceps. $\vec{\mathbf{F}}_1$ is the force on the half-forceps due to the plastic rod, and force $\vec{\mathbf{F}}_P$ is the force on the half-forceps from the pin joint. $\vec{\mathbf{F}}_P$ does not exert any torque about point P.



$$\sum \tau = F_T d_T \cos\theta - F_1 d_1 \cos\theta = 0 \;\rightarrow\; F_1 = F_T\frac{d_T}{d_1} = (11.0\text{ N})\frac{8.50\text{ cm}}{2.70\text{ cm}} = 34.6\text{ N}$$

The force that the forceps exerts on the rod is the opposite of $\vec{\mathbf{F}}_1$, and so is also $\boxed{34.6\text{ N}}$.

18. The beam is in equilibrium, and so the net force and net torque on the beam must be zero. From the free-body diagram for the beam, calculate the net torque (counterclockwise positive) about the wall support point to find $\vec{\mathbf{F}}_T$, and calculate the net force in both the *x* and *y* directions to find the components of $\vec{\mathbf{F}}_W$.



$$\sum \tau = F_T L \sin 40^\circ - mg\, L/2 = 0 \;\rightarrow$$

$$F_T = \frac{mg}{2\sin 40^\circ} = \frac{(27\text{ kg})\left(9.80\,\text{m/s}^2\right)}{2\sin 40^\circ} = 205.8\text{ N} \approx \boxed{2.1\times10^2\text{ N}}$$

$$\sum F_x = F_{\mathrm{W}x} - F_{\mathrm{T}}\cos 40^\circ = 0 \quad \rightarrow \quad F_{\mathrm{W}x} = F_{\mathrm{T}}\cos 40^\circ = (205.8\ \mathrm{N})\cos 40^\circ = 157.7\ \mathrm{N}$$

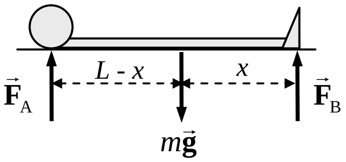
$$\sum F_y = F_{\mathrm{W}y} + F_{\mathrm{T}}\sin 40^\circ - mg = 0 \quad \rightarrow$$

$$F_{\mathrm{W}y} = mg - F_{\mathrm{T}}\sin 40^\circ = (27\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) - (205.8\ \mathrm{N})\sin 40^\circ = 132.3\ \mathrm{N}$$

$$F_{\mathrm{W}} = \sqrt{F_{\mathrm{W}x}^2 + F_{\mathrm{W}x}^2} = \sqrt{(157.7\ \mathrm{N})^2 + (132.3\ \mathrm{N})^2} = 205.8\ \mathrm{N} \approx \boxed{2.1\times 10^2\ \mathrm{N}}$$

$$\theta = \tan^{-1}\frac{F_{\mathrm{W}y}}{F_{\mathrm{W}x}} = \tan^{-1}\frac{132.3}{157.7} = \boxed{40^\circ}$$

19. The person is in equilibrium, and so both the net torque and net force must be zero. From the free-body diagram, calculate the net torque about the center of gravity, with counterclockwise torques as positive. Use that calculation to find the location of the center of gravity, a distance x from the feet.

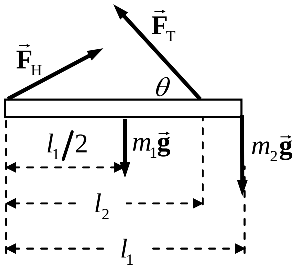


$$\sum \tau = F_{\mathrm{B}}x - F_{\mathrm{A}}(L-x) = 0$$

$$x = \frac{F_{\mathrm{A}}}{F_{\mathrm{A}} + F_{\mathrm{B}}}L = \frac{m_{\mathrm{A}}g}{m_{\mathrm{A}}g + m_{\mathrm{B}}g}L = \frac{m_{\mathrm{A}}}{m_{\mathrm{A}} + m_{\mathrm{B}}}L = \frac{35.1\ \mathrm{kg}}{31.6\ \mathrm{kg} + 35.1\ \mathrm{kg}}(1.72\ \mathrm{m}) = \boxed{9.05\times 10^{-1}\mathrm{m}}$$

The center of gravity is about 90.5 cm from the feet.

20. The beam is in equilibrium. Use the conditions of equilibrium to calculate the tension in the wire and the forces at the hinge. Calculate torques about the hinge, and take counterclockwise torques to be positive.



$$\sum \tau = (F_{\mathrm{T}}\sin\theta)l_2 - m_1 g\, l_1/2 - m_2 g l_1 = 0 \quad \rightarrow$$

$$F_{\mathrm{T}} = \frac{\frac{1}{2}m_1 g l_1 + m_2 g l_1}{l_2\sin\theta} = \frac{\frac{1}{2}(155\ \mathrm{N})(1.70\ \mathrm{m}) + (245\ \mathrm{N})(1.70\ \mathrm{m})}{(1.35\ \mathrm{m})(\sin 35.0^\circ)}$$

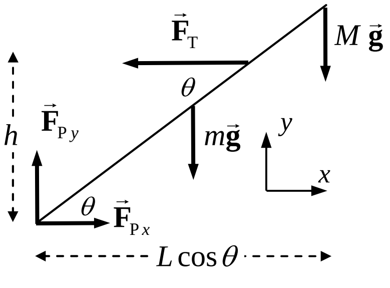
$$= 708.0\ \mathrm{N} \approx \boxed{7.08\times 10^2\ \mathrm{N}}$$

$$\sum F_x = F_{\mathrm{H}x} - F_{\mathrm{T}}\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{H}x} = F_{\mathrm{T}}\cos\theta = (708\ \mathrm{N})\cos 35.0^\circ = 579.99\ \mathrm{N} \approx \boxed{5.80\times 10^2\ \mathrm{N}}$$

$$\sum F_y = F_{\mathrm{H}y} + F_{\mathrm{T}}\sin\theta - m_1 g - m_2 g = 0 \quad \rightarrow$$

$$F_{\mathrm{H}y} = m_1 g + m_2 g - F_{\mathrm{T}}\sin\theta = 155\ \mathrm{N} + 245\ \mathrm{N} - (708\ \mathrm{N})\sin 35.0^\circ = -6.092\ \mathrm{N} \approx \boxed{-6\ \mathrm{N}(\mathrm{down})}$$

21. (*a*) The pole is in equilibrium, and so the net torque on it must be zero. From the free-body diagram, calculate the net torque about the lower end of the pole, with counterclockwise torques as positive. Use that calculation to find the tension in the cable. The length of the pole is *L*.


$\vec{\mathbf{F}}_\mathrm{T}$
$M\vec{\mathbf{g}}$
θ
h
$\vec{\mathbf{F}}_{\mathrm{P}y}$
$m\vec{\mathbf{g}}$
y
x
θ
$\vec{\mathbf{F}}_{\mathrm{P}x}$
$L\cos\theta$

$$\sum \tau = F_{\mathrm{T}}h - mg(L/2)\cos\theta - MgL\cos\theta = 0$$

$$F_{\mathrm{T}} = \frac{(m/2 + M)gL\cos\theta}{h}$$

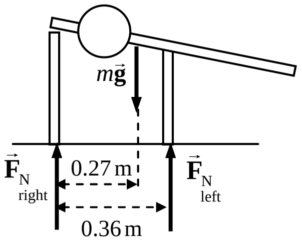
$$= \frac{(6.0\ \mathrm{kg} + 21.5\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(7.50\ \mathrm{m})\cos 37^\circ}{3.80\ \mathrm{m}} = 424.8\ \mathrm{N} \approx \boxed{4.25\times 10^2\ \mathrm{N}}$$

(*b*) The net force on the pole is also zero since it is in equilibrium. Write Newton's 2nd law in both the *x* and *y* directions to solve for the forces at the pivot.

$$\sum F_x = F_{\text{P}x} - F_{\text{T}} = 0 \ \rightarrow \ F_{\text{P}x} = F_{\text{T}} = \boxed{4.25\times10^2\ \text{N}}$$

$$\sum F_y = F_{\text{P}y} - mg - Mg = 0 \ \rightarrow \ F_{\text{P}y} = \left(m+M\right)g = \left(33.5\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = \boxed{3.28\times10^2\ \text{N}}$$

22. The man is in equilibrium, so the net force and the net torque on him are both zero. From the force diagram, write an expression for the net torque about a vertical axis through his right hand, with counterclockwise torques as positive. Also write an expression for the net force in the vertical direction.



$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{N}}$ right
0.27 m
$\vec{\mathbf{F}}_{\text{N}}$ left
0.36 m

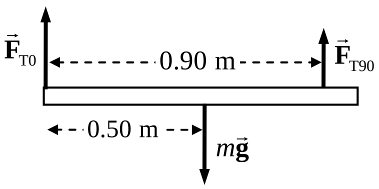
$$\sum \tau = F_{\underset{\text{left}}{\text{N}}}\left(0.36\ \text{m}\right) - mg\left(0.27\ \text{m}\right) = 0 \ \rightarrow$$

$$F_{\underset{\text{left}}{\text{N}}} = mg\frac{0.27}{0.36} = \left(72\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\frac{0.27}{0.36} = 529.2\ \text{N} \approx \boxed{5.3\times10^2\ \text{N}}$$

$$\sum F_y = F_{\underset{\text{left}}{\text{N}}} + F_{\underset{\text{right}}{\text{N}}} - mg = 0 \ \rightarrow$$

$$F_{\underset{\text{right}}{\text{N}}} = mg - F_{\underset{\text{left}}{\text{N}}} = \left(72\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) - 529.2\ \text{N} = \boxed{1.8\times10^2\ \text{N}}$$

23. (*a*) The meter stick is in equilibrium, so the net torque and the net force are both zero. From the force diagram, write an expression for the net torque about the 90-cm mark, with counterclockwise torques as positive.



$\vec{\mathbf{F}}_{\text{T0}}$
0.90 m
$\vec{\mathbf{F}}_{\text{T90}}$
0.50 m
$m\vec{\mathbf{g}}$

$$\sum \tau = mg\left(0.40\ \text{m}\right) - F_{\text{T0}}\left(0.90\ \text{m}\right) = 0 \ \rightarrow$$

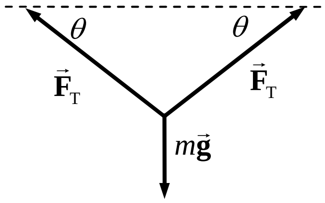
$$F_{\text{T0}} = mg\frac{0.40}{0.90} = \left(0.180\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\frac{0.40}{0.90} = \boxed{0.78\ \text{N}}$$

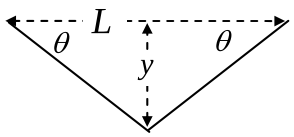
(*b*) Write Newton's 2nd law for the vertical direction with a net force of 0 to find the other tension.

$$\sum F_y = F_{\text{T0}} + F_{\text{T90}} - mg = 0 \ \rightarrow$$

$$F_{\text{T90}} = mg - F_{\text{T0}} = \left(0.180\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) - 0.78\ \text{N} = \boxed{0.98\ \text{N}}$$

$\boxed{24.}$ Since the backpack is midway between the two trees, the angles in the diagram are equal. Write Newton's 2nd law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the downward vertical force. The angle is determined by the distance between the trees and the amount of sag at the midpoint, as illustrated in the second diagram.





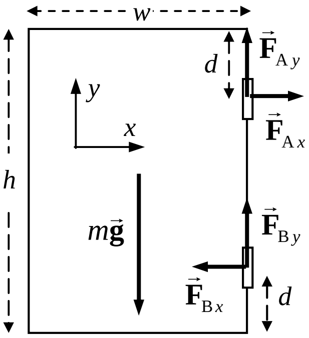
(*a*) $\theta = \tan^{-1}\dfrac{y}{L/2} = \tan^{-1}\dfrac{1.5\ \text{m}}{3.8\ \text{m}} = 21.5^{\circ}$

$$\sum F_y = 2F_{\text{T}}\sin\theta_1 - mg = 0 \ \rightarrow$$

$$F_{\text{T}} = \frac{mg}{2\sin\theta_1} = \frac{\left(19\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{2\sin 21.5^{\circ}} = \boxed{2.5\times10^2\ \text{N}}$$

$$(b)\quad \theta = \tan^{-1}\frac{y}{L/2} = \tan^{-1}\frac{0.15\text{ m}}{3.8\text{ m}} = 2.26^\circ \qquad F_\text{T} = \frac{mg}{2\sin\theta_1} = \frac{(19\text{ kg})(9.80\text{ m/s}^2)}{2\sin 2.26^\circ} = \boxed{2.4\times10^3\text{ N}}$$

25. The forces on the door are due to gravity and the hinges. Since the door is in equilibrium, the net torque and net force must be zero. Write the three equations of equilibrium. Calculate torques about the bottom hinge, with counterclockwise torques as positive. From the statement of the problem, $F_{\text{A}y} = F_{\text{B}y} = \frac{1}{2}mg$.


w
d
$\vec{\mathbf{F}}_{\text{A}y}$
y
x
$\vec{\mathbf{F}}_{\text{A}x}$
h
$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{B}y}$
$\vec{\mathbf{F}}_{\text{B}x}$
d

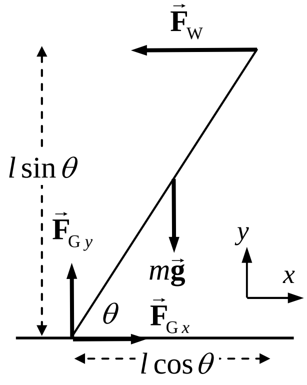
$$\sum\tau = mg\frac{w}{2} - F_{Ax}(h-2d) = 0$$

$$F_{Ax} = \frac{mgw}{2(h-2d)} = \frac{(13.0\text{ kg})(9.80\text{ m/s}^2)(1.30\text{ m})}{2(2.30\text{ m}-0.80\text{ m})} = \boxed{55.2\text{ N}}$$

$$\sum F_x = F_{Ax} - F_{Bx} = 0 \;\rightarrow\; F_{Bx} = F_{Ax} = \boxed{55.2\text{ N}}$$

$$\sum F_y = F_{Ay} + F_{By} - mg = 0 \;\rightarrow\; F_{Ay} = F_{By} = \tfrac{1}{2}mg = \tfrac{1}{2}(13.0\text{ kg})(9.8\text{ m/s}^2) = \boxed{63.7\text{ N}}$$

26. Write the conditions of equilibrium for the ladder, with torques taken about the bottom of the ladder, and counterclockwise torques as positive.


$\vec{\mathbf{F}}_\text{W}$
$l\sin\theta$
$\vec{\mathbf{F}}_{\text{G}y}$
$m\vec{\mathbf{g}}$
y
x
θ
$\vec{\mathbf{F}}_{\text{G}x}$
$l\cos\theta$

$$\sum\tau = F_\text{W} l\sin\theta - mg\frac{l}{2}\cos\theta = 0 \;\rightarrow\; F_\text{W} = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

$$\sum F_x = F_{\text{G}x} - F_\text{W} = 0 \;\rightarrow\; F_{\text{G}x} = F_\text{W} = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

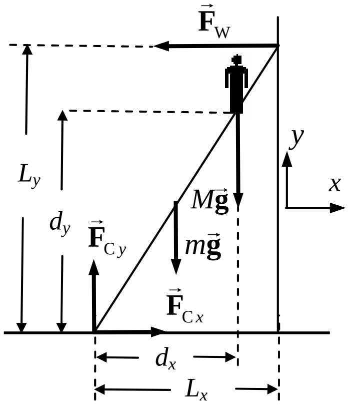
$$\sum F_y = F_{\text{G}y} - mg = 0 \;\rightarrow\; F_{\text{G}y} = mg$$

For the ladder to not slip, the force at the ground $F_{\text{G}x}$ must be less than or equal to the maximum force of static friction.

$$F_{\text{G}x} \le \mu F_\text{N} = \mu F_{\text{G}y} \;\rightarrow\; \tfrac{1}{2}\frac{mg}{\tan\theta} \le \mu mg \;\rightarrow\; \frac{1}{2\mu} \le \tan\theta \;\rightarrow\; \theta \ge \tan^{-1}(1/2\mu)$$

Thus the minimum angle is $\boxed{\theta_\text{min} = \tan^{-1}(1/2\mu)}$.

27. The ladder is in equilibrium, so the net torque and net force must be zero. By stating that the ladder is on the verge of slipping, the static frictional force at the ground, $F_{\text{C}x}$ is at its maximum value and so $F_{\text{C}x} = \mu_s F_{\text{C}y}$. Since the person is standing 70% of the way up the ladder, the height of the ladder is $L_y = d_y/0.7 = 2.8\text{ m}/0.7 = 4.0\text{ m}$. The width of the ladder is $L_x = d_x/0.7 = 2.1\text{ m}/0.7 = 3.0\text{ m}$. Torques are taken about the point of contact of the ladder with the ground, and counterclockwise torques are taken as positive. The three conditions of equilibrium are as follows.


$\vec{\mathbf{F}}_\text{W}$
y
L_y
x
$M\vec{\mathbf{g}}$
d_y
$\vec{\mathbf{F}}_{\text{C}y}$
$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{C}x}$
d_x
L_x

$$\sum F_x = F_{\text{C}x} - F_\text{W} = 0 \;\rightarrow\; F_{\text{C}x} = F_\text{W}$$

$$\sum F_y = F_{\mathrm{G}y} - Mg - mg = 0 \quad \rightarrow$$

$$F_{\mathrm{G}y} = (M+m)\,g = (67.0\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) = 656.6\ \mathrm{N}$$

$$\sum \tau = F_{\mathrm{W}} L_y - mg\left(\tfrac{1}{2}L_x\right) - Mgd_x = 0$$

Solving the torque equation gives

$$F_{\mathrm{W}} = \frac{\frac{1}{2}mL_x + Md_x}{L_y}\,g = \frac{\frac{1}{2}(12.0\ \mathrm{kg})(3.0\ \mathrm{m}) + (55.0\ \mathrm{kg})(2.1\ \mathrm{m})}{4.0\ \mathrm{m}}\left(9.80\ \mathrm{m/s^2}\right) = 327.1\ \mathrm{N}\,.$$

The coefficient of friction then is found to be

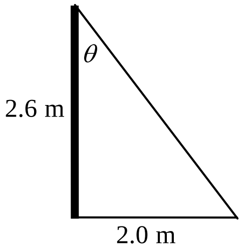
$$\mu_s = \frac{F_{\mathrm{G}x}}{F_{\mathrm{G}y}} = \frac{327.1\ \mathrm{N}}{656.6\ \mathrm{N}} = \boxed{0.50}\,.$$

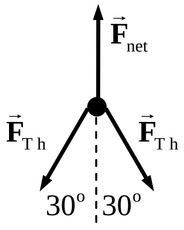
28. If the lamp is just at the point of tipping, then the normal force will be acting at the edge of the base, 12 cm from the lamp stand pole. We assume the lamp is in equilibrium and just on the verge of tipping, and is being pushed sideways at a constant speed. Take torques about the center of the base, with counterclockwise torques positive. Also write Newton's 2nd law for both the vertical and horizontal directions.

$$\sum F_y = F_{\mathrm{N}} - mg = 0 \;\rightarrow\; F_{\mathrm{N}} = mg \qquad \sum F_x = F_{\mathrm{P}} - F_{\mathrm{fr}} = 0 \;\rightarrow\; F_{\mathrm{P}} = F_{\mathrm{fr}} = \mu F_{\mathrm{N}} = \mu mg$$

$$\sum \tau = F_{\mathrm{N}}(0.12\ \mathrm{m}) - F_{\mathrm{P}}x = 0 \;\rightarrow\; x = \frac{F_{\mathrm{N}}}{F_{\mathrm{P}}}(0.12\ \mathrm{m}) = \frac{mg}{\mu mg}(0.12\ \mathrm{m}) = \frac{0.12\ \mathrm{m}}{0.20} = \boxed{0.60\ \mathrm{m}}$$

29. First consider the triangle made by the pole and one of the wires (first diagram). It has a vertical leg of 2.6 m, and a horizontal leg of 2.0 m. The angle that the tension (along the wire) makes with the vertical is $\theta = \tan^{-1}\frac{2.0}{2.6} = 37.6^\circ$. The part of the tension that is parallel to the ground is therefore $F_{\mathrm{T\,h}} = F_{\mathrm{T}}\sin\theta$. Now consider a top view of the pole, showing only force parallel to the ground (second diagram). The horizontal parts of the tension lie as the sides of an equilateral triangle, and so each make a 30° angle with the tension force of the net. Write the equilibrium equation for the forces along the direction of the tension in the net.



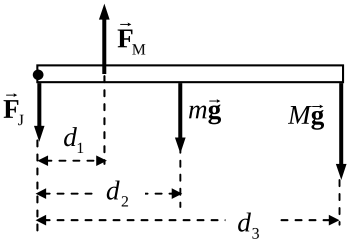


$\vec{\mathbf{F}}_{\mathrm{net}}$
$\vec{\mathbf{F}}_{\mathrm{T\,h}}$
$\vec{\mathbf{F}}_{\mathrm{T\,h}}$
30°
30°

$$\sum F = F_{\mathrm{net}} - 2F_{\mathrm{T\,h}}\cos 30^\circ = 0 \quad \rightarrow$$

$$F_{\mathrm{net}} = 2F_{\mathrm{T}}\sin\theta\cos 30^\circ = 2(95\ \mathrm{N})\sin 37.6^\circ\cos 30^\circ = \boxed{1.0\times 10^2\ \mathrm{N}}$$

30. The arm is in equilibrium. Take torques about the elbow joint (the dot in the free-body diagram), so that the force at the elbow joint does not enter the calculation. Counterclockwise torques are positive. The mass of the lower arm is $m = 2.0\ \mathrm{kg}$, and the mass of the load is M. It is given that $F_{\mathrm{M}} = 450\ \mathrm{N}$.

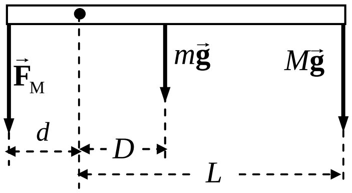


$\vec{\mathbf{F}}_{\mathrm{M}}$
$\vec{\mathbf{F}}_{\mathrm{J}}$
$m\vec{\mathbf{g}}$
$M\vec{\mathbf{g}}$
d_1
d_2
d_3

$$\sum \tau = F_{\mathrm{M}}d_1 - mgd_2 - Mgd_3 = 0 \quad \rightarrow$$

$$M = \frac{F_{\mathrm{M}}d_1 - mgd_2}{gd_3} = \frac{(450\ \mathrm{N})(0.060\ \mathrm{m}) - (2.0\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(0.15\ \mathrm{m})}{\left(9.80\ \mathrm{m/s^2}\right)(0.35\ \mathrm{m})} = \boxed{7.0\ \mathrm{kg}}$$

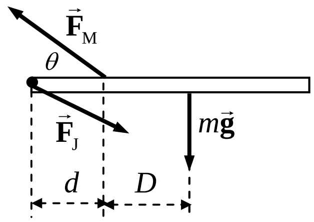
31. Calculate the torques about the elbow joint (the dot in the free body diagram). The arm is in equilibrium. Counterclockwise torques are positive.


$\vec{\mathbf{F}}_\mathrm{M}$
$m\vec{\mathbf{g}}$
$M\vec{\mathbf{g}}$
d
D
L

$$\sum \tau = F_\mathrm{M} d - mgD - MgL = 0$$

$$F_\mathrm{M} = \frac{mD + ML}{d} g = \frac{(2.8\ \mathrm{kg})(0.12\ \mathrm{m}) + (7.3\ \mathrm{kg})(0.300\ \mathrm{m})}{0.025\ \mathrm{m}}\left(9.8\ \mathrm{m/s^2}\right) = \boxed{9.9\times10^2\ \mathrm{N}}$$

32. (*a*) Calculate the torques about the elbow joint (the dot in the free-body diagram). The arm is in equilibrium. Take counterclockwise torques as positive.


$\vec{\mathbf{F}}_\mathrm{M}$
θ
$\vec{\mathbf{F}}_\mathrm{J}$
$m\vec{\mathbf{g}}$
d
D

$$\sum \tau = \left(F_\mathrm{M} \sin\theta\right) d - mgD = 0 \quad \rightarrow$$

$$F_\mathrm{M} = \frac{mgD}{d\sin\theta} = \frac{(3.3\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(0.24\ \mathrm{m})}{(0.12\ \mathrm{m})\sin 15^\circ} = \boxed{2.5\times10^2\ \mathrm{N}}$$

(*b*) To find the components of F_J, write Newton's 2nd law for both the *x* and *y* directions. Then combine them to find the magnitude.

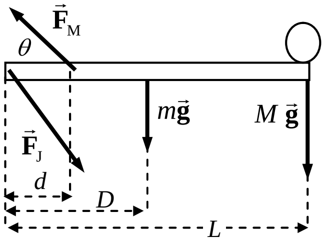
$$\sum F_x = F_{\mathrm{J}x} - F_\mathrm{M}\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{J}x} = F_\mathrm{M}\cos\theta = (250\ \mathrm{N})\cos 15^\circ = 241\ \mathrm{N}$$

$$\sum F_y = F_\mathrm{M}\sin\theta - mg - F_{\mathrm{J}y} = 0 \quad \rightarrow$$

$$F_{\mathrm{J}y} = F_\mathrm{M}\sin\theta - mg = (250\ \mathrm{N})\sin 15^\circ - (3.3\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) = 32\ \mathrm{N}$$

$$\mathrm{F_J} = \sqrt{F_{\mathrm{J}x}^2 + F_{\mathrm{J}y}^2} = \sqrt{(241\ \mathrm{N})^2 + (32\ \mathrm{N})^2} = 243.5\ \mathrm{N} \approx \boxed{2.4\times10^2\ \mathrm{N}}$$

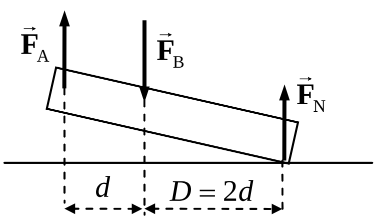
33. Calculate the torques about the shoulder joint, which is at the left end of the free-body diagram of the arm. Since the arm is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques to be positive. The force due to the shoulder joint is drawn, but it does not exert any torque about the shoulder joint.


$\vec{\mathbf{F}}_\mathrm{M}$
θ
$\vec{\mathbf{F}}_\mathrm{J}$
$m\vec{\mathbf{g}}$
$M\,\vec{\mathbf{g}}$
d
D
L

$$\sum \tau = F_m d\sin\theta - mgD - MgL = 0$$

$$F_m = \frac{mD + ML}{d\sin\theta} g = \frac{(3.3\ \mathrm{kg})(0.24\ \mathrm{cm}) + (15\ \mathrm{kg})(0.52\ \mathrm{m})}{(0.12\ \mathrm{m})\sin 15^\circ}\left(9.8\ \mathrm{m/s^2}\right) = \boxed{2.7\times10^3\ \mathrm{N}}$$

34. There will be a normal force upwards at the ball of the foot, equal to the person's weight $\left(F_\mathrm{N} = mg\right)$. Calculate torques about a point on the floor directly below the leg bone (and so in line with the leg bone force, $\vec{\mathbf{F}}_\mathrm{B}$). Since the foot is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques as positive.

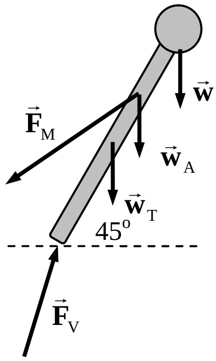

$\vec{\mathbf{F}}_\mathrm{A}$
$\vec{\mathbf{F}}_\mathrm{B}$
$\vec{\mathbf{F}}_\mathrm{N}$
d
$D = 2d$

$$\sum \tau = F_\mathrm{N}(2d) - F_\mathrm{A} d = 0 \quad \rightarrow$$

$$F_\mathrm{A} = 2F_\mathrm{N} = 2mg = 2(72\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) = \boxed{1.4\times10^3\ \mathrm{N}}$$

The net force in the *y* direction must be zero. Use that to find F_B.

$$\sum F_y = F_{\text{N}} + F_{\text{A}} - F_{\text{B}} = 0 \ \rightarrow \ F_{\text{B}} = F_{\text{N}} + F_{\text{A}} = 2mg + mg = 3mg = \boxed{2.1\times10^3\,\text{N}}$$



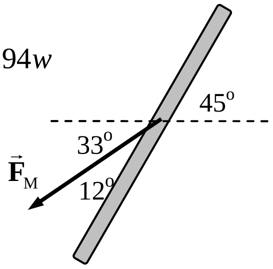
$\vec{\mathbf{w}}_{\text{H}}$
$\vec{\mathbf{F}}_{\text{M}}$
$\vec{\mathbf{w}}_{\text{A}}$
$\vec{\mathbf{w}}_{\text{T}}$
45°
$\vec{\mathbf{F}}_{\text{V}}$

35. Figures 9-14 (b) and (c) are redrawn here with the person 45° from the horizontal, instead of the original 30°. The distances are all the same as in the original problem. We still assume that the back muscles pull at a 12° angle to the spine. The 18° angle from the original problem becomes 33°. Torques are taken about the same point at the base of the spine, with counterclockwise torques as positive.

$$\sum \tau = (0.48\text{ m})F_{\text{M}}\sin 12^\circ - (0.72\text{ m})(w_{\text{H}})\sin 45^\circ$$
$$-(0.48\text{ m})(w_{\text{A}})\sin 45^\circ - (0.36\text{ m})(w_{\text{T}})\sin 45^\circ = 0$$

As in the original problem, $w_{\text{H}} = 0.07w$, $w_{\text{A}} = 0.12w$, $w_{\text{T}} = 0.46w$. With this, the torque equation gives the following result.

$$F_{\text{M}} = \frac{[(0.72\text{ m})(0.07) + (0.48\text{ m})(0.12) + (0.36\text{ m})(0.46)]}{(0.48\text{ m})\sin 12^\circ} w\sin 45^\circ = 1.94w$$



Take the sum of the forces in the vertical direction, set equal to zero.

$$\sum F_y = F_{\text{V}y} - F_{\text{M}}\sin 33^\circ - 0.07w - 0.12w - 0.46w = 0 \ \rightarrow \ F_{\text{V}y} = 1.71w$$

Take the sum of the forces in the horizontal direction, set equal to zero.

$$\sum F_x = F_{\text{V}x} - F_{\text{M}}\cos 33^\circ = 0 \ \rightarrow \ F_{\text{V}y} = 1.63w$$

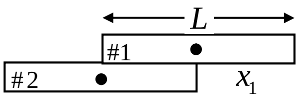
The final result is

$$F_{\text{V}} = \sqrt{F_{\text{V}x}^2 + F_{\text{V}y}^2} = \boxed{2.4w}$$

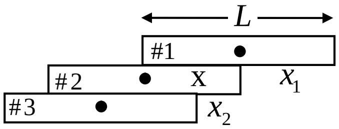
This compares to 2.5*w* for the more bent position.

36. From Section 9-4: "An object whose CG is above its base of support will be stable if a vertical line projected downward from the CG falls within the base of support." For the tower, the base of support is a circle of radius 3.5 m. If the top is 4.5 m off center, then the CG will be 2.25 m off center, and a vertical line downward from the CG will be 2.25 m from the center of the base. Thus the tower is in $\boxed{\text{stable equilibrium}}$. To be unstable, the CG has to be more than 3.5 m off center, and thus the top must be more than 7.0 m off center. Thus the top will have to lean $\boxed{2.5\text{ m}}$ further to reach the verge of instability.

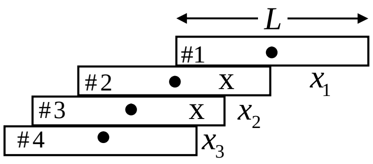
37. (*a*) The maximum distance for brick #1 to remain on brick #2 will be reached when the CM of brick #1 is directly over the edge of brick #2. Thus brick #1 will overhang brick #2 by $x_1 = L/2$.



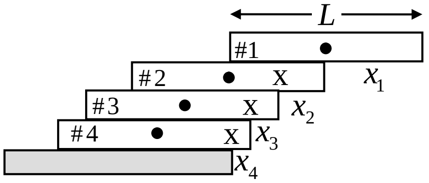
The maximum distance for the top two bricks to remain on brick #3 will be reached when the center of mass of the top two bricks is directly over the edge of brick #3. The CM of the top two bricks is (obviously) at the point labeled x on brick #2, a distance of $L/4$ from the right edge of brick #2. Thus $x_2 = L/4$.



The maximum distance for the top three bricks to remain on brick #4 will be reached when the center of mass of the top three bricks is directly over the edge of brick #4. The CM of the top three bricks is at the point labeled x on brick #3, and is found relative to the center of brick # 3 by



$\text{CM} = \dfrac{m(0) + 2m(L/2)}{3m} = L/3$, or $L/6$ from the right edge of brick #3. Thus $x_3 = L/6$.



The maximum distance for the four bricks to remain on a tabletop will be reached when the center of mass of the four bricks is directly over the edge of the table. The CM of all four bricks is at the point labeled x on brick #4, and is found relative to the center of brick #4 by $\text{CM} = \dfrac{m(0) + 3m(L/2)}{4m} = 3L/8$, or $L/8$ from the right edge of brick #4. Thus $x_4 = L/8$.

(*b*) From the last diagram, the distance from the edge of the tabletop to the right edge of brick #1 is

$$x_4 + x_3 + x_2 + x_1 = (L/8) + (L/6) + (L/4) + (L/2) = 25L/24 > L$$

Since this distance is greater than *L*, the answer is $\boxed{\text{yes}}$, the first brick is completely beyond the edge of the table.

(*c*) From the work in part (a), we see that the general formula for the total distance spanned by *n* bricks is

$$x_1 + x_2 + x_3 + \cdots x_n = (L/2) + (L/4) + (L/6) + \cdots + (L/2n) = \boxed{\sum_{i=1}^{n} \frac{L}{2i}}$$

(*d*) The arch is to span 1.0 m, so the span from one side will be 0.50 m. Thus we must solve $\sum_{i=1}^{n} \dfrac{0.30\,\text{m}}{2i} \geq 0.50\,\text{m}$. Evaluation of this expression for various values of *n* shows that 15 bricks will span a distance of 0.498 m, and that 16 bricks will span a distance of 0.507 m. Thus it takes 16 bricks for each half-span, plus 1 brick on top and 1 brick as the base on each side (as in Fig. 9-67(b)), for a total of $\boxed{\text{35 bricks}}$.

38. The amount of stretch can be found using the elastic modulus in Eq. 9-4.

$$\Delta L = \frac{1}{E}\frac{F}{A}L_0 = \frac{1}{5\times10^9\ \text{N/m}^2}\frac{275\ \text{N}}{\pi\left(5.00\times10^{-4}\right)^2}(0.300\ \text{m}) = \boxed{2.10\times10^{-2}\,\text{m}}$$

$\boxed{39.}$ (*a*) $\text{Stress} = \dfrac{F}{A} = \dfrac{mg}{A} = \dfrac{(25000\ \text{kg})(9.8\,\text{m/s}^2)}{1.2\text{m}^2} = 2.042\times10^5\ \text{N/m}^2 \approx \boxed{2.0\times10^5\ \text{N/m}^2}$

(*b*) $\text{Strain} = \dfrac{\text{Stress}}{\text{Young's Modulus}} = \dfrac{2.042\times10^5\ \text{N/m}^2}{50\times10^9\ \text{N/m}^2} = \boxed{4.1\times10^{-6}}$

40. The change in length is found from the strain.

$$\text{Strain} = \frac{\Delta L}{L_0} \quad \rightarrow \quad \Delta L = L_0\,(\text{Strain}) = (9.6\ \text{m})\left(4.1\times10^{-6}\right) = \boxed{3.9\times10^{-5}\,\text{m}}$$

41. (*a*) $\text{Stress} = \dfrac{F}{A} = \dfrac{mg}{A} = \dfrac{(2100\ \text{kg})(9.8\,\text{m/s}^2)}{0.15\text{m}^2} = 1.372\times10^5\ \text{N/m}^2 \approx \boxed{1.4\times10^5\ \text{N/m}^2}$

(*b*) $\text{Strain} = \dfrac{\text{Stress}}{\text{Young's Modulus}} = \dfrac{1.372\times10^5\ \text{N/m}^2}{200\times10^9\ \text{N/m}^2} = 6.86\times10^{-7} \approx \boxed{6.9\times10^{-7}}$

(*c*) $\Delta L = (\text{Strain})(L_o) = (6.86\times10^{-7})(9.50\text{ m}) = \boxed{6.5\times10^{-6}\text{ m}}$

42. The change in volume is given by Eq. 9-7. We assume the original pressure is atmospheric pressure, $1.0\times10^5\text{ N/m}^2$.

$$\Delta V = -V_0\frac{\Delta P}{B} = -(1000\text{ cm}^3)\frac{(2.6\times10^6\text{ N/m}^2 - 1.0\times10^5\text{ N/m}^2)}{1.0\times10^9\text{ N/m}^2} = -2.5\text{ cm}^3$$

$$V = V_0 + \Delta V = 1000\text{ cm}^3 - 2.5\text{ cm}^3 = \boxed{997\text{ cm}^3}$$

43. The Young's Modulus is the stress divided by the strain.

$$\text{Young's Modulus} = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L_o} = \frac{(13.4\text{ N})\Big/\left[\pi\left(\tfrac{1}{2}\times8.5\times10^{-3}\text{ m}\right)^2\right]}{(3.7\times10^{-3}\text{ m})/(15\times10^{-2}\text{ m})} = \boxed{9.6\times10^6\text{ N/m}^2}$$

44. The relationship between pressure change and volume change is given by Eq. 9-7.

$$\Delta V = -V_0\frac{\Delta P}{B} \rightarrow \Delta P = -\frac{\Delta V}{V_0}B = -(0.10\times10^{-2})(90\times10^9\text{ N/m}^2) = \boxed{9.0\times10^7\text{ N/m}^2}$$

$$\frac{\Delta P}{P_{\text{atm}}} = \frac{9.0\times10^7\text{ N/m}^2}{1.0\times10^5\text{ N/m}^2} = \boxed{9.0\times10^2}$$

45. The percentage change in volume is found by multiplying the relative change in volume by 100. The change in pressure is 199 times atmospheric pressure, since it increases from atmospheric pressure to 200 times atmospheric pressure.

$$100\frac{\Delta V}{V_o} = -100\frac{\Delta P}{B} = -100\frac{199(1.0\times10^5\text{ N/m}^2)}{90\times10^9\text{ N/m}^2} = \boxed{-2\times10^{-2}\%}$$

The negative sign indicates that the interior space got smaller.

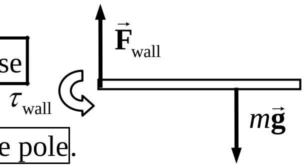
46. Elastic potential energy is given by $PE_{\text{elastic}} = \tfrac{1}{2}k(\Delta x)^2 = \tfrac{1}{2}F\Delta x$. The force is found from Eq. 9-4, using ΔL as Δx.

$$PE_{\text{elastic}} = \tfrac{1}{2}F\Delta x = \tfrac{1}{2}\left(\frac{EA}{L_0}\Delta L\right)\Delta L = \tfrac{1}{2}\frac{(2.0\times10^6\text{ N/m}^2)(0.50\times10^{-4}\text{ m}^2)}{(3.0\times10^{-3}\text{ m})}(1.0\times10^{-3}\text{ m})^2$$

$$= \boxed{1.7\times10^{-2}\text{ J}}$$

47. (*a*) The torque due to the sign is the product of the weight of the sign and the distance of the sign from the wall.

$$\tau = mgd = (5.1\text{ kg})(9.80\text{ m/s}^2)(2.2\text{ m}) = \boxed{1.1\times10^2\text{ m}\bullet\text{N , clockwise}}$$



(*b*) Since the wall is the only other object that can put force on the pole (ignoring the weight of the pole), then $\boxed{\text{the wall must put a torque on the pole}}$. The torque due to the sign is clockwise, so the torque due to the wall must be counterclockwise. See the diagram. Also note that the wall must put an upward force on the pole as well, so that the net force on the pole will be zero.

(*c*) The torque on the rod can be considered as the wall pulling horizontally to the left on the top left corner of the rod and pushing horizontally to the right at the bottom left corner of the rod. The reaction forces to these put a shear on the wall at the point of contact. Also, since the wall is pulling upwards on the rod, the rod is pulling down on the wall at the top surface of contact, causing tension. Likewise the rod is pushing down on the wall at the bottom surface of contact, causing compression. Thus $\boxed{\text{all three are present}}$.

48. Set the compressive strength of the bone equal to the stress of the bone.

$$\text{Compressive Strength} = \frac{F_{\max}}{A} \quad \rightarrow \quad F_{\max} = \left(170\times10^{6}\ \text{N/m}^2\right)\left(3.0\times10^{-4}\,\text{m}^2\right) = \boxed{5.1\times10^{4}\,\text{N}}$$

49. (*a*) The maximum tension can be found from the ultimate tensile strength of the material.

$$\text{Tensile Strength} = \frac{F_{\max}}{A} \quad \rightarrow$$

$$F_{\max} = \left(\text{Tensile Strength}\right)A = \left(500\times10^{6}\ \text{N/m}^2\right)\pi\left(5.00\times10^{-4}\,\text{m}\right)^2 = \boxed{393\ \text{N}}$$

(*b*) To prevent breakage, $\boxed{\text{thicker strings}}$ should be used, which will increase the cross-sectional area of the strings, and thus increase the maximum force. Breakage occurs because when the strings are hit by the ball, they stretch, increasing the tension. The strings are reasonably tight in the normal racket configuration, so when the tension is increased by a particularly hard hit, the tension may exceed the maximum force.

50. (*a*) Compare the stress on the bone to the compressive strength to see if the bone breaks.

$$\text{Stress} = \frac{F}{A} = \frac{3.6\times10^{4}\,\text{N}}{3.6\times10^{-4}\,\text{m}^2} = 1.0\times10^{8}\ \text{N/m}^2 < \text{Compressive Strength of bone}$$

$\boxed{\text{The bone will not break.}}$

(*b*) The change in length is calculated from Eq. 9-4.

$$\Delta L = \frac{L_0}{E}\frac{F}{A} = \left(\frac{0.22\ \text{m}}{15\times10^{9}\ \text{N/m}^2}\right)\left(1.0\times10^{8}\ \text{N/m}^2\right) = \boxed{1.5\times10^{-3}\,\text{m}}$$

51. (*a*) The area can be found from the ultimate tensile strength of the material.

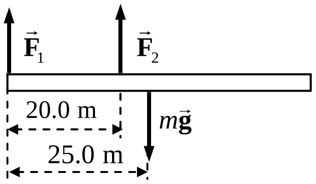
$$\frac{\text{Tensile Strength}}{\text{Safety Factor}} = \frac{F}{A}$$

$$A = F\left(\frac{\text{Safety Factor}}{\text{Tensile Strength}}\right) = \left(320\ \text{kg}\right)\left(9.8\,\text{m/s}^2\right)\frac{7.0}{500\times10^{6}\ \text{N/m}^2} = \boxed{4.4\times10^{-5}\,\text{m}^2}$$

(*b*) The change in length can be found from the stress-strain relationship, equation (9-5).

$$\frac{F}{A} = E\frac{\Delta L}{L_0} \quad \rightarrow \quad \Delta L = \frac{L_0 F}{AE} = \frac{\left(7.5\ \text{m}\right)\left(320\ \text{kg}\right)\left(9.8\,\text{m/s}^2\right)}{\left(4.4\times10^{-5}\,\text{m}^2\right)\left(200\times10^{9}\ \text{N/m}^2\right)} = \boxed{2.7\times10^{-3}\,\text{m}}$$

52. For each support, to find the minimum cross-sectional area with a safety factor means that $\frac{F}{A} = \frac{\text{Strength}}{\text{Safety Factor}}$, where either the tensile or compressive strength is used, as appropriate for each force. To find the force on each support, use the conditions of equilibrium for the beam. Take torques about the left end of the beam, calling counterclockwise



torques positive, and also sum the vertical forces, taking upward forces as positive.

$$\sum \tau = F_2\left(20.0\text{ m}\right) - mg\left(25.0\text{ m}\right) = 0 \quad \rightarrow \quad F_2 = \tfrac{25.0}{20.0}mg = 1.25mg$$

$$\sum F_y = F_1 + F_2 - mg = 0 \quad \rightarrow \quad F_1 = mg - F_2 = mg - 1.25mg = -0.25mg$$

Notice that the forces on the supports are the opposite of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. So the force on support # 1 is directed upwards, which means that support # 1 is in tension. The force on support # 2 is directed downwards, so support # 2 is in compression.

$$\frac{F_1}{A_1} = \frac{\text{Tensile Strength}}{8.5} \quad \rightarrow$$

$$A_1 = 8.5\frac{\left(0.25mg\right)}{\text{Tensile Strength}} = 8.5\frac{\left(0.25\right)\left(2.6\times10^3\text{kg}\right)\left(9.80\text{ m/s}^2\right)}{40\times10^6\text{ N/m}^2} = \boxed{1.4\times10^{-3}\text{m}^2}$$

$$\frac{F_2}{A_2} = \frac{\text{Compressive Strength}}{8.5} \quad \rightarrow$$

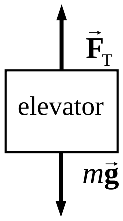
$$A_1 = 8.5\frac{\left(1.25mg\right)}{\text{Compressive Strength}} = 8.5\frac{\left(1.25\right)\left(2.6\times10^3\text{kg}\right)\left(9.80\text{ m/s}^2\right)}{35\times10^6\text{ N/m}^2} = \boxed{7.7\times10^{-3}\text{m}^2}$$

53. The diameter can be found from the ultimate shear strength of the material.

$$\frac{\text{Shear Strength}}{\text{Safety Factor}} = \frac{F}{A} = \frac{F}{\pi\left(d/2\right)^2}$$

$$d = \sqrt{\frac{4F}{\pi}\left(\frac{\text{Safety Factor}}{\text{Shear Strength}}\right)} = \sqrt{\frac{4\left(3200\text{ N}\right)}{\pi}\frac{6.0}{170\times10^6\text{ N/m}^2}} = \boxed{1.2\times10^{-2}\text{m}}$$

54. See the free-body diagram. The largest tension will occur when the elevator has an upward acceleration. Use that with the maximum tension to calculate the diameter of the bolt, *d*. Write Newton's second law for the elevator to find the tension.



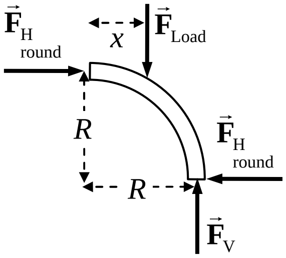
$$\sum F_y = F_\text{T} - mg = ma \quad \rightarrow$$

$$F_\text{T} = mg + ma = m\left(g + a\right) = m\left(9.8\text{ m/s}^2 + 1.2\text{ m/s}^2\right) = 11.0m$$

$$\frac{F_\text{T}}{A} = \frac{F_\text{T}}{\pi\left(d/2\right)^2} = \tfrac{1}{7}\left(\text{Tensile strength}\right) \quad \rightarrow$$

$$d = \sqrt{\frac{28F_\text{T}}{\pi\left(\text{Tensile strength}\right)}} = \sqrt{\frac{28\left(11.0\right)\left(3.1\times10^3\text{kg}\right)}{\pi\left(500\times10^6\text{ N/m}^2\right)}} = \boxed{2.5\times10^{-2}\text{m}}$$

55. Draw free-body diagrams similar to Figures 9-31(a) and 9-31(b) for the forces on the right half of a round arch and a pointed arch. The load force is placed at the same horizontal position on each arch. For each half-arch, take torques about the lower right hand corner, with counterclockwise as positive.


$\vec{\mathbf{F}}_{\text{H round}}$
x
$\vec{\mathbf{F}}_\text{Load}$
R
$\vec{\mathbf{F}}_{\text{H round}}$
R
$\vec{\mathbf{F}}_\text{V}$

For the round arch:

$$\sum \tau = F_\text{Load}\left(R - x\right) - F_{\underset{\text{round}}{\text{H}}}R = 0 \quad \rightarrow \quad F_{\underset{\text{round}}{\text{H}}} = F_\text{Load}\frac{R - x}{R}$$

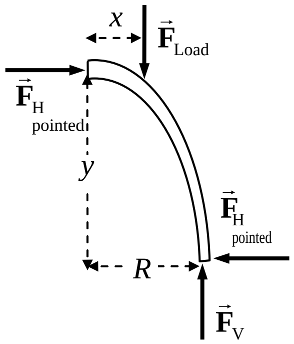
For the pointed arch:

$$\sum\tau = F_{\text{Load}}\left(R-x\right) - F_{\substack{\text{H}\\ \text{pointed}}}\,y = 0 \;\rightarrow\; F_{\substack{\text{H}\\ \text{pointed}}} = F_{\text{Load}}\frac{R-x}{y}$$

Solve for y, given that $F_{\substack{\text{H}\\ \text{pointed}}} = \frac{1}{3}F_{\substack{\text{H}\\ \text{round}}}$.

$$F_{\substack{\text{H}\\ \text{pointed}}} = \tfrac{1}{3}F_{\substack{\text{H}\\ \text{round}}} \;\rightarrow\; F_{\text{Load}}\frac{R-x}{y} = \tfrac{1}{3}F_{\text{Load}}\frac{R-x}{R} \;\rightarrow$$

$$y = 3R = 3\left(\tfrac{1}{2}8.0\text{ m}\right) = \boxed{12\text{ m}}$$

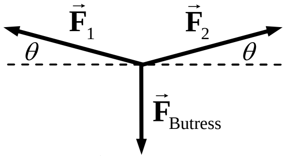


x
$\vec{\mathbf{F}}_{\text{Load}}$
$\vec{\mathbf{F}}_{\text{H}}$ pointed
y
$\vec{\mathbf{F}}_{\text{H}}$ pointed
R
$\vec{\mathbf{F}}_{\text{V}}$

56. Write Newton's 2nd law for the horizontal direction.

$$\sum F_x = F_2\cos\theta - F_1\cos\theta = 0 \;\rightarrow\; F_2 = F_1$$

Thus the two forces are the same size. Now write Newton's 2nd law for the vertical direction.



$\vec{\mathbf{F}}_1$
$\vec{\mathbf{F}}_2$
θ
θ
$\vec{\mathbf{F}}_{\text{Butress}}$

$$\sum F_y = F_1\sin\theta + F_1\sin\theta - F_{\text{butress}} = 0 \;\rightarrow\; F_1 = \frac{F_{\text{butress}}}{2\sin\theta} = \frac{4.3\times10^5\text{ N}}{2\left(\sin 5^{\circ}\right)} = \boxed{2.5\times10^6\text{ N}}$$

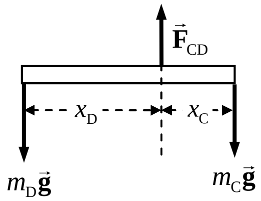
57. Each crossbar in the mobile is in equilibrium, and so the net torque about the suspension point for each crossbar must be 0. Counterclockwise torques will be taken as positive. The suspension point is used so that the tension in the suspension string need not be known initially. The net vertical force must also be 0.

The bottom bar:

$$\sum\tau = m_{\text{D}}gx_{\text{D}} - m_{\text{C}}gx_{\text{C}} = 0 \;\rightarrow$$

$$m_{\text{C}} = m_{\text{D}}\frac{x_{\text{D}}}{x_{\text{C}}} = m_{\text{D}}\frac{17.50\text{ cm}}{5.00\text{ cm}} = 3.50m_{\text{D}}$$

$$\sum F_y = F_{\text{CD}} - m_{\text{C}}g - m_{\text{D}}g = 0 \;\rightarrow\; F_{\text{CD}} = \left(m_{\text{C}} + m_{\text{D}}\right)g = 4.50m_{\text{D}}g$$



$\vec{\mathbf{F}}_{\text{CD}}$
x_{D}
x_{C}
$m_{\text{D}}\vec{\mathbf{g}}$
$m_{\text{C}}\vec{\mathbf{g}}$

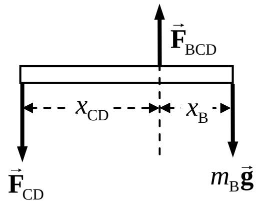
The middle bar:

$$\sum\tau = F_{\text{CD}}x_{\text{CD}} - m_{\text{B}}gx_{\text{B}} = 0 \;\rightarrow\; F_{\text{CD}} = m_{\text{B}}g\frac{x_{\text{B}}}{x_{\text{CD}}} \;\rightarrow\; 4.50m_{\text{D}}g = m_{\text{B}}g\frac{x_{\text{B}}}{x_{\text{CD}}}$$

$$m_{\text{D}} = \frac{m_{\text{B}}}{4.50}\frac{x_{\text{B}}}{x_{\text{CD}}} = \frac{\left(0.885\text{ kg}\right)\left(5.00\text{ cm}\right)}{\left(4.50\right)\left(15.00\text{ cm}\right)} = 0.06555 \approx \boxed{6.56\times10^{-2}\text{ kg}}$$

$$m_{\text{C}} = 3.50m_{\text{D}} = \left(3.50\right)\left(0.06555\text{ kg}\right) = \boxed{2.29\times10^{-1}\text{ kg}}$$

$$\sum F_y = F_{\text{BCD}} - F_{\text{CD}} - m_{\text{B}}g = 0 \;\rightarrow\; F_{\text{BCD}} = F_{\text{CD}} + m_{\text{B}}g = \left(4.50m_{\text{D}} + m_{\text{B}}\right)g$$



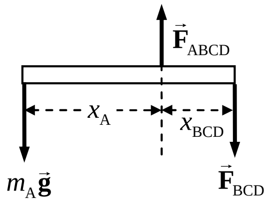
$\vec{\mathbf{F}}_{\text{BCD}}$
x_{CD}
x_{B}
$\vec{\mathbf{F}}_{\text{CD}}$
$m_{\text{B}}\vec{\mathbf{g}}$

The top bar:

$$\sum\tau = m_{\text{A}}gx_{\text{A}} - F_{\text{BCD}}x_{\text{BCD}} = 0 \;\rightarrow$$

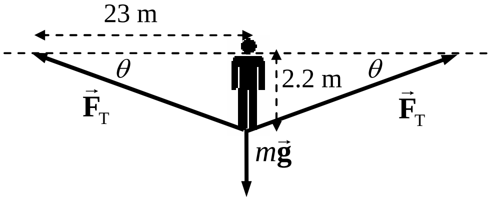
$$m_{\text{A}} = \frac{\left(4.50m_{\text{D}} + m_{\text{B}}\right)gx_{\text{BCD}}}{gx_{\text{A}}} = \left(4.50m_{\text{D}} + m_{\text{B}}\right)\frac{x_{\text{BCD}}}{x_{\text{A}}}$$

$$= \left[\left(4.50\right)\left(0.06555\text{ kg}\right) + 0.885\text{ kg}\right]\frac{7.50\text{ cm}}{30.00\text{ cm}} = \boxed{2.94\times10^{-1}\text{kg}}$$



$\vec{\mathbf{F}}_{\text{ABCD}}$
x_{A}
x_{BCD}
$m_{\text{A}}\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{BCD}}$

58. From the free-body diagram (not to scale), write the force equilibrium condition for the vertical direction.

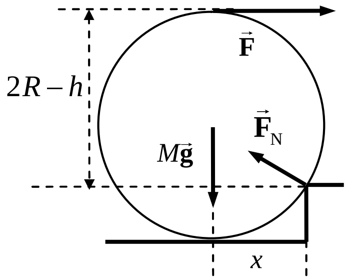


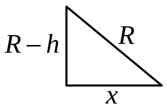
$$\sum F_y = 2T\sin\theta - mg = 0$$

$$T = \frac{mg}{2\sin\theta} = \frac{(60.0\text{ kg})(9.80\text{ m/s}^2)}{2\left(\dfrac{2.2\text{ m}}{23\text{ m}}\right)} = \boxed{3.1\times10^3\text{ N}}$$

It is not possible to increase the tension so that there is no sag. There must always be a vertical component of the tension to balance the gravity force. The larger the tension gets, the smaller the sag angle will be, however.

59. (*a*) If the wheel is just lifted off the lowest level, then the only forces on the wheel are the horizontal pull, its weight, and the contact force $\vec{\mathbf{F}}_N$ at the corner. Take torques about the corner point, for the wheel just barely off the ground, being held in equilibrium. The contact force at the corner exerts no torque and so does not enter the calculation. The pulling force has a lever arm of $R + R - h = 2R - h$, and gravity has a lever arm of x, found from the triangle shown.



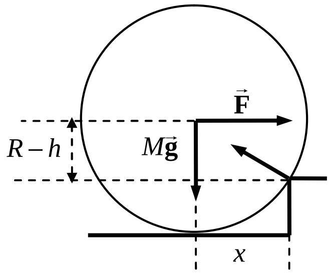


$$x = \sqrt{R^2 - (R-h)^2} = \sqrt{h(2R-h)}$$

$$\sum\tau = Mgx - F(2R-h) = 0 \quad\rightarrow$$

$$F = \frac{Mgx}{2R-h} = Mg\frac{\sqrt{h(2R-h)}}{2R-h} = \boxed{Mg\sqrt{\frac{h}{2R-h}}}$$

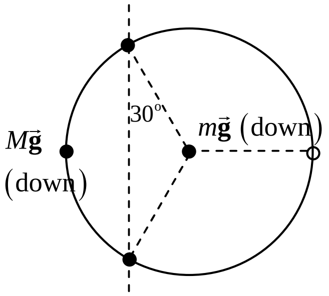
. (*b*) The only difference is that now the pulling force has a lever arm of $R-h$.

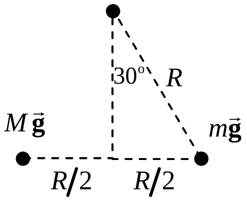


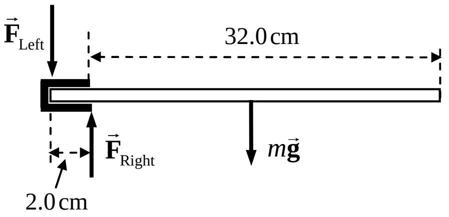
$$\sum\tau = Mgx - F(R-h) = 0 \quad\rightarrow$$

$$F = \frac{Mgx}{R-h} = \boxed{Mg\frac{\sqrt{h(2R-h)}}{R-h}}$$

60. The mass is to be placed symmetrically between two legs of the table. When enough mass is added, the table will rise up off of the third leg, and then the normal force on the table will all be on just two legs. Since the table legs are equally spaced, the angle marked in the diagram is 30°. Take torques about a line connecting the two legs that remain on the floor, so that the normal forces cause no torque. It is seen from the second diagram (a portion of the first diagram but enlarged) that the two forces are equidistant from the line joining the two legs on the floor. Since the lever arms are equal, then the torques will be equal if the forces are equal. Thus, to be in equilibrium, the two forces must be the same. If the force on the edge of the table is any bigger than the weight of the table, it will tip. Thus $\boxed{M > 25\text{ kg}}$ will cause the table to tip.







61. (*a*) The weight of the shelf exerts a downward force and a clockwise torque about the point where the shelf touches the wall. Thus there must be an upward force and a counterclockwise torque exerted by the slot for the shelf to be in equilibrium. Since any force exerted by the slot will have a short lever arm relative to the point where the shelf touches the wall, the upward force must be larger than the gravity force. Accordingly, there then must be a downward force exerted by the slot at its left edge, exerting no torque, but balancing the vertical forces.

(*b*) Calculate the values of the three forces by first taking torques about the left end of the shelf, with the net torque being zero, and then sum the vertical forces, with the sum being zero.

$$\sum \tau = F_{\text{Right}}\left(2.0\times10^{-2}\,\text{m}\right) - mg\left(17.0\times10^{-2}\,\text{m}\right) = 0 \quad \rightarrow$$

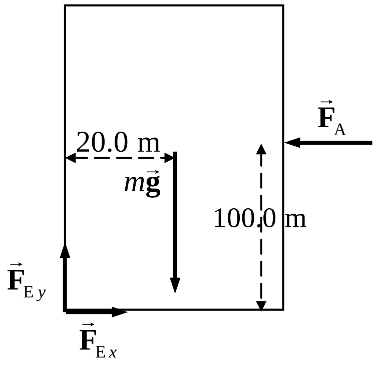
$$F_{\text{Right}} = \left(5.0\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)\left(\frac{17.0\times10^{-2}\,\text{m}}{2.0\times10^{-2}\,\text{m}}\right) = 416.5\,\text{N} \approx \boxed{4.2\times10^2\,\text{N}}$$

$$\sum F_y = F_{\text{Right}} - F_{\text{Left}} - mg \quad \rightarrow$$

$$F_{\text{Left}} = F_{\text{Right}} - mg = 416.5\text{ N} - \left(5.0\text{ kg}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{3.7\times10^2\,\text{N}}$$

(*c*) The torque exerted by the support about the left end of the rod is

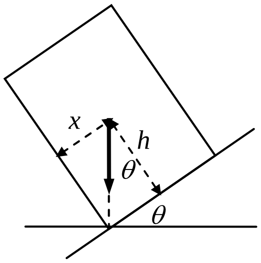
$$\tau = F_{\text{Right}}\left(2.0\times10^{-2}\,\text{m}\right) = \left(416.5\text{ N}\right)\left(2.0\times10^{-2}\,\text{m}\right) = \boxed{8.3\,\text{m}\bullet\text{N}}$$



62. Assume that the building has just begun to tip, so that it is essentially vertical, but that all of the force on the building due to contact with the Earth is at the lower left corner, as shown in the figure. Take torques about that corner, with counterclockwise torques as positive.

$$\begin{aligned}\sum \tau &= F_A\left(100.0\text{ m}\right) - mg\left(20.0\text{ m}\right)\\ &= \left(950\,\text{N/m}^2\right)\left(200.0\text{ m}\right)\left(70.0\text{ m}\right)\left(100.0\text{ m}\right)\\ &\qquad - \left(1.8\times10^7\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(20.0\text{ m}\right)\\ &= -2.2\times10^9\,\text{m}\bullet\text{N}\end{aligned}$$

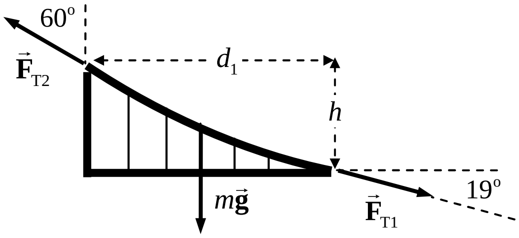
Since this is a negative torque, the building will tend to rotate clockwise, which means it will rotate back down to the ground. Thus $\boxed{\text{the building will not topple}}$.



63. The truck will not tip as long as a vertical line down from the CG is between the wheels. When that vertical line is at the wheel, it is in unstable equilibrium and will tip if the road is inclined any more. See the diagram for the truck at the tipping angle, showing the truck's weight vector.

$$\tan\theta = \frac{x}{h} \quad \rightarrow \quad \theta = \tan^{-1}\frac{x}{h} = \tan^{-1}\frac{1.2\text{ m}}{2.2\text{ m}} = \boxed{29^\circ}$$

64. Draw a force diagram for the cable that is supporting the right-hand section. The forces will be the tension at the left end, $\vec{F}_{T2}$, the tension at the right end, $\vec{F}_{T1}$, and the weight of the section, $m\vec{g}$. The weight acts at the midpoint of the horizontal span of the cable. The system is in equilibrium. Write Newton's 2nd law in both the *x* and *y* directions to find the tensions.


60°
$\vec{\mathbf{F}}_{\text{T2}}$
d_1
h
$m\vec{\mathbf{g}}$
19°
$\vec{\mathbf{F}}_{\text{T1}}$

$$\sum F_x = F_{\text{T1}}\cos 19^\circ - F_{\text{T2}}\sin 60^\circ = 0 \;\rightarrow$$

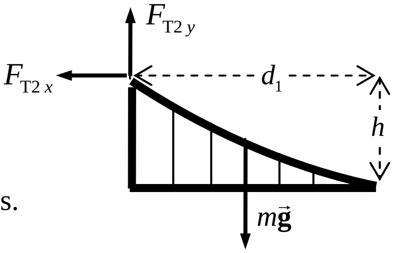
$$F_{\text{T2}} = F_{\text{T1}}\frac{\cos 19^\circ}{\sin 60^\circ}$$

$$\sum F_y = F_{\text{T2}}\cos 60^\circ - F_{\text{T1}}\sin 19^\circ - mg = 0 \;\rightarrow$$

$$F_{\text{T1}} = \frac{F_{\text{T2}}\cos 60^\circ - mg}{\sin 19^\circ} = \frac{F_{\text{T1}}\dfrac{\cos 19^\circ}{\sin 60^\circ}\cos 60^\circ - mg}{\sin 19^\circ} \;\rightarrow$$

$$F_{\text{T1}} = mg\frac{\sin 60^\circ}{\left(\cos 19^\circ\cos 60^\circ - \sin 19^\circ\sin 60^\circ\right)} = 4.539\,mg \approx \boxed{4.5\,mg}$$

$$F_{\text{T2}} = F_{\text{T1}}\frac{\cos 19^\circ}{\sin 60^\circ} = 4.539\frac{\cos 19^\circ}{\sin 60^\circ}mg = 4.956\,mg \approx \boxed{5.0\,mg}$$



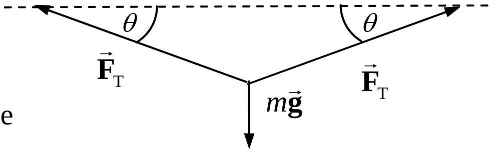
To find the height of the tower, take torques about the point where the roadway meets the ground, at the right side of the roadway. Note that then $\vec{\mathbf{F}}_{\text{T1}}$ will exert no torque. Take counterclockwise torques as positive. For purposes of calculating the torque due to $\vec{\mathbf{F}}_{\text{T2}}$, split it into *x* and *y* components.

$$\sum \tau = mg\left(\tfrac{1}{2}d_1\right) + F_{\text{T2}x}h - F_{\text{T2}y}d_1 = 0 \;\rightarrow$$

$$h = \frac{\left(F_{\text{T2}y} - \frac{1}{2}mg\right)}{F_{\text{T2}x}}d_1 = \frac{\left(F_{\text{T2}}\cos 60^\circ - \frac{1}{2}mg\right)}{F_{\text{T2}}\sin 60^\circ}d_1 = \frac{\left(4.956\,mg\cos 60^\circ - 0.50\,mg\right)}{4.956\,mg\sin 60^\circ}\left(343\text{ m}\right)$$

$$= \boxed{158\text{ m}}$$

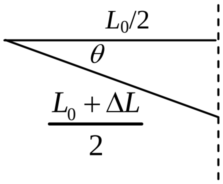
65. The radius of the wire can be determined from the relationship between stress and strain, expressed by equation (9-5).



$$\frac{F}{A} = E\frac{\Delta L}{L_0} \;\rightarrow\; A = \frac{FL_0}{E\Delta L} = \pi r^2 \;\rightarrow\; r = \sqrt{\frac{1}{\pi}\frac{F}{E}\frac{L_0}{\Delta L}}$$

Use the free-body diagram for the point of connection of the mass to the wire to determine the tension force in the wire.

$$\sum F_y = 2F_{\text{T}}\sin\theta - mg = 0 \;\rightarrow\; F_{\text{T}} = \frac{mg}{2\sin\theta} = \frac{\left(25\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)}{2\sin 12^\circ} = 589.2\text{ N}$$



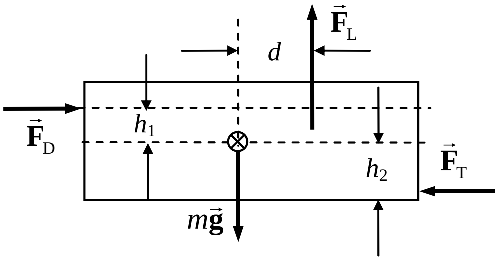
The fractional change in the length of the wire can be found from the geometry of the problem.

$$\cos\theta = \frac{L_0/2}{\dfrac{L_0+\Delta L}{2}} \;\rightarrow\; \frac{\Delta L}{L_0} = \frac{1}{\cos\theta} - 1 = \frac{1}{\cos 12^\circ} - 1 = 2.234\times 10^{-2}$$

Thus the radius is

$$r = \sqrt{\frac{1}{\pi}\frac{F_{\text{T}}}{E}\frac{L_0}{\Delta L}} = \sqrt{\frac{1}{\pi}\frac{589.2\text{ N}}{70\times 10^9\,\text{N/m}^2}\left(2.234\times 10^{-2}\right)} = \boxed{7.7\times 10^{-6}\,\text{m}}$$

66. The airplane is in equilibrium, and so the net force in each direction and the net torque are all equal to zero. First write Newton's 2nd law for both the horizontal and vertical directions, to find the values of the forces.



$$\sum F_x = F_D - F_T = 0 \quad \rightarrow \quad F_D = F_T = \boxed{5.0\times10^5\,\text{N}}$$

$$\sum F_y = F_L - mg = 0$$

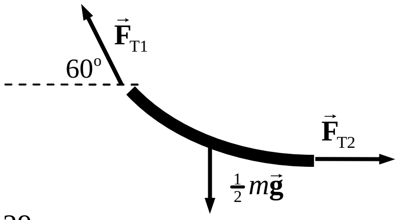
$$F_L = mg = \left(6.7\times10^4\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right) = \boxed{6.6\times10^5\,\text{N}}$$

Calculate the torques about the CM, calling counterclockwise torques positive.

$$\sum \tau = F_L d - F_D h_1 - F_T h_2 = 0$$

$$h_1 = \frac{F_L d - F_T h_2}{F_D} = \frac{\left(6.6\times10^5\,\text{N}\right)\left(3.2\text{ m}\right) - \left(5.0\times10^5\,\text{N}\right)\left(1.6\text{ m}\right)}{\left(5.0\times10^5\,\text{N}\right)} = \boxed{2.6\text{ m}}$$

67. Draw a free-body diagram for half of the cable. Write Newton's 2nd law for both the vertical and horizontal directions, with the net force equal to 0 in each direction.


$\vec{\mathbf{F}}_{\text{T1}}$
60°
$\vec{\mathbf{F}}_{\text{T2}}$
$\frac{1}{2}m\vec{\mathbf{g}}$

$$\sum F_y = F_{\text{T1}}\sin 60^\circ - \tfrac{1}{2}mg = 0 \quad \rightarrow \quad F_{\text{T1}} = \tfrac{1}{2}\frac{mg}{\sin 60^\circ} = 0.58mg$$

$$\sum F_x = F_{\text{T2}} - F_{\text{T1}}\cos 60^\circ = 0 \quad \rightarrow \quad F_{\text{T2}} = 0.58mg\left(\cos 60^\circ\right) = 0.29mg$$

So the results are:

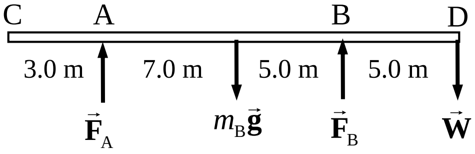
(*a*) $F_{\text{T2}} = \boxed{0.29mg}$

(*b*) $F_{\text{T1}} = \boxed{0.58mg}$

(*c*) The direction of the tension force is tangent to the cable at all points on the cable. Thus the direction of the tension force is $\boxed{\text{horizontal at the lowest point}}$, and is

$\boxed{60^\circ\text{above the horizontal at the attachment point}}$.

68. (*a*) For the extreme case of the beam being ready to tip, there would be no normal force at point A from the support. Use the free-body diagram to write the equation of rotational equilibrium under that condition to find the weight of the person, with $F_A = 0$. Take torques about the location of support B, and call counterclockwise torques positive. $\vec{\mathbf{W}}$ is the weight of the person.



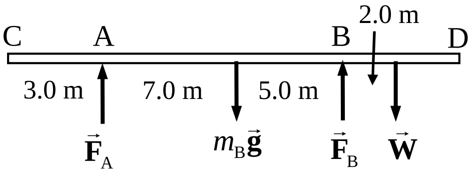
$$\sum \tau = m_B g\left(5.0\text{ m}\right) - W\left(5.0\text{ m}\right) = 0 \quad \rightarrow$$

$$W = m_B g = \boxed{550\text{ N}}$$

(*b*) With the person standing at point D, we have already assumed that $\boxed{F_A = 0}$. The net force in the vertical direction must also be zero.

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W = 550\text{ N} + 550\text{ N} = \boxed{1100\text{ N}}$$

(*c*) Now the person moves to a different spot, so the free-body diagram changes as shown. Again use the net torque about support B and then use the net vertical force.



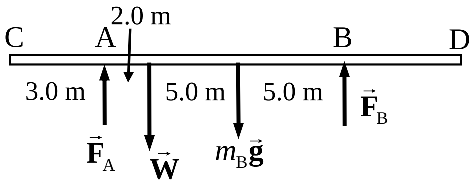
$$\sum \tau = m_B g(5.0\text{ m}) - W(2.0\text{ m}) - F_A(12.0\text{ m}) = 0$$

$$F_A = \frac{m_B g(5.0\text{ m}) - W(2.0\text{ m})}{12.0\text{ m}} = \frac{(550\text{ N})(3.0\text{ m})}{12.0\text{ m}}$$

$$= \boxed{140\text{ N}}$$

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W - F_A = 1100\text{ N} - 140\text{ N} = \boxed{960\text{ N}}$$

(*d*) Again the person moves to a different spot, so the free-body diagram changes again as shown. Again use the net torque about support B and then use the net vertical force.

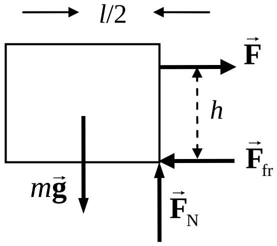


$$\sum \tau = m_B g(5.0\text{ m}) + W(10.0\text{ m}) - F_A(12.0\text{ m}) = 0$$

$$F_A = \frac{m_B g(5.0\text{ m}) + W(10.0\text{ m})}{12.0\text{ m}} = \frac{(550\text{ N})(5.0\text{ m}) + (550\text{ N})(10.0\text{ m})}{12.0\text{ m}} = \boxed{690\text{ N}}$$

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W - F_A = 1100\text{ N} - 690\text{ N} = \boxed{410\text{ N}}$$

69. If the block is on the verge of tipping, the normal force will be acting at the lower right corner of the block, as shown in the free-body diagram. The block will begin to rotate when the torque caused by the pulling force is larger than the torque caused by gravity. For the block to be able to slide, the pulling force must be as large as the maximum static frictional force. Write the equations of equilibrium for forces in the *x* and *y* directions and for torque with the conditions as stated above.



$$\sum F_y = F_N - mg = 0 \quad \rightarrow \quad F_N = mg$$

$$\sum F_x = F - F_{fr} = 0 \quad \rightarrow \quad F = F_{fr} = \mu_s F_N = \mu_s mg$$

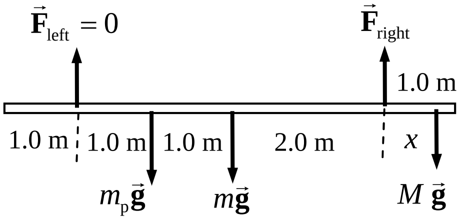
$$\sum \tau = mg\frac{l}{2} - Fh = 0 \quad \rightarrow \quad \frac{mgl}{2} = Fh = \mu_s mgh$$

Solve for the coefficient of friction in this limiting case, to find $\mu_s = \dfrac{l}{2h}$.

(*a*) If $\boxed{\mu_s < l/2h}$, then sliding will happen before tipping.

(*b*) If $\boxed{\mu_s > l/2h}$, then tipping will happen before sliding.

70. The limiting condition for the safety of the painter is the tension in the ropes. The ropes can only exert an upward tension on the scaffold. The tension will be least in the rope that is farther from the painter. The mass of the pail is m_p, the mass of the scaffold is m, and the mass of the painter is M.



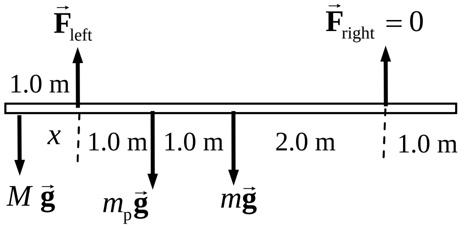
Find the distance to the right that the painter can walk before the tension in the left rope becomes zero. Take torques about the point where the right-side tension is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

$$\sum \tau = mg\left(2.0\,\text{m}\right) + m_{\text{p}}g\left(3.0\,\text{m}\right) - Mgx = 0 \quad \rightarrow$$

$$x = \frac{m\left(2.0\,\text{m}\right) + m_{\text{p}}\left(3.0\,\text{m}\right)}{M} = \frac{\left(25\text{ kg}\right)\left(2.0\text{ m}\right) + \left(4.0\text{ kg}\right)\left(3.0\text{ m}\right)}{60.0\text{ kg}} = 1.03\text{ m}$$

Since the maximum value for x is 1.0 m, the painter can walk to the right edge of the scaffold safely.

Now find the distance to the left that the painter can walk before the tension in the right rope becomes zero. Take torques about the point where the left-side tension is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.



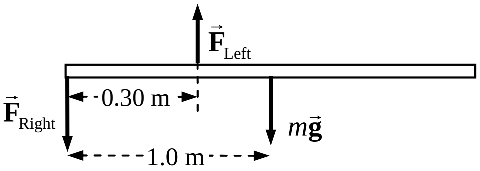
$\vec{\mathbf{F}}_{\text{left}}$
$\vec{\mathbf{F}}_{\text{right}} = 0$
1.0 m
x
1.0 m
1.0 m
2.0 m
1.0 m
$M\vec{\mathbf{g}}$
$m_{\text{p}}\vec{\mathbf{g}}$
$m\vec{\mathbf{g}}$

$$\sum \tau = Mgx - m_{\text{p}}g\left(1.0\,\text{m}\right) - mg\left(2.0\text{ m}\right) = 0 \quad \rightarrow$$

$$x = \frac{m\left(2.0\,\text{m}\right) + m_{\text{p}}\left(1.0\,\text{m}\right)}{M} = \frac{\left(25\text{ kg}\right)\left(2.0\text{ m}\right) + \left(4.0\text{ kg}\right)\left(1.0\text{ m}\right)}{60.0\text{ kg}} = 0.90\text{ m}$$

Thus the $\boxed{\text{left end}}$ is dangerous, and he can get within $\boxed{0.10\text{ m}}$ of the left end safely.

71. (*a*) The pole will exert a downward force and a clockwise torque about the woman's right hand. Thus there must be an upward force exerted by the left hand to cause a counterclockwise torque for the pole to have a net torque of zero. The force exerted by the right hand is then of such a magnitude and direction for the net vertical force on the pole to be zero.



$\vec{\mathbf{F}}_{\text{Left}}$
0.30 m
$\vec{\mathbf{F}}_{\text{Right}}$
$m\vec{\mathbf{g}}$
1.0 m

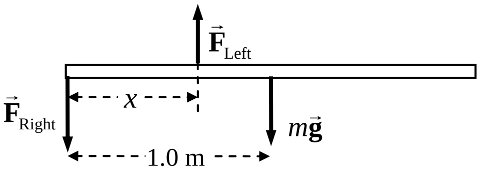
$$\sum \tau = F_{\text{Left}}\left(0.30\text{ m}\right) - mg\left(1.0\text{ m}\right) = 0 \quad \rightarrow$$

$$F_{\text{Left}} = mg\left(\frac{1.0\text{ m}}{0.30\text{ m}}\right) = \frac{\left(10.0\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)}{0.30} = 326.7\text{ N} \approx \boxed{3.3\times10^2\text{ N, upward}}$$

$$\sum F_y = F_{\text{Left}} - F_{\text{Right}} - mg = 0 \quad \rightarrow$$

$$F_{\text{Right}} = F_{\text{Left}} - mg = 326.7\text{ N} - \left(10.0\text{ kg}\right)\left(9.80\,\text{m/s}^2\right) = 228.7\text{ N} \approx \boxed{2.3\times10^2\text{ N, downward}}$$

(*b*) We see that the force due to the left hand is larger than the force due to the right hand, since both the right hand and gravity are downward. Set the left hand force equal to 150 N and calculate the location of the left hand, by setting the net torque equal to zero.



$\vec{\mathbf{F}}_{\text{Left}}$
x
$\vec{\mathbf{F}}_{\text{Right}}$
$m\vec{\mathbf{g}}$
1.0 m

$$\sum \tau = F_{\text{Left}}x - mg\left(1.0\text{ m}\right) = 0 \quad \rightarrow$$

$$x = \frac{mg}{F_{\text{Left}}}\left(1.0\text{ m}\right) = \frac{98.0\text{ N}}{150\text{ N}}\left(1.0\text{ m}\right) = \boxed{0.65\text{ m}}$$

As a check, calculate the force due to the right hand.

$$F_{\text{Right}} = F_{\text{Left}} - mg = 150\text{ N} - 98.0\text{ N} = 52\text{ N} \quad \text{OK}$$

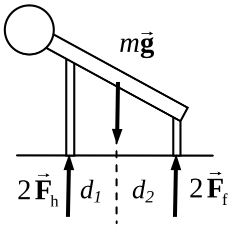
(*c*) Follow the same procedure, setting the left hand force equal to 85 N.

$$\sum \tau = F_{\text{Left}} x - mg\left(1.0\text{ m}\right) = 0 \;\; \rightarrow \;\; x = \frac{mg}{F_{\text{Left}}}\left(1.0\text{ m}\right) = \frac{98.0\text{ N}}{85\text{ N}}\left(1.0\text{ m}\right) = 1.153\text{ m} \approx \boxed{1.2\text{ m}}$$

$$F_{\text{Right}} = F_{\text{Left}} - mg = 85\text{ N} - 98.0\text{ N} = -13\text{ N} \quad \text{OK}$$

Note that now the force due to the right hand must be pulling upwards.

$\boxed{72.}$ The man is in equilibrium, so the net force and the net torque on him must be zero. We represent the force on the hands as twice the force on one hand, and the force on the feet as twice the force on one foot. Take torques about the point where his hands touch the ground, with counterclockwise as positive.



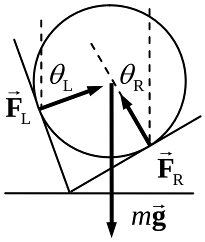
$$\sum \tau = 2F_{\text{f}}\left(d_1 + d_2\right) - mgd_1 = 0$$

$$F_{\text{f}} = \frac{mgd_1}{2\left(d_1 + d_2\right)} = \frac{\left(75\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)\left(0.40\text{ m}\right)}{2\left(1.35\text{ m}\right)} = 109\text{ N} \approx \boxed{1.1\times 10^2\text{ N}}$$

$$\sum F_y = 2F_{\text{h}} + 2F_{\text{f}} - mg = 0$$

$$F_{\text{h}} = \tfrac{1}{2}mg - F_{\text{f}} = \tfrac{1}{2}\left(75\text{ kg}\right)\left(9.8\,\text{m/s}^2\right) - 109\text{ N} = 259\text{ N} \approx \boxed{2.6\times 10^2\text{ N}}$$

73. The force on the sphere from each plane will be normal to the sphere, and so perpendicular to the plane at the point of contact. Use Newton's 2nd law in both the horizontal and vertical directions to determine the magnitudes of the forces.



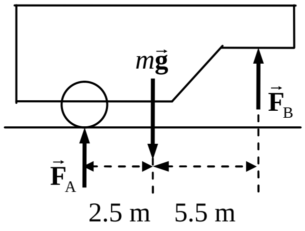
$$\sum F_x = F_{\text{L}}\sin\theta_{\text{L}} - F_{\text{R}}\sin\theta_{\text{R}} = 0 \;\; \rightarrow \;\; F_{\text{R}} = F_{\text{L}}\frac{\sin\theta_{\text{L}}}{\sin\theta_{\text{R}}} = F_{\text{L}}\frac{\sin 70^\circ}{\sin 30^\circ}$$

$$\sum F_y = F_{\text{L}}\cos\theta_{\text{L}} + F_{\text{R}}\cos\theta_{\text{R}} - mg = 0 \;\; \rightarrow \;\; F_{\text{L}}\left(\cos 70^\circ + \frac{\sin 70^\circ}{\sin 30^\circ}\cos 30^\circ\right) = mg$$

$$F_{\text{L}} = \frac{mg}{\left(\cos 70^\circ - \dfrac{\sin 70^\circ}{\sin 30^\circ}\cos 30^\circ\right)} = \frac{\left(20\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(\cos 70^\circ - \dfrac{\sin 70^\circ}{\sin 30^\circ}\cos 30^\circ\right)} = 99.51\,\text{N} \approx \boxed{1.0\times 10^2\text{ N}}$$

$$F_{\text{R}} = F_{\text{L}}\frac{\sin 70^\circ}{\sin 30^\circ} = \left(99.51\text{ N}\right)\frac{\sin 70^\circ}{\sin 30^\circ} = 187.0\,\text{N} \approx \boxed{1.9\times 10^2\text{ N}}$$

74. To find the normal force exerted on the road by the trailer tires, take the torques about point B, with counterclockwise torques as positive.



$$\sum \tau = mg\left(5.5\text{ m}\right) - F_{\text{A}}\left(8.0\text{ m}\right) = 0 \;\; \rightarrow$$

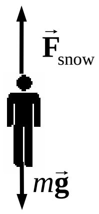
$$F_{\text{A}} = mg\left(\frac{5.5\text{ m}}{8.0\text{ m}}\right) = \left(2200\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)\left(\frac{5.5\text{ m}}{8.0\text{ m}}\right) = 14{,}823\text{ N}$$

$$\approx \boxed{1.5\times 10^4\text{ N}}$$

The net force in the vertical direction must be zero.

$$\sum F_y = F_{\text{B}} + F_{\text{A}} - mg = 0 \;\; \rightarrow \;\; F_{\text{B}} = mg - F_{\text{A}} = \left(2200\text{ kg}\right)\left(9.80\,\text{m/s}^2\right) - 14{,}823\text{ N} = \boxed{6.7\times 10^3\text{ N}}$$

75. Assume a constant acceleration as the person is brought to rest, with up as the positive direction. Use Eq. 2-11c to find the acceleration. From the acceleration, find the average force of the snow on the person, and compare the force per area to the strength of body tissue.



$$v^2 = v_0^2 - 2a\left(x - x_0\right) \;\rightarrow\; a = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{0 - \left(60\,\text{m/s}\right)^2}{2\left(-1.0\text{ m}\right)} = 1800\,\text{m/s}^2$$

$$\frac{F}{A} = \frac{ma}{A} = \frac{\left(75\text{ kg}\right)\left(1800\,\text{m/s}^2\right)}{0.30\text{m}^2} = 4.5\times10^5\text{ N/m}^2 < \text{ Tissue strength} = 5\times10^5\text{ N/m}^2$$

Since the average force on the person is less than the strength of body tissue, the person may escape serious injury. Certain parts of the body, such as the legs if landing feet first, may get more than the average force, though, and so still sustain injury.

76. The mass can be calculated from the equation for the relationship between stress and strain. The force causing the strain is the weight of the mass suspended from the wire.

$$\frac{\Delta L}{L_0} = \frac{1}{E}\frac{F}{A} = \frac{mg}{EA} \;\rightarrow\; m = \frac{EA}{g}\frac{\Delta L}{L_0} = \left(200\times10^9\text{ N/m}^2\right)\frac{\pi\left(1.0\times10^{-3}\text{m}\right)^2}{\left(9.80\,\text{m/s}^2\right)}\frac{0.030}{100} = \boxed{19\text{ kg}}$$

77. (*a*) From Example 7-6, the total force of the ground on one leg for a "stiff-legged landing" is $2.1\times10^5\,\text{N}/2 = 1.05\times10^5\,\text{N}$. The stress in the tibia bone is the force divided by the cross-sectional area of the bone.

$$\text{Stress} = \frac{F}{A} = \frac{1.05\times10^5\,\text{N}}{3.0\times10^{-4}\,\text{m}^2} = \boxed{3.5\times10^8\text{ N/m}^2}$$

(*b*) Since the stress from above is greater than the compressive strength of bone $\left(1.7\times10^8\text{ N/m}^2\right)$, the $\boxed{\text{bone will break}}$.

(*c*) From Example 7-6, the total force of the ground on one leg for a "bent-legged landing" is $4.9\times10^3\,\text{N}/2 = 2.45\times10^3\,\text{N}$. The stress in the tibia bone is the force divided by the cross-sectional area of the bone.

$$\text{Stress} = \frac{F}{A} = \frac{2.45\times10^3\,\text{N}}{3.0\times10^{-4}\,\text{m}^2} = \boxed{8.2\times10^6\text{ N/m}^2}$$

Since the stress from above is less than the compressive strength of bone $\left(1.7\times10^8\text{ N/m}^2\right)$, the $\boxed{\text{bone will not break}}$.

78. The number of supports can be found from the compressive strength of the wood. Since the wood will be oriented longitudinally, the stress will be parallel to the grain.

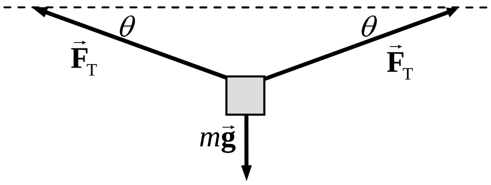
$$\frac{\text{Compressive Strength}}{\text{Safety Factor}} = \frac{\text{Load force on supports}}{\text{Area of supports}} = \frac{\text{Weight of roof}}{\left(\#\text{ supports}\right)\left(\text{area per support}\right)}$$

$$\left(\#\text{ supports}\right) = \frac{\text{Weight of roof}}{\left(\text{area per support}\right)}\frac{\text{Safety Factor}}{\text{Compressive Strength}}$$

$$= \frac{\left(1.26\times10^4\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(0.040\text{ m}\right)\left(0.090\text{ m}\right)}\frac{12}{\left(35\times10^6\text{ N/m}^2\right)} = 12\text{ supports}$$

Since there are to be 12 supports, there will be $\boxed{\text{6 supports on each side}}$. That means there will be 5 gaps on each side between the supports, and so $\text{Spacing} = \dfrac{10.0\text{ m}}{5\text{ gaps}} = \boxed{2.0\,\text{m/gap}}$.

79. The tension in the string when it breaks is found from the ultimate strength of nylon under tension, from Table 9-2.



$$\frac{F_\text{T}}{A} = \text{Tensile Strength} \rightarrow$$

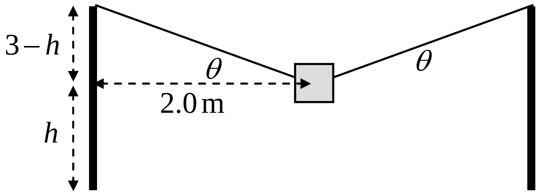
$$F_\text{T} = A(\text{Tensile Strength})$$

$$= \pi\left(5.00\times10^{-4}\,\text{m}\right)^2\left(500\times10^6\,\text{N/m}^2\right) = 392.7\text{ N}$$

From the force diagram for the box, we calculate the angle of the rope relative to the horizontal from Newton's 2nd law in the vertical direction. Note that since the tension is the same throughout the string, the angles must be the same so that the object does not accelerate horizontally.

$$\sum F_y = 2F_\text{T}\sin\theta - mg = 0 \rightarrow$$

$$\theta = \sin^{-1}\frac{mg}{2F_\text{T}} = \sin^{-1}\frac{(25\text{ kg})\left(9.80\,\text{m/s}^2\right)}{2(392.7\text{ N})} = 18.18^\circ$$



To find the height above the ground, consider the second diagram.

$$\tan\theta = \frac{3.00\,\text{m} - h}{2.00\,\text{m}} \rightarrow h = 3.00\,\text{m} - 2.00\,\text{m}(\tan\theta) = 3.00\,\text{m} - 2.00\,\text{m}\left(\tan 18.18^\circ\right) = \boxed{2.34\text{ m}}$$

80. The maximum compressive force in a column will occur at the bottom. The bottom layer supports the entire weight of the column, and so the compressive force on that layer is mg. For the column to be on the verge of buckling, the weight divided by the area of the column will be the compressive strength of the material. The mass of the column is its volume (area x height) times its density.

$$\frac{mg}{A} = \text{Compressive Strength} = \frac{hA\rho g}{A} \rightarrow h = \frac{\text{Compressive Strength}}{\rho g}$$

Note that the area of the column cancels out of the expression, and so the height does not depend on the cross-sectional area of the column.

(*a*) $h_\text{steel} = \dfrac{\text{Compressive Strength}}{\rho g} = \dfrac{500\times10^6\,\text{N/m}^2}{\left(7.8\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{6.5\times10^3\,\text{m}}$

(*b*) $h_\text{granite} = \dfrac{\text{Compressive Strength}}{\rho g} = \dfrac{170\times10^6\,\text{N/m}^2}{\left(2.7\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{6.4\times10^3\,\text{m}}$

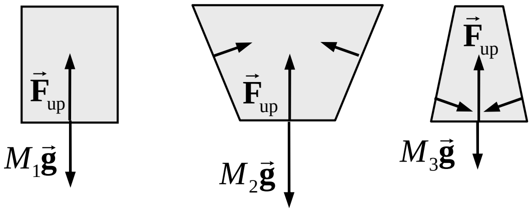
CHAPTER 10: Fluids

Answers to Questions

1. Density is the ratio of mass to volume. A high density may mean that lighter molecules are packed more closely together and thus a given amount of mass is occupying a smaller volume, making a higher density. An atom of gold weighs less than an atom of lead, because gold has a lower atomic mass, but the density of gold is higher than that of lead.

2. The air pressure inside the cabin of an airplane is lower than normal sea-level air pressure, evidenced by the sensation in the ears as the plane descends and the cabin reaches normal air pressure. If the container was opened at normal air pressure before the flight, then the pressure inside the container is normal air pressure. During the flight, while the pressure outside the container is lower than that inside the container, the difference in pressure causes a force from the inside towards the outside, and so the contents may get forced out of the container.

3. Let the first container have a mass of water M_1. The force of gravity on the water, acting downward, is thus $M_1 g$. The side walls only exert a force perpendicular to the walls, which is horizontal for the first container. Thus there must also be a net upward force of $F_{\text{up}} = M_1 g$ on the water to keep it at rest. This is the normal force from the bottom of the container. This upward force is the pressure at the bottom of the container times the area of the container, and is the same for all three containers. The second container has more water, and so has a larger downward force of gravity on it $\left(M_2 g > F_{\text{up}}\right)$, but the same upward force from the bottom of the container. Thus there must be additional upward force on the water. That upward force comes from the normal force of the slanted sides of the container pushing on the water. That force has an upward component – just enough to add to the normal force and balance the force of gravity. Similarly, the third container has less water, and so has a smaller downward force of gravity $\left(M_3 g < F_{\text{up}}\right)$. With the same upward normal force from the bottom, there must be more downward force on the water for it to be at rest. That downward force comes from the normal force of the slanted sides of the container pushing on the water. That force has a downward component – just enough to add to the gravity force and balance the force. The key to this problem is that the force on a container due to the hydrostatic pressure is always perpendicular to the surface of the water. According to Newton's 3rd law, the container will push back on the water in the exact opposite direction, also perpendicular to the surface of the container.


$\vec{\mathbf{F}}_{\text{up}}$
$M_1\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{up}}$
$M_2\vec{\mathbf{g}}$
$\vec{\mathbf{F}}_{\text{up}}$
$M_3\vec{\mathbf{g}}$

4. The sharp end of the pin (with a smaller area) will pierce the skin when pushed with a certain minimum force, while the same force applied in pushing the blunt end of the pen (with a larger area) into the skin does not pierce the skin. Thus it is pressure (force per unit area) that determines whether or not the skin is pierced.

5. The boiling water makes a relatively large quantity of steam inside the can. The gas inside the can (including the steam) will be at atmospheric pressure, but will be much warmer than the surroundings. When the gas in the sealed can cools, the steam condenses and the pressure drops greatly. This lowering of pressure on the inside means that the outside air pressure is higher than the pressure in the can, and thus the outside air pressure crushes the can.

   From a microscopic viewpoint, the water molecules are moving very fast when boiled to a vapor. Many of the water molecules escape the can, but the remaining ones can hold the can in its original shape because their high speeds mean they hit the walls with a large force and balance the force caused by the outside air pressure. After the lid is put on and the can is cooled, the water vapor molecules slow down (some will condense), and no gas can enter from outside the can. The slow molecules are not moving as fast and so put less force on the inside walls. The greater force from the outside air pressure then crushes the can.

6. The blood pressure exerted by the heart is to be measured. If the blood pressure is measured at a location *h* lower than the heart, the blood pressure will be higher than the pressure at the heart, due to the effects of gravity, by an amount ρgh. Likewise, if the blood pressure is measured at a location *h* higher than the heart, the blood pressure will be lower than the pressure at the heart, again due to the effects of gravity, by an amount ρgh.

7. Since the ice floats, the density of ice must be less than that of the water. The mass of the ice displaces a water volume equal to its weight, whether it is solid or liquid. Thus as the ice melts, the level in the glass stays the same. The ice displaces its melted volume.

8. The density of ice is greater than that of alcohol, so the ice cube will not float in a glass of alcohol. The ice cube will sink in the alcohol.

9. Both products have gas dissolved in them (the carbonation process), making their density lower than that of water. The Coke has a significant amount of sugar dissolved in it, increasing its density and making it greater than that of water. The Diet Coke does not have the dissolved sugar, and so its density remains less than that of water . Thus the Coke sinks, and the Diet Coke floats.

10. Iron ships are not solid iron. If they were, then they would sink. But the ships have quite a bit of open space in their volume (the volume between the deck and the hull, for instance), making their overall density less than that of water. The total mass of iron divided by the total volume of the boat is less than the density of water, and so the boat floats.

11. Refer to the diagram in the textbook. The pressure at the surface of both containers of liquid is atmospheric pressure. The pressure in each tube would thus be atmospheric pressure at the level of the surface of the liquid in each container. The pressure in each tube will decrease with height by an amount ρgh. Since the portion of the tube going into the lower container is longer than the portion of the tube going into the higher container, the pressure at the highest point on the right side is lower than the pressure at the highest point on the left side. This pressure difference causes liquid to flow from the left side of the tube to the right side of the tube. And as noted in the question, the tube must be filled with liquid before this argument can be made.

12. Since sand is denser than water, adding a given volume of sand (equal to the area of the barge times the depth of added sand) to the barge will require that an even greater volume of water be displaced to support the added weight. Thus the extra height of the barge caused by adding the sand will be more than compensated for by the extra depth to which the barge has to be submerged in order to float. Removing sand would have the opposite effect – the barge would get higher.

13. If you assume that the air is incompressible, then the answer is yes. The full balloon will have more weight (more downward force), due to the mass of the air in the balloon. The full balloon will also have an upward buoyant force on it, equal to the weight of the air displaced by the balloon. Since the balloon is both containing air and floating in air, the weight of the air inside the balloon is the same magnitude as the buoyant force. Thus the empty balloon will have the same apparent weight as the filled balloon.

    However, the air inside the balloon is compressed slightly compared to the outside air, and so has a higher density. This higher density means that the weight of the air inside the balloon is higher than the weight of the air it displaces, and so the filled balloon has a higher apparent weight than the empty balloon.

14. As the balloon rises, the air pressure outside the balloon will decrease and be lower than the pressure inside the balloon. The excess inside air pressure will cause the balloon to expand, lowering the pressure inside but stretching the balloon in the process. If, at launch, the material of the balloon were already stretched to the limit, the expansion of the balloon due to the decreasing outside air pressure would cause the balloon to burst. Thus the balloon is only filled to a fraction of the maximum volume.

15. (a) The boat displaces enough water to equal the weight of the boat. If the boat is removed from the water, the water will no longer be displaced and thus the water level will lower.
    (b) While the anchor is in the boat, the water displaced has a weight equal to that of the boat and the anchor together. If the anchor is placed on the shore, then less water will need to be displaced, and the water level will lower.
    (c) While the anchor is in the boat, the water displaced has a weight equal to that of the boat and the anchor together. If the anchor is dropped into the pool, the water displaced is equal to the weight of the boat (which will float) and the weight of a volume of water equal to the volume of the anchor (which will sink). Since the anchor is more dense than the water, it takes more water displacement to hold up the anchor (while in the boat) than is displaced when the anchor is in the water. Thus the water level will lower when the anchor is thrown overboard.

16. Salt water has a higher density than fresh water. Thus you have to displace less salt water to equal your weight than you do in fresh water. You then float "higher" in the salt water.

17. The papers will move toward each other. Bernoulli's principle says that as the speed of the gas flow increases, the pressure decreases (when there is no appreciable change in height). So as the air passes between the papers, the air pressure between the papers is lowered. The air pressure on the outside of the papers is then greater than that between the papers, and so the papers are pushed together.

18. As the car drives through the air, the air inside the car is stationary with respect to the top, but the outside air is moving with respect to the top. There is no appreciable change in height between the two sides of the canvas top. By Bernoulli's principle, the outside air pressure near the canvas top will be less than the inside air pressure. That difference in pressure results in a force that makes the top bulge outward.

19. The roofs are actually pushed off from the inside. By Bernoulli's principle, the fast moving winds of the tornado or hurricane cause the air pressure above the roof to be quite low, but the pressure inside the house is still near normal levels. There is no appreciable change in height between the two sides of the roof. This pressure difference, combined with the large surface area of the roof, gives a very large force which can push the roof off the house. That is why it is advised to open some windows if

a tornado is imminent, so that the pressure inside the house can somewhat equalize with the outside pressure.

20. It is possible. Due to viscosity, some of the air near the train will be pulled along at a speed approximately that of the train. By Bernoulli's principle, that air will be at a lower pressure than air further away from the train. That difference in pressure results in a force towards the train, which could push a lightweight child towards the train.

21. Water will not flow from the holes when the cup and water are in free fall. The acceleration due to gravity is the same for all falling objects (ignoring friction), and so the cup and water would fall together. For the water to flow out of the holes while falling would mean that the water would have an acceleration larger than the acceleration due to gravity. Another way to consider the situation is that there will no longer be a pressure difference between the top and bottom of the cup of water, since the lower water molecules don't need to hold up the upper water molecules.

22. The lift generated by a wind depends on the speed of the air relative to the wing. For example, a model in a wind tunnel will have lift even though the model isn't moving relative to the ground. By taking off into the wind, the speed of the air relative to the wing is the sum of the plane's speed and the wind speed. This allows the plane to take off at a lower ground speed, requiring a shorter runway.

23. As the stream of water falls, its vertical speed is faster away from the faucet than close to it, due to the acceleration caused by gravity. Since the water is essentially incompressible, Eq. 10-4b applies, which says that a faster flow has a smaller cross-sectional area. Thus the faster moving water has a narrower stream.

24. When the two ships are moving parallel to each other, water between them starts to move with them due to viscosity. There will be more water moving along with the ships in between them then on their outside sides. According to Bernoulli's principle, this moving water is at a lower pressure than stationary water, further away from the ship. Each ship will thus experience a net force towards the other ship and be drawn in towards the other ship. Thus they risk colliding.

Solutions to Problems

1. The mass is found from the density of granite and the volume of granite.

$$m=\rho V=\left(2.7\times10^{3}\ \mathrm{kg/m^{3}}\right)\left(10^{8}\ \mathrm{m^{3}}\right)=2.7\times10^{11}\ \mathrm{kg}\approx\boxed{3\times10^{11}\ \mathrm{kg}}$$

2. The mass is found from the density of air and the volume of air.

$$m=\rho V=\left(1.29\ \mathrm{kg/m^{3}}\right)\left(4.8\ \mathrm{m}\right)\left(3.8\ \mathrm{m}\right)\left(2.8\ \mathrm{m}\right)=\boxed{66\ \mathrm{kg}}$$

3. The mass is found from the density of gold and the volume of gold.

$$m=\rho V=\left(19.3\times10^{3}\ \mathrm{kg/m^{3}}\right)\left(0.60\ \mathrm{m}\right)\left(0.28\ \mathrm{m}\right)\left(0.18\ \mathrm{m}\right)=\boxed{5.8\times10^{2}\ \mathrm{kg}}\quad\left(\sim1300\ \mathrm{lb}\right)$$

4. Assume that your density is that of water, and that your mass is 75 kg.

$$V=\frac{m}{\rho}=\frac{75\ \mathrm{kg}}{1.00\times10^{3}\ \mathrm{kg/m^{3}}}=\boxed{7.5\times10^{-2}\ \mathrm{m^{3}}}=75\ \mathrm{L}$$

5. To find the specific gravity of the fluid, take the ratio of the density of the fluid to that of water, noting that the same volume is used for both liquids.

$$SJ_{\text{fluid}} = \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \frac{(m/V)_{\text{fluid}}}{(m/V)_{\text{water}}} = \frac{m_{\text{fluid}}}{m_{\text{water}}} = \frac{88.78\text{ g} - 35.00\text{ g}}{98.44\text{ g} - 35.00\text{ g}} = \boxed{0.8477}$$

6. The specific gravity of the mixture is the ratio of the density of the mixture to that of water. To find the density of the mixture, the mass of antifreeze and the mass of water must be known.

$$m_{\text{antifreeze}} = \rho_{\text{antifreeze}} V_{\text{antifreeze}} = SG_{\text{antifreeze}} \rho_{\text{water}} V_{\text{antifreeze}} \qquad m_{\text{water}} = \rho_{\text{water}} V_{\text{water}}$$

$$SG_{\text{mixture}} = \frac{\rho_{\text{mixture}}}{\rho_{\text{water}}} = \frac{m_{\text{mixture}}/V_{\text{mixture}}}{\rho_{\text{water}}} = \frac{m_{\text{antifreeze}} + m_{\text{water}}}{\rho_{\text{water}} V_{\text{mixture}}} = \frac{SG_{\text{antifreeze}} \rho_{\text{water}} V_{\text{antifreeze}} + \rho_{\text{water}} V_{\text{water}}}{\rho_{\text{water}} V_{\text{mixture}}}$$

$$= \frac{SG_{\text{antifreeze}} V_{\text{antifreeze}} + V_{\text{water}}}{V_{\text{mixture}}} = \frac{(0.80)(5.0\text{ L}) + 4.0\text{ L}}{9.0\text{ L}} = \boxed{0.89}$$

7. (*a*) The pressure exerted on the floor by the chair leg is caused by the leg pushing down on the floor. That downward push is the reaction to the normal force of the floor on the leg, and the normal forced is equal to the weight of the leg. Thus the pressure is

$$P_{\text{chair}} = \frac{W_{leg}}{A} = \frac{\frac{1}{4}(60\text{ kg})(9.8\,\text{m/s}^2)}{(0.020\text{ cm}^2)\left(\frac{1\text{ m}}{100\text{ cm}}\right)^2} = 7.35\times10^7\text{ N/m}^2 \approx \boxed{7\times10^7\text{ N/m}^2}.$$

(*b*) The pressure exerted by the elephant is

$$P_{\text{elephant}} = \frac{W_{\text{elephant}}}{A} = \frac{(1500\text{ kg})(9.8\,\text{m/s}^2)}{(800\text{ cm}^2)\left(\frac{1\text{ m}}{100\text{ cm}}\right)^2} = 1.84\times10^5\text{ N/m}^2 \approx \boxed{2\times10^5\text{ N/m}^2}.$$

Note that the chair pressure is larger than the elephant pressure by a factor of about 400.

8. From Equation 10-3b, the pressure difference is

$$\Delta P = \rho g \Delta h = (1.05\times10^3\text{ kg/m}^3)(9.80\,\text{m/s}^2)(1.60\,\text{m}) = 1.646\times10^4\text{ N/m}^2\left(\frac{1\text{ mm-Hg}}{133\,\text{N/m}^2}\right)$$

$$= \boxed{124\,\text{mm-Hg}}$$

9. (*a*) The total force of the atmosphere on the table will be the air pressure times the area of the table.

$$F = PA = (1.013\times10^5\text{ N/m}^2)(1.6\text{ m})(2.9\text{ m}) = \boxed{4.7\times10^5\,\text{N}}$$

(*b*) Since the atmospheric pressure is the same on the underside of the table (the height difference is minimal), the upward force of air pressure is the same as the downward force of air on the top of the table, $\boxed{4.7\times10^5\,\text{N}}$.

10. The pressure difference on the lungs is the pressure change from the depth of water

$$\Delta P = \rho g \Delta h \;\rightarrow\; \Delta h = \frac{\Delta P}{\rho g} = \frac{(85\,\text{mm-Hg})\left(\frac{133\,\text{N/m}^2}{1\text{ mm-Hg}}\right)}{(1.00\times10^3\text{ kg/m}^3)(9.80\,\text{m/s}^2)} = 1.154\text{ m} \approx \boxed{1.2\text{ m}}$$

11. The sum of the force exerted by the pressure in each tire is equal to the weight of the car.

$$mg = 4PA \quad \rightarrow \quad m = \frac{4PA}{g} = \frac{4\left(2.40\times10^5\ \mathrm{N/m^2}\right)\left(220\,\mathrm{cm^2}\right)\left(\dfrac{1\ \mathrm{m^2}}{10^4\,\mathrm{cm^2}}\right)}{\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{2.2\times10^3\,\mathrm{kg}}$$

12. The force exerted by the gauge pressure will be equal to the weight of the vehicle.

$$mg = PA = P\left(\pi r^2\right) \quad \rightarrow$$

$$m = \frac{P\pi r^2}{g} = \frac{\pi\left(17.0\ \mathrm{atm}\right)\left(\dfrac{1.013\times10^5\ \mathrm{N/m^2}}{1\ \mathrm{atm}}\right)\left(0.140\ \mathrm{m}\right)^2}{\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{1.08\times10^4\,\mathrm{kg}}$$

13. The height is found from Eq. 10-3a, using normal atmospheric pressure.

$$P = \rho g h \quad \rightarrow \quad h = \frac{P}{\rho g} = \frac{1.013\times10^5\ \mathrm{N/m^2}}{\left(0.79\times10^3\ \mathrm{kg/m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{13\ \mathrm{m}}$$

14. (*a*) The absolute pressure is given by Eq. 10-3c, and the total force is the absolute pressure times the area of the bottom of the pool.

$$P = P_0 + \rho g h = 1.013\times10^5\ \mathrm{N/m^2} + \left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)\left(2.0\ \mathrm{m}\right)$$

$$= \boxed{1.21\times10^5\ \mathrm{N/m^2}}$$

$$F = PA = \left(1.21\times10^5\ \mathrm{N/m^2}\right)\left(22.0\ \mathrm{m}\right)\left(8.5\ \mathrm{m}\right) = \boxed{2.3\times10^7\,\mathrm{N}}$$

(*b*) The pressure against the side of the pool, near the bottom, will be the same as the pressure at the bottom, $P = \boxed{1.21\times10^5\ \mathrm{N/m^2}}$

15. If the atmosphere were of uniform density, then the pressure at any height *h* would be $P = P_0 - \rho g h$. At the top of the uniform atmosphere, the pressure would be 0. Thus solve for the height at which the pressure becomes 0, using a density of half of sea-level atmospheric density.

$$P = P_0 - \rho g h = 0 \quad \rightarrow \quad h = \frac{P_0}{\rho g} = \frac{\left(1.013\times10^5\ \mathrm{N/m^2}\right)}{\frac{1}{2}\left(1.29\,\mathrm{kg/m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{1.60\times10^4\,\mathrm{m}}$$

16. The pressure at points a and b are equal since they are the same height in the same fluid. If they were unequal, the fluid would flow. Calculate the pressure at both a and b, starting with atmospheric pressure at the top surface of each liquid, and then equate those pressures.

$$P_a = P_b \quad \rightarrow \quad P_0 + \rho_{\mathrm{oil}} g h_{\mathrm{oil}} = P_0 + \rho_{\mathrm{water}} g h_{\mathrm{water}} \quad \rightarrow \quad \rho_{\mathrm{oil}} h_{\mathrm{oil}} = \rho_{\mathrm{water}} h_{\mathrm{water}} \quad \rightarrow$$

$$\rho_{\mathrm{oil}} = \frac{\rho_{\mathrm{water}} h_{\mathrm{water}}}{h_{\mathrm{oil}}} = \frac{\left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(0.272\ \mathrm{m} - 0.0941\ \mathrm{m}\right)}{\left(0.272\ \mathrm{m}\right)} = \boxed{6.54\times10^2\ \mathrm{kg/m^3}}$$

17. (*a*) The gauge pressure is given by Eq. 10-3a. The height is the height from the bottom of the hill to the top of the water tank.

$$P_G = \rho g h = \left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)\left[5.0\ \mathrm{m} + \left(110\ \mathrm{m}\right)\sin 58^\circ\right] = \boxed{9.6\times10^5\ \mathrm{N/m^2}}$$

11. The sum of the force exerted by the pressure in each tire is equal to the weight of the car.

$$mg = 4PA \;\rightarrow\; m = \frac{4PA}{g} = \frac{4\left(2.40\times10^{5}\ \text{N/m}^2\right)\left(220\,\text{cm}^2\right)\left(\dfrac{1\ \text{m}^2}{10^4\,\text{cm}^2}\right)}{\left(9.80\,\text{m/s}^2\right)} = \boxed{2.2\times10^{3}\,\text{kg}}$$

12. The force exerted by the gauge pressure will be equal to the weight of the vehicle.

$$mg = PA = P\left(\pi r^2\right) \;\rightarrow$$

$$m = \frac{P\pi r^2}{g} = \frac{\pi\left(17.0\ \text{atm}\right)\left(\dfrac{1.013\times10^{5}\ \text{N/m}^2}{1\ \text{atm}}\right)\left(0.140\ \text{m}\right)^2}{\left(9.80\,\text{m/s}^2\right)} = \boxed{1.08\times10^{4}\,\text{kg}}$$

13. The height is found from Eq. 10-3a, using normal atmospheric pressure.

$$P = \rho g h \;\rightarrow\; h = \frac{P}{\rho g} = \frac{1.013\times10^{5}\ \text{N/m}^2}{\left(0.79\times10^{3}\ \text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{13\ \text{m}}$$

14. (*a*) The absolute pressure is given by Eq. 10-3c, and the total force is the absolute pressure times the area of the bottom of the pool.

$$P = P_0 + \rho g h = 1.013\times10^{5}\ \text{N/m}^2 + \left(1.00\times10^{3}\ \text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(2.0\ \text{m}\right)$$

$$= \boxed{1.21\times10^{5}\ \text{N/m}^2}$$

$$F = PA = \left(1.21\times10^{5}\ \text{N/m}^2\right)\left(22.0\ \text{m}\right)\left(8.5\ \text{m}\right) = \boxed{2.3\times10^{7}\,\text{N}}$$

(*b*) The pressure against the side of the pool, near the bottom, will be the same as the pressure at the bottom, $P = \boxed{1.21\times10^{5}\ \text{N/m}^2}$

15. If the atmosphere were of uniform density, then the pressure at any height *h* would be $P = P_0 - \rho g h$. At the top of the uniform atmosphere, the pressure would be 0. Thus solve for the height at which the pressure becomes 0, using a density of half of sea-level atmospheric density.

$$P = P_0 - \rho g h = 0 \;\rightarrow\; h = \frac{P_0}{\rho g} = \frac{\left(1.013\times10^{5}\ \text{N/m}^2\right)}{\frac{1}{2}\left(1.29\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{1.60\times10^{4}\,\text{m}}$$

16. The pressure at points a and b are equal since they are the same height in the same fluid. If they were unequal, the fluid would flow. Calculate the pressure at both a and b, starting with atmospheric pressure at the top surface of each liquid, and then equate those pressures.

$$P_a = P_b \;\rightarrow\; P_0 + \rho_{\text{oil}} g h_{\text{oil}} = P_0 + \rho_{\text{water}} g h_{\text{water}} \;\rightarrow\; \rho_{\text{oil}} h_{\text{oil}} = \rho_{\text{water}} h_{\text{water}} \;\rightarrow$$

$$\rho_{\text{oil}} = \frac{\rho_{\text{water}} h_{\text{water}}}{h_{\text{oil}}} = \frac{\left(1.00\times10^{3}\ \text{kg/m}^3\right)\left(0.272\ \text{m} - 0.0941\ \text{m}\right)}{\left(0.272\ \text{m}\right)} = \boxed{6.54\times10^{2}\ \text{kg/m}^3}$$

17. (*a*) The gauge pressure is given by Eq. 10-3a. The height is the height from the bottom of the hill to the top of the water tank.

$$P_{\text{G}} = \rho g h = \left(1.00\times10^{3}\ \text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left[5.0\ \text{m} + \left(110\ \text{m}\right)\sin 58^{\circ}\right] = \boxed{9.6\times10^{5}\ \text{N/m}^2}$$

(*b*) The water would be able to shoot up to the top of the tank (ignoring any friction).

$$h = 5.0\text{ m} + (110\text{ m})\sin 58^\circ = \boxed{98\text{ m}}$$

18. The minimum gauge pressure would cause the water to come out of the faucet with very little speed. This means the gauge pressure needed must be enough to hold the water at this elevation. Use Eq. 10-3a.

$$P_G = \rho g h = (1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(38\text{ m}) = \boxed{3.7\times10^5\text{ N/m}^2}$$

19. The pressure in the tank is atmospheric pressure plus the pressure difference due to the column of mercury, as given in Eq. 10-3c.

(*a*) $P = P_0 + \rho g h = 1.04\,\text{bar} + \rho_{\text{Hg}} g h$

$$= (1.04\,\text{bar})\left(\frac{1.00\times10^5\text{ N/m}^2}{1\text{bar}}\right) + (13.6\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.280\text{ m}) = \boxed{1.41\times10^5\text{ N/m}^2}$$

(*b*) $$P = (1.04\,\text{bar})\left(\frac{1.00\times10^5\text{ N/m}^2}{1\text{bar}}\right) + (13.6\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(-0.042\text{ m}) = \boxed{9.84\times10^4\text{ N/m}^2}$$

20. (*a*) The mass of water in the tube is the volume of the tube times the density of water.

$$m = \rho V = \rho\pi r^2 h = (1.00\times10^3\text{ kg/m}^3)\pi(0.30\times10^{-2}\text{ m})^2(12\text{ m}) = 0.3393\,\text{kg} \approx \boxed{0.34\,\text{kg}}$$

(*b*) The net force exerted on the lid is the gauge pressure of the water times the area of the lid. The gauge pressure is found from Eq. 10-3b.

$$F = P_{\text{gauge}}A = \rho g h \pi R^2 = (1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(12\text{ m})\pi(0.21\text{m})^2 = \boxed{1.6\times10^4\text{ N}}$$

21. From section 9-5, the change in volume due to pressure change is $\frac{\Delta V}{V_0} = -\frac{\Delta P}{B}$, where B is the bulk modulus of the water, given in Table 9-1. The pressure increase with depth for a fluid of constant density is given by $\Delta P = \rho g \Delta h$, where Δh is the depth of descent. If the density change is small, then we can use the initial value of the density to calculate the pressure change, and so $\Delta P \cong \rho_0 g \Delta h$. Finally, consider a constant mass of water. That constant mass will relate the volume and density at the two locations by $M = \rho V = \rho_0 V_0$. Combine these relationships and solve for the density deep in the sea, ρ.

$$\rho V = \rho_0 V_0 \quad \rightarrow$$

$$\rho = \frac{\rho_0 V_0}{V} = \frac{\rho_0 V_0}{V_0 + \Delta V} = \frac{\rho_0 V_0}{V_0 + \left(-V_0\frac{\Delta P}{B}\right)} = \frac{\rho_0}{1 - \frac{\rho_0 g h}{B}} = \frac{1025\,\text{kg/m}^3}{1 - \frac{(1025\,\text{kg/m}^3)(9.80\text{ m/s}^2)(6.0\times10^3\text{m})}{2.0\times10^9\text{ N/m}^2}}$$

$$= 1057\,\text{kg/m}^3 \approx \boxed{1.06\times10^3\text{ kg/m}^3}$$

$$\rho/\rho_0 = \frac{1057}{1025} = 1.03$$

The density at the 6 km depth is about $\boxed{\text{3\% larger}}$ than the density at the surface.

22. The difference in the actual mass and the apparent mass is the mass of the water displaced by the rock. The mass of the water displaced is the volume of the rock times the density of water, and the volume of the rock is the mass of the rock divided by its density. Combining these relationships yields an expression for the density of the rock.

$$m_{\text{actual}} - m_{\text{apparent}} = \Delta m = \rho_{\text{water}} V_{\text{rock}} = \rho_{\text{water}} \frac{m_{\text{rock}}}{\rho_{\text{rock}}} \quad \rightarrow$$

$$\rho_{\text{rock}} = \rho_{\text{water}} \frac{m_{\text{rock}}}{\Delta m} = \left(1.00\times10^3 \text{ kg/m}^3\right) \frac{9.28 \text{ kg}}{9.28 \text{ kg} - 6.18 \text{ kg}} = \boxed{2.99\times10^3 \text{ kg/m}^3}$$

23. If the aluminum is floating, then the net force on it is zero. The buoyant force on the aluminum must be equal to its weight. The buoyant force is equal to the weight of the mercury displaced by the submerged aluminum.

$$F_{\text{buoyant}} = m_{\text{Al}} g \quad \rightarrow \quad \rho_{\text{Hg}} g V_{\text{submerged}} = \rho_{\text{Al}} g V_{\text{total}} \quad \rightarrow$$

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Hg}}} = \frac{2.70\times10^3 \text{ kg/m}^3}{13.6\times10^3 \text{ kg/m}^3} = \boxed{0.199} \approx 20\%$$

24. (*a*) When the hull is submerged, both the buoyant force and the tension force act upward on the hull, and so their sum is equal to the weight of the hull. The buoyant force is the weight of the water displaced.

$$T + F_{\text{buoyant}} = mg \quad \rightarrow$$

$$T = mg - F_{\text{buoyant}} = m_{\text{hull}} g - \rho_{\text{water}} V_{\text{sub}} g = m_{\text{hull}} g - \rho_{\text{water}} \frac{m_{\text{hull}}}{\rho_{\text{hull}}} g = m_{\text{hull}} g \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{hull}}}\right)$$

$$= \left(1.8\times10^4 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(1 - \frac{1.00\times10^3 \text{ kg/m}^3}{7.8\times10^3 \text{ kg/m}^3}\right) = 1.538\times10^5 \text{ N} \approx \boxed{1.5\times10^5 \text{ N}}$$

(*b*) When the hull is completely out of the water, the tension in the crane's cable must be equal to the weight of the hull.

$$T = mg = \left(1.8\times10^4 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right) = 1.764\times10^5 \text{ N} \approx \boxed{1.8\times10^5 \text{ N}}$$

25. The buoyant force of the balloon must equal the weight of the balloon plus the weight of the helium in the balloon plus the weight of the load. For calculating the weight of the helium, we assume it is at 0°C and 1 atm pressure. The buoyant force is the weight of the air displaced by the volume of the balloon.

$$F_{\text{buoyant}} = \rho_{\text{air}} V_{\text{balloon}} g = m_{\text{He}} g + m_{\text{balloon}} g + m_{\text{cargo}} g \quad \rightarrow$$

$$m_{\text{cargo}} = \rho_{\text{air}} V_{\text{balloon}} - m_{\text{He}} - m_{\text{balloon}} = \rho_{\text{air}} V_{\text{balloon}} - \rho_{\text{He}} V_{\text{balloon}} - m_{\text{balloon}} = \left(\rho_{\text{air}} - \rho_{\text{He}}\right) V_{\text{balloon}} - m_{\text{balloon}}$$

$$= \left(1.29 \text{ kg/m}^3 - 0.179 \text{ kg/m}^3\right) \tfrac{4}{3}\pi \left(7.35 \text{ m}\right)^3 - 930 \text{ kg} = \boxed{920 \text{ kg}} = 9.0\times10^3 \text{ N}$$

26. The difference in the actual mass and the apparent mass is the mass of the water displaced by the legs. The mass of the water displaced is the volume of the legs times the density of water, and the volume of the legs is the mass of the legs divided by their density. The density of the legs is assumed to be the same as that of water. Combining these relationships yields an expression for the mass of the legs.

$$m_{\text{actual}} - m_{\text{apparent}} = \Delta m = \rho_{\text{water}} V_{\text{legs}} = \rho_{\text{water}} \frac{m_{\text{legs}}}{\rho_{\text{legs}}} = 2m_{\text{leg}} \quad \rightarrow$$

$$m_{\text{leg}} = \frac{\Delta m}{2} = \frac{\left(78\text{ kg} - 54\text{ kg}\right)}{2} = \boxed{12\text{ kg}}$$

27. The apparent weight is the actual weight minus the buoyant force. The buoyant force is weight of a mass of water occupying the volume of the metal sample.

$$m_{\text{apparent}} g = m_{\text{metal}} g - F_{\text{B}} = m_{\text{metal}} g - V_{\text{metal}} \rho_{\text{H}_2\text{O}} g = m_{\text{metal}} g - \frac{m_{\text{metal}}}{\rho_{\text{metal}}} \rho_{\text{H}_2\text{O}} g \quad \rightarrow$$

$$m_{\text{apparent}} = m_{\text{metal}} - \frac{m_{\text{metal}}}{\rho_{\text{metal}}} \rho_{\text{H}_2\text{O}} \quad \rightarrow$$

$$\rho_{\text{metal}} = \frac{m_{\text{metal}}}{\left(m_{\text{metal}} - m_{\text{apparent}}\right)} \rho_{\text{H}_2\text{O}} = \frac{63.5\,\text{g}}{\left(63.5\,\text{g} - 55.4\,\text{g}\right)} \left(1000\,\text{kg/m}^3\right) = 7840\,\text{kg/m}^3$$

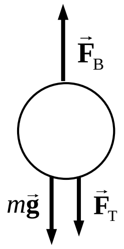
Based on the density value, the metal is probably $\boxed{\text{iron or steel}}$.

28. The difference in the actual mass and the apparent mass of the aluminum is the mass of the air displaced by the aluminum. The mass of the air displaced is the volume of the aluminum times the density of air, and the volume of the aluminum is the actual mass of the aluminum divided by the density of aluminum. Combining these relationships yields an expression for the actual mass.

$$m_{\text{actual}} - m_{\text{apparent}} = \rho_{\text{air}} V_{\text{Al}} = \rho_{\text{air}} \frac{m_{\text{actual}}}{\rho_{\text{Al}}} \quad \rightarrow$$

$$m_{\text{actual}} = \frac{m_{\text{apparent}}}{1 - \dfrac{\rho_{\text{air}}}{\rho_{\text{Al}}}} = \frac{2.0000\text{ kg}}{1 - \dfrac{1.29\,\text{kg/m}^3}{2.70 \times 10^3\text{ kg/m}^3}} = \boxed{2.0010\text{ kg}}$$

29. There are three forces on the chamber: the weight of the chamber, the tension in the cable, and the buoyant force. See the free-body diagram.



(*a*) The buoyant force is the weight of water displaced by the chamber.

$$F_{\text{buoyant}} = \rho_{\text{H}_2\text{O}} V_{\text{chamber}} g = \rho_{\text{H}_2\text{O}} \tfrac{4}{3}\pi R_{\text{chamber}}^3 g$$
$$= \left(1.025 \times 10^3\text{ kg/m}^3\right) \tfrac{4}{3}\pi \left(2.60\,\text{m}\right)^3 \left(9.80\,\text{m/s}^2\right)$$
$$= 7.3953 \times 10^5\,\text{N} \approx \boxed{7.40 \times 10^5\,\text{N}}$$

(b) To find the tension, use Newton's 2nd law for the stationary chamber.

$$F_{\text{buoyant}} = mg + F_{\text{T}} \quad \rightarrow$$

$$F_{\text{T}} = F_{\text{buoyant}} - mg = 7.3953 \times 10^5\,\text{N} - \left(7.44 \times 10^4\text{kg}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{1.04 \times 10^4\,\text{N}}$$

30. (*a*) The buoyant force is the weight of the water displaced, using the density of sea water.

$$F_{\text{buoyant}} = m_{\substack{\text{water}\\\text{displaced}}} g = \rho_{\text{water}} V_{\text{displaced}} g$$
$$= \left(1.025 \times 10^3\text{ kg/m}^3\right)\left(65.0\text{ L}\right)\left(\frac{1 \times 10^{-3}\text{m}^3}{1\text{ L}}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{653\,\text{N}}$$

(*b*) The weight of the diver is $m_{\text{diver}} g = (68.0\text{ kg})(9.80\,\text{m/s}^2) = 666\text{ N}$. Since the buoyant force is not as large as her weight, $\boxed{\text{she will sink}}$, although it will be very gradual since the two forces are almost the same.

31. (*a*) The difference in the actual mass and the apparent mass of the aluminum ball is the mass of the liquid displaced by the ball. The mass of the liquid displaced is the volume of the ball times the density of the liquid, and the volume of the ball is the mass of the ball divided by its density. Combining these relationships yields an expression for the density of the liquid.

$$m_{\text{actual}} - m_{\text{apparent}} = \Delta m = \rho_{\text{liquid}} V_{\text{ball}} = \rho_{\text{liquid}} \frac{m_{\text{ball}}}{\rho_{\text{Al}}} \rightarrow$$

$$\rho_{\text{liquid}} = \frac{\Delta m}{m_{\text{ball}}} \rho_{\text{Al}} = \frac{(3.40\text{ kg} - 2.10\text{ kg})}{3.40\text{ kg}} (2.70\times10^3\text{ kg/m}^3) = \boxed{1.03\times10^3\text{ kg/m}^3}$$

(*b*) Generalizing the relation from above, we have $\boxed{\rho_{\text{liquid}} = \frac{m_{\text{object}} - m_{\text{apparent}}}{m_{\text{object}}} \rho_{\text{object}}}$

32. The difference in the actual mass and the apparent mass is the mass of the alcohol displaced by the wood. The mass of the alcohol displaced is the volume of the wood times the density of the alcohol, the volume of the wood is the mass of the wood divided by the density of the wood, and the density of the alcohol is its specific gravity times the density of water.

$$m_{\text{actual}} - m_{\text{apparent}} = \rho_{\text{alc}} V_{\text{wood}} = \rho_{\text{alc}} \frac{m_{\text{actual}}}{\rho_{\text{wood}}} = \text{SG}_{\text{alc}} \rho_{H_2O} \frac{m_{\text{actual}}}{\rho_{\text{wood}}} \rightarrow$$

$$\frac{\rho_{\text{wood}}}{\rho_{H_2O}} = \text{SG}_{\text{wood}} = \text{SG}_{\text{alc}} \frac{m_{\text{actual}}}{(m_{\text{actual}} - m_{\text{apparent}})} = (0.79) \frac{0.48\,\text{kg}}{(0.48\,\text{kg} - 0.047\,\text{kg})} = \boxed{0.88}$$

$\boxed{33.}$ The buoyant force on the ice is equal to the weight of the ice, since it floats.

$$F_{\text{buoyant}} = W_{\text{ice}} \rightarrow m_{\substack{\text{seawater}\\\text{submerged}}} g = m_{\text{ice}} g \rightarrow m_{\substack{\text{seawater}\\\text{submerged}}} = m_{\text{ice}} \rightarrow$$

$$\rho_{\text{seawater}} V_{\text{seawater}} = \rho_{\text{ice}} V_{\text{ice}} \rightarrow (SG)_{\text{seawater}} \rho_{\text{water}} V_{\substack{\text{submerged}\\\text{ice}}} = (SG)_{\text{ice}} \rho_{\text{water}} V_{\text{ice}} \rightarrow$$

$$(SG)_{\text{seawater}} V_{\substack{\text{submerged}\\\text{ice}}} = (SG)_{\text{ice}} V_{\text{ice}} \rightarrow$$

$$V_{\substack{\text{submerged}\\\text{ice}}} = \frac{(SG)_{\text{ice}}}{(SG)_{\text{seawater}}} V_{\text{ice}} = \frac{0.917}{1.025} V_{\text{ice}} = 0.895 V_{\text{ice}}$$

Thus the fraction above the water is $V_{\text{above}} = V_{\text{ice}} - V_{\text{submerged}} = 0.105\ V_{\text{ice}}$ or $\boxed{10.5\%}$

34. For the combination to just barely sink, the total weight of the wood and lead must be equal to the total buoyant force on the wood and the lead.

$$F_{\text{weight}} = F_{\text{buoyant}} \rightarrow m_{\text{wood}} g + m_{\text{Pb}} g = V_{\text{wood}} \rho_{\text{water}} g + V_{\text{Pb}} \rho_{\text{water}} g \rightarrow$$

$$m_{\text{wood}} + m_{\text{Pb}} = \frac{m_{\text{wood}}}{\rho_{\text{wood}}} \rho_{\text{water}} + \frac{m_{\text{Pb}}}{\rho_{\text{Pb}}} \rho_{\text{water}} \rightarrow m_{\text{Pb}} \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{Pb}}}\right) = m_{\text{wood}} \left(\frac{\rho_{\text{water}}}{\rho_{\text{wood}}} - 1\right) \rightarrow$$

$$m_{\text{Pb}} = m_{\text{wood}} \frac{\left(\dfrac{\rho_{\text{water}}}{\rho_{\text{wood}}} - 1\right)}{\left(1 - \dfrac{\rho_{\text{water}}}{\rho_{\text{Pb}}}\right)} = m_{\text{wood}} \frac{\left(\dfrac{1}{SG_{\text{wood}}} - 1\right)}{\left(1 - \dfrac{1}{SG_{\text{Pb}}}\right)} = (5.25\,\text{kg}) \frac{\left(\dfrac{1}{0.50} - 1\right)}{\left(1 - \dfrac{1}{11.3}\right)} = \boxed{5.76\,\text{kg}}$$

35. Use Eq. 10-4b, the equation of continuity for an incompressible fluid, to compare blood flow in the aorta and in the major arteries.

$$(Av)_{\text{aorta}} = (Av)_{\text{arteries}} \quad \rightarrow$$

$$v_{\text{arteries}} = \frac{A_{\text{aorta}}}{A_{\text{arteries}}} v_{\text{aorta}} = \frac{\pi (1.2\,\text{cm})^2}{2.0\,\text{cm}^2} (40\,\text{cm/s}) = 90.5\,\text{cm/s} \approx \boxed{0.9\,\text{m/s}}$$

36. We apply the equation of continuity at constant density, Eq. 10-4b.

Flow rate out of duct = Flow rate into room

$$A_{\text{duct}} v_{\text{duct}} = \pi r^2 v_{\text{duct}} = \frac{V_{\text{room}}}{t_{\substack{\text{to fill}\\ \text{room}}}} \quad \rightarrow \quad v_{\text{duct}} = \frac{V_{\text{room}}}{\pi r^2 t_{\substack{\text{to fill}\\ \text{room}}}} = \frac{(9.2\text{ m})(5.0\text{ m})(4.5\text{ m})}{\pi (0.15\text{ m})^2 (16\,\text{min}) \left(\dfrac{60\text{ s}}{1\text{ min}}\right)} = \boxed{3.1\,\text{m/s}}$$

37. Bernoulli's equation is evaluated with $v_1 = v_2 = 0$. Let point 1 be the initial point, and point 2 be the final point.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow \quad P_1 + \rho g y_1 = P_2 + \rho g y_2 \quad \rightarrow$$

$$P_2 - P_1 = \rho g (y_1 - y_2) \quad \rightarrow \quad \Delta P = -\rho g \Delta y$$

But a change in *y* coordinate is the opposite of the change in depth which is what is represented in Eq. 10-3b. So our final result is $\Delta P = \rho g \Delta h$, Eq. 10-3b.

38. We may apply Torricelli's theorem., Eq. 10-6.

$$v_1 = \sqrt{2g(y_2 - y_1)} = \sqrt{2(9.80\,\text{m/s}^2)(4.6\,\text{m})} = \boxed{9.5\,\text{m/s}}$$

39. The volume flow rate of water from the hose, multiplied times the time of filling, must equal the volume of the pool.

$$(Av)_{\text{hose}} = \frac{V_{\text{pool}}}{t} \quad \rightarrow \quad t = \frac{V_{\text{pool}}}{A_{\text{hose}} v_{\text{hose}}} = \frac{\pi (3.05\,\text{m})^2 (1.2\,\text{m})}{\pi \left[\tfrac{1}{2}\left(\tfrac{5}{8}\right)'' \left(\dfrac{1\,\text{m}}{39.37''}\right)\right]^2 (0.40\,\text{m/s})} = 4.429 \times 10^5\,\text{s}$$

$$4.429 \times 10^5\,\text{s} \left(\frac{1\,\text{day}}{60 \times 60 \times 24\,\text{s}}\right) = \boxed{5.1\text{ days}}$$

40. Apply Bernoulli's equation with point 1 being the water main, and point 2 being the top of the spray. The velocity of the water will be zero at both points. The pressure at point 2 will be atmospheric pressure. Measure heights from the level of point 1.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow$$

$$P_1 - P_{\text{atm}} = \rho g y_2 = (1.00 \times 10^3\text{ kg/m}^3)(9.8\,\text{m/s}^2)(15\text{ m}) = \boxed{1.5 \times 10^5\text{ N/m}^2}$$

41. Use the equation of continuity (Eq. 10-4) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 10-5) to relate the pressure conditions at the two locations. We assume that the two locations are at the same height. Express the pressures as atmospheric pressure plus gauge pressure. We use subscript "1" for the larger diameter, and subscript "2" for the smaller diameter.

$$A_1v_1 = A_2v_2 \quad\rightarrow\quad v_2 = v_1\frac{A_1}{A_2} = v_1\frac{\pi r_1^2}{\pi r_2^2} = v_1\frac{r_1^2}{r_2^2}$$

$$P_0 + P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_0 + P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad\rightarrow$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 = P_2 + \tfrac{1}{2}\rho v_1^2\frac{r_1^4}{r_2^4} \quad\rightarrow\quad v_1 = \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho\left(\frac{r_1^4}{r_2^4} - 1\right)}} \quad\rightarrow$$

$$A_1v_1 = \pi r_1^2\sqrt{\frac{2\left(P_1 - P_2\right)}{\rho\left(\frac{r_1^4}{r_2^4} - 1\right)}} = \pi\left(3.0\times10^{-2}\,\text{m}\right)^2\sqrt{\frac{2\left(32.0\times10^3\,\text{Pa} - 24.0\times10^3\,\text{Pa}\right)}{\left(1.0\times10^3\,\text{kg/m}^3\right)\left(\frac{\left(3.0\times10^{-2}\,\text{m}\right)^4}{\left(2.0\times10^{-2}\,\text{m}\right)^4} - 1\right)}}$$

$$= \boxed{5.6\times10^{-3}\,\text{m}^3/\text{s}}$$

42. The pressure head can be interpreted as an initial height for the water, with a speed of 0 and atmospheric pressure. Apply Bernoulli's equation to the faucet location and the pressure head location to find the speed of the water at the faucet, and then calculate the volume flow rate. Since the faucet is open, the pressure there will be atmospheric as well.

$$P_{\text{faucet}} + \tfrac{1}{2}\rho v_{\text{faucet}}^2 + \rho g y_{\text{faucet}} = P_{\text{head}} + \tfrac{1}{2}\rho v_{\text{head}}^2 + \rho g y_{\text{head}} \quad\rightarrow$$

$$v_{\text{faucet}}^2 = \frac{2}{\rho}\left(P_{\text{head}} - P_{\text{faucet}}\right) + v_{\text{head}}^2 + 2g\left(y_{\text{head}} - y_{\text{faucet}}\right) = 2gy_{\text{head}} \quad\rightarrow$$

$$v_{\text{faucet}} = \sqrt{2gy_{\text{head}}}$$

$$\text{Volume flow rate} = Av = \pi r^2\sqrt{2gy_{\text{head}}} = \pi\left[\tfrac{1}{2}\left(1.85\times10^{-2}\,\text{m}\right)\right]^2\sqrt{2\left(9.80\,\text{m/s}^2\right)\left(15.0\,\text{m}\right)}$$

$$= \boxed{4.6\times10^{-3}\,\text{m}^3/\text{s}}$$

43. We assume that there is no appreciable height difference between the two sides of the roof. Then the net force on the roof due to the air is the difference in pressure on the two sides of the roof, times the area of the roof. The difference in pressure can be found from Bernoulli's equation.

$$P_{\text{inside}} + \tfrac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \tfrac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}} \quad\rightarrow$$

$$P_{\text{inside}} - P_{\text{outside}} = \tfrac{1}{2}\rho_{\text{air}}v_{\text{outside}}^2 = \frac{F_{\text{air}}}{A_{\text{roof}}} \quad\rightarrow$$

$$F_{\text{air}} = \tfrac{1}{2}\rho_{\text{air}}v_{\text{outside}}^2A_{\text{roof}} = \tfrac{1}{2}\left(1.29\,\text{kg/m}^3\right)\left(35\,\text{m/s}\right)^2\left(240\,\text{m}^2\right) = \boxed{1.9\times10^5\,\text{N}}$$

$\boxed{44.}$ The lift force would be the difference in pressure between the two wing surfaces, times the area of the wing surface. The difference in pressure can be found from Bernoulli's equation. We consider the two surfaces of the wing to be at the same height above the ground. Call the bottom surface of the wing point 1, and the top surface point 2.

$$P_1+\tfrac{1}{2}\rho v_1^2+\rho g y_1=P_2+\tfrac{1}{2}\rho v_2^2+\rho g y_2 \quad\rightarrow\quad P_1-P_2=\tfrac{1}{2}\rho\left(v_2^2-v_1^2\right)$$

$$F_{\text{lift}}=\left(P_1-P_2\right)\left(\text{Area of wing}\right)=\tfrac{1}{2}\rho\left(v_2^2-v_1^2\right)A$$

$$=\tfrac{1}{2}\left(1.29\,\text{kg}/\text{m}^3\right)\left[\left(260\,\text{m}/\text{s}\right)^2-\left(150\,\text{m}/\text{s}\right)^2\right]\left(78\,\text{m}^2\right)=\boxed{2.3\times10^6\,\text{N}}$$

45. The air pressure inside the hurricane can be estimated using Bernoulli's equation. Assume the pressure outside the hurricane is air pressure, the speed of the wind outside the hurricane is 0, and that the two pressure measurements are made at the same height.

$$P_{\text{inside}}+\tfrac{1}{2}\rho v_{\text{inside}}^2+\rho g y_{\text{inside}}=P_{\text{outside}}+\tfrac{1}{2}\rho v_{\text{outside}}^2+\rho g y_{\text{outside}} \quad\rightarrow$$

$$P_{\text{inside}}=P_{\text{outside}}-\tfrac{1}{2}\rho_{\text{air}}v_{\text{inside}}^2$$

$$=1.013\times10^5\,\text{Pa}-\tfrac{1}{2}\left(1.29\,\text{kg}/\text{m}^3\right)\left[\left(300\,\text{km}/\text{h}\right)\left(\frac{1000\,\text{m}}{\text{km}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)\right]^2$$

$$=\boxed{9.7\times10^4\,\text{Pa}}\approx0.96\,\text{atm}$$

46. Use the equation of continuity (Eq. 10-4) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 10-5) to relate the conditions at the street to those at the top floor. Express the pressures as atmospheric pressure plus gauge pressure.

$$A_{\text{street}}v_{\text{street}}=A_{\text{top}}v_{\text{top}} \quad\rightarrow$$

$$v_{\text{top}}=v_{\text{street}}\frac{A_{\text{street}}}{A_{\text{top}}}=\left(0.60\,\text{m}/\text{s}\right)\frac{\pi\left(5.0\times10^{-2}\,\text{m}\right)^2}{\pi\left(2.6\times10^{-2}\,\text{m}\right)^2}=2.219\,\text{m}/\text{s}\approx\boxed{2.2\,\text{m}/\text{s}}$$

$$P_0+P_{\substack{\text{gauge}\\\text{street}}}+\tfrac{1}{2}\rho v_{\text{street}}^2+\rho g y_{\text{street}}=P_0+P_{\substack{\text{gauge}\\\text{top}}}+\tfrac{1}{2}\rho v_{\text{top}}^2+\rho g y_{\text{top}} \quad\rightarrow$$

$$P_{\substack{\text{gauge}\\\text{top}}}=P_{\substack{\text{gauge}\\\text{street}}}+\tfrac{1}{2}\rho\left(v_{\text{street}}^2-v_{\text{top}}^2\right)+\rho g y\left(y_{\text{street}}-y_{\text{top}}\right)$$

$$=\left(3.8\,\text{atm}\right)\left(\frac{1.013\times10^5\,\text{Pa}}{\text{atm}}\right)+\tfrac{1}{2}\left(1.00\times10^3\,\text{kg}/\text{m}^3\right)\left[\left(0.60\,\text{m}/\text{s}\right)^2-\left(2.219\,\text{m}/\text{s}\right)^2\right]^2$$

$$+\left(1.00\times10^3\,\text{kg}/\text{m}^3\right)\left(9.8\,\text{m}/\text{s}^2\right)\left(-18\,\text{m}\right)$$

$$=2.063\times10^5\,\text{Pa}\left(\frac{1\,\text{atm}}{1.013\times10^5\,\text{Pa}}\right)\approx\boxed{2.0\,\text{atm}}$$

47. (*a*) Apply the equation of continuity and Bernoulli's equation at the same height to the wide and narrow portions of the tube.

$$A_2v_2=A_1v_1 \quad\rightarrow\quad v_2=v_1\frac{A_1}{A_2}$$

$$P_1+\tfrac{1}{2}\rho v_1^2=P_2+\tfrac{1}{2}\rho v_2^2 \quad\rightarrow\quad \frac{2\left(P_1-P_2\right)}{\rho}=v_2^2-v_1^2 \quad\rightarrow$$

$$\left(v_1\frac{A_1}{A_2}\right)^2-v_1^2=\frac{2\left(P_1-P_2\right)}{\rho} \quad\rightarrow\quad v_1^2\left(\frac{A_1^2}{A_2^2}-\frac{A_2^2}{A_2^2}\right)=\frac{2\left(P_1-P_2\right)}{\rho} \quad\rightarrow$$

$$v_1^2 = \frac{2A_2^2\left(P_1 - P_2\right)}{\rho\left(A_1^2 - A_2^2\right)} \rightarrow \boxed{v_1 = A_2\sqrt{\frac{2\left(P_1 - P_2\right)}{\rho\left(A_1^2 - A_2^2\right)}}}$$

(*b*) $v_1 = A_2\sqrt{\frac{2\left(P_1 - P_2\right)}{\rho\left(A_1^2 - A_2^2\right)}}$

$$= \pi\left[\tfrac{1}{2}\left(0.010\,\text{m}\right)\right]^2\sqrt{\frac{2\left(18\,\text{mm Hg}\right)\left(\frac{133\,\text{N/m}^2}{\text{mm Hg}}\right)}{\left(1000\,\text{kg/m}^3\right)\left(\pi^2\left[\tfrac{1}{2}\left(0.030\,\text{m}\right)\right]^4 - \pi^2\left[\tfrac{1}{2}\left(0.010\,\text{m}\right)\right]^4\right)}} = \boxed{0.24\,\text{m/s}}$$

48. Apply both Bernoulli's equation and the equation of continuity between the two openings of the tank. Note that the pressure at each opening will be atmospheric pressure.

$$A_2v_2 = A_1v_1 \rightarrow v_2 = v_1\frac{A_1}{A_2}$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow v_1^2 - v_2^2 = 2g\left(y_2 - y_1\right) = 2gh$$

$$v_1^2 - \left(v_1\frac{A_1}{A_2}\right) = 2gh \rightarrow v_1^2\left(1 - \frac{A_1^2}{A_2^2}\right) = 2gh \rightarrow \boxed{v_1 = \sqrt{\frac{2gh}{\left(1 - A_1^2/A_2^2\right)}}}$$

49. Use Bernoulli's equation to find the speed of the liquid as it leaves the opening, assuming that the speed of the liquid at the top is 0, and that the pressure at each opening is air pressure.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow v_1 = \sqrt{2g\left(h_2 - h_1\right)}$$

(*a*) Since the liquid is launched horizontally, the initial vertical speed is zero. Use Eq. 2-11(a) for constant acceleration to find the time of fall, with upward as the positive direction. Then multiply the time of fall times v_1, the (constant) horizontal speed.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_yt^2 \rightarrow 0 = h_1 + 0 - \tfrac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h_1}{g}}$$

$$\Delta x = v_1t = \sqrt{2g\left(h_2 - h_1\right)}\sqrt{\frac{2h_1}{g}} = \boxed{2\sqrt{\left(h_2 - h_1\right)h_1}}$$

(*b*) We seek some height h_1' such that $2\sqrt{\left(h_2 - h_1\right)h_1} = 2\sqrt{\left(h_2 - h_1'\right)h_1'}$.

$$2\sqrt{\left(h_2 - h_1\right)h_1} = 2\sqrt{\left(h_2 - h_1'\right)h_1'} \rightarrow \left(h_2 - h_1\right)h_1 = \left(h_2 - h_1'\right)h_1' \rightarrow$$

$$h_1'^2 - h_2h_1' + \left(h_2 - h_1\right)h_1 = 0 \rightarrow$$

$$h_1' = \frac{h_2 \pm \sqrt{h_2^2 - 4\left(h_2 - h_1\right)h_1}}{2} = \frac{h_2 \pm \sqrt{h_2^2 - 4h_1h_2 + 4h_1^2}}{2} = \frac{h_2 \pm \left(h_2 - 2h_1\right)}{2} = \frac{2h_2 - 2h_1}{2}, \frac{2h_1}{2}$$

$$\boxed{h_1' = h_2 - h_1}$$

50. Apply Eq. 10-8. Use the average radius to calculate the plate area.

$$F=\eta A\frac{v}{l}\quad\rightarrow\quad \eta=\frac{Fl}{Av}=\frac{\left(\dfrac{\tau}{r_{\text{inner}}}\right)\left(r_{\text{outer}}-r_{\text{inner}}\right)}{\left(2\pi r_{\text{avg}}h\right)\left(\omega r_{\text{inner}}\right)}$$

$$=\frac{\left(\dfrac{0.024\,\text{m}\bullet\text{N}}{0.0510\,\text{m}}\right)\left(0.20\times10^{-2}\,\text{m}\right)}{2\pi\left(0.0520\,\text{m}\right)\left(0.120\,\text{m}\right)\left(62\dfrac{\text{rev}}{\text{min}}\times\dfrac{2\pi\,\text{rad}}{\text{rev}}\times\dfrac{1\,\text{min}}{60\,\text{s}}\right)\left(0.0510\,\text{m}\right)}=\boxed{7.2\times10^{-2}\,\text{Pa}\bullet\text{s}}$$

51. From Poiseuille's equation, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus $Q/R^4=\frac{V}{t}\frac{1}{R^4}$ is constant. If the volume of water used to water the garden is to be same in both cases, then tR^4 is constant.

$$t_1R_1^4=t_2R_2^4\quad\rightarrow\quad t_2=t_1\left(\frac{R_1}{R_2}\right)^4=t_1\left(\frac{3/8}{5/8}\right)^4=0.13t_1$$

Thus the $\boxed{\text{time has been cut by 87\%}}$.

52. Use Poiseuille's equation to find the pressure difference.

$$Q=\frac{\pi R^4\left(P_2-P_1\right)}{8\eta L}\quad\rightarrow$$

$$\left(P_2-P_1\right)=\frac{8Q\eta L}{\pi R^4}=\frac{8\left[5.6\times10^{-3}\dfrac{\text{L}}{\text{min}}\times\dfrac{1\,\text{min}}{60\,\text{s}}\times\dfrac{1\times10^{-3}\,\text{m}^3}{1\,\text{L}}\right]\left(0.2\,\text{Pa}\bullet\text{s}\right)\left(5.5\times10^{-2}\,\text{m}\right)}{\pi\left(0.9\times10^{-3}\,\text{m}\right)^4}$$

$$=\boxed{4.0\times10^{3}\text{Pa}}$$

53. Use Poiseuille's equation to find the pressure difference.

$$Q=\frac{\pi R^4\left(P_2-P_1\right)}{8\eta L}\quad\rightarrow$$

$$\left(P_2-P_1\right)=\frac{8Q\eta L}{\pi R^4}=\frac{8\left(450\,\text{cm}^3/\text{s}\right)\left(10^{-6}\,\text{m}^3/\text{cm}^3\right)\left(0.20\,\text{Pa}\bullet\text{s}\right)\left(1.9\times10^{3}\,\text{m}\right)}{\pi\left(0.145\,\text{m}\right)^4}=985.1\,\text{Pa}\approx\boxed{990\,\text{Pa}}$$

54. Use Poiseuille's equation to find the radius, and then double the radius to the diameter.

$$Q=\frac{\pi R^4\left(P_2-P_1\right)}{8\eta L}\quad\rightarrow$$

$$d=2R=2\left[\frac{8\eta LQ}{\pi\left(P_2-P_1\right)}\right]^{1/4}=2\left[\frac{8\left(1.8\times10^{-5}\,\text{Pa}\bullet\text{s}\right)\left(21.0\,\text{m}\right)\left(\dfrac{9.0\times12.0\times4.0\,\text{m}^3}{600\,\text{s}}\right)}{\pi\left(0.71\times10^{-3}\,\text{atm}\right)\left(1.013\times10^{5}\,\text{Pa}/\text{atm}\right)}\right]^{1/4}=\boxed{0.11\,\text{m}}$$

55. The pressure drop per cm can be found from Poiseuille's equation, using a length of 1 cm. The volume flow rate is area of the aorta times the speed of the moving blood.

$$Q=\frac{\pi R^4\left(P_2-P_1\right)}{8\eta L} \quad \rightarrow$$

$$\frac{\left(P_2-P_1\right)}{L}=\frac{8\eta Q}{\pi R^4}=\frac{8\eta\pi R^2 v}{\pi R^4}=\frac{8\eta v}{R^2}=\frac{8\left(4\times10^{-3}\,\text{Pa}\bullet\text{s}\right)\left(0.4\,\text{m/s}\right)}{\left(1.2\times10^{-2}\,\text{m}\right)^2}=88.9\,\text{Pa/m}=\boxed{0.89\,\text{Pa/cm}}$$

56. From Poiseuille's equation, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus Q/R^4 is constant.

$$\frac{Q_{\text{final}}}{R^4_{\text{final}}}=\frac{Q_{\text{initial}}}{R^4_{\text{initial}}} \quad \rightarrow \quad R_{\text{final}}=\left(\frac{Q_{\text{final}}}{Q_{\text{initial}}}\right)^{1/4}R_{\text{initial}}=\left(0.25\right)^{1/4}R_{\text{initial}}=0.707R_{\text{initial}}$$

Thus the radius has been reduced by about $\boxed{29\%}$.

57. (*a*) $Re=\dfrac{2\overline{v}r\rho}{\eta}=\dfrac{2\left(0.40\,\text{cm/s}\right)\left(1.2\times10^{-2}\,\text{m}\right)\left(1.05\times10^{3}\,\text{kg/m}^3\right)}{4\times10^{-3}\,\text{Pa}\bullet\text{s}}=2520$

The flow is $\boxed{\text{turbulent}}$ at this speed.

(*b*) Since the velocity is doubled the Reynolds number will double to 5040. The flow is $\boxed{\text{turbulent}}$ at this speed.

58. The fluid pressure must be 18 torr higher than air pressure as it exits the needle, so that the blood will enter the vein. The pressure at the entrance to the needle must be higher than 18 torr, due to the viscosity of the blood. To produce that excess pressure, the blood reservoir is placed above the level of the needle. Use Poiseuille's equation to calculate the excess pressure needed due to the viscosity, and then use Eq. 10-3b to find the height of the blood reservoir necessary to produce that excess pressure.

$$Q=\frac{\pi R^4\left(P_2-P_1\right)}{8\eta_{\text{blood}}L} \quad \rightarrow \quad P_2=P_1+\frac{8\eta_{\text{blood}}LQ}{\pi R^4}=\rho_{\text{blood}}g\Delta h \quad \rightarrow$$

$$\Delta h=\frac{1}{\rho_{\text{blood}}g}\left(P_1+\frac{8\eta_{\text{blood}}LQ}{\pi R^4}\right)$$

$$=\frac{1}{\left(1.05\times10^{3}\,\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.80\,\text{m/s}^2\right)}\left(\begin{array}{c}\left(18\,\text{mm-Hg}\right)\left(\dfrac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right)+\\ \dfrac{8\left(4\times10^{-3}\,\text{Pa}\bullet\text{s}\right)\left(4.0\times10^{-2}\,\text{m}\right)\left(\dfrac{4.0\times10^{-6}\,\text{m}^3}{60\text{ s}}\right)}{\pi\left(0.20\times10^{-3}\,\text{m}\right)^4}\end{array}\right)$$

$$=\boxed{1.8\,\text{m}}$$

59. In Figure 10-35, we have $\gamma=F/2L$. Use this to calculate the force.

$$\gamma=\frac{F}{2L}=\frac{5.1\times10^{-3}\,\text{N}}{2\left(0.070\,\text{m}\right)}=\boxed{3.6\times10^{-2}\,\text{N/m}}$$

60. As in Figure 10-35, there are 2 surfaces being increased, and so $\gamma = F/2L$. Use this relationship to calculate the force.

$$\gamma = F/2L \quad \rightarrow \quad F = 2\gamma L = 2(0.025\,\mathrm{N/m})(0.182\,\mathrm{m}) = \boxed{9.1\times10^{-3}\,\mathrm{N}}$$

61. From Example 10-14, we have that $2\pi r\gamma\cos\theta = \frac{1}{6}mg$. The maximum mass will occur at $\theta = 0^{\circ}$.

$$2\pi r\gamma\cos\theta = \tfrac{1}{6}mg \quad \rightarrow \quad m_{\max} = \frac{12\pi r\gamma}{g} = \frac{12\pi(3.0\times10^{-5}\,\mathrm{m})(0.072\,\mathrm{N/m})}{9.80\,\mathrm{m/s^2}} = 8.3\times10^{-6}\,\mathrm{kg}$$

This is much less than the insect's mass, and so $\boxed{\text{the insect will not remain on top of the water}}$.

62. (*a*) We assume that the weight of the platinum ring is negligible. Then the surface tension is the force to lift the ring, divided by the length of surface that is being pulled. Surface tension will act at both edges of the ring, as in Figure 10-35 (b). Thus $\boxed{\gamma = \frac{F}{2(2\pi r)} = \frac{F}{4\pi r}}$

(*b*) $\gamma = \frac{F}{4\pi r} = \frac{8.40\times10^{-3}\,\mathrm{N}}{4\pi(2.8\times10^{-2}\,\mathrm{m})} = \boxed{2.4\times10^{-2}\,\mathrm{N/m}}$

63. The pressures for parts (*a*) and (*b*) stated in this problem are gauge pressures, relative to atmospheric pressure. The pressure change due to depth in a fluid is given by Eq. 10-3b, $\Delta P = \rho g\Delta h$.

(*a*) $$\Delta h = \frac{\Delta P}{\rho g} = \frac{(55\,\text{mm-Hg})\left(\frac{133\,\mathrm{N/m^2}}{1\,\text{mm-Hg}}\right)}{\left(1.00\frac{\mathrm{g}}{\mathrm{cm^3}}\times\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\times\frac{10^6\,\mathrm{cm^3}}{1\,\mathrm{m^3}}\right)(9.80\,\mathrm{m/s^2})} = \boxed{0.75\,\mathrm{m}}$$

(*b*) $$\Delta h = \frac{\Delta P}{\rho g} = \frac{(650\,\text{mm-H}_2\text{O})\left(\frac{9.81\,\mathrm{N/m^2}}{1\,\text{mm-H}_2\text{O}}\right)}{\left(1.00\frac{\mathrm{g}}{\mathrm{cm^3}}\times\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\times\frac{10^6\,\mathrm{cm^3}}{1\,\mathrm{m^3}}\right)(9.80\,\mathrm{m/s^2})} = \boxed{0.65\,\mathrm{m}}$$

(*c*) For the fluid to just barely enter the vein, the fluid pressure must be the same as the blood pressure.

$$\Delta h = \frac{\Delta P}{\rho g} = \frac{(18\,\text{mm-Hg})\left(\frac{133\,\mathrm{N/m^2}}{1\,\text{mm-Hg}}\right)}{\left(1.00\frac{\mathrm{g}}{\mathrm{cm^3}}\times\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\times\frac{10^6\,\mathrm{cm^3}}{1\,\mathrm{m^3}}\right)(9.80\,\mathrm{m/s^2})} = \boxed{0.24\,\mathrm{m}}$$

64. (*a*) The fluid in the needle is confined, and so Pascal's principle may be applied.

$$P_{\text{plunger}} = P_{\text{needle}} \quad \rightarrow \quad \frac{F_{\text{plunger}}}{A_{\text{plunger}}} = \frac{F_{\text{needle}}}{A_{\text{needle}}} \quad \rightarrow \quad F_{\text{needle}} = F_{\text{plunger}}\frac{A_{\text{needle}}}{A_{\text{plunger}}} = F_{\text{plunger}}\frac{\pi r_{\text{needle}}^2}{\pi r_{\text{plunger}}^2} = F_{\text{plunger}}\frac{r_{\text{needle}}^2}{r_{\text{plunger}}^2}$$

$$= (2.4\,\mathrm{N})\frac{(0.10\times10^{-3}\,\mathrm{m})^2}{(0.65\times10^{-2}\,\mathrm{m})^2} = \boxed{5.7\times10^{-4}\,\mathrm{N}}$$

(*b*) $$F_{\text{plunger}} = P_{\text{plunger}} A_{\text{plunger}} \left(18\,\text{mm-Hg}\right)\left(\frac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right)\pi\left(0.65\times10^{-2}\,\text{m}\right)^2 = \boxed{0.32\,\text{N}}$$

65. The force can be found by multiplying the pressure times the area of the pump cylinder.

$$F_i = P_i A = \left(2.10\times10^5\ \text{N/m}^2\right)\pi\left(0.015\ \text{m}\right)^2 = 1.5\times10^2\,\text{N}$$

$$F_f = P_f A = \left(3.10\times10^5\ \text{N/m}^2\right)\pi\left(0.015\ \text{m}\right)^2 = 2.2\times10^2\,\text{N}$$

The range of forces is $\boxed{1.5\times10^2\,\text{N} \le F \le 2.2\times10^2\,\text{N}}$

66. The pressure would be the weight of the ice divided by the area covered by the ice. The volume of the ice is represented by *V*, and its thickness by *d*. The volume is also the mass of the ice divided by the density of the ice.

$$P = \frac{F}{A} = \frac{mg}{V/d} = \frac{mgd}{V} = \frac{mgd}{m/\rho} = gd\rho = \left(9.80\,\text{m/s}^2\right)\left(3000\,\text{m}\right)\left(917\,\text{kg/m}^3\right) = \boxed{2.7\times10^7\,\text{Pa}}$$

$$= 2.7\times10^7\,\text{Pa}\left(\frac{1\ \text{atm}}{1.013\times10^5\,\text{Pa}}\right) \approx \boxed{270\,\text{atm}}$$

67. The change in pressure with height is given by Eq. 10-3b.

$$\Delta P = \rho g \Delta h \quad\to\quad \frac{\Delta P}{P_0} = \frac{\rho g \Delta h}{P_0} = \frac{\left(1.29\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(380\,\text{m}\right)}{1.013\times10^5\,\text{Pa}} = 0.047 \quad\to$$

$$\boxed{\Delta P = 0.047\,\text{atm}}$$

68. (*a*) The input pressure is equal to the output pressure.

$$P_{\text{input}} = P_{\text{output}} \quad\to\quad \frac{F_{\text{input}}}{A_{\text{input}}} = \frac{F_{\text{output}}}{A_{\text{output}}} \quad\to$$

$$A_{\text{input}} = A_{\text{output}}\frac{F_{\text{input}}}{F_{\text{output}}} = \pi\left(9.0\times10^{-2}\,\text{m}\right)^2 \frac{250\,\text{N}}{\left(970\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{6.7\times10^{-4}\,\text{m}^2}$$

(*b*) The work is the force needed to lift the car (its weight) times the vertical distance lifted.

$$W = mgh = \left(970\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(0.12\,\text{m}\right) = 1.141\times10^3\,\text{J} \approx \boxed{1.1\times10^3\,\text{J}}$$

(*c*) The work done by the input piston is equal to the work done in lifting the car.

$$W_{\text{input}} = W_{\text{output}} \quad\to\quad F_{\text{input}} d_{\text{input}} = F_{\text{output}} d_{\text{ouptut}} = mgh \quad\to$$

$$h = \frac{F_{\text{input}} d_{\text{input}}}{mg} = \frac{\left(250\,\text{N}\right)\left(0.13\,\text{m}\right)}{\left(970\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)} = 3.419\times10^{-3}\,\text{m} \approx \boxed{3.4\times10^{-3}\,\text{m}}$$

(*d*) The number of strokes is the full distance divided by the distance per stroke.

$$h_{\text{full}} = N h_{\text{stroke}} \quad\to\quad N = \frac{h_{\text{full}}}{h_{\text{stroke}}} = \frac{0.12\,\text{m}}{3.419\times10^{-3}\,\text{m}} = \boxed{35\,\text{strokes}}$$

(*e*) The work input is the input force times the total distance moved by the input piston.

$$W_{\text{input}} = N F_{\text{input}} d_{\text{input}} \quad\to\quad 35\left(250\,\text{N}\right)\left(0.13\,\text{m}\right) = \boxed{1.1\times10^3\,\text{J}}$$

Since the work input is equal to the work output, energy is conserved.

69. The change in pressure with height is given by Eq. 10-3b.

$$\Delta P = \rho g \Delta h \quad \rightarrow \quad \frac{\Delta P}{P_0} = \frac{\rho g \Delta h}{P_0} = \frac{\left(1.05\times10^3\ \mathrm{kg/m^3}\right)\left(9.80\ \mathrm{m/s^2}\right)\left(6\ \mathrm{m}\right)}{1.013\times10^5\ \mathrm{Pa}} = 0.609 \quad \rightarrow$$

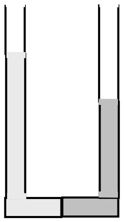
$$\boxed{\Delta P = 0.6\ \mathrm{atm}}$$

70. The pressure change due to a change in height is given by Eq. 10-3b. That pressure is the excess force on the eardrum, divided by the area of the eardrum.

$$\Delta P = \rho g \Delta h = \frac{F}{A} \quad \rightarrow$$

$$F = \rho g \Delta h A = \left(1.29\ \mathrm{kg/m^3}\right)\left(9.80\ \mathrm{m/s^2}\right)\left(950\ \mathrm{m}\right)\left(0.50\times10^{-4}\ \mathrm{m^2}\right) = \boxed{0.60\ \mathrm{N}}$$

71. The pressure at the top of each liquid will be atmospheric pressure, and the pressure at the place where the two fluids meet must be the same if the fluid is to be stationary. In the diagram, the darker color represents the water, and the lighter color represents the alcohol. Write the expression for the pressure at a depth for both liquids, starting at the top of each liquid with atmospheric pressure.



$$P_{\text{alcohol}} = P_0 + \rho_{\text{alcohol}} g \Delta h_{\text{alcohol}} = P_{\text{water}} = P_0 + \rho_{\text{water}} g \Delta h_{\text{water}} \quad \rightarrow$$

$$\rho_{\text{alcohol}} \Delta h_{\text{alcohol}} = \rho_{\text{water}} \Delta h_{\text{water}} \quad \rightarrow$$

$$\Delta h_{\text{water}} = \Delta h_{\text{alcohol}} \frac{\rho_{\text{alcohol}}}{\rho_{\text{water}}} = 18.0\ \mathrm{cm}\left(0.79\right) = \boxed{14.2\ \mathrm{cm}}$$

72. The buoyant force, equal to the weight of mantle displaced, must be equal to the weight of the continent. Let *h* represent the full height of the continent, and *y* represent the height of the continent above the surrounding rock.

$$W_{\text{continent}} = W_{\substack{\text{displaced}\\\text{mantle}}} \quad \rightarrow \quad A h \rho_{\text{continent}} g = A\left(h - y\right)\rho_{\text{mantle}} g \quad \rightarrow$$

$$y = h\left(1 - \frac{\rho_{\text{continent}}}{\rho_{\text{mantle}}}\right) = \left(35\ \mathrm{km}\right)\left(1 - \frac{2800\ \mathrm{kg/m^3}}{3300\ \mathrm{kg/m^3}}\right) = \boxed{5.3\ \mathrm{km}}$$

73. The force is the pressure times the surface area.

$$F = PA = \left(120\ \text{mm-Hg}\right)\left(\frac{133\ \mathrm{N/m^2}}{1\ \text{mm-Hg}}\right)\left(82\times10^{-4}\ \mathrm{m^2}\right) = 130.9\ \mathrm{N} \approx \boxed{130\ \mathrm{N}}$$

74. We assume that the air pressure is due to the weight of the atmosphere, with the area equal to the surface area of the Earth.

$$P = \frac{F}{A} \quad \rightarrow \quad F = PA = mg \quad \rightarrow$$

$$m = \frac{PA}{g} = \frac{4\pi R_{\text{Earth}}^2 P}{g} = \frac{4\pi\left(6.38\times10^6\ \mathrm{m}\right)^2\left(1.013\times10^5\ \mathrm{N/m^2}\right)}{9.80\ \mathrm{m/s^2}} = \boxed{5.29\times10^{18}\ \mathrm{kg}}$$

75. The pressure difference due to the lungs is the pressure change in the column of water.

$$\Delta P = \rho g \Delta h \quad \to \quad \Delta h = \frac{\Delta P}{\rho g} = \frac{(80\,\text{mm-Hg})\left(\dfrac{133\,\text{N/m}^2}{1\text{ mm-Hg}}\right)}{\left(1.00\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)} = 1.086\text{ m} \approx \boxed{1.1\text{ m}}$$

76. The buoyant force due to the fresh water must be the weight of displaced seawater, and would be the volume of the displacement times the density of sea water times the acceleration due to gravity. But the buoyant force on the ship is also the weight of displaced sea water.

$$F_{\text{buoyant}} = V_{\text{displaced}}\rho_{\substack{\text{sea}\\\text{water}}} g = m_{\text{fresh}} g \quad \to \quad m_{\text{fresh}} = \left(2650\,\text{m}^2\right)(8.50\,\text{m})\left(1025\,\text{kg/m}^3\right) = \boxed{2.31\times10^7\,\text{kg}}$$

This can also be expressed as a volume.

$$V_{\text{fresh}} = \frac{m_{\text{fresh}}}{\rho_{\text{fresh}}} = \frac{2.31\times10^7\,\text{kg}}{1.00\times10^3\,\text{kg/m}^3} = \boxed{2.31\times10^4\,\text{m}^3} = \boxed{2.31\times10^7\,\text{L}}$$

77. The buoyant force on the block of wood must be equal to the combined weight of the wood and copper.

$$\left(m_{\text{wood}} + m_{\text{Cu}}\right) g = V_{\text{wood}}\rho_{\text{water}} g = \frac{m_{\text{wood}}}{\rho_{\text{wood}}}\rho_{\text{water}} g \quad \to \quad m_{\text{wood}} + m_{\text{Cu}} = \frac{m_{\text{wood}}}{\rho_{\text{wood}}}\rho_{\text{water}} \quad \to$$

$$m_{\text{Cu}} = m_{\text{wood}}\left(\frac{\rho_{\text{water}}}{\rho_{\text{wood}}} - 1\right) = (0.50\,\text{kg})\left(\frac{1000\,\text{kg/m}^3}{600\,\text{kg/m}^3} - 1\right) = \boxed{0.33\,\text{kg}}$$

78. The buoyant force must be equal to the weight of the water displaced by the full volume of the logs, and must also be equal to the full weight of the raft plus the passengers. Let *N* represent the number of passengers.

weight of water displaced by logs = weight of people + weight of logs

$$10V_{\text{log}}\rho_{\text{water}} g = N m_{\text{person}} g + 10V_{\text{log}}\rho_{\text{log}} g \quad \to$$

$$N = \frac{10V_{\text{log}}\left(\rho_{\text{water}} - \rho_{\text{log}}\right)}{m_{\text{person}}} = \frac{10\pi r_{\text{log}}^2 l_{\text{log}}\left(\rho_{\text{water}} - SG_{\text{log}}\rho_{\text{water}}\right)}{m_{\text{person}}} = \frac{10\pi r_{\text{log}}^2 l_{\text{log}}\rho_{\text{water}}\left(1 - SG_{\text{log}}\right)}{m_{\text{person}}}$$

$$= \frac{10\pi(0.28\,\text{m})^2(6.1\,\text{m})\left(1000\,\text{kg/m}^3\right)(1-0.60)}{68\,\text{kg}} = 88.37$$

Thus $\boxed{88}$ people can stand on the raft without getting wet. When the 89th person gets on, the raft will sink.

79. The work done during each heartbeat is the force on the fluid times the distance that the fluid moves in the direction of the force.

$$W = F\Delta l = PA\Delta l = PV \quad \to$$

$$\text{Power} = \frac{W}{t} = \frac{PV}{t} = \frac{(105\,\text{mm-Hg})\left(\dfrac{133\,\text{N/m}^2}{1\text{ mm-Hg}}\right)\left(70\times10^{-6}\,\text{m}^3\right)}{\left(\dfrac{1}{70}\text{min}\right)\left(\dfrac{60\text{ s}}{\text{min}}\right)} = \boxed{1.1\,\text{W}}$$

80. The buoyant force on the rock is the force that would be on a mass of water with the same volume as the rock. Since the equivalent mass of water is accelerating upward, that same acceleration must be taken into account in the calculation of the buoyant force.

$$F_{\text{buoyant}} - m_{\text{water}} g = m_{\text{water}} a \quad \rightarrow$$

$$F_{\text{buoyant}} = m_{\text{water}}(g+a) = V_{\text{water}} \rho_{\text{water}} (g+a) = V_{\text{rock}} \rho_{\text{water}} (g+a)$$

$$= \frac{m_{\text{rock}}}{\rho_{\text{rock}}} \rho_{\text{water}} (g+a) = \frac{m_{\text{rock}}}{SG_{\text{rock}}}(g+a) = \frac{(3.0\,\text{kg})3.4\left(9.80\,\text{m/s}^2\right)}{2.7} = \boxed{37\,\text{N}}$$

For the rock to not sink, the upward buoyant force on the rock minus the weight of the rock must be equal to the net force on the rock.

$$F_{\text{buoyant}} - m_{\text{rock}} g = m_{\text{rock}} a \quad \rightarrow \quad F_{\text{buoyant}} = m_{\text{rock}}(g+a) = (3.0\,\text{kg})3.4\left(9.80\,\text{m/s}^2\right) = 100\,\text{N}$$

$\boxed{\text{The rock will sink}}$, because the buoyant force is not large enough to “float” the rock.

81. The pressure head can be interpreted as an initial height for the water, with a speed of 0 and atmospheric pressure. Apply Bernoulli’s equation to the faucet location and the pressure head location to find the speed of the water at the faucet. Since the faucet is open, the pressure there will be atmospheric as well.

$$P_{\text{faucet}} + \tfrac{1}{2}\rho v_{\text{faucet}}^2 + \rho g y_{\text{faucet}} = P_{\text{head}} + \tfrac{1}{2}\rho v_{\text{head}}^2 + \rho g y_{\text{head}} \quad \rightarrow$$

$$y_{\text{head}} = \frac{v_{\text{faucet}}^2}{2g} = \frac{(9.5\,\text{m/s})^2}{2\left(9.80\,\text{m/s}^2\right)} = \boxed{4.6\,\text{m}}$$

82. Apply both Bernoulli’s equation and the equation of continuity at the two locations of the stream, with the faucet being location 0 and the lower position being location 1. The pressure will be air pressure at both locations. The lower location has $y_1 = 0$ and the faucet is at height $y_0 = y$.

$$A_0 v_0 = A_1 v_1 \quad \rightarrow \quad v_1 = v_0 \frac{A_0}{A_1} = v_0 \frac{\pi (d_0/2)^2}{\pi (d_1/2)^2} = v_0 \frac{d_0^2}{d_1^2} \quad \rightarrow$$

$$P_0 + \tfrac{1}{2}\rho v_0^2 + \rho g y_0 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 \quad \rightarrow \quad v_0^2 + 2gy = v_1^2 = v_0^2 \frac{d_0^4}{d_1^4} \quad \rightarrow$$

$$\boxed{d_1 = d_0 \left(\frac{v_0^2}{v_0^2 + 2gy} \right)^{1/4}}$$

83. (*a*) We assume that the water is launched at ground level. Since it also lands at ground level, the level range formula from chapter 4 may be used.

$$R = \frac{v_0^2 \sin 2\theta}{g} \quad \rightarrow \quad v_0 = \sqrt{\frac{Rg}{\sin 2\theta}} = \sqrt{\frac{(8.0\,\text{m})\left(9.80\,\text{m/s}^2\right)}{\sin 70^\circ}} = 9.134\,\text{m/s} \approx \boxed{9.1\,\text{m/s}}$$

(*b*) The volume rate of flow is the area of the flow times the speed of the flow. Multiply by 4 for the 4 heads.

$$\text{Volume flow rate} = Av = 4\pi r^2 v = 4\pi \left(1.5\times10^{-3}\,\text{m}\right)^2 (9.134\,\text{m/s})$$

$$= 2.583\times10^{-4}\,\text{m}^3/\text{s} \left(\frac{1\,\text{L}}{1.0\times10^{-3}\,\text{m}^3} \right) \approx \boxed{0.26\,\text{L/s}}$$

(*c*) Use the equation of continuity to calculate the flow rate in the supply pipe.

$$(Av)_{\text{supply}} = (Av)_{\text{heads}} \quad \rightarrow \quad v_{\text{supply}} = \frac{(Av)_{\text{heads}}}{A_{\text{supply}}} = \frac{2.583\times10^{-4}\ \text{m}^3/\text{s}}{\pi\left(0.95\times10^{-2}\,\text{m}\right)^2} = \boxed{0.91\,\text{m/s}}$$

84. (*a*) We assume that the tube in the pail is about 4.0 cm below the surface of the liquid in the pail so that the pressure at that two tube ends is approximately the same. Apply Bernoulli's equation between the two ends of the tube.

$$P_{\text{sink}} + \tfrac{1}{2}\rho v_{\text{sink}}^2 + \rho g y_{\text{sink}} = P_{\text{pail}} + \tfrac{1}{2}\rho v_{\text{pail}}^2 + \rho g y_{\text{pail}} \quad \rightarrow$$

$$v_{\text{pail}} = \sqrt{2g\left(y_{\text{sink}} - y_{\text{pail}}\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(0.50\,\text{m}\right)} = 3.130\,\text{m/s} \approx \boxed{3.1\,\text{m/s}}$$

(*b*) The volume flow rate (at the pail end of the tube) times the time must equal the volume of water in the sink.

$$(Av)_{\text{pail}}\, t = V_{\text{sink}} \quad \rightarrow \quad t = \frac{V_{\text{sink}}}{(Av)_{\text{pail}}} = \frac{\left(0.48\,\text{m}^2\right)\left(4.0\times10^{-2}\,\text{m}\right)}{\pi\left(1.0\times10^{-2}\,\text{m}\right)^2\left(3.13\,\text{m/s}\right)} = \boxed{20\,\text{s}}$$

85. We assume that the speed of the water at the entry point to the siphon tube is zero, and we assume that the pressure at both ends of the siphon hose is the same.

$$P_{\text{top}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}} = P_{\text{bottom}} + \tfrac{1}{2}\rho v_{\text{bottom}}^2 + \rho g y_{\text{bottom}} \quad \rightarrow \quad v_{\text{bottom}} = \sqrt{2g\left(y_{\text{top}} - y_{\text{bottom}}\right)}$$

$$Av = \pi r_{\text{tube}}^2 v_{\text{bottom}} = \pi r_{\text{tube}}^2 \sqrt{2g\left(y_{\text{top}} - y_{\text{bottom}}\right)} = \pi\left(0.60\times10^{-2}\,\text{m}\right)^2 \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(0.64\,\text{m}\right)}$$

$$= \boxed{4.0\times10^{-4}\ \text{m}^3/\text{s}}$$

86. The upward force due to air pressure on the bottom of the wing must be equal to the weight of the airplane plus the downward force due to air pressure on the top of the wing. Bernoulli's equation can be used to relate the forces due to air pressure. We assume that there is no appreciable height difference between the top and the bottom of the wing.

$$P_{\text{top}}A + mg = P_{\text{bottom}}A \quad \rightarrow \quad \left(P_{\text{bottom}} - P_{\text{top}}\right) = \frac{mg}{A}$$

$$P_0 + P_{\text{bottom}} + \tfrac{1}{2}\rho v_{\text{bottom}}^2 + \rho g y_{\text{bottom}} = P_0 + P_{\text{top}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}}$$

$$v_{\text{top}}^2 = \frac{2\left(P_{\text{bottom}} - P_{\text{top}}\right)}{\rho} + v_{\text{bottom}}^2 \quad \rightarrow$$

$$v_{\text{top}} = \sqrt{\frac{2\left(P_{\text{bottom}} - P_{\text{top}}\right)}{\rho} + v_{\text{bottom}}^2} = \sqrt{\frac{2mg}{\rho A} + v_{\text{bottom}}^2} = \sqrt{\frac{2\left(2.0\times10^6\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.29\,\text{kg/m}^3\right)\left(1200\,\text{m}^2\right)} + \left(95\,\text{m/s}\right)^2}$$

$$= 185.3\,\text{m/s} \approx \boxed{190\,\text{m/s}}$$

87. From Poiseuille's equation, the viscosity can be found from the volume flow rate, the geometry of the tube, and the pressure difference. The pressure difference over the length of the tube is the same as the pressure difference due to the height of the reservoir, assuming that the open end of the needle is at atmospheric pressure.

$$Q = \frac{\pi R^4\left(P_2 - P_1\right)}{8\eta L} \ ; \ P_2 - P_1 = \rho_{\text{blood}} g h \quad \rightarrow$$

$$\eta = \frac{\pi R^4 \left(P_2 - P_1\right)}{8QL} = \frac{\pi R^4 \rho_{\text{blood}} g h}{8QL} = \frac{\pi\left(0.20\times10^{-3}\,\text{m}\right)^4\left(1.05\times10^3\,\text{kg}/\text{m}^3\right)\left(9.80\,\text{m}/\text{s}^2\right)\left(1.70\,\text{m}\right)}{8\left[4.1\frac{\text{cm}^3}{\text{min}}\times\frac{1\text{ min}}{60\text{ s}}\times\frac{10^{-6}\,\text{m}^3}{\text{cm}^3}\right]\left(3.8\times10^{-2}\,\text{m}\right)}$$

$$= \boxed{4.2\times10^{-3}\,\text{Pa}\bullet\text{s}}$$

88. From Poiseuille's equation, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus Q/R^4 is constant. Also, if the diameter is reduced by 15%, so is the radius.

$$\frac{Q_{\text{final}}}{R_{\text{final}}^4} = \frac{Q_{\text{initial}}}{R_{\text{initial}}^4} \quad \rightarrow \quad \frac{Q_{\text{final}}}{Q_{\text{initial}}} = \frac{R_{\text{final}}^4}{R_{\text{initial}}^4} = \left(0.85\right)^4 = 0.52$$

$\boxed{\text{The flow rate is 52\% of the original value.}}$

CHAPTER 11: Vibrations and Waves

Answers to Questions

1. The blades in an electric shaver vibrate, approximately in SHM.
The speakers in a stereo system vibrate, but usually in a very complicated way since many notes are being sounded at the same time.
A piano string vibrates when struck, in approximately SHM.
The pistons in a car engine oscillate, in approximately SHM.
The free end of a diving board oscillates after a diver jumps, in approximately SHM.

$\boxed{2.}$ The acceleration of a simple harmonic oscillator is zero whenever the oscillating object is at the equilibrium position.

3. The motion of the piston can be approximated as simple harmonic. First of all, the piston will have a constant period while the engine is running at a constant speed. The speed of the piston will be zero at the extremes of its motion – the top and bottom of the stroke – which is the same as in simple harmonic motion. There is a large force exerted on the piston at one extreme of its motion – the combustion of the fuel mixture – and simple harmonic motion has the largest force at the extremes of the motion. Also, as the crankshaft moves in a circle, its component of motion in one dimension is transferred to the piston. It is similar to Fig. 11-6.

4. Since the real spring has mass, the mass that is moving is greater than the mass at the end of the spring. Since $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, a larger mass means a smaller frequency. Thus the true frequency will be smaller than the "massless spring" approximation. And since the true frequency is smaller, the true period will be larger than the "massless spring" approximation. About 1/3 the mass of the spring contributes to the total mass value.

5. The maximum speed is given by $v_{\max} = A\sqrt{k/m}$. Various combinations of changing *A*, *k*, and/or *m* can result in a doubling of the maximum speed. For example, if *k* and *m* are kept constant, then doubling the amplitude will double the maximum speed. Or, if *A* and *k* are kept constant, then reducing the mass to one-fourth its original value will double the maximum speed. Note that changing either *k* or *m* will also change the frequency of the oscillator, since $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

6. The scale reading will oscillate with damped oscillations about an equilibrium reading of 5.0 kg, with an initial amplitude of 5.0 kg (so the range of readings is initially from 0.0 kg and 10.0 kg). Due to friction in the spring and scale mechanism, the oscillation amplitude will decrease over time, eventually coming to rest at the 5.0 kg mark.

$\boxed{7.}$ The period of a pendulum clock is inversely proportional to the square root of *g*, by Equation 11-11a, $T = 2\pi\sqrt{L/g}$. When taken to high altitude, the value of *g* will decrease (by a small amount), which means the period will increase. If the period is too long, the clock is running slow and so will lose time.

8. The tire swing approximates a simple pendulum. With a stopwatch, you can measure the period T of the tire swing, and then solve Equation 11-11a for the length, $L = \dfrac{gT^2}{4\pi^2}$.

9. To make the water "slosh", you must shake the water (and the pan) at the natural frequency for water waves in the pan. The water then is in resonance, or in a standing wave pattern, and the amplitude of oscillation gets large. That natural frequency is determined by the size of the pan – smaller pans will slosh at higher frequencies, corresponding to shorter wavelengths for the standing waves. The period of the shaking must be the same as the time it takes a water wave to make a "round trip" in the pan.

10. Some examples of resonance:
    Pushing a child on a playground swing – you always push at the frequency of the swing.
    Seeing a stop sign oscillating back and forth on a windy day.
    When singing in the shower, certain notes will sound much louder than others.
    Utility lines along the roadside can have a large amplitude due to the wind.
    Rubbing your finger on a wineglass and making it "sing".
    Blowing across the top of a bottle.
    A rattle in a car (see Question 11).

11. A rattle in a car is very often a resonance phenomenon. The car itself vibrates in many pieces, because there are many periodic motions occurring in the car – wheels rotating, pistons moving up and down, valves opening and closing, transmission gears spinning, driveshaft spinning, etc. There are also vibrations caused by irregularities in the road surface as the car is driven, such as hitting a hole in the road. If there is a loose part, and its natural frequency is close to one of the frequencies already occurring in the car's normal operation, then that part will have a larger than usual amplitude of oscillation, and it will rattle. This is why some rattles only occur at certain speeds when driving.

12. The frequency of a simple periodic wave is equal to the frequency of its source. The wave is created by the source moving the wave medium that is in contact with the source. If you have one end of a taut string in your hand, and you move your hand with a frequency of 2 Hz, then the end of the string in your hand will be moving at 2 Hz, because it is in contact with your hand. Then those parts of the medium that you are moving exert forces on adjacent parts of the medium and cause them to oscillate. Since those two portions of the medium stay in contact with each other, they also must be moving with the same frequency. That can be repeated all along the medium, and so the entire wave throughout the medium has the same frequency as the source.

$\boxed{13.}$ The speed of the transverse wave is measuring how fast the wave disturbance moves along the cord. For a uniform cord, that speed is constant, and depends on the tension in the cord and the mass density of the cord. The speed of a tiny piece of the cord is measuring how fast the piece of cord moves perpendicularly to the cord, as the disturbance passes by. That speed is not constant – if a sinusoidal wave is traveling on the cord, the speed of each piece of the cord will be given by the speed relationship of a simple harmonic oscillator (Equation 11-9), which depends on the amplitude of the wave, the frequency of the wave, and the specific time of observation.

14. From Equation 11-19b, the fundamental frequency of oscillation for a string with both ends fixed is $f_1 = \dfrac{v}{2L}$. The speed of waves on the string is given by Equation 11-13, $v = \sqrt{\dfrac{F_T}{m/L}}$. Combining

these two relationships gives $f_1 = \frac{1}{2}\sqrt{\frac{F_T}{mL}}$. By wrapping the string with wire, the mass of the string can be greatly increased without changing the length or the tension of the string, and thus the string has a low fundamental frequency.

15. If you strike the horizontal rod vertically, you will create primarily transverse waves. If you strike the rod parallel to its length, you will create primarily longitudinal waves.

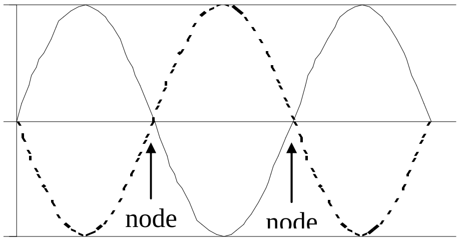
16. From Equation 11-14b, the speed of waves in a gas is given by $v = \sqrt{B/\rho}$. A decrease in the density due to a temperature increase therefore leads to a higher speed of sound. We expect the speed of sound to increase as temperature increases.

17. (*a*) Similar to the discussion in section 11-9 for spherical waves, as a circular wave expands, the circumference of the wave increases. For the energy in the wave to be conserved, as the circumference increases, the intensity has to decrease. The intensity of the wave is proportional to the square of the amplitude
    (*b*) The water waves will decrease in amplitude due to dissipation of energy from viscosity in the water (dissipative or frictional energy loss).

$\boxed{18.}$ Assuming the two waves are in the same medium, then they will both have the same speed. Since $v = f\lambda$, the wave with the smaller wavelength will have twice the frequency of the other wave. From Equation 11-18, the intensity of wave is proportional to the square of the frequency of the wave. Thus the wave with the shorter wavelength will transmit 4 times as much energy as the other wave.

19. The frequency must stay the same because the media is continuous – the end of one section of cord is physically tied to the other section of cord. If the end of the first section of cord is vibrating up and down with a given frequency, then since it is attached to the other section of cord, the other section must vibrate at the same frequency. If the two pieces of cord did not move at the same frequency, they would not stay connected, and then the waves would not pass from one section to another.

20. The string could be touched at the location of a node without disturbing the motion, because the nodes do not move. A string vibrating in three segments has 2 nodes in addition to the ones at the ends. See the diagram.



21. The energy of a wave is not localized at one point, because the wave is not localized at one point, and so to talk about the energy "at a node" being zero is not really a meaningful statement. Due to the interference of the waves the total energy of the medium particles at the nodes points is zero, but the energy of the medium is not zero at points of the medium that are not nodes. In fact, the anti-node points have more energy than they would have if only one of the two waves were present.

22. A major distinction between energy transfer by particles and energy transfer by waves is that particles must travel in a straight line from one place to another in order to transfer energy, but waves can diffract around obstacles. For instance, sound can be heard around a corner, while you cannot throw a ball around a corner. So if a barrier is placed between the source of the energy and the

location where the energy is being received, and energy is still received in spite of the barrier, it is a good indication that the energy is being carried by waves. If the placement of the barrier stops the energy transfer, it could be that the energy transfer is being carried out by particles. It could also be that the energy transfer is being carried out with waves whose wavelength is much smaller than the dimensions of the barrier.

Solutions to Problems

1. The particle would travel four times the amplitude: from $x = A$ to $x = 0$ to $x = -A$ to $x = 0$ to $x = A$. So the total distance $= 4A = 4(0.18\text{ m}) = \boxed{0.72\text{ m}}$.

2. The spring constant is the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{180\text{ N} - 75\text{ N}}{0.85\text{ m} - 0.65\text{ m}} = \frac{105\text{ N}}{0.20\text{ m}} = \boxed{5.3\times10^{2}\text{ N/m}}$$

3. The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{(68\text{ kg})(9.8\,\text{m/s}^2)}{5\times10^{-3}\,\text{m}} = 1.333\times10^{5}\text{ N/m}$$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.333\times10^{5}\text{ N/m}}{1568\text{ kg}}} = 1.467\text{ Hz} \approx \boxed{1.5\text{ Hz}}$$

4. (*a*) The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{(2.7\text{ kg})(9.80\,\text{m/s}^2)}{3.6\times10^{-2}\,\text{m}} = 735\,\text{N/m} \approx \boxed{7.4\times10^{2}\text{ N/m}}$$

(*b*) The amplitude is the distance pulled down from equilibrium, so $A = \boxed{2.5\times10^{-2}\,\text{m}}$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{735\,\text{N/m}}{2.7\text{ kg}}} = 2.626\,\text{Hz} \approx \boxed{2.6\text{ Hz}}$$

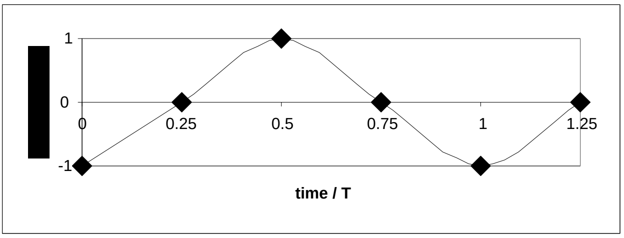
$\boxed{5.}$ The spring constant is the same regardless of what mass is hung from the spring.

$$f = \frac{1}{2\pi}\sqrt{k/m} \;\rightarrow\; \sqrt{k}/2\pi = f\sqrt{m} = \text{constant} \;\rightarrow\; f_1\sqrt{m_1} = f_2\sqrt{m_2} \;\rightarrow$$

$$f_2 = f_1\sqrt{m_1/m_2} = (3.0\text{ Hz})\sqrt{0.60\text{ kg}/0.38\text{ kg}} = \boxed{3.8\text{ Hz}}$$

6. The table of data is shown, along with the smoothed graph. Every quarter of a period, the mass moves from an extreme point to the equilibrium. The graph resembles a cosine wave (actually, the opposite of a cosine wave).

time	position
0	- A
T /4	0
T/2	A
3T/4	0
T	- A
5T/4	0



7. The relationship between frequency, mass, and spring constant is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

(*a*) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow k = 4\pi^2 f^2 m = 4\pi^2 \left(4.0\text{ Hz}\right)^2 \left(2.5\times10^{-4}\text{kg}\right) = 0.1579\,\text{N/m} \approx \boxed{0.16\,\text{N/m}}$

(*b*) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.1579\,\text{N/m}}{5.0\times10^{-4}\,\text{kg}}} = \boxed{2.8\text{ Hz}}$

8. The spring constant is the same regardless of what mass is attached to the spring.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \frac{k}{4\pi^2} = mf^2 = \text{constant} \rightarrow m_1 f_1^2 = m_2 f_1^2 \rightarrow$$

$$\left(m\text{ kg}\right)\left(0.88\text{ Hz}\right)^2 = \left(m\text{ kg} + 0.68\text{ kg}\right)\left(0.60\text{ Hz}\right)^2 \rightarrow m = \frac{\left(0.68\text{ kg}\right)\left(0.60\text{ Hz}\right)^2}{\left(0.88\text{ Hz}\right)^2 - \left(0.60\text{ Hz}\right)^2} = \boxed{0.59\text{ kg}}$$

9. (*a*) At equilibrium, the velocity is its maximum.

$$v_{\text{max}} = \sqrt{\frac{k}{m}}A = \omega A = 2\pi fA = 2\pi\left(3\text{ Hz}\right)\left(0.13\text{ m}\right) = 2.450\,\text{m/s} \approx \boxed{2.5\,\text{m/s}}$$

(*b*) From Equation (11-5), we find the velocity at any position.

$$v = \pm v_{\text{max}}\sqrt{1-\frac{x^2}{A^2}} = \pm\left(2.45\,\text{m/s}\right)\sqrt{1-\frac{\left(0.10\text{ m}\right)^2}{\left(0.13\text{ m}\right)^2}} = \pm1.565\,\text{m/s} \approx \boxed{\pm1.6\,\text{m/s}}$$

(*c*) $E_{\text{total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}\left(0.60\text{ kg}\right)\left(2.45\,\text{m/s}\right)^2 = 1.801\text{J} \approx \boxed{1.8\text{ J}}$

(*d*) Since the object has a maximum displacement at *t* = 0, the position will be described by the cosine function.

$$x = \left(0.13\text{ m}\right)\cos\left(2\pi\left(3.0\text{ Hz}\right)t\right) \rightarrow \boxed{x = \left(0.13\text{ m}\right)\cos\left(6.0\pi t\right)}$$

10. The relationship between the velocity and the position of a SHO is given by Equation (11-5). Set that expression equal to half the maximum speed, and solve for the displacement.

$$v = \pm v_{\text{max}}\sqrt{1-x^2/A^2} = \tfrac{1}{2}v_{\text{max}} \rightarrow \pm\sqrt{1-x^2/A^2} = \tfrac{1}{2} \rightarrow 1-x^2/A^2 = \tfrac{1}{4} \rightarrow x^2/A^2 = \tfrac{3}{4} \rightarrow$$

$$\boxed{x = \pm\sqrt{3}A/2 \approx 0.866A}$$

11. Since $F = -kx = ma$ for an object attached to a spring, the acceleration is proportional to the displacement (although in the opposite direction), as $a = -x\,k/m$. Thus the acceleration will have half its maximum value where the displacement has half its maximum value, at $\boxed{\pm\frac{1}{2}x_0}$

12. The spring constant can be found from the stretch distance corresponding to the weight suspended on the spring.

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{\left(2.62\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)}{0.315\text{ m}} = 81.5\,\text{N/m}$$

After being stretched further and released, the mass will oscillate. It takes one-quarter of a period for the mass to move from the maximum displacement to the equilibrium position.

$$\tfrac{1}{4}T=\tfrac{1}{4}2\pi\sqrt{m/k}=\frac{\pi}{2}\sqrt{\frac{2.62\text{ kg}}{81.5\,\text{N/m}}}=\boxed{0.282\text{ s}}$$

13. (*a*) The total energy of an object in SHM is constant. When the position is at the amplitude, the speed is zero. Use that relationship to find the amplitude.

$$E_{\text{tot}}=\tfrac{1}{2}mv^2+\tfrac{1}{2}kx^2=\tfrac{1}{2}kA^2\quad\rightarrow$$

$$A=\sqrt{\frac{m}{k}v^2+x^2}=\sqrt{\frac{3.0\text{ kg}}{280\,\text{N/m}}(0.55\,\text{m/s})^2+(0.020\text{ m})^2}=6.034\times10^{-2}\text{ m}\approx\boxed{6.0\times10^{-2}\text{ m}}$$

(*b*) Again use conservation of energy. The energy is all kinetic energy when the object has its maximum velocity.

$$E_{\text{tot}}=\tfrac{1}{2}mv^2+\tfrac{1}{2}kx^2=\tfrac{1}{2}kA^2=\tfrac{1}{2}mv_{\text{max}}^2\quad\rightarrow$$

$$v_{\text{max}}=A\sqrt{\frac{k}{m}}=\left(6.034\times10^{-2}\text{ m}\right)\sqrt{\frac{280\,\text{N/m}}{3.0\text{ kg}}}=0.5829\,\text{m/s}\approx\boxed{0.58\,\text{m/s}}$$

14. The spring constant is found from the ratio of applied force to displacement.

$$k=\frac{F}{x}=\frac{80.0\text{ N}}{0.200\text{ m}}=4.00\times10^2\text{ N/m}$$

Assuming that there are no dissipative forces acting on the ball, the elastic potential energy in the loaded position will become kinetic energy of the ball.

$$E_i=E_f\quad\rightarrow\quad\tfrac{1}{2}kx_{\text{max}}^2=\tfrac{1}{2}mv_{\text{max}}^2\quad\rightarrow\quad v_{\text{max}}=x_{\text{max}}\sqrt{\frac{k}{m}}=(0.200\text{ m})\sqrt{\frac{4.00\times10^2\text{ N/m}}{0.180\text{ kg}}}=\boxed{9.43\,\text{m/s}}$$

15. (*a*) The work done to compress a spring is stored as potential energy.

$$W=\tfrac{1}{2}kx^2\quad\rightarrow\quad k=\frac{2W}{x^2}=\frac{2(3.0\text{ J})}{(0.12\text{ m})^2}=416.7\,\text{N/m}\approx\boxed{4.2\times10^2\text{ N/m}}$$

(*b*) The distance that the spring was compressed becomes the amplitude of its motion. The maximum acceleration is given by $a_{\text{max}}=\frac{k}{m}A$. Solve this for the mass.

$$a_{\text{max}}=\frac{k}{m}A\quad\rightarrow\quad m=\frac{k}{a_{\text{max}}}A=\left(\frac{4.167\times10^2\text{ N/m}}{15\,\text{m/s}^2}\right)(0.12\text{ m})=3.333\text{ kg}\approx\boxed{3.3\text{ kg}}$$

16. The general form of the motion is $x=A\cos\omega t=0.45\cos6.40t$.

(*a*) The amplitude is $A=x_{\text{max}}=\boxed{0.45\text{ m}}$.

(*b*) The frequency is found by $\omega=2\pi f=6.40\text{ s}^{-1}\quad\rightarrow\quad f=\frac{6.40\text{ s}^{-1}}{2\pi}=1.019\,\text{Hz}\approx\boxed{1.02\,\text{Hz}}$

(*c*) The total energy is given by

$$E_{\text{total}}=\tfrac{1}{2}mv_{\text{max}}^2=\tfrac{1}{2}m(\omega A)^2=\tfrac{1}{2}(0.60\text{ kg})\left[(6.40\text{ s}^{-1})(0.45\text{ m})\right]^2=2.488\,\text{J}\approx\boxed{2.5\text{ J}}$$

(*d*) The potential energy is given by

$$E_{\text{potential}}=\tfrac{1}{2}kx^2=\tfrac{1}{2}m\omega^2x^2=\tfrac{1}{2}(0.60\text{ kg})\left(6.40\text{ s}^{-1}\right)^2(0.30\text{ m})^2=1.111\,\text{J}\approx\boxed{1.1\text{ J}}$$

The kinetic energy is given by

$$E_{\text{kinetic}} = E_{\text{total}} - E_{\text{potential}} = 2.488\text{ J} - 1.111\text{ J} = 1.377\text{ J} \approx \boxed{1.4\text{ J}}$$

17. If the energy of the SHO is half potential and half kinetic, then the potential energy is half the total energy. The total energy is the potential energy when the displacement has the value of the amplitude.

$$E_{\text{pot}} = \tfrac{1}{2}E_{\text{tot}} \quad\rightarrow\quad \tfrac{1}{2}kx^2 = \tfrac{1}{2}\left(\tfrac{1}{2}kA^2\right) \quad\rightarrow\quad \boxed{x = \pm\frac{1}{\sqrt{2}}A \approx \pm 0.707A}$$

18. If the frequencies and masses are the same, then the spring constants for the two vibrations are the same. The total energy is given by the maximum potential energy.

$$\frac{E_1}{E_2} = \frac{\tfrac{1}{2}kA_1^2}{\tfrac{1}{2}kA_2^2} = \left(\frac{A_1}{A_2}\right)^2 = 7.0 \quad\rightarrow\quad \frac{A_1}{A_2} = \sqrt{7.0} = \boxed{2.6}$$

19. (*a*) The general equation for SHM is Equation (11-8c), $y = A\cos\left(2\pi t/T\right)$. For the pumpkin,

$$\boxed{y = \left(0.18\text{ m}\right)\cos\left(\frac{2\pi t}{0.65\text{ s}}\right)}.$$

(*b*) The time to return back to the equilibrium position is one-quarter of a period.

$$t = \tfrac{1}{4}T = \tfrac{1}{4}\left(0.65\text{ s}\right) = \boxed{0.16\text{ s}}$$

(*c*) The maximum speed is given by the angular frequency times the amplitude.

$$v_{\max} = \omega A = \frac{2\pi}{T}A = \frac{2\pi}{0.65\text{ s}}\left(0.18\text{ m}\right) = \boxed{1.7\text{ m/s}}$$

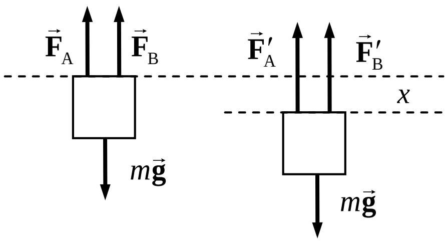
(*d*) The maximum acceleration is given by

$$a_{\max} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A = \frac{4\pi^2}{\left(0.65\text{ s}\right)^2}\left(0.18\text{ m}\right) = \boxed{17\text{ m/s}^2}.$$

The maximum acceleration is first attained at the release point of the pumpkin.

20. Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's 2nd law for vertical forces, choosing up as positive, gives this.

$$\sum F_y = F_{\text{A}} + F_{\text{B}} - mg = 0 \quad\rightarrow\quad F_{\text{A}} + F_{\text{B}} = mg$$


$\vec{\mathbf{F}}_{\text{A}}$
$\vec{\mathbf{F}}_{\text{B}}$
$m\vec{\mathbf{g}}$
$\vec{\mathbf{F}}'_{\text{A}}$
$\vec{\mathbf{F}}'_{\text{B}}$
x
$m\vec{\mathbf{g}}$

Now consider the second free-body diagram, in which the block is displaced a distance x from the equilibrium point. Each upward force will have increased by an amount $-kx$, since $x < 0$. Again write Newton's 2nd law for vertical forces.

$$\sum F_y = F_{net} = F'_{\text{A}} + F'_{\text{B}} - mg = F_{\text{A}} - kx + F_{\text{B}} - kx - mg = -2kx + \left(F_{\text{A}} + F_{\text{B}} - mg\right) = -2kx$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2k$. Thus the frequency of vibration is as follows.

$$f = \frac{1}{2\pi}\sqrt{k_{\text{effective}}/m} = \boxed{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

21. The equation of motion is $x = 0.38\sin 6.50t = A\sin\omega t$.

(*a*) The amplitude is $A = x_{\text{max}} = \boxed{0.38\text{ m}}$.

(*b*) The frequency is found by $\omega = 2\pi f = 6.50\text{ s}^{-1} \rightarrow f = \dfrac{6.50\text{ s}^{-1}}{2\pi} = \boxed{1.03\,\text{Hz}}$

(*c*) The period is the reciprocal of the frequency. $T = 1/f = 1/1.03\text{ Hz} = \boxed{0.967\text{ s}}$.

(*d*) The total energy is given by

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m(\omega A)^2 = \tfrac{1}{2}(0.300\text{ kg})\left[\left(6.50\text{ s}^{-1}\right)(0.38\text{ m})\right]^2 = 0.9151\text{ J} \approx \boxed{0.92\text{ J}}.$$

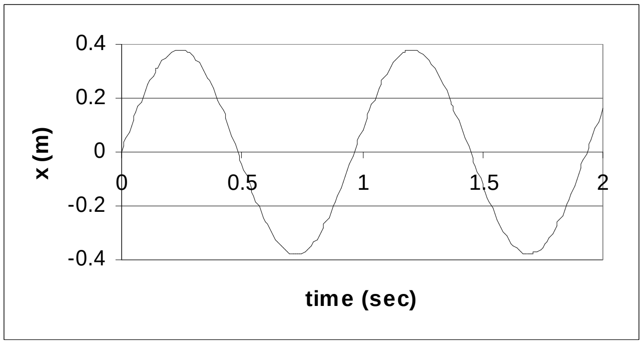
(*e*) The potential energy is given by

$$E_{\text{potential}} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(0.300\text{ kg})\left(6.50\text{ s}^{-1}\right)^2(0.090\text{m})^2 = 0.0513\text{J} \approx \boxed{5.1\times10^{-2}\text{ J}}.$$

The kinetic energy is given by

$$E_{\text{kinetic}} = E_{\text{total}} - E_{\text{potential}} = 0.9151\text{J} - 0.0513\text{J} = 0.8638\text{J} \approx \boxed{0.86\text{ J}}.$$

(*f*)



22. (*a*) For A, the amplitude is $A_{\text{A}} = \boxed{2.5\text{ m}}$. For B, the amplitude is $A_{\text{B}} = \boxed{3.5\text{ m}}$.

(*b*) For A, the frequency is 1 cycle every 4.0 seconds, so $f_{\text{A}} = \boxed{0.25\text{ Hz}}$. For B, the frequency is 1 cycle every 2.0 seconds, so $f_{\text{B}} = \boxed{0.50\text{ Hz}}$.

(*c*) For C, the period is $T_{\text{A}} = \boxed{4.0\text{ s}}$. For B, the period is $T_{\text{B}} = \boxed{2.0\text{ s}}$

(*d*) Object A has a displacement of 0 when $t = 0$, so it is a sine function.

$$x_{\text{A}} = A_{\text{A}}\sin\left(2\pi f_{\text{A}}t\right) \rightarrow \boxed{x_{\text{A}} = (2.5\text{ m})\sin\left(\frac{\pi}{2}t\right)}$$

Object B has a maximum displacement when $t = 0$, so it is a cosine function.

$$x_{\text{B}} = A_{\text{B}}\cos\left(2\pi f_{\text{B}}t\right) \rightarrow \boxed{x_{\text{B}} = (3.5\text{ m})\cos(\pi t)}$$

$\boxed{23.}$ (*a*) Find the period and frequency from the mass and the spring constant.

$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.755\text{ kg}/(124\text{ N/m})} = \boxed{0.490\text{ s}} \qquad f = 1/T = 1/(0.490\text{ s}) = \boxed{2.04\,\text{Hz}}$$

(*b*) The initial speed is the maximum speed, and that can be used to find the amplitude.

$$v_{\text{max}} = A\sqrt{k/m} \rightarrow A = v_{\text{max}}\sqrt{m/k} = (2.96\,\text{m/s})\sqrt{0.755\text{ kg}/(124\text{ N/m})} = \boxed{0.231\text{ m}}$$

(*c*) The maximum acceleration can be found from the mass, spring constant, and amplitude

$$a_{\text{max}} = Ak/m = (0.231\text{ m})(124\text{ N/m})/(0.755\text{ kg}) = \boxed{37.9\,\text{m/s}^2}$$

(*d*) Because the mass started at the equilibrium position of $x = 0$, the position function will be proportional to the sine function.

$$x = (0.231\text{ m})\sin[2\pi(2.04\text{ Hz})t] \rightarrow \boxed{x = (0.231\text{ m})\sin(4.08\pi t)}$$

(*e*) The maximum energy is the kinetic energy that the object has when at the equilibrium position.

$$E = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}(0.755\text{ kg})(2.96\text{ m/s})^2 = \boxed{3.31\text{ J}}$$

24. We assume that downward is the positive direction of motion. For this motion, we have $k = 305\text{ N/m}$, $A = 0.280\text{ m}$, $m = 0.260\text{ kg}$ and $\omega = \sqrt{k/m} = \sqrt{305\text{ N/m}/0.260\text{ kg}} = 34.250\text{ rad/s}$.

(*a*) Since the mass has a zero displacement and a positive velocity at $t = 0$, the equation is a sine function.

$$\boxed{y(t) = (0.280\text{ m})\sin[(34.3\text{ rad/s})t]}$$

(*b*) The period of oscillation is given by $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{34.25\text{ rad/s}} = 0.18345\text{ s}$. The spring will have its maximum extension at times given by the following.

$$t_{\text{max}} = \frac{T}{4} + nT = \boxed{4.59\times10^{-2}\text{ s} + n(0.183\text{ s}),\ n = 0,1,2,\cdots}$$

The spring will have its minimum extension at times given by the following.

$$t_{\text{min}} = \frac{3T}{4} + nT = \boxed{1.38\times10^{-1}\text{ s} + n(0.183\text{ s}),\ n = 0,1,2,\cdots}$$

25. If the block is displaced a distance x to the right in the diagram, then spring # 1 will exert a force $F_1 = -k_1x$, in the opposite direction to x. Likewise, spring # 2 will exert a force $F_2 = -k_2x$, in the same direction as F_1. Thus the net force on the block is $F = F_1 + F_2 = -k_1x - k_2x = -(k_1 + k_2)x$. The effective spring constant is thus $k = k_1 + k_2$, and the period is given by $T = 2\pi\sqrt{\dfrac{m}{k}} = \boxed{2\pi\sqrt{\dfrac{m}{k_1 + k_2}}}$.

26. The energy of the oscillator will be conserved after the collision. Thus

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(m + M)v_{\text{max}}^2 \rightarrow v_{\text{max}} = A\sqrt{k/(m + M)}$$

This speed is the speed that the block and bullet have immediately after the collision. Linear momentum in one dimension will have been conserved during the collision, and so the initial speed of the bullet can be found.

$$p_{\text{before}} = p_{\text{after}} \rightarrow mv_o = (m + M)v_{\text{max}}$$

$$v_o = \frac{m + M}{m}A\sqrt{\frac{k}{m + M}} = \frac{6.25\times10^{-1}\text{ kg}}{2.5\times10^{-2}\text{ kg}}(2.15\times10^{-1}\text{ m})\sqrt{\frac{7.70\times10^3\text{ N/m}}{6.25\times10^{-1}\text{ kg}}} = \boxed{597\text{ m/s}}$$

27. The period of the jumper's motion is $T = \dfrac{38.0\text{ s}}{8\text{ cycles}} = 4.75\text{ s}$. The spring constant can then be found from the period and the jumper's mass.

$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(65.0\text{ kg})}{(4.75\text{ s})^2} = 113.73\text{ N/m} \approx \boxed{114\text{ N/m}}$$

The stretch of the bungee cord needs to provide a force equal to the weight of the jumper when he is at the equilibrium point.

$$k\Delta x = mg \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(65.0\text{ kg})(9.80\,\text{m/s}^2)}{113.73\,\text{N/m}} = 5.60\text{ m}$$

Thus the unstretched bungee cord must be $25.0\text{ m} - 5.60\text{ m} = \boxed{19.4\text{ m}}$

28. (*a*) The period is given by $T = \dfrac{60\text{ s}}{36\text{ cycles}} = \boxed{1.7\,\text{s/cycle}}$.

(*b*) The frequency is given by $f = \dfrac{36\text{ cycles}}{60\text{ s}} = \boxed{0.60\text{ Hz}}$.

29. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. Solve for the length using a period of 2.0 seconds.

$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad L = \frac{T^2 g}{4\pi^2} = \frac{(2.0\text{ s})^2(9.8\,\text{m/s}^2)}{4\pi^2} = \boxed{0.99\text{ m}}$$

30. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad \frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{L/g_{\text{Mars}}}}{2\pi\sqrt{L/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} \quad \rightarrow$$

$$T_{\text{Mars}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} = (0.80\text{ s})\sqrt{\frac{1}{0.37}} = \boxed{1.3\text{ s}}$$

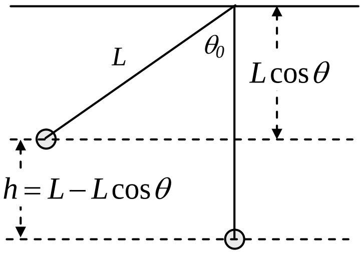
31. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$.

(*a*) $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{\dfrac{0.80\text{ m}}{9.8\,\text{m/s}^2}} = \boxed{1.8\text{ s}}$

(*b*) If the pendulum is in free fall, there is no tension in the string supporting the pendulum bob, and so no restoring force to cause oscillations. Thus there will be no period – the pendulum will not oscillate and so no period can be defined.

32. (*a*) The frequency can be found from the length of the pendulum, and the acceleration due to gravity.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\,\text{m/s}^2}{0.760\text{ m}}} = 0.57151\text{ Hz} \approx \boxed{0.572\text{ Hz}}$$



(*b*) To find the speed at the lowest point, use the conservation of energy relating the lowest point to the release point of the pendulum. Take the lowest point to be the zero level of gravitational potential energy.

$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad KE_{\text{top}} + PE_{\text{top}} = KE_{\text{bottom}} + PE_{\text{bottom}}$$

$$0 + mg(L - L\cos\theta_{\text{o}}) = \tfrac{1}{2}mv_{\text{bottom}}^2 + 0$$

$$v_{\text{bottom}} = \sqrt{2gL(1-\cos\theta_o)} = \sqrt{2(9.80\,\text{m/s}^2)(0.760\text{m})(1-\cos 12.0^\circ)} = \boxed{0.571\text{ m/s}}$$

(*c*) The total energy can be found from the kinetic energy at the bottom of the motion.

$$E_{\text{total}} = \tfrac{1}{2}mv^2_{\text{bottom}} = \tfrac{1}{2}(0.365\text{ kg})(0.571\text{m/s})^2 = \boxed{5.95\times10^{-2}\,\text{J}}$$

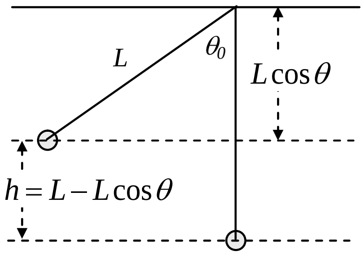
33. There are $(24\text{ h})(60\,\text{min/h})(60\,\text{s/min}) = 86,400\text{ s}$ in a day. The clock should make one cycle in exactly two seconds (a "tick" and a "tock"), and so the clock should make 43,200 cycles per day. After one day, the clock in question is 30 seconds slow, which means that it has made 15 less cycles than required for precise timekeeping. Thus the clock is only making 43,185 cycles in a day. Accordingly, the period of the clock must be decreased by a factor $\frac{43,185}{43,200}$.

$$T_{\text{new}} = \frac{43,185}{43,200}T_{\text{old}} \rightarrow 2\pi\sqrt{L_{\text{new}}/g} = \left(\frac{43,185}{43,200}\right)2\pi\sqrt{L_{\text{old}}/g} \rightarrow$$

$$L_{\text{new}} = \left(\frac{43,185}{43,200}\right)^2 L_{\text{old}} = \left(\frac{43,185}{43,200}\right)^2(0.9930\text{ m}) = 0.9923\text{ m}$$

Thus the pendulum should be $\boxed{\text{shortened by 0.7 mm}}$.

34. Use energy conservation to relate the potential energy at the maximum height of the pendulum to the kinetic energy at the lowest point of the swing. Take the lowest point to be the zero location for gravitational potential energy. See the diagram.



$$E_{\text{top}} = E_{\text{bottom}} \rightarrow KE_{\text{top}} + PE_{\text{top}} = KE_{\text{bottom}} + PE_{\text{bottom}} \rightarrow$$

$$0 + mgh = \tfrac{1}{2}mv^2_{\text{max}} \rightarrow v_{\text{max}} = \sqrt{2gh} = \boxed{\sqrt{2gL(1-\cos\theta_o)}}$$

35. The equation of motion for an object in SHM that has the maximum displacement at $t = 0$ is given by $x = A\cos(2\pi f\,t)$. For a pendulum, $x = L\theta$ and so $x_{\text{max}} = A = L\theta_{\text{max}}$, where θ must be measured in radians. Thus the equation for the pendulum's angular displacement is

$$L\theta = L\theta_{\text{max}}\cos(2\pi f\,t) \rightarrow \theta = \theta_{\text{max}}\cos(2\pi f\,t)$$

If both sides of the equation are multiplied by $180^\circ/\pi$ rad, then the angles can be measured in degrees. Thus the angular displacement of the pendulum can be written as below. Please note that the argument of the cosine function is still in radians.

$$\theta^\circ = \theta^\circ_{\text{max}}\cos(2\pi ft) = 15^\circ\cos(5.0\pi t)$$

(*a*) $\theta^\circ(t = 0.25\text{ s}) = 15^\circ\cos(5.0\pi(0.25)) = \boxed{-11^\circ}$

(*b*) $\theta^\circ(t = 1.6\text{ s}) = 15^\circ\cos(5.0\pi(1.6)) = \boxed{15^\circ}$ (here the time is exactly 4 periods)

(*c*) $\theta^\circ(t = 500\text{ s}) = 15^\circ\cos(5.0\pi(500)) = \boxed{15^\circ}$ (here the time is exactly 1250 periods)

36. The wave speed is given by $v = \lambda f$. The period is 3.0 seconds, and the wavelength is 6.5 m.

$$v = \lambda f = \lambda/T = (6.5\text{ m})/(3.0\text{ s}) = \boxed{2.2\,\text{m/s}}$$

37. The distance between wave crests is the wavelength of the wave.

$$\lambda = v/f = 343\,\text{m/s}/262\ \text{Hz} = \boxed{1.31\ \text{m}}$$

38. To find the wavelength, use $\lambda = v/f$.

AM: $\lambda_1 = \dfrac{v}{f_1} = \dfrac{3.00\times10^8\ \text{m/s}}{550\times10^3\,\text{Hz}} = 545\ \text{m}$ $\quad\lambda_2 = \dfrac{v}{f_2} = \dfrac{3.00\times10^8\ \text{m/s}}{1600\times10^3\,\text{Hz}} = 188\ \text{m}$ $\quad\boxed{\text{AM: 190 m to 550 m}}$

FM: $\lambda_1 = \dfrac{v}{f_1} = \dfrac{3.00\times10^8\ \text{m/s}}{88.0\times10^6\,\text{Hz}} = 3.41\ \text{m}$ $\quad\lambda_2 = \dfrac{v}{f_2} = \dfrac{3.00\times10^8\ \text{m/s}}{108\times10^6\,\text{Hz}} = 2.78\ \text{m}$ $\quad\boxed{\text{FM: 2.78 m to 3.41 m}}$

39. The elastic and bulk moduli are taken from Table 9-1 in chapter 9. The densities are taken from Table 10-1 in chapter 10.

(*a*) For water: $v = \sqrt{B/\rho} = \sqrt{\dfrac{2.0\times10^9\ \text{N/m}^2}{1.00\times10^3\ \text{kg/m}^3}} = \boxed{1.4\times10^3\ \text{m/s}}$

(*b*) For granite: $v = \sqrt{E/\rho} = \sqrt{\dfrac{45\times10^9\ \text{N/m}^2}{2.7\times10^3\ \text{kg/m}^3}} = \boxed{4.1\times10^3\ \text{m/s}}$

(*c*) For steel: $v = \sqrt{E/\rho} = \sqrt{\dfrac{200\times10^9\ \text{N/m}^2}{7.8\times10^3\ \text{kg/m}^3}} = \boxed{5.1\times10^3\ \text{m/s}}$

40. The speed of a longitudinal wave in a solid is given by $v = \sqrt{E/\rho}$. Call the density of the less dense material ρ_1, and the density of the more dense material ρ_2. The less dense material will have the higher speed, since the speed is inversely proportional to the square root of the density.

$$\frac{v_1}{v_2} = \frac{\sqrt{E/\rho_1}}{\sqrt{E/\rho_2}} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{2} = \boxed{1.41}$$

$\boxed{41.}$ To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, we have $v = \sqrt{\dfrac{F_T}{m/L}}$.

$$v = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_T}{m/L}} \rightarrow \Delta t = \frac{\Delta x}{\sqrt{\dfrac{F_T}{m/L}}} = \frac{28\ \text{m}}{\sqrt{\dfrac{150\ \text{N}}{(0.65\ \text{kg})/(28\ \text{m})}}} = \boxed{0.35\ \text{s}}$$

42. (*a*) The speed of the pulse is given by

$$v = \frac{\Delta x}{\Delta t} = \frac{2(620\ \text{m})}{16\ \text{s}} = 77.5\,\text{m/s} \approx \boxed{78\,\text{m/s}}$$

(*b*) The tension is related to the speed of the pulse by $v = \sqrt{\dfrac{F_T}{m/L}}$. The mass per unit length of the cable can be found from its volume and density.

$$\rho = \frac{m}{V} = \frac{m}{\pi\left(d/2\right)^2 L} \quad \rightarrow \quad \frac{m}{L} = \pi\rho\left(\frac{d}{2}\right)^2 = \pi\left(7.8\times10^3\ \mathrm{kg/m^3}\right)\left(\frac{1.5\times10^{-2}\,\mathrm{m}}{2}\right)^2 = 1.378\,\mathrm{kg/m}$$

$$v = \sqrt{\frac{F_T}{m/L}} \quad \rightarrow \quad F_T = v^2\frac{m}{L} = \left(77.5\,\mathrm{m/s}\right)^2\left(1.378\,\mathrm{kg/m}\right) = \boxed{8.3\times10^3\,\mathrm{N}}$$

43. The speed of the water wave is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 9-1, and ρ is the density of sea water, from Table 10-1. The wave travels twice the depth of the ocean during the elapsed time.

$$v = \frac{2L}{t} \quad \rightarrow \quad L = \frac{vt}{2} = \frac{t}{2}\sqrt{\frac{B}{\rho}} = \frac{3.0\ \mathrm{s}}{2}\sqrt{\frac{2.0\times10^9\ \mathrm{N/m^2}}{1.025\times10^3\ \mathrm{kg/m^3}}} = \boxed{2.1\times10^3\mathrm{m}}$$

44. (*a*) Both waves travel the same distance, so $\Delta x = v_1 t_1 = v_2 t_2$. We let the smaller speed be v_1, and the larger speed be v_2. The slower wave will take longer to arrive, and so t_1 is more than t_2.

$$t_1 = t_2 + 2.0\,\mathrm{min} = t_2 + 120\ \mathrm{s} \quad \rightarrow \quad v_1\left(t_2 + 120\ \mathrm{s}\right) = v_2 t_2 \quad \rightarrow$$

$$t_2 = \frac{v_1}{v_2 - v_1}\left(120\ \mathrm{s}\right) = \frac{5.5\,\mathrm{km/s}}{8.5\,\mathrm{km/s} - 5.5\mathrm{km/s}}\left(120\ \mathrm{s}\right) = 220\ \mathrm{s}$$

$$\Delta x = v_2 t_2 = \left(8.5\,\mathrm{km/s}\right)\left(220\ \mathrm{s}\right) = \boxed{1.9\times10^3\mathrm{km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius $1.9\times10^3\,\mathrm{km}$ from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

45. We assume that the earthquake wave is moving the ground vertically, since it is a transverse wave. An object sitting on the ground will then be moving with SHM, due to the two forces on it – the normal force upwards from the ground and the weight downwards due to gravity. If the object loses contact with the ground, then the normal force will be zero, and the only force on the object will be its weight. If the only force is the weight, then the object will have an acceleration of *g* downwards. Thus the limiting condition for beginning to lose contact with the ground is when the maximum acceleration caused by the wave is greater than *g*. Any larger downward acceleration and the ground would "fall" quicker than the object. The maximum acceleration is related to the amplitude and the frequency as follows.

$$a_{\max} = \omega^2 A > g \quad \rightarrow \quad A > \frac{g}{\omega^2} = \frac{g}{4\pi^2 f^2} = \frac{9.8\,\mathrm{m/s^2}}{4\pi^2\left(0.50\ \mathrm{Hz}\right)^2} = \boxed{0.99\ \mathrm{m}}$$

46. (*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. (11-16b) applies, stating that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{20\ \mathrm{km}}/I_{10\ \mathrm{km}} = \left(10\ \mathrm{km}\right)^2/\left(20\ \mathrm{km}\right)^2 = \boxed{0.25}$$

(*b*) The intensity is proportional to the square of the amplitude, and so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{20\ \mathrm{km}}/A_{10\ \mathrm{km}} = 10\ \mathrm{km}/20\ \mathrm{km} = \boxed{0.50}$$

$\boxed{47.}$ (*a*) Assuming spherically symmetric waves, the intensity will be inversely proportional to the square of the distance from the source. Thus Ir^2 will be constant.

$$I_{\text{near}}r_{\text{near}}^2 = I_{\text{far}}r_{\text{far}}^2 \quad \rightarrow$$

$$I_{\text{near}} = I_{\text{far}}\frac{r_{\text{far}}^2}{r_{\text{near}}^2} = \left(2.0\times10^6\ \text{W}/\text{m}^2\right)\frac{(48\ \text{km})^2}{(1\ \text{km})^2} = 4.608\times10^9\ \text{W}/\text{m}^2 \approx \boxed{4.6\times10^9\ \text{W}/\text{m}^2}$$

(*b*) The power passing through an area is the intensity times the area.

$$P = IA = \left(4.608\times10^9\ \text{W}/\text{m}^2\right)\left(5.0\,\text{m}^2\right) = \boxed{2.3\times10^{10}\,\text{W}}$$

48. From Equation (11-18), if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 2 \quad \rightarrow \quad A_2/A_1 = \sqrt{2} = \boxed{1.41}$$

The more energetic wave has the larger amplitude.

49. From Equation (11-18), if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

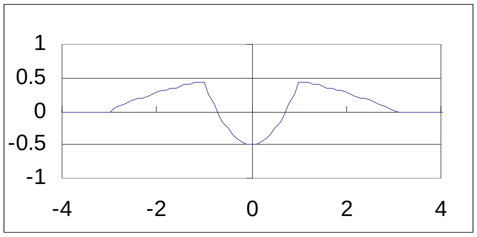
$$I_2/I_1 = P_2/P_1 = A_2^2/A_1^2 = 3 \quad \rightarrow \quad A_2/A_1 = \sqrt{3} = \boxed{1.73}$$

The more energetic wave has the larger amplitude.

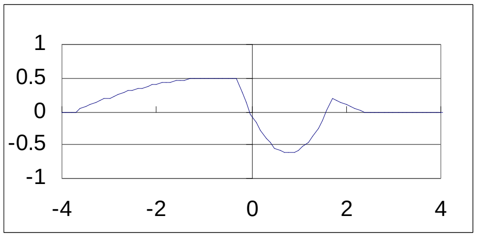
50. The bug moves in SHM as the wave passes. The maximum KE of a particle in SHM is the total energy, which is given by $E_{\text{total}} = \frac{1}{2}kA^2$. Compare the two KE maxima.

$$\frac{KE_2}{KE_1} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2.25\ \text{cm}}{3.0\ \text{cm}}\right)^2 = \boxed{0.56}$$

51. (*a*)



(*b*)



(*c*) The energy is all kinetic energy at the moment when the string has no displacement. There is no elastic potential energy at that moment. Each piece of the string has speed but no displacement.

52. The frequencies of the harmonics of a string that is fixed at both ends are given by $f_n = nf_1$, and so the first four harmonics are $\boxed{f_1 = 440\ \text{Hz}\ ,\ f_2 = 880\ \text{Hz}\ ,\ f_3 = 1320\ \text{Hz}\ ,\ f_4 = 1760\ \text{Hz}}$.

53. The fundamental frequency of the full string is given by $f_{\text{unfingered}} = \dfrac{v}{2L} = 294\ \text{Hz}$. If the length is reduced to 2/3 of its current value, and the velocity of waves on the string is not changed, then the new frequency will be

$$f_{\text{fingered}} = \frac{v}{2\left(\frac{2}{3}L\right)} = \frac{3}{2}\frac{v}{2L} = \left(\frac{3}{2}\right) f_{\text{unfingered}} = \left(\frac{3}{2}\right) 294\text{ Hz} = \boxed{441\text{ Hz}}$$

54. Four loops is the standing wave pattern for the 4th harmonic, with a frequency given by $f_4 = 4f_1 = 280\text{ Hz}$. Thus $\boxed{f_1 = 70\text{ Hz}, f_2 = 140\text{ Hz}, f_3 = 210\text{ Hz and } f_5 = 350\text{ Hz}}$ are all other resonant frequencies.

55. Adjacent nodes are separated by a half-wavelength, as examination of Figure 11-40 will show.

$$\lambda = \frac{v}{f} \rightarrow \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{92\,\text{m/s}}{2(475\text{ Hz})} = \boxed{9.7\times10^{-2}\,\text{m}}$$

56. Since $f_n = nf_1$, two successive overtones differ by the fundamental frequency, as shown below.

$$\Delta f = f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1 = 350\text{ Hz} - 280\text{ Hz} = \boxed{70\text{ Hz}}$$

$\boxed{57.}$ The speed of waves on the string is given by equation (11-13), $v = \sqrt{\frac{F_\text{T}}{m/L}}$. The resonant frequencies of a string with both ends fixed are given by equation (11-19b), $f_n = \frac{nv}{2L_{\text{vib}}}$, where L_{vib} is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$f_n = \frac{n}{2L_{\text{vib}}}\sqrt{\frac{F_\text{T}}{m/L}} \qquad f_1 = \frac{1}{2(0.62\text{ m})}\sqrt{\frac{520\text{ N}}{(3.6\times10^{-3}\text{kg})/(0.90\text{ m})}} = 290.77\,\text{Hz}$$

$$f_2 = 2f_1 = 581.54\,\text{Hz} \qquad f_3 = 3f_1 = 872.31\,\text{Hz}$$

So the three frequencies are $\boxed{290\text{ Hz}, 580\text{ Hz}, 870\text{ Hz}}$, to 2 significant figures.

58. From Equation (11-19b), $f_n = \frac{nv}{2L}$, we see that the frequency is proportional to the wave speed on the stretched string. From equation (11-13), $v = \sqrt{\frac{F_\text{T}}{m/L}}$, we see that the wave speed is proportional to the square root of the tension. Thus the frequency is proportional to the square root of the tension.

$$\sqrt{\frac{F_{\text{T}2}}{F_{\text{T}1}}} = \frac{f_2}{f_1} \rightarrow F_{\text{T}2} = \left(\frac{f_2}{f_1}\right)^2 F_{\text{T}1} = \left(\frac{200\text{ Hz}}{205\text{ Hz}}\right)^2 F_{\text{T}1} = 0.952\,F_{\text{T}1}$$

Thus the tension should be $\boxed{\text{decreased by }4.8\%}$.

59. The string must vibrate in a standing wave pattern to have a certain number of loops. The frequency of the standing waves will all be 60 Hz, the same as the vibrator. That frequency is also expressed by Equation (11-19b), $f_n = \frac{nv}{2L}$. The speed of waves on the string is given by Equation (11-13),

$v = \sqrt{\frac{F_T}{m/L}}$. The tension in the string will be the same as the weight of the masses hung from the end of the string, $F_T = mg$. Combining these relationships gives an expression for the masses hung from the end of the string.

(*a*) $f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{F_T}{m/L}} = \frac{n}{2L}\sqrt{\frac{mg}{(m/L)}} \rightarrow m = \frac{4L^2 f_n^2 (m/L)}{n^2 g}$

$$m_1 = \frac{4(1.50\text{ m})^2 (60\text{ Hz})^2 (3.9\times10^{-4}\text{ kg/m})}{1^2 (9.80\text{ m/s}^2)} = 1.289\text{ kg} \approx \boxed{1.3\text{ kg}}$$

(*b*) $m_2 = \frac{m_1}{2^2} = \frac{1.289\text{ kg}}{4} = \boxed{0.32\text{ kg}}$

(*c*) $m_5 = \frac{m_1}{5^2} = \frac{1.289\text{ kg}}{25} = \boxed{5.2\times10^{-2}\text{ kg}}$

60. The tension in the string is the weight of the hanging mass, $F_T = mg$. The speed of waves on the string can be found by $v = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{mg}{(m/L)}}$, and the frequency is given as $f = 60\text{ Hz}$. The wavelength of waves created on the string will thus be given by

$$\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{mg}{(m/L)}} = \frac{1}{60\text{ Hz}}\sqrt{\frac{(0.080\text{ kg})(9.80\text{ m/s}^2)}{(3.9\times10^{-4}\text{ kg/m})}} = 0.7473\text{ m}.$$

The length of the string must be an integer multiple of half of the wavelength for there to be nodes at both ends and thus form a standing wave. Thus $L = \lambda/2, \lambda, 3\lambda/2, \cdots$, and so on. This gives $L = 0.37\text{ m}$, 0.75 m, 1.12 m, 1.49 m as the possible lengths, and so there are $\boxed{4}$ standing wave patterns that may be achieved.

61. From the description of the water's behavior, there is an anti-node at each end of the tub, and a node in the middle. Thus one wavelength is twice the tube length.

$$v = \lambda f = (2L_{\text{tub}}) f = 2(0.65\text{ m})(0.85\text{ Hz}) = \boxed{1.1\text{ m/s}}$$

62. The speed in the second medium can be found from the law of refraction, Equation (11-20).

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \rightarrow v_2 = v_1\frac{\sin\theta_2}{\sin\theta_1} = (8.0\text{ km/s})\left(\frac{\sin 35^\circ}{\sin 47^\circ}\right) = \boxed{6.3\text{ km/s}}$$

63. The angle of refraction can be found from the law of refraction, Equation (11-20).

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \rightarrow \sin\theta_2 = \sin\theta_1\frac{v_2}{v_1} = \sin 34^\circ\frac{2.1\text{ m/s}}{2.8\text{ m/s}} = 0.419 \rightarrow \theta_2 = \sin^{-1}0.419 = \boxed{25^\circ}$$

64. The angle of refraction can be found from the law of refraction, Equation (11-20). The relative velocities can be found from the relationship given in the problem.

$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{331+0.60T_2}{331+0.60T_1} \quad\to\quad \sin\theta_2=\sin 25^\circ\frac{331+0.60(-10)}{331+0.60(10)}=\sin 25^\circ\frac{325}{337}=0.4076$$

$$\theta_2=\sin^{-1}0.4076=\boxed{24^\circ}$$

65. The angle of refraction can be found from the law of refraction, Equation (11-20). The relative velocities can be found from Equation (11-14a).

$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{\sqrt{E/\rho_2}}{\sqrt{E/\rho_1}}=\sqrt{\frac{\rho_1}{\rho_2}}=\sqrt{\frac{SG_1\rho_{\text{water}}}{SG_2\rho_{\text{water}}}}=\sqrt{\frac{SG_1}{SG_2}}$$

$$\sin\theta_2=\sin\theta_1\sqrt{\frac{SG_1}{SG_2}}=\sin 38^\circ\sqrt{\frac{3.6}{2.8}}=0.70 \quad\to\quad \theta_2=\sin^{-1}0.70=\boxed{44^\circ}$$

66. The error of 2° is allowed due to diffraction of the waves. If the waves are incident at the "edge" of the dish, they can still diffract into the dish if the relationship $\theta\approx\lambda/L$ is satisfied.

$$\theta\approx\frac{\lambda}{L} \quad\to\quad \lambda=L\theta=(0.5\text{ m})\left(2^\circ\times\frac{\pi\text{ rad}}{180^\circ}\right)=1.745\times10^{-2}\text{ m}\approx\boxed{2\times10^{-2}\text{ m}}$$

If the wavelength is longer than that, there will not be much diffraction, but "shadowing" instead.

67. The unusual decrease of water corresponds to a trough in Figure 11-24. The crest or peak of the wave is then one-half wavelength distant. The peak is 125 km away, traveling at 750 km/hr.

$$\Delta x=vt \quad\to\quad t=\frac{\Delta x}{v}=\frac{125\text{ km}}{750\text{ km/hr}}\left(\frac{60\text{ min}}{1\text{ hr}}\right)=\boxed{10\text{ min}}$$

68. Apply the conservation of mechanical energy to the car, calling condition # 1 to be before the collision and condition # 2 to be after the collision. Assume that all of the kinetic energy of the car is converted to potential energy stored in the bumper. We know that $x_1=0$ and $v_2=0$.

$$E_1=E_2 \quad\to\quad \tfrac{1}{2}mv_1^2+\tfrac{1}{2}kx_1^2=\tfrac{1}{2}mv_2^2+\tfrac{1}{2}kx_2^2 \quad\to\quad \tfrac{1}{2}mv_1^2=\tfrac{1}{2}kx_2^2 \quad\to$$

$$x_2=\sqrt{\frac{m}{k}}v_1=\sqrt{\frac{1500\text{ kg}}{550\times10^3\text{ N/m}}}(2.2\text{ m/s})=\boxed{0.11\text{ m}}$$

69. Consider the conservation of energy for the person. Call the unstretched position of the fire net the zero location for both elastic potential energy and gravitational potential energy. The amount of stretch of the fire net is given by *x*, measured positively in the downward direction. The vertical displacement for gravitational potential energy is given by the variable *y*, measured positively for the upward direction. Calculate the spring constant by conserving energy between the window height and the lowest location of the person. The person has no kinetic energy at either location.

$$E_{\text{top}}=E_{\text{bottom}} \quad\to\quad mgy_{\text{top}}=mgy_{\text{bottom}}+\tfrac{1}{2}kx_{\text{bottom}}^2$$

$$k=2mg\frac{(y_{top}-y_{\text{bottom}})}{x_{\text{bottom}}^2}=2(65\text{ kg})(9.8\text{ m/s}^2)\frac{[18\text{ m}-(-1.1\text{ m})]}{(1.1\text{ m})^2}=2.011\times10^4\text{ N/m}$$

(*a*) If the person were to lie on the fire net, they would stretch the net an amount such that the upward force of the net would be equal to their weight.

$$F=kx=mg \quad\rightarrow\quad x=\frac{mg}{k}=\frac{(65\ \text{kg})(9.8\,\text{m/s}^2)}{2.011\times10^4\ \text{N/m}}=\boxed{3.2\times10^{-2}\,\text{m}}$$

(*b*) To find the amount of stretch given a starting height of 35 m, again use conservation of energy. Note that $y_{\text{bottom}}=-x$, and there is no kinetic energy at the top or bottom positions.

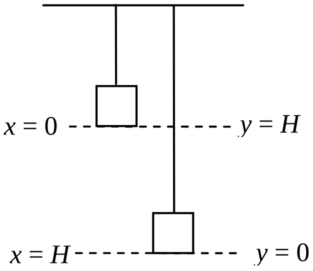
$$E_{\text{top}}=E_{\text{bottom}} \quad\rightarrow\quad mgy_{\text{top}}=mgy_{\text{bottom}}+\tfrac{1}{2}kx^2 \quad\rightarrow\quad x^2-2\frac{mg}{k}x-2\frac{mg}{k}y_{top}=0$$

$$x^2-2\frac{(65\ \text{kg})(9.8\,\text{m/s}^2)}{2.011\times10^4\ \text{N/m}}x-2\frac{(65\ \text{kg})(9.8\,\text{m/s}^2)}{2.011\times10^4\ \text{N/m}}(35\ \text{m})=0 \quad\rightarrow$$

$$x^2-0.06335x-2.2173=0 \quad\rightarrow\quad x=1.5211\ \text{m}\,,\ -1.458\ \text{m}$$

This is a quadratic equation. The solution is the positive root, since the net must be below the unstretched position. The result is $\boxed{1.5\,\text{m}}$.

70. Consider energy conservation for the mass over the range of motion from "letting go" (the highest point) to the lowest point. The mass falls the same distance that the spring is stretched, and has no KE at either endpoint. Call the lowest point the zero of gravitational potential energy. The variable "*x*" represents the amount that the spring is stretched from the equilibrium position.



$$E_{\text{top}}=E_{\text{bottom}}$$

$$\tfrac{1}{2}mv_{\text{top}}^2+mgy_{\text{top}}+\tfrac{1}{2}kx_{\text{top}}^2=\tfrac{1}{2}mv_{\text{bottom}}^2+mgy_{\text{bottom}}+\tfrac{1}{2}kx_{\text{bottom}}^2$$

$$0+mgH+0=0+0+\tfrac{1}{2}kH^2 \quad\rightarrow\quad \frac{k}{m}=\frac{2g}{H}=\omega^2 \quad\rightarrow\quad \omega=\sqrt{\frac{2g}{H}}$$

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{2g}{H}}=\frac{1}{2\pi}\sqrt{\frac{2(9.8\,\text{m/s}^2)}{0.33\ \text{m}}}=\boxed{1.2\ \text{Hz}}$$

71. (*a*) From conservation of energy, the initial kinetic energy of the car will all be changed into elastic potential energy by compressing the spring.

$$E_1=E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2+\tfrac{1}{2}kx_1^2=\tfrac{1}{2}mv_2^2+\tfrac{1}{2}kx_2^2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2=\tfrac{1}{2}kx_2^2 \quad\rightarrow$$

$$k=m\frac{v_1^2}{x_2^2}=(950\ \text{kg})\frac{(22\,\text{m/s})^2}{(5.0\ \text{m})^2}=1.8392\times10^4\ \text{N/m}\approx\boxed{1.8\times10^4\ \text{N/m}}$$

(*b*) The car will be in contact with the spring for half a period, as it moves from the equilibrium location to maximum displacement and back to equilibrium.

$$\tfrac{1}{2}T=\tfrac{1}{2}2\pi\sqrt{\frac{m}{k}}=\pi\sqrt{\frac{(950\ \text{kg})}{1.8392\times10^4\ \text{N/m}}}=\boxed{0.71\ \text{s}}$$

72. The frequency at which the water is being shaken is about 1 Hz. The sloshing coffee is in a standing wave mode, with anti-nodes at each edge of the cup. The cup diameter is thus a half-wavelength, or $\lambda=16\ \text{cm}$. The wave speed can be calculated from the frequency and the wavelength.

$$v=\lambda f=(16\ \text{cm})(1\ \text{Hz})=\boxed{16\,\text{cm/s}}$$

73. Relative to the fixed needle position, the ripples are moving with a linear velocity given by

$$v=\left(33\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\text{ s}}\right)\left(\frac{2\pi\left(0.108\text{ m}\right)}{1\text{ rev}}\right)=0.373\,\text{m/s}$$

This speed is the speed of the ripple waves moving past the needle. The frequency of the waves is

$$f=\frac{v}{\lambda}=\frac{0.373\,\text{m/s}}{1.70\times10^{-3}\,\text{m}}=\boxed{220\text{ Hz}}$$

74. The equation of motion is $x=0.650\cos 7.40t=A\cos\omega t$.

(*a*) The amplitude is $A=\boxed{0.650\text{ m}}$

(*b*) The frequency is given by $\omega=2\pi f=7.40\,\text{rad/s}\quad\rightarrow\quad f=\dfrac{7.40\,\text{rad/s}}{2\pi\text{ rad}}=1.177\,\text{Hz}\approx\boxed{1.18\text{ Hz}}$

(*c*) The total energy is given by

$$E_{\text{total}}=\tfrac{1}{2}kA^2=\tfrac{1}{2}m\omega^2A^2=\tfrac{1}{2}\left(2.00\,\text{kg}\right)\left(7.40\,\text{rad/s}\right)^2\left(0.650\text{ m}\right)^2=23.136\,\text{J}\approx\boxed{23.1\text{ J}}.$$

(*d*) The potential energy is found by

$$PE=\tfrac{1}{2}kx^2=\tfrac{1}{2}m\omega^2x^2=\tfrac{1}{2}\left(2.00\,\text{kg}\right)\left(7.40\,\text{rad/s}\right)^2\left(0.260\text{ m}\right)^2=3.702\,\text{J}\approx\boxed{3.70\text{ J}}.$$

The kinetic energy is found by

$$KE=E_{\text{total}}-PE=23.136\text{ J}-3.702\text{ J}=\boxed{19.4\text{ J}}.$$

75. The frequency of a simple pendulum is given by $f=\dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$. The pendulum is accelerating vertically which is equivalent to increasing (or decreasing) the acceleration due to gravity by the acceleration of the pendulum.

(*a*) $f_{\text{new}}=\dfrac{1}{2\pi}\sqrt{\dfrac{g+a}{L}}=\dfrac{1}{2\pi}\sqrt{\dfrac{1.50g}{L}}=\sqrt{1.50}\dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}=\sqrt{1.50}f=\boxed{1.22f}$

(*b*) $f_{\text{new}}=\dfrac{1}{2\pi}\sqrt{\dfrac{g+a}{L}}=\dfrac{1}{2\pi}\sqrt{\dfrac{0.5g}{L}}=\sqrt{0.5}\dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}=\sqrt{0.5}f=\boxed{0.71f}$

$\boxed{76.}$ The force of the man's weight causes the raft to sink, and that causes the water to put a larger upward force on the raft. This extra buoyant force is a restoring force, because it is in the opposite direction of the force put on the raft by the man. This is analogous to pulling down on a mass-spring system that is in equilibrium, by applying an extra force. Then when the man steps off, the restoring force pushes upward on the raft, and thus the raft – water system acts like a spring, with a spring constant found as follows.

$$k=\frac{F}{x}=\frac{\left(75\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)}{4.0\times10^{-2}\,\text{m}}=1.8375\times10^4\text{ N/m}$$

(*a*) The frequency of vibration is determined by the spring constant and the mass of the raft.

$$f_n=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{1.8375\times10^4\text{ N/m}}{220\text{ kg}}}=1.455\,\text{Hz}\approx\boxed{1.5\text{ Hz}}$$

(*b*) As explained in the text, for a vertical spring the gravitational potential energy can be ignored if the displacement is measured from the oscillator's equilibrium position. The total energy is thus

$$E_{\text{total}}=\tfrac{1}{2}kA^2=\tfrac{1}{2}\left(1.8375\times10^4\text{ N/m}\right)\left(4.0\times10^{-2}\,\text{m}\right)^2=14.7\,\text{J}\approx\boxed{15\text{ J}}.$$

77. (*a*) The overtones are given by $f_n = nf_1, n = 2,3,4\ldots$

$$\text{G}:\quad f_2 = 2\left(392\text{ Hz}\right) = \boxed{784\text{ Hz}} \qquad f_3 = 3\left(392\text{ Hz}\right) = \boxed{1180\text{ Hz}}$$

$$\text{A}:\quad f_2 = 2\left(440\text{ Hz}\right) = \boxed{880\text{ Hz}} \qquad f_3 = 3\left(440\text{ Hz}\right) = \boxed{1320\text{ Hz}}$$

(*b*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different masses for the strings.

$$\frac{f_G}{f_\text{A}} = \frac{v_G/\lambda}{v_A/\lambda} = \frac{v_G}{v_A} = \frac{\sqrt{\dfrac{F_\text{T}}{m_G/L}}}{\sqrt{\dfrac{F_\text{T}}{m_A/L}}} = \sqrt{\frac{m_A}{m_G}} \quad\rightarrow\quad \frac{m_G}{m_A} = \left(\frac{f_A}{f_G}\right)^2 = \left(\frac{440}{392}\right)^2 = \boxed{1.26}$$

(*c*) If the two strings have the same mass per unit length and the same tension, then the wave speed on both strings is the same. The frequency difference is then due to a difference in wavelength. For the fundamental, the wavelength is twice the length of the string.

$$\frac{f_G}{f_\text{A}} = \frac{v/\lambda_G}{v/\lambda_A} = \frac{\lambda_A}{\lambda_G} = \frac{2L_A}{2L_G} \quad\rightarrow\quad \frac{L_G}{L_A} = \frac{f_A}{f_G} = \frac{440}{392} = \boxed{1.12}$$

(*d*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different tensions for the strings.

$$\frac{f_G}{f_\text{A}} = \frac{v_G/\lambda}{v_A/\lambda} = \frac{v_G}{v_A} = \frac{\sqrt{\dfrac{F_{\text{T}G}}{m/L}}}{\sqrt{\dfrac{F_{\text{T}A}}{m/L}}} = \sqrt{\frac{F_{\text{T}G}}{F_{\text{T}A}}} \quad\rightarrow\quad \frac{F_{\text{T}G}}{F_{\text{T}AA}} = \left(\frac{f_G}{f_A}\right)^2 = \left(\frac{392}{440}\right)^2 = \boxed{0.794}$$

78. (*a*) Since the cord is not accelerating to the left or right, the tension in the cord must be the same everywhere. Thus the tension is the same in the two parts of the cord. The speed difference will then be due to the different mass densities of the two parts of the cord. Let the symbol μ represent the mass per unit length of each part of the cord.

$$\frac{v_\text{H}}{\text{v}_\text{L}} = \frac{\left(\sqrt{F_\text{T}/\mu}\right)_\text{H}}{\left(\sqrt{F_\text{T}/\mu}\right)_\text{L}} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

(*b*) The wavelength ratio is found as follows.

$$\frac{\lambda_\text{H}}{\lambda_\text{L}} = \frac{\left(v/f\right)_\text{H}}{\left(v/f\right)_\text{L}} = \frac{v_\text{H}}{v_\text{L}} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

The two frequencies must be the same for the cord to remain continuous at the boundary. If the two parts of the cord oscillate at different frequencies, the cord cannot stay in one piece, because the two parts would be out of phase with each other at various times.

(*c*) Since $\mu_\text{H} > \mu_\text{L}$, we see that $\lambda_\text{H} < \lambda_\text{L}$, and so the wavelength is greater in the $\boxed{\text{lighter cord}}$.

79. (*a*) The maximum speed is given by

$$v_\text{max} = 2\pi f\,A = 2\pi\left(264\text{ Hz}\right)\left(1.8\times10^{-3}\text{m}\right) = \boxed{3.0\,\text{m/s}}.$$

(*b*) The maximum acceleration is given by

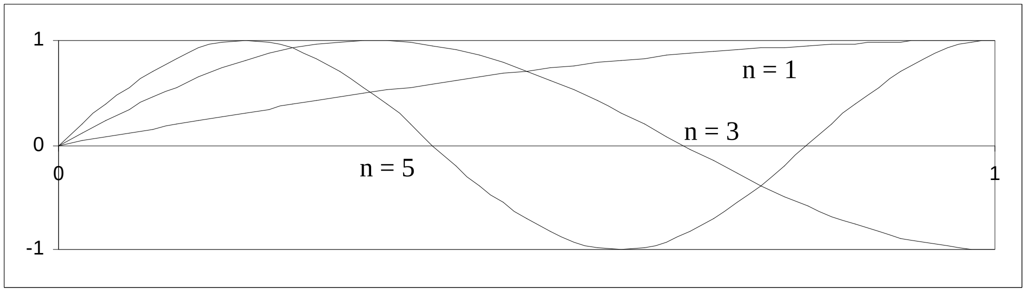
$$a_\text{max} = 4\pi^2 f^2 A = 4\pi^2\left(264\text{ Hz}\right)^2\left(1.8\times10^{-3}\text{m}\right) = \boxed{5.0\times10^3\text{ m/s}^2}.$$

80. For the pebble to lose contact with the board means that there is no normal force of the board on the pebble. If there is no normal force on the pebble, then the only force on the pebble is the force of gravity, and the acceleration of the pebble will be g downward, the acceleration due to gravity. This is the maximum downward acceleration that the pebble can have. Thus if the board's downward acceleration exceeds g, then the pebble will lose contact. The maximum acceleration and the amplitude are related by $a_{\max} = 4\pi^2 f^2 A$.

$$a_{\max} = 4\pi^2 f^2 A \le g \quad \rightarrow \quad A \le \frac{g}{4\pi^2 f^2} \le \frac{9.8\,\mathrm{m/s^2}}{4\pi^2 (1.5\ \mathrm{Hz})^2} \le \boxed{1.1\times10^{-1}\,\mathrm{m}}$$

81. For a resonant condition, the free end of the string will be an antinode, and the fixed end of the string will be a node. The minimum distance from a node to an antinode is $\lambda/4$. Other wave patterns that fit the boundary conditions of a node at one end and an antinode at the other end include $3\lambda/4$, $5\lambda/4$, … . See the diagrams. The general relationship is $L = (2n-1)\lambda/4$, $n = 1, 2, 3, \cdots$.

Solving for the wavelength gives $\boxed{\lambda = \frac{4L}{2n-1}\ ,\ n = 1, 2, 3, \cdots}$



82. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$, and so the length is $L = \frac{T^2 g}{4\pi^2}$.

(*a*) $$L_{\text{Austin}} = \frac{T^2 g_{\text{Austin}}}{4\pi^2} = \frac{(2.000\ \mathrm{s})^2 (9.793\,\mathrm{m/s^2})}{4\pi^2} = \boxed{0.9922\ \mathrm{m}}$$

(*b*) $$L_{\text{Paris}} = \frac{T^2 g_{\text{Paris}}}{4\pi^2} = \frac{(2.000\ \mathrm{s})^2 (9.809\,\mathrm{m/s^2})}{4\pi^2} = 0.9939\ \mathrm{m}$$

$$L_{\text{Paris}} - L_{\text{Austin}} = 0.9939\ \mathrm{m} - 0.9922\ \mathrm{m} = 0.0016\ \mathrm{m} = \boxed{1.6\ \mathrm{mm}}$$

(*c*) $$L_{\text{Moon}} = \frac{T^2 g_{\text{Moon}}}{4\pi^2} = \frac{(2.00\ \mathrm{s})^2 (1.62\,\mathrm{m/s^2})}{4\pi^2} = \boxed{0.164\ \mathrm{m}}$$

83. The spring, originally of length l_0, will be stretched downward to a new equilibrium length L when the mass is hung on it. The amount of downward stretch $L - l_0$ is found from setting the spring force upward on the mass equal to the weight of the mass: $k(L - l_0) = mg \quad \rightarrow \quad L = l_0 + mg/k$. The length of the pendulum is then $L = l_0 + mg/k$. The period of the vertical oscillations is given by $T_{\text{ver}} = 2\pi\sqrt{m/k}$, while the period of the pendulum oscillations is given by $T_{\text{pen}} = 2\pi\sqrt{L/g}$. Now compare the periods of the two motions.

$$\frac{T_{\text{pen}}}{T_{\text{ver}}}=\frac{2\pi\sqrt{\left(l_0+mg/k\right)/g}}{2\pi\sqrt{m/k}}=\sqrt{\frac{l_0+mg/k}{mg/k}}=\sqrt{1+\frac{l_0k}{mg}}>1 \;\rightarrow$$

$$\boxed{T_{\text{pen}}>T_{\text{ver}}\text{, by a factor of }\sqrt{1+\frac{l_0k}{mg}}}$$

84. Block *m* stays on top of block *M* (executing SHM relative to the ground) without slipping due to static friction. The maximum static frictional force on *m* is $F_{\substack{\text{fr}\\ \text{max}}}=\mu_s mg$. This frictional force causes block *m* to accelerate, so $ma_{\text{max}}=\mu_s mg \;\rightarrow\; a_{\text{max}}=\mu_s g$. Thus for the blocks to stay in contact without slipping, the maximum acceleration of block *M* is also $a_{\text{max}}=\mu_s g$. But an object in SHM has a maximum acceleration given by $a_{\text{max}}=\omega^2A=\dfrac{k}{M_{total}}A$. Equate these two expressions for the maximum acceleration.

$$a_{\text{max}}=\frac{k}{M_{total}}A=\mu_s g \;\rightarrow\; A=\frac{\mu_s g}{k}\left(M+m\right)=\frac{(0.30)\left(9.8\,\text{m/s}^2\right)}{130\,\text{N/m}}\left(6.25\text{ kg}\right)=\boxed{0.14\text{ m}}$$

85. The speed of the pulses is found from the tension and mass per unit length of the wire.

$$v=\sqrt{\frac{F_{\text{T}}}{m/L}}=\sqrt{\frac{255\text{ N}}{\left(0.123\text{ kg}\right)/\left(10.0\text{ m}\right)}}=143.985\,\text{m/s}$$

The total distance traveled by the two pulses will be the length of the wire. The second pulse has a shorter time of travel than the first pulse, by 20.0 ms.

$$L=d_1+d_2=vt_1+vt_2=vt_1+v\left(t_1-2.00\times10^{-2}\right)$$

$$t_1=\frac{L+2.00\times10^{-2}v}{2v}=\frac{\left(10.0\text{ m}\right)+2.00\times10^{-2}\left(143.985\,\text{m/s}\right)}{2\left(143.985\,\text{m/s}\right)}=4.4726\times10^{-2}\text{ s}$$

$$d_1=vt_1=\left(143.985\,\text{m/s}\right)\left(4.4726\times10^{-2}\text{ s}\right)=6.44\text{ m}$$

The two pulses meet $\boxed{6.44\text{ m}}$ from the end where the first pulse originated.

86. For the penny to stay on the block at all times means that there will be a normal force on the penny from the block, exerted upward. If down is taken to be the positive direction, then the net force on the penny is $F_{\text{net}}=mg-F_N=ma$. Solving for the magnitude of the normal force gives $F_N=mg-ma$. This expression is always positive if the acceleration is upwards ($a<0$), and so there is no possibility of the penny losing contact while accelerating upwards. But if a downward acceleration were to be larger than *g*, then the normal force would go to zero, since the normal force cannot switch directions $\left(F_N>0\right)$. Thus the limiting condition is $a_{\text{down}}=g$. This is the maximum value for the acceleration. For SHM, we also know that $a_{\text{max}}=\omega^2A=\dfrac{k}{M+m}A\approx\dfrac{k}{M}A$. Equate these two values for the acceleration.

$$a_{\text{max}}=\frac{k}{M}A=g \;\rightarrow\; \boxed{A=\frac{Mg}{k}}$$

87. The car on the end of the cable produces tension in the cable, and stretches the cable according to Equation (9-4), $\Delta L = \frac{1}{E}\frac{F}{A}L_o$, where E is Young's modulus. Rearrange this equation to see that the tension force is proportional to the amount of stretch, $F = \frac{EA}{L_o}\Delta L$, and so the effective spring constant is $k = \frac{EA}{L_o}$. The period of the bouncing can be found from the spring constant and the mass on the end of the cable.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mL_o}{EA}} = 2\pi\sqrt{\frac{(1200\text{ kg})(22\text{ m})}{\left(200\times10^9\text{ N/m}^2\right)\pi\left(3.2\times10^{-3}\text{m}\right)^2}} = \boxed{0.40\text{ s}}$$

88. From Equation (9-6) and Figure (9-22c), the restoring force on the top of the Jell-O is $F = \frac{GA}{L_o}\Delta L$, and is in the opposite direction to the displacement of the top from the equilibrium condition. Thus the "spring constant" for the restoring force is $k = \frac{GA}{L_o}$. If you were to look at a layer of Jell-O closer to the base, the displacement would be less, but so would the restoring force in proportion, and so we estimate all of the Jell-O as having the same spring constant. The frequency of vibration can be determined from the spring constant and the mass of the Jell-O.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{GA/L_o}{\rho V}} = \frac{1}{2\pi}\sqrt{\frac{GA/L_o}{\rho AL_o}} = \frac{1}{2\pi}\sqrt{\frac{G}{\rho L_o^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{520\text{ N/m}^2}{\left(1300\text{ kg/m}^3\right)\left(4.0\times10^{-2}\text{ m}\right)^2}} = \boxed{2.5\text{ Hz}}$$

CHAPTER 12: Sound

Answers to Questions

1. Sound exhibits several phenomena that give evidence that it is a wave. The phenomenon of interference is a wave phenomenon, and sound produces interference (such as beats). The phenomenon of diffraction is a wave phenomenon, and sound can be diffracted (such as sound being heard around corners). Refraction is a wave phenomenon, and sound exhibits refraction when passing obliquely from one medium to another.

2. Evidence that sound is a form of energy is found in the fact that sound can do work. A sound wave created in one location can cause the mechanical vibration of an object at a different location. For example, sound can set eardrums in motion, make windows rattle, or shatter a glass.

3. The child speaking into a cup creates sound waves which cause the bottom of the cup to vibrate. Since the string is tightly attached to the bottom of the cup, the vibrations of the cup are transmitted to longitudinal waves in the string. These longitudinal waves travel down the string, and cause the bottom of the receiver cup to vibrate. This relatively large vibrating surface moves the adjacent air, and generates sound waves from the bottom of the cup, traveling up into the cup. These waves are incident on the receiver's ear, and they hear the sound from the speaker.

4. If the frequency were to change, the two media could not stay in contact with each other. If one medium vibrates with a certain frequency, and the other medium vibrates with a different frequency, then particles from the two media initially in contact could not stay in contact with each other. But particles must be in contact in order for the wave to be transmitted from one medium to the other, and so the frequency does not change. Since the wave speed changes in passing from air into water, and the frequency does not change, we expect the wavelength to change. The wave travels about four times faster in water, so we expect the wavelength in water to be about four times longer than it is in air.

5. Listening to music while seated far away from the source of sound gives evidence that the speed of sound in air does not depend on frequency. If the speed were highly frequency dependent, then high and low sounds created at the same time at the source would arrive at your location at different times, and the music would sound very disjointed. The fact that the music "stays together" is evidence that the speed is independent of frequency.

6. The sound-production anatomy of a person includes various resonating cavities, such as the throat. The relatively fixed geometry of these cavities will determine the relatively fixed wavelengths of sound that a person can produce. Those wavelengths will have associated frequencies given by $f = v/\lambda$. The speed of sound is determined by the gas that is filling the resonant cavities. If the person has inhaled helium, then the speed of sound will be much higher than normal, since the speed of sound waves in helium is about 3 times that in air. Thus the person's frequencies will go up about a factor of 3. This is about a 1.5 octave shift, and so the person sounds very high pitched.

7. The basic equation determining the pitch of the organ pipe is either $f_{\text{closed}} = \frac{nv}{4L}, n = \text{odd integer}$, for a closed pipe, or $f_{\text{open}} = \frac{nv}{2L}, n = \text{integer}$, for an open pipe. In each case, the frequency is proportional to the speed of sound in air. Since the speed is a function of temperature, and the length of any

particular pipe is fixed, the frequency is also a function of temperature. Thus when the temperature changes, the resonant frequencies of the organ pipes change as well. Since the speed of sound increases with temperature, as the temperature increases, the pitch of the pipes increases as well.

8. A tube of a given length will resonate (permit standing waves) at certain frequencies. When a mix of frequencies is input to the tube, only those frequencies close to resonant frequencies will produce sound that persists, because standing waves are created for those frequencies. Frequencies far from resonant frequencies will not persist very long at all – they will "die out" quickly. If, for example, two adjacent resonances of a tube are at 100 Hz and 200 Hz, then sound input near one of those frequencies will persist and sound relatively loud. A sound input near 150 Hz would fade out quickly, and so have a reduced amplitude as compared to the resonant frequencies. The length of the tube can be chosen to thus "filter" certain frequencies, if those filtered frequencies are not close to resonant frequencies.

9. For a string with fixed ends, the fundamental frequency is given by $f = \frac{v}{2L}$ and so the length of string for a given frequency is $L = \frac{v}{2f}$. For a string, if the tension is not changed while fretting, the speed of sound waves will be constant. Thus for two frequencies $f_1 < f_2$, the spacing between the frets corresponding to those frequencies is given as follows.

$$L_1 - L_2 = \frac{v}{2f_1} - \frac{v}{2f_2} = \frac{v}{2}\left(\frac{1}{f_1} - \frac{1}{f_2}\right)$$

Now see table 12-3. Each note there would correspond to one fret on the guitar neck. Notice that as the adjacent frequencies get higher, the inter-frequency spacing also increases. The change from C to C# is 15 Hz, while the change from G to G# is 23 Hz. Thus their reciprocals get closer together, and so from the above formula, the length spacing gets closer together. Consider a numeric example.

$$L_{\text{C}} - L_{\text{C\#}} = \frac{v}{2}\left(\frac{1}{262} - \frac{1}{277}\right) = \frac{v}{2}\left(2.07\times10^{-4}\right) \qquad L_{\text{G}} - L_{\text{G\#}} = \frac{v}{2}\left(\frac{1}{392} - \frac{1}{415}\right) = \frac{v}{2}\left(1.41\times10^{-4}\right)$$

$$\frac{L_{\text{G}} - L_{\text{G\#}}}{L_{\text{C}} - L_{\text{C\#}}} = 0.68$$

The G to G# spacing is only about 68% of the C to C# spacing.

10. When you first hear the truck, you cannot see it. There is no straight line path from the truck to you. The sound waves that you are hearing are therefore arriving at your location due to diffraction. Long wavelengths are diffracted more than short wavelengths, and so you are initially only hearing sound with long wavelengths, which are low-frequency sounds. After you can see the truck, you are able to receive all frequencies being emitted by the truck, not just the lower frequencies. Thus the sound "brightens" due to your hearing more high frequency components.

11. The wave pattern created by standing waves does not "travel" from one place to another. The node locations are fixed in space. Any one point in the medium has the same amplitude at all times. Thus the interference can be described as "interference in space" – moving the observation point from one location to another changes the interference from constructive (anti-node) to destructive (node). To experience the full range from node to anti-node, the position of observation must change, but all observations could be made at the same time by a group of observers.

    The wave pattern created by beats does travel from one place to another. Any one point in the medium will at one time have a 0 amplitude (node) and half a beat period later, have a maximum amplitude (anti-node). Thus the interference can be described as “interference in time”. To experience the full range from constructive interference to destructive interference, the time of observation must change, but all observations could be made at the same position.

12. If the frequency of the speakers is lowered, then the wavelength will be increased. Each circle in the diagram will be larger, and so the points C and D will move farther apart.

13. So-called *active noise reduction* devices work on the principle of interference. If the electronics are fast enough to detect the noise, invert it, and create the opposite wave (180° out of phase with the original) in significantly less time than one period of the components of the noise, then the original noise and the created noise will be approximately in a destructive interference relationship. The person wearing the headphones will hear a net sound signal that is very low in intensity.

14. From the two waves shown, it is seen that the frequency of beating is higher in Figure (a) – the beats occur more frequently. The beat frequency is the difference between the two component frequencies, and so since (a) has a higher beat frequency, the component frequencies are further apart in (a).

15. There is no Doppler shift if the source and observer move in the same direction, with the same velocity. Doppler shift is caused by relative motion between source and observer, and if both source and observer move in the same direction with the same velocity, there is no relative motion.

16. If the wind is blowing but the listener is at rest with respect to the source, the listener will not hear a Doppler effect. We analyze the case of the wind blowing from the source towards the listener. The moving air (wind) has the same effect as if the speed of sound had been increased by an amount equal to the wind speed. The wavelength of the sound waves (distance that a wave travels during one period of time) will be increased by the same percentage that the wind speed is relative to the still-air speed of sound. Since the frequency is the speed divided by the wavelength, the frequency does not change, and so there is no Doppler effect to hear. Alternatively, the wind has the same effect as if the air were not moving but the source and listener were moving at the same speed in the same direction. See question 15 for a discussion of that situation.

$\boxed{17.}$ The highest frequency of sound will be heard at position C, while the child is swinging forward. Assuming the child is moving with SHM, then the highest speed is at the equilibrium point, point C. And to have an increased pitch, the relative motion of the source and detector must be towards each other. The child would also hear the lowest frequency of sound at point C, while swinging backwards.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted frequencies as correct to the number of digits shown, especially where other values might indicate that. For example, in problem 42, values of 350 Hz and 355 Hz are used. We took both of those values to have 3 significant figures. We treated the decibel values similarly. For example, in problem 11, we treated the value of 120 dB as having three significant figures.

1. The round trip time for sound is 2.0 seconds, so the time for sound to travel the length of the lake is 1.0 seconds. Use the time and the speed of sound to determine the length of the lake.

$$d = vt = \left(343\,\mathrm{m/s}\right)\left(1.0\ \mathrm{s}\right) = 343\ \mathrm{m} \approx \boxed{3.4\times10^{2}\,\mathrm{m}}$$

2. The round trip time for sound is 2.5 seconds, so the time for sound to travel the length of the lake is 1.25 seconds. Use the time and the speed of sound in water to determine the depth of the lake.

$$d = vt = (1560\,\text{m/s})(1.25\text{ s}) = 1950\text{ m} = \boxed{2.0\times 10^{3}\,\text{m}}$$

3. (*a*) $\lambda_{20\text{ Hz}} = \frac{v}{f} = \frac{343\,\text{m/s}}{20\text{ Hz}} = \boxed{17\text{ m}}$ $\quad\lambda_{20\text{ kHz}} = \frac{v}{f} = \frac{343\,\text{m/s}}{2.0\times 10^{4}\,\text{Hz}} = \boxed{1.7\times 10^{-2}\,\text{m}}$

So the range is from 17 cm to 17 m.

(*b*) $\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{10\times 10^{6}\,\text{Hz}} = \boxed{3.4\times 10^{-5}\,\text{m}}$

4. (*a*) For the fish, the speed of sound in seawater must be used.

$$d = vt \;\rightarrow\; t = \frac{d}{v} = \frac{1.0\times 10^{3}\text{ m}}{1560\,\text{m/s}} = \boxed{0.64\text{ s}}$$

(*b*) For the fishermen, the speed of sound in air must be used.

$$d = vt \;\rightarrow\; t = \frac{d}{v} = \frac{1.0\times 10^{3}\text{ m}}{343\,\text{m/s}} = \boxed{2.9\text{ s}}$$

5. The total time T is the time for the stone to fall (t_{down}) plus the time for the sound to come back to the top of the cliff (t_{up}): $T = t_{\text{up}} + t_{\text{down}}$. Use constant acceleration relationships for an object dropped from rest that falls a distance h in order to find t_{down}, with down as the positive direction. Use the constant speed of sound to find t_{up} for the sound to travel a distance h.

$$\text{down: } y = y_0 + v_0 t_{\text{down}} + \tfrac{1}{2}at_{\text{down}}^2 \;\rightarrow\; h = \tfrac{1}{2}gt_{\text{down}}^2 \qquad \text{up: } h = v_{\text{snd}}t_{\text{up}} \;\rightarrow\; t_{\text{up}} = \frac{h}{v_{\text{snd}}}$$

$$h = \tfrac{1}{2}gt_{\text{down}}^2 = \tfrac{1}{2}g\left(T - t_{\text{up}}\right)^2 = \tfrac{1}{2}g\left(T - \frac{h}{v_{\text{snd}}}\right)^2 \;\rightarrow\; h^2 - 2v_{\text{snd}}\left(\frac{v_{\text{snd}}}{g} + T\right)h + T^2v_{\text{snd}}^2 = 0$$

This is a quadratic equation for the height. This can be solved with the quadratic formula, but be sure to keep several significant digits in the calculations.

$$h^2 - 2(343\,\text{m/s})\left(\frac{343\,\text{m/s}}{9.80\,\text{m/s}^2} + 3.5\text{ s}\right)h + (3.5\text{ s})^2(343\,\text{m/s})^2 = 0 \;\rightarrow$$

$$h^2 - (26411\text{ m})h + 1.4412\times 10^{6}\,\text{m}^2 = 0 \;\rightarrow\; h = 26356\text{ m},\ 55\text{ m}$$

The larger root is impossible since it takes more than 3.5 sec for the rock to fall that distance, so the correct result is $h = \boxed{55\text{ m}}$.

6. The two sound waves travel the same distance. The sound will travel faster in the concrete, and thus take a shorter time.

$$d = v_{\text{air}}t_{\text{air}} = v_{\text{concrete}}t_{\text{concrete}} = v_{\text{concrete}}\left(t_{\text{air}} - 1.1\text{ s}\right) \;\rightarrow\; t_{\text{air}} = \frac{v_{\text{concrete}}}{v_{\text{concrete}} - v_{\text{air}}}1.1\text{ s}$$

$$d = v_{\text{air}}t_{\text{air}} = v_{\text{air}}\left(\frac{v_{\text{concrete}}}{v_{\text{concrete}} - v_{\text{air}}}1.1\text{ s}\right)$$

The speed of sound in concrete is obtained from Equation (11-14a), Table (9-1), and Table (10-1).

$$v_{\text{concrete}} = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{20\times10^9\ \text{N/m}^2}{2.3\times10^3\ \text{kg/m}^3}} = 2949\,\text{m/s}$$

$$d = v_{\text{air}}t_{\text{air}} = \left(343\,\text{m/s}\right)\left(\frac{2949\,\text{m/s}}{2949\,\text{m/s}-343\,\text{m/s}}1.1\ \text{s}\right) = 427\text{m} \approx \boxed{4.3\times10^2\,\text{m}}$$

7. The "5 second rule" says that for every 5 seconds between seeing a lightning strike and hearing the associated sound, the lightning is 1 mile distant. We assume that there are 5 seconds between seeing the lightning and hearing the sound.

(*a*) At 30°C, the speed of sound is $\left[331+0.60(30)\right]\text{m/s} = 349\,\text{m/s}$. The actual distance to the lightning is therefore $d = vt = \left(349\,\text{m/s}\right)\left(5\,\text{s}\right) = 1745\ \text{m}$. A mile is 1610 m.

$$\%\ \text{error} = \frac{1745-1610}{1745}(100) \approx \boxed{8\%}$$

(*b*) At 10°C, the speed of sound is $\left[331+0.60(10)\right]\text{m/s} = 337\,\text{m/s}$. The actual distance to the lightning is therefore $d = vt = \left(337\,\text{m/s}\right)\left(5\,\text{s}\right) = 1685\ \text{m}$. A mile is 1610 m.

$$\%\ \text{error} = \frac{1685-1610}{1685}(100) \approx \boxed{4\%}$$

$\boxed{8.}$ $$120\ \text{dB} = 10\log\frac{I_{120}}{I_0} \rightarrow I_{120} = 10^{12}I_0 = 10^{12}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = \boxed{1.0\,\text{W/m}^2}$$

$$20\ \text{dB} = 10\log\frac{I_{20}}{I_0} \rightarrow I_{20} = 10^{2}I_0 = 10^{2}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = \boxed{1.0\times10^{-10}\ \text{W/m}^2}$$

The pain level is 10^{10} times more intense than the whisper.

9. $$\beta = 10\log\frac{I}{I_0} = 10\log\frac{2.0\times10^{-6}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2} = \boxed{63\ \text{dB}}$$

10. From Example 12-4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus the sound level for one firecracker will be $95\ \text{dB} - 3\ \text{dB} = \boxed{92\ \text{dB}}$.

11. From Example 12-4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus, if two engines are shut down, the intensity will be cut in half, and the sound level will be 117 dB. Then, if one more engine is shut down, the intensity will be cut in half again, and the sound level will drop by 3 more dB, to a final value of $\boxed{114\ \text{dB}}$.

12. $$58\ \text{dB} = 10\log\left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} \rightarrow \left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} = 10^{5.8} = \boxed{6.3\times10^5}$$

$$95\ \text{dB} = 10\log\left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} \rightarrow \left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} = 10^{9.5} = \boxed{3.2\times10^9}$$

13. (*a*) According to Table 12-2, the intensity in normal conversation, when about 50 cm from the speaker, is about $3\times10^{-6}\ \mathrm{W/m^2}$. The intensity is the power output per unit area, and so the power output can be found. The area is that of a sphere.

$$I=\frac{P}{A}\ \rightarrow\ P=IA=I\left(4\pi r^2\right)=\left(3\times10^{-6}\ \mathrm{W/m^2}\right)4\pi\left(0.50\ \mathrm{m}\right)^2=9.425\times10^{-6}\,\mathrm{W}\approx\boxed{9\times10^{-6}\,\mathrm{W}}$$

(*b*) $$100\ \mathrm{W}\left(\frac{1\ \text{person}}{9.425\times10^{-6}\,\mathrm{W}}\right)=1.06\times10^{7}\approx\boxed{1\times10^{7}\,\text{people}}$$

14. (*a*) The energy absorbed per second is the power of the wave, which is the intensity times the area.

$$50\ \mathrm{dB}=10\log\frac{I}{I_0}\ \rightarrow\ I=10^5 I_0=10^5\left(1.0\times10^{-12}\ \mathrm{W/m^2}\right)=1.0\times10^{-7}\ \mathrm{W/m^2}$$

$$P=IA=\left(1.0\times10^{-7}\ \mathrm{W/m^2}\right)\left(5.0\times10^{-5}\,\mathrm{m^2}\right)=\boxed{5.0\times10^{-12}\,\mathrm{W}}$$

(*b*) $$1\ \mathrm{J}\left(\frac{1\ \mathrm{s}}{5.0\times10^{-12}\,\mathrm{J}}\right)\left(\frac{1\ \mathrm{yr}}{3.16\times10^{7}\,\mathrm{s}}\right)=\boxed{6.3\times10^{3}\,\mathrm{yr}}$$

15. The intensity of the sound is defined to be the power per unit area. We assume that the sound spreads out spherically from the loudspeaker.

(*a*) $$I_{250}=\frac{250\ \mathrm{W}}{4\pi\left(3.5\mathrm{m}\right)^2}=1.6\,\mathrm{W/m^2}\qquad \beta_{250}=10\log\frac{I_{250}}{I_0}=10\log\frac{1.6\,\mathrm{W/m^2}}{1.0\times10^{-12}\ \mathrm{W/m^2}}=\boxed{122\,\mathrm{dB}}$$

$$I_{40}=\frac{40\ \mathrm{W}}{4\pi\left(3.5\mathrm{m}\right)^2}=0.26\,\mathrm{W/m^2}\qquad \beta_{40}=10\log\frac{I_{40}}{I_0}=10\log\frac{0.26\,\mathrm{W/m^2}}{1.0\times10^{-12}\ \mathrm{W/m^2}}=\boxed{114\,\mathrm{dB}}$$

(*b*) According to the textbook, for a sound to be perceived as twice as loud as another means that the intensities need to differ by a factor of 10. That is not the case here – they differ only by a factor of $\frac{1.6}{0.26}\approx 6$. $\boxed{\text{The expensive amp will not sound twice as loud as the cheaper one.}}$

16. (*a*) Find the intensity from the 130 dB value, and then find the power output corresponding to that intensity at that distance from the speaker.

$$\beta=130\ \mathrm{dB}=10\log\frac{I_{2.8\mathrm{m}}}{I_0}\ \rightarrow\ I_{2.8\mathrm{m}}=10^{13}I_0=10^{13}\left(1.0\times10^{-12}\ \mathrm{W/m^2}\right)=10\,\mathrm{W/m^2}$$

$$P=IA=4\pi r^2 I=4\pi\left(2.8\ \mathrm{m}\right)^2\left(10\,\mathrm{W/m^2}\right)=985\,\mathrm{W}\approx\boxed{9.9\times10^{2}\,\mathrm{W}}$$

(*b*) Find the intensity from the 90 dB value, and then from the power output, find the distance corresponding to that intensity.

$$\beta=90\ \mathrm{dB}=10\log\frac{I}{I_0}\ \rightarrow\ I=10^9 I_0=10^9\left(1.0\times10^{-12}\ \mathrm{W/m^2}\right)=1.0\times10^{-3}\ \mathrm{W/m^2}$$

$$P=4\pi r^2 I\ \rightarrow\ r=\sqrt{\frac{P}{4\pi I}}=\sqrt{\frac{985\ \mathrm{W}}{4\pi\left(1.0\times10^{-3}\ \mathrm{W/m^2}\right)}}=\boxed{2.8\times10^{2}\,\mathrm{m}}$$

17. The intensity is proportional to the square of the amplitude.

$$2.0\text{ dB} = 10\log\frac{I_{2.0}}{I_0} = 10\log\frac{A_{2.0}^2}{A_0^2} = 20\log\frac{A_{2.0}}{A_0} \quad\rightarrow\quad \frac{A_{2.0}}{A_0} = 10^{0.1} = 1.259 \approx \boxed{1.3}$$

18. (*a*) The intensity is proportional to the square of the amplitude, so if the amplitude is tripled, the intensity will $\boxed{\text{increase by a factor of 9}}$.

(*b*) $\beta = 10\log I/I_0 = 10\log 9 = \boxed{9.5\,\text{dB}}$

19. The intensity is given by $I = 2\rho v\pi^2 f^2 A^2$. If the only difference in two sound waves is their frequencies, then the ratio of the intensities is the ratio of the square of the frequencies.

$$\frac{I_{2f}}{I_f} = \frac{(2f)^2}{f^2} = \boxed{4}$$

$\boxed{20.}$ The intensity is given by $I = 2\rho v\pi^2 f^2 A^2$, using the density of air and the speed of sound in air.

$$I = 2\rho v\pi^2 f^2 A^2 = 2\left(1.29\,\text{kg/m}^3\right)\left(343\,\text{m/s}\right)\pi^2\left(300\text{ Hz}\right)^2\left(1.3\times10^{-4}\,\text{m}\right)^2 = 13.28\,\text{W/m}^2$$

$$\beta = 10\log\frac{I}{I_0} = 10\log\frac{13.28\,\text{W/m}^2}{1.0\times10^{-12}\,\text{W/m}^2} = 131.2\,\text{dB} \approx \boxed{130\text{ dB}}$$

Note that this is above the threshold of pain.

21. From Figure 12-6, a 100-Hz tone at 50 dB has a loudness of about 20 phons. At 6000 Hz, 20 phons corresponds to about $\boxed{25\text{ dB}}$. Answers may vary due to estimation in the reading of the graph.

22. From Figure 12-6, at 30 dB the low frequency threshold of hearing is about $\boxed{150\text{ Hz}}$. There is no intersection of the threshold of hearing with the 30 dB level on the high frequency side of the chart, and so a 30 dB signal can be heard all the way up to the highest frequency that a human can hear, $\boxed{20{,}000\text{ Hz}}$.

23. (*a*) From Figure 12-6, at 100 Hz, the threshold of hearing (the lowest detectable intensity by the ear) is approximately $5\times10^{-9}\,\text{W/m}^2$. The threshold of pain is about $5\,\text{W/m}^2$. The ratio of highest to lowest intensity is thus $\dfrac{5\,\text{W/m}^2}{5\times10^{-9}\,\text{W/m}^2} = \boxed{10^9}$.

(*b*) At 5000 Hz, the threshold of hearing is about $10^{-13}\,\text{W/m}^2$, and the threshold of pain is about $10^{-1}\,\text{W/m}^2$. The ratio of highest to lowest intensity is $\dfrac{10^{-1}\,\text{W/m}^2}{10^{-13}\,\text{W/m}^2} = \boxed{10^{12}}$.

Answers may vary due to estimation in the reading of the graph.

24. For a vibrating string, the frequency of the fundamental mode is given by $f = \dfrac{v}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{F_T}{m/L}}$.

$$f = \frac{1}{2L}\sqrt{\frac{F_T}{m/L}} \quad \rightarrow \quad F_T = 4Lf^2 m = 4(0.32\,\text{m})(440\,\text{Hz})^2(3.5\times10^{-4}\,\text{kg}) = \boxed{87\,\text{N}}$$

$\boxed{25.}$ (*a*) If the pipe is closed at one end, only the odd harmonic frequencies are present, and are given by $f_n = \dfrac{nv}{4L} = nf_1, n = 1,3,5\cdots$.

$$f_1 = \frac{v}{4L} = \frac{343\,\text{m/s}}{4(1.12\,\text{m})} = \boxed{76.6\text{ Hz}}$$

$$f_3 = 3f_1 = \boxed{230\text{ Hz}} \quad f_5 = 5f_1 = \boxed{383\text{ Hz}} \quad f_7 = 7f_1 = \boxed{536\text{ Hz}}$$

(*b*) If the pipe is open at both ends, all the harmonic frequencies are present, and are given by $f_n = \dfrac{nv}{2L} = nf_1$.

$$f_1 = \frac{v}{2L} = \frac{343\,\text{m/s}}{2(1.12\,\text{m})} = \boxed{153\text{ Hz}}$$

$$f_2 = 2f_1 = \boxed{306\text{ Hz}} \quad f_3 = 3f_1 = \boxed{459\text{ Hz}} \quad f_4 = 4f_1 = \boxed{612\text{ Hz}}$$

26. (*a*) The length of the tube is one-fourth of a wavelength for this (one end closed) tube, and so the wavelength is four times the length of the tube.

$$f = \frac{v}{\lambda} = \frac{343\,\text{m/s}}{4(0.18\,\text{m})} = \boxed{480\text{ Hz}}$$

(*b*) If the bottle is one-third full, then the effective length of the air column is reduced to 12 cm.

$$f = \frac{v}{\lambda} = \frac{343\,\text{m/s}}{4(0.12\,\text{m})} = \boxed{710\text{ Hz}}$$

27. For a pipe open at both ends, the fundamental frequency is given by $f_1 = \dfrac{v}{2L}$, and so the length for a given fundamental frequency is $L = \dfrac{v}{2f_1}$.

$$L_{20\text{ Hz}} = \frac{343\,\text{m/s}}{2(20\text{ Hz})} = \boxed{8.6\text{ m}} \qquad L_{20\text{ kHz}} = \frac{343\,\text{m/s}}{2(20{,}000\text{ Hz})} = \boxed{8.6\times10^{-3}\,\text{m}}$$

28. For a fixed string, the frequency of the nth harmonic is given by $f_n = nf_1$. Thus the fundamental for this string is $f_1 = f_3/3 = 540\text{ Hz}/3 = 180\text{ Hz}$. When the string is fingered, it has a new length of 60% of the original length. The fundamental frequency of the vibrating string is also given by $f_1 = \dfrac{v}{2L}$, and v is a constant for the string, assuming its tension is not changed.

$$f_{1\,\text{fingered}} = \frac{v}{2L_{\text{fingered}}} = \frac{v}{2(0.60)L} = \frac{1}{0.60}f_1 = \frac{180\text{ Hz}}{0.60} = \boxed{300\text{ Hz}}$$

29. (*a*) We assume that the speed of waves on the guitar string does not change when the string is fretted. The fundamental frequency is given by $f = \frac{v}{2L}$, and so the frequency is inversely proportional to the length.

$$f \propto \frac{1}{L} \quad \rightarrow \quad fL = \text{constant}$$

$$f_\text{E} L_\text{E} = f_\text{A} L_\text{A} \quad \rightarrow \quad L_\text{A} = L_\text{E}\frac{f_\text{E}}{f_\text{A}} = \left(0.73\text{ m}\right)\left(\frac{330\text{ Hz}}{440\text{ Hz}}\right) = 0.5475\text{ m}$$

The string should be fretted a distance $0.73\text{ m} - 0.5475\text{ m} = 0.1825\,\text{m} \approx \boxed{0.18\text{ m}}$ from the nut of the guitar.

(*b*) The string is fixed at both ends and is vibrating in its fundamental mode. Thus the wavelength is twice the length of the string (see Fig. 12-7).

$$\lambda = 2L = 2\left(0.5475\text{ m}\right) = 1.095\,\text{m} \approx \boxed{1.1\text{ m}}$$

(*c*) The frequency of the sound will be the same as that of the string, $\boxed{440\text{ Hz}}$. The wavelength is given by the following.

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{440\text{ Hz}} = \boxed{0.78\text{ m}}$$

30. (*a*) At $T = 21^\circ\text{C}$, the speed of sound is given by $v = \left(331 + 0.60\left(21\right)\right)\text{m/s} = 343.6\,\text{m/s}$. For an open pipe, the fundamental frequency is given by $f = \frac{v}{2L}$.

$$f = \frac{v}{2L} \quad \rightarrow \quad L = \frac{v}{2f} = \frac{343.6\,\text{m/s}}{2\left(262\text{ Hz}\right)} = \boxed{0.656\text{ m}}$$

(*b*) The frequency of the standing wave in the tube is $\boxed{262\text{ Hz}}$. The wavelength is twice the length of the pipe, $\boxed{1.31\text{ m}}$

(*c*) The wavelength and frequency are the same in the air, because it is air that is resonating in the organ pipe. The frequency is $\boxed{262\text{ Hz}}$ and the wavelength is $\boxed{1.31\text{ m}}$

31. The speed of sound will change as the temperature changes, and that will change the frequency of the organ. Assume that the length of the pipe (and thus the resonant wavelength) does not change.

$$f_{20} = \frac{v_{20}}{\lambda} \qquad f_{5.0} = \frac{v_{5.0}}{\lambda} \qquad \Delta f = f_{5.0} - f_{20} = \frac{v_{5.0} - v_{20}}{\lambda}$$

$$\frac{\Delta f}{f} = \frac{\frac{v_{5.0} - v_{20}}{\lambda}}{\frac{v_{20}}{\lambda}} = \frac{v_{5.0}}{v_{20}} - 1 = \frac{331 + 0.60\left(5.0\right)}{331 + 0.60\left(20\right)} - 1 = -2.6\times10^{-2} = \boxed{-2.6\%}$$

32. A flute is a tube that is open at both ends, and so the fundamental frequency is given by $f = \frac{v}{2L}$, where L is the distance from the mouthpiece (antinode) to the first open side hole in the flute tube (antinode).

$$f = \frac{v}{2L} \quad \rightarrow \quad L = \frac{v}{2f} = \frac{343\,\text{m/s}}{2(294\ \text{Hz})} = \boxed{0.583\ \text{m}}$$

33. (*a*) At $T = 20^\circ\text{C}$, the speed of sound is $343\,\text{m/s}$. For an open pipe, the fundamental frequency is given by $f = \frac{v}{2L}$.

$$f = \frac{v}{2L} \quad \rightarrow \quad L = \frac{v}{2f} = \frac{343\,\text{m/s}}{2(294\ \text{Hz})} = \boxed{0.583\ \text{m}}$$

(*b*) The speed of sound in helium is $1005\ \text{m/s}$, from Table 12-1. Use this and the pipe's length to to find the pipe's fundamental frequency.

$$f = \frac{v}{2L} = \frac{1005\,\text{m/s}}{2(0.583\ \text{m})} = \boxed{862\ \text{Hz}}$$

34. (*a*) The difference between successive overtones for this pipe is 176 Hz. The difference between successive overtones for an open pipe is the fundamental frequency, and each overtone is an integer multiple of the fundamental. Since 264 Hz is not a multiple of 176 Hz, 176 Hz cannot be the fundamental, and so the pipe cannot be open. Thus it must be a $\boxed{\text{closed}}$ pipe.

(*b*) For a closed pipe, the successive overtones differ by twice the fundamental frequency. Thus 176 Hz must be twice the fundamental, so the fundamental is $\boxed{88.0\ \text{Hz}}$. This is verified since 264 Hz is 3 times the fundamental, 440 Hz is 5 times the fundamental, and 616 Hz is 7 times the fundamental.

35. (*a*) The difference between successive overtones for an open pipe is the fundamental frequency.

$$f_1 = 330\,\text{Hz} - 275\,\text{Hz} = \boxed{55\ \text{Hz}}$$

(*b*) The fundamental frequency is given by $f_1 = \frac{v}{2L}$. Solve this for the speed of sound.

$$v = 2Lf_1 = 2(1.80\ \text{m})(55\ \text{Hz}) = 198\,\text{m/s} \approx \boxed{2.0\times10^2\ \text{m/s}}$$

36. The difference in frequency for two successive harmonics is 40 Hz. For an open pipe, two successive harmonics differ by the fundamental, so the fundamental could be 40 Hz, with 240 Hz being the 6th harmonic and 280 Hz being the 7th harmonic. For a closed pipe, two successive harmonics differ by twice the fundamental, so the fundamental could be 20 Hz. But the overtones of a closed pipe are odd multiples of the fundamental, and both overtones are even multiples of 30 Hz. So the pipe must be an $\boxed{\text{open pipe}}$.

$$f = \frac{v}{2L} \quad \rightarrow \quad L = \frac{v}{2f} = \frac{343\,\text{m/s}}{2(40\ \text{Hz})} = \boxed{4.3\ \text{m}}$$

37. (*a*) The harmonics for the open pipe are $f_n = \frac{nv}{2L}$. To be audible, they must be below 20 kHz.

$$\frac{nv}{2L} < 2\times10^4\,\text{Hz} \quad \rightarrow \quad n < \frac{2(2.14\ \text{m})(2\times10^4\,\text{Hz})}{343\,\text{m/s}} = 249.6$$

Since there are 249 harmonics, there are $\boxed{248\ \text{overtones}}$

(*b*) The harmonics for the closed pipe are $f_n = \frac{nv}{4L}$, *n* odd. Again, they must be below 20 kHz.

$$\frac{nv}{4L} < 2\times10^4\,\text{Hz} \rightarrow n < \frac{4(2.14\text{ m})(2\times10^4\,\text{Hz})}{343\,\text{m/s}} = 499.1$$

The values of *n* must be odd, so *n* = 1, 3, 5, …, 499. There are 250 harmonics, and so there are $\boxed{\text{249 overtones}}$

38. The ear canal can be modeled as a closed pipe of length 2.5 cm. The resonant frequencies are given by $f_n = \frac{nv}{4L}$, *n* odd. The first several frequencies are calculated here.

$$f_n = \frac{nv}{4L} = \frac{n(343\,\text{m/s})}{4(2.5\times10^{-2}\,\text{m})} = n(3430\text{ Hz}),\ n\text{ odd}$$

$$\boxed{f_1 = 3430\text{ Hz} \quad f_3 = 10300\text{ Hz} \quad f_5 = 17200\text{ Hz}}$$

In the graph, the most sensitive frequency is between 3000 and 4000 Hz. This corresponds to the fundamental resonant frequency of the ear canal. The sensitivity decrease above 4000 Hz, but is seen to “flatten out” around 10,000 Hz again, indicating higher sensitivity near 10,000 Hz than at surrounding frequencies. This 10,000 Hz relatively sensitive region corresponds to the first overtone resonant frequency of the ear canal.

39. The beat period is 2.0 seconds, so the beat frequency is the reciprocal of that, 0.50 Hz. Thus the other string is off in frequency by $\boxed{\pm0.50\text{ Hz}}$. The beating does not tell the tuner whether the second string is too high or too low.

40. The beat frequency is the difference in the two frequencies, or $277\text{ Hz} - 262\text{ Hz} = \boxed{15\text{ Hz}}$. If the frequencies are both reduced by a factor of 4, then the difference between the two frequencies will also be reduced by a factor of 4, and so the beat frequency will be $\frac{1}{4}(15\text{ Hz}) = 3.75\,\text{Hz} \approx \boxed{3.8\text{ Hz}}$.

41. The 5000 Hz shrill whine is the beat frequency generated by the combination of the two sounds. This means that the brand X whistle is either 5000 Hz higher or 5000 Hz lower than the known-frequency whistle. If it were 5000 Hz lower, then it would be in the audible range for humans. Since it cannot be heard by humans, the brand X whistle must be 5000 Hz higher than the known frequency whistle. Thus the brand X frequency is $23.5\text{ kHz} + 5\text{ kHz} = \boxed{28.5\text{ kHz}}$

42. Since there are 4 beats/s when sounded with the 350 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 354 Hz. Since there are 9 beats/s when sounded with the 355 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 364 Hz. The common value is $\boxed{346\text{ Hz}}$.

43. The fundamental frequency of the violin string is given by $f = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F_T}{m/L}} = 294\text{ Hz}$. Change the tension to find the new frequency, and then subtract the two frequencies to find the beat frequency.

$$f' = \frac{1}{2L}\sqrt{\frac{(0.98)F_T}{m/L}} = \sqrt{0.98}\,\frac{1}{2L}\sqrt{\frac{F_T}{m/L}} = \sqrt{0.98}\,f$$

$$\Delta f = f - f' = f\left(1-\sqrt{0.98}\right) = (294\text{ Hz})\left(1-\sqrt{0.98}\right) = \boxed{3.0\text{ Hz}}$$

44. Beats will be heard because the difference in the speed of sound for the two flutes will result in two different frequencies. We assume that the flute at 25.0°C will accurately play the middle C.

$$f_1 = \frac{v_1}{2L} \rightarrow L = \frac{v_1}{2f_1} = \frac{[331+0.6(25.0)]\text{m/s}}{2(262\text{ Hz})} = 0.660\text{ m}$$

$$f_2 = \frac{v_2}{2L} = \frac{[331+0.6(5.0)]\text{m/s}}{2(0.660\text{ m})} = 253\text{ Hz} \qquad \Delta f = 262\text{ Hz} - 253\text{ Hz} = \boxed{9\text{ beats/sec}}$$

45. Tuning fork A must have a frequency of 3 Hz either higher or lower than the 441 Hz fork B. Tuning fork C must have a frequency of 4 Hz either higher or lower than the 441 Hz fork B.

$$\boxed{f_A = 438\text{ Hz or }444\text{ Hz} \qquad f_C = 437\text{ Hz or }445\text{ Hz}}$$

The possible beat frequencies are found by subtracting all possible frequencies of A and C.

$$\boxed{|f_A - f_C| = 1\text{ Hz or }7\text{ Hz}}$$

$\boxed{46.}$ (*a*) For destructive interference, the smallest path difference must be one-half wavelength. Thus the wavelength in this situation must be twice the path difference, or 1.00 m.

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{1.00\text{ m}} = \boxed{343\text{ Hz}}$$

(*b*) There will also be destructive interference if the path difference is 1.5 wavelengths, 2.5 wavelengths, etc.

$$\Delta L = 1.5\lambda \rightarrow \lambda = \frac{0.50\text{ m}}{1.5} = 0.333\text{ m} \rightarrow f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.33\text{ m}} = 1029\text{ Hz} \approx \boxed{1000\text{ Hz}}$$

$$\Delta L = 2.5\lambda \rightarrow \lambda = \frac{0.50\text{ m}}{2.5} = 0.20\text{ m} \rightarrow f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.20\text{ m}} = 1715\text{ Hz} \approx \boxed{1700\text{ Hz}}$$

47. The beat frequency is 3 beats per 2 seconds, or 1.5 Hz.

(*a*) The other string must be either $132\text{ Hz} - 1.5\text{ Hz} = \boxed{130.5\text{ Hz}}$ or $132\text{ Hz} + 1.5\text{ Hz} = \boxed{133.5\text{ Hz}}$.

(*b*) Since $f = \frac{v}{2L} = \frac{\sqrt{\frac{F_T}{m/L}}}{2L}$, we have $f \propto \sqrt{F_T} \rightarrow \frac{f}{\sqrt{F_T}} = \frac{f'}{\sqrt{F'_T}} \rightarrow F' = F_T\left(\frac{f'}{f}\right)^2$.

To change 130.5 Hz to 132 Hz: $F' = F_T\left(\frac{132}{130.5}\right)^2 = 1.023$, $\boxed{2.3\%\text{ increase}}$

To change 133.5 Hz to 132 Hz: $F' = F_T\left(\frac{132}{133.5}\right)^2 = 0.978$, $\boxed{2.2\%\text{ decrease}}$

48. To find the beat frequency, calculate the frequency of each sound, and then subtract the two frequencies.

$$f_{\text{beat}} = \left|f_1 - f_2\right| = \left|\frac{v}{\lambda_1} - \frac{v}{\lambda_2}\right| = \left(343\,\text{m/s}\right)\left|\frac{1}{2.64\text{ m}} - \frac{1}{2.76\text{ m}}\right| = 5.649 \approx \boxed{5.6\text{ Hz}}$$

$\boxed{49.}$ (*a*) Observer moving towards stationary source.

$$f' = \left(1+\frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1+\frac{30.0\,\text{m/s}}{343\,\text{m/s}}\right)\left(1550\text{ Hz}\right) = \boxed{1690\text{ Hz}}$$

(*b*) Observer moving away from stationary source.

$$f' = \left(1-\frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1-\frac{30.0\,\text{m/s}}{343\,\text{m/s}}\right)\left(1550\text{ Hz}\right) = \boxed{1410\text{ Hz}}$$

50. (*a*) Source moving towards stationary observer.

$$f' = \frac{f}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \frac{\left(1550\text{ Hz}\right)}{\left(1-\dfrac{32\,\text{m/s}}{343\,\text{m/s}}\right)} = \boxed{1710\text{ Hz}}$$

(*b*) Source moving away from stationary observer.

$$f' = \frac{f}{\left(1+\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \frac{\left(1550\text{ Hz}\right)}{\left(1+\dfrac{32\,\text{m/s}}{343\,\text{m/s}}\right)} = \boxed{1420\text{ Hz}}$$

51. (*a*) For the 15 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2000\text{ Hz}\right)\frac{1}{\left(1-\dfrac{15\,\text{m/s}}{343\,\text{m/s}}\right)} = \boxed{2091\text{ Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2000\text{ Hz}\right)\left(1+\frac{15\,\text{m/s}}{343\,\text{m/s}}\right) = \boxed{2087\text{ Hz}}$$

The frequency shifts are slightly different, with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$. The two frequencies are close, but they are not identical. To 3 significant figures they are the same.

(*b*) For the 150 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2000\text{ Hz}\right)\frac{1}{\left(1-\dfrac{150\,\text{m/s}}{343\,\text{m/s}}\right)} = \boxed{3.55\times10^3\,\text{Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2000\text{ Hz}\right)\left(1+\frac{150\,\text{m/s}}{343\,\text{m/s}}\right) = \boxed{2.87\times10^3\,\text{Hz}}$$

The difference in the frequency shifts is much larger this time, still with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$.

(*c*) For the 300 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2000\text{ Hz}\right)\frac{1}{\left(1-\dfrac{300\,\text{m/s}}{343\,\text{m/s}}\right)} = \boxed{16.0\times10^{3}\,\text{Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2000\text{ Hz}\right)\left(1+\frac{300\,\text{m/s}}{343\,\text{m/s}}\right) = \boxed{3.75\times10^{3}\text{ Hz}}$$

The difference in the frequency shifts is quite large, still with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$.

. The Doppler formulas are asymmetric, with a larger shift for the moving source than for the moving observer, when the two are getting closer to each other. As the source moves toward the observer with speeds approaching the speed of sound, the observed frequency tends towards infinity. As the observer moves toward the source with speeds approaching the speed of sound, the observed frequency tends towards twice the emitted frequency.

52. The frequency received by the stationary car is higher than the frequency emitted by the stationary car, by $\Delta f = 5.5\text{ Hz}$.

$$f_{\text{obs}} = f_{\text{source}} + \Delta f = \frac{f_{\text{source}}}{\left(1-\dfrac{v_{\text{source}}}{v_{\text{snd}}}\right)} \quad\rightarrow$$

$$f_{\text{source}} = \Delta f\left(\frac{v_{\text{snd}}}{v_{\text{source}}}-1\right) = \left(5.5\text{ Hz}\right)\left(\frac{343\,\text{m/s}}{15\,\text{m/s}}-1\right) = \boxed{120\text{ Hz}}$$

53. The moving object can be treated as a moving "observer" for calculating the frequency it receives and reflects. The bat (the source) is stationary.

$$f'_{\text{object}} = f_{\text{bat}}\left(1-\frac{v_{\text{object}}}{v_{\text{snd}}}\right)$$

Then the object can be treated as a moving source emitting the frequency f'_{object}, and the bat as a stationary observer.

$$f''_{\text{bat}} = \frac{f'_{\text{object}}}{\left(1+\dfrac{v_{\text{object}}}{v_{\text{snd}}}\right)} = f_{\text{bat}}\frac{\left(1-\dfrac{v_{\text{object}}}{v_{\text{snd}}}\right)}{\left(1+\dfrac{v_{\text{object}}}{v_{\text{snd}}}\right)} = f_{\text{bat}}\frac{\left(v_{\text{snd}}-v_{\text{object}}\right)}{\left(v_{\text{snd}}+v_{\text{object}}\right)}$$

$$= \left(5.00\times10^{4}\,\text{Hz}\right)\frac{343\,\text{m/s}-25.0\,\text{m/s}}{343\,\text{m/s}+25.0\,\text{m/s}} = \boxed{4.32\times10^{4}\,\text{Hz}}$$

54. The wall can be treated as a stationary "observer" for calculating the frequency it receives. The bat is flying toward the wall.

$$f'_{\text{wall}} = f_{\text{bat}}\frac{1}{\left(1-\dfrac{v_{\text{bat}}}{v_{\text{snd}}}\right)}$$

Then the wall can be treated as a stationary source emitting the frequency f'_{wall}, and the bat as a moving observer, flying toward the wall.

$$f''_{\text{bat}} = f'_{\text{wall}}\left(1+\frac{v_{\text{bat}}}{v_{\text{snd}}}\right) = f_{\text{bat}}\frac{1}{\left(1-\frac{v_{\text{bat}}}{v_{\text{snd}}}\right)}\left(1+\frac{v_{\text{bat}}}{v_{\text{snd}}}\right) = f_{\text{bat}}\frac{\left(v_{\text{snd}}+v_{\text{bat}}\right)}{\left(v_{\text{snd}}-v_{\text{bat}}\right)}$$

$$=\left(3.00\times10^{4}\,\text{Hz}\right)\frac{343\,\text{m/s}+5.0\,\text{m/s}}{343\,\text{m/s}-5.0\,\text{m/s}} = \boxed{3.09\times10^{4}\,\text{Hz}}$$

55. We assume that the comparison is to be made from the frame of reference of the stationary tuba. The stationary observers would observe a frequency from the moving tuba of

$$f_{\text{obs}} = \frac{f_{\text{source}}}{\left(1-\frac{v_{\text{source}}}{v_{\text{snd}}}\right)} = \frac{75\text{ Hz}}{\left(1-\frac{10.0\,\text{m/s}}{343\,\text{m/s}}\right)} = 77\text{ Hz} \qquad f_{\text{beat}} = 77\text{ Hz}-75\text{ Hz} = \boxed{2\text{ Hz}}.$$

56. The beats arise from the combining of the original 3.5 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts – one for the blood cells receiving the original signal (observer moving away from stationary source) and one for the detector receiving the reflected signal (source moving away from stationary observer).

$$f'_{\text{blood}} = f_{\text{original}}\left(1-\frac{v_{\text{blood}}}{v_{\text{snd}}}\right) \qquad f''_{\text{detector}} = \frac{f'_{\text{blood}}}{\left(1+\frac{v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1-\frac{v_{\text{blood}}}{v_{\text{snd}}}\right)}{\left(1+\frac{v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(v_{\text{snd}}-v_{\text{blood}}\right)}{\left(v_{\text{snd}}+v_{\text{blood}}\right)}$$

$$\Delta f = f_{\text{original}} - f''_{\text{detector}} = f_{\text{original}} - f_{\text{original}}\frac{\left(v_{\text{snd}}-v_{\text{blood}}\right)}{\left(v_{\text{snd}}+v_{\text{blood}}\right)} = f_{\text{original}}\frac{2v_{\text{blood}}}{\left(v_{\text{snd}}+v_{\text{blood}}\right)}$$

$$=\left(3.5\times10^{6}\,\text{Hz}\right)\frac{2\left(2.0\times10^{-2}\right)}{\left(1.54\times10^{3}\,\text{m/s}+2.0\times10^{-2}\right)} = \boxed{91\,\text{Hz}}$$

57. The maximum Doppler shift occurs when the heart has its maximum velocity. Assume that the heart is moving away from the original source of sound. The beats arise from the combining of the original 2.25 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts – one for the heart receiving the original signal (observer moving away from stationary source) and one for the detector receiving the reflected signal (source moving away from stationary observer).

$$f'_{\text{heart}} = f_{\text{original}}\left(1-\frac{v_{\text{heart}}}{v_{\text{snd}}}\right) \qquad f''_{\text{detector}} = \frac{f'_{\text{heart}}}{\left(1+\frac{v_{\text{heart}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1-\frac{v_{\text{heart}}}{v_{\text{snd}}}\right)}{\left(1+\frac{v_{\text{heart}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(v_{\text{snd}}-v_{\text{heart}}\right)}{\left(v_{\text{snd}}+v_{\text{heart}}\right)}$$

$$\Delta f = f_{\text{original}} - f''_{\text{detector}} = f_{\text{original}} - f_{\text{original}}\frac{\left(v_{\text{snd}}-v_{\text{blood}}\right)}{\left(v_{\text{snd}}+v_{\text{blood}}\right)} = f_{\text{original}}\frac{2v_{\text{blood}}}{\left(v_{\text{snd}}+v_{\text{blood}}\right)} \quad\rightarrow$$

$$v_{\text{blood}} = v_{\text{snd}} \frac{\Delta f}{2 f_{\text{original}} - \Delta f} = \left(1.54\times10^3 \text{ m/s}\right)\frac{500 \text{ Hz}}{2\left(2.25\times10^6 \text{ Hz}\right) - 500 \text{ Hz}} = \boxed{0.171 \text{ m/s}}$$

If instead we had assumed that the heart was moving towards the original source of sound, we would get $v_{\text{blood}} = v_{\text{snd}} \dfrac{\Delta f}{2 f_{\text{original}} + \Delta f}$. Since the beat frequency is much smaller than the original frequency, the Δf term in the denominator does not significantly affect the answer.

58. The Doppler effect occurs only when there is relative motion of the source and the observer along the line connecting them. In the first four parts of this problem, the whistle and the observer are not moving relative to each other and so there is no Doppler shift. The wind speed increases (or decreases) the velocity of the waves in the direction of the wind, and the wavelength of the waves by the same factor, while the frequency is unchanged.

(*a*), (*b*), (*c*), (*d*) $\boxed{f' = f = 570 \text{ Hz}}$

(*e*) The wind makes an effective speed of sound in air of 343 + 12.0 = 355 m/s, and the observer is moving towards a stationary source with a speed of 15.0 m/s.

$$f' = f\left(1 + \frac{v_{\text{obs}}}{v_{\text{sns}}}\right) = \left(570 \text{ Hz}\right)\left(1 + \frac{15.0 \text{ m/s}}{355 \text{ m/s}}\right) = \boxed{594 \text{ Hz}}$$

(*f*) Since the wind is not changing the speed of the sound waves moving towards the cyclist, the speed of sound is 343 m/s. The observer is moving towards a stationary source with a speed of 15.0 m/s.

$$f' = f\left(1 + \frac{v_{\text{obs}}}{v_{\text{sns}}}\right) = \left(570 \text{ Hz}\right)\left(1 + \frac{15.0 \text{ m/s}}{343 \text{ m/s}}\right) = \boxed{595 \text{ Hz}}$$

59. (*a*) We represent the Mach number by the symbol *M*.

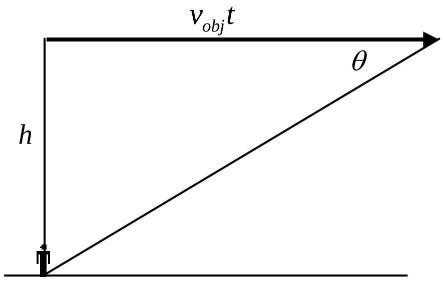
$$M = \frac{v_{\text{obj}}}{v_{\text{snd}}} \rightarrow v_{\text{obj}} = M v_{\text{snd}} = \left(0.33\right)\left(343 \text{ m/s}\right) = \boxed{110 \text{ m/s}}$$

(*b*) $M = \dfrac{v_{\text{obj}}}{v_{\text{snd}}} \rightarrow v_{\text{snd}} = \dfrac{v_{\text{obj}}}{M} = \dfrac{3000 \text{ km/h}}{3.2} = 937.5 \text{ km/h}\left(\dfrac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = \boxed{260 \text{ m/s}}$

60. (*a*) The angle of the shock wave front relative to the direction of motion is given by Eq. 12-7.

$$\sin\theta = \frac{v_{\text{snd}}}{v_{\text{obj}}} = \frac{v_{\text{snd}}}{2.3 v_{\text{snd}}} = \frac{1}{2.3} \rightarrow \theta = \sin^{-1}\frac{1}{2.3} = 25.77^\circ = \boxed{26^\circ}$$

(*b*) The displacement of the plane $\left(v_{obj}t\right)$ from the time it passes overhead to the time the shock wave reaches the observer is shown, along with the shock wave front. From the displacement and height of the plane, the time is found.



$$\tan\theta = \frac{h}{v_{\text{obj}}t} \rightarrow t = \frac{h}{v_{\text{obj}} \tan\theta}$$

$$= \frac{7100 \text{ m}}{\left(2.3\right)\left(310 \text{ m/s}\right)\tan 25.77^\circ} = 20.63 \text{ s} \approx \boxed{21 \text{ s}}$$

61. (*a*) The Mach number is the ratio of the object's speed to the speed of sound.

$$M=\frac{v_{\text{obs}}}{v_{\text{sound}}}=\frac{\left(1.5\times10^{4}\text{ km/hr}\right)\left(\dfrac{1\text{ m/s}}{3.6\text{ km/hr}}\right)}{35\text{ m/s}}=119.05\approx\boxed{120}$$

(*b*) Use Eq. 12.5 to find the angle.

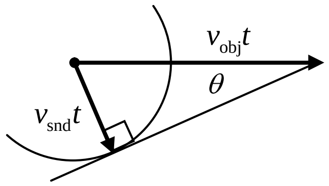
$$\theta=\sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}}=\sin^{-1}\frac{1}{M}=\sin^{-1}\frac{1}{119.05}=\boxed{0.48^{\circ}}$$

62. From Eq. 12-7, $\sin\theta=\dfrac{v_{\text{snd}}}{v_{\text{obj}}}$.

(*a*) $$\theta=\sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}}=\sin^{-1}\frac{343\text{ m/s}}{8500\text{ m/s}}=\boxed{2.3^{\circ}}$$

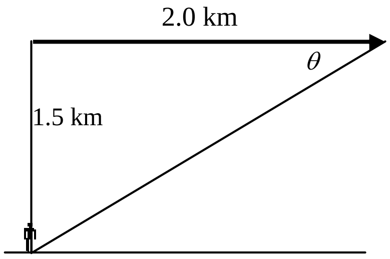
(*b*) $$\theta=\sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}}=\sin^{-1}\frac{1560\text{ m/s}}{8500\text{ m/s}}=\boxed{11^{\circ}}$$

63. Consider one particular wave as shown in the diagram, created at the location of the black dot. After a time *t* has elapsed from the creation of that wave, the supersonic source has moved a distance $v_{\text{obj}}t$, and the wave front has moved a distance $v_{\text{snd}}t$. The line from the position of the source at time *t* is tangent to all of the wave fronts, showing the location of the shock wave. A tangent to a circle at a point is perpendicular to the radius connecting that point to the center, and so a right angle is formed. From the right triangle, the angle θ can be defined.



$$\boxed{\sin\theta=\frac{v_{\text{snd}}t}{v_{\text{obj}}t}=\frac{v_{\text{snd}}}{v_{\text{obj}}}}$$

64. (*a*) The displacement of the plane from the time it passes overhead to the time the shock wave reaches the listener is shown, along with the shock wave front. From the displacement and height of the plane, the angle of the shock wave front relative to the direction of motion can be found, using Eq. 12-7.



$$\tan\theta=\frac{1.5\text{ km}}{2.0\text{ km}}\ \rightarrow\ \theta=\tan^{-1}\frac{1.5}{2.0}=\boxed{37^{\circ}}$$

(*b*) $$M=\frac{v_{\text{obj}}}{v_{\text{snd}}}=\frac{1}{\sin\theta}=\frac{1}{\sin37^{\circ}}=\boxed{1.7}$$

65. The minimum time between pulses would be the time for a pulse to travel from the boat to the maximum distance and back again. The total distance traveled by the pulse will be 400 m, at the speed of sound in fresh water, 1440 m/s.

$$d=vt\ \rightarrow\ t=\frac{d}{v}=\frac{400\text{ m}}{1440\text{ m/s}}=\boxed{0.28\text{ s}}$$

66. Each octave is a doubling of frequency. The number of octaves, *n*, can be found from the following.

$$20{,}000\text{ Hz} = 2^n\left(20\text{ Hz}\right) \rightarrow 1000 = 2^n \rightarrow \log 1000 = n\log 2 \rightarrow$$

$$n = \frac{\log 1000}{\log 2} = 9.97 \approx \boxed{10\text{ octaves}}$$

67. Assume that only the fundamental frequency is heard. The fundamental frequency of an open pipe is given by $f = \dfrac{v}{2L}$.

(*a*) $$f_{3.0} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(3.0\text{ m}\right)} = \boxed{57\text{ Hz}} \qquad f_{2.5} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(2.5\text{ m}\right)} = \boxed{69\text{ Hz}}$$

$$f_{2.0} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(2.0\text{ m}\right)} = \boxed{86\text{ Hz}} \qquad f_{1.5} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(1.5\text{ m}\right)} = 114.3\,\text{Hz} \approx \boxed{110\text{ Hz}}$$

$$f_{1.0} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(1.0\text{ m}\right)} = 171.5\text{ Hz} \approx \boxed{170\text{ Hz}}$$

(*b*) On a noisy day, there are a large number of component frequencies to the sounds that are being made – more people walking, more people talking, etc. Thus it is more likely that the frequencies listed above will be a component of the overall sound, and then the resonance will be more prominent to the hearer. If the day is quiet, there might be very little sound at the desired frequencies, and then the tubes will not have any standing waves in them to detect.

68. The single mosquito creates a sound intensity of $I_0 = 1\times10^{-12}\text{ W/m}^2$. Thus 1000 mosquitoes will create a sound intensity of 1000 times that of a single mosquito.

$$I = 1000 I_0 \qquad \beta = 10\log\frac{1000 I_0}{I_0} = 10\log 1000 = \boxed{30\text{ dB}}.$$

69. The two sound level values must be converted to intensities, then the intensities added, and then converted back to sound level.

$$I_{82}:\ 82\text{ dB} = 10\log\frac{I_{82}}{I_0} \rightarrow I_{82} = 10^{8.2} I_0 = 1.585\times10^8 I_0$$

$$I_{87}:\ 87\text{ dB} = 10\log\frac{I_{87}}{I_0} \rightarrow I_{87} = 10^{8.7} I_0 = 5.012\times10^8 I_0$$

$$I_{\text{total}} = I_{82} + I_{87} = \left(6.597\times10^8\right) I_0 \rightarrow$$

$$\beta_{\text{total}} = 10\log\frac{6.597\times10^8 I_0}{I_0} = 10\log 6.597\times10^8 = \boxed{88\text{ dB}}$$

70. The power output is found from the intensity, which is the power radiated per unit area.

$$105\text{ dB} = 10\log\frac{I}{I_0} \rightarrow I = 10^{10.5} I_0 = 10^{10.5}\left(1.0\times10^{-12}\text{ W/m}^2\right) = 3.162\times10^{-2}\text{ W/m}^2$$

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} \rightarrow P = 4\pi r^2 I = 4\pi\left(12.0\text{ m}\right)^2\left(3.162\times10^{-2}\text{ W/m}^2\right) = \boxed{57.2\,\text{W}}$$

71. Relative to the 1000 Hz output, the 15 kHz output is –10 dB.

$$-10\text{ dB} = 10\log\frac{P_{15\text{ kHz}}}{150\text{ W}} \;\rightarrow\; -1 = \log\frac{P_{15\text{ kHz}}}{150\text{ W}} \;\rightarrow\; 0.1 = \frac{P_{15\text{ kHz}}}{150\text{ W}} \;\rightarrow\; P_{15\text{ kHz}} = \boxed{15\text{ W}}$$

72. The 140 dB level is used to find the intensity, and the intensity is used to find the power. It is assumed that the jet airplane engine radiates equally in all directions.

$$\beta = 140\text{ dB} = 10\log\frac{I}{I_0} \;\rightarrow\; I = 10^{14} I_0 = 10^{14}\left(1.0\times10^{-12}\text{ W/m}^2\right) = 1.0\times10^{2}\text{ W/m}^2$$

$$P = IA = I\pi r^2 = \left(1.0\times10^{2}\text{ W/m}^2\right)\pi\left(2.0\times10^{-2}\right)^2 = \boxed{0.13\text{ W}}$$

73. The gain is given by $\beta = 10\log\frac{P_{\text{out}}}{P_{\text{in}}} = 10\log\frac{100\text{ W}}{1\times10^{-3}\text{ W}} = \boxed{50\,\text{dB}}$.

74. Call the frequencies of four strings of the violin f_{A}, f_{B}, f_{C}, f_{D} with f_{A} the lowest pitch. The mass per unit length will be named μ. All strings are the same length and have the same tension. For a string with both ends fixed, the fundamental frequency is given by $f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F_{\text{T}}}{\mu}}$.

$$f_{\text{B}} = 1.5 f_{\text{A}} \;\rightarrow\; \frac{1}{2L}\sqrt{\frac{F_{\text{T}}}{\mu_{\text{B}}}} = 1.5\frac{1}{2L}\sqrt{\frac{F_{\text{T}}}{\mu_{\text{A}}}} \;\rightarrow\; \mu_{\text{B}} = \frac{\mu_{\text{A}}}{(1.5)^2} = \boxed{0.44\mu_{\text{A}}}$$

$$f_{\text{C}} = 1.5 f_{\text{B}} = (1.5)^2 f_{\text{A}} \;\rightarrow\; \frac{1}{2L}\sqrt{\frac{F_{\text{T}}}{\mu_{\text{C}}}} = (1.5)^2\frac{1}{2L}\sqrt{\frac{F_{\text{T}}}{\mu_{\text{A}}}} \;\rightarrow\; \mu_{\text{C}} = \frac{\mu_{\text{A}}}{(1.5)^4} = \boxed{0.20\mu_{\text{A}}}$$

$$f_{\text{D}} = 1.5 f_{\text{C}} = (1.5)^4 f_{\text{A}} \;\rightarrow\; \frac{1}{2L}\sqrt{\frac{F_{\text{T}}}{\mu_{\text{D}}}} = (1.5)^4\frac{1}{2L}\sqrt{\frac{F_{\text{T}}}{\mu_{\text{A}}}} \;\rightarrow\; \mu_{\text{D}} = \frac{\mu_{\text{A}}}{(1.5)^8} = \boxed{0.039\mu_{\text{A}}}$$

75. (*a*) The wave speed on the string can be found form the length and the fundamental frequency.

$$f = \frac{v}{2L} \;\rightarrow\; v = 2Lf = 2(0.32\text{ m})(440\text{ Hz}) = 281.6 = \boxed{2.8\times10^{2}\text{ m/s}}$$

The tension is found from the wave speed and the mass per unit length.

$$v = \sqrt{\frac{F_{\text{T}}}{\mu}} \;\rightarrow\; F_{\text{T}} = \mu v^2 = \left(6.1\times10^{-4}\text{ kg/m}\right)(281.6\,\text{m/s})^2 = \boxed{48\text{ N}}$$

(*b*) The length of the pipe can be found from the fundamental frequency and the speed of sound.

$$f = \frac{v}{4L} \;\rightarrow\; L = \frac{v}{4f} = \frac{343\,\text{m/s}}{4(440\text{ Hz})} = \boxed{0.195\text{ m}}$$

(*c*) The first overtone for the string is twice the fundamental. $\boxed{880\text{ Hz}}$

The first overtone for the open pipe is 3 times the fundamental. $\boxed{1320\text{ Hz}}$

$\boxed{76.}$ The apparatus is a closed tube. The water level is the closed end, and so is a node of air displacement. As the water level lowers, the distance from one resonance level to the next corresponds to the distance between adjacent nodes, which is one-half wavelength.

$$\Delta L = \tfrac{1}{2}\lambda \rightarrow \lambda = 2\Delta L = 2(0.395\,\text{m} - 0.125\,\text{m}) = 0.540\text{ m}$$

$$f = \frac{v}{\lambda} = \frac{343\,\text{m/s}}{0.540\text{ m}} = \boxed{635\text{ Hz}}$$

77. The frequency of the guitar string is to be the same as the third harmonic ($n = 3$) of the closed tube. The resonance frequencies of a closed tube are given by $f_n = \frac{nv}{4L}$, $n = 1, 3, 5\cdots$, and the frequency of a stretched string is given by $f = \frac{1}{2L}\sqrt{\frac{F_T}{m/L}}$. Equate the two frequencies and solve for the tension.

$$\frac{1}{2L_{\text{string}}}\sqrt{\frac{F_T}{m/L_{\text{string}}}} = \frac{3v}{4L_{\text{tube}}} \rightarrow F_T = \frac{9v^2 m}{4L_{\text{string}}} = \frac{9(343\,\text{m/s})^2(2.10\times10^{-3}\,\text{kg})}{4(0.75\,\text{m})} = \boxed{7.4\times10^2\,\text{N}}$$

78. By anchoring the overpass to the ground in the middle, the center of the overpass is now a node point. This forces the lowest frequency for the bridge to be twice the fundamental frequency, and so now $\boxed{\text{the resonant frequency is 8.0 Hz}}$. Since the earthquakes don't do significant shaking above 6 Hz, this modification should be effective.

79. Since the sound is loudest at points equidistant from the two sources, the two sources must be in phase. The difference in distance from the two sources must be an odd number of half-wavelengths for destructive interference.

$$0.34\text{ m} = \lambda/2 \rightarrow \lambda = 0.68\text{ m} \qquad f = v/\lambda = 343\,\text{m/s}/0.68\text{ m} = \boxed{504\text{ Hz}}$$

$$0.34\text{ m} = 3\lambda/2 \rightarrow \lambda = 0.227\text{ m} \qquad f = v/\lambda = 343\,\text{m/s}/0.227\text{ m} = 1513\text{ Hz (out of range)}$$

80. The Doppler shift is 3.0 Hz, and the emitted frequency from both trains is 424 Hz. Thus the frequency received by the conductor on the stationary train is 427 Hz. Use this to find the moving train's speed.

$$f' = f\frac{v_{\text{snd}}}{(v_{\text{snd}} - v_{\text{source}})} \rightarrow v_{\text{source}} = \left(1 - \frac{f}{f'}\right)v_{\text{snd}} = \left(1 - \frac{424\text{ Hz}}{427\text{ Hz}}\right)(343\,\text{m/s}) = \boxed{2.41\,\text{m/s}}$$

81. As the train approaches, the observed frequency is given by $f'_{\text{approach}} = f\Big/\left(1 - \frac{v_{\text{train}}}{v_{\text{snd}}}\right)$. As the train recedes, the observed frequency is given by $f'_{\text{recede}} = f\Big/\left(1 + \frac{v_{\text{train}}}{v_{\text{snd}}}\right)$. Solve each expression for f, equate them, and then solve for v_{train}.

$$f'_{\text{approach}}\left(1 - \frac{v_{\text{train}}}{v_{\text{snd}}}\right) = f'_{\text{recede}}\left(1 + \frac{v_{\text{train}}}{v_{\text{snd}}}\right) \rightarrow$$

$$v_{\text{train}} = v_{\text{snd}}\frac{(f'_{\text{approach}} - f'_{\text{recede}})}{(f'_{\text{approach}} + f'_{\text{recede}})} = (343\,\text{m/s})\frac{(538\text{ Hz} - 486\text{ Hz})}{(538\text{ Hz} + 486\text{ Hz})} = \boxed{17\,\text{m/s}}$$

82. The sound is Doppler shifted up as the car approaches, and Doppler shifted down as it recedes. The observer is stationary in both cases. The octave shift down means that $f_{\text{approach}} = 2f_{\text{recede}}$.

$$f'_{\text{approach}} = f_{\text{engine}} \Big/ \left(1 - \frac{v_{\text{car}}}{v_{\text{snd}}}\right) \qquad f'_{\text{recede}} = f_{\text{engine}} \Big/ \left(1 + \frac{v_{\text{car}}}{v_{\text{snd}}}\right) \qquad f_{\text{approach}} = 2f_{\text{recede}} \rightarrow$$

$$f_{\text{engine}} \Big/ \left(1 - \frac{v_{\text{car}}}{v_{\text{snd}}}\right) = 2\, f_{\text{engine}} \Big/ \left(1 + \frac{v_{\text{car}}}{v_{\text{snd}}}\right) \rightarrow v_{\text{car}} = \frac{v_{\text{snd}}}{3} = \frac{343\,\text{m/s}}{3} = \boxed{114\,\text{m/s}}$$

83. For each pipe, the fundamental frequency is given by $f = \frac{v}{2L}$. Find the frequency of the shortest pipe.

$$f = \frac{v}{2L} = \frac{343\,\text{m/s}}{2(2.40\,\text{m})} = 71.46\,\text{Hz}$$

The longer pipe has a lower frequency. Since the beat frequency is 11 Hz, the frequency of the longer pipe must be 60.46 Hz. Use that frequency to find the length of the longer pipe.

$$f = \frac{v}{2L} \rightarrow L = \frac{v}{2f} = \frac{343\,\text{m/s}}{2(60.46\,\text{Hz})} = \boxed{2.84\,\text{m}}$$

84. (*a*) Since both speakers are moving towards the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears no beats.

(*b*) The observer will detect an increased frequency from the speaker moving towards him and a decreased frequency from the speaker moving away. The difference in those two frequencies will be the beat frequency that is heard.

$$f'_{\text{towards}} = f\frac{1}{\left(1 - \frac{v_{\text{train}}}{v_{\text{snd}}}\right)} \qquad f'_{\text{away}} = f\frac{1}{\left(1 + \frac{v_{\text{train}}}{v_{\text{snd}}}\right)}$$

$$f'_{\text{towards}} - f'_{\text{away}} = f\frac{1}{\left(1 - \frac{v_{\text{train}}}{v_{\text{snd}}}\right)} - f\frac{1}{\left(1 + \frac{v_{\text{train}}}{v_{\text{snd}}}\right)} = f\left[\frac{v_{\text{snd}}}{(v_{\text{snd}} - v_{\text{train}})} - \frac{v_{\text{snd}}}{(v_{\text{snd}} + v_{\text{train}})}\right]$$

$$(212\text{ Hz})\left[\frac{343\,\text{m/s}}{(343\,\text{m/s} - 10.0\,\text{m/s})} - \frac{343\,\text{m/s}}{(343\,\text{m/s} + 10.0\,\text{m/s})}\right] = \boxed{12\text{ Hz}}$$

(*c*) Since both speakers are moving away from the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears no beats.

85. The beats arise from the combining of the original 5.50 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts – one for the blood cells receiving the original frequency (observer moving away from stationary source) and one for the detector receiving the reflected frequency (source moving away from stationary observer).

$$f'_{\text{blood}} = f_{\text{original}}\left(1 - \frac{v_{\text{blood}}}{v_{\text{snd}}}\right) \qquad f''_{\text{detector}} = \frac{f'_{\text{blood}}}{\left(1 + \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1 - \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)}{\left(1 + \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{(v_{\text{snd}} - v_{\text{blood}})}{(v_{\text{snd}} + v_{\text{blood}})}$$

$$\Delta f = f_{\text{original}} - f''_{\text{detector}} = f_{\text{original}} - f_{\text{original}}\frac{\left(v_{\text{snd}} - v_{\text{blood}}\right)}{\left(v_{\text{snd}} + v_{\text{blood}}\right)} = f_{\text{original}}\frac{2v_{\text{blood}}}{\left(v_{\text{snd}} + v_{\text{blood}}\right)}$$

$$= \left(5.50\times10^{6}\,\text{Hz}\right)\frac{2\left(0.32\,\text{m/s}\right)}{\left(1.54\times10^{3}\,\text{m/s} + 0.32\,\text{m/s}\right)} = \boxed{2.29\times10^{3}\,\text{Hz}}$$

86. Use Eq. 12-4, which applies when both source and observer are in motion. There will be two Doppler shifts in this problem – first for the emitted sound with the bat as the source and the moth as the observer, and then the reflected sound with the moth as the source and the bat as the observer.

$$f'_{\text{moth}} = f_{\text{bat}}\frac{\left(v_{\text{snd}} + v_{\text{moth}}\right)}{\left(v_{\text{snd}} - v_{\text{bat}}\right)} \qquad f''_{\text{bat}} = f'_{\text{moth}}\frac{\left(v_{\text{snd}} + v_{\text{bat}}\right)}{\left(v_{\text{snd}} - v_{\text{moth}}\right)} = f_{\text{bat}}\frac{\left(v_{\text{snd}} + v_{\text{moth}}\right)}{\left(v_{\text{snd}} - v_{\text{bat}}\right)}\frac{\left(v_{\text{snd}} + v_{\text{bat}}\right)}{\left(v_{\text{snd}} - v_{\text{moth}}\right)}$$

$$= \left(51.35\text{ kHz}\right)\frac{\left(343+5.0\right)}{\left(343-6.5\right)}\frac{\left(343+6.5\right)}{\left(343-5.0\right)} = \boxed{54.9\text{ kHz}}$$

87. It is 70.0 ms from the start of one chirp to the start of the next. Since the chirp itself is 3.0 ms long, it is 67.0 ms from the end of a chirp to the start of the next. Thus the time for the pulse to travel to the moth and back again is 67.0 ms. The distance to the moth is half the distance that the sound can travel in 67.0 ms, since the sound must reach the moth and return during the 67.0 ms.

$$d = v_{\text{snd}}t = \left(343\,\text{m/s}\right)\tfrac{1}{2}\left(67.0\times10^{-3}\text{s}\right) = \boxed{11.5\text{ m}}$$

88. The Alpenhorn can be modeled as an open tube, and so the fundamental frequency is $f = \frac{v}{2L}$, and the overtones are given by $f_n = \frac{nv}{2L},\ n = 1, 2, 3, \cdots$.

$$f_1 = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(3.4\text{ m}\right)} = 50.44\,\text{Hz} \approx \boxed{50\text{ Hz}}$$

$$f_n = nf_1 = f_{F\#} \quad\rightarrow\quad n\left(50.44\,\text{Hz}\right) = 370\text{ Hz} \quad\rightarrow\quad n = \frac{370}{50.44} = 7.34$$

Thus the 7th harmonic, which is the $\boxed{\text{6th overtone}}$, is close to F sharp.

89. The walls of the room must be air displacement nodes, and so the dimensions of the room between two parallel boundaries corresponds to a half-wavelength of sound. Fundamental frequencies are then given by $f = \frac{v}{2L}$.

$$\text{Length: } f = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(5.0\text{ m}\right)} = \boxed{34\text{ Hz}} \qquad \text{Width: } f = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(4.0\text{ m}\right)} = \boxed{43\text{ Hz}}$$

$$\text{Height: } f = \frac{v}{2L} = \frac{343\,\text{m/s}}{2\left(2.8\text{ m}\right)} = \boxed{61\text{ Hz}}$$

90. (*a*) The "singing" rod is manifesting standing waves. By holding the rod at its midpoint, it has a node at its midpoint, and antinodes at its ends. Thus the length of the rod is a half wavelength.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{5100\,\text{m/s}}{1.80\text{ m}} = 2833\text{ Hz} = \boxed{2.8\times10^{3}\,\text{Hz}}$$

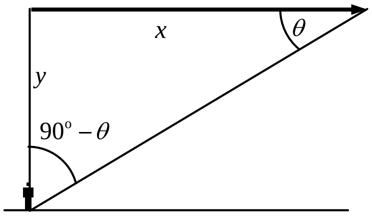
(*b*) The wavelength of sound in the rod is twice the length of the rod, $\boxed{1.80\text{ m}}$.

(*c*) The wavelength of the sound in air is determined by the frequency and the speed of sound in air.

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{2833\text{ Hz}} = \boxed{0.12\,\text{m}}$$

91. Eq. 11-18 gives the relationship between intensity and the displacement amplitude: $I = 2\pi^2 v\rho f^2 A^2$, where A is the displacement amplitude. Thus $I \propto A^2$, or $A \propto \sqrt{I}$. Since the intensity increased by a factor of 10^{12}, the amplitude would increase by a factor of the square root of the intensity increase, or $\boxed{10^6}$.

92. The angle between the direction of the airplane and the shock wave front is found from Eq. 12-5.



$$\sin\theta = \frac{v_{\text{snd}}}{v_{\text{obj}}} \rightarrow \theta = \sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\frac{1}{2.0} = 30^{\circ}$$

The distance that the plane has traveled horizontally from the observer is found from the time and the speed: $x = v_{\text{obj}}\,t$. The altitude is found from the angle and the horizontal distance.

$$\tan\theta = y/x \rightarrow y = x\tan\theta = v_{\text{obj}}t\tan 30^{\circ} = 2(343\,\text{m/s})(90\text{ s})\tan 30^{\circ} = \boxed{3.6\times10^4\,\text{m}}$$

93. The apex angle is 15°, so the shock wave angle is 7.5°. The angle of the shock wave is also given by $\sin\theta = v_{\text{wave}}/v_{\text{object}}$.

$$\sin\theta = v_{\text{wave}}/v_{\text{object}} \rightarrow v_{\text{object}} = v_{\text{wave}}/\sin\theta = 2.2\,\text{km/h}/\sin 7.5^{\circ} = \boxed{17\,\text{km/h}}$$

CHAPTER 13: Temperature and Kinetic Theory

Answers to Questions

1. Because the atomic mass of aluminum is smaller than that of iron, an atom of aluminum has less mass than an atom of iron. Thus 1 kg of aluminum will have more atoms than 1 kg of iron.

2. Properties of materials that can be exploited for the making of a thermometer include:
    i) Volume of a liquid (mercury thermometer)
    ii) Electrical resistance
    iii) Color (frequency) of emitted light from a heated object
    iv) Volume of a gas
    v) Expansion of a metal (bimetallic strip)

3. 1 C^o is larger than 1 F^o. There are 100 C^o between the freezing and boiling temperatures of water, while there are 180 F^o between the same two temperatures.

4. The following conclusions can be drawn:
    A and B are at the same temperature
    B and C are not at the same temperature
    A and C are not at the same temperature

$\boxed{5.}$ When heated, the aluminum expands more than the iron, because the expansion coefficient of aluminum is larger than that of iron. Thus the aluminum will be on the outside of the curve.

6. To be precise, L_0 is to be the initial length of the object. In practice, however, since the value of the coefficient of expansion is so small, there will be little difference in the calculation of ΔL caused by using either the initial or final length, unless the temperature change is quite large.

7. The coefficient of expansion is derived from a ratio of lengths: $\alpha = \frac{\Delta L}{L_0}\frac{1}{\Delta T}$. The length units cancel, and so the coefficient does not depend on the specific length unit used in its determination, as long as the same units are used for both ΔL and L_0.

8. The device controls the furnace by the expansion and contraction of the bimetallic strip. As the temperature increases, the strip coils more, and as the temperature decrease, the strip coils less. As the strip changes shape, it will move the liquid mercury switch. In the diagram, if the switch were tilted more to the right, the mercury would move and make contact between the heater wires, turning on the heater. By adjusting the temperature setting lever, the tilt of the mercury switch is changed, and a different amount of temperature change is needed to tilt the switch to the on (or off) position.

9. The steam pipe can have a large temperature change as the steam enters or leaves the pipe. If the pipe is fixed at both ends and the temperature changes significantly, there will be large thermal stresses which might break joints. The "U" in the pipe allows for expansion and contraction which is not possible at the fixed ends. This is similar to the joints placed in concrete roadway surfaces to allow expansion and contraction.

10. The lead floats in the mercury because $\rho_{\text{Hg}} > \rho_{\text{Pb}}$. As the substances are heated, the density of both substances will decrease due to volume expansion (see problem 17 for the derivation of this result). The density of the mercury decreases more upon heating than the density of the lead, because $\beta_{\text{Hg}} > \beta_{\text{Pb}}$. The net effect is that the densities get closer together, and so relatively more mercury will have to be displaced to hold up the lead, and the lead will float lower in the mercury.

11. The glass is the first to warm due to the hot water, and so the glass will initially expand a small amount. As the glass initially expands, the mercury level will decrease. As thermal equilibrium is reached, the mercury will expand more than the glass expands, since mercury has a larger coefficient of expansion than water, and the mercury level will rise to indicate the higher temperature.

12. If one part is heated or cooled more than another part, there will be more expansion or contraction of one part of the glass compared to an adjacent part. This causes internal stress forces which may exceed the maximum strength of the glass.

13. When Pyrex glass is heated or cooled, it will expand or contract much less than ordinary glass due to its small coefficient of linear expansion. The smaller changes in dimensions result in lower internal stresses than would be present in ordinary glass. Thus there is less of a chance of breaking the Pyrex by heating or cooling it.

$\boxed{14.}$ On a hot day, the pendulum will be slightly longer than at 20°C, due to thermal expansion. Since the period of a pendulum is proportional to the square root of its length, the period will be slightly longer on the hot day, meaning that the pendulum takes more time for one oscillation. Thus the clock will run slow.

15. The soda is mostly water. As water cools below 4°C it expands. There is more expansion of the soda as it cools below 4°C and freezes than there is available room in the can, and so the freezing soda pushes against the can surfaces hard enough to push them outward. Evidently the top and bottom of the can are the weakest parts.

16. When a small mass object collides with a stationary massive object, the speed of the small mass is not changed. But when a small mass object collides with a massive object moving in the opposite direction, the speed of the small object increases. For example, a tennis ball of a given speed that is struck with a racket will rebound with a greater speed than a tennis ball of the same speed bouncing off a wall. So as the gas molecules collide with the piston that is moving toward them, their speed increases. The microscopic increase in molecular speed is manifested macroscopically as a higher temperature.

    In a similar fashion, when the molecules collide with a piston that is moving away from them, they rebound with a reduced speed compared to their initial speed. This lower speed is manifested macroscopically as a lower temperature.

17. The buoyant force on the aluminum sphere is the weight of the water displaced by the sphere, which is the volume of the sphere times the density of water times g. As the substances are heated, the volume of the sphere increases and the density of the water decreases. Since the volume expansion coefficient of the water is almost three times larger than that of the aluminum, the fractional decrease in the water density is larger than the fractional increase in the aluminum volume. Thus the product of the volume of the sphere times the density of water decreases, and the buoyant force gets smaller.

18. Charles's law states that the volume of a fixed mass of gas increases proportionately to the absolute temperature, when the pressure is held constant. As the temperature increases, the molecules have more kinetic energy, and the average force exerted by a gas molecule colliding with the boundaries of the container is proportional to the kinetic energy. Thus the force exerted during the collisions increases. The pressure is the force per unit area, and so for the pressure to remain constant, the surface area of the boundaries must increase, which means the volume of the container must increase.

19. Gay-Lussac's law states that at constant volume, the absolute pressure of a gas is proportional to the absolute temperature. Kinetic molecular theory has a result that the average force exerted by gas particles as they collide with the container boundaries is proportional to the kinetic energy, assuming a fixed container size. For the pressure to increase, the force on the walls must increase, which means the kinetic energy must therefore increase. But the kinetic energy of the particles is proportional to the absolute temperature, and so for the pressure to increase, the temperature must also increase.

20. Since an N_2 molecule has less mass than an O_2 molecule, at the same temperature (and thus the same kinetic energy), N_2 molecules will have a larger speed on average than O_2 molecules. If we consider "launching" molecules of both types from the Earth's surface, the faster-moving N_2 will rise higher before stopping and falling back to Earth. Thus there will be proportionally more N_2 molecules at higher altitudes than at lower altitudes.

21. Because the escape velocity is much smaller for the Moon than for the Earth, most gas molecules even at low temperatures have a speed great enough to escape the Moon's gravity. Thus the atmosphere has "evaporated" over the long time of the Moon's existence.

22. Since the alcohol evaporates more quickly, the alcohol molecules escape "easier" than the water molecules. One explanation could be that the intermolecular forces (bonds) for alcohol are smaller than those for water. Another explanation could be that the alcohol molecules are moving more rapidly than the water molecules, indicating that alcohol molecules are less massive than water molecules. However, the simplest alcohol (CH_3OH) has a molecular mass higher than that of water, so mass is probably not the explanation.

$\boxed{23.}$ On a hot humid day, there is little evaporation from a human due to perspiration, because the air is already saturated with water vapor. Since perspiration is a major cooling mechanism, when it is restricted, humans will feel more uncomfortable. On a hot dry day, water molecules more easily evaporate into the air (taking their kinetic energy with them) and the body is cooled.

24. Liquids boil when their saturated vapor pressure equals the external pressure. For water, from Table 13-3, the saturated vapor pressure of water at $20^{\circ}C$ is about 0.023 atm. So if the external pressure is lowered to that level (about 2.3% of normal air pressure), the water will boil.

25. On a day when the relative humidity is high, the percentage of air molecules that are water (as opposed to N_2 or O_2 molecules) is increased. Since the molecular mass of water is less than that of either N_2 or O_2, the average mass of air molecules in a given volume will decrease, and thus the density will be lower for the humid air.

26. The water in the radiator of an overheated automobile engine is under pressure. Similar to a pressure cooker, that high pressure keeps the water in the liquid state even though the water is quite hot – hotter than $100^{\circ}C$. When the cap is opened, the pressure is suddenly lowered, and the superheated water boils quickly and violently. That hot steam can cause severe burns if it contacts the skin.

Also, the violent bursting forth of steam propels some of the overheated water out of the radiator as well, which can spray onto the person opening the cap and again cause serious burns.

27. Exhaled air contains water vapor, at a relatively high percentage. Since the air inside the lungs is quite warm, the partial pressure of water in the lungs can be high without saturating the air in the lungs, and condensation does not occur. But in the cold winter air, the air can hold very little water without condensation. Thus as the warm, water-laden exhaled air cools, the partial pressure of water vapor exceeds the saturated vapor pressure in the cold air, and some of the water will condense. The white cloud seen is due to the condensed water vapor.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that. For example, in problem 17, values of 25°C and –40°C are used. We took both of those values to have 2 significant figures in calculating the temperature change.

1. The number of atoms is found by dividing the mass of the substance by the mass of a single atom. Take the atomic mass of carbon to be 63.

$$N_{\text{Cu}} = \frac{3.4\times10^{-3}\,\text{kg}}{63\left(1.66\times10^{-27}\right)\text{kg/atom}} = \boxed{3.3\times10^{22}\,\text{atoms of Cu}}$$

2. The number of atoms in a pure substance can be found by dividing the mass of the substance by the mass of a single atom. Take the atomic mass of gold to be 197, and silver to be 108.

$$\frac{N_{\text{Au}}}{N_{\text{Ag}}} = \frac{\dfrac{2.65\times10^{-2}\,\text{kg}}{(197)\left(1.66\times10^{-27}\ \text{kg/atom}\right)}}{\dfrac{2.65\times10^{-2}\,\text{kg}}{(108)\left(1.66\times10^{-27}\ \text{kg/atom}\right)}} = \frac{108}{197} = 0.548 \quad \rightarrow \quad \boxed{N_{\text{Au}} = 0.548N_{\text{Ag}}}$$

Because a gold atom is heavier than a silver atom, there are fewer gold atoms in the given mass.

3. (*a*) $T\left({}^{\circ}\text{C}\right) = \frac{5}{9}\left[T\left({}^{\circ}\text{F}\right) - 32\right] = \frac{5}{9}\left[68 - 32\right] = \boxed{20^{\circ}\text{C}}$

(*b*) $T\left({}^{\circ}\text{F}\right) = \frac{9}{5}T\left({}^{\circ}\text{C}\right) + 32 = \frac{9}{5}(1800) + 32 = \boxed{3300^{\circ}\text{F}}$

4. High: $T\left({}^{\circ}\text{C}\right) = \frac{5}{9}\left[T\left({}^{\circ}\text{F}\right) - 32\right] = \frac{5}{9}\left[136 - 32\right] = \boxed{57.8^{\circ}\text{C}}$

Low: $T\left({}^{\circ}\text{C}\right) = \frac{5}{9}\left[T\left({}^{\circ}\text{F}\right) - 32\right] = \frac{5}{9}\left[-129 - 32\right] = \boxed{-89.4^{\circ}\text{C}}$

5. (*a*) $T\left({}^{\circ}\text{F}\right) = \frac{9}{5}T\left({}^{\circ}\text{C}\right) + 32 = \frac{9}{5}(-15) + 32 = \boxed{5^{\circ}\text{F}}$

(*b*) $T\left({}^{\circ}\text{C}\right) = \frac{5}{9}\left[T\left({}^{\circ}\text{F}\right) - 32\right] = \frac{5}{9}\left[-15 - 32\right] = \boxed{-26^{\circ}\text{C}}$

6. Assume that the temperature and the length are linearly related. The change in temperature per unit length change is as follows.

$$\frac{\Delta T}{\Delta L}=\frac{100.0^{\circ}\text{C}-0.0^{\circ}\text{C}}{22.85\text{ cm}-11.82\text{ cm}}=9.066\,\text{C}^{\circ}/\text{cm}$$

Then the temperature corresponding to length L is $T(L)=0.0^{\circ}\text{C}+(L-11.82\text{ cm})(9.066\,\text{C}^{\circ}/\text{cm})$.

(*a*) $T(16.70\text{ cm})=0.0^{\circ}\text{C}+(16.70\text{ cm}-11.82\text{ cm})(9.066\,\text{C}^{\circ}/\text{cm})=\boxed{44.2^{\circ}\text{C}}$

(*b*) $T(20.50\text{ cm})=0.0^{\circ}\text{C}+(20.50\text{ cm}-11.82\text{ cm})(9.066\,\text{C}^{\circ}/\text{cm})=\boxed{78.7^{\circ}\text{C}}$

7. When the concrete cools in the winter, it will contract, and there will be no danger of buckling. Thus the low temperature in the winter is not a factor in the design of the highway. But when the concrete warms in the summer, it will expand. A crack must be left between the slabs equal to the increase in length of the concrete as it heats from 20°C to 50°C.

$$\Delta L=\alpha L_0\Delta T=\left(12\times10^{-6}/\text{C}^{\circ}\right)(12\text{ m})\left(50^{\circ}\text{C}-20^{\circ}\text{C}\right)=\boxed{4.3\times10^{-3}\text{m}}$$

8. The increase in length of the table is given by Equation 13-1a.

$$\Delta L=\alpha L_0\Delta T=\left(0.2\times10^{-6}/\text{C}^{\circ}\right)(2.0\text{ m})\left(5.0\text{C}^{\circ}\right)=\boxed{2\times10^{-6}\text{m}}$$

For steel, $\Delta L=\alpha L_0\Delta T=\left(12\times10^{-6}/\text{C}^{\circ}\right)(2.0\text{ m})\left(5.0\text{C}^{\circ}\right)=\boxed{1.2\times10^{-4}\text{m}}$.

The change for Super Invar is approximately only 2% of the change for steel.

9. Take the 300 m height to be the height in January. Then the increase in the height of the tower is given by Equation 13-1a.

$$\Delta L=\alpha L_0\Delta T=\left(12\times10^{-6}/\text{C}^{\circ}\right)(300\text{ m})\left(25^{\circ}\text{C}-2^{\circ}\text{C}\right)=\boxed{8\times10^{-2}\text{m}}$$

$\boxed{10.}$ The rivet must be cooled so that its diameter becomes the same as the diameter of the hole.

$$\Delta L=\alpha L_0\Delta T\quad\rightarrow\quad L-L_0=\alpha L_0\left(T-T_0\right)$$

$$T=T_0+\frac{L-L_0}{\alpha L_0}=20^{\circ}\text{C}+\frac{1.869\text{ cm}-1.871\text{ cm}}{\left(12\times10^{-6}/\text{C}^{\circ}\right)(1.871\text{ cm})}=\boxed{-69^{\circ}\text{C}}$$

11. The density at 4°C is $\rho=\frac{M}{V}=\frac{1.00\times10^{3}\text{kg}}{1.00\,\text{m}^{3}}$. When the water is warmed, the mass will stay the same, but the volume will increase according to Equation 13-2.

$$\Delta V=\beta V_0\Delta T=\left(210\times10^{-6}/\text{C}^{\circ}\right)\left(1.00\text{ m}^{3}\right)\left(94^{\circ}\text{C}-4^{\circ}\text{C}\right)=1.89\times10^{-2}\,\text{m}^{3}$$

The density at the higher temperature is $\rho=\frac{M}{V}=\frac{1.00\times10^{3}\text{kg}}{1.00\,\text{m}^{3}+1.89\times10^{-2}\,\text{m}^{3}}=\boxed{981\,\text{kg}/\text{m}^{3}}$

12. The change in volume of the quartz is given by the volume expansion formula, Equation 13-2.

$$\Delta V=\beta V_0\Delta T=\left(1\times10^{-6}/\text{C}^{\circ}\right)\left(\tfrac{4}{3}\pi\left(\frac{8.75\text{ cm}}{2}\right)^{3}\right)\left(200^{\circ}\text{C}-30^{\circ}\text{C}\right)=\boxed{6.0\times10^{-2}\text{cm}^{3}}$$

$\boxed{13.}$ The amount of water that can be added to the container is the final volume of the container minus the final volume of the water. Also note that the original volumes of the water and the container are the same. We assume that the density of water is constant over the temperature change involved.

$$V_{\text{added}} = \left(V_0 + \Delta V\right)_{\text{container}} - \left(V_0 + \Delta V\right)_{\text{H}_2\text{O}} = \Delta V_{\text{container}} - \Delta V_{\text{H}_2\text{O}} = \left(\beta_{\text{container}} - \beta_{\text{H}_2\text{O}}\right) V_0 \Delta T$$

$$= \left(27\times10^{-6}/\text{C}^\circ - 210\times10^{-6}/\text{C}^\circ\right)\left(350.0\text{ mL}\right)\left(-80.0\,\text{C}^\circ\right) = \boxed{5.12\text{ mL}}$$

14. (*a*) The amount of water lost is the final volume of the water minus the final volume of the container. Also note that the original volumes of the water and the container are the same.

$$V_{\text{lost}} = \left(V_0 + \Delta V\right)_{\text{H}_2\text{O}} - \left(V_0 + \Delta V\right)_{\text{container}} = \Delta V_{\text{H}_2\text{O}} - \Delta V_{\text{container}} = \beta_{\text{H}_2\text{O}} V_0 \Delta T - \beta_{\text{container}} V_0 \Delta T$$

$$\beta_{\text{container}} = \beta_{\text{H}_2\text{O}} - \frac{V_{\text{lost}}}{V_0 \Delta T} = 210\times10^{-6}/\text{C}^\circ - \frac{\left(0.35\text{g}\right)\left(\dfrac{1\text{mL}}{0.98324\text{ g}}\right)}{\left(55.50\text{ mL}\right)\left(60^\circ\text{C} - 20^\circ\text{C}\right)} = \boxed{5.0\times10^{-5}/\text{C}^\circ}$$

(*b*) From Table 13-1, the most likely material is $\boxed{\text{copper}}$.

15. (*a*) The sum of the original diameter plus the expansion must be the same for both the plug and the ring.

$$\left(L_0 + \Delta L\right)_{\text{iron}} = \left(L_0 + \Delta L\right)_{\text{brass}} \quad\rightarrow\quad L_{\text{iron}} + \alpha_{\text{iron}} L_{\text{iron}} \Delta T = L_{\text{brass}} + \alpha_{\text{brass}} L_{\text{brass}} \Delta T$$

$$\Delta T = \frac{L_{\text{brass}} - L_{\text{iron}}}{\alpha_{\text{iron}} L_{\text{iron}} - \alpha_{\text{brass}} L_{\text{brass}}} = \frac{8.753\text{ cm} - 8.743\text{ cm}}{\left(12\times10^{-6}/\text{C}^\circ\right)\left(8.743\text{ cm}\right) - \left(19\times10^{-6}/\text{C}^\circ\right)\left(8.753\text{ cm}\right)}$$

$$= -163\text{C}^\circ = T_{\text{final}} - T_{\text{initial}} = T_{\text{final}} - 20^\circ\text{C} \quad\rightarrow\quad T_{\text{final}} = -143^\circ\text{C} \approx \boxed{-140^\circ\text{C}}$$

(*b*) Simply switch the initial values in the above calculation.

$$\Delta T = \frac{L_{\text{brass}} - L_{\text{iron}}}{\alpha_{\text{iron}} L_{\text{iron}} - \alpha_{\text{brass}} L_{\text{brass}}} = \frac{8.743\text{ cm} - 8.753\text{ cm}}{\left(12\times10^{-6}/\text{C}^\circ\right)\left(8.753\text{ cm}\right) - \left(19\times10^{-6}/\text{C}^\circ\right)\left(8.743\text{ cm}\right)} =$$

$$= 164\text{C}^\circ = T_{\text{final}} - T_{\text{initial}} = T_{\text{final}} - 20^\circ\text{C} \quad\rightarrow\quad T_{\text{final}} = 184^\circ\text{C} \approx \boxed{180^\circ\text{C}}$$

16. We model the vessel as having a constant cross-sectional area *A*. Then a volume V_0 of fluid will occupy a length L_0 of the tube, given that $V_0 = AL_0$. Likewise $V = AL$.

$\Delta V = V - V_0 = AL - AL_0 = A\Delta L$ and $\Delta V = \beta V_0 \Delta T = \beta A L_0 \Delta T$.

Equate the two expressions for ΔV, and get $A\Delta L = \beta A L_0 \Delta T \;\rightarrow\; \Delta L = \beta L_0 \Delta T$. But $\Delta L = \alpha L_0 \Delta T$, so we see that under the conditions of the problem, $\boxed{\alpha = \beta}$

17. (*a*) When a substance changes temperature, its volume will change by an amount given by Equation 13-2. This causes the density to change.

$$\Delta\rho = \rho - \rho_0 = \frac{M}{V} - \frac{M}{V_0} = \frac{M}{V_0 + \Delta V} - \frac{M}{V_0} = \frac{M}{V_0 + \beta V_0 \Delta T} - \frac{M}{V_0} = \frac{M}{V_0}\left(\frac{1}{1+\beta\Delta T} - 1\right)$$

$$= \rho_0\left(\frac{1}{1+\beta\Delta T} - \frac{1+\beta\Delta T}{1+\beta\Delta T}\right) = \rho_0\left(\frac{-\beta\Delta T}{1+\beta\Delta T}\right)$$

If we assume that $\beta\Delta T \ll 1$, then the denominator is approximately 1, so $\boxed{\Delta\rho = -\rho_0 \beta \Delta T}$.

(*b*) The fractional change in density is

$$\frac{\Delta\rho}{\rho_0}=\frac{-\rho_0\beta\Delta T}{\rho_0}=-\beta\Delta T=-\left(87\times10^{-6}/{}^\circ\mathrm{C}\right)\left(-40^\circ\mathrm{C}-25^\circ\mathrm{C}\right)=5.7\times10^{-3}$$

This is a $\boxed{0.57\%\text{ increase}}$.

18. Assume that each dimension of the plate changes according to Equation 13-1a.

$$\Delta A=A-A_0=\left(l+\Delta l\right)\left(w+\Delta w\right)-lw=lw+l\Delta w+w\Delta l+\Delta l\Delta w-lw=l\Delta w+w\Delta l+\Delta l\Delta w$$

Neglect the very small quantity $\Delta l\Delta w$.

$$\Delta A=l\Delta w+w\Delta l=l\left(\alpha w\Delta T\right)+w\left(\alpha\, l\Delta T\right)=\boxed{2\alpha\, l\, w\Delta T}$$

19. Consider a cubic solid, where the original length of each dimension of the cube is l.

$$\Delta V=V-V_0=\left(l+\Delta l\right)^3-l^3=l^3+3l^2\Delta l+3l\left(\Delta l\right)^2+\left(\Delta l\right)^3-l^3=3l^2\Delta l+3l\left(\Delta l\right)^2+\left(\Delta l\right)^3$$

Neglect the very small quantities involving Δl squared or cubed.

$$\Delta V=3l^2\Delta l=3l^2\alpha\, l\Delta T=3\alpha\, l^3\Delta T=3\alpha V_0\Delta T$$

But Equation 13-2 states $\Delta V=\beta V_0\Delta T$. Equate the two statements for ΔV.

$$\Delta V=3\alpha V_0\Delta T=\beta V_0\Delta T\quad\rightarrow\quad\boxed{\beta=3\alpha}$$

20. The pendulum has a period of $\tau_0=2\pi\sqrt{L_0/g}$ at 17°C, and a period of $\tau=2\pi\sqrt{L/g}$ at 25°C. Notice that $\tau>\tau_0$ since $L>L_0$. With every swing of the clock, the clock face will indicate that a time τ_0 has passed, but the actual amount of time that has passed is τ. Thus the clock face is "losing time" by an amount of $\Delta\tau=\tau-\tau_0$ every swing. The fractional loss is given by $\frac{\Delta\tau}{\tau_0}$, and the length at the higher temperature is given by

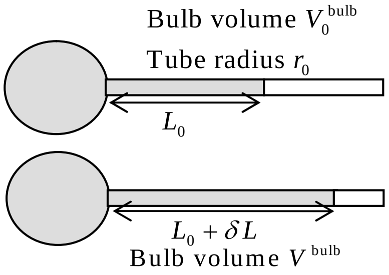
$$\frac{\Delta\tau}{\tau_0}=\frac{\tau-\tau_0}{\tau_0}=\frac{2\pi\sqrt{L/g}-2\pi\sqrt{L_0/g}}{2\pi\sqrt{L_0/g}}=\frac{\sqrt{L}-\sqrt{L_0}}{\sqrt{L_0}}=\frac{\sqrt{L_0+\Delta L}-\sqrt{L_0}}{\sqrt{L_0}}=\frac{\sqrt{L_0+\alpha L_0\Delta T}-\sqrt{L_0}}{\sqrt{L_0}}$$

$$=\sqrt{1+\alpha\Delta T}-1=\sqrt{1+\left(19\times10^{-6}/\mathrm{C}^\circ\right)\left(8\,\mathrm{C}^\circ\right)}-1=7.60\times10^{-5}$$

Thus the amount of time lost in any time period τ_0 is $\Delta\tau=\left(7.60\times10^{-5}\right)\tau_0$. For one year, we have the following.

$$\Delta\tau=\left(7.60\times10^{-5}\right)\left(3.16\times10^7\,\mathrm{s}\right)=2402\,\mathrm{s}\approx\boxed{40\text{ min}}$$

$\boxed{21.}$ (*a*) Consider the adjacent diagrams. The mercury expands due to the heat, as does the bulb volume. The volume of filled glass is equal to the volume of mercury at both temperatures. The value δL is the amount the thread of mercury moves. The additional length of the mercury column in the tube multiplied by the tube cross sectional area will be equal to the expansion of the volume of mercury, minus the expansion of the volume of the glass bulb. Since the tube volume is so much smaller than the bulb volume we can ignore any changes in the tube dimensions and in the mercury initially in the tube volume.


Bulb volume V_0^{bulb}
Tube radius r_0
L_0
$L_0+\delta L$
Bulb volume V^{bulb}

Original volume for glass bulb and Hg in bulb: V_0^{bulb}

Change in glass bulb volume: $\Delta V_{\text{glass}} = V_0^{\text{bulb}}\beta_{\text{glass}}\Delta T$

Change in Hg volume in glass bulb: $\Delta V_{\text{Hg}} = V_0^{\text{bulb}}\beta_{\text{Hg}}\Delta T$

Now find the additional volume of Hg, and use that to find the change in length of Hg in the tube.

$$(\delta L)\pi r_0^2 = \Delta V_{\text{Hg}} - \Delta V_{\text{glass}} = V_0^{\text{bulb}}\beta_{\text{Hg}}\Delta T - V_0^{\text{bulb}}\beta_{\text{glass}}\Delta T \quad \rightarrow$$

$$\delta L = \frac{V_0^{\text{bulb}}}{\pi r_0^2}\Delta T\left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right) = \frac{V_0^{\text{bulb}}}{\pi\left(d_0/2\right)^2}\Delta T\left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right) = \frac{4V_0^{\text{bulb}}}{\pi d_0^2}\Delta T\left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right)$$

$$= \frac{4\left(0.255\,\text{cm}^3\right)}{\pi\left(1.40\times10^{-2}\,\text{cm}\right)^2}\left(33.0^\circ\text{C} - 11.5^\circ\text{C}\right)\left[\left(180-9\right)\times10^{-6}/\text{C}^\circ\right] = \boxed{6.1\text{ cm}}$$

(*b*) The formula is quoted above: $\delta L = \dfrac{4V_0^{\text{bulb}}}{\pi d_0^2}\Delta T\left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right)$.

22. The change in radius with heating does not cause a torque on the rotating wheel, and so the wheel's angular momentum does not change. Also recall that for a cylindrical wheel rotating about its axis, the moment of inertia is $I = \frac{1}{2}mr^2$.

$$L_0 = L_{\text{final}} \quad\rightarrow\quad I_0\omega_0 = I_{\text{final}}\omega_{\text{final}} \quad\rightarrow\quad \omega_{\text{final}} = \frac{I_0\omega_0}{I_{\text{final}}} = \frac{\frac{1}{2}mr_0^2\omega_0}{\frac{1}{2}mr^2} = \frac{r_0^2\omega_0}{r^2}$$

$$\frac{\Delta\omega}{\omega} = \frac{\omega_{\text{final}} - \omega_0}{\omega_0} = \frac{\frac{r_0^2\omega_0}{r^2} - \omega_0}{\omega_0} = \frac{r_0^2}{r^2} - 1 = \frac{r_0^2}{\left(r_0 + \Delta r\right)^2} - 1 = \frac{r_0^2}{\left(r_0 + \alpha r_0\Delta T\right)^2} - 1 = \frac{r_0^2}{\left(r_0 + \alpha r_0\Delta T\right)^2} - 1$$

$$= \frac{1}{\left(1+\alpha\Delta T\right)^2} - 1 = \frac{1 - \left(1 + 2\alpha\Delta T + \left(\alpha\Delta T\right)^2\right)}{\left(1+\alpha\Delta T\right)^2} = \frac{-2\alpha\Delta T - \left(\alpha\Delta T\right)^2}{\left(1+\alpha\Delta T\right)^2} = -\alpha\Delta T\frac{2+\alpha\Delta T}{\left(1+\alpha\Delta T\right)^2}$$

Now assume that $\alpha\Delta T \ll 1$, and so $\dfrac{\Delta\omega}{\omega} = -\alpha\Delta T\dfrac{2+\alpha\Delta T}{\left(1+\alpha\Delta T\right)^2} \approx -2\alpha\Delta T$. Evaluate at the given values.

$$-2\alpha\Delta T = -2\left(25\times10^{-6}/\text{C}^\circ\right)\left(55\,\text{C}^\circ\right) = \boxed{-2.8\times10^{-3}}$$

23. The thermal stress must compensate for the thermal expansion. E is Young's modulus for the aluminum.

$$\text{Stress} = F/A = \alpha E\Delta T = \left(25\times10^{-6}/\text{C}^\circ\right)\left(70\times10^9\text{ N/m}^2\right)\left(35^\circ\text{C} - 15^\circ\text{C}\right) = \boxed{3.5\times10^7\text{ N/m}^2}$$

24. (*a*) Since the beam cannot shrink while cooling, the tensile stress must compensate in order to keep the length constant.

$$\text{Stress} = F/A = \alpha E\Delta T = \left(12\times10^{-6}/\text{C}^\circ\right)\left(200\times10^9\text{ N/m}^2\right)\left(60\text{C}^\circ\right) = \boxed{1.4\times10^8\text{ N/m}^2}$$

(*b*) The ultimate tensile strength of steel (from chapter 9) is $5\times10^8\text{ N/m}^2$, and so $\boxed{\text{the ultimate strength is not exceeded}}$. There would only be a safety factor of about 3.5.

(*c*) For concrete, repeat the calculation with the expansion coefficient and elastic modulus for concrete.

$$\text{Stress} = F/A = \alpha E \Delta T = \left(12\times10^{-6}/\text{C}^\circ\right)\left(20\times10^{9}\ \text{N/m}^2\right)\left(60\ \text{C}^\circ\right) = \boxed{1.4\times10^{7}\ \text{N/m}^2}$$

The ultimate tensile strength of concrete is $2\times10^{6}\ \text{N/m}^2$, and so $\boxed{\text{the concrete will fracture}}$.

25. (*a*) Calculate the change in temperature needed to increase the diameter of the iron band so that it fits over the barrel. Assume that the barrel does not change in dimensions.

$$\Delta L = \alpha L_0 \Delta T \quad \rightarrow \quad L - L_0 = \alpha L_0 \left(T - T_0\right)$$

$$T = T_0 + \frac{L - L_0}{\alpha L_0} = 20^\circ\text{C} + \frac{134.122\ \text{cm} - 134.110\ \text{cm}}{\left(12\times10^{-6}/\text{C}^\circ\right)\left(134.110\ \text{cm}\right)} = 27.457^\circ\text{C} \approx \boxed{27^\circ\text{C}}$$

(*b*) Since the band cannot shrink while cooling, the thermal stress must compensate in order to keep the length at a constant 132.122 cm. E is Young's modulus for the material.

$$\text{Stress} = F/A = \alpha E \Delta T \quad \rightarrow \quad F = AE\frac{\Delta L}{L_0} = AE\alpha\Delta T$$

$$= \left(7.4\times10^{-2}\,\text{m}\right)\left(6.5\times10^{-3}\,\text{m}\right)\left(100\times10^{9}\ \text{N/m}^2\right)\left(12\times10^{-6}/\text{C}^\circ\right)\left(7.457\text{C}^\circ\right) = \boxed{4.3\times10^{3}\,\text{N}}$$

26. Use the relationships $T(\text{K}) = T(^\circ\text{C}) + 273.15$ and $T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15$.

(*a*) $T(\text{K}) = T(^\circ\text{C}) + 273.15 = 86 + 273.15 = \boxed{359\ \text{K}}$

(*b*) $T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15 = \frac{5}{9}\left[78 - 32\right] + 273.15 = \boxed{299\ \text{K}}$

(*c*) $T(\text{K}) = T(^\circ\text{C}) + 273.15 = -100 + 273.15 = \boxed{173\ \text{K}}$

(*d*) $T(\text{K}) = T(^\circ\text{C}) + 273.15 = 5500 + 273.15 = \boxed{5773\ \text{K}}$

(*e*) $T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15 = \frac{5}{9}\left[-459 - 32\right] + 273.15 = \boxed{0.37\ \text{K}}$

27. Use the relationship that $T(\text{K}) = \frac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15$.

$$T(\text{K}) = \tfrac{5}{9}\left[T(^\circ\text{F}) - 32\right] + 273.15 \quad \rightarrow$$

$$T(^\circ\text{F}) = \tfrac{9}{5}\left[T(\text{K}) - 273.15\right] + 32 = \tfrac{9}{5}\left[0 - 273.15\right] + 32 = \boxed{-459.67^\circ\text{F}}$$

28. Use the relationship that $T(\text{K}) = T(^\circ\text{C}) + 273.15$.

(*a*) $T(\text{K}) = T(^\circ\text{C}) + 273.15 = 4270\ \text{K} \approx \boxed{4300\ \text{K}}$; $T(\text{K}) = T(^\circ\text{C}) + 273.15 = \boxed{15\times10^{6}\ \text{K}}$

(*b*) $\%\ \text{error} = \dfrac{\Delta T}{T(\text{K})}\times100 = \dfrac{273}{T(\text{K})}\times100$

4000°C: $\dfrac{273}{4000}\times100 \approx \boxed{7\%}$ $\qquad$ $15\times10^{6}\ ^\circ\text{C}$: $\dfrac{273}{15\times10^{6}}\times100 \approx \boxed{2\times10^{-3}\%}$

29. Assume the gas is ideal. Since the amount of gas is constant, the value of $\frac{PV}{T}$ is constant.

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \rightarrow V_2=V_1\frac{P_1}{P_2}\frac{T_2}{T_1}=\left(3.00\text{m}^3\right)\left(\frac{1.00\text{ atm}}{3.20\text{ atm}}\right)\frac{(273+38)\text{K}}{273\text{ K}}=\boxed{1.07\,\text{m}^3}$$

30. Assume the air is an ideal gas. Since the amount of air is constant, the value of $\frac{PV}{T}$ is constant.

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \rightarrow T_2=T_1\frac{P_2}{P_1}\frac{V_2}{V_1}=\left(293\text{ K}\right)\left(\frac{40\text{ atm}}{1\text{ atm}}\right)\left(\frac{1}{9}\right)=1300\text{ K}=1030^{\text{o}}\text{C}\approx\boxed{1000^{\text{o}}\text{C}}$$

31. Assume the oxygen is an ideal gas. From Example 13-10, the volume of one mole of oxygen gas at STP is $22.4\times10^{-3}\text{m}^3$. The mass of one mole of oxygen, with a molecular mass of 32.0 u, is 32.0 grams. Use these values to calculate the density of the oxygen gas.

$$\rho=\frac{M}{V}=\frac{32.0\times10^{-3}\text{kg}}{22.4\times10^{-3}\text{m}^3}=\boxed{1.43\,\text{kg}/\text{m}^3}$$

32. Assume that the nitrogen and carbon dioxide are ideal gases, and that the volume and temperature are constant for the two gases. From the ideal gas law, the value of $\frac{P}{n}=\frac{RT}{V}$ is constant. Also note that concerning the ideal gas law, the identity of the gas is unimportant, as long as the number of moles is considered.

$$\frac{P_1}{n_1}=\frac{P_2}{n_2} \rightarrow P_2=P_1\frac{n_2}{n_1}=\left(3.65\text{ atm}\right)\left(\frac{\dfrac{21.6\text{ kg CO}_2}{44\times10^{-3}\text{ kg CO}_2/\text{mol}}}{\dfrac{21.6\text{ kg N}_2}{28\times10^{-3}\text{ kg N}_2/\text{mol}}}\right)=\left(3.65\text{ atm}\right)\left(\frac{28}{44}\right)=\boxed{2.32\text{ atm}}$$

33. (*a*) Assume the nitrogen is an ideal gas. The number of moles of nitrogen is found from the atomic weight, and then the ideal gas law is used to calculate the volume of the gas.

$$n=\left(18.5\text{ kg}\right)\frac{1\text{ mole N}_2}{28.0\times10^{-3}\text{kg}}=660.71\text{ mol}$$

$$PV=nRT \rightarrow V=\frac{nRT}{P}=\frac{\left(660.71\text{ mol}\right)\left(8.315\,\text{J}/\text{mol}\bullet\text{K}\right)\left(273\text{ K}\right)}{1.013\times10^5\text{Pa}}=14.806\text{ m}^3$$

$$\approx\boxed{14.8\text{ m}^3}$$

(*b*) Hold the volume and temperature constant, and again use the ideal gas law.

$$n=\left(18.5\text{ kg}+15.0\text{ kg}\right)\frac{1\text{ mole N}_2}{28.0\times10^{-3}\text{kg}}=1196\text{ mol}$$

$$PV=nRT \rightarrow$$

$$P=\frac{nRT}{V}=\frac{\left(1196\text{ mol}\right)\left(8.315\,\text{J}/\text{mol}\bullet\text{K}\right)\left(273\text{ K}\right)}{14.806\text{ m}^3}=\boxed{1.83\times10^5\text{Pa}=1.82\,\text{atm}}$$

34. (*a*) Assume that the helium is an ideal gas, and then use the ideal gas law to calculate the volume. Absolute pressure must be used, even though gauge pressure is given.

$$PV = nRT \quad \rightarrow \quad V = \frac{nRT}{P} = \frac{(18.75\text{ mol})(8.315\,\text{J/mol}\bullet\text{K})(283\text{ K})}{(1.350\,\text{atm})(1.013\times10^5\text{ Pa/atm})} = \boxed{0.323\text{ m}^3}$$

(*b*) Since the amount of gas is not changed, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \quad T_2 = T_1\frac{P_2}{P_1}\frac{V_2}{V_1} = (283\text{ K})\left(\frac{2.00\text{ atm}}{1.350\text{ atm}}\right)\left(\frac{1}{2}\right) = 210\text{ K} = \boxed{-63^\circ\text{C}}$$

35. Assume the argon is an ideal gas. The number of moles of argon is found from the atomic weight, and then the ideal gas law is used to find the pressure.

$$n = (105.0\text{ kg})\frac{1\text{ mole Ar}}{39.95\times10^{-3}\,\text{kg}} = 2628\text{ mol}$$

$$PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V} = \frac{(2628\text{ mol})(8.315\,\text{J/mol}\bullet\text{k})(385\text{ K})}{(35.0\text{ L})(1.00\times10^{-3}\text{ m}^3/\text{L})} = \boxed{2.40\times10^8\,\text{Pa}}$$

36. Assume that the oxygen and helium are ideal gases, and that the volume and temperature are constant for the two gases. From the ideal gas law, the value of $\frac{P}{n} = \frac{RT}{V}$ is constant. Also note that concerning the ideal gas law, the identity of the gas is unimportant, as long as the number of moles is considered. Finally, gauge pressure must be changed to absolute pressure.

$$\frac{P_1}{n_1} = \frac{P_2}{n_2} \quad \rightarrow \quad n_2 = n_1\frac{P_2}{P_1} = (26.0\text{ kg O}_2)\left(\frac{1\text{ mole O}_2}{32\times10^{-3}\,\text{kg}}\right)\frac{(8.00\text{ atm})}{(9.70\text{ atm})} = 6.70\times10^2\,\text{moles}$$

$$(6.70\times10^2\,\text{moles})\left(\frac{4.0\times10^{-3}\,\text{kg}}{1\text{ mole He}}\right) = \boxed{2.68\text{ kg He}}$$

37. There are three forces to consider: the buoyant force upwards (which is the weight of the cold air displaced by the volume of the balloon), the downward weight of the hot air in the balloon, and the downward weight of the passengers and equipment. For the balloon to rise at constant speed, the buoyant force must equal the two weights.

$$F_{\text{buoyant}} = m_{\text{hot}}g + 2700\text{ N} \quad \rightarrow \quad V\rho_{\text{cold}}g = V\rho_{\text{hot}}g + 2700\text{ N}$$

The ideal gas law can be written in terms of the gas density ρ and the molecular mass M as follows.

$$PV = nRT = \frac{m}{M}RT \quad \rightarrow \quad \frac{PM}{R} = \frac{m}{V}T = \rho T$$

The gas inside and outside the balloon is air, and so M is the same for inside and outside. Also, since the balloon is open to the atmosphere, the pressure in the balloon is the same as the pressure outside the balloon. Thus the ideal gas law reduces to $\rho T = \text{constant} = (\rho T)_{\text{cold}} = (\rho T)_{\text{hot}}$.

$$V\rho_{\text{cold}}g = V\rho_{\text{hot}}g + 2700\text{ N} = V\rho_{\text{cold}}\frac{T_{\text{cold}}}{T_{\text{hot}}}g + 2700\text{ N} \quad \rightarrow$$

$$T_{\text{hot}} = \frac{V\rho_{\text{cold}}T_{\text{cold}}g}{(V\rho_{\text{cold}}g - 2700\text{ N})} = \frac{(1800\,\text{m}^3)(1.29\,\text{kg/m}^3)(273\,\text{K})(9.80\,\text{m/s}^2)}{\left[(1800\,\text{m}^3)(1.29\,\text{kg/m}^3)(9.80\,\text{m/s}^2) - 2700\text{ N}\right]}$$

$$= 309.8\,\text{K} \approx \boxed{37^\circ\text{C}}$$

One factor limiting the maximum altitude would be that as the balloon rises, the density of the air decreases, and thus the temperature required gets higher. Eventually the air would be too hot and the balloon fabric might be damaged.

38. Assume that the air is an ideal gas. The pressure and volume are held constant. From the ideal gas law, the value of $\frac{PV}{R}=nT$ is held constant.

$$n_1T_1=n_2T_2 \quad\rightarrow\quad \frac{n_2}{n_1}=\frac{T_1}{T_2}=\frac{(273+15)\,\mathrm{K}}{(273+38)\,\mathrm{K}}=\frac{288}{311}=0.926$$

Thus $1-0.926=0.074=\boxed{7.4\%}$ must be removed.

39. Assume the oxygen is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \quad\rightarrow\quad P_2=P_1\frac{V_1}{V_2}\frac{T_2}{T_1}=(2.45\ \mathrm{atm})\left(\frac{61.5\ \mathrm{L}}{48.8\ \mathrm{L}}\right)\frac{(273+50.0)\,\mathrm{K}}{(273+18.0)\,\mathrm{K}}=\boxed{3.43\ \mathrm{atm}}$$

40. Assume the helium is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant. We assume that since the outside air pressure decreases by 30%, the air pressure inside the balloon will also decrease 30%.

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \quad\rightarrow\quad \frac{V_2}{V_1}=\frac{P_1}{P_2}\frac{T_2}{T_1}=\left(\frac{1.0}{0.70}\right)\frac{(273+5.0)\,\mathrm{K}}{(273+20.0)\,\mathrm{K}}=\boxed{1.4\text{ times the original volume}}$$

41. At STP, 1 mole of ideal gas occupies 22.4 L.

$$\frac{1\ \mathrm{mole}}{22.4\ \mathrm{L}}\left(\frac{6.02\times10^{23}\,\mathrm{molecules}}{\mathrm{mole}}\right)\left(\frac{1\ \mathrm{L}}{10^{-3}\mathrm{m}^3}\right)=\boxed{2.69\times10^{25}\ \mathrm{molecules/m^3}}$$

42. We assume that the water is at 4°C so that its density is $1000\,\mathrm{kg/m^3}$.

$$1.000\ \mathrm{L}\left(\frac{10^{-3}\mathrm{m}^3}{1\ \mathrm{L}}\right)\left(\frac{1000\ \mathrm{kg}}{1\ \mathrm{m}^3}\right)\left(\frac{1\ \mathrm{mol}}{(15.9994+2\times1.00794)\times10^{-3}\,\mathrm{kg}}\right)=\boxed{55.51\ \mathrm{mol}}$$

$$55.51\ \mathrm{mol}\left(\frac{6.02\times10^{23}\,\mathrm{molecules}}{1\ \mathrm{mol}}\right)=\boxed{3.34\times10^{25}\,\mathrm{molecules}}$$

43. (*a*) Since the average depth of the oceans is very small compared to the radius of the Earth, the ocean's volume can be calculated as that of a spherical shell with surface area $4\pi R_{\mathrm{Earth}}^2$ and a thickness Δy. Then use the density of sea water to find the mass, and the molecular weight of water to find the number of moles.

$$\mathrm{Volume}=0.75\left(4\pi R_{\mathrm{Earth}}^2\right)\Delta y=0.75(4\pi)\left(6.38\times10^6\,\mathrm{m}\right)^2\left(3\times10^3\,\mathrm{m}\right)=1.15\times10^{18}\,\mathrm{m}^3$$

$$1.15\times10^{18}\,\mathrm{m}^3\left(\frac{1025\ \mathrm{kg}}{\mathrm{m}^3}\right)\left(\frac{1\ \mathrm{mol}}{18\times10^{-3}\,\mathrm{kg}}\right)=6.55\times10^{22}\,\mathrm{moles}\approx\boxed{7\times10^{22}\,\mathrm{moles}}$$

(*b*) $6.55\times10^{22}\,\text{moles}\left(\dfrac{6.02\times10^{23}\,\text{molecules}}{1\text{ mol}}\right)\approx\boxed{4\times10^{46}\,\text{molecules}}$

44. The net force on each side of the box will be the pressure difference between the inside and outside of the box, times the area of a side of the box. The outside pressure is 1 atmosphere. The ideal gas law is used to find the pressure inside the box, assuming that the mass of gas and the volume are constant.

$$\frac{P}{T}=\frac{nR}{V}=\text{constant}\;\rightarrow\;\frac{P_2}{T_2}=\frac{P_1}{T_1}\;\rightarrow\;P_2=P_1\frac{T_2}{T_1}=(1.00\text{ atm})\frac{(273+180)\text{K}}{(273+20)\text{K}}=1.55\text{ atm}$$

The area of a side of the box is given by

$$\text{Area}=L^2=\left[(\text{Volume of box})^{1/3}\right]^2=\left(5.1\times10^{-2}\,\text{m}^2\right)^{2/3}=1.4\times10^{-1}\,\text{m}^2$$

The net force on a side of the box is the pressure difference times the area.

$$F=(\Delta\,\text{Pressure})(\text{Area})=(0.55\text{ atm})\left(1.01\times10^5\,\text{Pa}\right)\left(1.4\times10^{-1}\,\text{m}^2\right)=\boxed{7.6\times10^3\,\text{N}}$$

45. We assume that the last breath Galileo took has been spread uniformly throughout the atmosphere since his death. Calculate the number of molecules in Galileo's last breath, and divide it by the volume of the atmosphere, to get "Galileo molecules/m^3". Multiply that factor times the size of a breath to find the number of Galileo molecules in one of our breaths.

$$PV=NkT\;\rightarrow\;N=\frac{PV}{kT}=\frac{\left(1.01\times10^5\,\text{Pa}\right)\left(2.0\times10^{-3}\,\text{m}^3\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)(300\text{ K})}=4.9\times10^{22}\,\text{molecules}$$

$$\text{Atmospheric volume}=4\pi R_{\text{Earth}}^2h=4\pi\left(6.38\times10^6\,\text{m}\right)^2\left(1.0\times10^4\,\text{m}\right)=5.1\times10^{18}\,\text{m}^3$$

$$\frac{\text{Galileo molecules}}{\text{m}^3}=\frac{4.9\times10^{22}\,\text{molecules}}{5.8\times10^{18}\,\text{m}^3}=9.6\times10^3\text{ molecules/m}^3$$

$$\frac{\#\text{ Galileo molecules}}{\text{breath}}=9.6\times10^3\,\frac{\text{molecules}}{\text{m}^3}\left(\frac{2.0\times10^{-3}\,\text{m}^3}{1\text{ breath}}\right)=\boxed{19\frac{\text{molecules}}{\text{breath}}}$$

46. (*a*) The average translational kinetic energy of a gas molecule is $\frac{3}{2}kT$.

$$KE_{\text{avg}}=\tfrac{3}{2}kT=\tfrac{3}{2}\left(1.38\times10^{-23}\,\text{J/K}\right)(273\text{ K})=\boxed{5.65\times10^{-21}\,\text{J}}$$

(*b*) The total translational kinetic energy is the average kinetic energy per molecule, times the number of molecules.

$$KE_{\text{total}}=N\left(KE_{\text{avg}}\right)=(2.0\text{ mol})\left(\frac{6.02\times10^{23}\,\text{molecules}}{1}\right)\tfrac{3}{2}\left(1.38\times10^{-23}\,\text{J/K}\right)(293\text{ K})$$

$$=\boxed{7.3\times10^3\,\text{J}}$$

$\boxed{47.}$ The rms speed is given by Equation 13-9, $v_{\text{rms}}=\sqrt{3kT/m}$. Helium has an atomic mass of 4.0.

$$v_{\text{rms}}=\sqrt{3kT/m}=\sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)(6000\text{ K})}{4.0\left(1.66\times10^{-27}\,\text{kg}\right)}}=6116\,\text{m/s}\approx\boxed{6\times10^3\,\text{m/s}}$$

48. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{373\text{ K}}{273\text{ K}}} = \boxed{1.17}$$

49. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. Since the rms speed is proportional to the square root of the absolute temperature, to double the rms speed without changing the mass, the absolute temperature must be multiplied by a factor of 4.

$$T_{\text{fast}} = 4T_{\text{slow}} = 4(273+20)\text{K} = 1172\text{ K} = \boxed{899^{\circ}\text{C}}$$

50. The rms speed is the square root of the mean (average) of the squares of the speeds.

$$v_{\text{rms}} = \sqrt{\frac{6^2+2^2+4^2+6^2+0^2+4^2+1^2+8^2+5^2+3^2+7^2+8^2}{12}} = \sqrt{\frac{320}{12}} = \boxed{5.2\text{ km/s}}$$

The average speed is $v_{\text{avg}} = \dfrac{6+2+4+6+0+4+1+8+5+3+7+8}{12} = \dfrac{54}{12} = 4.5\text{ km/s}$.

51. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = 1.010 = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} \rightarrow$$

$$T_2 = T_1(1.010)^2 = (293.2\text{ K})(1.010)^2 = 299.1\text{ K} = \boxed{25.9^{\circ}\text{C}}$$

52. From the ideal gas law, $PV = nRT$, if the volume and amount of gas are held constant, the temperature is proportional to the pressure, $PV = nRT \rightarrow P = \dfrac{nR}{V}T = (\text{constant})T$. Thus the temperature will be doubled . Since the rms speed is proportional to the square root of the temperature, $v_{\text{rms}} = \sqrt{3kT/m} = (\text{constant})\sqrt{T}$, the rms speed will be multiplied by a factor of $\sqrt{2} = \boxed{1.41}$.

53. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. The temperature can be found from the ideal gas law, $PV = NkT \rightarrow kT = PV/N$. The mass of the gas is the mass of a molecule times the number of molecules: $M = Nm$, and the density of the gas is the mass per unit volume, $\rho = \dfrac{M}{V}$. Combining these relationships gives the following.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3PV}{Nm}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}}$$

54. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \frac{\sqrt{3kT/m_2}}{\sqrt{3kT/m_1}} \rightarrow \boxed{\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \sqrt{\frac{m_1}{m_2}}}$$

55. The temperature of the nitrogen gas is found from the ideal gas law, and then the rms speed is found from the temperature.

$$PV = nRT \quad \rightarrow \quad T = \frac{PV}{nR} = \frac{(2.1\,\text{atm})(1.013\times10^5\ \text{Pa/atm})(8.5\,\text{m}^3)}{(1300\ \text{mol})(8.315\,\text{J/mol}\bullet\text{K})} = 167.3\ \text{K}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(167.3\ \text{K})}{28(1.66\times10^{-27}\,\text{kg})}} = 386.0\,\text{m/s} \approx \boxed{3.9\times10^2\ \text{m/s}}$$

56. (*a*) The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(273\ \text{K})}{32(1.66\times10^{-27}\,\text{kg})}} = \boxed{461\,\text{m/s}}$$

(*b*) Assuming that the particle has no preferred direction, then we have the following:

$$v_{\text{rms}}^2 = v_x^2 + v_y^2 + v_z^2 = 3v_x^2 \quad \rightarrow \quad v_x = v_{\text{rms}}/\sqrt{3}\,.$$

The time for one crossing of the room is then given by $t = d/v_x = \sqrt{3}d/v_{\text{rms}}$, and so the time for a round trip is $2\sqrt{3}d/v_{\text{rms}}$. Thus the number of back and forth round trips per second is the reciprocal of this time, $\frac{v_{\text{rms}}}{2\sqrt{3}d}$.

$$\#\ \text{round trips per sec} = \frac{v_{\text{rms}}}{2\sqrt{3}d} = \frac{461\,\text{m/s}}{2\sqrt{3}(7.0\ \text{m})} = \boxed{19\ \text{round trips per sec}}$$

57. Assume that nitrogen is an ideal gas, and that each molecule occupies the same cubical volume of L^3. Find the volume per molecule from the ideal gas law, and then the side length of that cubical molecular volume will be an estimate of the average distance between molecules.

$$PV = NkT \quad \rightarrow \quad \frac{V}{N} = \frac{kT}{P} = \frac{(1.38\times10^{-23}\ \text{J/K})(273\ \text{K})}{1.01\times10^5\,\text{Pa}} = 3.73\times10^{-26}\ \text{m}^3/\text{molecule}$$

$$L = \left(\frac{V}{N}\right)^{1/3} = (3.73\times10^{-26}\,\text{m}^3)^{1/3} = \boxed{3.34\times10^{-9}\,\text{m}}$$

58. It is stated in the text that the relationship $v_{\text{rms}} = \sqrt{3kT/m}$ is applicable to molecules within living cells at body temperature (37°C).

(*a*) For the amino acid: $v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(310\ \text{K})}{89(1.66\times10^{-27}\,\text{kg})}} = \boxed{2.9\times10^2\ \text{m/s}}$.

(*b*) For the protein: $v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(310\ \text{K})}{(50{,}000)(1.66\times10^{-27}\,\text{kg})}} = \boxed{12\,\text{m/s}}$

59. The pressure can be stated in terms of the ideal gas law, $P = NkT/V$. Substitute for the temperature from the expression for the rms speed, $v_{\text{rms}} = \sqrt{3kT/m} \rightarrow T = mv_{\text{rms}}^2/3k$. The mass of the gas is the mass of a molecule times the number of molecules: $M = Nm$, and the density of the gas is the mass per unit volume, $\rho = M/V$. Combining these relationships gives the following.

$$P = \frac{NkT}{V} = \frac{Nk}{V}\frac{mv_{\text{rms}}^2}{3k} = \tfrac{1}{3}\frac{Nm}{V}v_{\text{rms}}^2 = \tfrac{1}{3}\frac{M}{V}v_{\text{rms}}^2 = \tfrac{1}{3}\rho v_{\text{rms}}^2 \rightarrow \boxed{P = \tfrac{1}{3}\rho v_{\text{rms}}^2}$$

60. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{\left(v_{\text{rms}}\right)_{^{235}\text{UF}_6}}{\left(v_{\text{rms}}\right)_{^{235}\text{UF}_6}} = \frac{\sqrt{3kT/m_{^{235}\text{UF}_6}}}{\sqrt{3kT/m_{^{238}\text{UF}_6}}} = \sqrt{\frac{m_{^{238}\text{UF}_6}}{m_{^{235}\text{UF}_6}}} = \sqrt{\frac{238+6(19)}{235+6(19)}} = \sqrt{\frac{352}{349}} = \boxed{1.004}$$

61. (*a*) From Fig. 13-21, at atmospheric pressure, CO_2 can exist as $\boxed{\text{solid or vapor}}$.

(*b*) From Fig. 13-21, for CO_2 to exist as a liquid, $\boxed{5.11\ \text{atm} \le P \le 73\ \text{atm}}$ and $\boxed{-56.6^\circ\text{C} \le T \le 31^\circ\text{C}}$.

62. (*a*) From Fig. 13-20, water is $\boxed{\text{vapor}}$ when the pressure is 0.01 atm and the temperature is 90°C.

(*b*) From Fig. 13-20, water is $\boxed{\text{solid}}$ when the pressure is 0.01 atm and the temperature is –20°C.

63. From Table 13-3, if the temperature is 25°C, the saturated vapor pressure is 23.8 torr. If the relative humidity is 50%, then the partial pressure of water is half the saturated vapor pressure, or 11.9 torr. The dew point is the temperature at which the saturated vapor pressure is 11.9 torr, and from Table 13-3 that is between 10°C and 15°C. Since there is no entry for 11.9 torr, the temperature can be estimated by a linear interpolation. Between 10°C and 15°C, the temperature change per torr is as follows:

$$\frac{(15-10)\text{C}^\circ}{(12.8-9.21)\,\text{torr}} = 1.393\,\text{C}^\circ/\text{torr}.$$

Thus the temperature corresponding to 11.9 torr is

$$10^\circ\text{C} + \left[(11.9-9.21)\,\text{torr}\right]\left(1.393\,\text{C}^\circ/\text{torr}\right) = 13.7^\circ\text{C} \approx \boxed{14^\circ\text{C}}.$$

64. At the boiling temperature, the external air pressure equals the saturated vapor pressure. Thus from Table 13-3, the air pressure is $\boxed{526\ \text{torr}}$ or $\boxed{7.01\times10^4\,\text{Pa}}$ or $\boxed{0.692\ \text{atm}}$.

65. At the boiling temperature, the air pressure equals the saturated vapor pressure. The pressure of 0.72 atm is equal to $7.27\times10^4\,\text{Pa}$. From Table 13-3, the temperature is between 90°C and 100°C. Since there is no entry for $7.27\times10^4\,\text{Pa}$, the temperature can be estimated by a linear interpolation. Between 90°C and 100°C, the temperature change per Pa is as follows:

$$\frac{(100-90)\text{C}^\circ}{(10.1-7.01)\times10^4\text{Pa}} = 3.236\times10^{-4}\,\text{C}^\circ/\text{Pa}.$$

Thus the temperature corresponding to $7.27\times10^4\,\text{Pa}$ is

$$90^\circ\text{C} + \left[(7.27-7.01)\times10^4\text{Pa}\right]\left(3.236\times10^{-4}\,\text{C}^\circ/\text{Pa}\right) = 90.8^\circ\text{C} \approx \boxed{91^\circ\text{C}}.$$

66. The relative humidity of 40% with a partial pressure of 530 Pa of water gives a saturated vapor pressure of

$$0.40P_{\text{saturated}} = 530\text{ Pa} \quad\rightarrow\quad P_{\text{saturated}} = \frac{530\text{ Pa}}{0.40} = 1325\text{ Pa}$$

From Table 13-3, the temperature at which the saturated vapor pressure is 1325 Pa is between 10°C and 15°C. Since there is no entry for 1325 Pa, the temperature can be estimated by a linear interpolation. Between 10°C and 15°C, the temperature change per Pa is as follows:

$$\frac{(15-10)\text{C}^\circ}{(1.71-1.23)\times10^3\text{Pa}} = 1.042\times10^{-2}\text{ C}^\circ\big/\text{Pa}\ .$$

Thus the temperature corresponding to 1325 Pa is

$$10^\circ\text{C} + \left[(1325-1230)\text{Pa}\right]\left(1.042\times10^{-2}\text{ C}^\circ\big/\text{Pa}\right) = 10.99^\circ\text{C} \approx \boxed{11^\circ\text{C}}\ .$$

67. From Table 13-3, the saturated vapor pressure at 25°C is 3170 Pa. Since the relative humidity is 35%, the partial pressure of water is

$$P_{\text{water}} = 0.35P_{\text{saturated}} = 0.35(3170\text{ Pa}) = \boxed{1.1\times10^3\text{Pa}}$$

68. Since the water is boiling at 120°C, the saturated vapor pressure is the same as the pressure inside the pressure cooker. From Table 13-3, the pressure is $\boxed{1.99\times10^5\text{Pa} = 1.97\text{ atm}}$.

69. The total amount of water vapor that can be in the air can be found from the saturated vapor pressure in Table 13-3, using the ideal gas law. At 25°C, that pressure is $3.17\times10^3\text{Pa}$.

$$PV = nRT \quad\rightarrow\quad n = \frac{PV}{RT} = \frac{(3.17\times10^3\text{Pa})(680\text{ m}^3)}{(8.315\text{ J/mol}\bullet\text{K})(273+25)\text{K}} = 8.70\times10^2\text{ moles}$$

Since the relative humidity is only 80%, only 80% of the total possible water is in the air. Thus 20% of the total possible water can still evaporate into the air.

$$m_{\text{evaporate}} = 0.20(8.70\times10^2\text{ moles})\left(\frac{18\times10^{-3}\text{kg}}{1\text{ mole}}\right) = \boxed{3.1\text{ kg}}$$

70. The air is saturated at the lower temperature, so the vapor pressure of water is 872 Pa. The ideal gas law gives the following result for the change in volume of the given mass of air.

$$PV = nRT \quad\rightarrow\quad \frac{P}{nR} = \frac{T}{V} = \text{constant} \quad\rightarrow\quad \frac{T_1}{V_1} = \frac{T_2}{V_2}$$

Thus the vapor pressure of a given mass of air that moves from outside to inside is as follows.

$$P_{\text{in}} = \frac{nRT_{\text{in}}}{V_{\text{in}}} = \frac{nRT_{\text{out}}}{V_{\text{out}}} = P_{\text{out}} = 872\text{ Pa}$$

The saturated vapor pressure at the inside temperature is 3170 Pa, and so the relative humidity is as follows.

$$\text{rel. hum.} = \frac{872\text{ Pa}}{3170\text{ Pa}} = 0.275 = \boxed{27.5\%}$$

71. From Example 13-19, we have an expression for the time to diffuse a given distance. Divide the distance by the time to get the average speed.

$$t = \frac{\bar{C}}{\Delta C}\frac{(\Delta x)^2}{D} = \frac{\frac{1}{2}(1.00+0.40)\,\text{mol/m}^3}{(1.00-0.40)\,\text{mol/m}^3}\frac{(15\times10^{-6}\,\text{m})^2}{(95\times10^{-11}\,\text{m}^2/\text{s})} = 0.2763\text{ s} \approx \boxed{0.28\,\text{s}}$$

$$v_{\text{diffuse}} = \frac{\Delta x}{t} = \frac{15\times10^{-6}\,\text{m}}{0.2763\text{ s}} = \boxed{5.4\times10^{-5}\text{ m/s}}$$

The rms thermal speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(293\text{ K})}{75(1.66\times10^{-27}\text{ kg})}} = \boxed{3.1\times10^{2}\text{ m/s}}$$

$$\frac{v_{\text{diffuse}}}{v_{\text{rms}}} = \frac{5.4\times10^{-5}\text{ m/s}}{3.1\times10^{2}\text{ m/s}} = 1.7\times10^{-7}$$

The diffusion speed is several orders of magnitude smaller than the thermal speed.

72. (*a*) Use the ideal gas law to find the concentration of the oxygen. We assume that the air pressure is 1.00 atm, and so the pressure caused by the oxygen is 0.21 atm.

$$PV = nRT \quad \rightarrow$$

$$\frac{n}{V} = \frac{P}{RT} = \frac{(0.21\text{atm})(1.013\times10^5\text{ Pa/atm})}{(8.315\,\text{J/mol}\bullet\text{K})(293\text{ K})} = 8.732\,\text{mol/m}^3 \approx \boxed{8.7\,\text{mol/m}^3}$$

(*b*) Use Equation 13-10 to calculate the diffusion rate.

$$J = DA\frac{C_1 - C_2}{\Delta x} = (1\times10^{-5}\text{ m}^2/\text{s})(2\times10^{-9}\text{m}^2)\left(\frac{8.732\,\text{mol/m}^3 - 4.366\,\text{mol/m}^3}{2\times10^{-3}\text{m}}\right)$$

$$= 4.366\times10^{-11}\text{ mol/s} \approx \boxed{4\times10^{-11}\text{ mol/s}}$$

(*c*) From Example 13-19, we have an expression for the time to diffuse a given distance.

$$t = \frac{\bar{C}}{\Delta C}\frac{(\Delta x)^2}{D} = \frac{\frac{1}{2}(8.732\,\text{mol/m}^3 + 4.366\,\text{mol/m}^3)}{(8.732\,\text{mol/m}^3 - 4.366\,\text{mol/m}^3)}\frac{(2\times10^{-3}\text{m})^2}{1\times10^{-5}\text{ m}^2/\text{s}} = \boxed{0.6\text{ s}}$$

73. (*a*) At 34°C, the tape will expand from its calibration, and so will $\boxed{\text{read low}}$.

(*b*) $\frac{\Delta L}{L_0} = \alpha\Delta T = (12\times10^{-6}/^\circ\text{C})(34^\circ\text{C} - 20^\circ\text{C}) = 1.68\times10^{-4} \approx \boxed{1.7\times10^{-2}\text{ \%}}$

74. Since the glass does not expand, the measuring cup will contain 300 mL of hot water. Find the volume of water after it cools.

$$\Delta V = V_0\beta\Delta T = (300\text{ mL})(210\times10^{-6}/\text{C}^\circ)(20^\circ\text{C} - 80^\circ\text{C}) = -3.78\text{mL} \approx \boxed{-3.8\text{ mL}}$$

The volume of cool water is 3.8 mL less than the desired volume of 300 mL.

75. Assume the helium is an ideal gas. The volume of the cylinder is constant, and we assume that the temperature of the gas is also constant in the cylinder. From the ideal gas law, $PV = nRT$, under these conditions the amount of gas is proportional to the absolute pressure.

$$PV = nRT \quad \rightarrow \quad \frac{P}{n} = \frac{RT}{V} = \text{constant} \quad \rightarrow \quad \frac{P_1}{n_1} = \frac{P_2}{n_2} \quad \rightarrow \quad \frac{n_2}{n_1} = \frac{P_2}{P_1} = \frac{5\,\text{atm} + 1\,\text{atm}}{28\,\text{atm} + 1\,\text{atm}} = \frac{6}{29}$$

Thus $\boxed{6/29 = 0.207 \approx 21\%}$ of the original gas remains in the cylinder.

76. Assume the air is an ideal gas, and that the pressure is 1.0 atm.

$$PV = NkT \quad \rightarrow$$

$$N = \frac{PV}{kT} = \frac{\left(1.013\times10^{5}\,\text{Pa}\right)\left(6.5\times3.1\times2.5\right)\text{m}^{3}}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273+22\right)\text{K}} = 1.253\times10^{27}\,\text{molecules} \approx \boxed{1.3\times10^{27}\,\text{molecules}}$$

$$1.253\times10^{27}\,\text{molecules}\left(\frac{1\text{ mole}}{6.02\times10^{23}\,\text{molecules}}\right) = 2082\,\text{moles} \approx \boxed{2.1\times10^{3}\,\text{moles}}$$

$\boxed{77.}$ The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. Hydrogen atoms have a mass of 1 atomic mass unit.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(2.7\text{ K}\right)}{1\left(1.66\times10^{-27}\,\text{kg}\right)}} = \boxed{260\,\text{m/s}}$$

The pressure is found from the ideal gas law, $PV = NkT$.

$$PV = NkT \quad \rightarrow \quad P = \frac{NkT}{V} = \frac{(1)\left(1.38\times10^{-23}\,\text{J/K}\right)\left(2.7\text{ K}\right)}{1\text{ cm}^{3}\left(\frac{1\times10^{-6}\,\text{m}^{3}}{1\text{cm}^{3}}\right)} = 3.726\times10^{-17}\,\text{Pa}\left(\frac{1\text{ atm}}{1.01\times10^{5}\,\text{Pa}}\right)$$

$$= 3.689\times10^{-22}\,\text{atm} \approx \boxed{4\times10^{-22}\,\text{atm}}$$

78. Assume the gas is ideal at those low pressures, and use the ideal gas law.

$$PV = NkT \quad \rightarrow \quad \frac{N}{V} = \frac{P}{kT} = \frac{1\times10^{-12}\,\text{N/m}^{2}}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\text{ K}\right)} = \left(3\times10^{8}\,\frac{\text{molecules}}{\text{m}^{3}}\right)\left(\frac{10^{-6}\text{ m}^{3}}{1\text{ cm}^{3}}\right)$$

$$= \boxed{3\times10^{2}\,\frac{\text{molecules}}{\text{cm}^{3}}}$$

79. Assume that the air in the lungs is an ideal gas, that the amount of gas is constant, and that the temperature is constant. The ideal gas law then says that the value of PV is constant. The pressure a distance h below the surface of the water is discussed in chapter 10, and is given by $P = P_0 + \rho gh$, where P_0 is atmospheric pressure and ρ is the density of the water.

$$\left(PV\right)_{\text{surface}} = \left(PV\right)_{\text{submerged}} \quad \rightarrow \quad V_{\text{surface}} = V_{\text{submerged}}\frac{P_{\text{submerged}}}{P_{\text{surface}}} = V_{\text{submerged}}\frac{P_{\text{atm}}+\rho gh}{P_{\text{atm}}}$$

$$= \left(5.5\text{ L}\right)\frac{1.01\times10^{5}\,\text{Pa} + \left(1.0\times10^{3}\,\text{kg/m}^{3}\right)\left(9.8\,\text{m/s}^{2}\right)\left(10\,\text{m}\right)}{1.01\times10^{5}\,\text{Pa}} = \boxed{11\text{ L}}$$

This is obviously very dangerous, to have the lungs attempt to inflate to twice their volume. Thus it is $\boxed{\text{not advisable}}$ to quickly rise to the surface.

80. The temperature can be found from the rms speed by Equation 13-9, $v_{\text{rms}}=\sqrt{3kT/m}$.

$$v_{\text{rms}}=\sqrt{3kT/m}\ \rightarrow$$

$$T=\frac{mv_{\text{rms}}^2}{3k}=\frac{(28)\left(1.66\times10^{-27}\,\text{kg}\right)\left[4\times10^4\ \text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{3\left(1.38\times10^{-23}\ \text{J/K}\right)}=\boxed{1.4\times10^5\,\text{K}}$$

81. From the ideal gas law, if the volume and number of moles stay constant, then the ratio of pressure and temperature is constant.

$$PV=nRT\ \rightarrow\ \frac{P}{T}=\frac{nR}{V}=\text{constant}\ \rightarrow\ \frac{P_2}{T_2}=\frac{P_1}{T_1}\ \rightarrow$$

$$\frac{P_2}{P_1}=\frac{T_2}{T_1}=\frac{(273+360)\,\text{K}}{(273+110)\,\text{K}}=\frac{633\ \text{K}}{383\ \text{K}}=\boxed{1.65}$$

From the relationship $v_{\text{rms}}=\sqrt{3kT/m}$, if the mass per molecule stays constant, then the ratio of rms speed to the square root of the temperature is constant.

$$v_{\text{rms}}=\sqrt{3kT/m}\ \rightarrow\ \frac{v_{\text{rms}}}{\sqrt{T}}=\sqrt{\frac{3k}{m}}=\text{constant}\ \rightarrow\ \frac{(v_{\text{rms}})_2}{\sqrt{T_2}}=\frac{(v_{\text{rms}})_1}{\sqrt{T_1}}\ \rightarrow$$

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1}=\frac{\sqrt{T_2}}{\sqrt{T_1}}=\sqrt{\frac{633\ \text{K}}{383\ \text{K}}}=\boxed{1.29}$$

82. To do this problem, the "molecular weight" of air is needed. If we approximate air as 70% N_2 (molecular weight 28) and 30% O_2 (molecular weight 32), then the average molecular weight is

$$0.70(28)+0.30(32)=29$$

(*a*) Treat the air as an ideal gas. Assume that the pressure is 1.00 atm.

$$PV=nRT\ \rightarrow\ n=\frac{PV}{RT}=\frac{\left(1.01\times10^5\,\text{Pa}\right)\left(770\,\text{m}^3\right)}{(8.315\,\text{J/mol}\bullet\text{k})(293\ \text{K})}=3.192\times10^4\,\text{moles}$$

$$m=\left(3.192\times10^4\,\text{moles}\right)\left(29\times10^{-3}\ \text{kg/mol}\right)=925.7\text{kg}\approx\boxed{9.3\times10^2\,\text{kg}}$$

(*b*) Find the mass of air at the lower temperature, and then subtract the mass at the higher temperature.

$$n=\frac{PV}{RT}=\frac{\left(1.01\times10^5\,\text{Pa}\right)\left(770\,\text{m}^3\right)}{(8.315\,\text{J/mol}\bullet\text{k})(263\ \text{K})}=3.556\times10^4\,\text{moles}$$

$$m=\left(3.556\times10^4\,\text{moles}\right)\left(29\times10^{-3}\ \text{kg/mol}\right)=1031.2\,\text{kg}$$

The mass entering the house is $1031.2\ \text{kg}-925.7\ \text{kg}=105.5\ \text{kg}\approx\boxed{1.1\times10^2\,\text{kg}}$.

83. Since the pressure is force per unit area, if the pressure is multiplied by the surface area of the Earth, the force of the air is found. If we assume that the force of the air is due to its weight, then the mass of the air can be found. The number of molecules can then be found using the molecular mass of air (calculated in problem 82) and Avogadro's number.

$$P=\frac{F}{A}\ \rightarrow\ F=PA\ \rightarrow\ Mg=P4\pi R_{\text{Earth}}^2\ \rightarrow$$

$$M = \frac{4\pi R_{\text{Earth}}^2 P}{g} = \frac{4\pi\left(6.38\times10^{6}\,\text{m}\right)^2\left(1.01\times10^{5}\,\text{Pa}\right)}{9.80\,\text{m/s}^2} = 5.27\times10^{18}\,\text{kg}$$

$$N = 5.27\times10^{18}\,\text{kg}\left(\frac{1\text{ mole}}{29\times10^{-3}\,\text{kg}}\right)\left(\frac{6.02\times10^{23}\,\text{molecules}}{1\text{ mole}}\right) = \boxed{1.1\times10^{44}\text{ molecules}}$$

84. The temperature of the nitrogen gas is found from the ideal gas law, and then the rms speed is found from the temperature.

$$PV = nRT \quad\to\quad T = \frac{PV}{nR} = \frac{\left(4.2\,\text{atm}\right)\left(1.013\times10^{5}\text{ Pa/atm}\right)\left(7.6\,\text{m}^3\right)}{\left(1800\text{ mol}\right)\left(8.315\,\text{J/mol}\bullet\text{K}\right)} = 216\text{ K}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\text{ J/K}\right)\left(216\text{ K}\right)}{28\left(1.66\times10^{-27}\,\text{kg}\right)}} = 438.7\,\text{m/s} \approx \boxed{4.4\times10^{2}\text{ m/s}}$$

85. The amount of gas is the same under both sets of conditions, as is the temperature. Thus the ideal gas law gives $PV = nRT = \text{constant} \;\to\; (PV)_1 = (PV)_2$. Absolute pressure must be used instead of gauge pressure.

$$(PV)_{\substack{\text{in}\\\text{cylinder}}} = (PV)_{\substack{\text{flowing}\\\text{out}}} \quad\to$$

$$V_{\substack{\text{flowing}\\\text{out}}} = \frac{(PV)_{\substack{\text{in}\\\text{cylinder}}}}{P_{\substack{\text{flowing}\\\text{out}}}} = \frac{\left[\left(1.38\times10^{7}+1.01\times10^{5}\right)\text{Pa}\right]\left(16\,\text{L}\right)}{\left(1.01\times10^{5}\,\text{Pa}\right)} = 2202\,\text{L}$$

So there are 2202 L of oxygen to dispense, at a rate of 2.4 L/min.

$$t = \frac{2202\text{ L}}{2.4\,\text{L/min}} = 917.5\text{ min} \approx \boxed{15\text{ hours}}$$

86. (*a*) The iron floats in the mercury because $\rho_{\text{Hg}} > \rho_{\text{Fe}}$. As the substances are heated, the density of both substances will decrease due to volume expansion (see problem 17 for a detailed discussion of this effect). The density of the mercury decreases more upon heating than the density of the iron, because $\beta_{\text{Hg}} > \beta_{\text{Fe}}$. The net effect is that the densities get closer together, and so relatively more mercury will have to be displaced to hold up the iron, and the iron will float $\boxed{\text{lower}}$ in the mercury.

(*b*) The fraction of the volume submerged is $V_{\substack{\text{Hg}\\\text{displaced}}}\big/V_{\text{Fe}}$. Both volumes expand as heated. The subscript "displace" is dropped for convenience.

$$\text{fractional change} = \frac{V_{\text{Hg}}/V_{\text{Fe}} - V_{0\,\text{Hg}}/V_{0\,\text{Fe}}}{V_{0\,\text{Hg}}/V_{0\,\text{Fe}}} = \frac{\dfrac{V_{0\,\text{Hg}}\left(1+\beta_{\text{Hg}}\Delta T\right)}{V_{0\,\text{Fe}}\left(1+\beta_{\text{Fe}}\Delta T\right)} - V_{0\,\text{Hg}}/V_{0\,\text{Fe}}}{V_{0\,\text{Hg}}/V_{0\,\text{Fe}}} = \frac{\left(1+\beta_{\text{Hg}}\Delta T\right)}{\left(1+\beta_{\text{Fe}}\Delta T\right)} - 1$$

$$= \frac{1+\left(180\times10^{-6}/\text{C}^{\text{o}}\right)\left(25\,\text{C}^{\text{o}}\right)}{1+\left(35\times10^{-6}\right)\left(25\,\text{C}^{\text{o}}\right)} - 1 = \frac{1.0045}{1.000875} - 1 = 3.6\times10^{-3}$$

$$\%\text{ change} = \left(3.6\times10^{-3}\right)\left(100\right) = \boxed{0.36\,\%}$$

87. (*a*) Assume that a mass M of gasoline with volume V_0 at 0°C is under consideration, and so its density is $\rho_0 = M/V_0$. At a temperature of 32°C, the same mass has a volume $V = V_0\left(1+\beta\Delta T\right)$.

$$\rho = \frac{M}{V} = \frac{M}{V_0\left(1+\beta\Delta T\right)} = \frac{\rho_0}{1+\beta\Delta T} = \frac{0.68\times10^3\ \mathrm{kg/m^3}}{1+\left(950\times10^{-6}/\mathrm{C^o}\right)\left(38\mathrm{C^o}\right)} = 0.6563\times10^3\ \mathrm{kg/m^3}$$

$$\approx \boxed{0.66\times10^3\ \mathrm{kg/m^3}}$$

(*b*) Calculate the percentage change in the density.

$$\%\ \text{change} = \frac{\left(0.6563-0.68\right)\times10^3\ \mathrm{kg/m^3}}{0.68\times10^3\ \mathrm{kg/m^3}\,V}\times100 = \boxed{-3.5\%}$$

88. The original length of the steel band is $L_0 = 2\pi R_{\text{Earth}}$. At the higher temperature, the length of the band is $L = L_0 + \Delta L = 2\pi R = 2\pi\left(R_{\text{Earth}} + \Delta R\right)$. The change in radius, ΔR, would be the height above the Earth.

$$\Delta L = \alpha L_0 \Delta T = 2\pi\Delta R \quad \rightarrow$$

$$\Delta R = \frac{\alpha L_0 \Delta T}{2\pi} = \alpha R_{\text{Earth}}\Delta T = \left(12\times10^{-6}/\mathrm{C^o}\right)\left(6.38\times10^6\,\mathrm{m}\right)\left(45^\circ\mathrm{C}-25\ ^\circ\mathrm{C}\right) = \boxed{1.5\times10^3\,\mathrm{m}}$$

89. The gap will be the radius of the lid minus the radius of the jar. Also note that the original radii of the lid and the jar are the same.

$$r_{\text{gap}} = \left(r_0 + \Delta r\right)_{\text{lid}} - \left(r_0 + \Delta r\right)_{\text{jar}} = \Delta r_{\text{lid}} - \Delta r_{\text{jar}} = \left(\alpha_{\text{brass}} - \alpha_{\text{glass}}\right) r_0 \Delta T$$

$$= \left(19\times10^{-6}/\mathrm{C^o} - 9\times10^{-6}/\mathrm{C^o}\right)\left(4.0\ \mathrm{cm}\right)\left(40\,\mathrm{C^o}\right) = \boxed{1.6\times10^{-3}\mathrm{cm}}$$

90. The change in length is to be restricted to $\Delta L < 1.0\times10^{-6}\,\mathrm{m}$.

$$\Delta L = \alpha L_0 \Delta T \le 1.0\times10^{-6}\,\mathrm{m} \quad \rightarrow \quad \Delta T \le \frac{1.0\times10^{-6}\,\mathrm{m}}{\left(9\times10^{-6}/\mathrm{C^o}\right)\left(1.0\ \mathrm{m}\right)} \le 0.11\mathrm{C^o}$$

Thus the temperature would have to be controlled to within $\boxed{\pm0.11\mathrm{C^o}}$

91. (*a*) Treat the air as an ideal gas. Since the amount and temperature of the air are the same in both cases, the ideal gas law says $PV = nRT$ is a constant.

$$P_2V_2 = P_1V_1 \quad \rightarrow \quad V_2 = V_1\frac{P_1}{P_2} = \left(11.3\ \mathrm{L}\right)\frac{195\ \mathrm{atm}}{1.00\ \mathrm{atm}} = 2203.5\,\mathrm{L} \approx \boxed{2.20\times10^3\,\mathrm{L}}$$

(*b*) Before entering the water, the air coming out of the tank will be at 1.00 atm pressure, and so the person will be able to breathe $2203.5\,\mathrm{L}$ of air.

$$t = 2203.5\,\mathrm{L}\left(\frac{1\ \text{breath}}{2.0\ \mathrm{L}}\right)\left(\frac{1\ \text{min}}{12\ \text{breaths}}\right) = \boxed{92\ \text{min}}$$

(*c*) When the person is underwater, the temperature and pressure will be different. Use the ideal gas equation to relate the original tank conditions to the underwater breathing conditions. The amount of gas will be constant, so $PV/T = nR$ will be constant. The pressure a distance *h* below the surface of the water is discussed in chapter 10, and is given by $P = P_0 + \rho g h$, where P_0 is atmospheric pressure and ρ is the density of the sea water.

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \rightarrow V_2 = V_1\frac{P_1}{P_2}\frac{T_2}{T_1}$$

$$V_2 = (11.3\text{ L})\left[\frac{195\text{ atm}\left(1.01\times10^5\text{ Pa/atm}\right)}{1.01\times10^5\text{ Pa} + \left(1.025\times10^3\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)(20.0\text{ m})}\right]\left(\frac{283\text{ K}}{293\text{ K}}\right)$$

$$= 7.120\times10^2\text{ L} \qquad t = 7.120\times10^2\text{ L}\left(\frac{1\text{ breath}}{2.0\text{ L}}\right)\left(\frac{1\text{ min}}{12\text{ breaths}}\right) = \boxed{3.0\times10^1\text{min}}$$

92. The rms speed is given by Equation 13-9, $v_{\text{rms}} = \sqrt{3kT/m}$. Using the escape velocity as v_{rms}, solve for the temperature.

(*a*) For oxygen molecules: $T = \frac{mv_{\text{rms}}^2}{3k} = \frac{32\left(1.66\times10^{-27}\text{ kg}\right)\left(1.12\times10^4\text{ m/s}\right)^2}{3\left(1.38\times10^{-23}\text{ J/K}\right)} = 1.6\times10^5\text{K}$

(*b*) For helium atoms: $T = \frac{mv_{\text{rms}}^2}{3k} = \frac{4\left(1.66\times10^{-27}\text{ kg}\right)\left(1.12\times10^4\text{ m/s}\right)^2}{3\left(1.38\times10^{-23}\text{ J/K}\right)} = 2.0\times10^4\text{K}$

(*c*) Because the "escape temperature" is so high for oxygen, very few oxygen molecules ever escape the atmosphere. But helium, with one-eighth the mass, can escape at a much lower temperature. While the temperature of the Earth is not close to $2.0\times10^4\text{K}$ today, during the Earth's formation its temperature was possibly much hotter – presumably hot enough that helium was able to escape the atmosphere.

93. Following the development of the kinetic molecular theory in section 13-10 of the textbook, the tennis balls hitting the trash can lid are similar to the particles colliding with the walls of a container causing pressure. Quoting from the text, "the average force averaged over many collisions will be equal to the force exerted during one collision divided by the time between collisions." That average force must be the weight of the trash can lid in order to suspend it.

$$F_{\text{avg}} = M_{\text{lid}}g = \frac{2m_{\text{ball}}v_{\text{ball}}}{\Delta t} \rightarrow \Delta t = \frac{2m_{\text{ball}}v_{\text{ball}}}{M_{\text{lid}}g}$$

The above expression is "seconds per ball", so its reciprocal will be "balls per second".

$$\text{balls/s} = \frac{1}{\Delta t} = \frac{M_{\text{lid}}g}{2m_{\text{ball}}v_{\text{ball}}} = \frac{(1.0\text{ kg})\left(9.8\text{ m/s}^2\right)}{2(0.060\text{ kg})(12\text{ m/s})} = \boxed{6.8\text{ balls/s}}$$

94. The amount of gas and the temperature of the gas are constant. Then from the ideal gas law, the quantity $PV = nRT$ is constant. The pressure at the surface will be 1 atmosphere. The pressure a distance *h* below the surface of the water is discussed in chapter 10, and is given by $P = P_0 + \rho g h$, where P_0 is atmospheric pressure and ρ is the density of the water. The volume of a spherical bubble is given by $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$.

$$P_{\text{surface}}V_{\text{surface}} = P_{\text{depth}}V_{\text{depth}} \rightarrow P_{\text{surface}}\tfrac{1}{6}\pi d_{\text{surface}}^3 = P_{\text{depth}}\tfrac{1}{6}\pi d_{\text{depth}}^3 \rightarrow$$

$$d_{\text{surface}} = d_{\text{depth}}\left(\frac{P_{\text{depth}}}{P_{\text{surface}}}\right)^{1/3} = (3.00\text{ cm})\left(\frac{1.01\times10^5\,\text{Pa} + \left(1.0\times10^3\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)(14.0\text{ m})}{1.01\times10^5\,\text{Pa}}\right)^{1/3}$$

$$= \boxed{3.99\text{ cm}}$$

95. (*a*) At a temperature of 30°C, the saturated vapor pressure, from Table 13-6, is $4.24\times10^3\,\text{Pa}$. If the relative humidity is 40%, then the water vapor pressure is 40% of the saturated vapor pressure.

$$0.40\left(4.24\times10^3\,\text{Pa}\right) = \boxed{1.7\times10^3\,\text{Pa}}$$

(*b*) At a temperature of 5°C, the saturated vapor pressure, from Table 13-6, is $8.72\times10^2\,\text{Pa}$. If the relative humidity is 80%, then the water vapor pressure is 80% of the saturated vapor pressure.

$$0.80\left(8.72\times10^2\,\text{Pa}\right) = \boxed{7.0\times10^2\,\text{Pa}}$$

96. Assume that the water is an ideal gas, and that the temperature is constant. From Table 13-3, saturated vapor pressure at 90°C is $7.01\times10^4\,\text{Pa}$, and so to have a relative humidity of 10%, the vapor pressure will be $7.01\times10^3\,\text{Pa}$. Use the ideal gas law to calculate the amount of water.

$$PV = nRT \quad \rightarrow$$

$$n = \frac{PV}{RT} = \frac{\left(7.01\times10^3\,\text{Pa}\right)\left(7.0\,\text{m}^3\right)}{\left(8.315\,\text{J/mol}\bullet\text{K}\right)(273+90)\,\text{K}} = 16.26\text{ moles}\left(\frac{18\times10^{-3}\,\text{kg}}{1\text{ mole}}\right) = \boxed{0.29\text{ kg}}$$

97. The density is the mass divided by the volume. Let the original volume of the mass of iron be V_0, the original density $\rho_0 = M/V_0$. The volume of that same mass deep in the Earth is $V = V_0 + \Delta V$, and so the density deep in the Earth is $\rho = M/V = M/(V_0 + \Delta V)$. The change in volume is due to two effects: the increase in volume due to a higher temperature, $\Delta V_{\text{temp}} = \beta V_0 \Delta T$, and the decrease in volume due to a higher pressure, $\Delta V_{\text{pressure}} = -V_0 \Delta P/B$. So $\Delta V = \Delta V_{\text{temp}} + \Delta V_{\text{pressure}}$. The new density is then

$$\rho = M/V = \frac{M}{V_0 + \Delta V} = \frac{M}{V_0 + \Delta V_{\text{temp}} + \Delta V_{\text{pressure}}} = \frac{M}{V_0 + \beta V_0 \Delta T - V_0\,\Delta P/B} = \frac{M}{V_0}\frac{1}{\left(1 + \beta\Delta T - \Delta P/B\right)}$$

$$= \frac{\rho_0}{\left(1 + \beta\Delta T - \Delta P/B\right)}$$

$$= \frac{\rho_0}{\left[1 + \left(35\times10^{-6}/\text{C}^\circ\right)\left(2000\text{C}^\circ\right) - (5000\,\text{atm})\left(1.01\times10^5\,\text{Pa/atm}\right)/\left(90\times10^9\,\text{N/m}^2\right)\right]}$$

$$= \frac{\rho_0}{\left[1 + 0.07 - .00561\right]} = 0.9395\rho_0 \quad \rightarrow \quad \boxed{6\%\text{ decrease}}$$

98. (*a*) Assume the pressure and amount of gas are held constant, and so $P_0V_0 = nRT_0$ and $P_0V = nRT$. From these two expressions calculate the change in volume and relate it to the change in temperature.

$$V = V_0 + \Delta V \quad \rightarrow \quad \Delta V = V - V_0 = \frac{nRT}{P_0} - \frac{nRT_0}{P_0} = \frac{nR}{P_0}\left(T - T_0\right) = \frac{V_0}{T_0}\Delta T$$

But $\Delta V = \beta V_0 \Delta T$, and so $\Delta V = \dfrac{V_0}{T_0}\Delta T = \beta V_0 \Delta T \quad \rightarrow \quad \beta = \dfrac{1}{T_0}$

For $T_0 = 293\text{ K}$, $\beta = \dfrac{1}{T_0} = \dfrac{1}{293\text{ K}} = \boxed{3.4\times10^{-3}/\text{K}}$, which agrees well with Table 13-1.

(*b*) Assume the temperature and amount of gas are held constant, and so $P_0V_0 = nRT_0 = PV$. From these two expressions calculate change in volume and relate it to the change in pressure.

$$V = V_0 + \Delta V \quad \rightarrow$$

$$\Delta V = V - V_0 = \frac{nRT_0}{P} - \frac{nRT_0}{P_0} = nRT_0\left(\frac{1}{P} - \frac{1}{P_0}\right) = \frac{nRT_0}{P_0}\left(\frac{P_0 - P}{P}\right) = V_0\frac{1}{P}\left(-\Delta P\right)$$

But from chapter 9, $\Delta V = -V_0\dfrac{1}{B}\Delta P$ and so $\Delta V = V_0\dfrac{1}{P}\left(-\Delta P\right) = -V_0\dfrac{1}{B}\Delta P \quad \rightarrow \quad \boxed{B = P}$

99. Assume that the water vapor behaves like an ideal gas. At 20°C, the saturated vapor pressure is $2.33\times10^3\text{ Pa}$. Using the ideal gas law, find the number of moles of water in the air at both 95% and 30%. Subtract those mole amounts to find the amount of water that must be removed.

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT} \quad \rightarrow$$

$$n_1 - n_2 = \frac{V}{RT}\left(P_1 - P_2\right) = \frac{\left(95\text{ m}^2\right)\left(2.8\text{ m}\right)}{\left(8.315\,\text{J/mol}\bullet\text{k}\right)\left(293\text{ K}\right)}\left(2.33\times10^3\,\text{Pa}\right)\left(0.95 - 0.30\right) = 165\text{ mol}$$

$$165\text{ mol}\left(\frac{18\times10^{-3}\text{kg}}{1\text{ mol}}\right) = \boxed{3.0\text{ kg}}$$

CHAPTER 14: Heat

Answers to Questions

1. The work goes primarily into increasing the temperature of the orange juice, by increasing the average kinetic energy of the molecules comprising the orange juice.

2. When a hot object warms a cooler object, energy is transferred from the hot object to the cold object. Temperature does NOT flow. The temperature changes of the two objects are not necessarily equal in magnitude. Under certain circumstances, they can be equal in magnitude, however. In an ideal case, the amount of heat lost by the warmer object is the same as the amount of heat gained by the cooler object.

3. (a) Internal energy depends on both the number of molecules of material and the temperature of the material. Heat will flow naturally from the object with the higher temperature to the object with the lower temperature. The object with the high temperature may or may not be the object with the higher internal energy.
   (b) The two objects may consist of one with a higher temperature and smaller number of molecules, and the other with a lower temperature and a larger number of molecules. In that case it is possible for both objects to have the same internal energy, but heat will still flow from the object with the higher temperature to the one with the lower temperature.

4. The water will coat the plants, and so the water, not the plant, is in contact with the cold air. Thus as the air cools, the water cools before the plant does – the water insulates the plant. As the water cools, it releases energy, and raises the temperature of its surroundings, which includes the plant. Particularly if the water freezes, relatively large amounts of heat are released due to the relatively large heat of fusion for water.

5. Because the specific heat of water is quite large, it can contain a relatively large amount of thermal energy per unit mass with a relatively small increase in temperature. Since the water is a liquid, it is relatively easy to transport from one location to another, and so large quantities of energy can be moved from one place to another with relative simplicity by water.

6. The mechanism of evaporation of the water from the moist cloth jacket requires energy (the latent heat of vaporization), some of which will come from the interior of the canteen. This removal of energy from the interior helps to keep the interior of the canteen cool. Also, the metal canteen is a good thermal conductor, and so heat can transfer from the water to the cloth jacket to cool the water.

7. Steam at 100^{o}C contains more thermal energy than water at 100^{o}C. The difference is due to the latent heat of vaporization, which for water is quite high. As the steam touches the skin and condenses, a large amount of energy is released, causing more severe burns. And the condensed water is still at 100^{o}C , and so more burning can occur as that water cools.

8. Evaporation involves water molecules escaping the intermolecular bonds that hold the water together in the liquid state. It takes energy for the molecules to break those bonds (to overcome the bonding forces). This energy is the latent heat of vaporization. The most energetic molecules (those having the highest speed) are the ones that will be able to provide the most energy (from their kinetic energy) to be able to overcome the bonding forces. The slower moving molecules remain, lowering the average kinetic energy and thus lowering the internal energy and temperature of the liquid.

9. The potatoes will not cook faster if the water is boiling faster. The boiling water is the same temperature whether it is boiling fast or slow.

10. An ordinary fan does not cool the air directly. It actually warms the air slightly, because the motor used to power the fan will exhaust some heat into the air, and the increase in average kinetic energy of the air molecules caused by the fan blades pushing them means the air temperature increases slightly. The reason for using the fan is that it keeps air moving. The human body warms the air immediately around it, assuming the air is initially cooler than the body. If that warmed air stays in contact with the body, then the body will lose little further heat after the air is warmed. The fan, by circulating the air, removes the heated air from close to the body and replaces it with cooler air. Likewise, the body is also cooled by evaporation of water from the skin. As the relative humidity of the air close to the body increases, less water can be evaporated, and cooling by evaporation is decreased. The fan, by circulating the air, removes the humid air from close to the body and replaces it with less humid air, so that evaporation can continue.

11. Even though the temperature is high in the upper atmosphere, the density of gas particles is very low. There would be relatively very few collisions of high-temperature gas molecules with the animal to warm it. Instead, the animal would radiate heat to the rarified atmosphere. The emissivity of the animal is much greater than that of the rarified atmosphere, and so the animal will lose much more energy by radiation than it can gain from the atmosphere.

12. Snow, particularly at very low temperatures, has a low thermal conductivity because it has many tiny air pockets trapped in its structure – it might be described as “fluffy”. Since this “fluffy” snow has a low thermal conductivity, the snow will not conduct much heat away from an object covered in it.

13. We assume that the wet sand has been wetted fairly recently with water that is cooler than the sand’s initial temperature. Water has a higher heat capacity than sand, and so for equal masses of sand and water, the sand will cool more than the water warms as their temperatures move towards equilibrium. Thus the wet sand may actually be cooler than the dry sand. Also, if both the wet and dry sand are at a lower temperature than your feet, the sand with the water in it is a better thermal conductor and so heat will flow more rapidly from you into the wet sand than into the dry sand, giving more of a sensation of having touched something cold.

14. An object with “high heat content” does not have to have a high temperature. If a given amount of heat energy is transferred into equal-mass samples of two substances initially at the same temperature, the substance with the lower specific heat will have the higher final temperature. But both substances would have the same “heat content” relative to their original state. So an object with “high heat content” might be made of material with a very high specific heat, and therefore not necessarily be at a high temperature.

15. A hot-air furnace heats primarily by air convection. A return path (often called a “cold air return”) is necessary for the convective currents to be able to completely circulate. If the flow of air is blocked, then the convective currents and the heating process will be interrupted. Heating will be less efficient and less uniform if the convective currents are prevented from circulating.

16. A ceiling fan makes more of a “breeze” when it is set to blow the air down (usually called the “forward” direction by fan manufacturers). This is the setting for the summer, when the breeze will feel cooling since it accelerates evaporation from the skin. In the winter, the fan should be set to pull air up. This forces the warmer air at the top of the room to move out towards the walls and down. The relocation of warmer air keeps the room feeling warmer, and there is less “breeze” effect on the occupants of the room.

17. When the garment is fluffed up, it will have the most air trapped in its structure. The air has a low thermal conductivity, and the more the garment can be "fluffed", the more air it will trap, making it a better insulator. The "loft" value is similar to the R value of insulation, since the thicker the insulation, the higher the R value. The rate of thermal conduction is inversely proportional to the thickness of the conductor, so a thick conductor (high loft value) means a lower thermal conduction rate, and so a lower rate of losing body heat.

18. For all mechanisms of cooling, the rate of heat transfer from the hot object to the cold one is dependent on surface area. The heat sink with fins provides much more surface area than just a solid piece of metal, and so there is more cooling of the microprocessor chip. A major mechanism for cooling the heat sink is that of convection. More air is in contact with the finned heat sink than would be in contact with a solid piece of metal. There is often a cooling fan circulating air around that heat sink as well, so that heated air can continually be replaced with cool air to promote more cooling.

19. When there is a temperature difference in air, convection currents arise. Since the temperature of the land rises more rapidly than that of the water, the air above the land will be warmer than the air above the water. The warm air above the land will rise, and that rising warm air will be replaced by cooler air from over the body of water. The result is a breeze from the water towards the land.

20. We assume that the temperature in the house is higher than that under the house. Thus heat will flow through the floor out of the house. If the house sits directly on the ground or on concrete, the heat flow will warm the ground or concrete. Dirt and concrete are relatively poor conductors of heat, and so the thermal energy that goes into them will stay for a relatively long time, allowing their temperature to rise and thus reducing the heat loss through the floor. If the floor is over a crawlspace, then the thermal energy from the floor will be heating air instead of dirt or concrete. If that warmed air gets moved away by wind currents or by convection and replaced with colder air, then the temperature difference between the inside and outside will stay large, and more energy will leave through the floor, making the inside of the house cooler.

21. Air is a poorer conductor of heat than water by roughly a factor of 20, and so the rate of heat loss from your body to the air is roughly 20 times less than the rate of heat loss from your body to the water. Thus you lose heat quickly in the water, and feel cold. Another contributing factor is that water has a high heat capacity, and so as heat leaves your body and enters the water, the temperature rise for the water close to your body is small. Air has a smaller heat capacity, and so the temperature rise for the air close to your body is larger. This reduces the temperature difference between your body and the air, which reduces the rate of heat loss to the air as well.

22. A thermometer in the direct sunlight would gain thermal energy (and thus show a higher temperature) due to receiving radiation directly from the Sun. The emissivity of air is small, and so it does not gain as much energy from the Sun as the mercury and glass do. The thermometer is to reach its equilibrium temperature by heat transfer with the air, in order to measure the air temperature.

23. Premature babies have underdeveloped skin, and they can lose a lot of moisture through their skin by evaporation. For a baby in a very warm environment, like an incubator at $37^{o}C$, there will be a large evaporative effect. A significant increase in evaporation occurs at incubator temperatures, and that evaporation of moisture from the baby will cool the baby dramatically. Thus an incubator must have not only a high temperature but also a high humidity. Other factors might include radiative energy loss, blood vessels being close to the skin surface and so there is less insulation than a more mature baby, and low food consumption to replace lost energy.

$\boxed{24.}$ Shiny surfaces absorb very little of the radiation that is incident on them – they reflect it back towards the source. Thus the liner is silvered to reduce radiation energy transfer (both into and away from the substance in the thermos). The (near) vacuum between its two walls reduces the energy transfer by conduction. Vacuum is a very poor conductor of heat.

25. The overall R-value of the wall plus window is lower than R_1 and higher than R_2. The rate of heat transfer through the entire wall + window area will increase, but the total area and the temperature difference has not changed. Thus, since $\frac{Q}{t}=\frac{A}{R_{\text{effective}}}\left(T_1-T_2\right)$, for the rate to increase means the R-value had to drop from its original value. However, the rate of heat transfer will be lower than if the wall was totally glass, and so the final R-value must be higher than that of the glass.

26. (*a*) (1) Ventilation around the edges is cooling by convection.
    (2) Cooling through the frame is cooling by conduction.
    (3) Cooling through the glass panes is cooling by conduction and radiation.

    (*b*) Heavy curtains can reduce all three heat losses. The curtains will prevent air circulation around the edges of the windows, thus reducing the convection cooling. The curtains are more opaque than the glass, preventing the electromagnetic waves responsible for radiation heat transfer from reaching the glass. And the curtains provide another layer of insulation between the outdoors and the warm interior of the room, lowering the rate of conduction.

27. The thermal conductivity of the wood is about 2000 times less than that of the aluminum. Thus it takes a long time for energy from the wood to flow into your hand. Your skin temperature rises very slowly due to contact with the wood compared to contact with the aluminum, and so the sensation of heating is much less.

28. The Earth cools primarily by radiation. The clouds act as “insulation” in that they absorb energy from the radiating Earth, and reradiate some of it back to the Earth, reducing the net amount of radiant energy loss.

29. The emergency blanket is shiny (having a low emissivity) so that it reflects a person’s radiated energy back to them, keeping them warmer. Also, like any blanket, it can insulate and so reduce heat transfer by conduction.

30. Cities situated on the ocean have less temperature extremes because the oceans are a heat reservoir. Due to ocean currents, the temperature of the ocean in a locale will be fairly constant during a season. In the winter, the ocean temperature remains above freezing. Thus if the air and land near the ocean get colder than the oceans, the oceans will release thermal energy, moderating the temperature of the nearby region. Likewise, in the warm seasons, the ocean temperatures will be cooler than the surrounding land mass, which heats up more easily than the water. Then the oceans will absorb thermal energy from the surrounding areas, again moderating the temperature.

Solutions to Problems

1. The kcal is the heat needed to raise 1 kg of water by 1 C$^\text{o}$. Use the definition to find the heat needed.

$$\left(30.0\text{ kg}\right)\left(95^\text{o}\text{C}-15^\text{o}\text{C}\right)\frac{1\text{ kcal}}{\left(1\text{kg}\right)\left(1\text{C}^\text{o}\right)}\left(\frac{4186\text{ J}}{1\text{ kcal}}\right)=\boxed{1.0\times10^{7}\text{ J}}$$

2. The kcal is the heat needed to raise 1 kg of water by 1 C°. Use that definition to find the temperature change. Then the final temperature can be found.

$$\left(\frac{7700\text{ J}}{3.0\text{ kg}}\right)\left(\frac{1\text{ kcal}}{4186\text{ J}}\right)\frac{(1\text{kg})(1\text{C}^\circ)}{1\text{ kcal}}=0.61\text{C}^\circ \quad\rightarrow\quad \text{Final Temperature}=\boxed{10.6^\circ\text{C}}$$

$\boxed{3.}$ (*a*) $$2500\text{ Cal}\left(\frac{4.186\times10^{3}\text{ J}}{1\text{ Cal}}\right)=\boxed{1.0\times10^{7}\text{ J}}$$

(*b*) $$2500\text{ Cal}\left(\frac{1\text{ kWh}}{860\text{ Cal}}\right)=\boxed{2.9\text{ kWh}}$$

(*c*) At 10 cents per day, the food energy costs $\boxed{\$0.29\text{ per day}}$. It would be practically impossible to feed yourself in the United States on this amount of money.

4. Assume that we are at the surface of the Earth so that 1 kg has a weight of 2.20 lb.

$$1\text{Btu}=(1\text{ lb})(1^\circ\text{F})\left(\frac{0.454\text{ kg}}{1\text{ lb}}\right)\left(\frac{5/9^\circ\text{ C}}{1^\circ\text{F}}\right)\frac{1\text{ kcal}}{(1\text{ kg})(1\text{C}^\circ)}=\boxed{0.252\text{ kcal}}$$

$$0.252\text{ kcal}\left(\frac{4186\text{ J}}{1\text{ kcal}}\right)=\boxed{1055\text{J}}$$

5. The energy input is causing a certain rise in temperature, expressible as a number of Joules per hour per C°. Convert that to mass using the definition of kcal.

$$\left(\frac{3.2\times10^{7}\text{ J/h}}{35\text{ C}^\circ}\right)\left(\frac{1\text{ kcal}}{4186\text{ J}}\right)\frac{(1\text{kg})(1\text{C}^\circ)}{1\text{ kcal}}=\boxed{2.2\times10^{2}\text{ kg/h}}$$

6. The wattage rating is Joules per second. Note that 1 L of water has a mass of 1 kg.

$$\left[\left(2.50\times10^{-1}\text{L}\right)\left(\frac{1\text{ kg}}{1\text{ L}}\right)\left(40\text{C}^\circ\right)\right]\frac{1\text{ kcal}}{(1\text{ kg})(1\text{C}^\circ)}\left(\frac{4186\text{ J}}{\text{kcal}}\right)\left(\frac{1\text{ s}}{350\text{ J}}\right)=\boxed{1.2\times10^{2}\text{s}=2.0\text{ min}}$$

7. The energy generated by using the brakes must equal the car's initial kinetic energy, since its final kinetic energy is 0.

$$Q=\tfrac{1}{2}mv_0^2=\tfrac{1}{2}\left(1.2\times10^{3}\text{kg}\right)\left[(95\text{km/h})\left(\frac{1\text{m/s}}{3.6\text{km/h}}\right)\right]^{2}\left(\frac{1\text{ kcal}}{4186\text{ J}}\right)=\boxed{1.0\times10^{2}\text{kcal}}$$

8. The heat absorbed can be calculated from Eq. 14-2. Note that 1 L of water has a mass of 1 kg.

$$Q=mc\Delta T=\left[(16\text{ L})\left(\frac{1\times10^{-3}\text{m}^{3}}{1\text{ L}}\right)\left(\frac{1.0\times10^{3}\text{kg}}{1\text{ m}^{3}}\right)\right]\left(4186\text{J/kg}\bullet\text{C}^\circ\right)\left(90^\circ\text{C}-20^\circ\text{C}\right)=\boxed{4.7\times10^{6}\text{J}}$$

9. The specific heat can be calculated from Eq. 14-2.

$$Q=mc\Delta T \quad\rightarrow\quad c=\frac{Q}{m\Delta T}=\frac{1.35\times10^{5}\text{J}}{(5.1\text{ kg})\left(31.5^\circ\text{C}-18.0^\circ\text{C}\right)}=1961\text{ J/kg}\bullet\text{C}^\circ\approx\boxed{2.0\times10^{3}\text{ J/kg}\bullet\text{C}^\circ}$$

10. The heat absorbed by all three substances is given by Eq. 14-2, $Q = mc\Delta T$. Thus the amount of mass can be found as $m = \frac{Q}{c\Delta T}$. The heat and temperature change are the same for all three substances.

$$m_{\mathrm{Cu}} : m_{\mathrm{Al}} : m_{\mathrm{H_2O}} = \frac{Q}{c_{\mathrm{Cu}}\Delta T} : \frac{Q}{c_{\mathrm{Al}}\Delta T} : \frac{Q}{c_{\mathrm{H_2O}}\Delta T} = \frac{1}{c_{\mathrm{Cu}}} : \frac{1}{c_{\mathrm{Al}}} : \frac{1}{c_{\mathrm{H_2O}}} = \frac{1}{390} : \frac{1}{900} : \frac{1}{4186}$$

$$= \frac{4186}{390} : \frac{4186}{900} : \frac{4186}{4186} = \boxed{10.7 : 4.65 : 1}$$

$\boxed{11.}$ The heat gained by the glass thermometer must be equal to the heat lost by the water.

$$m_{\mathrm{glass}} c_{\mathrm{glass}} \left(T_{\mathrm{eq}} - T_{i\ \mathrm{glass}} \right) = m_{\mathrm{H_2O}} c_{\mathrm{H_2O}} \left(T_{i\,\mathrm{H_2O}} - T_{\mathrm{eq}} \right)$$

$$(35\ \mathrm{g})\left(0.20\,\mathrm{cal}/\mathrm{g{\bullet}C^o}\right)\left(39.2^{\circ}\mathrm{C} - 21.6^{\circ}\mathrm{C}\right) = (135\ \mathrm{g})\left(1.00\,\mathrm{cal}/\mathrm{g{\bullet}C^o}\right)\left(T_{i\,\mathrm{H_2O}} - 39.2^{\circ}\mathrm{C}\right)$$

$$T_{i\,\mathrm{H_2O}} = \boxed{40.1^{\circ}\mathrm{C}}$$

12. The heat lost by the copper must be equal to the heat gained by the aluminum and the water.

$$m_{\mathrm{Cu}} c_{\mathrm{Cu}} \left(T_{i\,\mathrm{Cu}} - T_{\mathrm{eq}} \right) = m_{\mathrm{Al}} c_{\mathrm{Al}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{Al}} \right) + m_{\mathrm{H_2O}} c_{\mathrm{H_2O}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{H_2O}} \right)$$

$$(0.245\ \mathrm{kg})\left(390\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right)\left(285^{\circ}\mathrm{C} - T_{\mathrm{eq}}\right) = \begin{bmatrix} (0.145\ \mathrm{kg})\left(900\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right) \\ +(0.825\ \mathrm{kg})\left(4186\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right) \end{bmatrix}\left(T_{\mathrm{eq}} - 12.0^{\circ}\mathrm{C}\right)$$

$$T_{\mathrm{eq}} = \boxed{19.1^{\circ}\mathrm{C}}$$

13. The heat lost by the horseshoe must be equal to the heat gained by the iron pot and the water. Note that 1 L of water has a mass of 1 kg.

$$m_{\mathrm{shoe}} c_{\mathrm{Fe}} \left(T_{i\,\mathrm{shoe}} - T_{\mathrm{eq}} \right) = m_{\mathrm{pot}} c_{\mathrm{Fe}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{pot}} \right) + m_{\mathrm{H_2O}} c_{\mathrm{H_2O}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{H_2O}} \right)$$

$$(0.40\ \mathrm{kg})\left(450\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right)\left(T_{i\,\mathrm{shoe}} - 25.0^{\circ}\mathrm{C}\right) = (0.30\ \mathrm{kg})\left(450\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right)\left(25.0\mathrm{C^o} - 20.0\mathrm{C^o}\right)$$
$$+(1.35\ \mathrm{kg})\left(4186\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right)\left(25.0\mathrm{C^o} - 20.0\mathrm{C^o}\right)$$

$$T_{i\,\mathrm{shoe}} = 186^{\circ}\mathrm{C} \approx \boxed{190^{\circ}\mathrm{C}}$$

14. The heat lost by the substance must be equal to the heat gained by the aluminum, water, and glass.

$$m_{\mathrm{x}} c_{\mathrm{x}} \left(T_{i\,\mathrm{x}} - T_{\mathrm{eq}} \right) = m_{\mathrm{Al}} c_{\mathrm{Al}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{Al}} \right) + m_{\mathrm{H_2O}} c_{\mathrm{H_2O}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{H_2O}} \right) + m_{\mathrm{glass}} c_{\mathrm{glass}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{glass}} \right)$$

$$c_{\mathrm{x}} = \frac{m_{\mathrm{Al}} c_{\mathrm{Al}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{Al}} \right) + m_{\mathrm{H_2O}} c_{\mathrm{H_2O}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{H_2O}} \right) + m_{\mathrm{glass}} c_{\mathrm{glass}} \left(T_{\mathrm{eq}} - T_{i\,\mathrm{glass}} \right)}{m_{\mathrm{x}} \left(T_{i\,\mathrm{x}} - T_{\mathrm{eq}} \right)}$$

$$= \frac{\left[(0.105\ \mathrm{kg})\left(900\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right) + (0.165\ \mathrm{kg})\left(4186\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right) + (0.017\ \mathrm{kg})\left(840\,\mathrm{J}/\mathrm{kg{\bullet}C^o}\right)\right]\left(22.5\mathrm{C^o}\right)}{(0.215\ \mathrm{kg})\left(330^{\circ}\mathrm{C} - 35.0^{\circ}\mathrm{C}\right)}$$

$$= \boxed{2.84 \times 10^{2}\ \mathrm{J}/\mathrm{kg{\bullet}C^o}}$$

15. The heat must warm both the water and the pot to 100°C. The heat is also the power times the time.

$$Q = Pt = \left(m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\right)\Delta T_{\text{H}_2\text{O}} \quad \rightarrow$$

$$t = \frac{\left(m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\right)\Delta T_{\text{H}_2\text{O}}}{P} = \frac{\left[(0.36\text{ kg})\left(900\,\text{J/kg}\bullet\text{C}^{\text{o}}\right) + (0.75\text{ kg})\left(4186\,\text{J/kg}\bullet\text{C}^{\text{o}}\right)\right]\left(92\text{C}^{\text{o}}\right)}{750\text{ W}}$$

$$= \boxed{425\text{ s}} \approx 7\text{ min}$$

16. The heat released by the 15 grams of candy in the burning is equal to the heat absorbed by the bomb, calorimeter, and water.

$$Q_{15} = \left[\left(m_{\text{bomb}} + m_{\text{cup}}\right)c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\right]\Delta T$$

$$= \left[(0.725\text{ kg} + 0.624\text{ kg})\left(0.22\,\text{kcal/kg}\bullet\text{C}^{\text{o}}\right) + (2.00\text{ kg})\left(1.00\,\text{kcal/kg}\bullet\text{C}^{\text{o}}\right)\right]\left(53.5^{\text{o}}\text{C} - 15.0^{\text{o}}\text{C}\right)$$

$$= 88.43\text{ kcal}$$

The heat released by 75 grams of the candy would be 5 times that released by the 15 grams.

$$Q_{75} = 5Q_{15} = 5(88.43\text{ kcal}) = 440\text{ kcal} = \boxed{440\text{ Cal}}$$

$\boxed{17.}$ The heat lost by the iron must be the heat gained by the aluminum and the glycerin.

$$m_{\text{Fe}}c_{\text{Fe}}\left(T_{i\,\text{Fe}} - T_{\text{eq}}\right) = m_{\text{Al}}c_{\text{Al}}\left(T_{\text{eq}} - T_{i\,\text{Al}}\right) + m_{\text{gly}}c_{\text{gly}}\left(T_{\text{eq}} - T_{i\,\text{gly}}\right)$$

$$(0.290\text{ kg})\left(450\,\text{J/kg}\bullet\text{C}^{\text{o}}\right)\left(142\text{C}^{\text{o}}\right) = (0.095\text{ kg})\left(900\,\text{J/kg}\bullet\text{C}^{\text{o}}\right)\left(28\text{C}^{\text{o}}\right) + (0.250\text{ kg})c_{\text{gly}}\left(28\text{C}^{\text{o}}\right)$$

$$c_{\text{gly}} = \boxed{2.3\times10^{3}\text{ J/kg}\bullet\text{C}^{\text{o}}}$$

18. We assume that all of the kinetic energy of the hammer goes into heating the nail.

$$KE = Q \quad \rightarrow \quad 10\left(\tfrac{1}{2}m_{\text{hammer}}v_{\text{hammer}}^2\right) = m_{\text{nail}}c_{\text{Fe}}\Delta T \quad \rightarrow$$

$$\Delta T = \frac{10\left(\tfrac{1}{2}m_{\text{hammer}}v_{\text{hammer}}^2\right)}{m_{\text{nail}}c_{\text{Fe}}} = \frac{5(1.20\text{ kg})(6.5\,\text{m/s})^2}{(0.014\text{ kg})\left(450\,\text{J/kg}\bullet\text{C}^{\text{o}}\right)} = 40.24\text{C}^{\text{o}} \approx \boxed{4.0\times10^{1}\text{C}^{\text{o}}}$$

19. 65% of the original potential energy of the aluminum goes to heating the aluminum.

$$0.65PE = Q \quad \rightarrow \quad 0.65m_{\text{Al}}gh = m_{\text{Al}}c_{\text{Al}}\Delta T \quad \rightarrow$$

$$\Delta T = \frac{0.65gh}{c_{\text{Al}}} = \frac{0.65\left(9.80\,\text{m/s}^2\right)(45\text{ m})}{\left(900\,\text{J/kg}\bullet\text{C}^{\text{o}}\right)} = \boxed{0.32\text{C}^{\text{o}}}$$

20. (*a*) Since $Q = mc\Delta T$ and $Q = C\Delta T$, equate these two expressions for Q and solve for C.

$$Q = mc\Delta T = C\Delta T \quad \rightarrow \quad \boxed{C = mc}$$

(*b*) For 1.0 kg of water: $C = mc = (1.0\text{ kg})\left(4186\,\text{J/kg}\bullet\text{C}^{\text{o}}\right) = \boxed{4.2\times10^{3}\text{ J/C}^{\text{o}}}$

(*c*) For 25 kg of water: $C = mc = (25\text{ kg})\left(4186\,\text{J/kg}\bullet\text{C}^{\text{o}}\right) = \boxed{1.0\times10^{5}\text{ J/C}^{\text{o}}}$

21. The silver must be heated to the melting temperature and then melted.

$$Q = Q_{\text{heat}} + Q_{\text{melt}} = mc\Delta T + mL_{\text{fusion}}$$

$$= (16.50\text{ kg})\left(230\,\text{J/kg}\bullet\text{C}^{\text{o}}\right)\left(961^{\text{o}}\text{C} - 20^{\text{o}}\text{C}\right) + (16.50\text{ kg})\left(0.88\times10^{5}\text{ J/kg}\right) = \boxed{5.0\times10^{6}\text{J}}$$

22. Assume that the heat from the person is only used to evaporate the water. Also, we use the heat of vaporization at room temperature (585 kcal/kg), since the person's temperature is closer to room temperature than 100°C.

$$Q = mL_{\text{vap}} \quad \rightarrow \quad m = \frac{Q}{L_{\text{vap}}} = \frac{180 \text{ kcal}}{585 \text{ kcal/kg}} = 0.308 \text{ kg} \approx \boxed{0.31 \text{ kg}} = 310 \text{ mL}$$

23. The oxygen is all at the boiling point, so any heat added will cause oxygen to evaporate (as opposed to raising its temperature). We assume that all the heat goes to the oxygen, and none to the flask.

$$Q = mL_{\text{vap}} \quad \rightarrow \quad m = \frac{Q}{L_{\text{vap}}} = \frac{2.80 \times 10^5 \text{ J}}{2.1 \times 10^5 \text{ J/kg}} = \boxed{1.3 \text{ kg}}$$

24. Assume that all of the heat lost by the ice cube in cooling to the temperature of the liquid nitrogen is used to boil the nitrogen, and so none is used to raise the temperature of the nitrogen. The boiling point of the nitrogen is $77\,\text{K} = -196^\circ\text{C}$.

$$m_{\text{ice}} c_{\text{ice}} \left(T_{i \text{ ice}} - T_{f \text{ ice}} \right) = m_{\text{nitrogen}} L_{\text{vap}} \quad \rightarrow$$

$$m_{\text{nitrogen}} = \frac{m_{\text{ice}} c_{\text{ice}} \left(T_{i \text{ ice}} - T_{f \text{ ice}} \right)}{L_{\text{vap}}} = \frac{\left(3.0 \times 10^{-2} \text{ kg}\right)\left(2100 \text{ J/kg}\bullet\text{C}^\circ\right)\left(0^\circ\text{C} - -196^\circ\text{C}\right)}{200 \times 10^3 \text{ J/kg}} = \boxed{6.2 \times 10^{-2} \text{ kg}}$$

25. The heat lost by the aluminum and 310 g of liquid water must be equal to the heat gained by the ice in warming in the solid state, melting, and warming in the liquid state.

$$m_{\text{Al}} c_{\text{Al}} \left(T_{i\text{Al}} - T_{\text{eq}} \right) + m_{\text{H}_2\text{O}} c_{\text{H}_2\text{O}} \left(T_{i\text{H}_2\text{O}} - T_{\text{eq}} \right) = m_{\text{ice}} \left[c_{\text{ice}} \left(T_{\text{melt}} - T_{i \text{ ice}} \right) + L_{\text{fusion}} + c_{\text{H}_2\text{O}} \left(T_{\text{eq}} - T_{\text{melt}} \right) \right]$$

$$m_{\text{ice}} = \frac{\left(0.095 \text{ kg}\right)\left(900 \text{ J/kg}\bullet\text{C}^\circ\right)\left(3.0\text{C}^\circ\right) + \left(0.31 \text{ kg}\right)\left(4186 \text{ J/kg}\bullet\text{C}^\circ\right)\left(3.0\text{C}^\circ\right)}{\left[\left(2100 \text{ J/kg}\bullet\text{C}^\circ\right)\left(8.5\text{C}^\circ\right) + 3.3 \times 10^5 \text{ J/kg} + \left(4186 \text{ J/kg}\bullet\text{C}^\circ\right)\left(17\text{C}^\circ\right)\right]} = \boxed{9.90 \times 10^{-3} \text{ kg}}$$

$\boxed{26.}$ (*a*) The heater must heat both the boiler and the water at the same time.

$$Q_1 = Pt_1 = \left(m_{\text{Fe}} c_{\text{Fe}} + m_{\text{H}_2\text{O}} c_{\text{H}_2\text{O}} \right) \Delta T \quad \rightarrow$$

$$t_1 = \frac{\left(m_{\text{Fe}} c_{\text{Fe}} + m_{\text{H}_2\text{O}} c_{\text{H}_2\text{O}} \right) \Delta T}{P} = \frac{\left[\left(230 \text{ kg}\right)\left(450 \text{ J/kg}\bullet\text{C}^\circ\right) + \left(830 \text{ kg}\right)\left(4186 \text{ J/kg}\bullet\text{C}^\circ\right)\right]\left(82\text{C}^\circ\right)}{5.2 \times 10^7 \text{ J/h}}$$

$$= 5.642\,\text{h} \approx \boxed{5.6 \text{ h}}$$

(*b*) Assume that after the water starts to boil, all the heat energy goes into boiling the water, and none goes to raising the temperature of the iron or the steam.

$$Q_2 = Pt_2 = m_{\text{H}_2\text{O}} L_{\text{vap}} \quad \rightarrow \quad t_2 = \frac{m_{\text{H}_2\text{O}} L_{\text{vap}}}{P} = \frac{\left(830 \text{ kg}\right)\left(22.6 \times 10^5 \text{ J/kg}\right)}{5.2 \times 10^7 \text{ J/h}} = 36.073 \text{ h}$$

Thus the total time is $t_1 + t_2 = 5.642 \text{ h} + 36.073 \text{ h} = 41.72 \text{ h} \approx \boxed{42 \text{ h}}$

27. We assume that the cyclist's energy is only going to evaporation, not any heating. Then the energy needed is equal to the mass of the water times the latent heat of vaporization for water. Note that 1 L of water has a mass of 1 kg. Also, we use the heat of vaporization at room temperature (585 kcal/kg), since the cyclist's temperature is closer to room temperature than 100°C.

$$Q = m_{\text{H}_2\text{O}} L_{\text{vap}} = \left(8.0 \text{ kg}\right)\left(585 \text{ kcal/kg}\right) = \boxed{4.7 \times 10^3 \text{ kcal}}$$

28. The heat lost by the steam condensing and then cooling to 20°C must be equal to the heat gained by the ice melting and then warming to 20°C.

$$m_{\text{steam}}\left[L_{\text{vap}}+c_{\text{H}_2\text{O}}\left(T_{i\,\text{steam}}-T_{\text{eq}}\right)\right]=m_{\text{ice}}\left[L_{\text{fus}}+c_{\text{H}_2\text{O}}\left(T_{\text{eq}}-T_{i\,\text{ice}}\right)\right]$$

$$m_{\text{steam}}=m_{\text{ice}}\frac{\left[L_{\text{fus}}+c_{\text{H}_2\text{O}}\left(T_{\text{eq}}-T_{i\,\text{ice}}\right)\right]}{\left[L_{\text{vap}}+c_{\text{H}_2\text{O}}\left(T_{i\,\text{steam}}-T_{\text{eq}}\right)\right]}=(1.00\text{ kg})\frac{\left[3.33\times10^5\text{ J/kg}+\left(4186\text{ J/kg}\bullet\text{C}^\circ\right)\left(20\text{C}^\circ\right)\right]}{\left[22.6\times10^5\text{ J/kg}+\left(4186\text{ J/kg}\bullet\text{C}^\circ\right)\left(80\text{C}^\circ\right)\right]}$$

$$=\boxed{1.61\times10^{-1}\text{kg}}$$

29. The heat lost by the aluminum and the water must equal the heat needed to melt the mercury and to warm the mercury to the equilibrium temperature.

$$m_{\text{Al}}c_{\text{Al}}\left(T_{i\,\text{Al}}-T_{\text{eq}}\right)+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_{i\,\text{H}_2\text{O}}-T_{\text{eq}}\right)=m_{\text{Hg}}\left[L_{\text{fusion}}+c_{\text{Hg}}\left(T_{\text{eq}}-T_{\text{melt}}\right)\right]$$

$$L_{\text{fusion}}=\frac{m_{\text{Al}}c_{\text{Al}}\left(T_{i\,\text{Al}}-T_{\text{eq}}\right)+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_{i\,\text{H}_2\text{O}}-T_{\text{eq}}\right)}{m_{\text{Hg}}}-c_{\text{Hg}}\left(T_{\text{eq}}-T_{\text{melt}}\right)$$

$$=\frac{\left[(0.620\text{ kg})\left(900\text{ J/kg}\bullet\text{C}^\circ\right)+(0.400\text{ kg})\left(4186\text{ J/kg}\bullet\text{C}^\circ\right)\right]\left(12.80^\circ\text{C}-5.06^\circ\text{C}\right)}{1.00\text{ kg}}$$

$$-\left(138\text{ J/kg}\bullet\text{C}^\circ\right)\left[5.06^\circ\text{C}-\left(-39.0^\circ\text{C}\right)\right]$$

$$=\boxed{1.12\times10^4\text{ J/kg}}$$

30. Assume that the kinetic energy of the bullet was all converted into heat which melted the ice.

$$\tfrac{1}{2}m_{\text{bullet}}v^2=Q=m_{\text{ice}}L_{\text{fusion}}\quad\rightarrow$$

$$m_{\text{ice}}=\frac{\frac{1}{2}m_{\text{bullet}}v^2}{L_{\text{fusion}}}=\frac{\frac{1}{2}\left(7.0\times10^{-2}\text{kg}\right)(250\text{ m/s})^2}{3.33\times10^5\text{ J/kg}}=\boxed{6.6\times10^{-3}\text{kg}}=6.6\text{ g}$$

31. Assume that all of the melted ice stays at 0°C, so that all the heat is used in melting ice, and none in warming water. The available heat is half of the original kinetic energy

$$\tfrac{1}{2}\left(\tfrac{1}{2}m_{\text{skater}}v^2\right)=Q=m_{\text{ice}}L_{\text{fusion}}\quad\rightarrow$$

$$m_{\text{ice}}=\frac{\frac{1}{4}m_{\text{skater}}v^2}{L_{\text{fusion}}}=\frac{\frac{1}{4}(54.0\text{ kg})(6.4\text{ m/s})^2}{3.33\times10^5\text{ J/kg}}=\boxed{1.7\times10^{-3}\text{kg}}=1.7\text{ g}$$

32. The kinetic energy of the bullet is assumed to warm the bullet and melt it.

$$\tfrac{1}{2}mv^2=Q=mc_{\text{Pb}}\left(T_{\text{melt}}-T_i\right)+mL_{\text{fusion}}\quad\rightarrow$$

$$v=\sqrt{2\left[c_{\text{Pb}}\left(T_{\text{melt}}-T_i\right)+L_{\text{fusion}}\right]}=\sqrt{2\left[\left(130\text{ J/kg}\bullet\text{C}^\circ\right)\left(327^\circ\text{C}-20^\circ\text{C}\right)+\left(0.25\times10^5\text{ J/kg}\right)\right]}$$

$$=\boxed{3.6\times10^2\text{ m/s}}$$

33. The heat conduction rate is given by Eq. 14-4.

$$\frac{Q}{t}=kA\frac{T_1-T_2}{l}=\left(200\text{ J/s}\bullet\text{m}\bullet\text{C}^\circ\right)\pi\left(1.0\times10^{-2}\text{m}\right)^2\frac{\left(460^\circ\text{C}-22^\circ\text{C}\right)}{0.33\text{m}}=\boxed{83\text{W}}$$

34. The heat conduction rate is given by Eq. 14-4.

$$\frac{Q}{t}=kA\frac{T_1-T_2}{l}=\left(0.84\,\mathrm{J/s{\bullet}m{\bullet}C^o}\right)\left(3.0\ \mathrm{m}^2\right)\frac{\left[15.0^\circ\mathrm{C}-\left(-5^\circ\mathrm{C}\right)\right]}{3.2\times10^{-3}\ \mathrm{m}}=\boxed{1.6\times10^4\,\mathrm{W}}$$

35. (*a*) The power radiated is given by Eq. 14-5. The temperature of the tungsten is $273\,\mathrm{K}+25\,\mathrm{K}=298\,\mathrm{K}$.

$$\frac{\Delta Q}{\Delta t}=e\sigma AT^4=\left(0.35\right)\left(5.67\times10^{-8}\ \mathrm{W/m^2{\bullet}K^4}\right)4\pi\left(0.22\ \mathrm{m}\right)^2\left(298\ \mathrm{K}\right)^4=\boxed{95\ \mathrm{W}}$$

(*b*) The net flow rate of energy is given by Eq. 14-6. The temperature of the surroundings is 268 K.

$$\frac{\Delta Q}{\Delta t}=e\sigma A\left(T_1^4-T_1^4\right)=\left(0.35\right)\left(5.67\times10^{-8}\ \mathrm{W/m^2{\bullet}K^4}\right)4\pi\left(0.22\ \mathrm{m}\right)^2\left[\left(298\ \mathrm{K}\right)^4-\left(268\ \mathrm{K}\right)^4\right]$$

$$=\boxed{33\ \mathrm{W}}$$

36. The distance can be calculated from the heat conduction rate, given by Eq. 14-4. The rate is given as a power (200 W = 200 J/s).

$$\frac{Q}{t}=P=kA\frac{T_1-T_2}{l}\quad\rightarrow\quad l=kA\frac{T_1-T_2}{P}=\left(0.2\,\mathrm{J/s{\bullet}m{\bullet}C^o}\right)\left(1.5\ \mathrm{m}^2\right)\frac{0.50\,\mathrm{C^o}}{200\ \mathrm{W}}=\boxed{8\times10^{-4}\mathrm{m}}$$

$\boxed{37.}$ This is a heat transfer by conduction, and so Eq. 14-4 is applicable.

$$\frac{Q}{t}=P=kA\frac{T_1-T_2}{l}=\left(0.84\,\mathrm{J/s{\bullet}m{\bullet}C^o}\right)\left(16\,\mathrm{m}^2\right)\frac{30^\circ\mathrm{C}-10^\circ\mathrm{C}}{0.12\ \mathrm{m}}=2.24\times10^3\,\mathrm{W}$$

If we assume that all of the energy from the light bulbs goes into this conduction, then:

$$2.24\times10^3\,\mathrm{W}\left(\frac{1\ \mathrm{bulb}}{100\ \mathrm{W}}\right)=22.4\ \mathrm{bulbs}\ \text{ and so }\ \boxed{23\ \mathrm{bulbs}}\ \text{ are needed.}$$

38. Eq. 14-7 gives the heat absorption rate for an object facing the Sun. The heat required to melt the ice is the mass of the ice times the latent heat of fusion for the ice. The mass is found by multiplying the volume of ice times its density.

$$\Delta Q=mL_f=\rho VL_f=\rho A\left(\Delta x\right)L_f\qquad\frac{\Delta Q}{\Delta t}=\left(1000\,\mathrm{W/m^2}\right)eA\cos\theta\ \rightarrow$$

$$\Delta t=\frac{\rho A\left(\Delta x\right)L_f}{\left(1000\,\mathrm{W/m^2}\right)eA\cos\theta}=\frac{\rho\left(\Delta x\right)L_f}{\left(1000\,\mathrm{W/m^2}\right)e\cos\theta}$$

$$=\frac{\left(9.17\times10^2\ \mathrm{kg/m^3}\right)\left(1.0\times10^{-2}\,\mathrm{m}\right)\left(3.33\times10^5\ \mathrm{J/kg}\right)}{\left(1000\,\mathrm{W/m^2}\right)\left(0.050\right)\cos30^\circ}=\boxed{7.1\times10^4\mathrm{s}}=20\ \mathrm{h}$$

39. For the temperature at the joint to remain constant, the heat flow in both rods must be the same. Note that the cross-sectional areas and lengths are the same. Use Eq. 14-4 for heat conduction.

$$\left(\frac{Q}{t}\right)_{\mathrm{Cu}}=\left(\frac{Q}{t}\right)_{\mathrm{Al}}\ \rightarrow\ k_{\mathrm{Cu}}A\frac{T_{\mathrm{hot}}-T_{\mathrm{middle}}}{l}=k_{\mathrm{Al}}A\frac{T_{\mathrm{middle}}-T_{\mathrm{cool}}}{l}\ \rightarrow$$

$$T_{\mathrm{middle}}=\frac{k_{\mathrm{Cu}}T_{\mathrm{hot}}+k_{\mathrm{Al}}T_{\mathrm{cool}}}{k_{\mathrm{Cu}}+k_{\mathrm{Al}}}=\frac{\left(380\,\mathrm{J/s{\bullet}m{\bullet}C^o}\right)\left(250^\circ\mathrm{C}\right)+\left(200\,\mathrm{J/s{\bullet}m{\bullet}C^o}\right)\left(0.0^\circ\mathrm{C}\right)}{380\,\mathrm{J/s{\bullet}m{\bullet}C^o}+200\,\mathrm{J/s{\bullet}m{\bullet}C^o}}=\boxed{1.6\times10^2\ {}^\circ\mathrm{C}}$$

40. (*a*) The cross-sectional area of the Earth, perpendicular to the Sun, is a circle of radius R_{Earth}, and so has an area of πR_{Earth}^2. Multiply this area times the solar constant to get the rate at which the Earth is receiving solar energy.

$$\frac{Q}{t} = \pi R_{\text{Earth}}^2 \left(\text{solar constant}\right) = \pi\left(6.38\times10^6\,\text{m}\right)^2\left(1350\,\text{W}/\text{m}^2\right) = \boxed{1.73\times10^{17}\,\text{W}}$$

(*b*) Use Eq. 14-5 to calculate the rate of heat output by radiation.

$$\frac{Q}{t} = e\sigma AT^4 \quad \rightarrow$$

$$T = \left(\frac{Q}{t}\frac{1}{\varepsilon\sigma A}\right)^{1/4} = \left[\left(1.7\times10^{17}\,\text{J}/\text{s}\right)\frac{1}{\left(1.0\right)\left(5.67\times10^{-8}\,\text{W}/\text{m}^2\text{•K}^4\right)4\pi\left(6.38\times10^6\,\text{m}\right)^2}\right]^{1/4}$$

$$= \boxed{278\text{ K} = 5^\circ\text{C}}$$

41. This is an example of heat conduction, and the temperature difference can be calculated by Eq. 14-4.

$$\frac{Q}{t} = P = kA\frac{T_1 - T_2}{l} \quad \rightarrow \quad \Delta T = \frac{Pl}{kA} = \frac{\left(95\text{ W}\right)\left(1.0\times10^{-3}\,\text{m}\right)}{\left(0.84\,\text{J}/\text{s•m•C}^\circ\right)4\pi\left(3.0\times10^{-2}\,\text{m}\right)^2} = \boxed{10\,\text{C}^\circ}$$

42. The conduction rates through the two materials must be equal. If they were not, the temperatures in the materials would be changing. Call the temperature at the boundary between the materials T_x.

$$\frac{Q}{t} = k_1 A\frac{T_1 - T_x}{l_1} = k_2 A\frac{T_x - T_2}{l_2} \quad \rightarrow \quad \frac{Q}{t}\frac{l_1}{k_1 A} = T_1 - T_x \ ; \ \frac{Q}{t}\frac{l_2}{k_2 A} = T_x - T_2$$

Add these two equations together, and solve for the heat conduction rate.

$$\frac{Q}{t}\frac{l_1}{k_1 A} + \frac{Q}{t}\frac{l_2}{k_2 A} = T_1 - T_x + T_x - T_2 \quad \rightarrow \quad \frac{Q}{t}\left(\frac{l_1}{k_1} + \frac{l_2}{k_2}\right)\frac{1}{A} = T_1 - T_2 \quad \rightarrow$$

$$\frac{Q}{t} = A\frac{\left(T_1 - T_2\right)}{\left(\frac{l_1}{k_1} + \frac{l_2}{k_2}\right)} = A\frac{\left(T_1 - T_2\right)}{\left(R_1 + R_2\right)} = \left(240\text{ ft}^2\right)\frac{\left(12\,\text{F}^\circ\right)}{\left(1+19\right)\text{ft}^2\text{•h•F}^\circ/\text{Btu}} = 144\,\text{Btu}/\text{h} \approx \boxed{1.4\times10^2\text{ Btu}/\text{h}}$$

This is about 42 Watts.

43. (*a*) We assume that $T_2 > T_1$. The conduction rates through the three materials must be equal. If they were not, the temperatures in the materials would be changing. Call the temperature at the boundary between the air and the left-most piece of glass, T_x, and the temperature at the boundary between the air and the right-most piece of glass, T_y. Write the conduction rate for each material separately, and solve for the temperature differences.

$$\frac{Q}{t} = k_1 A\frac{T_x - T_1}{l_1} = k_2 A\frac{T_y - T_x}{l_2} = k_3 A\frac{T_2 - T_y}{l_3} \quad \rightarrow$$

$$\frac{Q}{t}\frac{l_1}{k_1 A} = T_x - T_1 \ ; \ \frac{Q}{t}\frac{l_2}{k_2 A} = T_y - T_x \ ; \ \frac{Q}{t}\frac{l_3}{k_3 A} = T_2 - T_y$$

Add these three equations together, and solve for the heat conduction rate.

$$\frac{Q}{t}\frac{l_1}{k_1A}+\frac{Q}{t}\frac{l_2}{k_2A}+\frac{Q}{t}\frac{l_3}{k_3A}=T_x-T_1+T_y-T_x+T_2-T_y \rightarrow \frac{Q}{t}\left(\frac{l_1}{k_1}+\frac{l_2}{k_2}+\frac{l_3}{k_3}\right)\frac{1}{A}=T_2-T_1 \rightarrow$$

$$\boxed{\frac{Q}{t}=A\frac{\left(T_2-T_1\right)}{\left(l_1/k_1+l_2/k_2+l_3/k_3\right)}}$$

(*b*) For *n* materials placed next to one another, the expression would be

$$\frac{Q}{t}=A\frac{\left(T_2-T_1\right)}{\sum\limits_{i=1}^{n} l_i/k_i}= \rightarrow \boxed{\frac{Q}{t}=A\frac{\left(T_2-T_1\right)}{\sum\limits_{i=1}^{n} R_i}}$$

44. This is an example of heat conduction. The heat conducted is the heat released by the melting ice, $Q=m_{\text{ice}}L_{\text{fusion}}$. The area through which the heat is conducted is the total area of the six surfaces of the box, and the length of the conducting material is the thickness of the styrofoam. We assume that all of the heat conducted into the box goes into melting the ice, and none into raising the temperature inside the box. The time can then be calculated by Eq. 14-4.

$$\frac{Q}{t}=kA\frac{T_1-T_2}{l} \rightarrow t=\frac{m_{\text{ice}}L_{\text{fusion}}l}{kA\Delta T}$$

$$=\frac{\left(11.0\text{ kg}\right)\left(3.33\times10^5\text{ J/kg}\right)\left(1.5\times10^{-2}\text{ m}\right)}{2\left(0.023\text{ J/s}\bullet\text{m}\bullet\text{C}^\circ\right)\left[2\left(0.25\text{ m}\right)\left(0.35\text{ m}\right)+2\left(0.25\text{ m}\right)\left(0.55\text{ m}\right)+2\left(0.35\text{ m}\right)\left(0.55\text{ m}\right)\right]\left(32\text{ C}^\circ\right)}$$

$$=\boxed{4.5\times10^4\text{s}}\approx 12\text{ h}$$

45. The heat needed to warm the liquid can be calculated by Eq. 14-2.

$$Q=mc\Delta T=\left(0.20\text{ kg}\right)\left(1.00\,\text{kcal}/\text{kg}\bullet\text{C}^\circ\right)\left(37^\circ\text{C}-5^\circ\text{C}\right)=6.4\text{ kcal}=\boxed{6.4\text{ C}}$$

46. Since 30% of the heat generated is lost up the chimney, the heat required to heat the house is 70% of the heat provided by the coal.

$$2.0\times10^5\,\text{MJ}=0.70\left(30\times10^6\text{ MJ}/\text{kg}\right)\left(m\text{ kg}\right) \rightarrow m=\frac{2.0\times10^5\,\text{MJ}}{0.70\left(30\,\text{MJ/kg}\right)}=\boxed{9.5\times10^3\,\text{kg}}$$

47. The heat released can be calculated by Eq. 14-2. To find the mass of the water, use the density (of pure water).

$$Q=mc\Delta T=\rho Vc\Delta T=\left(1.0\times10^3\text{ kg}/\text{m}^3\right)\left(1.0\times10^3\text{m}\right)^3\left(4186\,\text{J}/\text{kg}\bullet\text{C}^\circ\right)\left(1\text{C}^\circ\right)=\boxed{4\times10^{15}\,\text{J}}$$

48. We assume that the initial kinetic energy of the bullet all goes into heating the wood and the bullet.

$$\tfrac{1}{2}m_{\text{bullet}}v_i^2=Q=m_{\text{bullet}}c_{\text{lead}}\Delta T_{\text{lead}}+m_{\text{wood}}c_{\text{wood}}\Delta T_{\text{wood}} \rightarrow$$

$$v_i=\sqrt{\frac{\left(m_{\text{bullet}}c_{\text{lead}}+m_{\text{wood}}c_{\text{wood}}\right)\Delta T}{\tfrac{1}{2}m_{\text{bullet}}}}$$

$$=\sqrt{\frac{\left[\left(0.015\text{ kg}\right)\left(130\,\text{J}/\text{kg}\bullet\text{C}^\circ\right)+\left(1.05\text{ kg}\right)\left(1700\,\text{J}/\text{kg}\bullet\text{C}^\circ\right)\right]\left(0.020\,\text{C}^\circ\right)}{\tfrac{1}{2}\left(0.015\text{ kg}\right)}}=\boxed{69\,\text{m}/\text{s}}$$

49. (*a*) Use Eq. 14-5 for total power radiated.

$$\frac{Q}{t}=e\sigma AT^4=e\sigma 4\pi R_{\text{Sun}}^2T^4=(1.0)\left(5.67\times10^{-8}\,\text{W/m}^2\bullet\text{K}^4\right)4\pi\left(7.0\times10^{8}\,\text{m}\right)^2(5500\text{ K})^4$$

$$=3.195\times10^{26}\,\text{W}\approx\boxed{3.2\times10^{26}\,\text{W}}$$

(*b*) Assume that the energy from the Sun is distributed symmetrically over a spherical surface with the Sun at the center.

$$\frac{P}{A}=\frac{Q/t}{4\pi R_{\text{Sun-Earth}}^2}=\frac{3.195\times10^{26}\,\text{W}}{4\pi\left(1.5\times10^{11}\,\text{m}\right)^2}=1.130\times10^{3}\,\text{W/m}^2\approx\boxed{1.1\times10^{3}\,\text{W/m}^2}$$

50. The temperature rise can be calculated from Eq. 14-2.

$$Q=mc\Delta T\quad\rightarrow\quad\Delta T=\frac{Q}{mc}=\frac{(0.80)(200\,\text{kcal/h})(1.00\text{ h})}{(70\text{ kg})\left(0.83\,\text{kcal/kg}\bullet\text{C}^{\circ}\right)}=\boxed{2.8\,\text{C}^{\circ}}$$

51. We assume that the starting speed of the boulder is zero, and that 50% of the original potential energy of the boulder goes to heating the boulder.

$$\tfrac{1}{2}PE=Q\quad\rightarrow\quad\tfrac{1}{2}(mgh)=mc_{\text{marble}}\Delta T\quad\rightarrow\quad\Delta T=\frac{\frac{1}{2}gh}{c_{\text{marble}}}=\frac{0.50\left(9.8\,\text{m/s}^2\right)(140\text{ m})}{860\,\text{J/kg}\bullet\text{C}^{\circ}}=\boxed{0.80\,\text{C}^{\circ}}$$

52. The heat lost by the lead must be equal to the heat gained by the water. Note that 1 L of water has a mass of 1 kg.

$$m_{\text{Pb}}c_{\text{Pb}}\left(T_{\text{i Pb}}-T_{\text{eq}}\right)=m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_{\text{eq}}-T_{i\,\text{H}_2\text{O}}\right)$$

$$(2.3\text{ kg})\left(130\,\text{J/kg}\bullet\text{C}^{\circ}\right)\left(T_{\text{i Pb}}-28.0^{\circ}\text{C}\right)=(2.5\text{ kg})\left(4186\,\text{J/kg}\bullet\text{C}^{\circ}\right)\left(8.0\,\text{C}^{\circ}\right)\quad\rightarrow$$

$$T_{\text{i Pb}}=308^{\circ}\text{C}\approx\boxed{310^{\circ}\text{C}}$$

$\boxed{53.}$ Use the heat conduction rate equation, Eq. 14-4.

(*a*) $$\frac{Q}{t}=kA\frac{T_1-T_2}{l}=\left(0.025\,\text{J/s}\bullet\text{m}\bullet\text{C}^{\circ}\right)\left(1.2\,\text{m}^2\right)\frac{\left[34^{\circ}\text{C}-\left(-20^{\circ}\text{C}\right)\right]}{3.5\times10^{-2}\,\text{m}}=\boxed{46\text{ W}}$$

(*b*) $$\frac{Q}{t}=kA\frac{T_1-T_2}{l}=\left(0.56\,\text{J/s}\bullet\text{m}\bullet\text{C}^{\circ}\right)\left(1.2\,\text{m}^2\right)\frac{\left[34^{\circ}\text{C}-\left(-20^{\circ}\text{C}\right)\right]}{5.0\times10^{-3}\,\text{m}}=\boxed{7.3\times10^{3}\text{ W}}$$

54. We assume that all of the heat provided by metabolism goes into evaporating the water. For the energy required for the evaporation of water, we use the heat of vaporization at room temperature (585 kcal/kg), since the runner's temperature is closer to room temperature than 100°C.

$$2.5\text{ h}\left(\frac{950\text{ kcal}}{1\text{ h}}\right)\left(\frac{\text{kg H}_2\text{O}}{585\text{ kcal}}\right)=\boxed{4.1\text{ kg}}$$

55. For an estimate of the heat conduction rate, use Eq. 14-4.

$$\frac{Q}{t}=kA\frac{T_1-T_2}{l}=\left(0.2\,\text{J/s}\bullet\text{m}\bullet\text{C}^{\circ}\right)\left(1.5\,\text{m}^2\right)\frac{\left(37^{\circ}\text{C}-34^{\circ}\text{C}\right)}{4.0\times10^{-2}\,\text{m}}=22.5\,\text{W}\approx\boxed{20\text{ W}}$$

This is only about 10% of the cooling capacity that is needed for the body. Thus convection cooling is clearly necessary.

56. (*a*) To calculate heat transfer by conduction, use Eq. 14-4 for all three areas – walls, roof, and windows. Each area has the same temperature difference.

$$\frac{Q_{\text{conduction}}}{t} = \left[\left(\frac{kA}{l}\right)_{\text{walls}} + \left(\frac{kA}{l}\right)_{\text{roof}} + \left(\frac{kA}{l}\right)_{\text{windows}}\right](T_1 - T_2)$$

$$= \left[\begin{array}{l} \dfrac{(0.023\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ)(410\,\text{m}^2)}{1.75\times10^{-1}\,\text{m}} + \dfrac{(0.12\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ)(280\,\text{m}^2)}{6.5\times10^{-2}\,\text{cm}} \\ + \dfrac{(0.84\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ)(33\,\text{m}^2)}{6.5\times10^{-3}\,\text{m}} \end{array}\right](33\,\text{C}^\circ)$$

$$= 1.596\times10^5\,\text{W} \approx \boxed{1.6\times10^5\,\text{W}}$$

(*b*) The energy being added must both heat the air and replace the energy lost by conduction, as considered above. The heat required to raise the temperature is given by Eq. 14-2, $Q_{\substack{\text{raise}\\ \text{temp}}} = m_{\text{air}}c_{\text{air}}(\Delta T)_{\text{warming}}$. The mass of the air can be found from the density of the air times its volume. The conduction heat loss is proportional to the temperature difference between the inside and outside, which varies from 20C° to 33C°. We will estimate the average temperature difference as 26.5°C and scale the answer from part (a) accordingly.

$$Q_{\text{added}} = Q_{\substack{\text{raise}\\ \text{temp}}} + Q_{\text{conduction}} = \rho_{\text{air}}Vc_{\text{air}}(\Delta T)_{\text{warming}} + \left(\frac{Q_{\text{conduction}}}{t}\right)(1800\text{ s})$$

$$= \left(1.29\frac{\text{kg}}{\text{m}^3}\right)(750\text{m}^3)\left(0.24\frac{\text{kcal}}{\text{kg}\bullet\text{C}^\circ}\right)\left(\frac{4186\text{ J}}{\text{kcal}}\right)(13^\circ\text{C})$$

$$+\left(1.596\times10^5\,\frac{\text{J}}{\text{s}}\right)\left(\frac{26.5^\circ\text{C}}{33^\circ\text{C}}\right)(1800\text{ s}) = \boxed{2.4\times10^8\,\text{J}}$$

(*c*) We assume a month is 30 days.

$$0.9Q_{\text{gas}} = \left(\frac{Q}{t}\right)_{\text{conduction}} t_{\text{month}} \quad \rightarrow$$

$$Q_{\text{gas}} = \frac{1}{0.9}\left(\frac{Q}{t}\right)_{\text{conduction}} t_{\text{month}} = \frac{1}{0.9}(1.596\times10^5\text{ J/s})(30\text{ d})\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 4.596\times10^{11}\,\text{J}$$

$$4.6\times10^{11}\,\text{J}\left(\frac{1\text{ kg}}{5.4\times10^7\,\text{J}}\right)\left(\frac{\$0.080}{\text{kg}}\right) = \boxed{\$680}$$

57. (*a*) The bullet will gain an amount of heat equal to 50% of its loss of kinetic energy. Initially assume that the phase of the bullet does not change, so that all of the heat causes a temperature increase.

$$\tfrac{1}{2}\left[\tfrac{1}{2}m(v_i^2 - v_f^2)\right] = Q = mc_{\text{Pb}}\Delta T \quad \rightarrow \quad \Delta T = \frac{\frac{1}{4}(v_i^2 - v_f^2)}{c_{\text{Pb}}} = \frac{(220\,\text{m/s})^2 - (160\,\text{m/s})^2}{4(130\,\text{J/kg}\bullet\text{C}^\circ)} = \boxed{44\,\text{C}^\circ}$$

(*b*) The final temperature of the bullet would be about 64°C, which is not above the melting temperature of lead, which is 327°C. Thus $\boxed{\text{none of the bullet will melt}}$.

58. (*a*) The rate of absorbing heat for an object facing the Sun is given by Eq. 14-7. The rise in temperature is related to the absorbed heat by Eq. 14-2. We assume that all absorbed heat raises the temperature of the leaf.

$$\Delta Q = mc\Delta T \qquad \frac{\Delta Q}{\Delta t} = \left(1000\,\mathrm{W/m^2}\right)eA\cos\theta \;\rightarrow$$

$$\frac{\Delta T}{\Delta t} = \frac{\left(1000\,\mathrm{W/m^2}\right)eA\cos\theta}{mc} = \frac{\left(1000\,\mathrm{W/m^2}\right)(0.85)\left(40\ \mathrm{cm^2}\right)\left(\dfrac{1\ \mathrm{m^2}}{1\times10^{4}\,\mathrm{cm^2}}\right)(1)}{\left(4.5\times10^{-4}\,\mathrm{kg}\right)\left(0.80\,\mathrm{kcal/kg{\bullet}C^o}\right)\left(\dfrac{4186\ \mathrm{J}}{1\ \mathrm{kcal}}\right)} = \boxed{2.3\,\mathrm{C^o/s}}$$

(*b*) We assume that the rate of heat loss by radiation must equal the rate of heat absorption of solar energy. Note that the area of the leaf that radiates is twice the area that absorbs heat energy.

$$\left(\frac{\Delta Q}{\Delta t}\right)_{\substack{\text{Solar}\\ \text{heating}}} = \left(\frac{\Delta Q}{\Delta t}\right)_{\text{radiation}} \;\rightarrow\; \left(1000\,\mathrm{W/m^2}\right)eA_{\text{absorb}}\cos\theta = e\sigma A_{\text{radiate}}\left(T_1^4 - T_2^4\right) \;\rightarrow$$

$$T_1 = \left[\frac{\left(1000\,\mathrm{W/m^2}\right)\cos\theta}{2\sigma} + T_2^4\right]^{1/4} = \left[\frac{\left(1000\,\mathrm{W/m^2}\right)(1)}{2\left(5.67\times10^{-8}\ \mathrm{W/m^2{\bullet}K^4}\right)} + \left(293\ \mathrm{K}\right)^4\right]^{1/4}$$

$$= 357\,\mathrm{K} = \boxed{84^\circ\mathrm{C}}$$

This is very hot, which indicates that the leaf must lose energy by other means than just radiation.

(*c*) The leaf can also lose heat by $\boxed{\text{conduction}}$ to the cooler air around it; by $\boxed{\text{convection}}$, as the wind continually moves cooler air over the surface of the leaf; and $\boxed{\text{evaporation}}$ of water.

59. The rate of energy absorption from the Sun must be equal to the rate of losing energy by radiation plus the rate of losing energy by evaporation if the leaf is to maintain a steady temperature. The latent heat of evaporation is taken be the value at 20°C, which is 2450 kJ/kg. Also note that the area of the leaf that radiates is twice the area that absorbs heat energy.

$$\left(\frac{\Delta Q}{\Delta t}\right)_{\substack{\text{Solar}\\ \text{heating}}} = \left(\frac{\Delta Q}{\Delta t}\right)_{\text{radiation}} + \left(\frac{\Delta Q}{\Delta t}\right)_{\text{evaporation}} \;\rightarrow$$

$$\left(1000\,\mathrm{W/m^2}\right)eA_{\text{absorb}}\cos\theta = e\sigma A_{\text{radiate}}\left(T_1^4 - T_2^4\right) + \frac{m_{\mathrm{H_2O}}L_{\text{evaporation}}}{\Delta t} \;\rightarrow$$

$$\frac{m_{\mathrm{H_2O}}}{\Delta t} = eA_{\text{absorb}}\,\frac{\left(1000\,\mathrm{W/m^2}\right)\cos\theta - 2\sigma\left(T_1^4 - T_2^4\right)}{L_{\text{evaporation}}}$$

$$= (0.85)\left(40\times10^{-4}\,\mathrm{m^2}\right)\frac{\left(1000\,\mathrm{W/m^2}\right)(1) - 2\left(5.67\times10^{-8}\ \mathrm{W/m^2{\bullet}K^4}\right)\left[\left(308\ \mathrm{K}\right)^4 - \left(293\ \mathrm{K}\right)^4\right]}{\left(2.45\times10^{6}\ \mathrm{J/kg}\right)}$$

$$= 1.1\times10^{-6}\ \mathrm{kg/s} = \boxed{4.1\,\mathrm{g/h}}$$

60. Assume that the final speed of the meteorite, as it completely melts, is 0, and that all of its initial kinetic energy was used in heating the iron to the melting point and then melting the iron.

$$\tfrac{1}{2}mv_i^2 = mc_{\mathrm{Fe}}\left(T_{\text{melt}} - T_i\right) + mL_{\text{fusion}} \;\rightarrow$$

$$v_i = \sqrt{2\left[c_{\mathrm{Fe}}\left(T_{\text{melt}} - T_i\right) + L_{\text{fusion}}\right]} = \sqrt{2\left[\left(450\,\mathrm{J/kg{\bullet}C^o}\right)\left(1808^\circ\mathrm{C} - -125^\circ\mathrm{C}\right) + 2.89\times10^{5}\ \mathrm{J/kg}\right]}$$

$$= \boxed{1.52\times10^{3}\ \mathrm{m/s}}$$

61. (*a*) We consider just the 30 m of crust immediately below the surface of the Earth, assuming that all the heat from the interior gets transferred to the surface, and so it all passes through this 30 m layer. This is a heat conduction problem, and so Eq. 14-4 is appropriate. The radius of the Earth is about $6.38\times10^{6}\,\mathrm{m}$.

$$\frac{Q}{t}=kA\frac{T_1-T_2}{l}\quad\rightarrow$$

$$Q_{\text{interior}}=kA\frac{T_1-T_2}{l}t=\left(0.80\,\mathrm{J/s{\bullet}m{\bullet}C^o}\right)4\pi R_{\text{Earth}}^2\frac{1.0\,\mathrm{C^o}}{30\ \mathrm{m}}\left[1\ \mathrm{day}\left(\frac{86,400\ \mathrm{s}}{\mathrm{day}}\right)\right]$$

$$=1.179\times10^{18}\,\mathrm{J}\approx\boxed{1.2\times10^{18}\,\mathrm{J}}$$

(*b*) The cross-sectional area of the Earth, perpendicular to the Sun, is a circle of radius R_{Earth}, and so has an area of πR_{Earth}^2. Multiply this area times the solar constant of $1350\,\mathrm{W/m^2}$ to get the amount of energy incident on the Earth from the Sun per second, and then convert to energy per day.

$$Q_{\text{Sun}}=\pi R_{\text{Earth}}^2\left(1350\,\mathrm{W/m^2}\right)\left[1\ \mathrm{day}\left(\frac{86,400\ \mathrm{s}}{\mathrm{day}}\right)\right]=1.492\times10^{22}\,\mathrm{J}$$

Thus $\dfrac{Q_{\text{interior}}}{Q_{\text{Sun}}}=\dfrac{1.179\times10^{18}\,\mathrm{J}}{1.492\times10^{22}\,\mathrm{J}}=7.902\times10^{-5}$, or $\boxed{Q_{\text{Sun}}=1.3\times10^{4}Q_{\text{interior}}}$.

62. Assume that the loss of kinetic energy is all turned into heat which changes the temperature of the squash ball.

$$KE_{\text{lost}}=Q\quad\rightarrow\quad\tfrac{1}{2}m\left(v_i^2-v_f^2\right)=mc\Delta T\quad\rightarrow\quad\Delta T=\frac{v_i^2-v_f^2}{2c}=\frac{\left(22\,\mathrm{m/s}\right)^2-\left(12\,\mathrm{m/s}\right)^2}{2\left(1200\,\mathrm{J/kg{\bullet}C^o}\right)}=\boxed{0.14\,\mathrm{C^o}}$$

63. The heat gained by the ice (to melt it and warm it) must be equal to the heat lost by the steam (in condensing and cooling).

$$mL_{\text{F}}+mc_{\text{H}_2\text{O}}\left(T_{\text{eq}}-0\right)=mL_{\text{V}}+mc_{\text{H}_2\text{O}}\left(100^\circ\mathrm{C}-T_{\text{eq}}\right)$$

$$T_{\text{eq}}=\frac{L_{\text{V}}-L_{\text{F}}}{2c_{\text{H}_2\text{O}}}+50^\circ\mathrm{C}=\frac{2260\,\mathrm{kJ/kg}-333\,\mathrm{kJ/kg}}{2\left(4.186\,\mathrm{kJ/kg{\bullet}C^o}\right)}+50^\circ\mathrm{C}=280^\circ\mathrm{C}$$

This answer is not possible. Because this answer is too high, the steam must not all condense, and none of it must cool below 100°C. Calculate the energy need to melt a kilogram of ice and warm it to 100°C.

$$Q=mL_{\text{F}}+mc_{\text{H}_2\text{O}}\left(T_{\text{eq}}-0\right)=\left(1\,\mathrm{kg}\right)\left[333\,\mathrm{kJ/kg}+\left(4.186\,\mathrm{kJ/kg{\bullet}C^o}\right)\left(100\mathrm{C^o}\right)\right]=751.6\,\mathrm{kJ}$$

Calculate the mass of steam that needs to condense in order to provide this much energy.

$$Q=mL_{\text{V}}\quad\rightarrow\quad m=\frac{Q}{L_{\text{V}}}=\frac{751.6\,\mathrm{kJ}}{2260\,\mathrm{kJ/kg}}=0.333\,\mathrm{kg}$$

Thus one-third of the original steam mass must condense to liquid at 100°C in order to melt the ice and warm the melted ice to 100°C. The final mixture will be at 100°C, with 1/3 of the total mass as steam, and 2/3 of the total mass as water.

64. The body's metabolism (blood circulation in particular) provides cooling by convection. If the metabolism has stopped, then heat loss will be by conduction and radiation, at a rate of 200 W, as given. The change in temperature is related to the body's heat loss by Eq. 14-2, $Q = mc\Delta T$.

$$\frac{Q}{t} = P = \frac{mc\Delta T}{t} \quad \rightarrow$$

$$t = \frac{mc\Delta T}{P} = \frac{(70\text{ kg})(3470\text{ J/kg}\bullet\text{C}^\circ)(36.6^\circ\text{C} - 35.6^\circ\text{C})}{200\text{ W}} = \boxed{1200\text{ s}} = 20\text{ min}$$

65. (*a*) The amount of heat energy required is given by Eq. 14-2. 1 L of water has a mass of 1 kg.

$$Q = mc\Delta T = (185\text{ kg})(4186\text{ J/kg}\bullet\text{C}^\circ)(50^\circ\text{C} - 10^\circ\text{C}) = 3.098\times10^7\text{ J} \approx \boxed{3.1\times10^7\text{ J}}$$

(*b*) The heat energy is the power input times the time.

$$Q = Pt \quad \rightarrow \quad t = \frac{Q}{P} = \frac{3.098\times10^7\text{ J}}{9.5\times10^3\text{ W}} = 3260\text{ s} \approx \boxed{3.3\times10^3\text{ s}} = 54\text{ min}$$

66. We assume that the light bulb emits energy by radiation, and so Eq. 14-6 applies. Use the data for the 60-W bulb to calculate the product $e\sigma A$ for the bulb, and then calculate the temperature of the 150-W bulb.

$$(Q/t)_{60\text{ W}} = e\sigma A\left(T^4_{60\text{ W}} - T^4_{\text{room}}\right) \quad \rightarrow$$

$$e\sigma A = \frac{(Q/t)_{60\text{ W}}}{\left(T^4_{60\text{ W}} - T^4_{\text{room}}\right)} = \frac{(0.90)(60\text{ W})}{[(273+65)\text{K}]^4 - [(273+18)\text{K}]^4} = 9.182\times10^{-9}\text{ W/K}^4$$

$$(Q/t)_{150\text{ W}} = e\sigma A\left(T^4_{150\text{ W}} - T^4_{\text{room}}\right) \quad \rightarrow$$

$$T_{150\text{ W}} = \left[\frac{(Q/t)_{150\text{ W}}}{e\sigma A} + T^4_{\text{room}}\right]^{1/4} = \left[\frac{(0.90)(150\text{ W})}{\left(9.182\times10^{-9}\text{ W/K}^4\right)} + (291\text{ K})^4\right]^{1/4}$$

$$= 385\text{ K} = 112^\circ\text{C} \approx \boxed{110^\circ\text{C}}$$

CHAPTER 15: The Laws of Thermodynamics

Answers to Questions

1. If water vapor condenses on the outside of a cold glass of water, the internal energy of the water vapor has decreased, by an amount equal to the heat of vaporization of the water vapor. Heat energy has left the water vapor, causing it to condense, and heat energy has entered the glass of water, and the air, causing them to get slightly warmer. No work is done, but heat is exchanged.

2. During compression, work is done on the gas. Assuming that there is no heat flow to or from the gas (since the process is quick), by conservation of energy (the first law of thermodynamics) the work done on the gas becomes internal energy of the gas, and so the temperature of the gas is increased. During expansion, work is done by the gas on its surroundings. Again assuming that there is no heat flow to or from the gas, by conservation of energy, the work is done by the gas at the expense of the internal energy of the gas, and so the temperature of the gas is decreased.

3. Since the process is isothermal, there is no change in the internal energy of the gas. Thus $\Delta U = Q - W = 0 \rightarrow Q = W$, and so the heat absorbed by the gas is equal to the work done by the gas. Thus 3700 J of heat was added to the gas.

4. It is possible for temperature (and thus internal energy) to remain constant in a system even though there is heat flow into or out of the system. By the first law of thermodynamics, there must be an equal amount of work done on or by the system, so that $\Delta U = Q - W = 0 \rightarrow Q = W$. The isothermal expansion or compression of a gas would be an example of this situation.

5. If the gas is compressed adiabatically, no heat enters or leaves from the gas. The compression means that work was done ON the gas. By the first law of thermodynamics, $\Delta U = Q - W$, since $Q = 0$, then $\Delta U = -W$. The change in internal energy is equal to the opposite of the work done by the gas, or is equal to the work done on the gas. Since positive work was done on the gas, the internal energy of the gas increased, and that corresponds to an increase in temperature. This is conservation of energy – the work done on the gas becomes internal energy of the gas particles, and the temperature increases accordingly.

6. Mechanical energy can be transformed completely into heat. As a moving object slides across a rough level floor and eventually stops, the mechanical energy of the moving object has been transformed completely into heat. Also, if a moving object were to be used to compress a frictionless piston containing an insulated gas, the kinetic energy of the object would become internal energy of the gas. A gas that expands adiabatically (without heat transfer) transforms internal energy into mechanical energy, by doing work on its surroundings at the expense of its internal energy. Of course, that is an ideal (reversible) process.

7. It is possible to warm the kitchen in the winter by having the oven door open. The oven heating elements radiate heat energy into the oven cavity, and if the oven door is open, the oven is just heating a bigger volume than usual. However, you cannot cool the kitchen by having the refrigerator door open. The refrigerator exhausts more heat than it removes from the refrigerated volume, so the room actually gets warmer with the refrigerator door open. If you could have the refrigerator exhaust into some other room, then the refrigerator would be similar to an air conditioner, and it could cool the kitchen, while heating up some other space.

8. This definition of efficiency is not useful, because with this definition, if the exhaust heat Q_L is less than the work done W (which is possible), the "efficiency" would exceed unity. Efficiency should be comparing to the heat input, not the heat output.

9. (*a*) In an internal combustion engine, the high temperature reservoir is the ignited gas-air mixture. The low temperature reservoir is the gases exhausted from the cylinder into the atmosphere.
   (*b*) In a steam engine, the high temperature reservoir is the heated, high-pressure steam from the boiler. The low temperature reservoir is the low-pressure steam from the exhaust.

10. The efficiency of a Carnot engine is given by Eq. 15-5, $e = 1 - \frac{T_L}{T_H}$. Both a decrease in T_L and an increase in T_H would cause the value of T_L / T_H to decrease, increasing the efficiency. Since $T_L < T_H$, the 10C° change is a larger percentage of change for T_L, and so will change the fraction more than the same numeric increase in the denominator. Note $e_1 = 1 - \frac{T_L - 10}{T_H} = \frac{T_H - T_L + 10}{T_H}$ and $e_2 = 1 - \frac{T_L}{T_H + 10} = \frac{T_H - T_L + 10}{T_H + 10}$. Both efficiencies have the same numerator, but e_2 has a larger denominator, and so $e_1 > e_2$.

11. To utilize the thermal energy in the ocean waters, a heat engine would need to be developed that operated between two different temperatures. If surface temperature water was to be both the source and the exhaust, then no work could be extracted. If the temperature difference between surface and deep ocean waters were to be used, there would be considerable engineering obstacles, high expense, and potential environmental difficulties involved in having a heat engine that connected surface water and deep ocean water. Likewise, if the difference in temperature between tropical water and arctic or Antarctic water were to be used, the same type of major difficulties would be involved because of the large distances involved.

12. (*a*) If a gas expands adiabatically, there is no heat transfer, and therefore $\Delta S = 0$ by Eq. 15-8, $\Delta S = Q/T$.
    (*b*) If a gas expands isothermally, there is no change in its internal energy, and the gas does work on its surroundings. Thus by the first law of thermodynamics, there must be heat flow into the gas, and so $\Delta S > 0$ – the entropy of the gas increases.

13. The adiabatic expansion results in no change in entropy, since there is no heat transfer. The isothermal expansion requires heat flow into the gas to compensate for the work that the gas does, and so the entropy of the gas $(\Delta S = Q/T)$ increases more for the isothermal expansion.

14. (*a*) The erosion of soil due to water flow over the ground.
    (*b*) The oxidation of various metals (copper, zinc, iron, etc.) when left exposed to the air.
    (*c*) The conversion of mechanical energy to heat energy by friction; i.e., a sliding object decreasing in speed and eventually stopping, and the surfaces of contact getting warmer.
    (*d*) A pile of compost decomposing.
    The reverse of these processes is not observed.

15. 1 kg of liquid iron has more entropy, because the atoms in liquid iron are less "ordered" than those in solid iron. Also, heat had to be added to solid iron in order to melt it, and $\Delta S = Q/T$.

$\boxed{16.}$ (*a*) If the lid is removed from a bottle of chlorine gas, the gas molecules will diffuse out of the mouth of the bottle, and eventually spread out uniformly in whatever volume to which they are confined.

(*b*) The reverse process, that of individual chlorine gas molecules in a closed volume spontaneously entering a small volume, never happens. The probability of the gas molecules all entering the bottle is infinitesimal compared to the probability of the gas molecules being uniformly spread throughout the room. The reverse process would require a spontaneous decrease in entropy.

(*c*) Some other examples of irreversibility: the shuffling of an ordered deck of cards; the diffusion of dye in a liquid; the toppling of buildings during an earthquake.

17. Any air conditioner-type heat engine will remove heat from the room (Q_L – the low temperature input). Work (W) is input to the device to enable it to remove heat from the low temperature region. By the 2nd law of thermodynamics (conservation of energy), there must be a high-temperature exhaust heat Q_H which is larger than Q_L . Perhaps the inventor has some clever method of having that exhaust heat move into a well-insulated heat "sink", like a container of water. But eventually the addition of that heat into the device will cause the device to heat up warmer than the room, and then heat will be transferred to the room. One very simple device that could do what is described in the problem would be a fan blowing over a large block of ice. Heat from the room will enter the ice; cool air from near the surface of the ice can be blown out of the box by a fan. But after the ice melts, the only end result is that the fan motor would heat the air.

18. (*a*) An empty perfume bottle is placed in a room containing perfume molecules, and all of the perfume molecules move into the bottle from various directions at the same time.

(*b*) Water on the sidewalk coalesces into droplets, are propelled upward, and rise into the air.

(*c*) Popcorn is placed in the refrigerator, and it "unpops", changing backed into uncooked kernels.

(*d*) A house got warmer in the winter while the outdoors got colder, due to heat moving from the outdoors to inside the house.

$\boxed{19.}$ While the state of the papers has gone from disorder to order, they did not do so spontaneously. An outside source (you) caused the increase in order. You had to provide energy to do this (through your metabolic processes), and in doing so, your entropy increased more than the entropy of the papers decreased. The overall effect is that the entropy of the universe increased, satisfying the second law of thermodynamics.

20. The first statement, "You can't get something for nothing," is a whimsical way of saying that energy is conserved. For instance, one way to write the 1st law is $W = Q - \Delta U$. This says that work done by a system must have a source – either heat is input to the system or the internal energy of the system is lowered. It "costs" energy – either heat energy or internal energy – to get work done. Another way to say this is that no heat engine can be built which puts out more energy in the form of work than it extracts in the form of heat or internal energy.

The second statement, "You can't even break even," reflects the fact that a consequence of the 2nd law is that there is no heat engine that is 100% efficient. Even though the 1st law is satisfied by an engine that takes in 100 J of heat and outputs 100 J of work, the 2nd law says that is impossible. If 100 J of heat were taken in, less than 100 J of work can be output from the heat engine, even if it is an ideal heat engine. Some energy will be "lost" as exhaust energy.

21. In an action movie, seeing a building or car go from an exploded state to an un-exploded state. In a movie with vehicle crashes, seeing two collided vehicles separate from each other, becoming un-wrecked as they separate. Watching someone “un-write” something on a piece of paper – moving a pen over paper, taking away written marks as the pen moves.

22. The synthesis of complex molecules from simple molecules does involve a decrease in entropy of the constituent molecules, since they have become more “structured” or “ordered”. However, the molecules are not a closed system. This process does not occur spontaneously or in isolation. The living organism in which the synthesis process occurs is part of the environment that must be considered for the overall change in entropy. The living organism will have an increase in entropy that is larger than the decrease in entropy of the molecules, and so overall, the second law is still satisfied, and the entropy of the entire system will increase.

Solutions to Problems

1. Use the first law of thermodynamics, Eq. 15-1, and the definition of internal energy, Eq. 14-1. Since the work is done by the gas, it is positive.

   (*a*) Since the temperature does not change, $\boxed{\Delta U = 0}$

   (*b*) $\Delta U = Q - W \;\; \rightarrow \;\; Q = \Delta U + W = 0 + 3.40\times10^{3}\,\text{J} = \boxed{3.40\times10^{3}\,\text{J}}$

2. (*a*) The work done by a gas at constant pressure is found from Eq. 15-3.

$$W = P\Delta V = \left(1\text{ atm}\right)\left(\frac{1.01\times10^{5}\,\text{Pa}}{1\text{ atm}}\right)\left(18.2\text{ m}^{3} - 12.0\text{ m}^{3}\right) = 6.262\times10^{5}\,\text{J} \approx \boxed{6.3\times10^{5}\,\text{J}}$$

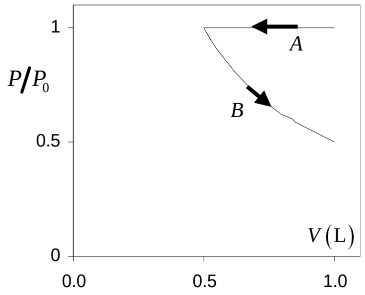
   (*b*) The change in internal energy is calculated from the first law of thermodynamics

$$\Delta U = Q - W = \left(1400\text{ kcal}\right)\left(\frac{4186\text{ J}}{1\text{ kcal}}\right) - 6.262\times10^{5}\,\text{J} = \boxed{5.2\times10^{6}\,\text{J}}$$

3. For the drawing of the graph, the pressure is given relative to the starting pressure, which is taken to be P_0.

   Segment *A* is the cooling at constant pressure.

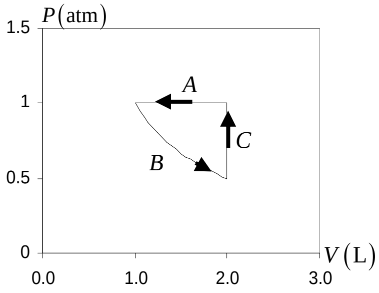
   Segment *B* is the isothermal expansion.

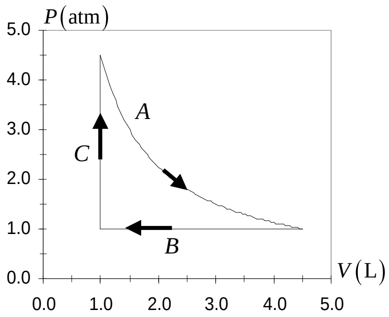


4. Segment *A* is the compression at constant pressure.

   Segment *B* is the isothermal expansion.

   Segment *C* is the pressure increase at constant volume.





5. Segment *A* is the isothermal expansion. Since the temperature and the amount of gas are constant, the quantity $PV = nRT$ is constant. Since the pressure is reduced by a factor of 4.5, the volume will increase by a factor of 4.5, to a final volume of 4.5 L.

Segment *B* is the compression at constant pressure.

Segment *C* is the pressure increase at constant volume.

6. (*a*) Since the container has rigid walls, there is no change in volume.

$$W = P\Delta V = \boxed{0\text{ J}}$$

(*b*) Use the first law of thermodynamics to find the change in internal energy.

$$\Delta U = Q - W = (-265\text{ kJ}) - 0 = \boxed{-265\text{ kJ}}$$

7. (*a*) Since the process is adiabatic, $Q = \boxed{0\text{ J}}$

(*b*) Use the first law of thermodynamics to find the change in internal energy.

$$\Delta U = Q - W = 0 - (-1850\text{ J}) = \boxed{1850\text{ J}}$$

(*c*) Since the internal energy is proportional to the temperature, a rise in internal energy means a $\boxed{\text{rise}}$ in temperature.

8. (*a*) Work is only done in the expansion at constant pressure, since there must be a volume change to have work done.

$$W = P\Delta V = (3.0\text{ atm})\left(\frac{1.01\times10^{5}\,\text{Pa}}{1\text{ atm}}\right)\left(660\times10^{-3}\,\text{L} - 400\times10^{-3}\,\text{L}\right)\frac{1\times10^{-3}\,\text{m}^3}{1\text{ L}} = \boxed{79\text{ J}}$$

(*b*) Use the first law of thermodynamics to find the heat flow. Notice that the temperature change over the entire process is 0, so there is no change in internal energy.

$$\Delta U = Q - W = 0 \quad\rightarrow\quad Q = W = \boxed{79\text{ J}}$$

9. Since the expansion is adiabatic, there is no heat flow into or out of the gas. Use the first law of thermodynamics to calculate the temperature change.

$$\Delta U = Q - W \quad\rightarrow\quad \tfrac{3}{2}nR\Delta T = 0 - W \quad\rightarrow$$

$$\Delta T = -\tfrac{2}{3}\frac{W}{nR} = -\frac{2(7500\text{ J})}{3(1.5\text{ mol})(8.315\,\text{J/mol}\bullet\text{K})} = -401\text{K} = \boxed{-4.0\times10^{2}\,\text{K}}$$

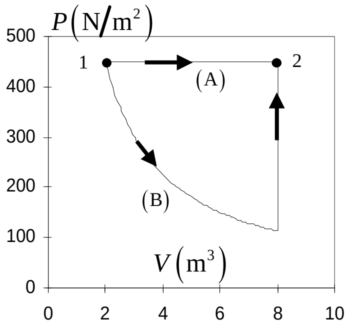
10. (*a*) No work is done during the first step, since the volume is constant. The work in the second step is given by $W = P\Delta V$.

$$W = P\Delta V = (1.4\text{ atm})\left(\frac{1.01\times10^{5}\,\text{Pa}}{1\text{ atm}}\right)(9.3\,\text{L} - 6.8\,\text{L})\frac{1\times10^{-3}\,\text{m}^3}{1\text{ L}} = \boxed{3.5\times10^{2}\,\text{J}}$$

(*b*) Since there is no overall change in temperature, $\Delta U = \boxed{0\text{ J}}$

(*c*) The heat flow can be found from the first law of thermodynamics.

$$\Delta U = Q - W \quad\rightarrow\quad Q = \Delta U + W = 0 + 3.5\times10^{2}\,\text{J} = \boxed{3.5\times10^{2}\,\text{J (into the gas)}}$$



11. (*a*,*c*) See diagram.

(*b*) The work done is found from Eq. 15-3.

$$W = P\Delta V = \left(455\,\mathrm{N/m^2}\right)\left(8.00\,\mathrm{m^3} - 2.00\,\mathrm{m^3}\right) = \boxed{2.73\times10^3\,\mathrm{J}}$$

The change in internal energy depends on the temperature change, which can be related to the ideal gas law, $PV = nRT$.

$$\Delta U = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}\left(nRT_2 - nRT_1\right) = \tfrac{3}{2}\left[\left(PV\right)_2 - \left(PV\right)_1\right]$$

$$= \tfrac{3}{2}P\Delta V = \tfrac{3}{2}W = \tfrac{3}{2}\left(2.73\times10^3\,\mathrm{J}\right) = \boxed{4.10\times10^3\,\mathrm{J}}$$

(*d*) The change in internal energy only depends on the initial and final temperatures. Since those temperatures are the same for process (B) as they are for process (A), the internal energy change is the same for process (B) as for process (A), $\boxed{4.10\times10^3\,\mathrm{J}}$.

12. For the path ac, use the first law of thermodynamics to find the change in internal energy.

$$\Delta U_{\mathrm{ac}} = Q_{\mathrm{ac}} - W_{\mathrm{ac}} = -63\ \mathrm{J} - \left(-35\ \mathrm{J}\right) = -28\ \mathrm{J}$$

Since internal energy only depends on the initial and final temperatures, this ΔU applies to any path that starts at a and ends at c. And for any path that starts at c and ends at a, $\Delta U_{\mathrm{ca}} = -\Delta U_{\mathrm{ac}} = 28\ \mathrm{J}$

(*a*) Use the first law of thermodynamics to find Q_{abc}.

$$\Delta U_{\mathrm{abc}} = Q_{\mathrm{abc}} - W_{\mathrm{abc}} \;\rightarrow\; Q_{\mathrm{abc}} = \Delta U_{\mathrm{abc}} + W_{\mathrm{abc}} = -28\,\mathrm{J} + \left(-48\ \mathrm{J}\right) = \boxed{-76\ \mathrm{J}}$$

(*b*) Since the work along path bc is 0, $W_{\mathrm{abc}} = W_{\mathrm{ab}} = P_{\mathrm{b}}\Delta V_{\mathrm{ab}} = P_{\mathrm{b}}\left(V_{\mathrm{b}} - V_{\mathrm{a}}\right)$. Also note that the work along path da is 0.

$$W_{\mathrm{cda}} = W_{\mathrm{cd}} = P_{\mathrm{c}}\Delta V_{\mathrm{cd}} = P_{\mathrm{c}}\left(V_{\mathrm{d}} - V_{\mathrm{c}}\right) = \tfrac{1}{2}P_{\mathrm{b}}\left(V_{\mathrm{a}} - V_{\mathrm{b}}\right) = -\tfrac{1}{2}W_{\mathrm{abc}} = -\tfrac{1}{2}\left(-48\ \mathrm{J}\right) = \boxed{24\ \mathrm{J}}$$

(*c*) Use the first law of thermodynamics to find Q_{abc}.

$$\Delta U_{\mathrm{cda}} = Q_{\mathrm{cda}} - W_{\mathrm{cda}} \;\rightarrow\; Q_{\mathrm{cda}} = \Delta U_{\mathrm{cda}} + W_{\mathrm{cda}} = 28\,\mathrm{J} + 24\ \mathrm{J} = \boxed{52\ \mathrm{J}}$$

(*d*) As found above, $U_{\mathrm{c}} - U_{\mathrm{a}} = \Delta U_{\mathrm{ca}} = -\Delta U_{\mathrm{ac}} = \boxed{28\ \mathrm{J}}$

(*e*) Since $U_{\mathrm{d}} - U_{\mathrm{c}} = 5\ \mathrm{J} \;\rightarrow\; U_{\mathrm{d}} = U_{\mathrm{c}} + 5\ \mathrm{J} \;\rightarrow\; \Delta U_{\mathrm{da}} = U_{\mathrm{a}} - U_{\mathrm{d}} = U_{\mathrm{a}} - U_{\mathrm{c}} - 5\ \mathrm{J} = \Delta U_{\mathrm{ca}} - 5\ \mathrm{J} = 23\ \mathrm{J}$.

Use the first law of thermodynamics to find Q_{da}.

$$\Delta U_{\mathrm{da}} = Q_{\mathrm{da}} - W_{\mathrm{da}} \;\rightarrow\; Q_{\mathrm{da}} = \Delta U_{\mathrm{da}} + W_{\mathrm{da}} = 23\ \mathrm{J} + 0 = \boxed{23\ \mathrm{J}}$$

13. We are given that $Q_{\mathrm{ac}} = -80\ \mathrm{J}$ and $W_{\mathrm{ac}} = -55\ \mathrm{J}$.

(*a*) Use the first law of thermodynamics to find $U_{\mathrm{a}} - U_{\mathrm{c}} = \Delta U_{\mathrm{ca}}$

$$\Delta U_{\mathrm{ca}} = -\Delta U_{\mathrm{ac}} = -\left(Q_{\mathrm{ac}} - W_{\mathrm{ac}}\right) = -\left(-80\,\mathrm{J} - -55\,\mathrm{J}\right) = \boxed{25\ \mathrm{J}}$$

(*b*) Use the first law of thermodynamics to find Q_{cda}.

$$\Delta U_{\mathrm{cda}} = Q_{\mathrm{cda}} - W_{\mathrm{cda}} \;\rightarrow\; Q_{\mathrm{cda}} = \Delta U_{\mathrm{cda}} + W_{\mathrm{cda}} = \Delta U_{\mathrm{ca}} + W_{\mathrm{cda}} = 25\ \mathrm{J} + 38\ \mathrm{J} = \boxed{63\ \mathrm{J}}$$

(*c*) Since the work along path bc is 0, $W_{\mathrm{abc}} = W_{\mathrm{ab}} = P_{\mathrm{a}}\Delta V_{\mathrm{ab}} = P_{\mathrm{a}}\left(V_{\mathrm{b}} - V_{\mathrm{a}}\right)$.

$$W_{\mathrm{abc}} = W_{\mathrm{ab}} = P_{\mathrm{a}}\Delta V_{\mathrm{ab}} = P_{\mathrm{a}}\left(V_{\mathrm{b}} - V_{\mathrm{a}}\right) = 2.5P_{\mathrm{d}}\left(V_{\mathrm{c}} - V_{\mathrm{d}}\right) = -2.5W_{\mathrm{cda}} = -2.5\left(38\ \mathrm{J}\right) = \boxed{-95\ \mathrm{J}}$$

(*d*) Use the first law of thermodynamics to find Q_{abc}

$$\Delta U_{\mathrm{abc}} = Q_{\mathrm{abc}} - W_{\mathrm{abc}} \;\rightarrow\; Q_{\mathrm{abc}} = \Delta U_{\mathrm{abc}} + W_{\mathrm{abc}} = -25\ \mathrm{J} - 95\,\mathrm{J} = \boxed{-120\ \mathrm{J}}$$

(*e*) Since $U_{\text{a}} - U_{\text{b}} = 10\text{ J} \rightarrow U_{\text{b}} = U_{\text{a}} - 10\text{ J}$, we have the following.

$$\Delta U_{\text{bc}} = U_{\text{c}} - U_{\text{b}} = U_{\text{c}} - \left(U_{\text{a}} - 10\text{ J}\right) = \Delta U_{\text{ac}} + 10\text{ J} = -25\text{ J} + 10\text{ J} = -15\text{ J}.$$

Use the first law of thermodynamics to find Q_{bc}.

$$\Delta U_{\text{bc}} = Q_{\text{bc}} - W_{\text{bc}} \rightarrow Q_{\text{bc}} = \Delta U_{\text{bc}} + W_{\text{bc}} = -15\text{ J} + 0 = \boxed{-15\text{ J}}$$

14. In Example 15-8, the total energy transformed was $1.15\times10^{7}\text{ J}$. We will subtract away the energy for 1 hour of desk work and add in the energy for 1 hour of running.

$$\text{Energy} = 1.15\times10^{7}\text{ J} + \left[-115\text{ J/s} + 1150\text{ J/s}\right]\left(3600\text{ s/h}\right) = \boxed{1.52\times10^{7}\text{ J}} \approx 3600\text{ Cal}$$

15. Follow the pattern set in Example 15-8. Find the average rate by dividing the total energy for the day by 24 hours.

$$\text{Avg. Energy} = \left[\begin{array}{l}\left(8.0\text{ h}\right)\left(70\text{ J/s}\right) + \left(8.0\text{ h}\right)\left(115\text{ J/s}\right) + \left(4.0\text{ h}\right)\left(230\text{ J/s}\right) \\ + \left(2.0\text{ h}\right)\left(115\text{ J/s}\right) + \left(1.5\text{ h}\right)\left(460\text{ J/s}\right) + \left(0.5\text{ h}\right)\left(1150\text{ J/s}\right)\end{array}\right] \Big/ 24\text{ h} = \boxed{1.62\times10^{2}\text{ W}}$$

16. From Table 15-2, the change in metabolic rate if one hour of sleeping is exchanged for light activity is an addition of 230 watts – 70 watts = 160 watts. Note that this increased rate is only applicable for one hour per day.

$$\left(160\frac{\text{J}}{\text{s}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ h}}{\text{day}}\right)\left(\frac{365\text{ day}}{1\text{ y}}\right)\left(\frac{1\text{ kg fat}}{4\times10^{7}\text{ J}}\right) = 5.256\text{ kg} \approx \boxed{5.3\text{ kg}}\left(\frac{2.20\text{ lb}}{1\text{ kg}}\right) = \boxed{12\text{ lbs}}$$

17. The efficiency of a heat engine is given by Eq. 15-4.

$$e = \frac{W}{Q_{\text{H}}} = \frac{W}{W + Q_{\text{L}}} = \frac{3200\text{ J}}{3200\text{ J} + 8200\text{ J}} = 0.28 = \boxed{28\%}$$

18. The efficiency of a heat engine is given by Eq. 15-4.

$$e = \frac{W}{Q_{\text{H}}} = \frac{9200\text{ J}}{\left(22.0\text{ kcal}\right)\left(4186\text{ J/kcal}\right)} = 0.10 = \boxed{10\%}$$

19. The maximum (or Carnot) efficiency is given by Eq. 15-5, with temperatures in Kelvins.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{\left(380 + 273\right)\text{K}}{\left(580 + 273\right)\text{K}} = 0.23 = \boxed{23\%}$$

20. The Carnot efficiency is given by Eq. 15-5, with temperatures in Kelvins.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} \rightarrow T_{\text{H}} = \frac{T_{\text{L}}}{1 - e} = \frac{\left(230 + 273\right)\text{K}}{1 - 0.28} = 699\text{K} = 426^{\circ}\text{C} \approx \boxed{430^{\circ}\text{C}}$$

21. The maximum (or Carnot) efficiency is given by Eq. 15-5, with temperatures in Kelvins.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{\left(350 + 273\right)\text{K}}{\left(625 + 273\right)\text{K}} = 0.306$$

Thus the total power generated can be found as follows.

$$\text{Actual Power} = \left(\text{Total Power}\right)\left(\text{max. eff.}\right)\left(\text{operating eff.}\right) \rightarrow$$

$$\text{Total Power} = \frac{\text{Actual Power}}{(\text{max. eff.})(\text{operating eff.})} = \frac{1.3\,\text{GW}}{(0.306)(0.75)} = 5.664\ \text{GW}$$

$$\text{Exhaust Power} = \text{Total Power} - \text{Actual Power} = 5.664\ \text{GW} - 1.3\ \text{GW} = 4.364\ \text{GW}$$

$$= \left(4.364\times10^{9}\ \text{J/s}\right)\left(3600\,\text{s/h}\right) = \boxed{1.6\times10^{13}\ \text{J/h}}$$

22. Calculate the Carnot efficiency for the given temperatures.

$$e_{\text{ideal}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{77\ \text{K}}{293\ \text{K}} = 0.7372 \approx \boxed{74\%}$$

23. This is a perfect Carnot engine, and so its efficiency is given by Eqs. 15-4 and 15-5. Equate these two expressions for the efficiency.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = \frac{W}{Q_{\text{H}}} \quad \rightarrow$$

$$T_{\text{L}} = T_{\text{H}}\left(1 - \frac{W}{Q_{\text{H}}}\right) = T_{\text{H}}\left(1 - \frac{W/t}{Q_{\text{H}}/t}\right) = \left[(570+273)\,\text{K}\right]\left(1 - \frac{4.4\times10^{5}\ \text{J/s}}{(680\,\text{kcal/s})(4186\,\text{J/kcal})}\right)$$

$$= 713\ \text{K} = \boxed{440^{\circ}\text{C}}$$

24. This is a perfect Carnot engine, and so its efficiency is given by Eqs. 15-4 and 15-5. Use these two expressions to solve for the rate of heat output.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{(45+273)\,\text{K}}{(210+273)\,\text{K}} = 0.3416 \qquad e = \frac{W}{Q_{\text{H}}} = \frac{W}{W+Q_{\text{L}}} \quad \rightarrow \quad Q_{\text{L}} = W\left(1/e - 1\right)$$

$$Q_{\text{L}}/t = W/t\left(1/e - 1\right) = (950\ \text{W})\left(1/0.3416 - 1\right) = \boxed{1.83\times10^{3}\,\text{W}}$$

25. The efficiency of a heat engine is given by Eq. 15-4.

$$e = \frac{W}{Q_{\text{H}}} = \frac{W}{W+Q_{\text{L}}} \quad \rightarrow \quad Q_{\text{L}} = W\left(1/e - 1\right) \quad \rightarrow$$

$$Q_{\text{L}}/t = W/t\left(1/e - 1\right) = (550\ \text{MW})\left(1/0.38 - 1\right) = \boxed{9.0\times10^{2}\,\text{MW}}$$

$\boxed{26.}$ Find the exhaust temperature from the original Carnot efficiency, and then recalculate the intake temperature for the new Carnot efficiency, using the same exhaust temperature.

$$e_1 = 1 - \frac{T_{\text{L}}}{T_{\text{H1}}} \quad \rightarrow \quad T_{\text{L}} = T_{\text{H1}}(1-e) = (550+273)\,\text{K}\,(1-0.28) = 592.6\ \text{K}$$

$$e_2 = 1 - \frac{T_{\text{L}}}{T_{\text{H2}}} \quad \rightarrow \quad T_{\text{H2}} = \frac{T_{\text{L}}}{1-e_2} = \frac{592.6\ \text{K}}{1-0.35} = 912\ \text{K} = 639^{\circ}\text{C} \approx \boxed{640^{\circ}\text{C}}$$

27. Find the intake temperature from the original Carnot efficiency, and then recalculate the exhaust temperature for the new Carnot efficiency, using the same intake temperature.

$$e_1 = 1 - \frac{T_{\text{L1}}}{T_{\text{H}}} \quad \rightarrow \quad T_{\text{H}} = \frac{T_{\text{L1}}}{1-e_1} = \frac{(350+273)\,\text{K}}{1-0.39} = 1021\,\text{K}$$

$$e_2 = 1 - \frac{T_{L2}}{T_H} \rightarrow T_{L2} = T_H(1-e_2) = (1021\text{ K})(1-0.49) = 521\text{ K} = 248^\circ\text{C} \approx \boxed{250^\circ\text{C}}$$

28. For each engine, the efficiency is given by $e = 0.60 e_{\text{Carnot}}$. Thus

$$e_1 = 0.6 e_{C-1} = 0.60\left(1 - \frac{T_{L1}}{T_{H1}}\right) = 0.60\left[1 - \frac{(440+273)\text{K}}{(670+273)\text{K}}\right] = 0.146$$

$$e_2 = 0.6 e_{C-2} = 0.60\left(1 - \frac{T_{L2}}{T_{H2}}\right) = 0.60\left[1 - \frac{(290+273)\text{K}}{(430+273)\text{K}}\right] = 0.119$$

For the first engine, the input heat is from the coal.

$W_1 = e_1 Q_{H1} = e_1 Q_{\text{coal}}$ and $Q_{L1} = Q_{H1} - W_1 = (1-e_1) Q_{\text{coal}}$.

For the second energy, the input heat is the output heat from the first engine.

$$W_2 = e_2 Q_{H2} = e_2 Q_{L1} = e_2 (1-e_1) Q_{\text{coal}}$$

Add the two work expressions together, and solve for Q_{coal}.

$$W_1 + W_2 = e_1 Q_{\text{coal}} + e_2(1-e_1) Q_{\text{coal}} = (e_1 + e_2 - e_1 e_2) Q_{\text{coal}}$$

$$Q_{\text{coal}} = \frac{W_1 + W_2}{e_1 + e_2 - e_1 e_2} \rightarrow Q_{\text{coal}}/t = \frac{(W_1+W_2)/t}{e_1 + e_2 - e_1 e_2}$$

Calculate the rate of coal use from the required rate of input energy, Q_{coal}/t.

$$Q_{\text{coal}}/t = \frac{1100\times10^6\text{ W}}{0.146 + 0.119 - (0.146)(0.119)} = 4.442\times10^9\text{ J/s}$$

$$(4.442\times10^9\text{ J/s})\left(\frac{1\text{ kg}}{2.8\times10^7\text{ J}}\right) = 158.6\text{ kg/s} \approx \boxed{1.6\times10^2\text{ kg/s}}$$

29. The coefficient of performance for a refrigerator is given by Eq. 15-6c, with temperatures in Kelvins.

$$\text{COP} = \frac{T_L}{T_H - T_L} = \frac{(-15+273)\text{K}}{(30+273)\text{K} - (-15+273)\text{K}} = \boxed{5.7}$$

30. The coefficient of performance for a refrigerator is given by Eq. 15-6c, with temperatures in Kelvins.

$$\text{COP} = \frac{T_L}{T_H - T_L} \rightarrow T_L = T_H\left(\frac{\text{COP}}{1+\text{COP}}\right) = [(24+273)\text{K}]\left(\frac{7.0}{8.0}\right) = 259.9\text{K} \approx \boxed{-13^\circ\text{C}}$$

31. The coefficient of performance for a refrigerator is given by Eq. 15-6c, with temperatures in Kelvins. Use that expression to find the temperature inside the refrigerator.

$$\text{COP} = \frac{T_L}{T_H - T_L} \rightarrow T_L = T_H \frac{\text{COP}}{1+\text{COP}} = [(29+273)\text{K}]\frac{5.0}{6.0} = 252\text{ K} = \boxed{-21^\circ\text{C}}$$

$\boxed{32.}$ The efficiency of a perfect Carnot engine is given by Eqs. 15-4 and 15-5. Equate these two expressions to solve for the work required.

$$e = 1 - \frac{T_L}{T_H} = \ ; \ e = \frac{W}{Q_H} \rightarrow 1 - \frac{T_L}{T_H} = \frac{W}{Q_H} \rightarrow W = Q_H\left(1 - \frac{T_L}{T_H}\right)$$

(*a*) $W = Q_{\mathrm{H}}\left(1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}\right) = 2800\ \mathrm{J}\left(1 - \frac{0+273}{22+273}\right) = \boxed{210\ \mathrm{J}}$

(*b*) $W = Q_{\mathrm{H}}\left(1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}\right) = 2800\ \mathrm{J}\left(1 - \frac{-15+273}{22+273}\right) = \boxed{350\ \mathrm{J}}$

33. The heat to be removed, Q_{L}, is the latent heat of fusion for the ice, so $Q_{\mathrm{L}} = mL_{\mathrm{fusion}} = V_{\mathrm{H2O}}\rho_{\mathrm{H2O}}L_{\mathrm{fusion}}$. The work done in one hour is 1.0 kilowatt-hour. The COP for a refrigerator is $\mathrm{COP} = \frac{Q_{\mathrm{L}}}{W}$, and so $Q_{\mathrm{L}} = (\mathrm{COP})W$. Equate the two expressions for Q_{L} and solve for the volume.

$$(\mathrm{COP})W = V_{\mathrm{H_2O}}\rho_{\mathrm{H_2O}}L_{\mathrm{fusion}} \quad \rightarrow$$

$$m = \frac{(\mathrm{COP})W}{\rho_{\mathrm{H_2O}}L_{\mathrm{fusion}}} = \frac{(7.0)\left(1.0\times10^{3}\,\mathrm{watt-hour}\right)\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{hour}}\right)}{\left(1.00\times10^{3}\ \mathrm{kg/m^3}\right)\left(3.33\times10^{5}\ \mathrm{J/kg}\right)}\left(\frac{1\ \mathrm{L}}{1.0\times10^{-3}\mathrm{m}^3}\right) = \boxed{76\ \mathrm{L}}$$

34. The COP for a heat pump is $\mathrm{COP} = \frac{Q_{\mathrm{H}}}{W}$ and the efficiency is $e = \frac{W}{Q_{\mathrm{H}}}$. Thus they are reciprocals of each other. So if the efficiency is 0.35, the COP is $\frac{1}{0.35} = \boxed{2.9}$.

35. Heat energy is taken away from the water, so the change in entropy will be negative. The heat transfer is the mass of the steam times the latent heat of vaporization.

$$\Delta S = \frac{Q}{T} = -\frac{mL_{\mathrm{vap}}}{T} = -\frac{(0.25\ \mathrm{kg})\left(22.6\times10^{5}\ \mathrm{J/kg}\right)}{(273+100)\,\mathrm{K}} = \boxed{-1.5\times10^{3}\ \mathrm{J/K}}$$

36. The heat added to the water is found from $\Delta Q = mc\Delta T$. Use the average temperature of 50°C in the approximate entropy calculation.

$$\Delta S = \frac{Q}{T} = \frac{mc\Delta T}{T} = \frac{(1.00\ \mathrm{kg})\left(4186\,\mathrm{J/kg{\bullet}C^{o}}\right)\left(100\mathrm{C^{o}}\right)}{(273+50)\,\mathrm{K}} = \boxed{1.30\times10^{3}\ \mathrm{J/K}}$$

37. Heat energy is taken away from the water, so the change in entropy will be negative. The heat taken away from the water is found from $\Delta Q = mL_{\mathrm{fusion}}$. Note that 1.00 m^3 of water has a mass of $1.00\times10^{3}\,\mathrm{kg}$.

$$\Delta S = \frac{Q}{T} = -\frac{mL_{\mathrm{fusion}}}{T} = -\frac{\left(1.00\times10^{3}\mathrm{kg}\right)\left(3.33\times10^{5}\ \mathrm{J/kg}\right)}{273\mathrm{K}} = \boxed{-1.22\times10^{6}\ \mathrm{J/K}}$$

38. There are three terms of entropy to consider. First, there is a loss of entropy from the water for the freezing process, ΔS_1. Second, there is a loss of entropy from that newly-formed ice as it cools to –10°C, ΔS_2. That process has an “average” temperature of –5°C. Finally, there is a gain of entropy by the “great deal of ice”, ΔS_3, as the heat lost from the original mass of water in steps 1 and 2 goes

into that great deal of ice. Since it is a large quantity of ice, we assume that its temperature does not change during the processes.

$$\Delta S_1 = \frac{Q_1}{T_1} = -\frac{mL_{\text{fusion}}}{T_1} = -\frac{\left(1.00\times10^3\,\text{kg}\right)\left(3.33\times10^5\text{ J/kg}\right)}{273\,\text{K}} = -1.2198\times10^6\text{ J}/\text{K}$$

$$\Delta S_2 = \frac{Q_2}{T_2} = -\frac{mc_{ice}\Delta T_2}{T_2} = -\frac{\left(1.00\times10^3\,\text{kg}\right)\left(2100\,\text{J/kg}\bullet\text{C}^\circ\right)\left(10\text{C}^\circ\right)}{\left(-5+273\right)\text{K}} = -7.8358\times10^4\text{ J}/\text{K}$$

$$\Delta S_3 = \frac{Q_3}{T_3} = \frac{-Q_1-Q_2}{T_3} = \frac{mL_{\text{fusion}}+mc_{ice}\Delta T_2}{T_3}$$

$$= \frac{\left(1.00\times10^3\,\text{kg}\right)\left[\left(3.33\times10^5\text{ J/kg}\right)+\left(2100\,\text{J/kg}\bullet\text{C}^\circ\right)\left(10\text{C}^\circ\right)\right]}{\left(-10+273\right)\text{K}} = 1.3460\times10^6\text{ J}/\text{K}$$

$$\Delta S = \Delta S_1+\Delta S_2+\Delta S_3 = -1.2198\times10^6\text{ J}/\text{K} - 7.8358\times10^4\text{ J}/\text{K} + 1.3460\times10^6\text{ J}/\text{K}$$

$$= 4.784\times10^4\text{ J}/\text{K} \approx \boxed{5\times10^4\text{ J}/\text{K}}$$

39. Energy has been made “unavailable” in the frictional stopping of the sliding box. We take that “lost” kinetic energy as the heat term of the entropy calculation.

$$\Delta S = Q/T = \tfrac{1}{2}mv_i^2/T = \tfrac{1}{2}\left(10.0\text{ kg}\right)\left(3.0\,\text{m/s}\right)^2/293\text{ K} = \boxed{0.15\,\text{J}/\text{K}}$$

Since this is a decrease in “availability”, the entropy of the universe has increased.

40. Take the energy transfer to use as the initial kinetic energy of the rock, because this energy becomes “unusable” after the collision – it is transferred to the environment. We assume that the rock and the environment are both at temperature T_0.

$$\Delta S = Q/T \quad \rightarrow \quad \boxed{\Delta S = KE/T_0}$$

$\boxed{41.}$ The same amount of heat that leaves the high temperature heat source enters the low temperature body of water.

$$\Delta S = \Delta S_1+\Delta S_2 = -\frac{Q}{T_{\text{high}}}+\frac{Q}{T_{\text{low}}} = Q\left(\frac{1}{T_{\text{low}}}-\frac{1}{T_{\text{high}}}\right) \quad \rightarrow$$

$$\frac{\Delta S}{t} = \frac{Q}{t}\left(\frac{1}{T_{\text{low}}}-\frac{1}{T_{\text{high}}}\right) = \left(7.50\,\text{cal/s}\right)\left(\frac{4.186\,\text{J}}{1\,\text{cal}}\right)\left(\frac{1}{\left(27+273\right)\text{K}}-\frac{1}{\left(240+273\right)\text{ K}}\right)$$

$$= \boxed{4.35\times10^{-2}\,\frac{\text{J/K}}{\text{s}}}$$

42. The same amount of heat that leaves the high temperature water will enter the low temperature water. Since the two masses of water are the same, the equilibrium temperature will be the midpoint between the two initial temperatures, 45°C. The cool water average temperature is $\left(30^\circ\text{C}+45^\circ\text{C}\right)/2 = 37.5^\circ\text{C}$, and the warm water average temperature is $\left(60^\circ\text{C}+45^\circ\text{C}\right)/2 = 52.5^\circ\text{C}$.

$$\Delta S = \Delta S_1+\Delta S_2 = -\frac{Q}{T_{\text{high}}}+\frac{Q}{T_{\text{low}}} = mc\Delta T\left(\frac{1}{T_{\text{low}}}-\frac{1}{T_{\text{high}}}\right)$$

$$=(1.0\text{ kg})\left(4186\,\text{J}/\text{kg}\bullet\text{C}^{\circ}\right)\left(15\text{C}^{\circ}\right)\left(\frac{1}{(37.5+273)\,\text{K}}-\frac{1}{(52.5+273)\ \text{K}}\right)=\boxed{9.3\,\text{J}/\text{K}}$$

43. The equilibrium temperature is found using calorimetry, from chapter 14. The heat lost by the aluminum is equal to the heat gained by the water.

$$m_{\text{Al}}c_{\text{Al}}\left(T_{i\text{Al}}-T_f\right)=m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_f-T_{i\text{H}_2\text{O}}\right)\quad\rightarrow$$

$$T_f=\frac{m_{\text{Al}}c_{\text{Al}}T_{i\text{Al}}+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}T_{i\text{H}_2\text{O}}}{m_{\text{Al}}c_{\text{Al}}+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}}$$

$$=\frac{(3.8\text{ kg})\left(900\,\text{J}/\text{kg}\bullet\text{C}^{\circ}\right)\left(30.0^{\circ}\text{C}\right)+(1.0\text{ kg})\left(4186\,\text{J}/\text{kg}\bullet\text{C}^{\circ}\right)\left(20^{\circ}\text{C}\right)}{(3.8\text{ kg})\left(900\,\text{J}/\text{kg}\bullet\text{C}^{\circ}\right)+(1.0\text{ kg})\left(4186\,\text{J}/\text{kg}\bullet\text{C}^{\circ}\right)}=24.5^{\circ}\text{C}$$

The amount of heat lost by the aluminum, and gained by the water, is

$$Q=m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_f-T_{i\text{H}_2\text{O}}\right)=(1.0\text{ kg})\left(4186\,\text{J}/\text{kg}\bullet\text{C}^{\circ}\right)\left(24.5^{\circ}\text{C}-20^{\circ}\text{C}\right)=1.9\times10^{4}\text{ J}$$

In calculating the entropy change, we will need to use estimates for the temperatures of the water and the aluminum since their temperatures are not constant. We will use their average temperatures.

$$\underset{\text{avg}}{T_{\text{H}_2\text{O}}}=\left(20^{\circ}\text{C}+24.5^{\circ}\text{C}\right)/2=22.25^{\circ}\text{C}\qquad \underset{\text{avg}}{T_{\text{Al}}}=\left(30^{\circ}\text{C}+24.5^{\circ}\text{C}\right)/2=27.25^{\circ}\text{C}$$

$$\Delta S=\Delta S_{\text{Al}}+\Delta S_{\text{H}_2\text{O}}=-\frac{Q}{\underset{\text{avg}}{T_{\text{Al}}}}+\frac{Q}{\underset{\text{avg}}{T_{\text{H}_2\text{O}}}}=\left(1.9\times10^{4}\text{ J}\right)\left(\frac{1}{(22.25+273)\text{K}}-\frac{1}{(27.25+273)\text{K}}\right)=\boxed{1.1\,\text{J}/\text{K}}$$

44. (*a*) $e_{\text{actual}}=W/Q_{\text{H}}=550\text{ J}/2200\text{ J}=0.250\qquad e_{\text{ideal}}=1-T_{\text{L}}/T_{\text{H}}=1-650\text{ K}/970\text{ K}=0.330$

Thus $e_{\text{actual}}/e_{\text{ideal}}=0.250/0.330=0.758\approx\boxed{76\%\text{ of ideal}}$

(*b*) The heat reservoirs do not change temperature during the operation of the engine. There is an entropy loss from the input reservoir, because it loses heat, and an entropy gain for the output reservoir, because it gains heat. Note that $Q_{\text{L}}=Q_{\text{H}}-W=2200\text{ J}-550\text{ J}=1650\text{ J}$.

$$\Delta S=\Delta S_{\text{input}}+\Delta S_{\text{output}}=-\frac{Q_{\text{H}}}{T_{\text{H}}}+\frac{Q_{\text{L}}}{T_{\text{L}}}=-\frac{2200\text{ J}}{970\text{ K}}+\frac{1650\text{ J}}{650\text{ K}}=\boxed{0.27\,\text{J}/\text{K}}$$

(*c*) For the Carnot engine, the exhaust energy will be $Q_{\text{L}}=Q_{\text{H}}\left(1-e_{\text{Carnot}}\right)=Q_{\text{H}}\,T_{\text{L}}/T_{\text{H}}$.

$$\Delta S=\Delta S_{\text{input}}+\Delta S_{\text{output}}=-\frac{Q_{\text{H}}}{T_{\text{H}}}+\frac{Q_{\text{L}}}{T_{\text{L}}}=-\frac{Q_{\text{H}}}{T_{\text{H}}}+\frac{Q_{\text{H}}\,T_{\text{L}}/T_{\text{H}}}{T_{\text{L}}}=-\frac{Q_{\text{H}}}{T_{\text{H}}}+\frac{Q_{\text{H}}}{T_{\text{H}}}=\boxed{0}$$

A numeric calculation might give a very small number due to not keeping all digits in the calculation.

45. When throwing two dice, there are 36 possible microstates.

(*a*) The possible microstates that give a total of 5 are: (1)(4) , (2)(3) , (3)(2), and (4)(1). Thus the probability of getting a 5 is $4/36=\boxed{1/9}$.

(*b*) The possible microstates that give a total of 11 are: (5)(6) and (6)(5). Thus the probability of getting an 11 is $2/36=\boxed{1/18}$.

46. A macrostate is a set of 5 cards from the deck, as given in the problem. For example, four aces and a king is a macrostate. Two jacks, two queens, and an ace is a macrostate. A microstate is a specific set of cards that meets the criterion of a certain macrostate. For example, the set (ace of spades, ace of clubs, ace of hearts, ace of diamonds, king of spades) is a microstate of the macrostate of 4 aces and a king. The problem then is asking for the relative number of microstates for the 4 given macrostates.
    (*a*) There are only 4 microstates for this macrostate, corresponding to the particular suit to which the king belongs.
    (*b*) Since every card is specified, there is only 1 microstate for this macrostate.
    (c) There are 6 possible jack pairs, (spade/club, spade/heart, spade/diamond, club/heart, club/diamond, and heart/diamond), 6 possible queen pairs, and 4 possible aces, so there are 6 x 6 x 4 = 144 card combinations or 144 microstates for this macrostate.
    (*d*) There are 52 possibilities for the first card, 48 possibilities for the second card, and so on. It is apparent that there are many more microstates for this macrostate than for any of the other listed macrostates.
    Thus in order of increasing probability, we have $\boxed{\text{(b), (a), (c), (d)}}$.

47. From the table below, we see that there are a total of $2^6 = 64$ microstates.

Macrostate	Possible Microstates (H = heads, T = tails)						Number of microstates
6 heads, 0 tails	H H H H H H						1
5 heads, 1 tails	H H H H H T	H H H H T H	H H H T H H	H H T H H H	H T H H H H	T H H H H H	6
4 heads, 2 tails	H H H H T T	H H H T H T	H H T H H T	H T H H H T	T H H H H T		15
	H H H T T H	H H T H T H	H T H H T H	T H H H T H	H H T T H H		
	H T H T H H	T H H T H H	H T T H H H	T H T H H H	T T H H H H		
3 heads, 3 tails	H H H T T T	H H T H T T	H T H H T T	T H H H T T	H H T T H T		20
	H T H T H T	T H H T H T	H T T H H T	T H T H H T	T T H H H T		
	T T T H H H	T T H T H H	T H T T H H	H T T T H H	T T H H T H		
	T H T H T H	H T T H T H	T H H T T H	H T H T T H	H H T T T H		
2 heads, 4 tails	T T T T H H	T T T H T H	T T H T T H	T H T T T H	H T T T T H		15
	T T T H H T	T T H T H T	T H T T H T	H T T T H T	T T H H T T		
	T H T H T T	H T T H T T	T H H T T T	H T H T T T	H H T T T T		
1 heads, 5 tails	T T T T T H	T T T T H T	T T T H T T	T T H T T T	T H T T T T	H T T T T T	6
0 heads, 6 tails	T T T T T T						1

(*a*) The probability of obtaining three heads and three tails is $\boxed{20/64}$.

(*b*) The probability of obtaining six heads is $\boxed{1/64}$.

48. The required area is $\left(22\dfrac{10^3\,\text{W}\bullet\text{h}}{\text{day}}\right)\left(\dfrac{1\text{ day}}{9\text{ h Sun}}\right)\left(\dfrac{1\text{ m}^2}{40\text{ W}}\right)=\boxed{61\text{ m}^2}$. A small house with 1000 ft^2 of floor space, and a roof tilted at 30^o, would have a roof area of $\left(1000\,\text{ft}^2\right)\left(\dfrac{1}{\cos 30^\text{o}}\right)\left(\dfrac{1\text{ m}}{3.28\text{ ft}}\right)^2=110\text{ m}^2$, which is about twice the area needed, and so $\boxed{\text{the cells would fit on the house}}$. But not all parts of the roof would have 9 hours of sunlight, so more than the minimum number of cells would be needed.

49. (*a*) Assume that there are no dissipative forces present, and so the energy required to pump the water to the lake is just the gravitational potential energy of the water.

$$U_{\text{grav}} = mgh = \left(1.00\times10^{5}\ \text{kg/s}\right)\left(10.0\ \text{h}\right)\left(9.80\ \text{m/s}^2\right)\left(135\ \text{m}\right) = 1.323\times10^{9}\ \text{W}\bullet\text{h}$$

$$\approx \boxed{1.32\times10^{6}\ \text{kWh}}$$

(*b*) $$\frac{\left(1.323\times10^{6}\ \text{kW}\bullet\text{h}\right)\left(0.75\right)}{14\ \text{h}} = \boxed{7.1\times10^{4}\ \text{kW}}$$

50. We assume that the electrical energy comes from the 100% effective conversion of the gravitational potential energy of the water.

$$W = mgh \quad \rightarrow$$

$$P = \frac{W}{t} = \frac{m}{t}gh = \rho\frac{V}{t}gh = \left(1.00\times10^{3}\ \text{kg/m}^3\right)\left(35\ \text{m}^3/\text{s}\right)\left(9.8\ \text{m/s}^2\right)\left(45\ \text{m}\right) = \boxed{1.5\times10^{7}\ \text{W}}$$

51. According to the heat figures provided by the inventor, the engine is 50% efficient:

$$e = \frac{W}{Q_{\text{H}}} = \frac{W/t}{Q_{\text{H}}/t} = \frac{1.50\ \text{MW}}{3.00\ \text{MW}} = 0.500$$

The ideal engine efficiency at the operating temperatures is given by Eq. 15-5.

$$e_{\text{ideal}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{215\ \text{K}}{425\ \text{K}} = 0.494$$

Thus his engine is not possible, even if it were ideal. So $\boxed{\text{yes}}$, there is something “fishy” about his claim. His engine is better than ideal.

52. (*a*) The work done at constant pressure is $W = P\Delta V$.

$$W = P\Delta V$$

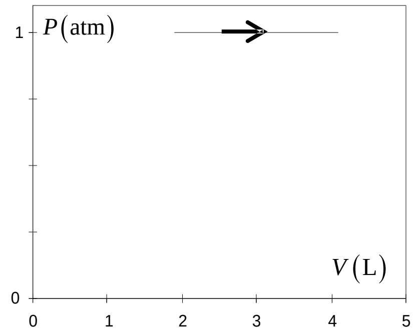
$$= \left(1.00\ \text{atm}\right)\left(1.01\times10^{5}\ \text{Pa/atm}\right)\left(4.1\ \text{m}^3 - 1.9\ \text{m}^3\right)$$

$$= 2.22\times10^{5}\ \text{J} \approx \boxed{2.2\times10^{5}\ \text{J}}$$

(*b*) Use the first law of thermodynamics.

$$\Delta U = Q - W = 5.30\times10^{5}\ \text{J} - 2.22\times10^{5}\ \text{J} = \boxed{3.1\times10^{5}\ \text{J}}$$

(*c*) See the adjacent graph.



53. (*a*) $$\text{Work/s} = \left(\frac{220\ \text{J}}{\text{cycle}\bullet\text{cylinder}}\right)\left(\frac{45\ \text{cycles}}{\text{s}}\right)\left(4\ \text{cylinders}\right) = 3.96\times10^{4}\ \text{J/s} \approx \boxed{4.0\times10^{4}\ \text{J/s}}$$

(*b*) $$e = \frac{W}{Q_{\text{H}}} \rightarrow Q_{\text{H}} = \frac{W}{e} \rightarrow Q_{\text{H}}/t = \frac{W/t}{e} = \frac{3.96\times10^{4}\ \text{J/s}}{0.25} = 1.584\times10^{5}\ \text{J/s} \approx \boxed{1.6\times10^{5}\ \text{J/s}}$$

(*c*) $$\left(1.0\ \text{L}\right)\left(\frac{35\times10^{6}\ \text{J}}{1\ \text{L}}\right)\left(\frac{1\ \text{s}}{1.584\times10^{5}\ \text{J}}\right) = \boxed{220\ \text{s}} = 3.7\ \text{min}$$

54. (*a*) The heat that must be removed from the water $\left(Q_{\text{L}}\right)$ is found in three parts – cooling the liquid water to the freezing point, freezing the liquid water, and then cooling the ice to the final temperatures.

$$Q_L = m\left(c_{\text{liquid}}\Delta T_{\text{liquid}} + L_{\text{fusion}} + c_{\text{ice}}\Delta T_{\text{ice}}\right)$$
$$= \left(0.50\text{ kg}\right)\begin{bmatrix}\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)\left(25\text{C}^\circ\right)+\left(3.33\times10^5\text{ J/kg}\right)\\ +\left(2100\,\text{J/kg}\bullet\text{C}^\circ\right)\left(17\text{C}^\circ\right)\end{bmatrix} = 2.367\times10^5\text{ J}$$

The Carnot efficiency can be used to find the work done by the refrigerator.

$$e = 1-\frac{T_L}{T_H} = \frac{W}{Q_H} = \frac{W}{W+Q_L} \quad\rightarrow$$
$$W = Q_L\left(\frac{T_H}{T_L}-1\right) = \left(2.367\times10^5\text{ J}\right)\left(\frac{\left(25+273\right)\text{K}}{\left(-17+273\right)\text{K}}-1\right) = 3.883\times10^4\text{ J} \approx \boxed{3.9\times10^4\text{ J}}$$

(*b*) Use the compressor wattage to calculate the time.

$$P = W/t \quad\rightarrow\quad t = W/P = 3.883\times10^4\text{ J}/210\text{ W} = 184.9\text{ s} \approx \boxed{3.1\text{ min}}$$

55. (*a*) Calculate the Carnot efficiency for an engine operated between the given temperatures.

$$e_{\text{ideal}} = 1-\frac{T_L}{T_H} = 1-\frac{\left(273+4\right)\text{K}}{\left(273+27\right)\text{K}} = 0.077 = \boxed{7.7\%}$$

(*b*) Such an engine might be feasible in spite of the low efficiency because of the large volume of "fuel" (ocean water) available. Ocean water would appear to be an "inexhaustible" source of heat energy.

(*c*) The pumping of water between radically different depths would probably move smaller sea-dwelling creatures from their natural location, perhaps killing them in the transport process. Mixing the water at different temperatures will also disturb the environment of sea-dwelling creatures. There is a significant dynamic of energy exchange between the ocean and the atmosphere, and so any changing of surface temperature water might affect at least the local climate, and perhaps also cause larger-scale climate changes.

56. Take the energy transfer to use as the initial kinetic energy of the cars, because this energy becomes "unusable" after the collision – it is transferred to the environment.

$$\Delta S = \frac{Q}{T} = \frac{2\left(\frac{1}{2}mv_i^2\right)}{T} = \frac{\left(1.1\times10^3\text{kg}\right)\left[\left(95\,\text{km/h}\right)\left(\frac{1\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{\left(20+273\right)\text{K}} = \boxed{2.6\times10^3\text{ J/K}}$$

57. (*a*) The equilibrium temperature is found using calorimetry, from chapter 14. The heat lost by the water is equal to the heat gained by the aluminum.

$$m_{H_2O}c_{H_2O}\left(T_{iH_2O}-T_f\right) = m_{Al}c_{Al}\left(T_f-T_{iAl}\right) \quad\rightarrow$$
$$T_f = \frac{m_{Al}c_{Al}T_{iAl}+m_{H_2O}c_{H_2O}T_{iH_2O}}{m_{Al}c_{Al}+m_{H_2O}c_{H_2O}}$$
$$= \frac{\left(0.12\text{ kg}\right)\left(900\,\text{J/kg}\bullet\text{C}^\circ\right)\left(15^\circ\text{C}\right)+\left(0.14\text{ kg}\right)\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)\left(50^\circ\text{C}\right)}{\left(0.12\text{ kg}\right)\left(900\,\text{J/kg}\bullet\text{C}^\circ\right)+\left(0.14\text{ kg}\right)\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)} = 44.55^\circ\text{C} \approx \boxed{45^\circ\text{C}}$$

(*b*) The amount of heat lost by the aluminum, and gained by the water, is

$$Q = m_{H_2O}c_{H_2O}\left(T_{iH_2O}-T_f\right) = \left(0.14\text{ kg}\right)\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)\left(50^\circ\text{C}-44.55^\circ\text{C}\right) = 3.194\times10^3\text{ J}$$

In calculating the entropy change, we need to use estimates for the temperatures of the water and the aluminum since their temperatures are not constant. We will use their average temperatures.

$$\underset{\text{avg}}{T_{\mathrm{H_2O}}} = \left(50^\circ\mathrm{C} + 44.55^\circ\mathrm{C}\right)/2 = 47.26^\circ\mathrm{C} \qquad \underset{\text{avg}}{T_{\mathrm{Al}}} = \left(15^\circ\mathrm{C} + 44.55^\circ\mathrm{C}\right)/2 = 29.78^\circ\mathrm{C}$$

$$\Delta S = \Delta S_{\mathrm{Al}} + \Delta S_{\mathrm{H_2O}} = -\frac{Q}{\underset{\text{avg}}{T_{\mathrm{H_2O}}}} + \frac{Q}{\underset{\text{avg}}{T_{\mathrm{Al}}}} = \left(3.194\times10^{3}\,\mathrm{J}\right)\left(\frac{1}{\left(29.78+273\right)\mathrm{K}} - \frac{1}{\left(47.26+273\right)\mathrm{K}}\right)$$

$$= 0.5758\,\mathrm{J/K} \approx \boxed{0.58\,\mathrm{J/K}}$$

58. The COP for an ideal heat pump is given by Eq. 15-7.

(*a*) $$\mathrm{COP} = \frac{Q_{\mathrm{H}}}{W} = \frac{Q_{\mathrm{H}}}{Q_{\mathrm{H}} - Q_{\mathrm{L}}} = \frac{T_{\mathrm{H}}}{T_{\mathrm{H}} - T_{\mathrm{L}}} = \frac{\left(24+273\right)\mathrm{K}}{18\ \mathrm{K}} = 16.5 \approx \boxed{17}$$

(*b*) $$\mathrm{COP} = \frac{Q_{\mathrm{H}}}{W} \rightarrow Q_{\mathrm{H}} = \left(W/t\right)\left(t\right)\left(\mathrm{COP}\right) = \left(1200\ \mathrm{W}\right)\left(3600\ \mathrm{s}\right)\left(16.5\right) = 7.128\times10^{7}\,\mathrm{J} \approx \boxed{7.1\times10^{7}\,\mathrm{J}}$$

59. The efficiency is given by $e = W/Q_{\mathrm{H}} = \dfrac{W/t}{Q_{\mathrm{H}}/t}$, and so the input power and the useful power are needed.

$$W/t = \left(25\ \mathrm{hp}\right)\left(746\ \mathrm{W/hp}\right) = 1.9\times10^{4}\ \mathrm{J/s}$$

$$Q_{\mathrm{H}}/t = \left(\frac{3.0\times10^{4}\,\mathrm{kcal}}{1\ \mathrm{gal}}\right)\left(\frac{1\ \mathrm{gal}}{41\ \mathrm{km}}\right)\left(\frac{90\ \mathrm{km}}{1\ \mathrm{h}}\right)\left(\frac{4186\ \mathrm{J}}{1\ \mathrm{kcal}}\right)\left(\frac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = 7.7\times10^{4}\ \mathrm{J/s}$$

$$e = \frac{W/t}{Q_{\mathrm{H}}/t} = \frac{1.9\times10^{4}\ \mathrm{J/s}}{7.7\times10^{4}\ \mathrm{J/s}} = 0.24 = \boxed{24\%}$$

60. Find the original intake temperature T_{H1} from the original Carnot efficiency, and then recalculate the intake temperature for the new Carnot efficiency, T_{H2}, using the same exhaust temperature.

$$e_1 = 1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H1}}} \rightarrow T_{\mathrm{H1}} = \frac{T_{\mathrm{L}}}{1-e_1} \qquad e_2 = 1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H2}}} \rightarrow T_{\mathrm{H2}} = \frac{T_{\mathrm{L}}}{1-e_2}$$

$$T_{\mathrm{H2}} - T_{\mathrm{H1}} = T_{\mathrm{L}}\left(\frac{1}{1-e_2} - \frac{1}{1-e_2}\right) = \left(273\ \mathrm{K} + 20\ \mathrm{K}\right)\left(\frac{1}{0.60} - \frac{1}{0.70}\right) = \boxed{70\,\mathrm{K}}$$

61. Note that there is NO work done as the gas goes from state A to state B or state D to state C, because there is no volume change. In general, the work done can be found from the “area” under the PV curve representing the process under consideration.

(*a*) $W_{\mathrm{ADC}} = \boxed{P_{\mathrm{A}}\left(V_{\mathrm{C}} - V_{\mathrm{A}}\right)}$

(*b*) $W_{\mathrm{ABC}} = \boxed{P_{\mathrm{C}}\left(V_{\mathrm{C}} - V_{\mathrm{A}}\right)}$

(*c*) $W_{\mathrm{AC}} = \boxed{\tfrac{1}{2}\left(P_{\mathrm{C}} + P_{\mathrm{A}}\right)\left(V_{\mathrm{C}} - V_{\mathrm{A}}\right)}$

62. (*a*) The exhaust heating rate is found from the delivered power and the efficiency. Use the output energy with the relationship $Q = mc\Delta T = \rho Vc\Delta T$ to calculate the volume of air that is heated.

$$e = W/Q_H = W/(Q_L + W) \;\rightarrow\; Q_L = W(1/e - 1) \;\rightarrow$$

$$Q_L/t = W/t(1/e - 1) = (8.5\times10^8\,\text{W})(1/0.33 - 1) = 1.726\times10^9\,\text{W}$$

$$Q_L = mc\Delta T \;\rightarrow\; Q_L/t = \frac{mc\Delta T}{t} = \frac{\rho Vc\Delta T}{t} \;\rightarrow\; V/t = \frac{(Q_L/t)}{\rho c\Delta T}$$

The change in air temperature is $7.0\,\text{C}^\circ$. The heated air is at a constant pressure of 1 atm.

$$V/t = \frac{(Q_L/t)t}{\rho c\Delta T} = \frac{(1.726\times10^9\,\text{W})(8.64\times10^4\,\text{s/day})}{(1.2\,\text{kg/m}^3)(1.0\times10^3\,\text{J/kg}\bullet\text{C}^\circ)(7.0\text{C}^\circ)}$$

$$= 1.775\times10^{10}\,\text{m}^3/\text{day}\left(\frac{10^{-9}\text{km}^3}{1\text{ m}^3}\right) \approx \boxed{18\text{ km}^3/\text{day}}$$

(*b*) If the air is 200 m thick, find the area by dividing the volume by the thickness.

$$A = \frac{\text{Volume}}{\text{thickness}} = \frac{18\,\text{km}^3}{0.2\text{ km}} = \boxed{90\text{ km}^2}$$

This would be a square of approximately 6 miles to a side. Thus the local climate for a few miles around the power plant might be heated significantly.

63. (*a*) The exhaust heating rate can be found from the delivered power *P* and the Carnot efficiency. Then use the relationship between energy and temperature change, $Q = mc\Delta T$, to calculate the temperature change of the cooling water.

$$e = 1 - \frac{T_L}{T_H} = \frac{W}{Q_H} = \frac{W}{Q_L + W} \;\rightarrow\; Q_L = W\frac{T_L}{T_H - T_L} \;\rightarrow\; Q_L/t = W/t\frac{T_L}{T_H - T_L} = P\frac{T_L}{T_H - T_L}$$

$$Q_L = mc\Delta T \;\rightarrow\; Q_L/t = \frac{m}{t}c\Delta T = \rho\frac{V}{t}c\Delta T$$

Equate the two expressions for Q_L/t, and solve for ΔT.

$$P\frac{T_L}{T_H - T_L} = \rho\frac{V}{t}c\Delta T \;\rightarrow\; \Delta T = \frac{P}{\rho\frac{V}{t}c}\frac{T_L}{T_H - T_L}$$

$$= \frac{9.8\times10^8\,\text{W}}{(1.0\times10^3\,\text{kg/m}^3)(37\,\text{m}^3/\text{s})(4186\,\text{J/kg}\bullet\text{C}^\circ)}\frac{285\text{ K}}{(625\text{ K} - 285\text{ K})} = 5.303\,\text{K} = \boxed{5.3\text{C}^\circ}$$

(*b*) The addition of heat per kilogram for the downstream water is $Q_L/t = c\Delta T$. We use the "average" temperature of the river water for the calculation: $T = T_0 + \frac{1}{2}\Delta T$. Now the entropy increase can be calculated.

$$\Delta S = \frac{\Delta Q}{T} = \frac{c\Delta T}{T_0 + \frac{1}{2}\Delta T} = \frac{(4186\,\text{J/kg}\bullet\text{C}^\circ)(5.303\,\text{K})}{\left[285 + \frac{1}{2}(5.303)\right]\text{K}} = \boxed{77\,\text{J/kg}\bullet\text{K}}$$

64. (*a*) Calculate the Carnot efficiency by $e = 1 - T_L/T_H$ and compare it to the 15% actual efficiency.

$$e_{\text{Carnot}} = 1 - T_L/T_H = 1 - (85 + 273)\,\text{K}/(495 + 273)\,\text{K} = 0.534 = 53.4\%$$

Thus the engine's relative efficiency is $e_{\text{actual}}/e_{\text{Carnot}} = 0.15/0.534 = 0.281 = \boxed{28\%}$

(*b*) Take the stated 100 hp as the useful power obtained from the engine. Use the efficiency to calculate the exhaust heat.

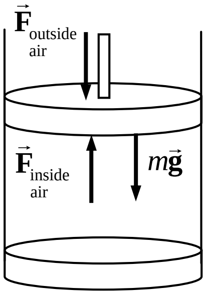
$$P = \frac{W}{t} = (100\text{ hp})\left(\frac{746\,\text{W}}{1\text{ hp}}\right) = \boxed{7.46\times10^{4}\,\text{W}}$$

$$e = \frac{W}{Q_{\text{H}}} = \frac{W}{Q_L + W} \quad \rightarrow$$

$$Q_L = W\left(\frac{1}{e} - 1\right) = Pt\left(\frac{1}{e} - 1\right) = \left(7.46\times10^{4}\text{ J/s}\right)(1\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1}{0.15} - 1\right)$$

$$= \boxed{1.5\times10^{9}\,\text{J}} = \left(1.5\times10^{9}\,\text{J}\right)\left(\frac{1\text{ kcal}}{4186\text{ J}}\right) = \boxed{3.6\times10^{5}\,\text{kcal}}$$

65. The net force on the piston must be 0, and so the weight of the piston must be equal to the net force exerted by the gas pressures on both sides of the piston. See the free-body diagram.


$\vec{\mathbf{F}}_{\substack{\text{outside}\\ \text{air}}}$
$\vec{\mathbf{F}}_{\substack{\text{inside}\\ \text{air}}}$
$m\vec{\mathbf{g}}$

$$\sum F = F_{\substack{\text{inside}\\ \text{air}}} - F_{\substack{\text{outside}\\ \text{air}}} - mg = 0 = P_{\text{inside}}A - P_{\text{outside}}A - mg = 0$$

$$P_{\text{inside}} = P_{\text{outside}} + \frac{mg}{A} = (1.0\text{ atm})\left(1.01\times10^{5}\,\frac{\text{Pa}}{\text{atm}}\right) + \frac{(0.10\text{ kg})\left(9.8\,\text{m/s}^{2}\right)}{0.080\text{ m}^{2}}$$

$$= 1.0101\times10^{5}\,\text{Pa} \approx 1\text{ atm}$$

We see that the weight of the piston is negligible compared to the pressure forces.

When the gas is heated, we assume that the inside pressure does not change. Since the weight of the piston does not change, and the outside air pressure does not change, the inside air pressure cannot change. Thus the expansion is at a constant pressure, and so the work done can be calculated. Use this with the first law of thermodynamics to find the heat required for the process.

$$U = \tfrac{3}{2}nRT = \tfrac{3}{2}PV \quad \rightarrow \quad \Delta U = \tfrac{3}{2}P\Delta V = Q - W$$

$$Q = \Delta U + W = \tfrac{3}{2}P\Delta V + P\Delta V = \tfrac{5}{2}P\Delta V = \tfrac{5}{2}PA\Delta y = 2.5\left(1.0\times10^{5}\,\text{Pa}\right)\left(0.080\text{ m}^{2}\right)\left(1.0\times10^{-2}\,\text{m}\right)$$

$$= \boxed{200\text{ J}}$$

66. (*a*) Multiply the power times the time times the mass per Joule relationship for the fat.

$$(95\,\text{J/s})(3600\,\text{s/h})(24\,\text{h/d})\left(1.0\text{ kg fat}/3.7\times10^{7}\,\text{J}\right) = 0.2218\,\text{kg/d} \approx \boxed{0.22\,\text{kg/d}}$$

(*b*) $1.0\text{ kg}\left(1\text{d}/0.2218\text{ kg}\right) = \boxed{4.5\text{ d}}$

67. The radiant energy is the heat to be removed at the low temperature. It can be related to the work necessary through the efficiency.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = \frac{W}{Q_{\text{H}}} = \frac{W}{W + Q_{\text{L}}} \quad \rightarrow \quad W = Q_{\text{L}}\left(\frac{T_{\text{H}}}{T_{\text{L}}} - 1\right) \quad \rightarrow \quad W/t = Q_{\text{L}}/t\left(\frac{T_{\text{H}}}{T_{\text{L}}} - 1\right)$$

$$(W/t)_{5300} = (5300\text{ W})\left(\frac{T_{\text{H}}}{T_{\text{L}}} - 1\right) \qquad (W/t)_{500} = (500\text{ W})\left(\frac{T_{\text{H}}}{T_{\text{L}}} - 1\right)$$

$$(W/t)_{\text{savings}} = (W/t)_{5300} - (W/t)_{500} = (5300\text{ W} - 500\text{ W})\left(\frac{(273+32)\text{K}}{(273+21)\text{K}} - 1\right) = \boxed{180\text{ W}}$$

68. To find the mass of water removed, find the energy that is removed from the low temperature reservoir from the work input and the Carnot efficiency. Then use the latent heat of vaporization to determine the mass of water from the energy required for the condensation. Note that the heat of vaporization used is that given in chapter 14 for evaporation at 20°C.

$$e = 1 - \frac{T_L}{T_H} = \frac{W}{Q_H} = \frac{W}{W + Q_L} \quad \rightarrow \quad Q_L = W\frac{T_L}{(T_H - T_L)} = mL_{\text{vapor}}$$

$$m = \frac{W}{L_{\text{vapor}}}\frac{T_L}{(T_H - T_L)} = \frac{(600\text{ W})(3600\text{ s})}{(2.45\times10^6\text{ J/kg})}\frac{(273+8)\text{K}}{17\text{ K}} = \boxed{14.6\text{ kg}}$$

CHAPTER 16: Electric Charge and Electric Field

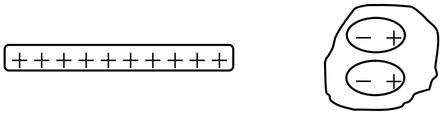
Answers to Questions

1. A plastic ruler is suspended by a thread and then rubbed with a cloth. As discussed in section 16-1, the ruler is negatively charged. Bring the charged comb close to the ruler. If the ruler is repelled by the comb, then the comb is negatively charged. If the ruler is attracted by the comb, then the comb is positively charged.

2. The clothing gets charged by frictional contact in the tumbling motion of the dryer. The air inside the dryer is dry, and so the clothes can sustain a relatively large static charge. That charged object will then polarize your clothing, and be attracted to you electrostatically.

3. Water is a polar molecule – it has a positive region and a negative region. Thus it is easily attracted to some other charged object, like an ion or electron in the air.

4. The positively charged rod slightly polarizes the molecules in the paper. The negative charges in the paper are slightly attracted to the part of the paper closest to the rod, while the positive charges in the paper are slightly repelled from the part of the paper closest to the rod. Since the opposite charges are now closer together and the like charges are now farther apart, there is a net attraction between the rod and the paper.



5. The plastic ruler has gained some electrons from the cloth and thus has a net negative charge. This charge polarizes the charge on the piece of paper, drawing positives slightly closer and repelling negatives slightly further away. This polarization results in a net attractive force on the piece of paper. A small amount of charge is able to create enough electric force to be stronger than gravity, and so the paper can be lifted.

   On a humid day this is more difficult because the water molecules in the air are polar. Those polar water molecules are able to attract some fraction of the free charges away from the plastic ruler. Thus the ruler has a smaller charge, the paper is less polarized, and there is not enough electric force to pick up the paper.

6. The net charge on the conductor is the unbalanced charge, or excess charge after neutrality has been established. The net charge is the sum of all of the positive and negative charges in the conductor. If a neutral conductor has extra electrons added to it, then the net charge is negative. If a neutral conductor has electrons removed from it, then the net charge is positive. If a neutral conductor has the same amount of positive and negative charge, then the net charge is zero.

   Free charges in a conductor refer to those electrons (usually 1 or 2 per atom) that are so loosely attracted to the nucleus that they are "free" to be moved around in the conductor by an external electric force. Neutral conductors have these free electrons.

7. For each atom in a conductor, only a small number of its electrons are free to move. For example, every atom of copper has 29 electrons, but only 1 or 2 from each atom are free to move easily. Also, not even all of the free electrons move. As electrons move toward a region, causing an excess of negative charge, that region then exerts a large repulsive force on other electrons, preventing them from all gathering in one place.

8. The force of gravity pulling down on the leaves, tending to return them to the vertical position.

9. The magnitude of the constant in Newton’s law is very small, while the magnitude of the constant in Coulomb’s law is quite large. Newton’s law says the gravitational force is proportional to the product of the two masses, while Coulomb’s law says the electrical force is proportional to the product of the two charges. Newton’s law only produces attractive forces, since there is only one kind of gravitational mass. Coulomb’s law produces both attractive and repulsive forces, since there are two kinds of electrical charge.

10. For the gravitational force, we don’t notice it because the force is very weak, due to the very small value of *G*, the gravitational constant, and the small value of ordinary masses. For the electric force, we don’t notice it because ordinary objects are electrically neutral to a very high degree. We notice our weight (the force of gravity) due to the huge mass of the Earth, making a significant gravity force. We notice the electric force when objects have a static charge (like static cling from the clothes dryer), creating a detectable electric force.

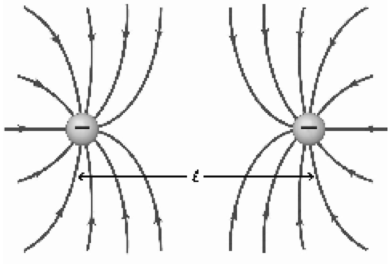
11. The electric force is conservative. You can “store” energy in it, and get the energy back. For example, moving a positive charge close to another stationary positive charge takes work (similar to lifting an object in the Earth’s gravitational field), but if the positive charge is then released, it will gain kinetic energy and move away from the “stored energy” location (like dropping an object in the Earth’s gravitational field). Another argument is that the mathematical form of Coulomb’s law is identical to that of Newton’s law of universal gravitation. We know that gravity is conservative, and so we would assume that the electric force is also conservative. There are other indications as well. If you move a charge around in an electric field, eventually returning to the starting position, the net work done will be 0 J. The work done in moving a charge around in an electric field is path independent – all that matters is the starting and ending locations. All of these are indications of a conservative force.

12. The charged plastic ruler has a negative charge residing on its surface. That charge polarizes the charge in the neutral paper, producing a net attractive force. When the piece of paper then touches the ruler, the paper can get charged by contact with the ruler, gaining a net negative charge. Then, since like charges repel, the paper is repelled by the comb.

13. The test charge creates its own electric field, and so the measured electric field is the sum of the original electric field plus the field of the test charge. By making the test charge small, the field that it causes is small, and so the actual measured electric field is not much different than the original field to measure.

14. A negative test charge could be used. For purposes of defining directions, the electric field might then be defined as the OPPOSITE of the force on the test charge, divided by the test charge. Equation (16-3) might be changed to $\vec{E} = -\vec{F}/q$, $q < 0$.

15.



16. The electric field is strongest to the right of the positive charge, because the individual fields from the positive charge and negative charge both are in the same direction (to the right) at that point, so they add to make a stronger field. The electric field is weakest to the left of the positive charge, because the individual fields from the positive charge and negative charge are in opposite directions at that point, and so they partially cancel each other. Another indication is the spacing of the field lines. The field lines are closer to each other to the right of the positive charge, and further apart to the left of the positive charge.

17. At point A, the net force on a positive test charge would be down and to the left, parallel to the nearby electric field lines. At point B, the net force on a positive test charge would be up and to the right, parallel to the nearby electric field lines. At point C, the net force on a positive test charge would be 0. In order of decreasing field strength, the points would be ordered A, B, C.

18. Electric field lines show the direction of the force on a test charge placed at a given location. The electric force has a unique direction at each point. If two field lines cross, it would indicate that the electric force is pointing in two directions at once, which is not possible.

19. From rule 1: A test charge would be either attracted directly towards or repelled directly away from a point charge, depending on the sign of the point charge. So the field lines must be directed either radially towards or radially away from the point charge.

    From rule 2: The magnitude of the field due to the point charge only depends on the distance from the point charge. Thus the density of the field lines must be the same at any location around the point charge, for a given distance from the point charge.

    From rule 3: If the point charge is positive, the field lines will originate from the location of the point charge. If the point charge is negative, the field lines will end at the location of the point charge.

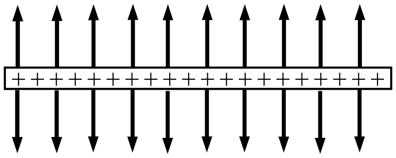
    Based on rules 1 and 2, the lines are radial and their density is constant for a given distance. This is equivalent to saying that the lines must be symmetrically spaced around the point charge.

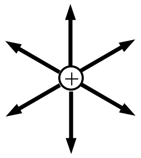
20. If the two charges are of opposite sign, then $\vec{\mathbf{E}} = 0$ at a point closer to the weaker charge, and on the opposite side of the weaker charge from the stronger charge. The fields due to the two charges are of opposite direction at such a point. If the distance between the two charges is l, then the point at which $\vec{\mathbf{E}} = 0$ is 2.41 l away from the weaker charge, and 3.41 l away from the stronger charge.

    If the two charges are the same sign, then $\vec{\mathbf{E}} = 0$ at a point between the two charges, closer to the weaker charge. The point is 41% of the distance from the weaker charge to the stronger charge.

21. We assume that there are no other forces (like gravity) acting on the test charge. The direction of the electric field line gives the direction of the force on the test charge. The acceleration is always parallel to the force by Newton's 2nd law, and so the acceleration lies along the field line. If the particle is at rest initially and then released, the initial velocity will also point along the field line, and the particle will start to move along the field line. However, once the particle has a velocity, it will not follow the field line unless the line is straight. The field line gives the direction of the acceleration, or the direction of the change in velocity.

22. Since the line of charge is infinitely long, it has no preferred left or right direction. Thus by symmetry, the lines must point radially out away from the center of the line. A side view and an end view are shown for a line of positive charge. As seen from the end view, the field is not uniform. As you move further away from the line of charge, the field lines get further apart, indicating that the field gets weaker as you move away from the line of charge.





23. Just because the electric flux through a closed surface is zero, the field need not be zero on the surface. For example, consider a closed surface near an isolated point charge, and the surface does not enclose the charge. There will be electric field lines passing through the surface, but the total electric flux through the surface will be zero since the surface does not enclose any charge. The same number of field lines will enter the volume enclosed by the surface as leave the volume enclosed by the surface.

    On the contrary, if $\vec{\mathbf{E}} = 0$ at all points on the surface, then there are no electric field lines passing through the surface, and so the flux through the surface is zero.

24. The electric flux depends only on the charge enclosed by the gaussian surface, not on the shape of the surface. Φ_E will be the same for the cube as for the sphere.

Solutions to Problems

1. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(3.60\times10^{-6}\,\mathrm{C}\right)^2}{\left(9.3\times10^{-2}\,\mathrm{m}\right)^2} = 13.47\,\mathrm{N} \approx \boxed{13\,\mathrm{N}}$$

2. Use the charge per electron to find the number of electrons.

$$\left(-30.0\times10^{-6}\,\mathrm{C}\right)\left(\frac{1\text{ electron}}{-1.602\times10^{-19}\,\mathrm{C}}\right) = \boxed{1.87\times10^{14}\,\text{electrons}}$$

3. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.602\times10^{-19}\,\mathrm{C}\right)\left(26\times1.602\times10^{-19}\,\mathrm{C}\right)}{\left(1.5\times10^{-12}\,\mathrm{m}\right)^2} = \boxed{2.7\times10^{-3}\,\mathrm{N}}$$

4. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.602\times10^{-19}\,\mathrm{C}\right)^2}{\left(5.0\times10^{-15}\,\mathrm{m}\right)^2} = \boxed{9.2\,\mathrm{N}}$$

5. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)\frac{\left(25\times10^{-6}\,\mathrm{C}\right)\left(3.0\times10^{-3}\,\mathrm{C}\right)}{\left(3.5\times10^{-1}\,\mathrm{m}\right)^2} = \boxed{5.5\times10^3\,\mathrm{N}}$$

6. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the distance is multiplied by a factor of 1/8, the force will be multiplied by a factor of 64.

$$F = 64F_0 = 64\left(3.2\times10^{-2}\,\mathrm{N}\right) = \boxed{2.0\ \mathrm{N}}$$

7. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the force is tripled, the distance has been reduced by a factor of $\sqrt{3}$.

$$r = \frac{r_0}{\sqrt{3}} = \frac{8.45\ \mathrm{cm}}{\sqrt{3}} = \boxed{4.88\ \mathrm{cm}}$$

8. Use the charge per electron and the mass per electron.

$$\left(-42\times10^{-6}\,\mathrm{C}\right)\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\,\mathrm{C}}\right) = 2.622\times10^{14} \approx \boxed{2.6\times10^{14}\,\text{electrons}}$$

$$\left(2.622\times10^{14}\,\mathrm{e^-}\right)\left(\frac{9.11\times10^{-31}\,\mathrm{kg}}{1\ \mathrm{e^-}}\right) = \boxed{2.4\times10^{-16}\,\mathrm{kg}}$$

9. Convert the kg of H_2O to moles, then to atoms, then to electrons. Oxygen has 8 electrons per atom, and hydrogen has 1 electron per atom.

$$1.0\ \mathrm{kg\ H_2O} = \left(1.0\ \mathrm{kg\ H_2O}\right)\left(\frac{1\ \mathrm{mole\ H_2O}}{1.8\times10^{-2}\,\mathrm{kg}}\right)\left(\frac{6.02\times10^{23}\,\text{molec.}}{1\ \text{mole}}\right)\left(\frac{10\ \mathrm{e}}{1\ \text{molec.}}\right)\left(\frac{-1.602\times10^{-19}\,\mathrm{C}}{\mathrm{e}}\right)$$

$$= \boxed{-5.4\times10^7\,\mathrm{C}}$$

10. Take the ratio of the electric force divided by the gravitational force.

$$\frac{F_\mathrm{E}}{F_\mathrm{G}} = \frac{k\dfrac{Q_1Q_2}{r^2}}{G\dfrac{m_1m_2}{r^2}} = \frac{kQ_1Q_2}{Gm_1m_2} = \frac{\left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.602\times10^{-19}\,\mathrm{C}\right)^2}{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(1.67\times10^{-27}\,\mathrm{kg}\right)} = \boxed{2.3\times10^{39}}$$

The electric force is about 2.3×10^{39} times stronger than the gravitational force for the given scenario.

11. (*a*) Let one of the charges be q, and then the other charge is $Q_\mathrm{T} - q$. The force between the charges is $F_\mathrm{E} = k\frac{q\left(Q_\mathrm{T}-q\right)}{r^2} = \frac{k}{r^2}\left(qQ_\mathrm{T} - q^2\right) = \frac{k}{r^2}Q_\mathrm{T}^2\left(\frac{q}{Q_\mathrm{T}} - \left(\frac{q}{Q_\mathrm{T}}\right)^2\right)$. If we let $x = \frac{q}{Q_\mathrm{T}}$, then $F_\mathrm{E} = \frac{k}{r^2}Q_\mathrm{T}^2\left(x - x^2\right)$, where $0 \le x \le 1$. A graph of $f\left(x\right) = x - x^2$ between the limits of 0 and 1

shows that the maximum occurs at $x = 0.5$, or $\boxed{q = 0.5Q_{\mathrm{T}}}$. Both charges are half of the total, and the actual maximized force is $F_{\mathrm{E}} = 0.25\dfrac{k}{r^2}Q_{\mathrm{T}}^2$.

(*b*) If one of the charges has all of the charge, and the other has no charge, then the force between them will be 0, which is the minimum possible force.

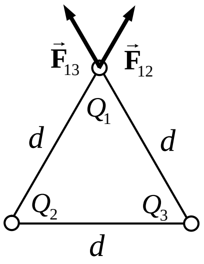
12. Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions, $k = 8.988\times10^9\ \mathrm{N\cdot m^2/C^2}$.

$$F_{+75} = -k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2} + k\frac{(75\mu\mathrm{C})(85\mu\mathrm{C})}{(0.70\,\mathrm{m})^2} = -147.2\,\mathrm{N} \approx \boxed{-1.5\times10^2\mathrm{N}}$$

$$F_{+48} = k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2} + k\frac{(48\mu\mathrm{C})(85\mu\mathrm{C})}{(0.35\,\mathrm{m})^2} = 563.5\,\mathrm{N} \approx \boxed{5.6\times10^2\mathrm{N}}$$

$$F_{-85} = -k\frac{(85\mu\mathrm{C})(75\mu\mathrm{C})}{(0.70\,\mathrm{m})^2} - k\frac{(85\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2} = -416.3\,\mathrm{N} \approx \boxed{-4.2\times10^2\mathrm{N}}$$

13. The forces on each charge lie along a line connecting the charges. Let the variable *d* represent the length of a side of the triangle, and let the variable *Q* represent the charge at each corner. Since the triangle is equilateral, each angle is 60°.



$$F_{12} = k\frac{Q^2}{d^2} \quad\rightarrow\quad F_{12x} = k\frac{Q^2}{d^2}\cos 60^\circ\ ,\ F_{12y} = k\frac{Q^2}{d^2}\sin 60^\circ$$

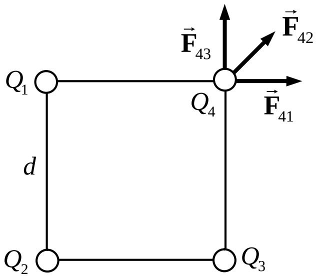
$$F_{13} = k\frac{Q^2}{d^2} \quad\rightarrow\quad F_{13x} = -k\frac{Q^2}{d^2}\cos 60^\circ\ ,\ F_{13y} = k\frac{Q^2}{d^2}\sin 60^\circ$$

$$F_{1x} = F_{12x} + F_{13x} = 0 \qquad F_{1y} = F_{12y} + F_{13y} = 2k\frac{Q^2}{d^2}\sin 60^\circ = \sqrt{3}k\frac{Q^2}{d^2}$$

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \sqrt{3}k\frac{Q^2}{d^2} = \sqrt{3}\left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(11.0\times10^{-6}\,\mathrm{C}\right)^2}{(0.150\,\mathrm{m})^2} = \boxed{83.7\,\mathrm{N}}$$

The direction of $\vec{\mathbf{F}}_1$ is in the $\boxed{y\text{-direction}}$. Also notice that it lies along the bisector of the opposite side of the triangle. Thus the force on the lower left charge is of magnitude $\boxed{83.7\,\mathrm{N}}$, and will point $\boxed{30^\circ\text{ below the } -x \text{ axis}}$. Finally, the force on the lower right charge is of magnitude $\boxed{83.7\,\mathrm{N}}$, and will point $\boxed{30^\circ\text{ below the } +x \text{ axis}}$.

14. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other three charges. The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 6.00 mC charge at each corner.



$$F_{41} = k\frac{Q^2}{d^2} \rightarrow F_{41x} = k\frac{Q^2}{d^2}, \; F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \rightarrow F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}, \; F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \rightarrow F_{43x} = 0, \; F_{43y} = k\frac{Q^2}{d^2}$$

Add the *x* and *y* components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right) = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q^2}{d^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

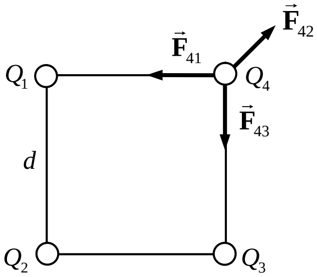
$$= \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(6.00\times10^{-3}\,\mathrm{C}\right)^2}{\left(0.100\,\mathrm{m}\right)^2}\left(\sqrt{2} + \frac{1}{2}\right) = \boxed{6.19\times10^7\ \mathrm{N}}$$

$$\theta = \tan^{-1}\frac{F_{4y}}{F_{4x}} = \boxed{45^\circ}$$ above the *x*-direction.

For each charge, the net force will be the magnitude determined above, and will lie along the line from the center of the square out towards the charge.

15. Determine the force on the upper right charge, and then the symmetry of the configuration says that the force on the lower left charge is the opposite of the force on the upper right charge. Likewise, determine the force on the lower right charge, and then the symmetry of the configuration says that the force on the upper left charge is the opposite of the force on the lower right charge.

The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 6.00 mC charge at each corner.



$$F_{41} = k\frac{Q^2}{d^2} \rightarrow F_{41x} = -k\frac{Q^2}{d^2}, \; F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \rightarrow F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}, \; F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \rightarrow F_{43x} = 0, \; F_{43y} = -k\frac{Q^2}{d^2}$$

Add the *x* and *y* components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = -k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(-1 + \frac{\sqrt{2}}{4}\right) = -0.64645k\frac{Q^2}{d^2} = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}\left(0.64645\right)\sqrt{2} = k\frac{Q^2}{d^2}\left(0.9142\right)$$

$$=\left(8.988\times10^{9}\ \mathrm{N\cdot m^{2}/C^{2}}\right)\frac{\left(6.00\times10^{-3}\,\mathrm{C}\right)^{2}}{\left(0.100\,\mathrm{m}\right)^{2}}\left(0.9142\right)=\boxed{2.96\times10^{7}\ \mathrm{N}}$$

$$\theta=\tan^{-1}\frac{F_{4y}}{F_{4x}}=\boxed{225^{\circ}}\ \text{from the }x\text{-direction, or exactly towards the center of the square.}$$

For each charge, the net force will be the magnitude of $\boxed{2.96\times10^{7}\ \mathrm{N}}$ and each net force will lie along the line from the charge inwards towards the center of the square.

16. Take the lower left hand corner of the square to be the origin of coordinates. Each charge will have a horizontal force on it due to one charge, a vertical force on it due to one charge, and a diagonal force on it due to one charge. Find the components of each force, add the components, find the magnitude of the net force, and the direction of the net force. At the conclusion of the problem is a diagram showing the net force on each of the two charges.

(*a*) $2Q$:
$$F_{2Qx}=k\frac{(2Q)Q}{l^{2}}+k\frac{(2Q)(4Q)}{2l^{2}}\cos45^{\circ}=k\frac{Q^{2}}{l^{2}}\left(2+2\sqrt{2}\right)=4.8284\frac{kQ^{2}}{l^{2}}$$

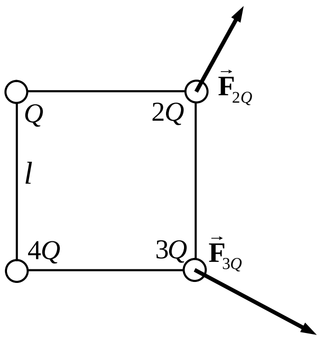
$$F_{2Qy}=k\frac{(2Q)(3Q)}{l^{2}}+k\frac{(2Q)(4Q)}{2l^{2}}\sin45^{\circ}=k\frac{Q^{2}}{l^{2}}\left(6+2\sqrt{2}\right)=8.8284\frac{kQ^{2}}{l^{2}}$$

$$F_{2Q}=\sqrt{F_{2Qx}^{2}+F_{2Qy}^{2}}=\boxed{10.1\frac{kQ^{2}}{l^{2}}}\qquad\theta_{2Q}=\tan^{-1}\frac{F_{2y}}{F_{2x}}=\tan^{-1}\frac{8.8284}{4.8284}=\boxed{61^{\circ}}$$

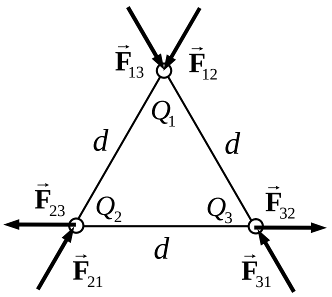
(*b*) $3Q$:
$$F_{3Qx}=k\frac{(3Q)(4Q)}{l^{2}}+k\frac{(3Q)Q}{2l^{2}}\cos45^{\circ}=k\frac{Q^{2}}{l^{2}}\left(12+\frac{3}{4}\sqrt{2}\right)=13.0607\frac{kQ^{2}}{l^{2}}$$

$$F_{3Qy}=-k\frac{(3Q)(2Q)}{l^{2}}-k\frac{(3Q)Q}{2l^{2}}\sin45^{\circ}=-k\frac{Q^{2}}{l^{2}}\left(6+\frac{3}{4}\sqrt{2}\right)=-7.0607\frac{kQ^{2}}{l^{2}}$$

$$F_{3Q}=\sqrt{F_{3Qx}^{2}+F_{3Qy}^{2}}=\boxed{14.8\frac{kQ^{2}}{l^{2}}}\qquad\theta_{3Q}=\tan^{-1}\frac{F_{3y}}{F_{3x}}=\tan^{-1}\frac{-7.0607}{13.0607}=\boxed{332^{\circ}}$$



17. The forces on each charge lie along a line connecting the charges. Let the variable d represent the length of a side of the triangle. Since the triangle is equilateral, each angle is 60°. First calculate the magnitude of each individual force.



$$F_{12} = k\frac{|Q_1Q_2|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.0\times10^{-6}\,\text{C}\right)\left(8.0\times10^{-6}\,\text{C}\right)}{\left(1.20\,\text{m}\right)^2}$$

$$= 0.1997\,\text{N} = F_{21}$$

$$F_{13} = k\frac{|Q_1Q_3|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.0\times10^{-6}\,\text{C}\right)\left(6.0\times10^{-6}\,\text{C}\right)}{\left(1.20\,\text{m}\right)^2}$$

$$= 0.1498\,\text{N} = F_{31}$$

$$F_{23} = k\frac{|Q_2Q_3|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(8.0\times10^{-6}\,\text{C}\right)\left(6.0\times10^{-6}\,\text{C}\right)}{\left(1.20\,\text{m}\right)^2} = 0.2996\,\text{N} = F_{32}$$

Now calculate the net force on each charge and the direction of that net force, using components.

$$F_{1x} = F_{12x} + F_{13x} = -\left(0.1997\,\text{N}\right)\cos 60^\circ + \left(0.1498\,\text{N}\right)\cos 60^\circ = -2.495\times10^{-2}\,\text{N}$$

$$F_{1y} = F_{12y} + F_{13y} = -\left(0.1997\,\text{N}\right)\sin 60^\circ - \left(0.1498\,\text{N}\right)\sin 60^\circ = -3.027\times10^{-1}\,\text{N}$$

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \boxed{0.30\,\text{N}} \qquad \theta_1 = \tan^{-1}\frac{F_{1y}}{F_{1x}} = \tan^{-1}\frac{-3.027\times10^{-1}\,\text{N}}{-2.495\times10^{-2}\,\text{N}} = \boxed{265^\circ}$$

$$F_{2x} = F_{21x} + F_{23x} = \left(0.1997\,\text{N}\right)\cos 60^\circ - \left(0.2996\,\text{N}\right) = -1.998\times10^{-1}\,\text{N}$$

$$F_{2y} = F_{21y} + F_{23y} = \left(0.1997\,\text{N}\right)\sin 60^\circ + 0 = 1.729\times10^{-1}\,\text{N}$$

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \boxed{0.26\,\text{N}} \qquad \theta_2 = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{1.729\times10^{-1}\,\text{N}}{-1.998\times10^{-1}\,\text{N}} = \boxed{139^\circ}$$

$$F_{3x} = F_{31x} + F_{32x} = -\left(0.1498\,\text{N}\right)\cos 60^\circ + \left(0.2996\,\text{N}\right) = 2.247\times10^{-1}\,\text{N}$$

$$F_{3y} = F_{31y} + F_{32y} = \left(0.1498\,\text{N}\right)\sin 60^\circ + 0 = 1.297\times10^{-1}\,\text{N}$$

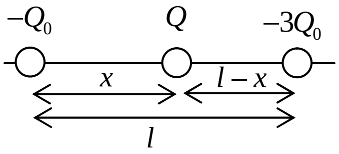
$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \boxed{0.26\,\text{N}} \qquad \theta_3 = \tan^{-1}\frac{F_{3y}}{F_{3x}} = \tan^{-1}\frac{1.297\times10^{-1}\,\text{N}}{2.247\times10^{-1}\,\text{N}} = \boxed{30^\circ}$$

18. Since the force is repulsive, both charges must be the same sign. Since the total charge is positive, both charges must be positive. Let the total charge be Q. Then if one charge is of magnitude q, then the other charge must be of magnitude $Q-q$. Write a Coulomb's law expression for one of the charges.

$$F = k\frac{q\left(Q-q\right)}{r^2} \quad\rightarrow\quad q^2 - Qq + \frac{Fr^2}{k} = 0 \quad\rightarrow$$

$$q = \frac{Q \pm \sqrt{Q^2 - \dfrac{4Fr^2}{k}}}{2} = \frac{\left(560\times10^{-6}\,\text{C}\right) \pm \sqrt{\left(560\times10^{-6}\,\text{C}\right)^2 - \dfrac{4\left(22.8\,\text{N}\right)\left(1.10\,\text{m}\right)^2}{\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}}}{2}$$

$$= \boxed{5.54\times10^{-4}\,\text{C}\ ,\ 5.54\times10^{-6}\,\text{C}} \qquad Q - q = \boxed{5.54\times10^{-6}\,\text{C}\ ,\ 5.54\times10^{-4}\,\text{C}}$$



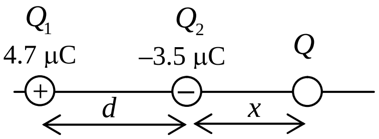
19. The negative charges will repel each other, and so the third charge must put an opposite force on each of the original charges. Consideration of the various possible configurations leads to the conclusion that the third charge must be positive and must be between the other two charges. See the diagram for the definition of variables. For each negative charge, equate the magnitudes of the two forces on the charge. Also note that $0 < x < l$.

$$\text{left: } k\frac{Q_0Q}{x^2} = k\frac{3Q_0^2}{l^2} \qquad \text{right: } k\frac{3Q_0Q}{(l-x)^2} = k\frac{3Q_0^2}{l^2} \quad \rightarrow$$

$$k\frac{Q_0Q}{x^2} = k\frac{3Q_0Q}{(l-x)^2} \quad \rightarrow \quad x = \frac{l}{\sqrt{3}+1} = 0.366l$$

$$k\frac{Q_0Q}{x^2} = k\frac{3Q_0^2}{l^2} \quad \rightarrow \quad Q = 3Q_0\frac{x^2}{l^2} = Q_0\frac{3}{\left(\sqrt{3}+1\right)^2} = 0.402Q_0$$

Thus the charge should be of magnitude $\boxed{0.40Q_0}$, and a distance $\boxed{0.37l \text{ from } -Q_0 \text{ towards } -3Q_0}$.



20. Assume that the negative charge is *d* = 18.5 cm to the right of the positive charge, on the *x*-axis. To experience no net force, the third charge *Q* must be closer to the smaller magnitude charge (the negative charge). The third charge cannot be between the charges, because it would experience a force from each charge in the same direction, and so the net force could not be zero. And the third charge must be on the line joining the other two charges, so that the two forces on the third charge are along the same line. See the diagram. Equate the magnitudes of the two forces on the third charge, and solve for *x* > 0.

$$\left|\vec{\mathbf{F}}_1\right| = \left|\vec{\mathbf{F}}_2\right| \quad \rightarrow \quad k\frac{Q_1|Q|}{(d+x)^2} = k\frac{|Q_2||Q|}{x^2} \quad \rightarrow \quad x = d\frac{\sqrt{|Q_2|}}{\left(\sqrt{Q_1}-\sqrt{|Q_2|}\right)}$$

$$x = d\frac{\sqrt{|Q_2|}}{\left(\sqrt{Q_1}-\sqrt{|Q_2|}\right)} = (18.5\,\text{cm})\frac{\sqrt{3.5\times10^{-6}\,\text{C}}}{\left(\sqrt{4.7\times10^{-6}\,\text{C}}-\sqrt{3.5\times10^{-6}\,\text{C}}\right)} = \boxed{116\,\text{cm}}$$

21. (*a*) If the force is repulsive, both charges must be positive since the total charge is positive. Call the total charge *Q*.

$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1(Q-Q_1)}{d^2} \quad \rightarrow \quad Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \frac{\left(90.0\times10^{-6}\,\text{C}\right) \pm \sqrt{\left(90.0\times10^{-6}\,\text{C}\right)^2 - 4\frac{(12.0\,\text{N})(1.06\,\text{m})^2}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}}}{2}$$

$$= \boxed{69.9\times10^{-6}\,\text{C}\,,\ 22.1\times10^{-6}\,\text{C}}$$

(*b*) If the force is attractive, then the charges are of opposite sign. The value used for F must then be negative. Other than that, the solution method is the same as for part (*a*).

$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1\left(Q - Q_1\right)}{d^2} \quad \rightarrow \quad Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \frac{\left(90.0\times10^{-6}\,\mathrm{C}\right) \pm \sqrt{\left(90.0\times10^{-6}\,\mathrm{C}\right)^2 - 4\frac{\left(-12.0\,\mathrm{N}\right)\left(1.06\,\mathrm{m}\right)^2}{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)}}}{2}$$

$$= \boxed{104.4\times10^{-6}\,\mathrm{C}\,,\ -14.4\times10^{-6}\,\mathrm{C}}$$

22. The spheres can be treated as point charges since they are spherical, and so Coulomb's law may be used to relate the amount of charge to the force of attraction. Each sphere will have a magnitude Q of charge, since that amount was removed from one sphere and added to the other, being initially uncharged.

$$F = k\frac{Q_1Q_2}{r^2} = k\frac{Q^2}{r^2} \quad \rightarrow \quad Q = r\sqrt{\frac{F}{k}} = \left(0.12\,\mathrm{m}\right)\sqrt{\frac{1.7\times10^{-2}\,\mathrm{N}}{8.988\times10^9\,\mathrm{N\cdot m^2/C^2}}}$$

$$= 1.650\times10^{-7}\,\mathrm{C}\left(\frac{1\ \text{electron}}{1.602\times10^{-19}\,\mathrm{C}}\right) = \boxed{1.0\times10^{12}\,\text{electrons}}$$

23. Use Eq. 16–3 to calculate the force.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} \quad \rightarrow \quad \vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(-1.602\times10^{-19}\,\mathrm{C}\right)\left(2360\,\mathrm{N/C}\ \text{east}\right) = \boxed{3.78\times10^{-16}\,\mathrm{N}\ \text{west}}$$

24. Use Eq. 16–3 to calculate the electric field.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{3.75\times10^{-14}\,\mathrm{N}\,\text{south}}{1.602\times10^{-19}\,\mathrm{C}} = \boxed{2.34\times10^5\ \mathrm{N/C}\ \text{south}}$$

25. Use Eq. 16–3 to calculate the electric field.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{8.4\,\mathrm{N}\,\text{down}}{-8.8\times10^{-6}\,\mathrm{C}} = \boxed{9.5\times10^5\ \mathrm{N/C}\ \text{up}}$$

26. Use Eq. 16–4a to calculate the electric field due to a point charge.

$$E = k\frac{Q}{r^2} = \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{33.0\times10^{-6}\,\mathrm{C}}{\left(2.00\times10^{-1}\,\mathrm{m}\right)^2} = \boxed{7.42\times10^6\ \mathrm{N/C}\ \text{up}}$$

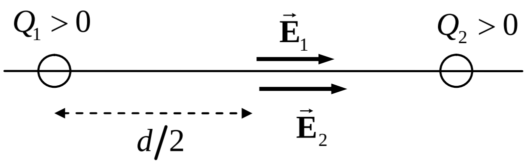
Note that the electric field points away from the positive charge.

27. Assuming the electric force is the only force on the electron, then Newton's 2nd law may be used to find the acceleration.

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \quad \rightarrow \quad a = \frac{|q|}{m}E = \frac{\left(1.602\times10^{-19}\,\text{C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)}750\,\text{N/C} = \boxed{1.32\times10^{14}\ \text{m/s}^2}$$

Since the charge is negative, the direction of the acceleration is $\boxed{\text{opposite to the field}}$.

$\boxed{28.}$ The electric field due to the negative charge will point toward the negative charge, and the electric field due to the positive charge will point away from the positive charge. Thus both fields point in the same direction, towards the negative charge, and so can be added.

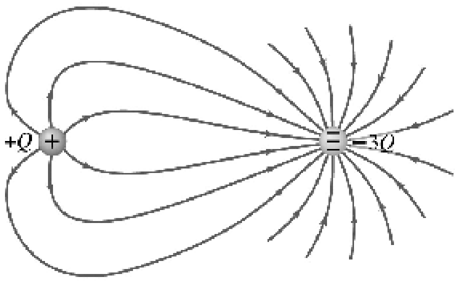


$$E = |E_1| + |E_2| = k\frac{|Q_1|}{r_1^2} + k\frac{|Q_2|}{r_2^2} = k\frac{|Q_1|}{(d/2)^2} + k\frac{|Q_2|}{(d/2)^2} = \frac{4k}{d^2}\left(|Q_1| + |Q_2|\right)$$

$$= \frac{4\left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(8.0\times10^{-2}\,\text{m}\right)^2}\left(8.0\times10^{-6}\,\text{C} + 7.0\times10^{-6}\,\text{C}\right) = \boxed{8.4\times10^{7}\ \text{N/C}}$$

The direction is $\boxed{\text{towards the negative charge}}$.

29.



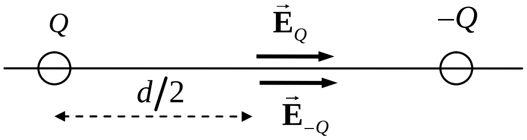
30. Assuming the electric force is the only force on the electron, then Newton's 2nd law may be used to find the electric field strength.

$$F_{net} = ma = qE \quad \rightarrow \quad E = \frac{ma}{q} = \frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(1\times10^{6}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.602\times10^{-19}\,\text{C}\right)} = 0.102\,\text{N/C} \approx \boxed{0.1\,\text{N/C}}$$

31. Since the electron accelerates from rest towards the north, the net force on it must be to the north. Assuming the electric force is the only force on the electron, then Newton's 2nd law may be used to find the electric field.

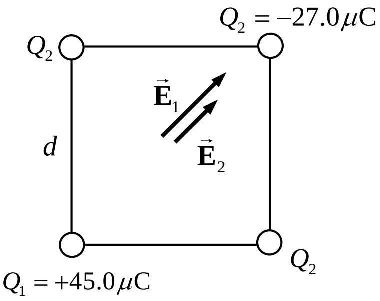
$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \quad \rightarrow \quad \vec{\mathbf{E}} = \frac{m}{q}\vec{\mathbf{a}} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(-1.602\times10^{-19}\,\text{C}\right)}\left(115\,\text{m/s}^2\ \text{north}\right) = \boxed{6.54\times10^{-10}\ \text{N/C south}}$$

32. The field due to the negative charge will point towards the negative charge, and the field due to the positive charge will point towards the negative charge. Thus the magnitudes of the two fields can be added together to find the charges.



$$E_{\text{net}} = 2E_Q = 2k\frac{Q}{(d/2)^2} = \frac{8kQ}{d^2} \;\rightarrow\; Q = \frac{Ed^2}{8k} = \frac{(745\,\text{N/C})(1.60\times10^{-1}\,\text{m})}{8(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)} = \boxed{2.65\times10^{-10}\,\text{C}}$$

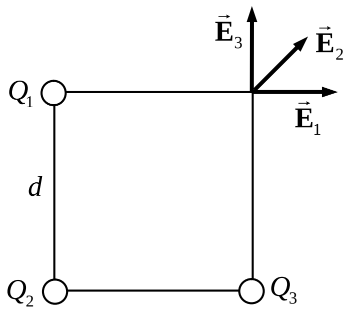
33. The field at the center due to the two negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, and so only the positive charge and the opposite negative charge need to be considered. The field due to the negative charge will point directly toward it, and the field due to the positive charge will point directly away from it. Accordingly, the two fields are in the same direction and can be added algebraically.



$$E = E_1 + E_2 = k\frac{Q_1}{d^2/2} + k\frac{|Q_2|}{d^2/2} = k\frac{Q_1 + |Q_2|}{d^2/2}$$

$$= (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(47.0+27.0)\times10^{-6}\,\text{C}}{(0.525\,\text{m})^2/2} = \boxed{4.70\times10^6\ \text{N/C at }45^\circ}$$

34. The field at the upper right corner of the square is the vector sum of the fields due to the other three charges. Let the variable d represent the 1.0 m length of a side of the square, and let the variable Q represent the charge at each of the three occupied corners.



$$E_1 = k\frac{Q}{d^2} \;\rightarrow\; E_{1x} = k\frac{Q}{d^2}\ ,\ E_{1y} = 0$$

$$E_2 = k\frac{Q}{2d^2} \;\rightarrow\; E_{2x} = k\frac{Q}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q}{4d^2}\ ,\ E_{2y} = k\frac{\sqrt{2}Q}{4d^2}$$

$$E_3 = k\frac{Q}{d^2} \;\rightarrow\; E_{3x} = 0\ ,\ E_{1y} = k\frac{Q}{d^2}$$

Add the x and y components together to find the total electric field, noting that $E_x = E_y$.

$$E_x = E_{1x} + E_{2x} + E_{3x} = k\frac{Q}{d^2} + k\frac{\sqrt{2}Q}{4d^2} + 0 = k\frac{Q}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right) = E_y$$

$$E = \sqrt{E_x^2 + E_y^2} = k\frac{Q}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q}{d^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$= (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.25\times10^{-6}\,\text{C})}{(1.00\,\text{m})^2}\left(\sqrt{2} + \frac{1}{2}\right) = \boxed{3.87\times10^4\ \text{N/C}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \boxed{45^\circ}\ \text{ from the } x\text{-direction.}$$

35. Choose the rightward direction to be positive. Then the field due to +*Q* will be positive, and the field due to –*Q* will be negative.

$$E = k\frac{Q}{(x+a)^2} - k\frac{Q}{(x-a)^2} = kQ\left(\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2}\right) = \boxed{\frac{-4kQxa}{(x^2-a^2)^2}}$$

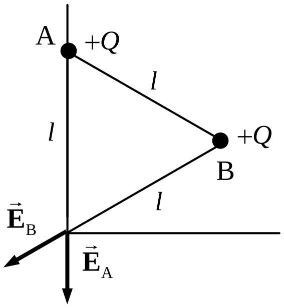
The negative sign means the field points to the $\boxed{\text{left}}$.

36. For the net field to be zero at point P, the magnitudes of the fields created by Q_1 and Q_2 must be equal. Also, the distance x will be taken as positive to the left of Q_1. That is the only region where the total field due to the two charges can be zero. Let the variable d represent the 12 cm distance, and note that $|Q_1| = \frac{1}{2}Q_2$.

$$\left|\vec{\mathbf{E}}_1\right| = \left|\vec{\mathbf{E}}_2\right| \rightarrow k\frac{|Q_1|}{x^2} = k\frac{Q_2}{(x+d)^2} \rightarrow$$

$$x = d\frac{\sqrt{|Q_1|}}{\left(\sqrt{Q_2} - \sqrt{|Q_1|}\right)} = d\frac{\sqrt{\frac{1}{2}Q_2}}{\left(\sqrt{Q_2} - \sqrt{\frac{1}{2}Q_2}\right)} = \frac{d}{\left(\sqrt{2}-1\right)} = \frac{12\,\text{cm}}{\sqrt{2}-1} = \boxed{29\,\text{cm}}$$

37. (*a*) The field due to the charge at A will point straight downward, and the field due to the charge at B will point along the line from A to the origin, 30° below the negative *x* axis.



$$E_A = k\frac{Q}{l^2} \rightarrow E_{Ax} = 0\,,\ E_{Ax} = -k\frac{Q}{l^2}$$

$$E_B = k\frac{Q}{l^2} \rightarrow E_{Bx} = -k\frac{Q}{l^2}\cos 30^\circ = -k\frac{\sqrt{3}Q}{2l^2}$$

$$E_{By} = -k\frac{Q}{l^2}\sin 30^\circ = -k\frac{Q}{2l^2}$$

$$E_x = E_{Ax} + E_{Bx} = -k\frac{\sqrt{3}Q}{2l^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{3Q}{2l^2}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4l^4} + \frac{9k^2Q^2}{4l^4}} = \sqrt{\frac{12k^2Q^2}{4l^4}} = \boxed{\frac{\sqrt{3}kQ}{l^2}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{-k\frac{3Q}{2l^2}}{-k\frac{\sqrt{3}Q}{2l^2}} = \tan^{-1}\frac{-3}{-\sqrt{3}} = \tan^{-1}\sqrt{3} = \boxed{240^\circ}$$

(*b*) Now reverse the direction of $\vec{\mathbf{E}}_A$

$$E_A = k\frac{Q}{l^2} \rightarrow E_{Ax} = 0\,,\ E_{Ax} = -k\frac{Q}{l^2}$$

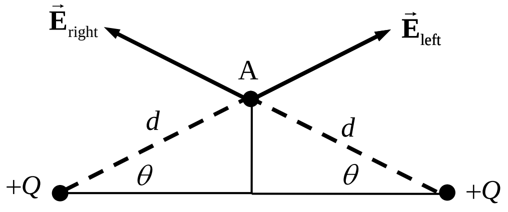
$$E_B = k\frac{Q}{l^2} \rightarrow E_{Bx} = k\frac{Q}{l^2}\cos 30^\circ = k\frac{\sqrt{3}Q}{2l^2}\,,\ E_{By} = k\frac{Q}{l^2}\sin 30^\circ = k\frac{Q}{2l^2}$$

$$E_x = E_{Ax} + E_{Bx} = k\frac{\sqrt{3}Q}{2l^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{Q}{2l^2}$$

$$E=\sqrt{E_x^2+E_y^2}=\sqrt{\frac{3k^2Q^2}{4l^4}+\frac{k^2Q^2}{4l^4}}=\sqrt{\frac{4k^2Q^2}{4l^4}}=\boxed{\frac{kQ}{l^2}}$$

$$\theta=\tan^{-1}\frac{E_y}{E_x}=\tan^{-1}\frac{k\dfrac{Q}{2l^2}}{-k\dfrac{\sqrt{3}Q}{2l^2}}=\tan^{-1}\frac{1}{-\sqrt{3}}=\boxed{330^\circ}$$

38. In each case, find the vector sum of the field caused by the charge on the left $\left(\vec{\mathbf{E}}_{\text{left}}\right)$ and the field caused by the charge on the right $\left(\vec{\mathbf{E}}_{\text{right}}\right)$



$\vec{\mathbf{E}}_{\text{right}}$
$\vec{\mathbf{E}}_{\text{left}}$
A
d
d
θ
θ
+Q
+Q

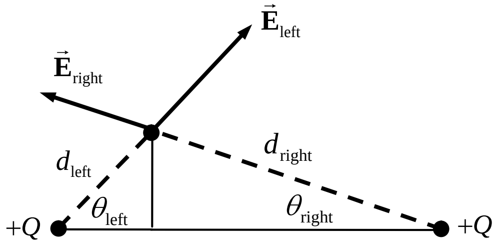
Point A: From the symmetry of the geometry, in calculating the electric field at point A only the vertical components of the fields need to be considered. The horizontal components will cancel each other.

$$\theta=\tan^{-1}\frac{5.0}{10.0}=26.6^\circ$$

$$d=\sqrt{(5.0\,\text{cm})^2+(10.0\,\text{cm})^2}=0.1118\,\text{m}$$

$$E_{\text{A}}=2\frac{kQ}{d^2}\sin\theta=2\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{7.0\times10^{-6}\,\text{C}}{(0.1118\,\text{m})^2}\sin26.6^\circ=\boxed{4.5\times10^6\ \text{N/C}}\qquad\theta_{\text{A}}=\boxed{90^\circ}$$

Point B: Now the point is not symmetrically placed, and so horizontal and vertical components of each individual field need to be calculated to find the resultant electric field.



$$\theta_{\text{left}}=\tan^{-1}\frac{5.0}{5.0}=45^\circ\qquad\theta_{\text{left}}=\tan^{-1}\frac{5.0}{15.0}=18.4^\circ$$

$$d_{\text{left}}=\sqrt{(5.0\,\text{cm})^2+(5.0\,\text{cm})^2}=0.0707\,\text{m}$$

$$d_{\text{right}}=\sqrt{(5.0\,\text{cm})^2+(15.0\,\text{cm})^2}=0.1581\,\text{m}$$

$$E_x=\left(\vec{\mathbf{E}}_{\text{left}}\right)_x+\left(\vec{\mathbf{E}}_{\text{right}}\right)_x=k\frac{Q}{d_{\text{left}}^2}\cos\theta_{\text{left}}-k\frac{Q}{d_{\text{right}}^2}\cos\theta_{\text{right}}$$

$$=\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.0\times10^{-6}\,\text{C}\right)\left[\frac{\cos45^\circ}{(0.0707\,\text{m})^2}-\frac{\cos18.4^\circ}{(0.1581\,\text{m})^2}\right]=6.51\times10^6\ \text{N/C}$$

$$E_y=\left(\vec{\mathbf{E}}_{\text{left}}\right)_y+\left(\vec{\mathbf{E}}_{\text{right}}\right)_y=k\frac{Q}{d_{\text{left}}^2}\sin\theta_{\text{left}}+k\frac{Q}{d_{\text{right}}^2}\sin\theta_{\text{right}}$$

$$=\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.0\times10^{-6}\,\text{C}\right)\left[\frac{\sin45^\circ}{(0.0707\,\text{m})^2}+\frac{\sin18.4^\circ}{(0.1581\,\text{m})^2}\right]=9.69\times10^6\ \text{N/C}$$

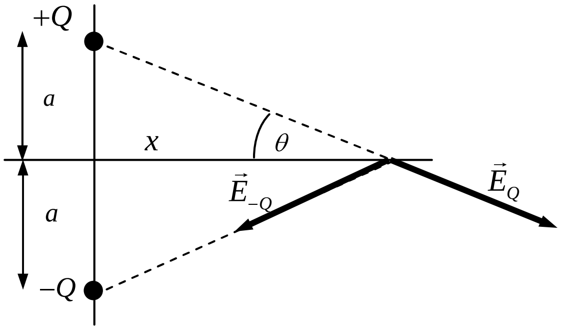
$$E_{\text{B}}=\sqrt{E_x^2+E_y^2}=\boxed{1.2\times10^7\ \text{N/C}}\qquad\theta_B=\tan^{-1}\frac{E_y}{E_x}=\boxed{56^\circ}$$

The results are consistent with Figure 16-31b. In the figure, the field at Point A points straight up, matching the calculations. The field at Point B should be to the right and vertical, matching the calculations. Finally, the field lines are closer together at Point B than at Point A, indicating that the field is stronger there, matching the calculations.

39. Both charges must be of the same sign so that the electric fields created by the two charges oppose each other, and so can add to zero. The magnitudes of the two electric fields must be equal.

$$E_1 = E_2 \;\rightarrow\; k\frac{Q_1}{(l/3)^2} = k\frac{Q_2}{(2l/3)^2} \;\rightarrow\; 9Q_1 = \frac{9Q_2}{4} \;\rightarrow\; \frac{Q_1}{Q_2} = \boxed{\frac{1}{4}}$$

40. From the diagram, we see that the *x* components of the two fields will cancel each other at the point P. Thus the net electric field will be in the negative *y*-direction, and will be twice the *y*-component of either electric field vector.



$$E_{\text{net}} = 2E\sin\theta = 2\frac{kQ}{x^2+a^2}\sin\theta$$

$$= \frac{2kQ}{x^2+a^2}\frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$= \boxed{\frac{2kQa}{\left(x^2+a^2\right)^{3/2}} \text{ in the negative } y \text{ direction}}$$

41. We assume that gravity can be ignored, which is proven in part (*b*).

(*a*) The electron will accelerate to the right. The magnitude of the acceleration can be found from setting the net force equal to the electric force on the electron. The acceleration is constant, so constant acceleration relationships can be used.

$$F_{\text{net}} = ma = |q|E \;\rightarrow\; a = \frac{|q|E}{m}$$

$$v^2 = v_0^2 + 2a\Delta x \;\rightarrow\; v = \sqrt{2a\Delta x} = \sqrt{2\frac{|q|E}{m}\Delta x}$$

$$= \sqrt{2\frac{\left(1.602\times10^{-19}\text{C}\right)\left(1.45\times10^4\text{ N/C}\right)}{\left(9.11\times10^{-31}\text{kg}\right)}\left(1.10\times10^{-2}\text{m}\right)} = \boxed{7.49\times10^6\text{ m/s}}$$

(*b*) The value of the acceleration caused by the electric field is compared to *g*.

$$a = \frac{|q|E}{m} = \frac{\left(1.602\times10^{-19}\text{C}\right)\left(1.45\times10^4\text{ N/C}\right)}{\left(9.11\times10^{-31}\text{kg}\right)} = 2.55\times10^{15}\text{ m/s}^2$$

$$\frac{a}{g} = \frac{2.55\times10^{15}\text{ m/s}^2}{9.80\text{ m/s}^2} = 2.60\times10^{14}$$

The acceleration due to gravity can be ignored compared to the acceleration caused by the electric field.

42. (*a*) The electron will experience a force in the opposite direction to the electric field. Since the electron is to be brought to rest, the electric field must be in the same direction as the initial velocity of the electron, and so is to the right.

(*b*) Since the field is uniform, the electron will experience a constant force, and therefore have a constant acceleration. Use constant acceleration relationships to find the field strength.

$$F = qE = ma \rightarrow a = \frac{qE}{m} \qquad v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{qE}{m}\Delta x \rightarrow$$

$$E = \frac{m\left(v^2 - v_0^2\right)}{2q\Delta x} = \frac{-mv_0^2}{2q\Delta x} = -\frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.0\times10^{6}\,\text{m/s}\right)^2}{2\left(-1.602\times10^{-19}\,\text{C}\right)\left(4.0\times10^{-2}\,\text{m}\right)} = \boxed{6.4\times10^{2}\,\text{N/C}}$$

43. Use Gauss's law to determine the enclosed charge.

$$\Phi_{\text{E}} = \frac{Q_{\text{encl}}}{\varepsilon_o} \rightarrow Q_{\text{encl}} = \Phi_{\text{E}}\varepsilon_o = \left(1.45\times10^{3}\,\text{N}\cdot\text{m}^2/\text{C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{1.28\times10^{-8}\,\text{C}}$$

44. (*a*) $\Phi_{\text{E}} = E_{\perp}A = E\pi r^2 = \left(5.8\times10^{2}\,\text{N/C}\right)\pi\left(1.8\times10^{-1}\,\text{m}\right)^2 = \boxed{59\,\text{N}\cdot\text{m}^2/\text{C}}$

(*b*) $\Phi_{\text{E}} = E_{\perp}A = \left(E\cos 45^{\circ}\right)\pi r^2 = \left(5.8\times10^{2}\,\text{N/C}\right)\left(\cos 45^{\circ}\right)\pi\left(1.8\times10^{-1}\,\text{m}\right)^2 = \boxed{42\,\text{N}\cdot\text{m}^2/\text{C}}$

(*c*) $\Phi_{\text{E}} = E_{\perp}A = \left(E\cos 90^{\circ}\right)\pi r^2 = \boxed{0}$

45. (*a*) Use Gauss's law to determine the electric flux.

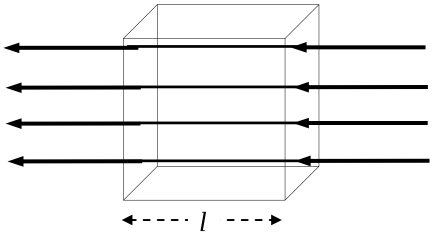
$$\Phi_{\text{E}} = \frac{Q_{\text{encl}}}{\varepsilon_o} = \frac{-1.0\times10^{-6}\,\text{C}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{-1.1\times10^{5}\,\text{N}\cdot\text{m}^2/\text{C}}$$

(*b*) Since there is no charge enclosed by surface A2, $\Phi_{\text{E}} = \boxed{0}$.

46. (*a*) Assuming that there is no charge contained within the cube, then the net flux through the cube is $\boxed{0}$. All of the field lines that enter the cube also leave the cube.

(*b*) There are four faces that have no flux through them, because none of the field lines pass through those faces. In the diagram shown, the left face has a positive flux and the right face has the opposite amount of negative flux.

$$\boxed{\begin{array}{l} \Phi_{\text{left}} = EA = El^2 = \left(6.50\times10^{3}\,\text{N/C}\right)l^2 \\ \Phi_{\text{right}} = -\left(6.50\times10^{3}\,\text{N/C}\right)l^2 \qquad \Phi_{\text{other}} = 0 \end{array}}$$



47. Equation 16-10 applies.

$$E = \frac{Q/A}{\varepsilon_0} \rightarrow Q = \varepsilon_0 EA = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(130\,\text{N/C}\right)\left(1.0\,\text{m}\right)^2 = \boxed{1.15\times10^{-9}\,\text{C}}$$

48. The electric field can be calculated by Eq. 16-4a, and that can be solved for the charge.

$$E = k\frac{Q}{r^2} \rightarrow Q = \frac{Er^2}{k} = \frac{\left(2.75\times10^{2}\,\text{N/C}\right)\left(3.50\times10^{-2}\,\text{m}\right)^2}{8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{3.75\times10^{-11}\,\text{C}}$$

This corresponds to about 2×10^{8} electrons. Since the field points toward the ball, the charge must be $\boxed{\text{negative}}$.

49. See Example 16-11 for a detailed discussion related to this problem.
    (*a*) Inside a solid metal sphere the electric field is $\boxed{0}$.
    (*b*) Inside a solid metal sphere the electric field is $\boxed{0}$.
    (*c*) Outside a solid metal sphere the electric field is the same as if all the charge were concentrated at the center as a point charge.

$$|E| = k\frac{|Q|}{r^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.50\times10^{-6}\,\text{C}\right)}{\left(3.10\,\text{m}\right)^2} = \boxed{3.27\times10^3\ \text{N/C}}$$

    (*d*) Same reasoning as in part (*c*).

$$|E| = k\frac{|Q|}{r^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.50\times10^{-6}\,\text{C}\right)}{\left(6.00\,\text{m}\right)^2} = \boxed{8.74\times10^2\ \text{N/C}}$$

    (*e*) The answers would be $\boxed{\text{no different}}$ for a thin metal shell.

50. See Figure 16-33 in the text for additional insight into this problem.
    (*a*) Inside the shell, the field is that of the point charge, $\boxed{E = k\frac{Q}{r^2}}$.
    (*b*) There is no field inside the conducting material: $\boxed{E = 0}$.
    (*c*) Outside the shell, the field is that of the point charge, $\boxed{E = k\frac{Q}{r^2}}$.
    (*d*) The shell does not affect the field due to *Q* alone, except in the shell material, where the field is 0. The charge *Q* does affect the shell – it polarizes it. There will be an induced charge of –*Q* uniformly distributed over the inside surface of the shell, and an induced charge of +*Q* uniformly distributed over the outside surface of the shell.

51. (*a*) The net force between the thymine and adenine is due to the following forces.

O – H attraction: $$F_{\text{OH}} = k\frac{(0.4e)(0.2e)}{\left(1.80\,\overset{\circ}{\text{A}}\right)^2} = \frac{0.08ke^2}{\left(1.80\,\overset{\circ}{\text{A}}\right)^2}$$

O – N repulsion: $$F_{\text{ON}} = k\frac{(0.4e)(0.2e)}{\left(2.80\,\overset{\circ}{\text{A}}\right)^2} = \frac{0.08ke^2}{\left(2.80\,\overset{\circ}{\text{A}}\right)^2}$$

N – N repulsion: $$F_{\text{NN}} = k\frac{(0.2e)(0.2e)}{\left(3.00\,\overset{\circ}{\text{A}}\right)^2} = \frac{0.04ke^2}{\left(3.00\,\overset{\circ}{\text{A}}\right)^2}$$

H – N attraction: $$F_{\text{HN}} = k\frac{(0.2e)(0.2e)}{\left(2.00\,\overset{\circ}{\text{A}}\right)^2} = \frac{0.04ke^2}{\left(2.00\,\overset{\circ}{\text{A}}\right)^2}$$

$$F_{\text{A-T}} = F_{\text{OH}} - F_{\text{ON}} - F_{\text{NN}} + F_{\text{HN}} = \left(\frac{0.08}{1.80^2} - \frac{0.08}{2.80^2} - \frac{0.04}{3.00^2} + \frac{0.04}{2.00^2}\right)\left(\frac{1}{1.0\times10^{-10}\,\text{m}}\right)^2\frac{ke^2}{d^2}$$

$$=(.02004)\frac{\left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(1.602\times10^{-19}\,\mathrm{C}\right)^2}{\left(1.0\times10^{-10}\,\mathrm{m}\right)^2}=4.623\times10^{-10}\,\mathrm{N}\approx\boxed{4.6\times10^{-10}\,\mathrm{N}}$$

(*b*) The net force between the cytosine and guanine is due to the following forces.

O – H attraction: $F_{\mathrm{OH}}=k\frac{(0.4e)(0.2e)}{\left(1.90\,\mathrm{\overset{o}{A}}\right)^2}=\frac{0.08ke^2}{\left(1.90\,\mathrm{\overset{o}{A}}\right)^2}$ (2 of these)

O – N repulsion: $F_{\mathrm{ON}}=k\frac{(0.4e)(0.2e)}{\left(2.90\,\mathrm{\overset{o}{A}}\right)^2}=\frac{0.08ke^2}{\left(2.90\,\mathrm{\overset{o}{A}}\right)^2}$ (2 of these)

H – N attraction: $F_{\mathrm{HN}}=k\frac{(0.2e)(0.2e)}{\left(2.00\,\mathrm{\overset{o}{A}}\right)^2}=\frac{0.04ke^2}{\left(2.00\,\mathrm{\overset{o}{A}}\right)^2}$

N – N repulsion: $F_{\mathrm{NN}}=k\frac{(0.2e)(0.2e)}{\left(3.00\,\mathrm{\overset{o}{A}}\right)^2}=\frac{0.04ke^2}{\left(3.00\,\mathrm{\overset{o}{A}}\right)^2}$

$$F_{\text{C-G}}=2F_{\mathrm{OH}}-2F_{\mathrm{ON}}-F_{\mathrm{NN}}+F_{\mathrm{HN}}=\left(2\frac{0.08}{1.90^2}-2\frac{0.08}{2.90^2}-\frac{0.04}{3.00^2}+\frac{0.04}{2.00^2}\right)\left(\frac{1}{1.0\times10^{-10}\,\mathrm{m}}\right)^2\frac{ke^2}{d^2}$$

$$=(.03085)\frac{\left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(1.602\times10^{-19}\,\mathrm{C}\right)^2}{\left(1.0\times10^{-10}\,\mathrm{m}\right)^2}=7.116\times10^{-10}\,\mathrm{N}\approx\boxed{7.1\times10^{-10}\,\mathrm{N}}$$

(*c*) For 10^5 pairs of molecules, we assume that half are A-T pairs and half are C-G pairs. We average the above results and multiply by 10^5.

$$F_{\mathrm{net}}=\tfrac{1}{2}10^5\left(F_{\text{A-T}}+F_{\text{C-G}}\right)=10^5\left(4.623\times10^{-10}\,\mathrm{N}+7.116\times10^{-10}\,\mathrm{N}\right)$$

$$=5.850\times10^{-5}\,\mathrm{N}\approx\boxed{6\times10^{-5}\,\mathrm{N}}$$

52. Set the magnitude of the electric force equal to the magnitude of the force of gravity and solve for the distance.

$$F_{\mathrm{E}}=F_{\mathrm{G}}\;\rightarrow\;k\frac{e^2}{r^2}=mg\;\rightarrow$$

$$r=e\sqrt{\frac{k}{mg}}=\left(1.602\times10^{-19}\,\mathrm{C}\right)\sqrt{\frac{\left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)}{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}}=\boxed{5.08\,\mathrm{m}}$$

53. Calculate the total charge on all electrons in 3.0 g of copper, and then compare the $38\mu\mathrm{C}$ to that value.

$$\text{Total electron charge}=3.0\,\mathrm{g}\left(\frac{1\text{ mole}}{63.5\,\mathrm{g}}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{\text{mole}}\right)\left(\frac{29\text{ e}}{\text{atoms}}\right)\left(\frac{1.602\times10^{-19}\,\mathrm{C}}{1\text{ e}}\right)$$

$$=1.32\times10^{5}\,\mathrm{C}$$

$$\text{Fraction lost} = \frac{38\times10^{-6}\,\text{C}}{1.32\times10^{5}\,\text{C}} = \boxed{2.9\times10^{-10}}$$

$\boxed{54.}$ Since the gravity force is downward, the electric force must be upward. Since the charge is positive, the electric field must also be upward. Equate the magnitudes of the two forces and solve for the electric field.

$$F_{\text{E}} = F_{\text{G}} \;\rightarrow\; qE = mg \;\rightarrow\; E = \frac{mg}{q} = \frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.602\times10^{-19}\,\text{C}\right)} = \boxed{1.02\times10^{-7}\ \text{N/C, up}}$$

55. Use Eq. 16-4a to calculate the magnitude of the electric charge on the Earth.

$$E = k\frac{Q}{r^2} \;\rightarrow\; Q = \frac{Er^2}{k} = \frac{\left(150\ \text{N/C}\right)\left(6.38\times10^{6}\,\text{m}\right)^2}{8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{6.8\times10^{5}\,\text{C}}$$

Since the electric field is pointing towards the Earth's center, the charge must be $\boxed{\text{negative}}$.

56. (*a*) From problem 55, we know that the electric field is pointed towards the Earth's center. Thus an electron in such a field would experience an upwards force of magnitude $F_{\text{E}} = eE$. The force of gravity on the electron will be negligible compared to the electric force.

$$F_{\text{E}} = eE = ma \;\rightarrow$$

$$a = \frac{eE}{m} = \frac{\left(1.602\times10^{-19}\,\text{C}\right)\left(150\,\text{N/C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)} = 2.638\times10^{13}\ \text{m/s}^2 \approx \boxed{2.6\times10^{13}\ \text{m/s}^2\text{, up}}$$

(*b*) A proton in the field would experience a downwards force of magnitude $F_{\text{E}} = eE$. The force of gravity on the proton will be negligible compared to the electric force.

$$F_{\text{E}} = eE = ma \;\rightarrow$$

$$a = \frac{eE}{m} = \frac{\left(1.602\times10^{-19}\,\text{C}\right)\left(150\,\text{N/C}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)} = 1.439\times10^{10}\ \text{m/s}^2 \approx \boxed{1.4\times10^{10}\ \text{m/s}^2\text{, down}}$$

(*c*) For the electron: $\dfrac{a}{g} = \dfrac{2.638\times10^{13}\ \text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{2.7\times10^{12}}$

For the proton: $\dfrac{a}{g} = \dfrac{1.439\times10^{10}\ \text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{1.5\times10^{9}}$

57. For the droplet to remain stationary, the magnitude of the electric force on the droplet must be the same as the weight of the droplet. The mass of the droplet is found from its volume times the density of water. Let *n* be the number of excess electrons on the water droplet.

$$F_{\text{E}} = |q|E = mg \;\rightarrow\; neE = \tfrac{4}{3}\pi r^3\rho g \;\rightarrow$$

$$n = \frac{4\pi r^3\rho g}{3eE} = \frac{4\pi\left(1.8\times10^{-5}\,\text{m}\right)^3\left(1.00\times10^{3}\ \text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)}{3\left(1.602\times10^{-19}\,\text{C}\right)\left(150\,\text{N/C}\right)} = 9.96\times10^{6} \approx \boxed{1.0\times10^{7}\,\text{electrons}}$$

58. There are four forces to calculate. Call the rightward direction the positive direction. The value of *k* is $8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$ and the value of *e* is $1.602\times10^{-19}\,\text{C}$.

$$F_{\text{net}} = F_{\text{CH}} + F_{\text{CN}} + F_{\text{OH}} + F_{\text{ON}} = \frac{k(0.40e)(0.20e)}{\left(1\times10^{-9}\,\text{m}\right)^2}\left[-\frac{1}{(0.30)^2}+\frac{1}{(0.40)^2}+\frac{1}{(0.18)^2}-\frac{1}{(0.28)^2}\right]$$

$$= 2.445\times10^{-10}\,\text{N} \approx \boxed{2.4\times10^{-10}\,\text{N}}$$

59. The electric force must be a radial force in order for the electron to move in a circular orbit.

$$F_{\text{E}} = F_{\text{radial}} \quad\to\quad k\frac{Q^2}{r_{\text{orbit}}^2} = \frac{mv^2}{r_{\text{orbit}}} \quad\to$$

$$r_{\text{orbit}} = k\frac{Q^2}{mv^2} = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\,\text{C}\right)^2}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.1\times10^6\,\text{m/s}\right)^2} = \boxed{2.1\times10^{-10}\,\text{m}}$$

60. Set the Coulomb electrical force equal to the Newtonian gravitational force on one of the bodies (the Moon).

$$F_{\text{E}} = F_{\text{G}} \quad\to\quad k\frac{Q^2}{r_{\text{orbit}}^2} = G\frac{M_{\text{Moon}}M_{\text{Earth}}}{r_{\text{orbit}}^2} \quad\to$$

$$Q = \sqrt{\frac{GM_{\text{Moon}}M_{\text{Earth}}}{k}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}} = \boxed{5.71\times10^{13}\,\text{C}}$$

$\boxed{61.}$ (*a*) The electron will experience a force in the opposite direction to the electric field. Thus the acceleration is in the opposite direction to the initial velocity. The force is constant, and so constant acceleration equations apply. To find the stopping distance, set the final velocity to 0.

$$F = eE = ma \quad\to\quad a = \frac{eE}{m} \qquad v^2 = v_0^2 + 2a\Delta x \quad\to$$

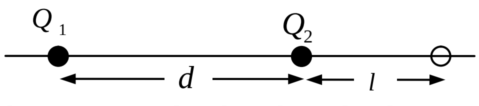
$$\Delta x = \frac{v^2 - v_0^2}{2a} = -\frac{mv_0^2}{2eE} = -\frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(21.5\times10^6\,\text{m/s}\right)^2}{2\left(-1.602\times10^{-19}\,\text{C}\right)\left(11.4\times10^3\,\text{N/C}\right)} = \boxed{0.115\,\text{m}}$$

(*b*) To return to the starting point, the velocity will reverse. Use that to find the time to return.

$$v = v_0 + at \quad\to$$

$$t = \frac{v - v_0}{a} = \frac{-v_0 - v_0}{a} = -\frac{2mv_0}{qE} = -\frac{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(21.5\times10^6\,\text{m/s}\right)}{\left(-1.602\times10^{-19}\,\text{C}\right)\left(11.4\times10^3\,\text{N/C}\right)} = \boxed{2.14\times10^{-8}\,\text{s}}$$

62. Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (Q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this means that *x* must be positive.



$$E = -k\frac{|Q_2|}{l^2} + k\frac{Q_1}{(l+d)^2} = 0 \quad\to\quad |Q_2|(l+d)^2 = Q_1 l^2 \quad\to$$

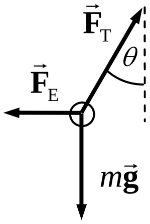
$$l=\frac{\sqrt{|Q_2|}}{\sqrt{Q_1}-\sqrt{|Q_2|}}d=\frac{\sqrt{5.0\times10^{-6}\,\mathrm{C}}}{\sqrt{2.5\times10^{-5}\,\mathrm{C}}-\sqrt{5.0\times10^{-6}\,\mathrm{C}}}(2.0\,\mathrm{m})=\boxed{\begin{array}{l}1.6\,\mathrm{m\ from\ }Q_2,\\2.6\,\mathrm{m\ from\ }Q_1\end{array}}$$

63. The sphere will oscillate sinusoidally about the equilibrium point, with an amplitude of 5.0 cm. The angular frequency of the sphere is given by $\omega=\sqrt{k/m}=\sqrt{126\,\mathrm{N/m}/0.800\,\mathrm{kg}}=12.5\,\mathrm{rad/s}$. The distance of the sphere from the table is given by $r=\left[0.150-0.050\cos(12.5t)\right]\mathrm{m}$. Use this distance and the charge to give the electric field value at the tabletop. That electric field will point upwards at all times, towards the negative sphere.

$$E=k\frac{|Q|}{r^2}=\frac{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(3.00\times10^{-6}\,\mathrm{C}\right)}{\left[0.150-0.050\cos(12.5t)\right]^2\,\mathrm{m}^2}=\frac{2.70\times10^4}{\left[0.150-0.050\cos(12.5t)\right]^2}\,\mathrm{N/C}$$

$$=\boxed{\frac{1.08\times10^7}{\left[3.00-\cos(12.5t)\right]^2}\,\mathrm{N/C},\ \text{upwards}}$$

64. The wires form two sides of an equilateral triangle, and so the two charges are separated by a distance $d=78$ cm and are directly horizontal from each other. Thus the electric force on each charge is horizontal. From the free-body diagram for one of the spheres, write the net force in both the horizontal and vertical directions and solve for the electric force. Then write the electric force by Coulomb's law, and equate the two expressions for the electric force to find the charge.



$$\sum F_y=F_\mathrm{T}\cos\theta-mg=0\ \rightarrow\ F_\mathrm{T}=\frac{mg}{\cos\theta}$$

$$\sum F_x=F_\mathrm{T}\sin\theta-F_\mathrm{E}=0\ \rightarrow\ F_\mathrm{E}=F_\mathrm{T}\sin\theta=\frac{mg}{\cos\theta}\sin\theta=mg\tan\theta$$

$$F_\mathrm{E}=k\frac{(Q/2)^2}{d^2}=mg\tan\theta\ \rightarrow\ Q=2d\sqrt{\frac{mg\tan\theta}{k}}$$

$$=2\left(7.8\times10^{-1}\,\mathrm{m}\right)\sqrt{\frac{\left(24\times10^{-3}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\tan30^\circ}{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)}}=6.064\times10^{-6}\,\mathrm{C}\approx\boxed{6.1\times10^{-6}\,\mathrm{C}}$$

65. The electric field at the surface of the pea is given by Equation (16-4a). Solve that equation for the charge.

$$E=k\frac{Q}{r^2}\ \rightarrow\ Q=\frac{Er^2}{k}=\frac{\left(3\times10^6\,\mathrm{N/C}\right)\left(3.75\times10^{-3}\,\mathrm{m}\right)^2}{8.988\times10^9\,\mathrm{N\cdot m^2/C^2}}=\boxed{5\times10^{-9}\,\mathrm{C}}$$

This corresponds to about 3 billion electrons.

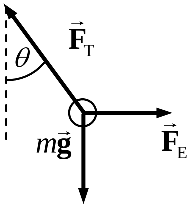
66. There will be a rightward force on Q_1 due to Q_2, given by Coulomb's law. There will be a leftward force on Q_1 due to the electric field created by the parallel plates. Let right be the positive direction.

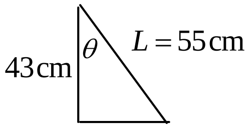
$$\sum F=k\frac{|Q_1Q_2|}{x^2}-|Q_1|E$$

$$=\left(8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(6.7\times10^{-6}\,\mathrm{C}\right)\left(1.8\times10^{-6}\,\mathrm{C}\right)}{\left(0.34\,\mathrm{m}\right)^2}-\left(6.7\times10^{-6}\,\mathrm{C}\right)\left(7.3\times10^{4}\ \mathrm{N/C}\right)$$

$$=\boxed{0.45\,\mathrm{N,\ right}}$$

$\boxed{67.}$ Since the electric field exerts a force on the charge in the same direction as the electric field, the charge is $\boxed{\text{positive}}$. Use the free-body diagram to write the equilibrium equations for both the horizontal and vertical directions, and use those equations to find the magnitude of the charge.





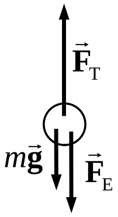
$$\theta=\cos^{-1}\frac{43}{55}=38.6^\circ$$

$$\sum F_x=F_\mathrm{E}-F_\mathrm{T}\sin\theta=0\ \rightarrow\ F_\mathrm{E}=F_\mathrm{T}\sin\theta=QE$$

$$\sum F_y=F_\mathrm{T}\cos\theta-mg=0\ \rightarrow\ F_\mathrm{T}=\frac{mg}{\cos\theta}\ \rightarrow\ QE=mg\tan\theta$$

$$Q=\frac{mg\tan\theta}{E}=\frac{\left(1.0\times10^{-3}\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\tan38.6^\circ}{\left(1.2\times10^{4}\ \mathrm{N/C}\right)}=\boxed{6.5\times10^{-7}\,\mathrm{C}}$$

68. The weight of the mass is only about 2 N. Since the tension in the string is more than that, there must be a downward electric force on the positive charge, which means that the electric field must be pointed $\boxed{\text{down}}$. Use the free-body diagram to write an expression for the magnitude of the electric field.



$$\sum F=F_\mathrm{T}-mg-F_\mathrm{E}=0\ \rightarrow\ F_\mathrm{E}=QE=F_\mathrm{T}-mg\ \rightarrow$$

$$E=\frac{F_\mathrm{T}-mg}{Q}=\frac{5.67\,\mathrm{N}-\left(0.210\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{3.40\times10^{-7}\mathrm{C}}=\boxed{1.06\times10^{7}\ \mathrm{N/C}}$$

69. To find the number of electrons, convert the mass to moles, the moles to atoms, and then multiply by the number of electrons in an atom to find the total electrons. Then convert to charge.

$$15\text{ kg Al}=\left(15\text{ kg Al}\right)\left(\frac{1\text{ mole Al}}{2.7\times10^{-2}\,\mathrm{kg}}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{1\text{ mole}}\right)\left(\frac{13\text{ electrons}}{1\text{ molecule}}\right)\left(\frac{-1.602\times10^{-19}\,\mathrm{C}}{\text{electron}}\right)$$

$$=\boxed{-7.0\times10^{8}\,\mathrm{C}}$$

The net charge of the bar is $\boxed{0\,\mathrm{C}}$, since there are equal numbers of protons and electrons.

70. (*a*) The force of sphere B on sphere A is given by Coulomb's law.

$$F_\mathrm{AB}=\boxed{\frac{kQ^2}{R^2},\ \text{away from B}}$$

(*b*) The result of touching sphere B to uncharged sphere C is that the charge on B is shared between the two spheres, and so the charge on B is reduced to $Q/2$. Again use Coulomb's law.

$$F_\mathrm{AB}=k\frac{QQ/2}{R^2}=\boxed{\frac{kQ^2}{2R^2},\ \text{away from B}}$$

(c) The result of touching sphere A to sphere C is that the charge on the two spheres is shared, and so the charge on A is reduced to $3Q/4$. Again use Coulomb's law.

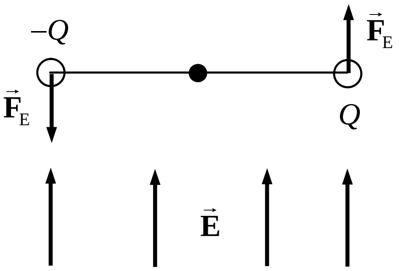
$$F_{\mathrm{AB}} = k\frac{(3Q/4)(Q/2)}{R^2} = \boxed{\frac{3kQ^2}{8R^2}\text{, away from B}}$$

71. On the *x*-axis, the electric field can only be zero at a location closer to the smaller magnitude charge. Thus the field will never be zero to the left of the midpoint between the two charges. Also, in between the two charges, the field due to both charges will point to the left, and so the total field cannot be zero. Thus the only place on the x-axis where the field can be zero is to the right of the negative charge, and so *x* must be positive. Calculate the field at point *P* and set it equal to zero.

$$E = k\frac{(-Q/2)}{x^2} + k\frac{Q}{(x+d)^2} = 0 \;\rightarrow\; 2x^2 = (x+d)^2 \;\rightarrow\; x = \boxed{\frac{d}{\sqrt{2}-1} \approx 2.41d}$$

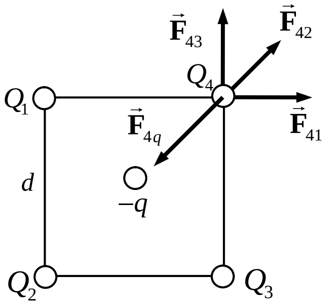
The field cannot be zero at any points off the *x*-axis. For any point off the *x*-axis, the electric fields due to the two charges will not be along the same line, and so they can never combine to give 0.

72. The electric field will put a force of magnitude $F_{\mathrm{E}} = QE$ on each charge. The distance of each charge from the pivot point is $L/2$, and so the torque caused by each force is $\tau = F_{\mathrm{E}} r_{\perp} = \frac{QEL}{2}$. Both torques will tend to make the rod rotate counterclockwise in the diagram, and so the net torque is $\tau_{\mathrm{net}} = 2\left(\frac{QEL}{2}\right) = \boxed{QEL}$.



$-Q$
$\vec{\mathbf{F}}_{\mathrm{E}}$
$\vec{\mathbf{F}}_{\mathrm{E}}$
Q
$\vec{\mathbf{E}}$

73. A negative charge must be placed at the center of the square. Let $Q = 8.0\,\mu\mathrm{C}$ be the charge at each corner, let -*q* be the magnitude of negative charge in the center, and let $d = 9.2\,\mathrm{cm}$ be the side length of the square. By the symmetry of the problem, if we make the net force on one of the corner charges be zero, the net force on each other corner charge will also be zero.



$$F_{41} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{41x} = k\frac{Q^2}{d^2},\; F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \;\rightarrow\; F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2},\; F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{43x} = 0,\; F_{43y} = k\frac{Q^2}{d^2}$$

$$F_{4q} = k\frac{qQ}{d^2/2} \;\rightarrow\; F_{4qx} = -k\frac{2qQ}{d^2}\cos 45^\circ = -k\frac{\sqrt{2}qQ}{d^2} = F_{4qy}$$

The net force in each direction should be zero.

$$\sum F_x = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 - k\frac{\sqrt{2}qQ}{d^2} = 0 \;\rightarrow\; q = Q\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = 7.66\times10^{-6}\,\mathrm{C}$$

So the charge to be placed is $-q = \boxed{-7.66\times10^{-6}\,\mathrm{C}}$.

This is an $\boxed{\text{unstable equilibrium}}$. If the center charge were slightly displaced, say towards the right, then it would be closer to the right charges than the left, and would be attracted more to the right. Likewise the positive charges on the right side of the square would be closer to it and would be attracted more to it, moving from their corner positions. The system would not have a tendency to return to the symmetric shape, but rather would have a tendency to move away from it if disturbed.

CHAPTER 17: Electric Potential

Answers to Questions

1. If two points are at the same potential, then no NET work was done in moving a test charge from one point to the other. Along some segments of the path, some positive work might have been done, but along other segments of the path, negative work would then have been done. And if the object was moved along an equipotential line, then no work would have been done along any segment of the path.

   Along any segment of the path where positive or negative work was done, a force would have to be exerted. If the object was moved along an equipotential line, then no force would have been exerted along any segment of the path.

   This is analogous to climbing up and then back down a flight of stairs to get from one point to another point on the same floor of a building. Gravitational potential increased while going up the stairs, and decreased while going down the stairs. A force was required both to go up the stairs and down the stairs. If instead you walked on the level from one point to another, then the gravitational potential was constant, and no force was need to change gravitational potential.

2. A negative charge will move toward a region of higher potential. A positive charge will move toward a region of lower potential. The potential energy of each will decrease.

3. (*a*) Electric potential, a scalar, is the electric potential energy per unit charge at a point in space. Electric field, a vector, is the electric force per unit charge at a point in space.
   (*b*) Electric potential energy is the work done against the electric force in moving a charge from a specified location of zero potential energy to some other location. Electric potential is the electric potential energy per unit charge.

4. The potential energy of the electron is proportional to the voltage used to accelerate it. Thus, if the voltage is multiplied by a factor of 4, then the potential energy is increased by a factor of 4 also. Then, by energy conservation, we assume that all of the potential energy is converted to kinetic energy during the acceleration process. Thus the kinetic energy has increased by a factor of 4 also. Finally, since the speed is proportional to the square root of kinetic energy, the speed must increase by a factor of 2.

5. The electric field is zero at the midpoint of the line segment joining the two equal positive charges. The electric field due to each charge is of the same magnitude at that location, because the location is equidistant from both charges, but the two fields are in the opposite direction. Thus the net electric field is zero there. The electric potential is never zero along that line, except at infinity. The electric potential due to each charge is positive, and so the total potential, which is the algebraic sum of the two potentials, is always positive.

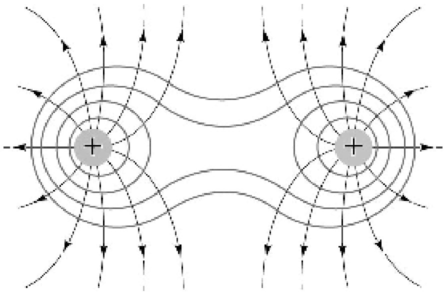
6. A negative particle will have its electric potential energy decrease if it moves from a region of low electric potential to one of high potential. By Eq. 17-3, if the charge is negative and the potential difference is positive, the change in potential energy will be negative, and so decrease.

7. The proton would gain half the kinetic energy as compared to the alpha particle. The alpha particle has twice the charge of the proton, and so has twice the potential energy for the same voltage. Thus the alpha will have twice the kinetic energy of the proton after acceleration.

8. There is no general relationship between the value of *V* and the value of $\vec{\mathbf{E}}$. Instead, the magnitude of $\vec{\mathbf{E}}$ is equal to the rate at which *V* decreases over a short distance. Consider the point midway between two positive charges. $\vec{\mathbf{E}}$ is 0 there, but *V* is high. Or, consider the point midway between two negative charges. $\vec{\mathbf{E}}$ is also 0 there, but *V* is low, because it is negative. Finally, consider the point midway between positive and negative charges of equal magnitude. There $\vec{\mathbf{E}}$ is not 0, because it points towards the negative charge, but *V* is zero.

9. Two equipotential lines cannot cross. That would indicate that a region in space had two different values for the potential. For example, if a 40-V line and a 50-V line crossed, then the potential at the point of crossing would be both 40 V and 50 V, which is impossible. Likewise, the electric field is perpendicular to the equipotential lines. If two lines crossed, the electric field at that point would point in two different directions simultaneously, which is not possible.

10. The equipotential lines are drawn so that they are perpendicular to the electric field lines where they cross.



11. The electric field would be zero in a region of space that has the same potential throughout. The electric field is related to the change in potential as you move from place to place. If the potential does not change, then the electric field is zero.

12. The orbit must be a circle. The gravitational potential (or potential energy) depends on the distance from the center of the Earth. If the potential is constant (equipotential line), then the distance from the center of the Earth must be constant, and so the orbit is a circle.

13. (*a*) *V* would decrease by 10 V at every location.
    (b) *E* is related to the change in electric potential. Decreasing the potential by 10 V everywhere would not affect the changes in potential from one location to another, and so would not affect *E*.

14. Any imbalance of charge that might exist would quickly be resolved. Suppose the positive plate, connected to the positive terminal of the battery, had more charge than the negative plate. Then negative charges from the negative battery terminal would be attracted to the negative plate by the more charged positive plate. This would only continue until the negative plate was as charged as the positive plate. If the negative plate became "over charged", then the opposite transfer of charge would take place, again until equilibrium was reached. Another way to explain the balance of charge is that neither the battery nor the capacitor can create or destroy charge. Since they were neutral before they were connected, they must be neutral after they are connected. The charge removed from one plate appears as excess on the other plate. This is true regardless of the conductor size or shape.

15. We meant that the capacitance did not depend on the amount of charge stored or on the potential difference between the capacitor plates. Changing the amount of charge stored or the potential difference will not change the capacitance.

Solutions to Problems

1. The work done by the electric field can be found from Eq. 17-2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \rightarrow W_{\text{ba}} = -qV_{\text{ba}} = -\left(-7.7\times10^{-6}\,\text{C}\right)\left(+55\text{ V}\right) = \boxed{4.2\times10^{-4}\,\text{J}}$$

2. The work done by the electric field can be found from Eq. 17-2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \rightarrow W_{\text{ba}} = -qV_{\text{ba}} = -\left(1.60\times10^{-19}\,\text{C}\right)\left(-1.80\times10^{2}\,\text{V}\right) = \boxed{2.88\times10^{-17}\,\text{J}}$$

$$= -\left(1\,e\right)\left(-180\text{ V}\right) = \boxed{1.80\times10^{2}\,\text{eV}}$$

3. The kinetic energy gained is equal to the work done on the electron by the electric field. The potential difference must be positive for the electron to gain potential energy. Use Eq. 17-2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \rightarrow W_{\text{ba}} = -qV_{\text{ba}} = -\left(-1.60\times10^{-19}\,\text{C}\right)\left(2.3\times10^{4}\,\text{V}\right) = \boxed{3.7\times10^{-15}\,\text{J}}$$

$$= -\left(-1\,e\right)\left(2.3\times10^{4}\,\text{V}\right) = \boxed{2.3\times10^{4}\,\text{eV}}$$

4. The kinetic energy gained by the electron is the work done by the electric force. Use Eq. 17-2b to calculate the potential difference.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} = -\frac{7.45\times10^{-17}\,\text{J}}{\left(-1.60\times10^{-19}\,\text{C}\right)} = \boxed{466\,\text{V}}$$

The electron moves from low potential to high potential, so $\boxed{\text{plate B}}$ is at the higher potential.

5. The magnitude of the electric field can be found from Eq. 17-4b.

$$E = \frac{V_{\text{ba}}}{d} = \frac{220\text{ V}}{5.8\times10^{-3}\,\text{m}} = \boxed{3.8\times10^{4}\text{ V/m}}$$

6. The magnitude of the voltage can be found from Eq. 17-4b.

$$E = \frac{V_{\text{ba}}}{d} \rightarrow V_{\text{ba}} = Ed = \left(640\,\text{V/m}\right)\left(11.0\times10^{-3}\,\text{m}\right) = \boxed{7.0\,\text{V}}$$

7. The distance between the plates can be found from Eq. 17-4b.

$$E = \frac{V_{\text{ba}}}{d} \rightarrow d = \frac{V_{\text{ba}}}{E} = \frac{45\text{ V}}{1500\,\text{V/m}} = \boxed{3.0\times10^{-2}\,\text{m}}$$

8. The gain of kinetic energy comes from a loss of potential energy due to conservation of energy, and the magnitude of the potential difference is the energy per unit charge.

$$\Delta V = \frac{\Delta\text{PE}}{q} = -\frac{\Delta\text{KE}}{q} = \frac{65.0\times10^{3}\,\text{eV}}{2e} = \boxed{-3.25\times10^{4}\,\text{V}}$$

The negative sign indicates that the helium nucleus had to go from a higher potential to a lower potential.

9. Find the distance corresponding to the maximum electric field, using Eq. 17-4b.

$$E=\frac{V_{\text{ba}}}{d} \rightarrow d=\frac{V_{\text{ba}}}{E}=\frac{200\text{ V}}{3\times10^{6}\text{ V/m}}=6.67\times10^{-5}\text{m}\approx\boxed{7\times10^{-5}\text{m}}$$

10. By the work energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by Eq. 17-2b.

$$W_{\text{external}}+W_{\text{electric}}=\text{KE}_{\text{final}}-\text{KE}_{\text{initial}} \rightarrow W_{\text{external}}-q\left(V_{\text{b}}-V_{\text{a}}\right)=\text{KE}_{\text{final}} \rightarrow$$

$$\left(V_{\text{b}}-V_{\text{a}}\right)=\frac{W_{\text{external}}-\text{KE}_{\text{final}}}{q}=\frac{15.0\times10^{-4}\text{J}-4.82\times10^{-4}\text{J}}{-8.50\times10^{-6}\text{C}}=\boxed{-1.20\times10^{2}\text{V}}$$

11. The kinetic energy of the electron is given in each case. Use the kinetic energy to find the speed.

(*a*) $\frac{1}{2}mv^{2}=\text{KE} \rightarrow v=\sqrt{\frac{2\text{KE}}{m}}=\sqrt{\frac{2\left(750\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{9.11\times10^{-31}\text{kg}}}=\boxed{1.6\times10^{7}\text{ m/s}}$

(*b*) $\frac{1}{2}mv^{2}=\text{KE} \rightarrow v=\sqrt{\frac{2\text{KE}}{m}}=\sqrt{\frac{2\left(3.2\times10^{3}\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{9.11\times10^{-31}\text{kg}}}=\boxed{3.4\times10^{7}\text{ m/s}}$

$\boxed{12.}$ The kinetic energy of the proton is given. Use the kinetic energy to find the speed.

$$\frac{1}{2}mv^{2}=\text{KE} \rightarrow v=\sqrt{\frac{2\text{KE}}{m}}=\sqrt{\frac{2\left(3.2\times10^{3}\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{1.67\times10^{-27}\text{ kg}}}=\boxed{7.8\times10^{5}\text{ m/s}}$$

13. The kinetic energy of the alpha particle is given. Use the kinetic energy to find the speed.

$$\frac{1}{2}mv^{2}=\text{KE} \rightarrow v=\sqrt{\frac{2\text{KE}}{m}}=\sqrt{\frac{2\left(5.53\times10^{6}\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{6.64\times10^{-27}\text{ kg}}}=\boxed{1.63\times10^{7}\text{ m/s}}$$

$\boxed{14.}$ Use Eq. 17-5 to find the potential.

$$V=\frac{1}{4\pi\varepsilon_{0}}\frac{Q}{r}=\left(8.99\times10^{9}\text{ N}\bullet\text{m}^{2}/\text{C}^{2}\right)\frac{4.00\times10^{-6}\text{C}}{1.50\times10^{-1}\text{m}}=\boxed{2.40\times10^{5}\text{V}}$$

15. Use Eq. 17-5 to find the charge.

$$V=\frac{1}{4\pi\varepsilon_{0}}\frac{Q}{r} \rightarrow Q=\left(4\pi\varepsilon_{0}\right)rV=\left(\frac{1}{8.99\times10^{9}\text{ N}\bullet\text{m}^{2}/\text{C}^{2}}\right)\left(0.15\text{ m}\right)\left(125\text{ V}\right)=\boxed{2.1\times10^{-9}\text{C}}$$

16. The work required is the difference in potential energy between the two locations. The test charge has potential energy due to each of the other charges, given in Conceptual Example 17-7 as $\text{PE}=k\frac{Q_{1}Q_{2}}{r}$. So to find the work, calculate the difference in potential energy between the two locations. Let Q represent the $35\mu\text{C}$ charge, let q represent the $0.50\mu\text{C}$ test charge, and let d represent the 32 cm distance.

$$\text{PE}_{\text{initial}}=\frac{kQq}{d/2}+\frac{kQq}{d/2} \qquad \text{PE}_{\text{final}}=\frac{kQq}{\left[d/2-0.12\text{ m}\right]}+\frac{kQq}{\left[d/2+0.12\text{ m}\right]}$$

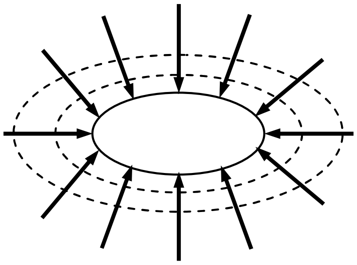
$$\text{Work} = \text{PE}_{\text{final}} - \text{PE}_{\text{initial}} = \frac{kQq}{[d/2 - 0.12\text{ m}]} + \frac{kQq}{[d/2 + 0.12\text{ m}]} - 2\left(\frac{kQq}{d/2}\right)$$

$$= kQq\left[\frac{1}{[0.16\,\text{m} - 0.12\,\text{m}]} + \frac{1}{[0.16\,\text{m} + 0.12\,\text{m}]} - \frac{2}{0.16\,\text{m}}\right]$$

$$= \left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(35\times10^{-6}\,\text{C}\right)\left(0.50\times10^{-6}\,\text{C}\right)\left(16.07\,\text{m}^{-1}\right) = \boxed{2.5\,\text{J}}$$

An external force needs to do positive work to move the charge.

17.



18. (*a*) The electric potential is given by Eq. 17-5.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{1.60\times10^{-19}\,\text{C}}{2.5\times10^{-15}\,\text{m}} = 5.754\times10^{5}\,\text{V} \approx \boxed{5.8\times10^{5}\,\text{V}}$$

(*b*) The potential energy of a pair of charges is derived in Conceptual Example 17.7.

$$\text{PE} = k\frac{Q_1Q_2}{r} = \left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\,\text{C}\right)^2}{2.5\times10^{-15}\,\text{m}} = \boxed{9.2\times10^{-14}\,\text{J}}$$

19. The potential at the corner is the sum of the potentials due to each of the charges, using Eq. 17-5.

$$V = \frac{k(3Q)}{L} + \frac{kQ}{\sqrt{2}L} + \frac{k(-2Q)}{L} = \frac{kQ}{L}\left(1 + \frac{1}{\sqrt{2}}\right) = \boxed{\frac{\sqrt{2}kQ}{2L}\left(\sqrt{2}+1\right)}$$

20. By energy conservation, all of the initial potential energy will change to kinetic energy of the electron when the electron is far away. The other charge is fixed, and so has no kinetic energy. When the electron is far away, there is no potential energy.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad \text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \quad\rightarrow\quad \frac{k(-e)(Q)}{r} = \tfrac{1}{2}mv^2 \quad\rightarrow$$

$$v = \sqrt{\frac{2k(-e)(Q)}{mr}} = \sqrt{\frac{2\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\,\text{C}\right)\left(-1.25\times10^{-7}\,\text{C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(0.325\,\text{m}\right)}}$$

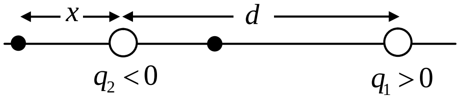
$$= \boxed{3.49\times10^{7}\ \text{m/s}}$$

21. By energy conservation, all of the initial potential energy of the charges will change to kinetic energy when the charges are very far away from each other. By momentum conservation, since the initial momentum is zero and the charges have identical masses, the charges will have equal speeds in opposite directions from each other as they move. Thus each charge will have the same kinetic energy.

$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; \text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \;\rightarrow\; \frac{kQ^2}{r} = 2\left(\tfrac{1}{2}mv^2\right) \;\rightarrow$$

$$v = \sqrt{\frac{kQ^2}{mr}} = \sqrt{\frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(9.5\times10^{-6}\,\text{C}\right)^2}{\left(1.0\times10^{-6}\,\text{kg}\right)\left(0.035\,\text{m}\right)}} = \boxed{4.8\times10^3\ \text{m/s}}$$

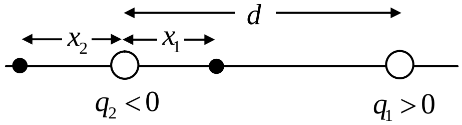
22. (*a*) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this is the point labeled as "*x*". Take to the right as the positive direction.



$$E = k\frac{|q_2|}{x^2} - k\frac{q_1}{(d+x)^2} = 0 \;\rightarrow\; |q_2|(d+x)^2 = q_1x^2 \;\rightarrow$$

$$x = \frac{\sqrt{|q_2|}}{\sqrt{q_1} - \sqrt{|q_2|}}d = \frac{\sqrt{2.0\times10^{-6}\,\text{C}}}{\sqrt{3.0\times10^{-6}\,\text{C}} - \sqrt{2.0\times10^{-6}\,\text{C}}}(5.0\,\text{cm}) = \boxed{22\,\text{cm left of } q_2}$$

(*b*) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge, any point where the potential is zero must be closer to the negative charge. So consider locations between the charges (position x_1) and to the left of the negative charge (position x_2) as shown in the diagram.

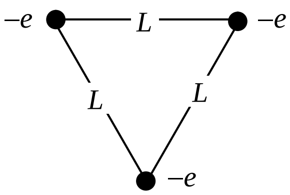


$$V_{\text{location 1}} = \frac{kq_1}{(d-x_1)} + \frac{kq_2}{x_1} = 0 \;\rightarrow\; x_1 = \frac{q_2 d}{(q_2 - q_1)} = \frac{\left(-2.0\times10^{-6}\,\text{C}\right)(5.0\,\text{cm})}{\left(-5.0\times10^{-6}\,\text{C}\right)} = 2.0\,\text{cm}$$

$$V_{\text{location 2}} = \frac{kq_1}{(d+x_2)} + \frac{kq_2}{x_2} = 0 \;\rightarrow\; x_2 = -\frac{q_2 d}{(q_1 + q_2)} = -\frac{\left(-2.0\times10^{-6}\,\text{C}\right)(5.0\,\text{cm})}{\left(1.0\times10^{-6}\,\text{C}\right)} = 10.0\,\text{cm}$$

So the two locations where the potential is zero are $\boxed{\text{2.0 cm from the negative charge towards the positive charge, and 10.0 cm from the negative charge away from the positive charge.}}$

23. Let the side length of the equilateral triangle be *L*. Imagine bringing the electrons in from infinity one at a time. It takes no work to bring the first electron to its final location, because there are no other charges present. Thus $W_1 = 0$. The work done in bringing in the second electron to its final location is equal to the charge on the electron times the potential (due to the first electron) at the final location of the second electron.



Thus $W_2 = (-e)\left(-\dfrac{ke}{L}\right) = \dfrac{ke^2}{L}$. The work done in bringing the third electron to its final location is equal to the charge on the electron times the potential (due to the first two electrons). Thus $W_3 = (-e)\left(-\dfrac{ke}{L} - \dfrac{ke}{L}\right) = \dfrac{2ke^2}{L}$. The total work done is the sum $W_1 + W_2 + W_3$.

$$W = W_1 + W_2 + W_3 = 0 + \frac{ke^2}{L} + \frac{2ke^2}{L} = \frac{3ke^2}{L} = \frac{3\left(8.99\times10^{9}\ \mathrm{N\bullet m^2/C^2}\right)\left(1.60\times10^{-19}\,\mathrm{C}\right)^2}{\left(1.0\times10^{-10}\,\mathrm{m}\right)}$$

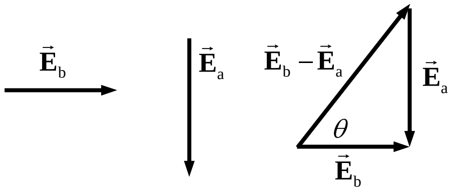
$$= \boxed{6.9\times10^{-18}\,\mathrm{J}}$$

24. (*a*) The potential due to a point charge is given by Eq. 17.5

$$V_{\mathrm{ba}} = V_{\mathrm{b}} - V_{\mathrm{a}} = \frac{kq}{r_{\mathrm{b}}} - \frac{kq}{r_{\mathrm{a}}} = kq\left(\frac{1}{r_{\mathrm{b}}} - \frac{1}{r_{\mathrm{a}}}\right)$$

$$= \left(8.99\times10^{9}\ \mathrm{N\bullet m^2/C^2}\right)\left(-3.8\times10^{-6}\,\mathrm{C}\right)\left(\frac{1}{0.88\,\mathrm{m}} - \frac{1}{0.72\,\mathrm{m}}\right) = \boxed{8.6\times10^{3}\,\mathrm{V}}$$

(*b*) The magnitude of the electric field due to a point charge is given by Eq. 16-4a. The direction of the electric field due to a negative charge is towards the charge, so the field at point a will point downward, and the field at point b will point to the right. See the vector diagram.


$\vec{\mathbf{E}}_{\mathrm{b}}$
$\vec{\mathbf{E}}_{\mathrm{a}}$
$\vec{\mathbf{E}}_{\mathrm{b}} - \vec{\mathbf{E}}_{\mathrm{a}}$
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θ
$\vec{\mathbf{E}}_{\mathrm{b}}$

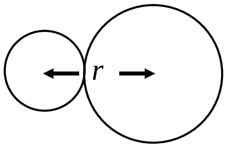
$$\vec{\mathbf{E}}_{\mathrm{b}} = \frac{k\left|q\right|}{r_{\mathrm{b}}^2}\ \text{right} = \frac{\left(8.99\times10^{9}\ \mathrm{N\bullet m^2/C^2}\right)\left(3.8\times10^{-6}\,\mathrm{C}\right)}{\left(0.88\,\mathrm{m}\right)^2}\ \text{right} = 4.4114\times10^{4}\ \mathrm{V/m},\ \text{right}$$

$$\vec{\mathbf{E}}_{\mathrm{a}} = \frac{k\left|q\right|}{r_{\mathrm{a}}^2}\ \text{down} = \frac{\left(8.99\times10^{9}\ \mathrm{N\bullet m^2/C^2}\right)\left(3.8\times10^{-6}\,\mathrm{C}\right)}{\left(0.72\,\mathrm{m}\right)^2}\ \text{down} = 6.5899\times10^{4}\ \mathrm{V/m},\ \text{down}$$

$$\left|\vec{\mathbf{E}}_{\mathrm{b}} - \vec{\mathbf{E}}_{\mathrm{a}}\right| = \sqrt{E_{\mathrm{a}}^2 + E_{\mathrm{b}}^2} - \sqrt{\left(4.4114\times10^{4}\ \mathrm{V/m}\right)^2 + \left(6.5899\times10^{4}\ \mathrm{V/m}\right)^2} = \boxed{7.9\times10^{4}\ \mathrm{V/m}}$$

$$\theta = \tan^{-1}\frac{E_{\mathrm{a}}}{E_{\mathrm{b}}} = \tan^{-1}\frac{65899}{44114} = \boxed{56^{\circ}}$$

25. We assume that all of the energy the proton gains in being accelerated by the voltage is changed to potential energy just as the proton's outer edge reaches the outer radius of the silicon nucleus.



$$\mathrm{PE}_{\mathrm{initial}} = \mathrm{PE}_{\mathrm{final}} \quad\rightarrow\quad eV_{\mathrm{initial}} = k\frac{e\left(14e\right)}{r} \quad\rightarrow$$

$$V_{\mathrm{initial}} = k\frac{14e}{r} = \left(8.99\times10^{9}\ \mathrm{N\bullet m^2/C^2}\right)\frac{\left(14\right)\left(1.60\times10^{-19}\,\mathrm{C}\right)}{\left(1.2+3.6\right)\times10^{-15}\mathrm{m}} = \boxed{4.2\times10^{6}\,\mathrm{V}}$$

26. Use Eq. 17-2b and Eq. 17-5.

$$V_{\mathrm{BA}} = V_{\mathrm{B}} - V_{\mathrm{A}} = \left(\frac{kq}{d-b} + \frac{k\left(-q\right)}{b}\right) - \left(\frac{kq}{b} + \frac{k\left(-q\right)}{d-b}\right) = kq\left(\frac{1}{d-b} - \frac{1}{b} - \frac{1}{b} + \frac{1}{d-b}\right)$$

$$= 2kq\left(\frac{1}{d-b} - \frac{1}{b}\right) = \boxed{\frac{2kq\left(2b-d\right)}{b\left(d-b\right)}}$$

27. (*a*) The electric potential is found from Eq. 17-5.

$$V_{\text{initial}} = k\frac{q_{\text{p}}}{r_{\text{atom}}} = \left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\,\text{C}\right)}{0.53\times10^{-10}\text{m}} \approx \boxed{27\,\text{V}}$$

(*b*) The kinetic energy can be found from the fact that the magnitude of the net force on the electron, which is the attraction by the proton, is causing circular motion.

$$\left|F_{\text{net}}\right| = \frac{m_{\text{e}}v^2}{r_{\text{atom}}} = k\frac{q_{\text{p}}\left|q_{\text{e}}\right|}{r_{\text{atom}}^2} \ \rightarrow\ m_{\text{e}}v^2 = k\frac{e^2}{r_{\text{atom}}} \ \rightarrow\ \text{KE} = \tfrac{1}{2}m_{\text{e}}v^2 = \tfrac{1}{2}k\frac{e^2}{r_{\text{atom}}}$$

$$= \tfrac{1}{2}\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(0.53\times10^{-10}\text{m}\right)} = 2.171\times10^{-18}\,\text{J} \approx \boxed{2.2\times10^{-18}\,\text{J}}$$

$$= 2.171\times10^{-18}\,\text{J}\times\frac{1\text{eV}}{1.60\times10^{-19}\text{J}} = 13.57\,\text{eV} \approx \boxed{14\,\text{eV}}$$

(*c*) The total energy of the electron is the sum of its KE and PE. The PE is found from Eq. 17-2a, and is negative since the electron's charge is negative.

$$E_{\text{total}} = \text{PE} + \text{KE} = -eV + \tfrac{1}{2}m_{\text{e}}v^2 = -k\frac{e^2}{r_{\text{atom}}} + \tfrac{1}{2}k\frac{e^2}{r_{\text{atom}}} = -\tfrac{1}{2}k\frac{e^2}{r_{\text{atom}}}$$

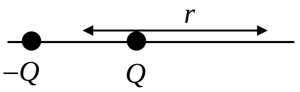
$$= -2.171\times10^{-18}\,\text{J} \approx \boxed{-2.2\times10^{-18}\,\text{J}} \approx \boxed{-14\,\text{eV}}$$

(*d*) If the electron is taken to infinity at rest, both its PE and KE would be 0. The amount of energy needed by the electron to have a total energy of 0 is just the opposite of the answer to part (c), $\boxed{2.2\times10^{-18}\text{J}}$ or $\boxed{14\,\text{eV}}$.

28. The dipole moment is the product of the magnitude of one of the charges times the separation distance.

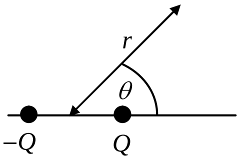
$$p = Ql = \left(1.60\times10^{-19}\,\text{C}\right)\left(0.53\times10^{-10}\,\text{m}\right) = \boxed{8.5\times10^{-30}\,\text{C}\bullet\text{m}}$$

29. The potential due to the dipole is given by Eq. 17-6b.



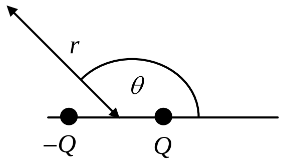
(*a*) $$V = \frac{kp\cos\theta}{r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(4.8\times10^{-30}\,\text{C}\bullet\text{m}\right)\cos0}{\left(1.1\times10^{-9}\text{m}\right)^2}$$

$$= \boxed{3.6\times10^{-2}\,\text{V}}$$



(*b*) $$V = \frac{kp\cos\theta}{r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(4.8\times10^{-30}\,\text{C}\bullet\text{m}\right)\cos45^\circ}{\left(1.1\times10^{-9}\text{m}\right)^2}$$

$$= \boxed{2.5\times10^{-2}\,\text{V}}$$



(*c*) $$V = \frac{kp\cos\theta}{r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(4.8\times10^{-30}\,\text{C}\bullet\text{m}\right)\cos135^\circ}{\left(1.1\times10^{-9}\text{m}\right)^2}$$

$$= \boxed{-2.5\times10^{-2}\,\text{V}}$$

30. We assume that $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ are equal in magnitude, and that each makes a 52° angle with $\vec{\mathbf{p}}$. The magnitude of $\vec{\mathbf{p}}_1$ is also given by $p_1 = qd$, where q is the charge on the hydrogen atom, and d is the distance between the H and the O.

$$p = 2p_1 \cos 52^\circ \quad \rightarrow \quad p_1 = \frac{p}{2\cos 52^\circ} = qd \quad \rightarrow$$

$$q = \frac{p}{2d\cos 52^\circ} = \frac{6.1\times10^{-30}\,\mathrm{C\bullet m}}{2\left(0.96\times10^{-10}\,\mathrm{m}\right)\cos 52^\circ} = \boxed{5.2\times10^{-20}\,\mathrm{C}}$$

This is about 0.32 times the charge on an electron.

31. The capacitance is found from Eq. 17-7.

$$Q = CV \quad \rightarrow \quad C = \frac{Q}{V} = \frac{2.5\times10^{-3}\,\mathrm{C}}{850\,\mathrm{V}} = \boxed{2.9\times10^{-6}\,\mathrm{F}}$$

32. The voltage is found from Eq. 17-7.

$$Q = CV \quad \rightarrow \quad V = \frac{Q}{C} = \frac{16.5\times10^{-8}\,\mathrm{C}}{9.5\times10^{-9}\,\mathrm{F}} = \boxed{17.4\,\mathrm{V}}$$

33. The capacitance is found from Eq. 17-7.

$$Q = CV \quad \rightarrow \quad C = \frac{Q}{V} = \frac{95\times10^{-12}\,\mathrm{C}}{120\,\mathrm{V}} = \boxed{7.9\times10^{-13}\,\mathrm{F}}$$

34. We assume the capacitor is fully charged, according to Eq. 17-7.

$$Q = CV = \left(7.00\times10^{-6}\,\mathrm{F}\right)\left(12.0\,\mathrm{V}\right) = \boxed{8.4\times10^{-5}\,\mathrm{C}}$$

35. The area can be found from Eq. 17-8.

$$C = \frac{\varepsilon_0 A}{d} \quad \rightarrow \quad A = \frac{Cd}{\varepsilon_0} = \frac{\left(0.20\,\mathrm{F}\right)\left(2.2\times10^{-3}\,\mathrm{m}\right)}{\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)} = \boxed{5.0\times10^{7}\,\mathrm{m^2}}$$

36. Let Q_1 and V_1 be the initial charge and voltage on the capacitor, and let Q_2 and V_2 be the final charge and voltage on the capacitor. Use Eq. 17-7 to relate the charges and voltages to the capacitance.

$$Q_1 = CV_1 \qquad Q_2 = CV_2 \qquad Q_2 - Q_1 = CV_2 - CV_1 = C\left(V_2 - V_1\right) \quad \rightarrow$$

$$C = \frac{Q_2 - Q_1}{V_2 - V_1} = \frac{18\times10^{-6}\,\mathrm{C}}{24\,\mathrm{V}} = \boxed{7.5\times10^{-7}\,\mathrm{F}}$$

37. The desired electric field is the value of V/d for the capacitor. Combine Eq. 17-7 and Eq. 17-8 to find the charge.

$$Q = CV = \frac{\varepsilon_0 AV}{d} = \varepsilon_0 AE = \left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(35.0\times10^{-4}\,\mathrm{m^2}\right)\left(8.50\times10^{5}\,\mathrm{V/m}\right)$$

$$= \boxed{2.63\times10^{-8}\,\mathrm{C}}$$

38. Combine Eq. 17-7 and Eq. 17-8 to find the area.

$$Q = CV = \frac{\varepsilon_0 AV}{d} = \varepsilon_0 AE \quad\rightarrow\quad A = \frac{Q}{\varepsilon_0 E} = \frac{\left(5.2\times10^{-6}\,\text{C}\right)}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(\dfrac{2000\,\text{V}}{10^{-3}\text{m}}\right)} = \boxed{0.29\ \text{m}^2}$$

39. From Eq. 17-4a, the voltage across the capacitor is the magnitude of the electric field times the separation distance of the plates. Use that with Eq. 17-7.

$$Q = CV = CEd \quad\rightarrow\quad E = \frac{Q}{Cd} = \frac{\left(72\times10^{-6}\,\text{C}\right)}{\left(0.80\times10^{-6}\,\text{F}\right)\left(2.0\times10^{-3}\,\text{m}\right)} = \boxed{4.5\times10^{4}\ \text{V/m}}$$

40. After the first capacitor is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected together, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the final potential difference to find the value of the second capacitor.

$$Q_{\text{Total}} = C_1 V_{1_{\text{initial}}} \qquad Q_{1_{\text{final}}} = C_1 V_{\text{final}} \qquad Q_{2_{\text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{1_{\text{final}}} + Q_{2_{\text{final}}} = \left(C_1 + C_2\right) V_{\text{final}} \quad\rightarrow\quad C_1 V_{1_{\text{initial}}} = \left(C_1 + C_2\right) V_{\text{final}} \quad\rightarrow$$

$$C_2 = C_1\left(\frac{V_{1_{\text{initial}}}}{V_{\text{final}}} - 1\right) = \left(7.7\times10^{-6}\,\text{F}\right)\left(\frac{125\,\text{V}}{15\,\text{V}} - 1\right) = \boxed{5.6\times10^{-5}\,\text{F}}$$

41. The total charge on the combination of capacitors is the sum of the charges on the two individual capacitors, since there is no battery connected to them to supply additional charge. The voltage across each capacitor must be the same after they are connected, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the fact of equal potentials to find the charge on each capacitor and the common potential difference.

$$Q_{1_{\text{initial}}} = C_1 V_{1_{\text{initial}}} \qquad Q_{2_{\text{initial}}} = C_2 V_{2_{\text{initial}}} \qquad Q_{1_{\text{final}}} = C_1 V_{\text{final}} \qquad Q_{2_{\text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{1_{\text{initial}}} + Q_{2_{\text{initial}}} = Q_{1_{\text{final}}} + Q_{2_{\text{final}}} = C_1 V_{1_{\text{initial}}} + C_2 V_{2_{\text{initial}}} = C_1 V_{\text{final}} + C_2 V_{\text{final}} \quad\rightarrow$$

$$V_{\text{final}} = \frac{C_1 V_{1_{\text{initial}}} + C_2 V_{2_{\text{initial}}}}{C_1 + C_2} = \frac{\left(2.50\times10^{-6}\,\text{F}\right)\left(875\,\text{V}\right) + \left(6.80\times10^{-6}\,\text{F}\right)\left(652\,\text{V}\right)}{\left(9.30\times10^{-6}\,\text{F}\right)}$$

$$= 711.95\,\text{V} \approx \boxed{712\,\text{V}} = V_1 = V_2$$

$$Q_{1_{\text{final}}} = C_1 V_{\text{final}} = \left(2.50\times10^{-6}\,\text{F}\right)\left(711.95\right) = \boxed{1.78\times10^{-3}\text{C}}$$

$$Q_{2_{\text{final}}} = C_2 V_{\text{final}} = \left(6.80\times10^{-6}\,\text{F}\right)\left(711.95\right) = \boxed{4.84\times10^{-3}\text{C}}$$

42. Use Eq. 17-9 to calculate the capacitance with a dielectric.

$$C = K\varepsilon_0 \frac{A}{d} = (2.2)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\frac{\left(5.5\times10^{-2}\,\mathrm{m}\right)^2}{\left(1.8\times10^{-3}\,\mathrm{m}\right)} = \boxed{3.3\times10^{-11}\,\mathrm{F}}$$

$\boxed{43.}$ Use Eq. 17-9 to calculate the capacitance with a dielectric.

$$C = K\varepsilon_0 \frac{A}{d} = (7)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\frac{\pi\left(5.0\times10^{-2}\,\mathrm{m}\right)^2}{\left(3.2\times10^{-3}\,\mathrm{m}\right)} = \boxed{1.5\times10^{-10}\,\mathrm{F}}$$

44. The initial charge on the capacitor is $Q_{\text{initial}} = C_{\text{initial}}V$. When the mica is inserted, the capacitance changes to $C_{\text{final}} = KC_{\text{initial}}$, and the voltage is unchanged since the capacitor is connected to the same battery. The final charge on the capacitor is $Q_{\text{final}} = C_{\text{final}}V$.

$$\Delta Q = Q_{\text{final}} - Q_{\text{initial}} = C_{\text{final}}V - C_{\text{initial}}V = (K-1)C_{\text{initial}}V = (7-1)\left(3.5\times10^{-9}\,\mathrm{F}\right)(22\,\mathrm{V})$$
$$= \boxed{4.6\times10^{-7}\,\mathrm{C}}$$

45. The capacitance is found from Eq. 17-7, with the voltage given by Eq. 17-4 (ignoring the sign).

$$Q = CV = C(Ed) \;\rightarrow\; C = \frac{Q}{Ed} = \frac{0.775\times10^{-6}\,\mathrm{C}}{\left(8.24\times10^{4}\,\mathrm{V/m}\right)\left(1.95\times10^{-3}\,\mathrm{m}\right)} = \boxed{4.82\times10^{-9}\,\mathrm{F}}$$

The plate area is found from Eq. 17-9.

$$C = K\varepsilon_0\frac{A}{d} \;\rightarrow\; A = \frac{Cd}{K\varepsilon_0} = \frac{\left(4.82\times10^{-9}\,\mathrm{F}\right)\left(1.95\times10^{-3}\,\mathrm{m}\right)}{(3.75)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)} = \boxed{0.283\,\mathrm{m^2}}$$

46. The stored energy is given by Eq. 17-10.

$$\mathrm{PE} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(2.2\times10^{-9}\,\mathrm{F}\right)(650\,\mathrm{V})^2 = \boxed{4.6\times10^{-4}\,\mathrm{J}}$$

47. The capacitance can be found from the stored energy using Eq. 17-10.

$$\mathrm{PE} = \tfrac{1}{2}CV^2 \;\rightarrow\; C = \frac{2(\mathrm{PE})}{V^2} = \frac{2(1200\,\mathrm{J})}{\left(5.0\times10^{3}\,\mathrm{V}\right)^2} = \boxed{9.6\times10^{-5}\,\mathrm{F}}$$

48. The two charged plates form a capacitor. Use Eq. 17-8 to calculate the capacitance, and Eq. 17-10 for the energy stored in the capacitor.

$$C = \frac{\varepsilon_0 A}{d} \qquad \mathrm{PE} = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{Q^2 d}{\varepsilon_0 A} = \tfrac{1}{2}\frac{\left(4.2\times10^{-4}\,\mathrm{C}\right)^2\left(1.5\times10^{-3}\,\mathrm{m}\right)}{\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(8.0\times10^{-2}\,\mathrm{m}\right)^2} = \boxed{2.3\times10^{3}\,\mathrm{J}}$$

49. (*a*) Use Eq. 17-8 to estimate the capacitance.

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\pi\left(4.5\,\mathrm{in}\times.0254\,\mathrm{m/in}\right)^2}{\left(5.0\times10^{-2}\,\mathrm{m}\right)} = 7.265\times10^{-12}\,\mathrm{F} \approx \boxed{7\times10^{-12}\,\mathrm{F}}$$

(*b*) Use Eq. 17-7 to estimate the charge on each plate.

$$Q = CV = \left(7.265\times10^{-12}\,\text{F}\right)\left(9\,\text{V}\right) = 6.54\times10^{-11}\,\text{C} \approx \boxed{7\times10^{-11}\,\text{C}}$$

(*c*) Use Eq. 17-4b to estimate the (assumed uniform) electric field between the plates. The actual location of the field measurement is not critical in this approximation.

$$E = \frac{V}{d} = \frac{9\,\text{V}}{5.0\times10^{-2}\,\text{m}} = 180\,\text{V/m} \approx \boxed{200\,\text{V/m}}$$

(*d*) By energy conservation, the work done by the battery to charge the plates is the potential energy stored in the capacitor, given by Eq. 17-10.

$$\text{PE} = \tfrac{1}{2}QV = \tfrac{1}{2}\left(6.54\times10^{-11}\,\text{C}\right)\left(9\,\text{V}\right) = 2.94\times10^{-10}\,\text{J} \approx \boxed{3\times10^{-10}\,\text{J}}$$

(*e*) If a dielectric is inserted, the $\boxed{\text{capacitance}}$ changes, and so the $\boxed{\text{charge}}$ on the capacitor and the $\boxed{\text{energy}}$ stored also change. The electric field does not change because it only depends on the battery voltage and the plate separation.

50. (*a*) From Eq. 17-8 for the capacitance, $C = \varepsilon_0 \dfrac{A}{d}$, since the separation of the plates is doubled, the capacitance is reduced to half its original value. Then from Eq. 17-10 for the energy, $\text{PE} = \tfrac{1}{2}\dfrac{Q^2}{C}$, since the charge is constant and the capacitance is halved, the energy is doubled. So the energy stored changes by a factor of $\boxed{2}$.

(*b*) The work done to separate the plates is the source of the increase of stored energy. So the work is the change in the stored energy.

$$\text{PE}_{\text{final}} - \text{PE}_{\text{initial}} = 2\text{PE}_{\text{initial}} - \text{PE}_{\text{initial}} = \text{PE}_{\text{initial}} = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{Q^2}{\varepsilon_0 \dfrac{A}{d}} = \boxed{\frac{Q^2 d}{2\varepsilon_0 A}}$$

51. (*a*) The energy stored in the capacitor is given by Eq. 17-10, $\text{PE} = \tfrac{1}{2}CV^2$. Assuming the capacitance is constant, then if the potential difference is doubled, the stored energy is $\boxed{\text{multiplied by 4}}$.

(*b*) Now we assume the potential difference is constant, since the capacitor remains connected to a battery. Then the energy stored in the capacitor is given by $\text{PE} = \tfrac{1}{2}QV$, and so the stored energy is $\boxed{\text{multiplied by 2}}$.

52. (*a*) Before the two capacitors are connected, all of the stored energy is in the first capacitor. Use Eq. 17-10.

$$\text{PE} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(2.70\times10^{-6}\,\text{F}\right)\left(12.0\,\text{V}\right)^2 = 1.944\times10^{-4}\,\text{J} \approx \boxed{1.94\times10^{-4}\,\text{J}}$$

(*b*) After the first capacitor is disconnected from the battery, the total charge must remain constant, and the voltage across each capacitor must be the same, since each capacitor plate is connected to a corresponding plate on the other capacitor by a connecting wire which always has a constant potential. Use the total charge and the fact of equal potentials to find the charge on each capacitor, and then calculate the stored energy.

$$Q_{\text{total}} = C_1 V_{\text{initial}} = \left(2.70\times10^{-6}\,\text{F}\right)\left(12.0\,\text{V}\right) = 3.24\times10^{-5}\,\text{C}$$

$$Q_1 = C_1 V \quad Q_2 = C_2 V \quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_{\text{total}} - Q_1}{C_2} \quad \rightarrow$$

$$Q_1 = Q_{\text{total}} \frac{C_1}{C_1 + C_2} = \left(3.24\times10^{-5}\,\text{C}\right)\frac{\left(2.70\times10^{-6}\,\text{F}\right)}{\left(6.70\times10^{-6}\,\text{F}\right)} = 1.3057\times10^{-5}\,\text{C}$$

$$\text{Q}_2 = Q_{\text{total}} - Q_1 = 3.24\times10^{-5}\,\text{C} - 1.3057\times10^{-5}\,\text{C} = 1.9343\times10^{-5}\,\text{C}$$

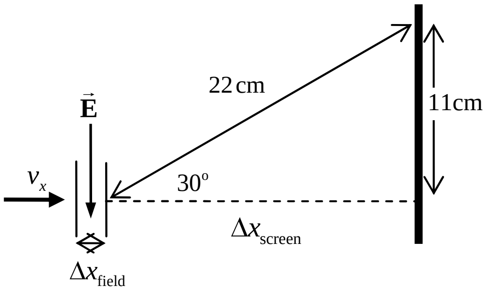
$$\text{PE}_{\text{total}} = \text{PE}_1 + \text{PE}_2 = \tfrac{1}{2}\frac{Q_1^2}{C_1} + \tfrac{1}{2}\frac{Q_2^2}{C_2} = \tfrac{1}{2}\left[\frac{\left(1.3057\times10^{-5}\,\text{C}\right)^2}{\left(2.70\times10^{-6}\,\text{F}\right)} + \frac{\left(1.9343\times10^{-5}\,\text{C}\right)^2}{\left(4.00\times10^{-6}\,\text{F}\right)}\right]$$

$$= \tfrac{1}{2}\left[\frac{\left(1.3057\times10^{-5}\,\text{C}\right)^2}{\left(2.70\times10^{-6}\,\text{F}\right)} + \frac{\left(1.9343\times10^{-5}\,\text{C}\right)^2}{\left(4.00\times10^{-6}\,\text{F}\right)}\right] = \boxed{7.83\times10^{-5}\,\text{J}}$$

(*c*) Subtract the two energies to find the change.

$$\Delta\text{PE} = \text{PE}_{\text{final}} - \text{PE}_{\text{initial}} = 7.83\times10^{-5}\,\text{J} - 1.944\times10^{-4}\,\text{J} = \boxed{-1.16\times10^{-4}\,\text{J}}$$

53. Consider three parts to the electron's motion. First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron v_x can be found from the accelerating potential V. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron an upward velocity, v_y. We assume that there is very little upward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the top of the screen, moving at an angle of approximately 30°.



Acceleration:

$$\text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \quad \rightarrow \quad eV = \tfrac{1}{2} m v_x^2 \quad \rightarrow \quad v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

$$\text{time in field: } \Delta x_{\text{field}} = v_x t_{\text{field}} \quad \rightarrow \quad t_{\text{field}} = \frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y = eE = ma_y \quad \rightarrow \quad a_y = \frac{eE}{m} \qquad v_y = v_0 + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \quad \rightarrow \quad t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\dfrac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \quad \rightarrow$$

$$E=\frac{\Delta y_{\text{screen}}mv_x^2}{\Delta x_{\text{screen}}e\Delta x_{\text{field}}}=\frac{\Delta y_{\text{screen}}m\frac{2eV}{m}}{\Delta x_{\text{screen}}e\Delta x_{\text{field}}}=\frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}\Delta x_{\text{field}}}=\frac{2\left(7.0\times10^3\,\text{V}\right)\left(0.11\,\text{m}\right)}{\left[\left(0.22\text{ m}\right)\cos30^\circ\right]\left(0.028\,\text{m}\right)}$$

$$=2.89\times10^5\text{ V/m}\approx\boxed{2.9\times10^5\text{ V/m}}$$

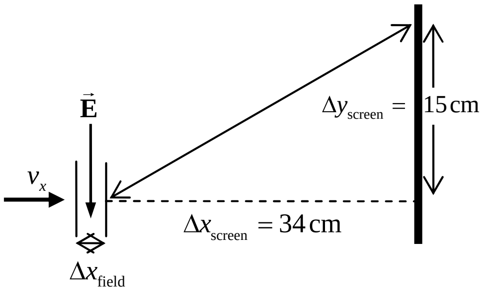
As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y=v_0t_{\text{field}}+\tfrac{1}{2}a_yt_{\text{field}}^2=0+\tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2=\frac{eE\left(\Delta x_{\text{field}}\right)^2}{2m\left(\frac{2eV}{m}\right)}=\frac{E\left(\Delta x_{\text{field}}\right)^2}{4V}$$

$$=\frac{\left(2.89\times10^5\text{ V/m}\right)\left(2.8\times10^{-2}\,\text{m}\right)^2}{4\left(7000\,\text{V}\right)}=8\times10^{-3}\,\text{m}$$

This is about 7% of the total 11 cm vertical deflection, and so for an estimation, our approximation is acceptable.

54. If there were no deflecting field, the electrons would hit the center of the screen. If an electric field of a certain direction moves the electrons towards one extreme of the screen, then the opposite field will move the electrons to the opposite extreme of the screen. So we solve for the field to move the electrons to one extreme of the screen. Consider three parts to the electron's motion, and see the diagram, which is a top view. First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron v_x can be found from the accelerating potential V. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron a leftward velocity, v_y. We assume that there is very little leftward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the left edge of the screen.


$\vec{\mathbf{E}}$
$\Delta y_{\text{screen}}=15\,\text{cm}$
v_x
$\Delta x_{\text{screen}}=34\,\text{cm}$
Δx_{field}

Acceleration:

$$\text{PE}_{\text{initial}}=\text{KE}_{\text{final}}\;\rightarrow\;eV=\tfrac{1}{2}mv_x^2\;\rightarrow\;v_x=\sqrt{\frac{2eV}{m}}$$

Deflection:

$$\text{time in field: }\Delta x_{\text{field}}=v_xt_{\text{field}}\;\rightarrow\;t_{\text{field}}=\frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y=eE=ma_y\;\rightarrow\;a_y=\frac{eE}{m}\qquad v_y=v_0+a_yt_{\text{field}}=0+\frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}}=v_xt_{\text{screen}}\;\rightarrow\;t_{\text{screen}}=\frac{\Delta x_{\text{screen}}}{v_x}\qquad\Delta y_{\text{screen}}=v_yt_{\text{screen}}=v_y\frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}}=\frac{v_y}{v_x}=\frac{\frac{eE\Delta x_{\text{field}}}{mv_x}}{v_x}=\frac{eE\Delta x_{\text{field}}}{mv_x^2}\;\rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} m v_x^2}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m \dfrac{2eV}{m}}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{2V \Delta y_{\text{screen}}}{\Delta x_{\text{screen}} \Delta x_{\text{field}}} = \frac{2\left(6.0\times10^{3}\,\text{V}\right)\left(0.15\,\text{m}\right)}{\left(0.34\text{ m}\right)\left(0.026\,\text{m}\right)}$$

$$= 2.04\times10^{5}\ \text{V/m} \approx 2.0\times10^{5}\ \text{V/m}$$

As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_0 t_{\text{field}} + \tfrac{1}{2} a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE\left(\Delta x_{\text{field}}\right)^2}{2m\left(\dfrac{2eV}{m}\right)} = \frac{E\left(\Delta x_{\text{field}}\right)^2}{4V}$$

$$= \frac{\left(2.04\times10^{5}\ \text{V/m}\right)\left(2.6\times10^{-2}\,\text{m}\right)^2}{4\left(6000\,\text{V}\right)} = 6\times10^{-3}\,\text{m}$$

This is about 4% of the total 15 cm vertical deflection, and so for an estimation, our approximation is acceptable. And so the field must vary from $\boxed{+2.0\times10^{5}\ \text{V/m}\ \text{ to }\ -2.0\times10^{5}\ \text{V/m}}$

55. (*a*) The electron was accelerated through a potential difference –6.3 kV in gaining 6.3 keV of KE. The proton is accelerated through the opposite potential difference as the electron, and has the exact opposite charge. Thus the proton gains the same KE, $\boxed{6.3\text{ keV}}$.

(*b*) Both the proton and the electron have the same KE. Use that to find the ratio of the speeds.

$$\tfrac{1}{2} m_{\text{p}} v_{\text{p}}^2 = \tfrac{1}{2} m_{\text{e}} v_{\text{e}}^2 \quad \rightarrow \quad \frac{v_{\text{e}}}{v_{\text{p}}} = \sqrt{\frac{m_{\text{p}}}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}}} = \boxed{42.8}$$

56. (*a*) The energy is related to the charge and the potential difference by Eq. 17-3.

$$\Delta\text{PE} = q\Delta V \quad \rightarrow \quad \Delta V = \frac{\Delta\text{PE}}{q} = \frac{4.2\times10^{6}\,\text{J}}{4.0\,\text{C}} = 1.05\times10^{6}\,\text{V} \approx \boxed{1.1\times10^{6}\,\text{V}}$$

(*b*) The energy (as heat energy) is used to raise the temperature of the water and boil it. Assume that room temperature is 20°C.

$$Q = mc\Delta T + mL_{\text{f}} \quad \rightarrow$$

$$m = \frac{Q}{c\Delta T + L_{\text{f}}} = \frac{4.2\times10^{6}\,\text{J}}{\left(4186\dfrac{\text{J}}{\text{kg}\bullet{}^{\circ}\text{C}}\right)\left(80\,\text{C}^{\circ}\right) + \left(22.6\times10^{5}\ \dfrac{\text{J}}{\text{kg}}\right)} = \boxed{1.6\,\text{kg}}$$

57. The energy density is given by Eq. 17-11.

$$\text{Energy density} = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(150\,\text{V/m}\right)^2 = \boxed{1.0\times10^{-7}\ \text{J/m}^3}$$

58. The electric force on the electron must be the same magnitude as the weight of the electron. The magnitude of the electric force is the charge on the electron times the magnitude of the electric field. The electric field is the potential difference per meter: $E = V/d$.

$$F_E = mg \quad \rightarrow \quad eV/d = mg \quad \rightarrow$$

$$V = \frac{mgd}{e} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(3.0\times10^{-2}\,\text{m}\right)}{1.60\times10^{-19}\text{C}} = \boxed{1.7\times10^{-12}\,\text{V}}$$

Since it takes such a tiny voltage to balance gravity, the thousands of volts in a television set are more than enough (by many orders of magnitude) to move electrons upward against the force of gravity.

59. The energy in the capacitor, given by Eq. 17-10, is the heat energy absorbed by the water, given by Eq. 14-2.

$$\text{PE} = Q_{\text{heat}} \quad \rightarrow \quad \tfrac{1}{2}CV^2 = mc\Delta T \quad \rightarrow$$

$$V = \sqrt{\frac{2mc\Delta T}{C}} = \sqrt{\frac{2(2.5\,\text{kg})\left(4186\dfrac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)\left(95^\circ\text{C} - 21\text{C}^\circ\right)}{4.0\,\text{F}}} = 622\,\text{V} \approx \boxed{620\,\text{V}}$$

60. Since the capacitor is disconnected from the battery, the charge on it cannot change. The capacitance of the capacitor is increased by a factor of *K*, the dielectric constant.

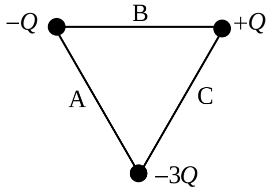
$$Q = C_{\text{initial}}V_{\text{initial}} = C_{\text{final}}V_{\text{final}} \quad \rightarrow \quad V_{\text{final}} = V_{\text{initial}}\frac{C_{\text{initial}}}{C_{\text{final}}} = V_{\text{initial}}\frac{C_{\text{initial}}}{KC_{\text{initial}}} = (24.0\,\text{V})\frac{1}{2.2} = \boxed{11\,\text{V}}$$

61. Combine Eq. 17-7 with Eq. 17-8 and Eq. 17-4.

$$Q = CV = \frac{\varepsilon_0 A}{d}V = \varepsilon_0 A\frac{V}{d} = \varepsilon_0 AE = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(56\times10^{-4}\,\text{m}^2\right)\left(3.0\times10^{6}\,\text{V/m}\right)$$

$$= \boxed{1.5\times10^{-7}\,\text{C}}$$

62. Use Eq. 17-5 to find the potential due to each charge. Since the triangle is equilateral, the 30-60-90 triangle relationship says that the distance from a corner to the midpoint of the opposite side is $\sqrt{3}L/2$.

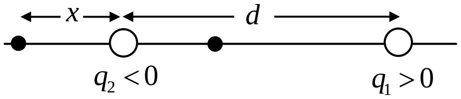


$$V_{\text{A}} = \frac{k(-Q)}{L/2} + \frac{k(-3Q)}{L/2} + \frac{k(Q)}{\sqrt{3}L/2} = \frac{2kQ}{L}\left(-4 + \frac{1}{\sqrt{3}}\right) = \boxed{-6.85\frac{kQ}{L}}$$

$$V_{\text{B}} = \frac{k(-Q)}{L/2} + \frac{k(Q)}{L/2} + \frac{k(-3Q)}{\sqrt{3}L/2} = -\frac{6kQ}{\sqrt{3}L} = \boxed{-3.46\frac{kQ}{L}}$$

$$V_{\text{C}} = \frac{k(Q)}{L/2} + \frac{k(-3Q)}{L/2} + \frac{k(-Q)}{\sqrt{3}L/2} = -\frac{2kQ}{L}\left(2 + \frac{1}{\sqrt{3}}\right) = \boxed{-5.15\frac{kQ}{L}}$$

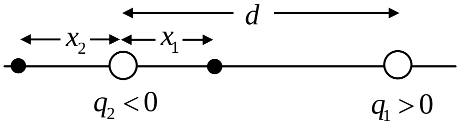
63. (*a*) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this is the point labeled as "*x*". Take to the right as the positive direction.



$$E = k\frac{|q_2|}{x^2} - k\frac{q_1}{(d+x)^2} = 0 \quad \rightarrow \quad |q_2|(d+x)^2 = q_1 x^2 \quad \rightarrow$$

$$x=\frac{\sqrt{|q_2|}}{\sqrt{q_1}-\sqrt{|q_2|}}d=\frac{\sqrt{2.6\times10^{-6}\,\mathrm{C}}}{\sqrt{3.4\times10^{-6}\,\mathrm{C}}-\sqrt{2.6\times10^{-6}\,\mathrm{C}}}(1.6\,\mathrm{cm})=\boxed{11.1\,\mathrm{cm\ left\ of\ }q_2}$$

(*b*) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge, any point where the potential is zero must be closer to the negative charge. So consider locations between the charges (position x_1) and to the left of the negative charge (position x_2) as shown in the diagram.



$$V_{\text{location 1}}=\frac{kq_1}{(d-x_1)}+\frac{kq_2}{x_1}=0\ \rightarrow\ x_1=\frac{q_2 d}{(q_2-q_1)}=\frac{(-2.6\times10^{-6}\,\mathrm{C})(1.6\,\mathrm{cm})}{(-6.0\times10^{-6}\,\mathrm{C})}=0.69\,\mathrm{cm}$$

$$V_{\text{location 2}}=\frac{kq_1}{(d+x_2)}+\frac{kq_2}{x_2}=0\ \rightarrow\ x_2=-\frac{q_2 d}{(q_1+q_2)}=-\frac{(-2.6\times10^{-6}\,\mathrm{C})(1.6\,\mathrm{cm})}{(0.8\times10^{-6}\,\mathrm{C})}=5.2\,\mathrm{cm}$$

So the two locations where the potential is zero are 0.7 cm from the negative charge towards the positive charge, and 5.2 cm from the negative charge away from the positive charge.

64. The voltage across the capacitor stays constant since the capacitor remains connected to the battery. The capacitance changes according to Eq. 17-9.

$$C_{\text{initial}}=\frac{\varepsilon_0 A}{d}=2600\times10^{-12}\,\mathrm{F}\qquad C_{\text{final}}=K\frac{\varepsilon_0 A}{d}=KC_{\text{initial}}=5C_{\text{initial}}$$

$$Q_{\text{final}}-Q_{\text{initial}}=C_{\text{final}}V-C_{\text{initial}}V=(5C_{\text{intial}}-C_{\text{initial}})V=4C_{\text{initial}}V=4(2600\times10^{-12}\,\mathrm{F})(9.0\,\mathrm{V})$$

$$=\boxed{9.4\times10^{-8}\,\mathrm{C}}$$

65. To find the angle, the horizontal and vertical components of the velocity are needed. The horizontal component can be found using conservation of energy for the initial acceleration of the electron. That component is not changed as the electron passes through the plates. The vertical component can be found using the vertical acceleration due to the potential difference of the plates, and the time the electron spends between the plates.

Horizontal:

$$\mathrm{PE}_{\text{inital}}=\mathrm{KE}_{\text{final}}\ \rightarrow\ qV=\tfrac{1}{2}mv_x^2\qquad t=\frac{\Delta x}{v_x}$$

Vertical:

$$F_{\mathrm{E}}=qE_y=ma=m\frac{(v_y-v_{y0})}{t}\ \rightarrow\ v_y=\frac{qE_y t}{m}=\frac{qE_y\Delta x}{mv_x}$$

Combined:

$$\tan\theta=\frac{v_y}{v_x}=\frac{\dfrac{qE_y\Delta x}{mv_x}}{v_x}=\frac{qE_y\Delta x}{mv_x^2}=\frac{qE_y\Delta x}{2qV}=\frac{E_y\Delta x}{2V}=\frac{\left(\dfrac{250\,\mathrm{V}}{0.013\,\mathrm{m}}\right)(0.065\,\mathrm{m})}{2(5{,}500\,\mathrm{V})}=0.1136$$

$$\theta=\tan^{-1}0.1136=\boxed{6.5^{\circ}}$$

66. There is no other source of charge except for the original capacitor. Thus the total charge must remain at Q_0. Also, since the plates of the one capacitor are connected via equipotential wires to the plate of the other capacitor, the two capacitors must have the same voltage across their plates. Use the total charge and fact of equal potentials to find the charge on each capacitor and the common potential difference.

$$Q_0 = Q_1 + Q_2 \quad Q_1 = C_1V \quad Q_2 = C_2V \quad Q_0 = C_1V + C_2V = \left(C_1 + C_2\right)V \;\rightarrow\; V = \boxed{\frac{Q_0}{C_1 + C_2}}$$

$$Q_1 = C_1V = \boxed{Q_0\frac{C_1}{C_1 + C_2}} \qquad Q_2 = C_2V = \boxed{Q_0\frac{C_2}{C_1 + C_2}}$$

67. Use Eq. 17-8 for the capacitance.

$$C = \frac{\varepsilon_0 A}{d} \;\rightarrow\; d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(1.0\times10^{-4}\,\mathrm{m^2}\right)}{\left(1\,\mathrm{F}\right)} = \boxed{9\times10^{-16}\,\mathrm{m}}$$

$\boxed{\text{No}}$, this is not practically achievable. The gap would have to be smaller than the radius of a proton.

68. Since the E-field points downward, the surface of the Earth is a lower potential than points above the surface. Call the surface of the Earth 0 volts. Then a height of 2.00 m has a potential of 300 V. We also call the surface of the Earth the 0 location for gravitational PE. Write conservation of energy relating the charged spheres at 2.00 m (where their speed is 0) and at ground level (where their electrical and gravitational potential energies are 0).

$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; mgh + qV = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{2\left(gh + \frac{qV}{m}\right)}$$

$$v_+ = \sqrt{2\left[\left(9.80\,\mathrm{m/s^2}\right)\left(2.00\,\mathrm{m}\right) + \frac{\left(6.5\times10^{-4}\,\mathrm{C}\right)\left(300\,\mathrm{V}\right)}{\left(0.540\,\mathrm{kg}\right)}\right]} = 6.3184\ \mathrm{m/s}$$

$$v_- = \sqrt{2\left[\left(9.80\,\mathrm{m/s^2}\right)\left(2.00\,\mathrm{m}\right) + \frac{\left(-6.5\times10^{-4}\,\mathrm{C}\right)\left(300\,\mathrm{V}\right)}{\left(0.540\,\mathrm{kg}\right)}\right]} = 6.2030\ \mathrm{m/s}$$

$$v_+ - v_- = 6.3184\,\mathrm{m/s} - 6.2030\,\mathrm{m/s} = \boxed{0.12\,\mathrm{m/s}}$$

69. (*a*) Use Eq. 17-10 to calculate the stored energy.

$$\mathrm{PE} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(5.0\times10^{-8}\,\mathrm{F}\right)\left(3.0\times10^{4}\,\mathrm{V}\right)^2 = 22.5\,\mathrm{J} \sim \boxed{23\,\mathrm{J}}$$

(*b*) The power is the energy converted per unit time.

$$P = \frac{\text{Energy}}{\text{time}} = \frac{0.12\left(22.5\,\mathrm{J}\right)}{8.0\times10^{-6}\,\mathrm{s}} = \boxed{3.4\times10^{5}\,\mathrm{W}}$$

70. (*a*) Use Eq. 17-8.

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(110\times10^{6}\,\mathrm{m^2}\right)}{\left(1500\,\mathrm{m}\right)} = 6.49\times10^{-7}\,\mathrm{F} \approx \boxed{6.5\times10^{-7}\,\mathrm{F}}$$

(*b*) Use Eq. 17-7.

$$Q = CV = \left(6.49\times10^{-7}\,\mathrm{F}\right)\left(3.5\times10^{7}\,\mathrm{V}\right) = 22.715\,\mathrm{C} \approx \boxed{23\,\mathrm{C}}$$

(*c*) Use Eq. 17-10.

$$\mathrm{PE} = \tfrac{1}{2}QV = \tfrac{1}{2}\left(22.715\,\mathrm{C}\right)\left(3.5\times10^{7}\,\mathrm{V}\right) = \boxed{4.0\times10^{8}\,\mathrm{J}}$$

71. The kinetic energy of the electrons (provided by the UV light) is converted completely to potential energy at the plate since they are stopped. Use energy conservation to find the emitted speed, taking the 0 of PE to be at the surface of the barium.

$$\mathrm{KE}_{\text{initial}} = \mathrm{PE}_{\text{final}} \quad\rightarrow\quad \tfrac{1}{2}mv^2 = qV \quad\rightarrow$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2\left(-1.60\times10^{-19}\,\mathrm{C}\right)\left(-3.02\,\mathrm{V}\right)}{9.11\times10^{-31}\,\mathrm{kg}}} = \boxed{1.03\times10^{6}\,\mathrm{m/s}}$$

72. Let d_1 represent the distance from the left charge to point b, and let d_2 represent the distance from the right charge to point b. Let *Q* represent the positive charges, and let *q* represent the negative charge that moves. The change in potential energy is given by Eq. 17-3, $\mathrm{PE_b} - \mathrm{PE_a} = qV_{\mathrm{ba}} = q\left(V_\mathrm{b} - V_\mathrm{a}\right)$.

$$d_1 = \sqrt{12^2 + 14^2}\,\mathrm{cm} = 18.44\ \mathrm{cm} \qquad d_2 = \sqrt{14^2 + 24^2}\,\mathrm{cm} = 27.78\ \mathrm{cm}$$

$$\mathrm{PE_b} - \mathrm{PE_a} = q\left(V_\mathrm{b} - V_\mathrm{a}\right) = q\left[\left(\frac{kQ}{0.1844\,\mathrm{m}} + \frac{kQ}{0.2778\,\mathrm{m}}\right) - \left(\frac{kQ}{0.12\,\mathrm{m}} + \frac{kQ}{0.24\,\mathrm{m}}\right)\right]$$

$$= kQq\left[\left(\frac{1}{0.1844\,\mathrm{m}} + \frac{1}{0.2778\,\mathrm{m}}\right) - \left(\frac{1}{0.12\,\mathrm{m}} + \frac{1}{0.24\,\mathrm{m}}\right)\right]$$

$$= \left(8.99\times10^{9}\right)\left(-1.5\times10^{-6}\,\mathrm{C}\right)\left(33\times10^{-6}\,\mathrm{C}\right)\left(-3.477\,\mathrm{m}^{-1}\right) = 1.547\,\mathrm{J} \approx \boxed{1.5\,\mathrm{J}}$$

73. Use Eq. 17-7 with Eq. 17-9 to find the charge.

$$Q = CV = K\varepsilon_0\frac{A}{d}V = \left(3.7\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\frac{\pi\left(0.55\times10^{-2}\,\mathrm{m}\right)^2}{\left(0.15\times10^{-3}\,\mathrm{m}\right)}\left(12\,\mathrm{V}\right) = \boxed{2.5\times10^{-10}\,\mathrm{C}}$$

74. (*a*) Use Eq. 17-5 to calculate the potential due to the charges. Let the distance between the charges be *d*.

$$V_{\text{mid}} = \frac{kQ_1}{\left(d/2\right)} + \frac{kQ_2}{\left(d/2\right)} = \frac{2k}{d}\left(Q_1 + Q_2\right) = \frac{2\left(8.99\times10^{9}\,\mathrm{N\bullet m^2/C^2}\right)}{\left(0.23\,\mathrm{m}\right)}\left(4.5\times10^{-6}\,\mathrm{C} - 8.2\times10^{-6}\,\mathrm{C}\right)$$

$$= \boxed{-2.9\times10^{5}\,\mathrm{V}}$$

(*b*) Use Eq. 16-4b to calculate the electric field. Note that the field due to each of the two charges will point to the left, away from the positive charge and towards the negative charge. Find the magnitude of the field using the absolute value of the charges.

$$E_{\text{mid}} = \frac{kQ_1}{\left(d/2\right)^2} + \frac{k\left|Q_2\right|}{\left(d/2\right)^2} = \frac{4k}{d^2}\left(Q_1 + \left|Q_2\right|\right)$$

$$= \frac{4\left(8.99\times10^{9}\ \mathrm{N\bullet m^2/C^2}\right)}{\left(0.23\,\mathrm{m}\right)^2}\left(4.5\times10^{-6}\,\mathrm{C}+8.2\times10^{-6}\,\mathrm{C}\right)=\boxed{8.6\times10^{6}\ \mathrm{V/m},\ \text{left}}$$

75. (*a*) Use Eq. 17-7 and Eq. 17-8 to calculate the charge.

$$Q=CV=\frac{\varepsilon_0 A}{d}V=\frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(2.0\times10^{-4}\,\mathrm{m^2}\right)}{\left(5.0\times10^{-4}\,\mathrm{m}\right)}\left(12\,\mathrm{V}\right)=4.248\times10^{-11}\,\mathrm{C}$$

$$\approx \boxed{4.2\times10^{-11}\mathrm{C}}$$

(*b*) The charge does not change. Since the capacitor is not connected to a battery, no charge can flow to it or from it. Thus $Q=4.248\times10^{-11}\,\mathrm{C}\ \approx \boxed{4.2\times10^{-11}\mathrm{C}}$.

(*c*) The separation distance was multiplied by a factor of 1.5. The capacitance is inversely proportional to the separation distance, so the capacitance was divided by a factor of 1.5. Since $Q=CV$, if the charge does not change and the capacitance is divided by a factor of 1.5, then the voltage must have increased by a factor of 1.5.

$$V_{\text{final}}=1.5V_{\text{initial}}=1.5\left(12\,\mathrm{V}\right)=\boxed{18\,\mathrm{V}}$$

(*d*) The work done is the change in energy stored in the capacitor.

$$W=PE_{\text{final}}-PE_{\text{initial}}=\tfrac{1}{2}QV_{\text{final}}-\tfrac{1}{2}QV_{\text{initial}}=\tfrac{1}{2}Q\left(V_{\text{final}}-V_{\text{initial}}\right)$$

$$=\tfrac{1}{2}\left(4.248\times10^{-11}\mathrm{C}\right)\left(18\,\mathrm{V}-12\,\mathrm{V}\right)=\boxed{1.3\times10^{-10}\,\mathrm{J}}$$

76. The energy stored in the capacitor is given by Eq. 17-10. The final energy is half the initial energy. Find the final charge, and then subtract the final charge from the initial charge to find the charge lost.

$$E_{\text{final}}=\tfrac{1}{2}E_{\text{initial}}\ \rightarrow\ \tfrac{1}{2}\frac{Q_{\text{final}}^2}{C}=\tfrac{1}{2}\tfrac{1}{2}\frac{Q_{\text{initial}}^2}{C}\ \rightarrow\ Q_{\text{final}}=\frac{1}{\sqrt{2}}Q_{\text{initial}}$$

$$Q_{\text{lost}}=Q_{\text{initial}}-Q_{\text{final}}=Q_{\text{initial}}\left(1-\frac{1}{\sqrt{2}}\right)=CV\left(1-\frac{1}{\sqrt{2}}\right)=\left(2.5\times10^{-6}\,\mathrm{F}\right)\left(6.0\,\mathrm{V}\right)\left(0.2929\right)$$

$$=\boxed{4.4\times10^{-6}\,\mathrm{C}}$$

77. (*a*) We assume that Q_2 is held at rest. The energy of the system will then be conserved, with the initial potential energy of the system all being changed to kinetic energy after a very long time.

$$E_{\text{initial}}=E_{\text{final}}\ \rightarrow\ \mathrm{PE}_{\text{initial}}=\mathrm{KE}_{\text{final}}\ \rightarrow\ \frac{kQ_1Q_2}{r}=\tfrac{1}{2}m_1v_1^2\ \rightarrow$$

$$v_1=\sqrt{\frac{2kQ_1Q_2}{m_1 r}}=\sqrt{\frac{2\left(8.99\times10^{9}\ \mathrm{N\bullet m^2/C^2}\right)\left(5.0\times10^{-6}\,\mathrm{C}\right)^2}{\left(1.5\times10^{-6}\,\mathrm{kg}\right)\left(4.0\times10^{-2}\,\mathrm{m}\right)}}=\boxed{2.7\times10^{3}\ \mathrm{m/s}}$$

(*b*) In this case, both the energy and the momentum of the system will be conserved. Since the initial momentum is zero, the magnitudes of the momenta of the two charges will be equal.

$$p_{\text{initial}}=p_{\text{final}}\ \rightarrow\ 0=m_1v_1+m_2v_2\ \rightarrow\ v_2=-v_1\frac{m_1}{m_2}$$

$$E_{\text{initial}}=E_{\text{final}}\ \rightarrow\ \mathrm{PE}_{\text{initial}}=\mathrm{KE}_{\text{final}}\ \rightarrow$$

$$\frac{kQ_1Q_2}{r}=\tfrac{1}{2}m_1v_1^2+\tfrac{1}{2}m_2v_2^2=\tfrac{1}{2}\left[m_1v_1^2+m_2\left(-v_1\frac{m_1}{m_2}\right)^2\right]=\tfrac{1}{2}\frac{m_1}{m_2}\left(m_1+m_2\right)v_1^2$$

$$v_1=\sqrt{\frac{2kQ_1Q_2m_2}{m_1\left(m_1+m_2\right)r}}=\sqrt{\frac{2\left(8.99\times10^9\ \mathrm{N\bullet m^2/C^2}\right)\left(5.0\times10^{-6}\,\mathrm{C}\right)^2\left(2.5\times10^{-6}\,\mathrm{kg}\right)}{\left(1.5\times10^{-6}\,\mathrm{kg}\right)\left(4.0\times10^{-2}\,\mathrm{m}\right)\left(4.0\times10^{-6}\,\mathrm{kg}\right)}}$$

$$=\boxed{2.2\times10^3\ \mathrm{m/s}}$$

78. Calculate $V_{\mathrm{AB}}=V_{\mathrm{A}}-V_{\mathrm{B}}$. Represent the 0.10 m distance by the variable *d*.

$$V_{\mathrm{AB}}=V_{\mathrm{A}}-V_{\mathrm{B}}=\left(\frac{kq_1}{d}+\frac{kq_2}{\sqrt{2}d}\right)-\left(\frac{kq_2}{d}+\frac{kq_1}{\sqrt{2}d}\right)=\frac{k}{d}\left(q_1-q_2\right)\left(1-\frac{1}{\sqrt{2}}\right)$$

$$=\frac{\left(8.99\times10^9\ \mathrm{N\bullet m^2/C^2}\right)}{0.1\,\mathrm{m}}\left(4.8\times10^{-6}\,\mathrm{C}\right)\left(1-\frac{1}{\sqrt{2}}\right)=\boxed{1.3\times10^5\,\mathrm{V}}$$

CHAPTER 18: Electric Currents

Answers to Questions

1. Ampere-hours measures charge. The ampere is a charge per unit time, and the hour is a time, so the product is charge. 1 Ampere-hour of charge is 3600 Coulombs of charge.

2. In the circuit (not in the battery), electrons flow from high potential energy (at the negative terminal) to low potential energy (at the positive terminal). Inside the battery, the chemical reaction does work on the electrons to take them from low potential energy to high potential energy (to the negative terminal). A more chemical description says that the chemical reaction taking place at the negative electrode leaves electrons behind on the terminal, and the positive ions created at the negative electrode pull electrons off the positive electrode.

3. Battery energy is what is being “used up”. As charges leave the battery terminal, they have a relatively high potential energy. Then as the charges move through the flashlight bulb, they lose potential energy. The battery uses a chemical reaction to replace the potential energy of the charges, by lowering the battery’s chemical potential energy. When a battery is “used up”, it is unable to give potential energy to charges.

4. The one terminal of the battery (usually the negative one) is connected to the metal chassis, frame, and engine block of the car. This means that all voltages used for electrical devices in the car are measured with respect to the car’s frame. Also, since the frame is a large mass of metal, it can supply charges for current without significantly changing its electrical potential.

5. Water flows immediately from the spout because there is already water in the spout when you turn on the faucet. Since water is (essentially) incompressible, a push on the water from some distant location (at the valve or pump) causes all the water to move almost immediately, pushing the water out of the spout. In a battery, a similar phenomenon exists in that the wires are “full” of charges (free electrons). The voltage “push” from the battery pushes all the electrons along the wire almost immediately, and the first ones at the other end of the wire are the beginnings of the electric current.

6. Resistance is given by the relationship $R = \rho L/A$. If the ratio of resistivity to area is the same for both the copper wire and the aluminum wire, then the resistances will be the same. Thus if $\rho_{\mathrm{Cu}}/A_{\mathrm{Cu}} = \rho_{\mathrm{Al}}/A_{\mathrm{Al}}$ or $A_{\mathrm{Al}}/A_{\mathrm{Cu}} = \rho_{\mathrm{Al}}/\rho_{\mathrm{Cu}}$, the resistances will be the same.

   Also, resistance changes with temperature. By having the two wires at different temperatures, it might be possible to have their resistance the same.

7. We assume that the voltage is the same in both cases. Then if the resistance increases, the power delivered to the heater will decrease according to $P = V^2/R$. If the power decreases, the heating process will slow down.

8. Resistance is given by the relationship $R = \rho L/A$. Thus, to minimize the resistance, you should have a small length and a large cross-sectional area. Likewise, to maximize the resistance, you should have a large length and a small cross-sectional area.
   (*a*) For the least resistance, connect the wires to the faces that have dimensions of $2a$ by $3a$, which maximizes the area ($6a^2$) and minimizes the length (a).

(*b*) For the greatest resistance, connect the wires to the faces that have dimensions of *a* by 2*a*, which minimizes the area ($2a^2$) and minimizes the length (3*a*).

9. To say that $P = V^2/R$ indicates a decrease in power as resistance increases implies that the voltage is constant. To say that $P = I^2R$ indicates an increase in power as resistance increases implies that the current is constant. Only one of those can be true for any given situation. If the resistance changes and the voltage is constant, then the current must also change. Likewise, if the resistance changes and the current is constant, then the voltage must also change.

10. When a light bulb burns out, its filament burns in two. Since the filament is part of the conducting path for the electricity flowing through the bulb, once the filament is broken, current can no longer flow through the bulb, and so it no longer gives off any light.

$\boxed{11.}$ When a light bulb is first turned on, it will be cool and the filament will have a lower resistance than when it is hot. This lower resistance means that there will be more current through the bulb while it is cool. This momentary high current will make the filament quite hot. If the temperature is too high, the filament will vaporize, and the current will no longer be able to flow in the bulb.

12. Assuming that both light bulbs have the same voltage, then since $P = IV$, the higher power bulb will draw the most current. Likewise assuming that both light bulbs have the same voltage, since $P = V^2/R$, the higher power bulb will have the lower resistance. So the 100 W bulb will draw the most current, and the 75 W bulb will have the higher resistance.

$\boxed{13.}$ Transmission lines have resistance, and therefore will change some electrical energy to thermal energy (heat) as the electrical energy is transmitted. We assume that the resistance of the transmission lines is constant. Then the "lost" power is given by $P_{\text{lost}} = I^2R$, where I is the current carried by the transmission lines. The transmitted power is given by $P_{\text{trans}} = IV$, where V is the voltage across the transmission lines. For a given value of $P_{\text{trans}} = IV$, the higher the voltage is, the lower the current has to be. As the current is decreased, $P_{\text{lost}} = I^2R$ is also decreased, and so there is a lower amount of power lost.

14. The 15-A fuse is blowing because the circuit is carrying more than 15 A of current. The circuit is probably designed to only carry 15 A, and so there might be a "short" or some other malfunction causing the current to exceed 15 A. Replacing the 15-A fuse with a 25-A fuse will allow more current to flow and thus make the wires carrying the current get hotter. A fire might result, or damage to certain kinds of electrical equipment. The blown fuse is a warning that something is wrong with the circuit.

15. At only 10 Hz, the metal filament in the wire will go on and off 20 times per second. (It has a maximum magnitude of current at the maximum current in each direction.) The metal filament has time to cool down and get dim during the low current parts of the cycle, and your eye can detect this. At 50 or 60 Hz, the filament never cools enough to dim significantly.

16. There are several factors which can be considered. As the voltage reverses with each cycle of AC, the potential energy of the electrons is raised again. Thus with each "pass" through the light, the electrons lose their potential energy, and then get it back again. Secondly, the heating of the filament (which causes the light) does not depend on the direction of the current, but only that a current exists, so the light occurs as the electrons move in both directions. Also, at 60 Hz, the current peaks 120

times per second. The small amount of time while the magnitude of the current is small is not long enough for the hot metal filament to cool down, so it stays lit the entire cycle. Finally, the human eye sees anything more rapid than about 20 Hz as continuous, even if it is not. So even if the light was to go dim during part of the cycle, our eyes would not detect it.

17. When the toaster is first turned on, the Nichrome wire is at room temperature. The wire starts to heat up almost immediately. Since the resistance increases with temperature, the resistance will be increasing as the wire heats. Assuming the voltage supplied is constant, then the current will be decreasing as the resistance increases.

18. Current is NOT used up in a resistor. The same current flows into the resistor as flows out of the resistor. If that were not the case, there would be either an increase or decrease in the charge of the resistor, but the resistor actually stays neutral, indicating equal flow in and out. What does get "used up" is potential energy. The charges that come out of a resistor have lower potential energy than the charges that go into the resistor. The amount of energy decrease per unit time is given by I^2R.

19. (*a*) With the batteries connected in series, the voltage across the bulb is higher, and so more current will flow. That will make the bulb glow brighter.
    (*b*) With the batteries connected in parallel, each battery would only have to provide half the total current to light the bulb, meaning that the device could be on longer before the batteries "ran down", as compared to a single battery. Also, if one battery should fail, the other battery can still provide the current to light the bulb.

Solutions to Problems

1. Use the definition of current, Eq. 18-1.

$$I = \frac{\Delta Q}{\Delta t} \quad \rightarrow \quad 1.30\,\text{A} = \frac{1.30\,\text{C}}{\text{s}} \times \frac{1\text{ electron}}{1.60\times10^{-19}\text{C}} = \boxed{8.13\times10^{18}\text{ electrons/s}}$$

2. Use the definition of current, Eq. 18-1.

$$I = \frac{\Delta Q}{\Delta t} \quad \rightarrow \quad \Delta Q = I\Delta t = (6.7\,\text{A})(5.0\,\text{h})(3600\,\text{s/h}) = \boxed{1.2\times10^{5}\,\text{C}}$$

3. Use the definition of current, Eq. 18-1.

$$I = \frac{\Delta Q}{\Delta t} = \frac{(1200\text{ ions})(1.60\times10^{-19}\text{ C/ion})}{3.5\times10^{-6}\text{s}} = \boxed{5.5\times10^{-11}\,\text{A}}$$

4. Use Eq. 18-2a for resistance.

$$R = \frac{V}{I} = \frac{120\,\text{V}}{4.2\,\text{A}} = \boxed{29\,\Omega}$$

5. Use Eq. 18-2b for the voltage.

$$V = IR = (0.25\,\text{A})(3800\,\Omega) = \boxed{950\,\text{V}}$$

6. (*a*) Use Eq. 18-2a for resistance.

$$R=\frac{V}{I}=\frac{120\,\text{V}}{7.5\,\text{A}}=\boxed{16\,\Omega}$$

(*b*) Use the definition of current, Eq. 18-1.

$$I=\frac{\Delta Q}{\Delta t}\quad\rightarrow\quad \Delta Q=I\Delta t=(7.5\,\text{A})(15\,\text{min})(60\,\text{s/min})=\boxed{6.8\times10^{3}\,\text{C}}$$

7. (*a*) Use Eq. 18-2b to find the current.

$$V=IR\quad\rightarrow\quad I=\frac{V}{R}=\frac{240\,\text{V}}{9.6\,\Omega}=\boxed{25\,\text{A}}$$

(*b*) Use the definition of current, Eq. 18-1.

$$I=\frac{\Delta Q}{\Delta t}\quad\rightarrow\quad \Delta Q=I\Delta t=(25\,\text{A})(50\,\text{min})(60\,\text{s/min})=\boxed{7.5\times10^{4}\,\text{C}}$$

$\boxed{8.}$ Find the current from the voltage and resistance, and then find the number of electrons from the current.

$$V=IR\quad\rightarrow\quad I=\frac{V}{R}=\frac{9.0\,\text{V}}{1.6\,\Omega}=5.625\,\text{A}$$

$$5.625\,\text{A}=5.625\frac{\text{C}}{\text{s}}\times\frac{1\text{ electron}}{1.60\times10^{-19}\text{C}}\times\frac{60\,\text{s}}{1\,\text{min}}=\boxed{2.1\times10^{21}\text{ electrons/min}}$$

9. Find the potential difference from the resistance and the current.

$$R=\left(2.5\times10^{-5}\,\Omega/\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)=1.0\times10^{-6}\,\Omega$$

$$V=IR=(2800\,\text{A})\left(1.0\times10^{-6}\,\Omega\right)=\boxed{2.8\times10^{-3}\,\text{V}}$$

10. (*a*) If the voltage drops by 15%, and the resistance stays the same, then by Eq. 18-2b, $V=IR$, the current will also drop by 15%.

$$I_{\text{final}}=0.85I_{\text{initial}}=0.85(6.50\,\text{A})=5.525\,\text{A}\approx\boxed{5.5\,\text{A}}$$

(*b*) If the resistance drops by 15% (the same as being multiplied by 0.85), and the voltage stays the same, then by Eq. 18-2b, the current must be divided by 0.85.

$$I_{\text{final}}=\frac{I_{\text{initial}}}{0.85}=\frac{6.50\,\text{A}}{0.85}=7.647\,\text{A}\approx\boxed{7.6\,\text{A}}$$

11. (*a*) Use Eq. 18-2a to find the resistance.

$$R=\frac{V}{I}=\frac{12\,\text{V}}{0.60\,\text{A}}=\boxed{20\,\Omega}$$

(*b*) An amount of charge ΔQ loses a potential energy of $(\Delta Q)V$ as it passes through the resistor. The amount of charge is found from Eq. 18-1.

$$\Delta\text{PE}=(\Delta Q)V=(I\Delta t)V=(0.60\,\text{A})(60\,\text{s})(12\,\text{V})=\boxed{430\,\text{J}}$$

12. Use Eq. 18-3 to find the diameter, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{L}{A} = \rho\frac{4L}{\pi d^2} \quad \rightarrow \quad d = \sqrt{\frac{4L\rho}{\pi R}} = \sqrt{\frac{4(1.00\,\text{m})(5.6\times10^{-8}\,\Omega\text{•m})}{\pi(0.32\,\Omega)}} = \boxed{4.7\times10^{-4}\,\text{m}}$$

13. Use Eq. 18-3 to calculate the resistance, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{L}{A} = \rho\frac{4L}{\pi d^2} = (1.68\times10^{-8}\,\Omega\text{•m})\frac{4(3.5\,\text{m})}{\pi(1.5\times10^{-3}\,\text{m})^2} = \boxed{3.3\times10^{-2}\,\Omega}$$

14. Use Eq. 18-3 to calculate the resistances, with the area as $A = \pi r^2 = \pi d^2/4$, and so

$$R = \rho\frac{L}{A} = \rho\frac{4L}{\pi d^2}.$$

$$\frac{R_{\text{Al}}}{R_{\text{Cu}}} = \frac{\rho_{\text{Al}}\dfrac{4L_{\text{Al}}}{\pi d_{\text{Al}}^2}}{\rho_{\text{Cu}}\dfrac{4L_{\text{Cu}}}{\pi d_{\text{Cu}}^2}} = \frac{\rho_{\text{Al}}L_{\text{Al}}d_{\text{Cu}}^2}{\rho_{\text{Cu}}L_{\text{Cu}}d_{\text{Al}}^2} = \frac{(2.65\times10^{-8}\,\Omega\text{•m})(10.0\,\text{m})(2.5\,\text{mm})^2}{(1.68\times10^{-8}\,\Omega\text{•m})(20.0\,\text{m})(2.0\,\text{mm})^2} = \boxed{1.2}$$

15. Use Eq. 18-3 to express the resistances, with the area as $A = \pi r^2 = \pi d^2/4$, and so

$$R = \rho\frac{L}{A} = \rho\frac{4L}{\pi d^2}.$$

$$R_{\text{W}} = R_{\text{Cu}} \quad \rightarrow \quad \rho_{\text{W}}\frac{4L}{\pi d_{\text{W}}^2} = \rho_{\text{Cu}}\frac{4L}{\pi d_{\text{Cu}}^2} \quad \rightarrow$$

$$d_{\text{W}} = d_{\text{Cu}}\sqrt{\frac{\rho_{\text{W}}}{\rho_{\text{Cu}}}} = (2.5\,\text{mm})\sqrt{\frac{5.6\times10^{-8}\,\Omega\text{•m}}{1.68\times10^{-8}\,\Omega\text{•m}}} = \boxed{4.6\,\text{mm}}$$

The diameter of the tungsten should be 4.6 mm.

16. Since the resistance is directly proportional to the length, the length of the long piece must be 4.0 times the length of the short piece.

$$L = L_{\text{short}} + L_{\text{long}} = L_{\text{short}} + 4.0L_{\text{short}} = 5.0L_{\text{short}} \quad \rightarrow \quad L_{\text{short}} = 0.20L\,,\ L_{\text{long}} = 0.80L$$

Make the cut at $\boxed{\text{20\% of the length of the wire}}$.

$$L_{\text{short}} = 0.20L\,,\ L_{\text{long}} = 0.80L \quad \rightarrow \quad R_{\text{short}} = 0.2R = \boxed{2.0\,\Omega}\,,\ R_{\text{long}} = 0.8R = \boxed{8.0\,\Omega}$$

17. Use Eq. 18-4 multiplied by L/A so that it expresses resistance instead of resistivity.

$$R = R_0\left[1+\alpha(T-T_0)\right] = 1.15R_0 \quad \rightarrow \quad 1+\alpha(T-T_0) = 1.15 \quad \rightarrow$$

$$T - T_0 = \frac{0.15}{\alpha} = \frac{0.15}{.0068(\text{C}^\circ)^{-1}} = \boxed{22\,\text{C}^\circ}$$

So raise the temperature by $22\,\text{C}^\circ$ to a final temperature of 42°C.

18. Use Eq. 18-4 for the resistivity.

$$\rho_{\text{T Cu}} = \rho_{\text{0 Cu}}\left[1+\alpha_{\text{Cu}}\left(T-T_0\right)\right] = \rho_{\text{0 W}} \quad \rightarrow$$

$$T = T_0 + \frac{1}{\alpha_{\text{Cu}}}\left(\frac{\rho_{\text{0 W}}}{\rho_{\text{0 Cu}}}-1\right) = 20^{\circ}\text{C} + \frac{1}{0.0068\left(\text{C}^{\circ}\right)^{-1}}\left(\frac{5.6\times10^{-8}\,\Omega\bullet\text{m}}{1.68\times10^{-8}\,\Omega\bullet\text{m}}-1\right) = 363.1^{\circ}\text{C} \approx \boxed{360^{\circ}\text{C}}$$

$\boxed{19.}$ Use Eq. 18-4 multiplied by L/A so that it expresses resistances instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] \quad \rightarrow$$

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0}-1\right) = 20^{\circ}\text{C} + \frac{1}{0.0060\left(\text{C}^{\circ}\right)^{-1}}\left(\frac{140\,\Omega}{12\,\Omega}-1\right) = 1798^{\circ}\text{C} \approx \boxed{1800^{\circ}\text{C}}$$

20. Calculate the voltage drop by combining Ohm's Law (Eq. 18-2b) with the expression for resistance, Eq. 18-3.

$$V = IR = I\frac{\rho L}{A} = I\frac{4\rho L}{\pi d^2} = \left(12\,\text{A}\right)\frac{4\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\left(26\,\text{m}\right)}{\pi\left(1.628\times10^{-3}\,\text{m}\right)^2} = \boxed{2.5\,\text{V}}$$

21. In each case calculate the resistance by using Eq. 18-3 for resistance.

(a) $$R_x = \frac{\rho L_x}{A_{yz}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(1.0\times10^{-2}\,\text{m}\right)}{\left(2.0\times10^{-2}\,\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)} = 3.75\times10^{-4}\,\Omega \approx \boxed{3.8\times10^{-4}\,\Omega}$$

(b) $$R_y = \frac{\rho L_y}{A_{xz}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(2.0\times10^{-2}\,\text{m}\right)}{\left(1.0\times10^{-2}\,\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)} = \boxed{1.5\times10^{-3}\,\Omega}$$

(c) $$R_z = \frac{\rho L_z}{A_{xy}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)}{\left(1.0\times10^{-2}\,\text{m}\right)\left(2.0\times10^{-2}\,\text{m}\right)} = \boxed{6.0\times10^{-3}\,\Omega}$$

22. The wires have the same resistance and the same resistivity.

$$R_{\text{long}} = R_{\text{short}} \quad \rightarrow \quad \frac{\rho L_{\text{long}}}{A_1} = \frac{\rho L_{\text{short}}}{A_2} \quad \rightarrow \quad \frac{(4)\,2L_{\text{short}}}{\pi d_{\text{long}}^2} = \frac{4L_{\text{short}}}{\pi d_{\text{short}}^2} \quad \rightarrow \quad \boxed{\frac{d_{\text{long}}}{d_{\text{short}}} = \sqrt{2}}$$

23. The original resistance is $R_0 = V/I_0$, and the high temperature resistance is $R = V/I$, where the two voltages are the same. The two resistances are related by Eq. 18-4, multiplied by L/A so that it expresses resistance instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] \quad \rightarrow \quad T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0}-1\right) = T_0 + \frac{1}{\alpha}\left(\frac{V/I}{V/I_0}-1\right) = T_0 + \frac{1}{\alpha}\left(\frac{I_0}{I}-1\right)$$

$$= 20.0^{\circ}\text{C} + \frac{1}{0.00429\left(\text{C}^{\circ}\right)^{-1}}\left(\frac{0.4212\,\text{A}}{0.3618\,\text{A}}-1\right) = \boxed{58.3^{\circ}\text{C}}$$

24. (*a*) Calculate each resistance separately using Eq. 18-3, and then add the resistances together to find the total resistance.

$$R_{\text{Cu}} = \frac{\rho_{\text{Cu}} L}{A} = \frac{4\rho_{\text{Cu}} L}{\pi d^2} = \frac{4\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\left(5.0\,\text{m}\right)}{\pi\left(1.0\times10^{-3}\,\text{m}\right)^2} = 0.10695\,\Omega$$

$$R_{\text{Al}} = \frac{\rho_{\text{Al}} L}{A} = \frac{4\rho_{\text{Al}} L}{\pi d^2} = \frac{4\left(2.65\times10^{-8}\,\Omega\bullet\text{m}\right)\left(5.0\,\text{m}\right)}{\pi\left(1.0\times10^{-3}\,\text{m}\right)^2} = 0.16870\,\Omega$$

$$R_{\text{total}} = R_{\text{Cu}} + R_{\text{Al}} = 0.10695\,\Omega + 0.16870\,\Omega = 0.27565\,\Omega \approx \boxed{0.28\,\Omega}$$

(*b*) The current through the wire is the voltage divided by the total resistance.

$$I = \frac{V}{R_{\text{total}}} = \frac{85\times10^{-3}\,\text{V}}{0.27565\,\Omega} = 0.30836\,\text{A} \approx \boxed{0.31\,\text{A}}$$

(*c*) For each segment of wire, Ohm's Law is true. Both wires have the current found in (*b*) above.

$$V_{\text{Cu}} = IR_{\text{Cu}} = \left(0.30836\,\text{A}\right)\left(0.10695\,\Omega\right) \approx \boxed{0.033\,\text{V}}$$

$$V_{\text{Al}} = IR_{\text{Al}} = \left(0.30836\,\text{A}\right)\left(0.16870\Omega\right) \approx \boxed{0.052\,\text{V}}$$

Notice that the total voltage is 85 mV.

25. The total resistance is to be 4700 ohms $\left(R_{\text{total}}\right)$ at all temperatures. Write each resistance in terms of Eq. 18-4 (with $T_0 = 0^{\circ}\text{C}$), multiplied by L/A to express resistance instead of resistivity.

$$R_{\text{total}} = R_{0\text{C}}\left[1+\alpha_{\text{C}}T\right] + R_{0\text{N}}\left[1+\alpha_{\text{N}}T\right] = R_{0\text{C}} + R_{0\text{C}}\alpha_{\text{C}}T + R_{0\text{N}} + R_{0\text{N}}\alpha_{\text{N}}T$$
$$= R_{0\text{C}} + R_{0\text{N}} + \left(R_{0\text{C}}\alpha_{\text{C}} + R_{0\text{N}}\alpha_{\text{N}}\right)T$$

For the above to be true, the terms with a temperature dependence must cancel, and the terms without a temperature dependence must add to R_{total}. Thus we have two equations in two unknowns.

$$0 = \left(R_{0\text{C}}\alpha_{\text{C}} + R_{0\text{N}}\alpha_{\text{N}}\right)T \;\rightarrow\; R_{0\text{N}} = -\frac{R_{0\text{C}}\alpha_{\text{C}}}{\alpha_{\text{N}}}$$

$$R_{\text{total}} = R_{0\text{C}} + R_{0\text{N}} = R_{0\text{C}} - \frac{R_{0\text{C}}\alpha_{\text{C}}}{\alpha_{\text{N}}} = \frac{R_{0\text{C}}\left(\alpha_{\text{N}} - \alpha_{\text{C}}\right)}{\alpha_{\text{N}}} \;\rightarrow$$

$$R_{0\text{C}} = R_{\text{total}}\frac{\alpha_{\text{N}}}{\left(\alpha_{\text{N}} - \alpha_{\text{C}}\right)} = \left(4700\,\Omega\right)\frac{0.0004\left(\text{C}^{\circ}\right)^{-1}}{0.0004\left(\text{C}^{\circ}\right)^{-1} + 0.0005\left(\text{C}^{\circ}\right)^{-1}} = \boxed{2100\,\Omega}$$

$$R_{0\text{N}} = R_{\text{total}} - R_{0\text{C}} = \boxed{2600\,\Omega}$$

26. Use Eq. 18-6b to find the resistance from the voltage and the power.

$$P = \frac{V^2}{R} \;\rightarrow\; R = \frac{V^2}{P} = \frac{\left(240\,\text{V}\right)^2}{3300\,\text{W}} = \boxed{17\,\Omega}$$

27. Use Eq. 18-5 to find the power from the voltage and the current.

$$P = IV = \left(0.32\,\text{A}\right)\left(3.0\,\text{V}\right) = \boxed{0.96\,\text{W}}$$

28. Use Eq. 18-6b to find the voltage from the power and the resistance.

$$P=\frac{V^2}{R} \rightarrow V=\sqrt{RP}=\sqrt{(2700\,\Omega)(0.25\,\text{W})}=\boxed{26\,\text{V}}$$

29. Use Eq. 18-6b to find the resistance, and Eq. 18-5 to find the current.

(*a*) $$P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P}=\frac{(120\,\text{V})^2}{75\,\text{W}}=192\,\Omega\approx\boxed{190\,\Omega}$$

$$P=IV \rightarrow I=\frac{P}{V}=\frac{75\,\text{W}}{120\,\text{V}}=0.625\,\text{A}\approx\boxed{0.63\,\text{A}}$$

(*b*) $$P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P}=\frac{(120\,\text{V})^2}{440\,\text{W}}=32.73\,\Omega\approx\boxed{33\,\Omega}$$

$$P=IV \rightarrow I=\frac{P}{V}=\frac{440\,\text{W}}{120\,\text{V}}=3.667\,\text{A}\approx\boxed{3.7\,\text{A}}$$

30. (*a*) Use Eq. 18-5 to find the current.

$$P=IV \rightarrow I=\frac{P}{V}=\frac{110\,\text{W}}{115\,\text{V}}=\boxed{0.96\,\text{A}}$$

(*b*) Use Eq. 18-6b to find the resistance.

$$P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P}=\frac{(115\,\text{V})^2}{110\,\text{W}}\approx\boxed{120\,\Omega}$$

31. (*a*) Since $P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P}$ says that the resistance is inversely proportional to the power for a constant voltage, we predict that the 850 W setting has the higher resistance.

(*b*) $$R=\frac{V^2}{P}=\frac{(120\,\text{V})^2}{850\,\text{W}}=\boxed{17\,\Omega}$$

(*c*) $$R=\frac{V^2}{P}=\frac{(120\,\text{V})^2}{1250\,\text{W}}=\boxed{12\,\Omega}$$

32. The power (and thus the brightness) of the bulb is proportional to the square of the voltage, according to Eq. 18-6a, $P=\frac{V^2}{R}$. Since the resistance is assumed to be constant, if the voltage is cut in half from 240 V to 120V, the power will be reduced by a factor of 4. Thus the bulb will appear only about $\boxed{\text{1/4 as bright}}$ in the United States as in Europe.

$\boxed{33.}$ To find the kWh of energy, multiply the kilowatts of power consumption by the number of hours in operation.

$$\text{Energy}=P(\text{in kW})t(\text{in h})=(550\,\text{W})\left(\frac{1\text{kW}}{1000\,\text{W}}\right)(15\,\text{min})\left(\frac{1\text{h}}{60\,\text{min}}\right)=0.1375\,\text{kWh}\approx\boxed{0.14\,\text{kWh}}$$

To find the cost of the energy used in a month, multiply times 4 days per week of usage, times 4 weeks per month, times the cost per kWh.

$$\text{Cost} = \left(0.1375\frac{\text{kWh}}{\text{d}}\right)\left(\frac{4\,\text{d}}{1\,\text{week}}\right)\left(\frac{4\,\text{week}}{1\,\text{month}}\right)\left(\frac{9.0\,\text{cents}}{\text{kWh}}\right) \approx \boxed{20\,\text{cents/month}}$$

34. To find the cost of the energy, multiply the kilowatts of power consumption by the number of hours in operation times the cost per kWh.

$$\text{Cost} = (25\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(365\,\text{day})\left(\frac{24\,\text{h}}{1\,\text{day}}\right)\left(\frac{\$0.095}{\text{kWh}}\right) \approx \boxed{\$21}$$

35. (*a*) Calculate the resistance from Eq. 18-2a, and the power from Eq. 18-5.

$$R = \frac{V}{I} = \frac{3.0\,\text{V}}{0.45\,\text{A}} = 6.667\,\Omega \approx \boxed{6.7\,\Omega} \qquad P = IV = (0.45\,\text{A})(3.0\,\text{V}) = 1.35\,\text{W} \approx \boxed{1.4\,\text{W}}$$

(*b*) If four D-cells are used, the voltage will be doubled to 6.0 V. Assuming that the resistance of the bulb stays the same (by ignoring heating effects in the filament), the power that the bulb would need to dissipate is given by Eq. 18-6b, $P = \frac{V^2}{R}$. A doubling of the voltage means the power is increased by a factor of $\boxed{4}$. This should not be tried because the bulb is probably not rated for such a high wattage. The filament in the bulb would probably burn out, and the glass bulb might even explode if the filament burns violently.

36. The A•h rating is the amount of charge that the battery can deliver. The potential energy of the charge is the charge times the voltage.

$$\text{PE} = QV = (85\,\text{A•h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)(12\,\text{V}) = \boxed{3.7\times10^{6}\,\text{J}}$$

37. Each bulb will draw an amount of current found from Eq. 18-5.

$$P = IV \quad \rightarrow \quad I_{\text{bulb}} = \frac{P}{V}$$

The number of bulbs to draw 15 A is the total current divided by the current per bulb.

$$I_{\text{total}} = nI_{\text{bulb}} = n\frac{P}{V} \quad \rightarrow \quad n = \frac{VI_{\text{total}}}{P} = \frac{(120\,\text{V})(15\,\text{A})}{100\,\text{W}} = \boxed{18\ \text{bulbs}}$$

38. Find the power dissipated in the cord by Eq. 18-6a, using Eq. 18-3 for the resistance.

$$P = I^2R = I^2\rho\frac{2L}{A} = (15.0\,\text{A})^2\left(1.68\times10^{-8}\,\Omega\text{•m}\right)\frac{8(2.7\,\text{m})}{\pi\left(0.129\times10^{-2}\,\text{m}\right)^2} = \boxed{16\,\text{W}}$$

39. Find the current used to deliver the power in each case, and then find the power dissipated in the resistance at the given current.

$$P = IV \quad \rightarrow \quad I = \frac{P}{V} \qquad P_{\text{dissipated}} = I^2R = \frac{P^2}{V^2}R$$

$$P_{\substack{\text{dissipated}\\ 12{,}000\,\text{V}}} = \frac{\left(6.20\times10^{5}\,\text{W}\right)^2}{\left(1.2\times10^{4}\,\text{V}\right)^2}(3.0\,\Omega) = 8008\,\text{W}$$

$$P_{\substack{\text{dissipated}\\ 50{,}000\,\text{V}}}=\frac{\left(6.20\times10^{5}\,\text{W}\right)^{2}}{\left(5\times10^{4}\,\text{V}\right)^{2}}\left(3.0\,\Omega\right)=461\,\text{W}\qquad \text{difference}=8008\,\text{W}-461\,\text{W}=\boxed{7.5\times10^{3}\,\text{W}}$$

40. The water temperature rises by absorbing the heat energy that the electromagnet dissipates. Express both energies in terms of power, which is energy per unit time.

$$P_{\text{electric}}=P_{\substack{\text{to heat}\\ \text{water}}}\;\rightarrow\; IV=\frac{Q_{\text{heat water}}}{t}=\frac{mc\Delta T}{t}\;\rightarrow$$

$$\frac{m}{t}=\frac{IV}{c\Delta T}=\frac{\left(17.5\,\text{A}\right)\left(240\,\text{V}\right)}{\left(4186\,\text{J/kg}\bullet{}^{\circ}\text{C}\right)\left(7.50\,\text{C}^{\circ}\right)}=0.134\ \text{kg/s}\approx\boxed{0.13\,\text{kg/s}}$$

This is about 134 ml per second.

41. (*a*) By conservation of energy and the efficiency claim, 60% of the electrical power dissipated by the heater must be the rate at which energy is absorbed by the water.

$$0.6P_{\substack{\text{emitted by}\\ \text{electromagnet}}}=P_{\substack{\text{absorbed}\\ \text{by water}}}\;\rightarrow\;0.6\left(IV\right)=\frac{Q_{\text{heat water}}}{t}=\frac{mc\Delta T}{t}\;\rightarrow$$

$$I=\frac{mc\Delta T}{0.6Vt}=\frac{\left(0.120\,\text{kg}\right)\left(4186\,\text{J/kg}\right)\left(95^{\circ}\text{C}-25^{\circ}\text{C}\right)}{\left(0.6\right)\left(12\,\text{V}\right)\left(480\,\text{s}\right)}=10.17\,\text{A}\approx\boxed{10\,\text{A}}$$

(*b*) Use Ohm's Law to find the resistance of the heater.

$$V=IR\;\rightarrow\;R=\frac{V}{I}=\frac{12\,\text{V}}{10.17\,\text{A}}=\boxed{1.2\,\Omega}$$

42. Use Ohm's law and the relationship between peak and rms values.

$$I_{\text{peak}}=\sqrt{2}I_{\text{rms}}=\sqrt{2}\frac{V_{\text{rms}}}{R}=\sqrt{2}\frac{220\,\text{V}}{2.2\times10^{3}\,\Omega}=\boxed{0.14\,\text{A}}$$

43. Find the peak current from Ohm's law, and then find the rms current from the relationship between peak and rms values.

$$I_{\text{peak}}=\frac{V_{\text{peak}}}{R}=\frac{180\,\text{V}}{330\,\Omega}=0.54545\,\text{A}\simeq\boxed{0.55\,\text{A}}\qquad I_{\text{rms}}=I_{\text{peak}}/\sqrt{2}=\left(0.54545\,\text{A}\right)/\sqrt{2}=\boxed{0.39\,\text{A}}$$

44. (*a*) When everything electrical is turned off, no current will be flowing into the house, even though a voltage is being supplied. Since for a given voltage, the more resistance, the lower the current, a zero current corresponds to an infinite resistance.

(b) Assuming that the voltage is 120 V, use Eq. 18-6a to calculate the resistance.

$$P=\frac{V^{2}}{R}\;\rightarrow\;R=\frac{V^{2}}{P}=\frac{\left(120\,\text{V}\right)^{2}}{75\,\text{W}}=192\,\Omega\approx\boxed{1.9\times10^{2}\,\Omega}$$

45. The power and current can be used to find the peak voltage, and then the rms voltage can be found from the peak voltage.

$$\overline{P}=I_{\text{rms}}V_{\text{rms}}=\frac{I_{\text{peak}}}{\sqrt{2}}V_{\text{rms}}\;\rightarrow\;V_{\text{rms}}=\frac{\sqrt{2}\overline{P}}{I_{\text{peak}}}=\frac{\sqrt{2}\left(1500\,\text{W}\right)}{5.4\,\text{A}}=\boxed{3.9\times10^{2}\,\text{V}}$$

$\boxed{46.}$ Use the average power and rms voltage to calculate the peak voltage and peak current.

(*a*) $V_{\text{peak}} = \sqrt{2}V_{\text{rms}} = \sqrt{2}\left(660\,\text{V}\right) = 933.4\,\text{V} \approx \boxed{9.3\times10^{2}\text{V}}$

(b) $\overline{P} = I_{\text{rms}}V_{\text{rms}} = \dfrac{I_{\text{peak}}}{\sqrt{2}}V_{\text{rms}} \rightarrow I_{\text{peak}} = \dfrac{\sqrt{2}\overline{P}}{V_{\text{rms}}} = \dfrac{\sqrt{2}\left(1800\,\text{W}\right)}{660\,\text{V}} = \boxed{3.9\,\text{A}}$

47. (*a*) We assume that the 3.0 hp is the average power, so the maximum power is twice that, or 6.0 hp, as seen in Figure 18-22.

$$6.0\,\text{hp}\left(\frac{746\,\text{W}}{1\text{hp}}\right) = 4476\,\text{W} \approx \boxed{4.5\times10^{3}\text{W}}$$

(*b*) Use the average power and the rms voltage to find the peak current.

$$\overline{P} = I_{\text{rms}}V_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}}V_{\text{rms}} \rightarrow I_{\text{peak}} = \frac{\sqrt{2}\overline{P}}{V_{\text{rms}}} = \frac{\sqrt{2}\left[\frac{1}{2}\left(4476\,\text{W}\right)\right]}{240\,\text{V}} = \boxed{13\,\text{A}}$$

48. (*a*) The average power used can be found from the resistance and the rms voltage by Eq. 18-9c.

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} = \frac{\left(240\,\text{V}\right)^2}{34\,\Omega} = 1694\,\text{W} \approx \boxed{1.7\times10^{3}\,\text{W}}$$

(*b*) The maximum power is twice the average power, and the minimum power is 0.

$$P_{\text{max}} = 2\overline{P} = 2\left(1694\,\text{W}\right) \approx \boxed{3.4\times10^{3}\,\text{W}} \qquad P_{\text{min}} = \boxed{0\,\text{W}}$$

49. We follow exactly the derivation in Example 18-14, which results in an expression for the drift velocity.

$$v_{\text{d}} = \frac{I}{neA} = \frac{I}{\dfrac{N\left(1\text{ mole}\right)}{m\left(1\text{ mole}\right)}\rho_{\text{D}}e\pi\left(\frac{1}{2}d\right)^2} = \frac{4I\,m}{N\rho_{\text{D}}e\pi d^2}$$

$$= \frac{4\left(2.3\times10^{-6}\,\text{A}\right)\left(63.5\times10^{-3}\,\text{kg}\right)}{\left(6.02\times10^{23}\right)\left(8.9\times10^{3}\,\text{kg/m}^3\right)\left(1.60\times10^{-19}\,\text{C}\right)\pi\left(0.65\times10^{-3}\,\text{m}\right)^2} = \boxed{5.1\times10^{-10}\,\text{m/s}}$$

50. (a) Use Ohm's Law to find the resistance.

$$V = IR \rightarrow R = \frac{V}{I} = \frac{22.0\times10^{-3}\,\text{V}}{0.75\,\text{A}} = 0.02933\,\Omega \approx \boxed{0.029\,\Omega}$$

(b) Find the resistivity from Eq. 18-3.

$$R = \frac{\rho L}{A} \rightarrow \rho = \frac{RA}{L} = \frac{R\pi r^2}{L} = \frac{\left(0.02933\,\Omega\right)\pi\left(1.0\times10^{-3}\,\text{m}\right)^2}{\left(5.80\,\text{m}\right)} = \boxed{1.6\times10^{-8}\,\Omega\bullet\text{m}}$$

(c) Find the number of electrons per unit volume from Eq. 18-10.

$$I = neAv_{\text{d}} \rightarrow$$

$$n = \frac{I}{eAv_{\text{d}}} = \frac{I}{e\pi r^2 v_{\text{d}}} = \frac{\left(0.75\,\text{A}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\pi\left(1.0\times10^{-3}\,\text{m}\right)^2\left(1.7\times10^{-5}\,\text{m/s}\right)} = \boxed{8.8\times10^{28}\,\text{e}^-/\text{m}^3}$$

51. We are given a charge density and a speed (like the drift speed) for both types of ions. From that we can use Eq. 18-10 to determine the current per unit area. Both currents are in the same direction in terms of conventional current – positive charge moving north has the same effect as negative charge moving south – and so they can be added.

$$I = neAv_{\mathrm{d}} \quad \rightarrow$$

$$\frac{I}{A} = \left(nev_{\mathrm{d}}\right)_{\mathrm{He}} + \left(nev_{\mathrm{d}}\right)_{\mathrm{O}} = \left[\left(2.8\times10^{12}\ \mathrm{ions/m^3}\right)2\left(1.60\times10^{-19}\ \mathrm{C/ion}\right)\left(2.0\times10^{6}\ \mathrm{m/s}\right)\right] + \left[\left(7.0\times10^{11}\ \mathrm{ions/m^3}\right)\left(1.60\times10^{-19}\ \mathrm{C/ion}\right)\left(7.2\times10^{6}\ \mathrm{m/s}\right)\right]$$

$$= \boxed{2.6\ \mathrm{A/m^2}\text{, North}}$$

52. The magnitude of the electric field is the voltage change per unit meter.

$$|E| = \frac{\Delta V}{\Delta x} = \frac{70\times10^{-3}\ \mathrm{V}}{1.0\times10^{-8}\ \mathrm{m}} = \boxed{7.0\times10^{6}\ \mathrm{V/m}}$$

53. The speed is the change in position per unit time.

$$v = \frac{\Delta x}{\Delta t} = \frac{7.20\times10^{-2}\ \mathrm{m} - 3.40\times10^{-2}\ \mathrm{m}}{0.0063\ \mathrm{s} - 0.0052\ \mathrm{s}} = \boxed{35\ \mathrm{m/s}}$$

Two measurements are needed because there may be a time delay from the stimulation of the nerve to the generation of the action potential.

54. The energy required to transmit one pulse is equivalent to the energy stored by charging the axon capacitance to full voltage. In Example 18-15, the capacitance is estimated at $1.0\times10^{-8}\ \mathrm{F}$, and the potential difference is about 0.1 V.

$$E = \tfrac{1}{2}CV^2 = 0.5\left(10^{-8}\ \mathrm{F}\right)\left(0.1\ \mathrm{V}\right)^2 = \boxed{5\times10^{-11}\ \mathrm{J}}$$

The power is the energy per unit time, for 10,000 neutrons transmitting 100 pulses each per second.

$$P = \frac{E}{t} = \frac{\left(5\times10^{-11}\ \mathrm{J/neuron}\right)\left(1.0\times10^{4}\ \mathrm{neurons}\right)}{0.01\mathrm{s}} = \boxed{5\times10^{-5}\ \mathrm{W}}$$

55. The power is the work done per unit time. The work done to move a charge through a potential difference is the charge times the potential difference. The charge density must be multiplied by the surface area of the cell (the surface area of an open tube, length times circumference) to find the actual charge moved.

$$P = \frac{W}{t} = \frac{QV}{t} = \frac{Q}{t}V$$

$$= \left(3\times10^{-7}\ \frac{\mathrm{mol}}{\mathrm{m^2\bullet s}}\right)\left(6.02\times10^{23}\ \frac{\mathrm{ions}}{\mathrm{mol}}\right)\left(1.6\times10^{-19}\ \frac{\mathrm{C}}{\mathrm{ion}}\right)\left(0.10\ \mathrm{m}\right)\pi\left(20\times10^{-6}\ \mathrm{m}\right)\left(0.030\ \mathrm{V}\right)$$

$$= \boxed{5.4\times10^{-9}\ \mathrm{W}}$$

56. $\left(1.00\mathrm{A\bullet h}\right)\left(\frac{1\mathrm{C/s}}{1\mathrm{A}}\right)\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = \boxed{3600\ \mathrm{C}}$

57. Use Eq. 18-5 to calculate the current.

$$P = IV \quad \rightarrow \quad I = \frac{P}{V} = \frac{746\,\text{W}}{120\,\text{V}} = \boxed{6.22\,\text{A}}$$

58. The energy supplied by the battery is the energy consumed by the lights.

$$E_{\text{supplied}} = E_{\text{consumed}} \quad \rightarrow \quad Q\Delta V = Pt \quad \rightarrow$$

$$t = \frac{Q\Delta V}{P} = \frac{(95\,\text{A}\bullet\text{h})(3600\,\text{s/h})(12\,\text{V})}{92\,\text{W}} = 44609\,\text{s}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 12.39\,\text{h} \approx \boxed{12\,\text{h}}$$

59. Use Eq. 18-6 to express the resistance in terms of the power, and Eq. 18-3 to express the resistance in terms of the wire geometry.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} \qquad R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2} = 4\rho\frac{L}{\pi d^2}$$

$$4\rho\frac{L}{\pi d^2} = \frac{V^2}{P} \quad \rightarrow \quad d = \sqrt{\frac{4\rho LP}{\pi V^2}} = \sqrt{\frac{4(9.71\times10^{-8}\,\Omega\bullet\text{m})(5.4\,\text{m})(1500\,\text{W})}{\pi(110\,\text{V})^2}} = \boxed{2.9\times10^{-4}\,\text{m}}$$

60. From Eq. 18-2a, if $R = V/I$, then $G = I/V$

$$G = \frac{I}{V} = \frac{0.73\,\text{A}}{3.0\,\text{V}} = \boxed{0.24\,\text{S}}$$

61. To deliver 10 MW of power at 120 V requires a current of $I = \frac{P}{V} = \frac{10^7\,\text{W}}{120\,\text{V}} = 83{,}300\,\text{A}$. Calculate the power dissipated in the resistors using the current and the resistance.

$$P = I^2R = I^2\rho\frac{L}{A} = I^2\rho\frac{L}{\pi r^2} = 4I^2\rho\frac{L}{\pi d^2} = 4(83{,}300\,\text{A})^2(1.68\times10^{-8}\,\Omega\bullet\text{m})\frac{2(1.0\,\text{m})}{\pi(5.0\times10^{-3}\,\text{m})^2}$$

$$= 1.187\times10^7\,\text{W}$$

$$\text{Cost} = (\text{Power})(\text{time})(\text{rate per kWh}) = (1.187\times10^7\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(1\,\text{h})\left(\frac{\$0.10}{\text{kWh}}\right)$$

$$= \$1187 \approx \boxed{\$1{,}200\text{ per hour per meter}}$$

62. (*a*) Calculate the total kWh used per day, and then multiply by the number of days and the cost per kWh.

$$(1.8\,\text{kW})(3.0\,\text{h/d}) + 4(0.1\,\text{kW})(6.0\,\text{h/d}) + (3.0\,\text{kW})(1.4\,\text{h/d}) + (2.0\,\text{kWh/d})$$

$$= 14.0\,\text{kWh/d}$$

$$\text{Cost} = (14.0\,\text{kWh/d})(30\,\text{d})\left(\frac{\$0.105}{\text{kWh}}\right) = \boxed{\$44.10\text{ per month}}$$

(*b*) The energy required by the household is 35% of the energy that needs to be supplied by the power plant.

$$\text{Household Energy} = 0.35(\text{coal mass})(\text{coal energy per mass}) \quad \rightarrow$$

$$\text{coal mass} = \frac{\text{Household Energy}}{(0.35)(\text{coal energy per mass})} = \frac{(14.0\,\text{kWh/d})(365\,\text{d})\left(\frac{1000\,\text{W}}{\text{kW}}\right)\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)}{(0.35)\left(7000\frac{\text{kcal}}{\text{kg}}\right)\left(\frac{4186\,\text{J}}{1\,\text{kcal}}\right)}$$

$$= 1794\,\text{kg} \approx \boxed{1800\,\text{kg of coal}}$$

63. The length of the new wire is half the length of the original wire, and the cross-sectional area of the new wire is twice that of the original wire. Use Eq. 18-3.

$$R_0 = \rho\frac{L_0}{A_0} \qquad L = \tfrac{1}{2}L_0 \qquad A = 2A_0 \qquad R = \rho\frac{L}{A} = \rho\frac{\tfrac{1}{2}L_0}{2A_0} = \tfrac{1}{4}\rho\frac{L_0}{A_0} = \tfrac{1}{4}R_0$$

The new resistance is $\boxed{\text{one-fourth}}$ of the original resistance.

64. (*a*) Use Eq. 18-6 to relate the power to the voltage for a constant resistance.

$$P = \frac{V^2}{R} \quad \rightarrow \quad \frac{P_{105}}{P_{117}} = \frac{(105\,\text{V})^2/R}{(117\,\text{V})^2/R} = \frac{(105\,\text{V})^2}{(117\,\text{V})^2} = 0.805 \text{ or a } \boxed{19.5\%\text{ decrease}}$$

(*b*) The lower power output means that the resistor is generating less heat, and so the resistor's temperature would be lower. The lower temperature results in a lower value of the resistance, which would increase the power output at the lower voltages. Thus the decrease would be somewhat smaller than the value given in the first part of the problem.

65. Assume that we have a meter of wire, carrying 35 A of current, and dissipating 1.8 W of heat. The power dissipated is $P_R = I^2R$, and the resistance is $R = \frac{\rho L}{A}$.

$$P_R = I^2R = I^2\frac{\rho L}{A} = I^2\frac{\rho L}{\pi r^2} = I^2\frac{4\rho L}{\pi d^2} \quad \rightarrow$$

$$d = \sqrt{I^2\frac{4\rho L}{P_R\pi}} = 2I\sqrt{\frac{\rho L}{P_R\pi}} = 2(35\,\text{A})\sqrt{\frac{(1.68\times10^{-8}\,\Omega\text{•m})(1.0\,\text{m})}{(1.8\,\text{W})\pi}} = \boxed{3.8\times10^{-3}\,\text{m}}$$

66. (*a*) The angular frequency is $\omega = 210\,\text{rad/s}$.

$$f = \frac{\omega}{2\pi} = \frac{210\,\text{rad/s}}{2\pi} = \boxed{33.4\,\text{Hz}}$$

(*b*) The maximum current is 1.80 A.

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{1.80\,\text{A}}{\sqrt{2}} = \boxed{1.27\,\text{A}}$$

(*c*) For a resistor, $V = IR$.

$$V = IR = (1.80\,\text{A})(\sin 210t)(42.0\,\Omega) = \boxed{(75.6\sin 210t)\,\text{V}}$$

67. (*a*) The power delivered to the interior is 65% of the power drawn from the source.

$$P_{\text{interior}} = 0.65P_{\text{source}} \quad \rightarrow \quad P_{\text{source}} = \frac{P_{\text{interior}}}{0.65} = \frac{950\,\text{W}}{0.65} = 1462\,\text{W} \approx \boxed{1500\,\text{W}}$$

(*b*) The current drawn is current from the source, and so the source power is used to calculate the current.

$$P_{\text{source}} = IV_{\text{source}} \quad \rightarrow \quad I = \frac{P_{\text{source}}}{V_{\text{source}}} = \frac{1462\,\text{W}}{120\,\text{V}} = 12.18\,\text{A} \approx \boxed{12\,\text{A}}$$

68. The volume of wire is unchanged by the stretching. The volume is equal to the length of the wire times its cross-sectional area, and so since the length was increased by a factor of 3, the area was decreased by a factor of 3. Use Eq. 18-3.

$$R_0 = \rho\frac{L_0}{A_0} \qquad L = 3L_0 \qquad A = \tfrac{1}{3}A_0 \qquad R = \rho\frac{L}{A} = \rho\frac{3L_0}{\frac{1}{3}A_0} = 9\rho\frac{L_0}{A_0} = 9R_0 = \boxed{9.00\,\Omega}$$

69. The long, thick conductor is labeled as conductor number 1, and the short, thin conductor is labeled as number 2. The power transformed by a resistor is given by Eq. 18-6, $P = \frac{V^2}{R}$,and both have the same voltage applied.

$$R_1 = \rho\frac{L_1}{A_1} \qquad R_2 = \rho\frac{L_2}{A_2} \qquad L_1 = 2L_2 \qquad A_1 = 4A_2 \quad \left(\text{diameter}_1 = 2\text{diameter}_2\right)$$

$$\frac{P_1}{P_2} = \frac{V_1^2/R_1}{V_2^2/R_2} = \frac{R_2}{R_1} = \frac{\rho\frac{L_2}{A_2}}{\rho\frac{L_1}{A_1}} = \frac{L_2}{L_1}\frac{A_1}{A_2} = \frac{1}{2}\times 4 = 2 \qquad \boxed{P_1 : P_2 = 2:1}$$

70. The heater must heat $124\,\text{m}^3$ of air per hour from 5°C to 20°C, and also replace the heat being lost at a rate of 850 kcal/h. Use Eq. 14-2 to calculate the energy needed to heat the air. The density of air is found in Table 10-1.

$$Q = mc\Delta T \quad \rightarrow \quad \frac{Q}{t} = \frac{m}{t}c\Delta T = \left(124\frac{\text{m}^3}{\text{h}}\right)\left(1.29\frac{\text{kg}}{\text{m}^3}\right)\left(0.17\frac{\text{kcal}}{\text{kg}\bullet\text{C}^\circ}\right)\left(15\text{C}^\circ\right) = 408\frac{\text{kcal}}{\text{h}}$$

$$\text{Power required} = 408\frac{\text{kcal}}{\text{h}} + 850\frac{\text{kcal}}{\text{h}} = 1258\frac{\text{kcal}}{\text{h}}\left(\frac{4186\,\text{J}}{\text{kcal}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 1463\,\text{W} \approx \boxed{1500\,\text{W}}$$

71. (*a*) Use Eq. 18-6.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} = \frac{\left(240\,\text{V}\right)^2}{2200\,\text{W}} = 26.18\,\Omega \approx \boxed{26\,\Omega}$$

(*b*) Only 75% of the heat from the oven is used to heat the water.

$$0.75\left(P_{\text{oven}}\right)t = \text{Heat absorbed by water} = mc\Delta T \quad \rightarrow$$

$$t = \frac{mc\Delta T}{0.75\left(P_{\text{oven}}\right)} = \frac{\left(0.120\,\text{L}\right)\left(\frac{1\,\text{kg}}{1\,\text{L}}\right)\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)\left(85\,\text{C}^\circ\right)}{0.75\left(2200\,\text{W}\right)} = 25.88\,\text{s} \approx \boxed{26\,\text{s}}$$

(*c*) $\frac{11\,\text{cents}}{\text{kWh}}(2.2\,\text{kW})(25.88\,\text{s})\frac{1\,\text{h}}{3600\,\text{s}} = \boxed{0.17\text{ cents}}$

72. (*a*) The horsepower required is the power dissipated by the frictional force, since we are neglecting the energy used for acceleration.

$$P = Fv = (240\,\text{N})(45\,\text{km/hr})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/hr}}\right) = 3000\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right) = \boxed{4.0\,\text{hp}}$$

(*b*) The charge available by each battery is $Q = 95\,\text{A}\bullet\text{h} = 95\,\text{C/s}\bullet 3600\,\text{s} = 3.42\times10^5\,\text{C}$, and so the total charge available is 24 times that. The potential energy of that charge is the charge times the voltage. That energy must be delivered (batteries discharged) in a certain amount of time to produce the 3000 W necessary. The speed of the car times the discharge time is the range of the car between recharges.

$$P = \frac{\text{PE}}{t} = \frac{QV}{t} \rightarrow t = \frac{QV}{P} = \frac{d}{v} \rightarrow$$

$$d = vt = v\frac{QV}{P} = v\frac{QV}{Fv} = \frac{QV}{F} = \frac{24(3.42\times10^5\,\text{C})(12\,\text{V})}{240\,\text{N}} = \boxed{410\,\text{km}}$$

73. The mass of the wire is the density of copper times the volume of the wire, and the resistance of the wire is given by Eq. 18-3. We represent the mass density by ρ_m and the resistivity by ρ.

$$R = \rho\frac{L}{A} \rightarrow A = \frac{\rho L}{R} \qquad m = \rho_\text{m}LA = \rho_\text{m}L\frac{\rho L}{R} \rightarrow$$

$$L = \sqrt{\frac{mR}{\rho_\text{m}\rho}} = \sqrt{\frac{(0.018\,\text{kg})(12.5\,\Omega)}{(8.9\times10^3\,\text{kg/m}^3)(1.68\times10^{-8}\,\Omega\bullet\text{m})}} = 38.79\,\text{m} \approx \boxed{38.8\,\text{m}}$$

$$A = \frac{\rho L}{R} = \pi\left(\tfrac{1}{2}d\right)^2 \rightarrow d = \sqrt{\frac{4\rho L}{\pi R}} = \sqrt{\frac{4(1.68\times10^{-8}\,\Omega\bullet\text{m})(38.79\,\text{m})}{\pi(12.5\,\Omega)}} = \boxed{2.58\times10^{-4}\,\text{m}}$$

$\boxed{74.}$ Use Eq. 18-6.

(*a*) $P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{12\,\Omega} = \boxed{1200\,\text{W}}$

(*b*) $P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{140\,\Omega} = 103\,\text{W} \approx \boxed{100\,\text{W}}$

75. Model the protons as moving in a continuous beam of cross-sectional area *A*. Then by Eq. 18-10, $I = neAv_\text{d}$. The variable *n* is the number of protons per unit volume, so $n = \frac{N}{Al}$, where *N* is the number of protons in the beam and *l* is the circumference of the ring. The "drift" velocity in this case is the speed of light.

$$I = neAv_\text{d} = \frac{N}{Al}eAv_\text{d} = \frac{N}{l}ev_\text{d} \rightarrow$$

$$N=\frac{Il}{ev_{\mathrm{d}}}=\frac{\left(11\times10^{-3}\,\mathrm{A}\right)\left(6300\,\mathrm{m}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}=\boxed{1.4\times10^{12}\,\text{protons}}$$

76. (*a*) The power is given by $P=IV$.

$$P=IV=\left(12\,\mathrm{A}\right)\left(220\,\mathrm{V}\right)=2640\,\mathrm{W}\approx\boxed{2600\,\mathrm{W}}$$

(*b*) The power dissipated is given by $P_R=I^2R$, and the resistance is $R=\frac{\rho L}{A}$.

$$P_R=I^2R=I^2\frac{\rho L}{A}=I^2\frac{\rho L}{\pi r^2}=I^2\frac{4\rho L}{\pi d^2}=\left(12\,\mathrm{A}\right)^2\frac{4\left(1.68\times10^{-8}\,\Omega\bullet\mathrm{m}\right)\left(15\,\mathrm{m}\right)}{\pi\left(1.628\times10^{-3}\,\mathrm{m}\right)^2}=17.433\,\mathrm{W}$$

$$\approx\boxed{17\,\mathrm{W}}$$

(*c*) $$P_R=I^2\frac{4\rho L}{\pi d^2}=\left(12\,\mathrm{A}\right)^2\frac{4\left(1.68\times10^{-8}\,\Omega\bullet\mathrm{m}\right)\left(15\,\mathrm{m}\right)}{\pi\left(2.053\times10^{-3}\,\mathrm{m}\right)^2}=10.962\,\mathrm{W}\approx\boxed{11\,\mathrm{W}}$$

(*d*) The savings is due to the power difference.

$$\text{Savings}=\left(17.433\,\mathrm{W}-10.962\,\mathrm{W}\right)\left(\frac{1\,\mathrm{kW}}{1000\,\mathrm{W}}\right)\left(30\,\mathrm{d}\right)\left(\frac{12\,\mathrm{h}}{1\,\mathrm{d}}\right)\left(\frac{\$0.12}{1\,\mathrm{kWh}}\right)$$

$$=\$0.2795/\text{month}\approx\boxed{28\text{ cents per month}}$$

77. The resistance can be calculated from the power and voltage, and then the diameter of the wire can be calculated from the resistance.

$$P=\frac{V^2}{R}\ \rightarrow\ R=\frac{V^2}{P}\qquad R=\frac{\rho L}{A}=\frac{\rho L}{\pi\left(\tfrac{1}{2}d\right)^2}\ \rightarrow\ \frac{V^2}{P}=\frac{\rho L}{\pi\left(\tfrac{1}{2}d\right)^2}\ \rightarrow$$

$$d=\sqrt{\frac{4\rho LP}{\pi V^2}}=\sqrt{\frac{4\left(100\times10^{-8}\,\Omega\bullet\mathrm{m}\right)\left(3.8\,\mathrm{m}\right)\left(95\,\mathrm{W}\right)}{\pi\left(120\,\mathrm{V}\right)^2}}=\boxed{1.8\times10^{-4}\,\mathrm{m}}$$

78. Use Eq. 18-6 for the power in each case, assuming the resistance is constant.

$$\frac{P_{13.8\,\mathrm{V}}}{P_{12.0\,\mathrm{V}}}=\frac{\left(V^2/R\right)_{13.8\,\mathrm{V}}}{\left(V^2/R\right)_{12.0\,\mathrm{V}}}=\frac{13.8^2}{12.0^2}=1.3225=\boxed{32\%\text{ increase}}$$

79. (*a*) The current can be found from Eq. 18-5.

$$I=P/V\qquad I_A=P_A/V_A=40\,\mathrm{W}/120\,\mathrm{V}=\boxed{0.33\,\mathrm{A}}\qquad I_B=P_B/V_B=40\,\mathrm{W}/12\,\mathrm{V}=\boxed{3.3\,\mathrm{A}}$$

(*b*) The resistance can be found from Eq. 18-6.

$$R=\frac{V^2}{P}\qquad R_A=\frac{V_A^2}{P_A}=\frac{\left(120\,\mathrm{V}\right)^2}{40\,\mathrm{W}}=\boxed{360\,\Omega}\qquad R_B=\frac{V_B^2}{P_B}=\frac{\left(12\,\mathrm{V}\right)^2}{40\,\mathrm{W}}=\boxed{3.6\,\Omega}$$

(*c*) The charge is the current times the time.

$$Q=It\qquad Q_A=I_At=\left(0.33\,\mathrm{A}\right)\left(3600\,\mathrm{s}\right)=\boxed{1200\,\mathrm{C}}$$

$$Q_B=I_Bt=\left(3.3\,\mathrm{A}\right)\left(3600\,\mathrm{s}\right)=\boxed{12{,}000\,\mathrm{C}}$$

(*d*) The energy is the power times the time, and the power is the same for both bulbs.

$$E = Pt \quad E_A = E_B = (40\,\mathrm{W})(3600\,\mathrm{s}) = \boxed{1.44\times10^{5}\,\mathrm{J}}$$

(*e*) $\boxed{\text{Bulb B}}$ requires a larger current, and so should have larger diameter connecting wires to avoid overheating the connecting wires.

80. The current in the wire can be found by $I = P/V$.

(*a*) $$P_R = I^2R = \frac{P^2}{V^2}R = \frac{P^2}{V^2}\frac{\rho L}{A} = \frac{P^2}{V^2}\frac{\rho L}{\pi r^2} = \frac{P^2}{V^2}\frac{4\rho L}{\pi d^2}$$
$$= \frac{(2250\,\mathrm{W})^2}{(120\,\mathrm{V})^2}\frac{4(1.68\times10^{-8}\,\Omega\bullet\mathrm{m})(25.0\,\mathrm{m})}{\pi(2.59\times10^{-3}\,\mathrm{m})^2} = \boxed{28.0\,\mathrm{W}}$$

(*b*) $$P_R = \frac{P^2}{V^2}\frac{4\rho L}{\pi d^2} = \frac{(2250\,\mathrm{W})^2}{(120\,\mathrm{V})^2}\frac{4(1.68\times10^{-8}\,\Omega\bullet\mathrm{m})(25.0\,\mathrm{m})}{\pi(4.12\times10^{-3}\,\mathrm{m})^2} = \boxed{11.1\,\mathrm{W}}$$

81. Eq. 18-3 can be used. The area to be used is the cross-sectional area of the pipe.

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi\left(r^2_{\text{outside}} - r^2_{\text{inside}}\right)} = \frac{(1.68\times10^{-8}\,\Omega\bullet\mathrm{m})(10.0\,\mathrm{m})}{\pi\left[(2.50\times10^{-2}\,\mathrm{m})^2 - (1.50\times10^{-2}\,\mathrm{m})^2\right]} = \boxed{1.34\times10^{-4}\,\Omega}$$

82. The volume of wire $(\text{Volume} = \text{length}\times\text{cross-sectional area})$ remains constant as the wire is stretched from the original length of L_0 to the final length of $2L_0$. Thus the cross-sectional area changes from A_0 to $\frac{1}{2}A_0$. Use Eq. 18-3 for the resistance and 18-6 for the power dissipated.

$$R_0 = \frac{\rho L_0}{A_0} \quad R = \frac{\rho L}{A} = \frac{\rho 2L_0}{\frac{1}{2}A_0} = 4\frac{\rho L_0}{A_0} = 4R_0$$

$$P_0 = \frac{V^2}{R_0} \quad P = \frac{V^2}{R} = \frac{V^2}{4R_0} = \frac{1}{4}\frac{V^2}{R_0} = \frac{1}{4}P_0$$

The $\boxed{\text{power is reduced by a factor of 4}}$.

83. The resistance of the filament when the flashlight is on is $R = \frac{V}{I} = \frac{3.2\,\mathrm{V}}{0.20\,\mathrm{A}} = 16\,\Omega$. That can be used with a combination of Eqs. 18-3 and 18-4 to find the temperature.

$$R = R_0\left[1 + \alpha(T - T_0)\right] \quad \rightarrow$$

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20^\circ\mathrm{C} + \frac{1}{0.0045(\mathrm{C}^\circ)^{-1}}\left(\frac{16\,\Omega}{1.5\,\Omega} - 1\right) = 2168^\circ\mathrm{C} \approx \boxed{2200^\circ\mathrm{C}}$$

CHAPTER 19: DC Circuits

Answers to Questions

1. The birds are safe because they are not grounded. Both of their legs are essentially at the same voltage (the only difference being due to the small resistance of the wire between their feet), and so there is no current flow through their bodies since the potential difference across their legs is very small. If you lean a metal ladder against the power line, you are making essentially a short circuit from the high potential wire to the low potential ground. A large current will flow at least momentarily, and that large current will be very dangerous to anybody touching the ladder.

2. If the lights are connected in parallel, if one bulb burns out, the rest of the string stays lit. That makes it easy to tell which light has gone out. A parallel string is more complicated to assemble than a series string, since two wires must be attached from bulb to bulb.

   If the lights are connected in series, if one bulb burns out, all of the bulbs will go out. That makes it difficult to tell which light has gone out. A series string is simpler to assemble than one in parallel, since only one wire must be attached from bulb to bulb. A "blinker bulb" can make the entire string flash on and off by cutting off the current.

3. If 20 of the 6-V lamps were connected in series and then connected to the 120 V line, there would be a voltage drop of 6 V for each of the lamps, and they would not burn out due to too much voltage. Being in series, if one of the bulbs went out for any reason, they would all turn off.

4. If the lightbulbs are in series, each will have the same current. The power dissipated by the bulb as heat and light is given by $P = I^2R$. Thus the bulb with the higher resistance $\left(R_2\right)$ will be brighter. If the bulbs are in parallel, each will have the same voltage. The power dissipated by the bulb as heat and light is given by $P = V^2/R$. Thus the bulb with the lower resistance $\left(R_1\right)$ will be brighter.

5. The outlets are connected in parallel to each other, because you can use one outlet without using the other. If they were in series, both outlets would have to be used at the same time to have a completed circuit. Also, both outlets supply the same voltage to whatever device is plugged in to the outlet, which indicates that they are wired in parallel to the voltage source.

6. The power output from a resistor is given by $P = V^2/R$. To maximize this value, the voltage needs to be as large as possible and the resistance as small as possible. That can be accomplished by putting the two batteries in series, and then connecting the two resistors in parallel to each other, across the full 2-battery voltage.

7. The power supplied by the battery is the product of the battery voltage times the total current flowing from the battery. With the two resistors in series, the current is half that with a single resistor. Thus the battery has to supply half the power for the two series resistors than for the single resistor.

8. There is more current flowing in the room's wiring when both bulbs are on, yet the voltage remains the same. Thus the resistance of the room's circuit must have decreased. Also, since the bulbs are in parallel, adding two resistors in parallel always results in a net resistance that is smaller than either of the individual resistances.

9. No, the sign of the battery’s emf does not depend on the direction of the current through the battery. The sign of the battery’s emf depends on the direction you go through the battery in applying the loop rule. If you go from negative pole to positive pole, the emf is added. If you go from positive pole to negative pole, the emf is subtracted.

   But the terminal voltage does depend on the direction of the current through the battery. If current is flowing through the battery in the normal orientation (leaving the positive terminal, flowing through the circuit, and arriving at the negative terminal) then there is a voltage drop across the internal resistance, and the terminal voltage is less than the emf. If the current flows in the opposite sense (as in charging the battery), then there is a voltage rise across the terminal resistance, and the terminal voltage is higher than the emf.

10. (*a*) stays the same
    (*b*) increases
    (*c*) decreases
    (*d*) increases
    (*e*) increases
    (*f*) decreases
    (*g*) decreases
    (*h*) increases
    (*i*) stays the same

11. Batteries are connected in series to increase the voltage available to a device. For instance, if there are two 1.5-V batteries in series in a flashlight, the potential across the bulb will be 3.0 V. The batteries need not be nearly identical.

    Batteries are connected in parallel to increase the total amount of current available to a device. The batteries need to be nearly identical. If they are not, the larger voltage batteries will recharge the smaller voltage batteries.

12. The terminal voltage of a battery can exceed its emf if the battery is being charged – if current is passing through the battery “backwards” from positive pole to negative pole. Then the terminal voltage is the emf of the battery plus the voltage drop across the internal resistance.

13. Refer to Figure 19-2. Connect the battery to a known resistance *R*, and measure the terminal voltage V_{ab}. The current in the circuit is given by Ohm’s law to be $I = \frac{V_{\text{ab}}}{R}$. It is also true that $V_{\text{ab}} = \mathsf{E} - Ir$ and so the internal resistance can be calculated by $r = \frac{\mathsf{E} - V_{\text{ab}}}{I} = R\frac{\mathsf{E} - V_{\text{ab}}}{V_{\text{ab}}}$.

14. The formulas are “opposite” each other in a certain sense. When connected in series, resistors add linearly but capacitors add reciprocally, according to the following rules.

$$R_{\underset{\text{series}}{\text{eq}}} = R_1 + R_2 + R_3 + \cdots \qquad \frac{1}{C_{\underset{\text{series}}{\text{eq}}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

    The series resistance is always larger than any one component resistance, and the series capacitance is always smaller than any one component capacitance. When connected in parallel, resistors add reciprocally but capacitors add linearly, according to the following rules.

$$\frac{1}{R_{\underset{\text{parallel}}{\text{eq}}}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots \qquad C_{\underset{\text{parallel}}{\text{eq}}}=C_1+C_2+C_3+\cdots$$

The parallel resistance is always smaller than any one component resistance, and the parallel capacitance is always larger than any one component capacitance.

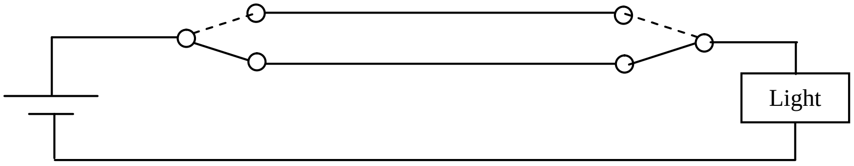
One way to consider the source of this difference is that the voltage across a resistor is proportional to the resistance, by $V = IR$, but the voltage across a capacitor is inversely proportion to the capacitance, by $V = Q/C$.

$\boxed{15.}$ The energy stored in a capacitor network can be calculated by $PE = \frac{1}{2}CV^2$. Since the voltage for the capacitor network is the same in this problem for both configurations, the configuration with the highest equivalent capacitance will store the most energy. The parallel combination has the highest equivalent capacitance, and so stores the most energy. Another way to consider this is that the total stored energy is the sum of the quantity $PE = \frac{1}{2}CV^2$ for each capacitor. Each capacitor has the same capacitance, but in the parallel circuit, each capacitor has a larger voltage than in the series circuit. Thus the parallel circuit stores more energy.

$\boxed{16.}$ The soles of your shoes are made of a material which has a relatively high resistance, and there is relatively high resistance flooring material between your shoes and the literal ground (the Earth). With that high resistance, a malfunctioning appliance would not be able to cause a large current flow through your body. The resistance of bare feet is much less than that of shoes, and the feet are in direct contact with the ground, so the total resistance is much lower and so a larger current would flow through your body.

17. As the sound wave is incident on the diaphragm, the diaphragm will oscillate with the same frequency as the sound wave. That oscillation will change the gap between the capacitor plates, causing the capacitance to change. As the capacitance changes, the charge state of the capacitor will change. For instance, if the plates move together, the capacitance will increase, and charge will flow to the capacitor. If the plates move apart, the capacitance will decrease, and charge will flow away from the capacitor. This current will change the output voltage. Thus if the wave has a frequency of 200 Hz, the capacitance and thus the current and thus the output voltage will all change with a frequency of 200 Hz.

18. The following is one way to accomplish the desired task.



In the current configuration, the light would be on. If either switch is moved, the light will go out. But then if either switch were moved again, the light would come back on.

19. The total energy supplied by the battery is greater than the total energy stored by the capacitor. The extra energy was dissipated in the form of heat in the resistor while current was flowing. That energy will not be “recovered” during the discharging process.

20. An analog voltmeter has a high value resistor in series with the galvanometer, so that the actual voltage drop across the galvanometer is small. An analog ammeter has a very low value resistor in parallel with the galvanometer, so that the actual current through the galvanometer is small. Another significant difference is how they are connected to a circuit. A voltmeter goes in parallel with the component being measured, while an ammeter goes in series with the component being measured.

21. If you mistakenly use an ammeter where you intend to use a voltmeter, you are inserting a short in parallel with some resistance. That means that the resistance of the entire circuit has been lowered, and all of the current will flow through the low-resistance ammeter. Ammeters usually have a fairly small current limit, and so the ammeter might very likely get damaged in such a scenario. Also, if the ammeter is inserted across a voltage source, the source will provide a large current, and again the meter will almost certainly be damaged, or at least disabled by burning out a fuse.

22. An ideal ammeter would have zero resistance so that there was no voltage drop across it, and so it would not affect a circuit into which it was placed in parallel. A zero resistance will not change the resistance of a circuit if placed in series, and hence would not change the current in the circuit. An ideal voltmeter would have infinite resistance so that it would draw no current, and thus would not affect a circuit into which it was placed in parallel. An infinite resistance will not change the resistance of a circuit if placed in parallel, and hence would not change the current in the circuit.

23. The behavior of the circuit is the same in either case. The same current will flow in the circuit with the elements rearranged, because the current in a series circuit is the same for all elements in the circuit.

24. When the voltmeter is connected to the circuit, it reduces the resistance of that part of the circuit. That will make the (resistor + voltmeter) combination a smaller fraction of the total resistance of the circuit than the resistor was alone, which means that it will have a smaller fraction of the total voltage drop across it.

25. The voltmeter, if it has a particularly high internal resistance, will measure the emf of the battery, which is 1.5 V. But if the battery has a high internal resistance (indicating that the battery is "used up", then its terminal voltage, which is the voltage supplied to the flashlight bulb, can be quite low when more current is drawn from the battery by a low resistance like the bulb.

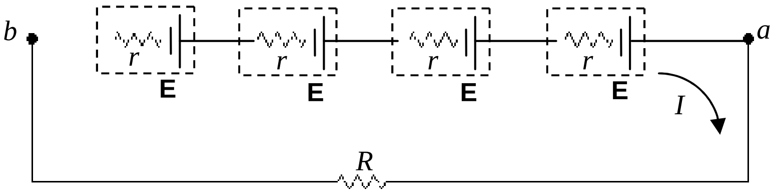
Solutions to Problems

1. See Figure 19-2 for a circuit diagram for this problem. Using the same analysis as in Example 19-1, the current in the circuit is $I=\dfrac{\mathbf{E}}{R+r}$. Use Eq. 19-1 to calculate the terminal voltage.

(*a*) $V_{\mathrm{ab}}=\mathbf{E}-Ir=\mathbf{E}-\left(\dfrac{\mathbf{E}}{R+r}\right)r=\dfrac{\mathbf{E}\left(R+r\right)-\mathbf{E}r}{R+r}=\mathbf{E}\dfrac{R}{R+r}=\left(8.50\,\mathrm{V}\right)\dfrac{81.0\,\Omega}{\left(81.0+0.900\right)\Omega}=\boxed{8.41\,\mathrm{V}}$

(*b*) $V_{\mathrm{ab}}=\mathbf{E}\dfrac{R}{R+r}=\left(8.50\,\mathrm{V}\right)\dfrac{810\,\Omega}{\left(810+0.900\right)\Omega}=\boxed{8.49\,\mathrm{V}}$

2. See the circuit diagram below. The current in the circuit is I. The voltage V_{ab} is given by Ohm's law to be $V_{\text{ab}} = IR$. That same voltage is the terminal voltage of the series EMF.



$$V_{\text{ab}} = (\mathbf{E} - Ir) + (\mathbf{E} - Ir) + (\mathbf{E} - Ir) + (\mathbf{E} - Ir) = 4(\mathbf{E} - Ir) \quad \text{and} \quad V_{\text{ab}} = IR$$

$$4(\mathbf{E} - Ir) = IR \quad \to \quad r = \frac{\mathbf{E} - \frac{1}{4}IR}{I} = \frac{(1.5\,\text{V}) - \frac{1}{4}(0.45\,\text{A})(12\,\Omega)}{0.45\,\text{A}} = \boxed{0.33\,\Omega}$$

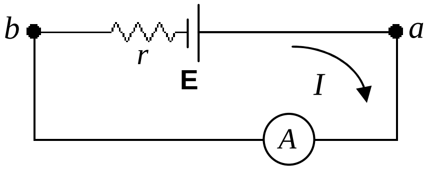
3. See Figure 19-2 for a circuit diagram for this problem. Use Eq. 19-1.

$$V_{\text{ab}} = \mathbf{E} - Ir \quad \to \quad r = \frac{\mathbf{E} - V_{\text{ab}}}{I} = \frac{12.0\,\text{V} - 8.4\,\text{V}}{75\,\text{A}} = \boxed{0.048\,\Omega}$$

$$V_{\text{ab}} = IR \quad \to \quad R = \frac{V_{\text{ab}}}{I} = \frac{8.4\,\text{V}}{75\,\text{A}} = \boxed{0.11\,\Omega}$$

4. We take the low-resistance ammeter to have no resistance. The circuit is shown. The terminal voltage will be 0 volts.

$$V_{\text{ab}} = \mathbf{E} - Ir = 0 \quad \to \quad r = \frac{\mathbf{E}}{I} = \frac{1.5\,\text{V}}{22\,\text{A}} = \boxed{0.068\,\Omega}$$



$\boxed{5.}$ For the resistors in series, use Eq. 19-3.

$$R_{\text{eq}} = R_1 + R_2 + R_3 + R_4 = 4(240\,\Omega) = \boxed{960\,\Omega}$$

For the resistors in parallel, use Eq. 19-4.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{4}{240\,\Omega} \quad \to \quad R_{\text{eq}} = \frac{240\,\Omega}{4} = \boxed{60\,\Omega}$$

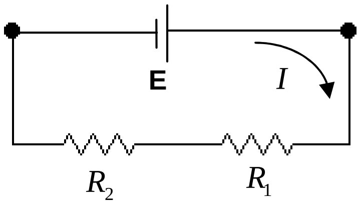
6. (*a*) For the resistors in series, use Eq. 19-3, which says the resistances add linearly.

$$R_{\text{eq}} = 3(45\,\Omega) + 3(75\,\Omega) = \boxed{360\,\Omega}$$

(*b*) For the resistors in parallel, use Eq. 19-4, which says the resistances add reciprocally.

$$\frac{1}{R_{\text{eq}}} = \frac{3}{45\,\Omega} + \frac{3}{75\,\Omega} = \frac{3(75\,\Omega) + 3(45\,\Omega)}{(75\,\Omega)(45\,\Omega)} \quad \to \quad R_{\text{eq}} = \frac{(75\,\Omega)(45\,\Omega)}{3(75\,\Omega) + 3(45\,\Omega)} = \boxed{9.4\,\Omega}$$

7. The equivalent resistance is the sum of the two resistances: $R_{\text{eq}} = R_1 + R_2$. The current in the circuit is then the voltage divided by the equivalent resistance: $I = \frac{V}{R_{\text{eq}}} = \frac{V}{R_1 + R_2}$. The voltage across the 2200-Ω resistor is given by Ohm's law.



$$V_{2200} = IR_2 = \frac{V}{R_1 + R_2} R_2 = V\frac{R_2}{R_1 + R_2} = (12.0\,\text{V})\frac{2200\,\Omega}{650\,\Omega + 2200\,\Omega} = \boxed{9.3\,\text{V}}$$

8. The possible resistances are each resistor considered individually, the series combination, and the parallel combination.

series: $R_{\text{eq}} = R_1 + R_2 = 25\,\Omega + 35\,\Omega = 60\,\Omega$

parallel: $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} \rightarrow R_{\text{eq}} = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{(25\,\Omega)(35\,\Omega)}{60\,\Omega} = 15\,\Omega$

$\boxed{25\,\Omega\text{ , }35\,\Omega\text{ , }60\,\Omega\text{ , }15\,\Omega}$

9. (*a*) The maximum resistance is made by combining the resistors in series.

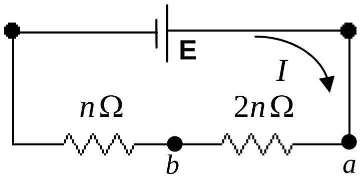
$$R_{\text{eq}} = R_1 + R_2 + R_3 = 680\,\Omega + 940\,\Omega + 1200\,\Omega = \boxed{2820\,\Omega}$$

(*b*) The minimum resistance is made by combining the resistors in parallel.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \rightarrow$$

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{680\,\Omega} + \frac{1}{940\,\Omega} + \frac{1}{1200\,\Omega}\right)^{-1} = \boxed{3.0\times10^2\,\Omega}$$

10. Connecting $3n$ of the resistors in series, where n is an integer, will enable you to make a voltage divider with a 4.0 V output.



$$R_{\text{eq}} = 3n(1.0\,\Omega) \qquad I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{3n}$$

$$V_{\text{ab}} = 2n(1.0\,\Omega)I = 2n(1.0\,\Omega)\frac{\mathcal{E}}{3n} = \frac{2}{3}\mathcal{E} = \frac{2}{3}(6.0\,\text{V}) = 4.0\,\text{V}$$

11. The resistors can all be connected in series.

$$R_{\text{eq}} = R + R + R = 3(240\,\Omega) = \boxed{720\,\Omega}$$

The resistors can all be connected in parallel.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow R_{\text{eq}} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3} = \frac{240\,\Omega}{3} = \boxed{80\,\Omega}$$

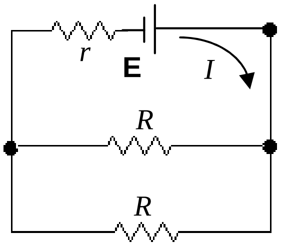
Two resistors in series can be placed in parallel with the third.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R+R} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \rightarrow R_{\text{eq}} = \frac{2R}{3} = \frac{2(240\,\Omega)}{3} = \boxed{160\,\Omega}$$

Two resistors in parallel can be placed in series with the third.

$$R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = R + \frac{R}{2} = \frac{3}{2}(240\,\Omega) = \boxed{360\,\Omega}$$

12. The resistance of each bulb can be found from its power rating.



$$P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P}=\frac{(12.0\,\text{V})^2}{3.0\,\text{W}}=48\,\Omega$$

Find the equivalent resistance of the two bulbs in parallel.

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R}+\frac{1}{R}=\frac{2}{R} \rightarrow R_{\text{eq}}=\frac{R}{2}=\frac{48\,\Omega}{2}=24\,\Omega$$

The terminal voltage is the voltage across this equivalent resistance. Use that to find the current drawn from the battery.

$$V_{\text{ab}}=IR_{\text{eq}} \rightarrow I=\frac{V_{\text{ab}}}{R_{\text{eq}}}=\frac{V_{\text{ab}}}{R/2}=\frac{2V_{\text{ab}}}{R}$$

Finally, use the terminal voltage and the current to find the internal resistance, as in Eq. 19-1.

$$V_{\text{ab}}=\mathsf{E}-Ir \rightarrow r=\frac{\mathsf{E}-V_{\text{ab}}}{I}=\frac{\mathsf{E}-V_{\text{ab}}}{\left(\frac{2V_{\text{ab}}}{R}\right)}=R\frac{\mathsf{E}-V_{\text{ab}}}{2V_{\text{ab}}}=(48\,\Omega)\frac{12.0\,\text{V}-11.8\,\text{V}}{2(11.8\,\text{V})}=\boxed{0.4\,\Omega}$$

13. (*a*) Each bulb should get one-eighth of the total voltage, but let us prove that instead of assuming it. Since the bulbs are identical, the net resistance is $R_{\text{eq}}=8R$. The current flowing through the bulbs is then $V_{\text{tot}}=IR_{\text{eq}} \rightarrow I=\frac{V_{\text{tot}}}{R_{\text{eq}}}=\frac{V_{\text{tot}}}{8R}$. The voltage across one bulb is found from Ohm's law.

$$V=IR=\frac{V_{\text{tot}}}{8R}R=\frac{V_{\text{tot}}}{8}=\frac{110\,\text{V}}{8}=13.75\,\text{V}\approx\boxed{14\,\text{V}}$$

(*b*) $I=\frac{V_{\text{tot}}}{8R} \rightarrow R=\frac{V_{\text{tot}}}{8I}=\frac{110\,\text{V}}{8(0.50\,\text{A})}=27.5\,\Omega\approx\boxed{28\,\Omega}$

$$P=I^2R=(0.50\,\text{A})^2(27.5\,\Omega)=6.875\,\text{W}\approx\boxed{6.9\,\text{W}}$$

14. We model the resistance of the long leads as a single resistor *r*. Since the bulbs are in parallel, the total current is the sum of the current in each bulb, and so $I=8I_R$. The voltage drop across the long leads is $V_{\text{leads}}=Ir=8I_Rr=8(0.24\,\text{A})(1.6\,\Omega)=3.072\,\text{V}$. Thus the voltage across each of the parallel resistors is $V_R=V_{\text{tot}}-V_{\text{leads}}=110\,\text{V}-3.072\,\text{V}=106.9\,\text{V}$. Since we have the current through each resistor, and the voltage across each resistor, we calculate the resistance using Ohm's law.

$$V_R=I_RR \rightarrow R=\frac{V_R}{I_R}=\frac{106.9\,\text{V}}{0.24\,\text{A}}=445.4\,\Omega=\boxed{450\,\Omega}$$

The total power delivered is $P=V_{\text{tot}}I$, and the "wasted" power is I^2r. The fraction wasted is the ratio of those powers.

$$\text{fraction wasted}=\frac{I^2r}{IV_{\text{tot}}}=\frac{Ir}{V_{\text{tot}}}=\frac{8(0.24\,\text{A})(1.6\,\Omega)}{110\,\text{V}}=\boxed{0.028}$$

So about 3% of the power is wasted.

15. Each bulb will get one-eighth of the total voltage, and so $V_{\text{bulb}} = \frac{V_{\text{tot}}}{8}$. Use that voltage and the power dissipated by each bulb to calculate the resistance of a bulb.

$$P_{\text{bulb}} = \frac{V_{\text{bulb}}^2}{R} \rightarrow R = \frac{V_{\text{bulb}}^2}{P} = \frac{V_{\text{tot}}^2}{64P} = \frac{(110\,\text{V})^2}{64(7.0\,\text{W})} = \boxed{27\,\Omega}$$

16. To fix this circuit, connect another resistor in parallel with the 480-Ω resistor so that the equivalent resistance is the desired 320 Ω.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_2 = \left(\frac{1}{R_{\text{eq}}} - \frac{1}{R_1}\right)^{-1} = \left(\frac{1}{320\,\Omega} - \frac{1}{480\,\Omega}\right)^{-1} = \boxed{960\,\Omega}$$

17. (*a*) The equivalent resistance is found by combining the 820 Ω and 680 Ω resistors in parallel, and then adding the 470 Ω resistor in series with that parallel combination.

$$R_{\text{eq}} = \left(\frac{1}{820\,\Omega} + \frac{1}{680\,\Omega}\right)^{-1} + 470\,\Omega = 372\,\Omega + 470\,\Omega = 842\,\Omega \approx \boxed{840\,\Omega}$$

(*b*) The current delivered by the battery is $I = \frac{V}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{842\,\Omega} = 1.425\times10^{-2}\,\text{A}$. This is the current in the 470 Ω resistor. The voltage across that resistor can be found by Ohm's law.

$$V_{470} = IR = (1.425\times10^{-2}\,\text{A})(470\,\Omega) = \boxed{6.7\,\text{V}}$$

Thus the voltage across the parallel combination must be $12.0\,\text{V} - 6.7\,\text{V} = \boxed{5.3\,\text{V}}$. This is the voltage across both the 820 Ω and 680 Ω resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

$$V_{\text{parallel}} = IR_{\text{parallel}} = (1.425\times10^{-2}\,\text{A})(372\,\Omega) = 5.3\,\text{V}$$

$\boxed{18.}$ The resistance of each bulb can be found by using Eq. 18-6, $P = V^2/R$. The two individual resistances are combined in parallel. We label the bulbs by their wattage.

$$P = V^2/R \rightarrow \frac{1}{R} = \frac{P}{V^2}$$

$$R_{\text{eq}} = \left(\frac{1}{R_{75}} + \frac{1}{R_{40}}\right)^{-1} = \left(\frac{75\,\text{W}}{(110\,\text{V})^2} + \frac{40\,\text{W}}{(110\,\text{V})^2}\right)^{-1} = 105.2\,\Omega \approx \boxed{110\,\Omega}$$

19. (*a*) Note that adding resistors in series always results in a larger resistance, and adding resistors in parallel always results in a smaller resistance. Closing the switch adds another resistor in parallel with R_3 and R_4, which lowers the net resistance of the parallel portion of the circuit, and thus lowers the equivalent resistance of the circuit. That means that more current will be delivered by the battery. Since R_1 is in series with the battery, its voltage will increase. Because of that increase, the voltage across R_3 and R_4 must decrease so that the total voltage drops around the loop are equal to the battery voltage. Since there was no voltage across R_2 until the switch was closed, its voltage will increase. To summarize:

$$\boxed{V_1 \text{ and } V_2 \text{ increase ; } V_3 \text{ and } V_4 \text{ decrease}}$$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance. Thus

$$\boxed{I_1 \text{ and } I_2 \text{ increase ; } I_3 \text{ and } I_4 \text{ decrease}}$$

(*c*) Since the battery voltage does not change and the current delivered by the battery increases, the power delivered by the battery, found by multiplying the voltage of the battery by the current delivered, $\boxed{\text{increases}}$.

(*d*) Before the switch is closed, the equivalent resistance is R_3 and R_4 in parallel, combined with R_1 in series.

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125\,\Omega + \left(\frac{2}{125\,\Omega}\right)^{-1} = 187.5\,\Omega$$

The current delivered by the battery is the same as the current through R_1.

$$I_{\text{total}} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{22.0\,\text{V}}{187.5\,\Omega} = 0.1173\,\text{A} = I_1$$

The voltage across R_1 is found by Ohm's law.

$$V_1 = IR_1 = (0.1173\,\text{A})(125\,\Omega) = 14.66\,\text{V}$$

The voltage across the parallel resistors is the battery voltage less the voltage across R_1.

$$V_{\text{p}} = V_{\text{battery}} - V_1 = 22.0\,\text{V} - 14.66\,\text{V} = 7.34\,\text{V}$$

The current through each of the parallel resistors is found from Ohm's law.

$$I_3 = \frac{V_{\text{p}}}{R_2} = \frac{7.34\,\text{V}}{125\,\Omega} = 0.0587\,\text{A} = I_4$$

Notice that the current through each of the parallel resistors is half of the total current, within the limits of significant figures. The currents before closing the switch are as follows.

$$\boxed{I_1 = 0.117\,\text{A} \quad I_3 = I_4 = 0.059\,\text{A}}$$

After the switch is closed, the equivalent resistance is R_2, R_3, and R_4 in parallel, combined with R_1 in series. Do a similar analysis.

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125\,\Omega + \left(\frac{3}{125\,\Omega}\right)^{-1} = 166.7\,\Omega$$

$$I_{\text{total}} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{22.0\,\text{V}}{166.7\,\Omega} = 0.1320\,\text{A} = I_1 \qquad V_1 = IR_1 = (0.1320\,\text{A})(125\,\Omega) = 16.5\,\text{V}$$

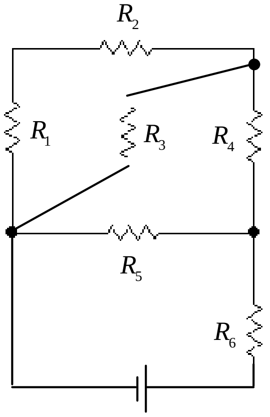
$$V_{\text{p}} = V_{\text{battery}} - V_1 = 22.0\,\text{V} - 16.5\,\text{V} = 5.5\,\text{V} \qquad I_2 = \frac{V_{\text{p}}}{R_2} = \frac{5.5\,\text{V}}{125\,\Omega} = 0.044\,\text{A} = I_3 = I_4$$

Notice that the current through each of the parallel resistors is one third of the total current, within the limits of significant figures. The currents after closing the switch are as follows.

$$\boxed{I_1 = 0.132\,\text{A} \quad I_2 = I_3 = I_4 = 0.044\,\text{A}}$$

$\boxed{\text{Yes}}$, the predictions made in part (*b*) are all confirmed.

20. The resistors have been numbered in the accompanying diagram to help in the analysis. R_1 and R_2 are in series with an equivalent resistance of $R_{12} = R + R = 2R$. This combination is in parallel with R_3, with an equivalent resistance of $R_{123} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \frac{2}{3}R$. This combination is in series with R_4, with an equivalent resistance of $R_{1234} = \frac{2}{3}R + R = \frac{5}{3}R$. This combination is in parallel with R_5, with an equivalent resistance of $R_{12345} = \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1} = \frac{5}{8}R$. Finally, this combination is in series with R_6, and we calculate the final equivalent resistance.



$$R_{\text{eq}} = \tfrac{5}{8}R + R = \tfrac{13}{8}R = \tfrac{13}{8}(2.8\,\text{k}\Omega) = 4.55\,\text{k}\Omega \approx \boxed{4.6\,\text{k}\Omega}$$

21. We label identical resistors from left to right as R_{left}, R_{middle}, and R_{right}. When the switch is opened, the equivalent resistance of the circuit increases from $\frac{3}{2}R + r$ to $2R + r$. Thus the current delivered by the battery decreases, from $\frac{\mathbf{E}}{\frac{3}{2}R + r}$ to $\frac{\mathbf{E}}{2R + r}$. Note that this is LESS than a 50% decrease.

(*a*) Because the current from the battery has decreased, the voltage drop across R_{left} will decrease, since it will have less current than before. The voltage drop across R_{right} decreases to 0, since no current is flowing in it. The voltage drop across R_{middle} will increase, because even though the total current has decreased, the current flowing through R_{middle} has increased since before the switch was opened, only half the total current was flowing through R_{middle}. To summarize:

$$\boxed{V_{\text{left}} \text{ decreases ; } V_{\text{middle}} \text{ increases ; } V_{\text{right}} \text{ goes to 0}}\,.$$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance.

$$\boxed{I_{\text{left}} \text{ decreases ; } I_{\text{middle}} \text{ increases ; } I_{\text{right}} \text{ goes to 0}}$$

(*c*) Since the current from the battery has decreased, the voltage drop across *r* will decrease, and thus the $\boxed{\text{terminal voltage increases}}$.

(*d*) With the switch closed, the equivalent resistance is $\frac{3}{2}R + r$. Thus the current in the circuit is $I_{\text{closed}} = \frac{\mathbf{E}}{\frac{3}{2}R + r}$, and the terminal voltage is given by Eq. 19-1.

$$V_{\substack{\text{terminal}\\\text{closed}}} = \mathbf{E} - I_{\text{closed}}r = \mathbf{E} - \frac{\mathbf{E}}{\frac{3}{2}R + r}r = \mathbf{E}\left(1 - \frac{r}{\frac{3}{2}R + r}\right) = (15.0\,\text{V})\left(1 - \frac{0.50\,\Omega}{\frac{3}{2}(5.50\,\Omega) + 0.50\,\Omega}\right)$$

$$= \boxed{14.1\,\text{V}}$$

(*e*) With the switch open, the equivalent resistance is $2R + r$. Thus the current in the circuit is $I_{\text{closed}} = \frac{\mathbf{E}}{2R + r}$, and again the terminal voltage is given by Eq. 19-1.

$$V_{\substack{\text{terminal}\\ \text{closed}}} = \mathcal{E} - I_{\text{closed}} r = \mathcal{E} - \frac{\mathcal{E}}{2R+r} r = \mathcal{E}\left(1 - \frac{r}{2R+r}\right) = (15.0\,\text{V})\left(1 - \frac{0.50\,\Omega}{2(5.50\,\Omega)+0.50\,\Omega}\right)$$
$$= \boxed{14.3\,\text{V}}$$

22. Find the maximum current and resulting voltage for each resistor under the power restriction.

$$P = I^2R = \frac{V^2}{R} \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}\,,\; V = \sqrt{RP}$$
$$I_{1800} = \sqrt{\frac{0.5\,\text{W}}{1.8\times10^3\,\Omega}} = 0.0167\,\text{A} \qquad V_{1800} = \sqrt{(0.5\,\text{W})(1.8\times10^3\,\Omega)} = 30\,\text{V}$$
$$I_{2800} = \sqrt{\frac{0.5\,\text{W}}{2.8\times10^3\,\Omega}} = 0.0134\,\text{A} \qquad V_{2800} = \sqrt{(0.5\,\text{W})(2.8\times10^3\,\Omega)} = 37.4\,\text{V}$$
$$I_{2100} = \sqrt{\frac{0.5\,\text{W}}{2.1\times10^3\,\Omega}} = 0.0154\,\text{A} \qquad V_{2100} = \sqrt{(0.5\,\text{W})(2.1\times10^3\,\Omega)} = 32.4\,\text{V}$$

The parallel resistors have to have the same voltage, and so the voltage across that combination is limited to 32.4 V. That would require a current given by Ohm's law and the parallel combination of the two resistors.

$$I_{\text{parallel}} = \frac{V_{\text{parallel}}}{R_{\text{parallel}}} = V_{\text{parallel}}\left(\frac{1}{R_{2800}} + \frac{1}{R_{2100}}\right) = (32.4\,\text{V})\left(\frac{1}{2800\,\Omega} + \frac{1}{2100\,\Omega}\right) = 0.027\,\text{A}$$

This is more than the maximum current that can be in R_{1800}. Thus the maximum current that R_{1800} can carry, $0.0167\,\text{A}$, is the maximum current for the circuit. The maximum voltage that can be applied across the combination is the maximum current times the equivalent resistance. The equivalent resistance is the parallel combination of R_{2800} and R_{2100} added to R_{1800}.

$$V_{\text{max}} = I_{\text{max}} R_{\text{eq}} = I_{\text{max}}\left[R_{1800} + \left(\frac{1}{R_{2800}} + \frac{1}{R_{2100}}\right)^{-1}\right] = (0.0167\,\text{A})\left[1800\,\Omega + \left(\frac{1}{2800\,\Omega} + \frac{1}{2100\,\Omega}\right)^{-1}\right]$$
$$= (0.0167\,\text{A})\left[1800\,\Omega + \left(\frac{1}{2800\,\Omega} + \frac{1}{2100\,\Omega}\right)^{-1}\right] = \boxed{50\,\text{V}}$$

23. All of the resistors are in series, so the equivalent resistance is just the sum of the resistors. Use Ohm's law then to find the current, and show all voltage changes starting at the negative pole of the battery and going counterclockwise.

$$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{9.0\,\text{V}}{(8.0+12.0+2.0)\,\Omega} = 0.409\,\text{A} \approx \boxed{0.41\,\text{A}}$$
$$\sum \text{voltages} = 9.0\,\text{V} - (8.0\,\Omega)(0.409\,\text{A}) - (12.0\,\Omega)(0.409\,\text{A}) - (2.0\,\Omega)(0.409\,\text{A})$$
$$= 9.0\,\text{V} - 3.27\,\text{V} - 4.91\,\text{V} - 0.82\,\text{V} = \boxed{0.00\,\text{V}}$$

24. Apply Kirchhoff's loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$-I(1.0\,\Omega)+18\,\text{V}-I(6.6\;\Omega)-12\,\text{V}-I(2.0\,\Omega)=0 \;\rightarrow\; I=\frac{6\,\text{V}}{9.6\;\Omega}=0.625\text{A}$$

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

$$18\,\text{V battery:}\;\; V_{\text{terminal}}=-I(1.0\,\Omega)+18\,\text{V}=-(0.625\,\text{A})(1.0\,\Omega)+18\,\text{V}=\boxed{17.4\,\text{V}}$$

$$12\,\text{V battery:}\;\; V_{\text{terminal}}=I(2.0\,\Omega)+12\,\text{V}=(0.625\,\text{A})(2.0\,\Omega)+12\,\text{V}=\boxed{13.3\,\text{V}}$$

25. From Example 19-8, we have $I_1=-0.87\,\text{A}$, $I_2=2.6\,\text{A}$, $I_3=1.7\,\text{A}$. If another significant figure had been kept, the values would be $I_1=-0.858\,\text{A}$, $I_2=2.58\,\text{A}$, $I_3=1.73\,\text{A}$. We use those results.

(*a*) To find the potential difference between points a and d, start at point a and add each individual potential difference until reaching point d. The simplest way to do this is along the top branch.

$$V_{\text{ad}}=V_{\text{d}}-V_{\text{a}}=-I_1(30\,\Omega)=-(0.858\,\text{A})(30\,\Omega)=\boxed{-25.7\,\text{V}}$$

Slight differences will be obtained in the final answer depending on the branch used, due to rounding. For example, using the bottom branch, we get the following.

$$V_{\text{ad}}=V_{\text{d}}-V_{\text{a}}=\mathsf{E}_1-I_2(21\Omega)=80\,\text{V}-(2.58\,\text{A})(21\Omega)=-25.8\,\text{V}$$

(*b*) For the 80-V battery, the terminal voltage is the potential difference from point g to point e. For the 45-V battery, the terminal voltage is the potential difference from point d to point b.

$$80\,\text{V battery:}\;\; V_{\text{terminal}}=\mathsf{E}_1-I_2r=80\,\text{V}-(2.58\,\text{A})(1.0\,\Omega)=\boxed{77.4\,\text{V}}$$

$$45\,\text{V battery:}\;\; V_{\text{terminal}}=E_2-I_3r=45\,\text{V}-(1.73\,\text{A})(1.0\,\Omega)=\boxed{43.3\,\text{V}}$$

26. To find the potential difference between points a and b, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$-IR+\mathsf{E}-IR-IR+\mathsf{E}-IR=0 \;\rightarrow\; I=\frac{\mathsf{E}}{2R}$$

$$V_{\text{ab}}=V_{\text{a}}-V_{\text{b}}=-IR+\mathsf{E}-IR=\mathsf{E}-2IR=\mathsf{E}-2\frac{\mathsf{E}}{2R}R=\boxed{0\,\text{V}}$$

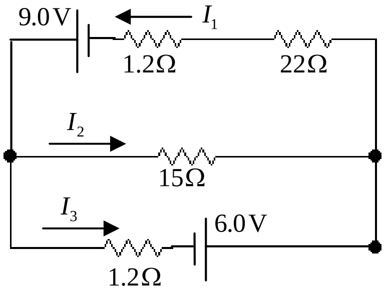
27. Because there are no resistors in the bottom branch, it is possible to write Kirchhoff loop equations that only have one current term, making them easier to solve. To find the current through R_1, go around the outer loop counterclockwise, starting at the lower left corner.

$$V_3-I_1R_1+V_1=0 \;\rightarrow\; I_1=\frac{V_3+V_1}{R_1}=\frac{6.0\,\text{V}+9.0\,\text{V}}{22\,\Omega}=\boxed{0.68\,\text{A}\text{, left}}$$

To find the current through R_2, go around the lower loop counterclockwise, starting at the lower left corner.

$$V_3-I_2R_2=0 \;\rightarrow\; I_2=\frac{V_3}{R_2}=\frac{6.0\,\text{V}}{15\,\Omega}=\boxed{0.40\,\text{A}\text{, left}}$$

28. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the left of the circuit.



$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the outer loop, starting at the lower left corner, and progressing counterclockwise.

$$-I_3(1.2\,\Omega) + 6.0\,\text{V} - I_1(22\,\Omega) - I_1(1.2\,\Omega) + 9.0\,\text{V} = 0 \quad \rightarrow$$

$$15 = 23.2I_1 + 1.2I_3$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the lower left corner, and progressing counterclockwise.

$$-I_3(1.2\,\Omega) + 6.0\,\text{V} + I_2(15\,\Omega) = 0 \quad \rightarrow \quad 6 = -15I_2 + 1.2I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$15 = 23.2I_1 + 1.2I_3 = 23.2(I_2 + I_3) + 1.2I_3 = 23.2I_2 + 24.4I_3 \;\; ; \;\; 6 = -15I_2 + 1.2I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

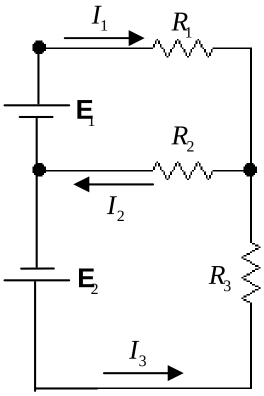
$$6 = -15I_2 + 1.2I_3 \quad \rightarrow \quad I_2 = \frac{-6 + 1.2I_3}{15}$$

$$15 = 23.2I_2 + 24.4I_3 = 23.2\left(\frac{-6 + 1.2I_3}{15}\right) + 24.4I_3 \quad \rightarrow \quad 225 = -138 + 27.84I_3 + 366I_3 \quad \rightarrow$$

$$I_3 = \frac{363}{393.84} = 0.9217\,\text{A} \;\; ; \;\; I_2 = \frac{-6 + 1.2I_3}{15} = \frac{-6 + 1.2(0.9217)}{15} = -0.3263\,\text{A} \approx \boxed{0.33\,\text{A}\,,\ \text{left}}$$

$$I_1 = I_2 + I_3 = 0.5954\,\text{A} \approx \boxed{0.60\,\text{A}\,,\ \text{left}}$$

$\boxed{29.}$ There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.



$$I_2 = I_1 + I_3 \quad \rightarrow \quad I_1 = I_2 - I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$E_1 - I_1R_1 - I_2R_2 = 0 \quad \rightarrow \quad 9 = 25I_1 + 18I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$E_2 - I_3R_3 - I_2R_2 = 0 \quad \rightarrow \quad 12 = 35I_3 + 18I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 25I_1 + 18I_2 = 25(I_2 - I_3) + 18I_2 = 43I_2 - 25I_3 \;\; ; \;\; 12 = 35I_3 + 18I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

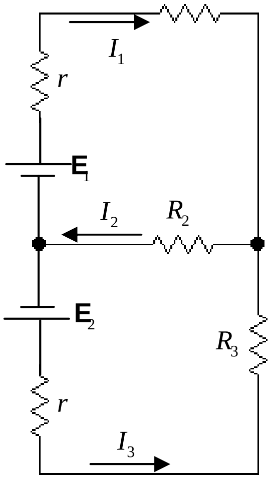
$$12 = 35I_3 + 18I_2 \quad \rightarrow \quad I_2 = \frac{12-35I_3}{18}$$

$$9 = 43I_2 - 25I_3 = 43\left(\frac{12-35I_3}{18}\right) - 25I_3 \quad \rightarrow \quad 162 = 516 - 1505I_3 - 450I_3 \quad \rightarrow$$

$$I_3 = \frac{354}{1955} = 0.1811\,\text{A} \approx \boxed{0.18\,\text{A}\text{, up}} \;\; ; \;\; I_2 = \frac{12-35I_3}{18} = 0.3145\,\text{A} \approx \boxed{0.31\,\text{A}\text{, left}}$$

$$I_1 = I_2 - I_3 = 0.1334\,\text{A} \approx \boxed{0.13\,\text{A}\text{, right}}$$

30. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.



$$I_2 = I_1 + I_3 \quad \rightarrow \quad I_1 = I_2 - I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$\mathcal{E}_1 - I_1 r - I_1 R_1 - I_2 R_2 = 0 \quad \rightarrow \quad 9 = 26I_1 + 18I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$\mathcal{E}_2 - I_3 r - I_3 R_3 - I_2 R_2 = 0 \quad \rightarrow \quad 12 = 36I_3 + 18I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 26I_1 + 18I_2 = 26\left(I_2 - I_3\right) + 18I_2 = 44I_2 - 25I_3 \;\; ; \;\; 12 = 36I_3 + 18I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$12 = 36I_3 + 18I_2 \quad \rightarrow \quad I_2 = \frac{12-36I_3}{18} = \frac{2-6I_3}{3}$$

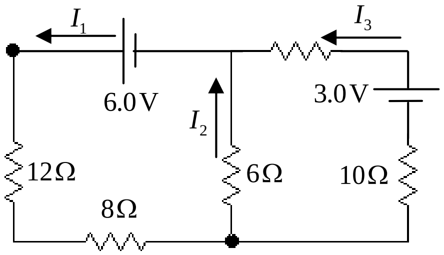
$$9 = 44I_2 - 25I_3 = 44\left(\frac{2-6I_3}{3}\right) - 25I_3 \quad \rightarrow \quad 27 = 88 - 264I_3 - 75I_3 \quad \rightarrow$$

$$I_3 = \frac{61}{339} = 0.1799\,\text{A} \approx \boxed{0.18\,\text{A}\text{, up}} \;\; ; \;\; I_2 = \frac{2-6I_3}{3} = 0.3069\,\text{A} \approx \boxed{0.31\,\text{A left}}$$

$$I_1 = I_2 - I_3 = 0.127\,\text{A} \approx \boxed{0.13\,\text{A}\text{, right}}$$

To two significant figures, the currents do not change from problem 29 with the addition of internal resistances.

31. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.



$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$6.0\,\text{V} - I_1(12\,\Omega) - I_1(8\,\Omega) - I_2(6\,\Omega) = 0 \;\rightarrow\; 6 = 20I_1 + 6I_2 \;\rightarrow\; 3 = 10I_1 + 3I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0\,\text{V} - I_3(2\,\Omega) + I_2(6.0\,\Omega) - I_3(10\,\Omega) = 0 \;\rightarrow\; 3 = -6I_2 + 12I_3 \;\rightarrow\; 1 = -2I_2 + 4I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$3 = 10I_1 + 3I_2 = 10(I_2 + I_3) + 3I_2 = 13I_2 + 10I_3 \;\;;\;\; 1 = -2I_2 + 4I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$1 = -2I_2 + 4I_3 \;\rightarrow\; I_2 = \frac{4I_3 - 1}{2}$$

$$3 = 13I_2 + 10I_3 = 13\left(\frac{4I_3 - 1}{2}\right) + 10I_3 \;\rightarrow\; 6 = 52I_3 - 13 + 20I_3 \;\rightarrow$$

$$I_3 = \frac{19}{72} = 0.2639\,\text{A} \approx 0.26\,\text{A} \;\;;\;\; I_2 = \frac{4I_3 - 1}{2} = 0.0278\,\text{A} \approx 0.028\,\text{A}$$

$$I_1 = I_2 + I_3 = 0.2917\,\text{A} \approx 0.29\,\text{A}$$

The current in each resistor is as follows:

$$\boxed{2\,\Omega\text{: 0.26 A} \quad 6\,\Omega\text{: 0.028 A} \quad 8\,\Omega\text{: 0.29 A} \quad 10\,\Omega\text{: 0.26 A} \quad 12\,\Omega\text{: 0.29 A}}$$

32. Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise.

$$12.0\,\text{V} - I_2(1.0\,\Omega) - I_2(10\,\Omega) - I_1(12\,\Omega) + 12.0\,\text{V} - I_2(1.0\,\Omega) - I_1(8.0\,\Omega) = 0 \;\rightarrow$$

$$24 = 11I_2 + 21I_1 = 0$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\,\text{V} - I_2(1.0\,\Omega) - I_2(10\,\Omega) + I_3(18\,\Omega) + I_3(1.0\,\Omega) - 6.0\,\text{V} + I_3(15\,\Omega) = 0 \;\rightarrow$$

$$6 = 11I_2 - 34I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 11I_2 + 21I_1 = 11I_2 + 21(I_2 + I_3) = 32I_2 + 21I_3 \;\;;\;\; 6 = 11I_2 - 34I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6 = 11I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{11}$$

$$24 = 32I_2 + 21I_3 = 32\left(\frac{6 + 34I_3}{11}\right) + 21I_3 \;\rightarrow\; 264 = 192 + 1088I_3 + 231I_3 \;\rightarrow\; 72 = 1319I_3 \;\rightarrow$$

$$I_3 = \frac{72}{1319} = 0.05459\,\text{A} \approx \boxed{0.055\,\text{A}} \;\;;\;\; I_2 = \frac{6 + 34I_3}{11} = 0.714\,\text{A} \approx \boxed{0.71\,\text{A}} \;\;;\;\; I_1 = I_2 + I_3 = \boxed{0.77\,\text{A}}$$

Also find the terminal voltage of the 6.0-V battery.

$$V_{\text{terminal}} = \mathsf{E} - I_3 r = 6.0\,\text{V} - (0.0546\,\text{A})(1.0\,\Omega) = \boxed{5.9\,\text{V}}$$

33. Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise.

$$12.0\,\text{V} - I_2(1.0\,\Omega) - I_2(10\,\Omega) + 12.0\,\text{V} - I_2(1.0\,\Omega) - I_1(8.0\,\Omega) = 0 \quad \rightarrow$$

$$24 = 11I_2 + 9I_1 = 0$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\,\text{V} - I_2(1.0\,\Omega) - I_2(10\,\Omega) + I_3(18\,\Omega) + I_3(1.0\,\Omega) - 6.0\,\text{V} + I_3(15\,\Omega) = 0 \quad \rightarrow$$

$$6 = 11I_2 - 34I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 11I_2 + 9I_1 = 11I_2 + 9(I_2 + I_3) = 20I_2 + 9I_3 \ ; \ 6 = 11I_2 - 34I_3$$

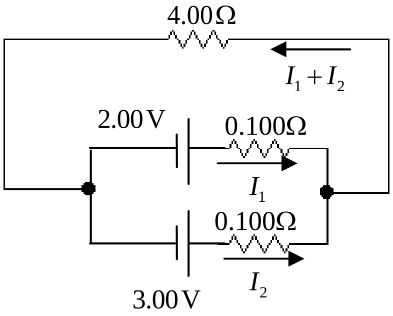
Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6 = 11I_2 - 34I_3 \quad \rightarrow \quad I_2 = \frac{6 + 34I_3}{11}$$

$$24 = 20I_2 + 9I_3 = 20\left(\frac{6 + 34I_3}{11}\right) + 9I_3 \quad \rightarrow \quad 264 = 120 + 680I_3 + 99I_3 \quad \rightarrow \quad 144 = 779I_3 \quad \rightarrow$$

$$I_3 = \frac{144}{779} = 0.1849\,\text{A} \ ; \ I_2 = \frac{6 + 34I_3}{11} = 1.1170\,\text{A} \ ; \ I_1 = I_2 + I_3 = \boxed{1.30\,\text{A}}$$

34. Define I_1 to be the current to the right through the 2.00 V battery, and I_2 to be the current to the right through the 3.00 V battery. At the junction, they combine to give current $I = I_1 + I_2$ to the left through the top branch. Apply Kirchhoff's loop rule first to the upper loop, and then to the outer loop, and solve for the currents.



$$2.00\,\text{V} - I_1(0.100\,\Omega) - (I_1 + I_2)(4.00\,\Omega) = 0 \quad \rightarrow$$

$$2.00 - 4.100I_1 - 4.00I_2 = 0$$

$$3.00\,\text{V} - I_2(0.100\,\Omega) - (I_1 + I_2)(4.00\,\Omega) = 0 \quad \rightarrow$$

$$3.00 - 4.00I_1 - 4.100I_2 = 0$$

Solve the first equation for I_2, and substitute into the second equation to solve for the currents.

$$2.00-4.100I_1-4.00I_2=0 \quad\to\quad I_2=\frac{2.00-4.100I_1}{4.00}$$

$$3.00-4.00I_1--4.100I_2=3.00-4.00I_1-4.100\left(\frac{2.00-4.100I_1}{4.00}\right)=0$$

Multiply by 4

$$12.00-16.00I_1-4.100\left(2.00-4.100I_1\right)=12.00-16.00I_1-8.20+16.81I_1=0$$

$$I_1=-\frac{3.80}{0.81}=-4.691\,\text{A} \qquad I_2=\frac{2.00-4.100I_1}{4.00}=\frac{2.00-4.100\left(-4.691\right)}{4.00}=5.308\,\text{A}$$

The voltage across R is its resistance times $I=I_1+I_2$.

$$V_R=R\left(I_1+I_2\right)=\left(4.00\,\Omega\right)\left(-4.691\,\text{A}+5.308\,\text{A}\right)=2.468\,\text{V}\approx\boxed{2.47\,\text{V}}$$

Note that the top battery is being charged – the current is flowing through it from positive to negative.

35. (*a*) Capacitors in parallel add according to Eq. 19-5.

$$C_{\text{eq}}=C_1+C_2+C_3+C_4+C_5+C_6=6\left(4.7\times10^{-6}\,\text{F}\right)=\boxed{2.82\times10^{-5}\,\text{F}}=28.2\,\mu\text{F}$$

(*b*) Capacitors in series add according to Eq. 19-6.

$$C_{\text{eq}}=\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}+\frac{1}{C_5}+\frac{1}{C_6}\right)^{-1}=\left(\frac{6}{4.7\times10^{-6}\,\text{F}}\right)^{-1}=\frac{4.7\times10^{-6}\,\text{F}}{6}=\boxed{7.8\times10^{-7}\,\text{F}}$$
$$=0.78\,\mu\text{F}$$

36. The maximum capacitance is found by connecting the capacitors in parallel.

$$C_{\text{max}}=C_1+C_2+C_3=3.2\times10^{-9}\,\text{F}+7.5\times10^{-9}\,\text{F}+1.00\times10^{-8}\,\text{F}=\boxed{2.07\times10^{-8}\,\text{F in parallel}}$$

The minimum capacitance is found by connecting the capacitors in series.

$$C_{\text{min}}=\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}=\left(\frac{1}{3.2\times10^{-9}\,\text{F}}+\frac{1}{7.5\times10^{-9}\,\text{F}}+\frac{1}{1.00\times10^{-8}\,\text{F}}\right)^{-1}=\boxed{1.83\times10^{-9}\,\text{F in series}}$$

37. The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{\text{eq}}=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}+C_3=\left(\frac{1}{3.00\times10^{-6}\,\text{F}}+\frac{1}{4.00\times10^{-6}\,\text{F}}\right)^{-1}+2.00\times10^{-6}\,\text{F}=\boxed{3.71\times10^{-6}\,\text{F}}=3.71\,\mu\text{F}$$

38. The full voltage is across the $2.00\,\mu\text{F}$ capacitor, and so $\boxed{V_{2.00}=26.0\,\text{V}}$. To find the voltage across the two capacitors in series, find their equivalent capacitance and the charge stored. That charge will be the same for both of the series capacitors. Finally, use that charge to determine the voltage on each capacitor. Notice that the sum of the voltages across the series capacitors is 26.0 V.

$$C_{\text{eq}}=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}=\left(\frac{1}{3.00\times10^{-6}\,\text{F}}+\frac{1}{4.00\times10^{-6}\,\text{F}}\right)^{-1}=1.714\times10^{-6}\,\text{F}$$

$$Q_{\text{eq}}=C_{\text{eq}}V=\left(1.714\times10^{-6}\,\text{F}\right)\left(26.0\,\text{V}\right)=4.456\times10^{-5}\,\text{C}$$

$$V_{3.00}=\frac{Q_{\text{eq}}}{C}=\frac{4.456\times10^{-5}\,\text{C}}{3.00\times10^{-6}\,\text{F}}=\boxed{14.9\,\text{V}} \qquad V_{4.00}=\frac{Q_{\text{eq}}}{C}=\frac{4.456\times10^{-5}\,\text{C}}{4.00\times10^{-6}\,\text{F}}=\boxed{11.1\,\text{V}}$$

39. To reduce the net capacitance, another capacitor must be added in series.

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2} \quad\rightarrow\quad \frac{1}{C_2}=\frac{1}{C_{\text{eq}}}-\frac{1}{C_1}=\frac{C_1-C_{\text{eq}}}{C_1C_{\text{eq}}} \quad\rightarrow$$

$$C_2=\frac{C_1C_{\text{eq}}}{C_1-C_{\text{eq}}}=\frac{\left(4.8\times10^{-9}\,\text{F}\right)\left(2.9\times10^{-9}\,\text{F}\right)}{\left(4.8\times10^{-9}\,\text{F}\right)-\left(2.9\times10^{-9}\,\text{F}\right)}=\boxed{7.3\times10^{-9}\,\text{F}}$$

$\boxed{\text{Yes}}$, an existing connection needs to be broken in the process. One of the connections of the original capacitor to the circuit must be disconnected in order to connect the additional capacitor in series.

40. Capacitors in parallel add linearly, and so adding a capacitor in parallel will increase the net capacitance without removing the $5.0\,\mu\text{F}$ capacitor.

$$5.0\,\mu\text{F}+C=16\mu\text{F} \quad\rightarrow\quad C=\boxed{11.0\mu\text{F connected in parallel}}$$

41. The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{\text{eq}}=C_1+\left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}=C_1+\left(\frac{C_3}{C_2C_3}+\frac{C_2}{C_2C_3}\right)^{-1}=C_1+\left(\frac{C_2+C_3}{C_2C_3}\right)^{-1}=\boxed{C_1+\frac{C_2C_3}{C_2+C_3}}$$

42. For each capacitor, the charge is found by multiplying the capacitance times the voltage. For C_1, the full 45.0 V is across the capacitance, so $Q_1=C_1V=\left(22.6\times10^{-6}\,\text{F}\right)\left(45.0\,\text{V}\right)=1.02\times10^{-3}\,\text{C}$. The equivalent capacitance of the series combination of C_2 and C_3 has the full 45.0 V across it, and the charge on the series combination is the same as the charge on each of the individual capacitors.

$$C_{\text{eq}}=\left(\frac{1}{C}+\frac{1}{C/2}\right)^{-1}=\frac{C}{3} \qquad Q_{\text{eq}}=C_{eq}V=\tfrac{1}{3}\left(22.6\times10^{-6}\,\text{F}\right)\left(45.0\,\text{V}\right)=3.39\times10^{-4}\,\text{C}$$

To summarize: $\boxed{Q_1=1.02\times10^{-3}\,\text{C} \qquad Q_2=Q_3=3.39\times10^{-4}\,\text{C}}$

43. Capacitors in series have the same charge, so $Q_3=24.0\mu\text{C}$. The voltage on a capacitor is the charge on the capacitor divided by the capacitance.

$$V_2=\frac{Q_2}{C_2}=\frac{24.0\mu\,\text{C}}{16.0\mu\,\text{F}}=1.50\,\text{V} \;;\; V_3=\frac{Q_3}{C_3}=\frac{24.0\mu\,\text{C}}{16.0\mu\,\text{F}}=1.50\,\text{V}$$

Adding the two voltages together gives $V=V_2+V_3=3.00\,\text{V}$. This is also V_1. The charge on C_1 is found from the capacitance and the voltage: $Q_1=C_1V_1=\left(16.0\mu\,\text{F}\right)\left(3.00\,\text{V}\right)=48.0\mu\,\text{C}$.

$$\boxed{\begin{array}{l} Q_1=48.0\mu\,\text{C},\ V_1=3.00\,\text{V} \qquad Q_2=24.0\mu\,\text{C},\ V_2=1.50\,\text{V} \qquad Q_3=24.0\mu\,\text{C},\ V_3=1.50\,\text{V} \\ V=3.00\,\text{V} \end{array}}$$

44. Find the equivalent capacitance, and then calculate the stored energy using Eq. 17-10, $\text{PE}=\tfrac{1}{2}CV^2$. The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{\text{eq}}=C+\left(\frac{1}{C}+\frac{1}{C}\right)^{-1}=\tfrac{3}{2}C \qquad \text{PE}=\tfrac{1}{2}C_{\text{eq}}V^2=\tfrac{1}{2}\tfrac{3}{2}\left(7.2\times10^{-6}\,\text{F}\right)\left(78\,\text{V}\right)^2=\boxed{3.3\times10^{-2}\,\text{J}}$$

45. When the capacitors are connected in series, they each have the same charge as the net capacitance.

(*a*) $Q_1 = Q_2 = Q_{\text{eq}} = C_{eq}V = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} V = \left(\frac{1}{0.40\times10^{-6}\,\mu\text{F}} + \frac{1}{0.60\times10^{-6}\,\mu\text{F}}\right)^{-1} (9.0\,\text{V})$

$= 2.16\times10^{-6}\,\text{C}$

$V_1 = \frac{Q_1}{C_1} = \frac{2.16\times10^{-6}\,\text{C}}{0.40\times10^{-6}\,\text{F}} = \boxed{5.4\,\text{V}} \qquad V_2 = \frac{Q_2}{C_2} = \frac{2.16\times10^{-6}\,\text{C}}{0.60\times10^{-6}\,\text{F}} = \boxed{3.6\,\text{V}}$

(*b*) $Q_1 = Q_2 = Q_{\text{eq}} = 2.16\times10^{-6}\,\text{C} \approx \boxed{2.2\times10^{-6}\,\text{C}}$

When the capacitors are connected in parallel, they each have the full potential difference.

(*c*) $V_1 = \boxed{9.0\,\text{V}} \quad V_2 = \boxed{9.0\,\text{V}} \quad Q_1 = C_1V_1 = (0.40\times10^{-6}\,\text{F})(9.0\,\text{V}) = \boxed{3.6\times10^{-6}\,\text{C}}$

$Q_2 = C_2V_2 = (0.60\times10^{-6}\,\text{F})(9.0\,\text{V}) = \boxed{5.4\times10^{-6}\,\text{C}}$

46. (*a*) The two capacitors are in parallel. Both capacitors have their high voltage plates at the same potential (the middle plate), and both capacitors have their low voltage plates at the same potential (the outer plates, which are connected).

(*b*) The capacitance of two capacitors in parallel is the sum of the individual capacitances.

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{d_1} + \frac{\varepsilon_0 A}{d_2} = \boxed{\varepsilon_0 A\left(\frac{1}{d_1} + \frac{1}{d_2}\right)}$$

47. The energy stored by a capacitor is given by Eq. 17-10, $\text{PE} = \frac{1}{2}CV^2$.

$\text{PE}_{\text{final}} = 3\text{PE}_{\text{initial}} \rightarrow \frac{1}{2}C_{\text{final}}V_{\text{final}}^2 = 3\frac{1}{2}C_{\text{initial}}V_{\text{initial}}^2$

One simple way to accomplish this is to have $C_{\text{final}} = 3C_{\text{initial}}$ and $V_{\text{final}} = V_{\text{initial}}$. In order to keep the voltage the same for both configurations, any additional capacitors must be connected in $\boxed{\text{parallel}}$ to the original capacitor. In order to triple the capacitance, we recognize that capacitors added in parallel add linearly. Thus if a capacitor of value $2C = \boxed{500\,\text{pF}}$ were connected in parallel to the original capacitor, the final capacitance would be 3 times the original capacitance with the same voltage, and so the potential energy would triple.

48. Capacitors in series each store the same amount of charge, and so the capacitance on the unknown capacitor is 125 pC. The voltage across the 185-pF capacitor is $V_{185} = \frac{Q_{185}}{C_{185}} = \frac{125\times10^{-12}\,\text{C}}{185\times10^{-12}\,\text{F}} = 0.676\,\text{V}$.

Thus the voltage across the unknown capacitor is $25\,\text{V} - V_{185} = 25\,\text{V} - 0.676\text{V} = 24.324\,\text{V}$. The capacitance can be calculated from the voltage across and charge on that capacitor.

$$C = \frac{Q}{V} = \frac{125\times10^{-12}\,\text{C}}{24.324\,\text{V}} = \boxed{5.14\,\text{pF}}$$

49. From Eq. 19-7, the product RC is equal to the time constant.

$$\tau = RC \rightarrow R = \frac{\tau}{C} = \frac{3.0\,\text{s}}{3.0\times10^{-6}\,\text{F}} = \boxed{1.0\times10^{6}\,\Omega}$$

$\boxed{50.}$ (*a*) From Eq. 19-7. the product *RC* is equal to the time constant.

$$\tau = RC \rightarrow C = \frac{\tau}{R} = \frac{35.0\times10^{-6}\,\text{s}}{15.0\times10^{3}\,\Omega} = \boxed{2.33\times10^{-9}\,\text{F}}$$

(*b*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \mathcal{E}\left(1-e^{-t/\tau}\right) \rightarrow e^{-t/\tau} = \left(1-\frac{V_C}{\mathcal{E}}\right) \rightarrow -\frac{t}{\tau} = \ln\left(1-\frac{V_C}{\mathcal{E}}\right) \rightarrow$$

$$t = -\tau\ln\left(1-\frac{V_C}{\mathcal{E}}\right) = -\left(35.0\times10^{-6}\,\text{s}\right)\ln\left(1-\frac{8.0\,\text{V}}{24.0\,\text{V}}\right) = \boxed{1.42\times10^{-5}\,\text{s}}$$

51. The voltage of the discharging capacitor is given by $V_\text{C} = V_0 e^{-t/RC}$. The capacitor voltage is to be $0.010V_0$.

$$V_\text{C} = V_0 e^{-t/RC} \rightarrow 0.010V_0 = V_0 e^{-t/RC} \rightarrow 0.010 = e^{-t/RC} \rightarrow \ln(0.010) = -\frac{t}{RC} \rightarrow$$

$$t = -RC\ln(0.010) = -\left(6.7\times10^{3}\,\Omega\right)\left(3.0\times10^{-6}\,\text{F}\right)\ln(0.010) = \boxed{9.3\times10^{-2}\,\text{s}}$$

52. (*a*) With the switch open, the resistors are in series with each other. Apply the loop rule clockwise around the left loop, starting at the negative terminal of the source, to find the current.

$$V - IR_1 - IR_2 = 0 \rightarrow I = \frac{V}{R_1+R_2} = \frac{24\,\text{V}}{8.8\,\Omega+4.4\,\Omega} = 1.818\,\text{A}$$

The voltage at point a is the voltage across the $4.4\,\Omega$-resistor.

$$V_a = IR_2 = (1.818\,\text{A})(4.4\,\Omega) = \boxed{8.0\,\text{V}}$$

(*b*) With the switch open, the capacitors are in series with each other. Find the equivalent capacitance. The charge stored on the equivalent capacitance is the same value as the charge stored on each capacitor in series.

$$\frac{1}{C_\text{eq}} = \frac{1}{C_1}+\frac{1}{C_2} \rightarrow C_\text{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{(0.48\,\mu\text{F})(0.24\,\mu\text{F})}{(0.48\,\mu\text{F}+0.24\,\mu\text{F})} = 0.16\,\mu\text{F}$$

$$Q_\text{eq} = VC_\text{eq} = (24.0\,\text{V})(0.16\,\mu\text{F}) = 3.84\,\mu\text{C} = Q_1 = Q_2$$

The voltage at point b is the voltage across the $0.24\,\mu\text{F}$-capacitor.

$$V_b = \frac{Q_2}{C_2} = \frac{3.84\,\mu\text{C}}{0.24\,\mu\text{F}} = \boxed{16\,\text{V}}$$

(*c*) The switch is now closed. After equilibrium has been reached a long time, there is no current flowing in the capacitors, and so the resistors are again in series, and the voltage of point a must be 8.0 V. Point b is connected by a conductor to point a, and so point b must be at the same potential as point a, $\boxed{8.0\,\text{V}}$. This also means that the voltage across C_2 is 8.0 V, and the voltage across C_1 is 16 V.

(*d*) Find the charge on each of the capacitors, which are no longer in series.

$$Q_1 = V_1C_1 = (16\,\text{V})(0.48\,\mu\text{F}) = 7.68\,\mu\text{C}$$

$$Q_2 = V_2C_2 = (8.0\,\text{V})(0.24\,\mu\text{F}) = 1.92\,\mu\text{C}$$

When the switch was open, point b had a net charge of 0, because the charge on the negative plate of C_1 had the same magnitude as the charge on the positive plate of C_2. With the switch closed, these charges are not equal. The net charge at point b is the sum of the charge on the negative plate of C_1 and the charge on the positive plate of C_2.

$$Q_b = -Q_1 + Q_2 = -7.68\mu\text{C} + 1.92\mu\text{C} = -5.76\mu\text{C}$$

Thus $\boxed{5.76\mu\text{C}}$ of charge has passed through the switch, from right to left.

53. The resistance is the full-scale voltage multiplied by the sensitivity.

$$R = V_{\substack{\text{full-}\\\text{scale}}}(\text{sensitivity}) = (250\,\text{V})(30{,}000\,\Omega/V) = \boxed{7.5\times10^{6}\Omega}$$

54. The full-scale current is the reciprocal of the sensitivity.

$$I_{\substack{\text{full-}\\\text{scale}}} = \frac{1}{20{,}000\,\Omega/V} = \boxed{5\times10^{-5}\text{A}} \text{ or } 50\mu\text{A}$$

$\boxed{55.}$ (*a*) To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. See Figure 19-30 for a circuit diagram.

$$V_{\text{shunt}} = V_G \;\rightarrow\; (I_{\text{full}} - I_G)R_{\text{shunt}} = I_G R_G \;\rightarrow$$

$$R_{\text{shunt}} = \frac{I_G R_G}{(I_{\text{full}} - I_G)} = \frac{(50\times10^{-6}\,\text{A})(30\,\Omega)R_G}{(30\,\text{A} - 50\times10^{-6}\,\text{A})} = \boxed{5.0\times10^{-5}\Omega}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full-scale voltage corresponds to the full scale current of the galvanometer. See Figure 19-31 for a circuit diagram.

$$V_{\text{full scale}} = I_G(R_{\text{ser}} + R_G) \;\rightarrow\; R_{\text{ser}} = \frac{V_{\text{full scale}}}{I_G} - R_G = \frac{250\,\text{V}}{50\times10^{-6}\,\text{A}} - 30\,\Omega = \boxed{5.0\times10^{6}\Omega}$$

56. (*a*) The current for full-scale deflection of the galvanometer is

$$I_G = \frac{1}{\text{sensitivity}} = \frac{1}{35{,}000\,\Omega/\text{V}} = 2.857\times10^{-5}\,\text{A}$$

To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. The total current is to be 2.0 A. See Figure 19-30 for a circuit diagram.

$$I_G r_G = I_s R_s \;\rightarrow$$

$$R_s = \frac{I_G}{I_s}r_G = \frac{I_G}{I_{\text{full}} - I_G}r_G = \frac{2.857\times10^{-5}\,\text{A}}{2.0\text{A} - 2.857\times10^{-5}\,\text{A}}(20\,\Omega) = 2.857\times10^{-4}\,\Omega$$

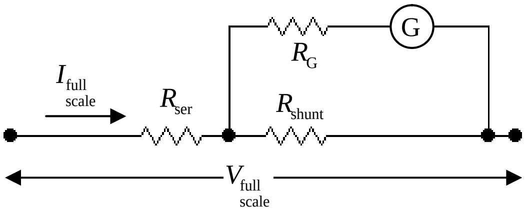
$$\approx \boxed{2.9\times10^{-4}\,\Omega \text{ in parallel}}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full scale voltage corresponds to the full scale current of the galvanometer. See Figure 19-31 for a circuit diagram. The total current must be the full-scale deflection current.

$$V_{\text{full}} = I_{\text{G}}\left(r_{\text{G}} + R\right) \quad \rightarrow$$

$$R = \frac{V_{\text{full}}}{I_{\text{G}}} - r_{\text{G}} = \frac{1.0\text{V}}{2.857\times10^{-5}\,\text{A}} - 20.0\,\Omega = 34981\,\Omega \approx \boxed{35\,\text{k}\Omega \text{ in series}}$$

57. To make a voltmeter, a resistor R_{ser} must be placed in series with the existing meter so that the desired full scale voltage corresponds to the full scale current of the galvanometer. We know that 10 mA produces full scale deflection of the galvanometer, so the voltage drop across the total meter must be 10 V when the current through the meter is 10 mA.


G
R_{G}
$I_{\substack{\text{full}\\\text{scale}}}$
R_{ser}
R_{shunt}
$V_{\substack{\text{full}\\\text{scale}}}$

$$V_{\substack{\text{full}\\\text{scale}}} = I_{\substack{\text{full}\\\text{scale}}} R_{\text{eq}} = I_{\substack{\text{full}\\\text{scale}}}\left[R_{\text{ser}} + \left(\frac{1}{R_{\text{G}}} + \frac{1}{R_{\text{shunt}}}\right)^{-1}\right] \quad \rightarrow$$

$$R_{\text{ser}} = \frac{V_{\substack{\text{full}\\\text{scale}}}}{I_{\substack{\text{full}\\\text{scale}}}} - \left(\frac{1}{R_{\text{G}}} + \frac{1}{R_{\text{shunt}}}\right)^{-1} = \frac{10\,\text{V}}{10\times10^{-3}\,\text{A}} - \left(\frac{1}{30\,\Omega} + \frac{1}{0.2\,\Omega}\right)^{-1} = 999.8\,\Omega \approx \boxed{1.0\times10^{3}\,\Omega}$$

The sensitivity is $\frac{1.0\times10^{3}\,\Omega}{10\,\text{V}} = \boxed{100\,\Omega/\text{V}}$

58. If the voltmeter were ideal, then the only resistance in the circuit would be the series combination of the two resistors. The current can be found from the battery and the equivalent resistance, and then the voltage across each resistor can be found.

$$R_{\text{tot}} = R_1 + R_2 = 38\,\text{k}\Omega + 27\text{k}\Omega = 65\,\text{k}\Omega \qquad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{65\times10^{3}\,\Omega} = 6.923\times10^{-4}\,\text{A}$$

$$V_1 = IR_1 = \left(6.923\times10^{-4}\,\text{A}\right)\left(38\times10^{3}\,\Omega\right) = 26.31\,\text{V}$$

$$V_2 = IR_2 = \left(6.923\times10^{-4}\,\text{A}\right)\left(27\times10^{3}\,\Omega\right) = 18.69\,\text{V}$$

Now put the voltmeter in parallel with the $38\ \text{k}\Omega$ resistor. Find its equivalent resistance, and then follow the same analysis as above.

$$R_{\text{eq}} = \left(\frac{1}{38\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega}\right)^{-1} = 27.1\text{k}\Omega$$

$$R_{\text{tot}} = R_{\text{eq}} + R_2 = 27.1\text{k}\Omega + 27\text{k}\Omega = 54.1\text{k}\Omega \qquad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{54.1\times10^{3}\,\Omega} = 8.318\times10^{-4}\,\text{A}$$

$$V_1 = V_{\text{eq}} = IR_{\text{eq}} = \left(8.318\times10^{-4}\,\text{A}\right)\left(27.1\times10^{3}\,\Omega\right) = 22.54\text{V} \approx \boxed{22.5\,\text{V}}$$

$$\%\ \text{error} = \frac{26.31\,\text{V} - 22.54\text{V}}{26.31\,\text{V}} \times 100 = \boxed{14\% \text{ too low}}$$

And now put the voltmeter in parallel with the $27\ \text{k}\Omega$ resistor, and repeat the process.

$$R_{\text{eq}} = \left(\frac{1}{27\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega}\right)^{-1} = 21.0\,\text{k}\Omega$$

$$R_{\text{tot}} = R_{\text{eq}} + R_1 = 21.0\,\text{k}\Omega + 38\,\text{k}\Omega = 59.0\,\text{k}\Omega \qquad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{59.0\times10^{3}\,\Omega} = 7.627\times10^{-4}\,\text{A}$$

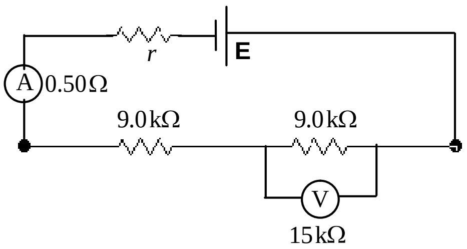
$$V_2 = V_{\text{eq}} = IR_{\text{eq}} = \left(7.627\times10^{-4}\,\text{A}\right)\left(21.0\times10^{3}\,\Omega\right) = 16.02\,\text{V} \approx \boxed{16.0\,\text{V}}$$

$$\%\ \text{error} = \frac{18.69\,\text{V} - 16.02\,\text{V}}{18.69\,\text{V}}\times100 = \boxed{14\%\ \text{too low}}$$

59. The total resistance with the ammeter present is $R_{\text{eq}} = 1293\,\Omega$. The voltage supplied by the battery is found from Ohm's law to be $V_{\text{battery}} = IR_{\text{eq}} = \left(5.25\times10^{-3}\,\text{A}\right)\left(1293\,\Omega\right) = 6.788\,\text{V}$. When the ammeter is removed, we assume that the battery voltage does not change. The equivalent resistance changes to $R'_{\text{eq}} = 1230\,\Omega$, and the new current is again found from Ohm's law.

$$I = \frac{V_{\text{battery}}}{R'_{\text{eq}}} = \frac{6.788\,\text{V}}{1230\,\Omega} = \boxed{5.52\times10^{-3}\,\text{A}}$$

60. Find the equivalent resistance for the entire circuit, and then find the current drawn from the source. That current will be the ammeter reading. The ammeter and voltmeter symbols in the diagram below are each assumed to have resistance.



$$R_{\text{eq}} = 1.0\,\Omega + 0.50\,\Omega + 9000\,\Omega + \frac{\left(9000\,\Omega\right)\left(15000\,\Omega\right)}{\left(9000\,\Omega + 15000\,\Omega\right)}$$

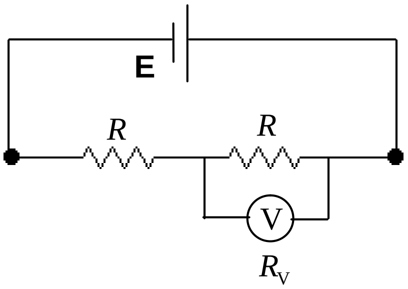
$$= 14626.5\,\Omega \approx 14630\,\Omega$$

$$I_{\text{source}} = \frac{\mathsf{E}}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{14630\,\Omega} = \boxed{8.20\times10^{-4}\,\text{A}}$$

The voltmeter reading will be the source current times the equivalent resistance of the resistor–voltmeter combination.

$$V_{\text{meter}} = I_{\text{source}}R_{\text{eq}} = \left(8.20\times10^{-4}\,\text{A}\right)\frac{\left(9000\,\Omega\right)\left(15000\,\Omega\right)}{\left(9000\,\Omega + 15000\,\Omega\right)} = \boxed{4.61\,\text{V}}$$

61. The sensitivity of the voltmeter is 1000 ohms per volt on the 3.0 volt scale, so it has a resistance of 3000 ohms. The circuit is shown in the diagram. Find the equivalent resistance of the meter–resistor parallel combination and the entire circuit.



$$R_{\text{p}} = \left(\frac{1}{R} + \frac{1}{R_{\text{V}}}\right)^{-1} = \frac{R_{\text{V}}R}{R_{\text{V}} + R} = \frac{\left(3000\,\Omega\right)\left(9400\,\Omega\right)}{3000\,\Omega + 9400\,\Omega} = 2274\,\Omega$$

$$R_{\text{eq}} = R + R_{\text{p}} = 2274\,\Omega + 9400\,\Omega = 11674\,\Omega$$

Using the meter reading of 2.0 volts, calculate the current into the parallel combination, which is the current delivered by the battery. Use that current to find the EMF of the battery.

$$I = \frac{V}{R_p} = \frac{2.0\,\text{V}}{2274\,\Omega} = 8.795\times10^{-4}\,\text{A}$$

$$\mathsf{E} = IR_{\text{eq}} = \left(8.795\times10^{-4}\,\text{A}\right)\left(11674\,\Omega\right) = 10.27\,\text{V} \approx \boxed{10\,\text{V}}$$

62. We know from Example 19-15 that the voltage across the resistor without the voltmeter connected is 4.0 V. Thus the minimum acceptable voltmeter reading is 97% of that: (0.97)(4.0 V) = 3.88 V. The voltage across the other resistor would then be 4.12 V, which is used to find the current in the circuit.

$$I = \frac{V_2}{R_2} = \frac{4.12\,\text{V}}{15,000\,\Omega} = 2.747\times10^{-4}\,\text{A}$$

This current is used with the voltmeter reading to find the equivalent resistance of the meter–resistor combination, from which the voltmeter resistance can be found.

$$R_{\text{comb}} = \frac{V_{\text{comb}}}{I} = \frac{3.88\,\text{V}}{2.747\times10^{-4}\,\text{A}} = 14,120\,\Omega$$

$$\frac{1}{R_{\text{comb}}} = \frac{1}{R_1} + \frac{1}{R_{\text{meter}}} \rightarrow \frac{1}{R_{\text{meter}}} = \frac{1}{R_{\text{comb}}} - \frac{1}{R_1} \rightarrow$$

$$R_{\text{meter}} = \frac{R_1 R_{\text{comb}}}{R_1 - R_{\text{comb}}} = \frac{\left(15,000\,\Omega\right)\left(14,120\,\Omega\right)}{15,000\,\Omega - 14,120\,\Omega} = 240,700\,\Omega \approx \boxed{240\,\text{k}\Omega}$$

63. By calling the voltmeter "high resistance," we can assume it has no current passing through it. Write Kirchhoff's loop rule for the circuit for both cases, starting with the negative pole of the battery and proceeding counterclockwise.

Case 1: $V_{\text{meter}} = V_1 = I_1 R_1 \quad \mathsf{E} - I_1 r - I_1 R_1 = 0 \rightarrow \mathsf{E} = I_1\left(r + R_1\right) = \frac{V_1}{R_1}\left(r + R_1\right)$

Case 2: $V_{\text{meter}} = V_2 = I_2 R_2 \quad \mathsf{E} - I_2 r - I_2 R_2 = 0 \rightarrow \mathsf{E} = I_2\left(r + R_2\right) = \frac{V_2}{R_2}\left(r + R_2\right)$

Solve these two equations for the two unknowns of E and r.

$$\mathsf{E} = \frac{V_1}{R_1}\left(r + R_1\right) = \frac{V_2}{R_2}\left(r + R_2\right) \rightarrow$$

$$r = R_1 R_2\left(\frac{V_2 - V_1}{V_1 R_2 - V_2 R_1}\right) = \left(35\,\Omega\right)\left(9.0\,\Omega\right)\left(\frac{8.1\,\text{V} - 9.7\,\text{V}}{\left(9.7\,\text{V}\right)\left(9.0\,\Omega\right) - \left(8.1\,\text{V}\right)\left(35\,\Omega\right)}\right) = 2.569\,\Omega \approx \boxed{2.6\,\Omega}$$

$$\mathsf{E} = \frac{V_1}{R_1}\left(r + R_1\right) = \frac{9.7\,\text{V}}{35\,\Omega}\left(2.569\,\Omega + 35\,\Omega\right) = 10.41\,\text{V} \approx \boxed{10.4\,\text{V}}$$

64. We can use a voltage divider circuit, as discussed in Example 19-3. The current to be passing through the body is given by Ohm's law.

$$I = \frac{V_{\text{body}}}{R_{\text{body}}} = \frac{0.25\,\text{V}}{2,000\,\Omega} = 1.25\times10^{-4}\,\text{A}$$

This is the current in the entire circuit. Use this to find the resistor to put in series with the body.

$$V_{\text{battery}} = I\left(R_{\text{body}} + R_{\text{series}}\right) \rightarrow$$

$$R_{\text{series}} = \frac{V_{\text{battery}}}{I} - R_{\text{body}} = \frac{9.0\,\text{V}}{1.25\times10^{-4}\,\text{A}} - 2000\,\Omega = 70000\,\Omega = \boxed{70\,\text{k}\Omega}$$

65. (*a*) Since $P = V^2/R$ and the voltage is the same for each combination, the power and resistance are inversely related to each other. So for the $\boxed{\text{50 W output, use the higher-resistance filament}}$. For the $\boxed{\text{100 W output, use the lower-resistance filament}}$. For the $\boxed{\text{150 W output, use the filaments in parallel}}$.

(*b*) $P = V^2/R \;\rightarrow\; R = \dfrac{V^2}{P}$ $\quad R_1 = \dfrac{(120\,\text{V})^2}{50\,\text{W}} = 288\,\Omega \approx \boxed{290\,\Omega}$ $\quad R_2 = \dfrac{(120\,\text{V})^2}{100\,\text{W}} = 144\,\Omega \approx \boxed{140\,\Omega}$

As a check, the parallel combination of the resistors gives the following.

$$R_{\text{p}} = \frac{R_1R_2}{R_1+R_2} = \frac{(288\,\Omega)(144\,\Omega)}{288\,\Omega + 144\,\Omega} = 96\,\Omega \qquad P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{96\,\Omega} = 150\,\text{W}\,.$$

66. The voltage drop across the two wires is the 3.0 A current times their total resistance.

$$V_{\text{wires}} = IR_{\text{wires}} = (3.0\,\text{A})(0.0065\,\Omega/\text{m})(190\,\text{m})R_{\text{p}} = \boxed{3.7\,\text{V}}$$

Thus the voltage applied to the apparatus is $V = V_{\text{source}} - V_{\text{wires}} = 120\,\text{V} - 3.7\,\text{V} = 116.3\,\text{V} \approx \boxed{116\,\text{V}}$.

67. The equivalent resistance is the series sum of all the resistances. The current is found from Ohm's law.

$$I = \frac{\mathsf{E}}{R_{\text{eq}}} = \frac{220\,\text{V}}{3\times10^{4}\,\Omega} = 0.00733\,\text{A} \approx \boxed{7\times10^{-3}\text{A}}$$

This is about 7 milliamps, and 10 milliamps is considered to be a dangerous level in that it can cause sustained muscular contraction. The 7 milliamps could certainly be felt by the patient, and could be painful.

68. (*a*) When the capacitors are connected in parallel, their equivalent capacitance is found from Eq. 19-5. The stored energy is given by Eq. 17-10.

$$C_{\text{eq}} = C_1 + C_2 = 1.00\,\mu\text{F} \;;\; E = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}(1.00\times10^{-6}\,\text{F})(45\,\text{V})^2 = \boxed{1.01\times10^{-3}\text{J}}$$

(*b*) When the capacitors are connected in parallel, their equivalent capacitance is found from Eq. 19-6. The stored energy is again given by Eq. 17-10.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow\; C_{\text{eq}} = \frac{C_1C_2}{C_1+C_2} = \frac{(0.40\,\mu\text{F})(0.60\,\mu\text{F})}{1.00\,\mu\text{F}} = 0.24\,\mu\text{F}$$

$$E = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}(0.24\times10^{-6}\,\text{F})(45\,\text{V})^2 = \boxed{2.43\times10^{-4}\text{J}}$$

(*c*) The charge is found from Eq. 17.7, $Q = CV$.

$$Q_a = (C_{\text{eq}})_a V = (1.00\,\mu\text{F})(45\,\text{V}) = \boxed{45\,\mu\text{C}}$$

$$Q_b = (C_{\text{eq}})_b V = (0.24\,\mu\text{F})(45\,\text{V}) = 10.8\,\mu\text{C} \approx \boxed{11\,\mu\text{C}}$$

69. The capacitor will charge up to 63% of its maximum value, and then discharge. The charging time is the time for one heartbeat.

$$t_{\text{beat}} = \frac{1\,\text{min}}{72\,\text{beats}} \times \frac{60\,\text{s}}{1\,\text{min}} = 0.8333\,\text{s}$$

$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right) \rightarrow 0.63V_0 = V_0\left(1 - e^{-\frac{t_{\text{beat}}}{RC}}\right) \rightarrow e^{-\frac{t_{\text{beat}}}{RC}} = 0.37 \rightarrow \left(-\frac{t_{\text{beat}}}{RC}\right) = \ln(0.37) \rightarrow$$

$$R = -\frac{t_{\text{beat}}}{C\ln(0.37)} = -\frac{0.8333\,\text{s}}{\left(7.5\times10^{-6}\,\text{F}\right)(-0.9943)} = \boxed{1.1\times10^{-5}\,\Omega}$$

70. (*a*) Apply Ohm's law to find the current.

$$I = \frac{V_{\text{body}}}{R_{\text{body}}} = \frac{110\,\text{V}}{950\,\Omega} = 0.116\,\text{A} \approx \boxed{0.12\,\text{A}}$$

(*b*) The description of "alternative path to ground" is a statement that the $45\,\Omega$ path is in parallel with the body. Thus the full 110 V is still applied across the body, and so the current is the same: $\boxed{0.12\,\text{A}}$.

(*c*) If the current is limited to a total of 1.5 A, then that current will get divided between the person and the parallel path. The voltage across the body and the parallel path will be the same, since they are in parallel.

$$V_{\text{body}} = V_{\text{alterante}} \rightarrow I_{\text{body}}R_{\text{body}} = I_{\text{alternate}}R_{\text{alternate}} = \left(I_{\text{total}} - I_{\text{body}}\right)R_{\text{alternate}} \rightarrow$$

$$I_{\text{body}} = I_{\text{total}}\frac{R_{\text{alternate}}}{\left(R_{\text{body}} + R_{\text{alternate}}\right)} = (1.5\,\text{A})\frac{45\,\Omega}{950\,\Omega + 45\,\Omega} = 0.0678\,\text{A} \approx \boxed{68\,\text{mA}}$$

This is still a very dangerous current.

71. (*a*) If the ammeter shows no current with the closing of the switch, then points B and D must be at the same potential, because the ammeter has some small resistance. Any potential difference between points B and D would cause current to flow through the ammeter. Thus the potential drop from A to B must be the same as the drop from A to D. Since points B and D are at the same potential, the potential drop from B to C must be the same as the drop from D to C. Use these two potential relationships to find the unknown resistance.

$$V_{\text{BA}} = V_{\text{DA}} \rightarrow I_3R_3 = I_1R_1 \rightarrow \frac{R_3}{R_1} = \frac{I_1}{I_3}$$

$$V_{\text{CB}} = V_{\text{CD}} \rightarrow I_3R_x = I_1R_2 \rightarrow R_x = R_2\frac{I_1}{I_3} = \boxed{R_2\frac{R_3}{R_1}}$$

(*b*) $R_x = R_2\dfrac{R_3}{R_1} = (972\,\Omega)\left(\dfrac{42.6\,\Omega}{630\,\Omega}\right) = \boxed{65.7\,\Omega}$

72. From the solution to problem 71, the unknown resistance is given by $R_x = R_2 \frac{R_3}{R_1}$. We use that with Eq. 18-3 to find the length of the wire.

$$R_x = R_2 \frac{R_3}{R_1} = \frac{\rho L}{A} = \frac{\rho L}{\pi (d/2)^2} = \frac{4\rho L}{\pi d^2} \rightarrow$$

$$L = \frac{R_2 R_3 \pi d^2}{4R_1 \rho} = \frac{(46.0\,\Omega)(3.48\,\Omega)\pi(9.20\times10^{-4}\,\text{m})^2}{4(38.0\,\Omega)(10.6\times10^{-8}\,\Omega\bullet\text{m})} = \boxed{26.4\,\text{m}}$$

73. There are eight values of effective capacitance that can be obtained from the four capacitors.

All four in parallel: $C_{\text{eq}} = C + C + C + C = \boxed{4C}$

All four in series: $C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}\right)^{-1} = \boxed{\tfrac{1}{4}C}$

(Three in parallel) in series with one:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C+C+C} + \frac{1}{C} = \frac{1}{3C} + \frac{1}{C} = \frac{4}{3C} \rightarrow C_{\text{eq}} = \boxed{\tfrac{3}{4}C}$$

(Two in parallel) in series with (two in parallel):

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C+C} + \frac{1}{C+C} = \frac{1}{2C} + \frac{1}{2C} = \frac{2}{2C} \rightarrow C_{\text{eq}} = \boxed{C}$$

(Two in parallel) in series with (two in series)

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C+C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{2C} + \frac{2}{C} = \frac{5}{2C} \rightarrow C_{\text{eq}} = \boxed{\tfrac{2}{5}C}$$

(Two in series) in parallel with (two in series)

$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} + \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} = \left(\frac{2}{C}\right)^{-1} + \left(\frac{2}{C}\right)^{-1} = \frac{C}{2} + \frac{C}{2} = C \quad \text{not a new value}$$

(Three in series) in parallel with one.

$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C} + \frac{1}{C}\right)^{-1} + C = \left(\frac{3}{C}\right)^{-1} + C = \boxed{\tfrac{4}{3}C}$$

(Two in series) in parallel with (two in parallel)

$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} + C + C = \left(\frac{2}{C}\right)^{-1} + 2C = \boxed{\tfrac{5}{2}C}$$

((Two in series) in parallel with one) in series with one

$$\frac{1}{C_{\text{eq}}} = \left[\left(\frac{1}{C} + \frac{1}{C}\right)^{-1} + C\right]^{-1} + \frac{1}{C} = \left[\frac{C}{2} + C\right]^{-1} + \frac{1}{C} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \rightarrow C_{\text{eq}} = \boxed{\tfrac{3}{5}C}$$

74. (*a*) From the diagram, we see that one group of 4 plates is connected together, and the other group of 4 plates is connected together. This common grouping shows that the capacitors are connected $\boxed{\text{in parallel}}$.

(*b*) Since they are connected in parallel, the equivalent capacitance is the sum of the individual capacitances. The variable area will change the equivalent capacitance.

$$C_{\text{eq}} = 7C = 7\varepsilon_0 \frac{A}{d}$$

$$C_{\min} = 7\varepsilon_0 \frac{A_{\min}}{d} = 7\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(2.0\times10^{-4}\,\text{m}^2\right)}{\left(1.5\times10^{-3}\,\text{m}\right)} = 8.3\times10^{-12}\,\text{F}$$

$$C_{\max} = 7\varepsilon_0 \frac{A_{\max}}{d} = 7\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(9.5\times10^{-4}\,\text{m}^2\right)}{\left(1.5\times10^{-3}\,\text{m}\right)} = 3.9\times10^{-11}\,\text{F}$$

And so the range is $\boxed{\text{from } 8.3\,\text{pF to } 39\,\text{pF}}$.

75. The terminal voltage and current are given for two situations. Apply Eq. 19-1 to both of these situations, and solve the resulting two equations for the two unknowns.

$$V_1 = \mathsf{E} - I_1 r \ ;\ V_2 = \mathsf{E} - I_2 r \ \rightarrow\ \mathsf{E} = V_1 + I_1 r = V_2 + I_2 r \ \rightarrow$$

$$r = \frac{V_2 - V_1}{I_1 - I_2} = \frac{47.3\,\text{V} - 40.8\,\text{V}}{7.40\,\text{A} - 2.20\,\text{A}} = \boxed{1.25\,\Omega}\ ;\ \mathsf{E} = V_1 + I_1 r = 40.8\,\text{V} + \left(7.40\,\text{A}\right)\left(1.25\,\Omega\right) = \boxed{50.1\,\text{V}}$$

76. One way is to connect *N* resistors in series. If each resistor watt can dissipate 0.5 W, then it will take 7 resistors in series to dissipate 3.5 W. Since the resistors are in series, each resistor will be 1/7 of the total resistance.

$$R = \frac{R_{\text{eq}}}{7} = \frac{2200\,\Omega}{7} = 314\,\Omega \approx 310\,\Omega$$

So connect $\boxed{7\ 310\,\Omega \text{ resistors, each rated at ½ W, in series}}$.

Or, the resistors could be connected in parallel. Again, if each resistor watt can dissipate 0.5 W, then it will take 7 resistors in parallel to dissipate 3.5 W. Since the resistors are in parallel, the equivalent resistance will be 1/7 of each individual resistance.

$$\frac{1}{R_{\text{eq}}} = 7\left(\frac{1}{R}\right) \ \rightarrow\ R = 7R_{\text{eq}} = 7\left(2200\,\Omega\right) = 15.4\,\text{k}\Omega$$

So connect $\boxed{7\ 15.4\,\text{k}\Omega \text{ resistors, each rated at ½ W, in parallel}}$.

77. There are two answers because it is not known which direction the given current is flowing through the $4.0\,\text{k}\Omega$ resistor.

Assume the current is to the right. The voltage across the $4.0\,\text{k}\Omega$ resistor is given by Ohm's law as $V = IR = \left(3.50\times10^{-3}\,\text{A}\right)\left(4000\,\Omega\right) = 14\,\text{V}$. The voltage drop across the $8.0\,\text{k}\Omega$ must be the same, and the current through it is $I = \frac{V}{R} = \frac{14\,\text{V}}{8000\,\Omega} = 1.75\times10^{-3}\,\text{A}$. The total current in the circuit is the sum of the two currents, and so $I_{\text{tot}} = 5.25\times10^{-3}\,\text{A}$. That current can be used to find the terminal voltage of the battery. Write a loop equation, starting at the negative terminal of the unknown battery and going clockwise.

$$V_{\text{ab}} - \left(5000\,\Omega\right)I_{\text{tot}} - 14.0\,\text{V} - 12.0\,\text{V} - \left(1.0\,\Omega\right)I_{\text{tot}} \ \rightarrow$$

$$V_{\text{ab}} = 26.0\text{V} + \left(5001\Omega\right)\left(5.25\times10^{-3}\,\text{A}\right) = \boxed{52.3\,\text{V}}$$

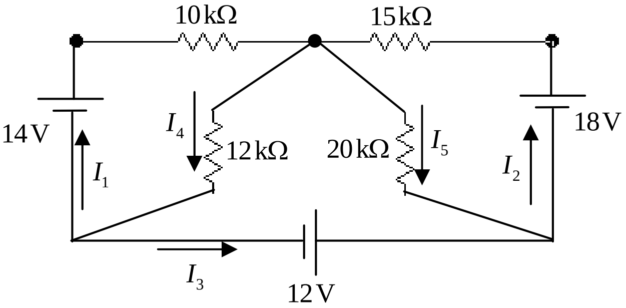
If the current is to the left, then the voltage drop across the parallel combination of resistors is still 14.0 V, but with the opposite orientation. Again write a loop equation, starting at the negative terminal of the unknown battery and going clockwise. The current is now to the left.

$$V_{ab}+(5000\,\Omega)I_{tot}+14.0\,\text{V}-12.0\,\text{V}+(1.0\,\Omega)I_{tot}\rightarrow$$
$$V_{ab}=-2.0\text{V}-(5001\Omega)(5.25\times10^{-3}\,\text{A})=\boxed{-28.3\,\text{V}}$$

78. The potential difference is the same on each half of the capacitance, so it can be treated as two capacitors in parallel. Let the original plate area be *A* and the distance between the plates be *d*. Each of the "new" capacitors has half the area of the original capacitor.

$$C_0=\varepsilon_0\frac{A}{d}\qquad C_1=K_1\varepsilon_0\frac{A/2}{d}\qquad C_2=K_2\varepsilon_0\frac{A/2}{d}$$
$$C_{new}=C_1+C_2=K_1\varepsilon_0\frac{A/2}{d}+K_2\varepsilon_0\frac{A/2}{d}=\varepsilon_0\frac{A}{d}\left(\frac{K_1}{2}+\frac{K_2}{2}\right)=\boxed{C_0\left(\frac{K_1+K_2}{2}\right)}$$

79. There are three loops, and so there are three loop equations necessary to solve the circuit. We identify five currents in the diagram, and so we need two junction equations to complete the analysis.



Lower left junction: $I_4=I_1+I_3$

Lower right junction: $I_2=I_3+I_5$

Left loop, clockwise: $14\,\text{V}-10000I_1-12000I_4=0$

Right loop, counterclockwise: $18\,\text{V}-15000I_2-20000I_5=0$

Bottom loop, counterclockwise: $12\,\text{V}+20000I_5-12000I_4=0$

$$14-10000I_1-12000(I_1+I_3)=0\quad\rightarrow\quad14-22000I_1-12000I_3=0$$
$$18-15000(I_3+I_5)-20000I_5=0\quad\rightarrow\quad18-15000I_3-35000I_5=0$$
$$12+20000I_5-12000(I_1+I_3)=0\quad\rightarrow\quad12-12000I_1-12000I_3+20000I_5=0$$

$$I_1=\frac{14-12000I_3}{22000}\qquad I_5=\frac{18-15000I_3}{35000}$$
$$12-12000\left[\frac{14-12000I_3}{22000}\right]-12000I_3+20000\left[\frac{18-15000I_3}{35000}\right]=0$$
$$12-\tfrac{12}{22}(14-12000I_3)-12000I_3+\tfrac{20}{35}[18-15000I_3]=0$$
$$12-\tfrac{6}{11}14+\tfrac{4}{7}18=\left(12000+\tfrac{4}{7}15000-\tfrac{6}{11}12000\right)I_3$$
$$14.65=14026I_3\quad\rightarrow\quad I_3=\frac{14.65}{14026}=1.044\times10^{-3}\,\text{A}$$
$$I_1=\frac{14-12000I_3}{22000}=\frac{14-12000(1.044\times10^{-3})}{22000}=\boxed{6.7\times10^{-5}\,\text{A, upward}}$$

80. To build up a high voltage, the cells will have to be put in series. 120 V is needed from a series of 0.80 V cells. Thus $\frac{120\,\text{V}}{0.80\,\text{V/cell}} = 150\text{ cells}$ are needed to provide the desired voltage. Since these cells are all in series, their current will all be the same at 350 mA. To achieve the higher current desired, banks made of 150 cells each can be connected in parallel. Then their voltage will still be at 120 V, but the currents would add making a total of $\frac{1.00\,\text{A}}{350\times10^{-3}\,\text{A/bank}} = 2.86\text{ banks} \approx 3\text{ banks}$. So the total number of cells is $\boxed{450\text{ cells}}$. The panel area is $450\text{ cells}\left(9.0\times10^{-4}\,\text{m}^2/\text{cell}\right) = \boxed{0.405\,\text{m}^2}$. The cells should be wired in $\boxed{\text{3 banks of 150 cells in series per bank, with the banks in parallel}}$. This will produce 1.05 A at 120 V. To optimize the output, $\boxed{\text{always have the panel pointed directly at the sun}}$.

81. (*a*) If the terminal voltage is to be 3.0 V, then the voltage across R_1 will be 9.0 V. This can be used to find the current, which then can be used to find the value of R_2.

$$V_1 = IR_1 \rightarrow I = \frac{V_1}{R_1} \qquad V_2 = IR_2 \rightarrow$$

$$R_2 = \frac{V_2}{I} = R_1\frac{V_2}{V_1} = (10.0\,\Omega)\frac{3.0\,\text{V}}{9.0\,\text{V}} = 3.333\,\Omega \approx \boxed{3.3\,\Omega}$$

(*b*) If the load has a resistance of $7.0\,\Omega$, then the parallel combination of R_2 and the load must be used to analyze the circuit. The equivalent resistance of the circuit can be found and used to calculate the current in the circuit. Then the terminal voltage can be found from Ohm's law, using the parallel combination resistance.

$$R_{2+\text{load}} = \frac{R_2 R_{\text{load}}}{R_2 + R_{\text{load}}} = \frac{(3.33\,\Omega)(7.0\,\Omega)}{10.33\,\Omega} = 2.26\,\Omega \qquad R_{eq} = 2.26\,\Omega + 10.0\,\Omega = 12.26\,\Omega$$

$$I = \frac{V}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{12.26\,\Omega} = 0.9788\,\text{A} \qquad V_{\text{T}} = IR_{2+\text{load}} = (0.9788\,\text{A})(2.26\,\Omega) = 2.21\,\text{V} \approx \boxed{2.2\,\text{V}}$$

The presence of the load has affected the terminal voltage significantly.

82. (*a*) The light will first flash when the voltage across the capacitor reaches 90.0 V.

$$V = \mathcal{E}_0\left(1 - e^{-\frac{t}{RC}}\right)$$

$$t = -RC\ln\left(1 - \frac{V}{\mathcal{E}_0}\right) = -\left(2.35\times10^6\,\Omega\right)\left(0.150\times10^{-6}\,\text{F}\right)\ln\left(1 - \frac{90}{105}\right) = \boxed{0.686\,\text{s}} \rightarrow$$

(*b*) We see from the equation that $t \propto R$, and so if R increases, the $\boxed{\text{time will increase}}$.

(*c*) The capacitor discharges through a very low resistance (the lamp), and so the discharge time constant is very short. Thus the flash is very brief.

(*d*) Once the lamp has flashed, the stored energy in the capacitor is gone, and there is no source of charge to maintain the lamp current. The lamp "goes out", the lamp resistance increases, and the capacitor starts to recharge. It charges again for about 0.686 seconds, and the process will repeat.

83. If the switches are both open, then the circuit is a simple series circuit. Use Kirchhoff's loop rule to find the current in that case.

$$6.0\,\text{V} - I\left(50\,\Omega + 20\,\Omega + 10\,\Omega\right) = 0 \;\rightarrow\; I = \frac{6.0\,\text{V}}{80\,\Omega} = 0.075\,\text{A}$$

If the switches are both closed, the 20-Ω resistor is in parallel with *R*. Apply Kirchhoff's loop rule to the outer loop of the circuit, with the 20-Ω resistor having the current found previously.

$$6.0\,\text{V} - I\left(50\,\Omega\right) - \left(0.075\,\text{A}\right)\left(20\,\Omega\right) = 0 \;\rightarrow\; I = \frac{6.0\,\text{V} - \left(0.075\,\text{A}\right)\left(20\,\Omega\right)}{50\,\Omega} = 0.090\,\text{A}$$

This is the current that flows into the parallel combination. Since 0.075 A is in the 20-Ω resistor, 0.015 A must be in *R*. The voltage drops across *R* and the 20-Ω resistor must be the same, since they are in parallel.

$$V_{20} = V_R \;\rightarrow\; I_{20}R_{20} = I_R R \;\rightarrow\; R = R_{20}\frac{I_{20}}{I_R} = \left(20\,\Omega\right)\frac{0.075\,\text{A}}{0.015\,\text{A}} = \boxed{100\,\Omega}$$

84. The current in the circuit can be found from the resistance and the power dissipated. Then the product of that current and the equivalent resistance is equal to the battery voltage.

$$P = I^2R \;\rightarrow\; I = \sqrt{\frac{P_{33}}{R_{33}}} = \sqrt{\frac{0.50\,\text{W}}{33\,\Omega}} = 0.123\,\text{A}$$

$$R_{\text{eq}} = 33\,\Omega + \left(\frac{1}{68\,\Omega} + \frac{1}{75\,\Omega}\right)^{-1} = 68.88\,\Omega \qquad V = IR_{\text{eq}} = \left(0.123\,\text{A}\right)\left(68.88\,\Omega\right) = 8.472\,\text{V} \approx \boxed{8.5\,\text{V}}$$

85. (*a*) The 12-Ω and the 30-Ω resistors are in parallel, with a net resistance $R_{1\text{-}2}$ as follows.

$$R_{1\text{-}2} = \left(\frac{1}{12\,\Omega} + \frac{1}{30\,\Omega}\right)^{-1} = 8.57\,\Omega$$

$R_{1\text{-}2}$ is in series with the 4.5-Ω resistor, for a net resistance $R_{1\text{-}2\text{-}3}$ as follows.

$$R_{1\text{-}2\text{-}3} = 4.5\,\Omega + 8.57\,\Omega = 13.07\,\Omega$$

That net resistance is in parallel with the 18-Ω resistor, for a final equivalent resistance as follows.

$$R_{\text{eq}} = \left(\frac{1}{13.07\,\Omega} + \frac{1}{18\,\Omega}\right)^{-1} = 7.57\,\Omega \approx \boxed{7.6\,\Omega}$$

(*b*) Find the current in the 18-Ω resistor by using Kirchhoff's loop rule for the loop containing the battery and the 18-Ω resistor.

$$\mathsf{E} - I_{18}R_{18} = 0 \;\rightarrow\; I_{18} = \frac{\mathsf{E}}{R_{18}} = \frac{6.0\,\text{V}}{18\,\Omega} = \boxed{0.33\,\text{A}}$$

(*c*) Find the current in $R_{1\text{-}2}$ and the 4.5-Ω resistor by using Kirchhoff's loop rule for the outer loop containing the battery and the resistors $R_{1\text{-}2}$ and the 4.5-Ω resistor.

$$\mathsf{E} - I_{1\text{-}2}R_{1\text{-}2} - I_{1\text{-}2}R_{4.5} = 0 \;\rightarrow\; I_{1\text{-}2} = \frac{\mathsf{E}}{R_{1\text{-}2} + R_{4.5}} = \frac{6.0\,\text{V}}{13.07\,\Omega} = 0.459\,\text{A}$$

This current divides to go through the 12-Ω and 30-Ω resistors in such a way that the voltage drop across each of them is the same. Use that to find the current in the 12-Ω resistor.

$$I_{1\text{-}2} = I_{12} + I_{30} \quad\rightarrow\quad I_{30} = I_{1\text{-}2} - I_{12}$$

$$V_{R_{12}} = V_{R_{30}} \quad\rightarrow\quad I_{12}R_{12} = I_{30}R_{30} = \left(I_{1\text{-}2} - I_{12}\right)R_{30} \quad\rightarrow$$

$$I_{12} = I_{1\text{-}2}\frac{R_{30}}{\left(R_{12}+R_{30}\right)} = \left(0.459\,\text{A}\right)\frac{30\,\Omega}{42\,\Omega} = \boxed{0.33\,\text{A}}$$

(*d*) The current in the 4.5-Ω resistor was found above to be $I_{1\text{-}2} = 0.459\,\text{A}$. Find the power accordingly.

$$P_{4.5} = I_{1\text{-}2}^2 R_{4.5} = \left(0.459\,\text{A}\right)^2\left(4.5\,\Omega\right) = \boxed{0.95\,\text{W}}$$

86. (*a*) We assume that the ammeter is ideal and so has 0 resistance, but that the voltmeter has resistance R_V. Then apply Ohm's law, using the equivalent resistance. We also assume the voltmeter is accurate, and so it is reading the voltage across the battery.

$$V = IR_\text{eq} = I\frac{1}{\dfrac{1}{R}+\dfrac{1}{R_\text{V}}} \quad\rightarrow\quad V\left(\frac{1}{R}+\frac{1}{R_\text{V}}\right) = I \quad\rightarrow\quad \frac{1}{R}+\frac{1}{R_\text{V}} = \frac{I}{V} \quad\rightarrow\quad \boxed{\frac{1}{R} = \frac{I}{V} - \frac{1}{R_\text{V}}}$$

(*b*) We now assume the voltmeter is ideal, and so has an infinite resistance, but that the ammeter has resistance R_A. We also assume that the voltmeter is accurate and so is reading the voltage across the battery.

$$V = IR_\text{eq} = I\left(R+R_\text{A}\right) \quad\rightarrow\quad R+R_\text{A} = \frac{V}{I} \quad\rightarrow\quad \boxed{R = \frac{V}{I} - R_\text{A}}$$

87. Write Kirchhoff's loop rule for the circuit, and substitute for the current and the bulb resistance based on the bulb ratings.

$$P_\text{bulb} = \frac{V_\text{bulb}^2}{R_\text{bulb}} \quad\rightarrow\quad R_\text{bulb} = \frac{V_\text{bulb}^2}{P_\text{bulb}} \qquad P_\text{bulb} = I_\text{bulb}V_\text{bulb} \quad\rightarrow\quad I_\text{bulb} = \frac{P_\text{bulb}}{V_\text{bulb}}$$

$$\mathcal{E} - I_\text{bulb}R - I_\text{bulb}R_\text{bulb} = 0 \quad\rightarrow$$

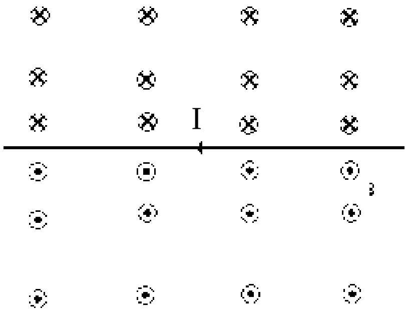
$$R = \frac{\mathcal{E}}{I_\text{bulb}} - R_\text{bulb} = \frac{\mathcal{E}}{P_\text{bulb}/V_\text{bulb}} - \frac{V_\text{bulb}^2}{P_\text{bulb}} = \frac{V_\text{bulb}}{P_\text{bulb}}\left(\mathcal{E} - V_\text{bulb}\right) = \frac{3.0\,\text{V}}{2.5\,\text{W}}\left(9.0\,\text{V} - 3.0\,\text{V}\right) = \boxed{7.2\,\Omega}$$

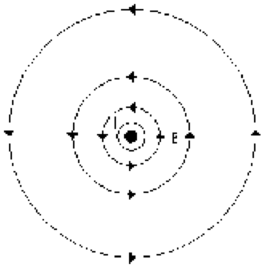
CHAPTER 20: Magnetism

Answers to Questions

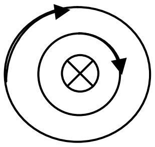
1. The Earth's magnetic field is not always parallel to the surface of the Earth – it may have a component perpendicular to the Earth's surface. The compass will tend to line up with the local direction of the magnetic field, and so one end of the compass will dip downward. The angle that the Earth's magnetic field makes with the horizontal is called the angle of dip.

2. The magnetic field lines circle around the wire and get weaker as you get farther away from the wire. To determine the direction, use the Right Hand Rule. Notice, if the wire was laid horizontally on this piece of paper in a left-right direction and the current was moving to the left, the magnetic field lines would be pointing into the paper above the wire and coming out of the paper below the wire. If the wire was placed perpendicular to this page with the current coming out of the end of the wire facing us, the magnetic fields would point around the wire in a counterclockwise direction.





3. The magnetic field lines form clockwise circles centered on the wire.



4. The right-hand rule shows us that if a current in a wire is pointing directly away from you while the wire is in a magnetic field that points from left to right, the force on the wire is pointed downward. See Figure 20-11(a).

5. A magnet will not attract just any metallic object. For example, while a magnet will attract paper clips and nails, it will not attract coins or pieces of aluminum foil. This is because magnets will only attract other ferromagnetic materials (iron, cobalt, nickel, gadolinium, and some of their oxides and alloys). Iron and its alloys are the only common materials. These ferromagnetic materials contain magnetic domains that can be made to temporarily align when a strong magnet is brought near. The alignment occurs in such a way that the north pole of the domain points toward the south pole of the strong magnet, and vice versa, which creates the attraction.

6. The bars cannot both be magnets. If both were magnets, then if like poles were placed close to each other, the bars would repel each other. Only one of the bars can be a magnet.

7. Typical current in a house circuit is 60 Hz AC. Due to the mass of the compass needle, its reaction to 60 Hz (changing direction back and forth at 60 complete cycles per second) will probably not be noticeable. A DC current in a single wire could affect a compass, depending on the relative orientation of the wire and the compass, the magnitude of the current, and the distance from the wire

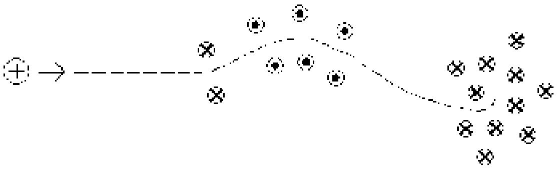
to the compass. A DC current being carried by two very close wires in opposite directions would not have much of an effect on the compass needle, since the two currents would cause magnetic fields that tended to cancel each other.

8. The magnetic force will be exactly perpendicular to the velocity, and so also perpendicular to the direction of motion. Since there is no component of force in the direction of motion, the work done by the magnetic force will be zero, and the kinetic energy of the particle will not change. The particle will change direction, but not change speed.

9. Use the right hand rule to determine the direction of the force on each particle. In the plane of the diagram, the magnetic field is coming out of the paper for points above the wire, and is going into the paper for points below the wire.
    Upper left hand particle: force down, toward the wire
    Upper right hand particle: force to the left, opposite of the direction of the current
    Lower right hand particle: force to the left, opposite of the direction of the current
    Lower left hand particle: force up, toward the wire

$\boxed{10.}$ Charge a experiences an initial upward force. By the right hand rule, charge a must be positive.
Charge b experiences no force, and so charge b must be uncharged.
Charge c experiences an initial downward force. By the right hand rule, charge c must be negative.

11. The right hand rule tell us that as the positive particle moves to the right, if it is deflected upward the magnetic field must be into the paper and if it is deflected downward the magnetic field must be out of the paper. Also, the stronger the magnetic field, the more tightly the charged particle will turn. The magnetic field at the first bend must be relatively small and pointed into the paper. The magnetic field at the second bend must be medium in size and pointed out of the paper. The magnetic field at the third bend must be relatively very large and pointed into the paper.



12. A magnet can attract an iron bar, so by Newton's third law, an iron bar can attract a magnet. Another consideration is that the iron has domains that can be made slightly magnetic by an external magnetic field, as opposed to a substance like plastic or wood.

13. Section 17-10 shows that a CRT television picture is created by shooting a beam of electrons at a screen, and wherever the electrons hit the screen it lights up for us to see. When you hold a strong magnet too close to the screen, the magnetic field puts a force on the moving electrons $\left(F = qvB\sin\theta\right)$, which causes them to bend away from where they were supposed to hit the screen. These errant electrons then cause parts of the screen to light up that aren't supposed to be lit and cause other parts to be slightly darker than they are supposed to be. The picture sometimes goes completely black where the field is strongest because at these points all of the electrons that were supposed to hit that particular spot have been deflected by the strong field.

$\boxed{14.}$ Put one end of one rod close to one end of another rod. The ends will either attract or repel. Continue trying all combinations of rods and ends until two ends repel each other. Then the two rods used in that case are the magnets.

15. No, you cannot set a resting electron into motion with a magnetic field (no matter how big the field is). A magnetic field can only put a force on a *moving* charge. Thus, with no force (which means no acceleration), the velocity of the electron will not change – it will remain at rest. However, you can set a resting electron into motion with an electric field. An electric field will put a force on *any* charged particle, moving or not. Thus, the electric force can cause the electron to accelerate from rest to a higher speed.

16. The charged particle will now travel in a helix. It will continue in its original circular path while it is traveling in the direction of the magnetic and electric fields, which is perpendicular to the plane of the original circle. For as long as the electric field stays on, the particle will move in a constantly elongating helix (the pitch of the helix will continue to increase), since the electric field will cause the charged particle to accelerate along the electric field lines.

17. Using the configuration shown in Fig. 20-48, the electric field puts a force on the charged particles and causes them to accelerate in a vertical direction from rest (the negative particles are accelerated upward and the positive particles are accelerated downward). Once the charged particles are moving, the magnetic field now also puts a force on the charged particles. Using the Right Hand Rule, both the upward-moving negative particles and the downward-moving positive particles experience a force out of the page, which is along the direction of the vessel. Thus, yes, both the positive and negative particles experience a force in the same direction.

18. The beam of electrons is deflected to the right. The magnetic field created by the current in the wire, *I*, is directed into the page on the side of the wire where the electron beam is located. The right hand rule tells us that the negative electrons will feel a force to the right as they move toward the wire in a downward magnetic field.

$\boxed{19.}$ A moving electric charge creates both electric and magnetic fields around it. The electric field is radial and it looks very similar to the typical electric field that surrounds a stationary electric charge. The magnetic field circles the charge in a plane that is perpendicular to the direction of motion and it looks very similar to the typical magnetic field that surrounds a current-carrying wire. Both of these fields decrease in strength as you go farther away from the moving charge in any direction.

20. Yes, there could be a non-zero magnetic field in this region of space, even though the charged particle travels in a straight line. One possible situation is that the charged particle is traveling in a direction parallel (or anti-parallel) to the magnetic field lines. In this situation, no magnetic force is exerted on the charged particle, since the $\sin\theta$ term in $F = qvB\sin\theta$ is zero. The other possible situation is that the charged particle is traveling through "crossed" electric and magnetic fields in such a way that the electric force and the magnetic force are canceling each other out. For example, if a positively charged particle were moving east in a region of space where the electric field was south (which creates a force toward the south) and the magnetic field was downward (which creates a magnetic field toward the north), and the magnitudes of E and B were chosen correctly, then the electric and magnetic forces would cancel out, allowing the charged particle to travel in a straight line.

21. No, a moving charged particle can be deflected sideways with an electric field that points perpendicular to the direction of the velocity, even when the magnetic field in the region is zero.

22. No, although when an electron is injected perpendicularly into a region of uniform magnetic field, the electron is forced to travel along a curve in the shape of a perfect circle, the electron would travel in a path that is only half of a circle and at that point it would come back out of the magnetic field and head back in the direction it came. Yes, if an electron were injected into the middle of a large

region of space occupied by a perpendicular magnetic field, the electron could travel in a closed circular path.

23. If the moving electrons are changing speed as they are being deflected, then an electric field is present. This will be the case whether or not a magnetic field exists in the same region of space. If the moving electrons are being deflected but they are not changing speed, then only a magnetic field is present.

24. Use a small loop of current-carrying wire that is able to pivot freely along a vertical axis (the faces of the loop should be pointing horizontally). Use the right hand rule to determine which face of the loop has a magnetic north pole pointing out of it. The north and magnetic south poles created by the loop will attempt to pivot to align with Earth's magnetic field. The north face of the loop will point toward the magnetic south pole of Earth (which is near the geographic North Pole).

25. Consider that the two wires are both horizontal, with the lower one carrying a current pointing east and with the upper one carrying a current pointing north. The magnetic field from the upper wire points downward on the east half of the lower wire and points upward on the west half of the lower wire. Using the right hand rule on the east half of the lower wire, where the magnetic field is downward, the force points to the north. Using the right hand rule on the west half of the lower wire, where the magnetic field is upward, the force points to the south. Thus, the lower wire experiences a counterclockwise torque about the vertical direction due to the magnetic forces from the upper wire. This torque is attempting to rotate the two wires so their currents are parallel. Likewise the upper wire would experience a clockwise torque tending to align the currents in the wires.

26. (*a*) The current in the lower wire is pointing in the opposite direction of the current in the upper wire. If the current in the lower wire is toward the north, then it creates a magnetic field pointing east at the upper wire. Using the Right Hand Rule, the current of the upper wire must be toward the south so that the magnetic force created on it is upward.
    (*b*) Yes, the upper wire is vertically stable. If the upper wire falls toward the lower wire slightly, then the magnetic force increases to push it back up to its stable position. If the upper wire rises away from the lower wire slightly, although the magnetic field decreases, gravity will pull it back into its stable position.

27. The magnetic domains in the unmagnetized piece of iron are initially pointing in random directions (which is why it appears to be unmagnetized). When the south pole of a strong external magnet is brought close to the random magnetic domains of the iron, many of the domains will rotate slightly so that their north poles are closer to the external south pole, which causes the unmagnetized iron to be attracted to the magnet. Of course, when the north pole of a strong external magnet is brought close to the random magnetic domains of the iron, many of the domains will rotate slightly so that their south poles are closer to the external north pole, which causes the unmagnetized iron to now be attracted to the magnet. Thus, either pole of a magnet will attract an unmagnetized piece of iron.

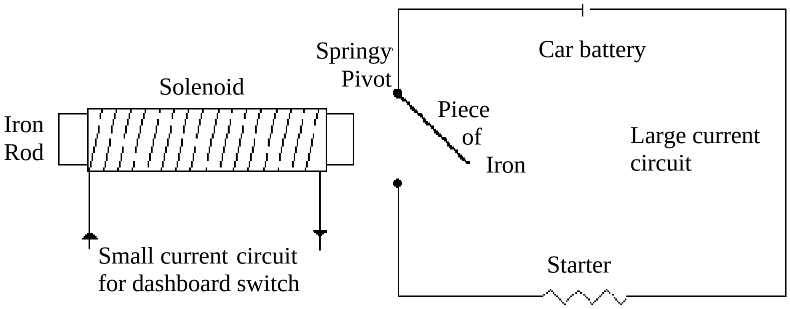
28. Initially, both the nail and the paper clip have all of their magnetic domains pointing in random directions (which is why they appear to be unmagnetized). Thus, when you bring them close to each other, they are not attracted to or repelled from each other. Once the nail is in contact with a magnet (let's say the north pole), though, many of the nail's domains will align in such a way that the end touching the magnet becomes a south pole, due to the strong attraction, and the opposite end of the nail then becomes a north pole. Now, when you bring the nail close to the paper clip, there are mainly north poles of nail domains close to the paper clip, which causes some of the domains in the paper clip to align in such a way that the end near the nail becomes a south pole, which is exactly the same procedure that the nail was "magnetized." Since the nail's domains are only partially aligned,

it will not be a strong magnet and thus the alignment of the paper clip's domains will be even weaker. The attraction of the paper clip to the nail will be weaker than the attraction of the nail to the magnet.

29. The two ions will come out of the velocity selector portion of the mass spectrometer at the same speed, since $v = E/B$ (where the charge *q* has canceled out). Once the charges reach the area of the mass spectrometer where there is only a magnetic field, then the difference in the ions' charges will have an effect. The equation for the mass spectrometer is $m = qBB'r/E$, which can be rearranged as $r = mE/qBB'$. Since every quantity in the equation is constant except *q*, the doubly ionized ion will hit the film at a half of the radius as the singly ionized ion.

30. The equation for the magnetic field inside of a solenoid is $B = \mu_0 NI/l$.
    (*a*) Nothing would happen to *B* if the loop diameter was doubled. The equation shows that the magnetic field is independent of the diameter of the solenoid.
    (*b*) *B* would decrease by a factor of 2. When the spacing between the loops is doubled, the number of loops per length, N/l, is cut in half, which decreases *B* by a factor of 2.
    (*c*) Nothing would happen to *B* if both the length and the number of loops were doubled. The magnetic field does depend on the number of loops per length, N/l, but if both the number of loops and the length of the solenoid are doubled, this ratio remains constant and the magnetic field doesn't change.

31. To design a relay, place the iron rod inside of a solenoid, then point one end of the solenoid/rod combination at the piece of iron on a pivot. A spring normally holds the piece of iron away from a switch, making an open circuit where current cannot flow. When the relay is activated with a small current, a relatively strong magnetic field is created inside the solenoid, which aligns most of the magnetic domains in the iron rod and produces a strong magnetic field at the end of the solenoid/rod combination. This magnetic field attracts the piece of iron on the springy pivot, which causes it to move toward the switch, connecting it and allowing current to flow through the large current circuit.



Solutions to Problems

1. (*a*) Use Eq. 20-1 to calculate the force with an angle of 90° and a length of 1 meter.

$$F = IlB\sin\theta \;\rightarrow\; \frac{F}{l} = IB\sin\theta = (8.40\,\mathrm{A})(0.90\,\mathrm{T})\sin 90^\circ = \boxed{7.6\,\mathrm{N/m}}$$

(*b*) $\frac{F}{l} = IB\sin\theta = (8.40\,\mathrm{A})(0.90\,\mathrm{T})\sin 45.0^\circ = \boxed{5.3\,\mathrm{N/m}}$

2. Use Eq. 20-1 to calculate the force.

$$F = IlB\sin\theta = (150\,\mathrm{A})(160\,\mathrm{m})\left(5.0\times10^{-5}\,\mathrm{T}\right)\sin 65^{\circ} = \boxed{1.1\,\mathrm{N}}$$

3. Use Eq. 20-2.

$$F_{\max} = IlB \quad\rightarrow\quad I = \frac{F_{\max}}{lB} = \frac{0.750\,\mathrm{N}}{(4.80\,\mathrm{m})\left(8.00\times10^{-2}\,\mathrm{T}\right)} = \boxed{1.95\,\mathrm{A}}$$

4. The dip angle is the angle between the Earth's magnetic field and the current in the wire. Use Eq. 20-1 to calculate the force.

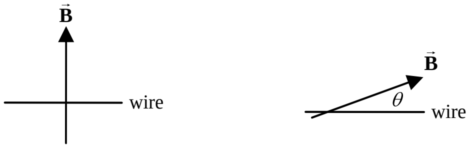
$$F = IlB\sin\theta = (4.5\,\mathrm{A})(1.5\,\mathrm{m})\left(5.5\times10^{-5}\,\mathrm{T}\right)\sin 38^{\circ} = \boxed{2.3\times10^{-4}\,\mathrm{N}}$$

5. Use Eq. 20-2. The length of wire in the B-field is the same as the diameter of the pole faces.

$$F_{\max} = IlB \quad\rightarrow\quad B = \frac{F_{\max}}{Il} = \frac{1.28\,\mathrm{N}}{(8.75\,\mathrm{A})(0.555\,\mathrm{m})} = \boxed{0.264\,\mathrm{T}}$$

6. To have the maximum force, the current must be perpendicular to the magnetic field, as shown in the first diagram. Use $\frac{F}{l} = 0.35\frac{F_{\max}}{l}$ to find the angle between the wire and the magnetic field, illustrated in the second diagram.

$$\frac{F}{l} = 0.35\frac{F_{\max}}{l} \quad\rightarrow\quad IB\sin\theta = 0.35IB \quad\rightarrow\quad \theta = \sin^{-1}0.35 = \boxed{20^{\circ}}$$



7. (*a*) By the right hand rule, the magnetic field must be pointing up, and so the top pole face must be a $\boxed{\text{South pole}}$.

(*b*) Use Eq. 20-2 to relate the maximum force to the current. The length of wire in the magnetic field is equal to the diameter of the pole faces.

$$F_{\max} = IlB \quad\rightarrow\quad I = \frac{F_{\max}}{lB} = \frac{\left(6.50\times10^{-2}\,\mathrm{N}\right)}{(0.100\,\mathrm{m})(0.16\,\mathrm{T})} = 4.063\,\mathrm{A} \approx \boxed{4.1\,\mathrm{A}}$$

(*c*) Multiply the maximum force by the sine of the angle between the wire and the magnetic field.

$$F = F_{\max}\sin\theta = \left(6.50\times10^{-2}\,\mathrm{N}\right)\sin 80.0^{\circ} = \boxed{6.40\times10^{-2}\,\mathrm{N}}$$

$\boxed{8.}$ The magnetic force must be equal in magnitude to the force of gravity on the wire. The maximum magnetic force is applicable since the wire is perpendicular to the magnetic field. The mass of the wire is the density of copper times the volume of the wire.

$$F_{\mathrm{B}} = mg \quad\rightarrow\quad IlB = \rho\pi\left(\frac{d}{2}\right)^{2} l\,g \quad\rightarrow$$

$$I = \frac{\rho\pi d^{2}g}{4B} = \frac{\left(8.9\times10^{3}\,\mathrm{kg/m^{3}}\right)\pi\left(1.00\times10^{-3}\,\mathrm{m}\right)^{2}\left(9.80\,\mathrm{m/s^{2}}\right)}{4\left(5.0\times10^{-5}\,\mathrm{T}\right)} = \boxed{1400\,\mathrm{A}}$$

This answer does not seem feasible. The current is very large, and the Joule heating in the thin copper wire would probably melt it.

9. In this scenario, the magnetic force is causing centripetal motion, and so must have the form of a centripetal force. The magnetic force is perpendicular to the velocity at all times for circular motion.

$$F_{\max} = qvB = m\frac{v^2}{r} \quad\rightarrow\quad B = \frac{mv}{qr} = \frac{\left(6.6\times10^{-27}\,\text{kg}\right)\left(1.6\times10^{7}\,\text{m/s}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(0.25\,\text{m}\right)} = \boxed{1.3\,\text{T}}$$

10. The maximum magnetic force as given in Eq. 20-4 can be used since the velocity is perpendicular to the magnetic field.

$$F_{\max} = qvB = \left(1.60\times10^{-19}\,\text{C}\right)\left(8.75\times10^{5}\,\text{m/s}\right)\left(0.75\,\text{T}\right) = \boxed{1.05\times10^{-13}\,\text{N}}$$

By the right hand rule, the force must be directed to the $\boxed{\text{North}}$.

11. Since the charge is negative, the answer is the OPPOSITE of the result given from the right hand rule applied to the velocity and magnetic field.
(*a*) left
(*b*) left
(*c*) upward
(*d*) inward into the paper
(*e*) no force
(*f*) downward

12. The right hand rule applied to the velocity and magnetic field would give the direction of the force. Use this to determine the direction of the magnetic field given the velocity and the force.
(*a*) downward
(*b*) inward into the paper
(*c*) right

13. The magnetic force will cause centripetal motion, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.

$$F_{\max} = qvB = m\frac{v^2}{r} \quad\rightarrow\quad r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.70\times10^{6}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.350\,\text{T}\right)} = \boxed{2.77\times10^{-5}\,\text{m}}$$

$\boxed{14.}$ The kinetic energy of the proton can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined.

$$\text{KE} = \tfrac{1}{2}mv^2 \quad\rightarrow\quad \sqrt{\frac{2\,\text{KE}}{m}} \qquad qvB = \frac{mv^2}{r} \quad\rightarrow$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2\,\text{KE}}{m}}}{qB} = \frac{\sqrt{2\,\text{KE}\,m}}{qB} = \frac{\sqrt{2\left(5.0\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(1.67\times10^{-27}\,\text{kg}\right)}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.20\,\text{T}\right)} = \boxed{1.6\,\text{m}}$$

15. The magnetic field can be found from Eq. 20-4, and the direction is found from the right hand rule. Remember that the charge is negative.

$$F_{\max} = qvB \quad\rightarrow\quad B = \frac{F_{\max}}{qv} = \frac{7.2\times10^{-13}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.9\times10^{6}\,\text{m/s}\right)} = \boxed{1.6\,\text{T}}$$

The direction would have to be $\boxed{\text{East}}$ for the right hand rule, applied to the velocity and the magnetic field, to give the proper direction of force.

16. The force on the electron due to the electric force must be the same magnitude as the force on the electron due to the magnetic force.

$$F_{\text{E}} = F_{\text{B}} \;\rightarrow\; qE = qvB \;\rightarrow\; v = \frac{E}{B} = \frac{8.8\times10^{3}\,\text{V/m}}{3.5\times10^{-3}\,\text{T}} = 2.514\times10^{6}\,\text{m/s} \approx \boxed{2.5\times10^{6}\,\text{m/s}}$$

If the electric field is turned off, the magnetic force will cause circular motion.

$$F_{\text{max}} = qvB = m\frac{v^2}{r} \;\rightarrow\; r = \frac{mv}{qB} = \frac{(9.11\times10^{-31}\,\text{kg})(2.514\times10^{6}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(3.5\times10^{-3}\,\text{T})} = \boxed{4.1\times10^{-3}\,\text{m}}$$

17. (*a*) The velocity of the ion can be found using energy conservation. The electrical potential energy of the ion becomes kinetic energy as it is accelerated. Then, since the ion is moving perpendicular to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path.

$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; qV = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{\frac{2qV}{m}}$$

$$F_{\text{max}} = qvB = m\frac{v^2}{r} \;\rightarrow$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{0.340\,\text{T}}\sqrt{\frac{2(6.6\times10^{-27}\,\text{kg})(2100\,\text{V})}{2(1.60\times10^{-19}\,\text{C})}} = \boxed{2.7\times10^{-2}\,\text{m}}$$

(*b*) The period can be found from the speed and the radius. Use the expressions for the radius and the speed from above.

$$v = \frac{2\pi r}{T} \;\rightarrow\; T = \frac{2\pi r}{v} = \frac{2\pi\frac{1}{B}\sqrt{\frac{2mV}{q}}}{\sqrt{\frac{2qV}{m}}} = \frac{2\pi m}{qB} = \frac{2\pi(6.6\times10^{-27}\,\text{kg})}{2(1.60\times10^{-19}\,\text{C})(0.340\,\text{T})} = \boxed{3.8\times10^{-7}\,\text{s}}$$

18. The velocity of each charged particle can be found using energy conservation. The electrical potential energy of the particle becomes kinetic energy as it is accelerated. Then, since the particle is moving perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path, and the radius can be determined in terms of the mass and charge of the particle.

$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; qV = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{\frac{2qV}{m}}$$

$$F_{\text{max}} = qvB = m\frac{v^2}{r} \;\rightarrow\; r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\frac{r_{\text{d}}}{r_{\text{p}}}=\frac{\dfrac{1}{B}\sqrt{\dfrac{2m_{\text{d}}V}{q_{\text{d}}}}}{\dfrac{1}{B}\sqrt{\dfrac{2m_{\text{p}}V}{q_{\text{p}}}}}=\frac{\sqrt{\dfrac{m_{\text{d}}}{m_{\text{p}}}}}{\sqrt{\dfrac{q_{\text{d}}}{q_{\text{p}}}}}=\frac{\sqrt{2}}{\sqrt{1}}=\sqrt{2}\;\rightarrow\;\boxed{r_{\text{d}}=2r_{\text{p}}}$$

$$\frac{r_{\alpha}}{r_{\text{p}}}=\frac{\dfrac{1}{B}\sqrt{\dfrac{2m_{\alpha}V}{q_{\alpha}}}}{\dfrac{1}{B}\sqrt{\dfrac{2m_{\text{p}}V}{q_{\text{p}}}}}=\frac{\sqrt{\dfrac{m_{\alpha}}{m_{\text{p}}}}}{\sqrt{\dfrac{q_{\alpha}}{q_{\text{p}}}}}=\frac{\sqrt{4}}{\sqrt{2}}=\sqrt{2}\;\rightarrow\;\boxed{r_{\alpha}=2r_{\text{p}}}$$

19. From Example 20-5, we have that $r=\frac{mv}{qB}$. Then, from chapter 5, the period for an object moving in uniform circular motion is given by $T=\frac{2\pi r}{v}$. Combine these two relationships.

$$T=\frac{2\pi r}{v}=\frac{2\pi\dfrac{mv}{qB}}{v}=\frac{2\pi m}{qB}$$

20. From Example 20-5, we have that $r=\frac{mv}{qB}$. The quantity mv is the momentum, p, and so $r=\frac{p}{qB}$. Thus $p=qBr$.

21. From Example 20-5, we have that $r=\frac{mv}{qB}$, and so $v=\frac{rqB}{m}$. The kinetic energy is given by

$$\text{KE}=\tfrac{1}{2}mv^2=\tfrac{1}{2}m\left(\frac{rqB}{m}\right)^2=\frac{r^2q^2B^2}{2m}\text{ and so we see that }\text{KE}\propto r^2.$$

22. The angular momentum of a particle moving in a circular path is given by $L=mvr$. From Example 20-5, we have that $r=\frac{mv}{qB}$, and so $v=\frac{rqB}{m}$. Combining these relationships gives the following.

$$L=mvr=m\frac{rqB}{m}r=qBr^2$$

23. The magnetic force produces an acceleration that is perpendicular to the original motion. If that perpendicular acceleration is small, it will produce a small deflection, and the original velocity can be assumed to always be perpendicular to the magnetic field. This leads to a constant perpendicular acceleration. The time that this (approximately) constant acceleration acts can be found from the original velocity *v* and the distance traveled *L*. The starting speed in the perpendicular direction will be zero.

$$F_{\perp}=ma_{\perp}=qvB\;\rightarrow\;a_{\perp}=\frac{qvB}{m}$$

$$d_\perp = v_{0\perp}t + \tfrac{1}{2}a_\perp t^2 = \tfrac{1}{2}\frac{qvB}{m}\left(\frac{L}{v}\right)^2 = \frac{qBL^2}{2mv} = \frac{\left(13.5\times10^{-9}\,\mathrm{C}\right)\left(5.00\times10^{-5}\,\mathrm{T}\right)\left(1.00\times10^{3}\,\mathrm{m}\right)^2}{2\left(3.40\times10^{-3}\,\mathrm{kg}\right)\left(160\,\mathrm{m/s}\right)}$$

$$= \boxed{6.2\times10^{-7}\,\mathrm{m}}$$

This small distance justifies the assumption of constant acceleration.

24. The magnetic force will produce centripetal acceleration. Use that relationship to calculate the speed. The radius of the Earth is $6.38\times10^{6}\,\mathrm{km}$, and the altitude is added to that.

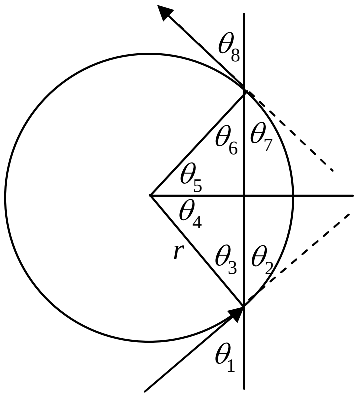
$$F_\mathrm{B} = qvB = m\frac{v^2}{r} \quad\rightarrow\quad v = \frac{qrB}{m} = \frac{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(6.385\times10^{6}\,\mathrm{m}\right)\left(0.40\times10^{-4}\,\mathrm{T}\right)}{238\left(1.66\times10^{-27}\,\mathrm{kg}\right)} = \boxed{1.0\times10^{8}\,\mathrm{m/s}}$$

Compare the size of the magnetic force to the force of gravity on the ion.

$$\frac{F_\mathrm{B}}{F_\mathrm{g}} = \frac{qvB}{mg} = \frac{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(1.0\times10^{8}\,\mathrm{m/s}\right)\left(0.40\times10^{-4}\,\mathrm{T}\right)}{238\left(1.66\times10^{-27}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{1.7\times10^{8}}$$

It is fine to ignore gravity – the magnetic force is more than 100 million times larger than gravity.

25. (*a*) In the magnetic field, the proton will move along an arc of a circle. The distance *x* in the diagram is a chord of that circle, and so the center of the circular path lies on the perpendicular bisector of the chord. That perpendicular bisector bisects the central angle of the circle which subtends the chord. Also recall that a radius is perpendicular to a tangent. In the diagram, $\theta_1 = \theta_2$ because they are vertical angles. Then $\theta_2 = \theta_4$, because they are both complements of θ_3, so $\theta_1 = \theta_4$. We have $\theta_4 = \theta_5$ since the central angle is bisected by the perpendicular bisector of the chord. $\theta_5 = \theta_7$ because they are both complements of θ_6, and $\theta_7 = \theta_8$ because they are vertical angles. Thus $\theta_1 = \theta_2 = \theta_4 = \theta_5 = \theta_7 = \theta_8$, and so in the textbook diagram, the angle at which the proton leaves is $\boxed{\theta = 45^\circ}$.



(*b*) The radius of curvature is given by $r = \dfrac{mv}{qB}$, and the distance *x* is twice the value of $r\cos\theta$.

$$x = 2r\cos\theta = 2\frac{mv}{qB}\cos\theta = 2\frac{\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(2.0\times10^{5}\,\mathrm{m/s}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(0.850\,\mathrm{T}\right)}\cos 45^\circ = \boxed{3.5\times10^{-3}\,\mathrm{m}}$$

26. We assume the jumper cable is a long straight wire, and use Eq. 20-6.

$$B_\mathrm{cable} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(65\,\mathrm{A}\right)}{2\pi\left(6.0\times10^{-2}\,\mathrm{m}\right)} = 2.167\times10^{-4}\,\mathrm{T} \approx \boxed{2.2\times10^{-4}\,\mathrm{T}}$$

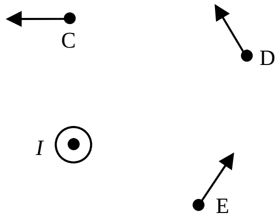
Compare this to the Earth's field of $0.5\times10^{-4}\,\mathrm{T}$.

$$B_\mathrm{cable}\big/B_\mathrm{Earth} = \frac{2.167\times10^{-4}\,\mathrm{T}}{0.5\times10^{-4}\,\mathrm{T}} = 4.33, \text{ so } \boxed{\text{the field of the cable is about 4 times that of the Earth.}}$$

27. We assume that the wire is long and straight, and use Eq. 20-6.

$$B_{\text{wire}}=\frac{\mu_0 I}{2\pi r}\quad\to\quad I=\frac{2\pi r B_{\text{wire}}}{\mu_0}=\frac{2\pi\left(0.25\,\text{m}\right)\left(0.55\times10^{-4}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}=\boxed{69\,\text{A}}$$

28. To find the direction, draw a radius line from the wire to the field point. Then at the field point, draw a perpendicular to the radius line, directed so that the perpendicular line would be part of a counterclockwise circle.



29. Since the force is attractive, the currents must be in the same direction, so the current in the second wire must also be upward. Use Eq. 20-7 to calculate the magnitude of the second current.

$$F_2=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}l_2\quad\to$$

$$I_2=\frac{2\pi}{\mu_0}\frac{F_2}{l_2}\frac{d}{I_1}=\frac{2\pi}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}\left(8.8\times10^{-4}\,\text{N/m}\right)\frac{7.0\times10^{-2}\,\text{m}}{24\,\text{A}}=\boxed{13\,\text{A upward}}$$

30. Since the currents are parallel, the force on each wire will be attractive, toward the other wire. Use Eq. 20-7 to calculate the magnitude of the force.

$$F_2=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}l_2=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi}\frac{\left(25\,\text{A}\right)^2}{\left(6.0\times10^{-2}\,\text{m}\right)}\left(35\,\text{m}\right)=\boxed{7.3\times10^{-2}\,\text{N, attractive}}$$

$\boxed{31.}$ For the experiment to be accurate to ±1.0%, the magnetic field due to the current in the cable must be less than or equal to 1.0% of the Earth's magnetic field. Use Eq. 20-6 to calculate the magnetic field due to the current in the cable.

$$B_{\text{cable}}=\frac{\mu_0 I}{2\pi r}\le 0.01B_{\text{Earth}}\quad\to\quad I\le\frac{2\pi r\left(0.01B_{\text{Earth}}\right)}{\mu_0}=\frac{2\pi\left(1.00\,\text{m}\right)\left(0.01\right)\left(0.5\times10^{-4}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}=2.5\,\text{A}$$

Thus the maximum allowable current is $\boxed{2.5\,\text{A}}$.

32. We assume that the power line is long and straight, and use Eq. 20-6.

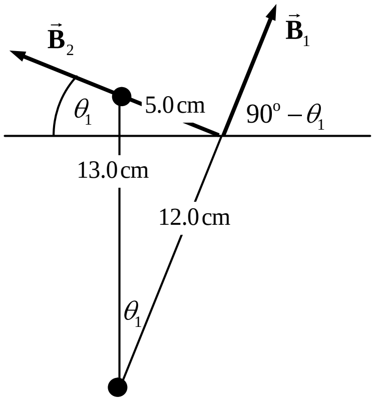
$$B_{\text{line}}=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(95\,\text{A}\right)}{2\pi\left(8.5\,\text{m}\right)}=2.235\times10^{-6}\,\text{T}\approx\boxed{2.2\times10^{-6}\,\text{T}}$$

Compare this to the Earth's field of $0.5\times10^{-4}\,\text{T}$.

$$B_{\text{line}}/B_{\text{Earth}}=\frac{2.235\times10^{-6}\,\text{T}}{0.5\times10^{-4}\,\text{T}}=0.0447$$

$\boxed{\text{The field of the cable is about 4\% that of the Earth.}}$

33. The field from each wire makes concentric circles about the source wire. The individual fields at point P are perpendicular to the lines from the wires to point P. We define $\vec{\mathbf{B}}_1$ as the field from the closer point, and $\vec{\mathbf{B}}_2$ as the field from the further point. Use Eq. 20-6 to calculate each individual field.



$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(25\,\mathrm{A}\right)}{2\pi\left(0.050\,\mathrm{m}\right)} = 1.0\times10^{-4}\,\mathrm{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(25\,\mathrm{A}\right)}{2\pi\left(0.120\,\mathrm{m}\right)} = 4.167\times10^{-5}\,\mathrm{T}$$

$$\theta_1 = \tan^{-1}\frac{5}{12} = 22.6^\circ \qquad \vec{\mathbf{B}}_{\rm net} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2$$

$$B_{\text{net}\,x} = B_1\cos\left(90^\circ - \theta_1\right) - B_2\cos\theta_1 = \left(1.0\times10^{-4}\,\mathrm{T}\right)\cos 67.4^\circ - \left(4.167\times10^{-5}\,\mathrm{T}\right)\cos 22.6^\circ$$

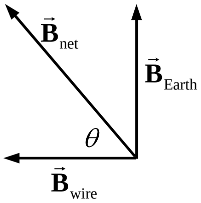
$$= -4.1\times10^{-8}\,\mathrm{T} \approx 0\,\mathrm{T}$$

$$B_{\text{net}\,y} = B_1\sin\left(90^\circ - \theta_1\right) + B_2\sin\theta_1 = \left(1.0\times10^{-4}\,\mathrm{T}\right)\sin 67.4^\circ + \left(4.167\times10^{-5}\,\mathrm{T}\right)\sin 22.6^\circ$$

$$= 1.1\times10^{-4}\,\mathrm{T}$$

$$B_{\rm net} = \boxed{1.1\times10^{-4}\,\mathrm{T}\ @\ 90^\circ}$$

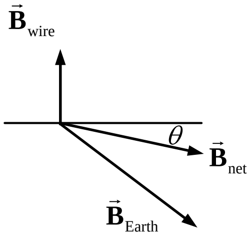
34. At the location of the compass, the magnetic field caused by the wire will point to the west, and the Earth's magnetic field points due North. The compass needle will point in the direction of the NET magnetic field.


$\vec{\mathbf{B}}_{\rm net}$
$\vec{\mathbf{B}}_{\rm Earth}$
θ
$\vec{\mathbf{B}}_{\rm wire}$

$$B_{\rm wire} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(35\,\mathrm{A}\right)}{2\pi\left(0.18\,\mathrm{m}\right)} = 3.89\times10^{-5}\,\mathrm{T}$$

$$\theta = \tan^{-1}\frac{B_{\rm Earth}}{B_{\rm wire}} = \tan^{-1}\frac{0.45\times10^{-4}\,\mathrm{T}}{3.89\times10^{-5}\,\mathrm{T}} = \boxed{49^\circ\ \text{N of W}}$$

35. The magnetic field due to the long horizontal wire points straight up at the point in question, and its magnitude is given by Eq. 20-6. The two fields are oriented as shown in the diagram. The net field is the vector sum of the two fields.


$\vec{\mathbf{B}}_{\rm wire}$
θ
$\vec{\mathbf{B}}_{\rm net}$
$\vec{\mathbf{B}}_{\rm Earth}$

$$B_{\rm wire} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(22.0\,\mathrm{A}\right)}{2\pi\left(0.200\,\mathrm{m}\right)} = 2.2\times10^{-5}\,\mathrm{T}$$

$$B_{\rm Earth} = 5.0\times10^{-5}\,\mathrm{T}$$

$$B_{\text{net}\,x} = B_{\rm Earth}\cos 37^\circ = 4.00\times10^{-5}\,\mathrm{T} \qquad B_{\text{net}\,y} = B_{\rm wire} - B_{\rm Earth}\sin 37^\circ = -0.81\times10^{-5}\,\mathrm{T}$$

$$B_{\rm net} = \sqrt{B_{\text{net}\,x}^2 + B_{\text{net}\,y}^2} = \sqrt{\left(4.00\times10^{-5}\,\mathrm{T}\right)^2 + \left(-0.81\times10^{-5}\,\mathrm{T}\right)^2} = \boxed{4.1\times10^{-5}\,\mathrm{T}}$$

$$\theta = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,x}} = \tan^{-1}\frac{-0.81\times10^{-5}\,\mathrm{T}}{4.00\times10^{-5}\,\mathrm{T}} = \boxed{11^\circ\ \text{below the horizontal}}$$

36. The stream of protons constitutes a current, whose magnitude is found by multiplying the proton rate times the charge of a proton. Then use Eq. 20-6 to calculate the magnetic field.

$$B_{\text{stream}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \text{T•m/A}\right)\left(1.5\times 10^{9}\ \text{protons/s}\right)\left(1.60\times 10^{-19}\ \text{C/proton}\right)}{2\pi\left(2.0\,\text{m}\right)} = \boxed{2.4\times 10^{-17}\,\text{T}}$$

37. (*a*) If the currents are in the same direction, the magnetic fields at the midpoint between the two currents will oppose each other, and so their magnitudes should be subtracted.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\left(4\pi\times 10^{-7}\ \text{T•m/A}\right)}{2\pi\left(1.0\times 10^{-2}\,\text{m}\right)}\left(I - 15\,\text{A}\right) = \left(2\times 10^{-5}\ \text{T/A}\right)\left(I - 15\,\text{A}\right)$$

(*b*) If the currents are in the opposite direction, the magnetic fields at the midpoint between the two currents will reinforce each other, and so their magnitudes should be added.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2} = \frac{\left(4\pi\times 10^{-7}\ \text{T•m/A}\right)}{2\pi\left(1.0\times 10^{-2}\,\text{m}\right)}\left(I + 15\,\text{A}\right) = \left(2\times 10^{-5}\ \text{T/A}\right)\left(I + 15\,\text{A}\right)$$

38. The fields created by the two wires will oppose each other, so the net field is the difference of the magnitudes of the two fields. The positive direction for the fields is taken to be into the paper, and so the closer wire creates a field in the positive direction, and the farther wire creates a field in the negative direction.

$$\begin{aligned} B_{\text{net}} &= \frac{\mu_0 I}{2\pi r_{\text{closer}}} - \frac{\mu_0 I}{2\pi r_{\text{farther}}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_{\text{closer}}} - \frac{1}{r_{\text{closer}}}\right) \\ &= \frac{\left(4\pi\times 10^{-7}\ \text{T•m/A}\right)\left(25.0\,\text{A}\right)}{2\pi}\left(\frac{1}{\left(0.10\,\text{m} - 0.0014\,\text{m}\right)} - \frac{1}{\left(0.10\,\text{m} + 0.0014\,\text{m}\right)r_{\text{closer}}}\right) \\ &= 1.400\times 10^{-6}\,\text{T} \approx \boxed{1.4\times 10^{-6}\,\text{T}} \end{aligned}$$

Compare this to the Earth's field of $0.5\times 10^{-4}\,\text{T}$.

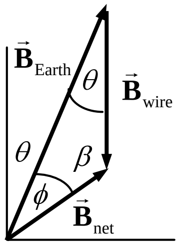
$$B_{\text{net}} / B_{\text{Earth}} = \frac{1.400\times 10^{-6}\,\text{T}}{0.5\times 10^{-4}\,\text{T}} = 0.028$$

$\boxed{\text{The field of the wires is about 3\% that of the Earth.}}$

39. The center of the third wire is 5.6 mm from the left wire, and 2.8 mm from the right wire. The force on the near (right) wire will attract the near wire, since the currents are in the same direction. The force on the far (left) wire will repel the far wire, since the currents oppose each other. Use Eq. 20-7 to calculate the force per unit length.

$$F_{\text{near}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}} l_{\text{near}} \quad\rightarrow\quad \frac{F_{\text{near}}}{l_{\text{near}}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}} = \frac{4\pi\times 10^{-7}\ \text{T•m/A}}{2\pi}\frac{\left(25.0\,\text{A}\right)^2}{2.8\times 10^{-3}\,\text{m}} = \boxed{4.5\times 10^{-2}\ \text{N/m, attract}}$$

$$F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}} l_{\text{far}} \quad\rightarrow\quad \frac{F_{\text{far}}}{l_{\text{far}}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}} = \frac{4\pi\times 10^{-7}\ \text{T•m/A}}{2\pi}\frac{\left(25.0\,\text{A}\right)^2}{5.6\times 10^{-3}\,\text{m}} = \boxed{2.2\times 10^{-2}\ \text{N/m, repel}}$$



$\vec{\mathbf{B}}_{\text{Earth}}$
θ
$\vec{\mathbf{B}}_{\text{wire}}$
θ
β
ϕ
$\vec{\mathbf{B}}_{\text{net}}$

40. The Earth's magnetic field is present at both locations in the problem, and we assume it is the same at both locations. The field east of a vertical wire must be pointing either due north or due south. The compass shows the direction of the net magnetic field, and it changes from 23° E of N to 55° E of N when taken inside. That is a "southerly" change (rather than a "northerly" change"), and so the field due to the wire must be pointing due south. See the diagram. For the angles, $\theta = 23^\circ$, $\theta+\phi=55^\circ$, and $\beta+\theta+\phi=180^\circ$ and so $\phi=32^\circ$ and $\beta=125^\circ$. Use the law of sines to find the magnitude of $\vec{\mathbf{B}}_{\text{wire}}$, and then use Eq. 20-6 to find the magnitude of the current.

$$\frac{B_{\text{wire}}}{\sin\phi}=\frac{B_{\text{Earth}}}{\sin\beta} \rightarrow B_{\text{wire}}=B_{\text{Earth}}\frac{\sin\phi}{\sin\beta}=\frac{\mu_0}{2\pi}\frac{I}{r} \rightarrow$$

$$I=B_{\text{Earth}}\frac{\sin\phi}{\sin\beta}\frac{2\pi}{\mu_0}r=\left(5.0\times10^{-5}\,\text{T}\right)\frac{\sin 32^\circ}{\sin 125^\circ}\frac{2\pi}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}\left(0.120\,\text{m}\right)=\boxed{19.4\,\text{A}}$$

Since the field due to the wire is due south, the current in the wire must be $\boxed{\text{down}}$.

41. The magnetic field at the loop due to the long wire is into the page, and can be calculated by Eq. 20-6. The force on the segment of the loop closest to the wire is towards the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is difficult to calculate the total force on either the left or right segments of the loop. Using the right hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, and so only the parallel segments need to be considered in the calculation. Use Eq. 20-7.

$$F_{\text{net}}=F_{\text{near}}-F_{\text{far}}=\frac{\mu_0}{2\pi}\frac{I_1I_2}{d_{\text{near}}}l_{\text{near}}-\frac{\mu_0}{2\pi}\frac{I_1I_2}{d_{\text{far}}}l_{\text{far}}=\frac{\mu_0}{2\pi}I_1I_2l\left(\frac{1}{d_{\text{near}}}-\frac{1}{d_{\text{far}}}\right)$$

$$=\frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi}\left(2.5\,\text{A}\right)^2\left(0.100\,\text{m}\right)\left(\frac{1}{0.030\,\text{m}}-\frac{1}{0.080\,\text{m}}\right)=\boxed{2.6\times10^{-6}\,\text{N, towards wire}}$$

42. (*a*) The magnetic force exerted on the lower wire must be upwards, and must be the same magnitude as the gravitational force on the lower wire. Consider a length L of the copper wire.

$$F_g=F_{\text{B}} \rightarrow m_{\text{Cu}}g=\frac{\mu_0}{2\pi}\frac{I_{\text{top}}I_{\text{Cu}}}{d}L_{\text{Cu}} \rightarrow I_{\text{Cu}}=\frac{2\pi dm_{\text{Cu}}g}{\mu_0I_{\text{top}}L_{\text{Cu}}}=\frac{2\pi d\rho_{\text{Cu}}L_{\text{Cu}}\pi r_{\text{Cu}}^2g}{\mu_0I_{\text{top}}L_{\text{Cu}}}$$

$$=\frac{2\pi d\rho_{\text{Cu}}\pi r_{\text{Cu}}^2g}{\mu_0I_{\text{top}}}=\frac{2\pi\left(0.15\,\text{m}\right)\left(8900\,\text{kg/m}^2\right)\pi\left(1.25\times10^{-3}\,\text{m}\right)^2\left(9.80\,\text{m/s}^2\right)}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(48\,\text{A}\right)}=\boxed{6700\,\text{A}}$$

The currents must be in the same direction for the magnetic force to be upward, so the current is to the $\boxed{\text{right}}$.

(*b*) If the wire is displaced slightly downward, the magnetic force would be less than the force of gravity, so the object would continue to move downward and it would fall. Thus the lower wire is $\boxed{\text{not in stable equilibrium}}$.

(*c*) Because the wires are the same distance apart, the current must be the same, $\boxed{6700\,\text{A}}$. The current in the upper wire must now be to the $\boxed{\text{left}}$ so that the magnetic force is upwards. If the wire is now displaced upwards, the magnetic force decreases, and the object starts to fall, moving toward its original position. If the wire is displaced downwards, the magnetic force increases, and the object would move back toward its original position. Thus the wire $\boxed{\text{is in stable equilibrium}}$.

43. The magnetic fields created by the individual currents will be at right angles to each other. The field due to the top wire will be to the right, and the field due to the bottom wire will be out of the page. Since they are at right angles, the net field is the hypotenuse of the two individual fields.

$$B_{\text{net}} = \sqrt{\left(\frac{\mu_0 I_{\text{top}}}{2\pi r_{\text{top}}}\right)^2 + \left(\frac{\mu_0 I_{\text{bottom}}}{2\pi r_{\text{bottom}}}\right)^2} = \frac{\mu_0}{2\pi r}\sqrt{I_{\text{top}}^2 + I_{\text{bottom}}^2} = \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi(0.100\,\text{m})}\sqrt{(20.0\,\text{A})^2 + (5.0\,\text{A})^2}$$

$$= \boxed{4.12\times10^{-5}\,\text{T}}$$

44. (*a*) Use Eq. 20-6 to calculate the field due to a long straight wire.

$$B_{\text{A at B}} = \frac{\mu_0 I_{\text{A}}}{2\pi r_{\text{A to B}}} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(2.0\,\text{A})}{2\pi(0.15\,\text{m})} = 2.667\times10^{-6}\,\text{T} \approx \boxed{2.7\times10^{-6}\,\text{T}}$$

(*b*) $$B_{\text{B at A}} = \frac{\mu_0 I_{\text{B}}}{2\pi r_{\text{B to A}}} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(4.0\,\text{A})}{2\pi(0.15\,\text{m})} = 5.333\times10^{-6}\,\text{T} \approx \boxed{5.3\times10^{-6}\,\text{T}}$$

(*c*) The two fields are not equal and opposite. Each individual field is due to a single wire, and has no dependence on the other wire. The magnitude of current in the second wire has nothing to do with the value of the field caused by the first wire.

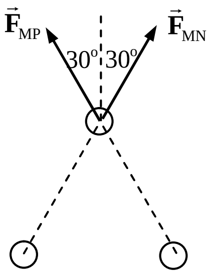
(*d*) Use Eq. 20-7 to calculate the force due to one wire on another. The forces are attractive since the currents are in the same direction.

$$\frac{F_{\text{on A due to B}}}{l_{\text{A}}} = \frac{F_{\text{on B due to A}}}{l_{\text{B}}} = \frac{\mu_0}{2\pi}\frac{I_{\text{A}} I_{\text{B}}}{d_{\text{A to B}}} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})}{2\pi}\frac{(2.0\,\text{A})(4.0\,\text{A})}{(0.15\,\text{m})}$$

$$= 1.067\times10^{-5}\,\text{N/m} \approx \boxed{1.1\times10^{-5}\,\text{N/m}}$$

These two forces per unit length are equal and opposite because they are a Newton's Third Law pair of forces.

45. The forces on wire M due to the other wires are repelling forces, away from the other wires. Use Eq. 20-7 to calculate the force per unit length on wire M due to each of the other wires, and then add the force vectors together. The horizontal parts of the forces cancel, and the sum is vertical.

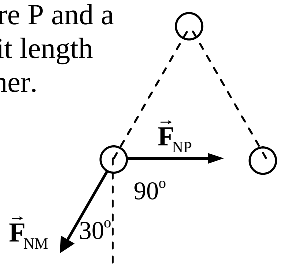


$$\frac{F_{\text{M net y}}}{l_{\text{M}}} = \frac{F_{\text{MN}}}{l_{\text{M}}}\cos 30^\circ + \frac{F_{\text{MP}}}{l_{\text{M}}}\cos 30^\circ$$

$$= \frac{\mu_0}{2\pi}\frac{I_{\text{M}} I_{\text{N}}}{d_{\text{MN}}}(1\,\text{m})\cos 30^\circ + \frac{\mu_0}{2\pi}\frac{I_{\text{M}} I_{\text{P}}}{d_{\text{MP}}}(1\,\text{m})\cos 30^\circ$$

$$= 2\frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})}{2\pi}\frac{(8.00\,\text{A})^2}{(0.038\,\text{m})}\cos 30^\circ = \boxed{5.8\times10^{-4}\,\text{N/m},\ 90^\circ}$$

The forces on wire N due to the other wires are an attractive force towards wire P and a repelling force away from wire M. Use Eq. 20-7 to calculate the force per unit length on wire N due to each of the other wires, and then add the force vectors together. From the symmetry, we expect the net force to lie exactly between the two individual force vectors, which is 60° below the horizontal.


$\vec{\mathbf{F}}_{\mathrm{NP}}$
90°
$\vec{\mathbf{F}}_{\mathrm{NM}}$
30°

$$F_{\mathrm{N\,net\,y}} = -F_{\mathrm{NM}}\cos 30^{\circ} = -\frac{\mu_0}{2\pi}\frac{I_{\mathrm{M}}I_{\mathrm{N}}}{d_{\mathrm{MN}}}\cos 30^{\circ}$$

$$= -\frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}{2\pi}\frac{\left(8.00\,\mathrm{A}\right)^2}{\left(0.038\,\mathrm{m}\right)}\cos 30^{\circ} = -2.917\times10^{-4}\,\mathrm{N/m}$$

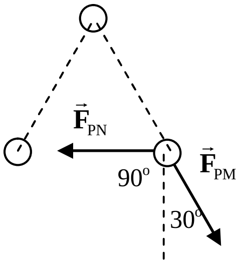
$$F_{\mathrm{N\,net\,x}} = -F_{\mathrm{NM}}\sin 30^{\circ} + F_{\mathrm{NP}}$$

$$= -\frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}{2\pi}\frac{\left(8.00\,\mathrm{A}\right)^2}{\left(0.038\,\mathrm{m}\right)}\sin 30^{\circ} + \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}{2\pi}\frac{\left(8.00\,\mathrm{A}\right)^2}{\left(0.038\,\mathrm{m}\right)}$$

$$= 1.684\times10^{-5}\,\mathrm{N/m}$$

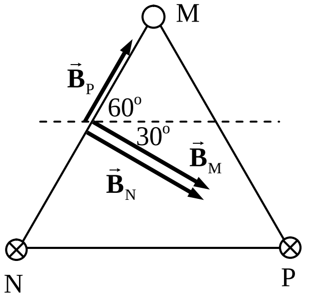
$$F_{\mathrm{N\,net}} = \sqrt{F^2_{\mathrm{N\,net\,x}} + F^2_{\mathrm{N\,net\,y}}} = \boxed{3.4\times10^{-4}\,\mathrm{N/m}} \qquad \theta = \tan^{-1}\frac{F_{\mathrm{N\,net\,y}}}{F_{\mathrm{N\,net\,x}}} = \tan^{-1}\frac{-2.917}{1.684} = \boxed{300^{\circ}}$$

The forces on wire P due to the other wires are an attractive force towards wire N and a repelling force away from wire M. Use Eq. 20-7 to calculate the force per unit length on wire P due to each of the other wires, and then add the force vectors together. From symmetry, this is just a mirror image of the previous solution, and so the net force is as follows.


$\vec{\mathbf{F}}_{\mathrm{PN}}$
$\vec{\mathbf{F}}_{\mathrm{PM}}$
90°
30°

$$F_{\mathrm{P\,net}} = \boxed{3.4\times10^{-4}\,\mathrm{N/m}} \qquad \theta = \boxed{240^{\circ}}$$

46. There will be three magnetic fields to sum – one from each wire. Each field will point perpendicularly to the line connecting the wire to the midpoint. The two fields due to M and N are drawn slightly separated from each other, but should be collinear. The magnitude of each field is given by Eq. 29-6.


M
$\vec{\mathbf{B}}_{\mathrm{P}}$
60°
30°
$\vec{\mathbf{B}}_{\mathrm{M}}$
$\vec{\mathbf{B}}_{\mathrm{N}}$
N
P

$$\vec{\mathbf{B}}_{\mathrm{net}} = \vec{\mathbf{B}}_{\mathrm{M}} + \vec{\mathbf{B}}_{\mathrm{N}} + \vec{\mathbf{B}}_{\mathrm{P}}$$

$$B_{\mathrm{M}} = B_{\mathrm{N}} = \frac{\mu_0}{2\pi}\frac{I}{r_{\mathrm{M}}} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}{2\pi}\frac{8.00\,\mathrm{A}}{0.019\,\mathrm{m}} = 8.421\times10^{-5}\,\mathrm{T}$$

$$B_{\mathrm{P}} = \frac{\mu_0}{2\pi}\frac{I}{r_{\mathrm{P}}} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}{2\pi}\frac{8.00\,\mathrm{A}}{\sqrt{3}\left(0.019\,\mathrm{m}\right)} = 4.862\times10^{-5}\,\mathrm{T}$$

$$B_{\mathrm{net}\,x} = 2B_{\mathrm{M}}\cos 30^{\circ} + B_{\mathrm{P}}\cos 60^{\circ} = 2\left(8.421\times10^{-5}\,\mathrm{T}\right)\cos 30^{\circ} + \left(4.862\times10^{-5}\,\mathrm{T}\right)\cos 60^{\circ}$$

$$= 1.702\times10^{-4}\,\mathrm{T}$$

$$B_{\mathrm{net}\,y} = -2B_{\mathrm{M}}\sin 30^{\circ} + B_{\mathrm{P}}\sin 60^{\circ} = -2\left(8.421\times10^{-5}\,\mathrm{T}\right)\sin 30^{\circ} + \left(4.862\times10^{-5}\,\mathrm{T}\right)\sin 60^{\circ}$$

$$= -4.210\times10^{-5}\,\mathrm{T}$$

$$B_{\mathrm{net}} = \sqrt{B^2_{\mathrm{net}\,x} + B^2_{\mathrm{net}\,y}} = \sqrt{\left(1.702\times10^{-4}\,\mathrm{T}\right)^2 + \left(-4.210\times10^{-5}\,\mathrm{T}\right)} = \boxed{1.75\times10^{-4}\,\mathrm{T}}$$

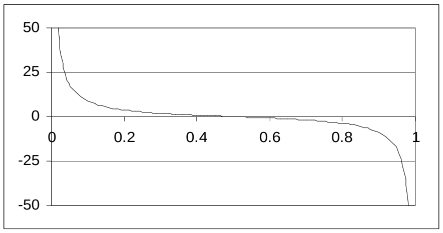
$$\theta_{\text{net}} = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,y}} = \tan^{-1}\frac{-4.210\times10^{-5}\,\text{T}}{1.702\times10^{-4}\,\text{T}} = \boxed{-14^\circ}$$

The net field points slightly below the horizontal direction.

47. The left wire will cause a field on the *x*-axis that points in the *y*-direction, and the right wire will cause a field on the *x*-axis that points in the negative *y*-direction. The distance from the left wire to a point on the *x*-axis is *x*, and the distance from the right wire is $d - x$.

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi x} - \frac{\mu_0 I}{2\pi(d-x)} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{d-x}\right)$$

A graph of the above function is shown here.



48. Use Eq. 20-8 for the field inside a solenoid.

$$B = \mu_0 IN/l = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(2.0\,\text{A})(420)}{0.12\,\text{m}} = \boxed{8.8\times10^{-3}\,\text{T}}$$

49. Use Eq. 20-8 for the field inside a solenoid.

$$B = \mu_0 IN/l \;\rightarrow\; I = \frac{Bl}{\mu_0 N} = \frac{(0.385\,\text{T})(0.300\,\text{m})}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(975)} = \boxed{94.3\,\text{A}}$$

50. The field due to the solenoid is given by Eq. 20-8. Since the field due to the solenoid is perpendicular to the current in the wire, Eq. 20-2 can be used to find the force on the wire segment.

$$F = I_{\text{wire}} l_{\text{wire}} B_{\text{solenoid}} = I_{\text{wire}} l_{\text{wire}} \frac{\mu_0 I_{\text{solenoid}} N}{l_{\text{solenoid}}} = (22\,\text{A})(0.030\text{m})\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(33\,\text{A})(550)}{(0.15\,\text{m})}$$

$$= \boxed{0.10\,\text{N}}$$

51. Since the mass of copper is fixed and the density is fixed, the volume is fixed, and we designate it as $V_{\text{Cu}} = m_{\text{Cu}}\rho_{\text{Cu}} = L_{\text{Cu}}A_{\text{Cu}}$. We call the fixed voltage V_0. The magnetic field in the solenoid is given by Eq. 20-8.

$$B = \frac{\mu_0 IN}{L_{\text{sol}}} = \mu_0 V_0 \frac{N}{R_{\text{Cu}} L_{\text{sol}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{\frac{L_{\text{Cu}}}{A_{\text{Cu}}} L_{\text{sol}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{L_{\text{sol}}} \frac{A_{\text{Cu}}}{L_{\text{Cu}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{L_{\text{sol}}} \frac{m_{\text{Cu}}\rho_{\text{Cu}}}{L_{\text{Cu}}^2}$$

$$= \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{N}{L_{\text{sol}} L_{\text{Cu}}^2}$$

The number of turns of wire is the length of wire divided by the circumference of the solenoid.

$$N = \frac{L_{\text{Cu}}}{2\pi r_{\text{sol}}} \;\rightarrow\; B = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{N}{L_{\text{sol}} L_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{\frac{L_{\text{Cu}}}{2\pi r_{\text{sol}}}}{L_{\text{sol}} L_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{2\pi\rho_{\text{R Cu}}} \frac{1}{L_{\text{sol}} r_{\text{sol}} L_{\text{Cu}}}$$

The first factor in the expression for B is made of constants, so we have $B \propto \dfrac{1}{L_{\text{sol}} r_{\text{sol}} L_{\text{Cu}}}$. Thus we want the wire to be $\boxed{\text{short and fat}}$. Also the radius of the solenoid should be small and the length of the solenoid small.

52. (*a*) Each loop of wire produces a field along its axis, similar to Figure 20-9. For path 1, with all the loops taken together, that symmetry leads to a magnetic field that is the same anywhere along the path, and parallel to the path. One side of every turn of the wire is enclosed by path 1, and so the enclosed current is NI. Apply Eq. 20-9.

$$\sum B_{\|}\Delta l = \mu_0 NI \rightarrow B(2\pi R) = \mu_0 NI \rightarrow \boxed{B = \mu_0 NI/2\pi R}$$

(*b*) For path 2, each loop of wire pierces the enclosed area twice – once going up and once going down. Ampere's law takes the direction of the current into account, and so the net current through the area enclosed by path 2 is 0.

$$\sum B_{\|}\Delta l = \mu_0 NI \rightarrow B(2\pi R) = 0 \rightarrow \boxed{B = 0}$$

(*c*) The field inside a toroid is $\boxed{\text{not uniform}}$. As seen in the result to part (a), the field varies inversely as the radius of the toroid, and so $\boxed{B \propto \dfrac{1}{R}}$. The field is strongest at the inside wall of the toroid, and weakest at the outside wall.

53. (*a*) Ampere's Law says that along a closed path, $\sum B_{\|}\Delta l = \mu_0 I_{\text{encl}}$. For the path choose a circle of radius *r*, centered on the center of the wire, greater than the radius of the inner wire and less than the radius of the outer cylindrical braid. Because the wire is long and straight, the magnetic field is tangent to the chosen path, and so $B_{\|} = B$. The current enclosed by the path is *I*.

$$\mu_0 I_{\text{encl}} = \sum B_{\|}\Delta l = \sum B\Delta l = B\sum \Delta l = B(2\pi r) \rightarrow B = \frac{\mu_0 I}{2\pi r}$$

(*b*) We make a similar argument, but now choose the path to be a circle of radius *r*, greater than the radius of the outer cylindrical braid. Because the wire and the braid are long and straight, the magnetic field is tangent to the chosen path, and so $B_{\|} = B$. The current enclosed by the path is zero, since there are two equal but oppositely directed currents.

$$\mu_0 I_{\text{encl}} = \sum B_{\|}\Delta l = \sum B\Delta l = B\sum \Delta l = B(2\pi r) \rightarrow B = \frac{\mu_0 I_{\text{encl}}}{2\pi r} = 0$$

54. If the face of the loop of wire is parallel to the magnetic field, the angle between the perpendicular to the loop and the magnetic field is 90°. Use Eq. 20-10 to calculate the magnetic field strength.

$$\tau = NIAB\sin\theta \rightarrow B = \frac{\tau}{NIA\sin\theta} = \frac{0.325\,\text{m}\bullet\text{N}}{(1)(6.30\,\text{A})(0.220\,\text{m})^2\sin 90^\circ} = \boxed{1.07\,\text{T}}$$

55. In section 20-10, it is shown that the deflection of the galvanometer needle is proportional to the product of the current and the magnetic field. Thus if the magnetic field is decreased to 0.860 times its original value, the current must be increased by dividing the original value by 0.860 to obtain the same deflection.

$$(IB)_{\text{initial}} = (IB)_{\text{final}} \rightarrow I_{\text{final}} = \frac{I_{\text{initial}}B_{\text{initial}}}{B_{\text{final}}} = \frac{(53.0\,\mu\text{A})\,B_{\text{initial}}}{0.860B_{\text{initial}}} = \boxed{61.6\,\mu\text{A}}$$

56. From the galvanometer discussion in section 20-10, the amount of deflection is proportional to the ratio of the current and the spring constant: $\phi \propto \frac{I}{k}$. Thus if the spring constant decreases by 25%, the current can decrease by 25% to produce the same deflection. The new current will be 75% of the original current.

$$I_{\text{final}} = 0.75 I_{\text{initial}} = 0.75(36\mu\text{A}) = \boxed{27\mu\text{A}}$$

57. From section 20-9, we see that the torque is proportional to the current, so if the current drops by 12%, the output torque will also drop by 12%. Thus the final torque is 0.88 times the initial torque.

58. The magnetic dipole moment is defined in Eq. 20-11 as $M = NIA$. The number of turns, N, is 1. The current is the charge per unit time passing a given point, which on the average is the charge on the electron divided by the period of the circular motion, $I = e/T$. If we assume the electron is moving in a circular orbit of radius *r*, then the area is πr^2. The period of the motion is the circumference of the orbit divided by the speed, $T = 2\pi r/v$. Finally, the angular momentum of an object moving in a circle is given by $L = mrv$. Combine these relationships to find the magnetic moment.

$$M = NIA = \frac{e}{T}\pi r^2 = \frac{e}{2\pi r/v}\pi r^2 = \frac{e\pi r^2 v}{2\pi r} = \frac{erv}{2} = \frac{emrv}{2m} = \frac{e}{2m}mrv = \frac{e}{2m}L$$

59. (*a*) The torque is given by Eq. 20-10. The angle is the angle between the B-field and the perpendicular to the coil face.

$$\tau = NIAB\sin\theta = 9(7.20\,\text{A})\left[\pi(0.080\,\text{m})^2\right](5.50\times10^{-5}\,\text{T})\sin 34^{\circ} = \boxed{4.01\times10^{-5}\,\text{m}\bullet\text{N}}$$

(*b*) In Example 20-12 it is stated that if the coil is free to turn, it will rotate toward the orientation so that the angle is 0. In this case, that means the north edge of the coil will rise, so that a perpendicular to its face will be parallel with the Earth's magnetic field.

$\boxed{60.}$ The radius and magnetic field values can be used to find the speed of the protons. The electric field is then found from the fact that the magnetic force must be the same magnitude as the electric force for the protons to have straight paths.

$$qvB = mv^2/r \quad\rightarrow\quad v = qBr/m \qquad F_{\text{E}} = F_{\text{B}} \quad\rightarrow\quad qE = qvB \quad\rightarrow$$

$$E = vB = qB^2r/m = \frac{(1.60\times10^{-19}\,\text{C})(0.566\,\text{T})^2(5.10\times10^{-2}\,\text{cm})}{1.67\times10^{-27}\,\text{kg}} = \boxed{1.57\times10^{6}\,\text{V/m}}$$

The direction of the electric field must be perpendicular to both the velocity and the magnetic field, and must be in the opposite direction to the magnetic force on the protons.

61. The magnetic force on the ions causes them to move in a circular path, so the magnetic force is a centripetal force. This results in the ion mass being proportional to the path's radius of curvature.

$$qvB = mv^2/r \quad\rightarrow\quad m = qBr/v \quad\rightarrow\quad m/r = qB/v = \text{constant} = 76\ \ \text{u}/22.8\ \ \text{cm}$$

$$\frac{m_{21.0}}{21.0\ \ \text{cm}} = \frac{76\ \ \text{u}}{22.8\ \ \text{cm}} \rightarrow m_{21.0} = 70\ \ \text{u} \qquad \frac{m_{21.6}}{21.6\ \ \text{cm}} = \frac{76\ \ \text{u}}{22.8\ \ \text{cm}} \rightarrow m_{21.6} = 72\ \ \text{u}$$

$$\frac{m_{21.9}}{21.9\ \ \text{cm}} = \frac{76\ \ \text{u}}{22.8\ \ \text{cm}} \rightarrow m_{21.9} = 73\ \ \text{u} \qquad \frac{m_{22.2}}{22.2\ \ \text{cm}} = \frac{76\ \ \text{u}}{22.8\ \ \text{cm}} \rightarrow m_{22.2} = 74\ \ \text{u}$$

The other masses are 70 u, 72 u, 73 u, and 74 u.

62. The location of each line on the film is twice the radius of curvature of the ion. The radius of curvature can be found from the expression given in section 20-11.

$$m=\frac{qBB'r}{E}\quad\rightarrow\quad r=\frac{mE}{qBB'}\quad\rightarrow\quad 2r=\frac{2mE}{qBB'}$$

$$2r_{12}=\frac{2\left(12\right)\left(1.67\times10^{-27}\,\text{kg}\right)\left(2.48\times10^{4}\,\text{V/m}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.68\,\text{T}\right)^{2}}=1.344\times10^{-2}\,\text{m}$$

$$2r_{13}=1.455\times10^{-2}\,\text{m}\qquad 2r_{14}=1.567\times10^{-2}\,\text{m}$$

The distances between the lines are

$$2r_{13}-2r_{12}=1.455\times10^{-2}\,\text{m}-1.344\times10^{-2}\,\text{m}=1.11\times10^{-3}\,\text{m}\approx\boxed{1.1\times10^{-3}\,\text{m}}$$

$$2r_{14}-2r_{13}=1.567\times10^{-2}\,\text{m}-1.455\times10^{-2}\,\text{m}=1.12\times10^{-3}\,\text{m}\approx\boxed{1.1\times10^{-3}\,\text{m}}$$

If the ions are doubly charged, the value of q in the denominator of the expression would double, and so the actual distances on the film would be halved. Thus the distances between the lines would also be halved.

$$2r_{13}-2r_{12}=7.277\times10^{-3}\,\text{m}-6.718\times10^{-3}\,\text{m}=5.59\times10^{-4}\,\text{m}\approx\boxed{5.6\times10^{-4}\,\text{m}}$$

$$2r_{14}-2r_{13}=7.837\times10^{-3}\,\text{m}-7.277\times10^{-3}\,\text{m}=5.60\times10^{-4}\,\text{m}\approx\boxed{5.6\times10^{-4}\,\text{m}}$$

63. The particles in the mass spectrometer follow a semicircular path as shown in Fig. 20-39. A particle has a displacement of $2r$ from the point of entering the semicircular region to where it strikes the film. So if the separation of the two molecules on the film is 0.50 mm, the difference in radii of the two molecules is 0.25 mm. The mass to radius ratio is the same for the two molecules.

$$qvB=mv^{2}/r\quad\rightarrow\quad m=qBr/v\quad\rightarrow\quad m/r=\text{constant}$$

$$\left(\frac{m}{r}\right)_{\text{CO}}=\left(\frac{m}{r}\right)_{\text{N}_2}\quad\rightarrow\quad\frac{28.0106\ \ \text{u}}{r}=\frac{28.0134\ \ \text{u}}{r+2.5\times10^{-4}\,\text{m}}\quad\rightarrow\quad r=2.5\ \ \ \text{m}$$

64. The velocity of the ions is found using energy conservation. The electrical potential energy of the ions becomes kinetic energy as they are accelerated. Then, since the ions move perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ions to move in a circular path.

$$qvB=\frac{mv^{2}}{R}\quad\rightarrow\quad v=\frac{qBR}{m}\qquad qV=\tfrac{1}{2}mv^{2}=\tfrac{1}{2}m\left(\frac{qBR}{m}\right)^{2}=\frac{q^{2}B^{2}R^{2}}{2m}\quad\rightarrow\quad m=\frac{qR^{2}B^{2}}{2V}$$

65. The field inside the solenoid is given by Eq. 20-8 with μ_0 replaced by the permeability of the iron.

$$B=\frac{\mu NI}{l}=\frac{3000\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(430\right)\left(25\,\text{A}\right)}{\left(1.0\,\text{m}\right)}=\boxed{41\text{T}}$$

66. The field inside the solenoid is given by Eq. 20-8 with μ_0 replaced by the permeability of the iron.

$$B=\frac{\mu NI}{l}\quad\rightarrow\quad\mu=\frac{Bl}{NI}=\frac{\left(2.2\,\text{T}\right)\left(0.38\,\text{m}\right)}{\left(640\right)\left(48\,\text{A}\right)}=\boxed{2.7\times10^{-5}\,\text{T}\bullet\text{m/A}}\approx22\mu_0$$

67. The magnetic force produces centripetal acceleration.

$$qvB = mv^2/r \quad \rightarrow \quad mv = p = qBr \quad \rightarrow \quad B = \frac{p}{qr} = \frac{4.8\times10^{-16}\ \text{kg}\bullet\text{m/s}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.0\times10^{3}\,\text{m}\right)} = \boxed{3.0\,\text{T}}$$

The magnetic field must point upward to cause an inward-pointing (centripetal) force that steers the protons clockwise.

68. The kinetic energy is used to determine the speed of the particles, and then the speed can be used to determine the radius of the circular path, since the magnetic force is causing centripetal acceleration.

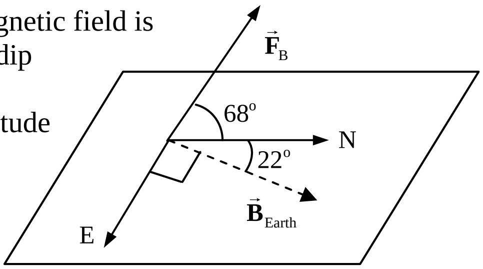
$$\text{KE} = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2\,\text{KE}}{m}} \qquad qvB = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2\,\text{KE}}{m}}}{qB} = \frac{\sqrt{2m\,\text{KE}}}{qB}$$

$$\frac{r_\text{p}}{r_\text{e}} = \frac{\dfrac{\sqrt{2m_p\,\text{KE}}}{qB}}{\dfrac{\sqrt{2m_e\,\text{KE}}}{qB}} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\text{kg}}} = \boxed{42.8}$$

69. We assume that the horizontal component of the Earth's magnetic field is pointing due north. The Earth's magnetic field also has the dip angle of 22°. The angle between the magnetic field and the eastward current is 90°. Use Eq. 20-1 to calculate the magnitude of the force.

$$F = IlB\sin\theta = \left(330\,\text{A}\right)\left(15\,\text{m}\right)\left(5.0\times10^{-5}\,\text{T}\right)\sin 90^\circ$$

$$= \boxed{0.25\,\text{N}}$$



Using the right hand rule with the eastward current and the Earth's magnetic field, the force on the wire is $\boxed{\text{northerly and 68° above the horizontal}}$.

70. The airplane is a charge moving in a magnetic field. Since it is flying perpendicular to the magnetic field, Eq. 20-4 applies.

$$F_\text{max} = qvB = \left(1550\times10^{-6}\,\text{C}\right)\left(120\,\text{m/s}\right)\left(5.0\times10^{-5}\,\text{T}\right) = \boxed{9.3\times10^{-6}\,\text{N}}$$

71. The magnetic force must be equal in magnitude to the weight of the electron.

$$mg = qvB \quad \rightarrow \quad v = \frac{mg}{qB} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(0.50\times10^{-4}\text{T}\right)} = \boxed{1.1\times10^{-6}\ \text{m/s}}$$

The magnetic force must point upwards, and so by the right hand rule and the negative charge of the electron, the electron must be moving $\boxed{\text{west}}$.

$\boxed{72.}$ (*a*) The velocity of the ions is found using energy conservation. The electrical potential energy of the ions becomes kinetic energy as they are accelerated. Then, assuming the ions move perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path.

$$\text{PE}_\text{initial} = \text{KE}_\text{final} \quad \rightarrow \quad qV = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2qV}{m}} \qquad qvB = \frac{mv^2}{r} \quad \rightarrow$$

$$r=\frac{mv}{qB}=\frac{m\sqrt{\frac{2qV}{m}}}{qB}=\sqrt{\frac{2mV}{qB^2}}=\sqrt{\frac{2\left(6.6\times10^{-27}\,\text{kg}\right)\left(2400\,\text{V}\right)}{2\left(1.60\times10^{-191}\,\text{C}\right)\left(0.240\,\text{T}\right)^2}}=4.146\times10^{-2}\,\text{m}\approx\boxed{4.1\times10^{-2}\,\text{m}}$$

(*b*) The period is the circumference of the circular path divided by the speed.

$$T=\frac{2\pi r}{v}=\frac{2\pi\sqrt{\frac{2mV}{qB^2}}}{\sqrt{\frac{2qV}{m}}}=\frac{2\pi m}{qB}=\frac{2\pi\left(6.6\times10^{-27}\,\text{kg}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(0.240\,\text{T}\right)}=\boxed{5.5\times10^{-7}\,\text{s}}$$

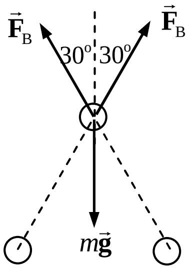
73. The accelerating force on the bar is due to the magnetic force on the current. If the current is constant, the magnetic force will be constant, and so constant acceleration kinematics can be used.

$$v^2=v_0^2+2a\Delta x\ \rightarrow\ a=\frac{v^2-0}{2\Delta x}=\frac{v^2}{2\Delta x}$$

$$F_{\text{net}}=ma=ILB\ \rightarrow\ I=\frac{ma}{LB}=\frac{m\left(\frac{v^2}{2\Delta x}\right)}{LB}=\frac{mv^2}{2\Delta xLB}=\frac{\left(1.5\times10^{-3}\,\text{kg}\right)\left(28\,\text{m/s}\right)^2}{2\left(1.0\,\text{m}\right)\left(0.22\,\text{m}\right)\left(1.7\,\text{T}\right)}=\boxed{1.6\,\text{A}}$$

Using the right hand rule, for the force on the bar to be in the direction of the acceleration shown in Fig. 20-65, the magnetic field must be $\boxed{\text{down}}$.

74. Each of the bottom wires will repel the top wire since each bottom current is opposite to the top current. The repelling forces will be along the line directly from the bottom wires to the top wires. Only the vertical components of those forces will be counteracting gravity. Use Eq. 20-7 to calculate the magnetic forces. The mass of the wire is its density times its volume. The length of the wire is represented by *L*.



$$mg=2F_{\text{B}}\cos30^{\circ}\ \rightarrow\ \rho_{\text{Cu}}L\pi\left(\tfrac{1}{2}d_{\text{Cu}}\right)^2g=2\frac{\mu_0}{2\pi}\frac{I_{\text{bottom}}I_{\text{Cu}}}{d}L\cos30^{\circ}\ \rightarrow$$

$$I_{\text{Cu}}=\frac{\rho_{\text{Cu}}\pi^2\left(\frac{1}{2}d_{\text{Cu}}\right)^2dg}{\mu_0I_{\text{bottom}}}=\frac{\left(8900\,\text{kg/m}^3\right)\pi^2\left(5.0\times10^{-4}\,\text{m}\right)^2\left(0.038\text{m}\right)\left(9.80\,\text{m/s}^2\right)}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(95\,\text{A}\right)\left(\cos30^{\circ}\right)}=\boxed{79\,\text{A}}$$

75. (*a*) There will be one force on the rod, due to the magnetic force on the charge carriers in the rod. That force is of magnitude $F_{\text{B}}=IlB$, and by Newton's 2nd law is equal to the mass of the rod times its acceleration. That force is constant, so the acceleration will be constant, and constant acceleration kinematics can be used.

$$F_{\text{net}}=F_{\text{B}}=IlB=ma\ \rightarrow\ a=\frac{IlB}{m}=\frac{v-v_0}{t}=\frac{v}{t}\ \rightarrow\ v=\boxed{\frac{ILB}{m}t}$$

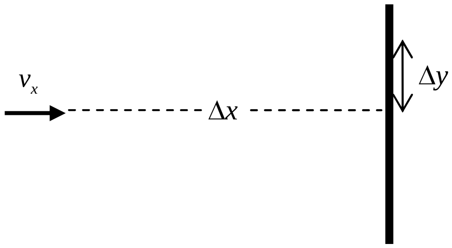
(*b*) Now the net force is the vector sum of the magnetic force and the force of kinetic friction.

$$F_{\text{net}}=F_{\text{B}}-F_{\text{fr}}=IlB-\mu_{\text{k}}F_{\text{N}}=IlB-\mu_{\text{k}}mg=ma\ \rightarrow$$

$$a=\frac{IlB}{m}-\mu_{\text{k}}g=\frac{v-v_0}{t}=\frac{v}{t}\ \rightarrow\ v=\boxed{\left(\frac{ILB}{m}-\mu_{\text{k}}g\right)t}$$

(*c*) Using the right hand rule, we find that the force on the rod is to the east, and the rod moves $\boxed{\text{east}}$.

76. The speed of the electrons is found by assuming the energy supplied by the accelerating voltage becomes kinetic energy of the electrons. We assume that those electrons are initially directed horizontally, and that the television set is oriented so that the electron velocity is perpendicular to the Earth's magnetic field, resulting in the largest possible force. Finally, we assume that the magnetic force on the electrons is small enough that the electron velocity is essentially perpendicular to the Earth's field for the entire trajectory. This results in a constant acceleration for the electrons.



(*a*) Acceleration:

$$\mathrm{PE}_{\text{initial}} = \mathrm{KE}_{\text{final}} \quad \rightarrow \quad eV = \tfrac{1}{2}mv_x^2 \quad \rightarrow \quad v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

$$\text{time in field: } \Delta x_{\text{field}} = v_x t_{\text{field}} \quad \rightarrow \quad t_{\text{field}} = \frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y = qv_x B_{\text{Earth}} = ma_y \quad \rightarrow \quad a_y = \frac{qv_x B_{\text{Earth}}}{m} = \frac{e\sqrt{\frac{2eV}{m}}B_{\text{Earth}}}{m} = \sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}$$

$$\Delta y = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}\sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \tfrac{1}{2}\sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}\left(\Delta x\right)^2\frac{m}{2eV}$$

$$= \sqrt{\frac{e}{8mV}}B_{\text{Earth}}\left(\Delta x\right)^2 = \sqrt{\frac{1.60\times10^{-19}\,\mathrm{C}}{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(2000\,\mathrm{V}\right)}}\left(0.5\times10^{-4}\,\mathrm{T}\right)\left(0.22\,\mathrm{m}\right)^2$$

$$= 8.02\times10^{-3}\,\mathrm{m} \approx \boxed{8\,\mathrm{mm}}$$

(*b*) $$\Delta y = \sqrt{\frac{e}{8mV}}B_{\text{Earth}}\left(\Delta x\right)^2 = \sqrt{\frac{1.60\times10^{-19}\,\mathrm{C}}{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(30{,}000\,\mathrm{V}\right)}}\left(0.5\times10^{-4}\,\mathrm{T}\right)\left(0.22\,\mathrm{m}\right)^2$$

$$= 2.07\times10^{-3}\,\mathrm{m} \approx \boxed{2\,\mathrm{mm}}$$

Note that the deflection is significantly smaller than the horizontal distance traveled, and so the assumptions made above are verified.

77. (*a*) The frequency of the voltage must match the frequency of circular motion of the particles, so that the electric field is synchronized with the circular motion. The radius of each circular orbit is given in Example 20-5 as $r = \frac{mv}{qB}$. For an object moving in circular motion, the period is given by $T = \frac{2\pi r}{v}$, and the frequency is the reciprocal of the period.

$$T = \frac{2\pi r}{v} \quad \rightarrow \quad f = \frac{v}{2\pi r} = \frac{v}{2\pi\frac{mv}{qB}} = \boxed{\frac{Bq}{2\pi m}}$$

In particular we note that this frequency is independent of the radius, and so the same frequency can be used throughout the acceleration.

(*b*) For a small gap, the electric field across the gap will be approximately constant and uniform as the particles cross the gap. If the motion and the voltage are synchronized so that the maximum voltage occurs when the particles are at the gap, the particles receive an energy increase of $\text{KE} = qV_0$ as they pass each gap. The energy gain from one revolution will include the passing of 2 gaps, so the total KE increase is $\boxed{2qV_0}$.

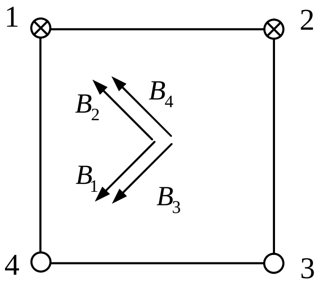
(*c*) The maximum kinetic energy will occur at the outside of the cyclotron.

$$v = \frac{rqB}{m}$$

$$\text{KE}_{\text{max}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m\left(\frac{r_{\text{max}}qB}{m}\right)^2 = \tfrac{1}{2}\frac{r_{\text{max}}^2 q^2 B^2}{m} = \tfrac{1}{2}\frac{(2.0\,\text{m})^2\left(1.60\times10^{-19}\,\text{C}\right)^2(0.50\,\text{T})^2}{1.67\times10^{-27}\,\text{kg}}$$

$$= 7.66\times10^{-12}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{MeV}}{10^6\,\text{eV}}\right) = \boxed{48\,\text{MeV}}$$

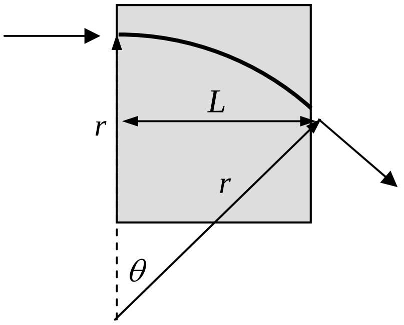
78. Each individual wire would cause a magnetic field given by Eq. 20-6 with the appropriate distance. The four individual magnetic fields are shown in the diagram. Note that all four magnitudes are the same. Also $\vec{\mathbf{B}}_1 = \vec{\mathbf{B}}_3$ and $\vec{\mathbf{B}}_2 = \vec{\mathbf{B}}_4$, but in the diagram they are shown as slightly displaced from each other. The vertical components all cancel, and so the net magnetic field is 4 times the horizontal component of one of the individual fields.



$$B_1 = B_2 = B_3 = B_4 = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{\mu_0}{2\pi}\frac{I}{l/\sqrt{2}}$$

$$B_{\text{net}} = 4\left(\frac{\mu_0}{2\pi}\frac{I}{l/\sqrt{2}}\right)\cos 45^\circ = 4\left(\frac{\mu_0}{2\pi}\frac{I}{l/\sqrt{2}}\right)\frac{\sqrt{2}}{2} = \boxed{\frac{2\mu_0 I}{\pi l},\ \text{left}}$$

79. The protons will follow a circular path as they move through the region of magnetic field, with a radius of curvature given in Example 20-5 as $r = \dfrac{mv}{qB}$. Fast-moving protons will have a radius of curvature that is too large and so they will exit above the second tube. Likewise, slow-moving protons will have a radius of curvature that is too small and so they will exit below the second tube. Since the exit velocity is perpendicular to the radius line from the center of curvature, the bending angle can be calculated.



$$\sin\theta = \frac{L}{r} \quad \rightarrow$$

$$\theta = \sin^{-1}\frac{L}{r} = \sin^{-1}\frac{LqB}{mv} = \sin^{-1}\frac{\left(5.0\times10^{-2}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{C}\right)(0.33\,\text{T})}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.0\times10^7\,\text{m/s}\right)} = \sin^{-1}0.158 = \boxed{9.1^\circ}$$

80. (*a*) The electromagnet must dissipate 4.0 kW of power if the power supply is to supply that power. Use that power with the voltage across the electromagnet to find the resistance of the electromagnet. Then calculate how much copper wire is needed to provide that resistance. The copper wire is a square of side length *d*, and the copper wire has length *L*.

$$P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P} \qquad R=\rho_{\text{Cu}}\frac{L}{A}=\rho_{\text{Cu}}\frac{L}{d^2} \qquad N=\frac{L}{2\pi r}$$

$$N=\frac{L}{2\pi r}=\frac{\frac{Rd^2}{\rho_{\text{Cu}}}}{2\pi r}=\frac{Rd^2}{2\pi r\rho_{\text{Cu}}}=\frac{V^2d^2}{2\pi r\rho_{\text{Cu}}P}=\frac{(120\,\text{V})^2\left(1.6\times10^{-3}\,\text{m}\right)^2}{2\pi(0.6\,\text{m})\left(1.68\times10^{-7}\,\Omega\bullet\text{m}\right)\left(4.0\times10^3\,\text{W}\right)}$$

$$=\boxed{14.6\text{ turns}}$$

(*b*) To calculate the magnetic field at the center of the coil, the current must be known, and it can be found from the power and the voltage since $P=IV$.

$$B=\frac{\mu_0NI}{2r}=\frac{\mu_0NP}{2rV}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(14.6)\left(4.0\times10^3\,\text{W}\right)}{2(0.6\,\text{m})(120\,\text{V})}=\boxed{5.1\times10^{-4}\,\text{T}}$$

(*c*) It would appear from $B=\frac{\mu_0NI}{2r}$ that an increase in *N* would increase *B*, but the increase in *N* must arise from a longer wire, which has more resistance, and so the current is lower. We need to express *B* in terms of the number of turns alone.

$$B=\frac{\mu_0NI}{2r}=\frac{\mu_0N\frac{V}{R}}{2rV}=\frac{\mu_0N}{2rR}=\frac{\mu_0}{2rR}\frac{Rd^2}{2\pi r\rho_{\text{Cu}}}=\frac{\mu_0d^2}{4\pi r^2\rho_{\text{Cu}}}=\text{constant}$$

$\boxed{\text{The magnetic field will remain constant.}}$

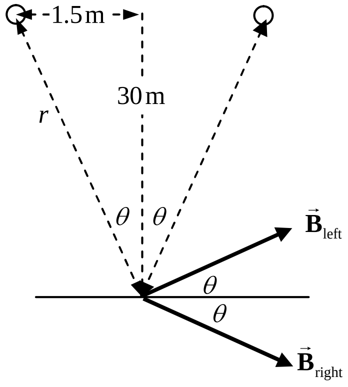
81. From problem 80, with one turn of wire, we have $B=\frac{\mu_0I}{2r}$. Use the radius of the Earth for *r*.

$$B=\frac{\mu_0I}{2r} \rightarrow I=\frac{2rB}{\mu_0}=\frac{2\left(6.38\times10^6\,\text{m}\right)\left(1\times10^{-4}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}=\boxed{1\times10^9\,\text{A}}$$

82. From Eq. 18-9(a) we calculate the rms current, given the power and the rms voltage, assuming that each wire provides half the power.

$$P=V_{\text{rms}}I_{\text{rms}} \rightarrow I_{\text{rms}}=\frac{P}{V_{\text{rms}}} \rightarrow$$

$$I_{\text{max}}=\frac{\sqrt{2}P}{V_{\text{rms}}}=\frac{\sqrt{2}\left(20\times10^6\,\text{W}\right)}{10\times10^3\,\text{V}}=2830\,\text{A}$$


1.5 m
30 m
r
θ
θ
$\vec{\mathbf{B}}_{\text{left}}$
θ
θ
$\vec{\mathbf{B}}_{\text{right}}$

From the geometry of the problem, we can calculate the magnetic field that is expected. Note that the drawing is NOT to scale.

$$r=\sqrt{(30\,\text{m})^2+(1.5\,\text{m})^2}=30.04\,\text{m} \qquad \theta=\tan^{-1}\frac{1.5}{30}=2.86^\circ$$

$$B_{\text{net}} = B_{\text{left}\,x} + B_{\text{right}\,x} = 2\frac{\mu_0}{2\pi}\frac{I}{r}\cos\theta$$

$$= 2\frac{\left(4\pi\times10^{-7}\ \text{T•m/A}\right)}{2\pi}\frac{2830\,\text{A}}{30.04\,\text{m}}\cos 2.86^{\circ} \approx \boxed{4\times10^{-5}\,\text{T}}$$

This is $\boxed{\text{comparable in magnitude}}$ to the Earth's field.

83. (*a*) For the beam of electrons to be undeflected, the magnitude of the magnetic force must equal the magnitude of the electric force. We assume that the magnetic field will be perpendicular to the velocity of the electrons so that the maximum magnetic force is obtained.

$$F_{\text{B}} = F_{\text{E}} \;\rightarrow\; qvB = qE \;\rightarrow\; B = \frac{E}{v} = \frac{10{,}000\,\text{V/m}}{4.8\times10^{6}\,\text{m/s}} = 2.083\times10^{-3}\,\text{T} \simeq \boxed{2.1\times10^{-3}\,\text{T}}$$

(*b*) Since the electric field is pointing up, the electric force is down. Thus the magnetic force must be up. Using the right hand rule with the negative electrons, the magnetic field must be $\boxed{\text{out}}$ of the plane of the plane formed by the electron velocity and the electric field.

(*c*) If the electric field is turned off, then the magnetic field will cause a centripetal force, moving the electrons in a circular path. The frequency is the reciprocal of the period of the motion.

$$qvB = \frac{mv^2}{r} \;\rightarrow\; v = \frac{qBr}{m}$$

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\frac{qBr}{m}}{2\pi r} = \frac{qB}{2\pi m} = \frac{qE}{2\pi mv} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(10{,}000\,\text{V/m}\right)}{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)\left(4.8\times10^{6}\,\text{m/s}\right)} = \boxed{5.8\times10^{7}\,\text{Hz}}$$

84. The speed of the proton can be calculated based on the radius of curvature of the (almost) circular motion. From that the kinetic energy can be calculated.

$$qvB = \frac{mv^2}{r} \;\rightarrow\; v = \frac{qBr}{m} \qquad \text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m}$$

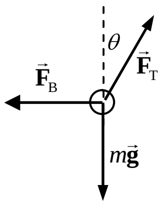
$$\Delta\text{KE} = \frac{q^2B^2}{2m}\left(r_2^2 - r_1^2\right) = \frac{\left(1.60\times10^{-19}\,\text{C}\right)^2\left(0.010\,\text{T}\right)^2}{2\left(1.67\times10^{-27}\,\text{kg}\right)}\left[\left(8.5\times10^{-3}\,\text{m}\right)^2 - \left(10.0\times10^{-3}\,\text{m}\right)^2\right]$$

$$= \boxed{-2.1\times10^{-20}\,\text{J}} \text{ or } -0.13\,\text{eV}$$

85. The field inside a solenoid is given by Eq. 20-8.

$$B = \mu_0 IN/l \;\rightarrow\; N = \frac{Bl}{\mu_0 I} = \frac{\left(0.30\,\text{T}\right)\left(0.32\,\text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T•m/A}\right)\left(5.7\,\text{A}\right)} = \boxed{1.3\times10^{4}\,\text{turns}}$$

86. There are three forces on each wire – its weight, the magnetic force of repulsion from the other wire, and the tension in the attached string. See the diagram. The magnetic force is given by Eq. 20-7. The mass of the wire is its density times its volume. The length of the wire is represented by *L*. The net force in both the vertical and horizontal directions is zero.



$$F_{\text{B}} = F_{\text{T}}\sin\theta \qquad F_{\text{T}}\cos\theta = mg \;\rightarrow\; F_{\text{T}} = \frac{mg}{\cos\theta} \;\rightarrow$$

$$F_\text{B} = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = \rho_\text{Al}\pi r_\text{Al}^2 Lg\tan\theta$$

$$F_\text{B} = \frac{\mu_0}{2\pi}\frac{I^2}{S}L \quad\rightarrow\quad \frac{\mu_0}{2\pi}\frac{I^2}{S}L = \rho_\text{Al}\pi r_\text{Al}^2 Lg\tan\theta \quad\rightarrow$$

$$I = \sqrt{\frac{2\pi S\rho_\text{Al}\pi r_\text{Al}^2 g\tan\theta}{\mu_0}} = \sqrt{\frac{2\pi\left[2\left(0.50\,\text{m}\right)\sin 3^\circ\right]\left(2700\,\text{kg/m}^3\right)\pi\left(2.5\times10^{-4}\,\text{m}\right)^2\left(9.8\,\text{m/s}^2\right)\tan 3^\circ}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}}$$

$$= \boxed{8.4\,\text{A}}$$

87. The centripetal force is caused by the magnetic field, and is given by Eq. 20-3.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \quad\rightarrow$$

$$r = \frac{mv_\perp}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.0\times10^6\,\text{m/s}\right)\sin 45^\circ}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.23\,\text{T}\right)} = 5.251\times10^{-5}\,\text{m} \approx \boxed{5.3\times10^{-5}\,\text{m}}$$

The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi\frac{mv_\perp}{qB}}{v_\perp} = \frac{2\pi m}{qB}$$

$$p = v_\parallel T = v\cos 45^\circ\left(\frac{2\pi m}{qB}\right) = \left(3.0\times10^6\,\text{m/s}\right)\cos 45^\circ\,\frac{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.23\,\text{T}\right)} = \boxed{3.3\times10^{-4}\,\text{m}}$$

CHAPTER 21: Electromagnetic Induction and Faraday's Law

Answers to Questions

1. The advantage of using many turns (N = large number) in Faraday's experiments is that the emf and induced current are proportional to N, which makes it easier to experimentally measure those quantities.

2. Magnetic flux is proportional to the total number of magnetic field lines passing through an enclosed loop area: $\Phi_B = BA\cos\theta$, and so the flux is proportional to the magnitude of the magnetic field. Thus, they also have different units (magnetic field = Tesla = T; magnetic flux = Tm^2 = Wb). Another difference is that magnetic field is a vector (size and direction), while magnetic flux is a scalar (size).

3. Yes, a current is induced in the ring when you bring the south pole toward the ring. An emf and current are induced in the ring due to the changing magnetic flux (as the magnet gets closer to the ring, more magnetic field lines are going through the ring). No, a current is not induced in the ring when the magnet is held steady within the ring. An emf and current are not induced in the ring since the magnetic flux through the ring is not changing while the magnet is held steady. Yes, a current is induced in the ring when you withdraw the magnet. An emf and current are induced in the ring due to the changing magnetic flux (as you pull the magnet out of the ring toward you, fewer magnetic field lines are going through the ring). Using Lenz's law and the Right Hand Rule, the direction of the induced current when you bring the south pole toward the ring is clockwise. In this case, the number of magnetic field lines coming through the loop and pointing toward you is increasing (remember, magnetic field lines point toward the south pole of the magnet). The induced current in the loop is going to try to oppose this change in flux and will attempt to create magnetic field lines through the loop that point away from you. A clockwise induced current will provide this opposing magnetic field. Using Lenz's law and the Right Hand Rule again, the direction of the induced current when you withdraw the south pole from the ring is counterclockwise. In this case, the number of magnetic field lines coming through the loop and pointing toward you is decreasing. The induced current in the loop is going to try to oppose this change in flux and will attempt to create more magnetic field lines through the loop that point toward you. A counterclockwise induced current will provide this opposing magnetic field.

4. For the ring on the left side of the current-carrying wire there is no induced current. As the ring moves along parallel to the wire, the magnetic flux through the ring does not change, which means there is no induced emf and no induced current. For the ring on the right side of the current-carrying wire, the induced current is clockwise. As the ring moves away from the wire, the magnetic flux through the ring is decreasing (there are fewer magnetic field lines pointing into the loop). In an attempt to oppose this decrease (due to Lenz's law), an emf and current will be induced in the ring in a clockwise direction (using the Right Hand Rule).

5. (*a*) Yes. As the battery is connected to the front loop and current starts to flow, it will create an increasing magnetic field that points away from you and down through the two loops. The second loop will try to oppose this increase in magnetic flux through it and an emf and current will be induced.
   (*b*) The induced current in the second loop starts to flow as soon as the current in the front loop starts to increase and create a magnetic field (basically, immediately upon the connection of the battery to the front loop).

(*c*) The current in the second loop stops flowing as soon as the current in the front loop becomes steady. Once the battery has increased the current in the front loop from zero to its steady-state value, then the magnetic field it creates is also steady. Since the magnetic flux through the second loop is no longer changing, the induced current goes to zero.

(*d*) The induced current in the second loop is counterclockwise. Since the increasing clockwise current in the front loop is causing an increase in the number of magnetic field lines down through the second loop, Lenz's law states that the second loop will attempt to oppose this change in flux. To oppose this change, the Right Hand Rule says that a counterclockwise current will be induced in the second loop.

(*e*) Yes. Since both loops carry currents and create magnetic fields while the current in the front loop is increasing from the battery, these two magnetic fields will interact and put a force on each of the loops.

(*f*) The force between the two loops will repel each other. The front loop is creating a magnetic field pointed toward the second loop. This changing magnetic field induces a current in the second loop to oppose the increasing magnetic field and this induced current creates a magnetic field pointing toward the front loop. These two magnetic fields will act like two north poles pointing at each other and repel.

6. (*a*) The induced current in R_A is to the right as coil B is moved toward coil A. As B approaches A, the magnetic flux through coil A increases (there are now more magnetic field lines in coil A pointing to the left). Coil A attempts to oppose this increase in flux, and the induced emf creates a current to produce a magnetic field pointing to the right through the center of the coil. A current through R_A to the right will produce this opposing field.

(*b*) The induced current in R_A is to the left as coil B is moved away from coil A. As B recedes from A, the magnetic flux through coil A decreases (there are now fewer magnetic field lines in coil A pointing to the left). Coil A attempts to oppose this decrease in flux, and the induced emf creates a current to produce a magnetic field pointing to the left through the center of the coil. A current through R_A to the left will produce this opposing field.

(*c*) The induced current in R_A is to the left as R_B in coil B is increased. As R_B increases, the current in coil B decreases, which also decreases the magnetic field coil B produces. As the magnetic field from coil B decreases, the magnetic flux through coil A decreases (there are now fewer magnetic field lines in coil A pointing to the left). Coil A attempts to oppose this decrease in flux, and the induced emf creates a current to produce a magnetic field pointing to the left through the center of the coil. A current through R_A to the left will produce this opposing field.

7. As the signal in the wire varies in time, it creates changing magnetic fields that emanate from the wire. If there were external magnetic fields nearby, these could interact with the signal wire's magnetic fields and cause interference or noise in the signal. With the "shield" in place around the signal wire carrying the return current, the net current in the wire, as seen by the outside world, would be zero and, thus, the wire would not emanate a magnetic field. In turn, the external magnetic fields would not have any signal field to interact with and the interference and noise in the signal would be reduced.

8. One advantage of placing the two insulated wires carrying ac close together is that whatever magnetic fields are created by the changing current moving one way in one wire is approximately cancelled out by the magnetic field created by the current moving in the opposite direction in the second wire. Also, since large loops of wire in a circuit can generate a large self-induced back emf, by placing the two wires close to each other, or even twisting them about each other, the effective area of the current loop is decreased and the induced current is minimized.

9. As the refrigerator motor first starts up, there is a small back emf in the circuit (back emf is proportional to the rotation speed of the motor), which allows for a large amount of current to flow to the refrigerator. This large current overtaxes the source and diminishes the current to all of the devices on the circuit, including the lights. As the refrigerator motor speeds up to its normal operational speed, the back emf increases to its normal level and the current delivered to the refrigerator is now limited to its usual amount. This current is no longer enough to overburden the source and all of the devices in the room go back to normal. Thus, it appears that the lights flicker just when the refrigerator motor first starts up. A heater, on the other hand, draws a large amount of current (it is a very low-resistance device) at all times. The source is then continually overtaxed and the other devices on the same circuit remain dimmed all the while that the heater is operating. In an ideal situation, the source could provide any amount of current to the whole circuit in either situation. In reality, though, the higher currents in the wires causes bigger losses of energy along the way to the devices and the lights dim.

10. Figure 21-17 shows that the induced current in the upper armature segment points into the page. This can be shown using a Right Hand Rule: The charges in the top metal armature segment are moving in the direction of the velocity shown with the green arrow (up and to the right) and these moving charges are in a magnetic field shown with the blue arrows (to the right), and then the Right Hand Rule says that the charges experience a force into the page producing the induced current. This induced current is also in the magnetic field. Using another Right Hand Rule, a current-carrying wire, with the current going into the page (as in the upper armature segment), in a magnetic field pointed to the right, will experience a force in a downward direction. Thus, there is a counter-clockwise torque on the armature while it is rotation in a clockwise direction. The back emf is opposing the motion of the armature during its operation.

11. Yes, eddy current brakes will work on metallic wheels, such as copper and aluminum. Eddy current brakes do not need to act on ferromagnetic wheels. The external magnetic field of the eddy brake just needs to interact with the "free" conduction electrons in the metal wheels to make it work. First, the magnetic field creates eddy currents in the moving metal wheel using the free conduction electrons (the Right Hand Rule says moving charges in a magnetic field will experience a magnetic force, making them move, and creating an eddy current). This eddy current is also in the braking magnetic field. The Right Hand Rule says these currents will experience a force opposing the original motion of the piece of metal and the eddy current brake will begin to slow the wheel. Good conductors, such as copper and aluminum, have many free conduction electrons and will allow large eddy currents to be created, which in turn will provide good braking results.

12. As the pieces of conducting materials slide down the incline past permanent magnets, the "free" conduction electrons in the pieces are forced to create eddy currents (the Right Hand Rule says moving charges in a magnetic field will experience a magnetic force, making them move, and creating an eddy current). These eddy currents are not created in the non-metallic pieces. These eddy currents are also moving through the magnetic fields of the permanent magnets and the Right Hand Rule says these currents will experience a magnetic force opposing the original motion of the pieces of metal. Thus, the metallic pieces will be slowed as they slide down the incline, while the non-metallic pieces will continue to accelerate the entire time. Since the metallic pieces have been slowed, they will basically just drop off of the end of the incline and you would put a "metal bin" directly below the end to catch the metal pieces. The non-metallic pieces, on the other hand, will come off the end of the incline with a high speed and overshoot the "metal bin" and be caught in a "non-metal bin" sitting farther away from the end.

13. The slots cut into the piece of pivoted metal bar confine the induced eddy currents to many very small loops, instead of just one large loop. These smaller current loops have less magnetic flux due to their smaller areas, which creates less emf and smaller eddy currents. The smaller eddy currents then experience a smaller opposing force to the motion of the metal bar. Thus, the slotted bar falls more quickly through the magnetic field.

14. As you try to move the aluminum sheet out of the magnetic field, areas of the sheet that used to be in B = 0 regions start to gain magnetic flux. This changing flux will induce currents in the "free" conduction electrons of the aluminum to oppose the change. These eddy currents are then acted on by the magnetic field and the resulting force opposes the motion of the aluminum sheet. Thus, it requires some amount of force to remove the sheet from between the poles.

$\boxed{15.}$ As a magnet falls through a metal tube, an increase in the magnetic flux is created in the areas ahead of it in the tube. This flux change induces a current to flow around the tube walls to create an opposing magnetic field in the tube (Lenz's law). This induced magnetic field pushes against the falling magnet and causes it to slow down. The opposing magnetic field cannot cause the magnet to actually come to a stop, since then the flux would become a constant and the induced current would disappear and so would the opposing magnetic field. Thus, the magnet reaches a state of equilibrium and falls at a constant terminal velocity.

16. As the metal bar enters (or leaves) the magnetic field during the swinging motion, areas of the metal bar experience a change in magnetic flux. This changing flux induces eddy currents with the "free" conduction electrons in the metal bar. These eddy currents are then acted on by the magnetic field and the resulting force opposes the motion of the swinging metal bar. This opposing force acts on the bar no matter which direction it is swinging through the magnetic field, thus damping the motion during both the to and fro portions of the swing.

17. To determine the ratio of turns on the two coils of a transformer without taking it apart, apply a known ac input source voltage to one pair of leads and carefully measure the output voltage across the other two leads. Then, $V_{\text{source}}/V_{\text{output}} = N_s/N_o$, which provides us with the ratio of turns on the two coils. To determine which leads are paired with which, you could use an ohmmeter, since the two source wires are in no way electrically connected to the two output wires. If the resistance between two wires is very small, then those two wires are a pair. If the resistance between two wires is infinite, then those two wires are not a pair.

18. Higher voltages, such as 600 V or 1200 V, would be dangerous if they were used in household wires! Such a large potential difference between household wires and anything that is grounded (other wires, people, etc.) would more easily cause electrical breakdown of the air and then much more sparking would occur. Basically, this would supply each of the charges in the household wires with much more energy than the lower voltages, which would allow them to arc to other conductors. This would increase the possibility of more short circuits and accidental electrocutions.

19. When 120 V dc is applied to the transformer, there is no induced back emf that would usually occur with 120 V ac. This means that the 120 V dc encounters much less resistance than the 120 V ac, which would cause too much current to flow through the transformer. This large amount of current would overheat the coils, which are usually wound with many loops of very fine wire, and could melt the insulation and burn out or short out the transformer.

20. (*a*) To create the largest amount of mutual inductance with two flat circular coils of wire, you would place them face-to-face and very close to each other. This way, almost all of the magnetic flux from one coil also goes through the other coil.

(*b*) To create the least amount of mutual inductance with two flat circular coils, you would place them with their faces at right angles. This way, almost none of the magnetic flux from one coil goes through the other coil.

21. (*a*) No. Although the current through an *LR* circuit is described by $I = \frac{\mathsf{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$, we can substitute $\frac{\mathsf{E}}{R} = I_{\max}$. Thus, a given fraction of a maximum possible current, $I/I_{\max}$, is equal to $1 - e^{-\frac{t}{L/R}}$ and is independent of the battery emf.

(*b*) Yes. Since $I = \frac{\mathsf{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$, if a given value of the current is desired, it is dependent on the value of the battery emf.

22. (*a*) Yes.

(*b*) Yes. The rms voltages across either an inductor or a capacitor of an *LRC* circuit can be greater than the rms source voltage because the different voltages are out of phase with each other. At any given instant, the voltage across either the inductor or the capacitor could be negative, for example, thus allowing for a very large positive voltage on the other device. (The rms voltages, however, are always positive by definition.)

23. (*a*) The frequency of the source emf does not affect the impedance of a pure resistance. The impedance of a pure resistance is independent of the source emf frequency.

(*b*) The impedance of a pure capacitance varies inversely with the frequency of the source emf according to $X_C = 1/2\pi fC$. As the source frequency gets very small, the impedance of the capacitor gets very large, and as the source frequency gets very large, the impedance of the capacitor gets very small.

(*c*) The impedance of a pure inductance varies directly with the frequency of the source emf according to $X_L = 2\pi fL$. Thus, as the source frequency gets very small, the impedance of the inductor gets very small and as the source frequency gets very large, the impedance of the inductor gets very large.

(*d*) The impedance of an *LRC* circuit, with a small *R*, near resonance is very sensitive to the frequency of the source emf. If the frequency is set at resonance exactly, where $X_L = X_C$, then the *LRC* circuit's impedance is very small and equal to *R*. The impedance increases rapidly as the source frequency is either increased or decreased a small amount from resonance.

(*e*) The impedance of an *LRC* circuit, with a small R, very far from resonance depends on whether the source frequency is much higher or much lower than the resonance frequency. If the source frequency is much higher than resonance, then the impedance is directly proportional to the frequency of the source emf. Basically, at extremely high frequencies, the circuit impedance is equal to $2\pi fL$. If the source frequency is much lower than resonance (nearly zero), then the impedance is inversely proportional to the frequency of the source emf. Basically, at extremely low frequencies, the circuit impedance is equal to $1/2\pi fC$.

$\boxed{24.}$ The way to make the impedance of an *LRC* circuit a minimum is to make the resistance very small and make the reactance of the capacitor equal to the reactance of the inductor: $X_L = X_C$, or

$$2\pi fL = \frac{1}{2\pi fC} \quad \rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}}.$$

Solutions to Problems

$\boxed{1.}$ The average induced emf is given by Eq. 21-2b.

$$\mathsf{E} = -N\frac{\Delta\Phi_B}{\Delta t} = -2\frac{38\,\text{Wb} - (-50\,\text{Wb})}{0.42\,\text{s}} = \boxed{-420\,\text{V}}$$

2. As the coil is pushed into the field, the magnetic flux through the coil increases into the page. To oppose this increase, the flux produced by the induced current must be out of the page, so the induced current is $\boxed{\text{counterclockwise}}$.

3. As the magnet is pushed into the coil, the magnetic flux increases to the right. To oppose this increase, flux produced by the induced current must be to the left, so the induced current in the resistor will be $\boxed{\text{from right to left}}$.

4. We assume the plane of the coil is perpendicular to the magnetic field. The magnitude of the average induced emf is given by Eq. 21-2a.

$$\mathsf{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.048\,\text{m})^2(0 - 1.10\,\text{T})}{0.15\,\text{s}} = \boxed{5.3\times10^{-2}\,\text{V}}$$

5. The flux changes because the loop rotates. The angle changes from 0° to 90°. The magnitude of the average induced emf is given by Eq. 21-2a.

$$\mathsf{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{AB\Delta\cos\theta}{\Delta t} = -\frac{\pi(0.060\,\text{m})^2\,1.5\,\text{T}\left(\cos 90^\circ - \cos 0^\circ\right)}{0.20\,\text{s}}$$

$$= -\frac{\pi(0.060\,\text{m})^2\,1.5\,\text{T}(0-1)}{0.20\,\text{s}} = \boxed{8.5\times10^{-2}\,\text{V}}$$

6. We choose up as the positive direction. The average induced emf is given by Eq. 21-2a.

$$\mathsf{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.051\,\text{m})^2(-0.25\,\text{T} - 0.63\,\text{T})}{0.15\,\text{s}} = \boxed{4.8\times10^{-2}\,\text{V}}$$

7. (*a*) When the plane of the loop is perpendicular to the field lines, the flux is given by the maximum of Eq. 21-1.

$$\Phi_B = BA = B\pi r^2 = (0.50\,\text{T})\pi(0.075\,\text{m})^2 = \boxed{8.8\times10^{-3}\,\text{Wb}}$$

(*b*) The angle is $\theta = \boxed{55^\circ}$

(*c*) Use Eq. 21-1.

$$\Phi_B = BA\cos\theta = B\pi r^2 = (0.50\,\text{T})\pi(0.075\,\text{m})^2\cos 55^\circ = \boxed{5.1\times10^{-3}\,\text{Wb}}$$

8. (*a*) As the resistance is increased, the current in the outer loop will decrease. Thus the flux through the inner loop, which is out of the page, will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux out of the page, so the direction of the induced current will be $\boxed{\text{counterclockwise}}$.

(*b*) If the small loop is placed to the left, the flux through the small loop will be into the page and will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

9. (*a*) The increasing current in the wire will cause an increasing field out of the page through the loop. To oppose this increase, the induced current in the loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

(*b*) The decreasing current in the wire will cause a decreasing field out of the page through the loop. To oppose this decrease, the induced current in the loop will produce a flux out of the page, so the direction of the induced current will be $\boxed{\text{counterclockwise}}$.

(*c*) The decreasing current in the wire will cause a decreasing field into the page through the loop. To oppose this decrease, the induced current in the loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

(*d*) Because the current is constant, there will be no change in flux, so the induced current will be $\boxed{\text{zero}}$.

10. As the solenoid is pulled away from the loop, the magnetic flux to the right through the loop decreases. To oppose this decrease, the flux produced by the induced current must be to the right, so the induced current is $\boxed{\text{counterclockwise}}$ as viewed from the right end of the solenoid.

11. (*a*) The average induced emf is given by Eq. 21-2a.

$$\mathsf{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi\left(0.060\,\text{m}\right)^2\left(-0.45\,\text{T}-0.52\,\text{T}\right)}{0.18\,\text{s}} = \boxed{6.1\times10^{-2}\,\text{V}}$$

(*b*) The positive result for the induced emf means the induced field is away from the observer, so the induced current is $\boxed{\text{clockwise}}$.

12. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 21-3.

$$\mathsf{E} = BLv = \left(0.800\,\text{T}\right)\left(0.120\,\text{m}\right)\left(0.150\,\text{m/s}\right) = \boxed{1.44\times10^{-2}\,\text{V}}$$

(*b*) Because the upward flux is increasing, the induced flux will be into the page, so the induced current is clockwise. Thus the induced emf in the rod is down, which means that the electric field will be down. The electric field is the induced voltage per unit length.

$$E = \frac{\mathsf{E}}{l} = \frac{1.44\times10^{-2}\,\text{V}}{0.120\,\text{m}} = \boxed{0.120\,\text{V/m, down}}$$

13. (*a*) The magnetic flux through the loop is into the paper and decreasing, because the area is decreasing. To oppose this decrease, the induced current in the loop will produce a flux into the paper, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

(*b*) The average induced emf is given by Eq. 21-2a.

$$\left|\mathsf{E}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B\left|\Delta A\right|}{\Delta t} = \frac{\left(0.75\,\text{T}\right)\pi\left[\left(0.100\,\text{m}\right)^2-\left(0.030\,\text{m}\right)^2\right]}{0.50\,\text{s}}$$

$$= 4.288\times10^{-2}\,\text{V} \approx \boxed{4.3\times10^{-2}\,\text{V}}$$

(*c*) We find the average induced current from Ohm's law.

$$I=\frac{V}{R}=\frac{4.288\times10^{-2}\,\mathrm{V}}{2.5\,\Omega}=\boxed{1.7\times10^{-2}\,\mathrm{A}}$$

$\boxed{14.}$ (*a*) The velocity is found from Eq. 21-3.

$$\mathcal{E}=Blv\ \rightarrow\ v=\frac{\mathcal{E}}{Bl}=\frac{0.12\,\mathrm{V}}{(0.90\,\mathrm{T})(0.132\,\mathrm{m})}=\boxed{1.0\,\mathrm{m/s}}$$

(*b*) Because the outward flux is increasing, the induced flux will be into the page, so the induced current is clockwise. Thus the induced emf in the rod is down, which means that the electric field will be down. The magnitude of the electric field is the induced voltage per unit length.

$$E=\frac{\mathcal{E}}{l}=\frac{0.12\,\mathrm{V}}{0.132\,\mathrm{m}}=\boxed{0.91\,\mathrm{V/m},\ \text{down}}$$

15. As the loop is pulled from the field, the flux through the loop decreases, causing an induced EMF whose magnitude is given by Eq. 21-3, $\mathcal{E}=Blv$. Because the inward flux is decreasing, the induced flux will be into the page, so the induced current is clockwise, given by $I=\mathcal{E}/R$. Because this current in the left-hand side of the loop is in a downward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by $F=IlB$.

$$F=IlB=\frac{\mathcal{E}}{R}lB=\frac{Blv}{R}lB=\frac{B^2l^2v}{R}=\frac{(0.550\,\mathrm{T})^2(0.350\,\mathrm{m})^2(3.40\,\mathrm{m/s})}{0.230\,\Omega}=\boxed{0.548\,\mathrm{N}}$$

16. The emf induced in the short coil is given by Eq. 21-2b, where N is the number of loops in the short coil, and the flux change is measured over the area of the short coil. The magnetic flux comes from the B-field created by the solenoid. The field in a solenoid is given by Eq. 20-8, $B=\mu_0 IN_{\text{solenoid}}/l_{\text{solenoid}}$, and the changing current in the solenoid causes the field to change.

$$|\mathcal{E}|=\frac{N_{\text{short}}A_{\text{short}}\Delta B}{\Delta t}=\frac{N_{\text{short}}A_{\text{short}}\Delta\left(\frac{\mu_0 IN_{\text{solenoid}}}{l_{\text{solenoid}}}\right)}{\Delta t}=\frac{\mu_0 N_{\text{short}}N_{\text{solenoid}}A_{\text{short}}}{l_{\text{solenoid}}}\frac{\Delta I}{\Delta t}$$

$$=\frac{(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A})(10)(500)\pi(0.0125\,\mathrm{m})^2}{(0.25\,\mathrm{m})}\frac{(5.0\,\mathrm{A})}{(0.60\,\mathrm{s})}=\boxed{1.0\times10^{-4}\,\mathrm{V}}$$

17. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 21-3.

$$\mathcal{E}=Blv=(0.35\,\mathrm{T})(0.300\,\mathrm{m})(1.6\,\mathrm{m/s})=0.168\,\mathrm{V}\approx\boxed{0.17\,\mathrm{V}}$$

(*b*) Find the induced current from Ohm's law.

$$I=\frac{\mathcal{E}}{R}=\frac{0.168\,\mathrm{V}}{27.5\,\Omega}=6.11\times10^{-3}\,\mathrm{A}\approx\boxed{6.1\times10^{-3}\,\mathrm{A}}$$

(*c*) The induced current in the rod will be down. Because this current is in an upward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by Eq. 20-2.

$$F=IlB=(6.11\times10^{-3}\,\mathrm{A})(0.300\,\mathrm{m})(0.35\,\mathrm{T})\approx\boxed{6.4\times10^{-4}\,\mathrm{N}}$$

18. (*a*) There is an emf induced in the coil since the flux through the coil changes. The current in the coil is the induced emf divided by the resistance of the coil. The resistance of the coil is found from Eq. 18-3.

$$\left|\mathcal{E}\right| = \frac{NA_{\text{coil}}\Delta B}{\Delta t} \qquad R = \frac{\rho L}{A_{\text{wire}}}$$

$$I = \frac{\mathcal{E}}{R} = \frac{\dfrac{NA_{\text{coil}}\Delta B}{\Delta t}}{\dfrac{\rho L}{A_{\text{wire}}}} = \frac{NA_{\text{coil}}A_{\text{wire}}\,\Delta B/\Delta t}{\rho L}$$

$$= \frac{20\left[\pi\left(0.11\,\text{m}\right)^2\right]\left[\pi\left(1.3\times10^{-3}\,\text{m}\right)^2\right]\left(8.65\times10^{-3}\,\text{T/s}\right)}{\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)20\left(2\pi\right)\left(0.11\right)} = 0.1504\,\text{A} \approx \boxed{0.15\,\text{A}}$$

(*b*) The rate at which thermal energy is produced in the wire is the power dissipated in the wire.

$$P = I^2R = I^2\frac{\rho L}{A_{\text{wire}}} = \left(0.1504\,\text{A}\right)^2\frac{\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)20\left(2\pi\right)\left(0.11\right)}{\pi\left(1.3\times10^{-3}\,\text{m}\right)^2} = \boxed{9.9\times10^{-4}\,\text{W}}$$

19. The charge that passes a given point is the current times the elapsed time, $Q = I\Delta t$. The current will be the emf divided by the resistance, $I = \dfrac{\mathcal{E}}{R}$. The resistance is given by E. 18-3, $R = \dfrac{\rho L}{A_{\text{wire}}}$, and the emf is given by Eq. 21-2b. Combine these equations to find the charge during the operation.

$$\left|\mathcal{E}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{A_{\text{loop}}\left|\Delta B\right|}{\Delta t} \;;\; R = \frac{\rho L}{A_{\text{wire}}} \;;\; I = \frac{\mathcal{E}}{R} = \frac{\dfrac{A_{\text{loop}}\left|\Delta B\right|}{\Delta t}}{\dfrac{\rho L}{A_{\text{wire}}}} = \frac{A_{\text{loop}}A_{\text{wire}}\left|\Delta B\right|}{\rho L\Delta t}$$

$$Q = I\Delta t = \frac{A_{\text{loop}}A_{\text{wire}}\left|\Delta B\right|}{\rho L} = \frac{\pi r_{\text{loop}}^2\pi r_{\text{wire}}^2\left|\Delta B\right|}{\rho\left(2\pi\right)r_{\text{loop}}} = \frac{r_{\text{loop}}\pi r_{\text{wire}}^2\left|\Delta B\right|}{2\rho}$$

$$= \frac{\left(0.066\,\text{m}\right)\pi\left(1.125\times10^{-3}\,\text{m}\right)^2\left(0.750\,\text{T}\right)}{2\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)} = \boxed{5.86\,\text{C}}$$

20. We find the number of turns from Eq. 21-5. The factor multiplying the sine term is the peak output voltage.

$$\mathcal{E}_{\text{peak}} = NB\omega A \;\rightarrow\; N = \frac{\mathcal{E}_{\text{peak}}}{B\omega A} = \frac{24.0\,\text{V}}{\left(0.420\,\text{T}\right)2\pi\,\text{rad/rev}\left(60\,\text{rev/s}\right)\left(0.060\,\text{m}\right)^2} = 42.1 \approx \boxed{42\text{ loops}}$$

21. From Eq. 21-5, the induced voltage is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathcal{E}_1}{\omega_1} = \frac{\mathcal{E}_2}{\omega_2} \;\rightarrow\; \mathcal{E}_2 = \mathcal{E}_1\frac{\omega_2}{\omega_1} = \left(12.4\,\text{V}\right)\frac{2500\,\text{rpm}}{1100\,\text{rpm}} = \boxed{28.2\,\text{V}}$$

22. From Eq. 21-5, the peak voltage is $\mathcal{E}_{\text{peak}} = NB\omega A$. The rms voltage is the peak voltage divided by $\sqrt{2}$, and so $V_{\text{rms}} = \mathcal{E}_{\text{peak}}/\sqrt{2} = NB\omega A/\sqrt{2}$.

23. From Eq. 21-5, the peak voltage is $\mathcal{E}_{\text{peak}} = NB\omega A$. Solve this for the rotation speed.

$$\mathcal{E}_{\text{peak}} = NB\omega A \rightarrow \omega = \frac{\mathcal{E}_{\text{peak}}}{NBA} = \frac{120\,\text{V}}{320(0.650\,\text{T})(0.210\,\text{m})^2} = 13.08\,\text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{13.08\,\text{rad/s}}{2\pi\,\text{rad/rev}} = \boxed{2.08\,\text{rev/s}}$$

$\boxed{24.}$ (*a*) The rms voltage is found from the peak induced emf. The peak induced emf is calculated from Eq. 21-5.

$$\mathcal{E}_{\text{peak}} = NB\omega A$$

$$V_{\text{rms}} = \frac{\mathcal{E}_{\text{peak}}}{\sqrt{2}} = \frac{NB\omega A}{\sqrt{2}} = \frac{(450)(0.55\,\text{T})(2\pi\,\text{rad/rev})(120\,\text{rev/s})\pi(0.040\text{m})^2}{\sqrt{2}}$$

$$= 663.3\,\text{V} \approx \boxed{660\,\text{V}}$$

(*b*) To double the output voltage, you must $\boxed{\text{double the rotation frequency}}$ to 240 rev/s.

25. (*a*) The peak current is found from the rms current.

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}(70.0\,\text{A}) = \boxed{99.0\,\text{V}}$$

(*b*) The area can be found from Eq. 21-5.

$$\mathcal{E}_{\text{peak}} = NB\omega A = \sqrt{2}V_{\text{rms}} \rightarrow$$

$$A = \frac{\sqrt{2}V_{\text{rms}}}{NB\omega} = \frac{\sqrt{2}(150\,\text{V})}{(1000)(0.030\,\text{T})(85\,\text{rev/s})(2\pi\,\text{rad/rev})} = \boxed{1.3\times10^{-2}\,\text{m}^2}$$

26. When the motor is running at full speed, the back emf opposes the applied emf, to give the net across the motor.

$$\mathcal{E}_{\text{applied}} - \mathcal{E}_{\text{back}} = IR \rightarrow \mathcal{E}_{\text{back}} = \mathcal{E}_{\text{applied}} - IR = 120\,\text{V} - (8.20\,\text{A})(3.25\,\Omega) = \boxed{93.4\,\text{V}}$$

27. From Eq. 21-5, the induced voltage (back emf) is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathcal{E}_1}{\omega_1} = \frac{\mathcal{E}_2}{\omega_2} \rightarrow \mathcal{E}_2 = \mathcal{E}_1\frac{\omega_2}{\omega_1} = (72\,\text{V})\frac{2500\,\text{rpm}}{1800\,\text{rpm}} = \boxed{100\,\text{V}}$$

28. The magnitude of the back emf is proportional to both the rotation speed and the magnetic field, from Eq. 21-5. Thus $\dfrac{\mathcal{E}}{B\omega}$ is constant.

$$\frac{\mathcal{E}_1}{B_1\omega_1} = \frac{\mathcal{E}_2}{B_2\omega_2} \rightarrow B_2 = \frac{\mathcal{E}_2}{\omega_2}\frac{B_1\omega_1}{\mathcal{E}_1} = \frac{(65\,\text{V})}{(2500\,\text{rpm})}\frac{B_1(1000\,\text{rpm})}{(95\,\text{V})} = 0.27B_1$$

So $\boxed{\text{reduce the magnetic field to 27\% of its original value}}$.

29. The back emf is proportional to the rotation speed (Eq. 21-5). Thus if the motor is running at half speed, the back emf is half the original value, or 54 V. Find the new current from writing a loop equation for the motor circuit, from Figure 21-19.

$$\mathcal{E} - \mathcal{E}_{\text{back}} - IR = 0 \quad \rightarrow \quad I = \frac{\mathcal{E} - \mathcal{E}_{\text{back}}}{R} = \frac{120\,\text{V} - 54\,\text{V}}{5.0\,\Omega} = \boxed{13\,\text{A}}$$

30. We find the number of turns in the secondary from Eq. 21-6.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \quad \rightarrow \quad N_S = N_P \frac{V_S}{V_P} = (164)\frac{10{,}000\,\text{V rpm}}{120\,\text{V}} \approx \boxed{13{,}700 \text{ turns}}$$

31. Because $N_S < N_P$, this is a $\boxed{\text{step-down}}$ transformer. Use Eq. 21-6 to find the voltage ratio, and Eq. 21-7 to find the current ratio.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{120\,\text{turns}}{320\,\text{turns}} = \boxed{0.375} \qquad \frac{I_S}{I_P} = \frac{N_P}{N_S} = \frac{320\,\text{turns}}{120\,\text{turns}} = \boxed{2.67}$$

32. Use Eqs. 21-6 and 21-7 to relate the voltage and current ratios.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}\ ;\ \frac{I_S}{I_P} = \frac{N_P}{N_S} \quad \rightarrow \quad \frac{V_S}{V_P} = \frac{I_P}{I_S} \quad \rightarrow \quad \frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{25\,\text{V}}{120\,\text{V}} = \boxed{0.21}$$

33. We find the ratio of the number of turns from Eq. 21-6.

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{12000\,\text{V}}{240\,\text{V}} = \boxed{50}$$

If the transformer is connected backward, the role of the turns will be reversed:

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} \quad \rightarrow \quad \frac{1}{50} = \frac{V_S}{240\,\text{V}} \quad \rightarrow \quad V_S = \frac{1}{50}(240\,\text{V}) = \boxed{4.8\,\text{V}}$$

34. (*a*) Use Eqs. 21-6 and 21-7 to relate the voltage and current ratios.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}\ ;\ \frac{I_S}{I_P} = \frac{N_P}{N_S} \quad \rightarrow \quad \frac{V_S}{V_P} = \frac{I_P}{I_S} \quad \rightarrow \quad V_S = V_P \frac{I_P}{I_S} = (120\,\text{V})\frac{0.35\,\text{A}}{7.5\,\text{A}} = \boxed{5.6\,\text{V}}$$

(*b*) Because $V_S < V_P$, this is a $\boxed{\text{step-down}}$ transformer.

35. (*a*) We assume 100% efficiency, and find the input voltage from $P = IV$.

$$P = I_P V_P \quad \rightarrow \quad V_P = \frac{P}{I_P} = \frac{95\,\text{W}}{22\,\text{A}} = 4.318\,\text{V}$$

Since $V_P < V_S$, this is a $\boxed{\text{step-up}}$ transformer.

(*b*) $$\frac{V_S}{V_P} = \frac{12\,\text{V}}{4.318\,\text{V}} = \boxed{2.8}$$

$\boxed{36.}$ Use Eqs. 21-6 and 21-7 to relate the voltage and current ratios.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \quad \rightarrow \quad V_S = V_P \frac{N_S}{N_P} = (120\,\text{V})\frac{1340\,\text{turns}}{330\,\text{turns}} = \boxed{487\,\text{V}}$$

$$\frac{I_S}{I_P}=\frac{N_P}{N_S} \rightarrow I_P=I_S\frac{N_S}{N_P}=(15.0\,\text{A})\frac{1340\,\text{turns}}{330\,\text{turns}}=\boxed{60.9\,\text{A}}$$

37. (*a*) The current in the transmission lines can be found from Eq. 18-9a, and then the emf at the end of the lines can be calculated from Kirchhoff's loop rule.

$$P_{\text{town}}=V_{\text{rms}}I_{\text{rms}} \rightarrow I_{\text{rms}}=\frac{P_{\text{town}}}{V_{\text{rms}}}=\frac{30\times10^6\,\text{W}}{45\times10^3\,\text{V}}=667\,\text{A}$$

$$\mathcal{E}-IR-V_{\text{output}}=0 \rightarrow$$

$$\mathcal{E}=IR+V_{\text{output}}=\frac{P_{\text{town}}}{V_{\text{rms}}}R+V_{\text{rms}}=\frac{30\times10^6\,\text{W}}{45\times10^3\,\text{V}}(4.0\,\Omega)+45\times10^3\,\text{V}=47700\,\text{V}\approx\boxed{48\,\text{kV}(\text{rms})}$$

(*b*) The power loss in the lines is given by $P_{\text{loss}}=I_{\text{rms}}^2R$.

$$\text{Fraction wasted}=\frac{P_{\text{loss}}}{P_{\text{total}}}=\frac{P_{\text{loss}}}{P_{\text{town}}+P_{\text{loss}}}=\frac{I_{\text{rms}}^2R}{P_{\text{town}}+I_{\text{rms}}^2R}=\frac{(667\,\text{A})^2(4.0\,\Omega)}{30\times10^6\,\text{W}+(667\,\text{A})^2(4.0\,\Omega)}$$

$$=\boxed{0.056}$$

38. Without the transformers, we find the delivered current, which is the current in the transmission lines, from the delivered power, and the power lost in the transmission lines.

$$P_{\text{out}}=V_{\text{out}}I_{\text{line}} \rightarrow I_{\text{line}}=\frac{P_{\text{out}}}{V_{\text{out}}}=\frac{65000\,\text{W}}{120\,\text{V}}=541.7\,\text{A}$$

$$P_{\text{lost}}=I_{\text{line}}^2R_{\text{line}}=(541.7\,\text{A})^2\,2(0.100\,\Omega)=58690\,\text{W}$$

Thus there must be $65000\,\text{W}+58690\,\text{W}=123690\text{W}\approx124\,\text{kW}$ of power generated at the start of the process.

With the transformers, to deliver the same power at 120 V, the delivered current from the step-down transformer must still be 541.7 A. Using the step-down transformer efficiency, we calculate the current in the transmission lines, and the loss in the transmission lines.

$$P_{\text{out}}=0.99P_{\substack{\text{line}\\\text{end}}} \rightarrow V_{\text{out}}I_{\text{out}}=0.99V_{\text{line}}I_{\text{line}} \rightarrow I_{\text{line}}=\frac{V_{\text{out}}I_{\text{out}}}{0.99V_{\text{line}}}=\frac{(120\,\text{V})(541.7\,\text{A})}{(0.99)(1200\,\text{V})}=54.72\,\text{A}$$

$$P_{\text{lost}}=I_{\text{line}}^2R_{\text{line}}=(54.72\,\text{A})^2\,2(0.100\,\Omega)=599\,\text{W}$$

The power to be delivered is 65000 W. The power that must be delivered to the step-down transformer is $\dfrac{65000\,\text{W}}{0.99}=65657\,\text{W}$. The power that must be present at the start of the transmission must be $65657\,\text{W}+599\,\text{W}=66256\,\text{W}$ to compensate for the transmission line loss. The power that must enter the transmission lines from the 99% efficient step-up transformer is $\dfrac{66256\,\text{W}}{0.99}=66925\approx67\,\text{kW}$. So the power saved is $124\,\text{kW}-67\,\text{kW}=\boxed{54\,\text{kW}}$.

39. Find the induced emf from Eq. 21-9.

$$\mathcal{E}=-L\frac{\Delta I}{\Delta t}=-(0.180\,\text{H})\frac{(10.0\,\text{A}-25.0\,\text{A})}{0.350\,\text{s}}=\boxed{7.71\,\text{V}}$$

40. Because the current in increasing, the emf is negative. We find the self-inductance from Eq. 21-9.

$$\mathcal{E}=-L\frac{\Delta I}{\Delta t}\quad\rightarrow\quad L=-\mathcal{E}\frac{\Delta t}{\Delta I}=-(2.50\,\text{V})\frac{0.0120\,\text{s}}{0.0310\,\text{A}-(-0.0280\,\text{A})}=\boxed{0.508\,\text{H}}$$

41. Use the relationship for the inductance of a solenoid, as given in Example 21-14.

$$L=\frac{\mu_0N^2A}{l}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(10000)^2\,\pi\left(1.45\times10^{-2}\,\text{m}\right)^2}{0.60\,\text{m}}=\boxed{0.14\,\text{H}}$$

42. Use the relationship for the inductance of a solenoid, as given in Example 21-14.

$$L=\frac{\mu_0N^2A}{l}\quad\rightarrow\quad N=\sqrt{\frac{Ll}{\mu_0A}}=\sqrt{\frac{(0.130\,\text{H})(0.300\,\text{m})}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\pi\left(2.6\times10^{-2}\,\text{m}\right)^2}}\approx\boxed{3800\,\text{turns}}$$

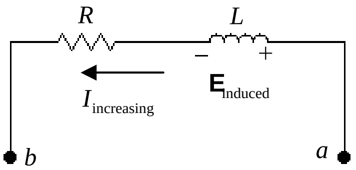
43. (*a*) The number of turns can be found from the inductance of a solenoid, which is derived in Example 21-14.

$$L=\frac{\mu_0N^2A}{l}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(2800)^2\,\pi(0.0125\,\text{m})^2}{(0.282\,\text{m})}=\boxed{1.71\times10^{-2}\,\text{H}}:$$

(*b*) Apply the same equation again, solving for the number of turns.

$$L=\frac{\mu_0N^2A}{l}\quad\rightarrow\quad N=\sqrt{\frac{Ll}{\mu_0A}}=\sqrt{\frac{\left(1.71\times10^{-2}\,\text{H}\right)(0.282\,\text{m})}{(1200)\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\pi(0.0125\,\text{m})^2}}\approx\boxed{81\,\text{turns}}$$

44. We draw the coil as two elements in series, and pure resistance and a pure inductance. There is a voltage drop due to the resistance of the coil, given by Ohm's law, and an induced emf due to the inductance of the coil, given by Eq. 21-9. Since the current is increasing, the inductance will create a potential difference to oppose the increasing current, and so there is a drop in the potential due to the inductance. The potential difference across the coil is the sum of the two potential drops.



$$V_{ab}=IR+L\frac{\Delta I}{\Delta t}=(3.00\,\text{A})(2.25\,\Omega)+(0.440\,\text{H})(3.50\,\text{A/s})=\boxed{8.29\,\text{V}}$$

45. We assume that both the solenoid and the coil have the same cross-sectional area. The magnetic field of the solenoid (which passes through the coil) is $B_1=\mu_0\frac{N_1I_1}{l}$. When the current in the solenoid changes, the magnetic field of the solenoid changes, and thus the flux through the coil changes, inducing an emf in the coil.

$$\mathcal{E}=-\frac{N_2A\Delta B}{\Delta t}=-\frac{N_2A\Delta\left(\mu_0\dfrac{N_1I_1}{l}\right)}{\Delta t}=-\mu_0\frac{N_1N_2A}{l}\frac{\Delta(I_1)}{\Delta t}$$

As in Eq. 21-8a, the mutual inductance is the proportinality constant in the above relationship.

$$\mathcal{E}=-\mu_0\frac{N_1N_2A}{l}\frac{\Delta(I_1)}{\Delta t}\quad\rightarrow\quad\boxed{M=\mu_0\frac{N_1N_2A}{l}}$$

46. The inductance of the solenoid is given by $B=\mu_0 NI/l$. The (constant) length of the wire is given by $l_{\text{wire}}=N\pi d_{\text{sol}}$, and so since $d_{\text{sol 2}}=2d_{\text{sol 1}}$, we also know that $N_1=2N_2$. The fact that the wire is tightly wound gives $l_{\text{sol}}=Nd_{\text{wire}}$. Find the ratio of the two inductances.

$$\frac{L_2}{L_1}=\frac{\dfrac{\mu_0\pi}{4}\dfrac{N_2^2}{l_{\text{sol 2}}}d_{\text{sol 2}}^2}{\dfrac{\mu_0\pi}{4}\dfrac{N_1^2}{l_{\text{sol 1}}}d_{\text{sol 1}}^2}=\frac{\dfrac{N_2^2}{l_{\text{sol 2}}}d_{\text{sol 2}}^2}{\dfrac{N_1^2}{l_{\text{sol 1}}}d_{\text{sol 1}}^2}=\frac{\dfrac{l_{\text{wire}}^2/\pi^2}{l_{\text{sol 2}}}}{\dfrac{l_{\text{wire}}^2/\pi^2}{l_{\text{sol 1}}}}=\frac{l_{\text{sol 1}}}{l_{\text{sol 2}}}=\frac{N_1 d_{\text{wire}}}{N_2 d_{\text{wire}}}=\frac{N_1}{N_2}=\boxed{2}$$

47. The magnetic energy in the field is derived from Eq. 21-10.

$$u=\frac{\text{Energy stored}}{\text{Volume}}=\tfrac{1}{2}\frac{B^2}{\mu_0}\quad\rightarrow$$

$$\text{Energy}=\tfrac{1}{2}\frac{B^2}{\mu_0}\left(\text{Volume}\right)=\tfrac{1}{2}\frac{B^2}{\mu_0}\pi r^2 L=\tfrac{1}{2}\frac{\left(0.80\,\text{T}\right)^2}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}\pi\left(0.010\,\text{m}\right)^2\left(0.36\,\text{m}\right)=\boxed{29\,\text{J}}$$

48. The initial energy stored in the inductor is found from an equation in Section 21-10.

$$U=\tfrac{1}{2}LI^2=\tfrac{1}{2}\left(60.0\times10^{-3}\,\text{H}\right)\left(50.0\times10^{-3}\,\text{A}\right)^2=\boxed{7.50\times10^{-5}\,\text{J}}$$

The final current is found from the final energy, which is 10 times the initial energy.

$$U_f=10U_0\quad\rightarrow\quad\tfrac{1}{2}LI_f^2=10\tfrac{1}{2}LI_0^2\quad\rightarrow\quad I_f=\sqrt{10}I_0$$

The current increases at a constant rate.

$$\frac{\Delta I}{\Delta t}=\frac{I_f-I_0}{\Delta t}=0.115\,\text{A/s}\quad\rightarrow$$

$$\Delta t=\frac{I_f-I_0}{0.115\,\text{A/s}}=\frac{\sqrt{10}I_0-I_0}{0.115\,\text{A/s}}=\frac{I_0}{0.115\,\text{A/s}}\left(\sqrt{10}-1\right)=\frac{\left(50.0\times10^{-3}\,\text{A}\right)}{0.115\,\text{A/s}}\left(\sqrt{10}-1\right)=\boxed{0.940\,\text{s}}$$

49. The magnetic energy in the field is derived from Eq. 21-10. The volume of a relatively thin spherical shell, like the first 10 km above the Earth's surface, is the surface area of the sphere times its thickness.

$$u=\tfrac{1}{2}\frac{B^2}{\mu_0}\quad\rightarrow$$

$$E=u\left(\text{volume}\right)=\tfrac{1}{2}\frac{B^2}{\mu_0}4\pi R_{\text{Earth}}^2 h=\tfrac{1}{2}\frac{\left(0.50\times10^{-4}\,\text{T}\right)^2}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}4\pi\left(6.38\times10^6\,\text{m}\right)^2\left(10^4\,\text{m}\right)=\boxed{5.1\times10^{15}\,\text{J}}$$

50. When the switch is initially closed, the inductor prevents current from flowing, and so the initial current is 0, as shown in Figure 21-34. If the current is 0, there is no voltage drop across the resistor (since $V_{\text{R}}=IR$, and so the entire battery voltage appears across the inductor. Apply Eq. 21-9 to find the initial rate of change of the current.

$$V=V_{\text{L}}=L\frac{\Delta I}{\Delta t}\quad\rightarrow\quad\frac{\Delta I}{\Delta t}=\boxed{\frac{V}{L}}$$

The maximum value of the current is reached after a long time, when there is no voltage across the inductor, and so the entire battery voltage appears across the resistor. Apply Ohm's law.

$$V = I_{\max} R \ \rightarrow \ I_{\max} = \frac{V}{R}.$$

Find the time to reach the maximum current if the rate of current change remained at $\frac{\Delta I}{\Delta t} = \frac{V}{L}$.

$$I_{\max} = I_0 + \frac{\Delta I}{\Delta t}(\text{elapsed time}) \ \rightarrow \ \text{elapsed time} = (I_{\max} - I_0)\frac{\Delta t}{\Delta I} = \left(\frac{V}{R} - 0\right)\frac{L}{V} = \boxed{\frac{L}{R}}$$

51. For an *LR* circuit, we have $I = I_{\max}\left(1 - e^{-t/\tau}\right)$. Solve for t.

$$I = I_{\max}\left(1 - e^{-t/\tau}\right) \ \rightarrow \ e^{-t/\tau} = 1 - \frac{I}{I_{\max}} \ \rightarrow \ t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right)$$

(*a*) $t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.9) = \boxed{2.3\tau}$

(*b*) $t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.99) = \boxed{4.6\tau}$

(*c*) $t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.999) = \boxed{6.9\tau}$

52. We use the inductance of a solenoid, as derived in Example 21-14: $L_{\text{sol}} = \frac{\mu_0 N^2 A}{l}$.

(*a*) Both solenoids have the same area and the same length. Because the wire in solenoid 1 is half as thick as the wire in solenoid 2, solenoid 1 will have twice the number of turns as solenoid 2.

$$\frac{L_1}{L_2} = \frac{\frac{\mu_0 N_1^2 A}{l}}{\frac{\mu_0 N_2^2 A}{l}} = \frac{N_1^2}{N_2^2} = \left(\frac{N_1}{N_2}\right)^2 = 2^2 = 4 \ \rightarrow \ \boxed{\frac{L_1}{L_2} = 4}$$

(*b*) To find the ratio of the time constants, both the inductance and resistance ratios need to be known. Since solenoid 1 has twice the number of turns as solenoid 2, the length of wire used to make solenoid 1 is twice that used to make solenoid 2, or $l_{\text{wire 1}} = 2l_{\text{wire 2}}$, and the diameter of the wire in solenoid 1 is half that in solenoid 2, or $d_{\text{wire 1}} = \frac{1}{2}d_{\text{wire 2}}$. Use this to find their relative resistances, and then the ratio of time constants.

$$\frac{R_1}{R_2} = \frac{\frac{\rho l_{\text{wire 1}}}{A_{\text{wire 1}}}}{\frac{\rho l_{\text{wire 2}}}{A_{\text{wire 2}}}} = \frac{\frac{l_{\text{wire 1}}}{\pi\left(d_{\text{wire 1}}/2\right)^2}}{\frac{l_{\text{wire 2}}}{\pi\left(d_{\text{wire 2}}/2\right)^2}} = \frac{\frac{l_{\text{wire 1}}}{d_{\text{wire 1}}^2}}{\frac{l_{\text{wire 2}}}{d_{\text{wire 2}}^2}} = \frac{l_{\text{wire 1}}}{l_{\text{wire 2}}}\left(\frac{d_{\text{wire 2}}}{d_{\text{wire 1}}}\right)^2 = 2(2)^2 = 8 \ \rightarrow \ \frac{R_1}{R_2} = 8$$

$$\frac{\tau_1}{\tau_2} = \frac{L_1/R_1}{L_2/R_2} = \frac{L_1}{L_2}\frac{R_2}{R_1} = 4\left(\frac{1}{8}\right) = \frac{1}{2} \ \rightarrow \ \boxed{\frac{\tau_1}{\tau_2} = \frac{1}{2}}$$

53. The reactance of a capacitor is given by Eq. 21-12b, $X_C = \dfrac{1}{2\pi fC}$.

(*a*) $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60.0\,\text{Hz})(7.20\times10^{-6}\,\text{F})} = \boxed{368\,\Omega}$

(*b*) $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(1.00\times10^{6}\,\text{Hz})(7.20\times10^{-6}\,\text{F})} = \boxed{2.21\times10^{-2}\,\Omega}$

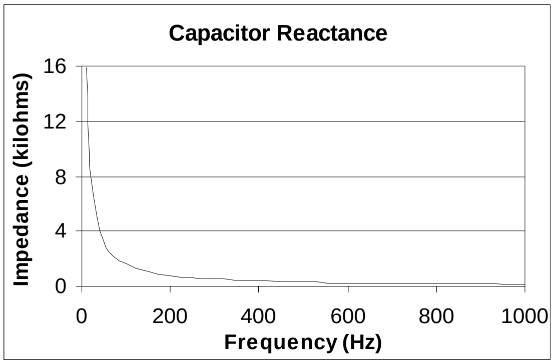
54. We find the frequency from Eq. 21-11b for the reactance of an inductor.

$$X_L = 2\pi fL \rightarrow f = \frac{X_L}{2\pi L} = \frac{660\,\Omega}{2\pi(22.0\times10^{-3}\,\text{H})} = \boxed{4770\,\text{Hz}}$$

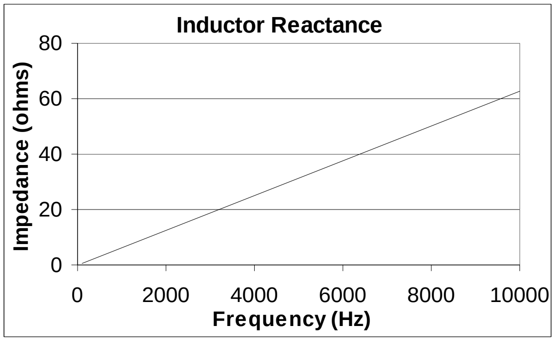
55. We find the frequency from Eq. 21-12b for the reactance of a capacitor.

$$X_C = \frac{1}{2\pi fC} \rightarrow f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi(6.70\times10^{3}\,\Omega)(2.40\times10^{-6}\,\text{F})} = \boxed{9.90\,\text{Hz}}$$

56. The impedance is $X_C = \dfrac{1}{2\pi fC}$.



57. The impedance is $X_L = 2\pi fL$.



58. We find the reactance from Eq. 21-11b, and the current from Ohm's law.

$$X_L = 2\pi fL = 2\pi(10.0\times10^{3}\,\text{Hz})(0.160\,\text{H}) = 10053\,\Omega \approx \boxed{10.0\,\text{k}\Omega}$$

$$V = IX_L \rightarrow I = \frac{V}{X_L} = \frac{240\,\text{V}}{10053\,\Omega} = \boxed{2.39\times10^{-2}\,\text{A}}$$

59. We find the reactance from Ohm's law, and the inductance by Eq. 21-11b.

$$V = IX_L \quad \rightarrow \quad X_L = \frac{V}{I}$$

$$X_L = 2\pi fL \quad \rightarrow \quad L = \frac{X_L}{2\pi f} = \frac{V}{2\pi fI} = \frac{240\,\text{V}}{2\pi\left(60.0\,\text{Hz}\right)12.8\,\text{A}} = \boxed{4.97\times10^{-2}\,\text{H}}$$

60. (*a*) We find the reactance from Eq. 21-12b.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\left(720\,\text{Hz}\right)\left(3.0\times10^{-8}\,\text{F}\right)} = 7368\,\Omega \approx \boxed{7400\,\Omega}$$

(*b*) We find the peak value of the current from Ohm's law.

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{V_{\text{rms}}}{X_C} = \sqrt{2}\frac{2.0\times10^3\,\text{V}}{7368\,\Omega} = \boxed{0.38\,\text{A}}$$

61. The impedance of the circuit is given by Eq. 21-15 without a capacitive reactance. The reactance of the inductor is given by Eq. 21-11b.

(*a*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2} = \sqrt{\left(30\times10^3\,\Omega\right)^2 + 4\pi^2\left(50\,\text{Hz}\right)^2\left(45\times10^{-3}\,\text{H}\right)^2} = \boxed{3.0\times10^4\,\Omega}$

(*b*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2} = \sqrt{\left(30\times10^3\,\Omega\right)^2 + 4\pi^2\left(3.0\times10^4\,\text{Hz}\right)^2\left(45\times10^{-3}\,\text{H}\right)^2}$

$= \boxed{3.1\times10^4\,\Omega}$

62. The impedance of the circuit is given by Eq. 21-15 without an inductive reactance. The reactance of the capacitor is given by Eq. 21-12b.

(*a*) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 C^2}} = \sqrt{\left(3.5\times10^3\,\Omega\right)^2 + \dfrac{1}{4\pi^2\left(60\,\text{Hz}\right)^2\left(4.0\times10^{-6}\,\text{F}\right)^2}}$

$= \boxed{3.6\times10^3\,\Omega}$

(*b*) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 C^2}} = \sqrt{\left(3.5\times10^3\,\Omega\right)^2 + \dfrac{1}{4\pi^2\left(60000\,\text{Hz}\right)^2\left(4.0\times10^{-6}\,\text{F}\right)^2}}$

$= \boxed{3.5\times10^3\,\Omega}$

63. We find the impedance from Eq. 21-14.

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{70\times10^{-3}\,\text{A}} = \boxed{1.7\times10^3\,\Omega}$$

64. Use Eq. 21-15, with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} \quad \rightarrow \quad R = \sqrt{Z^2 - X_L^2} = \sqrt{\left(235\,\Omega\right)^2 - \left(135\,\Omega\right)^2} = \boxed{192\,\Omega}$$

65. The total impedance is given by Eq. 21-15.

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{\left(8.70\times10^{3}\,\Omega\right)^{2} + \left[2\pi\left(1.00\times10^{4}\,\mathrm{Hz}\right)\left(2.20\times10^{-2}\,\mathrm{H}\right) - \frac{1}{2\pi\left(1.00\times10^{4}\,\mathrm{Hz}\right)\left(6.25\times10^{-9}\,\mathrm{F}\right)}\right]^{2}}$$

$$= 8777\,\Omega \approx \boxed{8.78\,\mathrm{k}\Omega}$$

The phase angle is given by Eq. 21-16a.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2\pi fL - \dfrac{1}{2\pi fC}}{R}$$

$$= \tan^{-1}\frac{2\pi\left(1.00\times10^{4}\,\mathrm{Hz}\right)\left(2.20\times10^{-2}\,\mathrm{H}\right) - \dfrac{1}{2\pi\left(1.00\times10^{4}\,\mathrm{Hz}\right)\left(6.25\times10^{-9}\,\mathrm{F}\right)}}{R}$$

$$= \tan^{-1}\frac{-1164\,\Omega}{8700\,\Omega} = \boxed{-7.62^{\circ}}$$

The voltage is lagging the current, or the current is leading the voltage.
The rms current is given by Eq. 21-18.

$$I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{Z} = \frac{725\,\mathrm{V}}{8777\,\Omega} = \boxed{8.26\times10^{-2}\,\mathrm{A}}$$

66. The impedance is given by Eq. 21-15 with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \left(2\pi fL\right)^2}\ .$$

$$Z_f = 2Z_{60} \ \rightarrow\ \sqrt{R^2 + 4\pi^2 f^2 L^2} = 2\sqrt{R^2 + 4\pi^2\left(60\,\mathrm{Hz}\right)^2 L^2} \ \rightarrow$$

$$R^2 + 4\pi^2 f^2 L^2 = 4\left[R^2 + 4\pi^2\left(60\,\mathrm{Hz}\right)^2 L^2\right] = 4R^2 + 16\pi^2\left(60\,\mathrm{Hz}\right)^2 L^2 \ \rightarrow$$

$$f = \sqrt{\frac{3R^2 + 16\pi^2\left(60\,\mathrm{Hz}\right)^2 L^2}{4\pi^2 L^2}} = \sqrt{\frac{3R^2}{4\pi^2 L^2} + 4\left(60\,\mathrm{Hz}\right)^2} = \sqrt{\frac{3\left(2500\,\Omega\right)^2}{4\pi^2\left(0.42\,\mathrm{H}\right)^2} + 4\left(60\,\mathrm{Hz}\right)^2}$$

$$= 1645\,\mathrm{Hz} \approx \boxed{1.6\,\mathrm{kHz}}$$

67. (*a*) The rms current is the rmv voltage divided by the impedance. The impedance is given by Eq. 21-15 with no capacitive reactance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \left(2\pi fL\right)^2}$.

$$I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{Z} = \frac{V_{\mathrm{rms}}}{\sqrt{R^2 + 4\pi^2 f^2 L^2}} = \frac{120\,\mathrm{V}}{\sqrt{\left(1.80\times10^{3}\,\Omega\right)^2 + 4\pi^2\left(60.0\,\mathrm{Hz}\right)^2\left(0.350\,\mathrm{H}\right)^2}}$$

$$= \frac{120\,\mathrm{V}}{1805\,\Omega} = \boxed{6.65\times10^{-2}\,\mathrm{A}}$$

(*b*) The phase angle is given by Eq. 21-16a with no capacitive reactance.

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{2\pi fL}{R} = \tan^{-1}\frac{2\pi\left(60.0\,\mathrm{Hz}\right)\left(0.350\,\mathrm{H}\right)}{1.80\times10^{3}\,\Omega} = \boxed{4.19^{\circ}}$$

The current is lagging the source voltage.

(*c*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\substack{\text{rms}\\ R}} = I_{\text{rms}}R = \left(6.65\times10^{-2}\,\text{A}\right)\left(1800\,\Omega\right) = 119.7\,\text{V} \approx \boxed{120\,\text{V}}$$

$$V_{\substack{\text{rms}\\ L}} = I_{\text{rms}}X_L = I_{\text{rms}}2\pi fL = \left(6.65\times10^{-2}\,\text{A}\right)2\pi\left(60.0\,\text{Hz}\right)\left(0.350\,\text{H}\right) = \boxed{8.77\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

68. (*a*) The rms current is the rmv voltage divided by the impedance. The impedance is given by Eq. 21-15 with no inductive reactance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{\left(2\pi fC\right)^2}}$.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 L^2}}} = \frac{120\,\text{V}}{\sqrt{\left(8.80\times10^3\,\Omega\right)^2 + \dfrac{1}{4\pi^2\left(60.0\,\text{Hz}\right)^2\left(1.80\times10^{-6}\,\text{F}\right)^2}}}$$

$$= \frac{120\,\text{V}}{8923\,\Omega} = \boxed{1.34\times10^{-2}\,\text{A}}$$

(*b*) The phase angle is given by Eq. 21-16a with no inductive reactance.

$$\phi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{-\dfrac{1}{2\pi fC}}{R} = \tan^{-1}\frac{-\dfrac{1}{2\pi\left(60.0\,\text{Hz}\right)\left(1.80\times10^{-6}\,\text{F}\right)}}{8.80\times10^3\,\Omega} = \boxed{-9.51^{\circ}}$$

The current is leading the source voltage.

(*c*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\substack{\text{rms}\\ R}} = I_{\text{rms}}R = \left(1.34\times10^{-2}\,\text{A}\right)\left(8.80\times10^3\,\Omega\right) = \boxed{118\,\text{V}}$$

$$V_{\substack{\text{rms}\\ C}} = I_{\text{rms}}X_C = I_{\text{rms}}\frac{1}{2\pi fC} = \left(1.34\times10^{-2}\,\text{A}\right)\frac{1}{2\pi\left(60.0\,\text{Hz}\right)\left(1.80\times10^{-6}\,\text{F}\right)} = \boxed{19.7\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

69. The resonant frequency is given by Eq. 21-19.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{\left(55.0\times10^{-6}\,\text{H}\right)\left(3500\times10^{-12}\,\text{F}\right)}} = \boxed{3.63\times10^5\,\text{Hz}}$$

70. (*a*) The resonant frequency is given by Eq. 21-19.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad\rightarrow\quad \frac{f_{580\,\text{kHz}}}{f_{1600\,\text{kHz}}} = \frac{580}{1600} = \frac{\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_{580\text{ kHz}}}}}{\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_{1600\text{ kHz}}}}} \quad\rightarrow$$

$$C_{1600\text{ kHz}} = \left(\frac{580}{1600}\right)^2 C_{580\text{ kHz}} = \left(\frac{580}{1600}\right)^2\left(2800\,\text{pF}\right) = 367.9\,\text{pF} \approx \boxed{370\,\text{pF}}$$

(*b*) The inductance can be found from the resonant frequency and the capacitance.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \rightarrow L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2\left(5.8\times10^{5}\,\text{Hz}\right)^2\left(2800\times10^{-12}\,\text{F}\right)} = \boxed{2.7\times10^{-5}\,\text{H}}$$

71. (*a*) We find the capacitance from the resonant frequency, Eq. 21-19.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \rightarrow C = \frac{1}{4\pi^2 L f_0^2} = \frac{1}{4\pi^2\left(14.8\times10^{-3}\,\text{H}\right)\left(3600\,\text{Hz}\right)^2} = \boxed{1.32\times10^{-7}\,\text{F}}$$

(*b*) At resonance the impedance is the resistance, so the current is given by Ohm's law.

$$I_{\text{peak}} = \frac{V_{\text{peak}}}{R} = \frac{150\,\text{V}}{4.40\,\Omega} = \boxed{34.1\,\text{A}}$$

72. (*a*) The clockwise current in the left-hand loop produces a magnetic field which is into the page within the loop and out of the page outside the loop. Thus the right-hand loop is in a magnetic field that is directed out of the page. Before the current in the left-hand loop reaches its steady state, there will be an induced current in the right-hand loop that will produce a magnetic field into the page to oppose the increase of the field from the left-hand loop. Thus the induced current will be $\boxed{\text{clockwise}}$.

(*b*) After a long time, the current in the left-hand loop is constant, so there will be $\boxed{\text{no induced current}}$ in the right-hand coil.

(*c*) If the second loop is pulled to the right, the magnetic field out of the page from the left-hand loop through the second loop will decrease. During the motion, there will be an induced current in the right-hand loop that will produce a magnetic field out of the page to oppose the decrease of the field from the left-hand loop. Thus the induced current will be $\boxed{\text{counterclockwise}}$.

$\boxed{73.}$ The electrical energy is dissipated because there is current flowing in a resistor. The power dissipation by a resistor is given by $P = I^2R$, and so the energy dissipated is $E = P\Delta t = I^2R\Delta t$. The current is created by the induced emf caused by the changing B-field. The emf is calculated by Eq. 21-2a.

$$\mathsf{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} \qquad I = \frac{\mathsf{E}}{R} = -\frac{A\Delta B}{R\Delta t}$$

$$E = P\Delta t = I^2R\Delta t = \frac{A^2\left(\Delta B\right)^2}{R^2\left(\Delta t\right)^2}R\Delta t = \frac{A^2\left(\Delta B\right)^2}{R\left(\Delta t\right)} = \frac{\left[\left(0.240\,\text{m}\right)^2\right]^2\left[\left(0-0.665\,\text{T}\right)\right]^2}{\left(5.20\,\Omega\right)\left(0.0400\,\text{s}\right)}$$

$$= \boxed{7.05\times10^{-3}\,\text{J}}$$

74. (*a*) Because $V_S < V_P$, this is a $\boxed{\text{step-down}}$ transformer.

(*b*) Assuming 100% efficiency, the power in both the primary and secondary is 45 W. Find the current in the secondary from the relationship $P = IV$.

$$P_S = I_S V_S \rightarrow I_S = \frac{P_S}{V_S} = \frac{45\,\text{W}}{12\,\text{V}} = \boxed{3.8\,\text{A}}$$

(*c*) $P_P = I_P V_P \rightarrow I_P = \dfrac{P_P}{V_P} = \dfrac{45\,\text{W}}{120\,\text{V}} = \boxed{0.38\,\text{A}}$

(*d*) Find the resistance of the bulb from Ohm's law. The bulb is in the secondary circuit.

$$V_S = I_S R \;\rightarrow\; R = \frac{V_S}{I_S} = \frac{12\,\text{V}}{3.75\,\text{A}} = \boxed{3.2\,\Omega}$$

75. Because there are perfect transformers, the power loss is due to resistive heating in the transmission lines. Since the town requires 50 MW, the power at the generating plant must be $\frac{50\,\text{MW}}{0.985} = 50.761$ MW. Thus the power lost in the transmission is 0.761 MW. This can be used to determine the current in the transmission lines.

$$P = I^2 R \;\rightarrow\; I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.761\times10^6\,\text{W}}{2(118\,\text{km})0.10\,\Omega/\text{km}}} = 179.6\,\text{A}$$

To produce 50.761 MW of power at 179.6 A requires the following voltage.

$$V = \frac{P}{I} = \frac{50.761\times10^6\,\text{W}}{179.6\,\text{A}} = 2.83\times10^5\,\text{V} \approx \boxed{280\,\text{kV}}$$

76. (*a*) From the efficiency of the transformer, we have $P_S = 0.80P_P$. Use this to calculate the current in the primary.

$$P_S = 0.80P_P = 0.80I_PV_P \;\rightarrow\; I_P = \frac{P_S}{0.80V_P} = \frac{75\,\text{W}}{0.80(110\,\text{V})} = 0.8523\,\text{A} \approx \boxed{0.85\,\text{A}}$$

(*b*) The voltage in both the primary and secondary is proportional to the number of turns in the respective coil. The secondary voltage is calculated from the secondary power and resistance since $P = V^2/R$.

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{V_P}{\sqrt{P_S R_S}} = \frac{110\,\text{V}}{\sqrt{(75\,\text{W})(2.4\,\Omega)}} = \boxed{8.2}$$

77. (*a*) The voltage drop across the lines is due to the resistance.

$$V_{\text{out}} = V_{\text{in}} - IR = 42000\,\text{V} - (740\,\text{A})(2)(0.80\,\Omega) = 40816\,\text{V} \approx \boxed{41\text{kV}}$$

(*b*) The power input is given by $P_{\text{in}} = IV_{\text{in}}$.

$$P_{\text{in}} = IV_{\text{in}} = (740\,\text{A})(42000\,\text{V}) = 3.108\times10^7\,\text{W} \approx \boxed{3.1\times10^7\,\text{W}}$$

(*c*) The power loss in the lines is due to the current in the resistive wires.

$$P_{\text{loss}} = I^2R = (740\,\text{A})^2(1.60\,\Omega) = 8.76\times10^5\,\text{W} \approx \boxed{8.8\times10^5\,\text{W}}$$

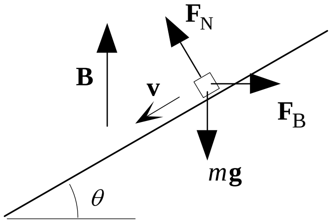
(*d*) The power output is given by $P_{\text{out}} = IV_{\text{out}}$.

$$P_{\text{out}} = IV_{\text{out}} = (740\,\text{A})(40816\,\text{V}) = 3.020\times10^7\,\text{W} \approx \boxed{3.0\times10^7\,\text{W}}.$$

This could also be found by subtracting the power lost from the input power.

$$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 3.108\times10^7\,\text{W} - 8.76\times10^5\,\text{W} = 3.020\times10^7\,\text{W} \approx \boxed{3.0\times10^7\,\text{W}}$$

78. A side view of the rail and bar is shown in the figure. From the discussion in Section 21-3, the emf in the bar is produced by the components of the magnetic field, the length of the bar, and the velocity of the bar, which are all mutually perpendicular. The magnetic field and the length of the bar are already perpendicular. The component of the velocity of the bar that is perpendicular to the magnetic field is $v\cos\theta$, and so the induced emf is given by the following.



$$\mathcal{E} = Blv\cos\theta$$

This produces a current in the wire, which can be found by Ohm's law. That current is pointing into the page on the diagram.

$$I = \frac{\mathcal{E}}{R} = \frac{Blv\cos\theta}{R}$$

Because the current is perpendicular to the magnetic field, the force on the wire from the magnetic field can be calculated from Eq. 20-2, and will be horizontal, as shown in the diagram.

$$F_B = IlB = \frac{Blv\cos\theta}{R}lB = \frac{B^2l^2v\cos\theta}{R}$$

For the wire to slide down at a steady speed, the net force along the rail must be zero. Write Newton's second law for forces along the rail, with up the rail being positive.

$$F_{\text{net}} = F_B\cos\theta - mg\sin\theta = 0 \quad \rightarrow \quad \frac{B^2l^2v\cos^2\theta}{R} = mg\sin\theta \quad \rightarrow$$

$$v = \frac{Rmg\sin\theta}{B^2l^2\cos^2\theta} = \frac{(0.60\,\Omega)(0.040\,\text{kg})\left(9.80\,\text{m/s}^2\right)\sin 6.0^\circ}{(0.55\,\text{T})^2(0.32\,\text{m})^2\cos^2 6.0^\circ} = \boxed{0.80\,\text{m/s}}$$

79. We find the current in the transmission lines from the power transmitted to the user, and then find the power loss in the lines.

$$P_\text{T} = I_\text{L}V \quad \rightarrow \quad I_\text{L} = \frac{P_\text{T}}{V} \qquad P_\text{L} = I_\text{L}^2R_\text{L} = \left(\frac{P_\text{T}}{V}\right)^2 R_\text{L} = \boxed{\frac{P_\text{T}^2R_\text{L}}{V^2}}$$

80. The induced current in the coil is the induced emf divided by the resistance. The induced emf is found from the changing flux by Eq. 21-2b. The magnetic field of the solenoid, which causes the flux, is given by Eq. 20-8. For the area used in Eq. 21-2b, the cross-sectional area of the solenoid (not the coil) must be used, because all of the magnetic flux is inside the solenoid.

$$I = \frac{\mathcal{E}_\text{ind}}{R} \qquad \left|\mathcal{E}_\text{ind}\right| = N_\text{coil}\frac{\Delta\Phi}{\Delta t} = N_\text{coil}A_\text{sol}\frac{\Delta B_\text{sol}}{\Delta t} \qquad B_\text{sol} = \mu_0\frac{N_\text{sol}I_\text{sol}}{l_\text{sol}}$$

$$I = \frac{N_\text{coil}A_\text{sol}\mu_0\dfrac{N_\text{sol}}{l_\text{sol}}\dfrac{\Delta I_\text{sol}}{\Delta t}}{R} = \frac{N_\text{coil}A_\text{sol}\mu_0}{R}\frac{N_\text{sol}}{l_\text{sol}}\frac{\Delta I_\text{sol}}{\Delta t}$$

$$= \frac{(150\,\text{turns})\pi(0.045\,\text{m})^2\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{12\,\Omega}\frac{(230\,\text{turns})}{(0.01\,\text{m})}\frac{2.0\,\text{A}}{0.10\,\text{s}} = \boxed{4.6\times10^{-2}\,\text{A}}$$

As the current in the solenoid increases, a magnetic field from right to left is created in the solenoid and the loop. The induced current will flow in such a direction as to oppose that field, and so must flow from $\boxed{\text{left to right}}$ through the resistor.

81. Putting an inductor in series with the device will protect it from sudden surges in current. The growth of current in an *LR* circuit is given is section 21-11.

$$I=\frac{V}{R}\left(1-e^{-tR/L}\right)=I_{\max}\left(1-e^{-tR/L}\right)$$

The maximum current is 55 mA, and the current is to have a value of 7.5 mA after a time of 120 microseconds. Use this data to solve for the inductance.

$$I=I_{\max}\left(1-e^{-tR/L}\right)\quad\rightarrow\quad e^{-tR/L}=1-\frac{I}{I_{\max}}\quad\rightarrow$$

$$L=-\frac{tR}{\ln\left(1-\dfrac{I}{I_{\max}}\right)}=-\frac{\left(1.2\times10^{-4}\,\mathrm{sec}\right)\left(150\,\Omega\right)}{\ln\left(1-\dfrac{7.5\,\mathrm{mA}}{55\,\mathrm{mA}}\right)}=0.1228\,\mathrm{H}\approx\boxed{0.12\,\mathrm{H\ in\ series}}$$

82. The emf is related to the flux change by Eq. 21-2b. The flux change is caused by the changing magnetic field.

$$\left|\mathcal{E}\right|=N\frac{\Delta\Phi_B}{\Delta t}=N\frac{A\Delta B}{\Delta t}\quad\rightarrow\quad\frac{\Delta B}{\Delta t}=\frac{\left|\mathcal{E}\right|}{NA}=\frac{120\,\mathrm{V}}{\left(25\right)\pi\left(6.25\times10^{-2}\,\mathrm{m}\right)^2}=\boxed{390\,\mathrm{T/s}}$$

83. We find the peak emf from Eq. 21-5.

$$\mathcal{E}_{\text{peak}}=NB\omega A=\left(155\right)\left(0.200\,\mathrm{T}\right)\left(2\pi\,\mathrm{rad/rev}\right)\left(120\,\mathrm{rev/s}\right)\left(6.60\times10^{-2}\,\mathrm{m}\right)^2=\boxed{102\,\mathrm{V}}$$

84. (*a*) The electric field energy density is given by Eq. 17-11, and the magnetic field energy density is given by Eq. 21-10.

$$u_E=\tfrac{1}{2}\varepsilon_0E^2=\tfrac{1}{2}\left(8.85\times10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\left(1.0\times10^4\,\mathrm{V/m}\right)$$

$$=4.425\times10^{-4}\,\mathrm{J/m^3}\approx\boxed{4.4\times10^{-4}\,\mathrm{J/m^3}}$$

$$u_B=\tfrac{1}{2}\frac{B^2}{\mu_0}=\tfrac{1}{2}\frac{\left(2.0\,\mathrm{T}\right)^2}{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}=1.592\times10^6\,\mathrm{J/m^3}\approx\boxed{1.6\times10^6\,\mathrm{J/m^3}}$$

$$\frac{u_B}{u_E}=\frac{1.592\times10^6\,\mathrm{J/m^3}}{4.425\times10^{-4}\,\mathrm{J/m^3}}=\boxed{3.6\times10^9}$$

The energy density in the magnetic field is 3.6 billion times greater than the energy density in the electric field.

(*b*) Set the two densities equal and solve for the magnitude of the electric field.

$$u_E=\tfrac{1}{2}\varepsilon_0E^2=u_B=\tfrac{1}{2}\frac{B^2}{\mu_0}\quad\rightarrow$$

$$E=\frac{B}{\sqrt{\varepsilon_0\mu_0}}=\frac{2.0\,\mathrm{T}}{\sqrt{\left(8.85\times10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}}=\boxed{6.0\times10^8\,\mathrm{V/m}}$$

85. The impedance is found from Eq. 21-14, and then the inductance is found from Eq. 21-11b.

$$Z=X_L=\frac{V_0}{I_0}\qquad X_L=2\pi fL\quad\rightarrow\quad L=\frac{X_L}{2\pi f}=\frac{V_0}{2\pi fI_0}=\frac{220\,\mathrm{V}}{2\pi\left(60.0\,\mathrm{Hz}\right)\left(5.8\,\mathrm{A}\right)}=\boxed{0.10\,\mathrm{H}}$$

86. For the current and voltage to be in phase, the net reactance of the capacitor and inductor must be zero, which means that the circuit is at resonance. Thus Eq. 21-19 applies.

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad\rightarrow\quad C=\frac{1}{4\pi^2 Lf_0^2}=\frac{1}{4\pi^2\left(0.130\,\mathrm{H}\right)\left(1360\,\mathrm{Hz}\right)^2}=\boxed{1.05\times10^{-7}\,\mathrm{F}}$$

87. The resistance of the coil is found from the dc current via Ohm's law.

$$V_{\mathrm{dc}}=I_{\mathrm{dc}}R \quad\rightarrow\quad R=\frac{V_{\mathrm{dc}}}{I_{\mathrm{dc}}}=\frac{36\,\mathrm{V}}{2.5\,\mathrm{A}}=14.4\,\Omega\approx\boxed{14\,\Omega}$$

The impedance of the coil is found from the ac current by Eq. 21-14, and then the inductance can be found from the impedance by Eq. 21-15a.

$$V_{\mathrm{ac}}=ZI_{\mathrm{ac}} \quad\rightarrow\quad Z=\frac{V_{\mathrm{ac}}}{I_{\mathrm{ac}}}=\sqrt{R^2+X_L^2}=\sqrt{R^2+4\pi^2 f^2L^2} \quad\rightarrow$$

$$L=\sqrt{\frac{\left(\frac{V_{\mathrm{ac}}}{I_{\mathrm{ac}}}\right)-R^2}{4\pi^2 f^2}}=\sqrt{\frac{\left(\frac{120\,\mathrm{V}}{3.8\,\mathrm{A}}\right)^2-\left(14.4\,\Omega\right)^2}{4\pi^2\left(60\,\mathrm{Hz}\right)^2}}=7.455\times10^{-2}\,\mathrm{H}\approx\boxed{7.5\times10^{-2}\,\mathrm{H}}$$

88. (*a*) The current is found from the voltage and impedance. The impedance is given by Eq. 21-15.

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}$$

$$=\sqrt{\left(2.0\,\Omega\right)^2+\left[2\pi\left(60\,\mathrm{Hz}\right)\left(0.135\,\mathrm{H}\right)-\frac{1}{2\pi\left(60\,\mathrm{Hz}\right)\left(20\times10^{-6}\,\mathrm{F}\right)}\right]^2}=81.76\,\Omega$$

$$I_{\mathrm{rms}}=\frac{V_{\mathrm{rms}}}{Z}=\frac{45\,\mathrm{V}}{81.76\,\Omega}=\boxed{0.55\,\mathrm{A}}$$

(*b*) Use Eq. 21-16a to find the phase angle.

$$\phi=\tan^{-1}\frac{X_L-X_C}{R}=\tan^{-1}\frac{2\pi fL-\frac{1}{2\pi fC}}{R}$$

$$=\tan^{-1}\frac{2\pi\left(60\,\mathrm{Hz}\right)\left(0.135\,\mathrm{H}\right)-\frac{1}{2\pi\left(60\,\mathrm{Hz}\right)\left(20\times10^{-6}\,\mathrm{F}\right)}}{2.0\,\Omega}=\tan^{-1}\frac{-81.7\,\Omega}{2.0\,\Omega}=\boxed{-89^\circ}$$

89. (*a*) From the text of the problem, the *Q* factor is the ratio of the voltage across the capacitor or inductor to the voltage across the resistor, at resonance. The resonant frequency is given by Eq. 21-19.

$$Q=\frac{V_L}{V_R}=\frac{I_{\mathrm{res}}X_L}{I_{\mathrm{res}}R}=\frac{2\pi f_{\mathrm{res}}L}{R}=\frac{2\pi\frac{1}{2\pi}\sqrt{\frac{1}{LC}}L}{R}=\boxed{\frac{1}{R}\sqrt{\frac{L}{C}}}$$

(*b*) Find the inductance from the resonant frequency, and the resistance from the *Q* factor.

$$f_{\text{res}}=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}\quad\rightarrow$$

$$L=\frac{1}{4\pi^2Cf_{\text{res}}^2}=\frac{1}{4\pi^2\left(1.0\times10^{-8}\,\text{F}\right)\left(1.0\times10^{6}\,\text{Hz}\right)^2}=2.533\times10^{-6}\,\text{H}\approx\boxed{2.5\times10^{-6}\,\text{H}}$$

$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}\quad\rightarrow\quad R=\frac{1}{Q}\sqrt{\frac{L}{C}}=\frac{1}{550}\sqrt{\frac{2.533\times10^{-6}\,\text{H}}{1.0\times10^{-8}\,\text{F}}}=\boxed{2.9\times10^{-2}\,\Omega}$$

CHAPTER 22: Electromagnetic Waves

Answers to Questions

1. If the direction of travel for the EM wave is north and the electric field oscillates east-west, then the magnetic field must oscillate up and down. For an EM wave, the direction of travel, the electric field, and the magnetic field must all be perpendicular to each other.

2. No, sound is not an electromagnetic wave. Sound is a mechanical (pressure) wave. The energy in the sound wave is actually oscillating the medium in which it travels (air, in this case). The energy in an EM wave is contained in the electric and magnetic fields and it does not need a medium in which to travel.

3. Yes, EM waves can travel through a perfect vacuum. The energy is carried in the oscillating electric and magnetic fields and no medium is required to travel. No, sound waves cannot travel through a perfect vacuum. A medium is needed to carry the energy of a mechanical wave such as sound and there is no medium in a perfect vacuum.

4. When you flip a light switch on, the electrons in the filament wire need to move to light the bulb, and these electrons need to receive energy to begin to move. It does take some time for the energy to travel through the wires from the switch to the bulb, but the time is extremely minimal, since the energy travels with the EM fields in the wire at nearly the speed of light. It also takes a while for the EM (light) waves to travel from the bulb to your eye, again at the speed of light, and so very little time passes. Some delay can usually be detected by your eyes due to the fact that the filament takes a little time to heat up to a temperature that emits visible light. Also, depending on the inductance of the circuit, a small amount of time could be added to the delay in your seeing the light go on (inductance acts like an electrical inertia). Thus, no, the light does not go on immediately when you flip the light switch, but the delay is very small.

5. The wavelengths of radio and TV signals are much longer than visible light. Radio waves are on the order of 3 m to 30,000 m. TV waves are on the order of 0.3 m to 3 m. Visible waves are on the order of 10^{-7} m.

6. It is not necessary to make the lead-in wires to your speakers the exact same length. Since energy in the wires travels at nearly the speed of light, the difference in time between the signals getting to the different speakers will be too small for your ears to detect. [Making sure the resistance of your speaker wires is correct is much more important.]

7. Wavelength of 10^3 km: Sub-radio waves (or very long radio waves; for example, ELF waves for submarine communication fall into this category). Wavelength of 1 km: Radio waves. Wavelength of 1 m: TV signals and microwaves. Wavelength of 1 cm: microwaves and satellite TV signals. Wavelength of 1 mm: microwaves and infrared waves. Wavelength of 1 μm: infrared waves.

8. Yes, radio waves can have the same frequencies as sound waves. These 20–20,000 Hz EM waves would have extremely long wavelengths (for example, ELF waves for submarine communication) when compared to the sound waves. A 5000 Hz sound wave has a wavelength of about 70 mm, while a 5000 Hz EM wave has a wavelength of about 60 km.

9. Two TV or two radio stations can be broadcast on the same carrier frequency, but if both signals are of similar strength in the same locality, the signals will be "scrambled". The carrier frequency is used by the receiver to distinguish between different stations. Once the receiver has locked on to a particular carrier frequency, its circuitry then does the work of demodulating the information being carried on that carrier frequency. If two stations had the same carrier frequency, the receiver would try to decipher both signals at once and you would get jumbled information instead of a clear signal.

10. The receiver antenna should also be vertical for obtaining the best reception. The oscillating carrier electric field is up-and-down, so a vertical antenna would "pick up" that signal better, since the electrons in the metal antenna would be forced to oscillate up and down along the entire length of the vertical antenna and creating a stronger signal. This is analogous to polarized light, as discussed in chapter 24.

11. Diffraction effects (the bending of waves around the edge of an object) are only evident when the size of the wavelength of the wave is larger than the size of the object. AM waves have wavelengths that are on the order of 300 m long, while FM waves have wavelengths on the order of 3 m long. Buildings and hills are much larger than FM waves, and so FM waves will not diffract around the buildings and hills. Thus the FM signal will not be received behind the hills or buildings. On the other hand, these objects are smaller than AM waves, and so the AM waves will diffract around them easily. The AM signal can be received behind the objects.

12. Cordless phones utilize EM waves when sending information back and forth between the phone (the part you hold up to your ear/mouth) and its base (where the base is sitting in your house and it is physically connected to the wire phone lines that lead outside to the phone company's network). These EM waves are usually very weak (you can't walk very far away from your house before you lose the signal) and use frequencies such as 49 MHz, 900 MHz, 2.4 GHz or 5 GHz. Cell phones utilize EM waves when sending information back and forth between the phone and the nearest tower in your geographical area (which could be miles away from your location). These EM waves need to be much stronger than cordless phone waves (the batteries are usually much more sophisticated and expensive) and use frequencies of 850 MHz or 1.2 GHz. [Cell phones also have many more channels than cordless phones, so more people can be talking using the same carrier frequency.]

13. Transmitting Morse code by flashing a flashlight on and off is an AM wave. The amplitude of the carrier wave is increasing/decreasing every time you turn the flashlight on and off. The frequency of the carrier wave is visible light, which is approximately 10^{14} Hz.

Solutions to Problems

1. The current in the wires must also be the displacement current in the capacitor. We find the rate at which the electric field is changing from

$$I_D = \varepsilon_0 A\left(\frac{\Delta E}{\Delta t}\right);$$

$$1.8\,\text{A} = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(0.0190\,\text{m}\right)^2\left(\frac{\Delta E}{\Delta t}\right), \text{ which gives } \boxed{7.94\times10^{14}\,\text{V/m•s.}}$$

2. The current in the wires is the rate at which charge is accumulating on the plates and must also be the displacement current in the capacitor. Because the location is outside the capacitor, we can use the expression for the magnetic field of a long wire:

$$B=\left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R}=\frac{\left(10^{-7}\,\mathrm{T{\bullet}m/A}\right)2\left(35.0\times10^{-3}\,\mathrm{A}\right)}{\left(0.100\,\mathrm{m}\right)}=\boxed{7.00\times10^{-8}\,\mathrm{T.}}$$

After the capacitor is fully charged, all currents will be zero, so the magnetic field will be $\boxed{\text{zero.}}$

3. The electric field is

$$E=cB=\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(17.5\times10^{-9}\,\mathrm{T}\right)=\boxed{5.25\,\mathrm{V/m.}}$$

4. The frequency of the two fields must be the same: $\boxed{80.0\text{ kHz.}}$
The rms strength of the electric field is

$$E_{\mathrm{rms}}=cB_{\mathrm{rms}}=\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(6.75\times10^{-9}\,\mathrm{T}\right)=\boxed{2.03\,\mathrm{V/m.}}$$

The electric field is perpendicular to both the direction of travel and the magnetic field, so the electric field oscillates along the $\boxed{\text{horizontal north-south line.}}$

5. The frequency of the microwave is

$$f=\frac{c}{\lambda}=\frac{\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{\left(1.60\times10^{-2}\,\mathrm{m}\right)}=\boxed{1.88\times10^{10}\,\mathrm{Hz.}}$$

6. The wavelength of the radar signal is

$$\lambda=\frac{c}{f}=\frac{\left(3.000\times10^{8}\,\mathrm{m/s}\right)}{\left(29.75\times10^{9}\,\mathrm{Hz}\right)}=\boxed{1.008\times10^{-2}\,\mathrm{m.}}$$

7. The wavelength of the wave is

$$\lambda=\frac{c}{f}=\frac{\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{\left(9.66\times10^{14}\,\mathrm{Hz}\right)}=3.11\times10^{-7}\,\mathrm{m}=\boxed{311\,\mathrm{nm.}}$$

This wavelength is just outside the violet end of the visible region, so it is $\boxed{\text{ultraviolet.}}$

$\boxed{8.}$ The frequency of the wave is

$$f=\frac{c}{\lambda}=\frac{\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{\left(650\times10^{-9}\,\mathrm{m}\right)}=\boxed{4.62\times10^{14}\,\mathrm{Hz.}}$$

This frequency is just outside the red end of the visible region, so it is $\boxed{\text{visible.}}$

9. The time for light to travel from the Sun to the Earth is

$$\Delta t=\frac{L}{c}=\frac{\left(1.50\times10^{11}\,\mathrm{m}\right)}{\left(3.00\times10^{8}\,\mathrm{m/s}\right)}=5.00\times10^{2}\,\mathrm{s}=\boxed{8.33\,\mathrm{min.}}$$

10. The radio frequency is

$$f=\frac{c}{\lambda}=\frac{\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{\left(49\,\mathrm{m}\right)}=\boxed{6.1\times10^{6}\,\mathrm{Hz.}}$$

11. We convert the units:

$$d = (4.2\ \text{ly})(3.00 \times 10^{8}\ \text{m/s})(3.16 \times 10^{7}\ \text{s/yr}) = \boxed{4.0 \times 10^{16}\ \text{m.}}$$

12. The eight-sided mirror would have to rotate 1/8 of a revolution for the succeeding mirror to be in position to reflect the light in the proper direction. During this time the light must travel to the opposite mirror and back. Thus the angular speed required is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\left(\dfrac{2\pi\ \text{rad}}{8}\right)}{\left(\dfrac{2L}{c}\right)} = \frac{(\pi\ \text{rad})c}{8L}$$

$$= \frac{(\pi\ \text{rad})(3.00 \times 10^{8}\ \text{m/s})}{8(3.5 \times 10^{3}\text{m})} = \boxed{3.4 \times 10^{3}\ \text{rad/s}}\ (3.2 \times 10^{4}\ \text{rev/min}).$$

$\boxed{13.}$ (*a*) If we assume the average distance of Earth to Mars, we have

$$\Delta t = \frac{L}{c} = \frac{(227.9 \times 10^{9}\text{m} - 149.6 \times 10^{9}\text{m})}{(3.000 \times 10^{8}\ \text{m/s})} = 261.0\text{s} = \boxed{4.350\text{min}.}$$

(*b*) If we assume the average distance of Earth to Saturn, we have

$$\Delta t = \frac{L}{c} = \frac{(1427 \times 10^{9}\text{m} - 149.6 \times 10^{9}\text{m})}{(3.000 \times 10^{8}\ \text{m/s})} = 4258\text{s} = \boxed{70.1\text{min}.}$$

$\boxed{14.}$ The distance that light travels in one year is

$$d = (3.00 \times 10^{8}\ \text{m/s})(3.156 \times 10^{7}\ \text{s/yr}) = \boxed{9.47 \times 10^{15}\ \text{m.}}$$

15. The mirror must rotate at a minimum rate of

$$\omega = \frac{\left(\frac{1}{6}\text{revolutions}\right)}{t}, \text{where } t = \frac{(2)(12\text{m})}{(3.0 \times 10^{8}\ \text{m/s})} = 8.0 \times 10^{-8}\ \text{s}.$$

Thus $\omega = \dfrac{\left(\frac{1}{6}\text{revolutions}\right)}{(8.0 \times 10^{-8}\ \text{s})} = \boxed{2.1 \times 10^{6}\ \text{revolutions/s.}}$

$\boxed{16.}$ If we ignore the time for the sound to travel to the microphone, the time difference is

$$\Delta t = t_{\text{radio}} - t_{\text{sound}} = \left(\frac{d_{\text{radio}}}{c}\right) - \left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right) = \left(\frac{3000\text{m}}{3.00 \times 10^{8}\ \text{m/s}}\right) - \left(\frac{50\text{m}}{343\text{m/s}}\right) = -0.14\text{s},$$

so the $\boxed{\text{person at the radio hears the voice 0.14 s sooner.}}$

17. The length of the pulse is $\Delta d = c\Delta t$, so the number of wavelengths in this length is

$$N = \frac{(c\Delta t)}{\lambda} = \frac{(3.00 \times 10^{8}\ \text{m/s})(32 \times 10^{-12}\text{s})}{(1062 \times 10^{-9}\text{m})} = \boxed{9040\ \text{wavelengths.}}$$

The time for the length of the pulse to be one wavelength is

$$\Delta t' = \frac{\lambda}{c} = \frac{\left(1062 \times 10^{-9}\,\text{m}\right)}{\left(3.00 \times 10^{8}\,\text{m/s}\right)} = 3.54 \times 10^{-15}\,\text{s} = \boxed{3.54\,\text{fs.}}$$

18. The energy per unit area per unit time is

$$S = \tfrac{1}{2} c\varepsilon_0 E_0^{\;2}$$
$$= \tfrac{1}{2}\left(3.00 \times 10^{8}\,\text{m/s}\right)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(21.8 \times 10^{-3}\,\text{V/m}\right)^2 = \boxed{6.31 \times 10^{-7}\,\text{W/m}^2.}$$

19. The energy per unit area per unit time is

$$S = \frac{cB_{\text{rms}}^{\;2}}{\mu_0} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)\left(40.3 \times 10^{-8}\,\text{T}\right)^2}{\left(4\pi \times 10^{-7}\,\text{T}\bullet\text{m/A}\right)} = 0.194\,\text{W/m}^2.$$

We find the time from

$$t = \frac{U}{AS} = \frac{\left(235\,\text{J}\right)}{\left(1.00 \times 10^{-4}\,\text{m}^2\right)\left(0.194\,\text{W/m}^2\right)} = 1.21 \times 10^{7}\,\text{s} = \boxed{140\ \text{days.}}$$

20. The energy per unit area per unit time is

$$S = c\varepsilon_0 E_{\text{rms}}^{\;2}$$
$$= \left(3.00 \times 10^{8}\ \text{m/s}\right)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.0386\ \text{V/m}\right)^2 = 3.9558 \times 10^{-6}\ \text{W/m}^2.$$

We find the energy transported from

$$U = AS = \left(1.00 \times 10^{-4}\,\text{m}^2\right)\left(3.9558 \times 10^{-6}\ \text{W/m}^2\right)\left(3600\ \text{s/h}\right) = \boxed{1.42 \times 10^{-6}\ \text{J/h.}}$$

21. Because the wave spreads out uniformly over the surface of the sphere, the power flux is

$$S = \frac{P}{A} = \frac{\left(1200\,\text{W}\right)}{\left[4\pi\left(10.0\,\text{m}\right)^2\right]} = \boxed{0.9549\,\text{W/m}^2.}$$

We find the rms value of the electric field from

$$S = c\varepsilon_0 E_{\text{rms}}^{\;2};$$
$$0.9549\,\text{W/m}^2 = \left(3.00 \times 10^{8}\ \text{m/s}\right)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)E_{\text{rms}}^{\;2},\ \text{which gives } E_{\text{rms}} = \boxed{19.0\,\text{V/m.}}$$

22. The energy per unit area per unit time is

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^{\;2};$$
$$\frac{\left(12.8 \times 10^{-3}\,\text{W}\right)}{\left[\tfrac{\pi}{4}\left(1.75 \times 10^{-3}\,\text{m}\right)^2\right]} = \left(3.00 \times 10^{8}\ \text{m/s}\right)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)E_{\text{rms}}^{\;2},$$

which gives $E_{\text{rms}} = \boxed{1420\,\text{V/m.}}$

The rms value of the magnetic field is

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{\left(1420\,\text{V/m}\right)}{\left(3.00 \times 10^{8}\ \text{m/s}\right)} = \boxed{4.72 \times 10^{-6}\ \text{T.}}$$

23. The radiation from the Sun has the same intensity in all directions, so the rate at which it reaches the Earth is the rate at which it passes through a sphere centered at the Sun:

$$P = S4\pi R^2 = \left(1350\,\mathrm{W/m^2}\right)4\pi\left(1.5\times10^{11}\,\mathrm{m}\right)^2 = \boxed{3.8\times10^{26}\,\mathrm{W.}}$$

24. (*a*) We find E using

$$E = cB = \left(3.0\times10^{8}\,\mathrm{m/s}\right)\left(2.5\times10^{-7}\,\mathrm{T}\right) = \boxed{75\,\mathrm{V/m.}}$$

(*b*) Average power per unit area is

$$\bar{I} = \frac{E_0B_0}{\left(2\mu_0\right)} = \frac{\left(75\,\mathrm{V/m}\right)\left(2.5\times10^{-7}\,\mathrm{T}\right)}{\left[2\left(4\pi\times10^{-7}\,\mathrm{N\bullet s^2/C^2}\right)\right]} = \boxed{7.5\,\mathrm{W/m^2}.}$$

25. (*a*) The energy emitted in each pulse is

$$U = Pt = \left(2.8\times10^{11}\,\mathrm{W}\right)\left(1.0\times10^{9}\,\mathrm{s}\right) = \boxed{280\,\mathrm{J.}}$$

(*b*) We find the rms electric field from

$$S = \frac{P}{A} = c\varepsilon_0 {E_{\mathrm{rms}}}^2;$$

$$\frac{\left(2.8\times10^{11}\,\mathrm{W}\right)}{\pi\left(2.2\times10^{-3}\,\mathrm{m}\right)^2} = \left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\bullet m^2}\right){E_{\mathrm{rms}}}^2,$$

which gives $E_{\mathrm{rms}} = \boxed{2.6\times10^{9}\,\mathrm{V/m.}}$

26. We find the pressure

$$P = \frac{\bar{I}}{cA} = \frac{100\,\mathrm{W}}{\left[\left(3.0\times10^{8}\,\mathrm{m/s}\right)4\pi\left(8.0\times10^{-2}\,\mathrm{m}\right)^2\right]} = \boxed{4.1\times10^{-6}\,\mathrm{N/m^2}.}$$

Supposing a 1.0 cm^2 fingertip, the force is

$$F = PA = \left(4.1\times10^{-6}\,\mathrm{N/m^2}\right)\left(1.0\times10^{-4}\,\mathrm{m^2}\right) = \boxed{4.1\times10^{-10}\,\mathrm{N.}}$$

27. (*a*) FM radio wavelengths are

$$\lambda = \frac{c}{f} = \frac{\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{\left(1.08\times10^{8}\,\mathrm{Hz}\right)} = \boxed{2.78\,\mathrm{m}}\text{ to}$$

$$\lambda = \frac{c}{f} = \frac{\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{\left(8.8\times10^{7}\,\mathrm{Hz}\right)} = \boxed{3.4\,\mathrm{m.}}$$

(*b*) AM radio wavelengths are

$$\lambda = \frac{c}{f} = \frac{\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{\left(1.7\times10^{6}\,\mathrm{Hz}\right)} = \boxed{176\,\mathrm{m}}\text{ to}$$

$$\lambda = \frac{c}{f} = \frac{\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{\left(5.35\times10^{5}\,\mathrm{Hz}\right)} = \boxed{561\,\mathrm{m.}}$$

28. The cell phone will receive signals of wavelength

$$\lambda = \frac{c}{f} = \frac{\left(3.0 \times 10^8 \text{ m/s}\right)}{\left(1.9 \times 10^9 \text{ Hz}\right)} = \boxed{0.16 \text{ m.}}$$

29. The frequencies are 940 kHz on the AM dial and 94 MHz on the FM dial. From $c = f\lambda$, we see that the lower frequency will have the longer wavelength: $\boxed{\text{the AM station.}}$

When we form the ratio of wavelengths, we get

$$\frac{\lambda_2}{\lambda_1} = \frac{f_1}{f_2} = \frac{\left(94 \times 10^6 \text{ Hz}\right)}{\left(940 \times 10^3 \text{ Hz}\right)} = \boxed{100.}$$

30. The wavelength of Channel 2 is

$$\lambda_2 = \frac{c}{f_2} = \frac{\left(3.00 \times 10^8 \text{ m/s}\right)}{\left(54.0 \times 10^6 \text{ Hz}\right)} = \boxed{5.56 \text{ m.}}$$

The wavelength of Channel 69 is

$$\lambda_{69} = \frac{c}{f_{69}} = \frac{\left(3.00 \times 10^8 \text{ m/s}\right)}{\left(806 \times 10^6 \text{ Hz}\right)} = \boxed{0.372 \text{ m.}}$$

31. The resonant frequency is given by

$$f_0^2 = \left(\frac{1}{2\pi}\right)^2 \left(\frac{1}{LC}\right).$$

When we form the ratio for the two stations, we get

$$\left(\frac{f_{02}}{f_{01}}\right)^2 = \frac{C_1}{C_2};$$

$$\left(\frac{1610 \text{ kHz}}{550 \text{ kHz}}\right)^2 = \frac{\left(2800 \text{ pF}\right)}{C_2}, \text{ which gives } C_2 = \boxed{327 \text{ pF.}}$$

32. We find the capacitance from the resonant frequency:

$$f_0 = \left(\frac{1}{2\pi}\right)\left(\frac{1}{LC}\right)^{\frac{1}{2}};$$

$$96.1 \times 10^6 \text{ Hz} = \left(\frac{1}{2\pi}\right)\left[\frac{1}{\left(1.8 \times 10^{-6} \text{ H}\right)C}\right]^{\frac{1}{2}}, \text{ which gives } C = 1.5 \times 10^{-12} \text{ F} = \boxed{1.5 \text{ pF.}}$$

$\boxed{33.}$ We find the inductance for the first frequency:

$$f_{01} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{L_1 C}\right)^{\frac{1}{2}};$$

$$8 \times 10^6\,\text{Hz} = \left(\frac{1}{2\pi}\right)\left[\frac{1}{L_1\left(840 \times 10^{-12}\,\text{F}\right)}\right]^{\frac{1}{2}}, \text{ which gives } L_1 = 3.89 \times 10^{-9}\,\text{H} = 3.89\,\text{nH}.$$

For the second frequency we have

$$f_{02} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{L_2 C}\right)^{\frac{1}{2}};$$

$$108 \times 10^6\,\text{Hz} = \left(\frac{1}{2\pi}\right)\left[\frac{1}{L_2\left(840 \times 10^{-12}\,\text{F}\right)}\right]^{\frac{1}{2}}, \text{ which gives } L_2 = 2.59 \times 10^{-9}\,\text{H} = 2.59\,\text{nH}.$$

Thus the range of inductances is $\boxed{2.59\,\text{nH} \le L \le 3.89\,\text{nH}.}$

34. (*a*) The minimum value of *C* corresponds to the higher frequency, so we have

$$f_{01} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{LC_1}\right)^{\frac{1}{2}};$$

$$15.0 \times 10^6\,\text{Hz} = \left(\frac{1}{2\pi}\right)\left[\frac{1}{L\left(82 \times 10^{-12}\,\text{F}\right)}\right]^{\frac{1}{2}}, \text{ which gives } L = 1.373 \times 10^{-6}\,\text{H} = \boxed{1.4\,\mu\text{H}.}$$

(*b*) The maximum value of *C* corresponds to the lower frequency, so we have

$$f_{02} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{LC_2}\right)^{\frac{1}{2}};$$

$$14.0 \times 10^6\,\text{Hz} = \left(\frac{1}{2\pi}\right)\left[\frac{1}{\left(1.373 \times 10^{-6}\,\text{H}\right)C_2}\right]^{\frac{1}{2}}, \text{ which gives } C_2 = 9.41 \times 10^{-11}\,\text{F} = \boxed{941\text{pF}.}$$

35. The rms electric field strength of the beam is given by

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}{}^2;$$

$$\frac{\left(10 \times 10^3\,\text{W}\right)}{\left(\pi\left(750\,\text{m}\right)^2\right)} = \left(3.0 \times 10^8\,\text{m/s}\right)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)E_{\text{rms}}{}^2,$$

which gives $E_{\text{rms}}{}^2 = 2.1314\,\text{V}^2/\text{m}^2$, and $E_{\text{rms}} = \boxed{1.5\,\text{V/m}.}$

36. To produce the voltage over the length of the antenna, we have

$$E_{\text{rms}} = \frac{V_{\text{rms}}}{d} = \frac{\left(1.00 \times 10^{-3}\,\text{V}\right)}{\left(1.60\,\text{m}\right)} = \boxed{7.14 \times 10^{-4}\,\text{V/m}.}$$

The rate of energy transport is

$$S = c\varepsilon_0 E_{\text{rms}}{}^2$$
$$= \left(3.00\times10^{8}\ \text{m/s}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)\left(7.14\times10^{-4}\ \text{V/m}\right)^2 = \boxed{1.35\times10^{-9}\ \text{W/m}^2.}$$

37. After the change occurred, we would find out when the change in radiation reached the Earth:

$$\Delta t = \frac{L}{c} = \frac{\left(1.50\times10^{14}\,\text{m}\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)} = 5.00\times10^{2}\,\text{s} = \boxed{8.33\,\text{min.}}$$

38. The length in space of a burst is

$$d = ct = \left(3.00\times10^{8}\ \text{m/s}\right)\left(10^{-8}\text{s}\right) = \boxed{3\,\text{m.}}$$

39. (*a*) The time for a signal to travel to the Moon is

$$\Delta t = \frac{L}{c} = \frac{\left(3.84\times10^{8}\,\text{m}\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)} = \boxed{1.28\,\text{s.}}$$

(*b*) The time for a signal to travel to Mars at the closest approach is

$$\Delta t = \frac{L}{c} = \frac{\left(78\times10^{9}\,\text{m}\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)} = 260\,\text{s} = \boxed{4.3\,\text{min.}}$$

40. The time consists of the time for the radio signal to travel to Earth and the time for the sound to travel from the loudspeaker:

$$t = t_{\text{radio}} + t_{\text{sound}} = \left(\frac{d_{\text{radio}}}{c}\right) + \left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right)$$
$$= \left(\frac{3.84\times10^{8}\,\text{m}}{3.00\times10^{8}\ \text{m/s}}\right) + \left(\frac{50\,\text{m}}{343\,\text{m/s}}\right) = \boxed{1.43\,\text{s.}}$$

Note that about 10% of the time is for the sound wave.

41. (*a*) The rms value of the associated electric field is found by

$$U = \varepsilon_0 E_{\text{rms}}{}^2 = 4\times10^{-14}\ \text{J/m}^3 = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)E_{\text{rms}}{}^2.$$

Thus $E_{\text{rms}}{}^2 = 4.52\times10^{-3}\ \text{V}^2/\text{m}^2$, and $E_{\text{rms}} = \boxed{0.07\,\text{V/m.}}$

(*b*) A comparable value is found by using the relation

$\overline{I} = \tfrac{1}{2}\varepsilon_0 c E^2 = \dfrac{P}{\left(4\pi r^2\right)}$. Solving for *r* yields

$$r^2 = \frac{P}{\left(2\pi\varepsilon_0 c E^2\right)} = \frac{10^{5}\,\text{W}}{\left[2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)\left(3\times10^{8}\ \text{m/s}\right)\left(0.1\text{V/m}\right)^2\right]}$$
$$= 5.995\times10^{7}\ \text{m}^2,\ \text{and}\ r = 7.7\times10^{3}\,\text{m} = \boxed{8\,\text{km.}}$$

$\boxed{42.}$ The light has the same intensity in all directions, so the energy per unit area per unit time over a sphere centered at the source is

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \frac{(95\,\text{W})}{4\pi(2.00\,\text{m})^2} = 1.89\,\text{W/m}^2.$$

We find the electric field from

$$S = \tfrac{1}{2}c\varepsilon_0 E_0{}^2;$$

$$1.89\,\text{W/m}^2 = \tfrac{1}{2}(3.00\times10^8\,\text{m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N•m}^2)E_0{}^2, \text{ which gives } E_0 = \boxed{37.7\ \text{V/m.}}$$

The magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{(37.7\,\text{V/m})}{(3.00\times10^8\,\text{m/s})} = \boxed{1.26\times10^{-7}\,\text{T.}}$$

43. The radiation from the Sun has the same intensity in all directions, so the rate at which it passes through a sphere centered at the Sun is

$$P = S4\pi R^2.$$

The rate must be the same for the two spheres, one containing the Earth and one containing Mars. When we form the ratio, we get

$$\frac{P_{\text{Mars}}}{P_{\text{Earth}}} = \left(\frac{S_{\text{Mars}}}{S_{\text{Earth}}}\right)\left(\frac{R_{\text{Mars}}}{R_{\text{Earth}}}\right)^2;$$

$$1 = \left(\frac{S_{\text{Mars}}}{1350\,\text{W/m}^2}\right)(1.52)^2, \text{ which gives } S_{\text{Mars}} = 584.3\,\text{W/m}^2.$$

We find the rms value of the electric field from

$$S_{\text{Mars}} = c\varepsilon_0 E_{\text{rms}}{}^2;$$

$$584.3\,\text{W/m}^2 = (3.00\times10^8\,\text{m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N•m}^2)E_{\text{rms}}{}^2, \text{which gives } E_{\text{rms}} = \boxed{469\,\text{V/m.}}$$

44. If we curl the fingers of our right hand from the direction of the electric field (south) into the direction of the magnetic field (west), our thumb points down, so the direction of the wave is $\boxed{\text{downward.}}$
We find the electric field from

$$S = \tfrac{1}{2}c\varepsilon_0 E_0{}^2;$$

$$560\,\text{W/m}^2 = \tfrac{1}{2}(3.00\times10^8\,\text{m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N•m}^2)E_0{}^2, \text{which gives } E_0 = \boxed{649\,\text{V/m.}}$$

The magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{(649\,\text{V/m})}{(3.00\times10^8\,\text{m/s})} = \boxed{2.16\times10^{-6}\,\text{T.}}$$

45. The wavelength of the AM radio signal is

$$\lambda = \frac{c}{f} = \frac{(3.0\times10^8\,\text{m/s})}{(10^6\,\text{Hz})} = 300\,\text{m}.$$

(*a*) $\tfrac{1}{2}\lambda = \tfrac{1}{2}(300\,\text{m}) = \boxed{150\,\text{m.}}$

(*b*) $\tfrac{1}{4}\lambda = \tfrac{1}{4}(300\,\text{m}) = \boxed{75\,\text{m.}}$

46. We find the magnetic field from

$$S = \tfrac{1}{2}\left(\frac{c}{\mu_0}\right)B_0^{\,2};$$

$$1.0\times10^{-4}\ \mathrm{W/m^2} = \tfrac{1}{2}\left[\frac{\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}\right]B_0^{\,2}\text{, which gives } B_0 = 9.15\times10^{-10}\,\mathrm{T}.$$

Because this field oscillates through the coil at $\omega = 2\pi f$, the maximum emf is

$$\xi_0 = NAB_0\omega = (380\,\text{turns})\pi(0.0011\mathrm{m})^2\left(9.15\times10^{-10}\ \mathrm{T}\right)2\pi\left(810\times10^{3}\mathrm{Hz}\right) = 6.73\times10^{-6}\,\mathrm{V}.$$

The ξ_{rms} is $\dfrac{\xi_0}{\sqrt{2}} = \dfrac{\left(6.73\times10^{-6}\mathrm{V}\right)}{\sqrt{2}} = \boxed{4.76\times10^{-6}\ \mathrm{V}.}$

47. (*a*) The energy received by the antenna is

$$U = IAt = \left(1.0\times10^{-13}\ \mathrm{W/m^2}\right)\pi\left(\frac{0.33\,\mathrm{m}}{2}\right)^2(6.0\,\mathrm{h})(3600\,\mathrm{s/h}) = \boxed{1.8\times10^{-10}\ \mathrm{J.}}$$

(*b*) The electric and magnetic field amplitudes are described by

$$\bar{I} = \left(\frac{1}{2}\right)\varepsilon_0 cE_0^{\,2} = \left(\frac{1}{2}\right)\left(\frac{c}{\mu_0}\right)B_0^{\,2}.$$

$$1.0\times10^{-13}\ \mathrm{W/m^2} = \left(\frac{1}{2}\right)\left(8.85\times10^{-12}\mathrm{C^2/N{\bullet}m^2}\right)\left(3\times10^{8}\ \mathrm{m/s}\right)E_0^{\,2}.$$

Solving for E_0 yields $\boxed{E_0 = 8.7\times10^{-6}\ \mathrm{V/m.}}$

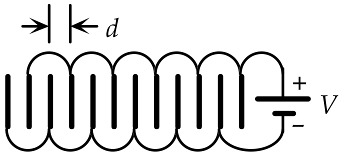
Substitution then gives $B_0 = \dfrac{E_0}{c} = \boxed{2.9\times10^{-14}\ \mathrm{T.}}$

48. To find the average output power we first find the average intensity.

$$\bar{I} = \left(\frac{1}{2}\right)\varepsilon_0 cE_0^{\,2} = \left(\frac{1}{2}\right)\left(8.85\times10^{-12}\mathrm{C^2/N{\bullet}m^2}\right)\left(3\times10^{8}\ \mathrm{m/s}\right)(0.12\,\mathrm{V/m})^2 = 1.9116\times10^{-5}\ \mathrm{W/m^2}.$$

Now $P = \bar{I}A = \left(1.9116\times10^{-5}\ \mathrm{W/m^2}\right)4\pi\left(15\times10^{3}\mathrm{m}\right)^2 = \boxed{54\,\mathrm{kW.}}$

49. (*a*) We see from the diagram that all positive plates are connected to the positive side of the battery, and that all negative plates are connected to the negative side of the battery, so the 11 capacitors are connected in $\boxed{\text{parallel.}}$



(*b*) For parallel capacitors, the total capacitance is the sum, so we have

$$C_{\min} = 11\left(\frac{\varepsilon_0 A_{\min}}{d}\right)$$

$$= \frac{11\left(8.85\times10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\left(1.0\times10^{-4}\,\mathrm{m^2}\right)}{\left(1.1\times10^{-3}\,\mathrm{m}\right)} = 8.9\times10^{-12}\,\mathrm{F} = 8.9\,\mathrm{pF};$$

$$C_{\max} = 11\left(\frac{\varepsilon_0 A_{\max}}{d}\right)$$

$$= \frac{11\left(8.85\times10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\left(9.0\times10^{-4}\,\mathrm{m^2}\right)}{\left(1.1\times10^{-3}\,\mathrm{m}\right)} = 80\times10^{-11}\,\mathrm{F} = 80\,\mathrm{pF}.$$

Thus the range is $\boxed{8.9\,\mathrm{pF} \le C \le 80\,\mathrm{pF}.}$

(*c*) The lowest resonant frequency requires the maximum capacitance.
We find the inductance for the lowest frequency:

$$f_{01} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{L_1 C_{\max}}\right)^{\frac{1}{2}};$$

$$550\times10^3\,\mathrm{Hz} = \left(\frac{1}{2\pi}\right)\left[\frac{1}{L_1\left(80\times10^{-12}\,\mathrm{F}\right)}\right]^{\frac{1}{2}},\text{ which gives } L_1 = 1.05\times10^{-3}\,\mathrm{H} = 1.05\,\mathrm{mH}.$$

We must check to make sure that the highest frequency can be reached.
We find the resonant frequency using this inductance and the minimum capacitance:

$$f_{0\max} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{L_1 C_{\min}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{2\pi}\right)\left[\frac{1}{\left(1.05\times10^{-3}\,\mathrm{H}\right)\left(8.9\times10^{-12}\,\mathrm{F}\right)}\right]^{\frac{1}{2}} = 1.64\times10^6\,\mathrm{Hz} = 1640\,\mathrm{kHz}.$$

Because this is greater than the highest frequency desired, the inductor will work.
We could also start with the highest frequency.
We find the inductance for the highest frequency:

$$f_{02} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{L_2 C_{\min}}\right)^{\frac{1}{2}};$$

$$1600\times10^3\,\mathrm{Hz} = \left(\frac{1}{2\pi}\right)\left[\frac{1}{L_2\left(8.9\times10^{-12}\,\mathrm{F}\right)}\right]^{\frac{1}{2}},\text{ which gives } L_2 = 1.22\times10^{-3}\,\mathrm{H} = 1.22\,\mathrm{mH}.$$

We must check to make sure that the lowest frequency can be reached.
We find the resonant frequency using this inductance and the maximum capacitance:

$$f_{0\min} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{L_2 C_{\max}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{2\pi}\right)\left[\frac{1}{\left(1.22\times10^{-3}\,\mathrm{H}\right)\left(80\times10^{-12}\,\mathrm{F}\right)}\right]^{\frac{1}{2}} = 509\times10^5\,\mathrm{Hz} = 509\,\mathrm{kHz}.$$

Because this is less than the lowest frequency desired, this inductor will also work.
Thus the range of inductances is $\boxed{1.05\,\text{mH} \le L \le 1.22\,\text{mH.}}$

50. Again using the relationship between average intensity and electric field strength,

$$\bar{I} = \left(\frac{1}{2}\right)\varepsilon_0 c E_0^{\ 2} = \left(\frac{1}{2}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3\times10^8\,\text{m/s}\right)\left(0.020\,\text{V/m}\right)^2 = 5.31\times10^{-7}\,\text{W/m}^2.$$

$$25\times10^3\,\text{W} = (5.31\times10^{-7}\,\text{W/m}^2)4\pi r^2.$$

Thus, to receive the transmission one should be within the radius $r = \boxed{61\text{ km.}}$

51. The light has the same intensity in all directions, so the energy per unit area per unit time over a sphere centered at the source is

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \tfrac{1}{2}c\varepsilon_0 E_0^{\ 2} = \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0,\ \text{which gives } E_0 = \left(\frac{\mu_0 c P_0}{2\pi r^2}\right)^{\frac{1}{2}}.$$

52. (*a*) The radio waves have the same intensity in all directions, so the energy per unit area per unit time over a sphere centered at the source with a radius of 100 m is

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \frac{\left(500\times10^3\,\text{W}\right)}{4\pi\left(100\,\text{m}\right)^2} = 0.398\,\text{W/m}^2.$$

Thus the power through the area is

$$P = SA = \left(0.398\,\text{W/m}^2\right)\left(1.0\,\text{m}^2\right) = \boxed{0.40\,\text{W.}}$$

(*b*) We find the rms value of the electric field from

$$S = c\varepsilon_0 E_{\text{rms}}^{\ 2};$$

$$0.398\,\text{W/m}^2 = \left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)E_{\text{rms}}^{\ 2},\ \text{which gives } E_{\text{rms}} = \boxed{12\,\text{V/m.}}$$

(*c*) The voltage over the length of the antenna is

$$E_{\text{rms}} = V_{\text{rms}} = E_{\text{rms}}d = \left(12\,\text{V/m}\right)\left(1.0\,\text{m}\right) = \boxed{12\,\text{V.}}$$

53. (*a*) The radio waves have the same intensity in all directions, so the energy per unit area per unit time over a sphere centered at the source with a radius of 100 km is

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \frac{\left(50\times10^3\,\text{W}\right)}{4\pi\left(100\times10^3\,\text{m}\right)^2} = 3.98\times10^{-7}\,\text{W/m}^2.$$

Thus the power through the area is

$$P = SA = \left(3.98\times10^{-7}\,\text{W/m}^2\right)\left(1.0\,\text{m}^2\right) = 3.98\times10^{-7}\,\text{W} = \boxed{0.40\,\mu\text{W.}}$$

(*b*) We find the rms value of the electric field from

$$S = c\varepsilon_0 E_{\text{rms}}^{\ 2};$$

$$3.98\times10^{-7}\,\text{W/m}^2 = \left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)E_{\text{rms}}^{\ 2},$$

which gives $E_{\text{rms}} = \boxed{0.012\,\text{V/m.}}$

(*c*) The voltage over the length of the antenna is

$$E_{\text{rms}} = V_{\text{rms}} = E_{\text{rms}}d = \left(0.012\,\text{V/m}\right)\left(1.0\,\text{m}\right) = \boxed{0.012\,\text{V.}}$$

54. The energy per unit area per unit time is

$$S = \tfrac{1}{2}c\varepsilon_0 {E_0}^2;$$

$$= \tfrac{1}{2}\left(3.00 \times 10^8\ \mathrm{m/s}\right)\left(8.85 \times 10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\left(3 \times 10^6\ \mathrm{V/m}\right)^2 = 1.20 \times 10^{10}\ \mathrm{W/m^2}.$$

The power output is

$$P = S4\pi r^2 = \left(1.20 \times 10^{10}\ \mathrm{W/m^2}\right)4\pi\left(1.0\,\mathrm{m}\right)^2 = \boxed{1.5 \times 10^{11}\,\mathrm{W.}}$$

CHAPTER 23: Light: Geometric Optics

Answers to Questions

1. (*a*) With a rough surface, the Moon would look just like it does now, because it has a rough surface. During the times of the month that we can see portions of the lit side, we see all parts of it reflecting back sunlight to us. Each and every spot is basically a point source of light, sending us reflections from any part that is lit by sunlight. This produces a relatively uniform color and intensity from all of the lit parts that we can see.

   (*b*) With a polished, mirror-like surface, the Moon would be mostly invisible to us, except for the one small spot that reflects an image of the Sun to us. We would also be able to see tiny bright dots scattered about on the Moon, as the light from stars is reflected back to us. We would also be able to see a small reflection of the lit portion of Earth at one spot. (This actually occurs right now in a diffuse way: Sunlight reflected from Earth to the Moon and then back to us gives an effect called "Earth shine," which can best be seen each month right after New Moon.)

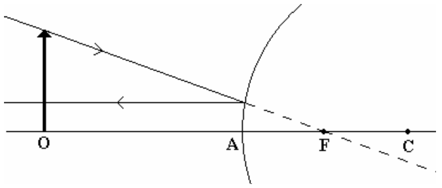
$\boxed{2.}$ No, it is not reasonable to be able to start all of the ships in a harbor on fire at once with a spherical mirror. It may be possible to start them on fire one at a time, though. Archimedes could focus the Sun's rays down to a single point with a spherical mirror, which would get this one point hot enough to start a fire, but it is not possible to do this over a large area like an entire harbor. The other problem is that the spherical mirror he had would only have one focal length, so he would have to be the same distance from each ship that he started on fire. If one ship was farther away than another, then he would have to move closer to the harbor to start the farther ship on fire. This would take a long time and he would have to move his mirror over large distances just to start the ships on fire one at a time. This would also require a very precisely ground and polished mirror that is huge and has an extremely long focal length, which would have been difficult in Archimedes time.

3. The mirror actually doesn't reverse right and left, either, just as it doesn't reverse up and down. When you are looking in a flat mirror and move your right hand, it is the image of your right hand that moves in the mirror. When we see our image, though, we imagine it as if it is another person looking back at us. If we saw this other person raise their left hand, we would see the hand that is on the right side of their body (from our point of view) move.

4. Yes. When a concave mirror produces a real image of a real object, this means d_i and d_o are positive. The magnification equation $\left(m=-d_i/d_o\right)$ then says that m is negative, which means the image is inverted. (If a virtual object is used, though, to make a real image, then d_o is negative and d_i is positive, which would give us a positive m and an upright image.)

5. When an object is placed in front of a spherical mirror and the resulting magnification is -3.0, the image is real and inverted. A negative magnification tells us that the image is inverted, when compared to the object. For a real object, the object distance is positive. Since the magnification is negative, the magnification equation $\left(m=-d_i/d_o\right)$ says that the image distance is also positive. The mirror is concave. A convex mirror cannot make an image of a real object that is magnified. The image is located on the reflecting side of the mirror. When the image distance is positive, as it is in this situation, the image is always located on the reflecting (real) side of the mirror.

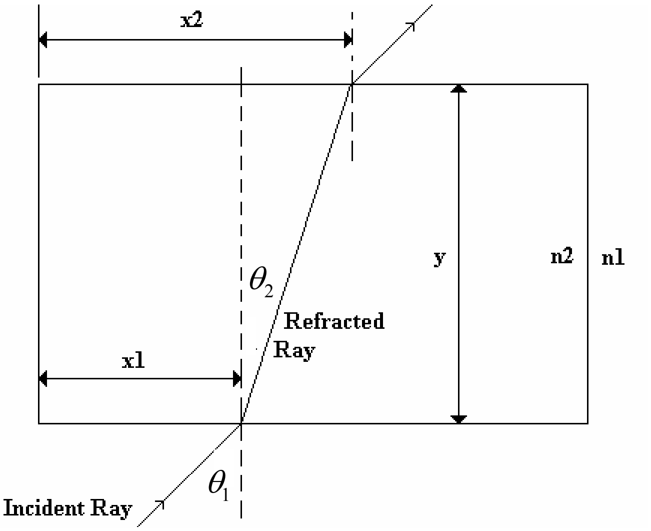
6. Ray 2 starts at the tip of the arrow (object) in front of the mirror and heads toward F (the focal point) on the other side of the mirror. When the ray encounters the reflecting surface of the mirror, it bounces back away from the mirror and parallel to the axis.



$\boxed{7.}$ The focal length of a plane mirror is infinity. The magnification of a plane mirror is +1. As a spherical mirror gets a larger and larger radius, the front surface gets more and more flat. The ultimate limit is that as the radius (and focal length) of the spherical mirror goes to infinity, the front surface becomes perfectly flat. For this mirror, the image height and object height are identical (as are the image distance and object distance) and the image is virtual, with a magnification of $m = +1$.

8. This effect is similar to diffuse reflection off of a rough surface. The ripply sea provides many different surface orientations, and so the light from the Moon reflects from that surface at many different angles. This makes it look like there is one continuous reflection along the surface of the water between you and the Moon, instead of just one specular reflection spot.

$\boxed{9.}$ When a light ray meets the boundary between two materials perpendicularly, both the angle of incidence and the angle of refraction are 0°. Snell's law shows this to be true for any combination of the indices of refraction of the two materials.

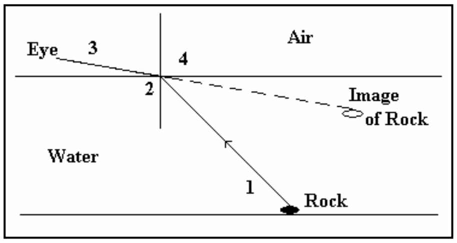


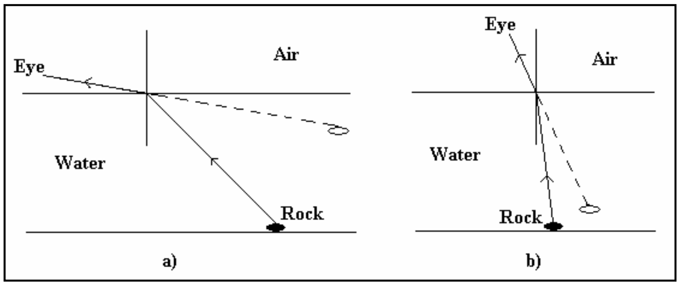
10. To determine the speed of light in a solid, transparent, rectangular object you would need to measure the incident and refracted angles. To do this, start by aiming a laser beam at a convenient angle at one side of the block (θ_1). Measure the distance from one end of the block to the point where the beam enters the block (x_1), measure the thickness of the block (y), and measure the distance from the same end of the block to the point where the beam exits the block (x_2). Using geometry, the refracted angle is $\tan^{-1}\frac{x_2 - x_1}{y}$. Use Snell's law to find the index of refraction of the block: $n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2}$. The index of refraction for air (n_1) is basically equal to 1. Then, we can finally use $v_2 = \frac{c}{n_2}$ to find the speed of light in the block.

11. When you look down into a lake or pool, you will underestimate its depth. Light that is shining on the bottom of the pool will reflect off diffusely in all directions. When these light rays reach the surface, they will be bent away from the normal, according to Snell's law. Some of these rays go to your eyes, which means that you can "see" the bottom of the pool. Your brain interprets these rays as if they all traveled in straight lines only and did not bend, and so you "see" the bottom of the pool as shallower than it actually is. In the first diagram: 1 – Light leaves the rock on the bottom of the pool. 2 – Light bends

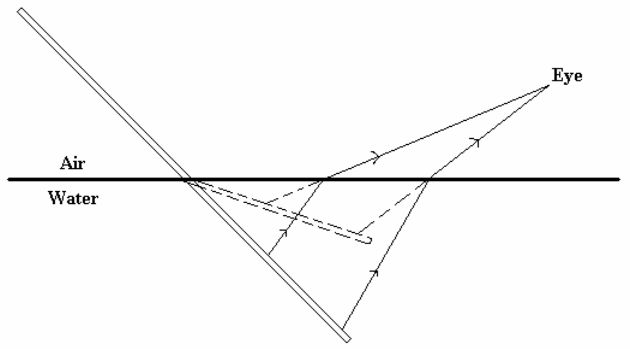
away from the normal as it leaves the water and enters the air. 3 – The light enters your eye. 4 – Your brain recreates the ray's path back to a shallower point.

As the viewing angle gets large (when the angle of refraction increases), the pool appears to get more shallow. Think about viewing a submerged object from two different angles: a) from nearly straight above and b) from a large viewing angle, in the second diagram. Your brain "draws" a very shallow angle into the water when the viewing angle is large, as compared to when the viewing angle is small, due to Snell's law.

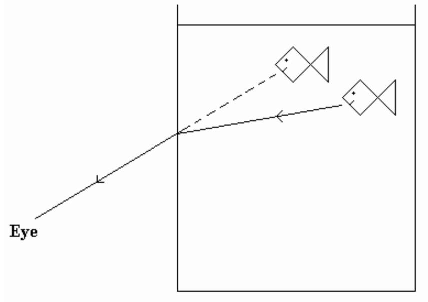




12. Your brain interprets the bending rays as if that the part of the stick that is underwater is shallower than it really is, so it looks bent. Each point on the stick that is underwater appears to you to be closer to the surface than it actually is.



13. The light rays leaving the fish bend away from the normal as they exit the tank and go into your eye. Thus, you see the image of the fish "above" (in this figure) the position of the real fish. Also, the image appears to be slightly closer to the wall of the aquarium. Since your brain doesn't know that light bends at the water/air interface, when your brain "follows" the ray back into the tank, it interprets the location of the fish as above its actual location.



14. One reason you can see a drop of transparent and colorless water on a surface is refraction. The light leaving the tabletop and going through the drop on the way to your eye will make that portion of the tabletop seem to be raised up, due to depth distortion and/or magnification due to the round drop. A second reason that you can see the drop is reflection. At certain angles, the light sources in the room will reflect off of the drop and give its location away.

15. When you are underwater and looking up out of the water at objects above, they all seem much smaller than they would look directly in air. As Conceptual Example 23-8 says, you would see the entire outside world compressed into a circle whose edge makes a 49° angle with the vertical. This compression will make objects above the water look smaller than they really are. See Figure 23-25.

16. A spherical mirror can have negative object distance if some other system of lenses and/or mirrors forms an image that lies behind the spherical mirror – that is, the image of the other optics lies on the non-reflecting side of the spherical mirror. Then, since the object for our spherical mirror is on the back side of the mirror, the convention is to label this a negative object distance.

17. (*a*) The fact that light rays from stars (including our Sun) bend toward the vertical direction as they pass through Earth's atmosphere makes sense because the index of refraction of the atmosphere is slightly greater than that of the vacuum of space. Thus, as the light rays from stars enter the atmosphere, they slow down and bend toward the vertical, according to Snell's law.
    (*b*) The apparent positions of stars in the sky are too high when compared to actual positions. Since the light rays bend toward the vertical as they come into the atmosphere and then into our eyes, as we "follow" these rays back into space, which doesn't take the bending into account, the image of the stars appears higher than the actual position.

18. To make a sharp image of an object that is very far away, the film of a camera must be placed at the focal length of the lens. Objects that are very far away have light rays coming into the camera that are basically parallel and these rays will be bent and focused to create an image at the focal point of the lens, which is where the film should be in order to record a focused image.

19. The mirror in Figure 23-46 is concave. The person is standing within the focal length of the mirror, which makes an upright and larger virtual image for a concave mirror. See Figure 23-16 and Example 23-3 to see a similar situation.

20. The camera lens moves farther from the film when the photographer refocuses. By moving closer to the object, the object distance has become smaller, which the thin lens equation says increases the image distance (and the image is now behind the film). To bring the image forward (and back to the film), you need to move the lens away from the film.

21. A diverging lens can actually form a real image if some other piece of optics forms a virtual object for the diverging lens. For a real object, though, a diverging lens cannot create a real image.

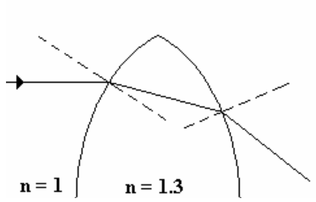
22. For a converging lens, with $d_o > f$, a real image will always be inverted, since the rays must cross the optical axis to create the image. See Figure 23-34 or Figure 23-37. For a converging lens, with $d_o < f$, a virtual image will always be upright, since the rays stay on the same side of the optical axis to create the image. See Figure 23-40. For a diverging lens, anywhere you put d_o, a virtual image will always be upright, since the rays stay on the same side of the optical axis to create the image. See Figure 23-36 or Figure 23-38.

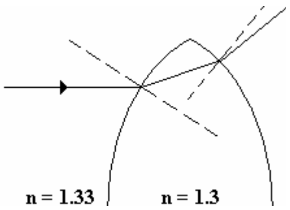
23. Yes, to say that light rays are "reversible" is consistent with the thin lens equation. Since $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$, if you interchange the image and object positions, the equation is unchanged.

24. Real images can be projected onto a screen. In this situation, real light rays actually focus at the position of the image, which can be seen on the screen. But virtual images cannot be projected onto a screen. In this situation, no real light rays are crossing at the position of the virtual image, and so they cannot be seen on a screen (recall a plane mirror's virtual image is actually behind the mirror, where no real light rays travel). Both real images and virtual images can be photographed. The lenses in the camera are designed to focus either diverging or converging light rays down onto the film (similar to a human eye, which can also focus either diverging or converging light rays down onto our "film").

25. (*a*) Yes, as a thin converging lens is moved closer to a nearby object, the position of the real image changes. In this situation, the object distance decreases and the thin lens equation says that the image distance increases. Thus, the position of the image moves farther away from the lens.
    (*b*) Yes, as a thin converging lens is moved closer to a nearby object, the size of the real image changes. In this situation, the object distance decreases and the thin lens equation says that the image distance increases, which means that the magnification $\left(m=-d_\text{i}/d_\text{o}\right)$ gets larger. Thus, the size of the image increases.

26. No. If a lens with $n = 1.3$ is a converging lens in air, it will become a diverging lens when placed in water, with $n = 1.33$. The figure on the left shows that as parallel light rays enter the lens when it is in air, at the first surface the ray is bent toward the normal and at the second surface the ray is bent away from the normal, which is a net converging situation. The figure on the right shows that as parallel light rays enter the lens when it is in water, at the first surface the ray is bent away from the normal and at the second surface the ray is bent toward the normal, which is a net diverging situation.





27. The nose of the dog will be magnified more than the tail. The dog's nose is closer to the converging lens, so the thin lens equation says that this smaller object distance will create a larger image distance. The magnification equation $\left(m=-d_\text{i}/d_\text{o}\right)$, in turn, tells us that with a smaller d_o and a larger d_i, the nose will have a larger magnification than the tail.

28. If the nose of the cat was closer to the converging lens than the focal point while the tail was farther away than the focal point, then the image of the nose would be virtual and the image of the tail would be real. The real image of the rear parts of the cat (from the tip of its tail to the focal point of the lens) will be spread out all the way to infinity on the other side of the lens. The front parts of the cat (from the focal point to the tip of its nose) will be spread out from infinity on the virtual side of the lens to nearly the focal point.

29. If the converging lens does not have a shorter focal length than the diverging lens, then the converging lens' image (which will be a virtual object for the diverging lens) will be too far away from the diverging lens and it will just form another virtual image. This situation does us no good in determining the focal length of the diverging lens, since we can't measure the distance from the lens to a virtual image (we can't project it on to a screen, for example).

30. You can have a virtual object any time that one lens creates a virtual image that another lens uses as an object.

31. No, the image point of an unsymmetrical lens does not change if you turn the lens around. The lensmaker's equation, $\frac{1}{f}=\left(n-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$, is symmetrical (even if the lens is not). Turning the lens around interchanges R_1 and R_2, which does not change the focal length or the image point.

32. To make a lens fatter in the middle, make the radii of the two convex sides smaller. Consider the

lensmaker's equation: $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$. If R_1 and R_2 decrease, the focal length decreases.

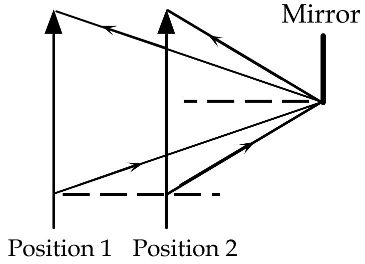
33. When an object is placed so that its image from one converging lens is placed at the focal point of a second converging lens, the final image is formed at infinity by the second lens. A real image is similar to a real object and when this image is placed at the focal point of another lens, the rays will head to the second lens in such a way as to be sent out on the other side of the lens in a direction parallel to the optical axis, which forms an image at infinity.

Solutions to Problems

1. For a flat mirror the image is as far behind the mirror as the object is in front, so the distance from object to image is

$$d_o + d_i = 2.5\,\text{m} + 2.5\,\text{m} = \boxed{5.0\,\text{m.}}$$

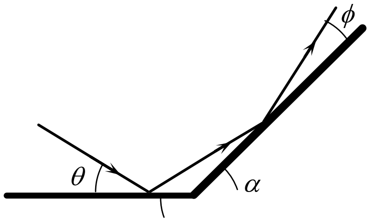
2. Because the angle of incidence must equal the angle of reflection, we see from the ray diagrams that the ray that reflects to your eye must be as far below the horizontal line to the reflection point on the mirror as the top is above the line, regardless of your position.



3. From the triangle formed by the mirrors and the first reflected ray, we have

$$\theta + \alpha + \phi = 180^\circ;$$

$$40^\circ + 135^\circ + \phi = 180^\circ, \text{ which gives } \phi = \boxed{5^\circ.}$$

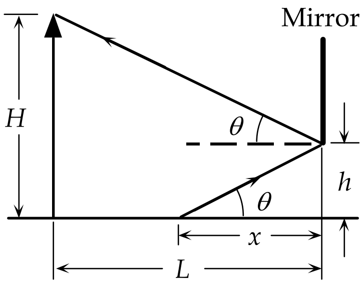


4. The angle of incidence is the angle of reflection. Thus we have

$$\tan\theta = \frac{(H-h)}{L} = \frac{h}{x};$$

$$\frac{(1.68\,\text{m} - 0.43\,\text{m})}{(2.10\,\text{m})} = \frac{(0.43\,\text{m})}{x},$$

which gives $x = 0.72\,\text{m} = \boxed{72\,\text{cm.}}$



5. Because the rays entering your eye are diverging from the image position behind the mirror, the diameter of the area on the mirror and the diameter of your pupil must subtend the same angle from the image:

$$\frac{D_{\text{mirror}}}{d_i} = \frac{D_{\text{pupil}}}{(d_o + d_i)};$$

$$\frac{D_{\text{mirror}}}{(90\,\text{cm})}=\frac{(5.5\,\text{mm})}{(90\,\text{cm}+90\,\text{cm})},\text{ which gives }D_{\text{mirror}}=2.75\,\text{mm}.$$

Thus the area on the mirror is

$$A_{\text{mirror}}=\tfrac{1}{4}\pi D_{\text{mirror}}{}^2=\tfrac{1}{4}\pi\left(2.75\times10^{-3}\,\text{m}\right)^2=\boxed{5.9\times10^{-6}\,\text{m}^2.}$$

6. For the first reflection at A the angle of incidence θ_1 is the angle of reflection. For the second reflection at B the angle of incidence θ_2 is the angle of reflection. We can relate these angles to the angle at which the mirrors meet, ϕ, by using the sum of the angles of the triangle ABC:

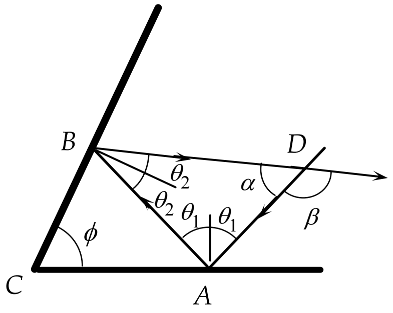
$$\phi+(90^\circ-\theta_1)+(90^\circ-\theta_2)=180^\circ,\text{ or }\phi=\theta_1+\theta_2.$$

In the same way, for the triangle ABD, we have

$$\alpha+2\theta_1+2\theta_2=180^\circ,\text{ or }\alpha=180^\circ-2(\theta_1+\theta_2)=180^\circ-2\phi.$$

At point D we see that the deflection is

$$\beta=180^\circ-\alpha=180^\circ-(180^\circ-2\phi)=2\phi.$$



7. The rays from the Sun will be parallel, so the image will be at the focal point. The radius is

$$r=2f=2(18.0\,\text{cm})=\boxed{36.0\,\text{cm.}}$$

8. To produce an image at infinity, the object will be at the focal point:

$$d_{\text{o}}=f=\frac{r}{2}=\frac{(23.0\,\text{cm})}{2}=\boxed{11.5\,\text{cm.}}$$

$\boxed{9.}$ The ball is a convex mirror with a focal length

$$f=\frac{r}{2}=\frac{(-4.5\,\text{cm})}{2}=-2.25\,\text{cm}.$$

We locate the image from

$$\left(\frac{1}{d_{\text{o}}}\right)+\left(\frac{1}{d_{\text{i}}}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(30.0\,\text{cm})}\right]+\left(\frac{1}{d_{\text{i}}}\right)=\frac{1}{(-2.25\,\text{cm})},\text{ which gives }d_{\text{i}}=-2.09\,\text{cm}.$$

The image is $\boxed{\text{2.09 cm behind the surface, virtual.}}$

The magnification is

$$m=-\frac{d_{\text{i}}}{d_{\text{o}}}=\frac{-(-2.09\,\text{cm})}{(30.0\,\text{cm})}=+0.070.$$

Because the magnification is positive, the image is $\boxed{\text{upright.}}$

10. We find the image distance from the magnification:

$$m=\frac{h_{\text{i}}}{h_{\text{o}}}=\frac{-d_{\text{i}}}{d_{\text{o}}};$$

$$+3=\frac{-d_\mathrm{i}}{(1.4\,\mathrm{m})},\text{ which gives } d_\mathrm{i}=-4.2\,\mathrm{m}.$$

We find the focal length from

$$\left(\frac{1}{d_\mathrm{o}}\right)+\left(\frac{1}{d_\mathrm{i}}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(1.4\,\mathrm{m})}\right]+\left[\frac{1}{(-4.2\,\mathrm{m})}\right]=\frac{1}{f},\text{ which gives } f=2.1\,\mathrm{m}.$$

The radius of the concave mirror is

$$r=2f=2(2.1\,\mathrm{m})=\boxed{4.2\,\mathrm{m.}}$$

$\boxed{11.}$ We find the image distance from the magnification:

$$m=\frac{h_\mathrm{i}}{h_\mathrm{o}}=\frac{-d_\mathrm{i}}{d_\mathrm{o}};$$

$$+4.5=\frac{-d_\mathrm{i}}{(2.20\,\mathrm{cm})},\text{ which gives } d_\mathrm{i}=-9.90\,\mathrm{cm}.$$

We find the focal length from

$$\left(\frac{1}{d_\mathrm{o}}\right)+\left(\frac{1}{d_\mathrm{i}}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(2.20\,\mathrm{cm})}\right]+\left[\frac{1}{(-9.90\,\mathrm{cm})}\right]=\frac{1}{f},\text{ which gives } f=2.83\,\mathrm{cm}.$$

Because the focal length is positive, the mirror is $\boxed{\text{concave}}$ with a radius of

$$r=2f=2(2.83\,\mathrm{cm})=\boxed{5.7\,\mathrm{cm.}}$$

12. To produce an upright image, we have $d_\mathrm{i}<0$. A smaller image means $|d_\mathrm{i}|<d_\mathrm{o}$, so $f<0$, which means the mirror is $\boxed{\text{convex.}}$

We find the image distance from the magnification:

$$m=\frac{h_\mathrm{i}}{h_\mathrm{o}}=\frac{-d_\mathrm{i}}{d_\mathrm{o}}.$$

$$+3=\frac{-d_\mathrm{i}}{(1.4\,\mathrm{m})}\text{ yields } d_\mathrm{i}=-4.2\,\mathrm{m}.$$

We find the focal length from

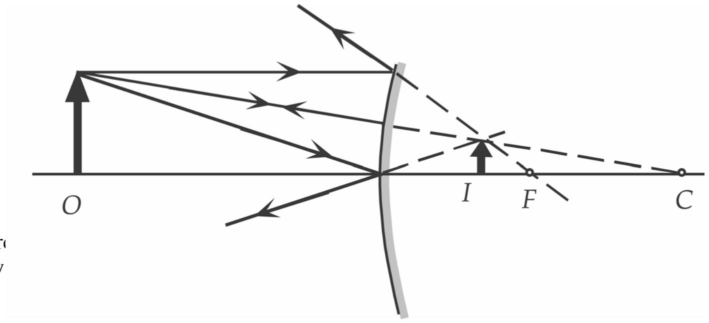
$$\frac{1}{f}=\frac{1}{d_\mathrm{o}}+\frac{1}{d_\mathrm{i}}=\left(\frac{1}{1.4\,\mathrm{m}}\right)+\left(\frac{1}{-4.2\,\mathrm{m}}\right),\text{ which gives } f=2.1\,\mathrm{m}.$$

The radius of the convex mirror is

$$r=2f=2(2.1\,\mathrm{m})=\boxed{4.2\,\mathrm{m.}}$$

13. (*a*) We see from the ray diagram that the image is behind the mirror, so it is virtual. We estimate the image distance as $\boxed{-7\,\mathrm{cm.}}$

(*b*) If we use a focal length of $-10\,\mathrm{cm}$, we



locate the image from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(20\,\text{cm})}\right]+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{(-10\,\text{cm})}, \text{ which gives } d_\text{i}=-6.7\,\text{cm}.$$

(*c*) We find the image size from the magnification:

$$m=\frac{h_\text{i}}{h_\text{o}}=\frac{-d_\text{i}}{d_\text{o}};$$

$$\frac{h_\text{i}}{(3.00\,\text{mm})}=-\frac{(-6.7\,\text{cm})}{(20\,\text{cm})}, \text{ which gives } h_\text{i}=\boxed{1.0\,\text{mm.}}$$

14. We find the image distance from the magnification, noting an upright image as in Fig. 23-46.

$$m=+0.5=\frac{h_\text{i}}{h_\text{o}}=\frac{-d_\text{i}}{d_\text{o}}=\frac{-d_\text{i}}{(3.0\,\text{m})}, \text{ which yields } d_\text{i}=\frac{(-3.0\,\text{m})}{(+0.5)}=-1.5\,\text{m}.$$

The thin lens equation gives the inverse focal length

$$\frac{1}{f}=\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\left[\frac{1}{(3.0\,\text{m})}\right]+\left[\frac{1}{(-1.5\,\text{m})}\right], \text{ so } f=-3.0\,\text{m}.$$

The radius of the mirror is $r=2f=2(-3.0\,\text{m})=\boxed{-6.0\,\text{m.}}$

15. (*a*) With $d_\text{i}=d_\text{o}$, we locate the object from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{o}}\right)=\frac{1}{f}, \text{ which gives } d_\text{o}=2f=r.$$

The object should be placed at the $\boxed{\text{center of curvature.}}$

(*b*) Because the image is in front of the mirror, $d_\text{i}>0$, it is $\boxed{\text{real.}}$

(*c*) The magnification is

$$m=\frac{-d_\text{i}}{d_\text{o}}=\frac{-d_\text{o}}{d_\text{o}}=-1.$$

Because the magnification is negative, the image is $\boxed{\text{inverted.}}$

(*d*) As found in part (*c*), $m=\boxed{-1.}$

16. We take the object distance to be ∞, and find the focal length from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

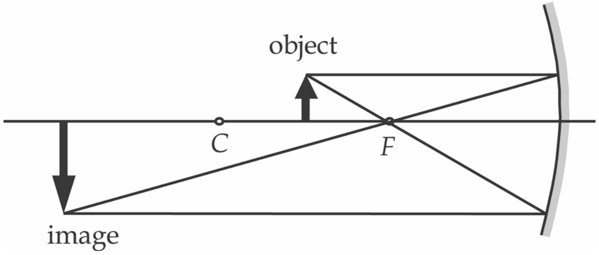
$$\left(\frac{1}{\infty}\right)+\left[\frac{1}{(-16.0\,\text{cm})}\right]=\frac{1}{f}, \text{ which gives } f=-16.0\,\text{cm}.$$

Because the focal length is negative, the mirror is $\boxed{\text{convex.}}$

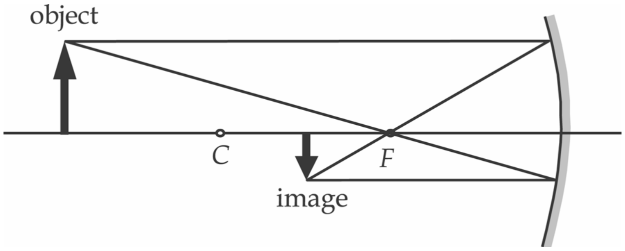
The radius is

$$r = 2f = 2(-16.0\,\text{cm}) = \boxed{-32.0\,\text{cm.}}$$

17.


object
C
F
image

$$|h_i| > |h_o|$$


object
C
F
image

$$|h_i| < |h_o|$$

We find the image distance from

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f} = \frac{2}{r}, \text{ which we can write as } d_i = \frac{rd_o}{(2d_o - r)}.$$

The magnification is

$$m = \frac{-d_i}{d_o} = \frac{-r}{(2d_o - r)}.$$

If $d_o > r$, then $(2d_o - r) > r$, so

$$|m| = \frac{r}{(2d_o - r)} = \frac{r}{(>r)} < 1.$$

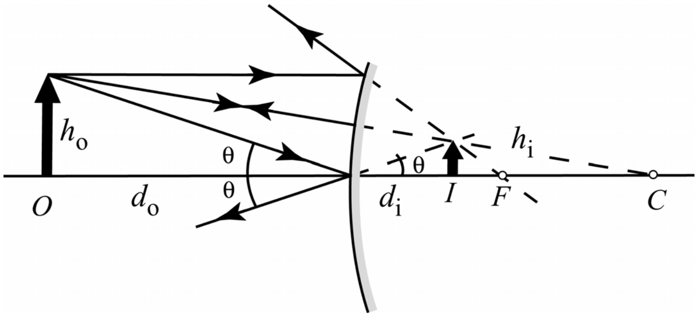
If $d_o < r$, then $(2d_o - r) < r$, so

$$|m| = \frac{r}{(2d_o - r)} = \frac{r}{(<r)} > 1.$$

18. From the ray that reflects from the center of the mirror, we have

$$\tan\theta = \frac{h_o}{d_o} = \frac{h_i}{d_i};$$

$$|m| = \frac{h_o}{h_i} = \frac{d_i}{d_o}.$$


h_o
θ
θ
θ
h_i
O
d_o
d_i
I
F
C

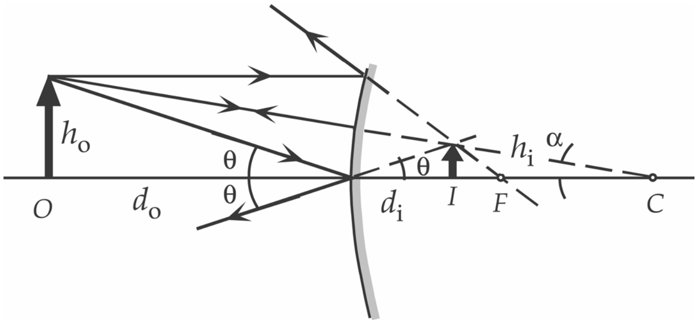
Because the image distance on the ray diagram is negative, we get

$$m = \frac{-d_\text{i}}{d_\text{o}}.$$

19. From the ray diagram, we see that

$$\tan\theta = \frac{h_\text{o}}{d_\text{o}} = \frac{h_\text{i}}{d_\text{i}};$$

$$\tan\alpha = \frac{h_\text{o}}{(d_\text{o} + r)} = \frac{h_\text{i}}{(r - d_\text{i})}.$$



When we divide the two equations, we get

$$\frac{(d_\text{o} + r)}{d_\text{o}} = \frac{(r - d_\text{i})}{d_\text{i}};$$

$$1 + \left(\frac{r}{d_\text{o}}\right) = \left(\frac{r}{d_\text{i}}\right) - 1, \text{ or}$$

$$\left(\frac{r}{d_\text{o}}\right) - \left(\frac{r}{d_\text{i}}\right) = -2;$$

$$\left(\frac{1}{d_\text{o}}\right) - \left(\frac{1}{d_\text{i}}\right) = \frac{-2}{r} = \frac{-1}{f}, \text{ with } f = \frac{r}{2}.$$

From the ray diagram, we see that $d_\text{i} < 0$.

If we consider *f* to be negative, we have

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f}.$$

20. We find the image distance from the magnification:

$$m = \frac{h_\text{i}}{h_\text{o}} = \frac{-d_\text{i}}{d_\text{o}};$$

$$+0.65 = \frac{-d_\text{i}}{(2.2\,\text{m})}, \text{ which gives } d_\text{i} = -1.43\,\text{m}.$$

We find the focal length from

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

$$\left[\frac{1}{(2.2\,\text{m})}\right] + \left[\frac{1}{(-1.43\,\text{m})}\right] = \frac{1}{f}, \text{ which gives } f = \boxed{-4.1\,\text{m.}}$$

21. (*a*) To produce a smaller image located behind the surface of the mirror requires a $\boxed{\text{convex mirror.}}$

(*b*) We find the image distance from the magnification:

$$m = \frac{h_\text{i}}{h_\text{o}} = \frac{-d_\text{i}}{d_\text{o}};$$

$$\frac{(3.5\,\text{cm})}{(4.5\,\text{cm})} = \frac{-d_\text{i}}{(28\,\text{cm})}, \text{ which gives } d_\text{i} = -21.8\,\text{cm}.$$

As expected, $d_i < 0$. The image is located $\boxed{\text{22 cm behind the surface.}}$

(*c*) We find the focal length from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(28\,\text{cm})}\right]+\left[\frac{1}{(-21.8\,\text{cm})}\right]=\frac{1}{f},\ \text{which gives } f=\boxed{-98\,\text{cm.}}$$

(*d*) The radius of curvature is

$$r=2f=2(-98\,\text{cm})=\boxed{-196\,\text{cm.}}$$

22. (*a*) To produce a larger image requires a $\boxed{\text{concave mirror.}}$
(*b*) The image will be $\boxed{\text{erect and virtual.}}$
(*c*) We find the image distance from the magnification:

$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o};$$

$$1.33=\frac{-d_i}{(20.0\,\text{cm})},\ \text{which gives } d_i=-26.6\,\text{cm}.$$

We find the focal length from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(20.0\,\text{cm})}\right]+\left[\frac{1}{(-26.6\,\text{cm})}\right]=\frac{1}{f},\ \text{which gives } f=80.6\,\text{cm}.$$

The radius of curvature is

$$r=2f=2(80.6\,\text{cm})=\boxed{161\,\text{cm.}}$$

23. (*a*) The speed in crown glass is

$$v=\frac{c}{n}=\frac{(3.00\times10^8\,\text{m/s})}{(1.52)}=\boxed{1.97\times10^8\,\text{m/s.}}$$

(*b*) The speed in Lucite is

$$v=\frac{c}{n}=\frac{(3.00\times10^8\,\text{m/s})}{(1.51)}=\boxed{1.99\times10^8\,\text{m/s.}}$$

(*c*) The speed in ethyl alcohol is $v=\frac{c}{n}=\frac{(3.00\times10^8\,\text{m/s})}{(1.36)}=\boxed{2.21\times10^8\,\text{m/s.}}$

24. We find the index of refraction from

$$v=\frac{c}{n};$$

$$2.29\times10^8\,\text{m/s}=\frac{(3.00\times10^8\,\text{m/s})}{n},\ \text{which gives } n=\boxed{1.31.}$$

25. We find the index of refraction from

$$v = \frac{c}{n};$$

$$0.89\,v_{\text{water}} = \frac{0.89c}{1.33} = \frac{c}{n}, \text{ which gives } n = \boxed{1.49.}$$

26. We find the angle of refraction in the glass from

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.00)\sin 63° = (1.58)\sin\theta_2, \text{ which gives } \theta_2 = \boxed{34°.}$$

27. We find the angle of refraction in the water from

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.33)\sin 42.5° = (1.00)\sin\theta_2, \text{ which gives } \theta_2 = \boxed{64.0°.}$$

28. We find the incident angle in the water from

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.33)\sin\theta_1 = (1.00)\sin 66.0°, \text{ which gives } \theta_1 = \boxed{40.6°.}$$

29. We find the incident angle in the air from

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.00)\sin\theta_1 = (1.33)\sin 31.0°, \text{ which gives } \theta_1 = 43.2°.$$

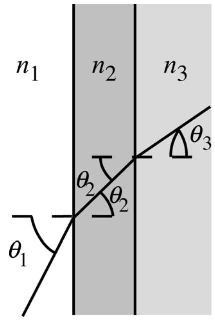
Thus the angle above the horizon is

$$90.0° - \theta_1 = 90.0° - 43.2° = \boxed{46.8°.}$$

30. (*a*) We find the angle in the glass from the refraction at the air–glass surface:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$(1.00)\sin 43.5° = (1.52)\sin\theta_2, \text{which gives } \theta_2 = \boxed{26.9°.}$$



(*b*) Because the surfaces are parallel, the refraction angle from the first surface is the incident angle at the second surface. We find the angle in the water from the refraction at the glass–water surface:

$$n_2 \sin\theta_2 = n_3 \sin\theta_3;$$

$$(1.52)\sin 26.9° = (1.33)\sin\theta_3, \text{ which gives } \theta_3 = \boxed{31.2°.}$$

(*c*) If there were no glass, we would have

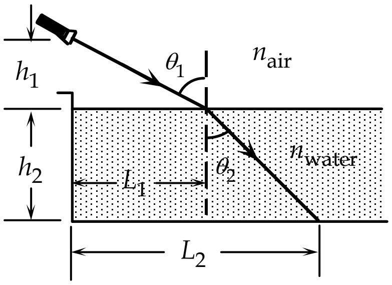
$$n_1 \sin\theta_1 = n_3 \sin\theta_3';$$

$$(1.00)\sin 43.5° = (1.33)\sin\theta_3', \text{ which gives } \theta_3' = \boxed{31.2°.}$$

Note that, because the sides are parallel, θ_3 is independent of the presence of the glass.

31. We find the angle of incidence from the distances:

$$\tan\theta_1 = \frac{L_1}{h_1} = \frac{(2.7\,\text{m})}{(1.3\,\text{m})} = 2.076, \text{ so } \theta_1 = 64.3°.$$



For the refraction from air into water, we have

$$n_{\text{air}} \sin\theta_1 = n_{\text{water}} \sin\theta_2;$$

$$(1.00)\sin 64.3° = (1.33)\sin\theta_2, \text{ which gives } \theta_2 = 42.6°.$$

We find the horizontal distance from the edge of the pool from

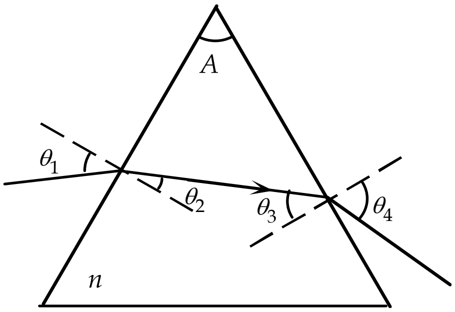
$$L = L_1 + L_2 = L_1 + h_2 \tan\theta_2$$

$$= 2.7\,\text{m} + (2.1\,\text{m})\tan 42.6° = \boxed{4.6\,\text{m.}}$$

32. For the refraction at the first surface, we have

$$n_{\text{air}} \sin\theta_1 = n \sin\theta_2;$$

$$(1.00)\sin 45.0° = (1.58)\sin\theta_2, \text{ which gives } \theta_2 = 26.6°.$$



We find the angle of incidence at the second surface from

$$(90° - \theta_2) + (90° - \theta_3) + A = 180°, \text{ which gives}$$

$$\theta_3 = A - \theta_2 = 60° - 26.6° = 33.4°.$$

For the refraction at the second surface, we have

$$n \sin\theta_3 = n_{\text{air}} \sin\theta_4;$$

$$(1.58)\sin 33.4° = (1.00)\sin\theta_4, \text{ which gives}$$

$$\theta_4 = \boxed{60.4° \text{ from the normal.}}$$

33. The angle of reflection is equal to the angle of incidence:

$$\theta_{\text{refl}} = \theta_1 = 2\theta_2.$$

For the refraction we have

$$n_{\text{air}} \sin\theta_1 = n_{\text{glass}} \sin\theta_2;$$

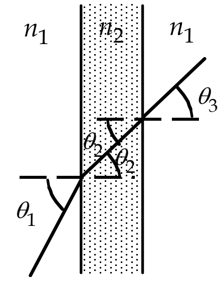
$$(1.00)\sin 2\theta_2 = (1.52)\sin\theta_2.$$

We use a trigonometric identity for the left-hand side:

$$\sin 2\theta_2 = 2\sin\theta_2 \cos\theta_2 = (1.52)\sin\theta_2, \text{ or } \cos\theta_2 = 0.760, \text{ so } \theta_2 = 40.5°.$$

Thus the angle of incidence is $\theta_1 = 2\theta_2 = \boxed{81.0°.}$

34. Because the surfaces are parallel, the angle of refraction from the first surface is the angle of incidence at the second. Thus for the refractions, we have



$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$

$$n_2 \sin\theta_2 = n_3 \sin\theta_3.$$

When we add the two equations, we get

$$n_1 \sin\theta_1 = n_1 \sin\theta_3, \text{ which gives } \theta_3 = \theta_1.$$

Because the ray emerges in the same index of refraction, it is undeviated.

35. Because the glass surfaces are parallel, the exit beam will be traveling in the same direction as the original beam. We find the angle inside the glass from

$$n_{\text{air}}\sin\theta = n\sin\phi.$$

If the angles are small, we use

$$\cos\phi \approx 1, \text{ and } \sin\phi \approx \phi, \text{ where } \phi \text{ is in radians.}$$

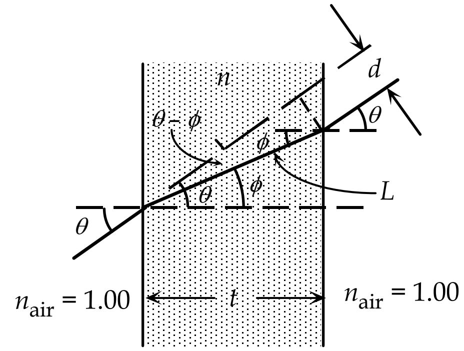
$$(1.00)\,\theta = n\phi, \text{ or } \phi = \frac{\theta}{n}.$$

We find the distance along the ray in the glass from

$$L = \frac{t}{\cos\phi} \approx t.$$

We find the perpendicular displacement from the original direction from

$$d = L\sin(\theta - \phi) \approx t(\theta - \phi) = t\left[\theta - \left(\frac{\theta}{n}\right)\right] = \frac{t\theta(n-1)}{n}.$$



36. When the light in the material with a higher index is incident at the critical angle, the refracted angle is 90°:

$$n_{\text{Lucite}}\sin\theta_1 = n_{\text{water}}\sin\theta_2;$$

$$(1.51)\sin\theta_1 = (1.33)\sin 90°, \text{which gives } \theta_1 = \boxed{61.7°.}$$

Because Lucite has the higher index, the light must start in $\boxed{\text{Lucite.}}$

37. When the light in the liquid is incident at the critical angle, the refracted angle is 90°:

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2;$$

$$n_{\text{liquid}}\sin 47.7° = (1.00)\sin 90°, \text{ which gives } n_{\text{liquid}} = \boxed{1.35.}$$

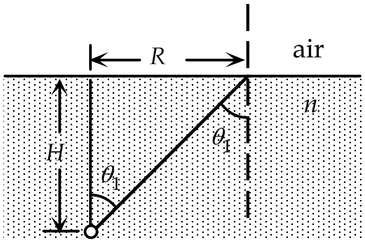
38. We find the critical angle for light leaving the water:

$$n\sin\theta_1 = \sin\theta_2;$$

$$(1.33)\sin\theta_{\text{C}} = \sin 90°, \text{ which gives } \theta_{\text{C}} = 48.8°.$$

If the light is incident at a greater angle than this, it will totally reflect. We see from the diagram that

$$R > H\tan\theta_{\text{C}} = (62.0\,\text{cm})\tan 48.8° = \boxed{70.7\,\text{cm.}}$$



$\boxed{39.}$ We find the angle of incidence from the distances:

$$\tan\theta_1 = \frac{L}{h} = \frac{(7.0\,\text{cm})}{(8.0\,\text{cm})} = 0.875, \text{ so } \theta_1 = 41.2°.$$

For the maximum incident angle for the refraction from liquid into air, we have

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2;$$

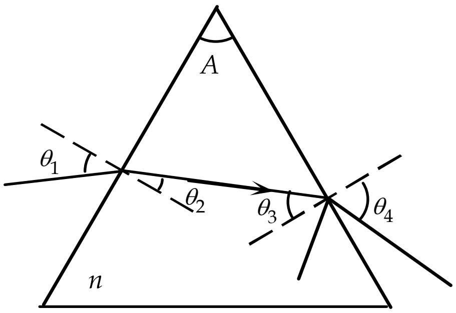
$$n_{\text{liquid}}\sin\theta_{1\max} = (1.00)\sin 90°, \text{ which gives } \sin\theta_{1\max} = \frac{1}{n_{\text{liquid}}}.$$

Thus we have

$$\sin\theta_1 \geq \sin\theta_{1\max} = \frac{1}{n_{\text{liquid}}};$$

$$\sin 41.2° = 0.659 \geq \frac{1}{n_{\text{liquid}}}, \text{ or } \boxed{n_{\text{liquid}} \geq 1.5.}$$

40. For the refraction at the first surface, we have



$$n_{\text{air}} \sin\theta_1 = n\sin\theta_2;$$

$$(1.00)\sin\theta_1 = n\sin\theta_2, \text{ which gives } \sin\theta_2 = \sin\frac{\theta_1}{n}.$$

We find the angle of incidence at the second surface from

$$(90° - \theta_2) + (90° - \theta_3) + A = 180°, \text{ which gives}$$

$$\theta_3 = A - \theta_2 = 75° - \theta_2.$$

For the refraction at the second surface, we have

$$n\sin\theta_3 = n_{\text{air}}\sin\theta_4 = (1.00)\sin\theta_4.$$

The maximum value of θ_4 before internal reflection takes place at the second surface is 90°. Thus for internal reflection to occur, we have

$$n\sin\theta_3 = n\sin(A - \theta_2) \geq 1.$$

When we expand the left-hand side, we get

$$n(\sin A\cos\theta_2 - \cos A\sin\theta_2) \geq 1.$$

If we use the result from the first surface to eliminate *n*, we get

$$\frac{\sin\theta_1(\sin A\cos\theta_2 - \cos A\sin\theta_2)}{(\sin\theta_2)} = \sin\theta_1\left(\frac{\sin A}{\tan\theta_2 - \cos A}\right) \geq 1, \text{ or}$$

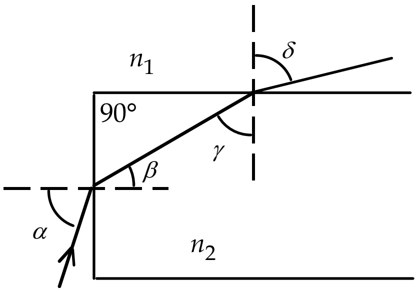
$$\frac{1}{\tan\theta_2} \geq \frac{\left[\left(\frac{1}{\sin\theta_1}\right) + \cos A\right]}{\sin A} = \frac{\left[\left(\frac{1}{\sin 45°}\right) + \cos 75°\right]}{\sin 75°} = 1.732, \text{ which gives}$$

$$\tan\theta_2 \leq 0.577, \text{ so } \theta_2 \leq 30°.$$

From the result for the first surface, we have

$$n_{\min} = \frac{\sin\theta_1}{\sin\theta_{2\max}} = \frac{\sin 45°}{\sin 30°} = 1.414, \text{ so } \boxed{n \geq 1.414.}$$

41. For the refraction at the side of the rod, we have



$$n_2\sin\gamma = n_1\sin\delta.$$

The minimum angle for total reflection $\gamma_{\min}$ occurs when $\delta = 90°$:

$$n_2\sin\gamma_{\min} = (1.00)(1) = 1, \text{ or } \sin\gamma_{\min} = \frac{1}{n_2}.$$

We find the maximum angle of refraction at the end of the rod from

$$\beta_{\max} = 90° - \gamma_{\min}.$$

Because the sine function increases with angle, for the refraction at the end of the rod, we have

$$n_1\sin\alpha_{\max} = n_2\sin\beta_{\max};$$

$$(1.00)\sin\alpha_{\max} = n_2\sin(90° - \gamma_{\min}) = n_2\cos\gamma_{\min}.$$

If we want total internal reflection to occur for any incident angle at the end of the fiber, the maximum value of α is 90°, so

$$n_2 \cos\gamma_{\text{min}} = 1.$$

When we divide this by the result for the refraction at the side, we get $\tan\gamma_{\text{min}} = 1,$ or $\gamma_{\text{min}} = 45°.$ Thus we have

$$n_2 \geq \frac{1}{\sin\gamma_{\text{min}}} = \frac{1}{\sin 45°} = 1.414.$$

42. (*a*) The ray enters normal to the first surface, so there is no deviation there. The angle of incidence is 45° at the second surface. When there is air outside the surface, we have

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$
$$n_1 \sin 45° = (1.00)\sin\theta_2.$$

For total internal reflection to occur, $\sin\theta_2 \geq 1$, so we have

$$n_1 \geq \frac{1}{\sin 45°} = \boxed{1.414.}$$

(*b*) When there is water outside the surface, we have

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$
$$(1.50)\sin 45° = (1.33)\sin\theta_2, \text{ which gives } \sin\theta_2 = 0.80.$$

Because $\sin\theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting.}}$

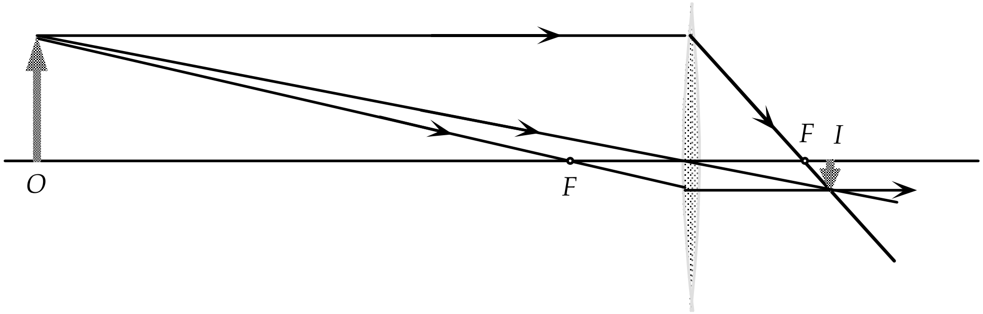
(*c*) For total reflection when there is water outside the surface, we have

$$n_1 \sin\theta_1 = n_2 \sin\theta_2;$$
$$n_1 \sin 45° = (1.33)\sin\theta_2.$$

For total internal reflection to occur, $\sin\theta_2 \geq 1,$ so we have

$$n_1 \geq \frac{1.33}{\sin 45°} = \boxed{1.88.}$$

43. (*a*) From the ray diagram, the object distance is about six focal lengths, or $\boxed{390 \text{ mm.}}$



(*b*) We find the object distance from

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f};$$
$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{78.0\,\text{mm}}\right) = \frac{1}{65.0\,\text{mm}}, \text{ which gives } d_o = 390\,\text{mm} = \boxed{39.0\,\text{cm.}}$$

44. (*a*) To form a real image from parallel rays requires a $\boxed{\text{converging lens.}}$

(*b*) We find the power of the lens from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f}=P,\ \text{when } f \text{ is in meters;}$$

$$\left(\frac{1}{\infty}\right)+\left(\frac{1}{0.185\,\text{m}}\right)=P=\boxed{5.41\,\text{D.}}$$

45. To form a real image from a real object requires a $\boxed{\text{converging lens.}}$
We find the focal length of the lens from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{275\,\text{cm}}\right)+\left(\frac{1}{48.3\,\text{cm}}\right)=\frac{1}{f},\ \text{which gives } f=\boxed{+41.1\,\text{cm.}}$$

Because $d_\text{i}>0,$ the image is $\boxed{\text{real.}}$

46. (*a*) The power of the lens is

$$P=\frac{1}{f}=\frac{1}{0.205\,\text{m}}=\boxed{4.88\,\text{D, converging.}}$$

(*b*) We find the focal length of the lens from

$$P=\frac{1}{f};$$

$$-6.25\,\text{D}=\frac{1}{f},\ \text{which gives } f=-0.160\,\text{m}=\boxed{-16.0\,\text{cm, diverging.}}$$

$\boxed{47.}$ (*a*) We locate the image from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{18\,\text{cm}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{24\,\text{cm}},\ \text{which gives } d_\text{i}=-72\,\text{cm.}$$

The negative sign means the image is $\boxed{\text{72 cm behind the lens (virtual).}}$

(*b*) We find the magnification from

$$m=\frac{-d_\text{i}}{d_\text{o}}=-\frac{(-72\,\text{cm})}{(18\,\text{cm})}=\boxed{+4.0.}$$

48. We find the image distance from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{0.140\,\text{m}}\right)+\left(\frac{1}{d_\text{i}}\right)=-5.5\,\text{D},$$

which gives $d_\text{i}=-0.076\,\text{m}=\boxed{\text{– 7.9 cm (virtual image behind the lens).}}$
We find the height of the image from

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o};$$

$$\frac{h_i}{(4.0\,\text{mm})} = -\frac{(-7.9\,\text{cm})}{(14.0\,\text{cm})},\ \text{which gives}\ h_i = \boxed{0.22\,\text{mm}\,(\text{upright}).}$$

49. (*a*) We find the image distance from

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f};$$

$$\left(\frac{1}{10.0\times10^3\,\text{mm}}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{80\,\text{mm}},\ \text{which gives}\ d_i = \boxed{81\,\text{mm}.}$$

(*b*) For an object distance of 3.0 m, we have

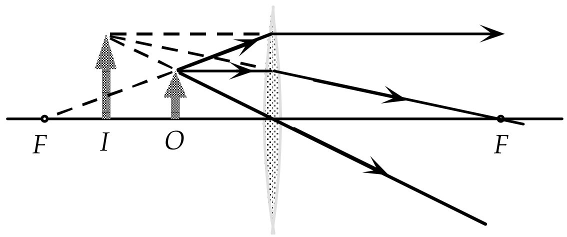
$$\left(\frac{1}{3.0\times10^3\,\text{mm}}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{80\,\text{mm}},\ \text{which gives}\ d_i = \boxed{82\,\text{mm}.}$$

(*c*) For an object distance of 1.0 m, we have

$$\left(\frac{1}{1.0\times10^3\,\text{mm}}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{80\,\text{mm}},\ \text{which gives}\ d_i = \boxed{87\,\text{mm}.}$$

(*d*) We find the smallest object distance from

$$\left(\frac{1}{d_{\text{omin}}}\right) + \left(\frac{1}{120\,\text{mm}}\right) = \frac{1}{80\,\text{mm}},\ \text{which gives}\ d_i = 240\,\text{mm} = \boxed{24\,\text{cm}.}$$



50. (*a*) We see that the image is behind the lens, so it is $\boxed{\text{virtual.}}$

(*b*) From the ray diagram we see that we need a $\boxed{\text{converging lens.}}$

(*c*) We find the image distance from the magnification:

$$m = \frac{-d_i}{d_o};$$

$$+2.5 = \frac{-d_i}{(8.0\,\text{cm})},\ \text{which gives}\ d_i = -20\,\text{cm}.$$

We find power of the lens from

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f} = P,\ \text{when } f \text{ is in meters;}$$

$$\left(\frac{1}{0.080\,\text{m}}\right) + \left[\frac{1}{(-0.20\,\text{m})}\right] = P = \boxed{7.5\,\text{D}.}$$

51. We find the image distance from

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f};$$

$$\frac{1}{1.5\,\text{m}} + \frac{1}{d_i} = 8.0\,\text{D},\ \text{which gives}\ d_i = 0.1364\,\text{m}.$$

(*a*) With $d_o = 0.5\,\text{m},$ the new image distance is determined by

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f};$$

$$\frac{1}{0.5\,\text{m}} + \frac{1}{d_i} = 8.0\,\text{D}, \text{ which gives } d_i = 0.1667\,\text{m}.$$

The image has moved $0.1667\,\text{m} - 0.1364\,\text{m} = 0.0303\,\text{m}$ or $\boxed{\text{3.0 cm away from the lens.}}$

(*b*) With $d_o = 2.5\,\text{m},$ the new image distance is determined by

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f};$$

$$\frac{1}{2.5\,\text{m}} + \frac{1}{d_i} = 8.0\,\text{D}, \text{ which gives } d_i = 0.1316\,\text{m}.$$

The image has moved $0.1316\,\text{m} - 0.1364\,\text{m} = -0.0048\,\text{m}$ or $\boxed{\text{0.5 cm toward the lens.}}$

52. $|h_i| = |h_o|$ when $d_i = d_o$. So we find d_o from $\frac{1}{d_o} + \frac{1}{d_o} = \frac{1}{f},$ which gives $d_o = 2f.$

An object placed two focal lengths away from a converging lens will produce a real image that is also two focal lengths from the lens, inverted, and the same size as the object. With $f = 25\,\text{cm},$ the object should be placed a distance of $2f = \boxed{50\,\text{cm}}$ from the lens.

53. We can relate the image and object distance from the magnification:

$$m = \frac{-d_i}{d_o}, \text{ or } d_o = \frac{-d_i}{m}.$$

We use this in the lens equation:

$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f};$$

$$-\left(\frac{m}{d_i}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f}, \text{ which gives } d_i = (1 - m)f.$$

(*a*) If the image is real, $d_i > 0$. With $f > 0,$ we see that $m < 1;$ thus $m = -2.00.$ The image distance is

$$d_i = [1 - (-2.00)](50.0\,\text{mm}) = 150\,\text{mm}.$$

The object distance is

$$d_o = \frac{-d_i}{m} = \frac{-(150\,\text{mm})}{(-2.00)} = \boxed{75.0\,\text{mm.}}$$

(*b*) If the image is virtual, $d_i < 0$. With $f > 0,$ we see that $m > 1;$ thus $m = +2.00.$ The image distance is

$$d_i = [1 - (+2.00)](50.0\,\text{mm}) = -50\,\text{mm}.$$

The object distance is

$$d_o = \frac{-d_i}{m} = \frac{-(-50\,\text{mm})}{(+2.00)} = \boxed{25.0\,\text{mm.}}$$

54. We can relate the image and object distance from the magnification:

$$m = \frac{-d_i}{d_o}, \text{ or } d_o = \frac{-d_i}{m}.$$

We use this in the lens equation:

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$-\left(\frac{m}{d_\text{i}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f},\ \text{which gives } d_\text{i}=(1-m)f.$$

(*a*) If the image is real, $d_\text{i}>0$. With $f<0$, we see that $m>1$; thus $m=+2.00$. The image distance is

$$d_\text{i}=\left[1-(+2.00)\right](-50.0\,\text{mm})=50.0\,\text{mm}.$$

The object distance is

$$d_\text{o}=\frac{-d_\text{i}}{m}=\frac{-(50.0\,\text{mm})}{(+2.00)}=\boxed{-25.0\,\text{mm}.}$$

The negative sign means the object is beyond the lens, so it would have to be an object formed by a preceding optical device.

(*b*) If the image is virtual, $d_\text{i}<0$. With $f<0$, we see that $m<1$; thus $m=-2.00$. The image distance is

$$d_\text{i}=\left[1-(-2.00)\right](-50.0\,\text{mm})=-150\,\text{mm}.$$

The object distance is

$$d_\text{o}=-\frac{d_\text{i}}{m}=-\frac{(-150\,\text{mm})}{(-2.00)}=\boxed{-75.0\,\text{mm}.}$$

The negative sign means the object is beyond the lens, so it would have to be an object formed by a preceding optical device.

55. (*a*) We find the image distance from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{1.20\times10^3\,\text{mm}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{135\,\text{mm}},\ \text{which gives } d_\text{i}=\boxed{152\,\text{mm}(\text{real, behind the lens}).}$$

We find the height of the image from

$$m=\frac{h_\text{i}}{h_\text{o}}=-\frac{d_\text{i}}{d_\text{o}};$$

$$\frac{h_\text{i}}{2.00\,\text{cm}}=\frac{-(152\,\text{mm})}{(1.20\times10^3\,\text{mm})},\ \text{which gives } h_\text{i}=\boxed{-0.254\,\text{cm}(\text{inverted}).}$$

(*b*) We find the image distance from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{1.20\times10^3\,\text{mm}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{(-135\,\text{mm})},$$

which gives $d_\text{i}=\boxed{-121\,\text{mm}(\text{virtual, in front of the lens}).}$

We find the height of the image from

$$m=\frac{h_\text{i}}{h_\text{o}}=-\frac{d_\text{i}}{d_\text{o}};$$

$$\frac{h_i}{(2.00\,\text{cm})}=\frac{-(-121\,\text{mm})}{(1.20\times10^3\,\text{mm})},\ \text{which gives}\ h_i=\boxed{+0.202\,\text{cm}\,(\text{upright}).}$$

56. For a real object and image, both d_o and d_i must be positive, so the magnification will be negative:

$$m=\frac{-d_i}{d_o};$$

$$-2.5=\frac{-d_i}{d_o},\ \text{or}\ d_i=2.5\,d_o.$$

We find the object distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{2.5\,d_o}\right)=\frac{1}{(+75\,\text{cm})},\ \text{which gives}\ d_o=105\,\text{cm}.$$

The image distance is

$$d_i=2.5\,d_o=2.5(105\,\text{cm})=262.5\,\text{cm}.$$

The distance between object and image is

$$L=d_o+d_i=105\,\text{cm}+262.5\,\text{cm}=\boxed{37\,\text{m}.}$$

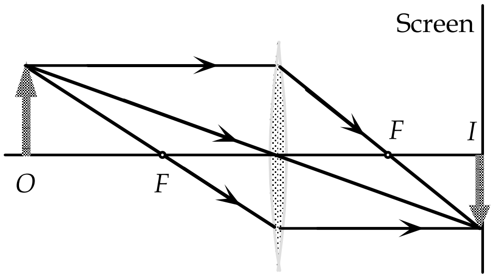
$\boxed{57.}$ The sum of the object and image distances must be the distance between object and screen:

$$d_o+d_i=L=66.0\,\text{cm}.$$

For the lens we have

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_o}\right)+\left[\frac{1}{(66.0\,\text{cm}-d_o)}\right]=\frac{1}{(12.5\,\text{cm})},$$



which gives a quadratic equation:

$$d_o^{\,2}-(66.0\,\text{cm})d_o+825\,\text{cm}^2=0.$$

From the quadratic formula, $d_o=49.2\,\text{cm}$ or $d_o=16.8\,\text{cm}$, so the lens should be placed $\boxed{49.2\,\text{cm or }16.8\,\text{cm}}$ from the object.

Note that in general the screen must be at least 4*f* from the object for an image to be formed on the screen.

58. We find the image formed by the refraction of the first lens:

$$\left(\frac{1}{d_{o1}}\right)+\left(\frac{1}{d_{i1}}\right)=\frac{1}{f_1};$$

$$\left(\frac{1}{36.0\,\text{cm}}\right)+\left(\frac{1}{d_{i1}}\right)=\frac{1}{28.0\,\text{cm}},\ \text{which gives}\ d_{i1}=+126\,\text{cm}.$$

This image is the object for the second lens. Because it is beyond the second lens, it has a negative object distance: $d_{o2}=16.5\,\text{cm}-126\,\text{cm}=-109.5\,\text{cm}$.

We find the image formed by the refraction of the second lens:

$$\left(\frac{1}{d_{o2}}\right)+\left(\frac{1}{d_{i2}}\right)=\frac{1}{f_2};$$

$$\left[\frac{1}{(-109.5\,\text{cm})}\right]+\left(\frac{1}{d_{i2}}\right)=\frac{1}{28.0\,\text{cm}},\ \text{which gives } d_{i2}=+22.3\,\text{cm}.$$

Thus the final image is $\boxed{\text{real, 22.3 cm beyond second lens.}}$

The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}$$

$$=\frac{(+126\,\text{cm})(+22.3\,\text{cm})}{(+36.0\,\text{cm})(-109.5\,\text{cm})}=\boxed{-0.713(\text{inverted}).}$$

59. The image of an infinite object formed by the refraction of the first lens will be at the focal point:

$$d_{i1}=f_1=+20.0\,\text{cm}.$$

This image is the object for the second lens. Because it is beyond the second lens, it has a negative object distance: $d_{o2}=14.0\,\text{cm}-20.0\,\text{cm}=-6.0\,\text{cm}.$

We find the image formed by the refraction of the second lens:

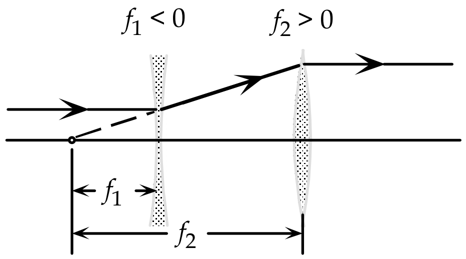
$$\left(\frac{1}{d_{o2}}\right)+\left(\frac{1}{d_{i2}}\right)=\frac{1}{f_2};$$

$$\left[\frac{1}{(-6.0\,\text{cm})}\right]+\left(\frac{1}{d_{i2}}\right)=\frac{1}{(-31.5\,\text{cm})},\ \text{which gives } d_{i2}=+7.4\,\text{cm}.$$

Thus the final image is $\boxed{\text{real, 7.4 cm beyond second lens.}}$

60. We see from the ray diagram that the image from the first lens will be a virtual image at its focal point. This is a real object for the second lens, and must be at the focal point of the second lens. If L is the separation of the lenses, the focal length of the first lens is

$$f_1=L-f_2=21.0\,\text{cm}-31.0\,\text{cm}=\boxed{-10.0\,\text{cm}.}$$



61. (*a*) The position of the image formed by the first lens is still 30.0 cm behind the first lens, A. This image becomes the object for the second lens, B, at a distance $d_{oB}=20.0\,\text{cm}-30.0\,\text{cm}=-10.0\,\text{cm}.$

The image formed by lens B is located a distance d_{iB} from lens B:

$$\frac{1}{d_{oB}}+\frac{1}{d_{iB}}=\frac{1}{f};$$

$$\frac{1}{-10.0\,\text{cm}}+\frac{1}{d_{iB}}=\frac{1}{25.0\,\text{cm}},$$

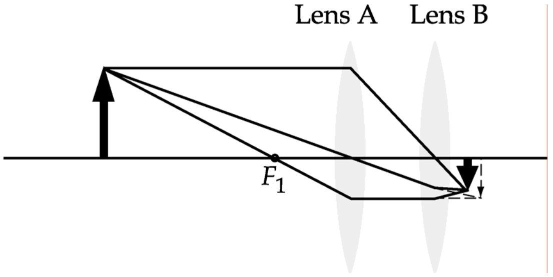
which gives $d_{iB}=\boxed{7.14\,\text{cm}}$ as the distance of the final image from lens B.

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{iA}}{d_{oA}}\right)\left(-\frac{d_{iB}}{d_{oB}}\right)=\frac{d_{iA}d_{iB}}{d_{oA}d_{oB}}$$

$$=\frac{(30.0\,\text{cm})(7.14\,\text{cm})}{(60.0\,\text{cm})(-10.0\,\text{cm})}=\boxed{-0.357.}$$

(*c*)



62. Working backwards, we use

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2}$$

with $d_{i2}=-\frac{1}{2}(30.0\,\text{cm})=-15.0\,\text{cm}$ and $f_2=20.0\,\text{cm}$:

$$\frac{1}{d_{o2}}+\frac{1}{-15.0\,\text{cm}}=\frac{1}{20.0\,\text{cm}} \text{ gives } d_{o2}=8.57\,\text{cm}.$$

This means that $d_{i1}=30.0\,\text{cm}-8.57\,\text{cm}=21.43\,\text{cm}$. Now we find d_{o1}:

$$\frac{1}{d_{o1}}+\frac{1}{d_{o2}}=\frac{1}{f_1};$$

$$\frac{1}{d_{o1}}+\frac{1}{21.43\,\text{cm}}=\frac{1}{15.0\,\text{cm}}, \text{ which gives } d_{o1}=50.0\,\text{cm}.$$

The original object is $\boxed{50.0\text{ cm}}$ to the left of the 15.0-cm-focal-length lens.

63. We find the image formed by the converging lens:

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1};$$

$$\frac{1}{33\,\text{cm}}+\frac{1}{d_{i1}}=\frac{1}{18\,\text{cm}}, \text{ which gives } d_{i1}=39.6\,\text{cm}.$$

(*a*) The image from the first lens becomes the object for the second lens, with $d_{o2}=12\,\text{cm}-39.6\,\text{cm}=-27.6\,\text{cm}$ (27.6 cm to the right of the second lens). Now we find the image formed by the second lens:

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2};$$

$$\frac{1}{-27.6\,\text{cm}}+\frac{1}{d_{i2}}=\frac{1}{-14\,\text{cm}} \text{ gives } d_{i2}=-28.4\,\text{cm},$$ which means that the final image is 28 cm to the left of the second lens, or $28\,\text{cm}-12\,\text{cm}=\boxed{16\,\text{cm to the left of the converging lens.}}$

(*b*) Now $d_{o2}=38\,\text{cm}-39.6\,\text{cm}=-1.6\,\text{cm}$, and

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \text{ yields}$$

$$\frac{1}{-1.6\,\text{cm}}+\frac{1}{d_{i2}}=\frac{1}{-14\,\text{cm}},\text{ which gives } d_{i2}=1.8\,\text{cm}.$$

Now the final image is $\boxed{\text{1.8 cm to the right of the diverging lens.}}$

64. (*a*) For the first lens, we have

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1};$$

$$\frac{1}{60.0\,\text{cm}}+\frac{1}{d_{i1}}=\frac{1}{20.0\,\text{cm}},\text{ so that } d_{i1}=30.0\,\text{cm}.$$

The image from the first lens becomes the object for the second lens, with $d_{o2}=25.0\,\text{cm}-30.0\,\text{cm}=-5.0\,\text{cm}$ (5.0 cm to the right of the diverging lens). Then for the second lens we have

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2};$$

$$\frac{1}{-5.0\,\text{cm}}+\frac{1}{d_{i2}}=\frac{1}{-10.0\,\text{cm}},\text{ which gives } d_{i2}=10\,\text{cm}.$$

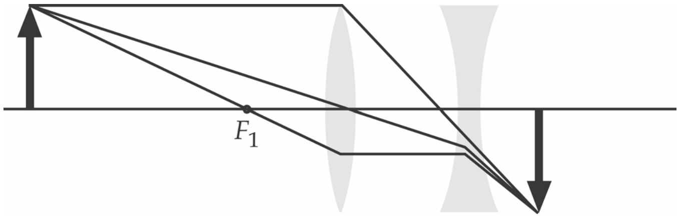
The final image is $\boxed{\text{10 cm to the right of the diverging lens.}}$

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}$$

$$=\frac{(30.0\,\text{cm})(10\,\text{cm})}{(60.0\,\text{cm})(-5.0\,\text{cm})}=\boxed{-1.0.}$$

(*c*)



65. We find the focal length by finding the image distance for an object very far away. For the first converging lens, we have

$$\left(\frac{1}{d_{o1}}\right)+\left(\frac{1}{d_{i1}}\right)=\frac{1}{f_C};$$

$$\left(\frac{1}{\infty}\right)+\left(\frac{1}{d_{i1}}\right)=\frac{1}{f_C},\text{ or, as expected, } d_{i1}=f_C.$$

The first image is the object for the second lens. If the first image is real, the second object distance is negative:

$$d_{o2}=-d_{i1}=-f_C.$$

For the second diverging lens, we have

$$\left(\frac{1}{d_{o2}}\right)+\left(\frac{1}{d_{i2}}\right)=\frac{1}{f_D};$$

$$\left[\frac{1}{(-f_C)}\right]+\left(\frac{1}{d_{i2}}\right)=\frac{1}{f_D}.$$

Because the second image must be at the focal point of the combination, we have

$$\left(\frac{-1}{f_C}\right)+\left(\frac{1}{f_T}\right)=\frac{1}{f_D},\text{ which gives }\frac{1}{f_D}=\left(\frac{1}{f_T}\right)-\left(\frac{1}{f_C}\right).$$

66. We find the focal length of the lens from

$$\frac{1}{f}=(n-1)\left[\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)\right]$$

$$=(1.52-1)\left\{\left[\frac{1}{(-34.2\,\text{cm})}\right]+\left[\frac{1}{(-23.8\,\text{cm})}\right]\right\},\text{ which gives }\boxed{f=-27.0\,\text{cm.}}$$

67. We find the index from the lensmaker's equation:

$$\frac{1}{f}=(n-1)\left[\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)\right];$$

$$\frac{1}{28.9\,\text{cm}}=(n-1)\left[\left(\frac{1}{31.0\,\text{cm}}\right)+\left(\frac{1}{31.0\,\text{cm}}\right)\right],\text{ which gives }n=\boxed{1.54.}$$

68. We find the radius from the lensmaker's equation:

$$\frac{1}{f}=(n-1)\left[\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)\right];$$

$$\frac{1}{(-23.4\,\text{cm})}=(1.50-1)\left[\left(\frac{1}{\infty}\right)+\left(\frac{1}{R_2}\right)\right],\text{ which gives }R_2=\boxed{-11.7\,\text{cm.}}$$

The negative sign indicates concave.

69. We find the focal length from the lensmaker's equation, using $n=1.51$ for Lucite:

$$\frac{1}{f}=(n-1)\left[\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)\right];$$

$$\frac{1}{f}=(1.51-1)\left[\left(\frac{1}{\infty}\right)+\left(\frac{1}{-18.4\,\text{cm}}\right)\right],\text{ which gives }f=\boxed{-36.1\,\text{cm.}}$$

70. We find the radius from the lensmaker's equation, with $R_1=R_2=R$:

$$\frac{1}{f}=(n-1)\left[\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)\right];$$

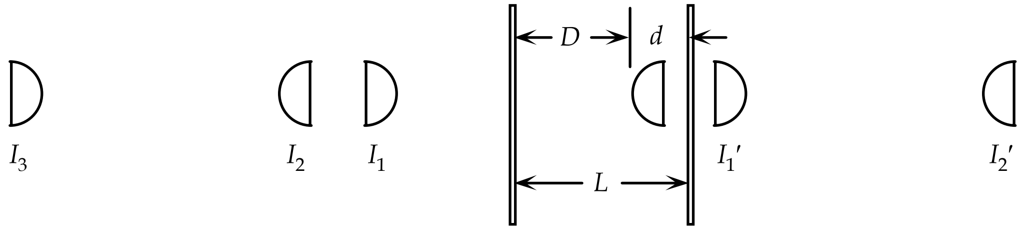
$$\frac{1}{25.0\,\text{cm}}=(1.52-1)\left(\frac{2}{R}\right),\text{ which gives }R=\boxed{26\,\text{cm.}}$$

71. We find the radius from the lensmaker's equation:

$$\frac{1}{f}=(n-1)\left[\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)\right];$$

$$+1.50\,\mathrm{D} = (1.56 - 1)\left[\left(\frac{1}{40.0\,\mathrm{cm}}\right) + \left(\frac{1}{R_2}\right)\right],\text{ which gives } R_2 = \boxed{5.6\,\mathrm{m}(\text{convex}).}$$

72. For a plane mirror each image is as far behind the mirror as the object is in front. Each reflection produces a front-to-back reversal. We show the three images and the two intermediate images that are not seen.



(*a*) The first image is from a single reflection, so it is

$$d_1 = 2D = 2(1.5\,\mathrm{m}) = \boxed{3.0\,\mathrm{m}}\text{ away.}$$

The second image is from two reflections, so it is

$$d_2 = L + d + D = 2.0\,\mathrm{m} + 0.5\,\mathrm{m} + 1.5\,\mathrm{m} = \boxed{4.0\,\mathrm{m}}\text{ away.}$$

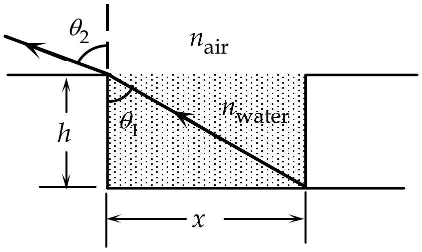
The third image is from three reflections, so it is

$$d_3 = 2L + D + D = 2(2.0\,\mathrm{m}) + 1.5\,\mathrm{m} + 1.5\,\mathrm{m} = \boxed{7.0\,\mathrm{m}}\text{ away.}$$

(*b*) We see from the diagram that

the first image is facing $\boxed{\text{toward you;}}$
the second image is facing $\boxed{\text{away from you;}}$
the third image is facing $\boxed{\text{toward you.}}$

73. We find the angle of incidence for the refraction from water into air:



$$n_{\text{water}}\sin\theta_1 = n_{\text{air}}\sin\theta_2;$$

$$(1.33)\sin\theta_1 = (1.00)\sin(90° - 14°),$$

which gives $\theta_1 = 47°$.

We find the depth of the pool from

$$\tan\theta_1 = \frac{x}{h};$$

$$\tan 47° = \frac{(5.50\,\mathrm{m})}{h},\text{ which gives } h = \boxed{5.2\,\mathrm{m}.}$$

74. At the critical angle, the refracted angle is 90°. For the refraction from plastic to air, we have

$$n_{\text{plastic}}\sin\theta_{\text{plastic}} = n_{\text{air}}\sin\theta_{\text{air}};$$

$$n_{\text{plastic}}\sin 37.3° = (1.00)\sin 90°,\text{ which gives } n_{\text{plastic}} = 1.65.$$

For the refraction from plastic to water, we have

$$n_{\text{plastic}}\sin\theta_{\text{plastic}}' = n_{\text{water}}\sin\theta_{\text{water}};$$

$$(1.65)\sin\theta_{\text{plastic}}' = (1.33)\sin 90°,\text{ which gives } \theta_{\text{plastic}}' = \boxed{53.7°.}$$

75. (*a*) As the radius of a sphere gets larger, the surface is flatter. The plane mirror can be considered a spherical mirror with an infinite radius, and thus $\boxed{f = \infty.}$

(*b*) When we use the mirror equation, we get

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

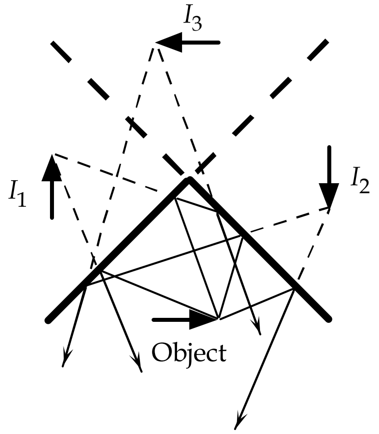
$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{\infty}=0,\text{ or }\boxed{d_i=-d_o.}$$

(*c*) For the magnification, we have

$$m=\frac{-d_i}{d_o}=\frac{-(-d_o)}{d_o}=\boxed{+1.}$$

(*d*) $\boxed{\text{Yes,}}$ these are consistent with the discussion on plane mirrors.

76. We show some of the rays from the tip of the arrow that form the three images. Single reflections form the two side images. Double reflections form the third image. The two reflections have reversed the orientation of the image.



77. We get an expression for the image distance from the lens equation:

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\frac{1}{d_i}=\left(\frac{1}{f}\right)-\left(\frac{1}{d_o}\right),\text{ or }d_i=\frac{fd_o}{(d_o-f)}.$$

If the lens is diverging, $f<0$. If we write $f=-|f|$, we get $d_i=\frac{-|f|d_o}{(d_o+|f|)}$.

For a real object, $d_o>0$.

All factors in the expression for d_i are positive, thus $d_i<0$, so the image is always virtual.

We can have a real image, $d_i>0$, if $d_o<0$, and $|d_o|<|f|$, so the denominator is still positive. Thus to have a real image from a diverging lens, the condition is $\boxed{0<-d_o<-f.}$

78. The two students $\boxed{\text{chose different signs for the magnification,}}$ i.e., one upright and one inverted.

The focal length of the concave mirror is $f=\frac{R}{2}=\frac{(40\,\text{cm})}{2}=20\,\text{cm}.$

We relate the object and image distances from the magnification:

$$m = \frac{-d_\text{i}}{d_\text{o}};$$

$$\pm 3 = \frac{-d_\text{i}}{d_\text{o}}, \text{ which gives } d_\text{i} = \mp 3d_\text{o}.$$

When we use this in the mirror equation, we get

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

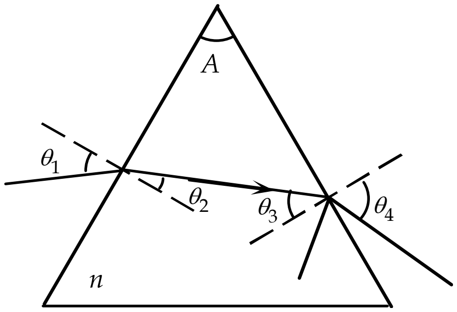
$$\left(\frac{1}{d_\text{o}}\right) + \left[\frac{1}{(\mp 3d_\text{o})}\right] = \frac{1}{f}, \text{ which gives } \boxed{d_\text{o} = \frac{2f}{3}, \frac{4f}{3} = 13.3\,\text{cm}, 26.7\,\text{cm}.}$$

The image distances are = – 40 cm (virtual, upright), and + 80 cm (real, inverted).

79. For the refraction at the second surface, we have

$$n \sin\theta_3 = n_\text{air} \sin\theta_4;$$

$$(1.50)\sin\theta_3 = (1.00)\sin\theta_4.$$



The maximum value of θ_4 before internal reflection takes place at the second surface is 90°. Thus for internal reflection not to occur, we have

$$(1.50)\sin\theta_3 \le 1.00;$$

$$\sin\theta_3 \le 0.667, \text{ so } \theta_3 \le 41.8°.$$

We find the refraction angle at the second surface from

$$(90° - \theta_2) + (90° - \theta_3) + A = 180°, \text{ which gives}$$

$$\theta_2 = A - \theta_3 = 72° - \theta_3.$$

Thus $\theta_2 \ge 72° - 41.8° = 30.2°$.

For the refraction at the first surface, we have

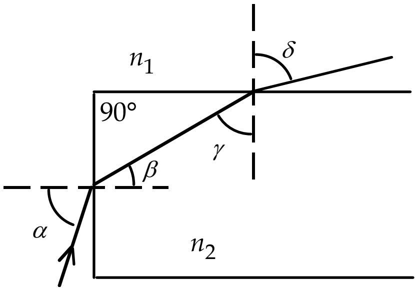
$$n_\text{air} \sin\theta_1 = n \sin\theta_2;$$

$$(1.00)\sin\theta_1 = (1.50)\sin\theta_2; \text{ which gives } \sin\theta_1 = (1.50)\sin\theta_2.$$

For the limiting condition, we have

$$\sin\theta_1 \ge (1.50)\sin 30.2° = 0.754, \text{ so } \theta_1 \boxed{\ge 49°.}$$

80. If total internal reflection fails at all, it fails for $\alpha \approx 90°$. So assume $\alpha = 90°$. Then



$$n_1 \sin\alpha = n_2 \sin\beta$$

becomes

$$n_1 = n_2 \sin\beta.$$

We combine this with

$$n_2 \sin\gamma = n_1 \sin\delta$$

as follows. Since $\beta + \gamma = 90°$,

$$\sin\beta = \cos\gamma = \sqrt{1 - \sin^2\gamma} = \sqrt{1 - \left(\frac{n_1}{n_2}\sin\delta\right)^2},$$

and so

$$n_1 = n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\delta};$$

$$1 = \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\delta};$$

$$\sin^2\delta = \left(\frac{n_2}{n_1}\right)^2 - 1.$$

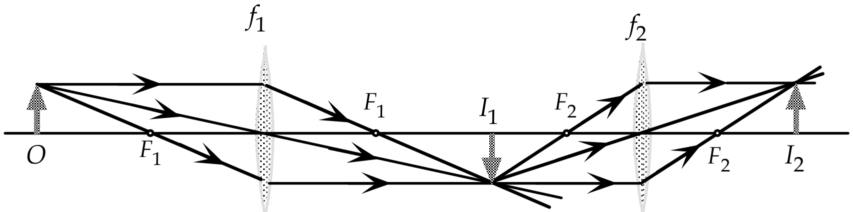
For total internal reflection to fail,

$$\sin\delta < 1;\ \sin^2\delta;$$

$$\left(\frac{n_2}{n_1}\right)^2 - 1 < 1;\ \left(\frac{n_2}{n_1}\right)^2 < 2.$$

This is not true when the rod $(n_2 = 1.54)$ is in air $(n_1 = 1.00)$, but it is when the rod is in water $(n_1 = 1.33)$. So with the rod in water, total internal reflection is not guaranteed.

81. (*a*)



We see that the image is real and upright, and estimate that it is 20 cm beyond the second lens.

(*b*) We find the image formed by the refraction of the first lens:

$$\left(\frac{1}{d_{\text{o}1}}\right) + \left(\frac{1}{d_{\text{i}1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{33\,\text{cm}}\right) + \left(\frac{1}{d_{\text{i}1}}\right) = \frac{1}{15\,\text{cm}},\ \text{which gives } d_{\text{i}1} = +27.5\,\text{cm}.$$

This image is the object for the second lens. Because it is in front of the second lens, it is a real object, with an object distance of $d_{\text{o}2} = 55\,\text{cm} - 27.5\,\text{cm} = 27.5\,\text{cm}$.

We find the image formed by the refraction of the second lens:

$$\left(\frac{1}{d_{\text{o}2}}\right) + \left(\frac{1}{d_{\text{i}2}}\right) = \frac{1}{f_2};$$

$$\left(\frac{1}{27.5\,\text{cm}}\right) + \left(\frac{1}{d_{\text{i}2}}\right) = \frac{1}{12\,\text{cm}},\ \text{which gives } d_{\text{i}2} = +21.3\,\text{cm}.$$

Thus the final image is $\boxed{\text{real, 21 cm beyond second lens.}}$

The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(\frac{-d_{\text{i}1}}{d_{\text{o}1}}\right)\left(\frac{-d_{\text{i}2}}{d_{\text{o}2}}\right) = \frac{d_{\text{i}1}d_{\text{i}2}}{d_{\text{o}1}d_{\text{o}2}}$$

$$= \frac{(+27.5\,\text{cm})(+21.3\,\text{cm})}{(+33\,\text{cm})(+27.5\,\text{cm})} = +0.65.$$

Thus the final image is $\boxed{\text{0.65 times the size of the original object.}}$

82. (*a*) We use the lens equation with $d_\text{o} + d_\text{i} = d_\text{T}$:

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{d_o}\right) + \left[\frac{1}{(d_\text{T} - d_\text{o})}\right] = \frac{1}{f}.$$

When we rearrange this, we get a quadratic equation for d_o:

$d_\text{o}^{\,2} - d_\text{T} d_\text{o} + d_\text{T} f = 0,$ which has the solution

$$d_\text{o} = \tfrac{1}{2}\left[d_\text{T} \pm \left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}\right].$$

If $d_\text{T} > 4f,$ we see that the term inside the square root $d_\text{T}^{\,2} - 4d_\text{T} f > 0,$ and $\left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}} < d_\text{T},$ so we get two real, positive solutions for d_o.

(*b*) If $d_\text{T} < 4f,$ we see that the term inside the square root $d_\text{T}^{\,2} - 4d_\text{T} f < 0,$ so there are no real solutions for d_o.

(*c*) When there are two solutions, the distance between them is

$$\Delta d = d_\text{o1} - d_\text{o2} = \tfrac{1}{2}\left[d_\text{T} + \left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}\right] - \tfrac{1}{2}\left[d_\text{T} + \left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}\right] = \boxed{\left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}.}$$

The image positions are given by

$$d_\text{i} = d_\text{T} - d_\text{o} = \tfrac{1}{2}\left[d_\text{T} \mp \left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}\right].$$

The ratio of image sizes is the ratio of magnifications:

$$m = \frac{m_2}{m_1} = \frac{\left(\dfrac{d_\text{i2}}{d_\text{o2}}\right)}{\left(\dfrac{d_\text{i1}}{d_\text{o1}}\right)} = \left(\frac{d_\text{i2}}{d_\text{o2}}\right)\left(\frac{d_\text{o1}}{d_\text{i1}}\right)$$

$$= \left\{\frac{\tfrac{1}{2}\left[d_\text{T} + \left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}\right]}{\tfrac{1}{2}\left[d_\text{T} - \left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}\right]}\right\}^2$$

$$= \boxed{\left\{\frac{\left[d_\text{T} + \left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}\right]}{\left[d_\text{T} - \left(d_\text{T}^{\,2} - 4d_\text{T} f\right)^{\frac{1}{2}}\right]}\right\}^2.}$$

83. We find the object distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{8.00\times 10^3\,\text{mm}}\right)=\frac{1}{105\,\text{mm}},\ \text{which gives } d_o=106\,\text{mm}=\boxed{0.106\,\text{m.}}$$

We find the size of the image from

$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o};$$

$$\frac{h_i}{(0.036\,\text{m})}=\frac{-(8.00\,\text{m})}{(0.106\,\text{m})},\ \text{which gives } |h_i|=\boxed{2.7\,\text{m.}}$$

84. We find the object distance from the required magnification (which is negative for a real object and a real image):

$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o};$$

$$\frac{-(2.7\times 10^3\,\text{mm})}{(36\,\text{mm})}=\frac{-(7.50\,\text{m})}{d_o},\ \text{which gives } d_o=0.10\,\text{m.}$$

We find the focal length of the lens from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{0.10\,\text{m}}\right)+\left(\frac{1}{7.50\,\text{m}}\right)=\frac{1}{f},\ \text{which gives } f=0.0987\,\text{m}=\boxed{+9.9\,\text{cm.}}$$

85. We get an expression for the image distance from the lens equation:

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\frac{1}{d_i}=\left(\frac{1}{f}\right)-\left(\frac{1}{d_o}\right),\ \text{or } d_i=\frac{fd_o}{(d_o-f)}.$$

The magnification is

$$m=\frac{-d_i}{d_o}=\frac{-f}{(d_o-f)}.$$

If the lens is converging, $f>0$.

For a real object, $d_o>0$.

When $d_o>f$, we have $(d_o-f)>0$, so all factors in the expressions for d_i and *m* are positive; thus $d_i>0$ (real), and $m<0$ (inverted).

When $d_o<f$, we have $(d_o-f)<0$, so the denominator in the expressions for d_i and *m* are negative; thus $d_i<0$ (virtual), and $m>0$ (upright).

For an object beyond the lens, $d_o > 0$.

When $-d_o > f$, we have $(d_o - f) < 0$, so both numerator and denominator in the expression for d_i are negative; thus $d_i > 0$, so the image is $\boxed{\text{real.}}$ The numerator in the expression for m is negative; thus $m > 0$, so the image is $\boxed{\text{upright.}}$

When $0 < -d_o < f$, we have $(d_o - f) < 0$, so we get the same result: $\boxed{\text{real and upright.}}$

86. We will use two equations:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \qquad (1)$$

and

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad (2)$$

From (1),

$$-\frac{1}{m} = -\frac{h_o}{h_i} = \frac{d_o}{d_i},$$

and from (2),

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}.$$

Therefore,

$$-\frac{h_o}{h_i} = d_o\left(\frac{1}{f} - \frac{1}{d_o}\right)$$

$$= \frac{d_o}{f} - 1; \text{ and so}$$

$$d_o = f\left(1 - \frac{h_o}{h_i}\right)$$

$$= (210\,\text{mm})\left(1 - \frac{1.75\,\text{m}}{-0.00825\,\text{m}}\right)$$

$$= 44{,}755\,\text{mm}.$$

The reporter was standing $\boxed{45\text{ m}}$ from the subject.

87. (*a*) Because the Sun is very far away, the image will be at the focal point. We find the size of the image from

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o};$$

$$\frac{h_i}{(1.4 \times 10^6\text{ km})} = \frac{-(28\,\text{mm})}{(1.5 \times 10^8\text{km})}, \text{ which gives } h_i = \boxed{-0.26\,\text{mm.}}$$

(*b*) For a 50 mm lens, we have

$$\frac{h_i}{(1.4 \times 10^6\text{ km})} = \frac{-(50\,\text{mm})}{(1.5 \times 10^8\text{km})}, \text{ which gives } h_i = \boxed{-0.47\,\text{mm.}}$$

(*c*) For a 135 mm lens, we have

$$\frac{h_\text{i}}{\left(1.4\times10^{6}\,\text{km}\right)}=\frac{-\left(135\,\text{mm}\right)}{\left(1.5\times10^{8}\,\text{km}\right)},\ \text{which gives}\ h_\text{i}=\boxed{-1.3\,\text{mm.}}$$

(*d*) The equations show that image height is directly proportional to focal length. Relative to the 50 mm lens, the magnifications of the other two lenses are $\frac{28\,\text{mm}}{50\,\text{mm}}=\boxed{0.56}$ for the 28 mm lens and $\frac{135\,\text{mm}}{50\,\text{mm}}=\boxed{2.7}$ for the 135 mm lens.

88. (*a*) We find the focal length of the lens from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{34.5\,\text{cm}}\right)+\left[\frac{1}{\left(-8.20\,\text{cm}\right)}\right]=\frac{1}{f},\ \text{which gives}\ f=\boxed{-10.8\,\text{cm}\ (\text{diverging}).}$$

The image is in front of the lens, so it is $\boxed{\text{virtual.}}$

(*b*) We find the focal length of the lens from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{34.5\,\text{cm}}\right)+\left[\frac{1}{\left(-41.5\,\text{cm}\right)}\right]=\frac{1}{f},\ \text{which gives}\ f=\boxed{+205\,\text{cm}\ (\text{converging}).}$$

The image is in front of the lens, so it is virtual.

89. If we let d_i represent the original image distance, then $d_\text{i}+10.0\,\text{cm}$ represents the new image distance. From the general equation

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}$$

we can then obtain two equations:

$$\frac{1}{60.0\,\text{cm}}+\frac{1}{d_\text{i}}=\frac{1}{f}\ \text{and}$$

$$\frac{1}{40.0\,\text{cm}}+\frac{1}{d_\text{i}+10.0\,\text{cm}}=\frac{1}{f}.$$

Setting the left sides equal, we solve

$$\frac{1}{60.0\,\text{cm}}+\frac{1}{d_\text{i}}=\frac{1}{40.0\,\text{cm}}+\frac{1}{d_\text{i}+10.0\,\text{cm}},$$

which gives $d_\text{i}=-40.0\,\text{cm}$ or $30.0\,\text{cm}$,

where the latter is the solution corresponding to a real image. Now we solve

$$\frac{1}{60.0\,\text{cm}}+\frac{1}{30.0\,\text{cm}}=\frac{1}{f}$$

to find that $f=\boxed{20.0\,\text{cm.}}$

90. Working backwards, we use

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2}$$

with $d_{i2}=17.0\,\text{cm}$ and $f_2=12.0\,\text{cm}$:

$$\frac{1}{d_{o2}}+\frac{1}{17.0\,\text{cm}}=\frac{1}{12.0\,\text{cm}}\ \text{gives}\ d_{o2}=40.8\,\text{cm}.$$

This means that $d_{i1}=30.0\,\text{cm}-40.8\,\text{cm}=-10.8\,\text{cm}$ (10.8 to the left of the diverging lens). So for the diverging lens,

$$\left[\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1};\right]$$

$$\frac{1}{25.0\,\text{cm}}+\frac{1}{-10.8\,\text{cm}}=\frac{1}{f_1},\ \text{which gives}\ f_1=\boxed{19\,\text{cm.}}$$

91. From the given information,

$$m=-\frac{1}{2}=-\frac{d_i}{d_o};$$

$$d_o=2d_i.$$

So for $d_o=15\,\text{cm}$, $\boxed{d_i=7.5\,\text{cm.}}$ Now we find the focal length:

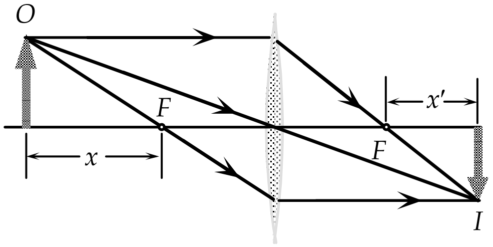
$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f};$$

$$\frac{1}{15\,\text{cm}}+\frac{1}{7.5\,\text{cm}}=\frac{1}{f},\ \text{which gives}\ f=5.0\,\text{cm}.$$

And since

$$f=\frac{r}{2},\ \text{it follows that}\ \boxed{r=10\,\text{cm.}}$$

92. (*a*) For the thin lens we have



$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(f+x)}\right]+\left[\frac{1}{(f+x')}\right]=\frac{1}{f},$$

which can be written as

$$2f+x+x'=\frac{(f+x)(f+x')}{f}$$

$$=f+(x+x')+\left(\frac{xx'}{f}\right),\ \text{or}\ xx'=f^2.$$

(*b*) For the standard form we have

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{45.0\,\text{cm}}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{32.0\,\text{cm}},\ \text{which gives}\ d_i=\boxed{+110.8\,\text{cm.}}$$

(*c*) For the Newtonian form we have

$$xx' = f^2;$$

$$(45.0\,\text{cm} - 32.0\,\text{cm})x' = (32.0\,\text{cm})^2, \text{which gives } x' = 78.7\,\text{cm}.$$

Thus the distance from the lens is

$$d_\text{i} = x' + f = 78.7\,\text{cm} + 32.0\,\text{cm} = \boxed{110.8\,\text{cm.}}$$

93. We find the focal length by finding the image distance for an object very far away.
For the first lens, we have

$$\left(\frac{1}{d_\text{o1}}\right) + \left(\frac{1}{d_\text{i1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{d_\text{i1}}\right) = \frac{1}{(10.0\,\text{cm})}, \text{ or, as expected, } d_\text{i1} = 10.0\,\text{cm}.$$

The first image is the object for the second lens. The first image is real, so the second object has a negative object distance:

$$d_\text{o2} = -d_\text{i1} = -10.0\,\text{cm}.$$

For the second lens, we have

$$\left(\frac{1}{d_\text{o2}}\right) + \left(\frac{1}{d_\text{i2}}\right) = \frac{1}{f_2};$$

$$\left[\frac{1}{(-10.0\,\text{cm})}\right] + \left(\frac{1}{d_\text{i2}}\right) = \frac{1}{(-20.0\,\text{cm})}, \text{ which gives } d_\text{i2} = +20.0\,\text{cm}.$$

Because the second image must be at the focal point of the combination, we have

$$\boxed{f = +20.0\,\text{cm (converging).}}$$

94. (*a*) We find the focal length by finding the image distance for an object very far away.
For the first lens, we have

$$\left(\frac{1}{d_\text{o1}}\right) + \left(\frac{1}{d_\text{i1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{d_\text{i1}}\right) = \frac{1}{f_1}, \text{ or, as expected, } d_\text{i1} = f_1.$$

The first image is the object for the second lens. If the first image is real, the second object is virtual:

$$d_\text{o2} = -d_\text{i1} = -f_1.$$

For the second lens, we have

$$\left(\frac{1}{d_\text{o2}}\right) + \left(\frac{1}{d_\text{i2}}\right) = \frac{1}{f_2};$$

$$\left[\frac{1}{(-f_1)}\right] + \left(\frac{1}{d_\text{i2}}\right) = \frac{1}{f_2}.$$

Because the second image must be at the focal point of the combination, we have $\left(\frac{-1}{f_1}\right)+\left(\frac{1}{f_T}\right)=\frac{1}{f_2}$, which gives $\frac{1}{f_T}=\left(\frac{1}{f_1}\right)+\left(\frac{1}{f_2}\right)$.

When we solve for f_T, we get

$$f_T=\frac{f_1f_2}{(f_1+f_2)}.$$

(*b*) If we use the intermediate result $\frac{1}{f_T}=\left(\frac{1}{f_1}\right)+\left(\frac{1}{f_2}\right)$, we see that

$$P=P_1+P_1.$$

CHAPTER 24: The Wave Nature of Light

Answers to Questions

1. Huygens' principle applies both to sound waves and water waves. Huygens' principle applies to all waves that form a wave crest. Both sounds and water waves can be represented in this way.

2. A piece of evidence that light is energy is that you can focus the light from the Sun with a magnifying glass on to a sheet of paper and burn a hole in it. You have added so much energy to the paper that its temperature rises to the point where it ignites.

3. There are certain situations where describing light as rays works well (for example, lenses) and there are other situations where describing light as waves works well (for example, diffraction). Actually, the ray model doesn't work at all when describing diffraction. Thus, there are always limitations to the "models" we use to describe nature and we need to realize what these are.

4. The main reason that we can hear sounds around corners, but not see around corners, is diffraction. Sound waves have very long wavelengths when compared to light waves, which makes diffraction effects much more obvious. Diffraction effects are very noticeable once the size of the object that the wave is diffracting around is about the same size as the wavelength of the wave. The wavelength of sound is on the order of 1 m, while the wavelength of light is on the order of 0.1 μm. A secondary reason is reflection. Sounds waves reflect off of walls very well in a specular manner, and so can bounce around corners, but light reflects off of the walls in a very diffuse manner.

$\boxed{5.}$ The wavelength of light in a medium such as water is decreased when compared to the wavelength in air. Thus, $d\sin\theta = m\lambda$ says that θ is decreased for a particular *m* and *d*. This means that the bright spots on the screen are more closely packed together in water than in air.

6. As red light is switched to blue light, the wavelength of the light is decreased. Thus, $d\sin\theta = m\lambda$ says that θ is decreased for a particular *m* and *d*. This means that the bright spots on the screen are more closely packed together with blue light than with red light.

$\boxed{7.}$ Destructive interference occurs when the path lengths of two rays of light from the same source differ by odd half-integers of the wavelength ($\lambda/2,\ 3\lambda/2,\ 5\lambda/2,\ \left(m+\tfrac{1}{2}\right)\lambda$, etc.). Under these conditions, the wave crests from one ray match up with the wave troughs from the other ray and cancellation occurs (destructive interference).

8. One reason was that the double-slit experiment allowed scientists to measure an actual wavelength, which was something that could not be done at the time with diffraction observations. Another reason was that using a particle model, you could explain diffraction in a qualitative way by talking about the particles bouncing off the edges of the diffracting object or the two edges of a single slit. When a second slit is added, the particle model had a much more difficult time explaining why there are now dark spots where the particles used to be able to strike with only one slit open.

9. Similarities between doing a double-slit experiment with sound and light: the sources must be coherent for the interference pattern to be observed; they both produce a pattern of high and low intensity at some distance away from the double slit (bright and dark for the light and loud and quiet for the sound); they both work best with a single-frequency source. Differences between doing a double-slit experiment with

sound and light: The slits for light must be extremely close together when compared to sound; you don't actually need slits for sound (just use two speakers).

$\boxed{10.}$ The reason you do not get an interference pattern from the two headlights of a distant car is that they are not coherent light sources (they have random phases). Thus, you cannot produce zones of destructive and constructive interference where the crests and troughs match up or the crests and crests match up. Also, the headlights are far enough apart that even if they were coherent, the interference pattern would be so tightly packed that it would not be observable with the unaided eye.

11. Basically there would be two overlapping single-slit diffraction patterns, with one being blue and the other being red. Wherever the two different patterns overlapped, there would be a purplish (red + blue) bright spot. There would not be a central bright spot, due to the different wavelengths of the two colors.

12. The two faces of the pane of glass are parallel to each other, so the white light rays that were separated into colors as they entered the front face all come out of the second face at the same angle as each other. When these rays reach your eyes, they all combine back together into white light. The separated colors in the prism reach the other face of the prism at all different angles (since the second face of the prism is not parallel to the first face), which means that all of these different colors leave the prism at all different angles. These light rays are still separated into colors when they reach your eyes, which allows you to see a rainbow of colors at different angles.

13. Since red light is bent less than violet light in glass (the index of refraction for red light in glass is less than the index of refraction for violet light in glass), the focal length of both a converging lens and a diverging lens is longer for red light and shorter for violet light.

14. By looking at the direction and the relative amount that the light rays bend at each interface, we can infer the relative sizes of the indices of refraction in the different materials (bends toward normal = faster material to slower material or smaller n material to larger n material; bends away from normal = slower material to faster material or larger n material to smaller n material). From the first material to the second material the ray bends toward the normal, thus it slows down and $n_1 < n_2$. From the second material to the third material the ray bends away from the normal, thus it speeds up and $n_2 > n_3$. Careful inspection shows that the ray in the third material does not bend back away from the normal as far as the ray was in the first material, thus the speed in the first material is the faster than in the third material and $n_1 < n_3$. Thus, the overall ranking of indices of refraction is: $n_1 < n_3 < n_2$.

15. As you squeeze your fingers together, you start to see vertical bright and dark bands that are aligned parallel to your fingers. The dark bands get wider as you continue to bring your fingers closer together, until at one point, when your fingers are still not actually touching, the dark bands seem to quickly jump in and darken the entire gap.

16. (*a*) When you immerse a single-slit diffraction apparatus in water, the wavelength of light gets smaller, due to the increase in the index of refraction, and the diffraction pattern gets more closely spaced. The equation $\sin\theta = \frac{\lambda}{D}$ for the half-width of the central maximum says that θ is decreased for a particular D when the wavelength decreases. And the equation for the locations of the minima, $\sin\theta = \frac{m\lambda}{D}$, also indicates that θ is decreased for a particular m and D when the wavelength decreases. This means that the bright spots on the screen are more closely spaced in water than in air.

(*b*) When you perform a single-slit diffraction experiment in vacuum, the wavelength of light gets slightly larger, due to the decrease in the index of refraction, and the diffraction pattern spreads farther apart. The equation $\sin\theta = \frac{\lambda}{D}$ for the half-width of the central maximum says that θ is increased for a particular *D* when the wavelength increases. And the equation for the locations of the minima, $\sin\theta = \frac{m\lambda}{D}$, also indicates that θ is increased for a particular *m* and *D* when the wavelength increases. This means that the bright spots on the screen are spread farther apart in vacuum than in air.

17. (*a*) When you increase the slit width in a single-slit diffraction experiment, the spacing of the fringes decreases. The equation for the location of the minima, $\sin\theta = \frac{m\lambda}{D}$, indicates that θ is decreased for a particular *m* and λ when the width *D* increases. This means that the bright spots on the screen are more closely packed together for a wider slit.

(*b*) When you increase the wavelength of light used in a single-slit diffraction experiment, the spacing of the fringes increases. The equation for the location of the minima, $\sin\theta = \frac{m\lambda}{D}$, indicates that θ is increased for a particular *m* and *D* when the wavelength increases. This means that the bright spots on the screen are spread further apart for a longer wavelength.

18. The interference pattern created by the diffraction grating with 10^4 lines/cm has bright maxima that are more sharply defined and narrower than the interference pattern created by the two slits 10^{-4} cm apart.

19. (*a*) The advantage of having many slits in a diffraction grating is that this makes the bright maxima in the interference pattern more sharply defined, brighter, and narrower.

(*b*) The advantage of having closely spaced slits in a diffraction grating is that this spreads out the bright maxima in the interference pattern and makes them easier to measure.

20. (*a*) The color at the top of the rainbow for the diffraction grating is violet. The equation $d\sin\theta = m\lambda$ says that θ is smallest (thus, the deviation from horizontal is smallest) for the shortest wavelength, for a given *d* and *m*. The wavelength of violet light (450 nm) is shorter than that of red light (700 nm).

(*b*) The color at the top of a rainbow for the prism is red. The index of refraction for transparent materials (like the glass that makes up the prism) is smaller for long (red) wavelengths and larger for short (violet) wavelengths. Since the red light encounters a smaller index of refraction as it goes through the prism, it doesn't slow as much as the violet light, which also means that it doesn't bend as much as the violet. If the red light is bent away from the horizontal direction least, it will appear at the top of the rainbow.

21. For the red and violet colors from different orders to overlap, the angles in the equation $d\sin\theta = m\lambda$ would need to be equal. Mathematically, this means: $\frac{(\mathrm{m}+1)(400\ \mathrm{nm})}{\mathrm{d}} < \frac{\mathrm{m}(700\ \mathrm{nm})}{\mathrm{d}}$. In other words, the m^{th} order 700 nm line overlaps the $(m+1)^{\text{th}}$ order 400 nm line. Reducing this equation, we get $m > 4/3$. So, starting with the 2nd order 700 nm line and the 3rd order 400 nm line, the observed spectra will always overlap at this and higher orders. This answer does not depend on *d*. As you can see from the above equation, the slit spacing cancels out.

22. Once the thickness of the film becomes more than a few wavelengths thick, several interference patterns become mixed together, and it is hard to see any individual effects. When the thickness of the film is only about 1 λ thick, then the reflections from the top and bottom surfaces of the film for each color have path differences of just one constructive interference (path difference = $\lambda/2$) and one destructive interference (path difference = λ) patterns. It is easy for our eyes to pick out these widely spaced bright colors that are separated by dark areas on the film. Once the film gets very thick, though, there are many constructive ($\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.) and many destructive (λ, 2λ, 3λ, etc.) path differences allowed. The resulting interference patterns are all closely spaced and overlapping, making it difficult for our eyes to distinguish between the bright and dark areas. As the film gets even thicker, the larger amount of overlap causes all the colors to run together, making it impossible to see the individual interference patterns.

23. There are many, many circular tracks on a CD and each track is made up of a series of pits and high spots. Light reflects very well off of the high spots and not the low spots. Thus, when you shine white light on a CD, each track is a slightly different distance from you, and as the light reflects off of each track to you, they each have a different path length. Thus, you'll basically see a colorful diffraction grating pattern. If a monochromatic light is used, you will see a single-color interference pattern. In other words, instead of seeing the full rainbow of colors spreading out from the center of the CD (as shown in Figure 24-56), there would just be several "spokes" of the same color as your source spreading out from the center of the CD. The spacings of these "spokes" can be used just like a diffraction grating to determine that the track spacing on the CD is approximately 1600 nm.

24. As you move farther away from the center of the curved piece of glass on top, the path differences change more rapidly due to the curvature. Thus, you get higher order interference patterns more closely spaced together. An air wedge has equally spaced interference patterns because as you move farther from the contact point of the flat piece of glass on top, the path differences change linearly.

25. If yellow/green light is getting reflected back to us, then the coating must be designed to transmit violet, blue, orange, and red light completely.

26. At the edge of the oil drop, the film is so thin that the path difference between the light reflecting off of the top surface and the light going through the oil and reflecting off of the bottom surface is so small that we can consider it to be zero. Thus, the two different rays of light must be in phase when they reach our eyes. We know that the phase of the light being reflected off of the top surface of the oil must have been flipped 180°, since the index of refraction of oil is greater than that of air. Thus, the reflection off of the bottom surface of the film (where it touches the water), must also have flipped the phase of the light 180°. This tells us that the index of refraction of the water is higher than that of the oil. Thus, we know that the index of refraction of the oil is greater than that of air and less than that of water: $1.00 < n < 1.33$.

$\boxed{27.}$ Polarization tells us that light is a transverse wave. Longitudinal waves cannot be polarized.

28. Polarized sunglasses completely (100%) block horizontally polarized glare and block all other polarizations of light 50%. Regular sunglasses just block 50-75% of all light coming in. The advantage of polarized sunglasses is the total elimination of glare. Even if regular sunglasses block a glare at 75%, the glare is so intense that it still makes it difficult for our eyes.

$\boxed{29.}$ To determine if a pair of sunglasses is polarizing or not, first find the glare (strong reflection) from a light source on a horizontal non-metallic surface (water, tiled floor, polished tabletop, top of car). Then look at the reflection through the sunglasses and rotate them. If they are polarizing sunglasses, then there should be orientations of the glasses that almost completely block out the glare and other orientations that let 50% of the glare through. If they are not polarizing sunglasses, the glare will be slightly diminished by the same amount at all orientations of the glasses, but it will never completely go away.

30. The first sheet of polarizer will diminish the intensity of the incoming non-polarized light by 50%: $I_1 = \frac{1}{2} I_0$. The next polarizer will diminish the light again, but this time only by a factor of $\cos^2 \theta$: $I_1 = I_1 \cos^2 30^\circ = \frac{3}{4} I_1 = \frac{3}{8} I_0$. The next polarizer will diminish the light by the same factor: $I_3 = I_2 \cos^2 30^\circ = \frac{3}{4} I_2 = \frac{9}{32} I_0$. The last polarizer will again diminish the light by the same factor: $I_4 = I_3 \cos^2 30^\circ = \frac{3}{4} I_3 = \frac{27}{128} I_0 = 0.211 I_0$. Thus, about 21% of the light gets through these four polarizers when each one is rotated by 30°.

31. If Earth had no atmosphere, the "color" of the sky would be black (and dotted with stars and planets) at all times. This is the condition of the sky that the astronauts found on the Moon, which has no atmosphere. The reason the sky is blue for Earth, is that the air molecules scatter light from the Sun in all directions, and preferentially scatter blue light down to the surface. If there were no air molecules to scatter the light from the Sun, the only light we would see would be from the stars/planets and directly from the Sun and the rest would be black.

32. If the atmosphere were 50% more dense, sunlight would be much redder than it is now. As the atmosphere increased in density, more and more of the blue light would be scattered away in all directions, making the light that reaches the ground very red. Think of the color of a deep red sunset, but this would be the color even at noon.

Solutions to Problems

1. For constructive interference, the path difference is a multiple of the wavelength:

$$d \sin\theta = m\lambda, \quad m = 0, 1, 2, 3, \ldots .$$

For the fifth order, we have

$$\left(1.6 \times 10^{-5}\,\text{m}\right)\sin 8.8^\circ = (5)\lambda, \text{ which gives } \lambda = \boxed{4.9 \times 10^{-7}\,\text{m.}}$$

2. For constructive interference, the path difference is a multiple of the wavelength:

$$d \sin\theta = m\lambda, \quad m = 0, 1, 2, 3, \ldots .$$

For the third order, we have

$$d \sin 18^\circ = (3)\left(610 \times 10^{-9}\,\text{m}\right), \text{ which gives } d = 5.9 \times 10^{-6}\,\text{m} = \boxed{5.9\,\mu\text{m.}}$$

3. For constructive interference, the path difference is a multiple of the wavelength:

$$d \sin\theta = m\lambda, \quad m = 0, 1, 2, 3, \ldots .$$

We find the location on the screen from

$$y = L \tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For adjacent fringes, $\Delta m = 1,$ so we have

$$\Delta y = \frac{L\lambda\,\Delta m}{d};$$

$$0.065\,\text{m} = \frac{(5.00\,\text{m})\lambda(1)}{(0.048\times10^{-3}\,\text{m})}, \text{ which gives } \lambda = \boxed{6.2\times10^{-7}\,\text{m.}}$$

The frequency is

$$f = \frac{c}{\lambda} = \frac{(3.00\times10^{8}\,\text{m/s})}{(6.24\times10^{-7}\,\text{m})} = \boxed{4.8\times10^{14}\,\text{Hz.}}$$

4. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots .$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For adjacent fringes, $\Delta m = 1,$ so we have

$$\Delta y = \frac{L\lambda\,\Delta m}{d};$$

$$= \frac{(3.6\,\text{m})(656\times10^{-9}\,\text{m})(1)}{(0.060\times10^{-3}\,\text{m})} = 3.9\times10^{-2}\,\text{m} = \boxed{3.9\text{ cm.}}$$

$\boxed{5.}$ For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots .$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For the fourth order we have

$$48\times10^{-3}\,\text{m} = \frac{(1.5\,\text{m})(680\times10^{-9}\,\text{m})(4)}{d}, \text{ which gives } d = 8.5\times10^{-5}\,\text{m} = \boxed{0.085\text{ mm.}}$$

6. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\quad m = 0, 1, 2, 3, \ldots .$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For the second order of the two wavelengths, we have

$$y_a = \frac{L\lambda_a m}{d} = \frac{(1.6\,\text{m})(480\times10^{-9}\,\text{m})(2)}{(0.54\times10^{-3}\,\text{m})} = 2.84\times10^{-3}\,\text{m} = 2.84\,\text{mm};$$

$$y_b = \frac{L\lambda_b m}{d} = \frac{(1.6\,\text{m})(620\times10^{-9}\,\text{m})(2)}{(0.54\times10^{-3}\,\text{m})} = 3.67\times10^{-3}\,\text{m} = 3.67\,\text{mm}.$$

Thus the two fringes are separated by $3.67\,\text{mm} - 2.84\,\text{mm} = \boxed{0.8\text{ mm.}}$

7. For constructive interference of the second order for the blue light, we have

$$d\sin\theta = m\lambda_b = (2)(460\,\text{nm}) = 920\,\text{nm}.$$

For destructive interference of the other light, we have

$$d\sin\theta = \left(m' + \tfrac{1}{2}\right)\lambda,\ m' = 0, 1, 2, 3, \ldots .$$

When the two angles are equal, we get

$$920\,\text{nm} = \left(m' + \tfrac{1}{2}\right)\lambda,\ m' = 0, 1, 2, 3, \ldots .$$

For the first three values of m', we get

$$920\,\text{nm} = \left(0 + \tfrac{1}{2}\right)\lambda,\ \text{which gives } \lambda = 1.84\times10^{3}\,\text{nm};$$

$$920\,\text{nm} = \left(1 + \tfrac{1}{2}\right)\lambda,\ \text{which gives } \lambda = 613\,\text{nm};$$

$$920\,\text{nm} = \left(2 + \tfrac{1}{2}\right)\lambda,\ \text{which gives } \lambda = 368\,\text{nm}.$$

The only one of these that is visible light is $\boxed{613\text{ nm.}}$

8. For destructive interference, the path difference is

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots;\ \text{or}$$

$$\sin\theta = \frac{\left(m + \tfrac{1}{2}\right)(2.5\,\text{cm})}{(5.0\,\text{cm})} = \left(m + \tfrac{1}{2}\right)(0.50),\ m = 0, 1, 2, 3, \ldots .$$

The angles for the first three regions of complete destructive interference are

$$\sin\theta_0 = \frac{\left(m + \tfrac{1}{2}\right)\lambda}{d} = \left(0 + \tfrac{1}{2}\right)(0.50) = 0.25,\ \theta_0 = 15°;$$

$$\sin\theta_1 = \frac{\left(m + \tfrac{1}{2}\right)\lambda}{d} = \left(1 + \tfrac{1}{2}\right)(0.50) = 0.75,\ \theta_1 = 49°;$$

$$\sin\theta_2 = \frac{\left(m + \tfrac{1}{2}\right)\lambda}{d} = \left(2 + \tfrac{1}{2}\right)(0.50) = 1.25,\ \text{therefore, no third region.}$$

We find the locations at the end of the tank from

$$y = L\tan\theta;$$

$$y_0 = (2.0\,\text{m})\tan 15° = 0.52\,\text{m};$$

$$y_1 = (2.0\,\text{m})\tan 49° = 2.3\,\text{m}.$$

Thus you could stand

$\boxed{\text{0.52 m, or 2.3 m away from the line perpendicular to the board midway between the openings.}}$

9. The 180° phase shift produced by the glass is equivalent to a path length of $\frac{1}{2}\lambda$. For constructive interference on the screen, the total path difference is a multiple of the wavelength:

$$\tfrac{1}{2}\lambda + d\sin\theta_{\text{max}} = m\lambda,\ m = 0, \pm 1, \pm 2, \pm 3, \ldots;\ \text{or}\ d\sin\theta = \left(m - \tfrac{1}{2}\right)\lambda,\ m = 0, \pm 1, \pm 2, \pm 3, \ldots.$$

For destructive interference on the screen, the total path difference is

$$\tfrac{1}{2}\lambda + d\sin\theta_{\text{max}} = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, \pm 1, \pm 2, \pm 3, \ldots;\ \text{or}\ d\sin\theta = m\lambda,\ m = 0, \pm 1, \pm 2, \pm 3, \ldots.$$

Thus the pattern is just the $\boxed{\text{reverse of the usual double-slit pattern.}}$

10. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots.$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta,\ \text{which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For the third order we have

$$12 \times 10^{-3}\,\text{m} = \frac{(3)(1.6\,\text{m})\left(500 \times 10^{-9}\,\text{m}\right)}{d},\ \text{which gives}\ d = 2.0 \times 10^{-4}\,\text{m}.$$

With the new wavelength, then, the second-order maximum is located a distance of

$$\begin{aligned} y &= \frac{mL\lambda}{d} \\ &= \frac{(2)(1.6\,\text{m})\left(650 \times 10^{-9}\,\text{m}\right)}{2.0 \times 10^{-4}\,\text{m}} = 0.010\,\text{m} \\ &= \boxed{10\,\text{mm}}\ \text{from the central maximum.} \end{aligned}$$

11. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots.$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta,\ \text{which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For adjacent fringes, $\Delta m = 1$, so we have

$$\begin{aligned} \Delta y &= \frac{L\lambda\,\Delta m}{d} \\ &= \frac{(5.0\,\text{m})\left(544 \times 10^{-9}\,\text{m}\right)(1)}{1.0 \times 10^{-3}\,\text{m}} = 2.7 \times 10^{-3}\,\text{m} = \boxed{2.7\,\text{mm.}} \end{aligned}$$

12. The presence of the water changes the wavelength: $\lambda_{\text{water}} = \dfrac{\lambda}{n_{\text{water}}} = \dfrac{480\,\text{nm}}{1.33} = 361\,\text{nm}.$

For constructive interference, the path difference is a multiple of the wavelength in the water:

$$d\sin\theta = m\lambda_{\text{water}},\ m = 0,1,2,3,\ldots.$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda_{\text{water}}}{d}\right) = \frac{mL\lambda_{\text{water}}}{d}.$$

For adjacent fringes, $\Delta m = 1$, so we have

$$\Delta y = \frac{L\lambda_{\text{water}}\,\Delta m}{d}$$

$$= \frac{(0.400\,\text{m})(361\times10^{-9}\,\text{m})(1)}{(6.00\times10^{-5}\,\text{m})} = 2.41\times10^{-3}\,\text{m} = \boxed{2.41\,\text{mm.}}$$

13. To change the center point from constructive interference to destructive interference, the phase shift produced by the introduction of the plastic must be an odd multiple of half a wavelength, corresponding to the change in the number of wavelengths in the distance equal to the thickness of the plastic. The minimum thickness will be for a shift of a half wavelength:

$$N = \left(\frac{t}{\lambda_{\text{plastic}}}\right) - \left(\frac{t}{\lambda}\right) = \left(\frac{tn_{\text{plastic}}}{\lambda}\right) - \left(\frac{t}{\lambda}\right) = \left(\frac{t}{\lambda}\right)(n_{\text{plastic}} - 1) = \tfrac{1}{2};$$

$$\left[\frac{t}{(640\,\text{nm})}\right](1.60 - 1) = \tfrac{1}{2}, \text{ which gives } t = \boxed{533\,\text{nm.}}$$

14. We find the speed of light from the index of refraction, $v = \dfrac{c}{n}$. For the change, we have

$$\frac{(v_{\text{red}} - v_{\text{violet}})}{v_{\text{violet}}} = \frac{\left[\left(\dfrac{c}{n_{\text{red}}}\right) - \left(\dfrac{c}{n_{\text{violet}}}\right)\right]}{\left(\dfrac{c}{n_{\text{violet}}}\right)}$$

$$= \frac{(n_{\text{violet}} - n_{\text{red}})}{n_{\text{red}}} = \frac{(1.665 - 1.617)}{(1.617)} = 0.030 = \boxed{3.0\%.}$$

$\boxed{15.}$ We find the angles of refraction in the glass from

$$n_1\sin\theta_1 = n_2\sin\theta_2;$$

$$(1.00)\sin 60.00° = (1.4820)\sin\theta_{2,450}, \text{ which gives } \theta_{2,450} = 35.76°;$$

$$(1.00)\sin 60.00° = (1.4742)\sin\theta_{2,700}, \text{ which gives } \theta_{2,700} = 35.98°.$$

Thus the angle between the refracted beams is

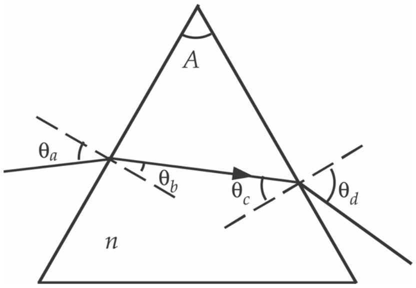
$$\theta_{2,700} - \theta_{2,450} = 35.98° - 35.76° = \boxed{0.22°.}$$

16. For the refraction at the first surface, we have

$$n_{\text{air}}\sin\theta_a = n\sin\theta_b;$$

$$(1.00)\sin 45° = (1.642)\sin\theta_{b1}, \text{ which gives } \theta_{b1} = 25.51°;$$

$$(1.00)\sin 45° = (1.619)\sin\theta_{b2}, \text{ which gives } \theta_{b2} = 25.90°.$$



We find the angle of incidence at the second surface from

$$(90° - \theta_b) + (90° - \theta_c) + A = 180°, \text{ which gives}$$

$$\theta_{c1} = A - \theta_{b1} = 60.00° - 25.51° = 34.49°;$$

$$\theta_{c2} = A - \theta_{b2} = 60.00° - 25.90° = 34.10°.$$

For the refraction at the second surface, we have

$$n\sin\theta_c = n_{\text{air}}\sin\theta_d;$$

$$(1.642)\sin 34.49° = (1.00)\sin\theta_{d1}, \text{ which gives } \boxed{\theta_{d1} = 68.4° \text{ from the normal;}}$$

$$(1.619)\sin 34.10° = (1.00)\sin\theta_{d2}, \text{ which gives } \boxed{\theta_{d2} = 65.2° \text{ from the normal.}}$$

17. We find the angle to the first minimum from

$$\sin\theta_{1\min} = \frac{m\lambda}{D} = \frac{(1)(580\times10^{-9}\,\text{m})}{(0.0440\times10^{-3}\,\text{m})} = 0.0132, \text{ so } \theta_{1\min} = 0.755°.$$

Thus the angular width of the central diffraction peak is

$$\Delta\theta_1 = 2\theta_{1\min} = 2(0.755°) = \boxed{1.51°.}$$

18. The angle from the central maximum to the first minimum is 17.5°.
We find the wavelength from

$$D\sin\theta_{1\min} = m\lambda;$$

$$(2.60\times10^{-6}\,\text{m})\sin(17.5°) = (1)\lambda, \text{ which gives } \lambda = 7.82\times10^{-7}\,\text{m} = \boxed{782\,\text{nm.}}$$

19. For constructive interference from the single slit, the path difference is

$$D\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 1, 2, 3, \ldots.$$

For the first fringe away from the central maximum, we have

$$(3.20\times10^{-6}\,\text{m})\sin\theta_1 = \left(\tfrac{3}{2}\right)(520\times10^{-9}\,\text{m}), \text{ which gives } \theta_1 = 14.1°.$$

We find the distance on the screen from

$$y_1 = L\tan\theta_1 = (10.0\,\text{m})\tan 14.1° = \boxed{2.51\,\text{m.}}$$

20. We find the angle to the first minimum from

$$\sin\theta_{1\min} = \frac{m\lambda}{D} = \frac{(1)(450\times10^{-9}\,\text{m})}{1.0\times10^{-3}\,\text{m}} = 0.00045.$$

We find the distance on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\sin\theta = (5.0\,\text{m})(0.00045) = 0.00225\,\text{m}.$$

Thus the width of the central maximum is

$$2y = 0.0045\,\text{m} = \boxed{0.45\,\text{cm.}}$$

21. The angle from the central maximum to the first bright fringe is 16°.
For constructive interference from the single slit, the path difference is

$$D\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 1,\ 2,\ 3,\ \ldots.$$

For the first fringe away from the central maximum, we have

$$D\sin\left(16^\circ\right) = \left(\tfrac{3}{2}\right)\left(653\times10^{-9}\,\text{m}\right),\ \text{which gives } D = 3.6\times10^{-6}\,\text{m} = \boxed{3.6\,\mu\text{m.}}$$

22. We find the angle to the first minimum from

$$\sin\theta_{1\min} = \frac{m\lambda}{D} = \frac{(1)\left(589\times10^{-9}\,\text{m}\right)}{\left(0.0348\times10^{-3}\,\text{m}\right)} = 0.0169,\ \text{so } \theta_{1\min} = 0.970^\circ.$$

We find the distance on the screen from

$$y_1 = L\tan\theta_1 = \left(2.30\,\text{m}\right)\tan 0.970^\circ = 3.89\times10^{-2}\,\text{m} = 3.89\,\text{cm}.$$

Thus the width of the peak is

$$\Delta y_1 = 2y_1 = 2\left(3.89\,\text{cm}\right) = \boxed{7.79\,\text{cm.}}$$

23. We find the angular half-width θ of the central maximum from

$$\sin\theta = \frac{\lambda}{D};$$

$$\sin\left(\frac{55.0^\circ}{2}\right) = \frac{440\times10^{-9}\,\text{m}}{D},\ \text{which gives } D = \boxed{9.53\times10^{-7}\,\text{m.}}$$

24. We find the angle to the first minimum from the distances:

$$\tan\theta_{1\min} = \tfrac{1}{2}\frac{\left(9.20\,\text{cm}\right)}{\left(255\,\text{cm}\right)} = 0.0180 = \sin\theta_{1\min},\ \text{because the angle is small.}$$

We find the slit width from

$$D\sin\theta_{1\min} = m\lambda;$$

$$D\left(0.0180\right) = (1)\left(415\times10^{-9}\,\text{m}\right),\ \text{which gives } D = 2.30\times10^{-5}\,\text{m} = \boxed{0.0230\,\text{mm.}}$$

25. Because the angles are small, we have

$$\tan\theta_{1\min} = \tfrac{1}{2}\frac{\left(\Delta y_1\right)}{L} = \sin\theta_{1\min}.$$

The condition for the first minimum is

$$D\sin\theta_{1\min} = \tfrac{1}{2}D\frac{\Delta y_1}{L} = \lambda.$$

If we form the ratio of the expressions for the two wavelengths, we get

$$\frac{\Delta y_{1b}}{\Delta y_{1a}} = \frac{\lambda_b}{\lambda_a};$$

$$\frac{\Delta y_{1b}}{\left(4.0\,\text{cm}\right)} = \frac{\left(420\,\text{nm}\right)}{\left(650\,\text{nm}\right)},\ \text{which gives } \Delta y_{1b} = \boxed{2.6\,\text{cm.}}$$

26. There will be no diffraction minima if the angle for the first minimum is greater than 90°. Thus the limiting condition is

$$D\sin\theta_{1\min} = m\lambda;$$

$$D_{\max}\sin 90° = (1)\lambda, \text{ or } \boxed{D_{\max} = \lambda.}$$

27. We find the angle for the second order from

$$d\sin\theta = m\lambda;$$

$$\left(1.45\times10^{-5}\,\text{m}\right)\sin\theta = (2)\left(560\times10^{-9}\,\text{m}\right), \text{ which gives } \sin\theta = 0.0772, \text{ so } \theta = \boxed{4.43°.}$$

28. We find the wavelength from

$$d\sin\theta = m\lambda;$$

$$\left[\frac{1}{(3500\,\text{lines/cm})}\right]\left(10^{-2}\,\text{m/cm}\right)\sin 28.0° = 3\lambda, \text{ which gives } \lambda = 4.47\times10^{-7}\,\text{m} = \boxed{447\,\text{nm.}}$$

29. We find the slit separation from

$$d\sin\theta = m\lambda;$$

$$d\sin 18.0° = (3)\left(630\times10^{-9}\,\text{m}\right), \text{ which gives } d = 6.12\times10^{-6}\,\text{m} = 6.12\times10^{-4}\,\text{cm}.$$

The number of lines/cm is

$$\frac{1}{d} = \frac{1}{\left(6.12\times10^{-4}\,\text{cm}\right)} = \boxed{1.64\times10^{3}\,\text{lines/cm.}}$$

30. Because the angle increases with wavelength, to have a complete order we use the largest wavelength. The maximum angle is 90°, so we have

$$d\sin\theta = m\lambda;$$

$$\left[\frac{1}{(8300\,\text{lines/cm})}\right]\left(10^{-2}\,\text{m/cm}\right)\sin 90° = m\left(700\times10^{-9}\,\text{m}\right), \text{ which gives } m = 1.70.$$

Thus only $\boxed{\text{one full order}}$ can be seen on each side of the central white line.

31. We find the slit separation from

$$d\sin\theta = m\lambda;$$

$$d\sin 15.5° = (1)\left(589\times10^{-9}\,\text{m}\right), \text{ which gives } d = 2.20\times10^{-6}\,\text{m} = \boxed{2.20\,\mu\text{m.}}$$

We find the angle for the fourth order from

$$d\sin\theta = m\lambda;$$

$$\left(2.20\times10^{-6}\,\text{m}\right)\sin\theta_4 = (4)\left(589\times10^{-9}\,\text{m}\right), \text{ which gives } \sin\theta_4 = 1.069, \text{ so there is } \boxed{\text{no fourth order.}}$$

32. We find the angles for the second order from

$$d\sin\theta = m\lambda \text{ with } m = 2.$$

$$\frac{1}{6.0\times10^{5}\,\text{lines/m}}\sin\theta_1 = 2\left(7.0\times10^{-7}\,\text{m}\right) \text{ gives } \sin\theta_1 = 0.84, \text{ so } \theta_1 = 57.1°.$$

$$\frac{1}{6.0\times10^{5}\,\text{lines/m}}\sin\theta_2 = 2\left(4.5\times10^{-7}\,\text{m}\right) \text{ gives } \sin\theta_2 = 0.54, \text{ so } \theta_2 = 32.7°.$$

Therefore, $\Delta\theta = 57.1° - 32.7° = \boxed{24°.}$

33. We find the wavelengths from

$$d\sin\theta = m\lambda;$$

$$\left[\frac{1}{(9700\,\text{lines/cm})}\right](10^{-2}\ \text{m/cm})\sin 31.2° = (1)\lambda_1,\ \text{which gives}\ \lambda_1 = 5.34\times10^{-7}\,\text{m} = \boxed{534\,\text{nm};}$$

$$\left[\frac{1}{(9700\,\text{lines/cm})}\right](10^{-2}\ \text{m/cm})\sin 36.4° = (1)\lambda_2,\ \text{which gives}\ \lambda_2 = 6.12\times10^{-7}\,\text{m} = \boxed{612\,\text{nm};}$$

$$\left[\frac{1}{(9700\,\text{lines/cm})}\right](10^{-2}\ \text{m/cm})\sin 47.5° = (1)\lambda_3,\ \text{which gives}\ \lambda_3 = 7.60\times10^{-7}\,\text{m} = \boxed{760\,\text{nm}.}$$

$\boxed{34.}$ The maximum angle is 90°, so we have

$$d\sin\theta = m\lambda;$$

$$\left[\frac{1}{(6000\,\text{lines/cm})}\right](10^{-2}\ \text{m/cm})\sin 90° = m(633\times10^{-9}\ \text{m}),\ \text{which gives}\ m = 2.63.$$

Thus $\boxed{\text{two orders}}$ can be seen on each side of the central white line.

35. Because the angle increases with wavelength, to have a full order we use the largest wavelength. The maximum angle is 90°, so we find the minimum separation from

$$d\sin\theta = m\lambda;$$

$$d_{\min}\sin 90° = (2)(750\times10^{-9}\ \text{m}),\ \text{which gives}\ d_{\min} = 1.50\times10^{-6}\ \text{m} = 1.50\times10^{-4}\ \text{cm}.$$

The maximum number of lines/cm is

$$\frac{1}{d_{\min}} = \frac{1}{(1.50\times10^{-4}\ \text{cm})} = \boxed{6.67\times10^{3}\ \text{lines/cm}.}$$

36. We find the angles for the first order from

$$d\sin\theta = m\lambda = \lambda;$$

$$\left[\frac{1}{(8500\,\text{lines/cm})}\right](10^{-2}\ \text{m/cm})\sin\theta_{410} = (410\times10^{-9}\ \text{m}),\ \text{which gives}$$

$$\sin\theta_{410} = 0.3485,\ \text{so}\ \theta_{410} = 20.4°;$$

$$\left[\frac{1}{(8500\,\text{lines/cm})}\right](10^{-2}\ \text{m/cm})\sin\theta_{750} = (750\times10^{-9}\ \text{m}),\ \text{which gives}$$

$$\sin\theta_{750} = 0.6375,\ \text{so}\ \theta_{750} = 39.6°.$$

The distances from the central white line on the screen are

$$y_{410} = L\tan\theta_{410} = (2.30\,\text{m})\tan 20.4° = 0.855\,\text{m};$$

$$y_{750} = L\tan\theta_{750} = (2.30\,\text{m})\tan 39.6° = 1.90\,\text{m}.$$

Thus the width of the spectrum is

$$y_{750} - y_{410} = 1.90\,\text{m} - 0.855\,\text{m} = \boxed{1.05\,\text{m}.}$$

37. We find the angle for the first order from

$$d\sin\theta = m\lambda = \lambda;$$

$$d\sin 21.5° = 6.328\times10^{-7}\,\text{m},\ \text{which gives}\ d = 1.73\times10^{-6}\,\text{m}.$$

The number of lines per meter is

$$\frac{1}{d}=\frac{1}{1.73\times10^{-6}\,\text{m}}=\boxed{5.79\times10^{5}\text{ lines/m.}}$$

38. Because the angles on each side of the central line are not the same, the incident light is not normal to the grating. We use the average angles:

$$\theta_1=\frac{(26°38'+26°48')}{2}=26°43'=26.72°;$$

$$\theta_2=\frac{(41°08'+41°19')}{2}=41°14'=41.23°.$$

We find the wavelengths from

$$d\sin\theta=m\lambda;$$

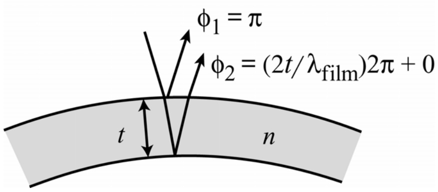
$$\left[\frac{1}{(9500\,\text{lines/cm})}\right](10^{-2}\text{ m/cm})\sin 26.72°=(1)\lambda_1,\text{ which gives }\lambda_1=4.73\times10^{-7}\,\text{m}=\boxed{473\,\text{nm;}}$$

$$\left[\frac{1}{(9500\,\text{lines/cm})}\right](10^{-2}\text{ m/cm})\sin 41.23°=(1)\lambda_2,\text{ which gives }\lambda_2=6.94\times10^{-7}\,\text{m}=\boxed{694\,\text{nm.}}$$

Note that the second wavelength is not visible.

39. We equate a path difference of one wavelength with a phase difference of 2π. With respect to the incident wave, the wave that reflects at the top surface from the higher index of the soap bubble has a phase change of

$$\phi_1=\pi.$$



With respect to the incident wave, the wave that reflects from the air at the bottom surface of the bubble has a phase change due to the additional path-length but no phase change on reflection:

$$\phi_2=\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi+0.$$

For constructive interference, the net phase change is

$$\phi=\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi-\pi=m2\pi,\ m=0,1,2,\ldots,\text{ or } t=\tfrac{1}{2}\lambda_{\text{film}}\left(m+\tfrac{1}{2}\right),\ m=0,1,2,\ldots.$$

The wavelengths in air that produce strong reflection are given by

$$\lambda=n\lambda_{\text{film}}=\frac{2nt}{\left(m+\frac{1}{2}\right)}=\frac{4(1.34)(120\,\text{nm})}{(2m+1)}=\frac{(643\,\text{nm})}{(2m+1)}.$$

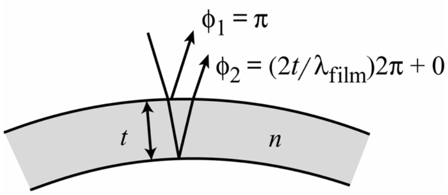
Thus we see that, for the light to be in the visible spectrum, the only value of m is 0:

$$\lambda=\frac{(643\,\text{nm})}{(0+1)}=643\,\text{nm},\text{ which is an }\boxed{\text{orange-red.}}$$

40. Between the 25 dark lines there are 24 intervals. When we add the half-interval at the wire end, we have 24.5 intervals, so the separation is

$$\frac{26.5\,\text{cm}}{24.5\,\text{intervals}}=\boxed{1.08\,\text{cm.}}$$

41. We equate a path difference of one wavelength with a phase difference of 2π. With respect to the incident wave, the wave that reflects at the top surface from the higher index of the soap bubble has a phase change of



$$\phi_1 = \pi.$$

With respect to the incident wave, the wave that reflects from the air at the bottom surface of the bubble has a phase change due to the additional path-length but no phase change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0.$$

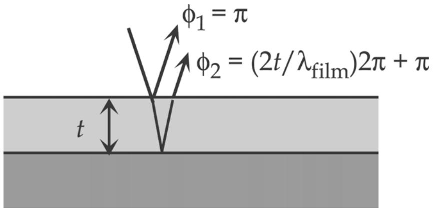
For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = \left(m - \tfrac{1}{2}\right)2\pi,\ m = 0, 1, 2, \ldots, \text{ or } t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{\lambda}{n}\right)m,\ m = 0, 1, 2, \ldots.$$

The minimum non-zero thickness is

$$t_{\text{min}} = \tfrac{1}{2}\left[\frac{(480\,\text{nm})}{(1.42)}\right](1) = \boxed{169\,\text{nm.}}$$

42. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of



$$\phi_1 = \pi.$$

With respect to the incident wave, the wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to the additional path-length and a phase change of π on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$$

For constructive interference, the net phase change is

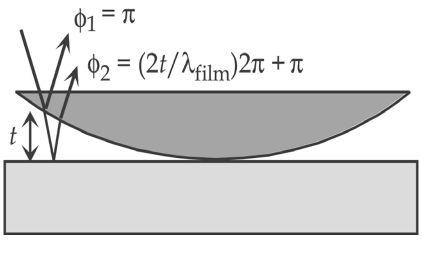
$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi - \pi = m2\pi,\ m = 1, 2, 3, \ldots, \text{ or } t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{\lambda}{n_{\text{film}}}\right)m,\ m = 1, 2, 3, \ldots.$$

The minimum non-zero thickness occurs for $m = 1$:

$$t_{\text{min}} = \frac{\lambda}{2n_{\text{film}}} = \frac{(570\,\text{nm})}{2(1.25)} = \boxed{228\,\text{nm.}}$$

570 nm is in the middle of the visible spectrum. The transmitted light will be stronger in the wavelengths at the ends of the spectrum, so the lens would emphasize the red and violet wavelengths.

43. The phase difference for the reflected waves from the path-length difference and the reflection at the bottom surface is



$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi.$$

For the dark rings, this phase difference must be an odd multiple of π, so we have

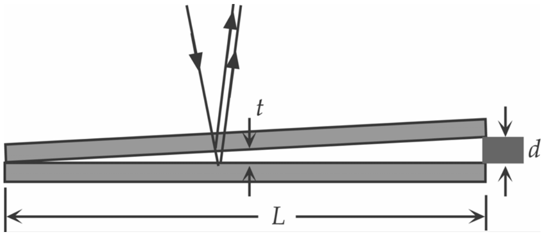
$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi = (2m+1)\pi,\ m = 0, 1, 2, \ldots, \text{ or}$$

$$t = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, \ldots .$$

Because $m = 0$ corresponds to the dark center, m represents the number of the ring. Thus the thickness of the lens is the thickness of the air at the edge of the lens:

$$t = \tfrac{1}{2}(31)(550\,\text{nm}) = 8.5\times10^3\ \text{nm} = \boxed{8.5\,\mu\text{m}.}$$

44. There is a phase difference for the reflected waves from the path-length difference, $\left(\frac{2t}{\lambda}\right)2\pi$, and the reflection at the bottom surface, π. For destructive interference, this phase difference must be an odd multiple of π, so we have



$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi = (2m+1)\pi,\ m = 0, 1, 2, \ldots, \text{ or}$$

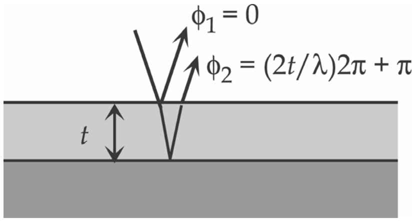
$$t = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, \ldots .$$

Because $m = 0$ corresponds to the edge where the glasses touch, $m+1$ represents the number of the fringe. Thus the thickness of the foil is

$$d = \tfrac{1}{2}(27)(670\,\text{nm}) = 9.05\times10^3\ \text{nm} = \boxed{9.05\,\mu\text{m}.}$$

45. With respect to the incident wave, the wave that reflects from the air at the top surface of the air layer has a phase change of

$$\phi_1 = 0.$$



With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the air layer has a phase change due to the additional path-length and a change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi.$$

For constructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi - 0 = m2\pi,\ m = 1, 2, 3, \ldots, \text{ or } t = \tfrac{1}{2}\lambda\left(m - \tfrac{1}{2}\right),\ m = 1, 2, 3, \ldots .$$

The minimum thickness is

$$t_{\min} = \tfrac{1}{2}(450\,\text{nm})\left(1 - \tfrac{1}{2}\right) = \boxed{113\,\text{nm}.}$$

For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda}\right)2\pi + \pi - 0 = (2m+1)\pi,\ m = 0, 1, 2, \ldots,\ \textit{or}\ t = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, \ldots .$$

The minimum non-zero thickness is

$$t_{\min} = \tfrac{1}{2}(450\,\text{nm})(1) = \boxed{225\,\text{nm}.}$$

46. The polarizing angle θ_p is found using

$$\tan\theta_\text{p} = \frac{n_2}{n_1}.$$

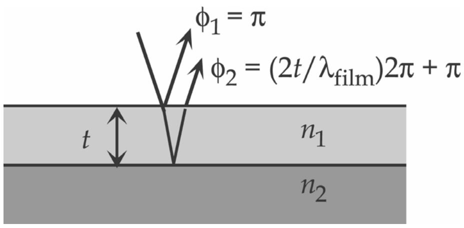
For an oil–diamond interface, $\tan\theta_\text{p} = \dfrac{2.42}{1.43}$, which gives $\theta_\text{p} = 59.4°$.

The material $\boxed{\text{does appear to be diamond.}}$

47. With respect to the incident wave, the wave that reflects from the top surface of the alcohol has a phase change of

$$\phi_1 = \pi.$$

With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the alcohol has a phase change due to the additional path-length and a phase change of π on reflection:



$$\phi_2 = \left(\frac{2t}{\lambda_\text{film}}\right)2\pi + \pi.$$

For constructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_\text{1film}}\right)2\pi + \pi - \pi = m_1 2\pi,\ m_1 = 1, 2, 3, \ldots, \text{ or } t = \tfrac{1}{2}\lambda_\text{1film}(m_1) = \tfrac{1}{2}\left(\frac{\lambda_1}{n_\text{film}}\right)(m_1),\ m_1 = 1, 2, 3, \ldots.$$

For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_\text{2film}}\right)2\pi + \pi - \pi = (2m_2 + 1)\pi,\ m_2 = 0, 1, 2, \ldots, \text{ or } t = \tfrac{1}{4}\left(\frac{\lambda_2}{n_\text{film}}\right)(2m_2 + 1),\ m_2 = 0, 1, 2, \ldots.$$

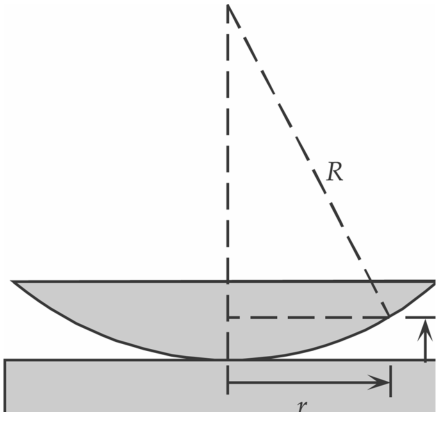
When we combine the two equations, we get

$$\tfrac{1}{2}\left(\frac{\lambda_1}{n_\text{film}}\right)(m_1) = \tfrac{1}{4}\left(\frac{\lambda_2}{n_\text{film}}\right)(2m_2 + 1), \text{ or } \frac{(2m_2 + 1)}{2m_1} = \frac{\lambda_1}{\lambda_2} = \frac{(640\,\text{nm})}{(512\,\text{nm})} = 1.25 = \frac{5}{4}.$$

Thus we see that $m_1 = m_2 = 2$, and the thickness of the film is

$$t = \tfrac{1}{2}\left(\frac{\lambda_1}{n_\text{film}}\right)(m_1) = \tfrac{1}{2}\left[\frac{(640\,\text{nm})}{(1.36)}\right](2) = \boxed{471\,\text{nm.}}$$

48. At a distance *r* from the center of the lens, the thickness of the air space is *y*, and the phase difference for the reflected waves from the path-length difference and the reflection at the bottom surface is



$$\phi = \left(\frac{2y}{\lambda}\right)2\pi + \pi.$$

For the dark rings, we have

$$\phi = \left(\frac{2y}{\lambda}\right)2\pi + \pi = (2m + 1)\pi,\ m = 0, 1, 2, \ldots, \text{ or}$$

$$y = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, \ldots.$$

Because $m = 0$ corresponds to the dark center, *m* represents the number of the ring. From the triangle in the diagram, we have

$$r^2 + (R - y)^2 = R^2, \text{ or } r^2 = 2yR - y^2 \approx 2yR, \text{ when } y \ll R,$$

which becomes

$$r^2 = 2\left(\tfrac{1}{2}m\lambda\right)R = m\lambda R,\ m = 0, 1, 2, \ldots.$$

When the apparatus is immersed in the liquid, the same analysis holds, if we use the wavelength in the liquid. If we form the ratio for the two conditions, we get

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{\lambda_1}{\lambda_2} = n, \text{ so}$$

$$n = \left(\frac{2.92\,\text{cm}}{2.48\,\text{cm}}\right)^2 = \boxed{1.39.}$$

49. One fringe shift corresponds to a change in path length of λ. The number of fringe shifts produced by a mirror movement of ΔL is

$$N = 2\frac{\Delta L}{\lambda};$$

$$644 = \frac{2\left(0.225 \times 10^{-3}\,\text{m}\right)}{\lambda}, \text{ which gives } \lambda = 6.99 \times 10^{-7}\,\text{m} = \boxed{699\,\text{nm.}}$$

50. One fringe shift corresponds to a change in path length of λ. The number of fringe shifts produced by a mirror movement of ΔL is

$$N = 2\frac{\Delta L}{\lambda};$$

$$272 = 2\frac{\Delta L}{\left(589\,\text{nm}\right)}, \text{ which gives } \Delta L = 8.01 \times 10^4\,\text{nm} = \boxed{80.1\,\mu\text{m.}}$$

$\boxed{51.}$ One fringe shift corresponds to a change in path length of λ. The number of fringe shifts produced by a mirror movement of ΔL is

$$N = 2\frac{\Delta L}{\lambda};$$

$$850 = 2\frac{\Delta L}{\left(589 \times 10^{-9}\,\text{m}\right)}, \text{ which gives } \Delta L = 2.50 \times 10^{-4}\,\text{m} = \boxed{0.250\,\text{mm.}}$$

52. One fringe shift corresponds to an effective change in path length of λ. The actual distance has not changed, but the number of wavelengths in the depth of the cavity has. If the cavity has a depth *d*, the number of wavelengths in vacuum is $\frac{d}{\lambda}$, and the number with the gas present is $\frac{d}{\lambda_{\text{gas}}} = \frac{n_{\text{gas}}d}{\lambda}$. Because the light passes through the cavity twice, the number of fringe shifts is

$$N = 2\left[\left(\frac{n_{\text{gas}}d}{\lambda}\right) - \left(\frac{d}{\lambda}\right)\right] = 2\left(\frac{d}{\lambda}\right)\left(n_{\text{gas}} - 1\right);$$

$$236 = 2\left[\frac{\left(1.30 \times 10^{-2}\,\text{m}\right)}{\left(610 \times 10^{-9}\,\text{m}\right)}\right]\left(n_{\text{gas}} - 1\right), \text{ which gives } n_{\text{gas}} = \boxed{1.00554.}$$

53. If the initial intensity is I_0, through the two sheets we have

$$I_1 = \tfrac{1}{2}I_0,$$

$$I_2 = I_1\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta, \text{ which gives}$$

$$\frac{I_2}{I_0} = \tfrac{1}{2}\cos^2\theta = \tfrac{1}{2}\cos^2 65° = \boxed{0.089.}$$

54. Because the light is coming from air to glass, we find the angle from the vertical from

$$\tan\theta_{\text{p}} = n_{\text{glass}} = 1.52, \text{ which gives } \boxed{\theta_{\text{p}} = 56.7°.}$$

55. Because the light is coming from water to diamond, we find the angle from the vertical from

$$\tan\theta_{\text{p}} = \frac{n_{\text{diamond}}}{n_{\text{water}}} = \frac{2.42}{1.33} = 1.82, \text{ which gives } \boxed{\theta_{\text{p}} = 61.2°.}$$

56. If I_0 is the intensity passed by the first Polaroid, the intensity passed by the second will be I_0 when the two axes are parallel. To reduce the intensity by half, we have

$$I = I_0\cos^2\theta = \tfrac{1}{2}I_0, \text{ which gives } \theta = \boxed{45°.}$$

57. If the initial intensity is I_0, through the two sheets we have

$$I_1 = \tfrac{1}{2}I_0,$$
$$I_2 = I_1\cos^2\theta; \text{ which means}$$
$$\frac{I_2}{I_0} = \tfrac{1}{2}\cos^2\theta.$$

(*a*) For $\frac{I_2}{I_0} = \tfrac{1}{3}$,

$$\tfrac{1}{3} = \tfrac{1}{2}\cos^2\theta \text{ gives } \theta = \boxed{35.3°.}$$

(*b*) For $\frac{I_2}{I_0} = \tfrac{1}{10}$,

$$\tfrac{1}{10} = \tfrac{1}{2}\cos^2\theta \text{ gives } \theta = \boxed{63.4°.}$$

58. If the initial intensity is I_0, through the two sheets we have

$$I_1 = I_0\cos^2\theta_1;$$
$$I_2 = I_1\cos^2\theta_2 = I_0\cos^2\theta_1\cos^2\theta_2;$$
$$0.15I_0 = I_0\cos^2\theta_1\cos^2 40°, \text{ which gives } \theta_1 = \boxed{60°.}$$

59. Through the successive sheets we have

$$I_1 = I_0\cos^2\theta_1,$$
$$I_2 = I_1\cos^2\theta_2, \text{ which gives}$$
$$I_2 = I_0\cos^2\theta_1\cos^2\theta_2 = I_0\left(\cos^2 19.0°\right)\left(\cos^2 38.0°\right) = 0.555I_0.$$

Thus the reduction is $\boxed{44.5\%.}$

60. If the light is coming from water to air, we find Brewster's angle from

$$\tan\theta_{\text{p}} = \frac{n_{\text{air}}}{n_{\text{water}}} = \frac{1.00}{1.33} = 0.752, \text{ which gives } \boxed{\theta_{\text{p}} = 36.9°.}$$

For the refraction at the critical angle from water to air, we have

$$n_{\text{air}}\sin\theta_1 = n_{\text{water}}\sin\theta_2;$$
$$(1.00)\sin 90° = (1.33)\sin\theta_{\text{c}}, \text{ which gives } \boxed{\theta_{\text{c}} = 48.8°.}$$

If the light is coming from air to water, we find Brewster's angle from

$$\tan\theta_p' = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} = 1.33, \text{ which gives } \boxed{\theta_p' = 53.1°.}$$

Thus $\theta_p + \theta_p' = 90.0°.$

61. When plane-polarized light hits a sheet oriented at an angle θ,

$$I_2 = I_1 \cos^2\theta.$$

For $\theta = 45°$,

$$\frac{I_2}{I_1} = \cos^2 45° = \tfrac{1}{2}.$$

So sheets two through five will each reduce the intensity by $\tfrac{1}{2}$.

Since the first sheet will reduce the intensity of the unpolarized incident light by $\tfrac{1}{2}$ as well, the intensity of the transmitted beam will be

$$I = I_0\left(\tfrac{1}{2}\right)^5 = \boxed{0.031 I_0.}$$

62. (*a*) For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots.$$

We find the location on the screen from

$$y = L\tan\theta.$$

For small angles, we have

$$\sin\theta \approx \tan\theta, \text{ which gives}$$

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For adjacent bands, $\Delta m = 1$, so we have

$$\Delta y = \frac{L\lambda\Delta m}{d};$$

$$2.0\times10^{-2}\,\text{m} = \frac{(4.0\,\text{m})(5.0\times10^{-7}\,\text{m})(1)}{d}, \text{ which gives } d = 1.0\times10^{-4}\,\text{m} = \boxed{0.10\,\text{mm.}}$$

(*b*) For destructive interference, the path difference is given by

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots.$$

Again we find the location on the screen from

$$y = L\tan\theta,$$

and again we use $\sin\theta \approx \tan\theta,$ this time to obtain

$$y = \frac{\left(m + \tfrac{1}{2}\right)L\lambda}{d}.$$

We are told that

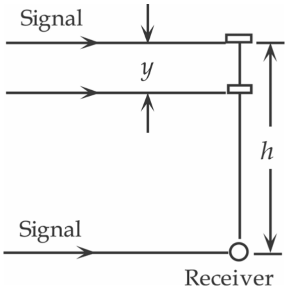
$$\frac{\left(5 + \tfrac{1}{2}\right)L\lambda_2}{d} = \frac{\left(4 + \tfrac{1}{2}\right)L\lambda_1}{d};$$

$$\left(5 + \tfrac{1}{2}\right)\lambda_2 = \left(4 + \tfrac{1}{2}\right)\left(5.0\times10^{-7}\,\text{m}\right),$$

which gives $\lambda_2 = \boxed{4.1\times10^{-7}\,\text{m.}}$

63. The wavelength of the signal is

$$\lambda = \frac{v}{f} = \frac{\left(3.00 \times 10^8 \text{ m/s}\right)}{\left(75 \times 10^6 \text{ Hz}\right)} = 4.00\,\text{m}.$$



(*a*) There is a phase difference between the direct and reflected signals from the path difference, $\left(\frac{h}{\lambda}\right)2\pi,$

and the reflection, π.
The total phase difference is

$$\phi = \left(\frac{h}{\lambda}\right)2\pi + \pi = \left[\frac{(118\,\text{m})}{(4.00\,\text{m})}\right]2\pi + \pi = 30(2\pi).$$

Thus the interference is $\boxed{\text{constructive.}}$

(*b*) When the plane is 22 m closer to the receiver, the phase difference is

$$\phi = \left[\frac{(h-y)}{\lambda}\right]2\pi + \pi$$

$$= \left[\frac{(118\,\text{m} - 22\,\text{m})}{(4.00\,\text{m})}\right]2\pi + \pi = 24(2\pi) + \pi.$$

Thus the interference is $\boxed{\text{destructive.}}$

64. We find the angles for the first order from

$d\sin\theta = m\lambda = \lambda;$

$$\frac{1}{3.00 \times 10^5 \text{ lines/m}}\sin\theta_\text{h} = 6.56 \times 10^{-7}\text{ m, which gives } \sin\theta_\text{h} = 0.197, \text{ so } \boxed{\theta_\text{h} = 11.3°;}$$

$$\frac{1}{3.00 \times 10^5 \text{ lines/m}}\sin\theta_\text{n} = 6.50 \times 10^{-7}\text{ m, which gives } \sin\theta_\text{n} = 0.195, \text{ so } \boxed{\theta_\text{n} = 11.2°;}$$

$$\frac{1}{3.00 \times 10^5 \text{ lines/m}}\sin\theta_\text{a} = 6.97 \times 10^{-7}\text{ m, which gives } \sin\theta_\text{a} = 0.209, \text{ so } \boxed{\theta_\text{a} = 12.1°.}$$

65. For constructive interference, the path difference is a multiple of the wavelength:

$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots.$

We find the location on the screen from

$y = L\tan\theta.$

For small angles, we have

$\sin\theta \approx \tan\theta,$ which gives

$$y = L\left(\frac{m\lambda}{d}\right) = \frac{mL\lambda}{d}.$$

For the second-order fringes we have

$$y_1 = \frac{2L\lambda_1}{d};$$

$$y_2 = \frac{2L\lambda_2}{d}.$$

When we subtract the two equations, we get

$$\Delta y = y_1 - y_2 = \left(\frac{2L}{d}\right)(\lambda_1 - \lambda_2);$$

$$1.33 \times 10^{-3}\ \text{m} = \left[\frac{2(1.70\,\text{m})}{(0.60 \times 10^{-3}\ \text{m})}\right](590\,\text{nm} - \lambda_2),\ \text{which gives } \lambda_2 = \boxed{355\,\text{nm.}}$$

66. The wavelength of the signal is

$$\lambda = \frac{v}{f} = \frac{(3.00 \times 10^8\ \text{m/s})}{(102.1 \times 10^6\ \text{Hz})} = 2.94\,\text{m}.$$

Because measurements are made far from the antennae, we can use the analysis for the double slit. For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots;$$

$$(8.0\,\text{m})\sin\theta_{1\max} = (1)(2.94\,\text{m}),\ \text{which gives } \theta_{1\max} = 22°;$$

$$(8.0\,\text{m})\sin\theta_{2\max} = (2)(2.94\,\text{m}),\ \text{which gives } \theta_{2\max} = 47°;$$

$$(8.0\,\text{m})\sin\theta_{3\max} = (3)(2.94\,\text{m}),\ \text{which gives } \sin\theta_{3\max} > 1,\ \text{so there is no third maximum.}$$

Because the interference pattern will be symmetrical above and below the midline and on either side of the antennae, the angles for maxima are $\boxed{22°, 47°, 133°, 158° \text{ above and below the midline.}}$

For destructive interference, the path difference is an odd multiple of half a wavelength:

$$d\sin\theta = \left(m - \tfrac{1}{2}\right)\lambda,\ m = 1, 2, 3, \ldots;\ \text{or}$$

$$(8.0\,\text{m})\sin\theta_{1\min} = \left(1 - \tfrac{1}{2}\right)(2.94\,\text{m}),\ \text{which gives } \theta_{1\min} = 11°;$$

$$(8.0\,\text{m})\sin\theta_{2\min} = \left(2 - \tfrac{1}{2}\right)(2.94\,\text{m}),\ \text{which gives } \theta_{2\min} = 33°;$$

$$(8.0\,\text{m})\sin\theta_{3\min} = \left(3 - \tfrac{1}{2}\right)(2.94\,\text{m}),\ \text{which gives } \theta_{3\min} = 67°;$$

$$(8.0\,\text{m})\sin\theta_{4\min} = \left(4 - \tfrac{1}{2}\right)(2.94\,\text{m}),\ \text{which gives } \sin\theta_{4\min} > 1,\ \text{so there is no fourth minimum.}$$

Because the interference pattern will be symmetrical above and below the midline and on either side of the antennae, the angles for minima are $\boxed{11°, 33°, 67°, 113°, 147°, 169° \text{ above and below the midline.}}$

67. The wavelength of the sound is

$$\lambda = \frac{v}{f} = \frac{(343\,\text{m/s})}{(750\,\text{Hz})} = 0.457\,\text{m}.$$

We find the angles of the minima from

$$D\sin\theta = m\lambda,\ m = 1, 2, 3, \ldots;$$

$$(0.88\,\text{m})\sin\theta_1 = (1)(0.457\,\text{m}),\ \text{which gives } \sin\theta_1 = 0.520,\ \text{so } \theta_1 = 31°;$$

$$(0.88\,\text{m})\sin\theta_2 = (2)(0.457\,\text{m}),\ \text{which gives} \sin\theta_2 = 1.04,\ \text{so there is no } \theta_2.$$

Thus the whistle would not be heard clearly at angles of $\boxed{31° \text{ on either side of the normal.}}$

68. The path difference between the top and bottom of the slit for the incident wave is

$$D\sin\theta_{\mathrm{i}}.$$

The path difference between the top and bottom of the slit for the diffracted wave is

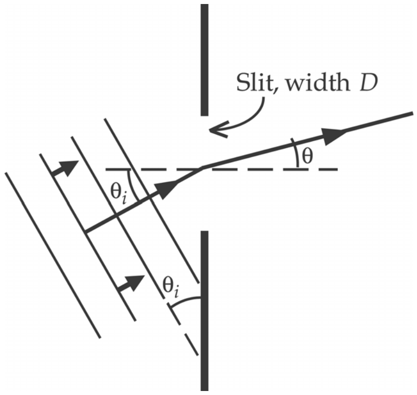
$$D\sin\theta.$$

When $\theta = \theta_{\mathrm{i}}$, the net path difference is zero, and there will be constructive interference. $\boxed{\text{There is a central maximum at } \theta = 30°.}$

When the net path difference is a multiple of a wavelength, there will be minima given by

$$(D\sin\theta_{\mathrm{i}}) - (D\sin\theta) = m\lambda,\ m = \pm1, \pm2, \ldots, \text{ or}$$

$$\boxed{\sin\theta = \sin 30° - \left(\frac{m\lambda}{D}\right), \text{ where } m = \pm1, \pm2, \ldots.}$$



69. The lines act like a grating. Assuming the first order, we find the separation of the lines from

$$d\sin\theta = m\lambda;$$

$$d\sin 51° = (1)(460\times10^{-9}\,\mathrm{m}), \text{ which gives } d = 5.9\times10^{-7}\,\mathrm{m} = \boxed{590\,\mathrm{nm}.}$$

70. Because the angle increases with wavelength, to miss a complete order we use the smallest wavelength. The maximum angle is 90°. We find the slit separation from

$$d\sin\theta = m\lambda;$$

$$d\sin 90° = (2)(400\times10^{-9}\,\mathrm{m}), \text{ which gives } d = 8.00\times10^{-7}\,\mathrm{m} = 8.00\times10^{-5}\,\mathrm{cm}.$$

The number of lines/cm is

$$\frac{1}{d} = \frac{1}{(8.00\times10^{-5}\,\mathrm{cm})} = \boxed{12{,}500\ \text{lines/cm}.}$$

71. Because the angle increases with wavelength, we compare the maximum angle for the second order with the minimum angle for the third order:

$$d\sin\theta = m\lambda, \text{ or } \sin\theta = \frac{\lambda}{d};$$

$$\sin\theta_{2\max} = \frac{(2)(750\,\mathrm{nm})}{d};$$

$$\sin\theta_{3\min} = \frac{(3)(400\,\mathrm{nm})}{d}.$$

When we divide the two equations, we get

$$\frac{\sin\theta_{3\min}}{\sin\theta_{2\max}} = \frac{(1200\,\mathrm{nm})}{(1500\,\mathrm{nm})} = 0.8.$$

Because the value of the sine increases with angle, this means $\theta_{3\min} < \theta_{2\max}$, so the orders overlap. To determine the overlap, we find the second-order wavelength that coincides with $\theta_{3\min}$:

$$(2)\lambda_2 = (3)(400\,\mathrm{nm}), \text{ which gives } \lambda_2 = 600\,\mathrm{nm}.$$

We find the third-order wavelength that coincides with $\theta_{2\max}$ from

$$(2)(750\,\mathrm{nm}) = (3)\lambda_3, \text{ which gives } \lambda_3 = 500\,\mathrm{nm}.$$

$\boxed{\text{Thus 600 nm to 750 nm of the second order overlaps with 400 nm to 500 nm of the third order.}}$

72. We find the angles for the first order from the distances:

$$\tan\theta_1 = \frac{y_1}{L} = \frac{(3.32\,\text{cm})}{(60.0\,\text{cm})} = 0.0553, \text{ so } \theta_1 = 3.17°;$$

$$\tan\theta_2 = \frac{y_2}{L} = \frac{(3.71\,\text{cm})}{(60.0\,\text{cm})} = 0.0618, \text{ so } \theta_2 = 3.54°.$$

We find the separation of lines from

$$d\sin\theta_1 = m\lambda_1;$$

$$d\sin 3.17° = (1)(589\times10^{-9}\,\text{m}), \text{ which gives } d = 1.066\times10^{-5}\,\text{m} = 1.066\times10^{-3}\,\text{cm}.$$

For the second wavelength we have

$$d\sin\theta_2 = m\lambda_2;$$

$$(1.06\times10^{-5}\,\text{m})\sin 3.54° = (1)\lambda_2, \text{ which gives } \lambda_2 = 6.58\times10^{-7}\,\text{m} = \boxed{658\,\text{nm.}}$$

The number of lines/cm is

$$\frac{1}{d} = \frac{1}{(1.066\times10^{-3}\,\text{cm})} = \boxed{938 \text{ lines/cm.}}$$

73. We find the angles for the first order from

$$d\sin\theta = m\lambda = \lambda;$$

$$\left[\frac{1}{(8600 \text{ lines/cm})}\right](10^{-2}\,\text{m/cm})\sin\theta_1 = 4.6\times10^{-7}\,\text{m}, \text{ which gives } \sin\theta_1 = 0.396, \text{ so } \theta_1 = 23.3°;$$

$$\left[\frac{1}{(8600 \text{ lines/cm})}\right](10^{-2}\,\text{m/cm})\sin\theta_2 = 6.8\times10^{-7}\,\text{m}, \text{ which gives } \sin\theta_2 = 0.585, \text{ so } \theta_2 = 35.8°.$$

The distances from the central white line on the screen are

$$y_1 = L\tan\theta_1 = (2.5\,\text{m})\tan 23.3° = 1.1\,\text{m};$$

$$y_2 = L\tan\theta_2 = (2.5\,\text{m})\tan 35.8° = 1.8\,\text{m}.$$

Thus the separation of the lines is

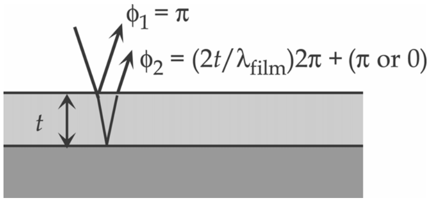
$$y_2 - y_1 = 1.8\,\text{m} - 1.1\,\text{m} = \boxed{0.7\,\text{m.}}$$

74. We equate a path difference of one wavelength with a phase difference of 2π. With respect to the incident wave, the wave that reflects at the top surface of the film has a phase change of

$$\phi_1 = \pi.$$

If we assume that the film has an index less than glass, the wave that reflects from the glass has a phase change due to the additional path-length and a phase change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$$


$\phi_1 = \pi$
$\phi_2 = (2t/\lambda_{\text{film}})2\pi + (\pi \text{ or } 0)$
t

For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi - \pi = \left(m - \tfrac{1}{2}\right)2\pi,\ m = 1, 2, \ldots, \text{ or } t = \tfrac{1}{2}\lambda_{\text{film}}\left(m - \tfrac{1}{2}\right) = \tfrac{1}{2}\left(\frac{\lambda}{n}\right)\left(m - \tfrac{1}{2}\right),\ m = 1, 2, \ldots.$$

For the minimum thickness, $m = 1,$ we have

$$150\,\text{nm} = \tfrac{1}{2}\left[\frac{(600\,\text{nm})}{(n)}\right]\left(1 - \tfrac{1}{2}\right),\text{ which gives } n = 1,\text{ so } \boxed{\text{no film with } n < n_{\text{glass}} \text{ is possible.}}$$

If we assume that the film has an index greater than glass, the wave that reflects from the glass has a phase change due to the additional path-length and no phase change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0.$$

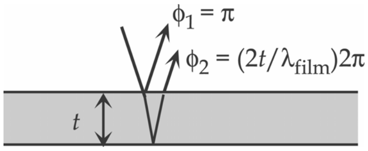
For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = \left(m - \tfrac{1}{2}\right)2\pi,\ m = 1, 2, \ldots,\text{ or } t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{m\lambda}{n}\right),\ m = 1, 2, \ldots.$$

For the minimum thickness, $m = 1$, we have

$$150\,\text{nm} = \tfrac{1}{2}\left(\frac{(1)(600\,\text{nm})}{n}\right),\text{ which gives } n = \boxed{2.00.}$$

75. With respect to the incident wave, the wave that reflects at the top surface of the film has a phase change of



$$\phi_1 = \pi.$$

The wave that reflects from the bottom surface has a phase change due to the additional path-length and no phase change on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0.$$

For destructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = \left(m - \tfrac{1}{2}\right)2\pi,\ m = 1, 2, \ldots,\text{ or } t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{m\lambda}{n}\right),\ m = 1, 2, \ldots.$$

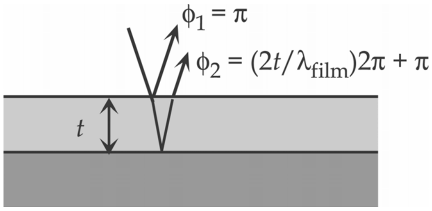
For the two wavelengths we have

$$t = \tfrac{1}{2}\left(\frac{m_1\lambda_1}{n}\right) = \tfrac{1}{2}\left(\frac{m_2\lambda_2}{n}\right),\text{ or } \frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} = \frac{640\,\text{nm}}{512\,\text{nm}} = 1.25 = \frac{5}{4}.$$

Thus $m_1 = 5,$ and $m_2 = 4.$ For the thickness we have

$$t = \tfrac{1}{2}\left(\frac{m_1\lambda_1}{n}\right) = \tfrac{1}{2}\left[\frac{(5)(512\,\text{nm})}{1.58}\right] = \boxed{810\,\text{nm.}}$$

76. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of



$$\phi_1 = \pi.$$

With respect to the incident wave, the wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to the additional path-length and a phase change of π on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$$

For destructive interference, this phase difference must be an odd multiple of π, so we have

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi - \pi = (2m+1)\pi,\ m = 0, 1, 2, \ldots,\ \text{or}\ t = \tfrac{1}{4}(2m+1)\lambda_{\text{film}},\ m = 0, 1, 2, \ldots.$$

Thus the minimum thickness is

$$t_{\min} = \tfrac{1}{4}\frac{\lambda}{n}.$$

(*a*) For the blue light we get

$$t_{\min} = \tfrac{1}{4}\frac{(450\,\text{nm})}{(1.38)} = \boxed{81.5\,\text{nm.}}$$

(*b*) For the red light we get

$$t_{\min} = \tfrac{1}{4}\frac{(700\,\text{nm})}{(1.38)} = \boxed{127\,\text{nm.}}$$

77. As explained in Example 24-8, the $\tfrac{1}{2}$-cycle phase change at the lower surface means that maximum destructive interference will occur when the thickness *t* is such that

$$2t = m\lambda,\ m = 0, 1, 2, \ldots.$$

We set $m = 1$ to find the smallest nonzero value of *t*:

$$t = \frac{\lambda}{2} = \frac{640\,\text{nm}}{2} = \boxed{320\,\text{nm.}}$$

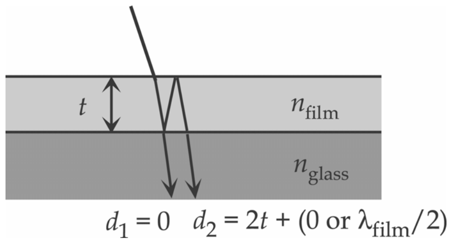
As also explained in Example 24-8, maximum constructive interference will occur when

$$2t = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \ldots.$$

We set $m = 0$ to find the smallest value of *t*:

$$t = \frac{\lambda}{4} = \frac{640\,\text{nm}}{4} = \boxed{160\,\text{nm.}}$$

78. If we consider the two rays shown in the diagram, we see that the second ray has reflected twice. If $n_{\text{film}} < n_{\text{glass}}$, the first reflection from the glass produces a shift equivalent to $\tfrac{1}{2}\lambda_{\text{film}}$, while the second reflection from the air produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift of



$$d_2 - d_1 = 2t + \tfrac{1}{2}\lambda_{\text{film}}.$$

For maxima, we have

$$2t + \tfrac{1}{2}\lambda_{\text{film}} = m\lambda_{\text{film}},\ m = 1, 2, 3, \ldots,\ \text{or}\ \boxed{t = \frac{\tfrac{1}{2}\left(m - \tfrac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots.}$$

For minima, we have

$$2t + \tfrac{1}{2}\lambda_{\text{film}} = \left(m + \tfrac{1}{2}\right)\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots,\ \text{or}\ \boxed{t = \frac{\tfrac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots.}$$

We see that for a film of zero thickness, that is, $t \ll \lambda_{\text{film}}$, there will be a minimum.

If $n_{\text{film}} > n_{\text{glass}}$, the first reflection from the glass produces no shift, while the second reflection from the air also produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift of

$$d_2 - d_1 = 2t.$$

For maxima, we have

$$2t = m\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots, \text{ or } \boxed{t = \frac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots .}$$

For minima, we have

$$2t = \left(m - \tfrac{1}{2}\right)\lambda_{\text{film}},\ m = 1, 2, 3, \ldots, \text{ or } \boxed{t = \frac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots .}$$

We see that for a film of zero thickness, that is, $t \ll \lambda_{\text{film}}$, there will be a maximum.

79. Because the light is coming from air to water, we find the angle from the vertical from

$$\tan\theta_{\text{p}} = n_{\text{water}} = 1.33, \text{ which gives } \theta_{\text{p}} = 53.1°.$$

Thus the angle above the horizon is $90.0° - 53.1° = \boxed{36.9°.}$

80. If the original intensity is I_0, the first Polaroid sheet will reduce the intensity of the original beam to

$$I_1 = \tfrac{1}{2}I_0.$$

If the axis of the second Polaroid sheet is oriented at an angle θ, the intensity is

$$I_2 = I_1\cos^2\theta.$$

(*a*) $I_2 = I_1\cos^2\theta = 0.25I_1$, which gives $\theta = \boxed{60°.}$

(*b*) $I_2 = I_1\cos^2\theta = 0.10I_1$, which gives $\theta = \boxed{72°.}$

(*c*) $I_2 = I_1\cos^2\theta = 0.010I_1$, which gives $\theta = \boxed{84°.}$

81. (*a*) $I_1 = \dfrac{I_0}{2}$;

$$I_2 = I_1\cos^2\theta_1 = \left(\frac{I_0}{2}\right)\cos^2 62°;$$

$$I_3 = I_2\cos^2\theta_2 = \left(\frac{I_0}{2}\right)\cos^2 62°\cos^2 28° = \boxed{0.086I_0.}$$

(*b*) If the third polarizer is placed in front of the other two, the last polarizer blocks whatever light comes through the middle one. Thus $\boxed{\text{no light gets transmitted.}}$

82. (*a*) Through the successive polarizers we have

$$I_1 = \tfrac{1}{2}I_0;$$

$$I_2 = I_1\cos^2\theta_2 = \tfrac{1}{2}I_0\cos^2\theta_2;$$

$$I_3 = I_2\cos^2\theta_3 = \tfrac{1}{2}I_0\cos^2\theta_2\cos^2\theta_3;$$

$$I_4 = I_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2\theta_2\cos^2\theta_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2 30°\cos^2 30°\cos^2 30° = \boxed{0.21I_0.}$$

(*b*) If we remove the second polarizer, we get

$$I_1 = \tfrac{1}{2}I_0;$$

$$I_3 = I_1\cos^2\theta_3' = \tfrac{1}{2}I_0\cos^2\theta_3';$$

$$I_4 = I_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2\theta_3'\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2 60°\cos^2 30° = 0.094I_0.$$

Thus we can decrease the intensity by removing either the $\boxed{\text{second or third polarizer.}}$

(*c*) If we remove the $\boxed{\text{second and third polarizers,}}$ we will have two polarizers with their axes perpendicular, so no light will be transmitted.

83. We will find the width of the central maximum by first finding the angular half-width θ, using

$$\sin\theta = \frac{\lambda}{D} = \frac{630\times 10^{-9}\,\text{m}}{0.010\,\text{m}} = 6.30\times 10^{-5}.$$

We find the distance on the Moon's surface from

$$y = L\tan\theta.$$

For small angles, we have

$\sin\theta \approx \tan\theta$, which gives

$$y = L\sin\theta = (3.80\times 10^{8}\,\text{m})(6.30\times 10^{-5})2.394\times 10^{4}\,\text{m}.$$

Thus the width of the central maximum is

$$2y = 4.788\times 10^{4}\,\text{m} = \boxed{47.9\,\text{km.}}$$

84. If the original intensity is I_0, the first polarizers will reduce the intensity to

$$I_1 = \tfrac{1}{2}I_0.$$

Each subsequent polarizer oriented at an angle θ to the preceding one will reduce the intensity as given by the equation

$$\text{I}_{\text{i+1}} = \text{I}_{\text{i}}\cos^2\theta.$$

So for *n* polarizers (including the first one),

$$\text{I}_{\text{f}} = \left(\tfrac{1}{2}\text{I}_0\right)\left(\cos^2\theta\right)^{\text{n}-1}.$$

We seek *n* such that

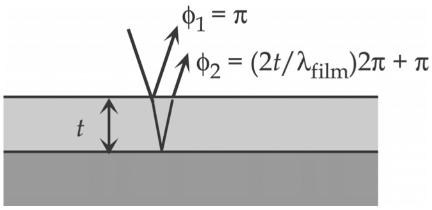
$$\tfrac{1}{4}I_0 = \left(\tfrac{1}{2}I_0\right)\left(\cos^2 10°\right)^{n-1};$$

$$\tfrac{1}{2} = \left(\cos 10°\right)^{2n-2};$$

$-\ln 2 = (2n-1)\ln(\cos 10°)$, which gives $n = 23.6$. So $\boxed{24}$ polarizers are needed for the intensity to drop below $\tfrac{1}{4}$ of its original value.

85. With respect to the incident wave, the wave that reflects from the top surface of the film has a phase change of

$$\phi_1 = \pi.$$



With respect to the incident wave, the wave that reflects from the glass $(n = 1.52)$ at the bottom surface of the film has a phase change due to the additional path-length and a phase change of π on reflection:

$$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi.$$

For constructive interference, the net phase change is

$$\phi = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi - \pi = m2\pi,\ m = 1, 2, 3, \ldots,\ \text{or}\ t = \tfrac{1}{2}\lambda_{\text{film}}m = \tfrac{1}{2}\left(\frac{\lambda}{n_{\text{film}}}\right)m,\ m = 1, 2, 3, \ldots.$$

The minimum non-zero thickness occurs for $m = 1$:

$$t_{\text{min}} = \frac{\lambda}{2n_{film}} = \frac{643\,\text{nm}}{2(1.34)} = \boxed{240\,\text{nm.}}$$

86. Destructive interference occurs when the path difference equals an odd number of half-wavelengths.

We write the equation

$$\sqrt{y^2 + (175\,\text{m})^2} - y = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots .$$

The wavelength λ is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\ \text{m/s}}{6.0 \times 10^6\,\text{Hz}} = 50\,\text{m}.$$

Solving:

$$\sqrt{y^2 + (175\,\text{m})^2} = y + \left(m + \tfrac{1}{2}\right)(50\,\text{m});$$

$$y^2 + (175\,\text{m})^2 = y^2 + 2\left(m + \tfrac{1}{2}\right)(50\,\text{m})y + \left[\left(m + \tfrac{1}{2}\right)(50\,\text{m})\right]^2;$$

$$y = \frac{(175\,\text{m})^2 - \left[\left(m + \tfrac{1}{2}\right)(50\,\text{m})\right]^2}{2\left(m + \tfrac{1}{2}\right)(50\,\text{m})}$$

For $m = 0, 1, 2,$ and 3, y equals 600 m, 167 m, 60 m, and 0, respectively. The first three points on the y axis at which there is destructive interference are at $\text{y} = \boxed{0, 60\,\text{m, and } 167\,\text{m.}}$

87. (*a*) For the refraction at the first surface, we have

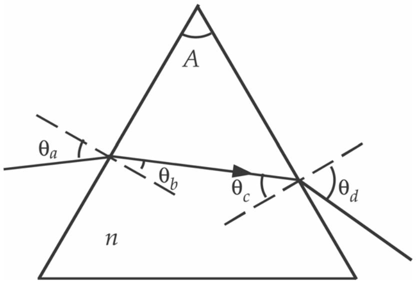
$$n_{\text{air}} \sin\theta_a = n\sin\theta_b;$$

$$(1.00)\sin 45° = (1.652)\sin\theta_{b1},\ \text{which gives}$$

$$\theta_{b1} = 25.34°;$$

$$(1.00)\sin 45° = (1.619)\sin\theta_{b2},\ \text{which gives}$$

$$\theta_{b2} = 25.90°.$$



We find the angle of incidence at the second surface from

$$(90° - \theta_b) + (90° - \theta_c) + A = 180°,\ \text{which gives}$$

$$\theta_{c1} = A - \theta_{b1} = 60.00° - 25.34° = 34.66°;$$

$$\theta_{c2} = A - \theta_{b2} = 60.00° - 25.90° = 34.10°$$

For the refraction at the second surface, we have

$$n\sin\theta_c = n_{\text{air}} \sin\theta_d;$$

$$(1.652)\sin 34.66° = (1.00)\sin\theta_{d1},\ \text{which gives } \theta_{d1} = 69.96°;$$

$$(1.619)\sin 34.10° = (1.00)\sin\theta_{d2},\ \text{which gives } \theta_{d2} = 65.20°.$$

The angle between the emerging beams is $69.96° - 65.20° = \boxed{4.8°.}$

(*b*) We find the angles for the first order from

$$d\sin\theta = m\lambda = \lambda;$$

$$\frac{1}{6200\,\text{lines/cm}}\left(10^{-2}\ \text{m/cm}\right)\sin\theta_1 = 420 \times 10^{-9}\,\text{m},\ \text{which gives } \sin\theta_1 = 0.2604,\ \text{so } \theta_1 = 15.09°.$$

$$\frac{1}{6200\,\text{lines/cm}}\left(10^{-2}\ \text{m/cm}\right)\sin\theta_2 = 650 \times 10^{-9}\,\text{m},\ \text{which gives } \sin\theta_2 = 0.4030,\ \text{so } \theta_2 = 23.77°.$$

The angle between the first-order maxima is $23.77° - 15.09° = \boxed{8.7°.}$

88. We use the lensmaker's equation to find the two focal lengths:

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

With $R_1 = \infty$ and $R_2 = 18.4\,\text{cm}$, this gives

$$f = \frac{18.4\,\text{cm}}{n-1};$$

$$f_{\text{red}} = \frac{18.4\,\text{cm}}{1.5106-1} = 36.04\,\text{cm};$$

$$f_{\text{yellow}} = \frac{18.4\,\text{cm}}{1.5226-1} = 35.21\,\text{cm}.$$

Now we find the two image distances.

$$\frac{1}{d_{\text{o}}} + \frac{1}{d_{\text{i}}} = \frac{1}{f};$$

$$\frac{1}{66.0\,\text{cm}} + \frac{1}{d_{\text{i-red}}} = \frac{1}{36.04\,\text{cm}}, \text{ which gives } \boxed{d_{\text{i-red}} = 79.4\,\text{cm.}}$$

$$\frac{1}{66.0\,\text{cm}} + \frac{1}{d_{\text{i-yellow}}} = \frac{1}{35.21\,\text{cm}}, \text{ which gives } \boxed{d_{\text{i-yellow}} = 75.5\,\text{cm.}}$$

CHAPTER 25: Optical Instruments

Answers to Questions

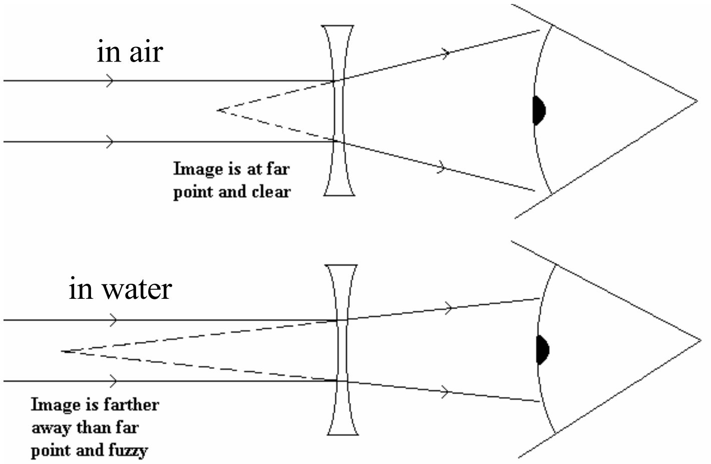
1. Stopping down a lens to a larger f-number means that the lens opening is smaller and only light rays coming through the central part of the lens are accepted. These rays form smaller circles of confusion, which means a greater range of object distances will be more sharply focused.

2. If a lens is stopped down too much (too big of an f-number), then diffraction will occur as light passes through the extremely small lens opening (the light will begin to "bend around" the edges of the stop). This diffraction will blur the image, especially around the edges of the film, and will lead to an image that is less sharp.

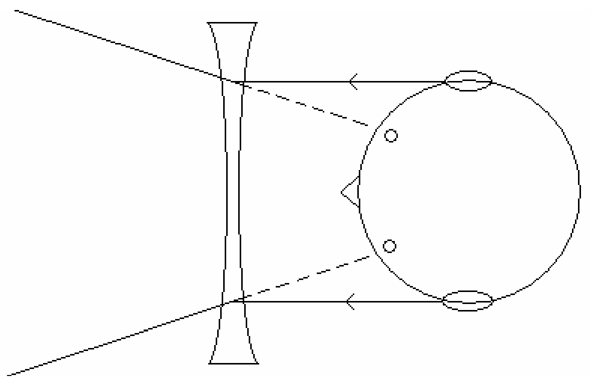
3. The lens equation $\left(\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\right)$ says that as an object gets closer to the lens (d_o decreases) and the focal length of the lens remains constant, the image distance d_i must get larger to create a focused image. Since we cannot move the film farther away from the lens, we need to move the lens farther away from the film. This is unlike the human eye, where the focal length of the lens is changed as the object distance changes.

4. As people get older, their eyes can no longer accommodate as well. It becomes harder for the muscles to change the shape of the lens, since the lens becomes less flexible with age. As people get older, their near point increases and becomes greater than the ideal value of 25 cm. They may still need the "regular" upper portion of their lenses so they can have a far point at infinity (most people need these at much younger ages), but now an older person will need a bifocal in the lower portion of their lenses to create a near point back at 25 cm for seeing close objects. Thus, as people get older, their far point is too small and their near point is too large. Bifocals can correct both of these problems.

5. No, a nearsighted person will not be able to see clearly if they wear their corrective lenses underwater. A nearsighted person has a far point that is closer than infinity and they wear corrective lenses to bring the image of a far away object to their far point so they can see it clearly. See the first pair of diagrams. The object is at infinity. In air, the image is at the far point. If the person's eyes and glasses are underwater, and since the index of refraction of glass is closer to that of water than to that of air, the glasses will not bend the light as much as they did in the air. Therefore, the image of the faraway object will now be at a position that is beyond the person's far point. The image will now be out of focus.


in air
Image is at far point and clear
in water
Image is farther away than far point and fuzzy

6. If a person's face appears narrower through the person's glasses, the lenses are diverging. If a person is wearing diverging lenses, then they are nearsighted. Since the rays from a person's ears will be bent away from the optical axis when they go through a diverging corrective lens, when that ray gets to our eyes, our brain follows that ray back through the lens without bending it (our brain doesn't know that light can bend) and places the ears closer to where the person's eyes actually are. Thus, diverging corrective lenses (for nearsighted people) make a person's head appear narrower when viewed through their glasses by an outside observer. In the diagram, a person viewing from the left would see an image of the ears about where the eyes should be.



7. To see far objects clearly, you want your eye muscles relaxed, which makes your lens relatively flat (large focal length). When you f-stop your eye down by closing your eyelids partially (squinting), you are only using the middle of your lens, where it is the most flat. This creates a smaller circle of confusion for the lens, which helps you see distant objects more clearly.

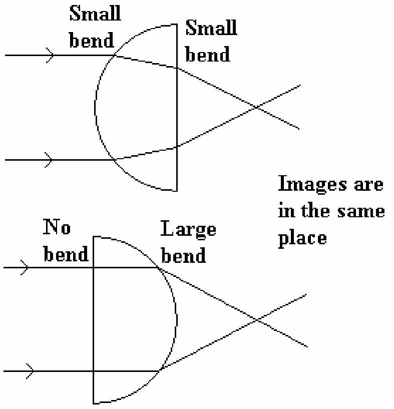
8. The images formed on our retinas of objects we look at are inverted and real. The implication of this is that our brains must flip this inverted image for us so that we can see things upright.

9. You can leave your eyes open while you are moving your head and still see clearly because your brain "refreshes" the image from the retina about 30 times a second. In other words, your brain and your retina work together in a manner similar to how a motion picture camera and film work together, which is not at all like how a still camera and film work together.

10. Yes, reading glasses are magnifiers. Similarity: they are both converging lenses. Difference: glasses try to put the image of the object at a person's near point so it is clear to see, while a magnifying lens tries to put the image of the object at infinity so it is incredibly large and it subtends as large of an angle as possible on our retina. Thus, the main difference is that the glasses need to have the object closer to the lens than the focal point, while the magnifying lens needs to have the object directly at the focal point.

11. A poor quality, inexpensive lens will not be correctly shaped to fix chromatic aberrations. Thus, the colors you see around the edges of these lenses are from all of the different colors of light getting focused at different points, instead of all being focused at the same point.

12. To minimize spherical aberrations of a planoconvex lens used to form a real image of an object at infinity, the convex side should face the object. In this situation, both surfaces bend the incoming parallel light rays a small amount each and nearly an equal amount each, which reduces the spherical aberrations. If the planar surface faced the object, then all of the bending of the light rays would take place only on the convex side where the rays emerge from the lens. Thus, with no bending on one face and all of the bending on the other face, the amount of bend is very different on the two faces and spherical aberrations will cause severe problems.



13. Spherical aberrations can be present in a simple lens. To correct this in a simple lens, usually many lenses are used together in combination to minimize the bending at each of the surfaces. Your eye minimizes spherical aberrations by also bending the light at many different interfaces as it makes its way through the different parts of the eye (cornea, aqueous humor, lens, vitreous humor, etc.), each with their own n. Also, the cornea is less curved at the edges than it is at the center and the lens is less dense at the edges than at the center. Both of these reduce spherical aberrations in the eye since they cause the rays at the outer edges to be bent less strongly. Curvature of field occurs when the focal plane is not flat. Our curved retina helps with this distortion, whereas a flat piece of film in a camera, for example, wouldn't be able to fix this. Distortion is a result of the variation of the magnification at different distances from the optical axis. This is most common in wide-angle lenses, where it must be corrected for. This is compensated in the human eye because it is a very small lens and our retina is curved. Chromatic aberrations are mostly compensated for in the human eye because the lens absorbs shorter wavelengths and the retina is not very sensitive to most blue and violet wavelengths where most chromatic aberrations occur.

14. Chromatic aberrations in lenses occur due to dispersion, where different colors (or wavelengths) of light are bent different amounts due to the fact that the index of refraction of most materials varies with wavelength. So, when light goes through a lens (a curved transparent material), not all of the different colors come out of the lens at the exact same angle, but they are all focused at different positions. A mirror, in contrast, reflects light off of a smooth metallic surface. This surface reflects all different colors of light at the exact same angle, thus there is no refraction, no dispersion and no chromatic aberrations. Of course, in most mirrors, there is a piece of glass that covers the metallic reflector. Since the two faces of this piece of glass are parallel to each other, even though refraction and dispersion take place inside the piece of glass, when the light emerges from the parallel face, all of the different colors are once again going in the same direction, and there is no dispersion and no chromatic aberration.

15. The Rayleigh criterion gives us the resolution limit for two objects a distance D apart when using light of wavelength λ: $\theta = \dfrac{1.22\lambda}{D}$, which can be interpreted as the smaller the angle, the better the resolution.

Looking at the two wavelengths given, $\theta_{\text{blue}} = \dfrac{1.22(450\,\text{nm})}{D}$ and $\theta_{\text{red}} = \dfrac{1.22(700\,\text{nm})}{D}$, we have

$$\frac{\theta_{\text{blue}}}{\theta_{\text{red}}} = \frac{\dfrac{1.22(450\,\text{nm})}{D}}{\dfrac{1.22(700\,\text{nm})}{D}} = \frac{450}{700} = 0.64.$$

This could be expressed as saying that the resolution with blue light is $\dfrac{1}{0.64} = 1.56$ times the resolution with red light.

16. A large reflecting mirror in a telescope has several advantages over a large refracting lens: One, the construction and grinding of a huge lens is very difficult (a mirror only needs one side to be ground and smoothed to perfection). Two, a huge lens is so heavy that it will sag under its own weight, especially since it needs to be held only along the edges of the lens in the telescope (mirrors can be well-supported along their entire back, non-reflecting, side). Three, a huge lens will have serious chromatic aberrations (a mirror has no chromatic aberrations).

17. To get the best resolution from a microscope, you should use blue/violet light. The Rayleigh criterion $\left(\theta = \dfrac{1.22\lambda}{D}\right)$ shows that the shorter wavelengths of light have higher resolution. (Of course, you would need very special lenses, since most glass absorbs blue/violet light very strongly.)

18. No, visible light cannot be used to "see" individual atoms. It is not possible to resolve the detail of objects smaller than the wavelength of the radiation being used to look at the object. The shortest wavelength of visible light is approximately $400\,\text{nm} = 4\times10^{-5}\,\text{m}$, which is too big to use to see atoms that are $\sim 10^{-8}\,\text{m}$ in diameter. Looking at the Rayleigh criterion, it gives: $\theta = \dfrac{1.22\lambda}{D} = \dfrac{1.22\left(4\times10^{-5}\,\text{m}\right)}{10^{-8}\,\text{m}} = 4880\,\text{rad}$. This is an impossible value, since our eyes have a value of about $5\times10^{-4}\,\text{rad}$ and microscopes can only improve this by about 1000X.

Solutions to Problems

1. From the definition of the *f*-stop, we have

$$f\text{-stop} = \frac{f}{D};$$

$$1.4 = \frac{(55\,\text{mm})}{D_{1.4}},\ \text{which gives } D_{1.4} = 39\,\text{mm};$$

$$22 = \frac{(55\,\text{mm})}{D_{22}},\ \text{which gives } D_{22} = 2.5\,\text{mm}.$$

Thus the range of diameters is $\boxed{2.5\,\text{mm} \le D \le 39\,\text{mm}.}$

2. We find the *f*-number from

$$f\text{-stop} = \frac{f}{D} = \frac{(14\,\text{cm})}{(6.0\,\text{cm})} = \boxed{\frac{f}{2.3}}.$$

3. The exposure is proportional to the area and the time:

$$\text{Exposure} \propto At \propto D^2t \propto \frac{t}{\left(f\text{-stop}\right)^2}.$$

Because we want the exposure to be the same, we have

$$\frac{t_1}{\left(f\text{-stop}_1\right)^2} = \frac{t_2}{\left(f\text{-stop}_2\right)^2};$$

$$\frac{\left[\left(\frac{1}{250}\right)\text{s}\right]}{(5.6)^2} = \frac{t_2}{(11)^2},\ \text{which gives } t_2 = \boxed{\left(\frac{1}{60}\right)\text{s}.}$$

4. The exposure is proportional to the area and the time:

$$\text{Exposure} \propto At \propto D^2t \propto \frac{t}{\left(f\text{-stop}\right)^2}.$$

Because we want the exposure to be the same, we have

$$\frac{t_1}{(f\text{-stop}_1)^2}=\frac{t_2}{(f\text{-stop}_2)^2};$$

$$\frac{\left[\left(\frac{1}{125}\right)\text{s}\right]}{(11)^2}=\frac{\left[\left(\frac{1}{1000}\right)\text{s}\right]}{(f\text{-stop}_2)^2},\text{ which gives } f\text{-stop}_2=\boxed{\frac{f}{4}.}$$

$\boxed{5.}$ For an object at infinity, the image will be in the focal plane, so we have $d_1 = f = 135\,\text{mm}$. When the object is at 1.2 m, we locate the image from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{1200\,\text{mm}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{135\,\text{cm}},\text{ which gives } d_\text{i}=152\,\text{mm}.$$

Thus the distance from the lens to the film must change by

$$d_i - f = 152\,\text{mm} - 135\,\text{mm} = \boxed{17\,\text{mm}.}$$

6. We find the object distances from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_\text{o1}}\right)+\left(\frac{1}{200.0\,\text{mm}}\right)=\frac{1}{200\,\text{mm}},\text{ which gives } d_\text{o1}=\infty;$$

$$\left(\frac{1}{d_\text{o2}}\right)+\left(\frac{1}{200.6\,\text{mm}}\right)=\frac{1}{200\,\text{mm}},\text{ which gives } d_\text{o1}=6.87\times10^3\,\text{mm}=6.87\,\text{m}.$$

Thus the range of object distances is $\boxed{6.87\,\text{m}\le d_\text{o}\le\infty.}$

7. The converging camera lens will form a real, inverted image. For the magnification, we have

$$m=\frac{h_\text{i}}{h_\text{o}}=-\frac{d_\text{i}}{d_\text{o}};$$

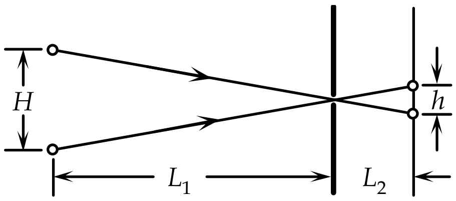
$$-\frac{(24\times10^{-3}\,\text{m})}{(28\,\text{m})}=-\frac{d_\text{i}}{(58\,\text{m})},\text{ or } d_\text{i}=4.97\times10^{-2}\,\text{m}.$$

We find the focal length of the lens from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{50}\,\text{m}\right)+\left(\frac{1}{4.97}\times10^{-2}\,\text{m}\right)=\frac{1}{f},\text{ which gives } f=5.0\times10^{-2}\,\text{m}=\boxed{50\,\text{mm}.}$$

8. From the similar triangles on the ray diagram, we see that



$$\frac{H}{L_1} = \frac{h}{L_2};$$

$$\frac{(2.0\,\text{cm})}{(100\,\text{cm})} = \frac{h}{(7.0\,\text{cm})},$$

which gives $h = 0.014\,\text{cm} = 1.4\,\text{mm}$.

To find the radius of each spot of the image, we consider the light going through a slit and find the distance from the central bright spot to the first dark spot. For destructive interference, the path-length difference to the first dark spot is

$$d\sin\theta = !\lambda;$$

We find the locations on the screen from

$$y = L_2 \tan\theta.$$

For small angles, we have

$\sin\theta \approx \tan\theta$, which gives

$$y = L_2\left(\frac{!\lambda}{d}\right) = \frac{!L_2\lambda}{d} = \frac{(7.0\times10^{-2}\,\text{m})(550\times10^{-9}\,\text{m})}{(1.0\times10^{-3}\,\text{m})} = 1.9\times10^{-5}\,\text{m} = 0.019\,\text{mm}.$$

Thus the diameter of the image spot is about 0.04 mm, which is much smaller than the separation of the spots, so they are easily resolvable.

9. We find the effective *f*-number for the pinhole:

$$f\text{-stop}_2 = \frac{f}{D} = \frac{(70\,\text{mm})}{(1.0\,\text{mm})} = \frac{f}{70}.$$

The exposure is proportional to the area and the time:

$$\frac{\text{Exposure} \propto At \propto D^2 t \propto t}{(f\text{-stop})^2}.$$

Because we want the exposure to be the same, we have

$$\frac{t_1}{(f\text{-stop}_1)^2} = \frac{t_2}{(f\text{-stop}_2)^2};$$

$$\frac{\left[\left(\frac{1}{250}\right)\text{s}\right]}{(11)^2} = \frac{t_2}{(70)^2},\ \text{which gives } t_2 = \boxed{\left(\frac{1}{6}\right)\text{s.}}$$

10. The length of the eyeball is the image distance for a far object, i.e., the focal length of the lens. We find the *f*-number from

$$f\text{-stop} = \frac{f}{D} = \frac{(20\,\text{mm})}{(8.0\,\text{mm})} = 2.5 \text{ or } \frac{f}{2.5}.$$

11. With the contact lens, an object at infinity would have a virtual image at the far point of the eye. We find the power of the lens from

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f} = P, \text{ when distances are in m;}$$

$$\left(\frac{1}{\infty}\right)+\left(\frac{1}{-0.17\,\text{m}}\right)=P=-5.9\,\text{D}.$$

To find the new near point, we have

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f}=P;$$

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{-0.12\,\text{m}}\right)=-5.9\,\text{D}, \text{ which gives } d_\text{o}=0.41\,\text{m}.$$

$\boxed{\text{Glasses would be better,}}$ because they give a near point of 32 cm from the eye.

12. With the lens, the screen placed 55 cm from the eye, or 53.2 cm from the lens, is to produce a virtual image 115 cm from the eye, or 113.2 cm from the lens.
We find the power of the lens from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f}=P, \text{ when distances are in m;}$$

$$\left(\frac{1}{0.532\,\text{m}}\right)+\left(\frac{1}{-1.132\,\text{m}}\right)=P=\boxed{+1.0\,\text{D.}}$$

13. With the glasses, an object at infinity would have its image 14 cm from the eye or $14\,\text{cm}-2\,\text{cm}=12\,\text{cm}$ from the lens; $d_\text{i}=-12\,\text{cm}$.

$$P=\frac{1}{f}=\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\left(\frac{1}{\infty}\right)+\left(\frac{1}{-0.12\,\text{m}}\right)=\boxed{-8.3\,\text{D}}$$

With the contact lenses, $\text{d}_\text{i}=-14\,\text{cm}$.

$$P=\frac{1}{f}=\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\left(\frac{1}{\infty}\right)+\left(\frac{1}{-0.14\,\text{m}}\right)=\boxed{-7.1\,\text{D}}$$

$\boxed{14.}$ The actual near point of the person is 45 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 45 cm from the eye, or 43 cm from the lens.
We find the power of the lens from

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f}=P, \text{ when distances are in m;}$$

$$\left(\frac{1}{0.23\,\text{m}}\right)+\left(\frac{1}{-0.43\,\text{m}}\right)=P=\boxed{+2.0\,\text{D.}}$$

15. (*a*) Since the diopter is negative, the lens is diverging, so it produces images closer than the object; thus the person is $\boxed{\text{nearsighted.}}$
(*b*) We find the far point by finding the image distance for an object at infinity:

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{i}}\right)=\frac{1}{f}=P;$$

$$\left(\frac{1}{\infty}\right)+\left(\frac{1}{d_\text{i}}\right)=-3.50\,\text{D}, \text{ which gives } d_\text{i}=-0.286\,\text{m}=-28.6\,\text{cm}.$$

Because this is the distance from the lens, the far point without glasses is
$28.6\,\text{cm} + 2.0\,\text{cm} = \boxed{30.6\,\text{cm.}}$

16. (*a*) We find the power of the lens for an object at infinity:

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f} = P;$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{-0.75\,\text{m}}\right) = P = \boxed{-1.3\,\text{D.}}$$

(*b*) To find the near point with the lens, we have

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f} = P;$$

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{-0.25\,\text{m}}\right) = -1.3\,\text{D}, \text{ which gives } d_\text{o} = 0.37\,\text{m} = \boxed{37\,\text{cm.}}$$

17. The 2.0 cm of a normal eye is the image distance for an object at infinity; thus it is the focal length of the lens of the eye. To find the length of the nearsighted eye, we find the image distance (distance from lens to retina) for an object at the far point of the eye:

$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{17\,\text{cm}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{2.0\,\text{cm}}, \text{which gives } d_\text{i} = 2.27\,\text{cm.}$$

Thus the difference is $2.27\,\text{cm} - 2.0\,\text{cm} = \boxed{0.3\,\text{cm.}}$

18. We find the far point of the eye by finding the image distance with the lens for an object at infinity:

$$\left(\frac{1}{d_\text{o1}}\right) + \left(\frac{1}{d_\text{i1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{d_\text{i1}}\right) = \frac{1}{-22.0\text{cm}}, \text{ which gives } d_\text{i1} = -22.0\,\text{cm} \text{ from the lens, or 23.8 cm from the eye.}$$

We find the focal length of the contact lens from

$$\left(\frac{1}{d_\text{o2}}\right) + \left(\frac{1}{d_\text{i2}}\right) = \frac{1}{f_2};$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{-23.8\,\text{cm}}\right) = \frac{1}{f_2}, \text{ which gives } f_2 = \boxed{-23.8\,\text{cm.}}$$

19. (*a*) We find the focal length of the lens for an object at infinity and the image on the retina:

$$\left(\frac{1}{d_\text{o1}}\right) + \left(\frac{1}{d_\text{i1}}\right) = \frac{1}{f_1};$$

$$\left(\frac{1}{\infty}\right) + \left(\frac{1}{2.0\,\text{cm}}\right) = \frac{1}{f_1}, \text{ which gives } f_1 = \boxed{2.0\,\text{cm.}}$$

(*b*) With the object 30 cm from the eye, we have

$$\left(\frac{1}{d_{o2}}\right)+\left(\frac{1}{d_{i2}}\right)=\frac{1}{f_2};$$

$$\left(\frac{1}{33\,\text{cm}}\right)+\left(\frac{1}{2.0\,\text{cm}}\right)=\frac{1}{f_2},\text{ which gives } f_2=\boxed{1.9\,\text{cm.}}$$

20. We find the object distance for an image at her eye's near point:

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f}=P;$$

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{-0.100\,\text{m}}\right)=-4.0\,\text{D},\text{ which gives } d_o=1.7\times10^{-2}\,\text{m}=\boxed{17\,\text{cm.}}$$

We find the object distance for an image at her eye's far point:

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f}=P;$$

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{-0.200\,\text{m}}\right)=-4.0\,\text{D},\text{ which gives } d_o=1.0\,\text{m}=\boxed{100\,\text{cm.}}$$

21. The magnification with the image at infinity is

$$M=\frac{N}{f}=\frac{(25\,\text{cm})}{(12\,\text{cm})}=\boxed{2.1\times.}$$

22. We find the focal length from

$$M=\frac{N}{f};$$

$$3.5=\frac{(25\,\text{cm})}{f},\text{ which gives } f=\boxed{7.1\,\text{cm.}}$$

23. (*a*) We find the focal length with the image at the near point from

$$M=1+\frac{N}{f_1};$$

$$2.5=\frac{1+(25\,\text{cm})}{f_1},\text{ which gives } f_1=\boxed{17\,\text{cm.}}$$

(*b*) If the eye is relaxed, the image is at infinity, so we have

$$M=\frac{N}{f_2};$$

$$2.5=\frac{(25\,\text{cm})}{f_2},\text{ which gives } f_2=\boxed{10\,\text{cm.}}$$

$\boxed{24.}$ Maximum magnification is obtained with the image at the near point. We find the object distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{-25.0\,\text{cm}}\right)=\frac{1}{9.00}\,\text{cm}, \text{ which gives } d_o=\boxed{6.6\text{ cm from the lens.}}$$

The magnification is

$$M=1+\frac{N}{f}=1+\frac{(25.0\,\text{cm})}{(9.00\,\text{cm})}=\boxed{3.78\times.}$$

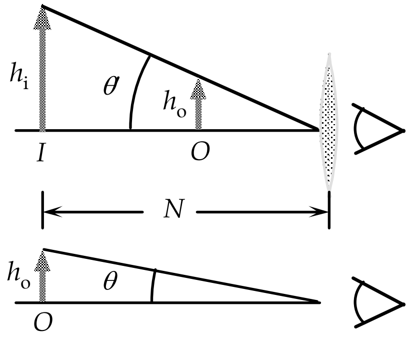
25. (*a*) The angular magnification with the image at the near point is

$$M=1+\frac{N}{f}=1+\frac{(25.0\,\text{cm})}{(9.50\,\text{cm})}=\boxed{3.63\times.}$$

(*b*) Because the object without the lens and the image with the lens are at the near point, the angular magnification is also the ratio of widths:

$$M=\frac{h_i}{h_o};$$

$$3.63=\frac{h_i}{(3.30\,\text{mm})}, \text{ which gives } h_i=\boxed{12.0\,\text{mm.}}$$



(*c*) We find the object distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{-25.0\,\text{cm}}\right)=\frac{1}{9.50}\,\text{cm},$$

which gives $d_o=\boxed{6.88\text{ cm from the lens.}}$

26. (*a*) We find the image distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

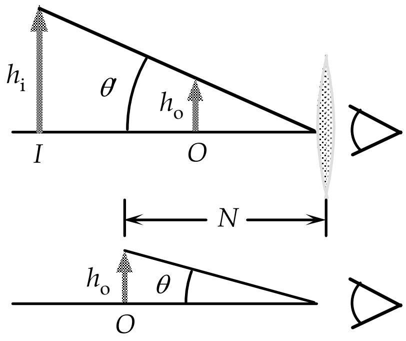
$$\left(\frac{1}{5.55\,\text{cm}}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{6.00}\,\text{cm},$$

which gives $d_i=\boxed{-74\,\text{cm.}}$

(*b*) From the diagram we see that the angular magnification is

$$M=\frac{\theta'}{\theta}=\frac{\left(\frac{h_o}{d_o}\right)}{\left(\frac{h_o}{N}\right)}=\frac{N}{d_o}$$

$$=\frac{(25\,\text{cm})}{(5.55\,\text{cm})}=\boxed{4.50\times.}$$



27. (*a*) We find the image distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{7.5\,\text{cm}}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{8.5}\text{cm, which gives } d_i=\boxed{-64\,\text{cm.}}$$

(*b*) The angular magnification is

$$M=\frac{\theta'}{\theta}=\frac{\left(\frac{h_o}{d_o}\right)}{\left(\frac{h_o}{N}\right)}=\frac{N}{d_o}$$

$$=\frac{(25\,\text{cm})}{(7.5\,\text{cm})}=\boxed{3.3\times.}$$

28. We find the focal length of the lens from

$$M=\frac{N}{f};$$

$$3.0=\frac{(25\,\text{cm})}{f},\text{ which gives } f=8.3\,\text{cm}.$$

(*a*) The magnification with the image at infinity is

$$M_1=\frac{N_1}{f}=\frac{(55\,\text{cm})}{(8.3\,\text{cm})}=\boxed{6.6\times.}$$

(*b*) The magnification with the image at infinity is

$$M_2=\frac{N_2}{f}=\frac{(16\,\text{cm})}{(8.3\,\text{cm})}=\boxed{1.9\times.}$$

Without the lens, the closest an object can be placed is the near point. A farther near point means a smaller angle subtended by the object without the lens, and thus greater magnification.

29. The magnification of the telescope is given by

$$M=-\frac{f_o}{f_e}=-\frac{(76\,\text{cm})}{(2.8\,\text{cm})}=\boxed{-27\times.}$$

For both object and image far away, the separation of the lenses is

$$L=f_o+f_e=76\,\text{cm}+2.8\,\text{cm}=\boxed{79\,\text{cm.}}$$

30. We find the focal length of the eyepiece from the magnification:

$$M=-\frac{f_o}{f_e};$$

$$-25=-\frac{(78\,\text{cm})}{f_e},\text{ which gives } f_e=\boxed{3.1\,\text{cm.}}$$

For both object and image far away, the separation of the lenses is

$$L=f_o+f_e=78\,\text{cm}+3.1\,\text{cm}=\boxed{81\,\text{cm.}}$$

31. We find the focal length of the objective from the magnification:

$$M = \frac{f_o}{f_e};$$

$$8.0 = \frac{f_o}{(2.8\,\text{cm})}, \text{ which gives } f_o = \boxed{22\,\text{cm.}}$$

32. We find the focal length of the eyepiece from the power:

$$f_e = \frac{1}{P} = \frac{1}{35}\,\text{D} = 0.029\,\text{m} = 2.9\,\text{cm}.$$

The magnification of the telescope is given by

$$M = -\frac{f_o}{f_e} = -\frac{(85\,\text{cm})}{(2.9\,\text{cm})} = \boxed{-29\times.}$$

33. For both object and image far away, we find the focal length of the eyepiece from the separation of the lenses:

$$L = f_o + f_e;$$

$$75.2\,\text{cm} = 74.5\,\text{cm} + f_e, \text{ which gives } f_e = 0.7\,\text{cm}.$$

The magnification of the telescope is given by

$$M = -\frac{f_o}{f_e} = -\frac{(75.2\,\text{cm})}{(0.7\,\text{cm})} = \boxed{-110\times.}$$

34. For both object and image far away, we find the (negative) focal length of the eyepiece from the separation of the lenses:

$$L = f_o + f_e;$$

$$32.8\,\text{cm} = 36.0\,\text{cm} + f_e, \text{ which gives } f_e = -3.2\,\text{cm}.$$

The magnification of the telescope is given by

$$M = -\frac{f_o}{f_e} = -\frac{(36.0\,\text{cm})}{(-3.2\,\text{cm})} = \boxed{11\times.}$$

$\boxed{35.}$ The reflecting mirror acts as the objective, with a focal length

$$f_o = \frac{r}{2} = \frac{(6.0\,\text{m})}{2} = 3.0\,\text{m}.$$

The magnification of the telescope is given by

$$M = -\frac{f_o}{f_e} = -\frac{(300\,\text{cm})}{(3.2\,\text{cm})} = \boxed{-94\times.}$$

36. We find the focal length of the mirror from

$$M = -\frac{f_o}{f_e};$$

$$-120 = -\frac{f_o}{(3.2\,\text{cm})}, \text{ which gives } f_o = 3.8\times10^2\,\text{cm} = \boxed{3.8\,\text{m.}}$$

The radius is

$$r = 2f_o = 2(3.8\,\text{m}) = \boxed{7.6\,\text{m.}}$$

37. For the magnification we have

$$M = -\frac{f_o}{f_e} = -170, \text{ or } f_o = 170 f_e.$$

For both object and image far away, we have

$$L = f_o + f_e;$$

$$1.25\,\text{m} = 170 f_e + f_e, \text{ which gives } f_e = 7.31\times10^{-3}\,\text{m} = \boxed{7.31\,\text{mm.}}$$

The focal length of the objective is

$$f_o = 170 f_e = 170\left(7.31\times10^{-3}\,\text{m}\right) = \boxed{1.24\,\text{m.}}$$

38. First find the image distance of the star from the objective mirror. The star is off at infinity, so this will be at the focal point to the left of the objective mirror.

$$d_{1i} = f_1 = \frac{r_1}{2} = \frac{3.0\,\text{m}}{2} = 1.5\,\text{m}$$

Then the object distance of the image of the star from the eyepiece mirror is

$$0.9\,\text{m} - 1.5\,\text{m} = -0.6\,\text{m}$$

The negative sign indicates the direction to the left of the eyepiece mirror.
Next find the image distance of the image of the star from the eyepiece mirror.

$$\frac{1}{d_{2i}} + \frac{1}{d_{2o}} = \frac{1}{f_2}$$

$$\frac{1}{d_{2i}} = \frac{1}{f_2} - \frac{1}{d_{2o}} = \frac{1}{\left(\frac{r_2}{2}\right)} - \frac{1}{d_{2o}} = \frac{1}{(-0.75\,\text{m})} - \frac{1}{(-0.6\,\text{m})}$$

Solve for d_{2i} to get

$$d_{2i} = \boxed{3\,\text{m.}}$$

39. The magnification of the microscope is

$$M = \frac{Nl}{f_o f_e} = \frac{(25\,\text{cm})(17.5\,\text{cm})}{(0.65\,\text{cm})(1.40\,\text{cm})} = \boxed{480\times.}$$

$\boxed{40.}$ We find the focal length of the eyepiece from the magnification of the microscope:

$$M = \frac{Nl}{f_o f_e};$$

$$620 = \frac{(25\,\text{cm})(17.5\,\text{cm})}{(0.40\,\text{cm})f_e}, \text{ which gives } f_e = \boxed{1.8\,\text{cm.}}$$

41. The total magnification is

$$M = \frac{Nl}{f_e f_o} = \frac{(25\,\text{cm})(17\,\text{cm})}{(2.5\,\text{cm})(0.28\,\text{cm})} = \boxed{610\times.}$$

42. (*a*) The total magnification is

$$M = M_o M_e = (59.0)(12.0) = \boxed{708\times.}$$

(*b*) With the final image at infinity, we find the focal length of the eyepiece from

$$M_{\text{e}} = \frac{N}{f_{\text{e}}};$$

$$12.0 = \frac{(25.0\,\text{cm})}{f_{\text{e}}}, \text{ which gives } f_{\text{e}} = \boxed{2.08\,\text{cm.}}$$

Because the image from the objective is at the focal point of the eyepiece, the image distance for the objective is

$$d_{\text{i}} = l - f_{\text{e}} = 20.0\,\text{cm} - 2.08\,\text{cm} = 17.9\,\text{cm}.$$

We find the object distance from the magnification:

$$M_{\text{o}} = \frac{d_{\text{i}}}{d_{\text{o}}};$$

$$59.0 = \frac{(17.9\,\text{cm})}{d_{\text{o}}}, \text{ which gives } d_{\text{o}} = 0.303\,\text{cm}.$$

We find the focal length of the objective from the lens equation:

$$\left(\frac{1}{d_{\text{o}}}\right) + \left(\frac{1}{d_{\text{i}}}\right) = \frac{1}{f_{\text{o}}};$$

$$\left(\frac{1}{0.289\,\text{cm}}\right) + \left(\frac{1}{17.9\,\text{cm}}\right) = \frac{1}{f_{\text{o}}}, \text{ which gives } f_{\text{o}} = \boxed{0.298\,\text{cm.}}$$

(*c*) We found the object distance: $d_{\text{o}} = \boxed{0.303\,\text{cm.}}$

43. (*a*) Because the image from the objective is at the focal point of the eyepiece, the image distance for the objective is

$$d_{\text{io}} = l - f_{\text{e}} = 16.0\,\text{cm} - 1.8\,\text{cm} = 14.2\,\text{cm}.$$

We find the object distance from the lens equation for the objective:

$$\left(\frac{1}{d_{\text{oo}}}\right) + \left(\frac{1}{d_{\text{io}}}\right) = \frac{1}{f_{\text{o}}};$$

$$\left(\frac{1}{d_{\text{oo}}}\right) + \left(\frac{1}{14.2\,\text{cm}}\right) = \frac{1}{0.80\text{cm}}, \text{ which gives } d_{\text{oo}} = \boxed{0.85\,\text{cm.}}$$

(*b*) With the final image at infinity, the magnification of the eyepiece is

$$M_{\text{e}} = \frac{N}{f} = \frac{(25.0\,\text{cm})}{(1.8\,\text{cm})} = 13.9\times.$$

The magnification of the objective is

$$M_{\text{o}} = \frac{d_{\text{io}}}{d_{\text{oo}}} = \frac{(14.2\,\text{cm})}{(0.85\,\text{cm})} = 16.7\times.$$

The total magnification is

$$M = M_{\text{o}}M_{\text{e}} = (16.7)(13.9) = \boxed{230\times.}$$

44. (*a*) We find the object distance from the lens equation for the eyepiece:

$$\left(\frac{1}{d_{\text{oe}}}\right) + \left(\frac{1}{d_{\text{ie}}}\right) = \frac{1}{f_{\text{e}}};$$

$$\left(\frac{1}{d_{\text{oe}}}\right)+\left(\frac{1}{-25\,\text{cm}}\right)=\frac{1}{1.8\text{cm}},\ \text{which gives } d_{\text{oe}}=1.7\,\text{cm}.$$

The image distance for the objective is

$$d_{\text{io}}=l-d_{\text{oe}}=16.0\,\text{cm}-1.7\,\text{cm}=14.3\,\text{cm}.$$

We find the object distance from the lens equation for the objective:

$$\left(\frac{1}{d_{\text{oo}}}\right)+\left(\frac{1}{d_{\text{io}}}\right)=\frac{1}{f_{\text{o}}};$$

$$\left(\frac{1}{d_{\text{oo}}}\right)+\left(\frac{1}{14.3\,\text{cm}}\right)=\frac{1}{0.80\text{cm}},\ \text{which gives } d_{\text{oo}}=\boxed{0.85\,\text{cm.}}$$

(*b*) With the final image at the near point, the magnification of the eyepiece is

$$M_{\text{e}}=1+\frac{N}{f}=1+\frac{(25.0\,\text{cm})}{(1.8\,\text{cm})}=14.9\times.$$

The magnification of the objective is

$$M_{\text{o}}=\frac{d_{\text{io}}}{d_{\text{oo}}}=\frac{(14.3\,\text{cm})}{(0.85\,\text{cm})}=16.9\times.$$

The total magnification is

$$M=M_{\text{o}}M_{\text{e}}=(16.9)(14.9)=\boxed{250\times.}$$

45. (*a*) We find the image distance from the lens equation for the objective:

$$\left(\frac{1}{d_{\text{oo}}}\right)+\left(\frac{1}{d_{\text{io}}}\right)=\frac{1}{f_{\text{o}}};$$

$$\left(\frac{1}{0.790\,\text{cm}}\right)+\left(\frac{1}{d_{\text{io}}}\right)=\frac{1}{0.740}\text{cm},\ \text{which gives } d_{\text{io}}=11.7\,\text{cm}.$$

For the relaxed eye, the image from the objective is at the focal point of the eyepiece: $d_{\text{oe}}=2.70\,\text{cm}$.

The distance between lenses is

$$l=d_{\text{io}}+d_{\text{oe}}=11.7\,\text{cm}-2.70\,\text{cm}=\boxed{14.4\,\text{cm.}}$$

(*b*) With the final image at infinity, the magnification of the eyepiece is

$$M_{\text{e}}=\frac{N}{f}=\frac{(25.0\,\text{cm})}{(2.70\,\text{cm})}=9.26\times.$$

The magnification of the objective is

$$M_{\text{o}}=\frac{d_{\text{io}}}{d_{\text{oo}}}=\frac{(11.7\,\text{cm})}{(0.790\,\text{cm})}=14.8\times.$$

The total magnification is

$$M=M_{\text{o}}M_{\text{e}}=(14.8)(9.26)=\boxed{137\times.}$$

46. (*a*) When lenses are in contact, the powers add:

$$P=P_1+P_2=\left(\frac{1}{-0.28\,\text{m}}\right)+\left(\frac{1}{0.23\,\text{m}}\right)=+0.776\,\text{D}.$$

It is a positive lens, and thus $\boxed{\text{converging.}}$

(*b*) The focal length is

$$f = \frac{1}{P} = \frac{1}{0.776\,\text{D}} = \boxed{1.3\,\text{m.}}$$

47. (*a*) We find the incident angle from

$$\sin\theta_1 = \frac{h_1}{R} = \frac{(1.0\,\text{cm})}{(12.0\,\text{cm})} = 0.0833, \text{ so } \theta_1 = 4.78°.$$

For the refraction at the curved surface, we have

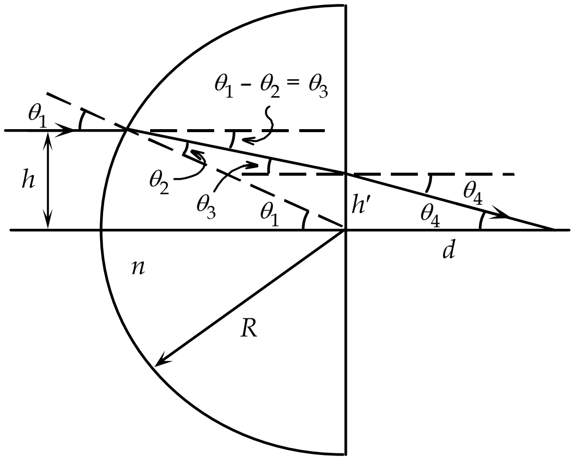
$$\sin\theta_1 = n\sin\theta_2;$$

$$\sin 4.78° = (1.50)\sin\theta_2,$$

which gives $\sin\theta_2 = 0.0556$, so $\theta_2 = 3.18°$.

We see from the diagram that

$$\theta_3 = \theta_1 - \theta_2 = 4.78° - 3.18° = 1.60°.$$



For the refraction at the flat face, we have

$$n\sin\theta_3 = \sin\theta_4;$$

$$(1.50)\sin 1.60° = \sin\theta_4,$$

which gives $\sin\theta_4 = 0.0419$, so $\theta_4 = 2.40°$.

We see from the diagram that

$$h_1' = h_1 - R\sin\theta_3$$
$$= 1.0\,\text{cm} - (12.0\,\text{cm})\sin 1.60° = 0.665\,\text{cm},$$

so the distance from the flat face to the point where the ray crosses the axis is

$$d_1 = \frac{h_1'}{\tan\theta_4} = \frac{(0.665\,\text{cm})}{\tan 2.40°} = \boxed{15.9\,\text{cm.}}$$

(*b*) We find the incident angle from

$$\sin\theta_1 = \frac{h_2}{R} = \frac{(4.0\,\text{cm})}{(12.0\,\text{cm})} = 0.333, \text{ so } \theta_1 = 19.47°.$$

For the refraction at the curved surface, we have

$$\sin\theta_1 = n\sin\theta_2;$$

$$\sin 19.47° = (1.50)\sin\theta_2, \text{ which gives } \sin\theta_2 = 0.222, \text{ so } \theta_2 = 12.84°.$$

We see from the diagram that

$$\theta_3 = \theta_1 - \theta_2 = 19.47° - 12.84° = 6.63°.$$

For the refraction at the flat face, we have

$$n\sin\theta_3 = \sin\theta_4;$$

$$(1.50)\sin 6.63° = \sin\theta_4, \text{ which gives } \sin\theta_4 = 0.173, \text{ so } \theta_4 = 9.97°.$$

We see from the diagram that

$$h_2' = h_2 - R\sin\theta_3 = 4.0\,\text{cm} - (12.0\,\text{cm})\sin 6.63° = 2.61\,\text{cm},$$

so the distance from the flat face to the point where the ray crosses the axis is

$$d_2 = \frac{h_2'}{\tan\theta_4} = \frac{(2.61\,\text{cm})}{\tan 9.97°} = \boxed{14.8\,\text{cm.}}$$

(*c*) The separation of the "focal points" is

$$\Delta d = d_1 - d_2 = 15.9\,\text{cm} - 14.8\,\text{cm} = \boxed{1.1\,\text{cm.}}$$

(*d*) When $h_2 = 4.0$ cm, the rays focus closer to the lens, so they will form a circle at the "focal point" for $h_1 = 1.0$ cm. We find the radius of this circle from similar triangles:

$$\frac{h_2'}{d_2} = \frac{r}{(d_1 - d_2)};$$

$$\frac{(2.61\,\text{cm})}{(14.8\,\text{cm})} = \frac{r}{(1.1\,\text{cm})}, \text{ which gives } r = \boxed{0.19\,\text{cm.}}$$

48. The minimum angular resolution is

$$\theta = \frac{1.22\lambda}{D} = \frac{(1.22)(550\times10^{-9}\,\text{m})}{(100\,\text{in})(0.0254\,\text{m/in})} = \boxed{2.64\times10^{-7}\,\text{rad} = (1.51\times10^{-5})^\circ.}$$

49. The angular resolution of the eye, which is the required resolution using the telescope, is

$$\theta_{\text{eye}} = \frac{d_{\text{eye}}}{L_{\text{eye}}} = \frac{(0.10\times10^{-3}\,\text{m})}{(25\times10^{-2}\,\text{m})} = 4.0\times10^{-4}\,\text{rad}.$$

The resolution without the telescope is

$$\theta = \frac{d}{L} = \frac{(7.0\,\text{km})}{(3.84\times10^{5}\,\text{km})} = 1.82\times10^{-5}\,\text{rad}.$$

If we ignore the inverted image, the magnification is

$$M = \frac{\theta_{\text{eye}}}{\theta} = \frac{f_{\text{o}}}{f_{\text{e}}};$$

$$\frac{(4.0\times10^{-4}\,\text{rad})}{(1.82\times10^{-5}\,\text{rad})} = \frac{(2.0\,\text{m})}{f_{\text{e}}}, \text{ which gives } f_{\text{e}} = 0.091\,\text{m} = \boxed{9.1\,\text{cm.}}$$

The resolution limit is

$$\theta = \frac{1.22\lambda}{D} = \frac{(1.22)(5.5\times10^{-7}\,\text{m})}{(0.11\,\text{m})} = \boxed{6.1\times10^{-6}\,\text{rad.}}$$

50. The minimum angular resolution is

$$\theta = \frac{1.22\lambda}{D}.$$

The distance between lines is the resolving power:

$$\text{RP} = f\theta = \frac{1.22\lambda f}{D} = 1.22\lambda(f\text{-stop}).$$

For $\frac{f}{2}$ we have

$$\text{RP}_2 = (1.22)(550\times10^{-9}\,\text{m})(2) = 1.34\times10^{-6}\,\text{m} = 1.34\times10^{-3}\,\text{mm}, \text{ so the resolution is}$$

$$\frac{1}{\text{RP}_2} = \frac{1}{(1.22\times10^{-3}\,\text{mm})} = \boxed{746\,\text{lines/mm.}}$$

For $\frac{f}{16}$ we have

$$\text{RP}_2 = (1.22)(550\times10^{-9}\,\text{m})(16.7) = 1.12\times10^{-5}\,\text{m} = 1.12\times10^{-2}\,\text{mm}, \text{ so the resolution is}$$

$$\frac{1}{\text{RP}_2} = \frac{1}{\left(1.02 \times 10^{-2}\ \text{mm}\right)} = \boxed{89\,\text{lines/mm.}}$$

51. The resolution of the telescope is

$$\theta = \frac{1.22\lambda}{D} = \frac{(1.22)\left(550 \times 10^{-9}\ \text{m}\right)}{(0.55\,\text{m})} = 1.22 \times 10^{-6}\ \text{rad.}$$

The separation of the stars is

$$d = L\theta = (15\,\text{ly})\left(9.46 \times 10^{15}\ \text{m/ly}\right)\left(1.22 \times 10^{-6}\ \text{rad}\right) = \boxed{1.7 \times 10^{11}\ \text{m.}}$$

52. (*a*) The resolution of the eye is

$$\theta = \frac{1.22\lambda}{D} = \frac{(1.22)\left(550 \times 10^{-9}\ \text{m}\right)}{\left(5.0 \times 10^{-3}\ \text{m}\right)} = 1.34 \times 10^{-4}\ \text{rad.}$$

We find the maximum distance from

$d = L\theta;$

$2.0\,\text{m} = L\left(1.34 \times 10^{-4}\ \text{rad}\right)$, which gives $L = 1.5 \times 10^4\ \text{m} = \boxed{15\,\text{km.}}$

(*b*) The angular separation is the resolution:

$\theta = 1.34 \times 10^{-4}\ \text{rad} = \left(7.68 \times 10^{-3}\right)^\circ = \boxed{0.46'.}$

Our answer is less than the real resolution because of the presence of aberrations.

53. The separation is $s = 400 \times 10^6$ m. The distance away is $L = 8 \times 10^{10}$ m.

The angular separation is

$$\theta = \frac{s}{L} = \frac{400 \times 10^6\ \text{m}}{8 \times 10^{10}\ \text{m}} = 0.005\,\text{rad.}$$

The angular resolution of the eye is

$$\theta_\text{e} = \frac{1.22\lambda}{D} = \frac{(1.22)\left(550 \times 10^{-9}\ \text{m}\right)}{(0.05\,\text{m})} = 1.34 \times 10^{-5}\ \text{rad.}$$

Since $\theta > \theta_\text{e}$, the answer is $\boxed{\text{yes.}}$

A person standing on Mars can resolve the Earth and its moon without a telescope.

54. For the diffraction from the crystal, we have

$m\lambda = 2d\sin\phi;\ m = 1, 2, 3, \ldots .$

For the first maximum, we get

$(1)(0.133\,\text{nm}) = 2(0.280\,\text{nm})\sin\phi$, which gives $\phi = \boxed{13.7^\circ.}$

$\boxed{55.}$ We find the spacing from

$m\lambda = 2d\sin\phi;\ m = 1, 2, 3, \ldots .$

$(2)(0.0973\,\text{nm}) = 2d\sin 23.4^\circ$, which gives $d = \boxed{0.245\,\text{nm.}}$

56. (*a*) For the diffraction from the crystal, we have

$m\lambda = 2d\sin\phi;\ m = 1, 2, 3, \ldots .$

When we form the ratio for the two orders, we get

$$\frac{m_2}{m_1} = \frac{(\sin\phi_2)}{(\sin\phi_1)};$$

$$\frac{2}{1} = \frac{(\sin\phi_2)}{(\sin 25.2°)}, \text{ which gives } \phi_2 = \boxed{58.4°.}$$

(*b*) We find the wavelength from

$$m_1\lambda = 2d\sin\phi_1;$$

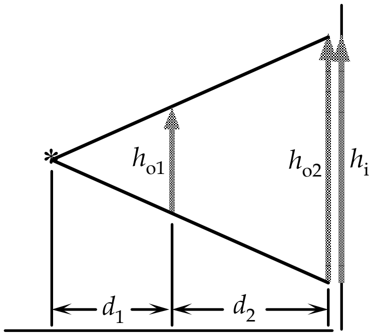
$$(1)\lambda = 2(0.24\,\text{nm})\sin 25.2°, \text{ which gives } \lambda = \boxed{0.20\,\text{nm.}}$$

57. (*a*) Because X-ray images are shadows, the image will be the same size as the object, so the magnification is $\boxed{1.}$

(*b*) The rays from the point source will not refract, so we can use similar triangles to compare the image size to the object size for the front of the body:

$$m_1 = \frac{h_i}{h_{o1}} = \frac{(d_1 + d_2)}{d_1} = \frac{(15\,\text{cm} + 25\,\text{cm})}{(15\,\text{cm})} = \boxed{2.7.}$$

For the back of the body, the image and object have the same size, so the magnification is $\boxed{1.}$



58. (*a*) The focal length of the lens is

$$f = \frac{1}{P} = \frac{1}{3.50\text{D}} = 0.286\,\text{m} = \boxed{28.6\,\text{cm.}}$$

(*b*) The lens produces a virtual image at his near point:

$$\left(\frac{1}{d_{o1}}\right) + \left(\frac{1}{d_{i1}}\right) = \frac{1}{f} = P;$$

$$\left(\frac{1}{0.23\,\text{m}}\right) + \left(\frac{1}{d_{i1}}\right) = +3.50\,\text{D}, \text{ which gives } d_{i1} = -1.18\,\text{m}, \text{ so his near point is } \boxed{120\,\text{cm.}}$$

(*c*) For Pam, we find the object distance that will have an image at her near point:

$$\left(\frac{1}{d_{o2}}\right) + \left(\frac{1}{d_{i2}}\right) = \frac{1}{f} = P;$$

$$\left(\frac{1}{d_{o2}}\right) + \left(\frac{1}{-0.23\,\text{m}}\right) = +3.50\text{D}, \text{ which gives } d_{o2} = -0.13\,\text{m}, \text{ which is } \boxed{15\text{ cm}} \text{ from her eyes.}$$

59. The exposure is proportional to the intensity, the area and the time:

$$\frac{\text{Exposure} \propto IAt \propto ID^2t \propto It}{(f\text{-stop})^2}.$$

With the same shutter speed, the time is constant.
Because we want the exposure to be the same, we have

$$\frac{I_1}{(f\text{-stop}_1)^2} = \frac{I_2}{(f\text{-stop}_2)^2};$$

$$\frac{I_1}{(5.6)^2}=\frac{I_2}{(22)^2},\text{ which gives }\boxed{I_2=16I_1.}$$

Note that we have followed convention to use multiples of 2.

60. When an object is very far away, the image will be at the focal point $d_i = f$. Thus the magnification is

$$m=-\frac{d_i}{d_o}=-\frac{f}{d_o},\text{ that is, proportional to } f.$$

61. For the magnification, we have

$$m=-\frac{h_i}{h_o}=-\frac{d_i}{d_o}=-1,\text{ so } d_i=d_o.$$

We find the object distance from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_o}\right)=\frac{1}{50\,\text{mm}},\text{ which gives } d_o=\boxed{100\,\text{mm.}}$$

The distance between the object and the film is

$$d=d_o+d_i=100\,\text{mm}+100\,\text{mm}=\boxed{200\,\text{mm.}}$$

62. The actual far point of the person is 155 cm. With the lens, an object far away is to produce a virtual image 155 cm from the eye, or 153 cm from the lens.

We find the power of the lens from

$$\left(\frac{1}{d_{o1}}\right)+\left(\frac{1}{d_{i1}}\right)=\frac{1}{f_1}=P_1,\text{ when distances are in m;}$$

$$\left(\frac{1}{\infty}\right)+\left(\frac{1}{-1.53\,\text{m}}\right)=P_1=\boxed{-0.65\,\text{D (upper part).}}$$

The actual near point of the person is 45 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 45 cm from the eye, or 43 cm from the lens.

We find the power of the lens from

$$\left(\frac{1}{d_{o2}}\right)+\left(\frac{1}{d_{i2}}\right)=\frac{1}{f_2}=P_2;$$

$$\left(\frac{1}{0.23\,\text{m}}\right)+\left(\frac{1}{-0.43\,\text{m}}\right)=P_2=\boxed{+2.0\,\text{D (lower part).}}$$

63. The maximum magnification is achieved with the image at the near point:

$$M_1=1+\frac{N_1}{f}=1+\frac{(15.0\,\text{cm})}{(8.0\,\text{cm})}=\boxed{2.9\times.}$$

For an adult we have

$$M_2=1+\frac{N_2}{f}=1+\frac{(25.0\,\text{cm})}{(8.0\,\text{cm})}=\boxed{4.1\times.}$$

The person with the normal eye (adult) sees more detail.

64. The magnification for a relaxed eye is

$$M=\frac{N}{f}=NP=(0.25\,\mathrm{m})(+4.0\,D)=\boxed{1.0\times.}$$

65. (*a*) The magnification of the telescope is given by

$$M=-\frac{f_\mathrm{o}}{f_\mathrm{e}}=-\frac{P_\mathrm{e}}{P_\mathrm{o}}=-\frac{(4.5\,\mathrm{D})}{(2.0\,\mathrm{D})}=\boxed{-2.25\times.}$$

(*b*) To get a magnification greater than 1, for the eyepiece we use the lens with the smaller focal length, or greater power: $\boxed{4.5\text{ D.}}$

66. To find the new near point, we have

$$\left(\frac{1}{d_{\mathrm{o}1}}\right)+\left(\frac{1}{d_{\mathrm{i}1}}\right)=\frac{1}{f_1}=P_1,\text{ when distances are in m;}$$

$$\left(\frac{1}{0.35\,\mathrm{m}}\right)+\left(\frac{1}{d_{\mathrm{i}1}}\right)=+2.5\,\mathrm{D},\text{ which gives }d_{\mathrm{i}1}=-2.8\,\mathrm{m}.$$

To give him a normal near point, we have

$$\left(\frac{1}{d_{\mathrm{o}2}}\right)+\left(\frac{1}{d_{\mathrm{i}2}}\right)=\frac{1}{f_2}=P_2;$$

$$\left(\frac{1}{0.25\,\mathrm{m}}\right)+\left(\frac{1}{-2.8\,\mathrm{m}}\right)=P_2=\boxed{+3.6\,\mathrm{D}.}$$

67. For the minimum aperture the angle subtended at the lens by the smallest feature is the angular resolution:

$$\theta=\frac{d}{L}=\frac{1.22\lambda}{D};$$

$$\frac{(5\times10^{-2}\,\mathrm{m})}{(25\times10^{3}\,\mathrm{m})}=\frac{(1.22)(550\times10^{-9}\,\mathrm{m})}{D},\text{ which gives }D=0.34\,\mathrm{m}=\boxed{34\,\mathrm{cm.}}$$

68. (*a*) When an object is very far away, the image will be at the focal point $d_{\mathrm{i}1}=f$.

From the magnification, we have

$$m=-\frac{h_\mathrm{i}}{h_\mathrm{o}}=-\frac{d_\mathrm{i}}{d_\mathrm{o}},\text{ so we see that }h_\mathrm{i}\text{ is proportional to }d_\mathrm{i}\,.$$

When the object and image are the same size, we get

$$-\frac{h_\mathrm{i}}{h_\mathrm{o}}=-1=-\frac{d_{\mathrm{i}2}}{d_{\mathrm{o}2}},\text{ so }d_{\mathrm{o}2}=d_{\mathrm{i}2}.$$

From the lens equation, we get

$$\left(\frac{1}{d_{\mathrm{o}2}}\right)+\left(\frac{1}{d_{\mathrm{i}2}}\right)=\frac{1}{f};$$

$$\left(\frac{1}{d_{\mathrm{i}2}}\right)+\left(\frac{1}{d_{\mathrm{i}2}}\right)=\frac{1}{f},\text{ which gives }d_{\mathrm{i}2}=2f.$$

The required exposure time is proportional to the area of the image on the film:

$$t\propto A\propto(h_\mathrm{i})^2\propto(d_\mathrm{i})^2.$$

When we form the ratio, we get

$$\frac{t_2}{t_1} = \left(\frac{d_{i2}}{d_{i1}}\right)^2 = \left(\frac{2f}{f}\right)^2 = 4.$$

(*b*) For the increased object distances, we have

$$\left(\frac{1}{d_{o3}}\right) + \left(\frac{1}{d_{i3}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{4f}\right) + \left(\frac{1}{d_{i3}}\right) = \frac{1}{f}, \text{ which gives } d_{i3} = \frac{4f}{3}; \text{ and } \frac{t_3}{t_1} = \left(\frac{4}{3}\right)^2 = 1.78.$$

$$\left(\frac{1}{d_{o4}}\right) + \left(\frac{1}{d_{i4}}\right) = \frac{1}{f};$$

$$\left(\frac{1}{5f}\right) + \left(\frac{1}{d_{i4}}\right) = \frac{1}{f}, \text{ which gives } d_{i4} = \frac{5f}{4}; \text{ and } \frac{t_4}{t_1} = \left(\frac{5}{4}\right)^2 = 1.56.$$

These increased exposures are less than the minimal adjustment on a typical camera, so they are negligible.

69. The focal length of the eyepiece is

$$f_e = \frac{1}{P_e} = \frac{1}{23\text{D}} = 4.3 \times 10^{-2}\text{ m} = 4.3\text{ cm}.$$

For both object and image far away, we find the focal length of the objective from the separation of the lenses:

$$L = f_o + f_e;$$

$$85\text{ cm} = f_o + 4.3\text{ cm}, \text{ which gives } f_o = 80.7\text{ cm}.$$

The magnification of the telescope is given by

$$M = -\frac{f_o}{f_e} = -\frac{(80.7\text{ cm})}{(4.3\text{ cm})} = \boxed{-19\times.}$$

70. The distance from the Earth to the Moon is 384×10^6 m.

$$s = L\theta = \frac{L(1.22\lambda)}{D} = \frac{(384 \times 10^6\text{ m})(1.22)(550 \times 10^{-9}\text{ m})}{(2.4\text{ m})} = \boxed{107\text{ m.}}$$

71. (*a*) The distance between the two lenses is $f_o + f_e = 4.0\text{ cm} + 44\text{ cm} = \boxed{48\text{ cm.}}$

The magnification is

$$M = -\frac{f_o}{f_e} = -\frac{(44\text{ cm})}{(4.0\text{ cm})} = \boxed{-11\times.}$$

The eyepiece is the lens with $f = \boxed{4.0\text{ cm.}}$

(*b*) The magnification is

$$M = -\frac{Nl}{f_e f_o} = \frac{(25\text{ cm})l}{(44\text{ cm})(4.0\text{ cm})} = 25.$$

Solve for l to get $l = \boxed{180\text{ cm.}}$

72. $M=-\dfrac{f_\text{o}}{f_\text{e}}=-8.0.$ Therefore, $f_\text{o}=8.0f_\text{e}.$

Since $f_\text{o}+f_\text{e}=28\,\text{cm},\ 8.0f_\text{e}+f_\text{e}=9.0f_\text{e}=28\,\text{cm}.$

Solving, $f_\text{e}=\dfrac{28\,\text{cm}}{9.0}=\boxed{3.1\,\text{cm}}$ and $f_\text{e}=\boxed{24.9\,\text{cm.}}$

73. The angular resolution must equal

$$\theta=\frac{s}{L}=\frac{(0.05\,\text{m})}{(130\times10^{3}\ \text{m})}=3.85\times10^{-7}.$$

Therefore,

$$\theta=\frac{1.22\,\lambda}{D}=\frac{(1.22)(550\times10^{-9}\ \text{m})}{D}=3.8\times10^{-7}.$$

Solve for D to get $D=\boxed{1.7\,\text{m.}}$

74. We find the focal lengths of the lens for the two colors:

$$\frac{1}{f_\text{red}}=(n_\text{red}-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$=(1.5106-1)\left[\left(\frac{1}{18.4\,\text{cm}}\right)+\left(\frac{1}{\infty}\right)\right]\text{ which gives } f_\text{red}=36.04\,\text{cm}.$$

$$\frac{1}{f_\text{orange}}=(n_\text{orange}-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$=(1.5226-1)\left[\left(\frac{1}{18.4\,\text{cm}}\right)+\left(\frac{1}{\infty}\right)\right]\text{ which gives } f_\text{orange}=35.21\,\text{cm}.$$

We find the image distances from

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_\text{ired}}\right)=\frac{1}{f_\text{red}};$$

$$\left(\frac{1}{66.0\,\text{cm}}\right)+\left(\frac{1}{\text{d}_\text{ired}}\right)=\frac{1}{36.04}\text{cm, which gives } d_\text{ired}=\boxed{79.4\,\text{cm.}}$$

$$\left(\frac{1}{d_\text{o}}\right)+\left(\frac{1}{d_\text{iorange}}\right)=\frac{1}{f_\text{orange}};$$

$$\left(\frac{1}{66.0\,\text{cm}}\right)+\left(\frac{1}{d_\text{iorange}}\right)=\frac{1}{35.21\,\text{cm}},\text{ which gives } d_\text{iorange}=\boxed{75.5\,\text{cm.}}$$

The images are 3.9 cm apart, an example of chromatic aberration.

CHAPTER 26: The Special Theory of Relativity

Answers to Questions

1. No. Since the windowless car in an exceptionally smooth train moving at a constant velocity is an inertial reference frame and all of the laws of physics are the same in any inertial frame, there is no way for you to tell if you are moving or not.

2. There is no way for you to tell the difference between absolute and relative motion. You always think that you are at rest and that other things around you are moving, but you really can't tell if maybe you are moving and everything else is at rest.

3. The ball will land on the roof of the railroad car (ignoring air resistance and assuming that the velocity of the train is constant). Both the ball and the car are already moving forward, so when the ball is thrown straight up into the air with respect to the car, it will continue to move forward at the same rate as the car and fall back down to land on the roof.

4. Whether you say the Earth goes around the Sun or the Sun goes around the Earth depends on your reference frame. It is valid to say either one, depending on which frame you choose. The laws of physics, though, won't be the same in each of these reference frames, since the Earth is accelerating as it goes around the Sun. The Sun is nearly an inertial reference frame, but the Earth is not.

$\boxed{5.}$ If you were in a spaceship traveling at $0.5c$ away from a star, its starlight would pass you at a speed of c. The speed of light is a constant in any reference frame, according to the 2nd postulate of special relativity.

6. The clocks are not at fault and they are functioning properly. Time itself is actually measured to pass more slowly in moving reference frames when compared to a rest frame.

7. Time actually passes more slowly in the moving reference frame. It is not just that it seems this way, it has actually been measured to pass more slowly, as predicted by special relativity.

8. If the young-looking astronaut has been traveling at extremely high speeds (relative to Earth) for many Earth years, since her clock will run slow compared to the clocks at rest on Earth, it is possible that her son could now be much older than she is after her trip. This is a twist on the twin paradox.

$\boxed{9.}$ You would not notice a change in your own heartbeat, mass, height or waistline. No matter how fast you are moving relative to Earth, you are at rest in your own reference frame. Thus, you would not notice any changes in your own characteristics. To observers on Earth, you are moving away at $0.5c$, which gives $\gamma = 0.87$. To these observers, it would appear that your heartbeat has slowed by a factor of 0.87, that your mass has increased by a factor of $1/0.87 = 1.15$, and that your waistline has decreased by a factor of 0.87 (all due to the relativity equations for time dilation, mass increase, and length contraction), but that your height would be unchanged (since there is no relative motion between you and Earth in that direction).

10. Time dilation and length contraction do occur at 90 km/h, but the effects are so small that they are not measurable.

11. If the speed of light was infinite, then time dilation and length contraction would not occur. The factor found in these equations, $\sqrt{1-v^2/c^2}$, would always be equal to zero at any speed v.

12. If the speed of light was 25 m/s, then we would see relativistic effects all the time, something like the opening Chapter figure or Figure 26-11 with question 14. Everything moving would look length contracted and time dilation and mass increase would have to be taken into account in all experiments. One of the most unusual changes for today's modern inhabitants of Earth would be that nothing would be able to move faster than 25 m/s, which is only about 56 mi/h.

13. As *v* gets very close to *c*, an outside observer should be able to show that $L = L_o\sqrt{1 - v^2/c^2}$ is getting smaller and smaller and that the limit as $v \to c$ is that $L \to 0$. This would show that *c* is a limiting speed, since nothing can get smaller than having a length of 0. Also, as *v* gets very close to *c*, an outside observer should be able to show that $\Delta t = \frac{\Delta t_o}{\sqrt{1 - v^2/c^2}}$ is getting longer and longer and that the limit as $v \to c$ is that $\Delta t \to \infty$. This would show that *c* is a limiting speed, since the slowest that time can pass is that it comes to a stop.

14. Mr. Tompkins and his bike would look very thin to the people standing on the sidewalk if *c* = 20 mi/h, due to the extreme length contraction that would occur. He would be the same height, though, since there is no relative motion in that direction between Mr. Tompkins and the people on the sidewalk.

15. No, the speed limit of *c* for an electron does not put an upper limit on the electron's momentum. As $v \to c$ in $p = \frac{m_o v}{\sqrt{1 - v^2/c^2}}$ then $p \to \infty$, which can be attributed to the fact that $m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$ goes to ∞ as $v \to c$ (the relativistic mass increase).

16. No, it is not possible for a non-zero rest mass particle to attain a speed of *c*. This is because its relativistic mass would increase to ∞ as $v \to c$, which means it would take an infinite amount of energy to increase the speed of this infinite mass particle to *c*.

17. No, $E = mc^2$ does not conflict with the conservation of energy, it actually completes it. Since this equation shows us that mass and energy are interconvertible, it says it is now necessary to include mass as a form of energy in the analysis of physical processes.

18. Yes. One way to describe the energy stored in the compressed spring is to say it is a mass increase (although it would be so small that it could not be measured). This mass will convert back to energy when the spring is uncompressed.

$\boxed{19.}$ Matter and energy are interconvertible (matter can be converted into energy and energy can be converted into matter), thus we should say "energy can neither be created nor destroyed."

20. No, our intuitive notion that velocities simply add is not completely wrong. Our intuition is based on our everyday experiences and at these everyday speeds our intuition is correct regarding how velocities add. Our intuition does break down, though, at very high speeds, where we have to take into account relativistic effects. Relativity does not contradict classical mechanics, but it is a more general theory where classical mechanics is a limiting case.

Solutions to Problems

$\boxed{1.}$ You measure the contracted length. We find the rest length from

$$L = L_0\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}};$$

$$28.2\,\text{m} = L_0\left[1 - (0.750)^2\right]^{\frac{1}{2}}, \text{ which gives } L_0 = \boxed{42.6\,\text{m.}}$$

2. We find the lifetime at rest from

$$\Delta t = \frac{\Delta t_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$4.76 \times 10^{-6}\,\text{s} = \frac{\Delta t_0}{\left\{1 - \left[\frac{(2.70 \times 10^8\,\text{m/s})}{(3.00 \times 10^8\,\text{m/s})}\right]^2\right\}^{\frac{1}{2}}}, \text{ which gives } \Delta t_0 = \boxed{2.07 \times 10^{-6}\,\text{s.}}$$

3. (*a*) $$\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left\{1 - \left[\frac{(20{,}000\,\text{m/s})}{(3.00 \times 10^8\,\text{m/s})}\right]^2\right\}^{\frac{1}{2}} = \boxed{1.00.}$$

(*b*) $$\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1 - (0.020)^2\right]^{\frac{1}{2}} = \boxed{0.9998.}$$

(*c*) $$\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1 - (0.200)^2\right]^{\frac{1}{2}} = \boxed{0.980.}$$

(*d*) $$\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1 - (0.95)^2\right]^{\frac{1}{2}} = \boxed{0.312.}$$

(*e*) $$\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1 - (0.98)^2\right]^{\frac{1}{2}} = \boxed{0.199.}$$

(*f*) $$\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = \left[1 - (0.999)^2\right]^{\frac{1}{2}} = \boxed{0.0447.}$$

4. You measure the contracted length:

$$L = L_0\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}$$

$$=(125\,\text{ly})\left\{1-\left[\frac{(2.50\times10^{8}\ \text{m/s})}{(3.00\times10^{8}\ \text{m/s})}\right]^{2}\right\}^{\frac{1}{2}}=\boxed{55.3\,\text{ly.}}$$

5. We determine the speed from the time dilation:

$$\Delta t=\frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$4.10\times10^{-8}\ \text{s}=\frac{(2.60\times10^{-8}\ \text{s})}{\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}}},\ \text{which gives } v=\boxed{0.773c.}$$

6. We determine the speed from the length contraction:

$$L=L_0\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}};$$

$$35\,\text{ly}=(82\,\text{ly})\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}},\ \text{which gives } v=\boxed{0.90c.}$$

7. We determine the speed from the length contraction:

$$L=L_0\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}};$$

$$25\,\text{ly}=(85\,\text{ly})\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}},\ \text{which gives } v=0.956c.$$

We then find the time from the speed and distance:

$$t=\frac{d}{v}=\frac{(25\,\text{y})c}{0.956c}=\boxed{26\,\text{y.}}$$

8. We determine the speed from the length contraction:

$$L=L_0\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}};$$

$$0.900\,\text{m}=(1.00\,\text{m})\left[1-\left(\frac{v}{c}\right)^{2}\right]^{\frac{1}{2}},\ \text{which gives } v=\boxed{0.436c.}$$

9. We convert the speed: $\frac{(40{,}000\,\text{km/h})}{(3.6\,\text{ks/h})}=1.11\times10^{4}\ \text{m/s}$.

The length contraction is given by

$$L = L_0\sqrt{1-\left(\frac{v}{c}\right)^2};$$

$$1-\frac{L}{L_0} = 1-\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$= 1-\sqrt{1-\left[\frac{(1.11\times10^4\ \text{m/s})}{(3.00\times10^8\ \text{m/s})}\right]^2}$$

$$= 7\times10^{-10}\ \text{or}\ \boxed{7\times10^{-8}\ \%.}$$

10. For a 1.00 percent change, the factor in the expressions for time dilation and length contraction must equal $1 - 0.0100 = 0.9900$:

$$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = 0.9900,\ \text{which gives}\ v = \boxed{0.141c.}$$

$\boxed{11.}$ In the Earth frame, the clock on the *Enterprise* will run slower.

(*a*) We find the elapsed time on the ship from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$5.0\,\text{yr} = \frac{\Delta t_0}{\left[1-(0.84)^2\right]^{\frac{1}{2}}},\ \text{which gives}\ \Delta t_0 = \boxed{2.7\,\text{yr.}}$$

(*b*) We find the elapsed time on the Earth from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{(5.0\,\text{yr})}{\left[1-(0.84)^2\right]^{\frac{1}{2}}} = \boxed{9.2\,\text{yr.}}$$

12. (*a*) To an observer on Earth, 10.6 ly is the rest length, so the time will be

$$t_{\text{Earth}} = \frac{L_0}{v} = \frac{(10.6\,\text{ly})}{0.960c} = \boxed{11.0\,\text{yr.}}$$

(*b*) We find the dilated time on the spacecraft from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$11.0\,\text{yr} = \frac{\Delta t_0}{\left[1-(0.960)^2\right]^{\frac{1}{2}}}, \text{ which gives } \Delta t_0 = \boxed{3.09\,\text{yr.}}$$

(*c*) To the spacecraft observer, the distance to the star is contracted:

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (10.6\,\text{ly})\left[1-(0.960)^2\right]^{\frac{1}{2}} = \boxed{2.97\,\text{ly.}}$$

(*d*) To the spacecraft observer, the speed of the spacecraft is

$$v = \frac{L}{\Delta t} = \frac{(2.97\,\text{ly})}{3.09\,\text{yr}} = \boxed{0.960c,} \text{ as expected.}$$

13. (*a*) You measure the contracted length. We find the rest length from

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}};$$

$$4.80\,\text{m} = L_0\left[1-(0.660)^2\right]^{\frac{1}{2}}, \text{ which gives } L_0 = \boxed{6.39\,\text{m.}}$$

Distances perpendicular to the motion do not change, so the rest height is $\boxed{1.25\text{ m.}}$

(*b*) We find the dilated time in the spacecraft from

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$20.0\,\text{s} = \frac{\Delta t_0}{\left[1-(0.660)^2\right]^{\frac{1}{2}}}, \text{ which gives } \Delta t_0 = \boxed{15.0\,\text{s.}}$$

(*c*) To your friend, you moved at the same relative speed: $\boxed{0.660c.}$

(*d*) She would measure the same time dilation: $\boxed{15.0\text{ s.}}$

14. In the Earth frame, the average lifetime of the pion will be dilated:

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}.$$

The speed as a fraction of the speed of light is

$$\frac{v}{c} = \frac{d}{c}\Delta t = \frac{d\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}{c}\Delta t_0;$$

$$\frac{v}{c} = \frac{(15\,\text{m})\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}{(3.00\times10^8\,\text{m/s})(2.60\times10^{-8}\,\text{s})},$$

which gives $v = \boxed{0.887c = 2.66\times10^8\,\text{m/s.}}$

$\boxed{15.}$ The momentum of the proton is

$$p=\frac{m_0v}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}=\frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(0.85\right)\left(3.00\times10^8\text{ m/s}\right)}{\left[1-\left(0.85\right)^2\right]^{\frac{1}{2}}}=\boxed{8.1\times10^{-19}\text{ kg.}}$$

16. We find the speed from

$$m=\frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$2m_0=\frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\text{ which gives }v=\boxed{0.866c.}$$

17. The initial momentum of the particle is

$$p=\frac{m_0v}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}};$$

$$=\frac{m_0\left(0.20c\right)}{\left[1-\left(0.20\right)^2\right]^{\frac{1}{2}}}.$$

We now find v which doubles the momentum:

$$\frac{m_0v}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}=2\left(\frac{m_0\left(0.20c\right)}{\left[1-\left(0.20\right)^2\right]^{\frac{1}{2}}}\right);$$

$$\frac{v^2}{1-\frac{v^2}{c^2}}=\frac{c^2}{6},\text{ which gives }v=\boxed{0.38c.}$$

18. (*a*) The momentum of the particle is

$$p=\frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$=\frac{m_0\left(0.10c\right)}{\sqrt{1-\left(0.10\right)^2}}=0.10050cm_0.$$

The classical formula gives $p=m_0v=0.10cm_0$.

The error is

$$\frac{0.10cm_0-0.10050cm_0}{0.10050cm_0}=-0.0050=\boxed{-0.50\%.}$$

(*b*) The momentum of the particle is

$$p = \frac{m_0(0.50c)}{\sqrt{1-(0.50)^2}} = 0.57735cm_0.$$

The classical formula gives $0.50cm_0$.

The error is

$$\frac{0.50cm_0 - 0.57735cm_0}{0.57735cm_0} = -0.13 = \boxed{-13\%.}$$

19. The proton's momenta are found using

$$p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}};$$

$$p_1 = \frac{m_0(0.45c)}{\sqrt{1-(0.45)^2}} = 0.5039cm_0;$$

$$p_2 = \frac{m_0(0.90c)}{\sqrt{1-(0.90)^2}} = 2.065cm_0;$$

$$p_3 = \frac{m_0(0.98c)}{\sqrt{1-(0.98)^2}} = 4.925cm_0.$$

(*a*) $\dfrac{p_2 - p_1}{p_1} = \dfrac{2.065cm_0 - 0.5039cm_0}{0.5039cm_0} = 3.10 = \boxed{310\%.}$

(*b*) $\dfrac{p_3 - p_2}{p_2} = \dfrac{4.925cm_0 - 2.065cm_0}{2.065cm_0} = 1.40 = \boxed{140\%.}$

20. We find the increase in mass from

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(4.82 \times 10^4\ \text{J})}{(3.00 \times 10^8\ \text{m/s})^2} = \boxed{5.36 \times 10^{-13}\ \text{kg.}}$$

Note that this is so small, most chemical reactions are considered to have mass conserved.

21. We find the loss in mass from

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(200\,\text{MeV})(1.60 \times 10^{-13}\ \text{J/MeV})}{(3.00 \times 10^8\ \text{m/s})^2} = \boxed{3.56 \times 10^{-28}\ \text{kg.}}$$

22. The rest energy of the electron is

$$E = m_0c^2 = (9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 = \boxed{8.20 \times 10^{-14}\ \text{J}}$$

$$= \frac{(8.20 \times 10^{-14}\ \text{J})}{(1.60 \times 10^{-13}\ \text{J/MeV})} = \boxed{0.511\text{MeV.}}$$

23. The rest mass of the proton is

$$m_0 = \frac{E}{c^2} = \frac{\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(2.9979 \times 10^8\ \text{m/s}\right)^2}{\left(1.60 \times 10^{-13}\ \text{J/MeV}\right)c^2} = \frac{938\text{MeV}}{c^2}.$$

24. We find the necessary mass conversion from

$$\Delta m = \frac{\Delta E}{c^2} = \frac{\left(8 \times 10^{19}\ \text{J}\right)}{\left(3.00 \times 10^8\ \text{m/s}\right)^2} = \boxed{9 \times 10^2\ \text{kg.}}$$

25. We find the energy equivalent of the mass from

$$E = mc^2 = \left(1.0 \times 10^{-3}\ \text{kg}\right)\left(3.00 \times 10^8\ \text{m/s}\right)^2 = \boxed{9.0 \times 10^{13}\ \text{J.}}$$

If this energy increases the gravitational energy, we have

$$E = mgh;$$

$$9.0 \times 10^{13}\ \text{J} = m\left(9.80\,\text{m/s}^2\right)\left(0.25 \times 10^3\ \text{m}\right),\ \text{which gives}\ m = \boxed{3.7 \times 10^{10}\ \text{kg.}}$$

26. If the kinetic energy is equal to the rest energy, we have

$$\text{KE} = \left(m - m_0\right)c^2 = m_0c^2,\ \text{or}\ m = 2m_0.$$

We find the speed from

$$m = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$2m_0 = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\ \text{which gives}\ v = 0.866c.$$

27. If the kinetic energy is 25% of the rest energy, we have

$$\text{KE} = \left(m - m_0\right)c^2 = 0.25m_0c^2,\ \text{or}\ m = 1.25m_0.$$

We find the speed from

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}};$$

$$1.25m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}},\ \text{which gives}\ v = \boxed{0.60c.}$$

28. (*a*) We find the work required from

$$W = \Delta\text{KE} = (m - m_0)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right)$$

$$= (938\,\text{MeV})\left(\left\{\frac{1}{\left[1-(0.997)^2\right]^{\frac{1}{2}}}\right\} - 1\right) = 11.2\times10^3\,\text{MeV} = \boxed{11.2\,\text{GeV}\left(2.23\times10^{-9}\,\text{J}\right).}$$

(*b*) The momentum of the proton is

$$p = mv = \frac{m_0v}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}$$

$$= \frac{\left(1.67\times10^{-27}\,\text{kg}\right)(0.997)\left(3.00\times10^8\,\text{m/s}\right)}{\left[1-(0.997)^2\right]^{\frac{1}{2}}} = \boxed{6.46\times10^{-18}\,\text{kg}\bullet\text{m/s}.}$$

29. The speed of the proton is

$$v = \frac{\left(2.60\times10^8\,\text{m/s}\right)}{\left(3.00\times10^8\,\text{m/s}\right)} = 0.867c.$$

The kinetic energy is

$$\text{KE} = (m - m_0)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right)$$

$$= (938\,\text{MeV})\left(\left\{\frac{1}{\left[1-(0.867)^2\right]^{\frac{1}{2}}}\right\} - 1\right) = \boxed{942\,\text{MeV}\left(1.51\times10^{-10}\,\text{J}\right).}$$

The momentum of the proton is

$$p = mv = m_0v\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\}$$

$$= \left(1.67\times10^{-27}\text{ kg}\right)\left(2.60\times10^{8}\text{ m/s}\right)\left\{\frac{1}{\left[1-(0.867)^2\right]^{\frac{1}{2}}}\right\} = \boxed{8.70\times10^{-19}\text{ kg}\bullet\text{m/s}\left(\frac{1.63\times10^{3}\text{ MeV}}{c}\right).}$$

30. The total energy of the proton is

$$E = \text{KE} + m_0c^2 = 750\text{ MeV} + 938\text{ MeV} = 1688\text{ MeV}.$$

The relation between the momentum and energy is

$$(pc)^2 = E^2 - \left(m_0c^2\right)^2;$$

$$p^2\left(3.00\times10^{8}\text{ m/s}\right)^2 = \left[(1688\text{ MeV})^2 - (938\text{ MeV})^2\right]\left(1.60\times10^{-13}\text{ J/MeV}\right)^2,$$

which gives $p = \boxed{7.48\times10^{-19}\text{ kg}\bullet\text{m/s.}}$

31. The kinetic energy acquired by the proton is

$$\text{KE} = qV = (1\text{e})(105\text{ MV}) = 105\text{ MeV}.$$

The mass of the proton is

$$m = m_0 + \frac{\text{KE}}{c^2} = \frac{938\text{ MeV}}{c^2} + \frac{(105\text{ MeV})}{c^2} = \frac{1043\text{ MeV}}{c^2}.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$\frac{1043\text{MeV}}{c^2} = \frac{\frac{938\text{MeV}}{c^2}}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\text{ which gives } v = \boxed{0.437c.}$$

32. The mass of the electron is

$$m = m_0 + \frac{\text{KE}}{c^2} = \frac{0.511\text{MeV}}{c^2} + \frac{(1.00\text{ MeV})}{c^2} = \frac{1.51\text{MeV}}{c^2}.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$\frac{1.51\text{MeV}}{c^2} = \frac{\frac{0.511\text{MeV}}{c^2}}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}},\text{ which gives } v = \boxed{0.941c.}$$

33. The kinetic energy acquired by the electron is

$$\text{KE} = qV = (1\text{e})(0.025\,\text{MV}) = 0.025\,\text{MeV}.$$

The mass of the electron is

$$m = m_0 + \frac{\text{KE}}{c^2} = \frac{0.511\,\text{MeV}}{c^2} + \frac{(0.025\,\text{MeV})}{c^2} = \frac{0.536\,\text{MeV}}{c^2}.$$

We find the speed from

$$m = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$\frac{0.536\,\text{MeV}}{c^2} = \frac{\left(\frac{0.511\,\text{MeV}}{c^2}\right)}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}, \text{ which gives } v = \boxed{0.30c.}$$

34. If M_0 is the rest mass of the new particle, for conservation of energy we have

$$2mc^2 = \frac{2m_0c^2}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} = M_0c^2, \text{ which gives } M_0 = \boxed{\frac{2m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}}.$$

Because energy is conserved, there was $\boxed{\text{no loss.}}$

The final particle is at rest, so the kinetic energy loss is the initial kinetic energy of the two colliding particles:

$$\text{KE}_{\text{loss}} = 2(m - m_0)c^2 = \boxed{2m_0c^2\left(\left\{\frac{1}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}\right\} - 1\right)}.$$

$\boxed{35.}$ (*a*) The total energy of the proton is

$$E = mc^2 = \text{KE} + m_0c^2 = \tfrac{1}{2}mc^2 + m_0c^2, \text{ which gives } m = 2m_0.$$

We find the speed from

$$m = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$2m_0 = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}, \text{ which gives } v = \boxed{0.866c.}$$

(*b*) $E = \text{KE} + m_0c^2 = \tfrac{1}{2}m_0c^2 + m_0c^2$, which gives $m = \tfrac{3}{2}m_0$.

We find the speed from

$$\tfrac{3}{2}m_0 = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}, \text{ which gives } v = \boxed{0.745c.}$$

36. The total energy of the electron is

$$E = mc^2 = \text{KE} + m_0c^2 = m_0c^2 + m_0c^2 = 2m_0c^2, \text{ which gives } m = 2m_0.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$2m_0 = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}, \text{ which gives } \left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}} = \tfrac{1}{2}, \text{ so } v = \boxed{0.866c.}$$

The momentum of the electron is

$$p = mv = \frac{m_0 v}{\left[1-\left(\frac{v}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(0.866\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(\frac{1}{2}\right)} = \boxed{4.73\times10^{-22}\text{ kg•m/s.}}$$

37. (*a*) The kinetic energy is

$$\text{KE} = \left(m - m_0\right)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right)$$

$$= \left(27{,}000\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)^2\left(\left\{\frac{1}{\left[1-\left(0.21\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right) = \boxed{5.5\times10^{19}\text{ J.}}$$

(*b*) When we use the classical expression, we get

$$\text{KE}_\text{c} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(27{,}000\text{ kg}\right)\left[\left(0.21\right)\left(3.00\times10^{8}\text{ m/s}\right)\right]^2 = 5.36\times10^{19}\text{ J.}$$

The error is

$$\frac{\left(5.36-5.5\right)}{\left(5.5\right)} = -0.033 = \boxed{-3.3\%.}$$

38. The speed of the proton is

$$v = \frac{\left(7.35 \times 10^{7}\ \text{m/s}\right)}{\left(3.00 \times 10^{8}\ \text{m/s}\right)} = 0.245c.$$

The kinetic energy is

$$\text{KE} = \left(m - m_0\right)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right)$$

$$= \left(938\,\text{MeV}\right)\left(\left\{\frac{1}{\left[1 - \left(0.245\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right) = \boxed{29.5\,\text{MeV}\left(4.7 \times 10^{-12}\ \text{J}\right).}$$

The momentum of the proton is

$$p = mv = \frac{m_0 v}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}$$

$$= \left(1.67 \times 10^{-27}\ \text{kg}\right)\left(7.35 \times 10^{7}\ \text{m/s}\right)\left\{\frac{1}{\left[1 - \left(0.245\right)^2\right]^{\frac{1}{2}}}\right\} = \boxed{1.3 \times 10^{-19}\ \text{kg}\bullet\text{m/s.}}$$

From the classical expressions, we get

$\text{KE}_\text{c} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(7.35 \times 10^{7}\ \text{m/s}\right)^2 = 4.51 \times 10^{-12}\ \text{J},$ with an error of

$$\frac{\left(4.51 - 4.7\right)}{\left(4.7\right)} = -0.05 = \boxed{-5\%.}$$

$p = mv = \left(1.67 \times 10^{-27}\ \text{kg}\right)\left(7.35 \times 10^{7}\ \text{m/s}\right) = 1.2 \times 10^{-19}\ \text{kg}\bullet\text{m/s},$ with an error of

$$\frac{\left(1.2 - 1.3\right)}{\left(1.3\right)} = -0.03 = \boxed{-3\%.}$$

39. If we ignore the recoil of the neptunium nucleus, the increase in kinetic energy is the kinetic energy of the alpha particle;

$$\text{KE}_\alpha = \left[m_\text{Am} - \left(m_\text{Np} + m_\alpha\right)\right]c^2;$$

$$5.5\,\text{MeV} = \left[241.05682\,\text{u} - \left(m_\text{Np} + 4.00260\,\text{u}\right)\right]c^2\left(\frac{931.5\text{MeV}}{\text{u}c^2}\right),$$

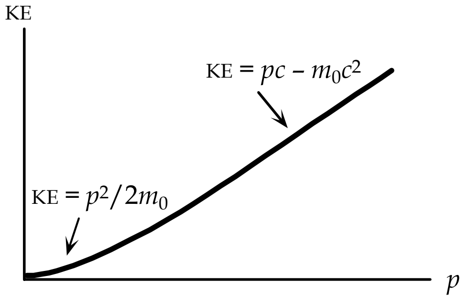
which gives $m_\text{Np} = \boxed{237.04832\,\text{u}.}$

$\boxed{40.}$ The increase in kinetic energy comes from the decrease in potential energy:

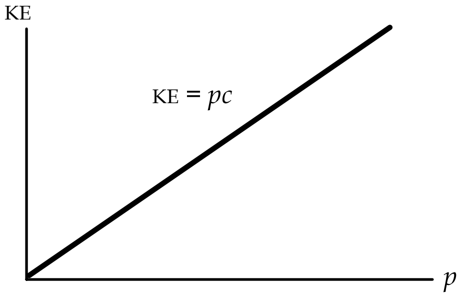
$$\text{KE} = (m - m_0)c^2 = m_0c^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right);$$

$$6.60 \times 10^{-14}\ \text{J} = (9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2\left(\left\{\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}\right\} - 1\right), \text{ which gives } v = \boxed{0.833c.}$$

41. (*a*)



(*b*)



42. The total energy of the proton is

$$E = mc^2 = \text{KE} + m_0c^2 = 998\,\text{GeV} + 0.938\,\text{GeV} = 999\,\text{GeV}, \text{ so the mass is } \frac{999\text{GeV}}{c^2}.$$

We find the speed from

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$\frac{999\text{GeV}}{c^2} = \frac{\left(\frac{0.938\text{GeV}}{c^2}\right)}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}, \text{ which gives } \left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}} = 9.39 \times 10^{-4}, \text{ so } v = 1.00c.$$

The magnetic force provides the radial acceleration:

$$qvB = \frac{mv^2}{r}, \text{ or}$$

$$B = \frac{mv}{qr} = \frac{m_0v}{qr\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$=\frac{\left(1.67\times10^{-27}\ \mathrm{kg}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{\left(1.6\times10^{-19}\ \mathrm{C}\right)\left(1.0\times10^{3}\ \mathrm{m}\right)\left(9.39\times10^{-4}\right)}=\boxed{3.3\,\mathrm{T.}}$$

Note that the mass is constant during the revolution.

$\boxed{43.}$ We find the speed in the frame of the Earth from

$$u=\frac{\left(v+u'\right)}{\left(1+\frac{vu'}{c^2}\right)}=\frac{\left(0.50c+0.50c\right)}{\left[1+\left(0.50\right)\left(0.50\right)\right]}=\boxed{0.80c.}$$

44. (*a*) In the reference frame of the second spaceship, the Earth is moving at 0.50*c*, and the first spaceship is moving at 0.50*c* relative to the Earth. Thus the speed of the first spaceship relative to the second is

$$u=\frac{\left(v+u'\right)}{\left(1+\frac{vu'}{c^2}\right)}=\frac{\left(0.50c+0.50c\right)}{\left[1+\left(0.50\right)\left(0.50\right)\right]}=\boxed{0.80c.}$$

(*b*) In the reference frame of the first spaceship, the Earth is moving at $-0.50c$, and the second spaceship is moving at $-0.50c$ relative to the Earth. Thus the speed of the second spaceship relative to the first is

$$u=\frac{\left(v+u'\right)}{\left(1+\frac{vu'}{c^2}\right)}=\frac{\left[-0.50c+\left(-0.50c\right)\right]}{\left[1+\left(-0.50\right)\left(-0.50\right)\right]}=\boxed{-0.80c,}\ \text{as expected.}$$

45. We take the positive direction in the direction of the first spaceship.

(*a*) In the reference frame of the Earth, the first spaceship is moving at $+0.71c$, and the second spaceship is moving at $+0.87c$ relative to the first. Thus the speed of the second spaceship relative to the Earth is

$$u=\frac{\left(v+u'\right)}{\left(1+\frac{vu'}{c^2}\right)}=\frac{\left(+0.71c+0.87c\right)}{\left[1+\left(0.71\right)\left(0.87\right)\right]}=\boxed{0.98c.}$$

(*b*) In the reference frame of the Earth, the first spaceship is moving at $+0.71c$, and the second spaceship is moving at $-0.87c$ relative to the first. Thus the speed of the second spaceship relative to the Earth is

$$u=\frac{\left(v+u'\right)}{\left(1+\frac{vu'}{c^2}\right)}=\frac{\left[+0.71c+\left(-0.87c\right)\right]}{\left[1+\left(0.71\right)\left(-0.87\right)\right]}=\boxed{-0.42c.}$$

46. We take the positive direction in the direction of the *Enterprise*. In the reference frame of the alien vessel, the Earth is moving at $-0.60c$, and the *Enterprise* is moving at $+0.90c$ relative to the Earth. Thus the speed of the *Enterprise* relative to the alien vessel is

$$u=\frac{\left(v+u'\right)}{\left(1+\frac{vu'}{c^2}\right)}=\frac{\left(-0.60c+0.90c\right)}{\left[1+\left(-0.60\right)\left(+0.90\right)\right]}=\boxed{0.65c.}$$

Note that the relative speed of the two vessels as seen on Earth is $0.90c-0.60c=0.30c$.

47. From the first pod's point of view, the spaceship and the second pod are moving in the same direction. Thus the speed of the second pod relative to the first is

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}} = \frac{0.60c + 0.70c}{1 + (0.60)(0.70)} = \boxed{0.92c.}$$

48. The speed of rocket B relative to Earth can be written as

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}},$$

where v is the speed of Rocket A relative to Earth and u' is the speed of Rocket B relative to Rocket A. With the given values, then,

$$0.95c = \frac{0.75c + u'}{1 + \frac{0.75u'}{c}}, \text{ which gives } u' = \boxed{0.70c.}$$

49. (*a*) To travelers on the spacecraft, the distance to the star is contracted:

$$L = L_0\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (4.3\,\text{ly})\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}.$$

Because the star is moving toward the spacecraft, to cover this distance in 4.0 yr, the speed of the star must be

$$v = \frac{L}{t} = \left(\frac{4.3\,\text{ly}}{4.0\,\text{yr}}\right)\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (1.075c)\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}, \text{ which gives } v = 0.73c.$$

Thus relative to the Earth-star system, the speed of the spacecraft is $\boxed{0.73c.}$

(*b*) According to observers on Earth, clocks on the spacecraft run slow:

$$t_{\text{Earth}} = \frac{t}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} = \frac{(4.0\,\text{yr})}{\left[1 - (0.73)^2\right]^{\frac{1}{2}}} = \boxed{5.9\,\text{yr.}}$$

Note that this agrees with the time found from distance and speed:

$$t_{\text{Earth}} = \frac{L_0}{v} = \frac{(4.3\,\text{ly})}{0.73c} = 5.9\,\text{yr}.$$

50. The electrostatic force provides the radial acceleration:

$$\frac{ke^2}{r^2} = \frac{mv^2}{r}.$$

Thus we find the speed from

$$v^2 = \frac{(9.0 \times 10^9\,\text{N•m}^2/\text{C}^2)(1.6 \times 10^{-19}\,\text{C})^2}{(9.11 \times 10^{-31}\,\text{kg})(0.5 \times 10^{-10}\,\text{m})},$$

which gives $v = 2 \times 10^6\,\text{m/s}$.

Because this is less than $0.1c$, the electron is $\boxed{\text{not relativistic.}}$

51. (*a*) We find the speed from

$$m = \frac{m_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$14{,}000 m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2, \text{ which gives}$$

$$\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}} = 7.14 \times 10^{-5}, \text{ or } \left(\frac{v}{c}\right)^2 = 1 - 5.10 \times 10^{-9}.$$

When we take the square root, we get

$$\frac{v}{c} = \left(1 - 5.10 \times 10^{-9}\right)^{\frac{1}{2}} \approx 1 - \tfrac{1}{2}\left(5.10 \times 10^{-9}\right) = 1 - 2.55 \times 10^{-9}.$$

Thus the speed is $\boxed{0.77\,\text{m/s less than } c.}$

(*b*) The contracted length of the tube is

$$L = L_0 \left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (3.0\,\text{km})\left(7.14 \times 10^{-5}\right) = 2.1 \times 10^{-4}\,\text{km} = \boxed{21\,\text{cm.}}$$

52. We find the mass change from the required energy:

$$E = Pt = m_0 c^2;$$

$$(100\,\text{W})\left(3.16 \times 10^7\,\text{s}\right) = m_0\left(3.00 \times 10^8\,\text{m/s}\right)^2, \text{ which gives } m_0 = \boxed{3.5 \times 10^{-8}\,\text{kg.}}$$

53. The minimum energy is required to produce the pair at rest:

$$E = 2m_0 c^2 = 2(0.511\,\text{MeV}) = \boxed{1.02\,\text{MeV}\left(1.64 \times 10^{13}\,\text{J}\right).}$$

54. The magnetic force provides the radial acceleration:

$$qvB = \frac{mv^2}{r}, \text{ or}$$

$$r = \frac{mv}{qB} = \frac{m_0 v}{qB\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(9.11 \times 10^{-31}\,\text{kg}\right)(0.92)\left(3.00 \times 10^8\,\text{m/s}\right)}{\left(1.6 \times 10^{-19}\,\text{C}\right)(1.8\,\text{T})\left[1 - (0.92)^2\right]^{\frac{1}{2}}}$$

$$= 2.2 \times 10^{-3}\,\text{m} = \boxed{2.2\,\text{mm.}}$$

55. Because the total energy of the muons becomes electromagnetic energy, we have

$$E = m_1c^2 + m_2c^2 = \frac{m_0}{\left[1-\left(\frac{v_1^2}{c^2}\right)\right]^{\frac{1}{2}}} + \frac{m_0}{\left[1-\left(\frac{v_2^2}{c^2}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(\frac{105.7\text{MeV}}{c^2}\right)(c^2)}{\left[1-(0.33)^2\right]^{\frac{1}{2}}} + \frac{\left(\frac{105.7\text{MeV}}{c^2}\right)(c^2)}{\left[1-(0.50)^2\right]^{\frac{1}{2}}} = \boxed{234\,\text{MeV.}}$$

56. The kinetic energy comes from the decrease in rest mass:

$$\text{KE} = \left[m_\text{n} - \left(m_\text{p} + m_e + m_\nu\right)\right]c^2$$

$$= \left[1.008665\,\text{u} - \left(1.00728\,\text{u} + 0.000549\,\text{u} + 0\right)\right]c^2\left(\frac{931.5\text{MeV}}{\text{u}c^2}\right) = \boxed{0.78\,\text{MeV.}}$$

57. (*a*) We find the rate of mass loss from

$$\frac{\Delta m}{\Delta t} = \frac{\left(\frac{\Delta E}{\Delta t}\right)}{c^2}$$

$$= \frac{\left(4\times10^{26}\text{ W}\right)}{\left(3\times10^8\text{ m/s}\right)^2} = \boxed{4.4\times10^9\text{ kg/s.}}$$

(*b*) We find the time from

$$\Delta t = \frac{\Delta m}{\text{rate}} = \frac{\left(5.98\times10^{24}\text{ kg}\right)}{\left(4.4\times10^9\text{ kg/s}\right)\left(3.16\times10^7\text{ s/yr}\right)} = \boxed{4.3\times10^7\text{ yr.}}$$

(*c*) We find the time for the Sun to lose all of its mass at this rate from

$$\Delta t = \frac{\Delta m}{\text{rate}} = \frac{\left(2.0\times10^{30}\text{ kg}\right)}{\left(4.4\times10^9\text{ kg/s}\right)\left(3.16\times10^7\text{ s/yr}\right)} = \boxed{1.4\times10^{13}\text{ yr.}}$$

58. The speed of the particle is

$$v = \frac{\left(2.24\times10^7\text{ m/s}\right)}{\left(3.00\times10^8\text{ m/s}\right)} = 0.747c.$$

We use the momentum to find the rest mass:

$$p = mv = \frac{m_0 v}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}};$$

$$3.0\times10^{-22}\,\text{kg•m/s} = \frac{m_0\left(0.747\right)\left(3.00\times10^8\text{ m/s}\right)}{\left[1-(0.747)^2\right]^{\frac{1}{2}}},$$

which gives $m_0 = 9.11\times10^{-31}$ kg.

Because the particle has a negative charge, it is $\boxed{\text{an electron.}}$

59. The binding energy is the energy required to provide the increase in rest mass:

$$\mathrm{KE} = \left[\left(2m_{\mathrm{p}} + 2m_{e}\right) - m_{\mathrm{He}}\right]c^{2}$$

$$= \left[2(1.00783\,\mathrm{u}) + 2(1.00867\,\mathrm{u}) - 4.00260\,\mathrm{u}\right]c^{2}\left(\frac{931.5\,\mathrm{MeV}}{\mathrm{u}c^{2}}\right) = \boxed{28.3\,\mathrm{MeV.}}$$

60. We convert the speed: $\dfrac{(110\,\mathrm{km/h})}{(3.6\,\mathrm{ks/h})} = 30.6\,\mathrm{m/s}.$

Because this is much smaller than c, the mass of the car is

$$m = \frac{m_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} \approx m_0\left[1 + \tfrac{1}{2}\left(\frac{v}{c}\right)^2\right].$$

The fractional change in mass is

$$\frac{(m - m_0)}{m_0} = \left[1 + \tfrac{1}{2}\left(\frac{v}{c}\right)^2\right] - 1 = \tfrac{1}{2}\left(\frac{v}{c}\right)^2$$

$$= \tfrac{1}{2}\left[\frac{(30.6\,\mathrm{m/s})}{(3.00\times10^{8}\,\mathrm{m/s})}\right]^2 = 5.19\times10^{-15} = \boxed{5.19\times10^{-13}\%.}$$

61. (*a*) The magnitudes of the momenta are equal:

$$p = mv = \frac{m_0 v}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}$$

$$= \frac{(1.67\times10^{-27}\,\mathrm{kg})(0.935)(3.00\times10^{8}\,\mathrm{m/s})}{\left[1-(0.935)^2\right]^{\frac{1}{2}}} = \boxed{1.32\times10^{-18}\,\mathrm{kg{\bullet}m/s.}}$$

(*b*) Because the protons are moving in opposite directions, the sum of the momenta is $\boxed{0.}$

(*c*) In the reference frame of one proton, the laboratory is moving at $0.935c$. The other proton is moving at $+0.935c$ relative to the laboratory. Thus the speed of the other proton relative to the first is

$$u = \frac{(v + u')}{\left(1 + \frac{vu'}{c^2}\right)} = \frac{\left[+0.935c + (+0.935c)\right]}{\left[1 + (+0.935)(+0.935)\right]} = 0.998c.$$

The magnitude of the momentum of the other proton is

$$p = mv = \frac{m_0 v}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}$$

$$= \frac{(1.67\times10^{-27}\,\mathrm{kg})(0.998)(3.00\times10^{8}\,\mathrm{m/s})}{\left[1-(0.998)^2\right]^{\frac{1}{2}}} = \boxed{7.45\times10^{-18}\,\mathrm{kg{\bullet}m/s.}}$$

62. The relation between energy and momentum is

$$E=\left(m_0{}^2c^4+p^2c^2\right)^{\frac{1}{2}}=c\left(m_0{}^2c^2+p^2\right)^{\frac{1}{2}}.$$

For the momentum, we have

$$p=mv=\frac{Ev}{c^2},\text{ or}$$

$$v=\frac{pc^2}{E}=\frac{pc}{\left(m_0{}^2c^2+p^2\right)^{\frac{1}{2}}}.$$

63. The kinetic energy that must be supplied is

$$\text{KE}=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}-m_0c^2$$

$$=\frac{m_0c^2}{\sqrt{1-(0.10)^2}}-m_0c^2=0.00504m_0c^2.$$

This is the amount of energy that would be liberated by converting $0.00504m_0$ of mass into energy, or

$$0.00504\left(5\times10^9\text{ kg}\right)=\boxed{2.5\times10^7\text{ kg.}}$$

64. (*a*) If the kinetic energy is 5.00 times as great as the rest energy, we have

$$\text{KE}=\left(m-m_0\right)c^2=5.00m_0c^2,\text{ or }m=6.00m_0.$$

We find the speed from

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}};$$

$$6.00m_0=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},\text{ which gives }v=\boxed{0.986c.}$$

(*b*) If the kinetic energy is 999 times as great as the rest energy,

$$\text{KE}=\left(m-m_0\right)c^2=999m_0c^2,\text{ or }m=1000m_0.$$

Then

$$1000m_0=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\text{ gives }v=\boxed{\left(1-5\times10^{-7}\right)c.}$$

65. To an observer in the barn reference frame, if the boy runs fast enough, the measured contracted length will be less than 12.0 m, so the observer can say that the two ends of the pole were simultaneously inside the barn. We find the necessary speed from

$$L=L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}};$$

$$12.0\,\text{m} = (15.0\,\text{m})\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}, \text{ which gives } v = 0.60c.$$

To the boy, the barn is moving and thus the length of the barn as he would measure it is less than the length of the pole:

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (12.0\,\text{m})\left[1-(0.60)^2\right]^{\frac{1}{2}} = 7.68\,\text{m}.$$

However, simultaneity is relative. Thus when the two ends are simultaneously inside the barn to the barn observer, those two events are not simultaneous to the boy. Thus he would claim that the observer in the barn determined that the ends of the pole were inside the barn at different times, which is also what the boy would say. It is $\boxed{\text{not possible}}$ in the boy's frame to have both ends of the pole inside the barn simultaneously.

66. We find the loss in mass from

$$\Delta m = \frac{\Delta E}{c^2} = \frac{484\times10^3\ \text{J}}{(3.00\times10^8\ \text{m/s})^2} = \boxed{5.38\times10^{-12}\ \text{kg.}}$$

One mole of hydrogen weighs $1\,\text{g} = 0.001$ kg.

One mole of oxygen weighs $16\,\text{g} = 0.016$ kg.

So the total mass of the reactants is 0.018 kg, and Δm as a percentage of that is

$$\frac{5.38\times10^{-12}\ \text{kg}}{0.018\ \text{kg}} = 3.0\times10^{-10} = \boxed{3.0\times10^{-8}\,\%.}$$

67. From one particle's point of view, the laboratory and the other particle are moving in the same direction. Thus the speed of one particle relative to the other is

$$u = \frac{(v+u')}{\left(1+\frac{vu'}{c^2}\right)} = \frac{(0.75c+0.75c)}{\left[1+(0.75)(0.75)\right]} = \boxed{0.96c.}$$

68. (*a*) Earth observers see the ship as contracted, as given by the equation

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = (25\,\text{m})\sqrt{1-(0.75)^2} = \boxed{17\,\text{m.}}$$

(*b*) Earth observers see the launch as dilated (lengthened) in time, as given by the equation

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{23\,\text{min}}{\sqrt{1-(0.75)^2}} = \boxed{35\,\text{min.}}$$

69. Every observer will measure the speed of a beam of light to be $\boxed{c.}$

Check: In the reference frame of the Earth, you are moving at $+0.85c$ and the beam is traveling at $-c$ relative to you. Thus the beam's speed relative to Earth is

$$u = \frac{v+u'}{1+vu'} = \frac{0.85c+(-c)}{1+(0.85)(-1)} = -c.$$

The beam's speed as a positive value, relative to Earth, is c.

70. The kinetic energy that must be supplied is

$$\mathrm{KE} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$

$$= \frac{m_0c^2}{\sqrt{1-(0.60)^2}} - m_0c^2 = 0.25m_0c^2.$$

This is the amount that would be liberated by converting $0.25m_0$ of mass into energy, or

$$0.25(150{,}000\,\mathrm{kg}) = \boxed{37{,}500\,\mathrm{kg.}}$$

From Earth's point of view the trip will take

$$\Delta t = \frac{d}{v} = \frac{(25\,\mathrm{y})c}{0.60c} = 41.7\,\mathrm{y}.$$

According to the astronauts, it will take less time:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}};$$

$$41.7\,\mathrm{y} = \frac{\Delta t_0}{\sqrt{1-(0.60)^2}},\ \text{which gives}\ \Delta t_0 = \boxed{33\,\mathrm{y.}}$$

71. The kinetic energy is

$$\mathrm{KE} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$

$$= \frac{(12{,}500\,\mathrm{kg})(3.00\times10^8\,\mathrm{m/s})^2}{\sqrt{1-(0.99)^2}} - (12{,}500\,\mathrm{kg})(3.00\times10^8\,\mathrm{m/s})^2 = \boxed{6.8\times10^{21}\,\mathrm{J.}}$$

We compare this with annual U.S. energy consumption:

$$\frac{6.8\times10^{21}\,\mathrm{J}}{10^{20}\,\mathrm{J}} = 68.$$

The spaceship's kinetic energy is about 68 times as great.

72. The passengers will see the trip distance shrunken as given by

$$L = L_0\sqrt{1-\frac{v^2}{c^2}},$$

and they will measure their speed to be

$$v = \frac{L}{\Delta t_0},\ \text{which means that}$$

$$v = \frac{L_0}{\Delta t_0}\sqrt{1-\frac{v^2}{c^2}}$$

$$= \frac{(6.0\,\mathrm{y})c}{1.0c}\sqrt{1-\frac{v^2}{c^2}},\ \text{which gives}\ v = 0.986c.$$

The work required to supply the necessary kinetic energy is

$$\text{KE} = \frac{m_0c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}} - m_0c^2$$

$$= \frac{(42{,}000\,\text{kg})\left(3.00 \times 10^{-8}\ \text{m/s}\right)^2}{\sqrt{1 - (0.986)^2}} - (42{,}000\,\text{kg})\left(3.00 \times 10^{8}\ \text{m/s}\right) = \boxed{1.83 \times 10^{22}\ \text{J.}}$$

73. (*a*) Because the spring is at rest on the spaceship, its period is

$$T = 2\pi\left(\frac{m}{k}\right)^{\frac{1}{2}} = 2\pi\left[\frac{(1.68\,\text{kg})}{(48.7\,\text{N/m})}\right]^{\frac{1}{2}} = \boxed{1.17\,\text{s.}}$$

(*b*) The oscillating mass is a clock. According to observers on Earth, clocks on the spacecraft run slow:

$$T_{\text{Earth}} = \frac{T}{\left[1 - \left(\dfrac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} = \frac{(1.17\,\text{s})}{\left[1 - (0.900)^2\right]^{\frac{1}{2}}} = \boxed{2.68\ \text{s.}}$$

74. The neutrino has no rest mass, so we have

$$E_v = \left(p_v{}^2c^2 + m_v{}^2c^4\right)^{\frac{1}{2}} = p_vc.$$

Because the pi meson decays at rest, momentum conservation tells us that the muon and neutrino have equal and opposite momenta:

$$p_\mu = p_v = p.$$

For energy conservation, we have

$$E_\pi = E_\mu + E_v;$$

$$m_\pi c^2 = \left(p_\mu{}^2c^2 + m_\mu{}^2c^4\right)^{\frac{1}{2}} + p_vc = \left(p^2c^2 + m_\mu{}^2c^4\right)^{\frac{1}{2}} + pc.$$

If we rearrange and square, we get

$$\left(m_\pi c^2 - pc\right)^2 = m_\pi{}^2c^4 - 2m_\pi c^2pc + p^2c^2 = p^2c^2 + m_\mu{}^2c^4, \text{ or } pc = \frac{\left(m_\pi{}^2c^2 - m_\mu{}^2c^2\right)}{2m_\pi}.$$

The kinetic energy of the muon is

$$\text{KE}_\mu = E_\mu - m_\mu c^2 = \left(m_\pi c^2 - pc\right) - m_\mu c^2 = m_\pi c^2 - m_\mu c^2 - \frac{\left(m_\pi{}^2c^2 - m_\mu{}^2c^2\right)}{2m_\pi}$$

$$= \frac{\left(2m_\pi{}^2 - 2m_\mu m_\pi - m_\pi{}^2 + m_\mu{}^2\right)c^2}{2m_\pi} = \frac{\left(m_\pi{}^2 - 2m_\mu m_\pi + m_\mu{}^2\right)c^2}{2m_\pi} = \frac{\left(m_\pi - m_\mu\right)^2c^2}{2m_\pi}.$$

CHAPTER 27: Early Quantum Theory and Models of the Atom

Answers to Questions

1. Bluish stars are the hottest, whitish-yellow stars are hot, and reddish stars are the coolest. This follows from Wien's law, which says that stars with the shortest wavelength peak in their spectrum have the highest temperatures.

2. The difficulty with seeing objects in the dark is that although all objects emit radiation, only a small portion of the electromagnetic spectrum can be detected by our eyes. Usually objects are so cool that they only give off very long wavelengths of light (infrared), which are too long for our eyes to detect.

3. No. The bulb, which is at a cooler temperature, has a redder tint to it than that of the hotter Sun. Wien's law says that the peak wavelength in the spectrum of emitted light is much longer for the cooler object, which makes it redder.

4. The red bulb used in black-and-white film dark rooms is a very "cool" filament, thus it does not emit much radiation at all in the range of visible wavelengths (and the small amount that it does emit in the visible region is not very intense), which means it will not develop the black-and-white film while still allowing a person to see what's going on. A red bulb will not work very well in a dark room that is used for color film. It will expose and develop the film during the process, especially at the red end of the spectrum, ruining the film.

5. Since the wavelength and the frequency of light are inversely proportional, and since the threshold wavelength of the light used in the photoelectric effect increased for the second metal, this means the frequency of incident light decreased for the second metal, which means that the work function of the second metal is smaller than the first metal. In other words, it is easier to knock the electrons out of the second metal, thus it has a smaller work function.

6. The wave theory of light predicts that if you use a very low frequency beam of light, but a very intense beam, eventually you would be able to eject electrons from the surface of a metal, as more and more energy is added to the metal. The fact that there is a cutoff frequency favors the particle theory of light, since it predicts that below a certain cutoff frequency of the incident beam of light that the metal will never eject an electron no matter how high the intensity of the beam. Experiments have conclusively shown that a cutoff frequency does exist, which strongly supports the particle theory of light.

7. UV light causes sunburns and visible light does not due to the higher frequency and energy carried by the UV photons. The UV photons can penetrate farther into the skin and once they are at this deeper level they can deposit a large amount of energy that can ionize atoms/molecules and cause damage to cells.

8. Yes, an X-ray photon that scatters from an electron does have its wavelength changed. The photon gives some of its energy to the electron during the collision and the electron recoils slightly. Thus, the photon's energy is less and its wavelength is longer after the collision, since the energy and wavelength are inversely proportional to each other $\left(E = hv = hc/\lambda\right)$.

9. In the photoelectric effect, the photons have only a few eV of energy, whereas in the Compton Effect, the photons have more than 1000 times greater energy and a correspondingly smaller wavelength. Also, in the photoelectric effect, the incident photons kick electrons completely out of the material and the

photons are absorbed in the material while the electrons are detected and studied, whereas in the Compton Effect, the incident photons just knock the electrons out of their atoms (but not necessarily out of the material) and then the photons are detected and studied.

10. (*a*) The energy of the light wave is spreading out spherically from the point source and the surface of this sphere is getting bigger as $4\pi r^2$. Thus, the intensity (energy/area) is decreasing proportionally as $1/r^2$.
    (*b*) As the particles are emitted from the point source in all directions, they spread out over a spherical surface that gets bigger as $4\pi r^2$. Thus, the intensity (number of particles/area) also decreases proportionally as $1/r^2$.
    Based on the above comments, we cannot use intensity of light from a point source to distinguish between the two theories.

11. (*a*) If a light source, such as a laser (especially an invisible one), was focused on a photocell in such a way that as a burglar opened a door or passed through a window that the beam was blocked from reaching the photocell, a burglar alarm could be triggered when the current in the ammeter went to 0 Amps.
    (*b*) If a light source was focused on a photocell in a smoke detector in such a way that as the density of smoke particles in the air became thicker and thicker, more and more of the light attempting to reach the photocell would be scattered and at some set minimum level the alarm would go off.
    (*c*) Hold up the photocell in the light conditions you want to know about and then read the ammeter. As the intensity of the light goes up, the number of photoelectrons goes up and the current increases. All you need to do is calibrate the ammeter to a known intensity of light.

12. We say that light has wave properties since we see it act like a wave when it is diffracted or refracted or exhibits interference with other light. We say that light has particle properties since we see it act like a particle in the photoelectric effect and Compton scattering.

13. We say that electrons have wave properties since we see them act like a wave when they are diffracted. We say that electrons have particle properties since we see them act like particles when they are bent by magnetic fields or accelerated and fired into materials where they scatter other electrons like billiard balls.

14.

Property	Photon	Electron
Mass	None	$9.11\times10^{-31}\text{kg}$
Charge	None	$-1.60\times10^{-19}\text{C}$
Speed	$3\times10^{8}\text{ m/s}$	$<3\times10^{8}\text{ m/s}$

$\boxed{15.}$ If an electron and a proton travel at the same speed, the proton has a shorter wavelength. Since $\lambda = h/mv$ and the mass of the proton is 1840 times larger than the mass of the electron, when they have the same speed then the wavelength of the proton is 1840 times smaller than that of the electron.

16. In Rutherford's planetary model of the atom, the Coulomb force keeps the electrons from flying off into space. Since the protons in the center are positively charged, the negatively charged electrons are attracted to the center by the Coulomb force and orbit around the center just like the planets orbiting a sun in a solar system due to the attractive gravitational force.

17. To tell if there is oxygen near the surface of the Sun, you need to collect the light that is coming from the Sun and spread it out using a diffraction grating or prism so you can see the spectrum of wavelengths. If

there is oxygen near the surface, we should be able to detect the atomic spectra (an absorption spectra) of the lines of oxygen in the spectrum.

18. Since the hydrogen gas is only at room temperature, almost all of the electrons will be in their ground state. As light excites these electrons up to higher energy levels, they all fall back down to the ground state, which is how the Lyman series is created. You need to heat up the gas to a temperature that is high enough where the electrons are in many different energy states and where they are falling down from extremely high energy states down to the mid-level energy states, which would create the other series.

19. The closely spaced energy levels in Figure 27-27 correspond to the different transitions of electrons from one energy state to another. When these transitions occur, they emit radiation that creates the closely spaced spectral lines shown in Figure 27-22.

20. It is possible for the de Broglie wavelength $\left(\lambda = h/p\right)$ of a particle to be bigger than the dimension of the particle. If the particle has a very small mass and a slow speed (like a low-energy electron or proton) then the wavelength may be larger than the dimension of the particle.

    It is also possible for the de Broglie wavelength of a particle to be smaller than the dimension of the particle if it has a large momentum and a moderate speed (like a baseball). There is no direct connection between the size of a particle and the size of the de Broglie wavelength of a particle. For example, you could also make the wavelength of a proton much smaller than the size of the proton by making it go very fast.

21. The electrons of a helium atom should be closer to their nucleus than the electrons in a hydrogen atom. This is because each of the electrons in the helium atom "sees" two positive charges at the nucleus, instead of just one positive charge for the hydrogen, and thus they are more strongly attracted.

22. Even though hydrogen only has one electron, it still has an infinite number of energy states for that one electron to occupy and each line in the spectrum represents a transition between two of those possible energy levels.

23. The Balmer series was discovered first, even though the Lyman series is brighter, because the Balmer series lines occur in the visible region of the electromagnetic spectrum. It was only later that the UV (Lyman) and IR (Paschen) regions were explored thoroughly.

24. When a photon is emitted by a hydrogen atom as the electron makes a transition from one energy state to a lower one, not only does the photon carry away energy and momentum, but to conserve momentum, the electron must also take away some momentum. If the electron carries away some momentum, then it must also carry away some of the available energy, which means that the photon takes away less energy than Equation 27-10 predicts.

25. Cesium will give a higher maximum kinetic energy for the ejected electrons. Since the incident photons bring in a given amount of energy, and in cesium less of this energy goes to releasing the electron from the material (the work function), it will give off electrons with a higher kinetic energy.

26. (*a*) No. It is possible that you could have many more IR photons in that beam than you have UV photons in the other beam. In this instance, even though each UV photon has more energy than each IR photon, the IR beam could be carrying more total energy than the UV beam.
    (*b*) Yes. A single IR photon will always have less energy than a single UV photon due to the inverse relationship between wavelength and frequency (or energy) of light. The long wavelength IR photon has a lower frequency and less energy than the short wavelength UV photon.

27. No, fewer electrons are emitted, but each one is emitted with higher kinetic energy, when the 400 nm light strikes the metal surface. Since the intensity (energy per unit time) is the same, but the 400 nm photons each have more energy than the 450 nm photons, then there are fewer photons hitting the surface per unit time. This means that fewer electrons will be ejected per unit time from the surface with the 400 nm light. The maximum kinetic energy of the electrons leaving the metal surface will be greater, though, since the incoming photons have shorter wavelengths and more energy per photon and it still takes the same amount of energy (the work function) to remove each electron. This "extra" energy goes into higher kinetic energy of the ejected electrons.

28. The spectral lines of hydrogen found at room temperature will be identical to some of the lines found at high temperature, but there will be many more lines at the higher temperature. These extra lines correspond to the electrons making transitions to the higher energy excited states that are only accessible at higher temperatures.

Solutions to Problems

Note: At the atomic scale, it is most convenient to have energies in electron-volts and wavelengths in nanometers. A useful expression for the energy of a photon in terms of its wavelength is

$$E = hf = hc/\lambda = \left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(10^{9}\,\text{nm/m}\right)/\left(1.60\times10^{-19}\,\text{J/eV}\right)\lambda\,;$$

$$E = \left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)/\lambda.$$

1. The velocity of the electron in the crossed fields is given by

$$v = \frac{E}{B}.$$

The radius of the path in the magnetic field is

$$r = \frac{mv}{eB} = \frac{mE}{eB^2}, \quad \text{or}$$

$$\frac{e}{m} = \frac{E}{rB^2} = \frac{\left(320\,\text{V/m}\right)}{\left(7.0\times10^{-3}\,\text{m}\right)\left(0.86\,\text{T}\right)^2} = \boxed{6.2\times10^4\ \text{C/kg.}}$$

2. The velocity of the electron in the crossed fields is given by

$$v = \frac{E}{B} = \frac{\left(1.88\times10^4\ \text{V/m}\right)}{\left(2.90\times10^{-3}\ \text{T}\right)} = \boxed{6.48\times10^6\ \text{m/s.}}$$

For the radius of the path in the magnetic field, we have

$$r = \frac{mv}{qB}$$

$$= \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(6.48\times10^6\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.90\times10^{-3}\ \text{T}\right)} = 1.27\times10^{-2}\ \text{m} = \boxed{1.27\,\text{cm.}}$$

$\boxed{3.}$ The force from the electric field must balance the weight:

$$qE = \frac{neV}{d} = mg;$$

$$\frac{n\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(340\,\mathrm{V}\right)}{\left(1.0\times10^{-2}\,\mathrm{m}\right)}=\left(2.8\times10^{-15}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right),\text{ which gives } n=\boxed{5.}$$

$\boxed{4.}$ We find the temperature for a peak wavelength of 440 nm:

$$T=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\lambda_{\mathrm{P}}}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(440\times10^{-9}\,\mathrm{m}\right)}=\boxed{6.59\times10^{3}\,\mathrm{K.}}$$

$\boxed{5.}$ (*a*) We find the peak wavelength from

$$\lambda_{\mathrm{P}}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(273\,\mathrm{K}\right)}=1.06\times10^{-5}\,\mathrm{m}=\boxed{10.6\,\mu\mathrm{m.}}$$

This wavelength is in the $\boxed{\text{near infrared.}}$

(*b*) We find the peak wavelength from

$$\lambda_{\mathrm{P}}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(3500\,\mathrm{K}\right)}=8.29\times10^{-7}\,\mathrm{m}=\boxed{829\,\mathrm{nm.}}$$

This wavelength is in the $\boxed{\text{infrared.}}$

(*c*) We find the peak wavelength from

$$\lambda_{\mathrm{P}}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(4\,\mathrm{K}\right)}=7.25\times10^{-4}\,\mathrm{m}=\boxed{0.73\,\mathrm{mm.}}$$

This wavelength is in the $\boxed{\text{microwave}}$ region.

(*d*) We find the peak wavelength from

$$\lambda_{\mathrm{P}}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(2.725\,\mathrm{K}\right)}=1.06\times10^{-2}\,\mathrm{m}=\boxed{1.06\text{ cm.}}$$

This wavelength is in the $\boxed{\text{microwave}}$ region.

6. (*a*) The temperature for a peak wavelength of 15.0 nm is

$$T=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\lambda_{\mathrm{P}}}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(18.0\times10^{-9}\,\mathrm{m}\right)}=\boxed{1.61\times10^{5}\,\mathrm{K.}}$$

(*b*) We find the peak wavelength from

$$\lambda_{\mathrm{P}}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T}=\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(2000\,\mathrm{K}\right)}=1.45\times10^{-6}\,\mathrm{m}=\boxed{1.45\,\mu\mathrm{m.}}$$

Note that this is not in the visible range.

7. Because the energy is quantized, $E = nhf$, the difference in energy between adjacent levels is

$$\Delta E=hf=\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(8.1\times10^{13}\,\mathrm{Hz}\right)=\boxed{5.4\times10^{-20}\,\mathrm{J}=0.34\,\mathrm{eV.}}$$

8. (*a*) The potential energy on the first step is

$$\mathrm{PE}_1=mgh=\left(68.0\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\left(0.200\,\mathrm{m}\right)=\boxed{133\,\mathrm{J.}}$$

(*b*) The potential energy on the second step is

$$\mathrm{PE}_2=mg2h=2\mathrm{PE}_1=2\left(133\,\mathrm{J}\right)=\boxed{267\,\mathrm{J.}}$$

(*c*) The potential energy on the third step is

$$\text{PE}_3 = mg3h = 3\text{PE}_1 = 3(133\,\text{J}) = \boxed{400\,\text{J.}}$$

(*d*) The potential energy on the *n*th step is

$$\text{PE}_\text{n} = mgnh = n\text{PE}_1 = n(133\,\text{J}) = \boxed{133n\,\text{J.}}$$

(*e*) The change in energy is

$$\Delta E = \text{PE}_2 - \text{PE}_6 = (2-6)(133\,\text{J}) = \boxed{-533\,\text{J.}}$$

9. We use a body temperature of $98°\text{F} = 37°\text{C} = 310\,\text{K}$. We find the peak wavelength from

$$\lambda_\text{P} = \frac{(2.90\times10^{-3}\,\text{m•K})}{T} = \frac{(2.90\times10^{-3}\,\text{m•K})}{(310\,\text{K})} = 9.4\times10^{-6}\,\text{m} = \boxed{9.4\,\mu\text{m.}}$$

10. The energy of the photon is

$$E = hf = (6.63\times10^{-34}\,\text{J•s})(88.5\times10^{6}\,\text{Hz}) = 5.87\times10^{-26}\,\text{J} = \boxed{3.67\times10^{-7}\,\text{eV.}}$$

11. The energy of the photons with wavelengths at the ends of the visible spectrum are

$$E_1 = hf_1 = \frac{hc}{\lambda_1} = \frac{(1.24\times10^{3}\,\text{eV•nm})}{(400\,\text{nm})} = 3.10\text{ eV};$$

$$E_2 = hf_2 = \frac{hc}{\lambda_2} = \frac{(1.24\times10^{3}\,\text{eV•nm})}{(750\,\text{nm})} = 1.65\text{ eV}.$$

Thus the range of energies is $\boxed{1.65\,\text{eV} < E < 3.10\text{ eV.}}$

12. We find the wavelength from

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(1.24\times10^{3}\,\text{eV•nm})}{(300\times10^{3}\,\text{eV})} = \boxed{4.13\times10^{-3}\text{ nm.}}$$

Significant diffraction occurs when the opening is on the order of the wavelength. Thus there would be $\boxed{\text{insignificant diffraction}}$ through the doorway.

13. We find the minimum frequency from

$$E_\text{min} = hf_\text{min};$$

$$(0.1\text{eV})(1.60\times10^{-19}\text{ J/eV}) = (6.63\times10^{-34}\,\text{J•s})f_\text{min},\ \text{which gives } f_\text{min} = \boxed{2.4\times10^{13}\,\text{Hz.}}$$

The maximum wavelength is

$$\lambda_\text{max} = \frac{c}{f_\text{min}} = \frac{(3.00\times10^{8}\text{ m/s})}{(2.4\times10^{13}\,\text{Hz})} = \boxed{1.2\times10^{-5}\,\text{m.}}$$

14. The momentum of the photon is

$$p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\text{ J•s})}{(6.00\times10^{-7}\text{ m})} = \boxed{1.1\times10^{-27}\text{ kg•m/s.}}$$

15. The momentum of the photon is

$$p = \frac{h}{\lambda} = \frac{\left(6.63 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(0.010 \times 10^{-9}\ \text{m}\right)} = \boxed{6.6 \times 10^{-23}\ \text{kg}\bullet\text{m/s.}}$$

16. The energy of a single photon is

$$E = hf = \frac{hc}{\lambda} = \frac{\left(6.63 \times 10^{-34}\ \text{J}\bullet\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{\left(550 \times 10^{-9}\ \text{m}\right)} = 3.62 \times 10^{-19}\ \text{J}.$$

Therefore, the number of photons required for an observable flash is calculated as

$$\frac{10^{-18}\ \text{J}}{3.62 \times 10^{-19}\ \text{J}} = 27.7.$$

Rounding up, we find that $\boxed{28}$ photons are required.

17. At the threshold frequency, the kinetic energy of the photoelectrons is zero, so we have

$$\text{KE} = hf - W = 0;$$

$$hf_{\text{min}} = W_0;$$

$$\left(6.63 \times 10^{-34}\ \text{J}\bullet\text{s}\right) f_{\text{min}} = 4.3 \times 10^{-19}\ \text{J},\ \text{which gives}\ f_{\text{min}} = \boxed{6.5 \times 10^{14}\ \text{Hz.}}$$

18. At the threshold wavelength, the kinetic energy of the photoelectrons is zero, so we have

$$\text{KE} = hf - W_0 = 0;$$

$$\frac{hc}{\lambda_{\text{max}}} = W_0,\ \text{or}$$

$$\lambda_{\text{max}} = \frac{\left(1.24 \times 10^{3}\ \text{eV}\bullet\text{nm}\right)}{\left(3.10\ \text{eV}\right)} = \boxed{400\,\text{nm.}}$$

19. The photon of visible light with the maximum energy has the least wavelength:

$$hf_{\text{max}} = \frac{\left(1.24 \times 10^{3}\ \text{eV}\bullet\text{nm}\right)}{\lambda_{\text{min}}} = \frac{\left(1.24 \times 10^{3}\ \text{eV}\bullet\text{nm}\right)}{\left(400\,\text{nm}\right)} = 3.10\ \text{eV}.$$

Electrons will not be emitted if this energy is less than the work function.
The metals with work functions greater than 3.10 eV are $\boxed{\text{copper and iron.}}$

20. (*a*) At the threshold wavelength, the kinetic energy of the photoelectrons is zero, so we have

$$\text{KE} = hf - W_0 = 0;$$

$$W_0 = \frac{hc}{\lambda_{\text{max}}} = \frac{\left(1.24 \times 10^{3}\ \text{eV}\bullet\text{nm}\right)}{\left(570\,\text{nm}\right)} = \boxed{2.18\ \text{eV.}}$$

(*b*) The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy:

$$\text{KE}_{\text{max}} = eV_0 = hf - W_0;$$

$$\left(1\text{e}\right)V_0 = \left[\frac{\left(1.24 \times 10^{3}\ \text{eV}\bullet\text{nm}\right)}{\left(400\,\text{nm}\right)}\right] - 2.18\ \text{eV} = 3.10\ \text{eV} - 2.18\ \text{eV} = 0.92\ \text{eV},$$

so the stopping voltage is $\boxed{0.92\ \text{V.}}$

21. The photon of visible light with the maximum energy has the minimum wavelength:

$$hf_{\max} = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\lambda_{\min}} = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\left(400\,\text{nm}\right)} = 3.10\text{ eV.}$$

The maximum kinetic energy of the photoelectrons is

$$\text{KE}_{\max} = hf - W_0 = 3.10\text{ eV} - 2.48\text{ eV} = \boxed{0.62\text{ eV.}}$$

22. The energy of the photon is

$$E = hf = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\lambda} = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\left(365\,\text{nm}\right)} = 3.40\text{ eV.}$$

The maximum kinetic energy of the photoelectrons is

$$\text{KE}_{\max} = hf - W_0 = 3.40\text{ eV} - 2.48\text{ eV} = \boxed{0.92\text{ eV.}}$$

We find the speed from

$$\text{KE}_{\max} = \tfrac{1}{2}mv^2;$$

$$\left(0.92\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right) = \tfrac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)v^2, \text{which gives } v = \boxed{5.7\times10^{5}\text{ m/s.}}$$

23. The energy of the photon is

$$hf = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\lambda} = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\left(285\,\text{nm}\right)} = 4.35\text{ eV.}$$

We find the work function from

$$\text{KE}_{\max} = hf - W_0;$$

$$1.40\text{ eV} = 4.35\text{ eV} - W_0, \text{which gives } W_0 = \boxed{2.95\text{ eV.}}$$

$\boxed{24.}$ The threshold wavelength determines the work function:

$$W_0 = \frac{hc}{\lambda_{\max}} = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\left(350\,\text{nm}\right)} = 3.54\text{ eV.}$$

(*a*) The energy of the photon is

$$hf = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\lambda} = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\left(280\,\text{nm}\right)} = 4.43\text{ eV.}$$

The maximum kinetic energy of the photoelectrons is

$$\text{KE}_{\max} = hf - W_0 = 4.43\text{ eV} - 3.54\text{ eV} = \boxed{0.89\text{ eV.}}$$

(*b*) Because the wavelength is greater than the threshold wavelength, the photon energy is less than the work function, so there will be $\boxed{\text{no ejected electrons.}}$

25. The energy required for the chemical reaction is provided by the photon:

$$E = hf = \frac{hc}{\lambda} = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\lambda} = \frac{\left(1.24\times10^{3}\text{ eV•nm}\right)}{\left(660\,\text{nm}\right)} = \boxed{1.88\text{ eV.}}$$

Each reaction takes place in a molecule, so we have

$$E = \frac{\left(1.88\text{ eV/molecule}\right)\left(6.02\times10^{23}\text{ molecules/mol}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{\left(4186\,\text{J/kcal}\right)}$$

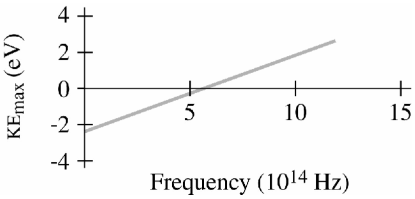
$$= \boxed{43.3\,\text{kcal/mol.}}$$

26. The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy:

$$\text{KE}_{\text{max}} = eV_0 = hf - W_0;$$

$$(1\text{e})(1.64\,\text{V}) = \left[\frac{(1.24\times10^3\,\text{eV}\bullet\text{nm})}{(230\,\text{nm})}\right] - W_0, \text{ which gives } W_0 = \boxed{3.75\text{ eV.}}$$

27.



If we use linear regression to find the slope of the line (using $y = mx + b$ form), we obtain $m = 4.16\times10^{-15}\,\text{eV}\bullet\text{s}$, and for the y-intercept we obtain $b = -2.30$ eV. Comparing this to the equation for the photoelectric effect,

$$\text{KE}_{\text{max}} = hf - W_0,$$

we find that $h = m$ and $W_0 = -b$.

(*a*) With the necessary unit conversion, $h = (4.16\times10^{-15}\,\text{eV}\bullet\text{s})(1.60\times10^{-19}\,\text{J/eV}) = \boxed{6.7\times10^{-34}\,\text{J}\bullet\text{s.}}$

(This is slightly higher than the best known value of $6.6\times10^{-34}\,\text{J}\bullet\text{s}$.)

(*b*) In the equation

$$\text{KE} = hf - W_0,$$

we set $\text{KE} = 0$ and find f:

$$0 = (4.16\times10^{-15}\,\text{eV}\bullet\text{s})f - (2.30\text{ eV}), \text{ which gives } f = \boxed{5.5\times10^{14}\,\text{Hz.}}$$

(*c*) $W_0 = \boxed{2.30\text{ eV.}}$

28. For the energy of the photon, we have

$$E = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.626\times10^{-34}\,\text{J}\bullet\text{s})(2.998\times10^8\,\text{m/s})}{(1.602\times10^{-19}\,\text{J/eV})\lambda} = \frac{(1.240\times10^{-6}\,\text{eV}\bullet\text{m})}{\lambda} = \frac{1.240\times10^3\,\text{eV}\bullet\text{nm}}{\lambda(\text{nm})}$$

29. (*a*) $$\frac{h}{m_0c} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{(9.11\times10^{-31}\,\text{kg})(3.00\times10^8\,\text{m/s})} = \boxed{2.43\times10^{-12}\,\text{m.}}$$

(*b*) $$\frac{h}{m_0c} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{(1.67\times10^{-27}\,\text{kg})(3.00\times10^8\,\text{m/s})} = \boxed{1.32\times10^{-15}\,\text{m.}}$$

(*c*) For the energy of the photon we have

$$E = hf = \frac{hc}{\lambda} = \frac{hc}{\left(\dfrac{h}{m_0c}\right)} = m_0c^2.$$

30. We find the Compton wavelength shift for a photon scattered from an electron:

$$\lambda' - \lambda = \left(\frac{h}{m_0 c}\right)(1 - \cos\theta).$$

(*a*) $\lambda'_a - \lambda = \left(2.43 \times 10^{-12}\ \text{m}\right)\left(1 - \cos 45°\right) = \boxed{7.12 \times 10^{-13}\ \text{m.}}$

(*b*) $\lambda'_b - \lambda = \left(2.43 \times 10^{-12}\ \text{m}\right)\left(1 - \cos 90°\right) = \boxed{2.43 \times 10^{-12}\ \text{m.}}$

(*c*) $\lambda'_c - \lambda = \left(2.43 \times 10^{-12}\ \text{m}\right)\left(1 - \cos 180°\right) = \boxed{4.86 \times 10^{-12}\ \text{m.}}$

31. (*a*) For the conservation of momentum for the head-on collision, we have

$$\frac{h}{\lambda} + 0 = -\left(\frac{h}{\lambda'}\right) + p_e, \text{ or } \frac{h}{\lambda'} = p_e - \left(\frac{h}{\lambda}\right).$$

Before			After	
hf	0	energy	hf'	$p_e^2/2m$
h/λ	0	momentum	$-h/\lambda'$	p_e

For energy conservation, we have

$$\left(\frac{hc}{\lambda}\right) + m_0c^2 = \left(\frac{hc}{\lambda'}\right) + \left[\left(m_0c^2\right)^2 + p_e^2c^2\right]^{\frac{1}{2}}$$

$$= \left(\frac{hc}{\lambda'}\right) + m_0c^2 + \left(\frac{p_e^2}{2m_0}\right),$$

where we have used the approximation $(1 + x)^{\frac{1}{2}} \approx 1 + \frac{x}{2}$, for $x \ll 1$.

When we use the result from momentum conservation, we get

$$\left(\frac{hc}{\lambda}\right) = p_ec - \left(\frac{hc}{\lambda}\right) + \left(\frac{p_e^2}{2m_0}\right), \text{ or}$$

$$\left[\frac{p_e^2}{2\left(9.11 \times 10^{-31}\ \text{kg}\right)}\right] + p_e\left(3.00 \times 10^8\ \text{m/s}\right) - 2\left[\frac{\left(6.63 \times 10^{-34}\ \text{J}\bullet\text{s}\right)\left(3.00 \times 10^8\ \text{m/s}\right)}{\left(0.100 \times 10^{-9}\ \text{m}\right)}\right] = 0.$$

When we solve this quadratic for p_e, the positive solution is

$$p_e = 1.295 \times 10^{-23}\ \text{kg}\bullet\text{m/s}.$$

The kinetic energy of the electron is

$$\text{KE} = \frac{p_e^2}{2m_0} = \frac{\left(1.295 \times 10^{-23}\ \text{kg}\bullet\text{m/s}\right)^2}{2\left(9.11 \times 10^{-31}\ \text{kg}\right)} = \boxed{9.21 \times 10^{-17}\ \text{J}}\left(576\ \text{eV}\right).$$

Because this is much less than $m_0c^2 = 0.511\,\text{MeV}$, we are justified in using the expansion.

(*b*) For the wavelength of the recoiling photon, we have

$$\frac{h}{\lambda'} = p_e - \left(\frac{h}{\lambda}\right);$$

$$\frac{\left(6.63 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\lambda'} = 1.295 \times 10^{-23}\ \text{kg}\bullet\text{m/s} - \left[\frac{\left(6.63 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(0.100 \times 10^{-9}\ \text{m}\right)}\right],$$

which gives $\lambda' = 1.05 \times 10^{-10}\,\text{m} = \boxed{0.105\,\text{nm.}}$

$\boxed{32.}$ The kinetic energy of the pair is

$$\text{KE} = hf - 2m_0c^2 = 3.84\,\text{MeV} - 2\left(0.511\,\text{MeV}\right) = \boxed{2.82\,\text{MeV.}}$$

33. The photon with the longest wavelength has the minimum energy to create the masses:

$$hf_{\text{min}} = \frac{hc}{\lambda_{\text{max}}} = 2m_0c^2;$$

$$\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\lambda_{\text{max}}} = 2\left(1.67\times10^{-27}\,\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right),\ \text{which gives}\ \lambda_{\text{max}} = \boxed{6.62\times10^{-16}\,\text{m.}}$$

34. The photon with minimum energy to create the masses is

$$hf_{\text{min}} = 2m_0c^2 = 2(207)(0.511\,\text{MeV}) = \boxed{212\,\text{MeV.}}$$

The wavelength is

$$\lambda = \frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\left(212\times10^{6}\,\text{eV}\right)} = 5.85\times10^{-6}\,\text{nm} = \boxed{5.85\times10^{-15}\,\text{m.}}$$

35. Since an electron and a positron have identical masses, the energy of each photon will consist of the total energy from one electron or positron, as given by

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}.$$

For $v < 0.001c$, this is essentially identical to

$$E = m_0c^2 = \left(\frac{0.51\,\text{MeV}}{c^2}\right)c^2 = \boxed{0.51\,\text{MeV.}}$$

The momentum of each photon is

$$p = \frac{E}{c} = \boxed{\frac{0.51\,\text{MeV}}{c}.}$$

36. The energy of the photon is

$$hf = 2\left(\text{KE} + m_0c^2\right) = 2\left(0.245\,\text{MeV} + 0.511\,\text{MeV}\right) = \boxed{1.51\,\text{MeV.}}$$

The wavelength is

$$\lambda = \frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\left(1.51\times10^{6}\,\text{eV}\right)} = 7.24\times10^{-4}\,\text{nm} = \boxed{8.20\times10^{-13}\,\text{m.}}$$

37. We find the wavelength from

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{(0.23\,\text{kg})(0.10\,\text{m/s})} = \boxed{2.9\times10^{-32}\,\text{m.}}$$

38. We find the wavelength from

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(6.5\times10^{4}\,\text{m/s}\right)} = \boxed{6.1\times10^{-12}\,\text{m.}}$$

39. We find the speed from

$$\lambda = \frac{h}{p} = \frac{h}{mv};$$

$$0.24\times10^{-9}\text{ m}=\frac{\left(6.63\times10^{-34}\text{ J•s}\right)}{\left(9.11\times10^{-31}\text{ kg}\right)v},\text{ which gives } v=3.03\times10^{6}\text{ m/s}.$$

Because this is much less than c, we can use the classical expression for the kinetic energy. The kinetic energy is equal to the potential energy change:

$$eV=\text{KE}=\tfrac{1}{2}mv^2=\tfrac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(3.03\times10^{6}\text{ m/s}\right)^2=4.19\times10^{-18}\text{ J}=26.2\text{eV}.$$

Thus the required potential difference is $\boxed{26\text{ V.}}$

40. With $\text{KE}=\dfrac{p^2}{2m_0}$, we have

$$\lambda=\frac{h}{p}=\frac{h}{\left[2m_0\left(\text{KE}\right)\right]^{\frac{1}{2}}}.$$

If we form the ratio for the two particles with equal wavelengths, we get

$$1=\left[\frac{m_{0\text{e}}\left(\text{KE}_\text{e}\right)}{\text{m}_{0\text{p}}\left(\text{KE}_\text{P}\right)}\right]^{\frac{1}{2}},\text{ or }\frac{\text{KE}_\text{e}}{\text{KE}_\text{p}}=\frac{m_{0\text{p}}}{m_{0\text{e}}}=\frac{\left(1.67\times10^{-27}\text{kg}\right)}{\left(9.11\times10^{-31}\text{kg}\right)}=\boxed{1.84\times10^{3}.}$$

41. (*a*) We find the momentum from

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}\text{J•s}}{5.0\times10^{-10}\text{m}}=\boxed{1.3\times10^{-24}\text{ kg•m/s.}}$$

(*b*) We find the speed from

$$\lambda=\frac{h}{p}=\frac{h}{mv};$$

$$5.0\times10^{-10}\text{m}=\frac{6.63\times10^{-34}\text{J•s}}{\left(9.11\times10^{-31}\text{kg}\right)v},\text{ which gives } v=\boxed{1.5\times10^{6}\text{ m/s.}}$$

(*c*) With $v<0.005c$, we can calculate KE classically:

$$\text{KE}=\tfrac{1}{2}mv^2$$

$$=\tfrac{1}{2}\left(9.11\times10^{-31}\text{kg}\right)\left(1.5\times10^{6}\text{ m/s}\right)^2=9.7\times10^{-19}\text{J},$$ which converted to electron-volts

equals $\dfrac{\left(9.7\times10^{-19}\right)}{\left(1.06\times10^{-19}\text{ J/eV}\right)}=6.0$ eV. This is the energy gained by an electron as it is accelerated through a potential difference of $\boxed{6.0\text{V.}}$

42. Because all the energies are much less than m_0c^2, we can use $\text{KE}=\dfrac{p^2}{2m_0}$, so

$$\lambda=\frac{h}{p}=\frac{h}{\left[2m_0\left(\text{KE}\right)\right]^{\frac{1}{2}}}=\frac{hc}{\left[2m_0c^2\left(\text{KE}\right)\right]^{\frac{1}{2}}}.$$

(*a*) $$\lambda=\frac{hc}{\left[2m_0c^2\left(\text{KE}\right)\right]^{\frac{1}{2}}}=\frac{\left(1.24\times10^{3}\text{eV•nm}\right)}{\left[2\left(0.511\times10^{6}\text{eV}\right)\left(10\text{ eV}\right)\right]^{\frac{1}{2}}}=\boxed{0.39\text{nm.}}$$

(*b*) $$\lambda = \frac{hc}{\left[2m_0c^2(\text{KE})\right]^{\frac{1}{2}}} = \frac{\left(1.24\times10^3\,\text{eV}\bullet\text{nm}\right)}{\left[2\left(0.511\times10^6\,\text{eV}\right)\left(100\text{ eV}\right)\right]^{\frac{1}{2}}} = \boxed{0.12\,\text{nm.}}$$

(*c*) $$\lambda = \frac{hc}{\left[2m_0c^2(\text{KE})\right]^{\frac{1}{2}}} = \frac{\left(1.24\times10^3\,\text{eV}\bullet\text{nm}\right)}{\left[2\left(0.511\times10^6\,\text{eV}\right)\left(1.0\times10^3\text{ eV}\right)\right]^{\frac{1}{2}}} = \boxed{0.039\,\text{nm.}}$$

43. With $\text{KE} = \frac{p^2}{2m_0}$, we have

$$\lambda = \frac{h}{p} = \frac{h}{\left[2m_0(\text{KE})\right]^{\frac{1}{2}}}.$$

If we form the ratio for the two particles with equal kinetic energies, we get

$$\frac{\lambda_\text{p}}{\lambda_\text{e}} = \left(\frac{m_{0\text{e}}}{m_{0\text{p}}}\right)^{\frac{1}{2}}.$$

Because $m_{0\text{p}} > m_{0\text{e}}, \lambda_\text{p} < \lambda_\text{e}$.

44. The kinetic energy is equal to the potential energy change:

$$\text{KE} = eV = (1\text{e})\left(20.0\times10^3\text{ V}\right) = 30.0\times10^3\text{ eV} = 0.0300\,\text{MeV}.$$

Because this is 6% of m_0c^2, the electron is relativistic. We find the momentum from

$$E^2 = \left[(\text{KE}) + m_0c^2\right]^2 = p^2c^2 + m_e{}^2c^4, \quad \text{or}$$

$$p^2c^2 = (\text{KE})^2 + 2(\text{KE})m_0c^2;$$

$$p^2c^2 = (0.0300\,\text{MeV})^2 + 2(0.0300\,\text{MeV})(0.511\,\text{MeV}), \text{ which gives } pc = 0.178\,\text{MeV}, \text{ or}$$

$$p = \frac{(0.178\,\text{MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)}{\left(3.00\times10^8\text{ m/s}\right)} = 9.47\times10^{-23}\text{ kg}\bullet\text{m/s}.$$

The wavelength is

$$\lambda = \frac{h}{p} = \frac{\left(6.63\times10^{-34}\text{ J}\bullet\text{s}\right)}{\left(9.47\times10^{-23}\text{ kg}\bullet\text{m/s}\right)} = \boxed{7.0\times10^{-12}\text{ m.}}$$

Because $\lambda \ll 5$ cm, $\boxed{\text{diffraction effects are negligible.}}$

45. For diffraction, the wavelength must be on the order of the opening. We find the speed from

$$\lambda = \frac{h}{p} = \frac{h}{mv};$$

$$10\,\text{m} = \frac{\left(6.63\times10^{-34}\text{ J}\bullet\text{s}\right)}{(1400\text{ kg})v}, \text{ which gives } v = \boxed{4.7\times10^{-38}\text{ m/s.}}$$

Not a good speed if you want to get somewhere.
At a speed of $30\,\text{m/s}$, $\lambda \ll 10\,\text{m}$, so there will be $\boxed{\text{no diffraction.}}$

$\boxed{46.}$ The kinetic energy of the electron is

$$\text{KE} = \frac{p^2}{2m_0} = \frac{h^2}{2m_0\lambda^2} = \frac{\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.20\times10^{-9}\ \text{m}\right)^2} = 6.03\times10^{-18}\ \text{J} = 37.7\,\text{eV}.$$

Because this must equal the potential energy change, the required voltage is $\boxed{37.7\ \text{V.}}$

47. The wavelength of the electron is

$$\lambda = \frac{h}{p} = \frac{h}{\left[2m_0\left(\text{KE}\right)\right]^{\frac{1}{2}}} = \frac{hc}{\left[2m_0c^2\left(\text{KE}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(1.24\times10^3\ \text{eV}\bullet\text{nm}\right)}{\left[2\left(0.511\times10^6\ \text{eV}\right)\left(2450\ \text{eV}\right)\right]^{\frac{1}{2}}} = \boxed{2.48\times10^{-2}\text{nm.}}$$

This is the maximum possible resolution, as discussed in Section 27-9.

48. The energy of a level is $E_n = -\frac{\left(13.6\ \text{eV}\right)}{n^2}$.

(*a*) The transition from $n = 1$ to $n' = 3$ is an $\boxed{\text{absorption,}}$ because the $\boxed{\text{final state,}}$ $n' = 3$, has a higher energy. The energy of the photon is

$$hf = E_{n'} - E_n = -\left(13.6\ \text{eV}\right)\left[\left(\frac{1}{3^2}\right) - \left(\frac{1}{1^2}\right)\right] = 12.1\ \text{eV}.$$

(*b*) The transition from $n = 6$ to $n' = 2$ is an $\boxed{\text{emission,}}$ because the $\boxed{\text{initial state,}}$ $n' = 2$, has a higher energy. The energy of the photon is

$$hf = -\left(E_{n'} - E_n\right) = \left(13.6\ \text{eV}\right)\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{6^2}\right)\right] = 3.0\ \text{eV}.$$

(*c*) The transition from $n = 4$ to $n' = 5$ is an $\boxed{\text{absorption,}}$ because the $\boxed{\text{final state,}}$ $n' = 5$, has a higher energy. The energy of the photon is

$$hf = E_{n'} - E_n = -\left(13.6\ \text{eV}\right)\left[\left(\frac{1}{5^2}\right) - \left(\frac{1}{4^2}\right)\right] = 0.31\ \text{eV}.$$

The photon for the transition from $\boxed{n = 1 \text{ to } n' = 3}$ has the largest energy.

49. To ionize the atom means removing the electron, or raising it to zero energy:

$$E_{\text{ion}} = 0 - E_n = \frac{\left(13.6\ \text{eV}\right)}{n^2} = \frac{\left(13.6\ \text{eV}\right)}{2^2} = \boxed{3.4\ \text{eV.}}$$

50. From $\Delta E = \frac{hc}{\lambda}$, we see that the third longest wavelength comes from the transition with the third smallest energy: $\boxed{n = 6 \text{ to } n' = 3.}$

51. Doubly ionized lithium is like hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2\left(13.6\ \text{eV}\right)}{n^2} = -\frac{3^2\left(13.6\ \text{eV}\right)}{n^2} = -\frac{\left(122\ \text{eV}\right)}{n^2}.$$

We find the energy needed to remove the remaining electron from

$$E = 0 - E_1 = 0 - \left[-\frac{(122\text{ eV})}{(1)^2}\right] = \boxed{122\text{ eV.}}$$

52. (*a*) For the jump from $n = 4$ to $n = 2$, we have

$$\lambda = \frac{(1.24 \times 10^3\,\text{eV}\bullet\text{nm})}{(E_4 - E_2)} = \frac{(1.24 \times 10^3\,\text{eV}\bullet\text{nm})}{[-0.85\text{ eV} - (-3.4\text{ eV})]} = \boxed{486\,\text{nm.}}$$

(*b*) For the jump from $n = 3$ to $n = 1$, we have

$$\lambda = \frac{(1.24 \times 10^3\,\text{eV}\bullet\text{nm})}{(E_3 - E_1)} = \frac{(1.24 \times 10^3\,\text{eV}\bullet\text{nm})}{[-1.5\text{ eV} - (-13.6\text{ eV})]} = \boxed{102\,\text{nm.}}$$

(*c*) The energy of the $n = 5$ level is

$$E_5 = \frac{-(13.6\text{ eV})}{5^2} = -0.54\text{ eV.}$$

For the jump from $n = 5$ to $n = 2$, we have

$$\lambda = \frac{(1.24 \times 10^3\,\text{eV}\bullet\text{nm})}{(E_5 - E_2)} = \frac{(1.24 \times 10^3\,\text{eV}\bullet\text{nm})}{[-0.54\text{ eV} - (-3.4\text{ eV})]} = \boxed{434\,\text{nm.}}$$

53. For the Rydberg constant we have

$$\begin{aligned} R &= \frac{2\pi^2 e^4 m k^2}{h^3 c} \\ &= \frac{2\pi(1.602177 \times 10^{-19}\,\text{C})^4 (9.109390 \times 10^{-31}\,\text{kg})(8.988 \times 10^9\,\text{N}\bullet\text{m}^2/\text{C}^2)^2}{(6.626076 \times 10^{-34}\,\text{J}\bullet\text{s})^3 (2.997925 \times 10^8\,\text{m/s})} \\ &= 1.0974 \times 10^7\,\text{m}^{-1}. \end{aligned}$$

54. The longest wavelength corresponds to the minimum energy, which is the ionization energy:

$$\lambda = \frac{(1.24\times10^3\,\text{eV}\bullet\text{nm})}{E_\text{ion}} = \frac{(1.24\times10^3\,\text{eV}\bullet\text{nm})}{(13.6\text{ eV})} = \boxed{91.2\,\text{nm.}}$$

55. The energy of the photon is

$$hf = E_\text{ion} + \text{KE} = 13.6\text{ eV} + 10.0\text{ eV} = 23.6\,\text{eV.}$$

We find the wavelength from

$$\lambda = \frac{(1.24 \times 10^3\,\text{eV}\bullet\text{nm})}{hf} = \frac{(1.24 \times 10^3\,\text{eV}\bullet\text{nm})}{(23.6\text{ eV})} = \boxed{52.5\,\text{nm.}}$$

56. Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\text{ eV})}{n^2} = -\frac{2^2(13.6\text{ eV})}{n^2} = -\frac{(54.4\text{ eV})}{n^2}.$$

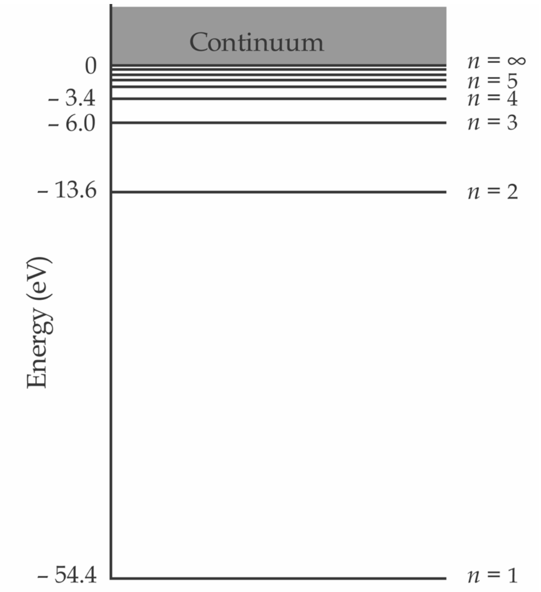
We find the energy of the photon from the $n = 6$ to $n = 2$ transition:

$$E = E_6 - E_2 = -(54.4\ \text{eV})\left[\left(\frac{1}{6^2}\right) - \left(\frac{1}{2^2}\right)\right] = 12.1\ \text{eV}.$$

Because this is the energy difference for the $n = 1$ to $n = 3$ transition in hydrogen, the photon can be absorbed by a hydrogen atom which will jump from $\boxed{n = 1 \text{ to } n = 3.}$

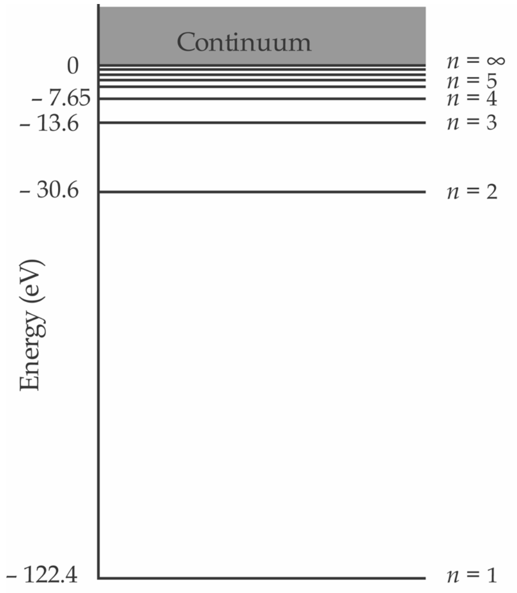
57. Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\ \text{eV})}{n^2} = -\frac{2^2(13.6\ \text{eV})}{n^2} = -\frac{(54.4\ \text{eV})}{n^2}.$$



58. Doubly ionized lithium is like hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\ \text{eV})}{n^2} = -\frac{3^2(13.6\ \text{eV})}{n^2} = -\frac{(122\ \text{eV})}{n^2}.$$



59. The potential energy for the ground state is

$$\text{PE} = -\frac{ke^2}{r_1} = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(0.529\times10^{-10}\ \text{m})} = -4.35\times10^{-18}\ \text{J} = \boxed{-27.2\ \text{eV}.}$$

The kinetic energy is

$$\text{KE} = E_1 - \text{PE} = -13.6\text{ eV} - (-27.2\text{ eV}) = \boxed{+13.6\text{ eV.}}$$

60. We find the value of n from

$$r_n = n^2 r_1;$$

$$1.00 \times 10^{-3}\text{ m} = n^2\left(0.529 \times 10^{-10}\text{ m}\right), \text{ which gives } n = \boxed{4.35 \times 10^3.}$$

The energy of this orbit is

$$E = -\frac{(13.6\text{ eV})}{n^2} = -\frac{(13.6\text{ eV})}{\left(4.35 \times 10^3\right)^2} = \boxed{-7.2 \times 10^{-7}\text{eV.}}$$

61. We find the velocity from the quantum condition:

$$mvr_1 = \frac{nh}{2\pi};$$

$$\left(9.11 \times 10^{-31}\text{kg}\right)v\left(0.529 \times 10^{-10}\text{m}\right) = \frac{(1)\left(6.63\times10^{-34}\text{J•s}\right)}{2\pi},$$

which gives $v = 2.18 \times 10^6\text{ m/s} = 7.3 \times 10^{-3}c.$

The relativistic factor is

$$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} \approx 1 - \tfrac{1}{2}\left(\frac{v}{c}\right)^2 = 1 - 2.7 \times 10^{-5}.$$

Because this is essentially 1, the use of nonrelativistic formulas is $\boxed{\text{justified.}}$

62. If we compare the two forces:

$$F_e = \frac{ke^2}{r^2}, \text{ and } F_g = \frac{Gm_em_p}{r^2},$$

we see that we can use the hydrogen expressions if we replace ke^2 with Gm_em_p. For the radius we get

$$r_1 = \frac{h^2}{4\pi^2 Gm_e^2 m_p}$$

$$= \frac{\left(6.63 \times 10^{-34}\text{ J•s}\right)^2}{4\pi^2\left(6.67 \times 10^{-11}\text{ N•m}^2/\text{kg}^2\right)\left(9.11 \times 10^{-31}\text{kg}\right)^2\left(1.67 \times 10^{-27}\text{ kg}\right)}$$

$$= \boxed{1.20 \times 10^{29}\text{ m.}}$$

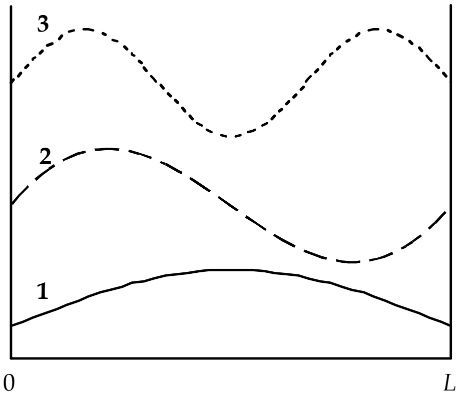
The ground state energy is

$$E_1 = -\frac{2\pi^2 G^2 m_e^3 m_p^2}{h^2}$$

$$= -\frac{2\pi^2\left(6.67 \times 10^{-11}\text{ N•m}^2/\text{kg}^2\right)^2\left(9.11 \times 10^{-31}\text{kg}\right)^3\left(1.67 \times 10^{-27}\text{ kg}\right)^2}{\left(6.63 \times 10^{-34}\text{ J•s}\right)^2}$$

$$= \boxed{-4.21 \times 10^{-97}\text{ J.}}$$

63. (*a*)



(*b*) From the diagram we see that the wavelengths are given by

$$\lambda_n = \frac{2L}{n}, \quad n = 1, 2, 3, \ldots .$$

so the momentum is

$$p_n = \frac{h}{\lambda} = \frac{nh}{2L}, \quad n = 1, 2, 3, \ldots .$$

Thus the kinetic energy is

$$\text{KE}_n = \frac{p_n^{\ 2}}{2m} = \frac{n^2h^2}{8mL^2}, \quad n = 1, 2, 3, \ldots .$$

(*c*) Because the potential energy is zero inside the box, the total energy is the kinetic energy. For the ground state energy, we get

$$E_1 = \text{KE}_1 = \frac{n^2h^2}{8mL^2}$$

$$= \frac{(1)^2\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.50\times10^{-10}\ \text{m}\right)^2} = 2.41\times10^{-17}\ \text{J} = \boxed{150\ \text{eV.}}$$

(*d*) For the baseball, we get

$$E_1 = \text{KE}_1 = \frac{n^2h^2}{8mL^2}$$

$$= \frac{(1)^2\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)^2}{8\left(0.140\,\text{kg}\right)\left(0.50\,\text{m}\right)^2} = \boxed{1.6\times10^{-66}\ \text{J.}}$$

We find the speed from

$$\text{KE}_1 = \tfrac{1}{2}mv^2;$$

$$1.6\times10^{-66}\ \text{J} = \tfrac{1}{2}\left(0.140\,\text{kg}\right)v^2, \text{ which gives } v = \boxed{4.7\times10^{-33}\ \text{m/s.}}$$

(*e*) We find the width of the box from

$$E_1 = \frac{n^2h^2}{8mL^2}$$

$$\left(22\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right) = \frac{(1)^2\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)L^2},$$

which gives $L = 1.3\times10^{-10}\ \text{m} = \boxed{0.13\,\text{nm.}}$

64. We find the peak wavelength from

$$\lambda_{\mathrm{P}} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(2.7\,\mathrm{K}\right)} = 1.1\times10^{-3}\,\mathrm{m} = \boxed{1.1\,\mathrm{mm.}}$$

65. To produce a photoelectron, the hydrogen atom must be ionized, so the minimum energy of the photon is 13.6 eV. We find the minimum frequency of the photon from

$$E_{\min} = hf_{\min};$$

$$\left(13.6\ \mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right) = \left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)f_{\min}, \text{which gives } f_{\min} = \boxed{3.28\times10^{15}\,\mathrm{Hz.}}$$

66. Because the energy is much less than m_0c^2, we can use $\mathrm{KE} = \frac{p^2}{2m_0}$, so the wavelength of the electron is

$$\lambda = \frac{h}{p} = \frac{h}{\left[2m_0\left(\mathrm{KE}\right)\right]^{\frac{1}{2}}} = \frac{hc}{\left[2m_0c^2\left(\mathrm{KE}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(1.24\times10^{3}\,\mathrm{eV{\bullet}nm}\right)}{\left[2\left(0.511\times10^{6}\,\mathrm{eV}\right)\left(85\ \mathrm{eV}\right)\right]^{\frac{1}{2}}} = 0.133\,\mathrm{nm.}$$

We find the spacing of the planes from

$$2d\sin\theta = m\lambda;$$

$$2d\sin38^\circ = \left(1\right)\left(0.133\,\mathrm{nm}\right), \text{ which gives } d = \boxed{0.108\,\mathrm{nm.}}$$

67. The energy of the photon is

$$E = hf = \frac{hc}{\lambda}$$

$$= \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{\left(12.2\times10^{-2}\,\mathrm{m}\right)} = 1.63\times10^{-24}\,\mathrm{J.}$$

Thus the rate at which photons are produced in the oven is

$$N = \frac{P}{E} = \frac{\left(760\,\mathrm{W}\right)}{\left(1.63\times10^{-24}\,\mathrm{J}\right)} = \boxed{4.66\times10^{26}\ \mathrm{photons/s.}}$$

68. The energy of the photon is

$$hf = \frac{\left(1.24\times10^{3}\,\mathrm{eV{\bullet}nm}\right)}{\lambda} = \frac{\left(1.24\times10^{3}\,\mathrm{eV{\bullet}nm}\right)}{\left(550\,\mathrm{nm}\right)} = 2.25\ \mathrm{eV.}$$

We find the intensity of photons from

$$I_{\text{photons}} = \frac{I}{hf} = \frac{\left(1400\,\mathrm{W/m^2}\right)}{\left(2.25\ \mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)} = \boxed{2.78\times10^{21}\ \mathrm{photons/s{\bullet}m^2.}}$$

69. The impulse on the wall is due to the change in momentum of the photons:

$$F\Delta t = \Delta p = np = \frac{nh}{\lambda}, \text{ or}$$

$$\frac{n}{\Delta t}=\frac{F\lambda}{h}=\frac{\left(5.5\times10^{-9}\,\mathrm{N}\right)\left(633\times10^{-9}\,\mathrm{m}\right)}{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)}=\boxed{5.3\times10^{18}\text{ photons/s.}}$$

70. The energy of the photon is

$$hf=\frac{\left(1.24\times10^{3}\,\mathrm{eV{\bullet}nm}\right)}{\lambda}=\frac{\left(1.24\times10^{3}\,\mathrm{eV{\bullet}nm}\right)}{\left(550\,\mathrm{nm}\right)}=2.25\text{ eV}.$$

Because the light radiates uniformly, the intensity at a distance L is

$I=\frac{P}{4\pi L^2}$, so the rate at which energy enters the pupil is

$$\frac{E}{t}=I\pi r^2=\frac{Pr^2}{4L^2}.$$

Thus the rate at which photons enter the pupil is

$$\frac{n}{t}=\frac{\left(\frac{E}{t}\right)}{hf}=\frac{Pr^2}{4L^2hf}$$

$$=\frac{(0.030)(100\,\mathrm{W})\left(2.0\times10^{-3}\,\mathrm{m}\right)^2}{4\left(1.0\times10^{3}\,\mathrm{m}\right)^2(2.25\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$$

$$=\boxed{8.3\times10^{6}\text{ photons/s.}}$$

71. Since an electron and a positron have identical masses, the energy of each photon will consist of the total energy from one electron or positron. Of that energy,

$$E=m_0c^2=\left(\frac{0.51\,\mathrm{MeV}}{c^2}\right)c^2=0.51\,\mathrm{MeV}$$

comes from the conversion of mass into energy. The rest of the 0.90 MeV comes from the particle’s kinetic energy: $0.90\,\mathrm{MeV}-0.51\,\mathrm{MeV}=\boxed{0.39\,\mathrm{MeV}}$ for each particle.

72. The required momentum is

$p=\frac{h}{\lambda}$, or

$$pc=\frac{hc}{\lambda}=\frac{\left(1.24\times10^{3}\,\mathrm{eV{\bullet}nm}\right)}{\left(5.0\times10^{-3}\,\mathrm{nm}\right)}=2.48\times10^{5}\,\mathrm{eV}=0.248\,\mathrm{MeV}.$$

(*a*) For the proton, $pc \ll m_0c^2$, so we can find the required kinetic energy from

$$\mathrm{KE}=\frac{p^2}{2m_0}=\frac{(pc)^2}{2m_0c^2}=\frac{(0.248\,\mathrm{MeV})^2}{2(938\,\mathrm{MeV})}=3.3\times10^{-5}\,\mathrm{MeV}=33\text{ eV}.$$

The potential difference to produce this kinetic energy is

$$V=\frac{\mathrm{KE}}{e}=\frac{(33\,\mathrm{eV})}{(1\mathrm{e})}=\boxed{33\,\mathrm{V}.}$$

(*b*) For the electron, pc is on the order of m_0c^2, so we can find the required kinetic energy from

$$\mathrm{KE}=\left[(pc)^2+\left(m_0c^2\right)^2\right]^{\frac{1}{2}}-m_0c^2$$

$$=\left[(0.248\,\text{MeV})^2+(0.511\,\text{MeV})^2\right]^{\frac{1}{2}}-0.511\,\text{MeV}=0.057\,\text{MeV}=57\,\text{keV}.$$

The potential difference to produce this kinetic energy is

$$V=\frac{\text{KE}}{e}=\frac{(57\,\text{KeV})}{(1\text{e})}=\boxed{57\,\text{kV.}}$$

73. If we ignore the recoil motion, at the closest approach the kinetic energy of both particles is zero. The potential energy of the two charges must equal the initial kinetic energy of the α particle:

$$\text{KE}=\frac{kZ_\alpha Z_{\text{Au}}e^2}{r_{\min}};$$

$$(4.8\,\text{MeV})\left(1.60\times10^{-13}\,\text{J/MeV}\right)=\frac{\left(9.00\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\,\text{C}\right)^2}{r_{\min}},$$

which gives $r_{\min}=\boxed{4.7\times10^{-14}\,\text{m.}}$

74. The decrease in mass occurs because a photon has been emitted:

$$\frac{\Delta m}{m_0}=\frac{\left(\frac{\Delta E}{c^2}\right)}{m_0}=\frac{\Delta E}{m_0c^2}=\frac{(-13.6\text{ eV})\left[\left(\frac{1}{1^2}\right)-\left(\frac{1}{3^2}\right)\right]}{\left(939\times10^6\text{ eV}\right)}=\boxed{-1.29\times10^{-8}.}$$

75. The ratio of the forces is

$$\frac{F_g}{F_e}=\frac{\left(\frac{Gm_em_p}{r^2}\right)}{\left(\frac{ke^2}{r^2}\right)}=\frac{Gm_em_p}{ke^2}$$

$$=\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(9.11\times10^{-31}\,\text{kg}\right)^2\left(1.67\times10^{-27}\,\text{kg}\right)}{\left(9.00\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}=\boxed{4.4\times10^{-40}.}$$

$\boxed{\text{Yes,}}$ the gravitational force may be safely ignored.

76. The potential difference produces a kinetic energy of 12.3 eV, so it is possible to provide this much energy to the hydrogen atom through collisions. From the ground state, the maximum energy of the atom is −13.6 eV + 12.3 eV = −1.3 eV. From the energy level diagram, we see that this means the atom could be excited to the *n* = 3 state, so the possible transitions when the atom returns to the ground state are *n* = 3 to *n* = 2, *n* = 3 to *n* = 1, and *n* = 2 to *n* = 1. For the wavelengths we have

$$\lambda_{3\to2}=\frac{\left(1.24\times10^3\text{ eV}\bullet\text{nm}\right)}{(E_3-E_2)}=\frac{\left(1.24\times10^3\text{eV}\bullet\text{nm}\right)}{\left[-1.5\text{ eV}-(-3.4\text{ eV})\right]}=\boxed{653\,\text{nm;}}$$

$$\lambda_{3\to1}=\frac{\left(1.24\times10^3\text{eV}\bullet\text{nm}\right)}{(E_3-E_1)}=\frac{\left(1.24\times10^3\text{eV}\bullet\text{nm}\right)}{\left[-1.5\text{ eV}-(-13.6\text{ eV})\right]}=\boxed{102\,\text{nm;}}$$

$$\lambda_{2\to1}=\frac{\left(1.24\times10^3\text{ eV}\bullet\text{nm}\right)}{(E_2-E_1)}=\frac{\left(1.24\times10^3\text{ eV}\bullet\text{nm}\right)}{\left[-3.4\text{ eV}-(-13.6\text{ eV})\right]}=\boxed{122\,\text{nm;}}$$

77. The stopping potential is the voltage that gives a potential energy change equal to the maximum kinetic energy:

$$\text{KE}_{\max} = eV_0 = \frac{hc}{\lambda} - W_0;$$

$$(1\text{e})(2.10\,\text{V})(1.60\times10^{-19}\,\text{J/eV}) = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{290\times10^{-9}\,\text{m}} - W_0,$$

which gives $W_0 = 3.50\times10^{-19}\,\text{J}$.

With the 440-nm light, then,

$$eV_0 = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{440\times10^{-9}\,\text{m}} - 3.50\times10^{-19}\,\text{J}$$

$$= 1.02\times10^{-19}\,\text{J} = \frac{1.02\times10^{-19}\,\text{J}}{1.60\times10^{-19}\,\text{J/eV}} = 0.64\text{ eV}.$$

The potential difference needed to cancel an electron kinetic energy of 0.64 eV is $\boxed{0.64\text{ V.}}$

78. (*a*) The electron has a charge *e*, so the potential difference produces a kinetic energy of *eV*. The shortest wavelength photon is produced when all the kinetic energy is lost and a photon emitted:

$$hf_{\max} = \frac{hc}{\lambda_0} = eV, \text{ which gives } \lambda_0 = \frac{hc}{eV}.$$

(*b*) $\lambda_0 = \dfrac{hc}{eV} = \dfrac{(1.24\times10^{3}\,\text{eV•nm})}{(30\,\text{keV})} = \boxed{0.041\,\text{nm.}}$

79. The energy of a photon in terms of the momentum is

$$E = hf = \frac{hc}{\lambda} = pc.$$

The rate at which photons are striking the sail is

$$\frac{N}{\Delta t} = \frac{IA}{E} = \frac{IA}{pc}.$$

Because the photons reflect from the sail, the change in momentum of a photon is

$$\Delta p = 2p.$$

The impulse on the sail is due to the change in momentum of the photons:

$F\Delta t = N\Delta p,$ or

$$F = \left(\frac{N}{\Delta t}\right)\Delta p = \left(\frac{IA}{pc}\right)(2p) = \frac{2IA}{c} = \frac{2(1000\,\text{W/m}^2)(1\times10^{3}\,\text{m})^2}{(3.00\times10^{8}\,\text{m/s})} = \boxed{7\text{ N.}}$$

80. The maximum kinetic energy of the photoelectrons is

$$\text{KE}_{\max} = \frac{hc}{\lambda} - W_0 = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{300\times10^{-9}\,\text{m}} - (2.2\text{ eV})(1.60\times10^{-19}\,\text{J/eV}) = 3.11\times10^{-19}\,\text{J}.$$

Since $p = mv$ and $\text{KE} = \frac{1}{2}mv^2$, $p = \sqrt{2m(\text{KE})}$, and so

$$\lambda_{\min} = \frac{h}{\sqrt{2m(\text{KE}_{\max})}} = \frac{6.63\times10^{-34}\,\text{J•s}}{\sqrt{2(9.11\times10^{-31}\,\text{kg})(3.11\times10^{-19}\,\text{J})}} = 8.8\times10^{-10}\,\text{m} = \boxed{0.88\,\text{nm.}}$$

81. With photon energy $hf = 6.0\text{ eV}$ and stopping potential $V_0 = 4.0\,\text{V}$, we write

$$eV_0 = hf - W_0;$$

$$4.0\text{ eV} = 6.0\text{ eV} - W_0, \text{ which gives } W_0 = 2.0\text{ eV}.$$

If the photons' wavelength is doubled, the energy is halved, from 6.0 eV to 3.0 eV. Now, with no stopping potential applied, we write

$$\text{KE}_{\text{max}} = 3.0\text{ eV} - 2.0\text{ eV} = \boxed{1.0\text{ eV.}}$$

If the photons' wavelength is tripled, the energy is reduced from 6.0 eV to 2.0 eV. And we write

$$\text{KE}_{\text{max}} = 2.0\text{ eV} - 2.0\text{ eV} = 0.$$

No electrons are emitted, and $\boxed{\text{no current}}$ flows.

82. We find the wavelength of the light from

$$d\sin\theta = m\lambda;$$

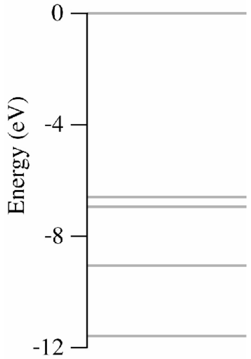
$$\left(0.010 \times 10^{-3}\,\text{m}\right)\sin 3.5° = (1)\lambda, \text{ which gives } \lambda = 6.10 \times 10^{-7}\,\text{m}.$$

Electrons with the same wavelength (and hence the same diffraction pattern) will have a velocity given by the equation

$$p = mv = \frac{h}{\lambda};$$

$$\left(9.11 \times 10^{-31}\text{kg}\right)v = \frac{6.63 \times 10^{-34}\,\text{J•s}}{6.10 \times 10^{-7}\,\text{m}}, \text{ which gives } v = \boxed{1.2 \times 10^{3}\,\text{m/s.}}$$

83. (*a*)



(*b*) Absorption of a 5.1 eV photon represents a transition from the $\boxed{\text{ground state}}$ to the state 5.1 eV above that, the third excited state. Possible photon emission energies are found by considering all the possible downward transitions that might occur as the electron makes its way back to the ground state:

$$5.1\text{ eV} - 4.7\text{ eV} = \boxed{0.4\text{ eV}}$$

$$4.7\text{ eV} - 2.5\text{ eV} = \boxed{2.2\text{ eV}}$$

$$2.5\text{ eV} - 0 = \boxed{2.5\text{ eV}}$$

but also

$$5.1\text{ eV} - 2.5\text{ eV} = \boxed{2.6\text{ eV}}$$

$$4.7\text{ eV} - 0 = \boxed{4.7\text{ eV}}$$

and

$$5.1\text{ eV} - 0 = \boxed{5.1\text{ eV.}}$$

84. (*a*) The stopping voltage is found from

$$eV_0 = \frac{hc}{\lambda} - W_0$$

$$= \frac{(6.63 \times 10^{-34}\,\text{J•s})(3.00 \times 10^{8}\,\text{m/s})}{(424 \times 10^{-9}\,\text{m})(1.60 \times 10^{-19}\,\text{J/eV})} - 2.28\,\text{eV} = 0.65\,\text{eV},\ \text{which gives } V_0 = \boxed{0.65\,\text{V.}}$$

(*b*) KE_{max} equals the energy found in part (a), namely 0.65 eV or

$$(0.65\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV}) = 1.04 \times 10^{-19}\,\text{J}.$$

The speed is determined by

$$\text{KE}_{\text{max}} = \tfrac{1}{2} m v_{\text{max}}^2;$$

$$1.04 \times 10^{-19}\,\text{J} = \tfrac{1}{2}(9.11 \times 10^{-31}\,\text{kg}) v_{\text{max}}^2,\ \text{which gives } v_{\text{max}} = \boxed{4.8 \times 10^{5}\,\text{m/s.}}$$

(*c*) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\,\text{J•s}}{(9.11 \times 10^{-31}\,\text{kg})(4.8 \times 10^{5}\,\text{m/s})} = 1.52 \times 10^{-9}\,\text{m} = \boxed{1.5\,\text{nm.}}$$

85. The electron acquires a kinetic energy of 96 eV. This is related to the speed by the equation

$$\text{KE} = \tfrac{1}{2} m v^2;$$

$$(96\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV}) = \tfrac{1}{2}(9.11 \times 10^{-31}\,\text{kg}) v^2,\ \text{which gives } v = 5.81\,\text{m/s}.$$

The ratio $\frac{e}{m}$ is then found using

$$\frac{e}{m} = \frac{v}{Br} = \frac{5.81\,\text{m/s}}{(3.67 \times 10^{-4}\,\text{T})(9.0 \times 10^{-2}\,\text{m})} = \boxed{1.8 \times 10^{11}\,\text{C/kg.}}$$

86. First we find the area of a sphere whose radius is the Earth–Sun distance:

$$A = 4\pi r^2 = 4\pi(150 \times 10^{9}\,\text{m})^2 = 2.83 \times 10^{23}\,\text{m}^2.$$

We multiply this by the given intensity to find the Sun's total power output:

$$(2.83 \times 10^{23}\,\text{m}^2)(1350\,\text{W/m}^2) = 3.82 \times 10^{26}\,\text{W}.$$

Multiplying by the number of seconds in a year gives the annual energy output

$$E = (3.82 \times 10^{26}\,\text{W})(3600\,\text{s/h})(24\,\text{h/d})(365.25\,\text{d/y}) = 1.20 \times 10^{34}\,\text{J/y}.$$

Finally we divide by the energy of one photon to find the number of photons per year:

$$\frac{E}{hf} = \frac{E\lambda}{hc} = \frac{(1.20 \times 10^{34}\,\text{J/y})(550 \times 10^{-9}\,\text{m})}{(6.63 \times 10^{-34}\,\text{J•s})(3.00 \times 10^{8}\,\text{m/s})} = \boxed{3.33 \times 10^{52}\,\text{photons/y.}}$$

87. We can use Bohr's analysis of the hydrogen atom, where we replace the proton mass with Earth's mass, the electron mass with the Moon's mass, and the electrostatic force $F_e = \frac{ke^2}{r^2}$ with the gravitational force $F_g = \frac{Gm_E m_M}{r^2}$. To account for the change in force, we replace ke^2 with $Gm_E m_M$.

With these replacements, the expression for the Bohr radius changes from

$$r_1 = \frac{h^2}{4\pi^2 mke^2}\ \text{to}\ r_1 = \frac{h^2}{4\pi^2 G m_E m_M^2},$$

while the expression for the lowest energy level changes from

$$E_1 = -\frac{2\pi^2 e^4 mk^2}{h^2} \text{ to } E_1 = -\frac{2\pi^2 G^2 m_E^2 m_M^3}{h^2}.$$

With $r_n = n^2 r_1$ and $E_n = \frac{E_1}{n^2}$, we find

$$r_n = \frac{n^2\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{4\pi^2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)\left(7.35\times10^{22}\,\text{kg}\right)^2} = \boxed{n^2\left(5.17\times10^{-129}\,\text{m}\right)}$$

and

$$E_n = -\frac{2\pi^2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)^2\left(5.98\times10^{24}\,\text{kg}\right)^2\left(7.35\times10^{22}\,\text{kg}\right)^3}{n^2\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}$$

$$= \boxed{-\frac{2.84\times10^{165}\,\text{J}}{n^2}.}$$

With the actual value $r_n = 384\times10^6\,\text{m}$, it follows that

$$n = \sqrt{\frac{384\times10^6\,\text{m}}{5.17\times10^{-129}\,\text{m}}} = 2.73\times10^{68}.$$

The difference between $n = 2.73\times10^{68}$ and $n+1 = 2.73\times10^{68}+1$ certainly makes no detectable difference in the values of r_n and E_n. Hence the quantization of energy and radius is $\boxed{\text{not apparent.}}$

CHAPTER 28: Quantum Mechanics of Atoms

Answers to Questions

1. (*a*) Both the matter wave Ψ and the string wave describe/represent the wave amplitude with respect to time and position. A matter wave Ψ is three-dimensional, but a string wave is two-dimensional. The string wave tells you the displacement of any part of the string from its equilibrium position at any given time, but for a matter wave the $|\Psi|^2$ represents the probability of finding the mass in a particular volume. The wave speed on the string depends on the string tension and mass density, but the Ψ wave speed depends on the energy.

   (*b*) Both the matter wave Ψ and the EM wave describe/represent the wave amplitude with respect to time and position, although the EM wave describes the amplitudes of the electric and magnetic fields (their strengths). Again, for the matter wave the $|\Psi|^2$ represents the probability of finding the mass in a particular volume. An EM wave can be polarized, but a matter wave cannot. The EM wave represents a vector field, but a matter wave is a scalar. The speed of the EM wave is *c*, but the matter wave's speed depends on the energy.

2. Bohr's model of the atom has the electrons in particular orbits with precise values of radii, but quantum mechanics gives a wide range of possible radii for each electron and only gives the probability of finding the electron at a particular radius. The uncertainty principle also says that both the position and the momentum of an electron cannot be known precisely, but Bohr's model does predict exact values for both position and momentum.

3. As a particle becomes more massive, the uncertainty of its momentum $(\Delta p = m\Delta v)$ becomes larger, which reduces the uncertainty of its position (due to $\Delta x = \hbar / \Delta p$). Thus, with a small uncertainty of its position, we can do a better job of predicting its future position.

4. Because of the very small value of $\hbar$, the uncertainties in both a baseball's momentum and position can be very small compared to typical macroscopic positions and momenta without being close to the limit imposed by the uncertainty principle. For instance, uncertainty in the baseball's position could be 10^{-10} m, and the uncertainty in the baseball's momentum $10^{-10}\ \text{kg}\bullet\text{m/s}$, and still easily satisfy the uncertainty principle. Yet we never try to measure the position or momentum of a baseball to that precision. For an electron, however, typical values of uncertainty in position and momentum can be close to the uncertainty principle limit, and so the relative uncertainty can be much higher for that small object.

5. No, it would not be possible to balance a very sharp needle precisely on its point. Both the center of mass of the needle and the point of the needle cannot be precisely known, especially when speed and momentum are small. If we don't know exactly where they are, but we need the center of mass directly above the point to balance, it is not possible. Also, the speed and momentum of both the center of mass and the point cannot be precisely known, especially if we think we know their positions exactly. Thus, if we do know that the center of mass and the point lined up exactly, then we couldn't also know that they are both not moving, and it would fall over.

6. No. The hot soup must warm up the thermometer when they come in contact, and so the final temperature of the thermometer and soup system will be less than the initial temperature of the soup. The thermometer reading will be lower than the original temperature of the soup. This is another

situation where making the measurement affects the physical situation and causes error or uncertainty to enter in the measurement.

7. Yes. The uncertainty principle tells us that the precision of a position measurement is dependent on the precision of the momentum ($\Delta x \Delta p = \hbar$). Thus, the uncertainty principle limits our ability to make a measurement of position. If it was possible to allow the uncertainty of the momentum to go to ∞ , then we could make an exact measurement of *x*.

8. If $\Delta x = 0$, then $\Delta p = \infty$. Thus, the uncertainty principle tells us that if we knew the position of an object precisely, then we would not know anything about the momentum.

9. Yes, air usually escapes when you check the air pressure in a tire, since some of the air needs to flow into the device you are using to check the pressure. The relationship to the uncertainty principle is that by making the measurement, you have affected the state of the system.

10. Yes, this is consistent with the uncertainty principle for energy $\left(\Delta E \Delta t = \hbar\right)$. The ground state energy can be fairly precisely known because the electrons remain in that state for a very long time. Thus, as $\Delta t \to \infty$, then $\Delta E \to 0$. For the excited states, the electrons do not remain in them for very long, thus the Δt in this case is relatively small, making the ΔE , the energy width, fairly large.

11. The quantum mechanical model predicts that the electron in the hydrogen atom will spend more time near the nucleus. The Bohr model predicts that the electron is always at some certain distance from the nucleus and that it can't get any closer if it is in the lowest energy level. Quantum mechanics predicts that the electron spends time both closer and farther from the nucleus (compared to Bohr), since it can be described as an "electron cloud" that is centered upon the Bohr radius. Thus, the quantum mechanical model predicts that the electron will spend more time near the nucleus. See Figure 28-6 for a good visual of this situation.

12. As the number of electrons goes up, so does the number of protons in the nucleus, which increases the attraction of the electrons to the center of the atom. Even though the outer electrons are partially screened by the inner electrons, they are all pulled closer to the very positive nuclei of the large Z atoms when compared to the low Z atoms. This keeps the atom from expanding in a "linear" fashion as larger numbers of electrons are added to the large Z atoms.

13. In helium, the electron falling down to the lowest energy level is interacting with another electron (which doesn't happen in hydrogen) and the electron is interacting with a more positive nucleus than in hydrogen. Thus, the emission spectrum in helium is very different than in hydrogen. The excited electrons are in very different environments, thus they have very different energy levels with different spacings. So, even though both atoms have such a ground state (n = 1, ℓ = 0, m_ℓ = 0), there are very different states above that ground state in which the electrons can exist. This then creates different emission spectral lines when the electrons return to the ground state.

14 The ground state configurations could only have half the number of electrons in each shell: $\ell = 0$ shells would only have 1 state, $\ell = 1$ shells would have 3 states, $\ell = 2$ shells would have 5 states, etc. With no spin number, the Pauli Exclusion Principle says that only one electron is allowed per state. Thus, we would need more shells to hold all of the electrons and the elements would need to be "stacked" differently in the periodic table, since 1 electron would fill the *s* shell and 3 electrons would fill the *p* shell and 5 electrons would fill the *d* shell, etc. The periodic table would look something like this:

$^{1}H_{1s}1$
$^{2}He_{2s}1$ $^{3}Li_{2p}1$ $^{4}Be_{2p}2$ $^{5}B_{2p}3$
$^{6}C_{3s}1$ $^{7}N_{3p}1$ $^{8}O_{3p}2$ $^{9}F_{3p}3$
$^{10}Ne_{4s}1$ $^{11}Na_{3d}1$ $^{12}Mg_{3d}2$ $^{13}Al_{3d}3$ $^{14}Si_{3d}4$ $^{15}P_{3d}5$ $^{16}S_{4p}1$ $^{17}Cl_{4p}2$ $^{18}Ar_{4p}3$
$^{19}K_{5s}1$ $^{20}Ca_{4d}1$

$\boxed{15.}$ (*a*) Allowed.
(*b*) Not allowed. Cannot have 8 electrons in the *p* shell.
(*c*) Allowed.

16. Uranium: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^2\ 5d^{10}\ 6p^6\ 4f^{14}\ 7s^2\ 5f^3\ 6d^1$.

$\boxed{17.}$ (*a*) Group II. Outer electrons are $3s^2$: magnesium.
(*b*) Group VIII. Outer electrons are $3s^2\ 3p^6$: argon.
(*c*) Group I. Outer electron is $4s^1$: potassium.
(*d*) Group VII. Outer electrons are $2s^2\ 2p^5$: fluorine.

$\boxed{18.}$ Both chlorine and iodine are in the same column of the periodic table (Group VII). Thus, both of their outer electron configurations are p^5 (5 electrons in their outer *p* shell), which makes them both halogens. Halogens lack one electron from a filled shell, so both of their outer electron orbit shapes are similar and can readily accept an additional electron from another atom. This makes their chemical reaction properties extremely similar.

19. Both sodium and potassium are in the same column of the periodic table (Group I). Thus, both of their outer electron configurations are s^1 (1 electron in their outer *s* shell), which makes them both alkali metals. Alkali metals have one lone electron in their outermost shell, so both of their outer electron orbit shapes are similar and they can readily share this electron with another atom (especially since this outermost electron spends a considerable amount of time very far away from the nucleus). This makes their chemical reaction properties extremely similar.

20. Neon is a "closed shell" atom (a noble gas) where all of the electron shells are completely full and spherically symmetric, and so its electrons are not readily ionized. Sodium is a Group I atom (an alkali metal) that has one lone electron in the outermost shell, and this electron spends a considerable amount of time far away from the nucleus and also shielded from the nucleus. The result is that sodium can be ionized much easier than neon, which the two ionization energies demonstrate very well.

21. As electrons are decelerated in their interactions with positive nuclei in a material, they radiate energy (called bremsstrahlung radiation). Because energy is conserved, the energy of the emitted photon must equal the loss of kinetic energy of the electron. An electron may lose all or part of its kinetic energy in such a collision. The continuous X-ray spectrum of Figure 28-11 is explained as being due to such bremsstrahlung collisions where varying amounts of energy are lost by the electrons. The shortest wavelength X-ray (λ_o) must be due to an electron that gives up all of its kinetic energy to produce one photon in a single collision. If X-rays were not photons but particles with mass, some of the kinetic energy would have to go into creating the mass of this particle, but since the λ_o corresponds directly to the kinetic energy of the incoming electrons, X-rays must be massless photons. Thus, this result is evidence that the photon theory of light is valid.

22. When we use the Bohr theory to predict the X-ray line wavelengths, we need to estimate the amount that the electron that is changing to a lower level is partially shielded by the other electron(s) in those lower

levels. So, for example, if we are looking at an electron changing into the $n = 1$ level, we estimate that the falling electron "sees" a nucleus of charge $Z - 1$, instead of just Z. The problem with this estimate is that the lower level electron(s) does not do a perfect job of shielding the changing electron, thus our calculated X-ray line wavelengths are slightly different from the measured ones.

23. To figure out which lines in an X-ray spectrum correspond to which transitions, you would use the Bohr model to estimate the differences in the sizes of the energy level jumps that the falling electrons will make. Then, you just need to match the $n = 2 \longrightarrow n = 1$ energy to the K_α line and the $n = 3 \longrightarrow n = 1$ energy to the K_β line and the $n = 3 \longrightarrow n = 2$ jump energy to the L_α line and so on.

24. The difference in energy between $n = 1$ and $n = 2$ levels is much bigger than between any other combination of energy levels. Thus, electron transitions between the $n = 1$ and $n = 2$ levels produce photons of extremely high energy and frequency, which means these photons have very short wavelengths (since frequency and wavelength are inversely proportional to each other).

25. Spontaneous emission occurs when an electron is in an excited state of an atom and it spontaneously (with no external stimulus) drops back down to a lower energy level. To do this, it emits a photon to carry away the excess energy. Stimulated emission occurs when an electron is in an excited state of an atom but a photon strikes the atom and causes or stimulates the electron to make its transition to a lower energy level sooner than it would have done so spontaneously. The stimulating photon has to have the same energy as the difference in energy levels of the transition.

26. Different: Laser light is coherent (all of the photons are in the same phase) and ordinary light is not coherent (all of the photons have random phases). Laser light is nearly a perfect plane wave, while ordinary light is spherically symmetric (which means that the intensity of laser light remains nearly constant as it moves away from the source, while the intensity of ordinary light drops off as $1/r^2$). Laser light is always monochromatic, while ordinary light can be monochromatic, but it usually is not. Similar: Both travel at c and both display wave-particle duality of photons.

27. The 0.0005 W laser beam's intensity is nearly constant as it travels away from its source due to it being approximately a plane wave. The street lamp's intensity, though, drops off as $1/r^2$ as it travels away from its source due to it being a spherically symmetric wave. So, at a distance, the intensity of the lamp light that is actually reaching the camera is much less than 1000 W. Also, the street lamp's intensity is spread out over many wavelengths, whereas the laser's intensity is all at one wavelength, and film can be more sensitive to certain wavelengths.

28. No, the intensity of light from a laser beam does not drop off as the inverse square of the distance. Laser light is much closer to being a plane wave rather than a spherically symmetric wave, and so its intensity is nearly constant along the entire beam.

Problem Solutions

Note: At the atomic scale, it is most convenient to have energies in electron-volts and wavelengths in nanometers. A useful expression for the energy of a photon in terms of its wavelength is

$$E = hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \mathrm{J\bullet s}\right)\left(3\times10^{8}\ \mathrm{m/s}\right)\left(10^{-9}\ \mathrm{nm/m}\right)}{\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)\lambda};$$

$$E = \frac{\left(1.24\times10^{3}\,\mathrm{eV{\bullet}nm}\right)}{\lambda}.$$

1. We find the wavelength of the neutron from

$$\lambda = \frac{h}{p} = \frac{h}{\left[2m_0\left(\text{KE}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)}{\left[2\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(0.025\,\mathrm{eV}\right)\left(1.6\times10^{-19}\,\mathrm{J/eV}\right)\right]^{\frac{1}{2}}} = 1.81\times10^{-10}\,\mathrm{m}.$$

The peaks of the interference pattern are given by

$$d\sin\theta = n\lambda,\ n = 1,\ 2, \ldots .$$

and the positions on the screen are

$$y = L\tan\theta.$$

For small angles, $\sin\theta = \tan\theta$, so we have

$$y = \frac{nL\lambda}{d}.$$

Thus the separation is

$$\Delta y = \frac{L\lambda}{d} = \frac{\left(1.0\,\mathrm{m}\right)\left(1.81\times10^{-10}\,\mathrm{m}\right)}{\left(0.50\,\mathrm{m}\right)} = \boxed{3.6\times10^{-7}\,\mathrm{m.}}$$

2. We find the wavelength of the bullet from

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)}{\left(3.0\times10^{-3}\,\mathrm{kg}\right)\left(220\,\mathrm{m/s}\right)} = 1.0\times10^{-33}\,\mathrm{m}.$$

The half-angle for the central circle of the diffraction pattern is given by

$$\sin\theta = \frac{1.22\lambda}{D}.$$

For small angles, $\sin\theta \approx \tan\theta$, so we have

$$r = L\tan\theta \approx L\sin\theta = \frac{1.22L\lambda}{D};$$

$$0.50\times10^{-2}\,\mathrm{m} = \frac{1.22L\left(1.0\times10^{-33}\,\mathrm{m}\right)}{\left(3.0\times10^{-3}\,\mathrm{m}\right)},\ \text{which gives } L = \boxed{1.2\times10^{28}\,\mathrm{m.}}$$

Diffraction effects are negligible for macroscopic objects.

$\boxed{3.}$ We find the uncertainty in the momentum:

$$\Delta p = m\Delta v = \left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(0.024\times10^{5}\,\mathrm{m/s}\right) = 4.00\times10^{-24}\,\mathrm{kg{\bullet}m/s}.$$

We find the uncertainty in the proton's position from

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)}{\left(4.00\times10^{-24}\,\mathrm{kg{\bullet}m/s}\right)} = 2.6\times10^{-11}\,\mathrm{m}.$$

Thus the accuracy of the position is $\boxed{\pm1.3\times10^{-11}\,\mathrm{m.}}$

4. We find the uncertainty in the momentum:

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(2.0 \times 10^{-8}\ \text{m}\right)} = 5.28 \times 10^{-27}\ \text{kg}\bullet\text{m/s}.$$

We find the uncertainty in the velocity from

$$\Delta p = m\,\Delta v;$$

$$5.28 \times 10^{-27}\ \text{kg}\bullet\text{m/s} = \left(9.11 \times 10^{-31}\ \text{kg}\right)\Delta v, \text{ which gives } \Delta v = \boxed{5.8 \times 10^{3}\ \text{m/s}.}$$

5. We find the minimum uncertainty in the energy of the state from

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(10^{-8}\ \text{s}\right)} = 1.1 \times 10^{-26}\ \text{J} = \boxed{6.6 \times 10^{-8}\ \text{eV}.}$$

6. We find the lifetime of the particle from

$$\Delta t \geq \frac{\hbar}{\Delta E} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(2.49\,\text{GeV}\right)\left(1.60 \times 10^{-10}\ \text{J/GeV}\right)} = \boxed{2.65 \times 10^{-25}\ \text{s}.}$$

7. We find the uncertainty in the energy of the muon from

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(2.2 \times 10^{-6}\ \text{s}\right)} = 4.8 \times 10^{-29}\ \text{J} = \boxed{3.0 \times 10^{-10}\ \text{eV}.}$$

Thus the uncertainty in the mass is

$$\Delta m = \frac{\Delta E}{c^2} = \boxed{3.0 \times 10^{-10}\ \text{eV}/c^2.}$$

$\boxed{8.}$ We find the uncertainty in the energy of the free neutron from

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(900\,\text{s}\right)} = 1.17 \times 10^{-37}\ \text{J}.$$

Thus the uncertainty in the mass is

$$\Delta m = \frac{\Delta E}{c^2} = \frac{\left(1.17 \times 10^{-37}\ \text{J}\right)}{\left(3.00 \times 10^{8}\ \text{m/s}\right)^2} = \boxed{1.30 \times 10^{-54}\ \text{kg}.}$$

9. The uncertainty in the velocity is

$$\Delta v = \left(\frac{0.055}{100}\right)\left(150\,\text{m/s}\right) = 0.0825\,\text{m/s}.$$

For the electron, we have

$$\Delta x \geq \frac{\hbar}{m\,\Delta v} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(0.0825\,\text{m/s}\right)} = \boxed{1.4 \times 10^{-3}\ \text{m}.}$$

For the baseball, we have

$$\Delta x \geq \frac{\hbar}{m\,\Delta v} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(0.140\,\text{kg}\right)\left(0.0825\,\text{m/s}\right)} = \boxed{9.1 \times 10^{-33}\ \text{m}.}$$

The uncertainty for the electron is greater by a factor of 1.5×10^{29}.

10. We use the radius as the uncertainty in position for the neutron. We find the uncertainty in the momentum from

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(1.0\times10^{-15}\ \text{m}\right)} = \boxed{1.055\times10^{-19}\ \text{kg}\bullet\text{m/s.}}$$

If we assume that the lowest value for the momentum is the least uncertainty, we estimate the lowest possible kinetic energy as

$$E = \frac{(\Delta p)^2}{2m} = \frac{\left(1.055\times10^{-19}\ \text{kg}\bullet\text{m/s}\right)^2}{2\left(1.67\times10^{-27}\ \text{kg}\right)} = 3.33\times10^{-12}\ \text{J} = \boxed{21\text{MeV.}}$$

11. We use the radius as the uncertainty in position for the electron. We find the uncertainty in the momentum from

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(1.0\times10^{-15}\ \text{m}\right)} = 1.055\times10^{-19}\ \text{kg}\bullet\text{m/s}.$$

If we assume that the lowest value for the momentum is the least uncertainty, we estimate the lowest possible energy as

$$E = (\text{KE}) + m_0c^2 = \left(p^2c^2 + m_0{}^2c^4\right)^{\frac{1}{2}} = \left[(\Delta p)^2c^2 + m_0{}^2c^4\right]^{\frac{1}{2}}$$

$$= \left[\left(1.055\times10^{-19}\ \text{kg}\bullet\text{m/s}\right)^2\left(3.00\times10^8\ \text{m/s}\right)^2 + \left(9.11\times10^{-31}\ \text{kg}\right)^2\left(3.00\times10^8\ \text{m/s}\right)^4\right]^{\frac{1}{2}}$$

$$= 3.175\times10^{-11}\ \text{J} \approx \boxed{200\,\text{MeV.}}$$

12. The momentum of the electron is

$$p = \left[2m(\text{KE})\right]^{\frac{1}{2}} = \left[2\left(9.11\times10^{-31}\ \text{kg}\right)(3.00\,\text{keV})\left(1.60\times10^{-16}\ \text{J/keV}\right)\right]^{\frac{1}{2}} = 2.96\times10^{-23}\ \text{kg}\bullet\text{m/s}.$$

When the energy is increased by 1.00%, the new momentum is

$$p' = \left[2m(1.0100\text{KE})\right]^{\frac{1}{2}} = \left[2m(\text{KE})\right]^{\frac{1}{2}}(1+0.0100)^{\frac{1}{2}} \approx p\left[1+\tfrac{1}{2}(0.0100)\right].$$

Thus the uncertainty in momentum is

$$\Delta p = p' - p = p\tfrac{1}{2}(0.0100) = \left(2.96\times10^{-23}\ \text{kg}\bullet\text{m/s}\right)^{\frac{1}{2}}(0.0100) = 1.48\times10^{-25}\ \text{kg}\bullet\text{m/s}.$$

We find the uncertainty in the electron's position from

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\left(1.055\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(1.48\times10^{-25}\ \text{kg}\bullet\text{m/s}\right)} = \boxed{7.13\times10^{-10}\ \text{m.}}$$

13. The value of ℓ can range from 0 to $n-1$. Thus for n = 6, we have $\boxed{\ell = 0, 1, 2, 3, 4, 5.}$

14. The value of m_ℓ can range from $-\ell$ to $+\ell$. Thus for $\ell = 3$, we have $\boxed{m_\ell = -3, -2, -1, 0, 1, 2, 3.}$

The possible values of m_s are $\boxed{-\tfrac{1}{2}, +\tfrac{1}{2}.}$

$\boxed{15.}$ The number of electrons in the subshell is determined by the value of ℓ. For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values. For each of these there are two values of m_s. Thus the total number for $\ell = 3$ is

$$N = 2(2\ell+1) = 2\left[2(3)+1\right] = \boxed{14\text{ electrons.}}$$

16. The value of ℓ can range from 0 to $n-1$. Thus for $n=4$, we have

$$\ell = 0, 1, 2, 3.$$

For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values. For each of these there are two values of m_s. Thus the total number for each ℓ is $2(2\ell+1)$.

The number of states is

$$N = 2(0+1) + 2(2+1) + 2(4+1) + 2(6+1) = \boxed{32\,\text{states.}}$$

We start with $\ell = 0$, and list the quantum numbers in the order (n, ℓ, m_ℓ, m_s);

$$\boxed{\begin{aligned}
&(4, 0, 0, -\tfrac{1}{2}),\ (4, 0, 0, +\tfrac{1}{2}),\ (4, 1, -1, -\tfrac{1}{2}),\ (4, 1, -1, +\tfrac{1}{2}),\ (4, 1, 0, -\tfrac{1}{2}),\ (4, 1, 0, +\tfrac{1}{2}),\\
&(4, 1, 1, -\tfrac{1}{2}),\ (4, 1, 1, +\tfrac{1}{2}),\ (4, 2, -2, -\tfrac{1}{2}),\ (4, 2, -2, +\tfrac{1}{2}),\ (4, 2, -1, -\tfrac{1}{2}),\ (4, 2, -1, +\tfrac{1}{2}),\\
&(4, 2, 0, -\tfrac{1}{2}),\ (4, 2, 0, +\tfrac{1}{2}),\ (4, 2, 1, -\tfrac{1}{2}),\ (4, 2, 1, +\tfrac{1}{2}),\ (4, 2, 2, -\tfrac{1}{2}),\ (4, 2, 2, +\tfrac{1}{2}),\\
&(4, 3, -3, -\tfrac{1}{2}),\ (4, 3, -3, +\tfrac{1}{2}),\ (4, 3, -2, -\tfrac{1}{2}),\ (4, 3, -2, +\tfrac{1}{2}),\ (4, 3, -1, -\tfrac{1}{2}),\ (4, 3, -1, +\tfrac{1}{2}),\\
&(4, 3, 0, -\tfrac{1}{2}),\ (4, 3, 0, +\tfrac{1}{2}),\ (4, 3, 1, -\tfrac{1}{2}),\ (4, 3, 1, +\tfrac{1}{2}),\ (4, 3, 2, -\tfrac{1}{2}),\ (4, 3, 2, +\tfrac{1}{2}),\\
&(4, 3, 3, -\tfrac{1}{2}),\ (4, 3, 3, +\tfrac{1}{2}).
\end{aligned}}$$

17. We start with the $n=1$ shell, and list the quantum numbers in the order (n, ℓ, m_ℓ, m_s);

(*a*) $\boxed{(1, 0, 0, -\tfrac{1}{2}),\ (1, 0, 0, +\tfrac{1}{2}),\ (2, 0, 0, -\tfrac{1}{2}),\ (2, 0, 0, +\tfrac{1}{2}),\ (2, 1, -1, -\tfrac{1}{2}),\ (2, 1, -1, +\tfrac{1}{2}).}$

Note that, without additional information, there are other possibilities for the last three electrons.

(*b*)

$$\boxed{\begin{aligned}
&(1, 0, 0, -\tfrac{1}{2}),\ (1, 0, 0, +\tfrac{1}{2}),\ (2, 0, 0, -\tfrac{1}{2}),\ (2, 0, 0, +\tfrac{1}{2}),\ (2, 1, -1, -\tfrac{1}{2}),\ (2, 1, -1, +\tfrac{1}{2}),\\
&(2, 1, 0, -\tfrac{1}{2}),\ (2, 1, 0, +\tfrac{1}{2}),\ (2, 1, -1, -\tfrac{1}{2}),\ (2, 1, -1, +\tfrac{1}{2}),\ (3, 0, 0, -\tfrac{1}{2}),\ (3, 0, 0, +\tfrac{1}{2}).
\end{aligned}}$$

18. We start with the $n=1$ shell, and list the quantum numbers in the order (n, ℓ, m_ℓ, m_s);

$$\boxed{\begin{aligned}
&(1, 0, 0, -\tfrac{1}{2}),\ (1, 0, 0, +\tfrac{1}{2}),\ (2, 0, 0, -\tfrac{1}{2}),\ (2, 0, 0, +\tfrac{1}{2}),\ (2, 1, -1, -\tfrac{1}{2}),\ (2, 1, -1, +\tfrac{1}{2}),\\
&(2, 1, 0, -\tfrac{1}{2}).
\end{aligned}}$$

Note that, without additional information, there are other possibilities for the last three electrons.

19. The value of ℓ can range from 0 to $n-1$. Thus for $\ell = 4$, we have $\boxed{n \geq 5.}$

For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$: $\boxed{m_\ell = -4, -3, -2, -1, 0, 1, 2, 3, 4.}$

There are two values of m_s: $\boxed{m_s = -\tfrac{1}{2}, +\tfrac{1}{2}.}$

20. The magnitude of the angular momentum depends on ℓ only:

$$L = \hbar\left[\ell(\ell+1)\right]^{\frac{1}{2}} = \left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)\left[(3)(3+1)\right]^{\frac{1}{2}} = \boxed{3.66\times10^{-34}\,\text{kg}\bullet\text{m}^2/\text{s.}}$$

21. The value of m_ℓ can range from $-\ell$ to $+\ell$, so we have $\boxed{\ell \geq 3.}$

The value of ℓ can range from 0 to $n-1$. Thus we have $\boxed{n \geq \ell + 1\,(\text{minimum}\,4).}$

There are two values of m_s: $\boxed{m_s = -\tfrac{1}{2}, +\tfrac{1}{2}.}$

22. From *spdfg*, we see that the "*g*" subshell has $\ell = 4,$ so the number of electrons is

$$N = 2(2\ell + 1) = 2\left[2(4) + 1\right] = \boxed{18 \text{ electrons.}}$$

23. (*a*) We start with hydrogen and fill the levels as indicated in the periodic table:

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^7 4s^2.}$$

Note that the $4s^2$ level is filled before the 3*d* level is started.

(*b*) For $Z = 36$ we have

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6.}$$

(*c*) For $Z = 38$ we have

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 5s^2.}$$

Note that the $5s^2$ level is filled before the 4*d* level is started.

24. (*a*) Selenium has $Z = 34$:

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^4.}$$

(*b*) Gold has $Z = 79$:

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^1.}$$

(*c*) Radium has $Z = 88$:

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6 7s^2.}$$

25. (*a*) The principal quantum number is $n = \boxed{6.}$

(*b*) The energy of the state is

$$E_6 = -\frac{(13.6\,\text{eV})}{n^2} = -\frac{(13.6\,\text{eV})}{6^2} = \boxed{-0.378\,\text{eV.}}$$

(*c*) The "*s*" subshell has $\ell = \boxed{0.}$ The magnitude of the angular momentum depends on ℓ only:

$$L = \hbar\left[\ell(\ell + 1)\right]^{\frac{1}{2}} = \left(1.055 \times 10^{-34}\,\text{J•s}\right)\left[(0)(0 + 1)\right]^{\frac{1}{2}} = \boxed{0.}$$

(*d*) For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$: $\boxed{m_\ell = 0.}$

26. The third electron in lithium is in the 2*s* subshell, which is outside the more tightly bound filled 1*s* shell. This makes it appear as if there is a "nucleus" with a net charge of $+1e$. Thus we use the energy of the hydrogen atom:

$$E_2 = -\frac{(13.6\,\text{eV})}{n^2} = -\frac{(13.6\,\text{eV})}{2^2} = -3.4\,\text{eV},$$

so the binding energy is $\boxed{3.4 \text{ eV.}}$

Our assumption of complete shielding of the nucleus by the 2*s* electrons is probably not correct.

27. In a filled subshell, we have $2(2\ell + 1)$ electrons. All of the m_ℓ values

$$-\ell, -\ell + 1, \ldots, 0, \ldots, \ell - 1, \ell$$

are filled, so their sum is zero. For each m_ℓ value, both values of m_s are filled, so their sum is also zero. Thus the total angular momentum is $\boxed{\text{zero.}}$

28. (*a*) The $4p \rightarrow 3p$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell \neq \pm 1.$
    (*b*) The $2p \rightarrow 1s$ transition is $\boxed{\text{allowed,}}$ $\Delta\ell = -1.$
    (*c*) The $3d \rightarrow 2d$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell \neq \pm 1.$
    (*d*) The $4d \rightarrow 3s$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell \neq \pm 1.$
    (*e*) The $4s \rightarrow 3p$ transition is $\boxed{\text{allowed,}}$ $\Delta\ell = +1.$

29. Photon emission means a jump to a lower state, so $n = 1, 2, 3, 4$, or 5. For a *d* subshell, $\ell = 2,$ and because $\Delta\ell = \pm 1,$ the new value of ℓ must be 1 or 3.
    (*a*) $\ell = 1$ corresponds to a *p* subshell, and $\ell = 3$ corresponds to an *f* subshell. Keeping in mind that $0 \leq \ell \leq n - 1,$ we find the following possible destination states: $2p, 3p, 4p, 5p, 4f, 5f$.
    (*b*) In a hydrogen atom, ℓ has no appreciable effect on energy, and so for energy purposes there are four possible destination states, corresponding to *n* = 2, 3, 4, and 5. Thus there are $\boxed{\text{four different photon wavelengths}}$ corresponding to four possible changes in energy.

$\boxed{30.}$ The shortest wavelength X-ray has the most energy, which is the maximum kinetic energy of the electron in the tube:

$$\lambda = \frac{\left(1.24 \times 10^3 \text{ eV•nm}\right)}{E} = \frac{\left(1.24 \times 10^3 \text{ eV•nm}\right)}{\left(33.5 \times 10^3 \text{ eV}\right)} = \boxed{0.37 \text{ nm.}}$$

The longest wavelength of the continuous spectrum would be at the limit of the X-ray region of the electromagnetic spectrum, generally on the order of $\boxed{\text{1 nm.}}$

31. The shortest wavelength X-ray has the most energy, which is the maximum kinetic energy of the electron in the tube:

$$E = \frac{\left(1.24 \times 10^3 \text{ eV•nm}\right)}{\lambda} = \frac{\left(1.24 \times 10^3 \text{ eV•nm}\right)}{\left(0.030 \text{ nm}\right)} = 4.1 \times 10^4 \text{ eV} = 41 \text{keV}.$$

Thus the operating voltage of the tube is $\boxed{41 \text{kV.}}$

32. The energy of the photon with the shortest wavelength must equal the maximum kinetic energy of an electron:

$$hf_0 = \frac{hc}{\lambda_0} = eV, \text{ or}$$

$$\lambda_0 = \frac{hc}{eV} = \frac{\left(6.63 \times 10^{-34} \text{ J•s}\right)\left(3 \times 10^8 \text{ m/s}\right)\left(10^{-9} \text{ nm/m}\right)}{\left(1.60 \times 10^{-19} \text{ J/eV}\right)\left(1\text{e}\right)V}$$

$$= \frac{\left(1.24 \times 10^3 \text{ V•nm}\right)}{V}.$$

33. With the shielding provided by the remaining $n = 1$ electron, we use the energies of the hydrogen atom with *Z* replaced by $Z - 1$. The energy of the photon is

$$hf = \Delta E = -\left(13.6 \text{eV}\right)\left(24 - 1\right)^2 \left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{1^2}\right)\right] = 6.90 \times 10^3 \text{ eV}.$$

The wavelength of the photon is

$$\lambda = \frac{\left(1.24\times10^{3}\text{ eV}\bullet\text{nm}\right)}{\Delta E} = \frac{\left(1.24\times10^{3}\text{ eV}\bullet\text{nm}\right)}{\left(6.90\times10^{3}\text{ eV}\right)} = \boxed{0.18\,\text{nm.}}$$

$\boxed{34.}$ With the shielding provided by the remaining $n=1$ electron, we use the energies of the hydrogen atom with Z replaced by $Z-1$. The energy of the photon is

$$hf = \Delta E = -\left(13.6\,\text{eV}\right)\left(26-1\right)^{2}\left[\left(\frac{1}{2^{2}}\right)-\left(\frac{1}{1^{2}}\right)\right] = 6.40\times10^{3}\text{ eV}.$$

The wavelength of the photon is

$$\lambda = \frac{\left(1.24\times10^{3}\text{ eV}\bullet\text{nm}\right)}{\Delta E} = \frac{\left(1.24\times10^{3}\text{ eV}\bullet\text{nm}\right)}{\left(6.40\times10^{3}\text{ eV}\right)} = \boxed{0.19\,\text{nm.}}$$

35. If we assume that the shielding is provided by the remaining $n=1$ electron, we use the energies of the hydrogen atom with Z replaced by $Z-1$. The energy of the photon is

$$hf = \Delta E = -\left(13.6\,\text{eV}\right)\left(42-1\right)^{2}\left[\left(\frac{1}{3^{2}}\right)-\left(\frac{1}{1^{2}}\right)\right] = 2.03\times10^{4}\text{ eV}.$$

The wavelength of the photon is

$$\lambda = \frac{\left(1.24\times10^{3}\text{ eV}\bullet\text{nm}\right)}{\Delta E} = \frac{\left(1.24\times10^{3}\text{ eV}\bullet\text{nm}\right)}{\left(2.03\times10^{4}\text{ eV}\right)} = \boxed{0.061\,\text{nm.}}$$

We do not expect perfect agreement because there is some $\boxed{\text{partial shielding provided by the } n=2 \text{ shell,}}$ which was ignored when we replaced Z by $Z-1$.

36. The K_{α} line is from the $n=2$ to $n=1$ transition. We use the energies of the hydrogen atom with Z replaced by $Z-1$. Thus we have

$$hf = \Delta E \propto \left(Z-1\right)^{2},\ \text{so}\ \lambda \propto \frac{1}{\left(Z-1\right)^{2}}.$$

When we form the ratio for the two materials, we get

$$\frac{\lambda_{\chi}}{\lambda_{\text{iron}}} = \frac{\left(Z_{\text{iron}}-1\right)^{2}}{\left(Z_{\chi}-1\right)^{2}};$$

$$\frac{\left(229\,\text{nm}\right)}{\left(194\,\text{nm}\right)} = \frac{\left(26-1\right)^{2}}{\left(Z_{\chi}-1\right)^{2}},\ \text{which gives}\ Z_{\chi}=24,$$

so the material is $\boxed{\text{chromium.}}$

37. The energy of a pulse is

$$E = P\,\Delta t = \left(0.68\,\text{W}\right)\left(28\times10^{-3}\text{ s}\right) = \boxed{0.019\,\text{J.}}$$

The energy of a photon is

$$hf = \frac{hc}{\lambda} = \frac{\left(1.24\times10^{3}\text{ eV}\bullet\text{nm}\right)}{\lambda} = \frac{\left(1.24\times10^{3}\text{ eV}\bullet\text{nm}\right)}{\left(640\,\text{nm}\right)} = 1.94\,\text{eV}.$$

Thus the number of photons in a pulse is

$$N=\frac{E}{hf}=\frac{(0.019\,\mathrm{J})}{(1.94\,\mathrm{eV})(1.6\times10^{-19}\,\mathrm{J/eV})}=\boxed{6.1\times10^{16}\text{ photons.}}$$

38. Intensity equals power per area.

(*a*) $I=\dfrac{P}{S}=\dfrac{P}{\pi r^2}=\dfrac{0.50\times10^{-3}\,\mathrm{W}}{\pi(1.5\times10^{-3}\,\mathrm{m})^2}=\boxed{71\,\mathrm{W/m^2}.}$

(*b*) $I=\dfrac{P}{S}=\dfrac{P}{4\pi r^2}=\dfrac{40\,\mathrm{W}}{4\pi(2.0\,\mathrm{m})^2}=\boxed{0.80\,\mathrm{W/m^2}.}$

The laser beam is more intense by a factor of $\dfrac{71\,\mathrm{W/m^2}}{0.80\,\mathrm{W/m^2}}=\boxed{89.}$

39. We find the angular half width of the beam from

$$\Delta\theta=\frac{1.22\lambda}{d}=\frac{1.22(694\times10^{-9}\,\mathrm{m})}{(3.0\times10^{-3}\,\mathrm{m})}=2.8\times10^{-4}\,\mathrm{rad},$$

so the angular width is $\theta=\boxed{5.6\times10^{-4}\,\mathrm{rad}.}$

(*a*) The diameter of the beam when it reaches the satellite is

$$D=r\theta=(300\times10^3\,\mathrm{m})(5.6\times10^{-4}\,\mathrm{rad})=\boxed{1.7\times10^2\,\mathrm{m}.}$$

(*b*) The diameter of the beam when it reaches the Moon is

$$D=r\theta=(384\times10^6\,\mathrm{m})(5.6\times10^{-4}\,\mathrm{rad})=\boxed{2.2\times10^5\,\mathrm{m}.}$$

40. Transition from the E_3' state to the E_2' state releases photons with energy 1.96 eV. The wavelength is determined by

$$\lambda=\frac{(1.24\times10^3\,\mathrm{eV{\bullet}nm})}{\Delta E}=\frac{(1.24\times10^3\,\mathrm{eV{\bullet}nm})}{(1.96\,\mathrm{eV})}=\boxed{633\,\mathrm{nm}.}$$

41. We can relate the momentum to the radius of the orbit from the quantum condition:

$$L=mvr=pr=n\hbar,\ \text{so}\ p=\frac{n\hbar}{r}=\frac{\hbar}{r_1}\ \text{for the ground state.}$$

If we assume that this is the uncertainty of the momentum, the uncertainty of the position is

$$\Delta x\ge\frac{\hbar}{\Delta p}=\frac{\hbar}{\left(\dfrac{\hbar}{r_1}\right)}=\boxed{r_1\text{, which is the Bohr radius.}}$$

42. (*a*) We find the minimum uncertainty in the energy of the state from

$$\Delta E\ge\frac{\hbar}{\Delta t}=\frac{(1.055\times10^{-34}\,\mathrm{J{\bullet}s})}{(10^{-8}\,\mathrm{s})}=1.1\times10^{-26}\,\mathrm{J}=\boxed{6.6\times10^{-8}\,\mathrm{eV}.}$$

Note that, because the ground state is stable, we associate the uncertainty with the excited state.

(*b*) The transition energy is

$$E = -(13.6\,\text{eV})\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{1^2}\right)\right] = 10.2\,\text{eV},$$

so we have

$$\frac{\Delta E}{E} = \frac{(6.6\times10^{-8}\,\text{eV})}{(10.2\,\text{eV})} = \boxed{6.5\times10^{-9}.}$$

(*c*) The wavelength of the line is

$$\lambda = (1.24\times10^{3}\,\text{eV•nm})E = \frac{(1.24\times10^{3}\,\text{eV•nm})}{(10.2\,\text{eV})} = \boxed{122\,\text{nm.}}$$

We find the width of the line from

$$\lambda + \Delta\lambda = \frac{(1.24\times10^{3}\,\text{eV•nm})}{(E+\Delta E)}$$

$$= \frac{\left[\dfrac{(1.24\times10^{3}\,\text{eV•nm})}{E}\right]}{\left[1+\left(\dfrac{\Delta E}{E}\right)\right]} \approx \lambda\left[1-\left(\frac{\Delta E}{E}\right)\right].$$

If we ignore the sign, we get

$$\Delta\lambda = \lambda\left(\frac{\Delta E}{E}\right) = (122\,\text{nm})(6.5\times10^{-9}) = \boxed{7.9\times10^{-7}\,\text{nm.}}$$

$\boxed{43.}$ The value of ℓ can range from 0 to $n-1$. Thus for $n=5$, we have $\ell \le 4$.

The smallest value of L is $\boxed{0.}$

The largest magnitude of L is

$$L = \hbar\left[\ell(\ell+1)\right]^{\frac{1}{2}} = (1.055\times10^{-34}\,\text{J•s})\left[(4)(4+1)\right]^{\frac{1}{2}} = \boxed{4.72\times10^{-34}\,\text{kg•m}^2/\text{s.}}$$

44. (*a*) We find the quantum number for the orbital angular momentum from

$$L = M_{\text{earth}}vR = \frac{M_{\text{earth}}2\pi R^2}{T} = \hbar\left[\ell(\ell+1)\right]^{\frac{1}{2}};$$

$$\frac{(5.98\times10^{24}\,\text{kg})2\pi(1.50\times10^{11}\,\text{m})^2}{(3.16\times10^{7}\,\text{s})} = (1.055\times10^{-34}\,\text{J•s})\left[\ell(\ell+1)\right]^{\frac{1}{2}},$$

which gives $\ell = \boxed{2.5\times10^{74}.}$

(*b*) The value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values, so the number of orientations is

$$N = 2\ell+1 = 2(2.5\times10^{74})+1 = \boxed{5.0\times10^{74}.}$$

45. (*a*) We find the wavelength of the bullet from

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$=\frac{\left(6.63\times10^{-34}\ \mathrm{J{\bullet}s}\right)}{\left(12\times10^{-3}\ \mathrm{kg}\right)\left(180\,\mathrm{m/s}\right)}=\boxed{3.1\times10^{-34}\ \mathrm{m.}}$$

(*b*) We find the uncertainty in the momentum component perpendicular to the motion:

$$\Delta p_y\geq\frac{\hbar}{\Delta y}=\frac{\left(1.055\times10^{-34}\ \mathrm{J{\bullet}s}\right)}{\left(0.60\times10^{-8}\ \mathrm{m}\right)}=\boxed{1.8\times10^{-32}\ \mathrm{kg{\bullet}m/s.}}$$

46. From the Bohr formula for the radius, we see that

$$r\propto\frac{1}{Z},\ \text{so}\ r_{\text{uranium}}=\frac{r_1}{Z}=\frac{\left(0.529\times10^{-10}\ \mathrm{m}\right)}{(92)}=\boxed{5.8\times10^{-13}\ \mathrm{m.}}$$

The innermost electron would “see” a nucleus with charge *Ze*.
Thus we use the energy of the hydrogen atom:

$$E_1=-\frac{(13.6\,\mathrm{eV})Z^2}{n^2}=-\frac{(13.6\,\mathrm{eV})(92)^2}{1^2}=-1.15\times10^5\ \mathrm{eV},$$

so the binding energy is $\boxed{115\ \mathrm{keV.}}$

47. If we assume that the rate at which heat is produced is 100% of the electrical power input, we have

$$P=IV=\frac{Q}{t}=mc\left(\frac{\Delta T}{t}\right);$$

$$\frac{(25\,\mathrm{mA})(95\,\mathrm{kV})(60\,\mathrm{s/min})}{(4186\,\mathrm{J/kcal})}=(0.085\,\mathrm{kg})(0.11\,\mathrm{kcal/kg\,C^\circ})\left(\frac{\Delta T}{t}\right),$$

which gives $\frac{\Delta T}{t}=\boxed{3.6\times10^3\ \mathrm{C^\circ/min.}}$

48. (*a*) Boron has $Z=4$, so the outermost electron has $n=2$. We use the Bohr result with an effective *Z*:

$$E_2=-\frac{(13.6\,\mathrm{eV})\left(Z_{\text{eff}}\right)^2}{n^2};$$

$$-8.26\,\mathrm{eV}=-\frac{(13.6\,\mathrm{eV})\left(Z_{\text{eff}}\right)^2}{2^2},\ \text{which gives}\ Z_{\text{eff}}=\boxed{1.56.}$$

Note that this indicates some shielding by the second electron in the $n=2$ shell.

(*b*) We find the average radius from

$$r=\frac{n^2r_1}{Z_{\text{eff}}}=\frac{2^2\left(0.529\times10^{-10}\ \mathrm{m}\right)}{(1.56)}=\boxed{1.4\times10^{-10}\ \mathrm{m.}}$$

49. The wavelength of the photon emitted for the transition from *n′* to *n* is

$$\frac{1}{\lambda}=\left(\frac{2\pi^2Z^2e^4mk^2}{h^3c}\right)\left[\left(\frac{1}{n'^2}\right)-\left(\frac{1}{n^2}\right)\right]=RZ^2\left[\left(\frac{1}{n'^2}\right)-\left(\frac{1}{n^2}\right)\right].$$

The K_α line is from the $n'=2$ to $n=1$ transition, and the other $n=1$ electron shields the nucleus, so the effective *Z* is $Z-1$:

$$\frac{1}{\lambda}=RZ_{\text{eff}}{}^2\left[\left(\frac{1}{n'^2}\right)-\left(\frac{1}{n^2}\right)\right]=R(Z-1)^2\left[\left(\frac{1}{2^2}\right)-\left(\frac{1}{1^2}\right)\right]=\tfrac{3}{4}R(Z-1)^2.$$

Thus we have

$\dfrac{1}{\sqrt{\lambda}} = \left(\tfrac{3}{4}R\right)^{\frac{1}{2}}\left(Z-1\right),$ which is the equation of a straight line, as in the Moseley plot, with $b=1.$

The value of *a* is

$$a = \left(\tfrac{3}{4}R\right)^{\frac{1}{2}} = \left[\tfrac{3}{4}\left(1.0974\times10^{7}\ \text{m}^{-1}\right)\right]^{\frac{1}{2}} = \boxed{2869\,\text{m}^{-\frac{1}{2}}.}$$

50. (*a*) For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell,$ or $2\ell+1$ values. For each of these there are two values of m_s. Thus the total number of states in a subshell is

$$N = 2\left(2\ell+1\right).$$

(*b*) For $\ell = 0, 1, 2, 3, 4, 5,$ and $6,$ $N = 2, 6, 10, 14, 18, 22,$ and $26,$ respectively.

51. For a given *n*, $0 \le \ell \le n-1.$ Since for each ℓ the number of possible states is $2\left(2\ell+1\right),$ the number of possible states for a given *n* is

$$\sum_{\ell=0}^{n-1} 2\left(2\ell+1\right) = 4\sum_{\ell=0}^{n-1}\ell + \sum_{\ell=0}^{n-1}2 = 4\left(\frac{n\left(n-1\right)}{2}\right) + 2n = 2n^{2}.$$

52. We find the wavelength of the electron from

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0\left(\text{KE}\right)}}$$

$$= \frac{6.63\times10^{-34}\ \text{J}{\bullet}\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)\left(45\times10^{3}\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}} = 5.79\times10^{-12}\,\text{m}.$$

The peaks of the interference pattern are given by

$$d\sin\theta = n\lambda,\ n = 1,\ 2,\ \ldots$$

and the positions on the screen are

$$y = L\tan\theta.$$

For small angles, $\sin\theta = \tan\theta,$ so we have

$$y = \frac{nL\lambda}{d}.$$

Thus the separation is

$$\Delta y = \frac{L\lambda}{d} = \frac{\left(0.350\,\text{m}\right)\left(5.79\times10^{-12}\,\text{m}\right)}{2.0\times10^{-6}\,\text{m}} = 1.0\times10^{-6}\,\text{m} = \boxed{1.0\,\mu\text{m}.}$$

53. According to the Bohr model,

$$L_{\text{Bohr}} = n\frac{h}{2\pi} = \boxed{2\hbar.}$$

According to quantum mechanics,

$L_{\text{QM}} = \sqrt{\ell\left(\ell+1\right)}\,\hbar_{\text{p}}$ and with $n=2,$ it follows that $\ell=0$ or $\ell=1,$ so that

$L_{\text{QM}} = \sqrt{0\left(0+1\right)}\,\hbar = \boxed{0}$ or

$$L_{\text{QM}} = \sqrt{1\left(1+1\right)}\,\hbar = \boxed{\sqrt{2}\,\hbar.}$$

54. We find the uncertainty in the position from

$$\Delta x = \frac{\hbar}{m\Delta v} = \frac{1.055\times10^{-34}\,\text{J}\bullet\text{s}}{(1100\,\text{kg})(0.22\,\text{m/s})} = \boxed{4.4\times10^{-37}\,\text{m.}}$$

55. The difference in measured energies is

$$\Delta E = E_2 - E_1 = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$$

$$= \left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(\frac{1}{489\times10^{-9}\,\text{m}} - \frac{1}{487\times10^{-9}\,\text{m}}\right) = 1.67\times10^{-21}\,\text{J}.$$

Then the lifetime of the excited state is determined by

$$\Delta E\Delta t \approx \hbar;$$

$$\left(1.67\times10^{-21}\,\text{J}\right)\Delta t \approx 1.055\times10^{-34}\,\text{J}\bullet\text{s},\ \text{which gives}\ \Delta t = \boxed{6.3\times10^{-14}\,\text{s.}}$$

56. We find the wavelength of the proton from

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0(\text{KE})}}$$

$$= \frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{\sqrt{2\left(1.67\times10^{-27}\,\text{kg}\right)(550\,\text{eV})\left(1.60\times10^{-19}\,\text{J/eV}\right)}} = 1.22\times10^{-12}\,\text{m}.$$

The peaks of the interference pattern are given by

$$d\sin\theta = n\lambda,\ n = 1, 2, \ldots$$

and the positions on the screen are

$$y = L\tan\theta.$$

For small angles, $\sin\theta = \tan\theta$, so we have

$$y = \frac{nL\lambda}{d}.$$

Thus the separation is

$$\Delta y = \frac{L\lambda}{d} = \frac{(28\,\text{m})\left(1.22\times10^{-12}\,\text{m}\right)}{0.70\times10^{-3}\,\text{m}} = 4.89\times10^{-8}\,\text{m} = \boxed{48.9\,\text{nm.}}$$

57. The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0(\text{KE})}},$$

so that for a given KE,

$$\Delta x = \lambda \propto \frac{1}{\sqrt{m_0}}.$$

Since $(\Delta x)(\Delta p)$ is constant,

$$\Delta p \propto \sqrt{m_0};$$

$$\frac{\Delta p_{\text{proton}}}{\Delta p_{\text{electron}}} = \frac{\sqrt{m_\text{p}}}{\sqrt{m_\text{e}}} = \frac{\sqrt{1.67\times10^{-27}\,\text{kg}}}{\sqrt{9.11\times10^{-31}\,\text{kg}}} = 42.8$$

$$\Delta p_{\text{proton}} : \Delta p_{\text{electron}} = 43:1.$$

58. Limiting the number of electron shells to six would mean that the periodic table stops with radon (Rn), since the next element, francium (Fr), begins filling the seventh shell. Including all elements up through radon means $\boxed{86}$ elements.

59. The de Broglie wavelength is determined by

$$\lambda = \frac{h}{mv};$$

$$0.50\,\mathrm{m} = \frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{(75.0\,\mathrm{kg})v}, \text{ which gives } v = \boxed{1.8\times10^{-35}\,\mathrm{m/s}.}$$

Since λ is comparable to the width of a typical doorway, $\boxed{\text{yes,}}$ you would notice diffraction effects. However, assuming that walking through the doorway requires travel through a distance of 0.2 m, the time Δt required will be determined by

$$v\Delta t = d$$

$$\left(1.8\times10^{-35}\,\mathrm{m/s}\right)\Delta t = 0.2\,\mathrm{m}, \text{ which gives } \Delta t \approx \boxed{10^{34}\,\mathrm{s}.}$$

60. Since

$$\frac{1}{\lambda} \propto Z^2\left(\frac{1}{n'^2} - \frac{1}{n^2}\right),$$

we seek Z and n'_{x} so that for every fourth value of n_{x} there is an n_{h} such that

$$\frac{1}{1} - \frac{1}{n_{\mathrm{h}}^2} = Z^2\left(\frac{1}{{n'_{\mathrm{x}}}^2} - \frac{1}{n_{\mathrm{x}}^2}\right).$$

This condition is met when $Z = 4$ and $n'_{\mathrm{x}} = 4$. The resulting spectral lines for $n_{\mathrm{x}} = 4, 8, 12, \ldots$ match the hydrogen Lyman series lines for $n = 1, 2, 3, \ldots$. The element is $\boxed{\text{beryllium.}}$

61. The K_α line is from the $n = 2$ to $n = 1$ transition. We use the energies of the hydrogen atom with Z replaced by $Z - 1$. Thus we have

$$\frac{1}{\lambda} = \left(\frac{2\pi^2 e^4 mk^2}{h^3 c}\right)(Z-1)^2\left(\frac{1}{n'^2} - \frac{1}{n^2}\right);$$

$$\frac{1}{0.154\times10^{-9}\,\mathrm{m}} = \left(1.097\times10^{7}\,\mathrm{m}^{-1}\right)(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right), \text{ which gives } Z \approx 29.$$

The element is $\boxed{\text{copper.}}$

62. We find the uncertainty in the momentum component perpendicular to the motion, when the width of the beam is constrained to a dimension D:

$$\Delta p_y \geq \frac{\left(\frac{h}{2\pi}\right)}{\Delta y} = \frac{\left(\frac{h}{2\pi}\right)}{D}.$$

The half angle of the beam is given by the direction of the velocity:

$$\sin\theta = \frac{v_y}{c}.$$

We assume that the angle is small: $\sin\theta \approx \theta,$ and we take the minimum uncertainty to be the perpendicular momentum, so we have

$$\theta \approx \frac{v_y}{v_x} = \frac{p_y}{p} = \frac{\left[\dfrac{\left(\dfrac{h}{2\pi}\right)}{D}\right]}{\left(\dfrac{h}{\lambda}\right)} = \frac{\lambda}{2\pi D}.$$

The angular spread is

$$2\theta \approx \frac{\lambda}{\pi D} \approx \frac{\lambda}{D}.$$

CHAPTER 29: Molecules and Solids

Answers to Questions

1. (*a*) The bond in an N_2 molecule is expected to be covalent.
   (*b*) The bond in the HCl molecule is expected to be ionic.
   (*c*) The bond between Fe atoms in a solid is expected to be metallic.

2. Ca has 2 *s* electrons in the outer shell and each Cl is only missing 1 electron from its outer shell. These three atoms share their electrons in such a way as to have filled outer shells. Look at Figures 29-4 and 29-5; each of the 2 outer electrons of Ca will fit into the "extra electron" position of the 2 Cl atoms, forming strong ionic bonds.

3. No, the H_2 molecule does not have a permanent dipole moment. The outer electrons are shared equally between the two atoms and they don't leave any unbalanced "polar" ends that are more positively or negatively charged. No, the O_2 molecule does not have a permanent dipole moment. The outer electrons are shared equally between the two atoms and they don't leave any unbalanced "polar" ends that are more positively or negatively charged. Yes, the H_2O molecule does have a permanent dipole moment. The electrons from the hydrogen atoms tend to spend more time around the 8 positively charged protons of the oxygen atom than the one positively charged proton in each hydrogen atom. This leaves the hydrogen atoms slightly positive and on one side of the molecule, while leaving the oxygen atom slightly negative on the other side of the molecule (see Figure 29-6). This creates a permanent dipole moment.

4. The H_3 atom has three electrons and only two of them can be in the $1s^2$ state (and only if they have opposite spins, according to the Pauli Exclusion Principle). Accordingly, the third electron cannot be in the $1s^2$ state, and so it is not held very tightly by the nucleus. This makes H_3 unstable. On the other hand, the H_3^+ ion only has two electrons and, if they have opposite spins, the Pauli Exclusion Principle will allow them to both be in the lower energy $1s^2$ state. This makes H_3^+ relatively more stable than H_3.

5. The four categories of energy contained in a molecule are: bond (electrostatic potential) energy, rotational kinetic energy, vibrational kinetic energy, and translational kinetic energy.

6. Yes, H_2^+ should be stable. The two positive nuclei will share the one negative electron. The electron will spend most of its time between the two positive nuclei (basically holding them together).

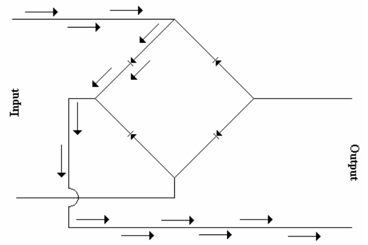
7. The electron configuration of carbon is $1s^2\ 2s^2\ 2p^2$. The inner two electrons are tightly and closely bound to the nucleus. The four remaining electrons are basically spread around the outside of the atom in four different directions (they repel each other). These four electrons can each form a simple hydrogen-like bond with four atoms that each have only one electron in an *s* orbital.

8. The freely roaming electrons in a metal are not completely free. They are still attracted to the positive nuclei in the metal, so they would need extra energy from outside the metal to be able to leave the metal. For example, in the photoelectron effect, a photon comes in and strikes an electron to knock it out of the metal. In general, a metal is neutral, so if one electron were to move away from the metal, then the metal would be positively charged and the electron would be attracted back to the metal.

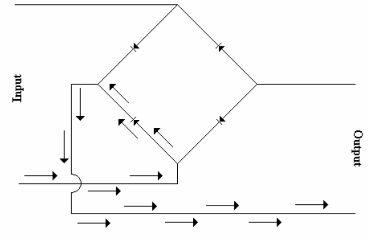
9. As you increase the temperature of a metal, the amount of disorder increases and this makes it more difficult for the conduction electrons to move through the metal. Thus, the resistivity increases with

increasing temperature. As you increase the temperature of a semiconductor, the small band gap between the valence band and the conduction band can now be "jumped" by the thermally excited electrons and this means there are more conduction electrons available to move through the semiconductor. Thus, the resistivity decreases with increasing temperature.

10. When the top branch of the Input circuit is at the high voltage (current is flowing in this branch during half of the cycle), then the bottom branch of the Output is at a high voltage. The current follows the path through the bridge in the first diagram.



When the bottom branch of the Input circuit is at the high voltage (current is flowing in this branch during the other half of the cycle), then the bottom branch of the Output is still at a high voltage. The current follows the path through the bridge in the second diagram.



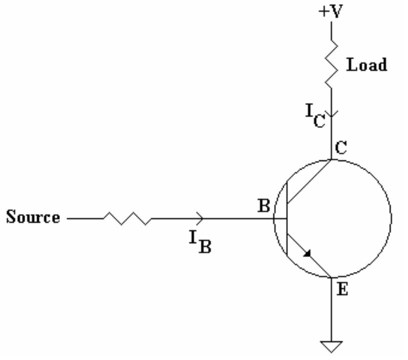
11. Look at Figure 29-28; it takes about 0.6V to get current to flow through the diode in the forward bias direction and it takes about 12 V to get current to flow through the diode in the reverse bias direction. Thus, to get the same current to flow in either direction:

$V_{\text{forward}} = IR_{\text{forward}}$ $\quad$ $V_{\text{reverse}} = IR_{\text{reverse}}$

$R_{\text{reverse}}/R_{\text{forward}} = 12\text{ V} / 0.6\text{ V} = 20$

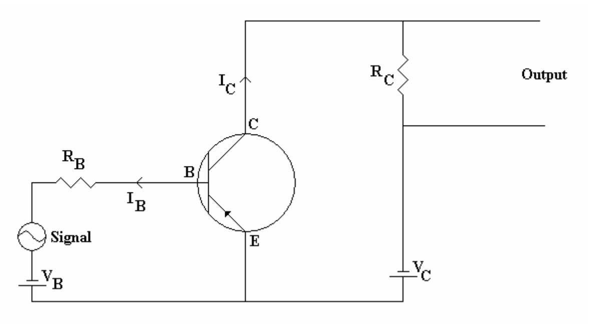
Thus, reverse bias resistance is approximately 20X larger than the forward bias resistance. This is very approximate based on estimates from reading the graph.

12. Connect the circuit shown in the adjacent diagram. When the source provides no current $\left(I_{\text{B}} = 0\right)$, which causes the CE resistance to be ∞, then no load current $\left(I_{\text{C}}\right)$ flows and the switch is OFF. When the source current is not 0, that causes the CE resistance to go to 0 Ohms. Then, load current flows and the switch is ON.



13. The main difference between *n*-type and *p*-type semiconductors is the type of atom used for the doping impurity. When a semiconductor such as Si or Ge, each atom of which has four electrons to share, is doped with an element that has five electrons to share (such as As or P), then it is an *n*-type semiconductor since an extra electron has been inserted into the lattice. When a semiconductor is doped with an element that has three electrons to share (such as Ga or In), then it is a *p*-type semiconductor since an extra hole (the lack of an electron) has been inserted into the lattice.

14. A *pnp* transistor could operate as an amplifier just like an *npn* transistor (see Figure 29-32), except that the two batteries need to be reversed, since we need to have the holes as the majority carriers (the current goes in the opposite direction as an *npn* transistor). The circuit would look like this:



15. In the circuit shown in Figure 29-32, the base-emitter is forward biased (the current will easily flow from the base to the emitter) and the base-collector is reverse biased (the current will not easily flow from the base to the collector).

16. Look at Figure 29-32; the input signal is amplified by the transistor with the help of the battery, $\mathcal{E}_{\mathrm{C}}$. When the CE resistance is huge $\left(I_{\mathrm{B}} = 0\right)$ the voltage output is 0 V, but when I_{B} is small, the CE resistance is tiny, which makes I_{C} large. Then the output voltage is large, powered by the battery, $\mathcal{E}_{\mathrm{C}}$.

17. Phosphorus is in the same column of the periodic table as arsenic and each of these atoms have 5 electrons to share when placed in a lattice. Since the phosphorus would be replacing a silicon atom that has 4 electrons to share, there would now be an extra electron in the lattice, making this an *n*-type semiconductor in which the phosphorus atoms are donors.

18. No, diodes and transistors do not obey Ohm's law. These devices are called "non-linear" devices, since the current that flows through them is not linearly proportional to the applied voltage.

19. No, a diode cannot be used to amplify a signal. A diode does let current flow through it in one direction easily (forward biased) and it does not let current flow through it in the other direction (reversed bias), but there is no way to connect a source of power to use it to amplify a signal (which is how a transistor amplifies a signal).

Solutions to Problems

Note: At the atomic scale, it is most convenient to have energies in electron-volts and wavelengths in nanometers. A useful expression for the energy of a photon in terms of its wavelength is

$$E = hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(10^{-9}\,\mathrm{nm/m}\right)}{\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)\lambda};$$

$$E = \frac{\left(1.24\times10^{3}\mathrm{eV{\bullet}nm}\right)}{\lambda}.$$

A factor that appears in the analysis of electron energies is

$$ke^2 = \left(9.00\times10^{9}\,\mathrm{N{\bullet}m^2/C^2}\right)\left(1.60\times10^{-19}\,\mathrm{C}\right)^2 = 2.30\times10^{-28}\,\mathrm{J{\bullet}m}.$$

$\boxed{1.}$ With the reference level at infinity, the binding energy of the two ions is

$$\text{Binding energy} = -\text{PE} = \frac{ke^2}{r}$$

$$= \frac{\left(2.30\times10^{-28}\,\text{J}\bullet\text{m}\right)}{\left(0.28\times10^{-9}\,\text{m}\right)} = 8.21\times10^{-19}\,\text{J} = \boxed{5.13\,\text{eV.}}$$

2. With the repulsion of the electron clouds, the binding energy is

$$\text{Binding energy} = -\text{PE} - \text{PE}_{\text{clouds}};$$

$$4.43\,\text{eV} = 5.13\,\text{eV} - \text{PE}_{\text{clouds}},\ \text{which gives}\ \text{PE}_{\text{clouds}} = \boxed{0.70\,\text{eV.}}$$

3. When the electrons are midway between the protons, each electron will have a potential energy due to the two protons:

$$\text{PE}_{\text{ep}} = \frac{-(2)(0.33)ke^2}{\left(\frac{r}{2}\right)} = -\frac{(4)(0.33)\left(2.30\times10^{-28}\,\text{J}\cdot\text{m}\right)}{\left(0.074\times10^{-9}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}$$

$$= -25.6\,\text{eV.}$$

The protons have a potential energy:

$$\text{PE}_{\text{pp}} = +\frac{ke^2}{r} = +\frac{\left(2.30\times10^{-28}\,\text{J}\cdot\text{m}\right)}{\left(0.074\times10^{-9}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = +19.4\,\text{eV.}$$

When the bond breaks, each hydrogen atom will be in the ground state with an energy $E_1 = -13.6\,\text{eV}$. Thus the binding energy is

$$\text{Binding energy} = 2E_1 - \left(2\text{PE}_{\text{ep}} + \text{PE}_{\text{pp}}\right) = 2(-13.6\,\text{eV}) - \left[2(-25.6\,\text{eV}) + 19.4\,\text{eV}\right] = \boxed{4.6\,\text{eV.}}$$

$\boxed{4.}$ We convert the units:

$$1\,\text{kcal/mol} = \frac{(1\,\text{kcal/mol})(4186\,\text{J/kcal})}{\left(6.02\times10^{23}\,\text{molecules/mol}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}$$

$$= \boxed{0.0435\,\text{eV/molecule.}}$$

For KCl we have

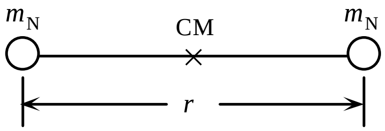
$$(4.43\,\text{eV/molecule})\left[\frac{(1\,\text{kcal/mol})}{(0.0434\,\text{eV/molecule})}\right] = \boxed{102\,\text{kcal/mol.}}$$

5. The neutral He atom has two electrons in the ground state, $n = 1,\ \ell = 0,\ m_\ell = 0$. Thus the two electrons have opposite spins, $m_s = \pm\frac{1}{2}$. If we try to form a covalent bond, we see that an electron from one of the atoms will have the same quantum numbers as one of the electrons on the other atom. From the exclusion principle, this is not allowed, so the electrons cannot be shared.
We consider the $\text{He}_2{}^+$ molecular ion to be formed from a neutral He atom and an He^+ ion. If the electron on the ion has a certain spin value, it is possible for one of the electrons on the atom to have the opposite spin. Thus the electron can be in the same spatial region as the electron on the ion, so a bond can be formed.

6. The units of $\dfrac{\hbar^2}{I}$ are

$$\frac{(\text{J}\bullet\text{s})^2}{(\text{kg}\bullet\text{m}^2)} = \frac{\text{J}^2}{(\text{kg}\bullet\text{m/s}^2)\text{m}} = \frac{\text{J}^2}{(\text{N}\bullet\text{m})} = \frac{\text{J}^2}{\text{J}} = \text{J}.$$

7. The moment of inertia of N_2 about its CM is



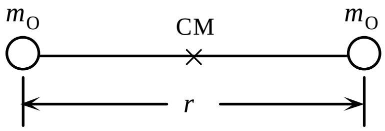
$$I = 2m_N\left(\frac{r}{2}\right)^2 = \frac{m_N r^2}{2}.$$

We find the bond length from

$$\frac{\hbar^2}{2I} = \frac{\hbar^2}{m_N r^2}$$

$$= \frac{(1.055\times10^{-34}\,\text{J}\bullet\text{s})^2}{(14)(1.66\times10^{-27}\,\text{kg})r^2},\text{ which gives } r = \boxed{1.10\times10^{-10}\,\text{m}.}$$

8. (*a*) The moment of inertia of O_2 about its CM is



$$I = 2m_O\left(\frac{r}{2}\right)^2 = \frac{m_O r^2}{2}.$$

We find the characteristic rotational energy from

$$\frac{\hbar^2}{2I} = \frac{\hbar^2}{m_O r^2}$$

$$= \frac{(1.055\times10^{-34}\,\text{J}\bullet\text{s})^2}{(16)(1.66\times10^{-27}\,\text{kg})(0.121\times10^{-9}\,\text{m})^2}$$

$$= 2.86\times10^{-23}\,\text{J} = \boxed{1.79\times10^{-4}\,\text{eV}.}$$

(*b*) The rotational energy is

$$E_{\text{rot}} = L(L+1)\left(\frac{\hbar^2}{2I}\right).$$

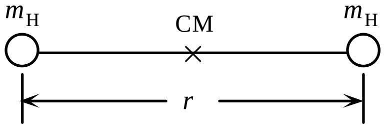
Thus the energy of the emitted photon from the $L = 2$ to $L = 1$ transition is

$$hf = \Delta E_{\text{rot}} = [(2)(2+1) - (1)(1+1)]\left(\frac{\hbar^2}{2I}\right) = 4\left(\frac{\hbar^2}{2I}\right) = 4(1.79\times10^{-4}\,\text{eV}) = \boxed{7.16\times10^{-4}\,\text{eV}.}$$

The wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{(1.24\times10^3\,\text{eV}\bullet\text{nm})}{(7.16\times10^{-4}\,\text{eV})} = 1.73\times10^6\,\text{nm} = \boxed{1.73\,\text{mm}.}$$

9. The moment of inertia of H_2 about its CM is



$$I = 2m_H\left(\frac{r}{2}\right)^2 = \frac{m_H r^2}{2}.$$

We find the characteristic rotational energy from

$$\frac{\hbar^2}{2I} = \frac{\hbar^2}{m_H r^2}$$

$$=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(0.074\times10^{-9}\,\text{m}\right)^2}$$

$$1.22\times10^{-23}\,\text{J}=7.61\times10^{-3}\,\text{eV}.$$

The rotational energy is

$$E_{\text{rot}}=L\left(L+1\right)\left(\frac{\hbar^2}{2I}\right).$$

Thus the energy of the emitted photon from the L to $L-1$ transition is

$$hf=\Delta E_{\text{rot}}=\left[\left(L\right)\left(L+1\right)-\left(L-1\right)\left(L\right)\right]\left(\frac{\hbar^2}{2I}\right)=2L\left(\frac{\hbar^2}{2I}\right).$$

(*a*) For the $L = 1$ to $L = 0$ transition, we get

$$hf=2L\left(\frac{\hbar^2}{2I}\right)=2\left(1\right)\left(7.61\times10^{-3}\,\text{eV}\right)=\boxed{1.52\times10^{-2}\,\text{eV.}}$$

The wavelength is

$$\lambda=\frac{c}{f}=\frac{hc}{hf}=\frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\left(1.52\times10^{-2}\,\text{eV}\right)}=8.16\times10^{4}\,\text{nm}=\boxed{0.082\,\text{mm.}}$$

(*b*) For the $L = 2$ to $L = 1$ transition, we get

$$hf=2L\left(\frac{\hbar^2}{2I}\right)=2\left(2\right)\left(7.61\times10^{-3}\,\text{eV}\right)=\boxed{3.04\times10^{-2}\,\text{eV.}}$$

The wavelength is

$$\lambda=\frac{c}{f}=\frac{hc}{hf}=\frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\left(3.04\times10^{-2}\,\text{eV}\right)}=4.08\times10^{4}\,\text{nm}=\boxed{0.041\text{mm.}}$$

(*c*) For the $L = 3$ to $L = 2$ transition, we get

$$hf=2L\left(\frac{\hbar^2}{2I}\right)=2\left(3\right)\left(7.61\times10^{-3}\,\text{eV}\right)=\boxed{4.56\times10^{-2}\,\text{eV.}}$$

The wavelength is

$$\lambda=\frac{c}{f}=\frac{hc}{hf}=\frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\left(4.56\times10^{-2}\,\text{eV}\right)}=2.72\times10^{4}\,\text{nm}=\boxed{0.027\,\text{mm.}}$$

10. We find the energies for the transitions from

$$\Delta E=hf=\frac{hc}{\lambda}=\frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\lambda}:$$

$$\Delta E_1=\frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\left(23.1\times10^{6}\,\text{nm}\right)}=5.37\times10^{-5}\,\text{eV};$$

$$\Delta E_2=\frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\left(11.6\times10^{6}\,\text{nm}\right)}=10.7\times10^{-5}\,\text{eV};$$

$$\Delta E_3=\frac{\left(1.24\times10^{3}\,\text{eV}\bullet\text{nm}\right)}{\left(7.71\times10^{6}\,\text{nm}\right)}=16.1\times10^{-5}\,\text{eV}.$$

The rotational energy is

$$E_{\text{rot}} = L(L+1)\left(\frac{\hbar^2}{2I}\right).$$

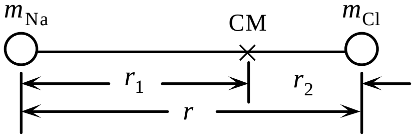
Thus the energy of the emitted photon from the L to $L-1$ transition is

$$hf = \Delta E_{\text{rot}} = \left[(L)(L+1) - (L-1)(L)\right]\left(\frac{\hbar^2}{2I}\right) = 2L\left(\frac{\hbar^2}{2I}\right).$$

Because $\Delta E_3 = 3\Delta E_1$, and $\Delta E_2 = 2\Delta E_1$, the three transitions must be from the L = 1, 2, and 3 states. We find the moment of inertia about the CM from

$$\Delta E_3 = \frac{2(3)\left(1.055\times10^{-34}\,\text{J•s}\right)^2}{2I},$$ which gives $I = 1.29\times10^{-45}\,\text{kg•m}^2$.

The positions of the atoms from the CM are



$$r_1 = \frac{\left[m_{\text{Na}}(0) + m_{\text{Cl}}r\right]}{\left(m_{\text{Na}} + m_{\text{Cl}}\right)}$$

$$= \frac{(35.5\,\text{u})r}{(23.0\,\text{u} + 35.5\,\text{u})} = 0.607r;$$

$$r_2 = r - r_1 = 0.393r.$$

We find the bond length from

$$I = m_{\text{Na}}r_1^2 + m_{\text{Cl}}r_2^2;$$

$$1.29\times10^{-45}\,\text{kg•m}^2 = \left[(23.0\,\text{u})(0.607r)^2 + (35.5\,\text{u})(0.393r)^2\right]\left(1.66\times10^{-27}\,\text{kg/u}\right),$$

which gives $r = \boxed{2.35\times10^{-10}\,\text{m.}}$

11. (*a*) The curve for $\text{PE} = \frac{1}{2}kx^2$ is shown in Figure 29–17 as a dotted line. This line crosses the $\text{PE} = 0$ axis at 0.120 nm. If we take the energy to be zero at the lowest point, the energy at the axis is 4.5 eV. Thus we have

$$\text{PE} = \tfrac{1}{2}kx^2;$$

$$(4.5\,\text{eV})\left(1.60\times10^{-19}\,\text{J/eV}\right) = \tfrac{1}{2}k\left[(0.120\,\text{nm} - 0.074\,\text{nm})\left(10^{-9}\,\text{m/nm}\right)\right]^2,$$ which gives

$$k = \boxed{680\,\text{N/m.}}$$

(*b*) For H_2 the reduced mass is

$$\mu = \frac{m_1m_2}{(m_1+m_2)} = \frac{(1\text{u})(1\text{u})}{(1\text{u}+1\text{u})} = 0.50\,\text{u} = 8.3\times10^{-28}\,\text{kg}.$$

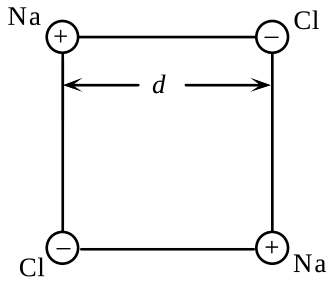
The fundamental wavelength is

$$\lambda = \frac{c}{f} = (2\pi c)\left(\frac{\mu}{k}\right)^{\frac{1}{2}}$$

$$= 2\pi\left(3.00\times10^8\,\text{m/s}\right)\left[\frac{\left(8.3\times10^{-28}\,\text{kg}\right)}{(680\,\text{N/m})}\right]^{\frac{1}{2}} = 2.1\times10^{-6}\,\text{m} = \boxed{2.1\mu\text{m.}}$$

12. From the figure we see that the distance between nearest neighbor Na ions is the diagonal of the cube:

$$D = d\sqrt{2} = (0.24\,\text{nm})\sqrt{2} = \boxed{0.34\,\text{nm.}}$$



13. Because each ion occupies a cell of side *s*, a molecule occupies two cells. Thus the density is

$$\rho = \frac{m_{\text{NaCl}}}{2s^3};$$

$$2.165\times10^3\,\text{kg/m}^3 = \frac{(58.44\,\text{u})(1.66\times10^{-27}\,\text{kg/u})}{2s^3},\ \text{which gives } s = 2.83\times10^{-10}\,\text{m} = \boxed{0.283\,\text{nm.}}$$

14. Because each ion occupies a cell of side *s*, a molecule occupies two cells. Thus the density is

$$\rho = \frac{m_{\text{KCl}}}{2s^3};$$

$$1.99\times10^3\,\text{kg/m}^3 = \frac{(39.1\,\text{u}+35.5\,\text{u})(1.66\times10^{-27}\,\text{kg/u})}{2s^3},\ \text{which gives } s = 3.15\times10^{-10}\,\text{m} = \boxed{0.315\,\text{nm.}}$$

15. The partially filled shell in Na is the 3*s* shell, which has 1 electron in it. The partially filled shell in Cl is the 2*p* shell which has 5 electrons in it. In NaCl the electron from the 3*s* shell in Na is transferred to the 2*p* shell in Cl, which results in filled shells for both ions. Thus when many ions are considered, the resulting bands are either completely filled (the valence band) or completely empty (the conduction band). Thus a large energy is required to create a conduction electron by raising an electron from the valence band to the conduction band.

16. The photon with the minimum frequency for conduction must have an energy equal to the energy gap:

$$E_g = hf = \frac{hc}{\lambda} = \frac{(1.24\times10^3\,\text{eV}\bullet\text{nm})}{(640\,\text{nm})} = \boxed{1.94\,\text{eV.}}$$

$\boxed{17.}$ The photon with the longest wavelength or minimum frequency for conduction must have an energy equal to the energy gap:

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{E_g} = \frac{(1.24\times10^3\,\text{eV}\bullet\text{nm})}{(1.1\,\text{eV})} = 1.1\times10^3\,\text{nm} = \boxed{1.1\,\mu\text{m.}}$$

18. The energy of the photon must be greater than or equal to the energy gap. Thus the longest wavelength that will excite an electron is

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{E_g} = \frac{(1.24\times10^3\,\text{eV}\bullet\text{nm})}{(0.72\,\text{eV})} = 1.7\times10^3\,\text{nm} = 1.7\,\mu\text{m.}$$

Thus the wavelength range is $\boxed{\lambda \le 1.7\,\mu\text{m.}}$

$\boxed{19.}$ The minimum energy provided to an electron must be equal to the energy gap:

$$E_g = 0.72\,\text{eV.}$$

Thus the maximum number of electrons is

$$N=\frac{hf}{E_g}=\frac{\left(760\times10^3\,\text{eV}\right)}{\left(0.72\,\text{eV}\right)}=\boxed{1.1\times10^6.}$$

20. (*a*) In the 2*s* shell of an atom, $\ell=0,$ so there are two states: $m_s=\pm\frac{1}{2}.$ When *N* atoms form bands, each atom provides 2 states, so the total number of states in the band is $\boxed{2N.}$

(*b*) In the 2*p* shell of an atom, $\ell=1,$ so there are three states from the m_ℓ values: $m_\ell=0,\pm1;$ each of which has two states from the m_s values: $m_s=\pm\frac{1}{2},$ for a total of 6 states. When *N* atoms form bands, each atom provides 6 states, so the total number of states in the band is $\boxed{6N.}$

(*c*) In the 3*p* shell of an atom, $\ell=1,$ so there are three states from the m_ℓ values: $m_\ell=0,\pm1;$ each of which has two states from the m_s values: $m_s=\pm\frac{1}{2},$ for a total of 6 states. When *N* atoms form bands, each atom provides 6 states, so the total number of states in the band is $\boxed{6N.}$

(*d*) In general, for a value of $\ell,$ there are $2\ell+1$ states from the m_ℓ values: $m_\ell=0,\pm1,\ldots,\pm\ell.$ For each of these there are two states from the m_s values: $m_s=\pm\frac{1}{2},$ for a total of $2\left(2\ell+1\right)$ states. When *N* atoms form bands, each atom provides $2\left(2\ell+1\right)$ states, so the total number of states in the band is $\boxed{2N\left(2\ell+1\right).}$

21. If we consider a mole of pure silicon $\left(28\,\text{g or } 6.02\times10^{23}\,\text{atoms}\right),$ the number of conduction electrons is

$$N_{\text{Si}}=\left[\frac{\left(28\times10^{-3}\,\text{kg}\right)}{\left(2330\,\text{kg/m}^3\right)}\right]\left(10^{16}\,\text{electrons/m}^3\right)=1.20\times10^{11}\ \text{conduction electrons.}$$

The additional conduction electrons provided by the doping is

$$N_{\text{doping}}=\frac{\left(6.02\times10^{23}\,\text{atoms}\right)}{10^6}=6.02\times10^{17}\ \text{added conduction electrons.}$$

Thus the density of conduction electrons has increased by

$$\frac{N_{\text{doping}}}{N_{\text{Si}}}=\frac{\left(6.02\times10^{17}\right)}{\left(1.20\times10^{11}\right)}=\boxed{5\times10^6.}$$

22. The photon will have an energy equal to the energy gap:

$$\lambda=\frac{c}{f}=\frac{hc}{hf}=\frac{hc}{E_g}=\frac{\left(1.24\times10^3\,\text{eV}\bullet\text{nm}\right)}{\left(1.4\,\text{eV}\right)}=8.9\times10^2\,\text{nm}=\boxed{0.89\,\mu\text{m.}}$$

23. The photon will have an energy equal to the energy gap:

$$E_g=hf=\frac{hc}{\lambda}=\frac{\left(1.24\times10^3\,\text{eV}\bullet\text{nm}\right)}{\left(650\,\text{nm}\right)}=\boxed{1.91\,\text{eV.}}$$

24. From the current-voltage characteristic, we see that a current of 12 mA means a voltage of 0.68 V across the diode. Thus the battery voltage is

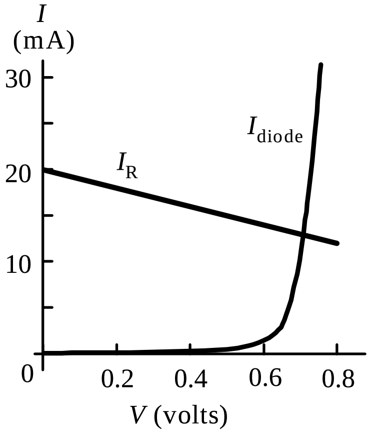
$$V_{\text{battery}}=V_{\text{diode}}+V_{\text{R}}=0.68\,\text{V}+\left(12\times10^{-3}\,\text{A}\right)\left(960\,\Omega\right)=\boxed{12.2\,\text{V.}}$$

25. The battery voltage is

$$V_{\text{battery}} = V_{\text{diode}} + V_{\text{R}};$$

$$2.0\,\text{V} = V(I) + I(0.100\,\text{k}\Omega), \text{ or } V(I) = 2.0\,\text{V} - I(0.100\,\text{k}\Omega).$$

This is a straight line which passes through the points (20 mA, 0 V) and (12 mA, 0.8 V), as drawn in the figure. Because $V(I)$ is represented by both curves, the intersection will give the current, which we see is $\boxed{13\text{ mA.}}$



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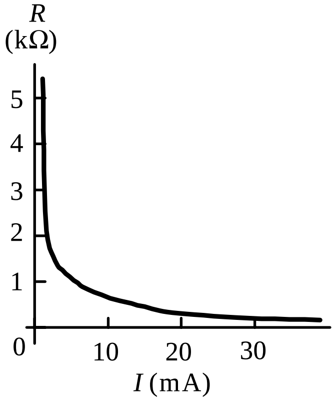


$\boxed{27.}$ (*a*) For a half-wave rectifier without a capacitor, the current is zero for half the time. Thus the average current is

$$I_{\text{av}} = \tfrac{1}{2}\frac{V_{\text{rms}}}{R} = \tfrac{1}{2}\frac{(120\,\text{V})}{(25\,\text{k}\Omega)} = \boxed{2.4\,\text{mA.}}$$

(*b*) For a full-wave rectifier without a capacitor, the current is positive all the time. Thus the average current is

$$I_{\text{av}} = \frac{V_{\text{rms}}}{R} = \frac{(120\,\text{V})}{(25\,\text{k}\Omega)} = \boxed{4.8\,\text{mA.}}$$

28. There will be a current in the resistor while the ac voltage varies from 0.6 V to 12.0 V rms. Because the 0.6 V is small, the voltage across the resistor will be almost sinusoidal, so the rms voltage across the resistor will be close to $12.0\,\text{V} - 0.6\,\text{V} = 11.4\,\text{V}$.

(*a*) For a half-wave rectifier without a capacitor, the current is zero for half the time. If we ignore the short times it takes to reach 0.6 V, this will also be true for the resistor. Thus the average current is

$$I_{\text{av}} = \tfrac{1}{2}\frac{V_{\text{rms}}}{R} = \tfrac{1}{2}\frac{(11.4\,\text{V})}{(0.150\,\text{k}\Omega)} = \boxed{38\,\text{mA.}}$$

(*b*) For a full-wave rectifier without a capacitor, the current is positive all the time. If we ignore the short times it takes to reach 0.6 V, this will also be true for the resistor. Thus the average current is

$$I_{\text{av}}=\frac{V_{\text{rms}}}{R}=\frac{(11.4\,\text{V})}{(0.150\,\text{k}\Omega)}=\boxed{76\,\text{mA.}}$$

29. (*a*) The time constant for the circuit is

$$\tau_1=RC_1=\left(21\times10^3\,\Omega\right)\left(25\times10^{-6}\,\text{F}\right)=0.53\text{s.}$$

Because there are two peaks per cycle, the period of the rectified voltage is

$$T=\frac{1}{2f}=\frac{1}{2(60\,\text{Hz})}=0.0083\text{s.}$$

Because $\tau_1 \gg T$, the voltage across the capacitor will be essentially constant during a cycle, so the voltage will be the peak voltage. Thus the average current is

$$I_{\text{av}}=\frac{V_0}{R}=\frac{\sqrt{2}(120\,\text{V})}{(21\text{k}\Omega)}=\boxed{8.1\text{mA}(\text{smooth}).}$$

(*b*) The time constant for the circuit is

$$\tau_2=RC_2=\left(21\times10^3\,\Omega\right)\left(0.10\times10^{-6}\,\text{F}\right)=0.0021\text{s.}$$

Because $\tau_2 < T$, the voltage across the capacitor will be rippled, so the average voltage will be close to the rms voltage. Thus the average current is

$$I_{\text{av}}=\frac{V_{\text{rms}}}{R}=\frac{(120\,\text{V})}{(21\text{k}\Omega)}=\boxed{5.7\,\text{mA}(\text{rippled}).}$$

30. The arrow at the emitter terminal, E, indicates the direction of current I_{E}. Because current into the transistor must equal current out of the transistor,

$$I_{\text{B}}+I_{\text{C}}=I_{\text{E}}.$$

31. For an electron confined within Δx, we find the uncertainty in the momentum from

$$\Delta p=\frac{\hbar}{\Delta x},$$

which we take to be the momentum of the particle. The kinetic energy of the electron is

$$\text{KE}=\frac{p^2}{2m}=\frac{\hbar^2}{2m(\Delta x)^2}.$$

When the two electrons are in separated atoms, we get

$$\text{KE}_1=\frac{2\hbar^2}{2m(\Delta x)^2}$$

$$=\frac{2\left(1.055\times10^{-34}\,\text{J•s}\right)^2}{2\left(9.11\times10^{-31}\text{kg}\right)\left(0.053\times10^{-9}\,\text{m}\right)^2}=4.35\times10^{-18}\,\text{J}=27.2\,\text{eV}.$$

When the electrons are in the molecule, we get

$$\text{KE}_2=\frac{2\hbar^2}{2m(\Delta x)^2}$$

$$=\frac{2\left(1.055\times10^{-34}\,\text{J•s}\right)^2}{2\left(9.11\times10^{-31}\text{kg}\right)\left(0.074\times10^{-9}\,\text{m}\right)^2}=2.23\times10^{-18}\,\text{J}=14.0\,\text{eV}.$$

Thus the binding energy is

$$\text{KE}_1 - \text{KE}_2 = 27.2\,\text{eV} - 14.0\,\text{eV} = \boxed{13\,\text{eV.}}$$

32. (*a*) We find the temperature from

$$\text{KE} = \tfrac{3}{2}kT;$$

$$(4.5\,\text{eV})(1.60\times10^{-19}\,\text{J/eV}) = \tfrac{3}{2}(1.38\times10^{-23}\,\text{J/K})T, \text{ which gives } T = \boxed{3.5\times10^{4}\,\text{K.}}$$

(*b*) We find the temperature from

$$\text{KE} = \tfrac{3}{2}kT;$$

$$(0.15\,\text{eV})(1.60\times10^{-19}\,\text{J/eV}) = \tfrac{3}{2}(1.38\times10^{-23}\,\text{J/K})T, \text{which gives } T = \boxed{1.2\times10^{3}\,\text{K.}}$$

33. (*a*) The potential energy for the point charges is

$$\text{PE} = -\frac{ke^2}{r} = -\frac{(2.30\times10^{-28}\,\text{J}\bullet\text{m})}{(0.27\times10^{-9}\,\text{m})} = -8.52\times10^{-19}\,\text{J} = \boxed{-5.3\,\text{eV.}}$$

(*b*) Because the potential energy of the ions is negative, 5.3 eV is released when the ions are brought together. A release of energy means that energy must be provided to return the ions to the state of free atoms. Thus the total binding energy of the KF ions is

$$\text{Binding energy} = 5.3\text{eV} + 4.07\,\text{eV} - 4.34\,\text{eV} = \boxed{5.0\,\text{eV.}}$$

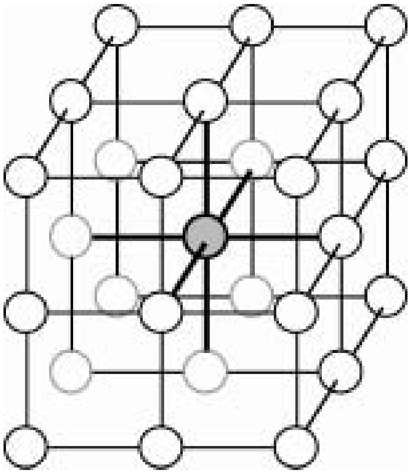
34. From the diagram of the cubic lattice, we see that an atom inside the cube is bonded to the six nearest neighbors. Because each bond is shared by two atoms, the number of bonds per atom is 3. We find the heat of fusion for argon from the energy required to break the bonds:

$$L_{\text{fusion}} = \left(\frac{\text{number of bonds}}{\text{atom}}\right)\left(\frac{\text{number of atoms}}{\text{kg}}\right)E_{\text{bond}}$$

$$= (3)\left[\frac{(6.02\times10^{23}\,\text{atoms/mol})}{(39.95\times10^{-3}\,\text{kg/mol})}\right]\times$$

$$(3.9\times10^{-3}\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})$$

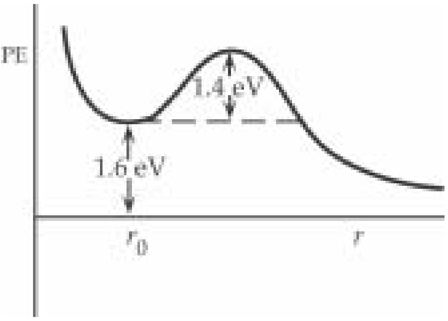
$$= \boxed{2.8\times10^{4}\,\text{J/kg.}}$$



35. An O_2 molecule can be treated as two point masses, 16 u each, and each a distance of $6.05\times10^{-11}\,\text{m}$ from the molecule's center of mass. Then

$$I = \sum mr^2 = 2\left[(16\,\text{u})(1.66\times10^{-27}\,\text{kg/u})\right](6.05\times10^{-11}\,\text{m})^2 = \boxed{1.94\times10^{-46}\,\text{kg}\bullet\text{m}^2.}$$

36. Since dissociation energy releases, the binding energy is positive and the diagram is similar to Fig. 29–10:



37. The photon with the minimum frequency for conduction must have an energy equal to the energy gap:

$$E_g = hf = \frac{hc}{\lambda} = \frac{1.24 \times 10^3\,\text{eV}\bullet\text{nm}}{226\,\text{nm}} = \boxed{5.49\,\text{eV.}}$$

38. To use silicon to filter the wavelengths, we want wavelengths below the IR to be able to cause the electron to be raised to the conduction band, so the photon is absorbed in the silicon. We find the shortest wavelength from

$$\lambda = \frac{c}{f} = \frac{hc}{hf} > \frac{hc}{E_g} = \frac{\left(1.24 \times 10^3\,\text{eV}\bullet\text{nm}\right)}{\left(1.14\,\text{eV}\right)} = 1.09 \times 10^3\,\text{nm} = \boxed{1.09\,\mu\text{m.}}$$

Because this is in the IR, the shorter wavelengths of visible light will excite the electron, so silicon $\boxed{\text{could be used}}$ as a window.

39. In a dielectric, Coulomb’s law becomes

$$F = \frac{ke^2}{r^2} = \frac{e^2}{4\pi K \epsilon_0 r^2}.$$

Thus where e^2 appears in an equation, we divide by *K*. If the “extra” electron is outside the arsenic ion, the effective *Z* will be 1, and we can use the hydrogen results.

(*a*) The energy of the electron is

$$E = -\frac{2\pi^2 Z^2 e^4 mk^2}{K^2 h^2 n^2} = -\frac{\left(13.6\,\text{eV}\right) Z^2}{K^2 n^2} = -\frac{\left(13.6\,\text{eV}\right)\left(1\right)^2}{\left(12\right)^2 \left(1\right)^2} = -0.094\,\text{eV}.$$

Thus the binding energy is $\boxed{0.094\text{ eV.}}$

(*b*) The radius of the electron orbit is

$$r = \frac{K^2 h^2 n^2}{4\pi^2 Z e^2 mk} = \frac{K^2 n^2 \left(0.0529\,\text{nm}\right)}{Z} = \frac{\left(12\right)\left(1\right)^2 \left(0.0529\,\text{nm}\right)}{\left(1\right)^2} = \boxed{0.63\,\text{nm.}}$$

Note that this result justifies the assumption that the electron is outside the arsenic ion.

40. The photon with the longest wavelength has the minimum energy, so the energy gap must be

$$E_g = \frac{hc}{\lambda} = \frac{\left(1.24 \times 10^3\,\text{eV}\bullet\text{nm}\right)}{\left(1000\,\text{nm}\right)} = \boxed{1.24\,\text{eV.}}$$

41. The photon with the minimum frequency for absorption must have an energy equal to the energy gap:

$$E_g = hf = \frac{hc}{\lambda} = \frac{1.24\times10^3\,\text{eV}\bullet\text{nm}}{1.92\times10^6\,\text{nm}} = \boxed{6.46\times10^{-4}\,\text{eV.}}$$

42. The energy gap is related to photon wavelength by

$$E_g = hf = \frac{hc}{\lambda}.$$

For a green LED,

$$E_g = \frac{1.24\times10^3\,\text{eV}\bullet\text{nm}}{525\,\text{nm}} = \boxed{2.36\,\text{eV.}}$$

For a blue LED,

$$E_g = \frac{1.24\times10^3\,\text{eV}\bullet\text{nm}}{465\,\text{nm}} = \boxed{2.67\,\text{eV.}}$$

43. (*a*) The current through the load resistor is

$$I_{\text{load}} = \frac{V_{\text{output}}}{R_{\text{load}}} = \frac{(130\,\text{V})}{(15.0\,\text{k}\Omega)} = 8.67\,\text{mA}.$$

At the minimum supply voltage the current through the diode will be zero, so the current through *R* is 8.67 mA, and the voltage across *R* is

$$V_{\text{R,min}} = I_{\text{R,min}}R = (8.67\,\text{mA})(1.80\,\text{k}\Omega) = 15.6\,\text{V}.$$

The minimum supply voltage is

$$V_{\text{min}} = V_{\text{R,min}} + V_{\text{output}} = 15.6\,\text{V} + 130\,\text{V} = 146\,\text{V}.$$

At the maximum supply voltage the current through the diode will be 120 mA, so the current through *R* is $120\,\text{mA} + 8.67\,\text{mA} = 128.7\,\text{mA}$, and the voltage across *R* is

$$V_{\text{R,max}} = I_{\text{R,max}}R = (128.7\,\text{mA})(1.80\,\text{k}\Omega) = 232\,\text{V}.$$

The maximum supply voltage is

$$V_{\text{max}} = V_{\text{R,max}} + V_{\text{output}} = 232\,\text{V} + 130\,\text{V} = 362\,\text{V}.$$

Thus the range of supply voltages is $\boxed{146\,\text{V} \le V \le 362\,\text{V.}}$

(*b*) At a constant supply voltage the voltage across *R* is $200\,\text{V} - 130\,\text{V} = 70\,\text{V}$, so the current in *R* is

$$I_{\text{R}} = \frac{(70\,\text{V})}{(1.80\,\text{k}\Omega)} = 38.9\,\text{mA}.$$

If there is no current through the diode, this current must be in the load resistor, so we have

$$R_{\text{load}} = \frac{(130\,\text{V})}{(38.9\,\text{mA})} = 3.34\,\text{k}\Omega.$$

If R_{load} is less than this, there will be a greater current through *R*, and thus the voltage across the load will drop and regulation will be lost. If R_{load} is greater than $3.34\,\text{k}\Omega$, the current through R_{load} will decrease and there will be current through the diode. The current through the diode is 38.9 mA when R_{load} is infinite, which is less than the maximum of 120 mA.

Thus the range for load resistance is $\boxed{3.34\,\text{k}\Omega \le R_{\text{load}} < \infty.}$

CHAPTER 30: Nuclear Physics and Radioactivity

Answers to Questions

1. Different isotopes of a given element have the same number of protons and electrons. Because they have the same number of electrons, they have almost identical chemical properties. Each isotope has a different number of neutrons from other isotopes of the same element. Accordingly, they have different atomic masses.

2. Identify the element based on the atomic number.
   (*a*) Uranium (A = 92)
   (*b*) Nitrogen (A = 7)
   (*c*) Hydrogen (A = 1)
   (*d*) Strontium (A = 38)
   (*e*) Berkelium (A = 97)

3. The number of protons is the same as the atomic number, and the number of protons is the mass number minus the number of protons.
   (*a*) Uranium: 92 protons, 140 neutrons
   (*b*) Nitrogen: 7 protons, 11 neutrons
   (*c*) Hydrogen: 1 proton, 0 neutrons
   (*d*) Strontium: 38 protons, 44 neutrons
   (*e*) Berkelium: 97 protons, 150 neutrons

4. With 88 nucleons and 50 neutrons, there must be 38 protons. This is the atomic number, and so the element is strontium. The nuclear symbol would be $^{88}_{38}\text{Sr}$.

5. The atomic mass of an element as shown in the periodic table is the average atomic mass of all naturally-occurring isotopes. For example, chlorine occurs as roughly 75% $^{35}_{17}\text{Cl}$ and 25% $^{36}_{17}\text{Cl}$, and so its atomic mass is about 35.5. This is the average atomic mass that a sample of naturally-occurring chlorine would have. Other smaller effects would include the fact that the masses of the nucleons are not exactly 1 atomic mass unit, and that some small fraction of the mass energy of the total set of nucleons is in the form of binding energy.

6. There must be some force holding the nucleus together. For all nuclei with atomic numbers greater than 1, there are protons packed very close to each other. If the only force present between the protons were the electrostatic force, the protons would repel each other and no nuclei would be stable. Since there are stable nuclei, there must be some other force stronger than the electrostatic force, holding the nucleus together – the strong nuclear force.

7. Similarities between the electromagnetic force and the strong nuclear force: They will both act on charged particles. Differences between the electromagnetic force and the strong nuclear force: The strong force is a short-range force, while the electromagnetic force is an infinite-range force. The strong force is only attractive, while the electromagnetic force can be either attractive or repulsive. The strong force acts between all nucleons no matter whether they are charged or neutral, while the electromagnetic force acts only on charged particles. The strong nuclear force is much stronger than the electromagnetic force.

8. Quoting from section 30-3, "… radioactivity was found in every case to be unaffected by the strongest physical and chemical treatments, including strong heating or cooling and the action of strong chemical reagents." Chemical reactions are a result of electron interactions, not nuclear processes. The absence of effects caused by chemical reactions is evidence that the radioactivity is not due to electron interactions. Another piece of evidence is the fact that the α-particle is much heavier than an electron and has a different charge than the electron, so it can't be an electron. Therefore it must be from the nucleus. Finally, the energies of the electrons or photons emitted from radioactivity are much higher than those corresponding to electron orbital transitions. All of these observations support radioactivity being a nuclear process.

9. For gamma decay: ${}^{64}_{29}\mathrm{Cu} \rightarrow {}^{64}_{29}\mathrm{Cu} + \gamma$. The resulting nuclide is still ${}^{64}_{29}\mathrm{Cu}$.
For beta emission: ${}^{64}_{29}\mathrm{Cu} \rightarrow {}^{64}_{30}\mathrm{Zn} + \mathrm{e}^- + \bar{\nu}$. The resulting nuclide is ${}^{64}_{30}\mathrm{Zn}$.
For positron emission: ${}^{64}_{29}\mathrm{Cu} \rightarrow {}^{64}_{28}\mathrm{Ni} + \mathrm{e}^+ + \nu$. The resulting nuclide is ${}^{64}_{28}\mathrm{Ni}$.

10. The ${}^{238}_{92}\mathrm{U}$ nucleus has 92 protons and 146 neutrons. It decays by α-decay (see appendix B), losing 2 protons and 2 neutrons. Thus the daughter nucleus has 144 neutrons. ${}^{238}_{92}\mathrm{U} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{234}_{90}\mathrm{Th}$. According to appendix B, it also can decay from an excited state via gamma radiation to a lower energy ${}^{238}_{92}\mathrm{U}$, which has 146 neutrons.

11. Gamma rays are neutrally charged, they are made up of high energy photons (travel at *c*), and they have no mass. Alpha rays are made up of helium nuclei, they are the most massive of these three particles, and they have a charge of +2e. Beta rays are made up of electrons or positrons, they can be either positively or negatively charged, and the particles are accompanied by either a neutrino or an anti-neutrino upon decay.

$\boxed{12.}$ (*a*) ${}^{24}_{11}\mathrm{Na} \rightarrow {}^{24}_{12}\mathrm{Mg} + \mathrm{e}^- + \bar{\nu}$ Magnesium-24 is formed.
(*b*) ${}^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + \mathrm{e}^+ + \nu$ Neon-22 is formed.
(*c*) ${}^{210}_{84}\mathrm{Po} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{206}_{82}\mathrm{Pb}$ Lead-206 is formed.

13. (*a*) ${}^{32}_{15}\mathrm{P} \rightarrow {}^{32}_{16}\mathrm{S} + \mathrm{e}^- + \bar{\nu}$ Sulfur-32 is formed.
(*b*) ${}^{35}_{16}\mathrm{S} \rightarrow {}^{35}_{17}\mathrm{Cl} + \mathrm{e}^- + \bar{\nu}$ Chlorine-35 is formed.
(*c*) ${}^{211}_{83}\mathrm{Bi} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{207}_{81}\mathrm{Tl}$ Thallium-207 is formed.

14. (*a*) ${}^{45}_{20}\mathrm{Ca} \rightarrow {}^{45}_{21}\mathrm{Sc} + \mathrm{e}^- + \bar{\nu}$ Scandium-45 is the missing nucleus.
(*b*) ${}^{58}_{29}\mathrm{Cu} \rightarrow {}^{58}_{29}\mathrm{Cu} + \gamma$ Copper-58 is the missing nucleus.
(*c*) ${}^{46}_{24}\mathrm{Cr} \rightarrow {}^{46}_{23}\mathrm{V} + \mathrm{e}^+ + \nu$ The positron and the neutrino are the missing particles.
(*d*) ${}^{234}_{94}\mathrm{Pu} \rightarrow {}^{230}_{92}\mathrm{U} + \alpha$ Uranium-230 is the missing nucleus.
(*e*) ${}^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + \mathrm{e}^- + \bar{\nu}$ The electron and the anti-neutrino are the missing particles.

15. The two "extra" electrons are no longer bound tightly to the nucleus (since the nucleus lost two positively charged protons), and so those extra electrons can move away from the nucleus, moving

towards a higher electric potential or some other positively charged object. They are not “emitted” during the decay, since they do not receive any of the kinetic energy from the decay. If they are close to each other, they would repel each other somewhat.

16. With β^- or β^+ decay, the number of protons in the nucleus changes. In β^- decay, the number of protons increases, and in β^+ decay, the number of protons decreases. Thus, the charge of the nucleus changes and when this occurs all of the energy levels of the atomic electrons change in size. Recall that both the energy values of the level and the Bohr radii are dependent on Z (the charge of the nucleus). Thus, after the decay, there is a lot of shuffling around of the electrons into the new and different energy levels. The electrons will change energies to occupy the new energy levels, and so many photons corresponding to those energy level changes will be emitted.

17. In alpha decay, assuming the energy of the parent nucleus is known, then the unknowns after the decay are the energies of the daughter nucleus and the alpha. These two values can be determined by energy and momentum conservation. Since there are two unknowns and two conditions, the values are uniquely determined. In beta decay, there are three unknown post-decay energies since there are three particles present after the decay. The conditions of energy and momentum conservation are not sufficient to exactly determine the energy of each particle, and so a range of possible values is possible.

18. In electron capture, the nucleus will effectively have a proton change to a neutron. This isotope will then lie to the left and above the original isotope. Since the process would only occur if it made the nucleus more stable, it must lie BELOW the line of stability in Fig. 30-2.

19. Neither hydrogen nor deuterium can emit an α particle. Hydrogen has only one nucleon (a proton) in its nucleus, and deuterium has only two nucleons (one proton and one neutron) in its nucleus. Neither one has the necessary four nucleons (two protons and two neutrons) to emit an α particle.

20. Many artificially produced radioactive isotopes are rare in nature because they have decayed away over time. If the half-lives of these isotopes are relatively short in comparison with the age of Earth (which is typical for these isotopes), then there won’t be any significant amount of these isotopes left to be found in nature.

21. After two months the sample will not have completely decayed. After one month half of the sample will remain, and after two months, one-fourth of the sample will remain. Each month half of the remaining atoms decay.

22. None of the elements with Z > 92 are stable because there is no number of neutrons that are capable of using the strong force in overcoming the electric repulsion of that many positive charges in such close proximity.

23. There are a total of 4 protons and 3 neutrons in the reactant particles. The α-particle has 2 protons and 2 neutrons, and so 2 protons and 1 neutron are in the other product particle. It must be Helium-3.

$${}^{6}_{3}\text{Li} + {}^{1}_{1}\text{p} \;\rightarrow\; {}^{4}_{2}\alpha + {}^{3}_{2}\text{He}$$

24. The technique of ${}^{14}_{6}\text{C}$ would not be used to measure the age of stone walls and tablets. Carbon-14 dating is only useful for measuring the age of objects that were living at some earlier time. Also, Carbon-14 dating is only useful for determining the age of objects less than about 60,000 years old. The stone from which the walls and tablets were made would be much older than 60,000 years.

25. In internal conversion, there is only one decay product (an electron) ejected from the nucleus, and so the electron would have a unique energy, equal to the kinetic energy that an emitted γ ray would have (minus the binding energy of the electron). In β decay, there are two decay products ejected from the nucleus (the electron and the neutrino), and the electron will have a range of possible energies, as discussed in question 17.

Solutions to Problems

Note: A useful expression for the energy of a photon in terms of its wavelength is

$$E = hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J•s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)\left(10^{-9}\ \text{nm/m}\right)}{\left(1.60\times10^{-19}\ \text{J/eV}\right)\lambda};$$

$$E = \frac{\left(1.24\times10^{3}\ \text{eV•nm}\right)}{\lambda}.$$

A factor that appears in the analysis of energies is

$$ke^2 = \left(9.00\times10^{9}\ \text{N•m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2 = 2.30\times10^{-28}\ \text{J•m} = 1.44\,\text{MeV•fm}.$$

1. We convert the units:

$$m = \frac{\left(139\,\text{MeV}/c^2\right)}{\left(931.5\,\text{MeV}/\text{u}c^2\right)} = \boxed{0.149\,\text{u}.}$$

2. The α particle is a helium nucleus:

$$r = \left(1.2\times10^{-15}\ \text{m}\right)A^{\frac{1}{3}} = \left(1.2\times10^{-15}\ \text{m}\right)\left(4\right)^{\frac{1}{3}} = \boxed{1.9\times10^{-15}\ \text{m}} = 1.9\,\text{fm}.$$

3. To find the rest mass of an α particle, we subtract the rest mass of the two electrons from the rest mass of a helium atom:

$$\begin{aligned} m_\alpha &= m_{\text{He}} - 2m_{\text{e}} \\ &= \left(4.002602\,\text{u}\right)\left(931.5\,\text{MeV}/\text{u}c^2\right) - 2\left(0.511\text{MeV}/c^2\right) = \boxed{3727\,\text{MeV}/c^2\,.} \end{aligned}$$

$\boxed{4.}$ (*a*) The radius of ^{64}Cu is

$$r = \left(1.2\times10^{-15}\ \text{m}\right)A^{\frac{1}{3}} = \left(1.2\times10^{-15}\ \text{m}\right)\left(64\right)^{\frac{1}{3}} = \boxed{4.8\times10^{-15}\ \text{m}} = 4.8\,\text{fm}.$$

(*b*) We find the value of *A* from

$$r = \left(1.2\times10^{-15}\ \text{m}\right)A^{\frac{1}{3}};$$

$$3.9\times10^{-15}\ \text{m} = \left(1.2\times10^{-15}\ \text{m}\right)A^{\frac{1}{3}},\ \text{which gives}\ A = \boxed{34.}$$

5. (*a*) The mass of a nucleus with mass number *A* is *A* u and its radius is

$$r = \left(1.2\times10^{-15}\ \text{m}\right)A^{\frac{1}{3}}.$$

Thus the density is

$$\rho = \frac{m}{V}$$

$$= A\frac{\left(1.66\times10^{-27}\ \text{kg/u}\right)}{\frac{4}{3}\pi r^3} = A\frac{\left(1.66\times10^{-27}\ \text{kg/u}\right)}{\frac{4}{3}\pi\left(1.2\times10^{-15}\ \text{m}\right)^3 A}$$

$$= \boxed{2.29\times10^{17}\ \text{kg/m}^3,}\ \text{independent of } A.$$

(*b*) We find the radius from

$$M = pV;$$

$$5.98\times10^{24}\ \text{kg} = \left(2.29\times10^{17}\ \text{kg/m}^3\right)\tfrac{4}{3}\pi R^3,\ \text{which gives } R = \boxed{180\,\text{m.}}$$

(*c*) For equal densities, we have

$$\rho = \frac{M_{\text{Earth}}}{\frac{4}{3}\pi R_{\text{Earth}}{}^3} = \frac{m_{\text{U}}}{\frac{4}{3}\pi r_{\text{U}}{}^3};$$

$$\frac{\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.38\times10^{6}\ \text{m}\right)^3} = \frac{(238\,\text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)}{r_{\text{U}}{}^3},\ \text{which gives } r_{\text{U}} = \boxed{2.58\times10^{-10}\,\text{m.}}$$

6. (*a*) The fraction of mass is

$$\frac{m_{\text{p}}}{\left(m_{\text{p}} + m_{\text{e}}\right)} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)}{\left(1.67\times10^{-27}\ \text{kg} + 9.11\times10^{-31}\ \text{kg}\right)} = \boxed{0.99945.}$$

(*b*) The fraction of volume is

$$\left(\frac{r_{\text{nucleus}}}{r_{\text{atom}}}\right)^3 = \left[\frac{\left(1.2\times10^{-15}\ \text{m}\right)}{\left(0.53\times10^{-10}\ \text{m}\right)}\right]^3 = \boxed{1.2\times10^{-14}.}$$

7. Electron mass is negligible compared to nucleon mass, and one nucleon weighs about 1.0 atomic mass unit. Therefore, in a 1.0-kg object,

$$N = \frac{(1.0\,\text{kg})\left(6.02\times10^{26}\ \text{u/kg}\right)}{1.0\,\text{u/nucleon}} \approx \boxed{6\times10^{26}\,\text{nucleons.}}$$

It does $\boxed{\text{not}}$ matter what the element is, because $\boxed{\text{the mass of one nucleon is essentially the same for all elements.}}$

8. We find the radii of the two nuclei from

$$R = r_0 A^{\frac{1}{3}};$$

$$R_\alpha = (1.2\,\text{fm})(4)^{\frac{1}{3}} = 1.9\,\text{fm};$$

$$R_{\text{U}} = (1.2\,\text{fm})(238)^{\frac{1}{3}} = 7.4\,\text{fm}.$$

If the two nuclei are just touching, the Coulomb potential energy must be the initial kinetic energy of the α particle:

$$\text{KE} = \text{PE} = \frac{Z_\alpha Z_{\text{U}} ke^2}{\left(R_\alpha + R_{\text{U}}\right)}$$

$$=\frac{(2)(92)(1.44\,\text{MeV}\bullet\text{fm})}{(1.9\,\text{fm}+7.4\,\text{fm})}=\boxed{28\,\text{MeV.}}$$

9. From Figure 30–1, we see that the average binding energy per nucleon at $A=40$ is $8.6\,\text{MeV}$. Thus the total binding energy for ^{40}Ca is

$$(40)(8.6\,\text{MeV})=\boxed{340\,\text{MeV.}}$$

10. (*a*) From Figure 30–1, we see that the average binding energy per nucleon at $A=238$ is $7.5\,\text{MeV}$. Thus the total binding energy for ^{238}U is

$$(238)(7.5\,\text{MeV})=\boxed{1.8\times10^{3}\,\text{MeV.}}$$

(*b*) From Figure 30–1, we see that the average binding energy per nucleon at $A=84$ is $8.7\,\text{MeV}$. Thus the total binding energy for ^{84}Kr is

$$(84)(8.7\,\text{MeV})=\boxed{730\,\text{MeV.}}$$

11. Deuterium consists of one proton and one neutron. We find the binding energy from the masses:

$$\text{Binding energy}=\left[m\left({}^{1}\text{H}\right)+m\left({}^{1}\text{n}\right)-m\left({}^{2}\text{H}\right)\right]c^{2}$$

$$=\left[(1.007825\,\text{u})+(1.008665\,\text{u})-(2.014102\,\text{u})\right]c^{2}\left(931.5\,\text{MeV}/\text{u}c^{2}\right)=\boxed{2.22\,\text{MeV.}}$$

$\boxed{12.}$ ^{14}N consists of seven protons and seven neutrons. We find the binding energy from the masses:

$$\text{Binding energy}=\left[7m\left({}^{1}\text{H}\right)+7m\left({}^{1}\text{n}\right)-m\left({}^{14}\text{N}\right)\right]c^{2}$$

$$=\left[7(1.007825\,\text{u})+7(1.008665\,\text{u})-(14.003074\,\text{u})\right]c^{2}\left(931.5\,\text{MeV}/\text{u}c^{2}\right)=104.7\,\text{MeV}.$$

Thus the binding energy per nucleon is

$$\frac{(104.7\,\text{MeV})}{14}=\boxed{7.48\,\text{MeV.}}$$

13. We find the binding energy of the last neutron from the masses:

$$\text{Binding energy}=\left[m\left({}^{39}\text{K}\right)+m\left({}^{1}\text{n}\right)-m\left({}^{40}\text{K}\right)\right]c^{2}$$

$$=\left[(38.963707\,\text{u})+(1.008665\,\text{u})-(39.963999\,\text{u})\right]c^{2}\left(931.5\,\text{MeV}/c^{2}\right)$$

$$=\boxed{7.799\,\text{MeV.}}$$

14. (*a*) ^{6}Li consists of three protons and three neutrons. We find the binding energy from the masses:

$$\text{Binding energy}=\left[3m\left({}^{1}\text{H}\right)+3m\left({}^{1}\text{n}\right)-m\left({}^{6}\text{Li}\right)\right]c^{2}$$

$$=\left[3(1.007825\,\text{u})+3(1.008665\,\text{u})-(6.015121\,\text{u})\right]c^{2}\left(931.5\,\text{MeV}/\text{u}c^{2}\right)$$

$$=\boxed{32.0\,\text{MeV.}}$$

Thus the binding energy per nucleon is

$$\frac{(32.0\,\text{MeV})}{6}=\boxed{5.33\,\text{MeV.}}$$

(*b*) $^{208}\mathrm{Pb}$ consists of 82 protons and 126 neutrons. We find the binding energy from the masses:

$$\begin{aligned}\text{Binding energy} &= \left[82m\left({}^{1}\mathrm{H}\right) + 126m\left({}^{1}\mathrm{n}\right) - m\left({}^{208}\mathrm{Pb}\right)\right]c^2 \\ &= \left[82(1.007825\,\mathrm{u}) + 126(1.008665\,\mathrm{u}) - (207.976627\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{Mev/u}c^2\right) \\ &= \boxed{1.64\,\mathrm{GeV.}}\end{aligned}$$

Thus the binding energy per nucleon is

$$\frac{1.64\,\mathrm{GeV}}{208} = \boxed{7.87\,\mathrm{MeV.}}$$

15. $^{23}\mathrm{Na}$ consists of 11 protons and 12 neutrons. We find the binding energy from the masses:

$$\begin{aligned}\text{Binding energy} &= \left[11m\left({}^{1}\mathrm{H}\right) + 12\left({}^{1}\mathrm{n}\right) - m\left({}^{23}\mathrm{Na}\right)\right]c^2 \\ &= \left[11(1.007825\,\mathrm{u}) + 12(1.008665\,\mathrm{u}) - (22.989770\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV/u}c^2\right) \\ &= 186.6\,\mathrm{MeV.}\end{aligned}$$

Then the average binding energy per nucleon is

$$\frac{186.6\,\mathrm{MeV}}{23} = \boxed{8.11\,\mathrm{MeV/nucleon.}}$$

Similarly, for $^{24}\mathrm{Na}$,

$$\begin{aligned}\text{Binding energy} &= \left[11m\left({}^{1}\mathrm{H}\right) + 12\left({}^{1}\mathrm{n}\right) - m\left({}^{24}\mathrm{Na}\right)\right]c^2 \\ &= \left[11(1.007825\,\mathrm{u}) + 13(1.008665\,\mathrm{u}) - (23.990963\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV/u}c^2\right) \\ &= 193.5\,\mathrm{MeV,}\end{aligned}$$

and the average binding force per nucleon is $\dfrac{193.5\,\mathrm{MeV}}{24} = \boxed{8.06\,\mathrm{MeV/nucleon.}}$

16. We find the required energy for separation from the masses.

(*a*) Removal of a proton creates an isotope of nitrogen:

$$\begin{aligned}\text{Energy(p)} &= \left[m\left({}^{15}\mathrm{N}\right) + m\left({}^{1}\mathrm{H}\right) - m\left({}^{16}\mathrm{O}\right)\right]c^2 \\ &= \left[(15.000108\,\mathrm{u}) + (1.007825\,\mathrm{u}) - (15.994915\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV/u}c^2\right) = \boxed{12.1\,\mathrm{MeV.}}\end{aligned}$$

(*b*) Removal of a neutron creates another isotope of oxygen:

$$\begin{aligned}\text{Energy(n)} &= \left[m\left({}^{15}\mathrm{O}\right) + m\left({}^{1}\mathrm{n}\right) - m\left({}^{16}\mathrm{O}\right)\right]c^2 \\ &= \left[(15.003065\,\mathrm{u}) + (1.008665\,\mathrm{u}) - (15.994915\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV/u}c^2\right) = \boxed{15.7\,\mathrm{MeV.}}\end{aligned}$$

The nucleons are held by the attractive strong nuclear force. It takes less energy to remove the proton because there is also the $\boxed{\text{repulsive electric force}}$ from the other protons.

17. (*a*) We find the binding energy from the masses:

$$\begin{aligned}\text{Binding energy} &= \left[2m\left({}^{4}\mathrm{He}\right) - m\left({}^{8}\mathrm{Be}\right)\right]c^2 \\ &= \left[2(4.002602\,\mathrm{u}) - (8.005305\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV/u}c^2\right) = -0.094\,\mathrm{MeV.}\end{aligned}$$

Because the binding energy is negative, the nucleus is unstable.

(*b*) We find the binding energy from the masses:

$$\text{Binding energy } = \left[3m\left({}^{4}\text{He}\right) - m\left({}^{12}\text{C}\right)\right]c^2$$

$$= \left[3(4.002602\,\text{u}) - (12.000000\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = +7.3\,\text{MeV}.$$

Because the binding energy is positive, the nucleus is $\boxed{\text{stable.}}$

18. The decay is ${}^{3}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{-1}\text{e} + \bar{\nu}$. When we add an electron to both sides to use atomic masses, we see that the mass of the emitted β particle is included in the atomic mass of ${}^{3}\text{He}$.
Thus the energy released is

$$Q = \left[m\left({}^{3}\text{H}\right) - m\left({}^{3}\text{He}\right)\right]c^2 = \left[(3.016049\,\text{u}) - (3.016029\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = 0.0186\,\text{MeV}$$

$$= \boxed{18.6\,\text{keV.}}$$

19. The decay is ${}^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e} + \bar{\nu}$. We take the electron mass to use the atomic mass of ${}^{1}\text{H}$. The kinetic energy of the electron will be maximum if no neutrino is emitted. If we ignore the recoil of the proton, the maximum kinetic energy is

$$\text{KE} = \left[m\left({}^{1}\text{n}\right) - m\left({}^{1}\text{H}\right)\right]c^2 = \left[(1.008665\,\text{u}) - (1.007825\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{0.783\,\text{MeV.}}$$

20. For the decay ${}^{11}_{6}\text{C} \rightarrow {}^{10}_{5}\text{B} + {}^{1}_{1}\text{p}$, we find the difference of the initial and the final masses:

$$\Delta m = m\left({}^{11}\text{C}\right) - m\left({}^{10}\text{B}\right) - m\left({}^{1}\text{H}\right)$$

$$= (11.011433\,\text{u}) - (10.012936\,\text{u}) - (1.007825\,\text{u}) = -0.0099318\,\text{u}.$$

Thus some additional energy would have to be added.

21. If ${}^{22}_{11}\text{Na}$ were a β^- emitter, the resulting nucleus would be ${}^{22}_{12}\text{Mg}$, which has too few neutrons relative to the number of protons to be stable. Thus we have a $\boxed{\beta^+ \text{ emitter.}}$
For the reaction ${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + \beta^+ + \nu$, if we add 11 electrons to both sides in order to use atomic masses, we see that we have two extra electron masses on the right. The kinetic energy of the β^+ will be maximum if no neutrino is emitted. If we ignore the recoil of the neon, the maximum kinetic energy is

$$\text{KE} = \left[m\left({}^{22}\text{Na}\right) - m\left({}^{22}\text{Ne}\right) - 2m(\text{e})\right]c^2$$

$$= \left[(21.994434\,\text{u}) - (21.991383\,\text{u}) - 2(0.00054858\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{1.82\,\text{MeV.}}$$

22. For each decay, we find the difference of the initial and the final masses:

(*a*) $\Delta m = m\left({}^{236}\text{U}\right) - m\left({}^{235}\text{U}\right) - m\left({}^{1}\text{n}\right)$

$$= (236.045562\,\text{u}) - (235.043924\,\text{u}) - (1.008665\,\text{u}) = -0.00703\,\text{u}.$$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

(*b*) $\Delta m = m\left({}^{16}\text{O}\right) - m\left({}^{15}\text{O}\right) - m\left({}^{1}\text{n}\right)$

$$= (15.994915\,\text{u}) - (15.003065\,\text{u}) - (1.008665\,\text{u}) = -0.0168\,\text{u}.$$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

(*c*) $\Delta m = m\left({}^{23}\text{Na}\right) - m\left({}^{22}\text{Na}\right) - m\left({}^{1}\text{n}\right)$

$= (22.989767\,\text{u}) - (21.994434\,\text{u}) - (1.008665\,\text{u}) = -0.0133\,\text{u}.$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

$\boxed{23.}$ We find the final nucleus by balancing the mass and charge numbers:

$Z(X) = Z(\text{U}) - Z(\text{He}) = 92 - 2 = 90;$

$A(X) = A(\text{U}) - A(\text{He}) = 238 - 4 = 234,$ so the final nucleus is ${}^{234}_{90}\text{Th}.$

If we ignore the recoil of the thorium, the kinetic energy of the α particle is

$\text{KE} = \left[m\left({}^{238}\text{U}\right) - m\left({}^{234}\text{Th}\right) - m\left({}^{4}\text{He}\right)\right]c^2;$

$4.20\,\text{MeV} = \left[(238.050783\,\text{u}) - m\left({}^{234}\text{Th}\right) - (4.002602\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right),$

which gives $m\left({}^{234}\text{Th}\right) = \boxed{234.04367\,\text{u.}}$

24. The kinetic energy of the electron will be maximum if no neutrino is emitted. If we ignore the recoil of the sodium, the maximum kinetic energy of the electron is

$\text{KE} = \left[m\left({}^{23}\text{Ne}\right) - m\left({}^{23}\text{Na}\right)\right]c^2$

$= \left[(22.9945\,\text{u}) - (22.989767\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{4.4\,\text{MeV.}}$

When the neutrino has all of the kinetic energy, the minimum kinetic energy of the electron is $\boxed{0.}$ The sum of the kinetic energy of the electron and the energy of the neutrino must be from the mass difference, so the energy range of the neutrino will be $\boxed{0 \le E_\nu \le 4.4\,\text{MeV.}}$

25. We use conservation of momentum:

$p\left({}^{4}\alpha\right) = p\left({}^{234}\chi\right);$

$\sqrt{2m_\alpha\left(\text{KE}_\alpha\right)} = \sqrt{2m_\chi\left(\text{KE}_\chi\right)};$

$\text{KE}_\chi = \dfrac{m_\alpha}{m_\chi}\left(\text{KE}_\alpha\right) = \dfrac{4}{234}(5.0\,\text{MeV}) = \boxed{0.085\,\text{MeV.}}$

26. The reaction is ${}^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni} + \beta^- + \nu$. The kinetic energy of the β^- will be maximum if no neutrino is emitted. If we ignore the recoil of the nickel, the maximum kinetic energy is

$\text{KE} = \left[m\left({}^{60}\text{Co}\right) - m\left({}^{60}\text{Ni}\right) - m(\text{e})\right]c^2$

$= \left[(59.933822\,\text{u}) - (59.930791\,\text{u}) - (0.000549\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{2.31\,\text{MeV.}}$

27. (*a*) We find the final nucleus by balancing the mass and charge numbers:

$Z(\chi) = Z(\text{P}) - Z(\text{e}) = 15 - (-1) = 16;$

$A(\chi) = A(\text{P}) - A(\text{e}) = 32 - 0 = 32,$ so the final nucleus is $\boxed{{}^{32}_{16}\text{S.}}$

(*b*) If we ignore the recoil of the sulfur, the maximum kinetic energy of the electron is

$\text{KE} = \left[m\left({}^{32}\text{P}\right) - m\left({}^{32}\text{S}\right)\right]c^2;$

$1.71\,\text{MeV} = \left[(31.973908\,\text{u}) - m\left({}^{32}\text{S}\right)\right]c^2\left(931.5\,\text{MeV/u}c^2\right),$

which gives $m\left({}^{32}\mathrm{S}\right)=\boxed{31.97207\,\mathrm{u}.}$

28. For alpha decay we have ${}^{218}_{84}\mathrm{Po} \rightarrow {}^{214}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$. The Q value is

$$Q=\left[m\left({}^{218}\mathrm{Po}\right)-m\left({}^{214}\mathrm{Pb}\right)-m\left({}^{4}\mathrm{He}\right)\right]c^2$$

$$=\left[(218.008965\,\mathrm{u})-(213.999798\,\mathrm{u})-(4.002602\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right)=\boxed{612\,\mathrm{MeV}.}$$

For beta decay we have ${}^{218}_{84}\mathrm{Po} \rightarrow {}^{218}_{85}\mathrm{At} + {}^{0}_{-1}\mathrm{e}$. The Q value is

$$Q=\left[m\left({}^{218}\mathrm{Po}\right)-m\left({}^{218}\mathrm{At}\right)\right]c^2$$

$$=\left[(218.008965\,\mathrm{u})-(218.00868\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right)=\boxed{0.27\,\mathrm{MeV}.}$$

29. For the electron capture ${}^{7}_{4}\mathrm{Be} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{7}_{3}\mathrm{Li} + v,$ we see that if we add three electron masses to both sides to use the atomic mass for Li, we use the captured electron for the atomic mass of Be.
We find the Q value from

$$Q=\left[m\left({}^{7}\mathrm{Be}\right)-m\left({}^{7}\mathrm{Li}\right)\right]c^2$$

$$=\left[(7.016928\,\mathrm{u})-(7.016003\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right)=\boxed{0.861\mathrm{MeV}.}$$

30. We find the energy from

$$E=\frac{h_c}{\lambda}=\frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{\left(1.00\times10^{-13}\mathrm{m}\right)\left(1.602\times10^{-19}\,\mathrm{J/eV}\right)}=\boxed{12.4\,\mathrm{MeV}.}$$

This is a γ ray from the nucleus. Electron transitions do not involve this much energy.

31. The kinetic energy of the β^+ particle will be maximum if no neutrino is emitted. If we ignore the recoil of the boron, the maximum kinetic energy is

$$\mathrm{KE}=\left[m\left({}^{11}\mathrm{C}\right)-m\left({}^{11}\mathrm{B}\right)-2m_{\mathrm{e}}\right]c^2$$

$$=\left[(11.011433\,\mathrm{u})-(11.009305\,\mathrm{u})-2(0.00054858)\right]c^2\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right)=\boxed{0.960\,\mathrm{MeV}.}$$

The sum of the kinetic energy of the β^+ particle and the energy of the neutrino must be from the mass difference, so the kinetic energy of the neutrino will range from $\boxed{0.960\text{ MeV to }0.}$

32. We find the recoil energy using the fact of conservation of momentum:

$$p_\gamma=\frac{E_\gamma}{c}=p_{\mathrm{K}};$$

$$\mathrm{KE}_{\mathrm{K}}=\frac{p_{\mathrm{K}}^{\ 2}}{2m}=\frac{\left(\dfrac{E_\gamma}{c}\right)^2}{2m}=\frac{(1.46\,\mathrm{MeV})^2}{2(40.0\,\mathrm{u})\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right)c^2}=2.86\times10^{-5}\,\mathrm{MeV}=\boxed{28.6\,\mathrm{eV}.}$$

$\boxed{33.}$ If we ignore the recoil of the lead, the kinetic energy of the α particle is

$$\mathrm{KE}=\left[m\left({}^{210}\mathrm{Po}\right)-m\left({}^{206}\mathrm{Pb}\right)-m\left({}^{4}\mathrm{He}\right)\right]c^2$$

$$=\left[(209.982848\,\mathrm{u})-(205.974440\,\mathrm{u})-(4.002602\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right)=\boxed{5.41\mathrm{MeV}.}$$

34. The decay is ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$. If the uranium nucleus is at rest when it decays, for momentum conservation we have

$$p_\alpha = p_{\text{Th}}.$$

Thus the kinetic energy of the thorium nucleus is

$$\text{KE}_{\text{Th}} = \frac{{p_{\text{Th}}}^2}{2m_{\text{Th}}} = \frac{{p_\alpha}^2}{2m_{\text{Th}}} = \left(\frac{m_\alpha}{m_{\text{Th}}}\right)\left(\text{KE}_\alpha\right) = \left(\frac{4\,\text{u}}{234\,\text{u}}\right)\left(4.20\,\text{MeV}\right) = \boxed{0.0718\,\text{MeV.}}$$

The Q value is the total kinetic energy produced:

$$\text{Q} = \text{KE}_\alpha + \text{KE}_{\text{Th}} = 4.20\,\text{MeV} + 0.0718 = \boxed{4.27\,\text{MeV.}}$$

35. For the positron-emission process

$${}^{A}_{Z+1}\chi \rightarrow {}^{A}_{Z}\chi' + \text{e}^+ + v,$$

we need to add $Z+1$ electrons to the nuclear mass of χ to be able to use the atomic mass. On the right-hand side we use Z electrons to be able to use the atomic mass of χ'. Thus we have 1 electron mass and the β-particle mass, which means that we must include 2 electron masses on the right-hand side.

The Q value will be

$$Q = \left[M_{\text{P}} - \left(M_{\text{D}} + 2m_{\text{e}}\right)\right]c^2 = \left(M_{\text{P}} - M_{\text{D}} - 2m_{\text{e}}\right)c^2.$$

$\boxed{36.}$ We find the decay constant from

$$\lambda N = \lambda N_0 e^{-\lambda t};$$

$$320\,\text{decays/min} = \left(1280\,\text{decays/min}\right)e^{-\lambda(4.6\,\text{h})},\ \text{which gives } \lambda = 0.301/\text{h}.$$

Thus the half-life is

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{\left(0.301/\text{h}\right)} = \boxed{2.3\,\text{h.}}$$

37. (*a*) The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{\left(4.5 \times 10^9\,\text{yr}\right)} = \boxed{1.5 \times 10^{-10}\,\text{yr.}}$$

(*b*) The half-life is

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{\left(8.2 \times 10^{-5}\,\text{s}^{-1}\right)} = 8450\,\text{s} = \boxed{2.3\,\text{h.}}$$

38. The activity of the sample is

$$\frac{\Delta N}{\Delta t} = \lambda N = \left(\frac{0.693}{T_{\frac{1}{2}}}\right)N = \left[\frac{0.693}{\left(5730\,\text{yr}\right)\left(3.16 \times 10^7\,\text{s/yr}\right)}\right]\left(3.1 \times 10^{20}\right) = \boxed{1.2 \times 10^9\,\text{decays/s.}}$$

39. We find the fraction remaining from

$$N = N_0 e^{-\lambda t};$$

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-\left[\frac{(0.693)(3.0\,\text{yr})(12\,\text{mo/yr})}{9\,\text{mo}}\right]} = \boxed{0.0625.}$$

40. The fraction left is

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n = \left(\tfrac{1}{2}\right)^6 = \boxed{0.015625.}$$

41. We find the number of nuclei from the activity of the sample:

$$\frac{\Delta N}{\Delta t} = \lambda N;$$

$$640\,\text{decays/s} = \left[\frac{(0.693)}{(4.468\times10^9\ \text{yr})(3.16\times10^7\ \text{s/yr})}\right]N,\ \text{which gives}\ N = \boxed{1.30\times10^{20}\ \text{nuclei.}}$$

42. Because only α particle decay changes the mass number (by 4), we have

$$N_\alpha = \frac{(235-207)}{4} = \boxed{7\,\alpha\ \text{particles.}}$$

An α particle decreases the atomic number by 2, while a β^- particle increases the atomic number by 1, so we have

$$N_\beta = \frac{[92-82-7(2)]}{(-1)} = \boxed{4\,\beta^-\ \text{particles.}}$$

$\boxed{43.}$ The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(8.02\,\text{days})(24\,\text{h/day})(3600\,\text{s/h})} = 1.000\times10^{-6}\ \text{s}^{-1}.$$

The initial number of nuclei is

$$N_0 = \left[\frac{(682\times10^{-6}\ \text{g})}{(131\,\text{g/mol})}\right](6.02\times10^{23}\ \text{atoms/mol}) = 3.134\times10^{18}\ \text{nuclei.}$$

(*a*) When $t=0,$ we get

$$\lambda N = \lambda N_0 e^{-\lambda t} = (1.000\times10^{-6}\ \text{s}^{-1})(3.134\times10^{18})e^0 = \boxed{3.13\times10^{12}\ \text{decays/s.}}$$

(*b*) When $t = 1.0\,\text{h},$ the exponent is

$$\lambda t = (1.000\times10^{-6}\ \text{s}^{-1})(1.0\,\text{h})(3600\,\text{s/h}) = 3.600\times10^{-3},$$

so we get

$$\lambda N = \lambda N_0 e^{-\lambda t} = (1.000\times10^{-6}\ \text{s}^{-1})(3.134\times10^{18})e^{-0.003600} = \boxed{3.12\times10^{12}\ \text{decays/s.}}$$

(*c*) When $t = 6$ months, the exponent is

$$\lambda t = (1.000\times10^{-6}\ \text{s}^{-1})(6\,\text{mo})(30.5\,\text{days/mo})(24\,\text{h/day})(3600\,\text{s/h}) = 15.81,$$

so we get

$$\lambda N = \lambda N_0 e^{-\lambda t} = (1.000\times10^{-6}\ \text{s}^{-1})(3.134\times10^{18})e^{-15.81} = \boxed{4.26\times10^5\ \text{decays/s.}}$$

44. The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(30.8\,\text{s})} = 0.0225\,\text{s}^{-1}.$$

(*a*) The initial number of nuclei is

$$N_0 = \frac{(8.8\times10^{-6}\,\text{g})}{(124\,\text{g/mol})}(6.02\times10^{23}\,\text{atoms/mol}) = \boxed{4.3\times10^{16}\,\text{nuclei.}}$$

(*b*) When $t = 2.0\,\text{min}$, the exponent is

$$\lambda t = (0.225\,\text{s}^{-1})(2.0\,\text{min})(60\,\text{s/min}) = 2.7,$$

so we get

$$N = N_0 e^{-\lambda t} = (4.3\times10^{16})e^{-2.7} = \boxed{2.9\times10^{15}\,\text{nuclei.}}$$

(*c*) The activity is

$$\lambda N = (0.0225\,\text{s}^{-1})(2.9\times10^{15}) = \boxed{6.5\times10^{13}\,\text{decays/s.}}$$

(*d*) We find the time from

$$\lambda N = \lambda N_0 e^{-\lambda t};$$

$$1\,\text{decay/s} = (0.0225\,\text{s}^{-1})(4.3\times10^{16})e^{-(0.0225/\text{s})t}, \text{ which gives } t = 1.53\times10^3\,\text{s} = \boxed{26\,\text{min.}}$$

45. We find the number of nuclei from

$$\text{Activity} = \lambda N;$$

$$2.0\times10^5\,\text{decays/s} = \left[\frac{0.693}{(1.28\times10^9\,\text{yr})(3.16\times10^7\,\text{s/yr})}\right]N, \text{ which gives } N = 1.17\times10^{12}\ \text{nuclei.}$$

The mass is

$$m = \frac{(1.17\times10^{22}\,\text{nuclei})(40.0\,\text{g/mol})}{6.02\times10^{23}\,\text{nuclei/mol}} = \boxed{0.77\,\text{g.}}$$

46. The number of nuclei is

$$N = \left[\frac{(9.7\times10^{-6}\,\text{g})}{(32\,\text{g/mol})}\right](6.02\times10^{23}\,\text{atoms/mol}) = 1.82\times10^{17}\ \text{nuclei.}$$

The activity is

$$\lambda N = \left[\frac{(0.693)}{(1.23\times10^6\,\text{s})}\right](1.82\times10^{17}) = \boxed{1.0\times10^{11}\,\text{decays/s.}}$$

47. We find the number of nuclei from

$$\text{Activity} = \lambda N;$$

$$2.65\times10^5\,\text{decays/s} = \left[\frac{(0.693)}{(7.55\times10^6\,\text{s})}\right]N, \text{ which gives } N = 2.89\times10^{12}\,\text{nuclei.}$$

The mass is

$$m = \left[\frac{(2.89\times10^{12}\,\text{nuclei})}{(6.02\times10^{23}\,\text{atoms/mol})}\right](35\,\text{g/mol}) = \boxed{1.68\times10^{-10}\,\text{g.}}$$

48. (*a*) The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{\left(1.59\times10^{5}\text{ yr}\right)\left(3.16\times10^{7}\text{ s/yr}\right)} = \boxed{1.38\times10^{-13}\text{ s}^{-1}.}$$

(*b*) The activity is

$$\lambda N = \left(1.38\times10^{-13}\text{ s}^{-1}\right)\left(7.50\times10^{19}\right) = 1.03\times10^{7}\text{ decays/s} = \boxed{6.21\times10^{8}\text{ decays/min.}}$$

49. We find the number of half-lives from

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)}{\left(\frac{\Delta N}{\Delta t}\right)_0} = \left(\tfrac{1}{2}\right)^n;$$

$$\frac{1}{10} = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log 10, \text{ which gives } n = 3.32.$$

Thus the half-life is

$$T_{\frac{1}{2}} = \frac{t}{n} = \frac{(8.6\text{ min})}{3.32} = \boxed{2.6\text{ min.}}$$

50. Because the fraction of atoms that are ^{14}C is so small, we use the atomic weight of ^{12}C to find the number of carbon atoms in 285 g:

$$N = \left[\frac{(285\text{ g})}{(12\text{ g/mol})}\right]\left(6.02\times10^{23}\text{ atoms/mol}\right) = 1.43\times10^{25}\text{ atoms.}$$

The number of ^{14}C nuclei is

$$N_{14} = \left(\frac{1.3}{10^{12}}\right)\left(1.43\times10^{25}\right) = 1.86\times10^{13}\text{ nuclei.}$$

The activity is

$$\lambda N = \left[\frac{0.693}{(5730\text{ yr})\left(3.16\times10^{7}\text{ s/yr}\right)}\right]\left(1.86\times10^{13}\right) = \boxed{71\text{ decays/s.}}$$

51. We find the number of nuclei from

$$\text{Activity} = \lambda N;$$

$$6.70\times10^{2}\text{ decays/s} = \left[\frac{(0.693)}{\left(1.28\times10^{9}\text{ s}\right)\left(3.16\times10^{7}\text{ s/yr}\right)}\right]N, \text{ which gives } N = 3.91\times10^{19}\text{ nuclei.}$$

The mass is

$$m = \left[\frac{\left(3.91\times10^{19}\text{ nuclei}\right)}{\left(6.02\times10^{23}\text{ atoms/mol}\right)}\right](40\text{ g/mol}) = 2.60\times10^{-3}\text{ g} = \boxed{2.60\text{ mg.}}$$

52. We assume that the elapsed time is much smaller than the half-life, so we can use a constant decay rate. Because ^{87}Sr is stable, and there was none present when the rocks were formed, every atom of ^{87}Rb that decayed is now an atom of ^{87}Sr. Thus we have

$$N_{\text{Sr}} = -\Delta N_{\text{Rb}} = \lambda N_{\text{Rb}}\,\Delta t, \text{ or}$$

$$\frac{N_{\text{Sr}}}{N_{\text{Rb}}} = \left(\frac{0.693}{T_{\frac{1}{2}}}\right)\Delta t;$$

$$0.0160 = \left(\frac{0.693}{\left(4.75\times10^{10}\ \text{yr}\right)}\right)\Delta t,\ \text{which gives}\ \Delta t = \boxed{1.1\times10^{9}\ \text{yr.}}$$

This is $\approx 2\%$ of the half-life, so our original assumption is valid.

53. The decay rate is

$$\frac{\Delta N}{\Delta t} = \lambda N.$$

If we assume equal numbers of nuclei decaying by α emission, we have

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)_{218}}{\left(\frac{\Delta N}{\Delta t}\right)_{214}} = \frac{\lambda_{218}}{\lambda_{214}} = \frac{T_{\frac{1}{2},214}}{T_{\frac{1}{2},218}}$$

$$= \frac{\left(1.6\times10^{-4}\ \text{s}\right)}{\left(3.1\,\text{min}\right)\left(60\,\text{s/min}\right)} = \boxed{8.6\times10^{-7}.}$$

54. The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{\left(53\,\text{days}\right)} = \frac{0.0131}{\text{day}} = 1.52\times10^{-7}\ \text{s}^{-1}.$$

(*a*) We find the number of half-lives from

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)}{\left(\frac{\Delta N}{\Delta t}\right)_0} = \left(\tfrac{1}{2}\right)^n;$$

$$\frac{\left(15\,\text{decays/s}\right)}{\left(450\,\text{decays/s}\right)} = \left(\tfrac{1}{2}\right)^n,\ \text{or}\ n\log 2 = \log 30,\ \text{which gives}\ n = 4.91.$$

Thus the elapsed time is

$$\Delta t = nT_{\frac{1}{2}} = \left(4.91\right)\left(53\,\text{days}\right) = \boxed{261\,\text{days}} \approx 8.5\,\text{months.}$$

(*b*) We find the number of nuclei from

$$\text{Activity} = \lambda N;$$

$$450\,\text{decays/s} = \left(1.52\times10^{-7}\ \text{s}^{-1}\right)N,\ \text{which gives}\ N = 2.97\times10^{9}\ \text{nuclei.}$$

The mass is

$$m = \left[\frac{\left(2.97\times10^{9}\ \text{nuclei}\right)}{\left(6.02\times10^{23}\ \text{atoms/mol}\right)}\right]\left(7\,\text{g/mol}\right) = \boxed{3.5\times10^{-17}\ \text{kg.}}$$

55. $\boxed{{}^{232}_{90}\mathrm{Th}} \rightarrow {}^{228}_{88}\mathrm{Ra} + {}^{4}_{2}\alpha$, $\boxed{{}^{228}_{88}\mathrm{Ra}} \rightarrow {}^{228}_{89}\mathrm{Ac} + \beta^{-}$,

$\boxed{{}^{228}_{89}\mathrm{Ac}} \rightarrow {}^{228}_{90}\mathrm{Th} + \beta^{-}$, $\boxed{{}^{228}_{90}\mathrm{Th}} \rightarrow {}^{224}_{88}\mathrm{Ra} + {}^{4}_{2}\alpha$,

$\boxed{{}^{224}_{88}\mathrm{Ra}} \rightarrow \boxed{{}^{220}_{86}\mathrm{Rn}} + {}^{4}_{2}\alpha$, $\boxed{{}^{235}_{92}\mathrm{U}} \rightarrow {}^{231}_{90}\mathrm{Th} + {}^{4}_{2}\alpha$,

$\boxed{{}^{231}_{90}\mathrm{Th}} \rightarrow {}^{231}_{91}\mathrm{Pa} + \beta^{-}$, $\boxed{{}^{231}_{91}\mathrm{Pa}} \rightarrow {}^{227}_{89}\mathrm{Ac} + {}^{4}_{2}\alpha$,

$\boxed{{}^{227}_{89}\mathrm{Ac}} \rightarrow {}^{227}_{90}\mathrm{Th} + \beta^{-}$, $\boxed{{}^{227}_{90}\mathrm{Th}} \rightarrow \boxed{{}^{223}_{88}\mathrm{Ra}} + {}^{4}_{2}\alpha$.

56. The decay constant is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(5730\,\mathrm{yr})} = 1.209\times10^{-4}/\mathrm{yr}.$$

Because the fraction of atoms that are ${}^{14}\mathrm{C}$ is so small, we use the atomic weight of ${}^{12}\mathrm{C}$ to find the number of carbon atoms in 290 g:

$$N = \left[\frac{(290\,\mathrm{g})}{(12\,\mathrm{g/mol})}\right]\left(6.02\times10^{23}\ \mathrm{atoms/mol}\right) = 1.45\times10^{25}\ \mathrm{atoms},$$

so the number of ${}^{14}\mathrm{C}$ nuclei in a sample from a living tree is

$$N_{14} = \left(1.3\times10^{-12}\right)\left(1.45\times10^{25}\right) = 1.89\times10^{13}\ \mathrm{nuclei}.$$

Because the carbon is being replenished in living trees, we assume that this number produced the activity when the club was made. We determine its age from

$$\lambda N = \lambda N_{14} e^{-\lambda t};$$

$$8.0\,\mathrm{decays/s} = \left[\frac{\left(1.209\times10^{-4}/\mathrm{yr}\right)}{\left(3.16\times10^{7}\ \mathrm{s/yr}\right)}\right]\left(1.89\times10^{13}\ \mathrm{nuclei}\right)e^{-\left(1.209\times10^{-4}/\mathrm{yr}\right)t},$$

which gives $t = \boxed{1.8\times10^{4}\ \mathrm{yr.}}$

57. The number of radioactive nuclei decreases exponentially:

$$N = N_0 e^{-\lambda t}.$$

Every radioactive nucleus that decays becomes a daughter nucleus, so we have

$$N_{\mathrm{D}} = N_0 - N = \boxed{N_0\left(1 - e^{-\lambda t}\right).}$$

58. We find the number of half-lives from

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)}{\left(\frac{\Delta N}{\Delta t}\right)_0} = \left(\tfrac{1}{2}\right)^n;$$

$$1.050\times10^{-2} = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log\left(\frac{1}{1.050\times10^{-2}}\right), \text{ which gives } n = 6.57.$$

Thus the half-life is

$$T_{\frac{1}{2}} = \frac{t}{n} = \frac{(4.00\,\text{h})}{6.57} = 0.609\,\text{h} = 36.5\,\text{min}.$$

From the Appendix we see that the isotope is $\boxed{{}^{211}_{82}\text{Pb.}}$

59. Because the carbon is being replenished in living trees, we assume that the amount of ^{14}C is constant until the wood is cut, and then it decays. We find the number of half-lives from

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n;$$

$$0.060 = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log(16.7), \text{ which gives } n = 4.06.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (4.06)(5730\,\text{yr}) = \boxed{2.3\times10^4\,\text{yr.}}$$

60. (*a*) We find the mass number from its radius:

$$r = (1.2\times10^{-15}\,\text{m})A^{\frac{1}{3}};$$

$$5.0\times10^3\,\text{m} = (1.2\times10^{-15}\,\text{m})A^{\frac{1}{3}}, \text{ which gives } A = \boxed{7.2\times10^{55}.}$$

(*b*) The mass of the neutron star is

$$m = A(1.66\times10^{-27}\,\text{kg/u}) = (7.2\times10^{55}\,\text{u})(1.66\times10^{-27}\,\text{kg/u}) = \boxed{1.2\times10^{29}\,\text{kg.}}$$

Note that this is about 6% of the mass of the Sun.

(*c*) The acceleration of gravity on the surface of the neutron star is

$$g = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11}\,\text{N•m}^2/\text{kg}^2)(1.2\times10^{29}\,\text{kg})}{(5.0\times10^3\,\text{m})^2} = \boxed{3.2\times10^{11}\,\text{m/s}^2.}$$

61. Because the tritium in water is being replenished, we assume that the amount is constant until the wine is made, and then it decays. We find the number of half-lives from

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n;$$

$$0.10 = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log(10), \text{ which gives } n = 3.32.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (3.32)(12.33\,\text{yr}) = \boxed{41\,\text{yr.}}$$

62. If we assume a body has 70 kg of water, the number of water molecules is

$$N_{\text{water}} = \left[\frac{(70\times10^3\,\text{g})}{(18\,\text{g/mol})}\right](6.02\times10^{23}\,\text{atoms/mol}) = 2.34\times10^{27}\text{ molecules.}$$

The number of protons in a water molecule (H_2O) is $2 + 8 = 10$, so the number of protons is

$$N_0 = 2.34\times10^{28}\text{ protons.}$$

If we assume that the time is much less than the half-life, the rate of decay is constant, so we have

$$\frac{\Delta N}{\Delta t}=\lambda N=\left(\frac{0.693}{T_{\frac{1}{2}}}\right)N;$$

$$\frac{(1\text{proton})}{\Delta t}=\left[\frac{(0.693)}{(10^{28}\text{ yr})}\right](2.34\times10^{28}\text{ protons}),\text{ which gives }\Delta t=\boxed{6\times10^{3}\text{ yr.}}$$

63. We find the number of half-lives from

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)}{\left(\frac{\Delta N}{\Delta t}\right)_0}=\left(\tfrac{1}{2}\right)^n;$$

$$1.00\times10^{-2}=\left(\tfrac{1}{2}\right)^n,\text{ or }n\log2=\log(100),\text{ which gives }n=6.65,\text{ so the time is }\boxed{6.65T_{\frac{1}{2}}.}$$

64. We find the number of ^{40}K nuclei from

$$\text{Activity}=\lambda N_{40};$$

$$60\,\text{decays/s}=\left[\frac{(0.693)}{(1.28\times10^{9}\text{ s})(3.16\times10^{7}\text{ s/yr})}\right]N_{40},\text{ which gives }N_{40}=3.5\times10^{18}\text{ nuclei.}$$

The mass of ^{40}K is

$$m_{40}=(3.5\times10^{18})(40\,\text{u})(1.66\times10^{-27}\text{ kg/u})=2.33\times10^{-7}\text{ kg}=\boxed{0.23\,\text{mg.}}$$

From the Appendix we have

$$N_{40}=(0.0117\%)N,\text{ and }N_{39}=(93.2581\%)N.$$

Thus the number of ^{39}K nuclei is

$$N_{39}=\left[\frac{(93.258\%)}{(0.0117\%)}\right](3.5\times10^{18}\text{ nuclei})=2.8\times10^{22}\text{ nuclei.}$$

The mass of ^{39}K is

$$m_{39}=(2.8\times10^{22})(39\,\text{u})(1.66\times10^{-27}\text{ kg/u})=1.81\times10^{-3}\text{ kg}=\boxed{1.8\,\text{g.}}$$

65. If the initial nucleus is at rest when it decays, for momentum conservation we have

$$p_\alpha=p_{\text{D}}.$$

Thus the kinetic energy of the daughter is

$$\text{KE}_{\text{D}}=\frac{P_{\text{D}}^{\,2}}{2m_{\text{D}}}=\frac{P_\alpha^{\,2}}{2m_{\text{D}}}=\left(\frac{m_\alpha}{m_{\text{D}}}\right)(\text{KE}_\alpha)=\left(\frac{A_\alpha}{A_{\text{D}}}\right)(\text{KE}_\alpha)=\left(\frac{4}{A_{\text{D}}}\right)(\text{KE}_\alpha).$$

Thus the fraction carried away by the daughter is

$$\frac{(\text{KE}_{\text{D}})}{(\text{KE}_\alpha+\text{KE}_{\text{D}})}=\frac{\left(\frac{4}{A_{\text{D}}}\right)(\text{KE}_\alpha)}{\left[(\text{KE}_\alpha)+\left(\frac{4}{A_{\text{D}}}\right)(\text{KE}_\alpha)\right]}=\frac{1}{\left[1+\left(\frac{A_{\text{D}}}{4}\right)\right]}.$$

For the decay of ^{226}Ra, the daughter has $A_{\text{D}}=222$, so we get

$$\text{fraction}_{\text{D}} = \frac{1}{\left[1+\left(\frac{222}{4}\right)\right]} = 0.018.$$

Thus the α particle carries away $1 - 0.018 = 0.982 = \boxed{98.2\%.}$

66. We see from the periodic chart that Sr is in the same column as $\boxed{\text{calcium.}}$

If strontium is ingested, the body will treat it chemically as if it were calcium, which means it will be $\boxed{\text{stored by the body in bones.}}$

We find the number of half-lives to reach a 1% level from

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n;$$

$$0.01 = \left(\tfrac{1}{2}\right)^n, \text{ or } n\log 2 = \log(100), \text{ which gives } n = 6.64.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (6.64)(29\,\text{yr}) = \boxed{193\,\text{yr.}}$$

The decay reactions are

$$\boxed{\begin{array}{l} {}^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + \bar{\nu},\ {}^{90}_{39}Y \text{ is radioactive;} \\ {}^{90}_{39}\text{Y} \rightarrow {}^{90}_{40}\text{Zr} + {}^{0}_{-1}\text{e} + \bar{\nu},\ {}^{90}_{40}\text{Zr is stable.} \end{array}}$$

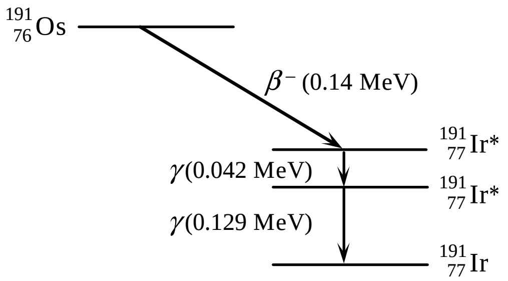
67. (*a*) We find the daughter nucleus by balancing the mass and charge numbers:

$$Z(X) = Z(\text{Os}) - Z(\text{e}^-) = 76 - (-1) = 77;$$

$$A(X) = A(\text{Os}) - A(\text{e}^-) = 191 - 0 = 191,$$

so the daughter nucleus is $\boxed{{}^{191}_{77}\text{Ir.}}$

(*b*) Because there is only one β energy, the β decay must be to the higher excited state.



68. (*a*) The number of nuclei is

$$N = \left(\frac{1.0\,\text{g}}{131\,\text{g/mol}}\right)(6.02\times10^{23}\text{ nuclei/mol}) = 4.60\times10^{21}\text{ nuclei.}$$

The activity is

$$\lambda N = \left[\frac{0.693}{(8.02\,\text{days})(8.64\times10^4\text{ s/day})}\right](4.60\times10^{21}) = \boxed{4.57\times10^{15}\text{ decays/s.}}$$

(*b*) The number of nuclei is

$$N = \left(\frac{1.0\,\text{g}}{238\,\text{g/mol}}\right)(6.02\times10^{23}\text{ nuclei/mol}) = 2.53\times10^{21}\text{ nuclei.}$$

The activity is

$$\lambda N = \left[\frac{0.693}{(4.47\times10^9\text{ yr})(3.16\times10^7\text{ s/yr})}\right](2.53\times10^{21}) = \boxed{1.24\times10^4\text{ decays/s.}}$$

69. From Figure 30–1, we see that the average binding energy per nucleon at $A = 29$ is 8.6 MeV. If we use the average atomic weight as the average number of nucleons for the two stable isotopes of copper, the total binding energy is

$$(63.5)(8.6\,\text{MeV}) = \boxed{550\,\text{MeV.}}$$

The number of atoms in a penny is

$$N = \left[\frac{\left(3\times10^{3}\,\text{g}\right)}{\left(63.5\,\text{g/mol}\right)}\right]\left(6.02\times10^{23}\,\text{atoms/mol}\right) = 2.84\times10^{22}\,\text{atoms.}$$

Thus the total energy needed is

$$\left(2.84\times10^{22}\right)\left(550\,\text{MeV}\right) = 1.57\times10^{25}\,\text{MeV} = \boxed{2.5\times10^{12}\,\text{J.}}$$

70. (*a*) $\Delta\left({}^{4}\text{He}\right) = m\left({}^{4}\text{He}\right) - A\left({}^{4}\text{He}\right) = 4.002602\,\text{u} - 4 = \boxed{0.002602\,\text{u}}$

$$= \left(0.002602\,\text{u}\right)\left(931.5\,\text{MeV/u}c^{2}\right) = \boxed{2.424\,\text{Mev}/c^{2}.}$$

(*b*) $\Delta\left({}^{12}\text{C}\right) = m\left({}^{12}\text{C}\right) - A\left({}^{12}\text{C}\right) = 12.000000\,\text{u} - 12 = \boxed{0.}$

(*c*) $\Delta\left({}^{107}\text{Ag}\right) = m\left({}^{107}\text{Ag}\right) - A\left({}^{107}\text{Ag}\right) = 106.905091\,\text{u} - 107 = \boxed{-0.094909\,\text{u}}$

$$= \left(-0.094909\,\text{u}\right)\left(931.5\,\text{MeV/u}c^{2}\right) = \boxed{-88.41\,\text{MeV}/c^{2}.}$$

(*d*) $\Delta\left({}^{235}\text{U}\right) = m\left({}^{235}\text{U}\right) - A\left({}^{235}\text{U}\right) = 235.043924\,\text{u} - 235 = \boxed{0.043924\,\text{u}}$

$$= \left(0.043924\,\text{u}\right)\left(931.5\,\text{MeV/u}c^{2}\right) = \boxed{40.92\,\text{MeV}/c^{2}.}$$

(*e*) From the Appendix we see that

$$\boxed{\begin{array}{l}\Delta \ge 0 \text{ for } 0 \le Z \le 8 \text{ and } Z \ge 85;\\ \Delta < 0 \text{ for } 9 \le Z \le 84.\end{array}}$$

71. (*a*) The usual fraction of ^{14}C is 1.3×10^{-12}. Because the fraction of atoms that are ^{14}C is so small, we use the atomic weight of ^{12}C to find the number of carbon atoms in 92 g:

$$N = \left[\frac{\left(92\,\text{g}\right)}{\left(12\,\text{g/mol}\right)}\right]\left(6.02\times10^{23}\,\text{atoms/mol}\right) = 4.62\times10^{24}\,\text{atoms.}$$

The number of ^{14}C nuclei in the sample is

$$N_{14} = \left(1.3\times10^{-12}\right)\left(4.62\times10^{24}\right) = 6.00\times10^{12}\,\text{nuclei.}$$

We find the number of half-lives from

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^{n};$$

$$\frac{1}{6.00\times10^{12}} = \left(\tfrac{1}{2}\right)^{n},\ \text{or}\ n\log2 = \log\left(6.00\times10^{12}\right),\ \text{which gives}\ n = 42.4.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (42.4)(5730\,\text{yr}) = \boxed{2.4\times10^{5}\,\text{yr.}}$$

(*b*) A similar calculation as above leads to $N_{14} = 1.83\times10^{13}$ nuclei, $n = 44.1$, and $t = 2.5\times10^{5}$ yr. Thus, sample amount has fairly little effect on the maximum age that can be measured, which is on the order of $\boxed{10^5\text{ yr.}}$

72. Because there are so many low-energy electrons available, this reaction would turn most of the protons into neutrons, which would eliminate chemistry, and thus life.
The Q-value of the reaction is

$$Q = \left[m\left({}^1\text{H}\right) - m\left({}^1\text{n}\right)\right]c^2 = \left[(1.007825\,\text{u}) - (1.008665\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = -0.782\,\text{MeV}.$$

The percentage increase in the proton's mass to make the Q-value = 0 is

$$\left(\frac{\Delta m}{m}\right)(100) = \left[\frac{\left(0.782\,\text{MeV}/c^2\right)}{\left(938.3\,\text{MeV}/c^2\right)}\right](100) = \boxed{0.083\%.}$$

73. The radius is given by

$$r = \frac{mv}{qB}$$

We set the two radii equal:

$$\frac{m_\alpha v_\alpha}{2eB} = \frac{m_\beta v_\beta}{eB};$$

$$m_\alpha v_\alpha = 2m_\beta v_\beta;$$

$$P_\alpha = 2P_\beta$$

The energies are given by

$$\text{KE} = \frac{p^2}{2m},$$

and therefore

$$\frac{\text{KE}_\alpha}{\text{KE}_\beta} = \frac{p_\alpha^2 m_\beta}{p_\beta^2 m_\alpha} = \frac{4m_\beta}{m_\alpha} = \frac{4(0.000549\,\text{u})}{4.002603\,\text{u}} = \boxed{\frac{5.49\times10^{-4}}{1}.}$$

74. We first find the number of ${}^{147}\text{Sm}$ nuclei from the mass and proportion information:

$$N = \frac{(0.15)(1.00\,\text{g})\left(6.02\times10^{23}\,\text{nuclei/mol}\right)}{147\,\text{g/mol}} = 6.14\times10^{20}\text{ nuclei.}$$

The activity level is determined by

$$\text{Activity level} = \lambda N;$$

$$120\,\text{s}^{-1} = \left(\frac{0.693}{T_{\frac{1}{2}}}\right)\left(6.14\times10^{20}\right),\ \text{which gives } T_{\frac{1}{2}} = 3.55\times10^{18}\,\text{s} = \boxed{1.1\times10^{11}\text{ yr.}}$$

75. Since amounts are not specified, we may suppose that there are 0.72 g of ${}^{235}\text{U}$ and $100.00 - 0.72 = 99.28\,\text{g}$ of ${}^{238}\text{U}$. Now we use $N = N_0e^{-\lambda t} = N_0e^{\frac{-0.693t}{T_{\frac{1}{2}}}}$.

(*a*) 1.0×10^9 years ago,

$$N_{0,235} = N_{235}e^{\frac{0.693t}{T_{\frac{1}{2}}}}$$

$$=\left(0.72\,\text{g}\right)e^{\frac{0.693\left(1.0\times10^9\right)}{\left(7.038\times10^8\right)}} = 1.93\,\text{g};$$

$$N_{0,238} = N_{238}e^{\frac{0.693t}{T_{\frac{1}{2}}}}$$

$$=\left(99.28\,\text{g}\right)e^{\frac{0.693\left(1.0\times10^9\right)}{\left(4.468\times10^9\right)}} = 115.94\,\text{g}.$$

The percentage of ^{235}U was

$$\frac{1.93}{1.93+115.94}\times100\% = \boxed{1.6\%.}$$

(b) In 100×10^6 years,

$$N_{235} = N_{0,235}e^{\frac{0.693t}{T_{\frac{1}{2}}}}$$

$$=\left(0.72\,\text{g}\right)e^{\frac{-0.693\left(100\times10^6\right)}{\left(7.038\times10^8\right)}} = 0.65\,\text{g};$$

$$N_{238} = N_{0,238}e^{\frac{-0.693t}{T_{\frac{1}{2}}}}$$

$$=\left(99.28\,\text{g}\right)e^{\frac{-0.693\left(100\times10^6\right)}{\left(4.468\times10^9\right)}} = 97.75\,\text{g}.$$

The percentage of ^{235}U will be

$$\frac{0.65}{0.65+97.75}\times100\% = \boxed{0.67\%.}$$

76. The mass of ^{40}K is

$$\left(400\times10^{-3}\,\text{g}\right)\left(0.000117\right) = 4.68\times10^{-5}\,\text{g}.$$

The number of nuclei is

$$N = \frac{\left(4.68\times10^{-5}\,\text{g}\right)\left(6.02\times10^{23}\,\text{atoms/mol}\right)}{40\,\text{g/mol}} = 7.04\times10^{17}\,\text{nuclei}.$$

The activity is

$$\lambda N = \frac{0.693\left(7.04\times10^{17}\right)}{\left(1.277\times10^9\,\text{yr}\right)\left(3.16\times10^7\,\text{s/yr}\right)} = \boxed{12.1\,\text{decays/s}.}$$

77. The mass of carbon 60,000 years ago was essentially 1.0 kg, for which the corresponding number of ^{12}C atoms would be

$$N_{12} = \frac{\left(1.0\times10^3\,\text{g}\right)\left(6.02\times10^{23}\,\text{atoms/mol}\right)}{12\,\text{g/mol}} = 5.02\times10^{25}\,\text{atoms}.$$

However, a small fraction will in fact be $^{14}\mathrm{C}$ atoms, namely

$$N_{14}=\left(5.02\times10^{25}\right)\left(1.3\times10^{-12}\right)=6.52\times10^{13}\,\text{atoms.}$$

The decay constant is

$$\lambda=\frac{0.693}{\mathrm{T}_{\frac{1}{2}}}=\frac{0.693}{\left(5730\,\text{yr}\right)\left(3.16\times10^{7}\,\text{s/yr}\right)}=3.83\times10^{-12}.$$

Activity today, finally, is given by

$$\text{Activity}=\lambda N_0e^{-\lambda t}=\left(3.83\times10^{-12}\right)\left(6.52\times10^{13}\right)e^{-\left(3.83\times10^{-12}\right)\left(60{,}000\right)\left(3.16\times10^{7}\right)}=\boxed{0.18\,\text{decays/s.}}$$

78. The mass number changes only with an α decay for which the change is -4.

If the mass number is $4n$, then the new number is $4n-4=4\left(n-1\right)=4n'$. Thus for each family, we have

$4n\rightarrow4n-4\rightarrow4n'$;

$4n+1\rightarrow4n-4+1\rightarrow4n'+1$;

$4n+2\rightarrow4n-4+2\rightarrow4n'+2$;

$4n+3\rightarrow4n-4+3\rightarrow4n'+3$.

Thus the daughter nuclides are always in the same family.

CHAPTER 31: Nuclear Energy; Effects and Uses of Radiation

Answers to Questions

1. (*a*) $\mathrm{n} + {}^{137}_{56}\mathrm{Ba} \rightarrow\ ? + \gamma$

   Conserve nucleon number: 1 + 137 = A + 0. Thus, A = 138. Conserve charge: 0 + 56 = Z + 0. Thus, Z = 56. This is ${}^{138}_{56}\mathrm{Ba}$ or Barium-138.

   (*b*) $\mathrm{n} + {}^{137}_{56}\mathrm{Ba} \rightarrow {}^{137}_{55}\mathrm{Cs}\ +\ ?$

   Conserve nucleon number: 1 + 137 = 137 + A. Thus, A = 1. Conserve charge: 0 + 56 = 55 + Z. Thus, Z = 1. This is ${}^{1}_{1}\mathrm{H}$ or p (a proton).

   (*c*) $\mathrm{d} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He}\ +\ ?$

   Conserve nucleon number: 2 + 2 = 4 + A. Thus, A = 0. Conserve charge: 1 + 1 = 2 + Z. Thus, Z = 0. This is γ or a gamma ray (a photon).

   $\boxed{(d)}$ $\alpha + {}^{197}_{79}\mathrm{Au} \rightarrow\ ?\ +\ \mathrm{d}$

   Conserve nucleon number: 4 + 197 = A + 2. Thus, A = 199. Conserve charge: 2 + 79 = Z + 1. Thus, Z = 80. This is ${}^{199}_{80}\mathrm{Hg}$ or Mercury-199.

2. The products have 16 protons and 33 nucleons, and so must have 17 neutrons. Thus the reactants must also have 16 protons and 17 neutrons as well. Since one reactant is a neutron, the other reactant must have 16 protons and 16 neutrons, for a total of 32 nucleons. That nucleus is ${}^{32}_{16}\mathrm{S}$.

$\boxed{3.}$ The reaction is ${}^{22}_{11}\mathrm{Na} + {}^{2}_{1}\mathrm{H} \rightarrow \alpha +\ ?$. The reactants have 12 protons and 24 nucleons, and so must have 12 neutrons. Thus the products must have 12 protons and 12 neutrons as well. Since the alpha has 2 protons and 2 neutrons, the other product must have 10 protons and 10 neutrons. That nucleus is ${}^{20}_{10}\mathrm{Ne}$.

$\boxed{4.}$ Neutrons are good projectiles for producing nuclear reactions because they are neutral and they are massive. If you want a particle to hit the nucleus with a lot of energy, a more massive particle is the better choice. A light electron would not be as effective. Using a positively charged projectile like an alpha or a proton means that the projectile will have to overcome the large electrical repulsion from the positively charged nucleus. Neutrons can penetrate directly to the nucleus and cause nuclear reactions.

5. The reaction is $\mathrm{p} + {}^{20}_{10}\mathrm{Ne} \rightarrow \alpha +\ ?$. The reactants have 11 protons and 21 nucleons, and so must have 10 neutrons. Thus the products must have 11 protons and 10 neutrons as well. Since the alpha has 2 protons and 2 neutrons, the other product must have 9 protons and 8 neutrons. That nucleus is ${}^{17}_{9}\mathrm{F}$.

6. Fission fragments are neutron-rich. They have Z values, though, in a range where stable nuclei should have nearly equal numbers of neutrons and protons. Thus, these fission fragments are unstable with more neutrons than protons. For them to become more stable they will emit β^- particles. The emission of a β^- particle essentially converts a neutron in the nucleus to a proton

(while ejecting an electron), which brings the nuclei closer to the stable numbers of neutrons and protons.

7. Yes, since the multiplication factor is greater than 1 ($f = 1.5$), a chain reaction can be sustained. The difference would be that the chain reaction would proceed more slowly and to make sure the chain reaction continued you would need to be very careful about leakage of the neutrons to the surroundings.

8. The critical mass of ${}^{239}_{94}\text{Pu}$ ($f = 2.9$) would be less than that for ${}^{235}_{92}\text{U}$ ($f = 2.5$). Since there are more neutrons created per Pu-239 nuclei decay we can have a smaller number of these nuclei and still create the needed chain reaction in this smaller volume of material (enough of the neutrons will strike other nuclei before escaping the smaller sample size).

9. The thermal energy from nuclear fission appears in the kinetic energy of the fission products (daughter nuclei and neutrons). In other words, the fission products are moving very fast (especially the neutrons, due to the conservation of momentum).

10. Uranium can't be enriched by chemical means because chemical reactions occur similarly with all of the isotopes of a given element. The number of neutrons in the nucleus does not influence the chemistry, which is primarily due to the valence electrons. Thus, trying to enrich uranium by chemical means, which means trying to increase the percent of ${}^{235}_{92}\text{U}$ in the sample versus ${}^{238}_{92}\text{U}$, is impossible.

11. First of all, the neutron can get close to the nucleus at such slow speeds due to the fact that it is neutral and will not be electrically repelled by either the electron cloud or the protons in the nucleus. Then, once it hits a nucleus, it is held due to the strong nuclear force. It adds more energy than just kinetic energy to the nucleus due to $E = mc^2$. This extra amount of mass-energy leaves the nucleus in an excited state. To decay back to a lower energy state, the nucleus will fission.

12. For a nuclear chain reaction to occur in a block of porous uranium (which would lead to an explosion), the neutrons being emitted by the decays must be slowed down. If the neutrons are too fast, they will pass through the block of uranium without interacting, effectively prohibiting a chain reaction. Water contains a much higher density of protons and neutrons than does air, and those protons and neutrons will slow down (moderate) the neutrons, enabling them to take part in nuclear reactions. Thus, if water is filling all of the porous cavities, the water will slow the neutrons, allowing them to be captured by other uranium nuclei, and allow the chain reaction to continue, which might lead to an explosion.

13. If the uranium is highly enriched, it won't matter that the ordinary water is a poorer moderator of neutrons than the heavy water. There will still be enough fissionable nuclei that even with the lower number of slowed (moderated) neutrons in ordinary water, the chain reaction will proceed. If the uranium is not highly enriched, there are not as many fissionable nuclei. If ordinary water is used, too many neutrons would be lost to absorption in the ordinary water and the relative amount of moderated neutrons would decrease. In that case heavy water is better than ordinary water for sustaining the chain reaction.

14. To get nuclei to fission, they must be bombarded by neutrons. This can be done using an external source, but then there is a large input of energy into the system, making it less efficient and so less useful. To make the fission reaction useful, it needs to be self-sustaining. To create such a chain reaction, each time a nuclei captures a neutron and fissions, one or more neutrons must be emitted in order to sustain the process.

15. Fossil fuels: Pros = inexpensive to build, the technology is already working. Cons = air pollution, greenhouse gas, limited supply of coal and oil. Fission: Pros = the technology is already working, no air pollution, no greenhouse gas. Cons = expensive to build, thermal pollution, disposal of radioactive waste, accidents are extremely dangerous. Fusion: Pros = no radioactive waste, no air pollution, no greenhouse gas. Cons = the technology is not yet working in a sustainable manner, expensive to build.

16. It is desirable to keep the radioactive materials as confined as possible to avoid accidental leakage and contamination. In the electric generating portion of the system, waste heat must be given off to the surroundings, and keeping all of the radioactive portions of the energy generation process as far away from this "release point" as possible is a major safety concern.

17. A heavy nucleus decays because it is neutron-rich, especially after neutron capture. With too many neutrons, when compared to protons, the nucleus is unstable and will fission into two daughter nuclei. These two daughter nuclei would still be neutron-rich and relatively unstable, and so the fission process is always accompanied by the emission of neutrons to alleviate this imbalance. Lighter nuclei are more stable with approximately equal numbers of protons and neutrons.

18. Gamma particles penetrate better than beta particles because they are neutral and have no mass. Thus, gamma particles do not interact with matter as easily or as often as beta particles, allowing them to better penetrate matter.

19. To ignite these fusion reactions the nuclei need to be given enough energy (higher temperature) to the point where the nucleus of one particle actually touches the nucleus of another particle during the collisions. To do this, the nuclei need to overcome their electrostatic repulsion in order to get close enough to fuse. Each of these nuclei has a single positive charge. The deuterium (d) has two neutrons and the tritium (t) has three neutrons. Electrostatic repulsion is proportional to the charge of each nucleus and inversely proportional to the square of the distance between the centers of the two nuclei. In both the d-d ignition and the d-t ignition, the charge of each nucleus is the same, and so the charge effect is the same in both ignitions. However, the distance between the centers of the two nuclei in the d-t ignition will be larger than in the d-d ignition (since the tritium nucleus is larger than the deuterium nucleus), which means there is less electrostatic repulsion to overcome in the d-t ignition. Thus a lower temperature will ignite the fusion process.

$\boxed{20.}$ The large amount of mass of a star creates an enormous gravitational attraction of all of that mass, which causes the gas to be compressed to a very high density. This high density creates a very high pressure and high temperature situation. The high temperatures give the gas particles a large amount of kinetic energy, which allows them to fuse when they collide. Thus, these conditions at the center of the Sun and other stars make the fusion process possible.

21. Stars, which include our Sun, maintain confinement of the fusion plasma with gravity. The huge amount of mass in a star creates an enormous gravitational attraction on the gas molecules and this attractive force overcomes the outward repulsive forces from electrostatics and radiation pressure.

22. Fission is the process in which heavier, less-stable nuclei break apart into two or more lighter, more-stable nuclei, which releases particles and energy. Fusion is the process in which lighter, less-stable nuclei combine with each other to create heavier, more-stable nuclei, which releases particles and energy.

23. Alpha particles cannot penetrate a person's skin, so they do no biological damage even when a person works with or touches an alpha emitter. But, even if dust-sized pieces of the alpha emitter are ingested, then they can do a large amount of ionization damage directly to the exposed cells in your body. Thus, strict rules for employees that prohibit any eating and drinking while working around alpha emitters is an extremely wise precaution. Also, machining of alpha emitters is usually strictly prohibited, due to the fact that this process might create fine dust particles of the radioactive substance that could be inhaled.

24. During a woman's child-bearing years (and earlier) it is possible for radiation damage to occur to the reproductive organs in such a way as to pass on the possible genetic defects to her children. The risk of this type of damage is greater than the risk of damage to just the woman. Thus once the child-bearing years are over, the acceptable dose goes up.

25. Radiation can kill or deactivate bacteria and viruses on medical supplies and even in food. Thus, the radiation will sterilize these things, making them safer for humans to use.

26. Absorbed dose is the amount of radiation energy deposited per mass. The SI unit is the Gray = Gy = 1 J/kg. Effective dose is the amount of biological damage that will occur due to the energy deposited per mass. The SI unit is the Sievert = Sv. The effective dose is related to the absorbed dose by the quality factor: effective dose = (absorbed dose)(quality factor).

27. Put a radioactive tracer in the liquid that flows through the pipe. Choose a tracer that emits particles that cannot penetrate the walls of the pipe. Follow the pipe with a Geiger counter until the tracer is detected on the outside of the pipe. This is where the leak is located.

Solutions to Problems

1. By absorbing a neutron, the mass number increases by one and the atomic number is unchanged. The product nucleus is $\boxed{{}^{28}_{13}\mathrm{Al}}$. Since the nucleus now has an "extra" neutron, it will decay by $\boxed{\beta^-}$, according to this reaction: ${}^{28}_{13}\mathrm{Al} \rightarrow {}^{28}_{14}\mathrm{Si} + \beta^- + \bar{\nu}_e$. Thus the product is $\boxed{{}^{28}_{14}\mathrm{Si}}$.

2. If the Q-value is positive, then no threshold energy is needed.

$$Q = 2m_{{}^2_1\mathrm{H}}c^2 - m_{{}^3_2\mathrm{He}}c^2 - m_{\mathrm{n}}c^2 = \left[2\left(2.014102\,\mathrm{u}\right) - 3.016029\,\mathrm{u} - 1.008665\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$
$$= 3.270\,\mathrm{MeV}$$

Thus $\boxed{\text{no threshold energy is required}}$.

$\boxed{3.}$ A "slow" neutron means that it has negligible kinetic energy. If the Q-value is positive, then the reaction is possible.

$$Q = m_{{}^{238}_{92}\mathrm{U}}c^2 + m_{\mathrm{n}}c^2 - m_{{}^{239}_{92}\mathrm{U}}c^2 = \left[238.050783\,\mathrm{u} + 1.008665\,\mathrm{u} - 239.054288\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$
$$= 4.807\,\mathrm{MeV}$$

Thus $\boxed{\text{the reaction is possible}}$.

4. The Q-value tells whether the reaction requires or releases energy.

$$Q = m_{\text{p}}c^2 + m_{{}^7_3\text{Li}}c^2 - m_{{}^4_2\text{He}}c^2 - m_\alpha c^2 = \left[1.007825\,\text{u} + 7.016004\,\text{u} - 2(4.002603)\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$= 17.35\,\text{MeV}$$

The reaction $\boxed{\text{releases } 17.35\,\text{MeV}}$.

5. The Q-value tells whether the reaction requires or releases energy.

$$Q = m_\alpha c^2 + m_{{}^9_4\text{Be}}c^2 - m_{{}^{12}_6\text{He}}c^2 - m_{\text{n}}c^2$$

$$= \left[4.002603\,\text{u} + 9.012182\,\text{u} - 12.000000\,\text{u} - 1.008665\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 5.701\,\text{MeV}$$

The reaction $\boxed{\text{releases } 5.701\,\text{MeV}}$.

6. (*a*) If the Q-value is positive, then no threshold energy is needed.

$$Q = m_{\text{n}}c^2 + m_{{}^{24}_{12}\text{Mg}}c^2 - m_{{}^{23}_{11}\text{Na}}c^2 - m_{\text{d}}c^2$$

$$= \left[1.008665\,\text{u} + 23.985042\,\text{u} - 22.989770\,\text{u} - 2.014102\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = -9.469\,\text{MeV}$$

Thus more energy is required if this reaction is to occur. The 10.00 MeV of kinetic energy is more than sufficient, and so $\boxed{\text{the reaction can occur}}$.

(*b*) $10.00\,\text{MeV} - 9.469\,\text{MeV} = \boxed{0.53\,\text{MeV of energy is released}}$

7. (*a*) If the Q-value is positive, then no threshold energy is needed.

$$Q = m_{\text{p}}c^2 + m_{{}^7_3\text{Li}}c^2 - m_{{}^4_2\text{He}}c^2 - m_\alpha c^2$$

$$= \left[1.007825\,\text{u} + 7.016004\,\text{u} - 2(4.002603\,\text{u})\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 17.35\,\text{MeV}$$

Since the Q-value is positive, $\boxed{\text{the reaction can occur}}$.

(*b*) The total KE of the products will be the Q-value plus the incoming kinetic energy.

$$\text{KE}_{\text{total}} = \text{KE}_{\text{reactants}} + Q = 2.500\,\text{MeV} + 17.35\,\text{MeV} = \boxed{19.85\,\text{MeV}}$$

8. (*a*) If the Q-value is positive, then no threshold energy is needed.

$$Q = m_\alpha c^2 + m_{{}^{14}_7\text{N}}c^2 - m_{{}^{17}_8\text{O}}c^2 - m_{\text{p}}c^2$$

$$= \left[4.002603\,\text{u} + 14.003074\,\text{u} - 16.999131\,\text{u} - 1.007825\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = -1.191\,\text{MeV}$$

Thus more energy is required if this reaction is to occur. The 7.68 MeV of kinetic energy is more than sufficient, and so $\boxed{\text{the reaction can occur}}$.

(*b*) The total KE of the products will be the Q-value plus the incoming kinetic energy.

$$\text{KE}_{\text{total}} = \text{KE}_{\text{reactants}} + Q = 7.68\,\text{MeV} - 1.191\,\text{MeV} = \boxed{6.49\,\text{MeV}}$$

9. $Q = m_\alpha c^2 + m_{{}^{16}_{8}\mathrm{O}}c^2 - m_{{}^{20}_{10}\mathrm{Ne}}$

$$= [4.002603\,\mathrm{u} + 15.994915\,\mathrm{u} - 19.992440\,\mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{4.730\,\mathrm{MeV}}$$

10. The Q-value tells whether the reaction requires or releases energy.

$Q = m_\mathrm{d}c^2 + m_{{}^{13}_{6}\mathrm{C}}c^2 - m_{{}^{14}_{7}\mathrm{N}}c^2 - m_\mathrm{n}c^2$

$$= [2.014102\,\mathrm{u} + 13.003355\,\mathrm{u} - 14.003074\,\mathrm{u} - 1.008665\,\mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 5.326\,\mathrm{MeV}$$

The total KE of the products will be the Q-value plus the incoming kinetic energy.

$$\mathrm{KE}_\mathrm{total} = \mathrm{KE}_\mathrm{reactants} + Q = 36.3\,\mathrm{MeV} + 5.326\,\mathrm{MeV} = \boxed{41.6\,\mathrm{MeV}}$$

11. The nitrogen-14 absorbs a neutron. Carbon-12 is a product. Thus the reaction is $\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + ?$. The reactants have 7 protons and 15 nucleons, which means 8 neutrons. Thus the products have 7 protons and 15 nucleons. The unknown product must be a proton. Thus the reaction is $\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + \mathrm{p}$.

$Q = m_\mathrm{n}c^2 + m_{{}^{14}_{7}\mathrm{N}}c^2 - m_{{}^{14}_{6}\mathrm{C}}c^2 - m_\mathrm{p}c^2$

$$= [1.008665\,\mathrm{u} + 14.003074\,\mathrm{u} - 14.003242\,\mathrm{u} - 1.007825\,\mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{0.626\,\mathrm{MeV}}$$

12. (*a*) The deuteron is ${}^{2}_{1}\mathrm{H}$, and so the reactants have 4 protons and 8 nucleons. Therefore the reactants have 4 neutrons. Thus the products must have 4 protons and 4 neutrons. That means that X must have 3 protons and 4 neutrons, and so X is $\boxed{{}^{7}_{3}\mathrm{Li}}$.

(*b*) This is called a "stripping" reaction because the lithium nucleus has "stripped" a neutron from the deuteron.

(*c*) The Q-value tells whether the reaction requires or releases energy.

$Q = m_\mathrm{d}c^2 + m_{{}^{6}_{3}\mathrm{Li}}c^2 - m_{{}^{7}_{3}\mathrm{Li}}c^2 - m_\mathrm{p}c^2$

$$= [2.014102\,\mathrm{u} + 6.015122\,\mathrm{u} - 7.016004\,\mathrm{u} - 1.007825\,\mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{5.025\,\mathrm{MeV}}$$

Since the Q-value is positive, the reaction is $\boxed{\text{exothermic}}$.

13. (*a*) This is called a "pickup" reaction because the helium has "picked up" a neutron from the carbon nucleus.

(*b*) The alpha is ${}^{4}_{2}\mathrm{He}$. The reactants have 8 protons and 15 nucleons, and so have 7 neutrons. Thus the products must also have 8 protons and 7 neutrons. The alpha has 2 protons and 2 neutrons, and so X must have 6 protons and 5 neutrons. Thus X is $\boxed{{}^{11}_{6}\mathrm{C}}$.

(*c*) The Q-value tells whether the reaction requires or releases energy.

$Q = m_{{}^{3}_{2}\mathrm{He}}c^2 + m_{{}^{12}_{6}\mathrm{C}}c^2 - m_{{}^{11}_{6}\mathrm{C}}c^2 - m_\alpha c^2$

$$=\left[3.016029\,\text{u}+12.000000\,\text{u}-11.011434\,\text{u}-4.002603\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2=\boxed{1.856\,\text{MeV}}$$

Since the Q-value is positive, the reaction is $\boxed{\text{exothermic}}$.

14. (*a*) The product has 16 protons and 16 neutrons. Thus the reactants must have 16 protons and 16 neutrons. Thus the missing nucleus has 15 protons and 16 neutrons, and so is $\boxed{{}^{31}_{15}\text{P}}$

(*b*) The Q-value tells whether the reaction requires or releases energy.

$$Q=m_{\text{p}}c^2+m_{{}^{31}_{15}\text{P}}c^2-m_{{}^{32}_{16}\text{S}}c^2$$

$$=\left[1.007825\,\text{u}+30.973762\,\text{u}-31.972071\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2=\boxed{8.864\,\text{MeV}}$$

15. We assume that all of the particles are essentially at rest, and so ignore conservation of momentum. To just make the fluorine nucleus, the Q-value plus the incoming KE should add to 0.

$$\text{KE}+Q=\text{KE}+m_{\text{p}}c^2+m_{{}^{18}_{8}\text{O}}c^2-m_{{}^{18}_{9}\text{F}}c^2-m_{\text{n}}c^2=0\;\rightarrow$$

$$m_{{}^{18}_{9}\text{F}}c^2=\text{KE}+m_{\text{p}}c^2+m_{{}^{18}_{8}\text{O}}c^2-m_{\text{n}}c^2=$$

$$=2.453\,\text{MeV}+\left[1.007825\,\text{u}+17.999160\,\text{u}-1.008665\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$=1.676789\times10^4\,\text{MeV}$$

$$m_{{}^{18}_{9}\text{F}}=\left(1.676788\times10^4\,\text{MeV}\right)c^2\left(\frac{1\,\text{u}}{931.5\,\text{MeV}/c^2}\right)=\boxed{18.000953\,\text{u}}$$

16. The Q-value gives the energy released in the reaction, assuming the initial kinetic energy of the neutron is very small.

$$Q=m_{\text{n}}c^2+m_{{}^{235}_{92}\text{U}}c^2-m_{{}^{88}_{38}\text{Sr}}c^2-m_{{}^{136}_{54}\text{Xe}}c^2-12m_{\text{n}}c^2$$

$$=\left[1.008665\,\text{u}+235.043923\,\text{u}-87.905614\,\text{u}-135.907220\,\text{u}-12\left(1.008665\,\text{u}\right)\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$=\boxed{126.5\,\text{MeV}}$$

17. The Q-value gives the energy released in the reaction, assuming the initial kinetic energy of the neutron is very small.

$$Q=m_{\text{n}}c^2+m_{{}^{235}_{92}\text{U}}c^2-m_{{}^{141}_{56}\text{Ba}}c^2-m_{{}^{92}_{36}\text{Kr}}c^2-3m_{\text{n}}c^2$$

$$=\left[1.008665\,\text{u}+235.043923\,\text{u}-140.914411\,\text{u}-91.926156\,\text{u}-3\left(1.008665\,\text{u}\right)\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$=\boxed{173.3\,\text{MeV}}$$

$\boxed{18.}$ The power released is the energy released per reaction times the number of reactions per second.

$$P = \frac{\text{energy}}{\text{reaction}} \times \frac{\#\text{ reactions}}{\text{s}} \rightarrow$$

$$\frac{\#\text{ reactions}}{\text{s}} = \frac{P}{\dfrac{\text{energy}}{\text{reaction}}} = \frac{200\times10^{6}\,\text{W}}{\left(200\times10^{6}\,\text{eV/reaction}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = \boxed{6\times10^{18}\text{ reactions/s}}$$

19. Compare the energy per fission with the rest mass energy.

$$\frac{\text{energy per fission}}{\text{rest mass energy } mc^2} = \frac{200\,\text{MeV}}{(235\,\text{u})\left(931.5\,\text{MeV}/c^2\right)c^2} = 9.1\times10^{-4} \approx \boxed{\frac{1}{1100}}$$

20. (*a*) The total number of nucleons for the reactants is 236, and so the total number of nucleons for the products must also be 236. The two daughter nuclei have a total of 231 nucleons, so $\boxed{5\text{ neutrons}}$ must be produced in the reaction: ${}^{235}_{92}\text{U} + \text{n} \rightarrow {}^{133}_{51}\text{Sb} + {}^{98}_{41}\text{Nb} + 5\text{n}$.

(*b*) $Q = m_{{}^{235}_{92}\text{U}}c^2 + m_{\text{n}}c^2 - m_{{}^{133}_{51}\text{Sb}}c^2 - m_{{}^{98}_{41}\text{Nb}}c^2 - 5m_{\text{n}}c^2$

$$= \left[235.043923\,\text{u} + 1.008665\,\text{u} - 132.915250\,\text{u} - 97.910328\,\text{u} - 5(1.008665\,\text{u})\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$= \boxed{171.1\,\text{MeV}}$$

21. We assume as stated in problems 18 and 19 that an average of 200 MeV is released per fission of a uranium nucleus.

$$\left(3\times10^{7}\,\text{J}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{MeV}}{1\times10^{6}\,\text{eV}}\right)\left(\frac{1\,\text{nucleus}}{200\,\text{MeV}}\right)\left(\frac{0.238\,\text{kg}}{6.02\times10^{23}\,\text{nuclei}}\right) = \boxed{3.7\times10^{-7}\,\text{kg }{}^{238}_{92}\text{U}}$$

22. Convert the power rating to a mass of uranium using the factor-label method.

$$950\frac{\text{J}}{\text{s}} \times \frac{1\text{ atom}}{200\times10^{6}\,\text{eV}} \times \frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} \times \frac{0.235\,\text{kg U}}{6.02\times10^{23}\,\text{atoms}} \times \frac{3.156\times10^{7}\,\text{s}}{1\,\text{y}} = \boxed{3.7\times10^{-4}\,\text{kg U}}$$

23. Since the reaction is 40% efficient, the fission needs to generate $(650/0.40)\,\text{MW}$ of power. Convert the power rating to a mass of uranium using the factor-label method. We assume 200 MeV is released per fission, as in other problems.

$$\frac{650\times10^{6}}{0.40}\frac{\text{J}}{\text{s}} \times \frac{1\text{ atom}}{200\times10^{6}\,\text{eV}} \times \frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} \times \frac{0.235\,\text{kg U}}{6.02\times10^{23}\,\text{atoms}} \times \frac{3.156\times10^{7}\,\text{s}}{1\,\text{y}} = 626\,\text{kg U} \approx \boxed{630\,\text{kg U}}$$

24. If the uranium splits into roughly equal fragments, each will have an atomic mass number of about half of 236, or 118. Each will have a nuclear charge of about half of 92, or 46. Calculate the electrical potential energy using a relationship developed in Ex. 17-7. The distance between the nuclei will be twice the radius of a nucleus, and the radius is given in Eq. 30-1.

$$\text{PE} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \left(9.0\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(46)^2\left(1.60\times10^{-19}\,\text{C}\right)^2}{2\left(1.2\times10^{-15}\,\text{m}\right)(118)^{1/3}}$$

$$=4.142\times10^{-11}\,\mathrm{J}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right)=2.59\times10^{8}\,\mathrm{eV}=\boxed{259\,\mathrm{MeV}}$$

This is about $\boxed{\text{30\% larger}}$ than the nuclear fission energy released.

$\boxed{25.}$ $\mathrm{KE}=\tfrac{3}{2}kT=\tfrac{3}{2}\left(1.38\times10^{-23}\,\mathrm{J/K}\right)\left(10^{7}\,\mathrm{K}\right)=\boxed{2.1\times10^{-16}\,\mathrm{J}}$

$$=\tfrac{3}{2}\left(1.38\times10^{-23}\,\mathrm{J/K}\right)\left(10^{7}\,\mathrm{K}\right)\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right)=\boxed{1300\,\mathrm{eV}}$$

26. The Q-value gives the energy released in the reaction.

$$Q=m_{{}^2_1\mathrm{H}}c^2+m_{{}^3_1\mathrm{H}}c^2-m_{{}^4_2\mathrm{He}}c^2-m_{\mathrm{n}}c^2$$

$$=\left[2.014102\,\mathrm{u}+3.016049\,\mathrm{u}-4.002603\,\mathrm{u}-1.008665\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2=\boxed{17.59\,\mathrm{MeV}}$$

27. Calculate the Q-value for the reaction ${}^2_1\mathrm{H}+{}^2_1\mathrm{H}\rightarrow{}^3_2\mathrm{He}+\mathrm{n}$

$$Q=2m_{{}^2_1\mathrm{H}}c^2-m_{{}^3_2\mathrm{He}}c^2-m_{\mathrm{n}}c^2$$

$$=\left[2\left(2.014102\,\mathrm{u}\right)-3.016029\,\mathrm{u}-1.008665\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2=\boxed{3.27\,\mathrm{MeV}}$$

28. For the reaction in Eq. 31-6a, if atomic masses are to be used, then one more electron needs to be added to the products side of the equation. Notice that charge is not balanced in the equation as written. The reaction is ${}^1_1\mathrm{H}+{}^1_1\mathrm{H}\rightarrow{}^2_1\mathrm{H}+\mathrm{e}^++\nu+\mathrm{e}^-$

$$Q=2m_{{}^1_1\mathrm{H}}c^2-m_{{}^2_1\mathrm{H}}c^2-m_{\mathrm{e}^+}c^2-m_{\mathrm{e}^-}c^2$$

$$=\left[2\left(1.007825\,\mathrm{u}\right)-2.014102\,\mathrm{u}-2\left(0.000549\,\mathrm{u}\right)\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2=\boxed{0.42\,\mathrm{MeV}}$$

For the reaction in Eq. 31-6b, use atomic masses since there would be two electrons on each side.

$$Q=m_{{}^1_1\mathrm{H}}c^2+m_{{}^2_1\mathrm{H}}c^2-m_{{}^3_2\mathrm{He}}c^2$$

$$=\left[1.007825\,\mathrm{u}+2.014102\,\mathrm{u}-3.016029\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2=\boxed{5.49\,\mathrm{MeV}}$$

For the reaction in Eq. 31-6c, use atomic masses since there would be two electrons on each side.

$$Q=2m_{{}^3_2\mathrm{He}}c^2-m_{{}^4_2\mathrm{He}}c^2-2m_{{}^1_1\mathrm{H}}c^2$$

$$=\left[2\left(3.016029\,\mathrm{u}\right)-4.002603-2\left(1.007825\,\mathrm{u}\right)\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2=\boxed{12.86\,\mathrm{MeV}}$$

29. Reaction 31-8a: $\dfrac{4.03\,\mathrm{MeV}}{2\left(2.014\,\mathrm{u}\right)}\times\dfrac{1\,\mathrm{u}}{1.66\times10^{-27}\,\mathrm{kg}}\times\dfrac{1\,\mathrm{kg}}{1000\,\mathrm{g}}=\boxed{6.03\times10^{23}\,\mathrm{MeV/g}}$

Reaction 31-8b: $\dfrac{3.27\,\mathrm{MeV}}{2\left(2.014\,\mathrm{u}\right)}\times\dfrac{1\,\mathrm{u}}{1.66\times10^{-27}\,\mathrm{kg}}\times\dfrac{1\,\mathrm{kg}}{1000\,\mathrm{g}}=\boxed{4.89\times10^{23}\,\mathrm{MeV/g}}$

Reaction 31-8c: $\dfrac{17.59\,\text{MeV}}{(2.014\,\text{u}+3.016\,\text{u})}\times\dfrac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}}\times\dfrac{1\,\text{kg}}{1000\,\text{g}}=\boxed{2.11\times10^{24}\,\text{MeV/g}}$

Uranium fission (200 MeV per nucleus):

$$\frac{200\,\text{MeV}}{(235\,\text{u})}\times\frac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}}\times\frac{1\,\text{kg}}{1000\,\text{g}}=\boxed{5.13\times10^{23}\,\text{MeV/g}}$$

All are within about a factor of 5 of each other.

30. Calculate the Q-value for the reaction ${}^{238}_{92}\text{U}+\text{n}\rightarrow{}^{239}_{92}\text{U}$

$$Q=m_{{}^{238}_{92}\text{U}}c^2+m_{\text{n}}c^2-m_{{}^{239}_{92}\text{U}}c^2$$

$$=\left[238.050783\,\text{u}+1.008665\,\text{u}-239.054288\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2=\boxed{4.807\,\text{MeV}}$$

31. The reaction of Eq. 31-8b consumes 2 deuterons and releases 3.27 MeV of energy. The amount of energy needed is the power times the elapsed time, and the energy can be related to the mass of deuterium by the reaction.

$$\left(950\frac{\text{J}}{\text{s}}\right)(1\text{ yr})\left(3.156\times10^{7}\,\frac{\text{s}}{\text{yr}}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{19}\,\text{J}}\right)\left(\frac{1\,\text{MeV}}{10^{6}\,\text{eV}}\right)\left(\frac{2\,\text{d}}{3.27\,\text{MeV}}\right)\left(\frac{2.01\times10^{-3}\text{kg}}{6.02\times10^{23}\,\text{d}}\right)$$

$$=3.8\times10^{-4}\,\text{kg}=\boxed{0.38\,\text{g}}$$

32. We assume that the reactants are at rest when they react, and so the total momentum of the system is 0. As a result, the momenta of the two products are equal in magnitude. The available energy of 17.59 MeV is much smaller than the masses involved, and so we use the non-relativistic relationship between momentum and kinetic energy, $\text{KE}=\dfrac{p^2}{2m}\rightarrow p=\sqrt{2m\text{KE}}$

$$\text{KE}_{{}^4_2\text{He}}+\text{KE}_{\text{n}}=\text{KE}_{\text{total}}=17.59\,\text{MeV}\qquad p_{{}^4_2\text{He}}=p_{\text{n}}\rightarrow\sqrt{2m_{{}^4_2\text{He}}\text{KE}_{{}^4_2\text{He}}}=\sqrt{2m_{\text{n}}\text{KE}_{\text{n}}}\rightarrow$$

$$m_{{}^4_2\text{He}}\text{KE}_{{}^4_2\text{He}}=m_{\text{n}}\text{KE}_{\text{n}}\rightarrow m_{{}^4_2\text{He}}\text{KE}_{{}^4_2\text{He}}=m_{\text{n}}\left(\text{KE}_{\text{total}}-\text{KE}_{{}^4_2\text{He}}\right)\rightarrow$$

$$\text{KE}_{{}^4_2\text{He}}=\frac{m_{\text{n}}}{m_{{}^4_2\text{He}}+m_{\text{n}}}\text{KE}_{\text{total}}=\left(\frac{1.008665}{4.002603+1.008665}\right)17.59\,\text{MeV}=3.54\,\text{MeV}\approx\boxed{3.5\,\text{MeV}}$$

$$\text{KE}_{\text{n}}=\text{KE}_{\text{total}}-\text{KE}_{{}^4_2\text{He}}=17.59\,\text{MeV}-3.54\,\text{MeV}=14.05\,\text{MeV}\approx\boxed{14\,\text{MeV}}$$

If the plasma temperature were significantly higher, then the approximation of 0 kinetic energy being brought into the reaction would not be reasonable. Thus the results would depend on plasma temperature. A higher plasma temperature would result in higher values for the energies.

33. Assume that the two reactions take place at equal rates, so they are both equally likely. Then from the reaction of 4 deuterons, there would be a total of 7.30 MeV of energy released, or 1.825 MeV per deuteron on the average. A total power of $\dfrac{1000\,\text{MW}}{0.30}=3333\,\text{MW}$ must be obtained from the fusion reactions to provide the required 1000 MW output, because of the 30% efficiency. We convert the power rating to a number of deuterons based on the energy released per reacting deuteron, and then convert that to an amount of water using the natural abundance of deuterium.

$$3333\,\mathrm{MW}\ \rightarrow\ \left[\left(3333\times10^{6}\,\frac{\mathrm{J}}{\mathrm{s}}\right)\left(\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right)\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right)\left(\frac{1\ \mathrm{d}}{1.825\,\mathrm{MeV}}\right)\left(\frac{1\ \mathrm{H\ atoms}}{0.000115\ \mathrm{d's}}\right)\times \left(\frac{1\,\mathrm{H_2O\ molecule}}{2\ \mathrm{H\ atoms}}\right)\left(\frac{0.018\ \mathrm{kg\,H_2O}}{6.02\times10^{23}\,\mathrm{molecules}}\right)\right]$$

$$=5342\,\mathrm{kg/h}\approx\boxed{5300\,\mathrm{kg/h}}$$

34. In Eq. 31-8a, 4.03 MeV of energy is released for every 2 deuterium atoms. The mass of water can be converted to a number of deuterium atoms.

$$\left(1.0\,\mathrm{kg\,H_2O}\right)\left(\frac{6.02\times10^{23}\mathrm{H_2O}}{0.018\,\mathrm{kg\,H_2O}}\right)\left(\frac{2\,\mathrm{H}}{1\mathrm{H_2O}}\right)\left(\frac{1.15\times10^{-4}\,\mathrm{d}}{1\mathrm{H}}\right)=7.692\times10^{21}\,\mathrm{d\ nuclei}\ \rightarrow$$

$$\left(7.692\times10^{21}\,\mathrm{d\ nuclei}\right)\left(\frac{4.03\times10^{6}\,\mathrm{eV}}{2\,\mathrm{d\,atoms}}\right)\left(\frac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)=\boxed{2.48\times10^{9}\,\mathrm{J}}$$

As compared to gasoline: $\dfrac{2.48\times10^{9}\,\mathrm{J}}{5\times10^{7}\,\mathrm{J}}=\boxed{\text{50 times more than gasoline}}$

35. (*a*) No carbon is consumed in this cycle because one $^{12}_{6}\mathrm{C}$ nucleus is required in the first step of the cycle, and one $^{12}_{6}\mathrm{C}$ nucleus is produced in the last step of the cycle. The net effect of the cycle can be found by adding all the reactants and all the products together, and canceling what appears on both sides of the reaction.

$$\begin{aligned}
^{12}_{6}\mathrm{C}+{}^{1}_{1}\mathrm{H}&\rightarrow {}^{13}_{7}\mathrm{N}+\gamma\\
^{13}_{7}\mathrm{N}&\rightarrow {}^{13}_{6}\mathrm{C}+\mathrm{e}^{+}+v\\
^{13}_{6}\mathrm{C}+{}^{1}_{1}\mathrm{H}&\rightarrow {}^{14}_{7}\mathrm{N}+\gamma\\
^{14}_{7}\mathrm{N}+{}^{1}_{1}\mathrm{H}&\rightarrow {}^{15}_{8}\mathrm{O}+\gamma\\
^{15}_{8}\mathrm{O}&\rightarrow {}^{15}_{7}\mathrm{N}+\mathrm{e}^{+}+v\\
^{15}_{7}\mathrm{N}+{}^{1}_{1}\mathrm{H}&\rightarrow {}^{12}_{6}\mathrm{C}+{}^{4}_{2}\mathrm{He}
\end{aligned}$$

$$^{12}_{6}\mathrm{C}+{}^{1}_{1}\mathrm{H}+{}^{13}_{7}\mathrm{N}+{}^{13}_{6}\mathrm{C}+{}^{1}_{1}\mathrm{H}+{}^{14}_{7}\mathrm{N}+{}^{1}_{1}\mathrm{H}+{}^{15}_{8}\mathrm{O}+{}^{15}_{7}\mathrm{N}+{}^{1}_{1}\mathrm{H}$$

$$\rightarrow\quad {}^{13}_{7}\mathrm{N}+\gamma+{}^{13}_{6}\mathrm{C}+\mathrm{e}^{+}+v+{}^{14}_{7}\mathrm{N}+\gamma+{}^{15}_{8}\mathrm{O}+\gamma+{}^{15}_{7}\mathrm{N}+\mathrm{e}^{+}+v+{}^{12}_{6}\mathrm{C}+{}^{4}_{2}\mathrm{He}$$

$$4\,{}^{1}_{1}\mathrm{H}\ \rightarrow\ {}^{4}_{2}\mathrm{He}+2\,\mathrm{e}^{+}+2v+3\gamma$$

There is a difference of one gamma ray in the process, as mentioned in the text.

(*b*) To use the values from Appendix B, we must be sure that number of electrons is balanced as well as the number of protons and electrons. The above "net" equation does not consider the electrons that neutral nuclei would have, because it does not conserve charge. What the above reaction really represents (ignoring the gammas and neutrinos) is the following.

$$4\,{}^{1}_{1}\mathrm{H}\ \rightarrow\ {}^{4}_{2}\mathrm{He}+2\,\mathrm{e}^{+}\quad\Rightarrow\quad 4\,{}^{1}_{1}\mathrm{p}\ \rightarrow\ 2\,{}^{1}_{1}\mathrm{p}+2\,{}^{1}_{0}\mathrm{n}+2\mathrm{e}^{+}$$

To use the values from Appendix B, we must add 4 electrons to each side of the reaction.

$$\left(4\,{}^{1}_{1}\mathrm{p}+4\mathrm{e}^{-}\right)\ \rightarrow\ \left(2\,{}^{1}_{1}\mathrm{p}+2\mathrm{e}^{-}+2\,{}^{1}_{0}\mathrm{n}\right)+2\mathrm{e}^{+}+2\mathrm{e}^{-}\quad\Rightarrow\quad 4\,{}^{1}_{1}\mathrm{H}\ \rightarrow\ {}^{4}_{2}\mathrm{He}+2\mathrm{e}^{+}+2\mathrm{e}^{-}$$

The energy produced in the reaction is the *Q*-value.

$$Q = 4m_{{}^1_1\mathrm{H}}c^2 - m_{{}^4_2\mathrm{He}}c^2 - 4m_\mathrm{e}$$

$$= \left[4(1.007825\,\mathrm{u}) - 4.002603\,\mathrm{u} - 4(0.000549\,\mathrm{u})\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{24.69\,\mathrm{MeV}}$$

(*c*) In some reactions extra electrons must be added in order to use the values from Appendix B. The first equation is electron-balanced, and so Appendix B can be used.

$$Q = m_{{}^{12}_6\mathrm{C}}c^2 + m_{{}^1_1\mathrm{H}}c^2 - m_{{}^{13}_7\mathrm{N}}c^2$$

$$= \left[12.000000\,\mathrm{u} + 1.007825\,\mathrm{u} - 13.005739\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{1.943\,\mathrm{MeV}}$$

The second equation needs to have another electron, so that ${}^{13}_7\mathrm{N} \rightarrow {}^{13}_6\mathrm{C} + \mathrm{e}^- + \mathrm{e}^+ + v$.

$$Q = m_{{}^{13}_7\mathrm{N}}c^2 - m_{{}^{13}_6\mathrm{C}}c^2 - 2m_\mathrm{e}c^2$$

$$= \left[13.005739\,\mathrm{u} - 13.003355\,\mathrm{u} - 2(0.000549\,\mathrm{u})\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{1.198\,\mathrm{MeV}}$$

The third equation is electron-balanced.

$$Q = m_{{}^{13}_6\mathrm{C}}c^2 + m_{{}^1_1\mathrm{H}}c^2 - m_{{}^{14}_7\mathrm{N}}c^2$$

$$= \left[13.003355\,\mathrm{u} + 1.007825\,\mathrm{u} - 14.003074\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{7.551\,\mathrm{MeV}}$$

The fourth equation is electron-balanced.

$$Q = m_{{}^{14}_7\mathrm{N}}c^2 + m_{{}^1_1\mathrm{H}}c^2 - m_{{}^{15}_8\mathrm{O}}c^2$$

$$= \left[14.003074\,\mathrm{u} + 1.007825\,\mathrm{u} - 15.003065\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{7.297\,\mathrm{MeV}}$$

The fifth equation needs to have another electron, so that ${}^{15}_8\mathrm{O} \rightarrow {}^{15}_7\mathrm{N} + \mathrm{e}^- + \mathrm{e}^+ + v$.

$$Q = m_{{}^{15}_8\mathrm{O}}c^2 - m_{{}^{15}_7\mathrm{N}}c^2 - 2m_\mathrm{e}c^2$$

$$= \left[15.003065\,\mathrm{u} - 15.000109\,\mathrm{u} - 2(0.000549\,\mathrm{u})\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{1.731\,\mathrm{MeV}}$$

The sixth equation is electron-balanced.

$$Q = m_{{}^{15}_7\mathrm{N}}c^2 + m_{{}^1_1\mathrm{H}}c^2 - m_{{}^{12}_6\mathrm{C}}c^2 - m_{{}^4_2\mathrm{He}}c^2$$

$$= \left[15.000109\,\mathrm{u} + 1.007825\,\mathrm{u} - 12.000000\,\mathrm{u} - 4.002603\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$

$$= \boxed{4.966\,\mathrm{MeV}}$$

The total is found as follows.

$$1.943\,\mathrm{MeV} + 1.198\,\mathrm{MeV} + 7.551\,\mathrm{MeV} + 7.297\,\mathrm{MeV} + 1.731\,\mathrm{MeV} + 4.966\,\mathrm{MeV}$$

$$= 24.69\,\mathrm{MeV}$$

(*d*) It takes a higher temperature for this reaction than for a proton-proton reaction because the reactants have to have more initial kinetic energy to overcome the Coulomb repulsion of one nucleus to another. In particular, the carbon and nitrogen nuclei have higher Z values leading to the requirement of a high temperature.

36. (*a*) We follow the method of Example 31-9. The reaction is ${}^{12}_{6}\text{C} + {}^{1}_{1}\text{H} \rightarrow {}^{13}_{7}\text{N} + \gamma$. We calculate the potential energy of the particles when they are separated by the sum of their radii. The radii are calculated from Eq. 30-1.

$$\text{KE}_{\text{total}} = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{C}}q_{\text{H}}}{r_{\text{C}}+r_{\text{H}}} = \left(9.0\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(6)(1)\left(1.60\times10^{-19}\text{C}\right)^2}{\left(1.2\times10^{-15}\text{m}\right)\left(1^{1/3}+12^{1/3}\right)}\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}}\right)$$
$$= 2.19\,\text{MeV}$$

For the d-t reaction, Example 31-9 shows $\text{KE}_{\text{total}} = 0.45\,\text{MeV}$. Find the ratio of the two energies.

$$\frac{\text{KE}_{\text{C}}}{\text{KE}_{\text{d-t}}} = \frac{2.19\,\text{MeV}}{0.45\,\text{MeV}} = 4.9$$

$\boxed{\text{The carbon reaction requires about 5 times more energy than the d-t reaction.}}$

(*b*) Since the kinetic energy is proportional to the temperature by $\overline{\text{KE}} = \tfrac{3}{2}kT$, since the kinetic energy has to increase by a factor of 5, so does the temperature. Thus we estimate $\boxed{T \approx 1.5\times10^{9}\,\text{K}}$.

37. Use Eq. 31-10b to relate Sv to Gy. From Table 31.1, the quality factor of gamma rays is 1, and so the number of Sv is equal to the number of Gy. Thus $4.0\,\text{Sv} = \boxed{4.0\,\text{Gy}}$.

38. Because the quality factor of alpha particles is 20 and the quality factor of X-rays is 1, it takes 20 times as many rads of X-rays to cause the same biological damage as compared to alpha particles. Thus the fifty rads of alpha particles is equivalent to $50\,\text{rad}\times20 = \boxed{1000\,\text{rad}}$ of X-rays.

39. The biological damage is measured by the effective dose, Eq. 31-10b.

$$75\,\text{rad fast neutrons}\times10 = x\,\text{rad slow neutrons}\times3 \ \rightarrow\ x = \frac{75\,\text{rad}\times10}{3} = \boxed{250\,\text{rad slow neutrons}}$$

40. A gray is 1 Joule per kg, according to Eq. 31-9.

$$2.0\frac{\text{J}}{\text{kg}}\times65\,\text{kg} = \boxed{130\,\text{J}}$$

41. The counting rate will be 85% of 25% of the activity.

$$\left(0.025\times10^{-6}\,\text{Ci}\right)\left(\frac{3.7\times10^{10}\ \text{decays/s}}{1\,\text{Ci}}\right)\left(\frac{1\beta}{1\,\text{decays}}\right)(0.25)(0.85) = 196.6\frac{\text{counts}}{\text{s}} \approx \boxed{200\,\text{counts/s}}$$

42. (*a*) Since the quality factor is 1, the effective dose (in rem) is the same as the absorbed dose (in rad). Thus the absorbed dose is $\boxed{\text{1 rad or 0.01 G}}$.

(*b*) A Gy is a J per kg.

$$(0.01\,\text{Gy})\left(\frac{1\,\text{J/kg}}{1\,\text{Gy}}\right)(0.25\,\text{kg})\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{p}}{1.2\times10^{6}\,\text{eV}}\right) = \boxed{1.3\times10^{10}\,\text{p}}$$

43. Let us start by approximating the decay rate as constant, and finding the time to administer 36 Gy. If that calculated time is significantly shorter than the half-life of the isotope, then our approximation is reasonable.

$$\text{dose} = \text{rate} \times \text{time} \quad \rightarrow \quad \text{time} = \frac{\text{dose}}{\text{rate}} = \frac{36\,\text{Gy}}{10 \times 10^{-3}\,\text{Gy/min}} = 3600\,\text{min} \times \frac{1\,\text{day}}{1440\,\text{min}} = \boxed{2.5\,\text{day}}$$

This is only about 17% of a half life, so our approximation is reasonable.

44. The two definitions of roentgen are 1.6×10^{12} ion pairs/g produced by the radiation, and $0.878 \times 10^{-2}\,\text{J/kg}$ deposited by the radiation. Start with the current definition, and relate them by the value of 35 eV per ion pair.

$$\left(0.878 \times 10^{-2}\,\text{J/kg}\right)\left(1\,\text{kg}/1000\,\text{g}\right)\left(1\,\text{eV}/1.60 \times 10^{-19}\,\text{J}\right)\left(1\,\text{ion pair}/35\,\text{eV}\right)$$

$$= 1.567 \times 10^{12}\,\text{ion pairs/g}$$

The two values are within about 2% of each other.

45. Each decay releases one gamma ray of energy 122 keV. Half of that energy is deposited in the body. The activity tells at what rate the gamma rays are released into the body. We assume the activity is constant.

$$\left(1.85 \times 10^{-6}\,\text{Ci}\right)\left(3.70 \times 10^{10}\,\frac{\text{gammas}}{\text{s}}\right)\left(86400\,\frac{\text{s}}{\text{day}}\right)\left(61\,\frac{\text{keV}}{\text{gamma}}\right)\left(1.60 \times 10^{-16}\,\frac{\text{J}}{\text{keV}}\right)\left(\frac{1}{70\,\text{kg}}\right)$$

$$= 8.25 \times 10^{-7}\,\frac{\text{J/kg}}{\text{day}} = \boxed{8.25 \times 10^{-7}\,\frac{\text{Gy}}{\text{day}}}$$

$\boxed{46.}$ Since the half-life is long (5730 yr) we can consider the activity as constant over a short period of time. Use the definition of the curie from section 31-5.

$$1.00 \times 10^{-6}\,\text{Ci} \times \frac{3.70 \times 10^{10}\,\text{decays/s}}{1\,\text{Ci}} = 3.70 \times 10^{4}\,\text{decays/s} = \frac{\Delta N}{\Delta t} = \frac{0.693}{T_{\frac{1}{2}}} N \quad \rightarrow$$

$$N = \frac{\Delta N}{\Delta t} \frac{T_{\frac{1}{2}}}{0.693} = \left(3.70 \times 10^{4}\,\text{decays/s}\right) \frac{5730\,\text{y}}{0.693} \left(3.156 \times 10^{7}\,\text{s/y}\right) = 9.655 \times 10^{15}\,\text{nuclei}$$

$$9.655 \times 10^{15}\,\text{nuclei} \left(\frac{0.014\,\text{kg}}{6.02 \times 10^{23}\,\text{nuclei}}\right) = \boxed{2.25 \times 10^{-10}\,\text{kg}}$$

47. (*a*) According to Appendix B, ${}^{131}_{53}\text{I}$ decays by beta decay.

$${}^{131}_{53}\text{I} \rightarrow {}^{131}_{54}\text{Xe} + \beta^{-} + \bar{\nu}$$

(*b*) The number of nuclei present is given by Eq. 30-4.

$$N = N_0 e^{-\lambda t} \quad \rightarrow \quad t = -\frac{\ln \frac{N}{N_0}}{\lambda} = -\frac{T_{1/2} \ln \frac{N}{N_0}}{0.693} = -\frac{(8.0\,\text{d}) \ln 0.10}{0.693} = 26.58\,\text{d} \approx \boxed{27\,\text{d}}$$

(*c*) The activity is given by $\frac{\Delta N}{\Delta t} = \lambda N$. This can be used to find the number of nuclei, and then the mass can be found.

$$\frac{\Delta N}{\Delta t}=\lambda N \quad \to$$

$$N=\frac{\Delta N/\Delta t}{\lambda}=\frac{\left(T_{1/2}\right)\left(\Delta N/\Delta t\right)}{0.693}=\frac{\left(8.0\,\text{d}\right)\left(86400\,\text{s/d}\right)\left(1\times10^{-3}\,\text{Ci}\right)\left(3.70\times10^{10}\,\text{decays/s}\right)}{0.693}$$

$$=3.69\times10^{13}\,\text{nuclei}$$

$$3.69\times10^{13}\,\text{nuclei}\left(\frac{0.131\,\text{kg}}{6.02\times10^{23}\,\text{nuclei}}\right)=\boxed{8.0\times10^{-12}\,\text{kg}}$$

48. The activity is converted to decays per day, then to energy per year, and finally to a dose per year. The potassium decays by gammas and betas, according to Appendix B. Gammas and betas have a quality factor of 1, so the number of Sv is the same as the number of Gy, and the number of rem is the same as the number of rad.

$$\left(2000\times10^{-12}\,\frac{\text{Ci}}{\text{L}}\right)\left(3.70\times10^{10}\,\frac{\text{decays/s}}{1\,\text{Ci}}\right)\left(12\,\text{hr}\,\frac{3600\,\text{s}}{\text{hr}}\right)\left(0.5\,\frac{\text{L}}{\text{day}}\right)=1.598\times10^{6}\,\frac{\text{decays}}{\text{day}}$$

$$\left(1.598\times10^{6}\,\frac{\text{decays}}{\text{day}}\right)\left(365\,\frac{\text{day}}{\text{yr}}\right)\left(0.15\times10^{6}\,\frac{\text{eV}}{\text{decay}}\right)\left(\frac{1.60\times10^{-19}\,\text{J}}{\text{eV}}\right)=1.400\times10^{-5}\,\frac{\text{J}}{\text{yr}}$$

(*a*) $$\left(1.400\times10^{-5}\,\frac{\text{J}}{\text{yr}}\right)\left(\frac{1}{50\,\text{kg}}\right)\left(\frac{1\,\text{Gy}}{1\,\text{J/kg}}\right)=2.80\times10^{-7}\,\frac{\text{Gy}}{\text{year}}\approx\boxed{3\times10^{-7}\,\frac{\text{Sv}}{\text{year}}}$$

$$\left(2.80\times10^{-7}\,\frac{\text{Gy}}{\text{year}}\right)\left(\frac{100\,\text{rad}}{1\,\text{Gy}}\right)=2.80\times10^{-5}\,\frac{\text{rad}}{\text{year}}\approx\boxed{3\times10^{-5}\,\frac{\text{rem}}{\text{year}}}$$

$$\text{fraction of allowed dose}=\frac{2.80\times10^{-5}\,\frac{\text{rem}}{\text{year}}}{100\times10^{-3}\,\frac{\text{rem}}{\text{year}}}\approx\boxed{3\times10^{-4}\,\text{times the allowed dose}}$$

(*b*) For the baby, the only difference is that the mass is 10 times smaller, so the effective dose is 10 times bigger. The results are as follows.

$$\boxed{3\times10^{-6}\,\frac{\text{Sv}}{\text{year}},\ 3\times10^{-4}\,\frac{\text{rem}}{\text{year}},\ 3\times10^{-4}\,\text{times the allowed dose}}$$

49. (*a*) The reaction has 86 protons and 222 total nucleons in the parent nucleus. Thus the alpha and the daughter nucleus must have a total of 86 protons and 222 total nucleons. The alpha has 2 protons and 4 total nucleons, so the daughter nucleus must have 84 protons and 218 total nucleons. That makes the daughter nucleus $\boxed{^{218}_{84}\text{Po}}$

(*b*) From figure 30-11, polonium-218 is $\boxed{\text{radioactive}}$. It decays via both $\boxed{\text{alpha and beta decay}}$, each with a half-life of $\boxed{\text{3.1 minutes}}$.

(*c*) The daughter nucleus is not a noble gas, so it is $\boxed{\text{chemically reacting}}$. It is in the same group as oxygen, so it might react with many other elements chemically.

(*d*) The activity is given by $\frac{\Delta N}{\Delta t}=\frac{0.693}{T_{\frac{1}{2}}}N$.

$$\frac{\Delta N}{\Delta t}=\frac{0.693}{T_{\frac{1}{2}}}N=\frac{0.693}{(3.8235\,\mathrm{d})(86400\,\mathrm{s/d})}\left(1.0\times10^{-9}\,\mathrm{g}\right)\frac{6.02\times10^{23}\,\text{nuclei}}{222\ \mathrm{g}}$$

$$=5.689\times10^{6}\ \text{decays/s}\approx\boxed{5.7\times10^{6}\,\mathrm{Bq}}$$

To find the activity after 1 month, use Eq. 30-5.

$$\frac{\Delta N}{\Delta t}=\left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\frac{0.693}{T_{1/2}}t}=\left(5.689\times10^{6}\ \text{decays/s}\right)e^{-\frac{0.693}{(3.8235\,\mathrm{d})}(30\ \mathrm{d})}=2.475\times10^{4}\ \text{decays/s}$$

$$\approx\boxed{2.5\times10^{5}\,\mathrm{Bq}}$$

50. The frequency is given in section 31-9 to be 42.58 MHz. Use that to find the wavelength.

$$c=f\lambda\ \rightarrow\ \lambda=\frac{c}{f}=\frac{2.998\times10^{8}\ \mathrm{m/s}}{42.58\times10^{6}\,\mathrm{Hz}}=\boxed{7.041\,\mathrm{m}}$$

This lies in the $\boxed{\text{radio wave}}$ portion of the spectrum.

51. (*a*) The reaction is ${}^{9}_{4}\mathrm{Be}+{}^{4}_{2}\mathrm{He}\ \rightarrow\ \mathrm{n}+?$. There are 6 protons and 13 nucleons in the reactants, and so there must be 6 protons and 13 nucleons in the products. The neutron is 1 nucleon, so the other product must have 6 protons ad 12 nucleons. Thus it is $\boxed{{}^{12}_{6}\mathrm{C}}$.

(*b*) $Q=m_{{}^{9}_{4}\mathrm{Be}}c^2+m_{{}^{4}_{2}\mathrm{He}}c^2-m_{\mathrm{n}}c^2-m_{{}^{12}_{6}\mathrm{U}}c^2$

$$=\left[9.012182\,\mathrm{u}+4.002603\,\mathrm{u}-1.008665\,\mathrm{u}-12.000000\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2=\boxed{5.701\,\mathrm{MeV}}$$

52. If $\overline{\mathrm{KE}}=kT$, then to get from kelvins to keV, the Boltzmann constant would be used. It must simply be put in the proper units.

$$k=1.381\times10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}\times\frac{1\,\mathrm{eV}}{1.602\times10^{-19}\,\mathrm{J}}\times\frac{1\,\mathrm{keV}}{1000\,\mathrm{eV}}=\boxed{8.62\times10^{-8}\ \mathrm{keV/K}}$$

$\boxed{53.}$ From Eq. 13-9, the average speed of a gas molecule (root mean square speed) is inversely proportional to the square root of the mass of the molecule, if the temperature is constant. We assume that the two gases are in the same environment and so at the same temperature. We use UF_6 molecules for the calculations.

$$\frac{v_{{}^{235}_{92}\mathrm{UF}_6}}{v_{{}^{238}_{92}\mathrm{UF}_6}}=\sqrt{\frac{m_{{}^{238}_{92}\mathrm{UF}_6}}{m_{{}^{235}_{92}\mathrm{UF}_6}}}=\sqrt{\frac{238+6(19)}{235+6(19)}}=\boxed{1.0043:1}$$

54. (*a*) We assume that the energy produced by the fission was 200 MeV per fission, as in problems 18 and 19.

$$\left(20\,\text{kilotons TNT}\right)\left(\frac{5\times10^{12}\,\mathrm{J}}{1\,\text{kiloton}}\right)\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right)\left(\frac{1\,\text{fission atom}}{200\,\mathrm{MeV}}\right)\left(\frac{0.235\,\mathrm{kg}}{6.02\times10^{23}\,\text{atom}}\right)$$

$$=1.220\,\mathrm{kg}\approx\boxed{1\,\mathrm{kg}}$$

(*b*) Use $E = mc^2$.

$$E = mc^2 \quad \rightarrow \quad m = \frac{E}{c^2} = \frac{(20\,\text{kilotons TNT})\left(\dfrac{5\times10^{12}\,\text{J}}{1\,\text{kiloton}}\right)}{\left(3.0\times10^{8}\,\text{m/s}\right)^2} = 1.11\times10^{-3}\,\text{kg} \approx \boxed{1\,\text{g}}$$

55. The effective dose (in rem) is equal to the actual dose (in rad) times the quality factor.

$$\text{dose}(\text{rem}) = \left(21\times10^{-3}\ \text{rad/yr X-ray, }\gamma\text{-ray}\right)(1) + \left(3.0\times10^{-3}\ \text{rad/yr}\right)(10) = \boxed{5.1\times10^{-2}\ \text{rem/yr}}$$

56. The oceans cover about 70% of the Earth, to an average depth of approximately 4 km. The density of the water is approximately $1000\,\text{kg/m}^3$. Find the volume of water using the surface area of the Earth. Then convert that volume of water to mass, to the number of water molecules, to the number of hydrogen atoms, and then finally to the number of deuterium atoms using the natural abundance of deuterium from Appendix B.

$$\text{Mass of water} = (\text{surface area})(\text{depth})(\text{density}) = 4\pi\left(6.38\times10^{6}\,\text{m}\right)^2(4000\,\text{m})\left(1000\,\text{kg/m}^3\right)$$

$$= 2.05\times10^{21}\,\text{kg water}$$

$$\left(2.05\times10^{21}\,\text{kg water}\right)\times\frac{6.02\times10^{23}\,\text{molecules}}{0.018\ \text{kg water}}\times\frac{2\ \text{H atoms}}{1\ \text{molecule}}\times\frac{0.000115\ \text{d atoms}}{1\ \text{H atom}}$$

$$= 1.58\times10^{43}\,\text{d} \approx \boxed{2\times10^{43}\,\text{d atoms}}$$

$$= 1.58\times10^{43}\,\text{d}\times\frac{2\times10^{-3}\,\text{kg}}{6.02\times10^{23}\,\text{d atoms}} \approx \boxed{5\times10^{16}\,\text{kg d}}$$

From Eqs. 31-8a and 31-8b, we see that, if the two reactions are carried out at the same rate, that 4 deuterons would produce 7.30 MeV of energy. Use that relationship to convert the number of deuterons in the oceans to energy.

$$\left(1.58\times10^{43}\,\text{d}\right)\times\frac{7.30\,\text{MeV}}{4\ \text{d}}\times\frac{1.60\times10^{-13}\,\text{J}}{1\,\text{MeV}} = 4.61\times10^{30}\,\text{J} \simeq \boxed{5\times10^{30}\,\text{J}}$$

57. Because the quality factor for gamma rays is 1, the dose in rem is equal in number to the dose in rad. Since the intensity falls off as the square of the distance, the exposure rate times the square of the distance will be constant.

$$\text{Allowed dose} = \frac{5\,\text{rem}}{\text{year}}\times\frac{1\,\text{rad}}{1\,\text{rem}}\frac{1\ \text{year}}{52\ \text{weeks}}\times\frac{1\,\text{week}}{40\,\text{hours}} = 2.404\times10^{-3}\,\frac{\text{rad}}{\text{hour}}$$

$$\left(2.404\times10^{-3}\,\frac{\text{rad}}{\text{hour}}\right)r^2 = \left(5.2\times10^{-2}\,\frac{\text{rad}}{\text{hour}}\right)(1\,\text{m})^2 \quad\rightarrow\quad r = \sqrt{\frac{\left(5.2\times10^{-2}\,\dfrac{\text{rad}}{\text{hour}}\right)(1\,\text{m})^2}{\left(2.404\times10^{-3}\,\dfrac{\text{rad}}{\text{hour}}\right)}} = \boxed{4.7\,\text{m}}$$

58. (*a*) The reaction is of the form ? → ${}^{4}_{2}\text{He} + {}^{222}_{86}\text{Rn}$. There are 88 protons and 226 nucleons as products, so there must be 88 protons and 226 nucleons as reactants. Thus the parent nucleus is ${}^{226}_{88}\text{Ra}$.

$${}^{226}_{88}\text{Ra} \rightarrow {}^{4}_{2}\text{He} + {}^{222}_{86}\text{Rn}$$

(*b*) If we ignore the KE of the daughter nucleus, then the KE of the alpha particle is the *Q*-value of the reaction.

$$\mathrm{KE}_\alpha = m_{^{226}_{88}\mathrm{Ra}}c^2 - m_{^4_2\mathrm{He}}c^2 - m_{^{222}_{86}\mathrm{Rn}}c^2$$

$$= \left[226.025403\,\mathrm{u} - 4.002603\,\mathrm{u} - 222.017570\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 4.872\,\mathrm{MeV}$$

(*c*) From momentum conservation, the momentum of the alpha particle will be equal in magnitude to the momentum of the alpha particle. At the energy above, the alpha particle is not relativistic, and so $\mathrm{KE}_\alpha = \dfrac{p_\alpha^2}{2m_\alpha} \rightarrow p_a = \sqrt{2m_a\mathrm{KE}_a}$.

$$p_\alpha = \sqrt{2m_a\mathrm{KE}_a} = \sqrt{2\left(4.003\,\mathrm{u}\right)\left(\frac{931.5\,\mathrm{MeV}/c^2}{1\,\mathrm{u}}\right)\left(4.872\,\mathrm{MeV}\right)} = 191\,\mathrm{MeV}/c$$

(*d*) Since $p_\alpha = p_{\text{daughter}}$, $\mathrm{KE}_{\text{daughter}} = \dfrac{p_{\text{daughter}}^2}{2m_{\text{daughter}}} = \dfrac{p_\alpha^2}{2m_{\text{daughter}}}$.

$$\mathrm{KE}_{\text{daughter}} = \frac{p_\alpha^2}{2m_{\text{daughter}}} = \frac{\left(191\,\mathrm{MeV}/c\right)^2}{2\left(222\,\mathrm{u}\right)\left(\dfrac{931.5\,\mathrm{MeV}/c^2}{1\,\mathrm{u}}\right)} = \boxed{8.82\times10^{-2}\,\mathrm{MeV}}$$

Thus we see that our original assumption of ignoring the KE of the daughter nucleus is valid. The KE of the daughter is less than 2% of the *Q*-value.

59. (*a*) The mass of fuel can be found by converting the power to energy to number of nuclei to mass.

$$\left(3400\times10^6\,\mathrm{J/s}\right)\left(1\,\mathrm{y}\right)\left(\frac{3.156\times10^7\mathrm{s}}{1\,\mathrm{y}}\right)\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\mathrm{J}}\right)\left(\frac{1\,\text{fission atom}}{200\,\mathrm{MeV}}\right)\left(\frac{0.235\,\mathrm{kg}}{6.02\times10^{23}\,\text{atom}}\right)$$

$$= 1.309\times10^3\,\mathrm{kg} \approx \boxed{1300\,\mathrm{kg}}$$

(*b*) The product of the first 5 factors above gives the number of U atoms that fission.

$$\#\text{Sr atoms} = 0.06\left(3400\times10^6\,\mathrm{J/s}\right)\left(1\,\mathrm{y}\right)\left(\frac{3.156\times10^7\mathrm{s}}{1\,\mathrm{y}}\right)\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\mathrm{J}}\right)\left(\frac{1\,\text{fission atom}}{200\,\mathrm{MeV}}\right)$$

$$= 2.01\times10^{26}\,\text{Sr atoms}$$

The activity is given by Eq. 30-3b.

$$\frac{\Delta N}{\Delta t} = \lambda N = \frac{0.693}{T_{1/2}}N = \frac{0.693}{\left(29\,\mathrm{yr}\right)\left(3.156\times10^7\,\mathrm{s/yr}\right)}\left(2.01\times10^{26}\right) = 1.522\times10^{17}\,\frac{\text{decays}}{\mathrm{s}}$$

$$= \left(1.522\times10^{17}\,\text{decays/s}\right)\left(\frac{1\,\mathrm{Ci}}{3.70\times10^{10}\,\text{decays/s}}\right) = \boxed{4.1\times10^6\,\mathrm{Ci}}$$

60. This “heat of combustion” is 26.2 MeV / 4 hydrogen atoms.

$$\frac{26.2\,\mathrm{MeV}}{4\ \text{H atoms}}\times\frac{10^6\,\mathrm{eV}}{1\,\mathrm{MeV}}\times\frac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\times\frac{1\,\text{H atom}}{1.008\,\mathrm{u}}\times\frac{1\,\mathrm{u}}{1.66\times10^{-27}\,\mathrm{kg}} = \boxed{6.26\times10^{14}\,\mathrm{J/kg}}$$

This is about $\boxed{2\times10^7}$ times the heat of combustion of coal.

61. (*a*) The energy is radiated uniformly over a sphere with a radius equal to the orbit radius of the Earth.

$$\left(1400\frac{\text{W}}{\text{m}^2}\right)4\pi\left(1.496\times10^{11}\,\text{m}\right)^2 = 3.937\times10^{26}\,\text{W} \approx \boxed{4.0\times10^{26}\,\text{W}}$$

(*b*) The reaction of Eq. 31-7 releases 26.7 MeV for every 4 protons consumed.

$$\left(3.937\times10^{26}\,\frac{\text{J}}{\text{s}}\right)\times\frac{4\text{ protons}}{26.7\,\text{MeV}}\times\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}} = 3.686\times10^{38}\text{ protons/s} \approx \boxed{3.7\times10^{38}\text{ protons/s}}$$

(*c*) Convert the Sun's mass to a number of protons, and then use the above result to estimate the Sun's lifetime.

$$2.0\times10^{30}\,\text{kg}\left(\frac{1\,\text{proton}}{1.673\times10^{-27}\text{kg}}\right)\left(\frac{1\,\text{s}}{3.686\times10^{38}\,\text{protons}}\right)\left(\frac{1\,\text{yr}}{3.156\times10^{7}}\right) \approx \boxed{1.0\times10^{11}\,\text{yr}}$$

62. (*a*) The energy released is given by the *Q*-value.

$$Q = 2m_{{}^{12}_{6}\text{C}}c^2 - m_{{}^{24}_{12}\text{Mg}}c^2 = \left[2\left(12.000000\,\text{u}\right) - 23.985042\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 13.93\,\text{MeV}$$

(*b*) The total kinetic energy of the two nuclei must equal their potential energy when separated by 6.0 fm.

$$2\text{KE} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} \quad\rightarrow$$

$$\text{KE} = \tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \tfrac{1}{2}\left(9.0\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left[(6)\left(1.60\times10^{-19}\text{C}\right)\right]^2}{6.0\times10^{-15}\text{m}} = \boxed{6.912\times10^{-13}\,\text{J}}$$

$$= 6.912\times10^{-13}\,\text{J}\left(1\text{eV}/1.60\times10^{-19}\,\text{J}\right)\left(1\,\text{MeV}/10^{6}\,\text{eV}\right) = \boxed{4.32\,\text{MeV}}$$

(*c*) The kinetic energy and temperature are related by Eq.13-8.

$$\text{KE} = \tfrac{3}{2}kT \;\rightarrow\; T = \tfrac{2}{3}\frac{\text{KE}}{k} = \tfrac{2}{3}\frac{6.912\times10^{-13}\,\text{J}}{1.38\times10^{-23}\ \text{J/K}} = \boxed{3.3\times10^{10}\,\text{K}}$$

63. (*a*) $\left(0.10\times10^{-6}\text{Ci}\right)\left(3.7\times10^{10}\text{ decays/s}\right) = \boxed{3700\,\text{decays/s}}$

(*b*) The beta particles have a quality factor of 1. We calculate the dose in gray and then convert to sieverts. The half life is over a billion years, so we assume the activity is constant.

$$\left(3700\,\text{decays/s}\right)\left(1.4\times10^{6}\ \text{eV/decay}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)\left(3.156\times10^{7}\ \text{s/y}\right)\left(\frac{1}{50\,\text{kg}}\right) =$$

$$= 5.23\times10^{-4}\ \text{J/kg/y} = 5.23\times10^{-4}\ \text{Gy/y} = \boxed{5.23\times10^{-4}\ \text{Sv/y}}$$

This is about $\frac{5.23\times10^{-4}\ \text{Sv/y}}{3.6\times10^{-3}\ \text{Sv/y}} = 0.15$ or $\boxed{\text{15\% of the background rate}}$.

64. The surface area of a sphere is $4\pi r^2$.

$$\frac{\text{Activity}}{\text{m}^2} = \frac{2.0\times10^{7}\,\text{Ci}}{4\pi r_{\text{Earth}}^2} = \frac{\left(2.0\times10^{7}\,\text{Ci}\right)\left(3.7\times10^{10}\text{ decays/s}\right)}{4\pi\left(6.38\times10^{6}\,\text{m}\right)^2} = \boxed{1.4\times10^{3}\,\frac{\text{decays/s}}{\text{m}^2}}$$

65. $Q = 3m_{{}^4_2\mathrm{He}}c^2 - m_{{}^{12}_6\mathrm{C}}c^2 = \left[3\left(4.002603\,\mathrm{u}\right) - 12.000000\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{7.274\,\mathrm{MeV}}$

66. Since the half-life is 30 years, we assume that the activity does not change during the 2.0 hours of exposure. We calculate the total energy absorbed, and then calculate the effective dose. The two energies can be added directly since the quality factor for both gammas and betas is about 1.

$$\text{Energy} = \left[\begin{array}{l}\left(1.0\times10^{-6}\,\mathrm{Ci}\right)\left(3.7\times10^{10}\,\dfrac{\text{decays}}{\mathrm{s}}\right)\left(2.0\,\mathrm{hr}\right)\left(3600\dfrac{\mathrm{s}}{1\,\mathrm{hr}}\right)\times\\ \left(850\times10^{3}\,\dfrac{\mathrm{eV}}{\text{decay}}\right)\left(1.60\times10^{-19}\,\dfrac{\mathrm{J}}{\mathrm{eV}}\right)\end{array}\right] = 3.367\times10^{-5}\,\mathrm{J}$$

$$\text{dose} = \frac{3.367\times10^{-5}\,\mathrm{J}}{75\,\mathrm{kg}}\times\frac{100\mathrm{rad}}{1\,\mathrm{J/kg}} = 4.83\times10^{-5}\,\mathrm{rad} \approx \boxed{4.8\times10^{-5}\,\mathrm{rem}}$$

67. The half life of the strontium isotope is 28.79 years. Use that with Eq. 30-5 to find the time for the activity to be reduced to 10% of its initial value.

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\frac{0.693}{T_{1/2}}t} \rightarrow 0.10\left(\frac{\Delta N}{\Delta t}\right)_0 = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\frac{0.693}{T_{1/2}}t} \rightarrow 0.10 = e^{-\frac{0.693}{T_{1/2}}t} \rightarrow$$

$$\ln\left(0.10\right) = -\frac{0.693}{T_{1/2}}t \rightarrow t = -\frac{T_{1/2}\ln\left(0.10\right)}{0.693} = -\frac{\left(28.79\,\mathrm{y}\right)\ln\left(0.10\right)}{0.693} \approx \boxed{96\,\mathrm{y}}$$

68. Source B is more dangerous than source A because of its higher energy. Since both sources have the same activity, they both emit the same number of gammas. Source B can deposit more energy per gamma and therefore cause more biological damage.

Source C is more dangerous than source B because the alphas have a quality factor up to 20 times larger than the gammas. Thus a number of alphas may have an effective dose up to 20 times higher than the effective dose of the same number of like-energy gammas.

So from most dangerous to least dangerous, the ranking of the sources is $\boxed{\mathrm{C-B-A}}$.

69. The whole-body dose can be converted into a number of decays, which would be the maximum number of nuclei that could be in the Tc sample. The quality factor of gammas is 1.

$$50\,\mathrm{mrem} = 50\,\mathrm{mrad} \rightarrow \left(50\times10^{-3}\,\mathrm{rad}\right)\times\frac{1\,\mathrm{J/kg}}{100\,\mathrm{rad}}\times\left(70\,\mathrm{kg}\right)\times\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}} = 2.19\times10^{17}\,\mathrm{eV}$$

$$\left(2.19\times10^{17}\,\mathrm{eV}\right)\times\frac{1\text{ effective}\,\gamma}{140\times10^{3}\mathrm{eV}}\times\frac{2\gamma\text{ decays}}{1\text{ effective }\gamma}\times\frac{1\text{ nucleus}}{1\gamma\text{ decay}} = 3.13\times10^{12}\,\text{nuclei}$$

This then is the total number of decays that will occur. The activity for this number of nuclei can be calculated from Eq. 30-3b.

$$\text{Activity} = \frac{\Delta N}{\Delta t} = \lambda N = \frac{0.693N}{T_{1/2}} = \frac{0.693\left(3.13\times10^{12}\,\text{decays}\right)}{\left(6\,\mathrm{h}\right)\left(3600\,\mathrm{s/h}\right)}\left(\frac{1\,\mathrm{Ci}}{3.70\times10^{10}\,\text{decays/s}}\right)$$

$$= 2.71\times10^{-3}\,\mathrm{Ci} \simeq \boxed{3\,\mathrm{mCi}}$$

CHAPTER 32: Elementary Particles

Answers to Questions

1. A reaction between two nucleons that would produce a π^- is: $\mathrm{p}+\mathrm{n} \rightarrow \mathrm{p}+\mathrm{p}+\pi^-$.

2. No, even if the proton has an extremely large kinetic energy (and, thus, an extremely large momentum), there is no way to conserve both energy and momentum during such a decay $\left(\mathrm{p} \rightarrow \mathrm{n}+\pi^+\right)$. Observed from the rest frame of the proton, the decay is energetically impossible, and so it is energetically impossible in every other reference frame as well. In the frame in which the proton is moving fast, the decay products must move very fast as well in order to conserve momentum, and with this constraint, there will still not be enough energy to make the decay energetically possible.

3. Antiatoms would be made up of antiprotons and antineutrons in the nucleus with positrons spinning around the nucleus in orbits. If antimatter and matter came into contact, all of the particle-antiparticle pairs would annihilate (convert matter and antimatter into energy) and release energy as gamma rays or other particles.

4. The photon is the particle that indicates the electromagnetic interaction.

5. Yes, if a neutrino is produced during a decay, the weak interaction is responsible. No, for example a weak interaction decay that does not produce a neutrino could produce a Z^0 instead.

6. Strong interaction forces are only seen at extremely short distances, but the rest of the time the weak interaction dominates. Also, the other decay products (electron and electron antineutrino) only interact via the weak force.

7. An electron takes part in the electromagnetic interaction (it is charged), the weak interaction, and the gravitational interaction (it has mass). A neutrino takes part in the weak interaction and the gravitational interaction (it has a small mass). A proton takes part in the strong interaction (baryon), the electromagnetic interaction (it is charged), the weak interaction, and the gravitational interaction (it has mass).

8. Charge and baryon number are conserved in the decays shown in Table 32-2. Here are a few examples.

   $\mathrm{W}^+ \rightarrow \mathrm{e}^+ + \nu_\mathrm{e}$: All components have B = 0, and so baryon number is conserved. Charge is also conserved.

   $\tau^- \rightarrow \mu^- + \overline{\nu}_\mu + \nu_\tau$: All components have B = 0, and so baryon number is conserved. Since the neutrinos are not charged, charge is also conserved.

   $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}$: The neutron and proton each have B = 1, and so baryon number is conserved. Since the charge on the left is equal to the charge on the right, charge is also conserved.

9. Since decays via the electromagnetic interaction are indicated by the production of photons, the decays in Table 32-2 that occur via the electromagnetic interaction are π^0, Σ^0, and η^0.
10. Since decays via the weak interaction are indicated by the production of neutrinos or W and Z bosons, the decays in Table 32-3 that occur via the weak interaction are: W, Z, muon, tau, pion, kaon, and neutron.

11. The decay times indicate that Σ^+ and Σ^- will decay via the weak interaction (although not to Λ^0). The decay times also indicate that the Σ^0 will decay via the electromagnetic interaction (verified by the production of a gamma).

12. Since the Δ baryon has B = 1, it must be made up of three quarks. Since the spin of the Δ baryon is 3/2, none of these quarks can be antiquarks. Thus, since the charges of quarks are either +2/3 or -1/3, the only charges we can create with this combination are q = -1 (= –1/3 – 1/3 – 1/3), 0 (= +2/3 – 1/3 – 1/3), +1 (= +2/3 + 2/3 – 1/3), and +2 (= +2/3 + 2/3 + 2/3).

13. Based on lifetimes shown in Table 32-4, the particle decays that occur via the electromagnetic interaction are J/ψ (3097) and Y (9460).

14. All of the particles in Table 32-4, except for J/ψ (3097) and Y (9460), decay via the weak interaction.

15. Baryons are made of a combination of three spin 1/2 quarks or spin -1/2 antiquarks. If a baryon has two quarks and one antiquark, or one quark and two antiquarks, then the magnitude of the spin will be 1/2. If a baryon has three quarks or three antiquarks, then the magnitude of the spin will be -1/2. If a meson has two quarks or two antiquarks, the magnitude of its spin will be 1. If a meson has a quark and an antiquark, the spin will be 0.

$\boxed{16.}$ If a neutrinolet was massless it would not interact via the gravitation force; if it had no electrical charge it would not interact via the electromagnetic force; if it had no color charge it would not interact via the strong force; and if it does not interact via the weak force, then it would not interact with matter at all and it would be very difficult to say that it even exists at all. However, this neutrinolet would perhaps be identical to a photon.

17. (*a*) It is not possible for a particle to be both a lepton and a baryon. A lepton is an elementary particle, not composed of quarks, while baryons are made up of three quarks.
    (*b*) Yes, it is possible for a particle to be both a baryon and a hadron. All baryons are spin = 1/2 hadrons.
    (*c*) No, it is not possible for a particle to be both a meson and a quark. A meson is made up of two quarks.
    (*d*) No, it is not possible for a particle to be both a hadron and a lepton. A lepton is an elementary particle, while a hadron is made up of three quarks.

18. No, it is not possible to find a particle that is made up of two quarks and no antiquarks. First of all, this would give the particle a fractional charge, which has never been observed. Second, the “color” of the particle would not be “white”. To have a “white” quark, requires a quark of one color and an antiquark with the corresponding anticolor.

    Yes, it is possible to find a particle that is made up of two quarks and two antiquarks. This combination of quarks/antiquarks would allow a particle to have a possible (non-fractional) charge and also allow it to be “white” in color. In Example 32-7, the comment is made that a π^0 is described as $\mathrm{u\bar{u}} + \mathrm{d\bar{d}}$.

19. In the nucleus, the strong interaction with the other nucleons does not allow the neutron to decay. When a neutron is free, the weak interaction is the dominant force and can cause the neutron to decay.

20. No, the reaction $e^- + p \rightarrow n + \overline{v}_e$ is not possible. The electron lepton number is not conserved: The reactants have $L_e = 1 + 0 = 1$, but the products have $L_e = 0 - 1 = -1$. Thus, this reaction is not possible.

21. The reaction $\Lambda^0 \rightarrow p^+ + e^- + \overline{v}_e$ proceeds via the weak force. We know this is the case since a neutrino is emitted.

Solutions to Problems

1. The total energy is given by Eq. 26-7a.

$$E = m_0c^2 + \text{KE} = 0.94\,\text{GeV} + 6.35\,\text{GeV} = \boxed{7.29\,\text{GeV}}$$

$\boxed{2.}$ Because the energy of the electrons is much greater than their rest mass, we have $\text{KE} = E = pc$. Combine that with Eq. 27-8 for the de Broglie wavelength.

$$E = pc\ ;\ p = \frac{h}{\lambda} \rightarrow E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.0\times10^{8}\,\text{m/s}\right)}{\left(35\times10^{9}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = \boxed{3.6\times10^{-17}\,\text{m}}$$

$\boxed{3.}$ The frequency is related to the magnetic field in Eq. 32-2.

$$f = \frac{qB}{2\pi m} \rightarrow B = \frac{2\pi mf}{q} = \frac{2\pi\left(1.67\times10^{-27}\,\text{kg}\right)\left(2.8\times10^{7}\,\text{Hz}\right)}{1.60\times10^{-19}\,\text{C}} = \boxed{1.8\,\text{T}}$$

4. The time for one revolution is the period of revolution, which is the circumference of the orbit divided by the speed of the protons. Since the protons have very high energy, their speed is essentially the speed of light.

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(1.0\times10^{3}\,\text{m}\right)}{3.0\times10^{8}\,\text{m/s}} = \boxed{2.1\times10^{-5}\,\text{s}}$$

5. Use Eq. 32-2 to calculate the frequency.

$$f = \frac{qB}{2\pi m} = \frac{2\left(1.60\times10^{-19}\,\text{C}\right)\left(1.7\,\text{T}\right)}{2\pi\left[4\left(1.67\times10^{-27}\,\text{kg}\right)\right]} = \boxed{1.3\times10^{7}\,\text{Hz}} = 13\,\text{MHz}$$

6. (*a*) The maximum kinetic energy is $\text{KE} = \frac{q^2B^2R^2}{2m} = \frac{1}{2}mv^2$. Compared to Example 32-2, the charge has been doubled and the mass has been multiplied by 4. These two effects cancel each other in the equation, and so the maximum kinetic energy is unchanged.

$$\text{KE} = \boxed{8.7\,\text{MeV}} \qquad v = \sqrt{\frac{2\,\text{KE}}{m}} = \sqrt{\frac{2\left(8.7\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{4\left(1.66\times10^{-27}\,\text{kg}\right)}} = \boxed{2.0\times10^{7}\,\text{m/s}}$$

(*b*) The maximum kinetic energy is $\mathrm{KE}=\dfrac{q^2B^2R^2}{2m}=\frac{1}{2}mv^2$. Compared to Example 32-2, the charge is unchanged and the mass has been multiplied by 2. Thus the kinetic energy will be half of what it was in Example 32.2

$$\mathrm{KE}=\boxed{4.3\,\mathrm{MeV}}\qquad v=\sqrt{\frac{2\,\mathrm{KE}}{m}}=\sqrt{\frac{2\left(4.3\times10^{6}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{\left(1.66\times10^{-27}\,\mathrm{kg}\right)}}=\boxed{2.0\times10^{7}\,\mathrm{m/s}}$$

The alpha and the deuteron have the same charge to mass ratio, and so move at the same speed.

(*c*) The frequency is given by $f=\dfrac{qB}{2\pi m}$. Since the charge to mass ratio of both the alpha and the deuteron is half that of the proton, the frequency for the alpha and the deuteron will both be half the frequency found in Example 32-2 for the proton.

$$\boxed{f=13\,\mathrm{MHz}}$$

7. From Eq. 30-1, the diameter of a nucleon is about $2.4\times10^{-15}\,\mathrm{m}$. The 30-MeV alpha particles and protons are not relativistic, so their momentum is given by $p=mv=\sqrt{2m\mathrm{KE}}$. The wavelength is given by Eq. 27-8, $\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{2m\mathrm{KE}}}$.

$$\lambda_\alpha=\frac{h}{\sqrt{2m_\alpha\mathrm{KE}}}=\frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{\sqrt{2(4)\left(1.66\times10^{-27}\,\mathrm{kg}\right)\left(30\times10^{6}\,\mathrm{eV}\right)\left(1.6\times10^{-19}\,\mathrm{J/eV}\right)}}=2.63\times10^{-15}\,\mathrm{m}$$

$$\lambda_\mathrm{p}=\frac{h}{\sqrt{2m_\mathrm{p}\mathrm{KE}}}=\frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{\sqrt{2\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(30\times10^{6}\,\mathrm{eV}\right)\left(1.6\times10^{-19}\,\mathrm{J/eV}\right)}}=5.24\times10^{-15}$$

The wavelength of the alpha particle is about the same as the diameter of a nucleon, while the wavelength of the proton is about twice the diameter of the nucleon. Thus the alpha particle will be better for picking out details in the nucleus.

8. The protons are accelerated twice during each revolution. During each acceleration the protons gain 55 keV of kinetic energy.

$$\frac{25\times10^{6}\,\mathrm{eV}}{2\left(55\times10^{3}\,\mathrm{eV/rev}\right)}=\boxed{227\text{ revolutions}}$$

9. Because the energy of the protons is much greater than their rest mass, we have $\mathrm{KE}=E=pc$. Combine that with Eq. 27-8 for the de Broglie wavelength.

$$E=pc\ ;\ p=\frac{h}{\lambda}\ \rightarrow\ E=\frac{hc}{\lambda}\ \rightarrow\ \lambda=\frac{hc}{E}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{\left(7.0\times10^{12}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}=\boxed{1.8\times10^{-19}\,\mathrm{m}}$$

10. (*a*) The magnetic field is found from the maximum kinetic energy as derived in Example 32-2.

$$\mathrm{KE}=\frac{q^2B^2R^2}{2m}\ \rightarrow\ B=\frac{\sqrt{2m\mathrm{KE}}}{qR}\ \rightarrow$$

$$B=\frac{\sqrt{2(2.014)\left(1.66\times10^{-27}\,\text{kg}\right)\left(12\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}}{\left(1.60\times10^{-19}\,\text{C}\right)(1.0\,\text{m})}=0.7082\,\text{T}\approx\boxed{0.71\,\text{T}}$$

(*b*) The cyclotron frequency is given by Eq. 32-2.

$$f=\frac{qB}{2\pi m}=\frac{\left(1.60\times10^{-19}\,\text{C}\right)(0.7082\,\text{T})}{2\pi(2.014)\left(1.66\times10^{-27}\,\text{kg}\right)}=5.394\times10^{6}\,\text{Hz}\simeq\boxed{5.4\,\text{MHz}}$$

(*c*) The deuteron will be accelerated twice per revolution, and so will gain energy equal to twice its charge times the voltage on each revolution.

$$\text{number of revolutions}=n=\frac{12\times10^{6}\,\text{eV}}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(22\times10^{3}\,\text{V}\right)}\left(1.60\times10^{-19}\ \text{J/eV}\right)$$

$$=273\text{ revolutions}\approx\boxed{270\text{ revolutions}}$$

(*d*) The time is the number of revolutions divided by the frequency (which is revolutions per second).

$$\Delta t=\frac{n}{f}=\frac{273\,\text{revolutions}}{5.394\times10^{6}\ \text{rev/s}}=\boxed{5.1\times10^{-6}\,\text{s}}$$

(*e*) If we use an average radius of half the radius of the cyclotron, then the distance traveled is the average circumference times the number of revolutions.

$$\text{distance}=\tfrac{1}{2}2\pi rn=\pi(1.0\,\text{m})(273)=\boxed{860\,\text{m}}$$

11. Because the energy of the protons is much greater than their rest mass, we have $\text{KE}=E=pc$. A relationship for the magnetic field is given in section 32-1.

$$v=\frac{qBr}{m}\ \rightarrow\ mv=qBr\ \rightarrow\ p=qBr\ \rightarrow\ \frac{E}{c}=qBr\ \rightarrow$$

$$B=\frac{E}{qrc}=\frac{\left(7.0\times10^{15}\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(4.25\times10^{3}\,\text{m}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}=\boxed{5.5\,\text{T}}$$

12. If the speed of the protons is *c*, then the time for one revolution is found from uniform circular motion. The number of revolutions is the total time divided by the time for one revolution. The energy per revolution is the total energy gained divided by the number of revolutions.

$$v=\frac{2\pi r}{T}\ \rightarrow\ T=\frac{2\pi r}{v}=\frac{2\pi r}{c}\qquad n=\frac{t}{T}=\frac{ct}{2\pi r}$$

$$\text{Energy/revolution}=\frac{\Delta E}{n}=\frac{(\Delta E)2\pi r}{ct}=\frac{\left(1.0\times10^{12}\,\text{eV}-150\times10^{9}\,\text{eV}\right)2\pi\left(1.0\times10^{3}\,\text{m}\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)(20\,\text{s})}$$

$$=8.9\times10^{5}\ \text{eV/rev}=\boxed{0.89\,\text{MeV/rev}}$$

13. Start with an expression from section 32-1, with *q* replaced by *e*.

$$v=\frac{eBr}{m}\ \rightarrow\ mv=eBr\ \rightarrow\ p=eBr$$

In the relativistic limit, $p=E/c$ and so $\frac{E}{c}=eBr$. To put the energy in electron volts, divide the energy by the charge of the object.

$$\frac{E}{c}=eBr \rightarrow \boxed{\frac{E}{e}=Brc}$$

14. Because the energy of the protons is much greater than their rest mass, we have $\text{KE}=E=pc$. Combine this with an expression from section 32-1.

$$v=\frac{qBr}{m} \rightarrow mv=qBr \rightarrow p=qBr \rightarrow \frac{E}{c}=qBr \rightarrow$$

$$B=\frac{E}{qrc}=\frac{\left(1.0\times10^{12}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.0\times10^{3}\,\text{m}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}=\boxed{3.3\,\text{T}}$$

15. The energy released is the difference in the mass energy between the products and the reactant.

$$\Delta E=m_{\pi^+}c^2-m_{\mu^+}c^2-m_{\nu_\mu}c^2=139.6\,\text{MeV}-105.7\,\text{MeV}-0=\boxed{33.9\,\text{MeV}}$$

16. The energy released is the difference in the mass energy between the products and the reactant.

$$\Delta E=m_{\Lambda^0}c^2-m_nc^2-m_{\pi^0}c^2=1115.7\,\text{MeV}-939.6\,\text{MeV}-135.0\,\text{MeV}=\boxed{41.1\,\text{MeV}}$$

17. The energy required is the mass energy of the two particles.

$$E=2m_nc^2=2\left(939.6\,\text{MeV}\right)=\boxed{1879.2\,\text{MeV}}$$

18. Use Eq. 32-3 to estimate the range of the force based on the mass of the mediating particle.

$$mc^2\approx\frac{hc}{2\pi d} \rightarrow d\approx\frac{hc}{2\pi mc^2}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.0\times10^{8}\,\text{m/s}\right)}{2\pi\left(497.7\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}\approx\boxed{4.0\times10^{-16}\,\text{m}}$$

19. Because the two protons are heading towards each other with the same speed, the total momentum of the system is 0. The minimum kinetic energy for the collision would result in all three particles at rest, and so the minimum kinetic energy of the collision must be equal to the mass energy of the π^0. Each proton will have half of that kinetic energy. From Table 32-2, the mass of the π^0 is $135.0\,\text{MeV}/c^2$.

$$2\left(\text{KE}_{\text{proton}}\right)=m_{\pi^0}c^2=135.0\,\text{MeV} \rightarrow \text{KE}_{\text{proton}}=\boxed{67.5\,\text{MeV}}$$

20. Because the two neutrons are heading towards each other with the same speed, the total momentum of the system is 0. The minimum kinetic energy for the collision would result in all four particles at rest, and so the minimum kinetic energy of the collision must be equal to the mass energy of the K^+K^- pair. Each neutron will have half of that kinetic energy. From Table 32-2, the mass of each of the K^+ and the K^- is $493.7\,\text{MeV}/c^2$.

$$2\left(\text{KE}_{\text{neutron}}\right)=2m_{\text{K}}c^2 \rightarrow \text{KE}_{\text{neutron}}=m_{\text{K}}c^2=\boxed{493.7\,\text{MeV}}$$

21. We use the average mass of $85\,\text{GeV}/c^2$.

$$mc^2\approx\frac{hc}{2\pi d} \rightarrow d\approx\frac{hc}{2\pi mc^2}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.0\times10^{8}\,\text{m/s}\right)}{2\pi\left(85\times10^{9}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}\approx\boxed{2.3\times10^{-18}\,\text{m}}$$

22. The energy of the two photons (assumed to be equal so that momentum is conserved) must be the combined rest mass energy of the proton and antiproton.

$$2m_0c^2 = 2hf = 2h\frac{c}{\lambda} \;\rightarrow\; \lambda = \frac{hc}{m_0c^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(938.3\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} \approx \boxed{1.32\times10^{-15}\,\text{m}}$$

23. (*a*) $\Lambda^0 \rightarrow \text{n} + \pi^-$ Charge conservation is violated, since $0 \neq 0 - 1$.
Strangeness is violated, since $-1 \neq 0 + 0$

(*b*) $\Lambda^0 \rightarrow \text{p} + \text{K}^-$ Energy conservation is violated, since
$1115.7\,\text{MeV}/c^2 < 938.3\,\text{MeV}/c^2 + 493.7\,\text{MeV}/c^2$

(*c*) $\Lambda^0 \rightarrow \pi^+ + \pi^-$ Baryon number conservation is violated, since $1 \neq 0 + 0$
Strangeness is violated, since $-1 \neq 0 + 0$

24. (*a*) The *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$$Q = m_{\Lambda^0}c^2 - \left(m_\text{p}c^2 + m_{\pi^-}c^2\right) = 1115.7\,\text{MeV} - \left(938.3\,\text{MeV} + 139.6\,\text{MeV}\right) = \boxed{37.8\,\text{MeV}}$$

(*b*) Energy conservation for the decay gives the following.

$$m_{\Lambda^0}c^2 = E_\text{p} + E_{\pi^-} \;\rightarrow\; E_{\pi^-} = m_{\Lambda^0}c^2 - E_\text{p}$$

Momentum conservation says that the magnitudes of the momenta of the two products are equal. Then convert that relationship to energy using $E^2 = p^2c^2 + m_0^2c^4$, with energy conservation.

$$p_\text{p} = p_{\pi^-} \;\rightarrow\; \left(p_\text{p}c\right)^2 = \left(p_{\pi^-}c\right)^2 \;\rightarrow$$

$$E_\text{p}^2 - m_\text{p}^2c^4 = E_{\pi^-}^2 - m_{\pi^-}^2c^4 = \left(m_{\Lambda^0}c^2 - E_\text{p}\right)^2 - m_{\pi^-}^2c^4$$

$$E_\text{p}^2 - m_\text{p}^2c^4 = \left(m_{\Lambda^0}^2c^4 - 2E_\text{p}m_{\Lambda^0}c^2 + E_\text{p}^2\right) - m_{\pi^-}^2c^4 \;\rightarrow$$

$$E_\text{p} = \frac{m_{\Lambda^0}^2c^4 + m_\text{p}^2c^4 - m_{\pi^-}^2c^4}{2m_{\Lambda^0}c^2} = \frac{\left(1115.7\,\text{MeV}\right)^2 + \left(938.3\,\text{MeV}\right)^2 - \left(139.6\,\text{MeV}\right)^2}{2\left(1115.7\,\text{MeV}\right)} = 943.7\,\text{MeV}$$

$$E_{\pi^-} = m_{\Lambda^0}c^2 - E_\text{p} = 1115.7\,\text{MeV} - 943.7\,\text{MeV} = 172.0\,\text{MeV}$$

$$\text{KE}_\text{p} = E_\text{p} - m_\text{p}c^2 = 943.7\,\text{MeV} - 938.3\,\text{MeV} = \boxed{5.4\,\text{MeV}}$$

$$\text{KE}_{\pi^-} = E_{\pi^-} - m_{\pi^-}c^2 = 172.0\,\text{MeV} - 139.6\,\text{MeV} = \boxed{32.4\,\text{MeV}}$$

25. (*a*) We work in the rest frame of the isolated electron, so that it is initially at rest. Energy conservation gives the following.

$$m_\text{e}c^2 = \text{KE}_\text{e} + m_\text{e}c^2 + E_\gamma \;\rightarrow\; \text{KE}_\text{e} = -E_\gamma \;\rightarrow\; \text{KE}_\text{e} = E_\gamma = 0$$

Since the photon has no energy, it does not exist, and so has not been emitted.

(*b*) For the photon exchange in Figure 32-7, the photon exists for such a short time that the uncertainty principle allows energy to not be conserved during the exchange.

$\boxed{26.}$ The total momentum of the electron and positron is 0, and so the total momentum of the two photons must be 0. Thus each photon has the same momentum, and so each photon also has the same energy. The total energy of the photons must be the total energy of the electron / positron pair.

$$E_{e^+/e^-\text{ pair}} = E_{\text{photons}} \rightarrow 2\left(m_0c^2 + \text{KE}\right) = 2hf = 2h\frac{c}{\lambda} \rightarrow$$

$$\lambda = \frac{hc}{m_0c^2 + \text{KE}} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(0.511\times10^{6}\text{eV} + 420\times10^{3}\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} = \boxed{1.34\times10^{-12}\text{m}}$$

27. Since the pion decays from rest, the momentum before the decay is zero. Thus the momentum after the decay is also zero, and so the magnitudes of the momenta of the positron and the neutrino are equal. We also treat the neutrino as massless. Use energy and momentum conservation along with the relativistic relationship between energy and momentum.

$$m_{\pi^+}c^2 = E_{e^+} + E_\nu \quad ; \quad p_{e^+} = p_\nu \rightarrow \left(p_{e^+}^2c^2\right) = \left(p_\nu^2c^2\right) \rightarrow E_{e^+}^2 - m_{e^+}^2c^4 = E_\nu^2$$

$$E_{e^+}^2 - m_{e^+}^2c^4 = \left(m_{\pi^+}c^2 - E_{e^+}\right)^2 = m_{\pi^+}^2c^4 - 2E_{e^+}m_{\pi^+}c^2 + E_{e^+}^2 \rightarrow 2E_{e^+}m_{\pi^+}c^2 = m_{\pi^+}^2c^4 + m_{e^+}^2c^4$$

$$E_{e^+} = \tfrac{1}{2}m_{\pi^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} \rightarrow \text{KE}_{e^+} + m_{e^+}c^2 = \tfrac{1}{2}m_{\pi^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} \rightarrow$$

$$\text{KE}_{e^+} = \tfrac{1}{2}m_{\pi^+}c^2 - m_{e^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} = \tfrac{1}{2}\left(139.6\,\text{MeV}\right) - 0.511\,\text{MeV} + \frac{\left(0.511\,\text{MeV}/c^2\right)\left(0.511\,\text{MeV}\right)}{2\left(139.6\,\text{MeV}/c^2\right)}$$

$$= \boxed{69.3\,\text{MeV}}$$

28. (*a*) For the reaction $\pi^- + \text{p} \rightarrow \text{n} + \eta^0$, the conservation laws are as follows.

Charge: $-1+1=0+0$	Charge is conserved.
Baryon number: $0+1=1+0$	Baryon number is conserved.
Lepton number: $0+0=0+0$	Lepton number is conserved.
Strangeness: $0+0=0+0$	Strangeness is conserved.

$\boxed{\text{The reaction is possible.}}$

(*b*) For the reaction $\pi^+ + \text{p} \rightarrow \text{n} + \pi^0$, the conservation laws are as follows.

Charge: $1+1\neq0+0$	Charge is NOT conserved.

$\boxed{\text{The reaction is forbidden, because charge is not conserved.}}$

(*c*) For the reaction $\pi^+ + \text{p} \rightarrow \text{p} + \text{e}^+$, the conservation laws are as follows.

Charge: $1+1=1+1$	Charge is conserved.
Baryon number: $0+1=1+0$	Baryon number is conserved.
Lepton number: $0+0\neq0+1$	Lepton number is NOT conserved.

$\boxed{\text{The reaction is forbidden, because lepton number is not conserved.}}$

(*d*) For the reaction $\text{p} \rightarrow \text{e}^+ + \nu_\text{e}$, the conservation laws are as follows.

Charge: $1=1+0$	Charge is conserved.
Baryon number: $1\neq0+0$	Baryon number NOT conserved.

$\boxed{\text{The reaction is forbidden, because baryon number is not conserved.}}$

(*e*) For the reaction $\mu^+ \rightarrow \text{e}^+ + \overline{\nu}_\mu$, the conservation laws are as follows.

Charge: $1=1+0$	Charge is conserved.
Baryon number: $0=0+0$	Baryon number is conserved.
Electron lepton number: $0\neq-1+0$	Lepton number is NOT conserved.

$\boxed{\text{The reaction is forbidden, because lepton number is not conserved.}}$

(*f*) For the reaction $\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$, the conservation laws are as follows.

Mass Energy: $938.3\,\mathrm{MeV}/c^2 < 939.6\,\mathrm{MeV}/c^2 + 0.511\,\mathrm{MeV}/c^2$

Mass energy is NOT conserved.

$\boxed{\text{The reaction is forbidden, because energy is not conserved.}}$

29. Since the Ξ^- decays from rest, the momentum before the decay is zero. Thus the momentum after the decay is also zero, and so the momenta of the Λ^0 and π^- are equal in magnitude. Use energy and momentum conservation along with the relativistic relationship between energy and momentum.

$$m_{\Xi^-}c^2 = E_{\Lambda^0} + E_{\pi^-} \rightarrow E_{\pi^-} = m_{\Xi^-}c^2 - E_{\Lambda^0}$$

$$p_{\Lambda^0} = p_{\pi^-} \rightarrow \left(p_{\Lambda^0}c\right)^2 = \left(p_{\pi^-}c\right)^2 \rightarrow$$

$$E_{\Lambda^0}^2 - m_{\Lambda^0}^2c^4 = E_{\pi^-}^2 - m_{\pi^-}^2c^4 = \left(m_{\Xi^-}c^2 - E_{\Lambda^0}\right)^2 - m_{\pi^-}^2c^4$$

$$E_{\Lambda^0}^2 - m_{\Lambda^0}^2c^4 = \left(m_{\Xi^-}^2c^4 - 2E_{\Lambda^0}m_{\Xi^-}c^2 + E_{\Lambda^0}^2\right) - m_{\pi^-}^2c^4 \rightarrow$$

$$E_{\Lambda^0} = \frac{m_{\Xi^-}^2c^4 + m_{\Lambda^0}^2c^4 - m_{\pi^-}^2c^4}{2m_{\Xi^-}c^2} = \frac{(1321.3\,\mathrm{MeV})^2 + (1115.7\,\mathrm{MeV})^2 - (139.6\,\mathrm{MeV})^2}{2(1321.3\,\mathrm{MeV})} = 1124.3\,\mathrm{MeV}$$

$$E_{\pi^-} = m_{\Xi^-}c^2 - E_{\Lambda^0} = 1321.3\,\mathrm{MeV} - 1124.3\,\mathrm{MeV} = 197.0\,\mathrm{MeV}$$

$$\mathrm{KE}_{\Lambda^0} = E_{\Lambda^0} - m_{\Lambda^0}c^2 = 1124.3\,\mathrm{MeV} - 1115.7\,\mathrm{MeV} = \boxed{8.6\,\mathrm{MeV}}$$

$$\mathrm{KE}_{\pi^-} = E_{\pi^-} - m_{\pi^-}c^2 = 197.0\,\mathrm{MeV} - 139.6\,\mathrm{MeV} = \boxed{57.4\,\mathrm{MeV}}$$

30. A π^+ could NOT be produced in the reaction $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{n} + \pi^+$. The pion has a mass energy of 139.6 MeV, and so the extra 100 MeV of energy could not create it. The *Q*-value for the reaction is $Q = 2m_\mathrm{p}c^2 - \left(2m_\mathrm{p}c^2 + m_{\pi^+}c^2\right) = -139.6\,\mathrm{MeV}$, and so more than 139.6 MeV of kinetic energy is needed. The minimum initial kinetic energy would produce the particles all moving together at the same speed, having the same total momentum as the incoming proton. We consider the products to be one mass $M = m_\mathrm{p} + m_\mathrm{n} + m_{\pi^+}$ since they all move together with the velocity. We use energy and momentum conservation, along with their relativistic relationship, $E^2 = p^2c^2 + m_0^2c^4$.

$$E_\mathrm{p} + m_\mathrm{p}c^2 = E_\mathrm{M} \quad ; \quad p_\mathrm{p} = p_\mathrm{M} \rightarrow \left(p_\mathrm{p}c\right)^2 = \left(p_\mathrm{M}c\right)^2 \rightarrow E_\mathrm{p}^2 - m_{\pi^-}^2c^4 = E_M^2 - M^2c^4 \rightarrow$$

$$E_\mathrm{p}^2 - m_\mathrm{p}^2c^4 = \left(E_\mathrm{p} + m_\mathrm{p}c^2\right)^2 - M^2c^4 = E_\mathrm{p}^2 + 2E_\mathrm{p}m_\mathrm{p}c^2 + m_\mathrm{p}^2c^4 - M^2c^4 \rightarrow$$

$$E_\mathrm{p} = \frac{M^2c^4 - 2m_\mathrm{p}^2c^4}{2m_\mathrm{p}c^2} = \mathrm{KE}_\mathrm{p} + m_\mathrm{p}c^2 \rightarrow$$

$$\mathrm{KE}_{\pi^-} = \frac{M^2c^4 - 2m_\mathrm{p}^2c^4}{2m_\mathrm{p}c^2} - m_pc^2 = \frac{M^2c^4}{2m_\mathrm{p}c^2} - 2m_pc^2$$

$$= \frac{(938.3\,\mathrm{MeV} + 939.6\,\mathrm{MeV} + 139.6\,\mathrm{MeV})^2}{2(938.3\,\mathrm{MeV})} - 2(938.3\,\mathrm{MeV}) = \boxed{292.4\,\mathrm{MeV}}$$

31. The two neutrinos must move in the opposite direction of the electron in order for the electron to have the maximum kinetic energy, and thus the total momentum of the neutrinos will be equal in magnitude to the momentum of the electron. Since a neutrino is (essentially) massless, we have that $E_\nu = p_\nu c$. We assume that the muon is at rest when it decays. Use conservation of energy and momentum, along with their relativistic relationship.

$$p_{\mathrm{e}^-} = p_{\bar{\nu}_\mathrm{e}} + p_{\nu_\mu}$$

$$m_{\mu^-}c^2 = E_{\mathrm{e}^-} + E_{\bar{\nu}_\mathrm{e}} + E_{\nu_\mu} = E_{\mathrm{e}^-} + p_{\bar{\nu}_\mathrm{e}}c + p_{\nu_\mu}c = E_{\mathrm{e}^-} + \left(p_{\bar{\nu}_\mathrm{e}} + p_{\nu_\mu}\right)c = E_{\mathrm{e}^-} + p_{\mathrm{e}^-}c \;\rightarrow$$

$$m_{\mu^-}c^2 - E_{\mathrm{e}^-} = p_{\mathrm{e}^-}c \;\rightarrow\; \left(m_{\mu^-}c^2 - E_{\mathrm{e}^-}\right)^2 = \left(p_{\mathrm{e}^-}c\right)^2 = E_{\mathrm{e}^-}^2 - m_{\mathrm{e}^-}^2c^4 \;\rightarrow$$

$$m_{\mu^-}^2c^4 - 2m_{\mu^-}c^2E_{\mathrm{e}^-} + E_{\mathrm{e}^-}^2 = E_{\mathrm{e}^-}^2 - m_{\mathrm{e}^-}^2c^4 \;\rightarrow\; E_{\mathrm{e}^-} = \frac{m_{\mu^-}^2c^4 + m_{\mathrm{e}^-}^2c^4}{2m_{\mu^-}c^2} = \mathrm{KE}_{\mathrm{e}^-} + m_{\mathrm{e}^-}c^2 \;\rightarrow$$

$$\mathrm{KE}_{\mathrm{e}^-} = \frac{m_{\mu^-}^2c^4 + m_{\mathrm{e}^-}^2c^4}{2m_{\mu^-}c^2} - m_{\mathrm{e}^-}c^2 = \frac{(105.7\,\mathrm{MeV})^2 + (0.511\,\mathrm{MeV})^2}{2(105.7\,\mathrm{MeV})} - (0.511\,\mathrm{MeV}) = \boxed{52.3\,\mathrm{MeV}}$$

32. The width of the peak was measured with a ruler and found to be about 9.5 mm, while the 200 MeV spacing on the graph was measured to be about 14 mm. Use a proportion to estimate the energy width, and then use the uncertainty principle to estimate the lifetime.

$$\frac{\Delta E}{200\,\mathrm{MeV}} = \frac{9.5\,\mathrm{mm}}{14\,\mathrm{mm}} \;\rightarrow\; \Delta E = (200\,\mathrm{MeV})\left(\frac{9.5\,\mathrm{mm}}{14\,\mathrm{mm}}\right) = \boxed{136\,\mathrm{MeV}}$$

$$\Delta E \approx \frac{h}{2\pi\Delta t} \;\rightarrow\; \Delta t \approx \frac{h}{2\pi\Delta E} = \frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{2\pi\left(136\times10^{6}\,\mathrm{eV}\right)\left(\dfrac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)} = \boxed{4.85\times10^{-24}\,\mathrm{s}}$$

33. Apply the uncertainty principle, which says that $\Delta E \approx \dfrac{h}{2\pi\Delta t}$.

$$\Delta E \approx \frac{h}{2\pi\Delta t} \;\rightarrow\; \Delta t \approx \frac{h}{2\pi\Delta E} = \frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{2\pi\left(88\times10^{3}\,\mathrm{eV}\right)\left(\dfrac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)} = \boxed{7.5\times10^{-21}\,\mathrm{s}}$$

34. We estimate the lifetime from the energy width and the uncertainty principle.

$$\Delta E \approx \frac{h}{2\pi\Delta t} \;\rightarrow\; \Delta t \approx \frac{h}{2\pi\Delta E} = \frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{2\pi\left(277\times10^{3}\,\mathrm{eV}\right)\left(\dfrac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)} = \boxed{2.38\times10^{-21}\,\mathrm{s}}$$

35. We find the energy width from the lifetime on Table 32-2 and the uncertainty principle.

(*a*) $$\Delta t = 5\times10^{-19}\,\mathrm{s} \quad \Delta E \approx \frac{h}{2\pi\Delta t} = \frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{2\pi\left(5\times10^{-19}\,\mathrm{s}\right)\left(\dfrac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)} = 1319\,\mathrm{eV} \approx \boxed{1.3\,\mathrm{keV}}$$

(*b*) $\Delta t = 7.4\times10^{-20}\,\text{s} \quad \Delta E \approx \dfrac{h}{2\pi\Delta t} = \dfrac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{2\pi\left(7.4\times10^{-20}\,\text{s}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 8912\,\text{eV} \approx \boxed{8.9\,\text{keV}}$

36. (*a*) For $\text{B}^- = \text{b}\,\overline{\text{u}}$, we have

Charge: $-1 = -\frac{1}{3} - \frac{2}{3}$

Spin: $0 = \frac{1}{2} - \frac{1}{2}$

Baryon number: $0 = \frac{1}{3} - \frac{1}{3}$

Strangeness: $0 = 0 + 0$

Charm: $0 = 0 + 0$

Bottomness: $-1 = -1 + 0$

Topness: $0 = 0 + 0$

(*b*) Because B^+ is the antiparticle of B^-, $\boxed{\text{B}^+ = \overline{\text{b}}\,\text{u}}$. The B^0 still must have a bottom quark, but must be neutral. Therefore $\boxed{\text{B}^0 = \text{b}\,\overline{\text{d}}}$. Because $\overline{\text{B}}^0$ is the antiparticle to B^0, we must have $\boxed{\overline{\text{B}}^0 = \overline{\text{b}}\,\text{d}}$.

$\boxed{37.}$ (*a*) Charge: $(0) = (+1) + (-1)$ Charge is conserved.

Energy: $1314.9\,\text{Mev}/c^2 > 1189.4\,\text{Mev}/c^2 + 139.6\,\text{Mev}/c^2 = 1329\,\text{Mev}/c^2$

Energy is not conserved.

$\boxed{\text{The decay is not possible, because energy is not conserved.}}$

(*b*) Charge: $(-1) = (0) + (-1) + (0)$ Charge is conserved.

Energy: $1672.5\,\text{Mev}/c^2 > 1192.6\,\text{Mev}/c^2 + 139.6\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 = 1332.2\,\text{Mev}/c^2$

Energy is conserved.

Lepton number: $(0) = (0) + (0) + (1)$ Lepton number is not conserved.

$\boxed{\text{The decay is not possible, because lepton number is not conserved.}}$

(*c*) Charge: $(0) = (0) + (0) + (0)$ Charge is conserved.

Energy: $1192.6\,\text{Mev}/c^2 > 1115.7\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 = 1115.7\,\text{Mev}/c^2$

Energy is conserved.

Lepton number: $(0) = (0) + (0) + (0)$ Lepton number is conserved.

Baryon number: $(0) = (0) + (0) + (0)$ Baryon number is conserved.

$\boxed{\text{The decay is possible.}}$

38. (*a*) The neutron has a baryon number of 1, so there must be three quarks. The charge must be 0, as must be the strangeness, the charm, the bottomness, and the topness. Thus $\boxed{n = \text{u}\,\text{u}\,\text{d}}$.

(*b*) The antineutron is the anti particle of the neutron, so $\boxed{\overline{n} = \overline{\text{u}}\,\overline{\text{u}}\,\overline{\text{d}}}$.

(*c*) The Λ^0 has a strangeness of -1, so it must contain an "s" quark. It is a baryon, so it must contain three quarks. And it must have charge, charm, bottomness, and topness equal to 0. Thus $\boxed{\Lambda^0 = \text{u}\,\text{d}\,\text{s}}$.

(*d*) The $\overline{\Sigma}^0$ has a strangeness of +1, so it must contain an $\overline{\mathrm{s}}$ quark. It is a baryon, so it must contain three quarks. And it must have charge, charm, bottomness, and topness equal to 0. Thus $\boxed{\overline{\Sigma}^0 = \overline{\mathrm{u}}\,\overline{\mathrm{d}}\,\overline{\mathrm{s}}}$.

39. (*a*) The combination $\mathrm{u\,u\,d}$ has charge = +1, baryon number = +1, and strangeness, charm, bottomness, and topness all equal to 0. Thus $\boxed{\mathrm{u\,u\,d = p}}$.

(*b*) The combination $\overline{\mathrm{u}}\,\overline{\mathrm{u}}\,\overline{\mathrm{s}}$ has charge = -1, baryon number = -1, strangenss = +1, and charm, bottomness, and topness all equal to 0. Thus $\boxed{\overline{\mathrm{u}}\,\overline{\mathrm{u}}\,\overline{\mathrm{s}} = \overline{\Sigma}^-}$.

(*c*) The combination $\overline{\mathrm{u}}\,\mathrm{s}$ has charge = -1, baryon number = 0, strangeness = -1, and charm, bottomness, and topness all equal to 0. Thus $\boxed{\overline{\mathrm{u}}\,\mathrm{s} = \mathrm{K}^-}$.

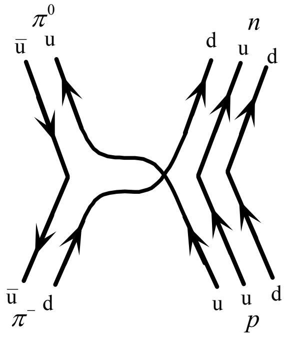
(*d*) The combination $\mathrm{d}\,\overline{\mathrm{u}}$ has charge = -1, baryon number = 0, and strangeness, charm, bottomness, and topness all equal to 0. Thus $\boxed{\mathrm{d}\,\overline{\mathrm{u}} = \pi^-}$

(*e*) The combination $\overline{\mathrm{c}}\,\mathrm{s}$ has charge = -1, baryon number = 0, strangeness = -1, charm = -1, and bottomness and topness of 0. Thus $\boxed{\overline{\mathrm{c}}\,\mathrm{s} = \mathrm{D}_S^-}$

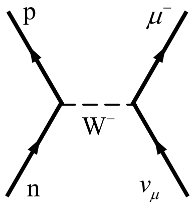
40. To form the D^0 meson, we must have a total charge of 0, a baryon number of 0, a strangeness of 0, and a charm of +1. We assume that there is no topness or bottomness. To get the charm, we must have a "c" quark, with a charge of $+\frac{2}{3}e$. To have a neutral meson, there must be another quark with a charge of $-\frac{2}{3}e$. To have a baryon number of 0, that second quark must be an antiquark. The only candidate with those properties is an anti-up quark. Thus $\boxed{D^0 = \mathrm{c}\,\overline{\mathrm{u}}}$.

41. To form the D_S^+ meson, we must have a total charge of +1, a baryon number of 0, a strangeness of +1, and a charm of +1. We assume that there is no topness or bottomness. To get the charm, we must have a "c" quark, with a charge of $+\frac{2}{3}e$. To have a total charge of +1, there must be another quark with a charge of $+\frac{1}{3}e$. To have a baryon number of 0, that second quark must be an antiquark. To have a strangeness of +1, the other quark must be an anti-strange. Thus $\boxed{D_S^+ = \mathrm{c}\,\overline{\mathrm{s}}}$.

42.



43. Since leptons are involved, the reaction $\mathrm{n}+\nu_{\mu} \rightarrow \mathrm{p}+\mu^{-}$ is a weak interaction. Since there is a charge change in the lepton, a W boson must be involved in the interaction.



44. The total energy is the sum of the kinetic energy and the mass energy. The wavelength is found from the relativistic momentum.

$$E=\mathrm{KE}+m_0c^2=25\times10^9\,\mathrm{eV}+938\times10^6\,\mathrm{eV}=2.59\times10^{10}\,\mathrm{eV}\approx\boxed{26\,\mathrm{GeV}}$$

$$\lambda=\frac{h}{p}=\frac{h}{\frac{\sqrt{E^2-\left(m_0c^2\right)^2}}{c}}=\frac{hc}{\sqrt{E^2-\left(m_0c^2\right)^2}}$$

$$=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)\left(3.00\times10^8\,\mathrm{m/s}\right)}{\sqrt{\left(2.59\times10^{10}\,\mathrm{eV}\right)^2-\left(938\times10^6\,\mathrm{eV}\right)^2}}\frac{1}{\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}=\boxed{4.8\times10^{-17}\,\mathrm{m}}$$

45. (*a*) At an energy of 1.0 TeV, the protons are moving at practically the speed of light. From uniform circular motion we find the time for the protons to complete one revolution of the ring. Then the total charge that passes any point in the ring during that time is the charge of the entire group of stored protons. The current is then the total charge divided by the period.

$$v=\frac{2\pi R}{T}\rightarrow T=\frac{2\pi R}{v}=\frac{2\pi R}{c}$$

$$I=\frac{Ne}{T}=\frac{Nec}{2\pi R}=\frac{\left(5.0\times10^{13}\,\mathrm{protons}\right)\left(1.60\times10^{-19}\,\mathrm{C/proton}\right)\left(3.0\times10^8\,\mathrm{m/s}\right)}{2\pi\left(1.0\times10^3\,\mathrm{m}\right)}=\boxed{0.38\,\mathrm{A}}$$

(*b*) The 1.0 TeV is equal to the KE of the proton beam.

$$\mathrm{KE}_{\mathrm{beam}}=\mathrm{KE}_{\mathrm{car}}\rightarrow \mathrm{KE}_{\mathrm{beam}}=\tfrac{1}{2}mv^2\rightarrow$$

$$v=\sqrt{\frac{2\mathrm{KE}_{\mathrm{beam}}}{m}}=\sqrt{\frac{2\left(1.0\times10^{12}\,\mathrm{eV/proton}\right)\left(5.0\times10^{13}\,\mathrm{protons}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{1500\,\mathrm{kg}}}=\boxed{103\,\mathrm{m/s}}$$

46. These protons will be moving at essentially the speed of light for the entire time of acceleration. The number of revolutions is the total gain in energy divided by the energy gain per revolution. Then the distance is the number of revolutions times the circumference of the ring, and the time is the distance of travel divided by the speed of the protons.

$$\mathrm{N}=\frac{\Delta E}{\Delta E/\mathrm{rev}}=\frac{\left(1.0\times10^{12}\,\mathrm{eV}-150\times10^9\,\mathrm{eV}\right)}{2.5\times10^6\,\mathrm{eV/rev}}=3.4\times10^5\,\mathrm{rev}$$

$$d=N\left(2\pi R\right)=\left(3.4\times10^5\right)2\pi\left(1.0\times10^3\,\mathrm{m}\right)=2.136\times10^9\,\mathrm{m}\approx\boxed{2.1\times10^9\,\mathrm{m}}$$

$$t=\frac{d}{c}=\frac{2.136\times10^9\,\mathrm{m}}{3.00\times10^8\,\mathrm{m/s}}=\boxed{7.1\,\mathrm{s}}$$

47. By assuming that the kinetic energy is approximately 0, the total energy released is the rest mass energy of the annihilating pair of particles.

(*a*) $E_{\text{total}} = 2m_0c^2 = 2(0.511\,\text{MeV}) = \boxed{1.022\,\text{MeV}}$

(*b*) $E_{\text{total}} = 2m_0c^2 = 2(938.3\,\text{MeV}) = \boxed{1876.6\,\text{MeV}}$

48. (*a*) For the reaction $\pi^- + \text{p} \rightarrow \text{K}^+ + \Sigma^-$, the conservation laws are as follows.

Charge: $-1+1=1-1$ Charge is conserved.
Baryon number: $0+1=0+1$ Baryon number is conserved.
Lepton number: $0+0=0+0$ Lepton number is conserved.
Strangeness: $0+0=1-1$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*b*) For the reaction $\pi^+ + \text{p} \rightarrow \text{K}^+ + \Sigma^+$, the conservation laws are as follows.

Charge: $1+1=1+1$ Charge is conserved.
Baryon number: $0+1=0+1$ Baryon number is conserved.
Lepton number: $0+0=0+0$ Lepton number is conserved.
Strangeness: $0+0=1-1$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*c*) For the reaction $\pi^- + \text{p} \rightarrow \Lambda^0 + \text{K}^0 + \pi^0$, the conservation laws are as follows.

Charge: $-1+1=0+0+0$ Charge is conserved.
Baryon number: $0+1=1+0+0$ Baryon number is conserved.
Lepton number: $0+0=0+0+0$ Lepton number is conserved.
Strangeness: $0+0=-1+1+0$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*d*) For the reaction $\pi^+ + \text{p} \rightarrow \Sigma^0 + \pi^0$, the conservation laws are as follows.

Charge: $1+1 \neq 0+0$ Charge is NOT conserved.

The reaction is not possible, because charge is not conserved.

(*e*) For the reaction $\pi^- + \text{p} \rightarrow \text{p} + \text{e}^- + \overline{\nu}_{\text{e}}$, the conservation laws are as follows.

Charge: $-1+1=1-1+0$ Charge is conserved.
Baryon number: $0+1=1+0+0$ Baryon number is conserved.
Lepton number: $0+0=0+1-1$ Lepton number is conserved.
Strangeness: $0+0=0+0+0$ Strangeness is conserved.

The reaction is possible, via the weak interaction.

49. (*a*) For the reaction $\pi^- + \text{p} \rightarrow \text{K}^0 + \text{p} + \pi^0$, the conservation laws are as follows.

Charge: $-1+1 \neq 0+1+0$ Charge is NOT conserved.

The reaction is not possible, because charge is not conserved.

(*b*) For the reaction $\text{K}^- + \text{p} \rightarrow \Lambda^0 + \pi^0$, the conservation laws are as follows.

Charge: $-1+1=0+0$ Charge is conserved.
Spin: $0+\frac{1}{2}=\frac{1}{2}+0$ Spin is conserved.
Baryon number: $0+1=1+0$ Baryon number is conserved.
Lepton number: $0+0=0+0$ Lepton number is conserved.

Strangeness: $-1+0=-1+0$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*c*) For the reaction $\mathrm{K}^+ + \mathrm{n} \to \Sigma^+ + \pi^0 + \gamma$, the conservation laws are as follows.

Charge: $1+0=1+0+0$ Charge is conserved.
Spin: $0+\frac{1}{2}=-\frac{1}{2}+0+1$ Spin is conserved.
Baryon number: $0+1=1+0+0$ Baryon number is conserved.
Lepton number: $0+0=0+0+0$ Lepton number is conserved.
Strangeness: $1+0\neq-1+0+0$ Strangeness is NOT conserved.

The reaction is not possible via the strong interaction because strangeness is not conserved. It is possible via the weak interaction.

(*d*) For the reaction $\mathrm{K}^+ \to \pi^0 + \pi^0 + \pi^+$, the conservation laws are as follows.

Charge: $1=0+0+1$ Charge is conserved.
Spin: $0=0+0+0$ Spin is conserved.
Baryon number: $0=0+0+0$ Baryon number is conserved.
Lepton number: $0=0+0+0$ Lepton number is conserved.
Strangeness: $1\neq0+0+0$ Strangeness is NOT conserved.

The reaction is not possible via the strong interaction because strangeness is not conserved. It is possible via the weak interaction.

(*e*) For the reaction $\pi^+ \to \mathrm{e}^+ + \nu_\mathrm{e}$, the conservation laws are as follows.

Charge: $1=1+0$ Charge is conserved.
Spin: $0=-\frac{1}{2}+\frac{1}{2}$ Spin is conserved.
Baryon number: $0=0+0$ Baryon number is conserved.
Lepton number: $0=-1+1$ Lepton number is conserved.
Strangeness: $0+0=0+0+0$ Strangeness is conserved.

The reaction is possible, via the weak interaction.

50. The π^- is the anti-particle of the π^+, so the reaction is $\boxed{\pi^- \to \mu^- + \bar{\nu}_\mu}$. The conservation rules are as follows.

Charge: $-1=-1+0$ Charge is conserved.
Baryon number: $0=0+0$ Baryon number is conserved.
Lepton number: $0=1-1$ Lepton number is conserved.
Strangeness: $0=0+0$ Strangeness is conserved.
Spin: $0=\frac{1}{2}-\frac{1}{2}$ Spin is conserved

51. Use Eq. 32-3 to estimate the mass of the particle based on the given distance.

$$mc^2 \approx \frac{hc}{2\pi d} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{2\pi\left(10^{-18}\,\mathrm{m}\right)}\left(\frac{1}{1.60\times10^{-19}\,\mathrm{J/eV}}\right) = 1.98\times10^{11}\,\mathrm{eV} \approx \boxed{200\,\mathrm{GeV}}$$

This value is of the same order of magnitude as the mass of the $\mathrm{W}^{\pm}$.

52. We assume that the interaction happens essentially at rest, so that there is no initial kinetic energy or momentum. Thus the momentum of the neutron and the momentum of the π^0 will have the same magnitude. From energy conservation we find the total energy of the π^0.

$$m_{\pi^-}c^2 + m_pc^2 = E_{\pi^0} + m_nc^2 + \mathrm{KE}_n \quad \rightarrow$$

$$E_{\pi^0} = m_{\pi^-}c^2 + m_pc^2 - \left(m_nc^2 + \mathrm{KE}_n\right) = 139.6\,\mathrm{MeV} + 938.3\,\mathrm{MeV} - \left(939.6\,\mathrm{MeV} + 0.60\,\mathrm{MeV}\right)$$

$$= 137.7\,\mathrm{MeV}$$

From momentum conservation, we can find the mass energy of the π^0. We utilize Eq. 26-10 to relate momentum and energy.

$$p_n = p_{\pi^0} \rightarrow \left(p_nc\right)^2 = \left(p_{\pi^0}c\right)^2 \rightarrow E_n^2 - m_n^2c^4 = E_{\pi^0}^2 - m_{\pi^0}^2c^4 \rightarrow m_{\pi^0}^2c^4 = E_{\pi^0}^2 - E_n^2 + m_n^2c^4 \rightarrow$$

$$m_{\pi^0}c^2 = \sqrt{E_{\pi^0}^2 - E_n^2 + m_n^2c^4} = \left[\left(137.7\,\mathrm{MeV}\right)^2 - \left(939.6\,\mathrm{MeV} + 0.60\,\mathrm{MeV}\right)^2 + \left(939.6\,\mathrm{MeV}\right)^2\right]^{1/2}$$

$$= 133.5\,\mathrm{MeV} \rightarrow m_{\pi^0} = \boxed{133.5\,\mathrm{MeV}/c^2}$$

The reference value is $133.5\,\mathrm{MeV}$.

53. The *Q*-value is the mass energy of the reactants minus the mass energy of the products. For the first reaction, $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p} + \pi^0$:

$$Q = 2m_pc^2 - \left(2m_pc^2 + m_{\pi^0}c^2\right) = -m_{\pi^0}c^2 = \boxed{-135.0\,\mathrm{MeV}}$$

For the second reaction, $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{n} + \pi^+$:

$$Q = 2m_pc^2 - \left(m_pc^2 + m_nc^2 + m_{\pi^+}c^2\right) = m_pc^2 - m_nc^2 - m_{\pi^+}c^2a$$

$$= 938.3\,\mathrm{MeV} - 939.6\,\mathrm{MeV} - 139.6\,\mathrm{MeV} = \boxed{-140.9\,\mathrm{MeV}}$$

54. The *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$$Q = m_{\pi^-}c^2 + m_pc^2 - \left(m_{\Lambda^0}c^2 + m_{K^0}c^2\right) = 139.6\,\mathrm{MeV} + 938.3\,\mathrm{MeV} - \left(1115.7\,\mathrm{MeV} + 497.7\,\mathrm{MeV}\right)$$

$$= \boxed{-535.5\,\mathrm{MeV}}$$

We consider the products to be one mass $M = m_{\Lambda^0} + m_{K^0} = 1613.4\,\mathrm{MeV}/c^2$ since they both have the same speed. Energy conservation gives the following: $E_{\pi^-} + m_pc^2 = E_M$. Momentum conservation says that the incoming momentum is equal to the outgoing momentum. Then convert that relationship to energy using the relativistic relationship that $E^2 = p^2c^2 + m_0^2c^4$.

$$p_{\pi^-} = p_M \rightarrow \left(p_{\pi^-}c\right)^2 = \left(p_Mc\right)^2 \rightarrow E_{\pi^-}^2 - m_{\pi^-}^2c^4 = E_M^2 - M^2c^4 \rightarrow$$

$$E_{\pi^-}^2 - m_{\pi^-}^2c^4 = \left(E_{\pi^-} + m_pc^2\right)^2 - M^2c^4 = E_{\pi^-}^2 + 2E_{\pi^-}m_pc^2 + m_p^2c^4 - M^2c^4 \rightarrow$$

$$E_{\pi^-} = \frac{M^2c^4 - m_{\pi^-}^2c^4 - m_p^2c^4}{2m_pc^2} = \mathrm{KE}_{\pi^-} + m_{\pi^-}c^2 \rightarrow$$

$$\mathrm{KE}_{\pi^-} = \frac{M^2c^4 - m_{\pi^-}^2c^4 - m_p^2c^4}{2m_pc^2} - m_{\pi^-}c^2$$

$$= \frac{\left(1613.4\,\mathrm{MeV}\right)^2 - \left(139.6\,\mathrm{MeV}\right)^2 - \left(938.3\,\mathrm{MeV}\right)^2}{2\left(938.3\,\mathrm{MeV}\right)} - \left(139.6\,\mathrm{MeV}\right) = \boxed{768.0\,\mathrm{MeV}}$$

55. The fundamental fermions are the quarks and electrons. In a water molecule there are 2 hydrogen atoms consisting of one electron and one proton each, and 1 oxygen atom, consisting of 8 electrons, 8 protons, and 8 neutrons. Thus there are 18 nucleons, consisting of 3 quarks each, and 10 electrons. The total number of fermions is thus $18\times3+10=\boxed{64\text{ fermions}}$.

56. Since there is no initial momentum, the final momentum must add to zero. Thus each of the pions must have the same momentum, and therefore the same kinetic energy. Use energy conservation to find the kinetic energy of each pion.

$$2m_\text{p}c^2=2\text{KE}_\pi+2m_\pi c^2 \quad\to\quad \text{KE}_\pi=m_\text{p}c^2-m_\pi c^2=938.3\,\text{MeV}-139.6\,\text{MeV}=\boxed{798.7\,\text{MeV}}$$

57. (*a*) First, from the uncertainty principle, Eq. 28-1. The energy is so high that we assume $E=pc$.

$$\Delta x\Delta p\approx\frac{h}{2\pi} \quad\to\quad \Delta x\frac{\Delta E}{c}\approx\frac{h}{2\pi} \quad\to$$

$$\Delta E\approx\frac{hc}{2\pi\Delta x}=\frac{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{2\pi\left(10^{-32}\,\text{m}\right)}\frac{\left(1\,\text{GeV}/10^{9}\,\text{eV}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}=\boxed{2.0\times10^{16}\,\text{GeV}}$$

Second, from de Broglie's wavelength formula. We take the de Broglie wavelength as the unification distance.

$$\lambda=\frac{h}{p}=\frac{h}{Ec} \quad\to$$

$$E=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(10^{-32}\,\text{m}\right)}\frac{\left(1\,\text{GeV}/10^{9}\,\text{eV}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}=\boxed{3.1\times10^{15}\,\text{GeV}}$$

Both energies are reasonably close to $10^{16}\,\text{GeV}$. This energy is the amount that could be violated in conservation of energy if the universe were the size of the unification distance.

(*b*) From Eq. 13-8, we have $E=\frac{3}{2}kT$.

$$E=\tfrac{3}{2}kT \quad\to\quad T=\frac{2E}{3k}=\frac{2\left(10^{25}\,\text{eV}\right)\left(1.6\times10^{-19}\,\text{J/eV}\right)}{3\left(1.38\times10^{-23}\,\text{J/K}\right)}=7.7\times10^{28}\,\text{K}\cong\boxed{10^{29}\,\text{K}}$$

58. The *Q*-value is the energy of the reactants minus the energy of the products. We assume that one of the initial protons is at rest, and that all four final particles have the same speed and therefore the same kinetic energy, since they all have the same mass. We consider the products to be one mass $M=4m_\text{p}$ since they all have the same speed.

$$Q=2m_\text{p}c^2-4m_\text{p}c^2=2m_\text{p}c^2-Mc^2=-2m_\text{p}c^2$$

Energy conservation gives the following, where KE_th is the threshold energy.

$$\left(\text{KE}_\text{th}+m_\text{p}c^2\right)+m_\text{p}c^2=E_M=\text{KE}_M+Mc^2$$

Momentum conservation says that the incoming momentum is equal to the outgoing momentum. Then convert that relationship to energy using the relativistic relationship that $E^2=p^2c^2+m_0^2c^4$.

$$p_\text{p}=p_\text{M} \quad\to\quad \left(p_\text{p}c\right)^2=\left(p_\text{M}c\right)^2 \quad\to\quad \left(\text{KE}_\text{th}+m_\text{p}c^2\right)^2-m_\text{p}^2c^4=\left(\text{KE}_M+Mc^2\right)^2-M^2c^4 \quad\to$$

$$\text{KE}_\text{th}^2+2\text{KE}_\text{th}m_\text{p}c^2+m_\text{p}^2c^4-m_\text{p}^2c^4=\text{KE}_\text{th}^2+4\text{KE}_\text{th}m_\text{p}c^2+4m_\text{p}^2c^4-\left(4m_\text{p}\right)^2c^4 \quad\to$$

$$2\mathrm{KE}_{\mathrm{th}} m_{\mathrm{p}} c^2 = 4\mathrm{KE}_{\mathrm{th}} m_{\mathrm{p}} c^2 + 4m_{\mathrm{p}}^2 c^4 - 16 m_{\mathrm{p}}^2 c^4 \rightarrow 2\mathrm{KE}_{\mathrm{th}} m_{\mathrm{p}} c^2 = 12 m_{\mathrm{p}}^2 c^4 \rightarrow$$

$$\mathrm{KE}_{\mathrm{th}} = 6 m_{\mathrm{p}} c^2 = 3\left|Q\right|$$

59. To find the length in the lab, we need to know the speed of the particle which is moving relativistically. Start with Eq. 26-6.

$$\mathrm{KE} = m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \rightarrow v = c\sqrt{1 - \frac{1}{\left(\frac{\mathrm{KE}}{m_0 c^2} + 1 \right)^2}} = c\sqrt{1 - \frac{1}{\left(\frac{450 \times 10^6\,\mathrm{eV}}{1777 \times 10^6\,\mathrm{eV}} + 1 \right)^2}} = 0.603c$$

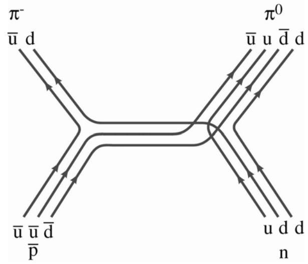
$$\Delta t_{\mathrm{lab}} = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{2.91 \times 10^{-13}\,\mathrm{s}}{\sqrt{1 - (0.603)^2}} = 3.65 \times 10^{-13}\,\mathrm{s}$$

$$\Delta x_{\mathrm{lab}} = v \Delta t_{\mathrm{lab}} = (0.603)\left(3.00 \times 10^8\,\mathrm{m/s}\right)\left(3.65 \times 10^{-13}\,\mathrm{s}\right) = \boxed{6.60 \times 10^{-5}\,\mathrm{m}}$$

60. (*a*) To conserve charge, the missing particle must be neutral. To conserve baryon number, the missing particle must be a meson. To conserve strangeness, charm, topness, and bottomness, the missing particle must be made of up and down quarks and antiquarks only. With all this information, the missing particle is $\boxed{\pi^0 = \mathrm{u}\,\overline{\mathrm{u}} + \mathrm{d}\,\overline{\mathrm{d}}}$.

(*b*) This is a weak interaction since one product is a lepton. To conserve charge, the missing particle must be neutral. To conserve the muon lepton number, the missing particle must be an antiparticle in the muon family. With this information, the missing particle is $\boxed{\overline{\nu}_\mu}$.

61. One $\overline{\mathrm{u}}$ quark from the antiproton joins with a d quark from the neutron to make the π^-. The other four quarks form the π^0.



62. A relationship between total energy and speed is given by Eq. 26-8.

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \rightarrow \sqrt{1 - v^2/c^2} = \frac{m_0 c^2}{E} \rightarrow 1 - \frac{v^2}{c^2} = \left(\frac{m_0 c^2}{E} \right)^2 \rightarrow$$

$$\frac{v}{c} = \sqrt{1 - \left(\frac{m_0 c^2}{E} \right)^2} = \sqrt{1 - \left(\frac{9.38 \times 10^8\,\mathrm{eV}}{7.0 \times 10^{12}\,\mathrm{eV}} \right)^2} = 1 \text{ (to 8 digits)} \rightarrow \boxed{v = c}$$

Use the binomial expansion to express the answer differently.

$$\frac{v}{c} = \sqrt{1 - \left(\frac{9.38 \times 10^8\,\mathrm{eV}}{7.0 \times 10^{12}\,\mathrm{eV}} \right)^2} \cong 1 - \tfrac{1}{2}\left(\frac{9.38 \times 10^8\,\mathrm{eV}}{7.0 \times 10^{12}\,\mathrm{eV}} \right)^2 = 1 - 9.0 \times 10^9 \rightarrow v = c\left(1 - 9.0 \times 10^9\right)$$

CHAPTER 33: Astrophysics and Cosmology

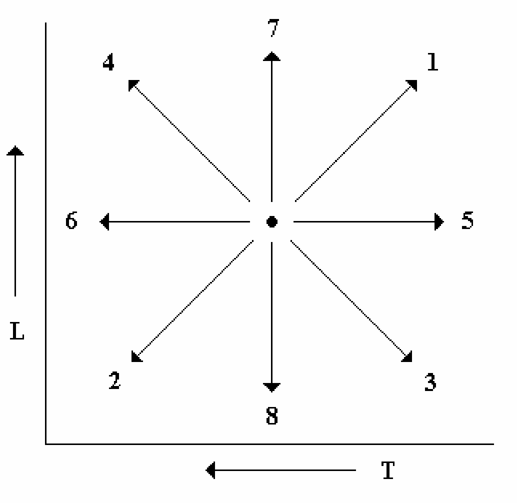
Answers to Questions

1. Long ago, without telescopes, it was difficult to see the individual stars in the Milky Way. The stars in this region of the sky were so numerous and so close together and so tiny that they all blended together to form a cloudy or milky stripe across the night sky. Now using more powerful telescopes, we can see the individual stars that make up the Milky Way galaxy.

2. When a star generates more energy than it radiates, its temperature increases, which produces a greater outward pressure that overcomes the inward gravitational forces and the star increases in size. When a star generates less energy than it radiates, its temperature decreases, which produces a smaller outward pressure and the star decreases in size as the inward gravitational forces start to overcome the outward radiation pressure.

3. A red giant star is extremely large in size, it has a very high luminosity, but it has a relatively cool surface temperature that causes it to be red in color.

4. See the H-R diagram:
    (1) Star is getting cooler and redder, it is increasing in size and mass (higher luminosity).
    (2) Star is getting hotter and whiter, it is decreasing in size and mass (lower luminosity).
    (3) Star is getting cooler and redder, it is decreasing in size and mass (lower luminosity).
    (4) Star is getting hotter and whiter, it is increasing in size and mass (higher luminosity).
    (5) Star is getting cooler and redder, it is increasing in size, but has same mass and luminosity.
    (6) Star is getting hotter and whiter, it is decreasing in size, but has same mass and luminosity.
    (7) Star has same temperature and color, but it is increasing in size and mass (higher luminosity).
    (8) Star has same temperature and color, but it is decreasing in size and mass (lower luminosity).



5. Although the H-R diagram only directly relates the surface temperature of a star to its absolute luminosity (and thus doesn't directly reveal anything about the core), the H-R diagram does provide clues regarding what is happening at the core of a star. Using the current model of stellar evolution and the H-R diagram, we can infer that the stars on the main sequence are fusing hydrogen nuclei to helium nuclei at the core and that stars in the red giant region are fusing helium and beryllium to make heavier nuclei such as carbon and that this red giant process will continue until fusion can no longer occur and the star will collapse.

6. The initial mass of a star determines its final destiny. If, after the red giant stage of a star's life, its mass is less than 1.4 solar masses, then the star cools as it shrinks and it becomes a white dwarf. If its mass is between 1.4 and 2-3 solar masses, then the star will condense down to a neutron star, which will eventually explode as a supernova and become a white dwarf. If its mass is greater than

2-3 solar masses, then the star will collapse even more than the neutron star and it will form a black hole.

7. Yes. Since the population of hotter stars on the H-R diagram is relatively low compared to other regions of the diagram, it seems to imply that these hotter stars stay on the main sequence for a shorter period of time and, thus, have shorter lives. (This also makes sense from a fuel standpoint, since these hotter stars will "burn" up their fuel more quickly.)

8. When measuring star parallaxes from the Moon, there are two different cases: (1) If you did the measurements two weeks apart (one at full moon and one at new moon), you would need to assume that the Earth did not move around the Sun very far, and then the *d* shown in Figure 33-12 would be the Earth-Moon distance instead of the Sun-Earth distance. (2) If you did the measurements six months apart and at full moon, then the *d* shown in Figure 33-12 would be the Sun-Earth distance plus the Earth-Moon distance instead of just the Sun-Earth distance. From Mars, then the *d* shown in Figure 33-12 would be the Sun-Mars distance instead of the Sun-Earth distance. You would also need to know the length of a Mars "year" so you could take your two measurements at the correct times.

9. Measure the period of the changing luminosity of a *Cepheid variable*, and then use the definite relationship between the period and absolute luminosity to determine the absolute luminosity (L) of the star. Then use $\ell = L/\left(4\pi d^2\right)$ to find the relationship between the apparent brightness $\left(\ell\right)$ and the distance to the star $\left(d\right)$. Once the apparent brightness is experimentally determined, we can then find the distance to the star.

10. A geodesic is the shortest distance between two points on a surface. Its role in General Relativity is to help us explain curved space-time. Light always travels by the shortest and most direct path between two points (a geodesic) and if this path is curved it means that space itself is curved. Seeing gravity as the cause of this curvature is a fundamental idea of General Relativity.

11. If the redshift was due to something besides expansion, then the Big Bang theory and the view of an expanding universe would be called into question, since the redshift is a major piece of evidence for the expanding universe. Also, the cosmic microwave background data would be conflicting evidence to this new data. Cosmic microwave background is seen as a confirmation of an expanding universe. Thus if there were some other explanation for the redshift, there might be two distinct schools of thought based on these two conflicting pieces of evidence.

12. No, just because everything appears to be moving away from us does not mean we are at the center of the universe. Here is an analogy. If we were sitting on the surface of a balloon and then more air was put into the balloon causing it to expand, notice that every other point on the balloon is now farther away from you. The points close to you are farther away because of the expansion of the rubber and the points on the other side of the balloon are farther away from you because the radius of the balloon is now larger.

$\boxed{13.}$ If you were located in a galaxy near the boundary of our observable universe, galaxies in the direction of the Milky Way would be receding from you. The outer "edges" of the observable universe are expanding at a faster rate than the points more "interior", thus, due to a relative velocity argument, the slower galaxies in the direction of the Milky Way would look like they are receding from your faster galaxy near the outer boundary. Also see Figure 33-20.

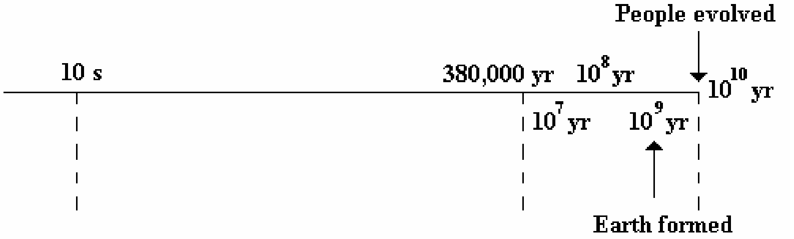
14. An explosion on Earth blows pieces out into the space around it, but the Big Bang was the start of the expansion of space itself. In an explosion on Earth, the pieces that are blown outward will slow down due to air resistance and the farther away they are the slower they will be moving and then they will eventually come to rest, but with the Big Bang, the farther away galaxies are from each other the faster they are moving away from each other. In an explosion on Earth, the pieces with the higher initial speeds end up farther away from the explosion before coming to rest, but the Big Bang appears to be relatively uniform where the farthest galaxies are moving fastest and the nearest galaxies are moving the slowest. An explosion on Earth would correspond to a closed universe, since the pieces would eventually stop, but we would not see a "big crunch" due to gravity as we would with an actual closed universe.

15. To "see" a black hole in space we need indirect evidence. If a large visible star or galaxy was rotating quickly around a non-visible gravitational companion, the non-visible companion could be a massive black hole. Also, as matter begins to accelerate toward a black hole, it will emit characteristic X-rays, which we could detect on Earth. Another way we could "see" a black hole is if it caused gravitational lensing of objects behind it. Then we would see stars and galaxies in the "wrong" place as their light is bent as it passes past the black hole on its way to Earth.

16. The Schwarzschild radius of a hydrogen atom in its ground state would be caused by a mass of:

$$R = 5.29\times10^{-11}\,\text{m} = \frac{2\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)M}{\left(3\times10^{8}\ \text{m/s}\right)^2} \quad\rightarrow\quad M = 3.57\times10^{16}\,\text{kg}$$

17. If we look closely at the top of the right-hand side of Figure 33-25:



The Earth formed 4 billion years ago and people evolved 1 million years ago. These events are both very close to the very end of the timeline shown in Figure 33-25.

18. The material contained in the universe is at some average temperature and, thus, it radiates a blackbody spectrum that is characteristic of this temperature. This temperature is now so low because of the expansion of the universe. When the universe was extremely small and the sum total energy of the universe was trapped in that extremely small volume, the average temperature of the universe was amazingly high. As this energy has been spread out over our expanding universe, the average temperature has decreased to its current low level.

19. Atoms were unable to exist until hundreds of thousands of years after the Big Bang because the temperature of the universe was still too high until that time. At those incredibly high temperatures, the free electrons and nuclei were moving so fast and had so much kinetic energy, and they had so many high energy photons colliding with them, that they could never combine together to form atoms. Once the universe cooled below 3000 K, this coupling could take place, and atoms were formed.

20. The universe would eventually collapse in on itself if it had a positive curvature. This would mean that there is so much mass in the universe that the gravitational attraction would pull all of the mass of the universe back into a "big crunch." In this situation, the average density of the universe would be greater than the critical density.

Solutions to Problems

1. See Figure 13-11 in the textbook. The parsec is the distance *D* when the angle ϕ is 1 second of arc.

$$\phi = 1'' \times \frac{1^\circ}{3600''} \times \frac{\pi \text{ rad}}{180^\circ} = 4.848 \times 10^{-6} \text{ rad}$$

$$\tan\phi = \frac{d}{D} \rightarrow D = \frac{d}{\tan\phi} = \frac{1.496 \times 10^{11} \text{ m}}{\tan\left(4.848 \times 10^{-6} \text{ rad}\right)} = 3.086 \times 10^{16} \text{ m}$$

$$D = 3.086 \times 10^{16} \text{ m}\left(\frac{1 \text{ ly}}{9.461 \times 10^{15} \text{ m}}\right) = \boxed{3.26 \text{ ly}}$$

2. Use the angle to calculate the distance in parsecs, and then convert to other units.

$$d\left(\text{pc}\right) = \frac{1}{\phi''} = \frac{1}{0.38''} = 2.63 \text{ pc}\left(\frac{3.26 \text{ ly}}{1 \text{ pc}}\right) = \boxed{8.6 \text{ ly}}$$

3. Convert the angle to seconds of arc, reciprocate to find the distance in parsecs, and then convert to light years.

$$\phi = \left(1.9 \times 10^{-4}\right)^\circ \left(\frac{3600''}{1^\circ}\right) = 0.684''$$

$$d\left(\text{pc}\right) = \frac{1}{\phi''} = \frac{1}{0.684} = 1.46 \text{ pc}\left(\frac{3.26 \text{ ly}}{1 \text{ pc}}\right) = \boxed{4.8 \text{ ly}}$$

$\boxed{4.}$ The reciprocal of the distance in parsecs is the angle in seconds of arc.

(*a*) $\phi'' = \frac{1}{d\left(\text{pc}\right)} = \frac{1}{36 \text{ pc}} = 0.02778'' \approx \boxed{\left(2.8 \times 10^{-2}\right)''}$

(*b*) $0.02778''\left(\frac{1^\circ}{3600''}\right) = \left(7.717 \times 10^{-6}\right)^\circ \approx \boxed{\left(7.7 \times 10^{-6}\right)^\circ}$

5. Convert the light years to parsecs, and then reciprocate the parsecs to find the parallax angle in seconds of arc.

$$55 \text{ ly}\left(\frac{1 \text{ pc}}{3.26 \text{ ly}}\right) = 16.87 \text{ pc} \approx \boxed{17 \text{ pc}} \qquad \phi = \frac{1}{16.87 \text{ pc}} = \boxed{0.059''}$$

6. The parallax angle is $\boxed{\text{smaller}}$ for the further star. Since $\tan\phi = \frac{d}{D}$, as the distance *D* to the star increases, the tangent decreases, so the angle decreases. And since for small angles, $\tan\phi \approx \phi$, we

have that $\phi \approx \dfrac{d}{D}$. Thus if the distance D is doubled, the angle ϕ will be $\boxed{\text{smaller by a factor of 2}}$.

7. Find the distance in light years. That value is also the time for light to reach us.

$$35\,\text{pc}\left(\frac{3.26\,\text{ly}}{1\,\text{pc}}\right)=114\,\text{ly}\approx 110\,\text{ly} \quad\rightarrow\quad \text{It takes light } \boxed{110\text{ years}} \text{ to reach us.}$$

8. (*a*) The apparent brightness is the solar constant, $\boxed{1.3\times10^{3}\ \text{W/m}^{2}}$.

(*b*) Use Eq. 33-1 to find the absolute luminosity.

$$l=\frac{L}{4\pi d^{2}} \quad\rightarrow\quad L=4\pi d^{2}l=4\pi\left(1.496\times10^{11}\,\text{m}^{2}\right)^{2}\left(1.3\times10^{3}\ \text{W/m}^{2}\right)=\boxed{3.7\times10^{26}\,\text{W}}$$

$\boxed{9.}$ The apparent brightness of an object is inversely proportional to the observer's distance from the object, given by $l=\dfrac{L}{4\pi d^{2}}$. To find the relative brightness at one location as compared to another, take a ratio of the apparent brightness at each location.

$$\frac{l_{\text{Jupiter}}}{l_{\text{Earth}}}=\frac{\dfrac{L}{4\pi d_{\text{Jupiter}}^{2}}}{\dfrac{L}{4\pi d_{\text{Earth}}^{2}}}=\frac{d_{\text{Earth}}^{2}}{d_{\text{Jupiter}}^{2}}=\left(\frac{d_{\text{Earth}}}{d_{\text{Jupiter}}}\right)^{2}=\left(\frac{1}{5.2}\right)^{2}=\boxed{3.7\times10^{-2}}$$

10. The angular width is the inverse tangent of the diameter of our Galaxy divided by the distance to the nearest galaxy. According to Figure 33-2, our Galaxy is about 100,000 ly in diameter.

$$\phi=\tan^{-1}\frac{\text{Galaxy diameter}}{\text{Distance to nearest galaxy}}=\tan^{-1}\frac{1.0\times10^{5}\,\text{ly}}{2.4\times10^{6}\,\text{ly}}=\boxed{4.2\times10^{-2}\,\text{rad}}\approx 2.4^{\circ}$$

$$\phi_{\text{Moon}}=\tan^{-1}\frac{\text{Moon diameter}}{\text{Distance to Moon}}=\tan^{-1}\frac{3.48\times10^{6}\,\text{m}}{3.84\times10^{8}\,\text{m}}=9.1\times10^{-3}\,\text{rad}\left(\approx 0.52^{\circ}\right)$$

$\boxed{\text{The galaxy width is about 4.5 times the moon width.}}$

11. The density is the mass divided by the volume.

$$\rho=\frac{M}{V}=\frac{M}{\frac{4}{3}\pi R^{3}}=\frac{1.99\times10^{30}\,\text{kg}}{\frac{4}{3}\pi\left(1.5\times10^{11}\,\text{m}\right)^{3}}=\boxed{1.4\times10^{-4}\ \text{kg/m}^{3}}$$

12. The angular width is the inverse tangent of the diameter of the Moon divided by the distance to the Sun.

$$\phi=\tan^{-1}\frac{\text{Moon diameter}}{\text{Distance to Sun}}=\tan^{-1}\frac{3.48\times10^{6}\,\text{m}}{1.496\times10^{11}\,\text{m}}=\boxed{2.33\times10^{-5}\,\text{rad}}\approx\left(1.33\times10^{-3}\right)^{\circ}\approx 4.79''$$

$\boxed{13.}$ The density is the mass divided by the volume.

$$\rho=\frac{M}{V}=\frac{M_{\text{Sun}}}{\frac{4}{3}\pi R_{\text{Earth}}^{3}}=\frac{1.99\times10^{30}\,\text{kg}}{\frac{4}{3}\pi\left(6.38\times10^{6}\,\text{m}\right)^{3}}=\boxed{1.83\times10^{9}\ \text{kg/m}^{3}}$$

Since the volumes are the same, the ratio of the densities is the same as the ratio of the masses.

$$\frac{\rho}{\rho_{\text{Earth}}}=\frac{M}{M_{\text{Earth}}}=\frac{1.99\times10^{30}\,\text{kg}}{5.98\times10^{24}\,\text{kg}}=\boxed{3.33\times10^{5}\text{ times larger}}$$

14. The density of the neutron star is its mass divided by its volume. Use the proton to calculate the density of nuclear matter.

$$\rho_{\substack{\text{neutron}\\\text{star}}}=\frac{M}{V}=\frac{M_{\text{Sun}}}{\frac{4}{3}\pi R_{\text{Earth}}^{3}}=\frac{1.5\left(1.99\times10^{30}\,\text{kg}\right)}{\frac{4}{3}\pi\left(11000\,\text{m}\right)^{3}}=5.354\times10^{17}\,\text{kg}/\text{m}^{3}\approx\boxed{5.4\times10^{17}\,\text{kg}/\text{m}^{3}}$$

$$\frac{\rho_{\substack{\text{neutron}\\\text{star}}}}{\rho_{\substack{\text{white}\\\text{dwarf}}}}=\frac{5.354\times10^{17}\,\text{kg}/\text{m}^{3}}{1.83\times10^{9}\,\text{kg}/\text{m}^{3}}=\boxed{2.9\times10^{8}}\qquad\frac{\rho_{\substack{\text{neutron}\\\text{star}}}}{\rho_{\substack{\text{nuclear}\\\text{matter}}}}=\frac{5.354\times10^{17}\,\text{kg}/\text{m}^{3}}{\dfrac{1.673\times10^{-27}\,\text{kg}}{\frac{4}{3}\pi\left(1\times10^{-15}\,\text{m}\right)^{3}}}=\boxed{1.3}$$

15. From Equation 30-2, the Q-value is the mass energy of the reactants minus the mass energy of the products.

$${}^{4}_{2}\text{He}+{}^{4}_{2}\text{He}\;\rightarrow\;{}^{8}_{4}\text{Be}$$

$$Q=m_{\text{Be}}c^{2}-2m_{\text{He}}c^{2}=\left[2\left(4.002603\,\text{u}\right)-8.005305\,\text{u}\right]c^{2}\left(931.5\,\text{Mev}/c^{2}\right)=\boxed{-0.092\,\text{MeV}}$$

$${}^{4}_{2}\text{He}+{}^{8}_{4}\text{Be}\;\rightarrow\;{}^{12}_{6}\text{C}$$

$$Q=m_{\text{Be}}c^{2}-2m_{\text{He}}c^{2}=\left[4.002603\,\text{u}+8.005305\,\text{u}-12.000000\right]c^{2}\left(931.5\,\text{Mev}/c^{2}\right)$$

$$=\boxed{7.366\,\text{MeV}}$$

16. Wien's law says that the $\lambda_{\text{p}}T=\alpha$, where α is a constant, and so $\lambda_{\text{P1}}T_{1}=\lambda_{\text{P2}}T_{2}$. The Stefan-Boltzmann equation says that the power output of a star is given by $P=\beta AT^{4}$, where β is a constant, and A is the radiating area. The P in the Stefan-Boltzmann equation is the same as the luminosity L in this chapter. It is given that $l_{1}=l_{2}$ and $r_{1}=r_{2}$. We assign $\lambda_{\text{P1}}=800\,\text{nm}$ and $\lambda_{\text{P2}}=400\,\text{nm}$.

$$\lambda_{\text{P1}}T_{1}=\lambda_{\text{P2}}T_{2}\;\rightarrow\;\frac{T_{2}}{T_{1}}=\frac{\lambda_{\text{P1}}}{\lambda_{\text{P2}}}=2$$

$$l_{1}=l_{2}\;\rightarrow\;\frac{L_{1}}{4\pi d_{1}^{2}}=\frac{L_{2}}{4\pi d_{2}^{2}}\;\rightarrow\;\frac{d_{2}^{2}}{d_{1}^{2}}=\frac{L_{2}}{L_{1}}=\frac{P_{2}}{P_{1}}=\frac{\beta A_{2}T_{2}^{4}}{\beta A_{1}T_{1}^{4}}=\frac{4\pi r_{2}^{2}T_{2}^{4}}{4\pi r_{1}^{2}T_{1}^{4}}=\frac{T_{2}^{4}}{T_{1}^{4}}=\left(\frac{T_{2}}{T_{1}}\right)^{4}\;\rightarrow$$

$$\frac{d_{2}}{d_{1}}=\left(\frac{T_{2}}{T_{1}}\right)^{2}=\left(2\right)^{2}=4$$

$$\boxed{\text{The star with the peak at 400 nm is 4 times further away than the star with the peak at 800 nm.}}$$

17. Wien's law says that the $\lambda_{\text{p}}T=\alpha$, where α is a constant, and so $\lambda_{\text{P1}}T_{1}=\lambda_{\text{P2}}T_{2}$. or . The Stefan-Boltzmann equation says that the power output of a star is given by $P=\beta AT^{4}$, where β is a constant, and A is the radiating area. The P in the Stefan-Boltzmann equation is the same as the luminosity L in this chapter. It is given that $l_{1}/l_{2}=0.091$, $d_{1}=d_{2}$, $\lambda_{\text{P1}}=500\,\text{nm}$ and $\lambda_{\text{P2}}=700\,\text{nm}$.

$$\lambda_{P1}T_1 = \lambda_{P2}T_2 \rightarrow \frac{T_2}{T_1} = \frac{\lambda_{P1}}{\lambda_{P2}} = \frac{5}{7} \qquad l_1 = 0.091 l_2 \rightarrow \frac{L_1}{4\pi d_1^2} = 0.091\frac{L_2}{4\pi d_2^2} \rightarrow$$

$$1 = \frac{d_2^2}{d_1^2} = \frac{0.091L_2}{L_1} = \frac{0.091P_2}{P_1} = \frac{0.091A_2T_2^4}{A_1T_1^4} = \frac{(0.091)4\pi r_2^2T_2^4}{4\pi r_1^2T_1^4} = 0.091\frac{T_2^4}{T_1^4}\frac{r_2^2}{r_1^2} \rightarrow$$

$$\frac{r_1}{r_2} = \sqrt{0.091}\left(\frac{T_2}{T_1}\right)^2 = \sqrt{0.091}\left(\frac{5}{7}\right)^2 = 0.15$$

The ratio of the diameters is the same as the ratio of radii, so $\frac{D_1}{D_2} = \boxed{0.15}$

18. The Schwarzschild radius is given by $R = \frac{2GM}{c^2}$.

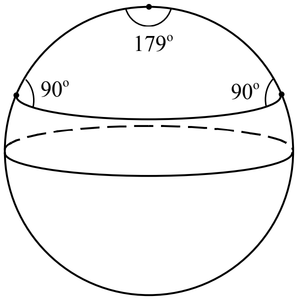
(*a*) $$R_{\text{Sun}} = \frac{2GM_{\text{Sun}}}{c^2} = \frac{2\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\,\text{kg}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^2} = 2950\,\text{m} = \boxed{2.95\,\text{km}}$$

(*b*) $$R_{\text{Earth}} = \frac{2GM_{\text{Earth}}}{c^2} = \frac{2\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^2} = 8.86\times10^{-3}\,\text{m} \approx \boxed{8.9\,\text{mm}}$$

19. The Schwarzschild radius is given by $R = \frac{2GM}{c^2}$.

$$R = \frac{2GM}{c^2} = \frac{2\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(3\times10^{41}\,\text{kg}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^2} = \boxed{4\times10^{14}\,\text{m}}$$

20. (*a*) For the vertices of the triangle we choose the North pole and two points on a latitude line on nearly opposite sides of the Earth, as shown on the diagram. Let the angle at the North pole be 179°.



(*b*) If the triangle is drawn on a small enough portion of the sphere that the portion is flat, then the sum of the angles will be 180°.

21. The limiting value for the angles in a triangle on a sphere is $\boxed{540^\circ}$. Imagine drawing an equilateral triangle near the north pole, enclosing the north pole. If that triangle were small, the surface would be approximately flat, and the each angle in the triangle would be 60°. Then imagine "stretching" each side of that triangle down towards the equator, while keeping sure that the north pole stayed inside the triangle. The angle at each vertex of the triangle would expand, with a limiting value of 180°. The three 180° angles in the triangle would add up to 540°.

22. Use Eq. 33-6, Hubble's law.

$$v = Hd \rightarrow d = \frac{v}{H} = \frac{(0.010)\left(3.00\times10^8\ \text{m/s}\right)}{2.2\times10^4\ \text{m/s/Mly}} = \boxed{140\,\text{Mly}} = 1.4\times10^8\,\text{ly}$$

23. Use Eq. 33-6, Hubble's law.

$$v = Hd \rightarrow d = \frac{v}{H} = \frac{3500\,\text{km/s}}{22\,\text{km/s/Mly}} = \boxed{160\,\text{Mly}} = 1.6\times10^{8}\,\text{ly}$$

24. Use Eq. 33-6, Hubble's law.

$$v = Hd = \left(22000\,\text{m/s/Mly}\right)\left(12000\,\text{Mly}\right) = 2.64\times10^{8}\,\text{m/s}\left(\frac{c}{3.00\times10^{5}\,\text{m/s}}\right) = \boxed{0.88\,c}$$

25. We find the velocity from Hubble's law (Eq. 33-6), and the observed wavelength from the Doppler shift, Eq. 33-3.

(*a*) $\frac{v}{c} = \frac{Hd}{c} = \frac{\left(22000\,\text{m/s/Mly}\right)\left(1.0\,\text{Mly}\right)}{3.00\times10^{8}\,\text{m/s}} = 7.33\times10^{-5}$

$$\lambda = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} = \left(656\,\text{nm}\right)\sqrt{\frac{1+7.33\times10^{-5}}{1-7.33\times10^{-5}}} = 656.05\,\text{nm} \approx \boxed{656\,\text{nm}}$$

(*b*) $\frac{v}{c} = \frac{Hd}{c} = \frac{\left(22000\,\text{m/s/Mly}\right)\left(1.0\times10^{2}\,\text{Mly}\right)}{3.00\times10^{8}\,\text{m/s}} = 7.33\times10^{-3}$

$$\lambda = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} = \left(656\,\text{nm}\right)\sqrt{\frac{1+7.33\times10^{-3}}{1-7.33\times10^{-3}}} = 660.83\,\text{nm} \approx \boxed{661\,\text{nm}}$$

(*c*) $\frac{v}{c} = \frac{Hd}{c} = \frac{\left(22000\,\text{m/s/Mly}\right)\left(1.0\times10^{4}\,\text{Mly}\right)}{3.00\times10^{8}\,\text{m/s}} = 0.733$

$$\lambda = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} = \left(656\,\text{nm}\right)\sqrt{\frac{1+0.733}{1-0.733}} = 1671.3\,\text{nm} \approx \boxed{1670\,\text{nm}}$$

$\boxed{26.}$ Use Eqs. 33-4 and 33-5a to solve for the speed of the galaxy.

$\Delta\lambda = 610\,\text{nm} - 434\,\text{nm} = 176\,\text{nm}$

$$z = \frac{\Delta\lambda}{\lambda_0} = \sqrt{\frac{1+v/c}{1-v/c}} - 1 \rightarrow \frac{v}{c} = \frac{\left(\frac{\Delta\lambda}{\lambda_0}+1\right)^2 - 1}{\left(\frac{\Delta\lambda}{\lambda_0}+1\right)^2 + 1}\;\frac{v}{c} = \frac{\left(\frac{176\,\text{nm}}{434\,\text{nm}}+1\right)^2 - 1}{\left(\frac{176\,\text{nm}}{434\,\text{nm}}+1\right)^2 + 1} = \frac{0.9755}{2.9755} = 0.327c \rightarrow$$

$v \approx \boxed{0.33c}$

Use Hubble's law, Eq. 33-6, to solve for the distance.

$$v = Hd \rightarrow d = \frac{v}{H} = \frac{\frac{v}{c}}{\frac{H}{c}} = \frac{0.3278}{\frac{\left(22000\,\text{m/s/Mly}\right)}{3.00\times10^{8}\,\text{m/s}}} = 4.47\times10^{3}\,\text{Mly} \approx \boxed{4.5\times10^{9}\,\text{ly}}$$

27. Use Eq. 33-35a to solve for the speed of the galaxy.

$$z=\sqrt{\frac{1+v/c}{1-v/c}}-1 \;\rightarrow\; \frac{v}{c}=\frac{(z+1)^2-1}{(z+1)^2+1}=\frac{1.6^2-1}{1.6^2+1}=0.44 \;\rightarrow\; v=\boxed{0.44c}$$

28. Use Eq. 33-35a to solve for the redshift parameter.

$$z=\sqrt{\frac{1+v/c}{1-v/c}}-1=\sqrt{\frac{1+0.50}{1-0.50}}-1=\boxed{0.73}$$

29. Eq. 33-3 states $\lambda=\lambda_0\sqrt{\frac{1+v/c}{1-v/c}}$.

$$\lambda=\lambda_0\sqrt{\frac{1+v/c}{1-v/c}}=\lambda_0\left(1+\frac{v}{c}\right)^{1/2}\left(1-\frac{v}{c}\right)^{-1/2}\approx\lambda_0\left(1+\tfrac{1}{2}\frac{v}{c}\right)\left(1-\left(-\tfrac{1}{2}\right)\frac{v}{c}\right)=\lambda_0\left(1+\tfrac{1}{2}\frac{v}{c}\right)^2$$

$$\lambda\approx\lambda_0\left(1+2\left(\tfrac{1}{2}\frac{v}{c}\right)\right)=\lambda_0\left(1+\frac{v}{c}\right)=\lambda_0+\lambda_0\frac{v}{c} \;\rightarrow\; \lambda-\lambda_0=\Delta\lambda=\lambda_0\frac{v}{c} \;\rightarrow\; \boxed{\frac{\Delta\lambda}{\lambda_0}=\frac{v}{c}}$$

30. Wien's law is given as Eq. 27-2 in chapter 27.

$$\lambda_{\text{P}}T=2.90\times10^{-3}\,\text{m}\bullet\text{K} \;\rightarrow\; \lambda_{\text{P}}=\frac{2.90\times10^{-3}\,\text{m}\bullet\text{K}}{T}=\frac{2.90\times10^{-3}\,\text{m}\bullet\text{K}}{2.7\,\text{K}}=\boxed{1.1\times10^{-3}\,\text{m}}$$

31. We use the proton as typical nuclear matter.

$$10^{-26}\,\frac{\text{kg}}{\text{m}^3}\times\frac{1\,\text{nucleon}}{1.67\times10^{-27}\,\text{kg}}=\boxed{6\,\text{nucleons}/\text{m}^3}$$

32. If the scale of the universe is inversely proportional to the temperature, then the scale times the temperature should be constant. If we call the current scale "1", and knowing the current temperature to be about 3 K, then the product of scale and temperature should be about 3. Use Figure 33-25 to estimate the temperature at various times. For purposes of illustration, we assume the universe has a current size of about $10^{10}\,\text{ly}$.

(*a*) At $t=10^6\,\text{yr}$, the temperature is about 1000 K. Thus the scale is found as follows.

$$(\text{Scale})(\text{Temperature})=3 \;\rightarrow\; \text{Scale}=\frac{3}{\text{Temperature}}=\frac{3}{1000}=\boxed{3\times10^{-3}}\approx3\times10^{7}\,\text{ly}$$

(*b*) At $t=1\,\text{s}$, the temperature is about $10^{10}\,\text{K}$.

$$\text{Scale}=\frac{3}{\text{Temperature}}=\frac{3}{10^{10}}=\boxed{3\times10^{-10}}\approx3\,\text{ly}$$

(*c*) At $t=10^{-6}\,\text{s}$, the temperature is about $10^{13}\,\text{K}$.

$$\text{Scale}=\frac{3}{\text{Temperature}}=\frac{3}{10^{13}}=\boxed{3\times10^{-13}}\approx3\times10^{-3}\,\text{ly}\approx3\times10^{13}\,\text{m}$$

(*d*) At $t=10^{-35}\,\text{s}$, the temperature is about $10^{27}\,\text{K}$.

$$\text{Scale}=\frac{3}{\text{Temperature}}=\frac{3}{10^{27}}=\boxed{3\times10^{-27}}\approx3\times10^{-17}\,\text{ly}\approx0.3\,\text{m}$$

33. The temperature corresponding to the average kinetic energy needed to produce a particle of mass m is given by $\frac{3}{2}kT = mc^2 \quad \rightarrow \quad T = \frac{2}{3}\frac{mc^2}{k}$.

(*a*) $$T = \frac{2}{3}\frac{mc^2}{k} = \frac{2}{3}\frac{\left(500\times10^6\ \mathrm{ev}/c^2\right)c^2\left(1.60\times10^{-19}\ \mathrm{J/ev}\right)}{1.38\times10^{-23}\ \mathrm{J/K}} = 3.86\times10^{12}\,\mathrm{K}$$

From Figure 33-25, this corresponds to a time of $\boxed{\sim10^{-5}\mathrm{s}}$.

(*b*) $$T = \frac{2}{3}\frac{mc^2}{k} = \frac{2}{3}\frac{\left(9500\times10^6\ \mathrm{ev}/c^2\right)c^2\left(1.60\times10^{-19}\ \mathrm{J/ev}\right)}{1.38\times10^{-23}\ \mathrm{J/K}} = 7.34\times10^{13}\,\mathrm{K}$$

From Figure 33-25, this corresponds to a time of $\boxed{\sim10^{-7}\mathrm{s}}$.

(*c*) $$T = \frac{2}{3}\frac{mc^2}{k} = \frac{2}{3}\frac{\left(100\times10^6\ \mathrm{ev}/c^2\right)c^2\left(1.60\times10^{-19}\ \mathrm{J/ev}\right)}{1.38\times10^{-23}\ \mathrm{J/K}} = 7.73\times10^{11}\,\mathrm{K}$$

From Figure 33-25, this corresponds to a time of $\boxed{\sim10^{-4}\mathrm{s}}$.

34. A: temperature increases, luminosity stays the same, size decreases
B: temperature stays the same, luminosity decreases, size decreases
C: temperature decreases, luminosity increases, size increases

35. The apparent luminosity is given by Eq. 33-1. Use that relationship to derive an expression for the absolute luminosity, and equate that for two stars.

$$l = \frac{L}{4\pi d^2} \quad \rightarrow \quad L = 4\pi d^2 l$$

$$L_{\substack{\text{distant}\\\text{star}}} = L_{\text{Sun}} \quad \rightarrow \quad 4\pi d^2_{\substack{\text{distant}\\\text{star}}} l_{\substack{\text{distant}\\\text{star}}} = 4\pi d^2_{\text{Sun}} l_{\text{Sun}} \quad \rightarrow$$

$$d_{\substack{\text{distant}\\\text{star}}} = d_{\text{Sun}}\sqrt{\frac{l_{\text{Sun}}}{l_{\substack{\text{distant}\\\text{star}}}}} = \left(1.5\times10^{11}\mathrm{m}\right)\sqrt{\frac{1}{10^{-11}}}\left(\frac{1\,\mathrm{ly}}{9.461\times10^{15}\mathrm{m}}\right) = \boxed{5\,\mathrm{ly}}$$

36. The angular momentum is the product of the rotational inertia and the angular velocity.

$$\left(I\omega\right)_{\text{initial}} = \left(I\omega\right)_{\text{final}} \quad \rightarrow$$

$$\omega_{\text{final}} = \omega_{\text{initial}}\left(\frac{I_{\text{initial}}}{I_{\text{final}}}\right) = \omega_{\text{initial}}\left(\frac{\frac{2}{5}MR^2_{\text{initial}}}{\frac{2}{5}MR^2_{\text{final}}}\right) = \omega_{\text{initial}}\left(\frac{R_{\text{initial}}}{R_{\text{final}}}\right)^2 = \left(1\,\mathrm{rev/month}\right)\left(\frac{7\times10^8\,\mathrm{m}}{1\times10^4\,\mathrm{m}}\right)^2$$

$$= 4.9\times10^9\ \mathrm{rev/month} = 4.9\times10^9\ \frac{\mathrm{rev}}{\mathrm{month}}\times\frac{1\,\mathrm{month}}{30\,\mathrm{d}}\times\frac{1\,\mathrm{d}}{24\,\mathrm{h}}\times\frac{1\,\mathrm{h}}{3600\,\mathrm{s}} = \boxed{1900\,\mathrm{rev/s}}$$

37. The rotational kinetic energy is given by $\frac{1}{2}I\omega^2$. The final angular velocity, from problem 36, is about $4.9\times10^9\ \mathrm{rev/month}$.

$$\frac{\mathrm{KE}_{\text{final}}}{\mathrm{KE}_{\text{initial}}} = \frac{\frac{1}{2}I_{\text{final}}\omega^2_{\text{final}}}{\frac{1}{2}I_{\text{initial}}\omega^2_{\text{initial}}} = \frac{\frac{2}{5}MR^2_{\text{final}}\omega^2_{\text{final}}}{\frac{2}{5}MR^2_{\text{initial}}\omega^2_{\text{initial}}} = \left(\frac{R_{\text{final}}\omega_{\text{final}}}{R_{\text{initial}}\omega_{\text{initial}}}\right)^2$$

$$=\left(\frac{\left(1\times10^{4}\,\text{m}\right)\left(4.9\times10^{9}\text{ rev/month}\right)}{\left(7\times10^{8}\,\text{m}\right)\left(1\,\text{rev/month}\right)}\right)^{2}=\boxed{4.9\times10^{9}}$$

38. The power output is the energy loss divided by the elapsed time.

$$P=\frac{\Delta\text{KE}}{\Delta t}=\frac{\frac{1}{2}I\omega^{2}\left(\text{fraction lost}\right)}{\Delta t}=\frac{\frac{1}{2}\frac{2}{5}MR^{2}\omega^{2}\left(\text{fraction lost}\right)}{\Delta t}$$

$$=\frac{1}{5}\frac{\left(1.5\right)\left(1.99\times10^{30}\text{kg}\right)\left(1.0\times10^{4}\text{m}\right)^{2}\left(2\pi\,\text{rad/s}\right)^{2}\left(1\times10^{-9}\right)}{\left(1\,\text{d}\right)\left(24\,\text{h/d}\right)\left(3600\,\text{s/h}\right)}=\boxed{2.7\times10^{25}\,\text{W}}$$

39. Use Newton's law of universal gravitation.

$$F=G\frac{m_1m_2}{r^2}=\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(3\times10^{41}\text{kg}\right)^2}{\left[\left(2\times10^{6}\text{ly}\right)\left(\frac{9.46\times10^{15}\text{m}}{1\,\text{ly}}\right)\right]^2}=1.68\times10^{28}\,\text{N}$$

$$\approx\boxed{2\times10^{28}\,\text{N}}$$

40. (*a*) Assume that the nucleons make up only 2% of the critical mass density.

$$\text{nucleon mass density}=0.02\left(10^{-26}\text{ kg/m}^3\right)$$

$$\text{nucleon number density}=\frac{0.02\left(10^{-26}\text{ kg/m}^3\right)}{1.67\times10^{-27}\text{ kg/nucleon}}=0.12\,\text{nucleon/m}^3$$

$$\text{neutrino number density}=10^{9}\left(\text{nucleon number density}\right)=1.2\times10^{8}\text{ neutrino/m}^3$$

$$\frac{0.98\left(10^{-26}\text{ kg/m}^3\right)}{1.2\times10^{8}\text{ neutrino/m}^3}=8.17\times10^{-35}\frac{\text{kg}}{\text{neutrino}}\times\frac{9.315\times10^{8}\text{ eV}/c^2}{1.66\times10^{-27}\text{kg}}=\boxed{46\,\text{eV}/c^2}$$

(*b*) Assume that the nucleons make up only 5% of the critical mass density.

$$\text{nucleon mass density}=0.05\left(10^{-26}\text{ kg/m}^3\right)$$

$$\text{nucleon number density}=\frac{0.05\left(10^{-26}\text{ kg/m}^3\right)}{1.67\times10^{-27}\text{ kg/nucleon}}=0.30\,\text{nucleon/m}^3$$

$$\text{neutrino number density}=10^{9}\left(\text{nucleon number density}\right)=3.0\times10^{8}\text{ neutrino/m}^3$$

$$\frac{0.95\left(10^{-26}\text{ kg/m}^3\right)}{3.0\times10^{8}\text{ neutrino/m}^3}=3.17\times10^{-35}\frac{\text{kg}}{\text{neutrino}}\times\frac{9.315\times10^{8}\text{ eV}/c^2}{1.66\times10^{-27}\text{kg}}=\boxed{18\,\text{eV}/c^2}$$

41. The temperature of each star can be found from Wien's law.

$$\lambda_{\text{P}}T=2.90\times10^{-3}\text{m}\bullet\text{K}\ \rightarrow$$

$$T_{600}=\frac{2.90\times10^{-3}\text{m}\bullet\text{K}}{600\times10^{-9}\text{m}}=4830\,\text{K}\qquad T_{400}=\frac{2.90\times10^{-3}\text{m}\bullet\text{K}}{400\times10^{-9}\text{m}}=7250\,\text{K}$$

The luminosity of each star can be found from the H-R diagram.

$$L_{600}\sim10^{26}\,\text{W}\qquad L_{400}\sim10^{27}\,\text{W}$$

The Stefan-Boltzmann equation says that the power output of a star is given by $P = \beta A T^4$, where β is a constant, and A is the radiating area. The P in the Stefan-Boltzmann equation is the same as the luminosity L. Form the ratio of the two luminosities.

$$\frac{L_{400}}{L_{600}} = \frac{\beta A_{400} T_{400}^4}{\beta A_{600} T_{600}^4} = \frac{\pi r_{400}^2 T_{400}^4}{\pi r_{600}^2 T_{600}^4} \rightarrow \frac{r_{400}}{r_{600}} = \sqrt{\frac{L_{400}}{L_{600}} \frac{T_{600}^2}{T_{400}^2}} = \sqrt{\frac{10^{27}\,\mathrm{W}}{10^{26}\,\mathrm{W}} \frac{(4830\,\mathrm{K})^2}{(7250\,\mathrm{K})^2}} = 1.4$$

The diameters are in the same ratio as the radii.

$$\frac{d_{400}}{d_{600}} = \boxed{1.4}$$

The luminosities are fairly subjective, since they are read from the H-R diagram. Different answers may arise from different readings of the H-R diagram.

42. The number of parsecs is the reciprocal of the angular resolution in seconds of arc.

$$100 \text{ parsec} = \frac{1}{\phi''} \rightarrow \phi = (0.01'')\left(\frac{1'}{60''}\right)\left(\frac{1^\circ}{60'}\right) \approx \boxed{\left(3\times10^{-6}\right)^\circ}$$

43. From Eq. 27-16, we see that the wavelengths from single-electron energy level transitions are inversely proportional to the square of the atomic number of the nucleus. Thus the lines from singly-ionized helium are usually one fourth the wavelength of the corresponding hydrogen lines. Because of their red shift, the lines have four times their usual wavelength. Use Eq. 33-3.

$$\lambda = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} \rightarrow 4\lambda_0 = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} \rightarrow 16 = \frac{1+v/c}{1-v/c} \rightarrow \frac{v}{c} = \frac{15}{17} \rightarrow v = \boxed{0.88c}$$

$\boxed{44.}$ From section 33-7, we can approximate the temperature – kinetic energy relationship by $\frac{3}{2}kT = \mathrm{KE}$.

$$\tfrac{3}{2}kT = \mathrm{KE} \rightarrow T = \tfrac{2}{3}\frac{\mathrm{KE}}{k} = \tfrac{2}{3}\frac{\left(1.8\times10^{12}\mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)}{1.38\times10^{-23}\ \mathrm{J/K}} = \boxed{1.4\times10^{16}\,\mathrm{K}}$$

From Figure 23-25, this is in the $\boxed{\text{hadron era}}$.

45. We assume that gravity causes a centripetal force on the gas. Solve for the speed of the rotating gas, and use Eq. 33-5b to find the Doppler shift.

$$F_{\text{gravity}} = F_{\text{centripetal}} \rightarrow G\frac{m_{\text{gas}} m_{\substack{\text{black}\\ \text{hole}}}}{r^2} = \frac{m_{\text{gas}} v_{\text{gas}}^2}{r} \rightarrow$$

$$v_{\text{gas}} = \sqrt{G\frac{m_{\substack{\text{black}\\ \text{hole}}}}{r}} = \sqrt{\left(6.67\times10^{-11}\ \mathrm{N{\bullet}m^2/kg^2}\right)\frac{\left(2\times10^9\right)\left(1.99\times10^{30}\mathrm{kg}\right)}{(60\,\mathrm{ly})\left(\frac{9.46\times10^{15}\,\mathrm{m}}{1\,\mathrm{ly}}\right)}} = 6.84\times10^5\ \mathrm{m/s}$$

$$z = \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c} = \frac{6.84\times10^5\ \mathrm{m/s}}{3.00\times10^8\ \mathrm{m/s}} \approx \boxed{2\times10^{-3}}$$

This is the Doppler shift as compared to the light coming from the center of the galaxy.

46. (*a*) Use Eqs. 33-4 and 33-5a to solve for the speed of the galaxy.

$$\Delta\lambda = 650\,\text{nm} - 434\,\text{nm} = 216\,\text{nm}$$

$$z = \frac{\Delta\lambda}{\lambda_0} = \sqrt{\frac{1+v/c}{1-v/c}} - 1 \;\rightarrow$$

$$\frac{v}{c} = \frac{\left(\dfrac{\Delta\lambda}{\lambda_0}+1\right)^2 - 1}{\left(\dfrac{\Delta\lambda}{\lambda_0}+1\right)^2 + 1} \quad \frac{v}{c} = \frac{\left(\dfrac{216\,\text{nm}}{434\,\text{nm}}+1\right)^2 - 1}{\left(\dfrac{216\,\text{nm}}{434\,\text{nm}}+1\right)^2 + 1} = \frac{1.243}{3.243} = 0.383c \;\rightarrow\; v \approx \boxed{0.383\,c}$$

(*b*) Use Hubble's law, Eq. 33-6, to solve for the distance.

$$v = Hd \;\rightarrow\; d = \frac{v}{H} = \frac{\frac{v}{c}}{\frac{H}{c}} = \frac{0.383}{\dfrac{(22000\,\text{m/s/Mly})}{3.00\times10^{8}\,\text{m/s}}} = 5.22\times10^{3}\,\text{Mly} \approx \boxed{5.2\times10^{9}\,\text{ly}}$$

$\boxed{47.}$ (*a*) Find the *Q*-value for this reaction. From Equation 30-2, the *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$${}^{12}_{6}\text{C} + {}^{12}_{6}\text{C} \;\rightarrow\; {}^{24}_{12}\text{Mg}$$

$$Q = 2m_{\text{C}}c^2 - m_{\text{Mg}}c^2 = \left[2(12.000000\,\text{u}) - 23.985042\,\text{u}\right]c^2\left(931.5\,\text{Mev}/c^2\right) = \boxed{13.93\,\text{MeV}}$$

(*b*) The total kinetic energy should be equal to the electrical potential energy of the two nuclei when they are just touching. The distance between the two nuclei will be twice the nuclear radius, from Eq. 30-1. Each nucleus will have half the total kinetic energy.

$$r = \left(1.2\times10^{-15}\,\text{m}\right)(A)^{1/3} = \left(1.2\times10^{-15}\,\text{m}\right)(12)^{1/3} \qquad \text{PE} = \frac{1}{4\pi\varepsilon_0}\frac{q^2_{\text{nucleus}}}{2r}$$

$$\text{KE}_{\text{nucleus}} = \tfrac{1}{2}\text{PE} = \tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q^2_{\text{nucleus}}}{2r}$$

$$= \tfrac{1}{2}\left(8.988\times10^{9}\,\text{N}\bullet\text{n}^2/\text{C}^2\right)\frac{(6)^2\left(1.60\times10^{-19}\,\text{C}\right)^2}{2\left(1.2\times10^{-15}\,\text{m}\right)(12)^{1/3}} \times \frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = \boxed{4.71\,\text{MeV}}$$

(*c*) We can approximate the temperature – kinetic energy relationship by $\tfrac{3}{2}kT = \text{KE}$.

$$\tfrac{3}{2}kT = \text{KE} \;\rightarrow\; T = \tfrac{2}{3}\frac{\text{KE}}{k} = \tfrac{2}{3}\frac{\left(4.71\times10^{6}\text{eV}\right)\left(\dfrac{1.60\times10^{-19}\,\text{J}}{1\,\text{eV}}\right)}{1.38\times10^{-23}\,\text{J/K}} = \boxed{3.6\times10^{10}\,\text{K}}$$

48. (*a*) Find the *Q*-value for this reaction. From Equation 30-2, the *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$${}^{16}_{8}\text{C} + {}^{16}_{8}\text{C} \;\rightarrow\; {}^{28}_{14}\text{Si} + {}^{4}_{2}\text{He}$$

$$Q = 2m_{\text{C}}c^2 - m_{\text{Si}}c^2 - m_{\text{He}}c^2 = \left[2(15.994915\,\text{u}) - 27.976927\,\text{u} - 4.002603\right]c^2\left(931.5\,\text{Mev}/c^2\right)$$

$$= \boxed{9.954\,\text{MeV}}$$

(*b*) The total kinetic energy should be equal to the electrical potential energy of the two nuclei when they are just touching. The distance between the two nuclei will be twice the nuclear radius, from Eq. 30-1. Each nucleus will have half the total kinetic energy.

$$r = \left(1.2\times10^{-15}\,\text{m}\right)\left(A\right)^{1/3} = \left(1.2\times10^{-15}\,\text{m}\right)\left(16\right)^{1/3} \qquad \text{PE} = \frac{1}{4\pi\varepsilon_0}\frac{q^2_{\text{nucleus}}}{2r}$$

$$\text{KE}_{\text{nucleus}} = \tfrac{1}{2}\text{PE} = \tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q^2_{\text{nucleus}}}{2r}$$

$$= \tfrac{1}{2}\left(8.988\times10^{9}\,\text{N}\bullet\text{n}^2/\text{C}^2\right)\frac{\left(8\right)^2\left(1.60\times10^{-19}\,\text{C}\right)^2}{2\left(1.2\times10^{-15}\,\text{m}\right)\left(16\right)^{1/3}}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = \boxed{7.61\,\text{MeV}}$$

(*c*) We can approximate the temperature – kinetic energy relationship by $\tfrac{3}{2}kT = \text{KE}$.

$$\tfrac{3}{2}kT = \text{KE} \;\rightarrow\; T = \tfrac{2}{3}\frac{\text{KE}}{k} = \tfrac{2}{3}\frac{\left(7.61\times10^{6}\text{eV}\right)\left(\dfrac{1.60\times10^{-19}\,\text{J}}{1\,\text{eV}}\right)}{1.38\times10^{-23}\,\text{J/K}} = \boxed{5.9\times10^{10}\,\text{K}}$$

49. We use the Sun's mass and given density to calculate the size of the Sun.

$$\rho = \frac{M}{V} = \frac{M}{\tfrac{4}{3}\pi r^3_{\text{Sun}}} \;\rightarrow$$

$$r_{\text{Sun}} = \left(\frac{3M}{4\pi\rho}\right)^{1/3} = \left[\frac{3\left(1.99\times10^{30}\,\text{kg}\right)}{4\pi\left(10^{-26}\,\text{kg/m}^3\right)}\right]^{1/3} = 3.62\times10^{18}\,\text{m}\left(\frac{1\,\text{ly}}{9.46\times10^{15}\,\text{m}}\right) = 382\,\text{ly} \approx \boxed{400\,\text{ly}}$$

$$\frac{r_{\text{Sun}}}{d_{\text{Earth-Sun}}} = \frac{3.62\times10^{18}\,\text{m}}{1.50\times10^{11}\text{m}} \approx \boxed{2\times10^{7}} \;;\; \frac{r_{\text{Sun}}}{r_{\text{galaxy}}} = \frac{382\,\text{ly}}{50{,}000\,\text{ly}} \approx \boxed{8\times10^{-3}}$$