Hall Effect

Advanced Placement Physics C

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Current Through the Conductor

The electric current through conductor is the rate at which charge carriers pass through a point in the conductor:

$$\boxed{I = \frac{dQ}{dt}} = \left(\frac{Q}{V}\right)\frac{dV}{dt} = [ne]\,[Av_d]$$

where

- Q/V is the amount of charges *per volume*, which is just the charge carrier density n times the elementary charge e
- dV/dt is the rate the volume of charges moves through the conductor, give by the cross-section area of the conductor A times the **drift velocity** v_d of the charge carrier

For simplicity, we *assume* that charge carriers are positive. While the opposite is true, the behavior will be *almost* identical.

Current Through the Conductor

$$\boxed{I = \frac{dQ}{dt} = neAv_d}$$

Quantity	**Symbol**	**SI Unit**
Current	I	A
Charge carrier density (carriers per volume)	n	/m^3
Elementary charge	e	C
Cross-section area of the conductor	A	m^2
Drift velocity of the charge carriers	v_d	m/s

The terms nev_d is also called the **current density** J, which has the unit *ampère per meter squared* (A/m^2).

Charge Carrier Density

Finding the charge carrier density n in a *conductor* requires some additional physical information about the material:

1. Divide the metals density ρ by the metal's molar mass M to find the *number of moles of atams per unit volume*
2. Multiply by Avagadro's number N_A to find *number of atoms per unit volume*
3. Multiply by *the number of free electrons per atom* k for that particular metal

Charge Carrier Density

Collecting all the terms from the last slide, we can see that the charge carrier density is given by:

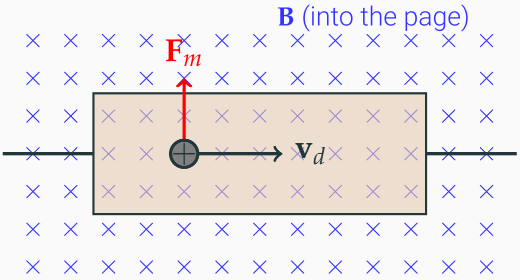
$$\boxed{n = \frac{\rho k N_A}{M}}$$

Quantity	Symbol	SI Unit
Charge carrier density	n	/m^3
Density of material	ρ	kg/m^3
Number of free electrons per atom	k	
Avogadro's number	N_A	/mol
Molar mass	M	kg/mol

For copper, $M = 63.54 \times 10^{-3}\,\mathrm{kg/mol}$, $\rho = 9.0 \times 10^3\,\mathrm{kg/m^3}$, $k = 1$ and therefore $n = 8.5 \times 10^{28}\,/\mathrm{m}^3$.

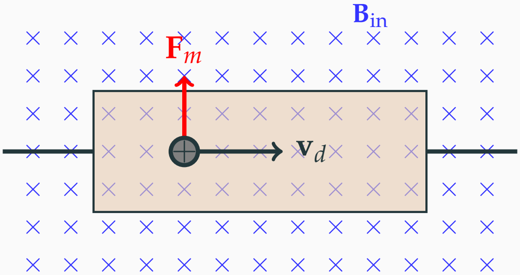
Hall Effect

When a current I flows through a conductor in a magnetic field $\mathbf{B}$, the magnetic field exerts a transverse (i.e. perpendicular to motion) magnetic force $\mathbf{F}_m$ on the moving charges which pushes them toward one side of the conductor.



This is most evident in a *thin*, *flat* conductor as illustrated.

Magnetic Force

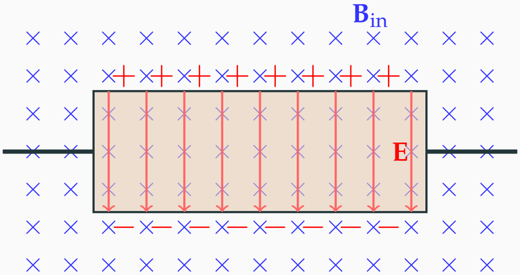


As the charges enter the magnetic field, $\mathbf{F}_m$ is directed toward the top:

$$\mathbf{F}_m = e\mathbf{v}_d \times \mathbf{B} = \frac{e\mathbf{I} \times \mathbf{B}}{neA} = \frac{\mathbf{I} \times \mathbf{B}}{nA}$$

leading to a surplus of positive charges on the top edge of the conductor, and negative charges on the bottom.

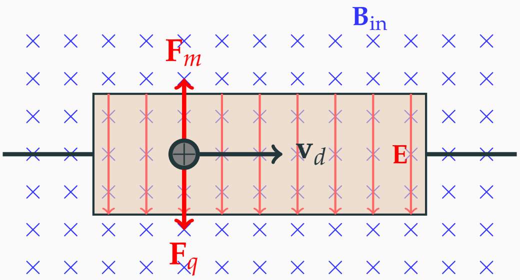
Hall Voltage



The charge imbalance on the conductor creates an electric field $\mathbf{E}$, pointing toward the bottom, and therefore a voltage across two sides of the conductor (width W), called the **Hall voltage**:

$$V_H = EW$$

Balancing Electrostatic & Magnetic Forces



Subsequently, charge carriers entering the magnetic field will experience both a magnetic force and an electrostatic force. At equilibrium, the two forces are balanced:

$$\mathbf{F}_m + \mathbf{F}_q = \mathbf{0}$$

Calculating Hall Voltage

The electrostatic force on the charge carrier can be expressed in terms of the Hall voltage V_H across the two sides of the plate:

$$F_q = eE = \frac{eV_H}{W}$$

Equating the magnitudes of electrostatic and magnetic forces, we can solve for the Hall voltage:

$$F_m = F_q \quad \rightarrow \quad \frac{IB}{nA} = \frac{eV_H}{W}$$

Hall Voltage

Cancelling terms and noting that the thickness of the conductor is

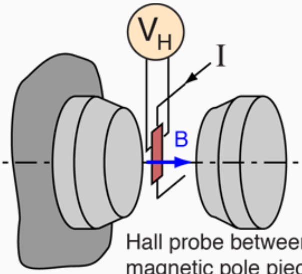
$$d = \frac{A}{W}$$

we find the expression for the Hall voltage V_H:

$$\boxed{V_H = \frac{IB}{ned}}$$

Hall Probe

Large magnetic fields ($\sim$ 1 T) is often measured using a **Hall probe**. A thin film Hall probe is placed in the magnetic field and the transverse voltage (usually measured in on the order of 10^{-6} V) is measured.



Hall probe between magnetic pole pieces

The polarity of the Hall voltage for a copper probe shows that electrons (negative charge) are the charge carriers.