Student #: ______________________ Student Name: ______________________

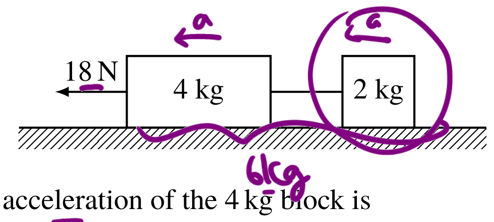
AP PHYSICS C: DYNAMICS

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

Questions 1–2

Two blocks, 4 kg and 2 kg, are connected by a string. An applied force F of magnitude 18 N pulls the blocks to the left.



1. The acceleration of the 4 kg block is

   (A) $2\,\mathrm{m/s^2}$
   (B) $3\,\mathrm{m/s^2}$
   (C) $4\,\mathrm{m/s^2}$
   (D) $4.5\,\mathrm{m/s^2}$
   (E) $6\,\mathrm{m/s^2}$

   B

   $F = ma$

   $a = \frac{F}{m} = \frac{18}{6} = 3$

2. The tension in the string between the blocks is

   (A) 4 N
   (B) 6 N
   (C) 12 N
   (D) 16 N
   (E) 18 N

   B

   N, T, Fg

   $T = ma$
   $= (2)(3)$

Questions 3–4

The position of a 2 kg object is described by the equation $x = 2t^2 - 3t^3$, where x is in meters and t is in seconds.

$F = ma$

3. The net force acting on the object at a time of 1 s is

   (A) -4 N
   (B) -8 N
   (C) -14 N
   (D) -20 N
   (E) -28 N

   E

   $v = \dot{x} = 4t - 9t^2$
   $a = \dot{v} = 4 - 18t$
   $a(1) = 4 - 18 = -14$

   $F(1) = m(a(1))$
   $= 2(-14)$

4. The net force acting on the object is positive from $t = 0$ until a time of

   (A) 0.11 s
   (B) 0.22 s
   (C) 0.44 s
   (D) 0.67 s
   (E) 1.0 s

   B

   $a = 4 - 18t = 0$
   $4 = 18t$
   $t = \frac{4}{18}$

Questions 5–6

An object of mass m moves along a straight line with a speed described by the equation $v = c + bt^3$.

5. The initial velocity of the mass is

   (A) c
   (B) $ct + bt^3$
   (C) $ct + bt^4$
   (D) bt^2
   (E) bt

   A

   $t = 0$

6. The net force acting on the mass at time T is

   (A) $3mbT$
   (B) $3mbT^2$
   (C) $3mbT^3$
   (D) $mc + 2mbT^2$
   (E) $mc^2 + 4mbT^4$

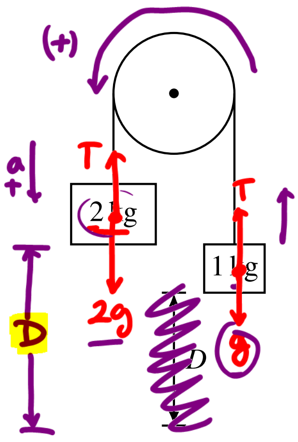
   B

   $F = ma$
   $a = \dot{v} = 3bt^2$
   $= m\frac{dv}{dt}$

   $F(t) = 3mbt^2$
   $F(T) = 3mbT^2$

Questions 7–8

A system consists of two blocks having masses of 2 kg and 1 kg. The blocks are connected by a string of negligible mass and hung over a light pulley, and then released from rest.



$\Sigma F = 2g - T + T - g$
$= g = ma$
3

$g = 3a \rightarrow a = \frac{g}{3}$

7. The acceleration of the 2 kg block is most nearly

   (A) $\frac{2}{9}g$
   (B) $\frac{1}{3}g$
   (C) $\frac{1}{2}g$
   (D) $\frac{2}{3}g$
   (E) g

   B

8. The speed of the 2 kg block after it has descended a distance D is most nearly

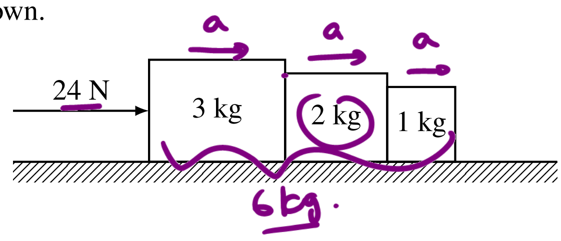
   (A) $\sqrt{\frac{4gD}{3}}$
   (B) $\sqrt{\frac{2gD}{3}}$
   (C) $\sqrt{\frac{gD}{3}}$
   (D) $\sqrt{\frac{gD}{2}}$
   (E) $\sqrt{\frac{4gD}{6}}$

   B

   $a = \frac{g}{3}$
   $v^2 = \cancel{v_0^2} + 2aD$
   $v = \sqrt{2aD} = \sqrt{\frac{2gD}{3}}$
   $\frac{g}{3}$

Questions 9–10

Three blocks of mass 3 kg, 2 kg, and 1 kg are pushed along a horizontal frictionless plane by a force of 24 N to the right, as shown.



9. The acceleration of the 2 kg block is

   (A) $144\,\mathrm{m/s^2}$
   (B) $72\,\mathrm{m/s^2}$
   (C) $12\,\mathrm{m/s^2}$
   (D) $6\,\mathrm{m/s^2}$
   (E) $4\,\mathrm{m/s^2}$

   E

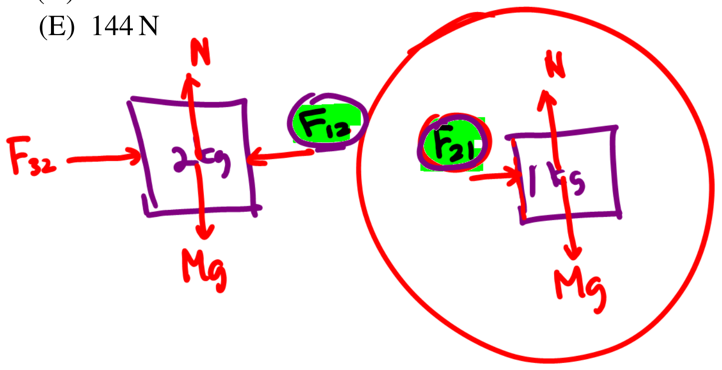
   $a = \frac{F}{m} = \frac{24}{6} = 4$

10. The force that the 2 kg block exerts on the 1 kg block is

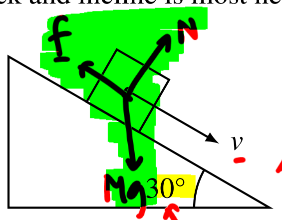
    (A) 2 N
    (B) 4 N
    (C) 6 N
    (D) 24 N
    (E) 144 N

    B

    $F_{21} = ma$
    $1 \quad 4$



11. A block of mass 4 kg slides down a rough incline with a constant speed. The angle the incline makes with the horizontal is 30°. The coefficient of friction acting between the block and incline is most nearly



(A) 0.1
(B) 0.2
(C) 0.3
(D) 0.4
(E) 0.6

E

$\Sigma F_y = 0$ $\quad Mg\cos\theta = N$

$\Sigma F_x = 0$ $\quad Mg\sin\theta - f = 0$ $\quad \mu N$

$Mg\sin\theta - \mu Mg\cos\theta = 0$

$\mu = \frac{\sin\theta}{\cos\theta}$ $\quad \boxed{\mu = \tan\theta}$ $\quad \mu = 0.58$

12. A ball is thrown straight up into the air, encountering air resistance as it rises. What forces, if any, act on the ball as it rises?

$D \propto v^2$ $\quad g$ - constant

(A) A decreasing gravitational force and an increasing force of air resistance
(B) An increasing gravitational force and an increasing force of air resistance
(C) A decreasing gravitational force and a decreasing force of air resistance
(D) A constant gravitational force and an increasing force of air resistance
(E) A constant gravitational force and a decreasing force of air resistance

E

13. A stone falls through the air toward the Earth's surface. The resistive force the air applies to the stone as it falls is given by the equation $F = cv$, where c is a positive constant and v is the speed of the stone. The acceleration of the ball is given by the equation

(A) $c - g$
(B) gcv/m
(C) $g + cv$
(D) $g - cv/m$
(E) cv/m

D

$F = cv$ $\quad Mg$

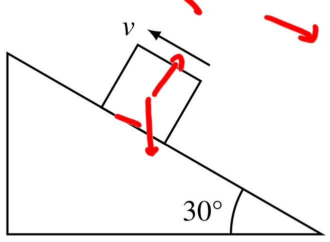
$\Sigma F = Mg - cv = ma$

$a = g - \frac{cv}{m}$

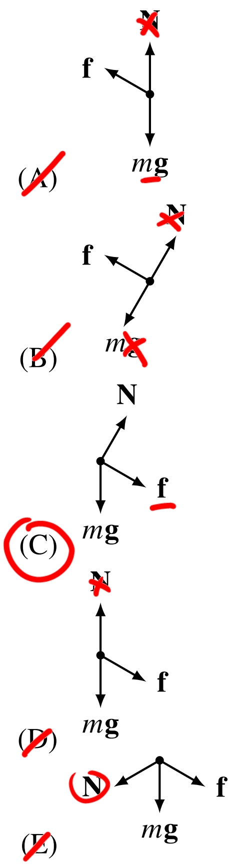
Questions 14–15

A 1 kg block is sliding up a rough 30° incline and is slowing down with an acceleration of −6 m/s². The direction up the ramp is positive.

(+)



14. Which of the following free body diagrams best represents the forces acting on the block as it slides up the plane?



C

15. The magnitude of the frictional force f between the block and the plane is most nearly

$M = 1$

(A) 1 N
(B) 2 N
(C) 3 N
(D) 4 N
(E) 5 N

A

(+) $\quad$ N $\quad$ f $\quad$ Mg

$\Sigma F = -Mg\sin\theta - f = Ma$ $\quad \sin 30° = \frac{1}{2}$

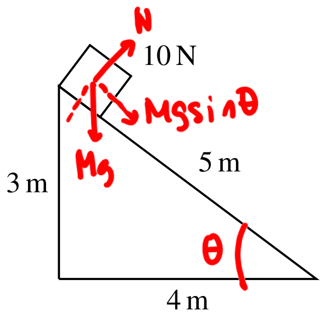
$-10\left(\frac{1}{2}\right) - f = -6$

$-5 - f = -6$ $\quad f = 1$

Questions 16–17

A 10 N block sits atop an inclined plane in the shape of a right triangle of sides 3 m, 4 m, and 5 m, as shown. The block is allowed to slide down the plane with negligible friction.

Mg $\quad$ (M=1)



N $\quad Mg\sin\theta$ $\quad$ Mg $\quad \theta$

$\sin\theta = \frac{3}{5}$ $\quad \cos\theta = \frac{4}{5}$

16. The acceleration of the block is most nearly

(A) 2 m/s²
(B) 4 m/s²
(C) 6 m/s²
(D) 10 m/s²
(E) 12 m/s²

C

$Mg\sin\theta = Ma$

$\frac{3}{5} g = a$ $\quad$ 10

17. The normal force exerted on the block by the plane is most nearly

(A) 2 N
(B) 4 N
(C) 6 N
(D) 8 N
(E) 10 N

D

$N = Mg\cos\theta$

$N = 10\left(\frac{4}{5}\right) = 8$

18. A constant force acts on a particle in such a way that the direction of the force is always perpendicular to its velocity. Which of the following is true of the particle's motion?

(A) The acceleration of the particle is increasing
(B) The acceleration of the particle is decreasing.
(C) The speed of the particle is increasing.
(D) The speed of the particle is constant.
(E) The speed of the particle is decreasing.

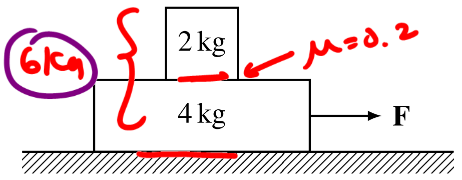
D

F_c $\quad v$

$a_c = \frac{v^2}{r}$ ← constant magnitude

Questions 19–20

A block of mass 2 kg rests on top of a larger block of mass 4 kg. The larger block slides without friction on a table, but the surface between the two blocks is not frictionless. The coefficient of friction between the two blocks is 0.2. A horizontal force **F** is applied to the 4 kg mass.



6kg $\quad \mu = 0.2$

19. What is the maximum force that can be applied such that there is no relative motion between the two blocks?

both blocks have the same acceleration.

(A) zero
(B) 1 N
(C) 2 N
(D) 4 N
(E) 12 N

$F = m_T a = (6)(2) = 12\text{ N}$

N $\quad$ 2kg $\quad F_s$ $\quad$ Mg

max. acceleration when F_s is also maximum.

$\max F_s = \mu N = \mu Mg$

$= (0.2)(2)(10)$

$= 4\text{ N}$

$a = \frac{F_s}{m} = \frac{4}{2} = \mu g = 2\text{ m/s}^2$

20. What is the acceleration of the 2 kg block relative to the 4 kg block if a force is applied to the 4-kg block that causes the 4 kg block to accelerate at 3 m/s² to the right?

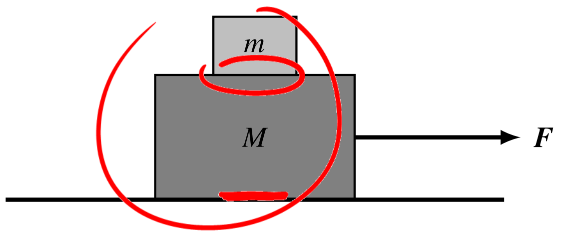
(A) 1 m/s² to the right
(B) 1 m/s² to the left
(C) 2 m/s² to the right
(D) 2 m/s² to the left
(E) zero

2kg → 2 m/s² $\quad$ 4kg → 3 m/s²

1 m/s² ← $\quad$ 2 $\quad$ 4

AP PHYSICS C: DYNAMICS
SECTION II
4 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.



1. A block with mass m sits on a block of mass M that is resting on a frictionless table, as shown below. The coefficient of friction between the blocks are $\mu_s = 0.30$ and $\mu_k = 0.20$.

   same acceleration!

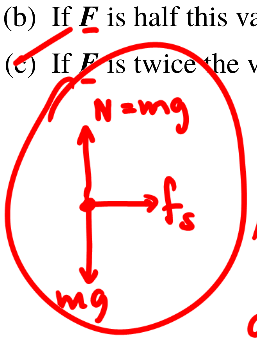
   (a) What is the maximum force $\boldsymbol{F}$ that can be applied if the block on the top is not to slide on the block on the bottom.

   (b) If $\boldsymbol{F}$ is half this value, find the acceleration of each block and the force of friction acting on each block.

   (c) If $\boldsymbol{F}$ is twice the value found in (a), find the acceleration of each block.

$\frac{F}{2}$

a) m:



$f_s = ma$ max. acceleration occurs at max f_s

$\mu_s N = ma$

$\mu_s mg = ma$

$a_{max} = \mu_s g = 3\ m/s^2$

$F = (M+m)a$

$\boxed{F = \mu_s(M+m)g}$

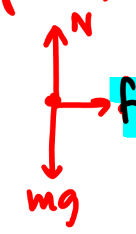
0.30

b) if F is half the original value → no slipping

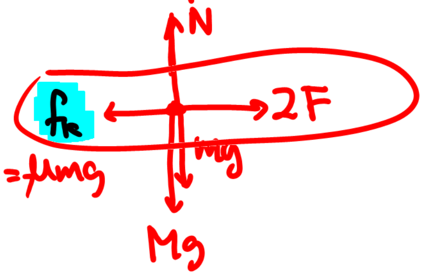
$\boxed{a = \frac{1}{2}\mu_s g}$

c) if applied force is 2F → the blocks will slip!!

m:



$f_k = \mu_k N = \mu_k mg$



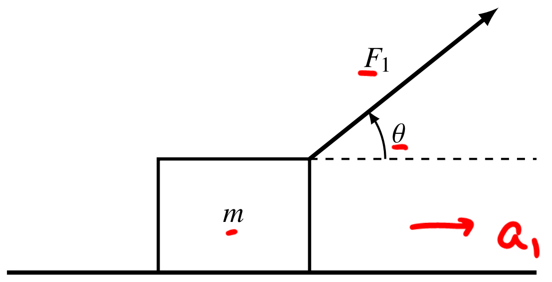
$\mu_k mg = ma$

top block: $\boxed{a = \mu_k g = 2\ m/s^2}$

bottom block: $2F - f_k = Ma$

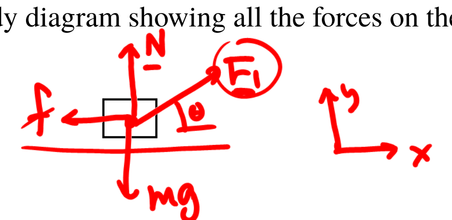
$2F - \mu_k mg = Ma$

bottom block: $\boxed{a = \frac{2F - \mu_k mg}{M}}$



2. A block of mass m is pulled along a rough horizontal surface by a constant applied force of magnitude F_1 that acts at an angle θ to the horizontal, as indicated above. The acceleration of the block is a_1. Express all algebraic answers in terms of m, F_1, θ, a_1, and fundamental constants. g

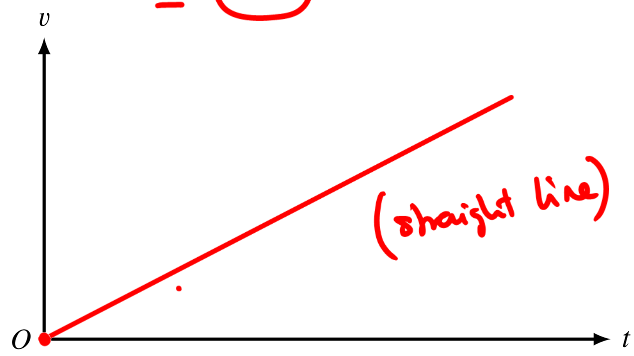
(a) On the figure below, draw and label a free-body diagram showing all the forces on the block.

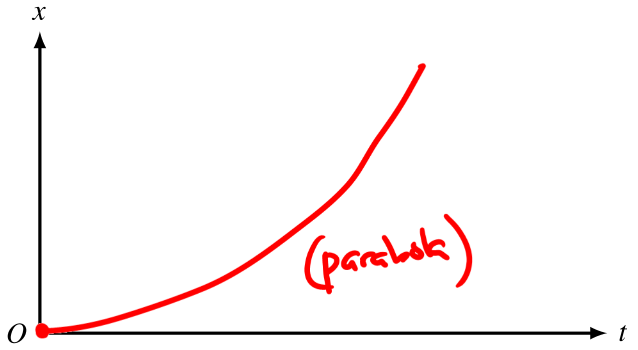


(b) Derive an expression for the normal force exerted by the surface on the block.

(c) Derive an expression for the coefficient of kinetic friction μ between the block and the surface.

(d) On the axes below, sketch graphs of the speed v and displacement x of the block as functions of time t if the block started from rest at $x = 0$ and $t = 0$.





(e) If the applied force is large enough, the block will lose contact with the surface. Derive an expression for the magnitude of the greatest acceleration a_{max} that the block can have and still maintain contact with the ground.

just as it loses contact $N=0. \rightarrow f=0$

$F_1\sin\theta - mg = 0$

$F_1 = \frac{mg}{\sin\theta}$ $\rightarrow$ $F_1\cos\theta = ma$

$\frac{\cancel{m}g}{\sin\theta}\cos\theta = \cancel{m}a$

$\boxed{a = g\cot\theta}$

(b) Derive an expression for the normal force exerted by the surface on the block.

$\Sigma F_y = N + F_1\sin\theta - mg = 0$

$\boxed{N = mg - F_1\sin\theta}$

(c) Derive an expression for the coefficient of kinetic friction μ between the block and the surface.

$\Sigma F_x = ma \rightarrow F_1\cos\theta - f = ma$

$f = \mu N$
$= \mu(mg - F_1\sin\theta)$

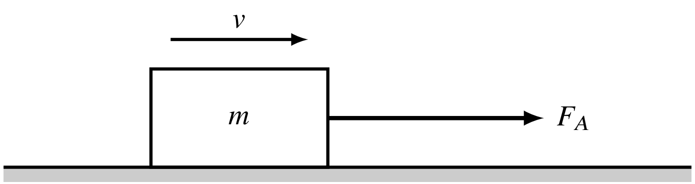
$F_1\cos\theta - \mu(mg - F_1\sin\theta) = ma$

$\mu(mg - F_1\sin\theta) = F_1\cos\theta - ma \rightarrow \boxed{\mu = \frac{F_1\cos\theta - ma}{mg - F_1\sin\theta}}$

$a = \frac{F_1\cos\theta - \mu(mg - F_1\sin\theta)}{m}$ constant!!

$\frac{F_1\cos\theta - \mu mg + F_1\sin\theta}{m}$

a maximum when $\mu mg = 0$.



3. A box of mass m initially at rest is acted upon by a constant applied force of magnitude F_A, as shown in the figure above. The friction between the box and the horizontal surface can be assumed to be negligible, but the box is subject to a drag force of magnitude kv where v is the speed of the box and k is a positive constant. Express all your answers in terms of the given quantities and fundamental constants, as appropriate.

   (a) The dot below represents the box. Draw and label the forces (not components) that act on the box.

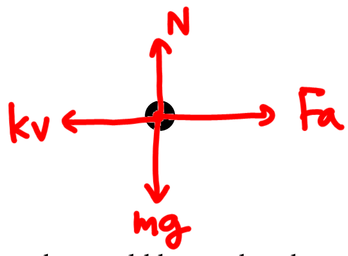
b) $F_{net} = ma$

$F_a - kv = ma$

$$\boxed{F_a - kv = m\frac{dv}{dt}}$$

c) $F_a - kv = 0$

$$\boxed{v = \frac{F_a}{k}}$$



   (b) Write, but do not solve, a differential equation that could be used to determine the speed v of the box as a function of time t. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

   (c) Determine the magnitude of the terminal velocity of the box. $\rightarrow a = 0$ $\quad \frac{dv}{dt} = 0$

   (d) Use the differential equation from part (b) to derive the equation for the speed v of the box as a function of time t. Assume that $v = 0$ at time $t = 0$.

   (e) On the axes below, sketch a graph of the speed v of the box as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

$F_a - kv = m\frac{dv}{dt}$

$\downarrow$

$dt = m\frac{dv}{F_a - kv}$

$\int_0^t dt = m\int_0^v \frac{dv}{F_a - kv}$

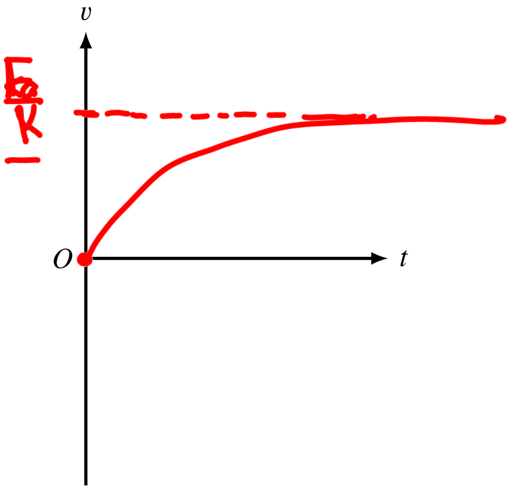
$t = \frac{-m}{k}\left(\ln\left[F_a - kv\right]\right)\Big|_0^v$

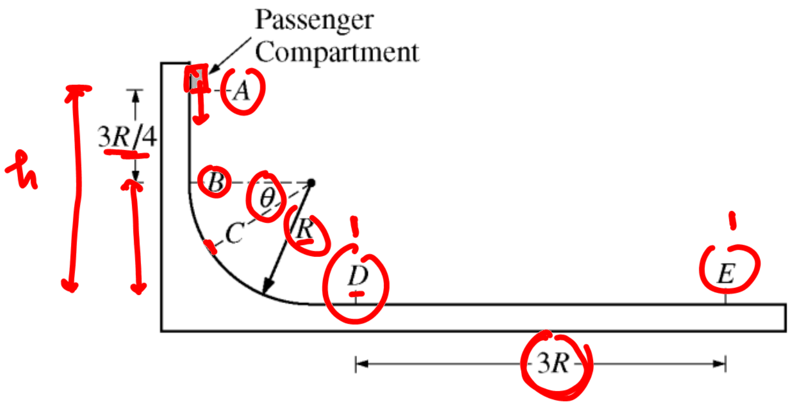
$\frac{-mt}{k} = \ln\frac{F_a - kv}{F_a}$

$e^{\frac{-mt}{k}} = \frac{F_a - kv}{F_a}$

$\downarrow$

$$\boxed{v(t) = \frac{F_a}{k}\left(1 - e^{-kt/m}\right)}$$



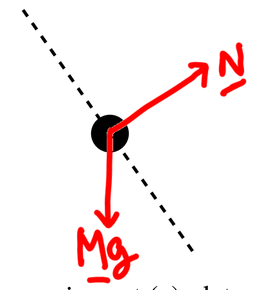


4. An amusement park ride features a passenger compartment of mass M that is released from rest at point A, as shown in the figure above, and moves along a track to point E. The compartment is in free fall between points A and B, which are a distance of $3R/4$ apart, then moves along the circular arc of radius R between points B and D. Assume the track is frictionless from point A to point D and the dimensions of the passenger compartment are negligible compared to R.

   (a) On the dot below that represents the passenger compartment, draw and label the forces (not components) that act on the passenger compartment when it is at point C, which is at an angle θ from point B.

$F_c = N - \underline{Mg\cos\theta}$

θ N mgcosθ Mg



c) $Mgh = \frac{1}{2}Mv^2$

$v = \sqrt{2gh}$

$\boxed{V = \sqrt{\frac{7gR}{2}}}$

   (b) In terms of θ and the magnitudes of the forces drawn in part (a), determine an expression for the magnitude of the centripetal force acting on the compartment at point C. If you need to draw anything besides what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

   (c) Derive an expression for the speed v_D of the passenger compartment as it reaches point D in terms of M, R, and fundamental constants, as appropriate.

A force acts on the compartment between points D and E and brings it to rest at point E.

   (d) If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction μ between the compartment and the track.

Now consider the case in which there is no friction between the compartment and the track, but instead the compartment is brought to rest by a braking force $-k\mathbf{v}$, where k is a constant and $\mathbf{v}$ is the velocity of the compartment. Express all algebraic answers to the following in terms of M, R, k, v_D, and fundamental constants, as appropriate.

$\sqrt{\frac{7}{2}gR}$

   (e) Write, but do NOT solve, the differential equation for $v(t)$.

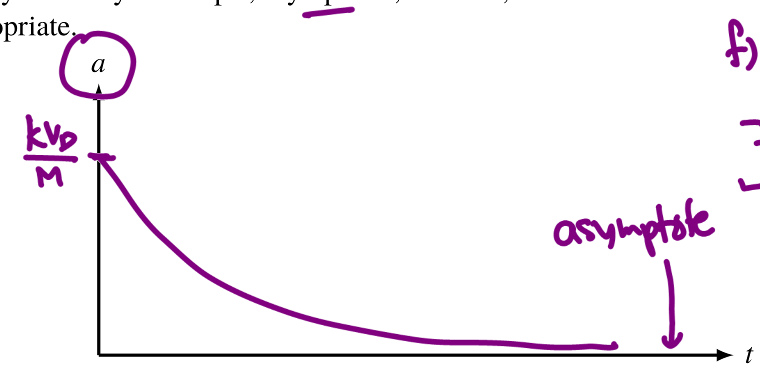
   (f) Solve the differential equation you wrote in part (e).

   (g) On the axes below, sketch a graph of the magnitude of the acceleration of the compartment as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

e) N $\vec{v}$ $k\vec{v}$ Mg

$-kv = ma$

$\boxed{-kv = m\frac{dv}{dt}}$



f) $-kv = m\frac{dv}{dt}$

$-\frac{k}{m}dt = \frac{dv}{v}$

$-\frac{kt}{m} + C = \ln v$

$V = e^{-\frac{kt}{m} + C}$

$\boxed{V = V_D e^{-\frac{kt}{m}}}$

$a = \frac{dv}{dt} = -\frac{kv_D}{m}e^{-\frac{kt}{m}}$ $|a| = \frac{kv_D}{m}e^{-\frac{kt}{m}}$

A force acts on the compartment between points D and E and brings it to rest at point E.

(d) If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction μ between the compartment and the track.

$N = mg$

$\mu N = \mu mg$

Mg

$\cancel{M}a = -\mu \cancel{M}g$

$a = -\mu g$

$\cancel{v^2} = v_D^2 + 2ax$

$v_D^2 = -2ax$

$\frac{7}{2}g\cancel{R} = 2\mu g(3\cancel{R}) \rightarrow \boxed{\mu = \frac{7}{12}}$

$W_{net} = \Delta K$

$-\mu Mg(3R) = -\frac{1}{2}Mv_D^2$

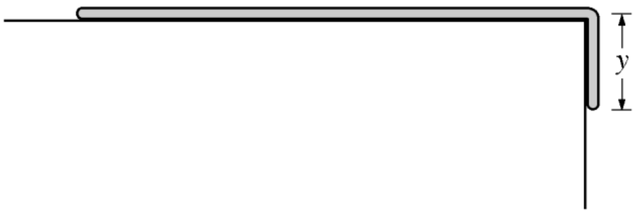
$\mu g(3R) = \frac{1}{2}\left(\frac{7}{2}gR\right)$

$\boxed{\mu = \frac{7}{12}}$

5. A block of mass $M/2$ rests on a frictionless horizontal table, as shown above. It is connected to one end of a string that passes over a massless pulley and has another block of mass $M/2$ hanging from its other end. The apparatus is released from rest.

(a) Derive an expression for the speed v_h of the hanging block as a function of the distance d it descends.

Now the block and pulley system is replaced by a uniform rope of length L and mass M, with one end of the rope hanging slightly over the edge of the frictionless table. The rope is released from rest, and at some time later there is a length y of rope hanging over the edge, as shown below. Express your answers to parts (b), (c), and (d) in terms of y, L, M, and fundamental constants.



(b) Determine an expression for the force of gravity on the hanging part of the rope as a function of y.

(c) Derive an expression for the work done by gravity on the rope as a function of y, assuming y is initially zero.

(d) Derive an expression for the speed v_r of the rope as a function of y.

(e) The hanging block and the right end of the rope are each allowed to fall a distance L (the length of the rope). The string is long enough that the sliding block does not hit the pulley. Indicate whether v_h from part (a) or v_r from part (d) is greater after the block and the end of the rope have traveled this distance. Justify your answer.

_______ v_h is greater. _______ v_r is greater. _______ The speeds are equal.