Topic 9: Fluid Mechanics

Advanced Placement Physics 2

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Olympiads School

Disclaimer

Fluid mechanics is part of the AP Physics 2 Exam, which does not require calculus. However, in the interest in completeness, *some* calculus will be shown when deriving equations.

What is a Fluid

- The *simple* (non-scientific) definition of a fliud is anything that *flows*, which covers most gases and liquids
- The *scientific* definition of a fluid is **any substance that deforms *continuously* under oblique stress**
    - When a force (stress) is applied to a solid, it deforms until all the forces are balanced, and the deformation stops (e.g. stretching a spring)
    - When a force is applied to a fluid, it continues to deform in shape as long as the force is present
    - Fluid is continuous: it will fill all available space without gaps

Density of a Fluid

The **density** ρ of a fluid is defined as the mass of the fluid m_{fluid} per unit volume V_{fluid} that it occupies:

$$\boxed{\rho = \frac{m_{\text{fluid}}}{V_{\text{fluid}}}}$$

- Unit for density is **kilograms per meter cubed** ($\mathrm{kg/m^3}$)
- Below Mach number $M \approx 0.3$, density can be assumed to be constant throughout the fluid
- May be dependent on temperature by basic thermal expansion

Viscosity of a Fluid

Viscosity μ measures how "thick" a fluid is. It relates the rate of deformation ($\Delta u / \Delta y$) of the fluid to the shear stress τ that it experiences:

$$\boxed{\tau = \mu \frac{\Delta u}{\Delta y}}$$

- e.g. honey is more viscous than water.
- Shear stress is defined as $\tau = F/A$, with a unit of **pascal** (Pa), which is the same as for pressure
- In AP Physics 2, we will mostly ignore viscous effects, as important as they are

Hydrostatics

The fluid **pressure** on its density and depth:

$$\boxed{p = p_0 + \rho_{\text{fluid}} g z}$$

where g is the acceleration due to gravity, z is the depth below the surface, and $p_0 = 1.01 \times 10^5$ Pa is the atmospheric pressure at the surface.

- Pressure is the same in all directions
- Pressure is defined as force per unit area, and the unit is **pascal**:

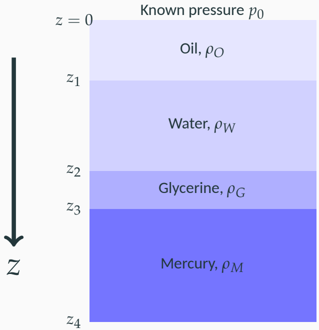
$$\boxed{p = \frac{F}{A}} \qquad 1\,\text{Pa} = 1\,\text{N}/\text{m}^2$$

Pascal's Principle

If force is applied somewhere on a container holding fluid, the pressure increases *everywhere* in the fluid, not just where the force is applied.

i.e. the pressure of the force will be transmitted into the fluid.

Pressure with Different Fluids



For the fluid surface to remain *static*, the fluid pressure on both side of the interface have to be equal. In this example:

$$
\begin{aligned}
p_1 - p_0 &= \rho_O g z_1 \\
p_2 - p_1 &= \rho_W g (z_2 - z_1) \\
p_3 - p_2 &= \rho_G g (z_3 - z_2) \\
p_4 - p_3 &= \rho_M g (z_4 - z_3) \\
\hline
p_4 - p_0 &= \sum \Delta p
\end{aligned}
$$

A Simple Example

Example 1: An aquarium is filled with water. The lateral wall of the aquarium is $40\,\mathrm{cm}$ long and $30\,\mathrm{cm}$ high. Using $10\,\mathrm{m/s^2}$ for the acceleration due to gravity, and $1\,\mathrm{g/cm^3}$ for density of water, the force on the lateral wall of the aquarium is:

(a) $36\,\mathrm{N}$

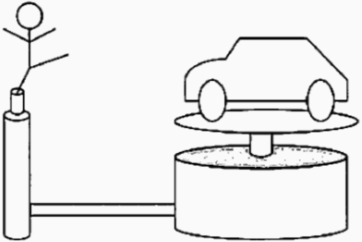
(b) $90\,\mathrm{N}$

(c) $180\,\mathrm{N}$

(d) $1500\,\mathrm{N}$

Example

Example 2: Consider the hydraulic jack in the diagram. A person stands on a piston that pushes down on a thin cylinder full of water. The cylinder is connected via pipes to a wide platform on top of which rests a 1-ton ($1000\,\mathrm{kg}$) car. The area of the platform under the car is $25\,\mathrm{m}^2$; the person stands on a $0.3\,\mathrm{m}^2$ piston. What is the lightest weight of a person who could successfully lift the car?

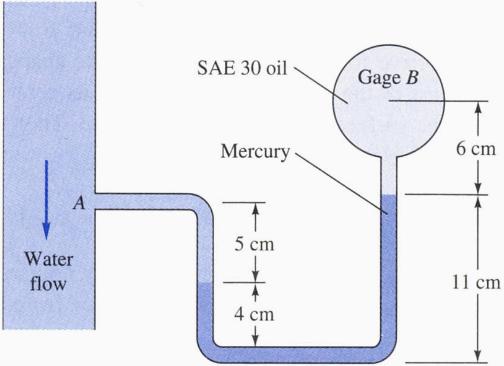


Believe it or not, there *is* someone who draws worse diagrams than Tim!

A "Manometer" Example

Example 3: Pressure gauge B is to measure the pressure at point A in a water flow, as shown in the figure on the right. If the pressure at B is $87\,\mathrm{kPa}$, estimate the pressure at A, in kPa. Assume all fluids are at $20\,°\mathrm{C}$. The densities of water, mercury and SAE 30 oil are, respectively:

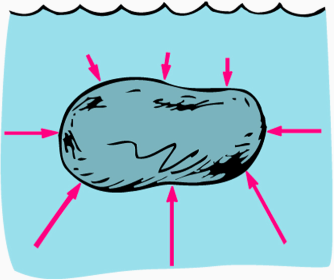
$$\rho_{\text{water}} = 1000\,\mathrm{kg/m^3}$$
$$\rho_{\text{Hg}} = 13\,600\,\mathrm{kg/m^3}$$
$$\rho_{\text{oil}} = 890\,\mathrm{kg/m^3}$$



Buoyancy: Everything Floats a Little

When an object is submerged inside a fluid

- The fluid exerts a pressure at the surface of the object
- By hydrostatics, the pressure is higher at the bottom than at the top



Buoyant Force

Using some basic calculus (well, depends on who you ask), we can find the pressure over the entire surface to find the total buoyant force $\mathbf{B}$ the fluid exerts on the object. The expression is surprisingly simple:

$$\boxed{\mathbf{B} = \rho_{\text{fluid}} g V \hat{\boldsymbol{k}} = m_{\text{fluid}} g \hat{\boldsymbol{k}}}$$

where ρ_{fluid} is the density of the displaced fluid, and V is the volume displaced. The direction of the force is upward. This equation is known as **Archimedes' principle**.

Buoyance force has a magnitude that equals to the weight of the fluid displaced by the submerged object, pointing upward.

Buoyancy

$$\boxed{\mathbf{B} = \rho_{\text{fluid}} g V \hat{\boldsymbol{k}} = m_{\text{fluid}} g \hat{\boldsymbol{k}}}$$

Buoyancy does not depend on:

- the mass of the immersed object, or
- the density of the immersed object

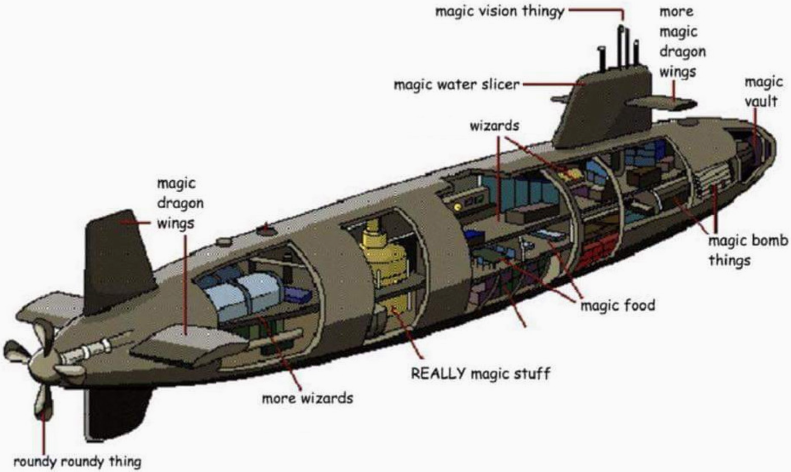
Objects immersed in a fluid have an “apparent weight” $\mathbf{W}'$ that is reduced by the buoyance force:

$$\mathbf{W}' = \mathbf{W} - \mathbf{B} = \rho' \mathbf{g} V$$

where $\rho' = \rho_{\text{obj}} - \rho_{\text{fluid}}$ is the relative density

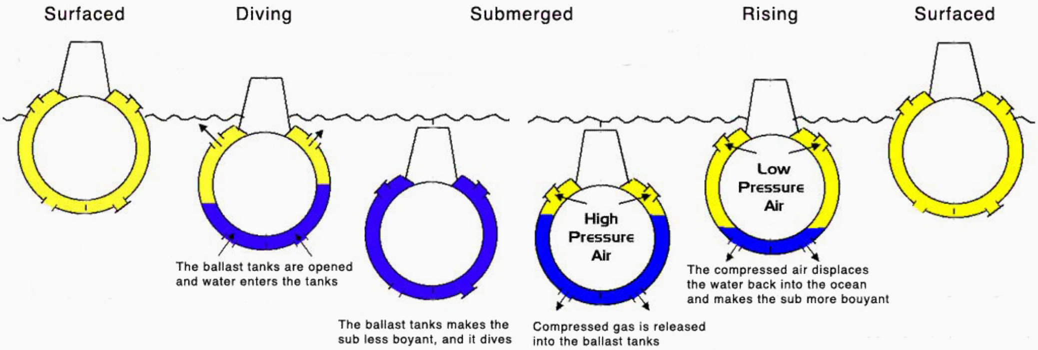
How Submarines Work

Like this:



How Submarines Work

Like all ships, a submarine does not naturally sink due to buoyancy. When a submarine submerges, water is pumped into the "ballast tanks" in the hull hull to make the submarine heavier.

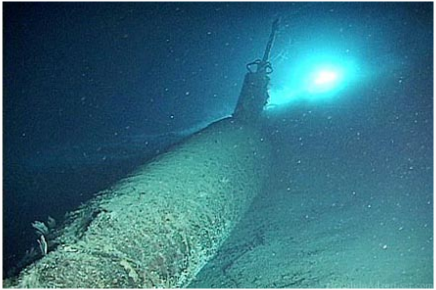


Example

Example 4: An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared with the force needed to hold the water just below the surface, what is the force needed to hold it at a deeper point?

(a) Larger
(b) The same
(c) Smaller
(d) Impossible to determine

Example



Example 5: A salvage ship tries to raise a sunken miniature submarine from the bottom of Lake Superior. The submarine and its contents have a mass of $72\,000\ \mathrm{kg}$ and a volume of $18.9\ \mathrm{m^3}$. What upward force must be applied to raise the submarine? The density of water is $1000\ \mathrm{kg/m^3}$.

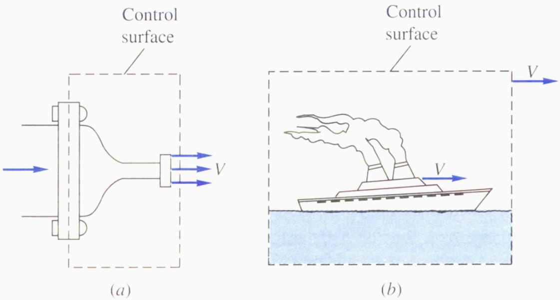
(a) $1.8 \times 10^5\ \mathrm{N}$
(b) $2.0 \times 10^5\ \mathrm{N}$
(c) $4.8 \times 10^5\ \mathrm{N}$
(d) $5.2 \times 10^5\ \mathrm{N}$

Fluid Flow

As important as it is to understand hydrostatics,
it's way more interesting when the fluid is moving!

Control Volume and Control Surfaces

A control volume “CV” is a fixed volume in which fluid is able to flow in and out of it. The surfaces of the control volume is called the control surface “CS”.



Navier-Stokes Equations

The governing equations in fluid mechanics is called **Navier-Stokes equations**, which is written in complicated vector and calculus symbols:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla (\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla p + \mathbf{f} + \mu \nabla^2 \mathbf{v}$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{v}) = -\nabla \cdot p + \frac{1}{Re\ Pr} \nabla q + \frac{1}{Re} \nabla \cdot (\tau \cdot \mathbf{v})$$

Even for university students experienced with calculus, solving these equations is still a daunting task. **But what do they actually mean?**

Continuity Equation

The first equation is called the **continuity equation**, which is the conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Meaning: The increase in fluid mass in a fixed volume containing a fluid is the amount of mass that flows in minus the volume that flows out of the volume.

Momentum Equation

The second equation is the **momentum equation**, which is the momentum-impulse theorem applied to fluids in a volume:

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \nabla(\rho\mathbf{v} \otimes \mathbf{v}) = -\nabla p + \mathbf{f} + \mu\nabla^2\mathbf{v}$$

Meaning: The increase in the total momentum of a fluid in a fixed volume is the sum of all the external forces (pressure, shear & body forces) applied to the fluids, plus the change in momentum through the flow of particles in and out of the volume.

Energy Equation

The third and final equation is the **energy equation**, which is the the work-kinetic energy theorem applied to fluids:

$$\frac{\partial e}{\partial t} + \nabla \cdot (e\mathbf{v}) = -\nabla \cdot p + \frac{1}{Re\ Pr} \nabla q + \frac{1}{Re} \nabla \cdot (\tau \cdot \mathbf{v})$$

Meaning: The increase in the internal energy of the fluid (i.e. the total kinetic energy of all the fluid particles) in a fixed volume is the work done by pressure forces, viscous forces (i.e. friction), plus the change in energy from the flow of fluid in and out of the volume.

Let’s Make Some Assumptions

For an “ideal fluid flow”, we make the following assumptions to simplify the Navier-Stokes equation:

1. The flow is **steady**
    - Flow is “time independent”, i.e. does not change with time
2. The flow is **inviscid**
    - The fluid has no viscosity
    - No friction between the fluid and the surrounding, and therefore
    - No shear stresses on the fluid
    - Only forces are pressure at the surface, and body forces from gravity
3. The flow is **incompressible**
    - Density is constant throughout

Assumptions for Ideal Fluid Flow

We will also assume that there is

- **no shaft work** done along the streamline
- **no heat transfer** along the streamline

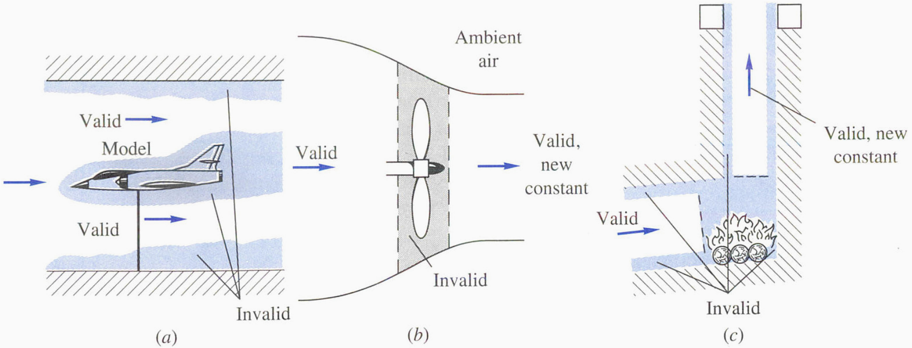
Then the N-S equations reduces to the **Bernoulli equation**

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2}$$

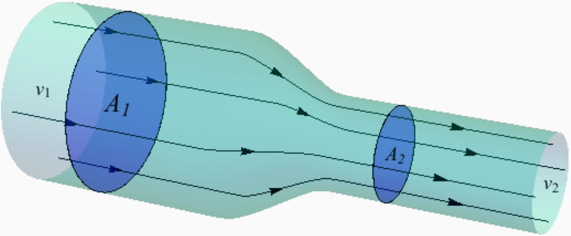
The term $\frac{1}{2}\rho v^2$ is called **dynamic pressure**, while $p + \rho g z$ is the **hydrostatic pressure**.

Bernoulli Equation

Regions where Bernoulli equation is valid:



Inlet Outlet Flow



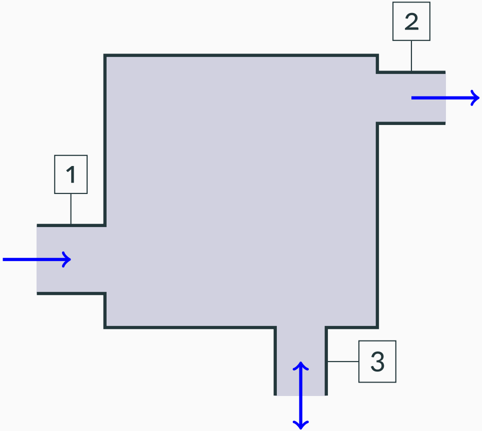
In this example, the mass flowing at the inlet is the same as the flow out of it:

$$\boxed{\rho_1 v_1 A_1 = \rho_2 v_2 A_2}$$

For constant fluid density, the ρ terms on both sides of the equation will cancel:

$$\boxed{v_1 A_1 = v_2 A_2}$$

Example: Multiple Inlet & Outlets

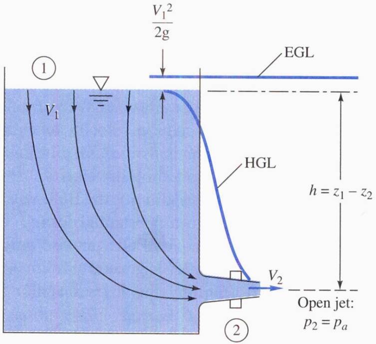


Example 6: Water at $20\,^\circ\mathrm{C}$ flows steadily through a closed tank, as shown in the figure. As section 1, $D_1 = 6\,\mathrm{cm}$ and the volume flow is $100\,\mathrm{m^3/h}$. At section 2, $D_2 = 5\,\mathrm{cm}$ and the average velocity is $8\,\mathrm{m/s}$. If $D_3 = 4\,\mathrm{cm}$, what is

1. the flow rate Q_3 in $\mathrm{m^3/h}$?
2. the average v_3 in $\mathrm{m/s}$?

Example

Example 7: Find a relation between the nozzle discharge velocity V and the tank free-surface height h. Assume frictionless flow.



The line labelled “EGL” is called the “energy grade line”, or the “Bernoulli head”, given by the equation $h_0 = z + p/\rho g + v^2/2g$. In the region where Bernoulli equation is valid, EGL is a constant.