WELCOME TO AP PHYSICS 1 & 2

Pre-requisites

- **Physics 11 and 12:** You will need to be comfortable with the topics covered in high-school level physics courses.
- **Vectors:** You need to be comfortable with vector operations, including addition and subtraction, multiplication/division by constants, as well as dot products and cross products.

If you already have a background in both differential and integral calculus, you may consider taking the AP Physics C exams instead.

Classroom Rules

- Treat each other with respect
- Raise your hands if you have a question. Don't wait too long
- E-mail me at `tleung@olympiadsmail.ca` for any questions related to physics and math and engineering
- Do ***not*** try to find me on social media

Topic 1: Kinematics

Advanced Placement Physics 1

Dr. Timothy Leung

June 29, 2020

Olympiads School

Vectors

Please refer to the handout to make sure that you are familiar with basic vector operations. We will be using a slightly more advanced notation method for this course.

Kinematics

Kinematics

Kinematics is a discipline within mechanics concerning the motion of bodies. It describes the relationship between

- Position
- Displacement
- Distance
- Velocity
- Speed
- Acceleration

Kinematics does not deal with the causes of motion.

Position

Position $\mathbf{x}$ describes the location of an object in a coordinate system. The origin of the coordinate system is called the “reference point”. The SI unit for position is **meter**, m.

$$\mathbf{x}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}$$

Vectors in 2D/3D Cartesian space are generally using the “IJK notation”

- $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ are **basis vectors** indicating the directions of the x, y and z axes. Basis vectors are **unit vectors** (i.e. length 1)
- The IJK notation does not explicitly give the magnitude or the direction of the vector (needs to be calculated using the Pythagorean theorem)

Displacement

Displacement $\Delta\mathbf{x}(t)$ is the change in position from the initial position $\mathbf{x}_0$ within the same coordinate system:

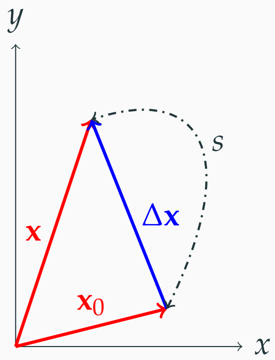
$$\Delta\mathbf{x}(t) = \mathbf{x} - \mathbf{x}_0 = (x - x_0)\hat{\boldsymbol{\imath}} + (y - y_0)\hat{\boldsymbol{\jmath}} + (z - z_0)\hat{\boldsymbol{k}}$$

- IJK notation makes vector addition and subtraction less prone to errors
- Since the reference point $\mathbf{x}_{\mathrm{ref}} = \mathbf{0}$, the position vector $\mathbf{x}$ is also its displacement from the reference point

Distance

Distance $s(t)$ is a quantity that is *related* to displacement.

- The length of the path taken by an object when it from $\mathbf{x}_0$ to $\mathbf{x}$
- A scalar quantity
- Always positive, i.e. $s \geq 0$
- Although the magnitude of the displacement vector is also a scalar, it is not necessarily the same as distance
- $s \geq |\Delta \mathbf{x}|$



Instantaneous Velocity

If position $\mathbf{x}$ is differentiable in time t, then velocity $\mathbf{v}$ can be found at any time t. The **instantaneous velocity** $\mathbf{v}$ of an object is the time rate of change of position:

$$\mathbf{v}(t) = \frac{d\mathbf{x}(t)}{dt}$$

Since $\mathbf{x}$ has x, y and z components in the $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ directions[1], we can take the time derivative in every component:

$$\mathbf{v}(t) = \frac{d\mathbf{x}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{\imath}} + \frac{dy}{dt}\hat{\boldsymbol{\jmath}} + \frac{dz}{dt}\hat{\boldsymbol{k}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\hat{\boldsymbol{k}}$$

[1]They are linearly independent!

Integrating Velocity to Get Position/Displacement

Conversely, if $\mathbf{v}(t)$ is the time rate of change of position $\mathbf{x}(t)$, then $\mathbf{x}$ is the time integral of $\mathbf{v}$:

$$\mathbf{x}(t) = \int \mathbf{v}(t)dt + \mathbf{x}_0$$

The constant of integration $\mathbf{x}_0$ is the *initial position* at $t = 0$. We can integrate each component to get $\mathbf{x}$:

$$\mathbf{x}(t) = \left(\int v_x \hat{\boldsymbol{\imath}} + \int v_y \hat{\boldsymbol{\jmath}} + \int v_z \hat{\boldsymbol{k}}\right) dt + \mathbf{x}_0$$

Average Velocity

Average velocity $\overline{\mathbf{v}}$ of an object is the finite change in position $\Delta\mathbf{x}$ over a *finite* time interval Δt:

$$\overline{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta t}$$

Like instantaneous velocity, we can find the x, y and z components of average velocity by separating components in each direction:

$$\overline{\mathbf{v}} = \frac{\Delta x}{\Delta t}\hat{\boldsymbol{\imath}} + \frac{\Delta y}{\Delta t}\hat{\boldsymbol{\jmath}} + \frac{\Delta z}{\Delta t}\hat{\boldsymbol{k}}$$

Instantaneous & Average Speed

Instantaneous speed v is the time rate of change of *distance*:

$$v = \frac{ds}{dt}$$

- Since distance of *any* path is always positive $s > 0$, instantaneous speed must also be positive
- Instantaneous speed v is the magnitude of the instantaneous velocity vector $\mathbf{v}$

Likewise, **average speed** is similar to average velocity: it is the distance travelled over a finite time interval.

$$\overline{v} = \frac{s}{\Delta t}$$

Instantaneous & Average Acceleration

In the same way that velocity is the time rate of change in position, **instantaneous acceleration** $\mathbf{a}(t)$ is the time rate of change in velocity:

$$\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = \frac{d^2\mathbf{x}(t)}{dt^2}$$

Likewise, **average acceleration** $\bar{\mathbf{a}}$ is the finite change in velocity $\Delta\mathbf{v}$ over a finite time interval Δt:

$$\bar{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{\mathbf{v} - \mathbf{v}_0}{\Delta t}$$

The unit for acceleration is $\mathrm{m/s^2}$.

Special Notation

Physicists and engineers often use a special notation when the derivative is taken with respect to *time*, by writing a dot above the variable:

- Velocity:

$$\mathbf{v}(t) = \dot{\mathbf{x}}$$

- Acceleration:

$$\mathbf{a}(t) = \dot{\mathbf{v}} = \ddot{\mathbf{x}}$$

We will use this notation whenever it is convenient

Integrating Acceleration to Get Velocity

Velocity $\mathbf{v}(t)$ is the time integral of acceleration $\mathbf{a}(t)$:

$$\mathbf{v}(t) = \int \mathbf{a}(t)dt + \mathbf{v}_0$$

Again, we can integrate each component of the vector independently:

$$\mathbf{v}(t) = \left(\int a_x \hat{\boldsymbol{\imath}} + \int a_y \hat{\boldsymbol{\jmath}} + \int a_z \hat{\boldsymbol{k}} \right) dt + \mathbf{v}_0$$

If You Are Curious

The time derivative of acceleration is called **jerk**, with a unit of $\mathrm{m/s^3}$:

$$\mathbf{j} = \frac{d\mathbf{a}}{dt} = \frac{d^2\mathbf{v}}{dt^2} = \frac{d^3\mathbf{x}}{dt^3}$$

The time derivative of jerk is **jounce**, or **snap**, with a unit of $\mathrm{m/s^4}$:

$$\mathbf{s} = \frac{d\mathbf{j}}{dt} = \frac{d^2\mathbf{a}}{dt^2} = \frac{d^3\mathbf{v}}{dt^3} = \frac{d^4\mathbf{x}}{dt^4}$$

The next two derivatives of snap are called **crackle** and **pop**, but these higher derivatives of position vector are rarely used. We will *not* be using them.

Acceleration as Functions of Velocity and Position

Acceleration may be expressed as functions of velocity and position rather than of time, if an object's motion is dominated by these forces:

- Gravitational or electrostatic forces: $a(x) = \dfrac{Gm_s}{x^2} \quad a(x) = \dfrac{kq_s}{x^2}$
- Spring force: $a(x) = -\dfrac{k}{m}x$
- Damping force: $a(v) = bv^n$ (b is a damping constant)
- Aerodynamic drag: $a(v) = \left[\dfrac{1}{2}\rho C_D A_{\mathrm{ref}}\right] v^2$

In these cases, solving for the motion quantities $x(t)$, $v(t)$ and $a(t)$ requires solving a differential equation (see kinematics handout).

Kinematic Equations

Kinematic Equations

While kinematic problems in AP Physics C exams often require calculus, these basic kinematic equations for <u>constant acceleration</u> are still a powerful tool.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

The variables of interests are:

- Initial position: x_0
- Position at time t: x
- Initial velocity: v_0
- Velocity at time t: v
- Acceleration (constant): a

Kinematic equations are sometimes called the "Big-five" or "Big-four" equations. Here, you will only be given three equations in your equation sheet. You will still be required to integrate when necessary.

Motion Graphs

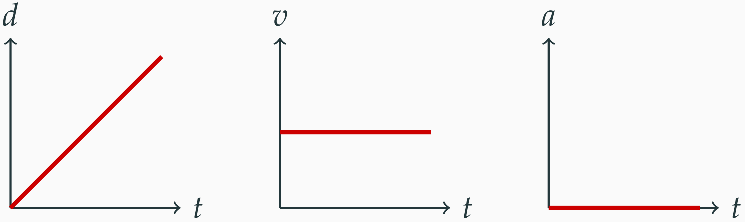
Motion Graphs

You should already be familiar with the basic motion graphs for 1D motion:

- Position vs. time $(x - t)$ graph
- Velocity vs. time $(v - t)$ graph
- Acceleration vs. time $(a - t)$ graph

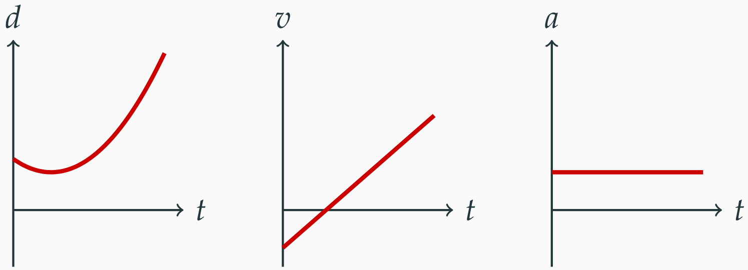
However, depending on the situation, it may be more useful to plot motion using other quantities as well.

Uniform Motion: Constant Velocity



- Constant velocity has a straight line in the $d - t$ graph
- The slope of the $d - t$ graph is the velocity v, which is constant
- The slope of the $v - t$ graph is the acceleration a, which is zero in this case

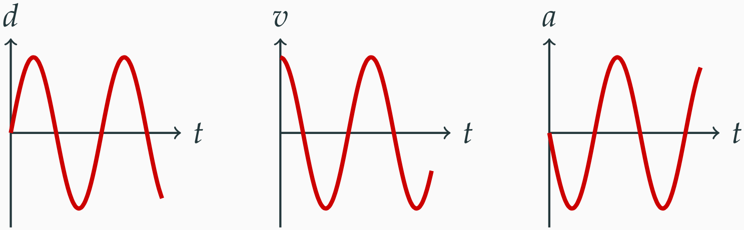
Uniform Acceleration: Constant Acceleration



- The $d - t$ graph for motion with constant acceleration is part of a *parabola*
    - If the parabola is *convex*, then acceleration is positive
    - If the parabola is *concave*, then acceleration is negative
- The $v - t$ graph is a straight line; its slope (a constant) is the acceleration

Simple Harmonic Motion

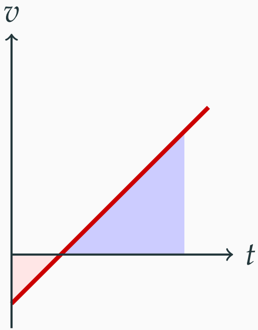
For **harmonic motions**, neither position, velocity nor acceleration are constant:



Bottom line: regardless of the type motion,

- The $v - t$ graph is the slope of the $d - t$ graph
- The $a - t$ graph is the slope of the $v - t$ graph

Area Under Motion Graphs



The area under the $v - t$ graph is the displacement $x - x_0$.

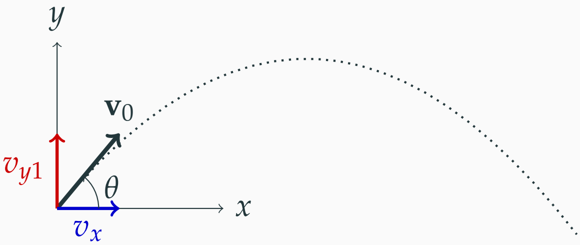
- Area *above* the time axis: $+$ displacement
- Area *below* the time axis: $-$ displacement

Likewise, the area under the $a - t$ graph is the change in velocity $v - v_0$.

Projectile Motion

Projectile Motion

A **projectile** is an object that is launched with an initial velocity of $\mathbf{v}_0$ along a parabolic trajectory and accelerates only due to gravity.



- x-axis is the *horizontal* direction, with the $(+)$ direction pointing *forward*
- y-axis is the *vertical* direction, with the $(+)$ direction pointing *up*
- The reference point is where the projectile is launched
- Consistent with the right-handed Cartesian coordinate system

Horizontal (x) Direction

No acceleration (i.e. $a_x = 0$) in the horizontal direction, therefore horizontal velocity component is constant. The kinematic equations reduce to:

$$x = v_x t = \left[v_0 \cos\theta \right] t$$

where x is the horizontal position at time t, v_0 is the magnitude of the initial velocity, $v_x = v_0 \cos\theta$ is its horizontal component.

Vertical (y) Direction

Constant acceleration due to gravity alone in the vertical direction, i.e. $a_y = -g$. (Acceleration is *negative* due to the way we defined the coordinate system.) The important equation is this one:

$$y = [v_0 \sin\theta]\, t - \frac{1}{2} g t^2$$

These two kinematic equations may also be useful:

$$v_y = [v_0 \sin\theta] - gt$$
$$v_y^2 = [v_0 \sin\theta]^2 - 2gy$$

Solving Projectile Motion Problems

Horizontal and vertical motions are independent of each other, but there are variables that are shared in both directions, namely:

- Time t
- Launch angle θ (above the horizontal)
- Initial speed v_0

When solving any projectile motion problems

- *Two* equations with *two* unknowns
- If an object lands on an incline, there will be a third equation describing the relationship between x and y

Symmetric Trajectory

A projectile's trajectory is symmetric if the object lands at the same height as when it launched.

- Time of flight
$$t_{\max} = \frac{2v_0 \sin\theta}{g}$$
- Range
$$R = \frac{v_0^2 \sin(2\theta)}{g}$$
- Maximum height
$$y_{\max} = \frac{v_0^2 \sin^2\theta}{2g}$$

The angle θ is measured above the the horizontal.

Maximum Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum range occurs at $\theta = 45^\circ$
- For a given initial speed v_0 and range R, launch angle θ is given by:

$$\theta_1 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

But there is another angle that *gives the same range*!

$$\theta_2 = 90^\circ - \theta_1$$

Relative Motion

Relative Motion

When expressing relative motion, the first subscript (A) represents the moving object, and the second subscript (B) represents the frame of reference:

$$\mathbf{v}_{AB}$$

If an airplane ("P") is traveling at $251\,\mathrm{km/h}$ [N] relative to Earth ("E"), its velocity is expressed as:

$$\mathbf{v}_{PE} = 251\,\mathrm{km/h}\ [\mathrm{N}]$$

Relative Motion

If the airplane flies in windy air ("A") we must consider the velocity of the airplane relative to air $\mathbf{v}_{PA}$ and the velocity of the air relative to Earth $\mathbf{v}_{AE}$. The velocity of the airplane relative to Earth is therefore

$$\mathbf{v}_{PE} = \mathbf{v}_{PA} + \mathbf{v}_{AE}$$

If an airplane is flying at a constant velocity of $253\,\mathrm{km/h}$ [S] relative to the air and the air velocity is $24\,\mathrm{km/h}$ [N], what is the velocity of the airplane relative to Earth?

Relative Motion

In classical mechanics, the equation for relative motion follows the **Galilean velocity addition rule**:

$$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$$

The velocity of A relative to reference frame C is the velocity of A relative to reference frame B, plus the velocity of B relative to C. If we add another frame of reference ("D"), the equation becomes:

$$\mathbf{v}_{AD} = \mathbf{v}_{AB} + \mathbf{v}_{BC} + \mathbf{v}_{CD}$$

Typical Problems

In an AP Physics C exam, questions involving kinematics usually appear in the multiple-choice section. The problems themselves are not very different compared to the Grade 12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use $g = 10\,\mathrm{m/s^2}$ in your calculations to make your lives simpler
- A lot of problems are *symbolic*, which means that they deal with the equations, not actual numbers
- Will be coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet