Topic 15: Light Waves and Optics

AP (2) and IBHL Physics

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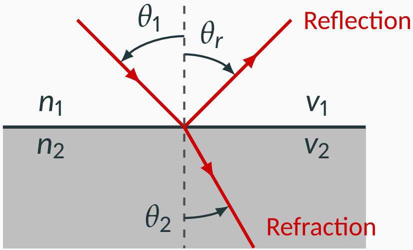
Olympiads School

Reflection and Refraction

Reflection and Refraction

We begin with a model of a beam of light traveling toward an interface between two "indexed material":



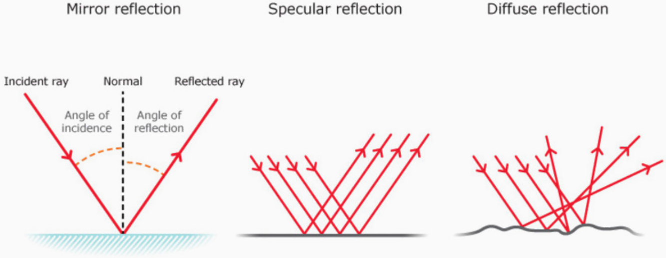
The **index of refraction** (or **refractive index**, **index**) of the two media is defined as the ratio of speeds of light in a vacuum c_0 and in the medium c:

$$n = \frac{c_0}{c} \geq 1$$

Reflection of Light

In the **law of reflection**, the incident ray, the reflected ray, and the normal to the surface of the mirror all lie in the same plane, and the angle of reflection θ_r is equal to the angle of incidence θ_1:

$$\boxed{\theta_r = \theta_1}$$



Specular Reflection Example

This photo of Lake Matheson shows specular reflection in the water of the lake with reflected images of Aoraki/Mt Cook (left) and Mt Tasman (right). The very still lake water provides a perfectly smooth surface for this to occur.

Intensity of Light Reflecting from

When the incident and reflected angles normal, i.e. $\theta_1 = \theta_r = 0°$, the reflected intensity of light I is related to the incident intensity I_0 by the indices of the two material (n_1 and n_2):

$$I = \left[\frac{n_1 - n_2}{n_1 + n_2}\right]^2 I_0$$

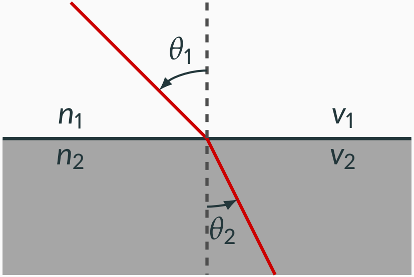
The intensity of a wave is the power P over the area A that the wave passes through:

$$I = \frac{P}{A}$$

The reflected intensity is lower than the incident, indicating that some of the energy from the incident wave is transmitted into the second medium.

Refraction of Light Through a Medium

Refraction occurs when light is transmitted from one medium to another at an oblique angle. The wave changes direction due to the difference in the speed of light in the two media.



Law of Refraction

Snell's law (or **law of refraction**) relates the refractive indices *n* of the two media to the directions of propagation in terms of the angles θ to the normal

$$\boxed{n_1 \sin \theta_1 = n_2 \sin \theta_2}$$

This equation holds for the refraction of any kind of wave incident on a boundary surface separating two media (e.g. surface ocean wave at two depths)

Index of Refraction

When light enters a new medium, the *frequency* remains the same: the atoms in the new medium would absorb and then radiate the light at the same frequency. However, the *speed* of the radiated wave is different, therefore a different *wavelength* is observed:

$$\boxed{\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}}$$

You can work this out using the relationship between wave speed, frequenc and wavelength: $v = f\lambda$

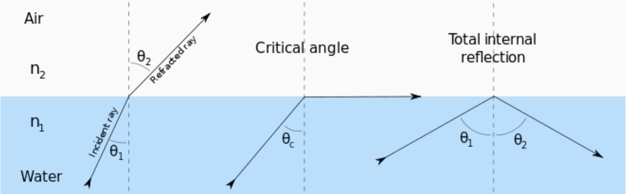
Index of Refraction of Common Materials

Material	n	Material	n
Vacuum	1	Ethanol	1.362
Air	1.000277	Glycerine	1.473
Water at 20 °C	1.33	Ice	1.31
Carbon disulfide	1.63	Polystyrene	1.59
Methylene iodide	1.74	Crown glass	1.50-1.62
Diamond	2.417	Flint glass	1.57-1.75

The values given are *approximate* and do not account for the small variation of index with light wavelength. That's called **dispersion**.

Total Internal Reflection

Total internal reflection ("TIR") can occur when light enters from a medium with high index to another with low index (i.e. $n_1 > n_2$).



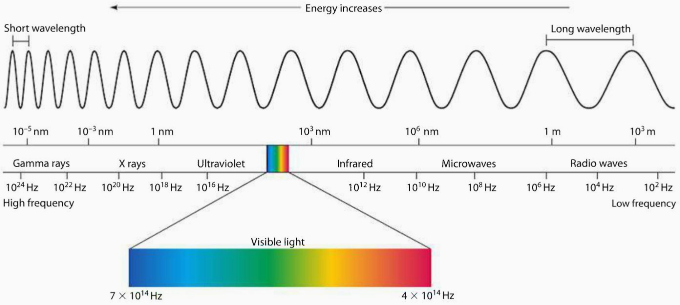
The **critical angle** can be found for when the refracted angle is 90°:

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

For water-air interface, θ_c=48.6°

Color of Light and Wavelength

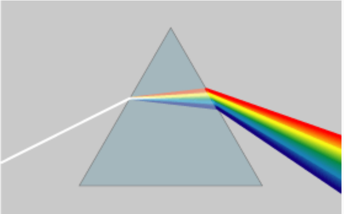
Human eyes perceive different frequencies of light as different colors. The visible spectrum of light range from about 380 nm (violet) to about 700 nm (red).



A good question to ask: where is *purple*?

Dispersion of Light

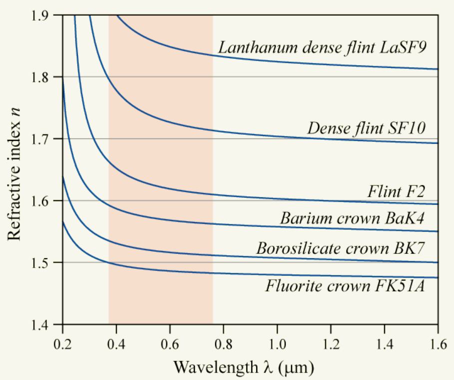
The index of refraction of a material varies slightly with wavelength λ. This is called **dispersion**. The dispersion of light through a prism is why we can see the rainbow colors from a beam of white light:



The index of refraction for shorter wavelengths is always higher than for longer wavelengths.

Wavelength Dependency of Index of Refraction

We can see that for different kinds of glass, the index of refraction can vary significantly through the visible spectrum.

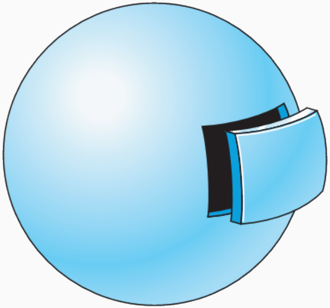


Lenses and Mirrors

Spherical Mirror

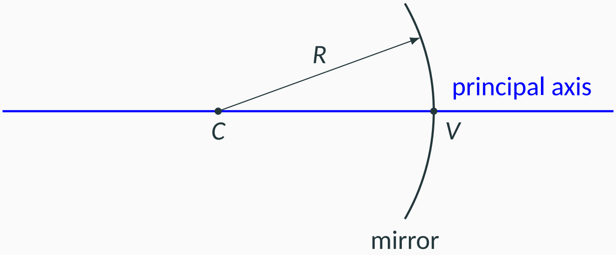
We can imagine a **spherical mirror** to be a sphere with smooth light-reflecting surfaces inside and out.

- A **concave mirror** is the surface *inside* the spherical mirror
- A **convex mirror** is the surface *outside* the spherical mirror



Spherical Mirror

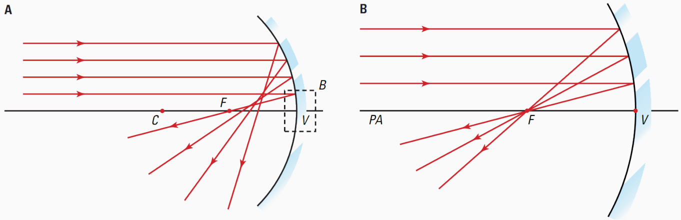
The cross-section diagram of the mirror:



- **Center of curvature** C: center of the imaginary sphere with the same radius as the mirror
- **Radius of curvature** R is any straight line from the center of curvature to the curved surface
- **Vertex** V is the geometric center of the mirror surface
- **Principal axis** “PA”: a straight that passes through V and C

Spherical vs. Parabolic Mirror

- Rays that are closed to the PA are called **paraxial rays**; they converge at a single point called the **focal point** *F*
- Rays that are far away from the PA are called **nonparaxial rays**; they converge to different points

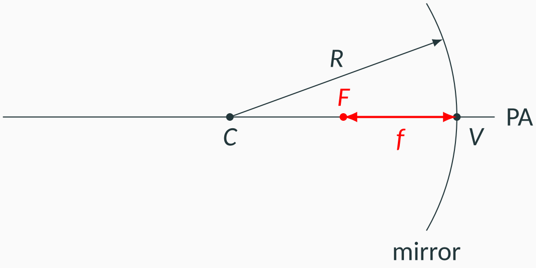


In order for all beams of light to converge to the focal point, a **parabolic mirror** must be used instead of a spherical mirror.

Focal Point and Focal Length

The focal point F is exactly half way between the center of curvature and the mirror, and the distance to the mirror is called the **focal length** f:

$$\boxed{f = \frac{1}{2}r}$$



Mirror Equation

In terms of the focal length, the **mirror equation** is expressed as

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

Quantity	**Symbol**	**SI Unit**
Distance to object	s	m
Distance to image	s'	m
Focal length	f	m

This equation is derived mathematically using basic geometry and the small-angle approximation.

Sign Convention for the Mirror Equation

When working with the mirror equation, we use the following sign convention:

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

s	$+$	object is in front of the mirror (real object)
	$-$	object is behind the mirror (virtual object)
s'	$+$	image is in front of the mirror (real image)
	$-$	image is in behind the mirror (virtual image)
R, f	$+$	center of curvature is in front of the mirror (concave mirror)
	$-$	center of curvature is behind the mirror (convex mirror)

Lateral Magnification

Using the sign convention as the mirror equation, the **lateral magnification** of the object is given by:

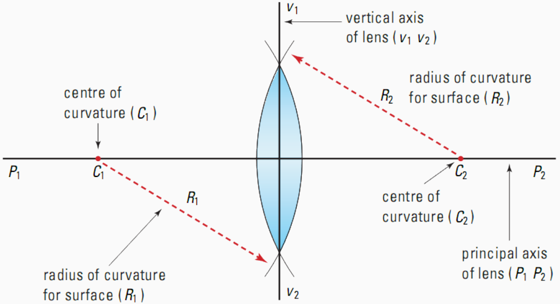
$$\boxed{m = \frac{h'}{h} = -\frac{s'}{s}}$$

Quantity	Symbol	SI Unit
Magnification factor	m	(no units)
Object & image height	h, h'	m (metres)
Distance to object & image	s, s'	m (metres)

- $m > 0$:image is upright; $m < 0$: image is inverted
- $|m| > 1$: image is enlarged; $|m| < 1$: image is reduced
- If both s and s' are positive (on the same side), then the image is inverted

Geometry of Convex Lenses

Here are some defining geometric properties for the **thin lens**.



The surface that light first passes through has the radius R_1 while the second surface has R_2.

Focal Length of the Thin Lens

The focal length f of a thin lens is defined using the **lens-makers' equation**:

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Quantity	**Symbol**	**SI Unit**
Focal length	f	(no units)
Index of refraction of the lens	n	m
Radii of curvature	R_1, R_2	m

Thin Lens Equation

The **thin-lens equation** is exactly the same as the mirror equation:

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

Quantity	Symbol	SI Unit
Distance to object	s	m
Distance to image	s'	m
Focal length	f	m

Sign Convention for the Thin Lens Equation

When working with the thin-lens equation, we use the following sign convention:

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

s	$+$	real object: for objects in front of the lens (incident side)
	$-$	virtual object: for objects behind the lens (transmission side)
s'	$+$	real image: behind the lens (transmission side)
	$-$	virtual image: in front of the lens (virtual image)
R, f	$+$	center of curvature is on the transmission side
	$-$	center of curvature on the incident side

Lateral Magnification

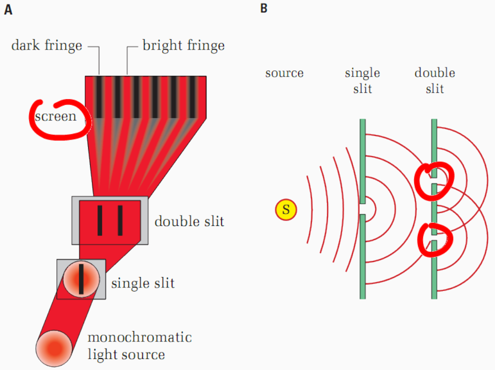
Likewise, the **lateral magnification** of the image is given by the same equation as the mirror, but this time using the sign convention for the thin lens:

$$\boxed{m = \frac{h'}{h} = -\frac{s'}{s}}$$

Quantity	**Symbol**	**SI Unit**
Magnification factor	m	(no units)
Object height	h	m
Image height	h'	m
Distance to object	s	m
Distance to image	s'	m

Interference of Light

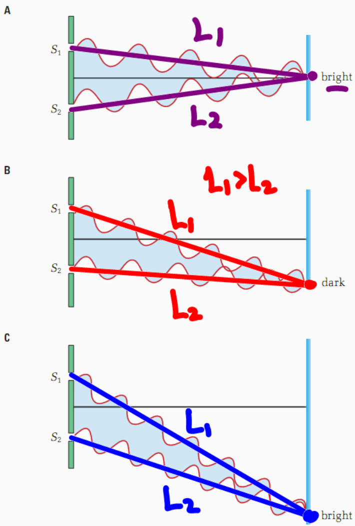
Thomas Young's Double-Slit Experiment



- **Monochromatic light** light with a single color (frequency); the light source can be a laser, LED , or gas lamp (most likely what Young used)
- **Slit:** an opening; also called an **aperture**
- The **screen** far away from the slits is also called the **projection**

Double-slit experiment showed that light causes interference, just like any other wave

Thomas Young's Double-Slit Experiment

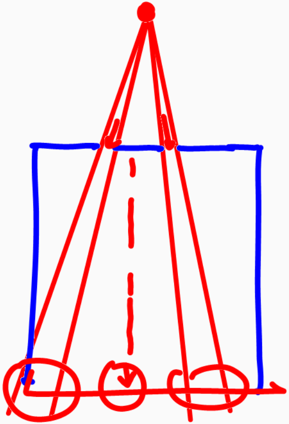


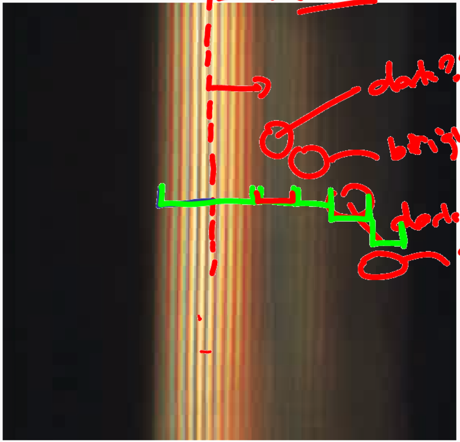
$L_1 = L_2 \rightarrow |\underbrace{L_1 - L_2}_{\Gamma \text{ - path difference}}| = 0$] constructive interference

- At **A**, the path from slits S_1 and S_2 are the same, therefore we have **constructive interference** and the projection is bright $\Gamma\ |L_1 - L_2| = \frac{\lambda}{2}$ ← out of phase by 180°
- At **B**, the path from S_1 and S_2 are diffed by half a wavelength, and therefore there is **destructive interference** and the projection is dark
- At **C**, the path from S_1 and S_2 are diffed by one wavelength, and therefore there is **constructive interference** again, and again, the projection is bright

$\Gamma = L_1 - L_2 = \lambda$ ← in phase

Interference Pattern: Bright and Dark Fringes

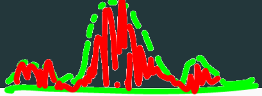




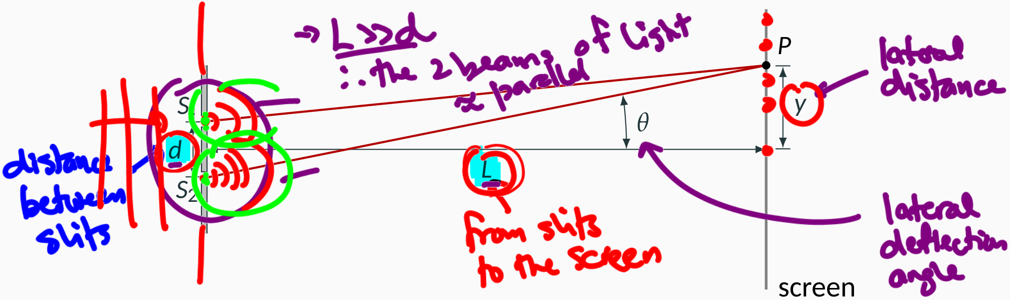
near center, fringes are similar in brightness.

The “bright fringes” are from constructive interference; the “dark fringes” are from destructive interference.

Geometry of the Two-Slit Interference



Two coherent (in phase) sources pass through two narrow slits (that can be treated as point sources). The light from the slits emerge at the screen *P* at a distance *L* away.



The two beams travel a slightly different distance:

- **Constructive interference:** path difference is an integer multiple of λ — n
- **Destructive interference:** path difference is a half-integer multiple of λ — $(n+\frac{1}{2})$

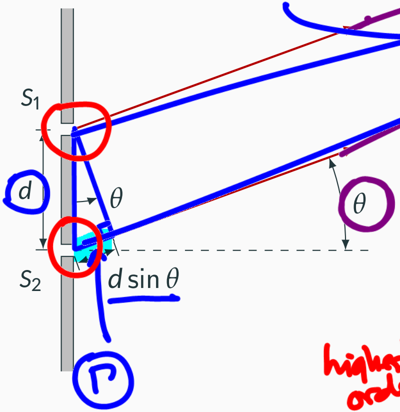
Geometry of the Two-Slit Interference

Using basic geometry, we can see that the path difference from the two slit to the projection is $d \sin\theta$.

$\Gamma = n\lambda$

Constructive maxima occur when:

$$n\lambda = \underbrace{d\sin\theta_n}_{\Gamma}$$



Destructive minima occur when:

$$\left(n+\frac{1}{2}\right)\lambda = \underbrace{d\sin\theta_n}_{\Gamma}$$

- $n = 0, 1, 2, 3 \ldots n_{max}$ is called the **order number**
- n_{max} can be found by setting $\sin\theta = 1$

highest order fringe

- The number of maxima is $2n_{max} + 1$

$-n_{max} \rightarrow +n_{max}$

Approximation of The Wavelength of Light

For small angles, we can apply the **small angle approximation** where

$$\sin\theta \approx \tan\theta \approx \theta$$

to find that the n-th bright fringe is located at y_n, which allows us to calculate the distance between fringes Δy:

distance between fringes.

bright $\rightarrow$ $\boxed{y_n \approx n\frac{\lambda L}{d}}$ $\rightarrow$ $\boxed{\Delta y \approx \frac{\lambda L}{d}}$ $\rightarrow$ $\lambda \approx \frac{d\,\Delta y}{L}$

This equation is used to estimate the wavelength of light based on the distances between bright fringes (or dark fringes).

dark $\rightarrow$ $y_n = \left(n+\frac{1}{2}\right)\frac{\lambda L}{d}$

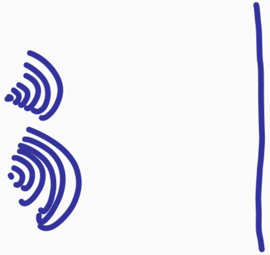
only valid for $\theta \ll 1$ (near center line)

Important Notes

- We usually apply the double-slit problem to light, but the problem can be applied to any wave (e.g. EM waves, sound waves, ocean waves) as well
- The sources don't actually need to be slits; any point source will do
- The projection/screen doesn't need to be a real screen either; it just has to be a line where wave intensity can be measured

Remember:

- integer multiple = constructive maxima
- half-integer multiple = destructive minima

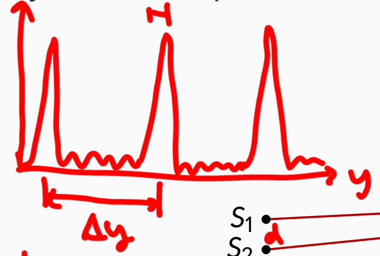


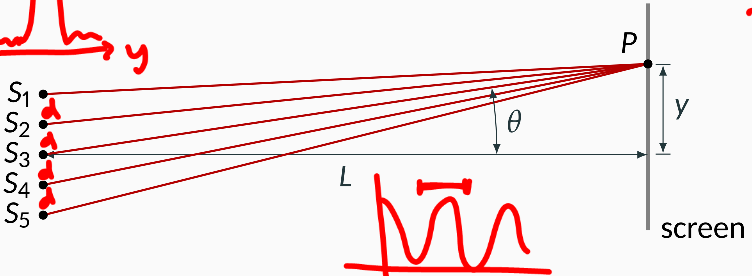
Interference of Multiple Equally Spaced Point sources

When there are multiple equally-spaced point sources (e.g. diffraction grating) we can just use the equation for two-slits:

$$\boxed{n\lambda = d\sin\theta} \rightarrow \boxed{y_n \approx \frac{n\lambda L}{d}} \rightarrow \boxed{\Delta y \approx \frac{\lambda L}{d}}$$

near center





The interference pattern gets sharper with increasing number of sources, and the bright fringes are narrower.

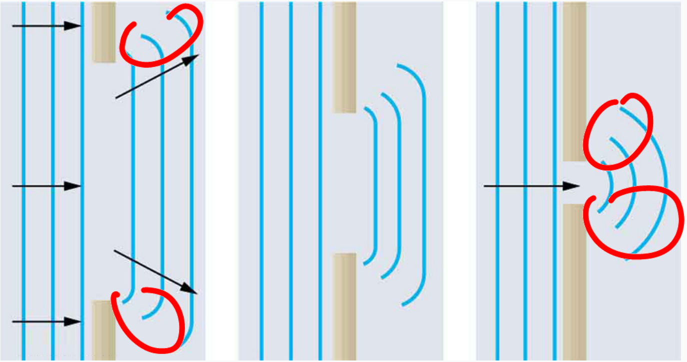
Diffraction

Diffraction of Waves

When a wave goes through an small opening, it **diffracts**. This happens with sound waves, ocean waves...and light.

The photo is from the Port of Alexandria in Egypt. The shape of the entire harbor is created because of diffraction of ocean wave.

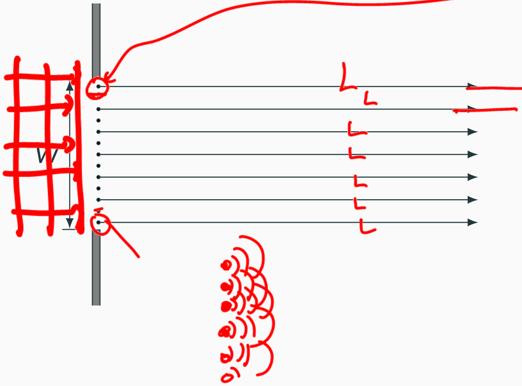
Diffraction of Waves



The smaller the opening (compared to the wavelength of the incoming wave) the greater the diffraction effects.

Geometry for Single-Slit Diffraction

To examine the geometry for the single-slit diffraction problem, we treat the wave passing through the slit of width W as an infinite series of point sources at the slit.



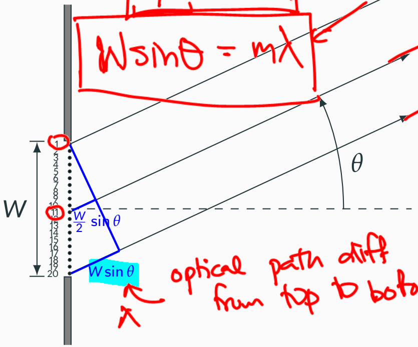
- Light from the wavelets traveling perpendicular to the aperture do not interfere with one another
- Therefore, there is a bright fringe at the middle, called the **central diffraction maximum**.

Geometry for Single-Slit Diffraction

Using the same analysis from the double-slit problem, we find that the path difference the wavelets at the top and bottom edges is $W \sin\theta$

$\boxed{\Gamma = m\lambda}$ ← destructive minima

$\boxed{W\sin\theta = m\lambda}$

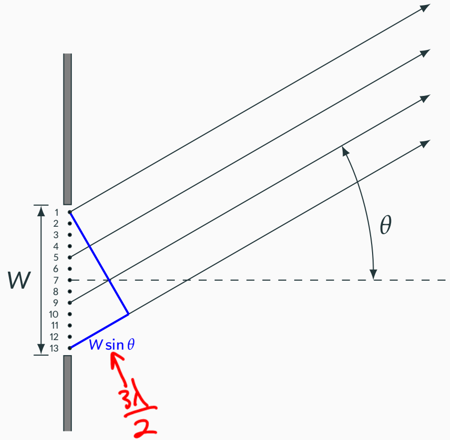


optical path diff from top to bottom.

- At some angle θ, the path difference between 1 and 20 will be an integer multiple of the wavelength ($m\lambda$)
- In this case, the path difference between 1 and 11 is a half-number multiple of the wavelength (i.e. destructive interference) and they cancel each other
- Similarly, 2 cancels 12, 3 cancels 13...

RESULT: Complete destructive interference

Geometry for Single-Slit Diffraction



- At some other angle θ, the path difference between the top and bottom is $W \sin\theta = \frac{3}{2}\lambda$
- 1 and 5 differ by $\frac{\lambda}{2}$, so they cancel (as do 2 and 6, 3 and 7, 4 and 8, 9 and 13)
- But some of the beams will not, so we have a "bright fringe" at the projection
- This bright fringe is not as bright as the central one because of the destructive interference

Dark and Bright Fringes

Destructive mimina exist on the screen at regular, whole-numbered intervals ($m = 1, 2, 3 \ldots$):

$$\boxed{m\lambda = W \sin\theta_m}$$

while bright fringes exist on the screen at regular, half-numbered intervals:

NOT constructive max, it's actually partial destructive interference

$$\boxed{\left(m + \frac{1}{2}\right)\lambda = W \sin\theta_m}$$

The equations look very similar to the double-slit equations for, but with dark and bright fringes in reverse, so be *very* careful when you use them!

Bright and Dark Fringes

The location of the bright fringes on the screen is determined by applying the small-angle approximation equation:

$$\boxed{y_m = \left(m + \frac{1}{2}\right)\frac{\lambda L}{W}} \leftarrow$$

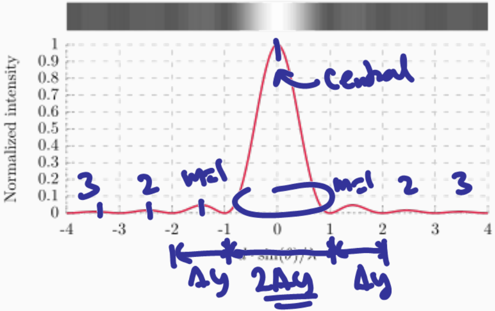
While for the dark fringes,

$$\boxed{y_m = \frac{m\lambda L}{W}} \leftarrow$$

Again, the equations look to be the reverse of the two-slit problem.

Single-Slit Diffraction, A Summary

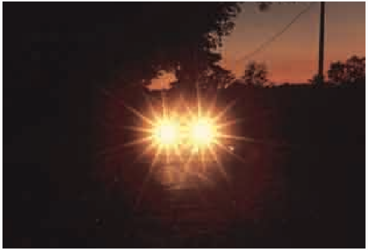
- Similar to the double-slit interference, single-slit diffraction projects a series of alternating bright fringes (“maxima”) and dark fringes (“minima”) in the far field
- The bright fringe in the middle (“central diffraction maximum”) is twice as wide and very bright
- Subsequent bright fringes on either side (“higher-order maxima”) are much dimmer because of the partial destructive interference



Optical Resolution

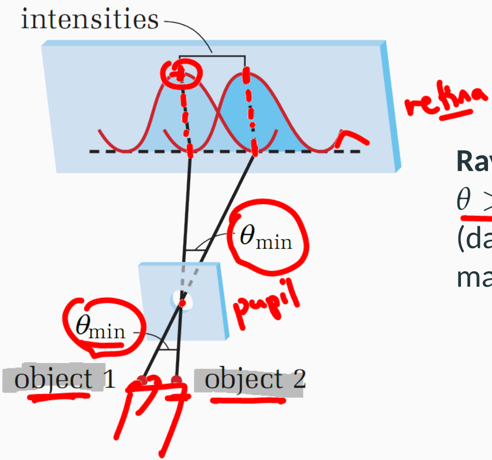
Optical Resolution

The ability of an optical instrument (e.g. the human eye, microscope, camera) to distinguish two distinct objects


When light from any object passes through an “optical instrument”, it *diffracts*, therefore “blurring” the object

Optical Resolution



Rayleigh limit: Two objects are resolved if the angle $\theta > \theta_{\min}$, where $\theta_{\min}$ is when the first minimum (dark fringe) from object 1 overlaps with the central maximum (bright fringe in the middle) from object 2.

Resolving Power

To resolve two objects, the minimum angle between rays from the two objects passing through an aperture is given by: D of the aperture.

Rectangular aperture:

$$\boxed{\theta_{\text{min}} = \frac{\lambda}{W}}$$

Circular aperture:

$$\boxed{\theta_{\text{min}} = \frac{1.22\lambda}{D}}$$

where W is the width of the aperture, and D is the diameter of the aperture. The angle θ_{min} is measured in **radians**.

* What is the optical path difference? * And how is the path difference related to wavelength?

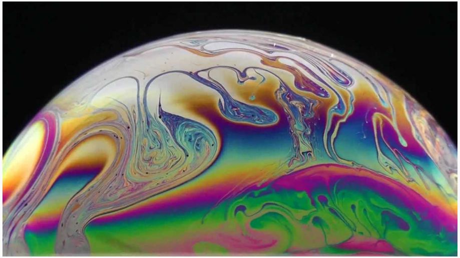
Thin-Film Interference

Thin-Film Interference

Thin-film interference occurs when light reflected/refracted at the upper & lower boundaries of a thin film of an indexed material interfere with one another

- "Indexed material" means a material that has a refractive index of $n > 1$
- The film is a few wavelengths in thickness
- The thickness determines whether the interference is constructive or destructive
- When white light is incident on the film, some colours are enhanced while others are reduced

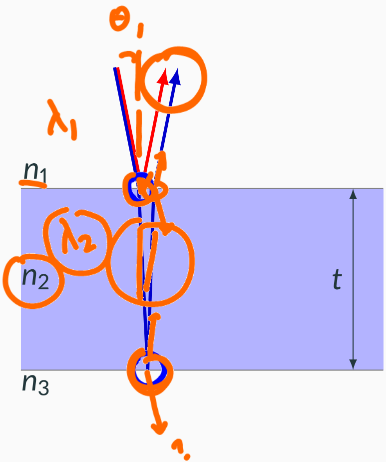
Thin-Film Interference



Examples:

- Soap bubbles
- Oil films on water
- Anti-reflection coatings on glasses and camera lenses

Optical Path Difference

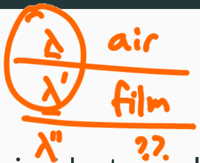


- Light hits the upper interface:
    - Some light is reflected into the first medium (red)
    - Some light is refracted, then reflected at the lower interface, then refracted into the first medium (blue)
- Assuming that the incident and refracted angles are small, then optical path difference Γ is just:

$$\Gamma = 2t$$

Same as single- and double-slits interference problems, Γ determines the condition for constructive and destructive interference

Optical Path Difference



The wavelength in the second medium λ' is related to the incident wavelength λ using the universal wave equation:

air

film

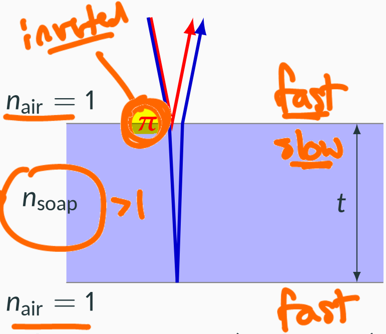
$$\lambda' = \frac{n_1}{n_2}\lambda \longrightarrow \boxed{\lambda' = \frac{\lambda}{n_{film}}}$$

air

film

If the first medium is air, use $n_1 = 1$ for simplicity

Soap Bubble



Light travels through air and strikes a soap film (with n_{soap}). On either side of the soap film is air.

- Upper interface: reflected light has a 180° (π radian) phase shift, as $n_{air} < n_{soap}$, i.e. light reflects from a fast to slow medium
- Lower interface: the reflected wave has no phase shift (slow to fast medium)

Soap Bubble

$$\boxed{\Gamma = \left(m + \frac{1}{2}\right)\lambda'}$$

Because of the phase shift, *constructive maxima* occurs if path difference Γ is a half-number multiple of wavelength:

$$\Gamma = (m + \tfrac{1}{2})\lambda'$$

$$\Gamma = 2t = \underbrace{\left(m + \frac{1}{2}\right)\lambda'}_{\lambda' = \frac{\lambda}{n}} \rightarrow \boxed{\frac{2n_{soap}t}{\cancel{\lambda}} = \left(m + \frac{1}{2}\right)\lambda}$$

while *destructive minima* if Γ is a whole-number multiple of wavelength:

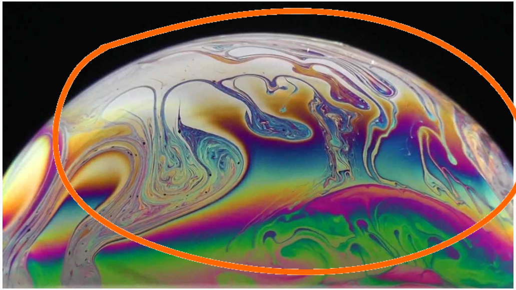
$$\boxed{\frac{2n_{soap}t}{\lambda} = m}$$

$$2nt = m\lambda$$

$$\Gamma = m\lambda'$$

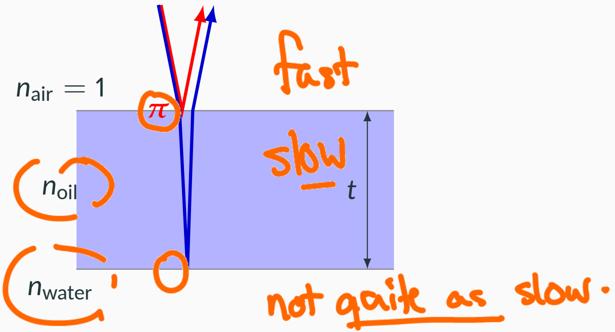
where $m = 0, 1, 2 \ldots$

Soap Bubble



- Because the condition for interference depends on the incident wavelength, when incident light is a white-light (broadband), some colours experience constructive interference, while other colours there is destructive interference
- The color pattern comes from the variations of thickness of the film (it is not constant!)

Oil Film on Water



Light travels through air and strikes a oil film (with n_{oil}). Below the oil film is water, which has a lower refractive index than oil ($n_{oil} > n_{water}$)

- Upper interface: Reflected light has a phase shift of 180° (π) because $n_{air} < n_{oil}$ (fast to slow medium)
- Lower interface: Reflected light has no phase shifts (slow to fast medium)

Oil Film on Water

Beause there is only one phase shift, so like the soap bubble, *constructive maxima* occurs if path difference is a half-number multiple of wavelength:

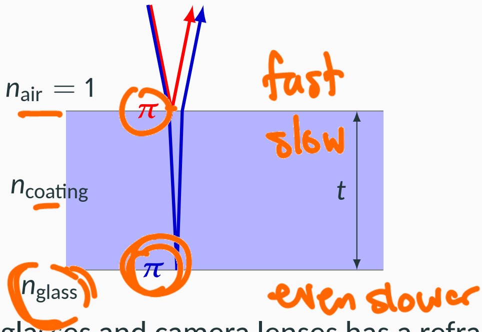
$$\cancel{\boxed{\frac{2n_{\text{oil}}t}{\lambda} = m + \frac{1}{2}}} \rightarrow \boxed{2nt = \left(m + \frac{1}{2}\right)\lambda}$$

while *destructive minima* if Γ is a whole-number multiple of wavelength:

$$\boxed{\frac{2n_{\text{oil}}t}{\lambda} = m} \rightarrow \boxed{2nt = m\lambda}$$

where $m = 0, 1, 2 \ldots$, and assuming that the refractive index of the oil film is *higher* than water.

Anti-Reflection Coating



Anti-reflective coating on eyeglasses and camera lenses has a refractive index lower than glass (but higher than air), i.e.:

$$n_{air} < n_{coating} < n_{glass}$$

Light reflects with a phase shift of π on both the upper and lower interfaces, because in both cases, the light is from fast to slow medium

Anti-Reflection Coating

The interference conditions for anti-reflection coating is *oppposite* to the soap bubble and oil film, because the phase shifts occur on both boundaries. *Constructive maxima* occurs if path difference is a whole-number multiple of wavelength:

$$\boxed{\frac{2n_{\text{film}}t}{\lambda} = m} \longrightarrow \boxed{2nt = m\lambda}$$

while *destructive minima* occurs if path difference is a half-number multiple of wavelength:

$$\boxed{\frac{2n_{\text{film}}t}{\lambda} = m + \frac{1}{2}} \longrightarrow \boxed{2nt = \left(m+\frac{1}{2}\right)\lambda}$$

where $m = 0, 1, 2 \ldots$