WELCOME TO AP PHYSICS C

AP Physics C Exams

There are two calculus-based AP Physics C exams, which are usually taken together on the same day, in the first or second week of May of each year.

- Mechanics
- Electricity and Magnetism

Topic 1: Kinematics

Advanced Placement Physics C

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Olympiads School

Kinematics

Position

Position ($\boldsymbol{x}(t)$) describes the location of an object in a predefined coordinate system.

$$\boxed{\boldsymbol{x}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}}$$

Vectors in 2D/3D Cartesian space are generally using the **IJK notation**

- $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ are **basis vectors** indicating the directions of the x, y and z axes. Basis vectors are **unit vectors** (i.e. length 1)
- The IJK notation does not explicitly give the magnitude or the direction of the vector (needs to be calculated using the Pythagorean theorem)

Displacement

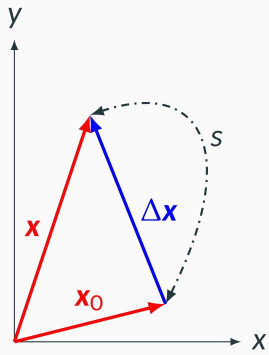
Displacement $\Delta\boldsymbol{x}(t)$ is the change in position from the initial position $\boldsymbol{x}_0$ within the same coordinate system:

$$\boxed{\Delta\boldsymbol{x}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_0 = (x - x_0)\hat{\boldsymbol{\imath}} + (y - y_0)\hat{\boldsymbol{\jmath}} + (z - z_0)\hat{\boldsymbol{k}}}$$

Distance

Distance $s(t)$ is a quantity that is *related* to displacement.

- The *length of the path* taken by an object when it travels from $\boldsymbol{x}_0$ to $\boldsymbol{x}(t)$
- A scalar quantity
- Always positive, i.e. $s \geq 0$
- Although the magnitude of the displacement vector is also a scalar, it is not necessarily the same as distance
- $s \geq |\Delta \boldsymbol{x}|$



Instantaneous & Average Velocity

Instantaneous velocity $\boldsymbol{v}(t)$ is the time rate of change in position. It is related to positiion $\boldsymbol{x}(t)$ by:

$$\boxed{\boldsymbol{v}(t) = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} \qquad \boldsymbol{x}(t) = \int \boldsymbol{v}(t)\mathrm{d}t + \boldsymbol{x}_0}$$

The constant of integration $\boldsymbol{x}_0$ evaluated at $t = 0$. Likewise, the **average velocity** $\bar{\boldsymbol{v}}(t)$ is the change in position $\Delta\boldsymbol{x}(t)$ over a finite time interval t:

$$\boxed{\bar{\boldsymbol{v}}(t) = \frac{\Delta\boldsymbol{x}}{t} = \frac{\boldsymbol{x}(t) - \boldsymbol{x}_0}{t} = \frac{\int_0^t \boldsymbol{v}\mathrm{d}t}{t}}$$

Instantaneous & Average Speed

Instantaneous speed v is the time rate of change of *distance*. It is the *magnitude* of instantaneous velocity (i.e. $v = |\mathbf{v}|$)

$$\boxed{v = \frac{\mathrm{d}s}{\mathrm{d}t}}$$

Since $s \geq 0$, instantaneous speed must also be positive, i.e. $v \geq 0$. **Average speed** $\bar{v}(t)$ is the distance $s(t)$ travelled over a finite time interval t:

$$\boxed{\bar{v} = \frac{s(t)}{t} = \frac{\int_0^t v\mathrm{d}t}{t}}$$

Instantaneous & Average Acceleration

In the same way that velocity is the time rate of change in position, **instantaneous acceleration** $\boldsymbol{a}(t)$ is related to instantaneous velocity by:

$$\boxed{\boldsymbol{a}(t) = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} \qquad \boldsymbol{v}(t) = \int \boldsymbol{a}(t)\mathrm{d}t + \boldsymbol{v}_0}$$

Likewise, **average acceleration** $\overline{\boldsymbol{a}}(t)$ is the finite change in velocity $\Delta\boldsymbol{v}(t)$ over a finite time interval t:

$$\boxed{\overline{\boldsymbol{a}}(t) = \frac{\Delta\boldsymbol{v}(t)}{t} = \frac{\boldsymbol{v}(t) - \boldsymbol{v}_0}{t} = \frac{\int_0^t \boldsymbol{a}\mathrm{d}t}{t}}$$

Note that acceleration only requires a *change* in velocity. It does *not* necessarily mean an object speeds up or slows down (e.g. uniform circular motion).

Special Notation When Differentiating With Time

Physicists and engineers use a special notation when the derivative is taken with respect to *time*, by writing a dot above the variable. For example:

$$v = \dot{x}$$

$$a = \dot{v} = \ddot{x}$$

We will use this notation whenever it is convenient

Linear Independence

The x, y and z components of $\boldsymbol{x}$ along the $\hat{\imath}$, $\hat{\jmath}$ and $\hat{\boldsymbol{k}}$ directions are *linearly independent*, therefore the time derivative and integral can be separated into components:

$$\boldsymbol{v}(t) = \frac{\mathrm{d}x}{\mathrm{d}t}\hat{\boldsymbol{\imath}} + \frac{\mathrm{d}y}{\mathrm{d}t}\hat{\boldsymbol{\jmath}} + \frac{\mathrm{d}z}{\mathrm{d}t}\hat{\boldsymbol{k}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\hat{\boldsymbol{k}}$$

$$\boldsymbol{a}(t) = \frac{\mathrm{d}v_x}{\mathrm{d}t}\hat{\boldsymbol{\imath}} + \frac{\mathrm{d}v_y}{\mathrm{d}t}\hat{\boldsymbol{\jmath}} + \frac{\mathrm{d}v_z}{\mathrm{d}t}\hat{\boldsymbol{k}} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}$$

If You Are Curious

The time derivative of acceleration is called **jerk**, with a unit of $\mathrm{m/s^3}$:

$$\boldsymbol{j}(t) = \frac{\mathrm{d}\boldsymbol{a}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{v}}{\mathrm{d}t^2} = \frac{\mathrm{d}^3\boldsymbol{x}}{\mathrm{d}t^3}$$

The time derivative of jerk is **jounce**, or **snap**, with a unit of $\mathrm{m/s^4}$:

$$\boldsymbol{s}(t) = \frac{\mathrm{d}\boldsymbol{j}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{a}}{\mathrm{d}t^2} = \frac{\mathrm{d}^3\boldsymbol{v}}{\mathrm{d}t^3} = \frac{\mathrm{d}^4\boldsymbol{x}}{\mathrm{d}t^4}$$

The next two derivatives of snap are called **crackle** and **pop**, but these higher derivatives of position vector are rarely used. We will *not* be using them.

Acceleration as Functions of Velocity and Position

Acceleration may be expressed as functions of velocity and position rather than of time, if motion is driven by these forces:

- Gravitational or electrostatic forces: $a(x) = \dfrac{Gm_s}{x^2} \quad a(x) = \dfrac{kq_1q_2}{mx^2}$
- Spring force: $a(x) = -\dfrac{k}{m}x$
- Damping force: $a(v) = -bv$
- Aerodynamic drag: $a(v) = \left[\dfrac{\rho C_D A}{2m}\right] v^2$

In these cases, solving for $x(t)$, $v(t)$ and $a(t)$ will require solving a differential equation (see handout).

Kinematic Equations

Kinematic Equations

While kinematic problems in AP Physics C exams often require calculus, these basic kinematic equations for constant acceleration are still a powerful tool.

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$
$$v = v_0 + a t$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

For AP Physics, you will only be given three kinematic equations in your equation sheet. You will still be required to integrate when acceleration is not constant.

Motion Graphs

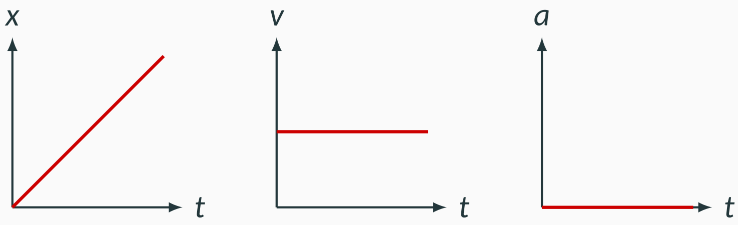
Motion Graphs

You should already be familiar with the basic motion graphs for 1D motion:

- Position vs. time graph
- Velocity vs. time graph
- Acceleration vs. time graph

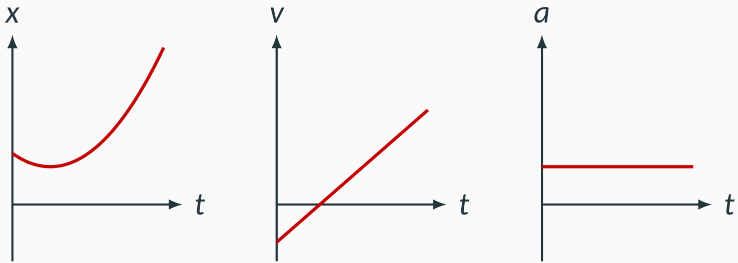
However, depending on the situation, it may be more useful to plot motion using other quantities as well.

Uniform Motion: Constant Velocity



- Constant velocity has a straight line in the $x - t$ graph
- The slope of the $x - t$ graph is the velocity v, which is constant
- The slope of the $v - t$ graph is the acceleration a, which is zero in this case

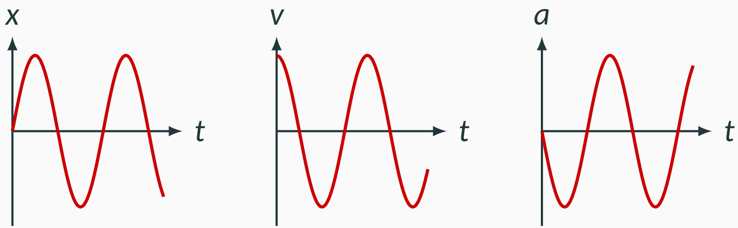
Uniform Acceleration: Constant Acceleration



- The $x - t$ graph for motion with constant acceleration is part of a *parabola*
  - If the parabola opens up, then acceleration is positive
  - If the parabola opens down, then acceleration is negative
- The $v - t$ graph is a straight line; its slope (a constant) is the acceleration

Simple Harmonic Motion

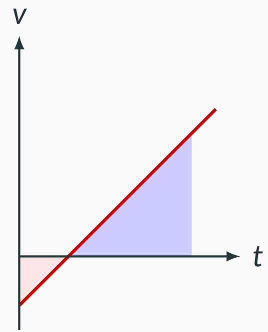
For **harmonic motions**, neither position, velocity nor acceleration are constant:



Bottom line: regardless of the type motion,

- The $v - t$ graph is the slope of the $x - t$ graph
- The $a - t$ graph is the slope of the $v - t$ graph

Area Under Motion Graphs



The area under the $v - t$ graph is the displacement $x - x_0$.

- Area *above* the time axis: $+$ displacement
- Area *below* the time axis: $-$ displacement

Likewise, the area under the $a - t$ graph is the change in velocity $v - v_0$.

Velocity Squared vs. Displacement

If velocity information is given as a function of position[1] then a motion graph can be plotted using this kinematic equation:

$$\underbrace{v^2}_{y} = \underbrace{v_0^2}_{b} + \underbrace{2a}_{m} \underbrace{(x - x_0)}_{x}$$

by plotting v^2 on the y-axis and displacement $\Delta x = x - x_0$ on the x-axis. The slope of the graph is $m = 2a$. The square of the initial velocity (v_0^2) is the y-intercept.

[1]Depends on experimental set up

Graphing “Linear” Functions

This concept extends to graphing other physical quantities not relating to motion:

- To find the index of refraction of a material using Snell’s law, plot $\sin\theta_i$ vs. $\sin\theta_2$ (rather than θ_1 vs. θ_2). The slope is the index n:

$$\underbrace{\sin\theta_1}_{y} = \underbrace{n}_{m}\underbrace{\sin\theta_2}_{x}$$

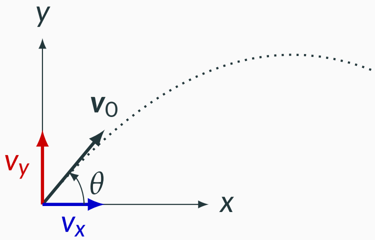
- To relate the period of oscillation of a simple pendulum to the length of the pendulum, plot T^2 vs. L:

$$\underbrace{T^2}_{y} = \underbrace{\frac{4\pi^2}{g}}_{m}\underbrace{L}_{x}$$

Projectile Motion

Projectile Motion

A **projectile** is an object that is launched with an initial velocity of $\mathbf{v}_0$ along a parabolic trajectory and accelerates only due to gravity.



- x-axis is the *horizontal* direction, with the $(+)$ direction pointing *forward*
- y-axis is the *vertical* direction, with the $(+)$ direction pointing *up*
- The reference point is where the projectile is launched
- Consistent with the right-handed Cartesian coordinate system
- The launch angle θ is measured above the the horizontal.

Horizontal (x) Direction

There is no acceleration (i.e. $a_x = 0$) along the horizontal direction, therefore horizontal velocity is constant. The kinematic equations reduce to:

$$x(t) = v_x t = \left[v_0 \cos\theta\right] t$$

where $x(t)$ is the horizontal position at time t, v_0 is the magnitude of the initial velocity, $v_x = v_0 \cos\theta$ is its horizontal component.

Vertical (y) Direction

Constant acceleration due to gravity alone along the vertical direction, i.e. $a_y = -g$. (Acceleration is *negative* due to the way we defined the coordinate system.) The important equation is this one:

$$y(t) = [v_0 \sin\theta]\, t - \frac{1}{2} g t^2$$

These two kinematic equations may also be useful:

$$v_y = [v_0 \sin\theta] - gt$$
$$v_y^2 = [v_0 \sin\theta]^2 - 2gy$$

Solving Projectile Motion Problems

Horizontal and vertical motions are independent of each other, but there are variables that are shared in both directions, namely:

- Time t
- Launch angle θ (above the horizontal)
- Initial speed v_0

When solving any projectile motion problems

- *Two* equations with *two* unknowns
- If an object lands on an incline, there will be a third equation describing the relationship between x and y

Symmetric Trajectory

A projectile's trajectory is symmetric if the object lands at the same height as when it launched. These equations are *not* provided in the AP Exam equation sheet, but it can save you a lot of time if you can use them, instead of deriving them during the exam.

- Time of flight

$$T = \frac{2v_0 \sin\theta}{g}$$

- Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum height

$$H = \frac{v_0^2 \sin^2\theta}{2g}$$

Maximum Range

$$R = \frac{v_0^2 \sin\left(2\theta\right)}{g}$$

- Maximum range occurs at $\theta = 45°$
- For a given initial speed v_0 and range R, launch angle θ is given by:

$$\theta_1 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

But there is another angle that *gives the same range*!

$$\theta_2 = 90° - \theta_1$$

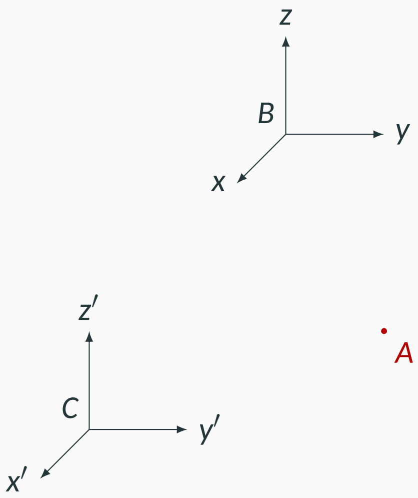
Relative Motion

Relative Motion

All motion quantities must be measured relative to a *frame of reference*

- **Frame of reference:** the *coordinate system* from which all physical measurements are made.
- In *classical* mechanics, the coordinate system is the Cartesian system
- There is no absolute motion/rest: all motions are relative
- **Principle of Relativity:** All laws of physics are equal in all inertial (non-accelerating) frames of reference

Relative Motion

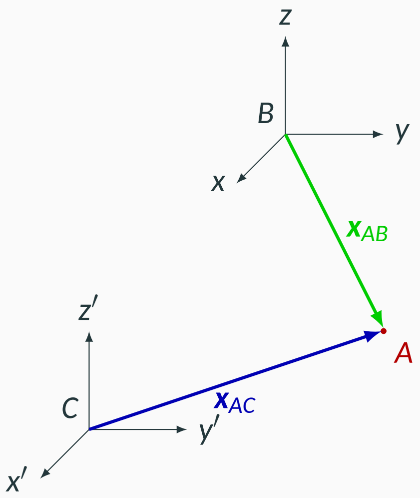


Two frames of reference

- B with axes x, y, z
- C with axes x', y', z'

The two reference frames may (or may not) be moving relative to each other. The motion of the two reference frames affect how motion of A is calculated.

Relative Motion

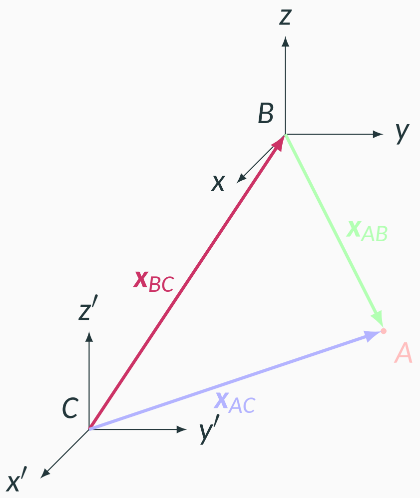


The position of A can be described by

- $\boldsymbol{x}_{AB}(t)$ (relative to frame B)
- $\boldsymbol{x}_{AC}(t)$ (relative to frame C)

It is obvious that $\boldsymbol{x}_{AB}(t)$ and $\boldsymbol{x}_{AC}(t)$ are different vectors

Relative Motion



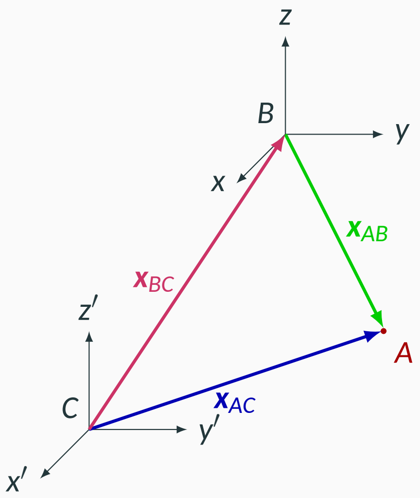
The position vector of the origins of the two reference frames is given by $\boldsymbol{x}_{BC}$

- The vector pointing from the origin of frame C to the origin of frame B
- If the two frames are moving relative to each other, then $\boldsymbol{x}_{BC}$ is also a function of time

Even without using vector notations, the relationship between the vectors is obvious:

$$\boxed{\boldsymbol{x}_{AC} = \boldsymbol{x}_{AB} + \boldsymbol{x}_{BC}}$$

Relative Motion



Starting from the definition of **relative position**:

$$\boxed{\boldsymbol{x}_{AC} = \boldsymbol{x}_{AB} + \boldsymbol{x}_{BC}}$$

Differentiating all terms with respect to time, we get the equation for **relative velocity**:

$$\boxed{\boldsymbol{v}_{AC} = \boldsymbol{v}_{AB} + \boldsymbol{v}_{BC}}$$

Differentiating with respect to time again, and we obtain the equation for **relative acceleration**:

$$\boxed{\boldsymbol{a}_{AC} = \boldsymbol{a}_{AB} + \boldsymbol{a}_{BC}}$$

Relative Velocity

In classical mechanics, the equation for relative velocities follows the **Galilean velocity addition rule**, which applies to speeds much less than the speed of light:

$$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$$

The velocity of A relative to reference frame C is the velocity of A relative to reference frame B, plus the velocity of B relative to C. If we add another reference frame D, the equation becomes:

$$\mathbf{v}_{AD} = \mathbf{v}_{AB} + \mathbf{v}_{BC} + \mathbf{v}_{CD}$$

Typical Problems

In an AP Physics C exam, questions involving only kinematics usually appear in the multiple-choice section. The problems themselves are not very different compared to the Grade 12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use $g = 10\,\mathrm{m/s^2}$ in your calculations to make your lives simpler
- Many problems are *symbolic*, which means that they deal with the equations, not actual numbers
- Will be coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet