Class 19A: Hall Effect

Advanced Placement Physics C

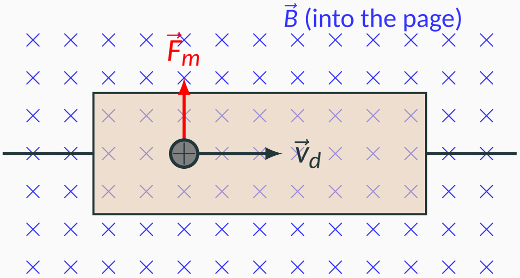
Dr. Timothy Leung

Fall 2021

Olympiads School

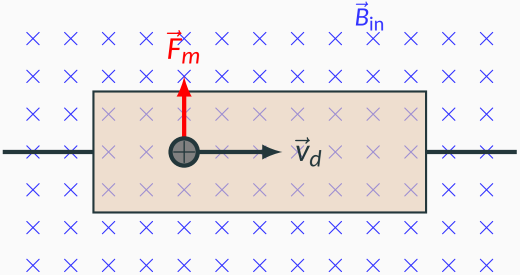
Hall Effect

When a current I flows through a conductor in a magnetic field $\vec{B}$, the magnetic field exerts a transverse (i.e. perpendicular to motion) magnetic force $\vec{F}_m$ on the moving charges which pushes them toward one side of the conductor.



This is most evident in a *thin*, *flat* conductor as illustrated.

Magnetic Force

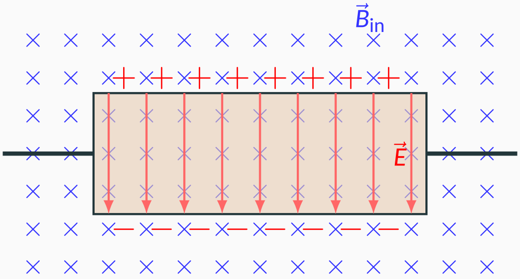


As the charges enter the magnetic field, $\vec{F}_m$ is directed toward the top:

$$\vec{F}_m = e\vec{v}_d \times \vec{B} = \frac{e\vec{I} \times \vec{B}}{neA} = \frac{\vec{I} \times \vec{B}}{nA}$$

leading to a surplus of positive charges on the top edge of the conductor, and negative charges on the bottom.

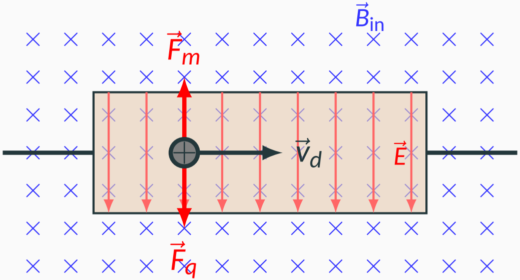
Hall Voltage



The charge imbalance on the conductor creates an electric field $\vec{E}$, pointing toward the bottom, and therefore a voltage across two sides of the conductor (width W), called the **Hall voltage**:

$$V_H = EW$$

Balancing Electrostatic & Magnetic Forces



Subsequently, charge carriers entering the magnetic field will experience both a magnetic force and an electrostatic force. At equilibrium, the two forces are balanced:

$$\vec{F}_m + \vec{F}_q = \vec{0}$$

Calculating Hall Voltage

The electrostatic force on the charge carrier can be expressed in terms of the Hall voltage V_H across the two sides of the plate:

$$F_q = eE = \frac{eV_H}{W}$$

Equating the magnitudes of electrostatic and magnetic forces, we can solve for the Hall voltage:

$$F_m = F_q \quad \rightarrow \quad \frac{IB}{nA} = \frac{eV_H}{W}$$

Hall Voltage

Cancelling terms and noting that the thickness of the conductor is

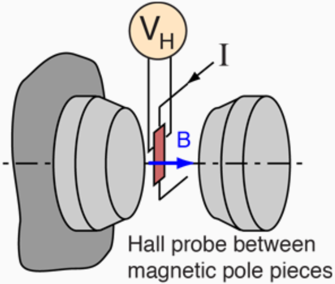
$$d = \frac{A}{W}$$

we find the expression for the Hall voltage V_H:

$$\boxed{V_H = \frac{IB}{ned}}$$

Hall Probe

Large magnetic fields ($\sim 1\,\mathrm{T}$) is often measured using a **Hall probe**. A thin film Hall probe is placed in the magnetic field and the transverse voltage (usually measured in on the order of 10^{-6} V) is measured.



The polarity of the Hall voltage for a copper probe shows that electrons (negative charge) are the charge carriers.