

# Topic 9: Fluid Mechanics

## Advanced Placement Physics 2

---

Dr. Timothy Leung

January 29, 2021

Olympiads School

# Disclaimer

Fluid mechanics is part of the AP Physics 2 Exam, which does not require calculus. However, in the interest in completeness, *some* calculus will be shown when deriving equations.

# What is a Fluid

- The *simple* (non-scientific) definition of a fluid is anything that *flows*, which covers most gases and liquids
- The *scientific* definition of a fluid is **any substance that deforms continuously under oblique stress**
  - When a force (stress) is applied to a solid, it deforms until all the forces are balanced, and the deformation stops (e.g. stretching a spring)
  - When a force is applied to a fluid, it continues to deform in shape as long as the force is present
  - Fluid is continuous: it will fill all available space without gaps

# Density of a Fluid

The **density**  $\rho$  of a fluid is defined as the mass of the fluid  $m_{\text{fluid}}$  per unit volume  $V_{\text{fluid}}$  that it occupies:

$$\rho = \frac{m_{\text{fluid}}}{V_{\text{fluid}}}$$

- Unit for density is **kilograms per meter cubed** ( $\text{kg}/\text{m}^3$ )
- Below Mach number  $M \approx 0.3$ , density can be assumed to be constant throughout the fluid
- May be dependent on temperature by basic thermal expansion

# Viscosity of a Fluid

**Viscosity**  $\mu$  measures how “thick” a fluid is. It relates the rate of deformation ( $\Delta u / \Delta y$ ) of the fluid to the shear stress  $\tau$  that it experiences:

$$\tau = \mu \frac{\Delta u}{\Delta y}$$

- e.g. honey is more viscous than water.
- Shear stress is defined as  $\tau = F/A$ , with a unit of **pascal** (Pa), which is the same as for pressure
- In AP Physics 2, we will mostly ignore viscous effects, as important as they are

# Hydrostatics

The fluid **pressure** on its density and depth:

$$p = p_0 + \rho_{\text{fluid}}gz$$

where  $g$  is the acceleration due to gravity,  $z$  is the depth below the surface, and  $p_0 = 1.01 \times 10^5 \text{ Pa}$  is the atmospheric pressure at the surface.

- Pressure is the same in all directions
- Pressure is defined as force per unit area, and the unit is **pascal**:

$$p = \frac{F}{A}$$

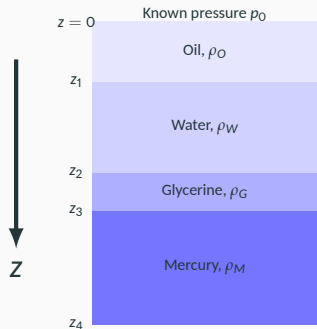
$$1 \text{ Pa} = 1 \text{ N/m}^2$$

# Pascal's Principle

If force is applied somewhere on a container holding fluid, the pressure increases *everywhere* in the fluid, not just where the force is applied.

i.e. the pressure of the force will be transmitted into the fluid.

# Pressure with Different Fluids



For the fluid surface to remain *static*, the fluid pressure on both side of the interface have to be equal. In this example:

$$p_1 - p_0 = \rho_O g z_1$$

$$p_2 - p_1 = \rho_W g (z_2 - z_1)$$

$$p_3 - p_2 = \rho_G g (z_3 - z_2)$$

$$p_4 - p_3 = \rho_M g (z_4 - z_3)$$

---

$$p_4 - p_0 = \sum \Delta p$$



## A Simple Example

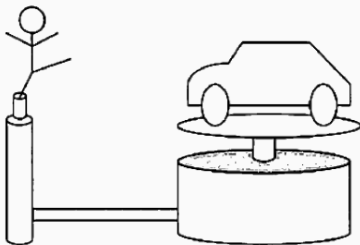
**Example 1:** An aquarium is filled with water. The lateral wall of the aquarium is 40 cm long and 30 cm high. Using  $10 \text{ m/s}^2$  for the acceleration due to gravity, and  $1 \text{ g/cm}^3$  for density of water, the force on the lateral wall of the aquarium is:

- (a) 36 N
- (b) 90 N
- (c) 180 N
- (d) 1500 N



## Example

**Example 2:** Consider the hydraulic jack in the diagram. A person stands on a piston that pushes down on a thin cylinder full of water. The cylinder is connected via pipes to a wide platform on top of which rests a 1-ton (1000 kg) car. The area of the platform under the car is  $25 \text{ m}^2$ ; the person stands on a  $0.3 \text{ m}^2$  piston. What is the lightest weight of a person who could successfully lift the car?



Believe it or not, there is someone who draws worse diagrams than Tim!

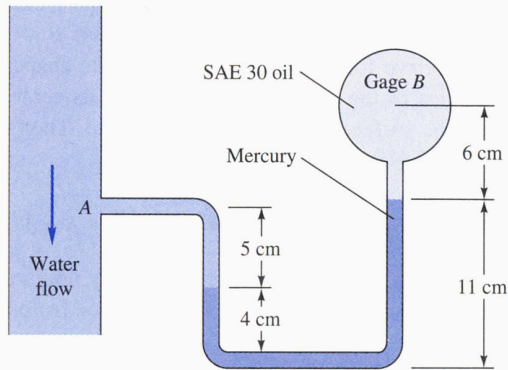
## A “Manometer” Example

**Example 3:** Pressure gauge *B* is to measure the pressure at point *A* in a water flow, as shown in the figure on the right. If the pressure at *B* is 87 kPa, estimate the pressure at *A*, in kPa. Assume all fluids are at 20 °C. The densities of water, mercury and SAE 30 oil are, respectively:

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

$$\rho_{\text{Hg}} = 13\,600 \text{ kg/m}^3$$

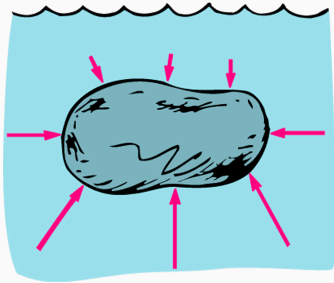
$$\rho_{\text{oil}} = 890 \text{ kg/m}^3$$



# Buoyancy: Everything Floats a Little

When an object is submerged inside a fluid

- The fluid exerts a pressure at the surface of the object
- By hydrostatics, the pressure is higher at the bottom than at the top



## Buoyant Force

Using some basic calculus (well, depends on who you ask), we can find the pressure over the entire surface to find the total buoyant force **B** the fluid exerts on the object. The expression is surprisingly simple:

$$\mathbf{B} = \rho_{\text{fluid}} g V \hat{\mathbf{k}} = m_{\text{fluid}} g \hat{\mathbf{k}}$$

where  $\rho_{\text{fluid}}$  is the density of the displaced fluid, and  $V$  is the volume displaced. The direction of the force is upward. This equation is known as **Archimedes' principle**.

**Buoyance force has a magnitude that equals to the weight of the fluid displaced by the submerged object, pointing upward.**

# Buoyancy

$$\mathbf{B} = \rho_{\text{fluid}} g V \hat{\mathbf{k}} = m_{\text{fluid}} g \hat{\mathbf{k}}$$

Buoyancy does not depend on:

- the mass of the immersed object, or
- the density of the immersed object

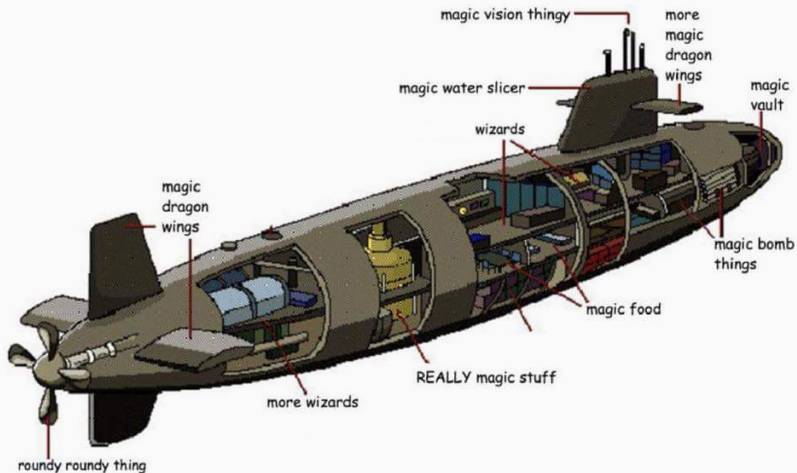
Objects immersed in a fluid have an “apparent weight”  $\mathbf{W}'$  that is reduced by the buoyance force:

$$\mathbf{W}' = \mathbf{W} - \mathbf{B} = \rho' g V$$

where  $\rho' = \rho_{\text{obj}} - \rho_{\text{fluid}}$  is the relative density

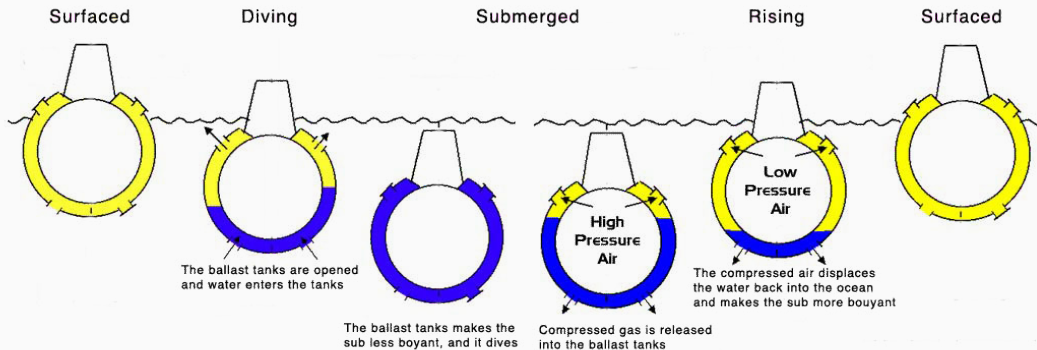
# How Submarines Work

Like this:



# How Submarines Work

Like all ships, a submarine does not naturally sink due to buoyancy. When a submarine submerges, water is pumped into the “ballast tanks” in the hull to make the submarine heavier.





## Example

**Example 4:** An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared with the force needed to hold the water just below the surface, what is the force needed to hold it at a deeper point?

- (a) Larger
- (b) The same
- (c) Smaller
- (d) Impossible to determine



## Example



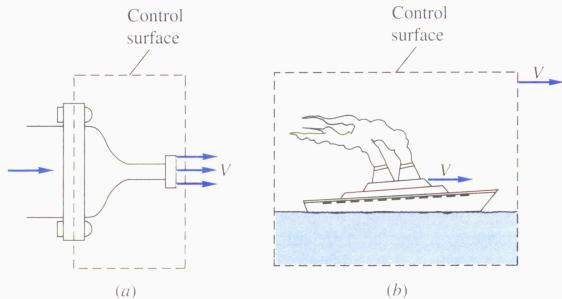
**Example 5:** A salvage ship tries to raise a sunken miniature submarine from the bottom of Lake Superior. The submarine and its contents have a mass of 72 000 kg and a volume of  $18.9 \text{ m}^3$ . What upward force must be applied to raise the submarine? The density of water is  $1000 \text{ kg/m}^3$ .

- (a)  $1.8 \times 10^5 \text{ N}$
- (b)  $2.0 \times 10^5 \text{ N}$
- (c)  $4.8 \times 10^5 \text{ N}$
- (d)  $5.2 \times 10^5 \text{ N}$

As important as it is to understand hydrostatics,  
it's way more interesting when the fluid is moving!

# Control Volume and Control Surfaces

A control volume “CV” is a fixed volume in which fluid is able to flow in and out of it. The surfaces of the control volume is called the control surface “CS”.



# Navier-Stokes Equations

The governing equations in fluid mechanics is called **Navier-Stokes equations**, which is written in complicated vector and calculus symbols:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla(\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla p + \mathbf{f} + \mu \nabla^2 \mathbf{v}$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{v}) = -\nabla \cdot p + \frac{1}{Re Pr} \nabla q + \frac{1}{Re} \nabla \cdot (\tau \cdot \mathbf{v})$$

Even for university students experienced with calculus, solving these equations is still a daunting task. **But what do they actually mean?**

# Continuity Equation

The first equation is called the **continuity equation**, which is the conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

**Meaning:** The increase in fluid mass in a fixed volume containing a fluid is the amount of mass that flows in minus the volume that flows out of the volume.

# Momentum Equation

The second equation is the **momentum equation**, which is the momentum-impulse theorem applied to fluids in a volume:

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla(\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g}$$

**Meaning:** The increase in the total momentum of a fluid in a fixed volume is the sum of all the external forces (pressure, shear & body forces) applied to the fluids, plus the change in momentum through the flow of particles in and out of the volume.

## Energy Equation

The third and final equation is the **energy equation**, which is the the work-kinetic energy theorem applied to fluids:

$$\rho \frac{\partial e}{\partial t} + p(\nabla \cdot \mathbf{v}) = \nabla \cdot (k \nabla T) + \Phi$$

**Meaning:** The increase in the internal energy of the fluid (i.e. the total kinetic energy of all the fluid particles) in a fixed volume is the work done by pressure forces, viscous forces (i.e. friction), plus the change in energy from the flow of fluid in and out of the volume.



# Let's Make Some Assumptions

For an “ideal fluid flow”, we make the following assumptions to simplify the Navier-Stokes equation:

1. The flow is **steady**

- Flow is “time independent”, i.e. does not change with time

2. The flow is **inviscid**

- The fluid has no viscosity
- No friction between the fluid and the surrounding, and therefore
- No shear stresses on the fluid
- Only forces are pressure at the surface, and body forces from gravity

3. The flow is **incompressible**

- Density is constant throughout

# Assumptions for Ideal Fluid Flow

We will also assume that there is

- **no shaft work** done along the streamline
- **no heat transfer** along the streamline

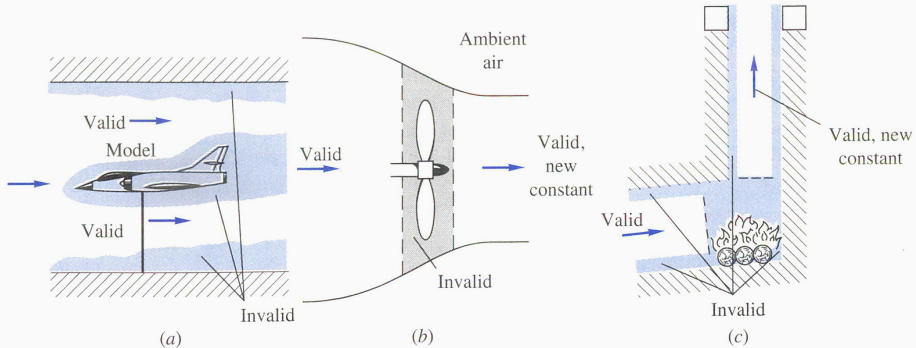
Then the N-S equations reduces to the **Bernoulli equation**

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$$

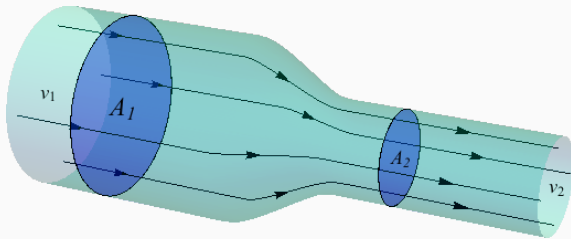
The term  $\frac{1}{2}\rho v^2$  is called **dynamic pressure**, while  $p + \rho g z$  is the **hydrostatic pressure**.

# Bernoulli Equation

Regions where Bernoulli equation is valid:



# Inlet Outlet Flow



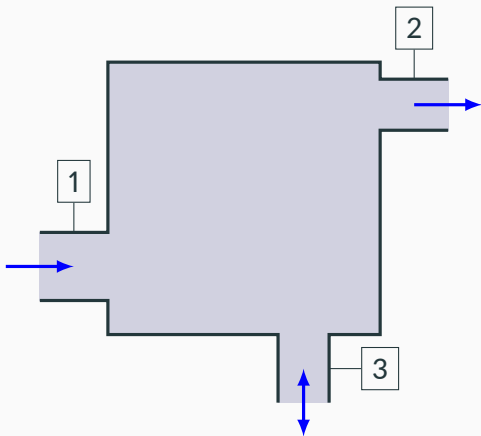
In this example, the mass flowing at the inlet is the same as the flow out of it:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

For constant fluid density, the  $\rho$  terms on both sides of the equation will cancel:

$$v_1 A_1 = v_2 A_2$$

## Example: Multiple Inlet & Outlets

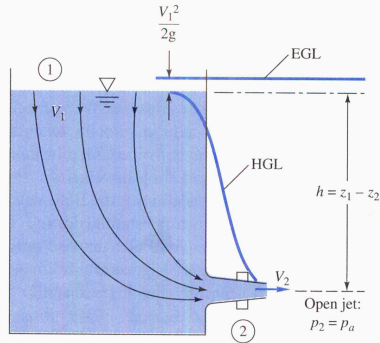


**Example 6:** Water at  $20^\circ\text{C}$  flows steadily through a closed tank, as shown in the figure. At section 1,  $D_1 = 6\text{ cm}$  and the volume flow is  $100\text{ m}^3/\text{h}$ . At section 2,  $D_2 = 5\text{ cm}$  and the average velocity is  $8\text{ m/s}$ . If  $D_3 = 4\text{ cm}$ , what is

1. the flow rate  $Q_3$  in  $\text{m}^3/\text{h}$ ?
2. the average  $v_3$  in  $\text{m/s}$ ?

# Example

**Example 7:** Find a relation between the nozzle discharge velocity  $V$  and the tank free-surface height  $h$ . Assume frictionless flow.



The line labelled “EGL” is called the “energy grade line”, or the “Bernoulli head”, given by the equation  $h_0 = z + p/\rho g + v^2/2g$ . In the region where Bernoulli equation is valid, EGL is a constant.