Student #: ____________________________ Student Name: ___________________________________

# AP PHYSICS C: MOMENTUM AND CENTER OF MASS

**Directions:** Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

**Note:** To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

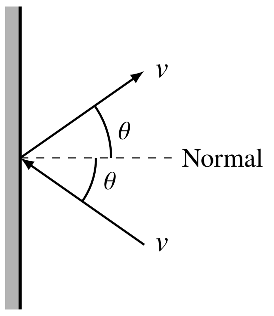
1. A toy train car of mass 3 kg rolls to the left at 2 m/s and collides with a 4 kg train car rolling to the right at 1 m/s. The two cars stick together. The velocity of the cars after the collision is

   (A) 2/7 m/s to the left
   (B) 2/7 m/s to the right
   (C) 4/7 m/s to the left
   (D) 4/7 m/s to the right
   (E) 9/7 m/s to the right

2. Two steel balls, one of mass $m$ and the other of mass $2m$, collide and rebound in a perfectly elastic collision. Which of the following is conserved in this elastic collision?

   (A) velocity only
   (B) momentum only
   (C) momentum and kinetic energy only
   (D) momentum, velocity, and kinetic energy
   (E) kinetic energy only

3. A rubber ball of mass $m$ strikes a wall with a speed $v$ at an angle $\theta$ below the normal line and rebounds from the wall at the same speed and angle above the normal line as shown. The magnitude of the change in momentum of the ball is

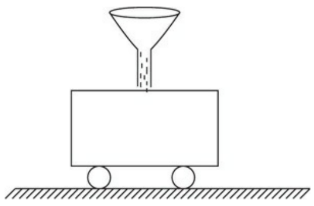



   (A) $mv$
   (B) $2mv$
   (C) $mv\cos\theta$
   (D) $2mv\cos\theta$
   (E) zero

4. Two blocks are connected by a compressed spring and rest on a frictionless surface. The blocks are released from rest and pushed apart by the compressed spring. If one mass is twice the mass of the other, which of the following is the same for both blocks?

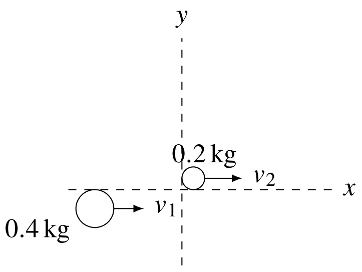
   (A) magnitude of momentum
   (B) acceleration
   (C) speed
   (D) kinetic energy
   (E) potential energy

5. A 1000 kg railroad car is rolling without friction on a horizontal track at a speed of 3.0 m/s. Sand is poured into the open top of the car for a time of 5.0 s. The speed of the car after 5.0 s is 1.0 m/s. The mass of the sand added to the car at the end of 5.0 s is



   (A) 500 kg
   (B) 1000 kg
   (C) 2000 kg
   (D) 3000 kg
   (E) 3500 kg

6. Two billiard balls are rolling to the right on a table as shown. The 0.4 kg ball is moving faster than the 0.2 kg ball, so it catches up and strikes it from behind at a slight angle. Immediately after the collision, the $y$-component of the 0.4 kg ball is 2 m/s downward. The $y$-component of the velocity of the 0.2 kg ball must be




   (A) 1 m/s upward
   (B) 2 m/s upward
   (C) 1 m/s downward
   (D) 2 m/s downward
   (E) 4 m/s upward

**Questions 7–8**. Two balls are on a horizontal billiard table. A 1.0 kg billiard ball moves downward along the $y$-axis with a speed of 16 m/s toward a 2.0 kg ball that is at rest. The balls collide at an angle, and move along the lines shown. After the collision, the 1.0 kg ball moves at 9 m/s along the $+x$-axis. The table below shows the $x$ and $y$ components of the momentum in $\mathrm{kg\cdot m/s}$ of the two balls before and after the collision.

| | $p_{1x}$ | $p_{1y}$ | $p_{2x}$ | $p_{2y}$ |
|---|---|---|---|---|
| **Before Collision** | 0 | −16 | 0 | 0 |
| **After Collision** | +9 | 0 | −9 | −16 |

7. Which of the following statements is true?

   (A) Momentum is conserved only in the $x$-direction in this collision.
   (B) Momentum is conserved only in the $y$-direction in this collision.
   (C) Momentum is conserved in both the $x$- and $y$-directions in this collision
   (D) The momentum of the 1.0 kg ball increases after the collision.
   (E) The momentum of the 2.0 kg ball decreases after the collision.

8. What is the speed of the 2.0 kg ball after the collision?

   (A) 16.0 m/s
   (B) 9.2 m/s
   (C) 7.5 m/s
   (D) 6.0 m/s
   (E) 5.0 m/s

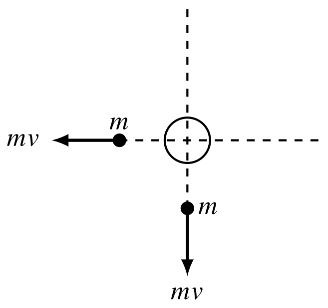
9. A 0.3 kg baseball at rest on a tee is struck by a bat. The ball leaves the at with a speed of 20 m/s at an angle of 45° above the horizontal. The magnitude of the impulse imparted to the baseball by the bat is most nearly
   (A) $2\,\mathrm{N\cdot s}$
   (B) $6\,\mathrm{N\cdot s}$
   (C) $12\,\mathrm{N\cdot s}$
   (D) $16\,\mathrm{N\cdot s}$
   (E) $20\,\mathrm{N\cdot s}$

10. Two ice skaters, a large man and a small woman, are initially at rest and holding each other's hands. They push away horizontally. Afterward, which of the following statements is true?
   (A) They have equal and opposite kinetic energies.
   (B) The have equal and opposite momenta.
   (C) The large man applies a greater force to the small woman.
   (D) The small woman applies a greater force to the large man.
   (E) They recoil with equal and opposite velocities.

11. A small mass $m$ is moving with a speed $v$ toward a stationary mass $M$. The speed of the center of mass of the system is
   (A) $\left(\dfrac{m}{m+M}\right)v$
   (B) $\left(\dfrac{m+M}{m}\right)v$
   (C) $\left(\dfrac{m}{M}\right)v$
   (D) $\left(1+\dfrac{m}{M}\right)v$
   (E) $\left(1+\dfrac{M}{3m}\right)v$

12. A known net force $F$ acts on an unknown mass for a known time $\Delta t$. From this information, you could determine the
   (A) change in kinetic energy of the object
   (B) change in velocity of the object
   (C) acceleration of the object
   (D) mass of the object
   (E) change in momentum of the object

**Questions 13–14**. An object has a mass $4m$. The object explodes into three pieces of mass $m$, $m$, and $2m$. The two pieces of mass $m$ move off at right angles to each other with the same momentum $mv$, as shown.

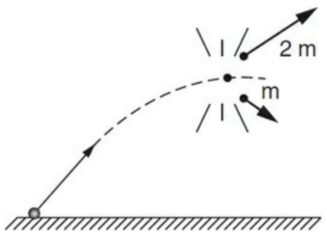



13. The speed of mass $2m$ after the explosion is
   (A) $2v$
   (B) $\sqrt{2}v$
   (C) $\dfrac{\sqrt{2}}{2}v$
   (D) $\dfrac{\sqrt{2}}{3}v$
   (E) $\dfrac{\sqrt{3}}{2}v$

14. The direction of velocity of mass $2m$ is
   (A) $\rightarrow$
   (B) $\swarrow$
   (C) $\downarrow$
   (D) $\nearrow$
   (E) $\uparrow$

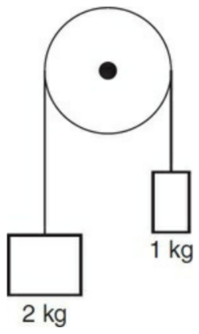
15. A 100 kg cannon sits at rest with a 1 kg cannonball in the barrel. The cannonball is fired with a speed of 50 m/s to the right, causing the cannon to recoil with a speed of 0.5 m/s to the left. The velocity of the center of mass of the cannon-cannonball system is
   (A) zero
   (B) 5 m/s to the right
   (C) 5 m/s to the left
   (D) 50 m/s to the right
   (E) 50 m/s to the left

**Questions 16–17**. A projectile is launched at an angle to the level ground as shown. At the top of the trajectory at point $P$, the projectile explodes into two pieces of mass $2m$ and $m$.



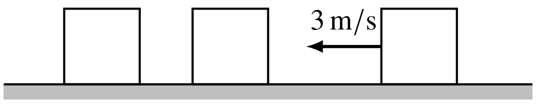

16. Which of the following arrows best represents the direction of the velocity of the center of mass of the projectile at point $P$ after the explosion?
   (A) $\leftarrow$
   (B) $\swarrow$
   (C) $\searrow$
   (D) $\rightarrow$
   (E) $\nearrow$

17. Which of the following statements is true of the center of mass of the projectile after the explosion?
   (A) The center of mass will continue on a parabolic path and land on the ground at the place where it would have landed had it not exploded.
   (B) The center of mass will alter its parabolic path and land on the ground farther from where it would have landed had it not exploded.
   (C) The center of mass will alter its parabolic path and land on the ground at a shorter distance than it would have landed had it not exploded.
   (D) The center of mass will fall straight downward from the point of explosion.
   (E) The center of mass will travel straight upward from the point of explosion.




18. A system consists of two blocks having masses of 2 kg and 1 kg. The blocks are connected by a string of negligible mass and hung over a light pulley, and then released from rest. When the speed of each block is $v$, the momentum of the center of mass of the system is
   (A) $(2\,\mathrm{kg}+1\,\mathrm{kg})v$
   (B) $(2\,\mathrm{kg}-1\,\mathrm{kg})v$
   (C) $1/3(2\,\mathrm{kg}+1\,\mathrm{kg})v$
   (D) $12(2\,\mathrm{kg}-1\,\mathrm{kg})v$
   (E) $(2\,\mathrm{kg})v$

**Questions 19–20**. Three identical masses can slide freely on a horizontal surface as shown. The first mass moves with a speed of 3.0 m/s toward the second and third masses, which are initially at rest. The first and second mass collide elastically, and then the second and third masses collide inelastically.




19. The speed of the second mass after the collision is

(A) zero
(B) 1.5 m/s
(C) 3.0 m/s
(D) 6.0 m/s
(E) 9.0 m/s

20. The speed of the second and third masses after they collide inelastically is

(A) zero
(B) 1.5 m/s
(C) 3.0 m/s
(D) 6.0 m/s
(E) 9.0 m/s

**Questions 21–22**. A 20 kg boy runs at a speed of 3.0 m/s and jumps onto a 40 kg sled on frictionless ice that is initially at rest. The boy and the sled then move together for a short time.

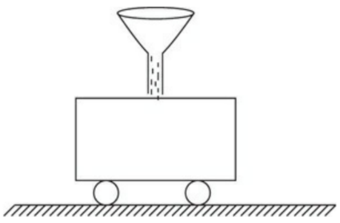
21. The speed of the boy and sled after he jumps on it is

(A) 0.5 m/s
(B) 0.8 m/s
(C) 1.0 m/s
(D) 1.5 m/s
(E) 2.0 m/s

22. While the boy and sled are moving, he jumps off the back of the sled in such a way the boy is at rest, and the sled continues to move forward. The speed of the sled after the boy jumps off is

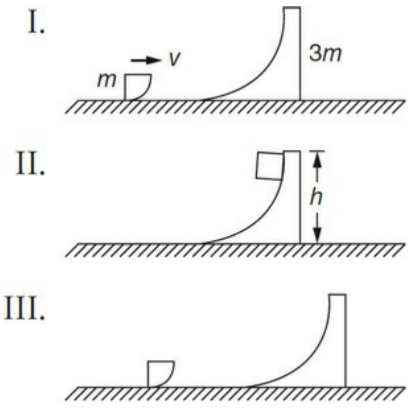
(A) 1.5 m/s
(B) 2.0 m/s
(C) 3.0 m/s
(D) 4.5 m/s
(E) 6.0 m/s

23. A 1000 kg (empty mass) railroad car is rolling without friction on a horizontal track at a speed of 2.0 m/s. Sand is poured into the open top of the car for the time interval from $t = 0$ to $t = 4.0$ s. The mass of the sand poured into the car as a function of time is $m(t) = 60t^2$. The velocity of the car at a time of 4.0 s is most nearly



(A) 1 m/s
(B) 2 m/s
(C) 3 m/s
(D) 4 m/s
(E) 5 m/s

**Questions 24–25**. A small block of mass $m$ slides on a horizontal frictionless surface toward a ramp of mass $3m$ which is also free to move on the surface. The small block slides up to a height $h$ on the ramp with no friction (Figure I), then they move together (Figure II), and the small block slides back down the ramp to the horizontal surface (Figure III). Both the block and the ramp continue to slide on the horizontal surface after they separate.




24. Which of the following is true regarding the conservation laws throughout this process?

(A) Kinetic energy is conserved from Figure I to Figure II.
(B) Momentum is conserved from Figure I to Figure III.
(C) Kinetic energy is conserved from Figure II to Figure III.
(D) Potential energy is conserved from Figure I to Figure II.
(E) Potential energy is conserved from Figure II to Figure III.

25. Which of the following is a true statement regarding Figure III?

(A) The small block is moving to the left and the ramp is moving to the right.
(B) The small block is moving to the right and the ramp is moving to the left.
(C) The small block is moving to the right and the ramp is moving to the right.
(D) The small block is moving to the left and the ramp is moving to the left.
(E) The small block and the large block are moving with the same velocity.

**Questions 26–27**. A remote controlled stunt car of mass 800 kg initially moving at 10 m/s is crashed into a rail car of mass $m$ that is initially at rest. The cars stick together, and the speed $v$ of both cars after the collision is given by $v = \dfrac{6}{t+1}$.

26. By considering the fact that the crash occurs at time $t = 0$, determine the mass $m$ of the rail car.

(A) 288 kg
(B) 445 kg
(C) 533 kg
(D) 698 kg
(E) 800 kg

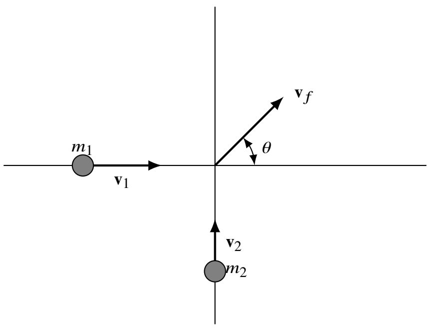
27. The magnitude of the resisting force acting on the cars as a function of time after the collision is

(A) $\dfrac{6m}{t+1}$
(B) $6m(t+1)$
(C) $6m(t+1)^2$
(D) $\dfrac{6m}{(t+1)^2}$
(E) $\dfrac{m(t+1)^2}{6}$

28. A moving object is changing its momentum during a time interval. If a graph of momentum vs. time is plotted, the net force acting on the mass at any time can be determined by finding the

(A) slope of line tangent to the graph at that time
(B) area under the graph
(C) $y$-intercept of the graph
(D) $x$-intercept of the graph
(E) change in slope of the graph from beginning to end

29. Two masses moving along the coordinates axes as shown collide at the origin and stick to each other. What is the angle $\theta$ that the final velocity that makes with the $x$-axis?




(A) $\tan^{-1}(v_2/v_1)$
(B) $\tan^{-1}[m_1v_1/(m_1+m_2)]$
(C) $\tan^{-1}(m_1v_2/m_2v_1)$
(D) $\tan^{-1}(m_2v_2^2/m_1v_1^1)$
(E) $\tan^{-1}(m_2v_2/m_1v_1)$

30. A mass traveling in the $+x$ direction collides with a mass at rest. Which of the following statements is true?

(A) After the collision, the two masses will move with parallel velocities
(B) After the collision, the masses will move with anti-parallel velocities
(C) After the collision, the masses will both move along the $x$-axis
(D) After the collision, the $y$-components of the velocities of the two particles will sum to zero.
(E) None of the above

31. A mass $m_1$ initially moving at speed $v_0$ collides with and sticks to a spring attached to a second, initially stationary mass $m_2$. The two masses continue to move to the right on a frictionless surface as the length of the spring oscillates. At the instant that the spring is maximally extended, the velocity of the first mass is


(A) $v_0$
(B) $m_1^2v_0/(m_1+m_2)^2$
(C) $m_2v_0/m_1$
(D) $m_1v_0/m_2$
(E) $m_1v_0/(m_1+m_2)$

**AP PHYSICS C: MOMENTUM, IMPULSE, COLLISIONS, AND CENTER OF MASS**
**SECTION II**
**5 Questions**

**Directions:** Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. A projectile is fired from the edge of a cliff 100 m high with an initial speed of 60 m/s at an angle of elevation of 45°.

   (a) Write equation for $x(t)$, $y(t)$, $v_x$ and $v_y$. Choose the origin of your coordinate system at the particle's original location.

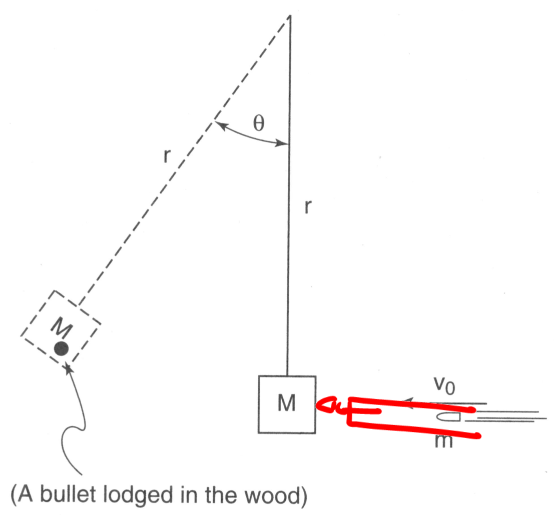
   (b) Calculate the location and velocity of the particle at time $t = 5$ s.

   Suppose the projectile experiences an internal explosion at time $t = 4$ s with an internal force purely in the $y$-direction, causing it to break into a 2 kg and a 1 kg fragment.

   (c) If the 2 kg fragment is 77 m above the height of the cliff at $t = 5$ s, what is the $y$-coordinate of the position of the 1 kg piece?

   (d) If the speed of the 2 kg fragment is 46 m/s and the fragment is falling at $t = 5$ s, what is the $y$-component of the velocity of the 1 kg fragment?

2. The Ballistic Pendulum. To determine the muzzle speed of a gun, a bullet is shot into a mass $M$ from a string as shown below, causing $M$ to swing upward through a maximum angle of $\theta$.




(a) What is the speed of $M$ the instant after the bullet lodges in it?

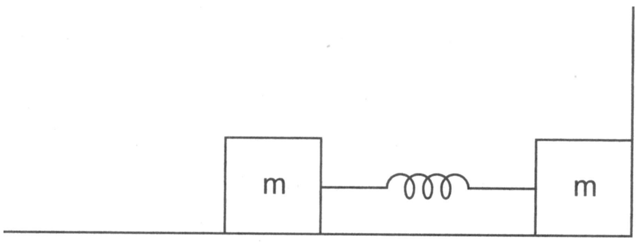
(b) What is the speed of the bullet before it hits $M$?

(c) What is the tension in the string at the highest point of the pendulum's swing (when the string makes an angle of $\theta$ with the vertical as shown)?

3. Two masses are connected by a spring (spring constant $k$) resting on a frictionless horizontal surface as shown. The right mass is initially in contact with a wall. A brief blow to the left block leaves it with an initial velocity $v_0$ to the right.

   (a) What is the maximum compression of the spring as the left block moves to the right?

   After the spring is maximally compressed, it eventually moves to the left, away from wall. As it moves away from the wall, it continues oscillating.



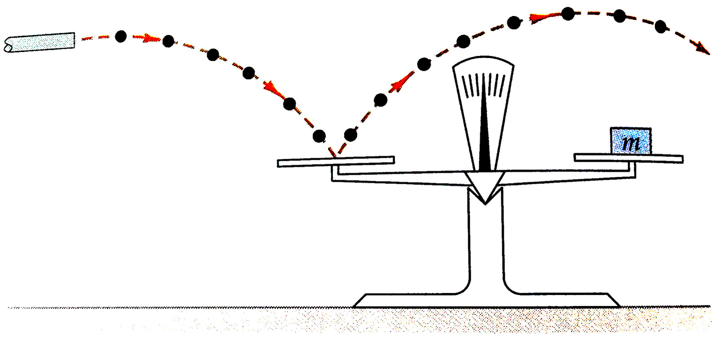

   (b) What is the net momentum of the two masses after they leave the wall?

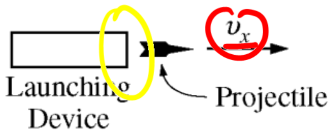
   (c) What is the total mechanical energy of the oscillating spring system?

   (d) What is the relative velocity of the two masses when the spring is maximally compressed?

   (e) What is the maximum compression of the spring after the two masses have left the wall? Compare the compression to the maximum compression calculated in part (a) and explain any similarities and differences.

4. A stream of glass beads, each with a mass of 0.5 g, comes out of a horizontal tube at 100 per second. The beads fall a distance of 0.5 m to a balance pan and bounce back to their original height as shown in the figure below. How much mass must be placed in the other pan of the balance to keep the pointer at zero?






5. A projectile is fired horizontally from a launching device, exiting with a speed $v_x$. While the projectile is in the launching device, the impulse imparted to it is $J_p$, and the average force on it is $F_{avg}$. Assume the force becomes zero just as the projectile reaches the end of the launching device. Express your answers to parts (a) and (b) in terms of $v_x$, $J_p$, $F_{avg}$, and fundamental constants, as appropriate.

   (a) Determine an expression for the time required for the projectile to travel the length of the launching device.

   (b) Determine an expression for the mass of the projectile.

   The projectile is fired horizontally into a block of wood that is clamped to a tabletop so that it cannot move. The projectile travels a distance $d$ into the block before it stops. Express all algebraic answers to the following in terms of $d$ and the given quantities previously indicated, as appropriate.

   (c) Derive an expression for the work done in stopping the projectile.

   (d) Derive an expression for the average force $F_b$ exerted on the projectile as it comes to rest in the block.

   Now a new projectile and block are used, identical to the first ones, but the block is not clamped to the table. The projectile is again fired into the block of wood and travels a new distance $d_n$ into the block while the block slides across the table a short distance $D$. Assume the following: the projectile enters the block with speed $v_x$, the average force $F_b$ between the projectile and the block has the same value as determined in part (d), the average force of friction between the table and the block is $f_T$, and the collision is instantaneous so the frictional force is negligible during the collision.

   ↳ only when the block travels after the projectile

   (e) Derive an expression for $d_n$ in terms of $d$, $D$, $f_T$, and $F_b$, as appropriate.

   (f) Derive an expression for $d_n$ in terms of $d$, the mass $m$ of the projectile, and the mass $M$ of the block.

projectile reaches the end of the launching device. Express your answers to parts (a) and (b) in terms of $v_x$, $J_p$, $F_{avg}$, and fundamental constants, as appropriate.

$\Delta t$

(a) Determine an expression for the time required for the projectile to travel the length of the launching device.

(b) Determine an expression for the mass of the projectile.

a) $\vec{J_p} = \int \vec{F}\,dt = F_{avg}\cdot\Delta t$

$\boxed{\Delta t = \frac{J_p}{F_{avg}}}$

b) $J_p = \Delta p = m\Delta v$ ← $v_x$

$\boxed{m = \frac{J_p}{v_x}}$


(c) Derive an expression for the work done in stopping the projectile.

(d) Derive an expression for the average force $F_b$ exerted on the projectile as it comes to rest in the block.

(c) $W = \Delta K = \cancel{K_f}^{\,0} - K_i = -\frac{1}{2}mv_x^2 = -\frac{1}{2}\left(\frac{J_p}{\cancel{v_x}}\right)v_x^{\cancel{2}} = \boxed{-\frac{1}{2}J_p\cdot v_x}$

(1) $\quad \frac{J_p}{v_x}$ part (b) (1) $\quad$ (1)

$K_i = \frac{1}{2}J_p\cdot v_x = F_b\cdot d$

d) $W = \int \vec{F}\cdot d\vec{x} = F_b\cdot \underbrace{\Delta x}_{d} = -F_b\cdot d = -\frac{1}{2}J_p\cdot v_x$ (1)

$\boxed{F_b = \frac{J_p\cdot v_x}{2d}}$ (1)

(e) Derive an expression for $d_n$ in terms of $d$, $D$, $f_T$, and $F_b$, as appropriate.

travels after

2 (f) Derive an expression for $d_n$ in terms of $d$, the mass $m$ of the projectile, and the mass $M$ of the block.

- Conservation of energy

$\boxed{K + W_{block} + W_{friction} = \cancel{K}^{\,0}}$

when projectile enters the block $\qquad$ when the block stops.

$K_i - F_b\cdot d_n - f_T\cdot D = 0$

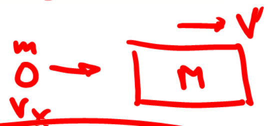
= work done to stop it

$F_b\cdot d - F_b\cdot d_n - f_T\cdot D = 0$

$F_b\cdot d_n = F_b\cdot d - f_T\cdot D$

$\boxed{d_n = \frac{F_b\cdot d - f_T\cdot D}{F_b}} \rightarrow \boxed{d_n = d - \frac{f_T}{F_b}\cdot D}$

f)




momentum
$mv_x = (M+m)V$

$V = \frac{mv_x}{M+m}$

energy
$\frac{1}{2}(M+m)V^2 = f_T\cdot D$

$\frac{1}{2}(M+m)\frac{m^2v_x^2}{(M+m)^2} = f_T\cdot D$

$\frac{mF_b\cdot d}{M+m} = f_T\cdot D = F_b d - F_b\cdot d_n$

$\boxed{d_n = d\left(1 - \frac{m}{M+m}\right)}$