WELCOME TO AP PHYSICS C

Pre-requisites

- **Physics 11 and 12:** You will need to be comfortable with the topics covered in high-school level physics courses.
- **Calculus:** Physics C exams are calculus based, and you will be required to do basic differentiation and integration. You don't need to be an expert, but basic knowledge is required. Differentiation and integration in the course are generally not difficult, but there are occasional challenges.
- **Vectors:** You need to be comfortable with vector operations, including addition and subtraction, multiplication/division by constants, as well as dot products and cross products.

AP Physics C Exams

There are two calculus-based AP Physics C exams, which are usually taken together on the same day, in the first or second week of May of each year.

- Mechanics
- Electricity and Magnetism

Topic 1: Kinematics

Advanced Placement Physics C

Dr. Timothy Leung

November 26, 2020

Olympiads School

Files for You to Download

There are a considerable number of files for you to download at the beginning of the course:

- `PhysAPC-courseOutline.pdf`–The course outline
- `PhysAPC-equationSheet.pdf`–Equation sheet for your exams
- `PhysAPC-01-kinematics.pdf`
- `PhysAPC-01a-vectorCalculus.pdf`–Vectors and calculus handout
- `PhysAPC-01b-kinematicsHandout.pdf`–Basic kinematics, expanded version
- `PhysAPC-01c-motionGraphs.pdf`–Handout on motion graphs
- `PhysAPC-01-Homework.pdf`–Homework problems for Topic 1

File Download

Please download/print the PDF file for the class slides before each class. There is no point copying notes that are already on the slides. Instead, focus on things that aren't necessarily on the slides. If you wish to print the slides, we recommend printing *four* slides per page.

Vectors and Calculus

Please refer to the handout to make sure that you are familiar with basic vector and calculus.

Kinematics

Kinematics

Kinematics is a discipline within mechanics concerning the motion of bodies. It describes the relationship between

- Position
- Displacement
- Distance
- Velocity
- Speed
- Acceleration

Kinematics does not deal with the causes of motion.

Position

Position $\mathbf{x}$ describes the location of an object in a coordinate system. The origin ofthe coordinate system is called the "reference point". The SI unit for position is **meter**, m.

$$\boxed{\mathbf{x}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}}$$

Vectors in 2D/3D Cartesian space are generally using the **IJK notation**

- $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are **basis vectors** indicating the directions of the x, y and z axes. Basis vectors are **unit vectors** (i.e. length 1)
- The IJK notation does not explicitly give the magnitude or the direction of the vector (needs to be calculated using the Pythagorean theorem)

Displacement

Displacement $\Delta\mathbf{x}(t)$ is the change in position from the initial position $\mathbf{x}_0$ within the same coordinate system:

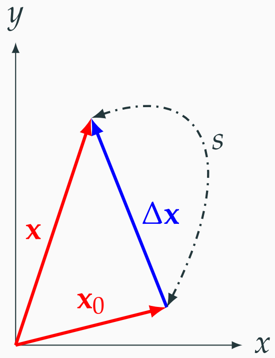
$$\boxed{\Delta\mathbf{x}(t) = \mathbf{x}(t) - \mathbf{x}_0 = (x - x_0)\hat{\boldsymbol{\imath}} + (y - y_0)\hat{\boldsymbol{\jmath}} + (z - z_0)\hat{\boldsymbol{k}}}$$

- IJK notation makes vector addition and subtraction less prone to errors
- Since the reference point $\mathbf{x}_{\text{ref}} = \mathbf{0}$, the position vector $\mathbf{x}$ is also its displacement from the reference point

Distance

Distance $s(t)$ is a quantity that is *related* to displacement. The unit for distance is also a meter (m).

- The length of the path taken by an object when it travels from $\mathbf{x}_0$ to $\mathbf{x}$
- A scalar quantity
- Always positive, i.e. $s \geq 0$
- Although the magnitude of the displacement vector is also a scalar, it is not necessarily the same as distance
- $s \geq |\Delta \mathbf{x}|$



Instantaneous Velocity

Since position $\mathbf{x}$ is differentiable in time t, then the **instantaneous velocity** can be found at any time by differentiating $\mathbf{x}$ with respect to t. The unit for velocity is **meters per second** (m / s):

$$\boxed{\mathbf{v}(t) = \frac{d\mathbf{x}}{dt}}$$

Since $\mathbf{x}$ has x, y and z components along the $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ directions that are linearly independent, we can take the time derivative in every component:

$$\mathbf{v}(t) = \frac{d\mathbf{x}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{\imath}} + \frac{dy}{dt}\hat{\boldsymbol{\jmath}} + \frac{dz}{dt}\hat{\boldsymbol{k}} = u\hat{\boldsymbol{\imath}} + v\hat{\boldsymbol{\jmath}} + w\hat{\boldsymbol{k}}$$

Integrating Velocity to Get Position/Displacement

Conversely, if instantaneous velocity $\mathbf{v}(t)$ is the time rate of change of position $\mathbf{x}(t)$, then $\mathbf{x}(t)$ is the time integral of $\mathbf{v}(t)$:

$$\boxed{\mathbf{x}(t) = \int \mathbf{v}(t) dt + \mathbf{x}_0}$$

The constant of integration $\mathbf{x}_0$ is the *initial position* at $t = 0$. We can integrate each component to get $\mathbf{x}$:

$$\mathbf{x}(t) = \left(\int u\hat{\boldsymbol{\imath}} + \int v\hat{\boldsymbol{\jmath}} + \int w\hat{\boldsymbol{k}} \right) dt + \mathbf{x}_0$$

Average Velocity

Average velocity $\overline{\mathbf{v}}$ of an object is the *finite* change in position $\Delta\mathbf{x}$ over a *finite* time interval Δt:

$$\boxed{\overline{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{\int \mathbf{v} dt}{\Delta t}}$$

Like instantaneous velocity, we can find the x, y and z components of average velocity by separating components in each direction:

$$\overline{\mathbf{v}} = \frac{\Delta x}{\Delta t}\hat{\boldsymbol{\imath}} + \frac{\Delta y}{\Delta t}\hat{\boldsymbol{\jmath}} + \frac{\Delta z}{\Delta t}\hat{\boldsymbol{k}}$$

The notation $\overline{v}$ means that v is averaged over *time*, while the notation $\langle v \rangle$ is used if v is the average of many particles (called an *ensemble average*)

Instantaneous & Average Speed

Instantaneous speed v is the time rate of change of *distance*.

$$\boxed{v = \frac{ds}{dt}}$$

- Since $s \geq 0$, instantaneous speed must also be positive $v \geq 0$
- Instantaneous speed is the magnitude of the instantaneous velocity vector, i.e. $v = |\mathbf{v}|$

Likewise, **average speed** is similar to average velocity: it is the distance travelled over a finite time interval.

$$\boxed{\overline{v} = \frac{s}{\Delta t} = \frac{\int v dt}{\Delta t}}$$

The SI unit of speed is also meters per second m/s.

Instantaneous & Average Acceleration

In the same way that velocity is the time rate of change in position, **instantaneous acceleration $\mathbf{a}(t)$** is the time rate of change in velocity, with a unit of **meters per second squared** ($\mathrm{m/s^2}$):

$$\boxed{\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{x}}{dt^2}}$$

Likewise, **average acceleration $\bar{\mathbf{a}}$** is the finite change in velocity $\Delta\mathbf{v}$ over a finite time interval Δt:

$$\bar{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{\mathbf{v} - \mathbf{v}_0}{\Delta t}$$

Note that acceleration only requires a *change* in velocity. It does *not* necessarily mean an object has to speed up or slow down.

Special Notation

Physicists and engineers use a special notation when the derivative is taken with respect to *time*, by writing a dot above the variable. For example:

$$\mathbf{v} = \dot{\mathbf{x}}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{x}}$$

We will use this notation whenever it is convenient

Integrating Acceleration to Get Velocity

Velocity $\mathbf{v}(t)$ is the time integral of acceleration $\mathbf{a}(t)$:

$$\boxed{\mathbf{v}(t) = \int \mathbf{a}(t)dt + \mathbf{v}_0}$$

Again, we can integrate each component of the vector independently:

$$\mathbf{v}(t) = \left(\int a_x \hat{\boldsymbol{\imath}} + \int a_y \hat{\boldsymbol{\jmath}} + \int a_z \hat{\boldsymbol{k}} \right) dt + \mathbf{v}_0$$

If You Are Curious

The time derivative of acceleration is called **jerk**, with a unit of $\mathrm{m/s^3}$:

$$\mathbf{j} = \frac{d\mathbf{a}}{dt} = \frac{d^2\mathbf{v}}{dt^2} = \frac{d^3\mathbf{x}}{dt^3}$$

The time derivative of jerk is **jounce**, or **snap**, with a unit of $\mathrm{m/s^4}$:

$$\mathbf{s} = \frac{d\mathbf{j}}{dt} = \frac{d^2\mathbf{a}}{dt^2} = \frac{d^3\mathbf{v}}{dt^3} = \frac{d^4\mathbf{x}}{dt^4}$$

The next two derivatives of snap are called **crackle** and **pop**, but these higher derivatives of position vector are rarely used. We will *not* be using them.

Acceleration as Functions of Velocity and Position

Acceleration may be expressed as functions of velocity and position rather than of time, if an object's motion is dominated by these forces:

- Gravitational or electrostatic forces: $a(x) = \dfrac{Gm_s}{x^2} \quad a(x) = \dfrac{kq_1q_2}{mx^2}$
- Spring force: $a(x) = -\dfrac{k}{m}x$
- Damping force: $a(v) = -bv$
- Aerodynamic drag: $a(v) = \left[\dfrac{1}{2}\rho C_D A_{\mathrm{ref}}\right] v^2$

In these cases, solving for $x(t)$, $v(t)$ and $a(t)$ will require solving a differential equation (see handout).

Kinematic Equations

Kinematic Equations

While kinematic problems in AP Physics C exams often require calculus, these basic kinematic equations for constant acceleration are still a powerful tool.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

The variables of interests are:

- Initial position: x_0
- Position at time t: x
- Initial velocity: v_0
- Instantaneous velocity: v
- Acceleration (constant): a

Kinematic equations are sometimes called the “Big-five” or “Big-four” equations. Here, you will only be given three equations in your equation sheet. You will still be required to integrate when acceleration is not constant.

Motion Graphs

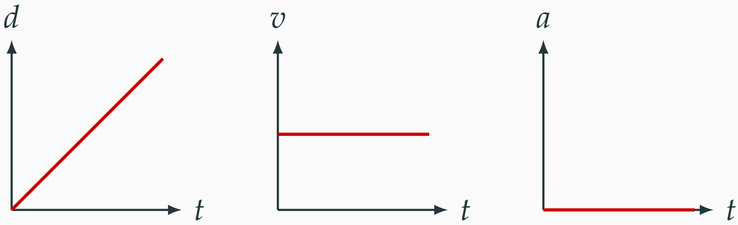
Motion Graphs

You should already be familiar with the basic motion graphs for 1D motion:

- Position vs. time ($x - t$) graph
- Velocity vs. time ($v - t$) graph
- Acceleration vs. time ($a - t$) graph

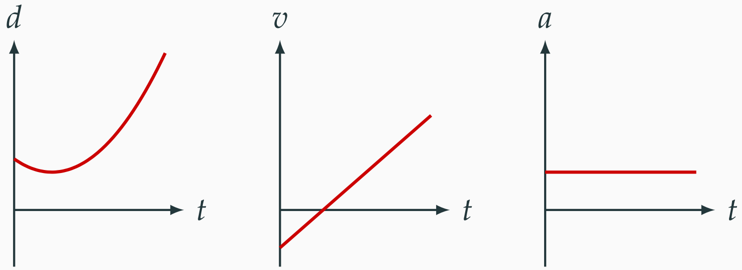
However, depending on the situation, it may be more useful to plot motion using other quantities as well.

Uniform Motion: Constant Velocity



- Constant velocity has a straight line in the $d - t$ graph
- The slope of the $d - t$ graph is the velocity v, which is constant
- The slope of the $v - t$ graph is the acceleration a, which is zero in this case

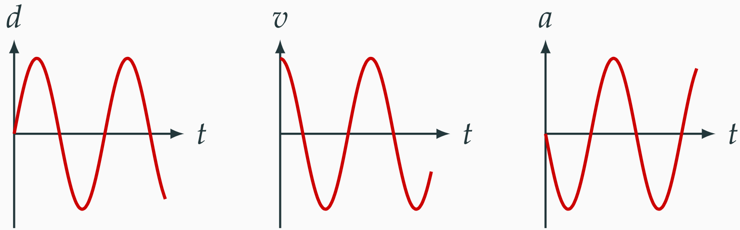
Uniform Acceleration: Constant Acceleration



- The $d - t$ graph for motion with constant acceleration is part of a *parabola*
  - If the parabola is *convex*, then acceleration is positive
  - If the parabola is *concave*, then acceleration is negative
- The $v - t$ graph is a straight line; its slope (a constant) is the acceleration

Simple Harmonic Motion

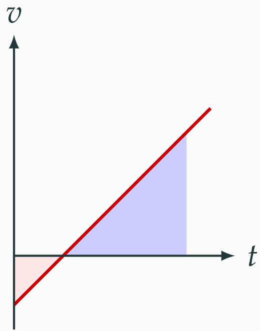
For **harmonic motions**, neither position, velocity nor acceleration are constant:



Bottom line: regardless of the type motion,

- The $v - t$ graph is the slope of the $d - t$ graph
- The $a - t$ graph is the slope of the $v - t$ graph

Area Under Motion Graphs



The area under the $v - t$ graph is the displacement $x - x_0$.

- Area *above* the time axis: $+$ displacement
- Area *below* the time axis: $-$ displacement

Likewise, the area under the $a - t$ graph is the change in velocity $v - v_0$.

Velocity Squared vs. Displacement

If velocity information is given as a function of position[1] then a motion graph can be plotted using this kinematic equation:

$$\underbrace{v^2}_{y} = \underbrace{v_0^2}_{b} + \underbrace{2a}_{m} \underbrace{(x - x_0)}_{x}$$

by plotting v^2 on the y-axis and displacement $\Delta x = x - x_0$ on the x-axis. The slope of the graph is $m = 2a$. The square of the initial velocity (v_0^2) is the y-intercept.

[1]Depends on experimental set up

Graphing “Linear” Functions

This concept extends to graphing other physical quantities not relating to motion:

- To find the index of refraction of a material using Snell’s law, plot $\sin\theta_i$ vs. $\sin\theta_2$ (rather than θ_1 vs. θ_2). The slope is the index n:

$$\underbrace{\sin\theta_1}_{y} = \underbrace{n}_{m}\underbrace{\sin\theta_2}_{x}$$

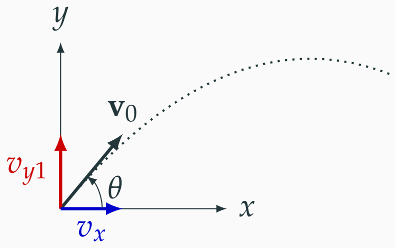
- To relate the period of oscillation of a simple pendulum to the length of the pendulum, plot T vs. $\sqrt{L}$:

$$\underbrace{T}_{y} = \underbrace{\frac{2\pi}{\sqrt{g}}}_{m}\underbrace{\sqrt{L}}_{x}$$

Projectile Motion

Projectile Motion

A **projectile** is an object that is launched with an initial velocity of $\mathbf{v}_0$ along a parabolic trajectory and accelerates only due to gravity.



- x-axis is the *horizontal* direction, with the $(+)$ direction pointing *forward*
- y-axis is the *vertical* direction, with the $(+)$ direction pointing *up*
- The reference point is where the projectile is launched
- Consistent with the right-handed Cartesian coordinate system

Horizontal (x) Direction

No acceleration (i.e. $a_x = 0$) in the horizontal direction, therefore horizontal velocity component is constant. The kinematic equations reduce to:

$$x = v_x t = \left[v_0 \cos\theta\right] t$$

where x is the horizontal position at time t, v_0 is the magnitude of the initial velocity, $v_x = v_0 \cos\theta$ is its horizontal component.

Vertical (y) Direction

Constant acceleration due to gravity alone in the vertical direction, i.e. $a_y = -g$. (Acceleration is *negative* due to the way we defined the coordinate system.) The important equation is this one:

$$y = [v_0 \sin\theta]\, t - \frac{1}{2} g t^2$$

These two kinematic equations may also be useful:

$$v_y = [v_0 \sin\theta] - gt$$
$$v_y^2 = [v_0 \sin\theta]^2 - 2gy$$

Solving Projectile Motion Problems

Horizontal and vertical motions are independent of each other, but there are variables that are shared in both directions, namely:

- Time t
- Launch angle θ (above the horizontal)
- Initial speed v_0

When solving any projectile motion problems

- *Two* equations with *two* unknowns
- If an object lands on an incline, there will be a third equation describing the relationship between x and y

Symmetric Trajectory

A projectile's trajectory is symmetric if the object lands at the same height as when it launched. These equations are *not* provided in the AP Exam equation sheet, but it can save you a lot of time if you can use them, instead of deriving them during the exam.

- Time of flight

$$t_{\max} = \frac{2v_0 \sin\theta}{g}$$

- Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum height

$$y_{\max} = \frac{v_0^2 \sin^2\theta}{2g}$$

The angle θ is measured above the the horizontal.

Maximum Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

- Maximum range occurs at $\theta = \dfrac{\pi}{2}$
- For a given initial speed v_0 and range R, launch angle θ is given by:

$$\theta_1 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

But there is another angle that *gives the same range*!

$$\theta_2 = 90° - \theta_1$$

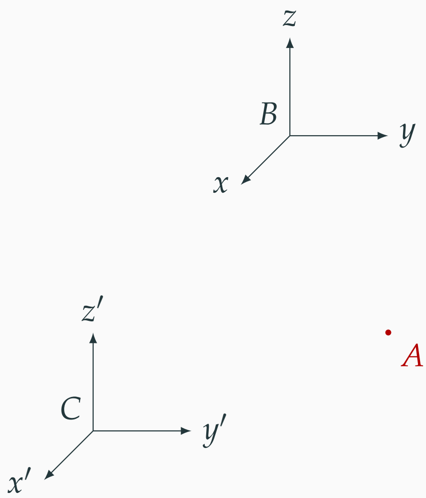
Relative Motion

Relative Motion

All motion quantities must be measured relative to a frame of reference

- A frame of reference is the coordinate system from which all physical measurements are made.
- In *classical* mechanics, the coordinate system is the Cartesian system
- There is no absolute motion/rest: all motions are relative
- All laws of physics are equal in all inertial (non-accelerating) frames of reference (principle of relativity)

Relative Motion

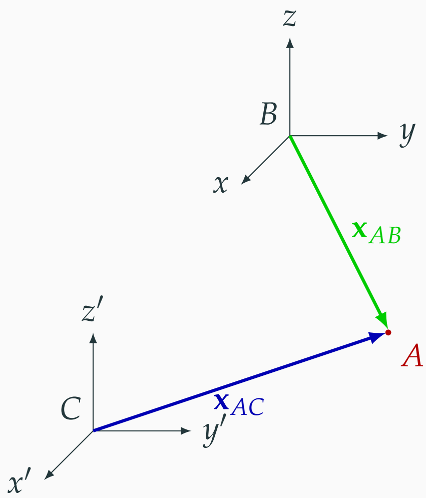


Two frames of reference

- B with axes x, y, z
- C with axes x', y', z'

The two reference frames may (or may not) be moving relative to each other. The motion of the two reference frames affect how motion of A is calculated.

Relative Motion

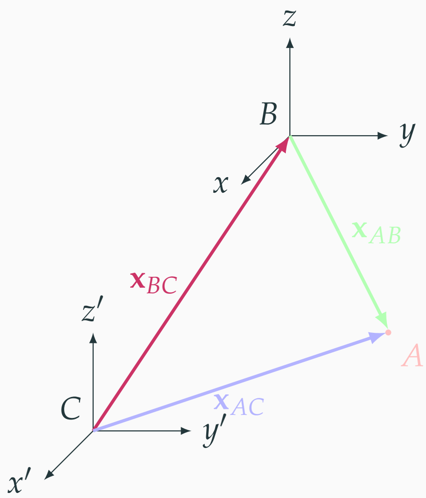


The position of A can be described by

- $\mathbf{x}_{AB}(t)$ (relative to frame B)
- $\mathbf{x}_{AC}(t)$ (relative to frame C)

It is obvious that $\mathbf{x}_{AB}(t)$ and $\mathbf{x}_{AC}(t)$ are different vectors

Relative Motion



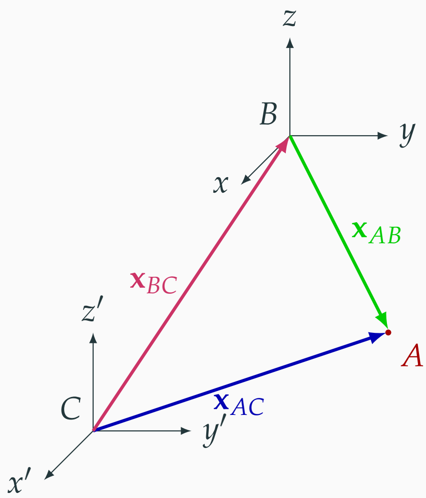
The position vector of the origins of the two reference frames is given by $\mathbf{x}_{BC}$

- The vector pointing from the origin of frame C to the origin of frame B
- If the two frames are moving relative to each other, then $\mathbf{x}_{BC}$ is also a function of time

Without using vector notations, the relationship between the vectors is obvious:

$$\boxed{\mathbf{x}_{AC} = \mathbf{x}_{AB} + \mathbf{x}_{BC}}$$

Relative Motion



Starting from the definition of **relative position**:

$$\boxed{\mathbf{x}_{AC} = \mathbf{x}_{AB} + \mathbf{x}_{BC}}$$

Differentiating all terms with respect with time, we get the equation for **relative velocity**:

$$\boxed{\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}}$$

Differentiating again, and we obtain the equation for **relative acceleration**:

$$\boxed{\mathbf{a}_{AC} = \mathbf{a}_{AB} + \mathbf{a}_{BC}}$$

Relative Velocity

In classical mechanics, the equation for relative velocities follows the **Galilean velocity addition rule**, which applies to speeds much less than the speed of light:

$$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$$

The velocity of A relative to reference frame C is the velocity of A relative to reference frame B, plus the velocity of B relative to C. If we add another reference frame D, the equation becomes:

$$\mathbf{v}_{AD} = \mathbf{v}_{AB} + \mathbf{v}_{BC} + \mathbf{v}_{CD}$$

Typical Problems

In an AP Physics C exam, questions involving kinematics usually appear in the multiple-choice section. The problems themselves are not very different compared to the Grade 12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use $g = 10\,\mathrm{m/s^2}$ in your calculations to make your lives simpler
- A lot of problems are *symbolic*, which means that they deal with the equations, not actual numbers
- Will be coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet