Topic 2: Dynamics

Advanced Placement Physics C

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Olympiads School

Dynamics

While we use **kinematics** to describe the motion of any object mathematically, we use **dynamics** to describe *what* causes motion (or more precisely, *what causes motion to change?*)

Laws of Motion

First Law of Motion

> **First Law: Every object will remain in its state of rest or uniform motion, until a net external force is applied to it.**

- Uniform motion means constant velocity; an object "at rest" is also in uniform motion with $v = 0$
- As long as an object moves in uniform motion, it must be that $F_{net} = 0$
- Common examples:
    - A hockey puck sliding on very smooth ice has gravity and normal force, but the net force is zero
    - A car traveling on a highway at 100 km/h has many forces acting on it, but the net force is zero
- This is a special case that assumes a constant mass.

Translational Equilibrium

If the net force on an object is zero ($\Sigma \mathbf{F} = \mathbf{0}$) then the object is in a *state of translational equilibrium*

- Dynamic equilibrium: the object is moving relative to us
- Static equilibrium: the object is not moving relative to us

Second Law of Motion

Second Law: **The acceleration of an object is proportional to, and along the direction of, the net external force.**

- Like the first law, this is also a "special case" that assumes a constant mass.
- The first two laws of motion can be summarized in the equation:

$$F_{\text{net}} = \Sigma F = ma$$

- For non-constant mass, net force is the rate of change of momentum p:

$$F_{\text{net}} = \frac{dp}{dt}$$

(More on this in a later topic!)

Mass

What is **mass** then? It is

- the property of an object that relates its acceleration to the force applied to it. Literally, it means:

$$m \equiv \frac{F_{\text{net}}}{a}$$

- Intrinsic to the object itself
- This is explicitly referred to as the object's **inertial mass**

Third Law of Motion

> **For every action there is always opposed and equal reaction: the mutual actions of two objects on each other are always equal, and directed to other object.**

For every action force on an object (B) due to another object (A), there is a reaction force which is equal in magnitude but opposite in direction, on object (A), due to object (B):

$$\boxed{\boldsymbol{F}_{AB} = -\boldsymbol{F}_{BA}}$$

- The action and reaction forces act on different objects!
- Third law is the natural consequence of the first and second law. Action/reaction forces are *internal* forces.

Forces

A **force** is the interaction between the objects.

- When there is interaction, then forces are created
- A "push" or a "pull"

There are two broad categories of forces:

- **Contact forces** act between two objects that are in contact with one another
- **Non-contact forces** act between two objects without them touching each other. They are also called "action-at-a-distance" force

Common Forces

Common Forces

Common everyday forces that we encounter in Physics 12 include:

- Weight (gravitational force) w (or F_G)
- Normal force N
- Friction (static f_s and kinetic f_k)
- Tension T
- Applied force F_a
- Spring force F_e
- Drag D (fluid resistance, then again in fluid mechanics)
- Buoyant force B (discussed in fluid mechanics)
- Electrostatic force F_E (discussed in E & M exam)
- Magnetic force F_M (discussed E & M exam)

Gravity

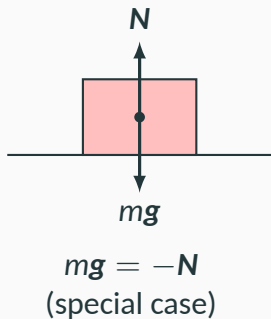Gravity is the force of attraction between all objects with mass

$$\boxed{w = mg}$$

- Near surface of Earth, use $g = 9.81 \, \text{m/s}^2$ (or $g = 10 \, \text{m/s}^2$ for your AP exam)
- You may be asked to find the value of g on some "unknown planet".
- w always points *down* (the direction of w is how down is defined)
- Based on the law of universal gravitation:

$$\boxed{F_g = \frac{Gm_1m_2}{r^2}} \quad \text{where} \quad G = 6.674 \times 10^{-11} \, \text{N} \cdot \text{m}^2/\text{kg}^2$$

Normal Force

N

mg

$m\mathbf{g} = -\mathbf{N}$
(special case)

- A force a surface exerts on another object that it is in contact with
- Always **perpendicular** to the contact surface
- **Special case:** When an object is on a horizontal surface with no additional applied force, the magnitude of the normal force is equal to the magnitude of the weight of the object, i.e. $N = mg$

Normal Force on a Stationary Slope

For this case, we define the *x*-axis to be along the slope, and *y*-axis to be perpendicular to the slope.
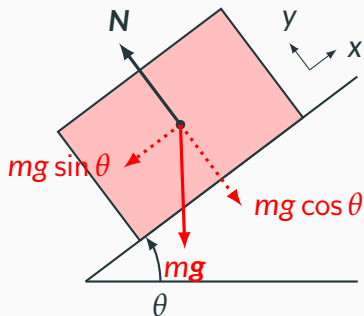
- On a stationary slope: $N = mg \cos \theta$
  - N decreases as ramp angle θ increases
- Weight has a component along the ramp ($mg \sin \theta$) that wants to slide the block down.

Friction

- A force that opposes the sliding of two surface against one another
- Always act in a direction that opposes motion or attempted motion
- Depends on:
    - Normal force N: The force the two surfaces are pressed against each other
    - Coefficient of friction (μ_s and μ_k): Smoothness of the surfaces, which itself depends on
        - The material(s) the surfaces are made of
        - The use of lubricants

μ is dependent on
surface-to-surface contact
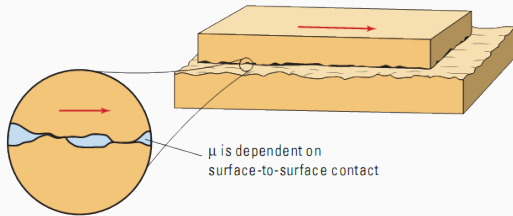
Static Friction

Static friction between the two surfaces is when there is no relative motion between them

- Increases with increasing applied force
- Maximum when the object is just about to move

$$f_s \leq \mu_s N$$

Quantity	Symbol	SI Unit
Magnitude of static friction	f_s	N
Static friction coefficient	μ_s	no units
Magnitude of normal force	N	N

Kinetic friction between two surfaces is when they are moving relative to each other. f_k is constant along the path of movement as long as **N** stays constant

$$\boxed{f_k = \mu_k N}$$

Quantity	Symbol	SI Unit
Magnitude of kinetic friction	f_k	N
Kinetic friction coefficient	μ_k	no units
Magnitude of normal force	N	N

Kinetic friction coefficient is always lower than the static coefficient, otherwise nothing will ever move:

$$\mu_k \leq \mu_s$$

Consider a simple case of a box being pulled along a level floor. The free-body diagram is simple (left). How do the magnitudes of the applied force F_a and friction f compare?
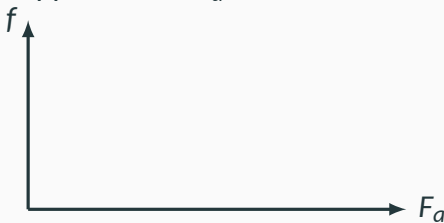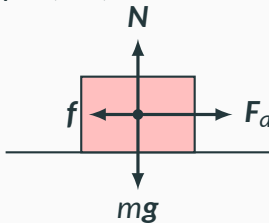
Drag

Drag (or **fluid resistance**) is the force opposing the motion of an object moving in a fluid, with magnitude:

$$D = \frac{1}{2}\rho V_\infty^2 A_{\text{ref}} C_d$$

Quantity	Symbol	SI Unit
Magnitude of drag force	D	N
Density of the fluid	ρ	kg/m^3
Free-stream fluid velocity	V_∞	m/s
Reference area	A_{ref}	m^2
Drag coefficient	C_d	(no unit)

Drag coefficient depends on the shape and surface smoothness of the object. For blunt objects ("bluff bodies") A_{ref} is the frontal area; for streamlined objects A_{ref} is the planform (top-view) area

Drag

In AP Physics you are *not* asked to know the drag equation. However, you should know that drag (air resistance) depends on the motion of the object and is not a constant.

Terminal Velocity

When we take drag force into account, we understand that the drag force increases as an object speeds up, and therefore a free-falling object does *not* accelerate infinitely. Instead it reaches a **terminal velocity**.

There is no air resistance just as the object *begins* to fall. Acceleration is due to gravity alone.

Drag increases as *v* increases. Magnitude of acceleration decreases, but the object continues to gather speed

Terminal velocity is reached when the drag force equals the object's weight. Not net force; no acceleration.

Tension in a Cable

**Tension** is the force exerted on and by a cable, rope, or string.

- You can't push on a rope
- Assume the cable/rope/string to be mass less
- Force can change direction when used with pulleys

Spring Force

The spring force F_e is the force a compressed or stretched spring exerts onto objects connected to it. It obeys Hooke's Law:

$$F_e = -kx$$

where x is the relative displacement of the ends of the spring.

Before dropping mass

Maximum compression mass at rest
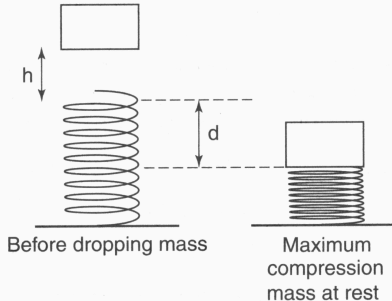
Free-Body Diagrams

Free-Body Diagrams

- Acceleration (if there is going to be any at all) depends on net force F_{net}
- Without a vector sum of all the forces, we cannot determine the magnitude, direction of the acceleration, or how acceleration will evolve in time
- We use **free-body diagrams** (FBD) to represent all the forces.
    - Very important in solving any dynamics problems
    - Don't try to save this step, even if the problem does not ask for it
    - Always draw FBD for solving classical mechanics problem

Free-Body Diagrams

In Grade 11/12 Physics, for rectilinear motion (no *rotational motion*), FBDs are usually drawn by assuming that all forces acting at the CM, represented by the "big dot". For example:
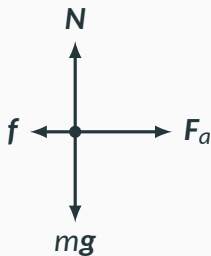

We can still use this method for rectilinear motion. However, this is not entirely correct. We notice that:

- Gravitational force *mg* acts at the CM, but
- Normal force *N*, friction *f* and applied force F_a act at the point of contact

Free-Body Diagrams

Instead, forces should be drawn at the point where they are applied. In the example below, a sphere rolling down a ramp should have weight $m\mathbf{g}$, normal force $\mathbf{N}$ and static friction $\mathbf{f}_s$ acting on it:
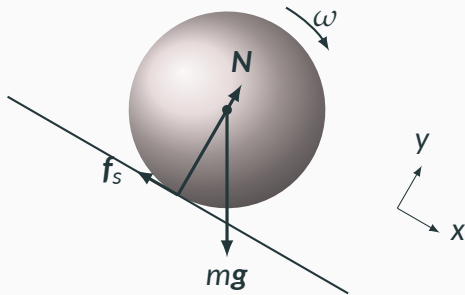

Once the FBD is drawn, decide on the axes to help you solve the motion. One of the axes should line up with the direction of motion. This guarantees that the *other* axis will not have any net force

Example Problem

A more difficult static problem may involve two surfaces with two different friction coefficients. For example, a ladder leaning on a wall. This problem cannot be solved without first understanding rotational motion, but we can still draw a FBD.

Example: A uniform ladder is 5.0 m long and weighs 400 N. The ladder rests against a slippery vertical wall, as shown in Figure. The inclination angle between the ladder and the rough floor is 53°. Find the reaction forces from the floor and from the wall on the ladder and the coefficient of static friction μ_s at the interface of the ladder with the floor that prevents the ladder from slipping.
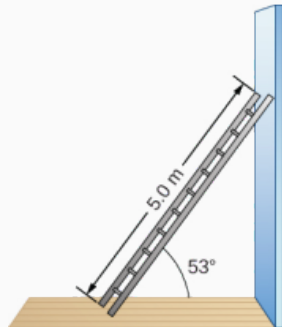
Multi-Body Problems

ApplyingThird Law on Connected Bodies

- The objects are connected by a cable or a solid linkage with negligible mass
- All objects (usually) have the same acceleration
- Require multiple free-body diagrams

Solving Connected-Bodies Problems

To solve a connected-bodies problem, you can follow these procedures:

1. Draw a FBD on each of the objects
2. Sum all the forces on all the objects along the direction of motion
   - Direction of motion is usually very obvious
   - All internal forces should cancel and do not figure into the acceleration of the system
3. Compute the acceleration of the entire system using second law of motion
   - Remember that (usually) every object has the same acceleration!
4. Go back to the FBD of each of the objects and compute the unknown forces (usually tension)

Different Types of Connected Bodies

Multiple objects pressed against one another. There may not be friction, but there are definitely action/reaction forces between the blocks.
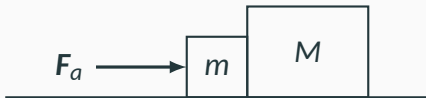

Or multiple objects stacked on top of one another. The contact surface between M and the floor may (or may not) have friction, while the surface between M and m must have a friction coefficient μ.
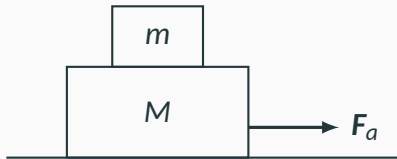
Pulley Problems

Example Problem: Atwood Machine

An **Atwood machine** is made of two objects connected by a rope that runs over a pulley. The pulley allows the direction of force and direction of motion to change between two objects.
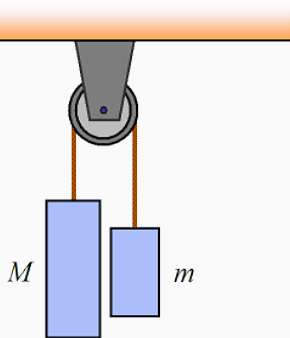
Example: The object on the left has a mass of M and the object on the right has a mass of m.

- What is the acceleration of the masses?
- What is the tension in the rope?

A More Typical Problem

More typically, an Atwood machine problem is one where two objects are sliding on a surface. These surfaces may have (or may not) have friction. In this example, two blocks are connected by a massless string over a frictionless pulley as shown in the diagram.

(a) Determine the acceleration of the blocks.
(b) Calculate the tension in the string.

A Slightly More Difficult Problem
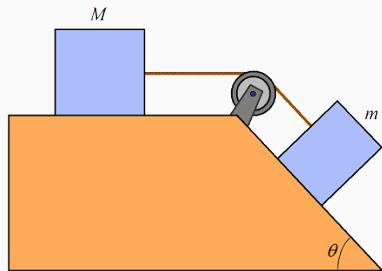
Two blocks of mass *m* and *M* are connected via pulley with a configuration as shown on the left. The coefficient of static friction is μ_s, between blocks and surface. What is the maximum mass *m* so that no sliding occurs?