Student #: ______________________ Student Name: ______________________________

AP PHYSICS C CLASS 10: HARMONIC MOTION

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\text{m/s}^2$ in all problems.

$\downarrow \omega = \sqrt{\frac{k\downarrow}{m}}$

1. A mass oscillates hozirontally on the end of a spring that obeys Hooke's law. Which of the following statements is true?

   $F_e = -k\vec{x}$

   (A) The amplitude of oscillation is equal to the potential energy of the spring. $\frac{1}{2}kA^2$
   (B) The kinetic energy of the oscillating mass is constant. X
   (C) Maximum potential energy occurs when the mass reaches the equilibrium position. amplitude
   (D) The potential energy of the spring at the amplitude is equal to the kinetic energy at the equilibrium position. U_e K
   (E) The kinetic energy of the spring at the amplitude is equal to the potential energy. X

   D

2. A "superball" is dropped from a height of 5.0 m above a floor. The ball bounces off the floor in a perfectly elastic collision so that it rises to the same height with each bounce. The motion of the ball can be described as

   completely elastic

   (A) harmonic motion with a period of 2 s
   (B) harmonic motion with a period of 1 s
   (C) harmonic motion with a period of 1/2 s
   (D) motion with a constant velocity X $g = 10\,\text{m/s}^2$!!
   (E) motion with a constant momentum mv

   A

   1s 1s

   $h = \frac{1}{2}gt^2 \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{(2)(5)}{10}} = 1\,\text{s}.$

3. An object oscillates in simple harmonic motion along the x-axis according to the equation $x = 6\cos(4t)$. The period of oscillation of the object is

   $\omega = 4$ rad/s.

   (A) 1/4 s
   (B) 4 s
   (C) $\pi/4$ s
   (D) $\pi/2$ s
   (E) 4π s

   D

   $f = \frac{\omega}{2\pi} \rightarrow T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

4. A mass hangs from two parallel springs, each with the same spring constant k. Compared to the period T of the same mass oscillating on one of the springs, the period of oscillation of the mass with both springs connected to it is

   (A) $T/2$
   (B) $T/\sqrt{2}$
   (C) T (unchanged)
   (D) $\sqrt{2}T$
   (E) $2T$

   B

   1 spring: $\omega = \sqrt{\frac{k}{m}} \rightarrow f = \frac{\omega}{2\pi}$

   $T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

   $k \rightarrow 2k$

   $T' = 2\pi\sqrt{\frac{m}{2k}} = \frac{T}{\sqrt{2}}$

5. Which of the following is generally true for an object in simple harmonic motion on a spring of constant k?

   (A) The greater the spring constant k, the greater the amplitude of the motion.
   (B) The greater the spring constant k, the greater the period of the motion.
   (C) The greater the spring constant k, the greater the frequency of the motion.
   (D) The lower the spring constant k, the greater the frequency of the motion.
   (E) The lower the spring constant k, the greater the kinetic energy of the motion.

   C

Questions 6–8

$-kx = ma$

$a = -\frac{k}{m}x$

ω^2

A harmonic oscillator follows the equation $\frac{d^2x}{dt^2} = -4x$. The spring constant k is 4 N/m.

a ω^2

6. The angular frequency ω of the harmonic motion is

   (A) zero
   (B) 2 rad/s
   (C) 4 rad/s
   (D) 8 rad/s
   (E) 16 rad/s

   B

   $\omega = \sqrt{4} = 2$

7. The mass m oscillating on the spring is

   (A) 1 kg
   (B) 2 kg
   (C) 4 kg
   (D) 8 kg
   (E) 16 kg

   A

   $\frac{k}{m} = 4 \rightarrow m = \frac{k}{4} = \frac{4}{4} = 1$

8. The period T of oscillation is

   (A) zero
   (B) $\pi/4$s
   (C) $\pi/2$s
   (D) π s
   (E) 2π s

   $f = \frac{\omega}{2\pi} \rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

9. A pendulum of length L has a period of 2 s on Earth. A planetary explorer takes the same pendulum of length L to another planet where its period is 1 s. The gravitational acceleration on the surface of this planet is most nearly

   (A) $8g$
   (B) $4g$
   (C) $2g$
   (D) $g/2$
   (E) $g/4$

   B

   $\omega = \sqrt{\frac{g}{L}}$

   $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

   $T \propto \frac{1}{\sqrt{g}}$

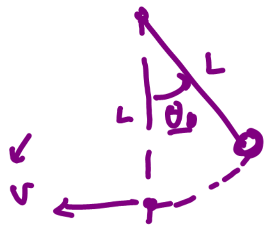
   $T' = \frac{1}{\sqrt{g'}} \rightarrow \frac{1}{2}T = \frac{1}{\sqrt{g'}}$

   $g' = 4g$

AP PHYSICS C: SIMPLE HARMONIC MOTION
SECTION II
6 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. A simple pendulum of length L is released from rest from an angle of θ_0.
   (a) Assuming the motion of the pendulum to be simple harmonic motion, find its speed as it passes through $\theta = 0$.
   (b) Using the conservation of energy, find this speed exactly.
   (c) Show that your results for (a) and (b) are the same when θ_0 is small.
   (d) Find the difference in your results for $\theta_0 = 0.20$ rad and $L = 1$ m.



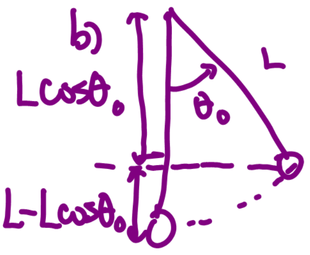
a) $\theta(t) = \theta_0 \cos(\omega t)$ ← small angle approx. $\omega = \sqrt{\frac{g}{L}}$

$\frac{d\theta}{dt} = -\theta_0\omega\sin(\omega t)$ max $\dot{\theta} = \theta_0\omega = \theta_0\sqrt{\frac{g}{L}}$

$v = L\dot{\theta} = \theta_0\sqrt{gL}$

assumes θ_0 is small.

b)



$\cancel{m}g(L - L\cos\theta) = \frac{1}{2}\cancel{m}v^2$

$v = \boxed{\sqrt{2gL(1-\cos\theta_0)}}$ ← exact, applies to ALL θ_0

c) for small θ_0 $\cos\theta_0 = 1 - \frac{\theta_0^2}{2!} + \frac{\theta_0^4}{4!} + \cdots$

$v \approx \sqrt{2gL\left(1 - \left(1 - \frac{\theta_0^2}{2!} + \frac{\theta_0^4}{4!}\right)\right)}$

$\approx \sqrt{2gL\left(\frac{\theta_0^2}{2} - \cdots\right)} = \sqrt{gL\theta_0^2} = \theta_0\sqrt{gL}$ ← same as (a).

d) $\theta_0\sqrt{gL} = \ldots$

$\sqrt{2gL(1-\cos\theta_0)} = \_\_\_$

a) $F_{net} = \cancel{ma}$

$\cancel{-k}x = ma$

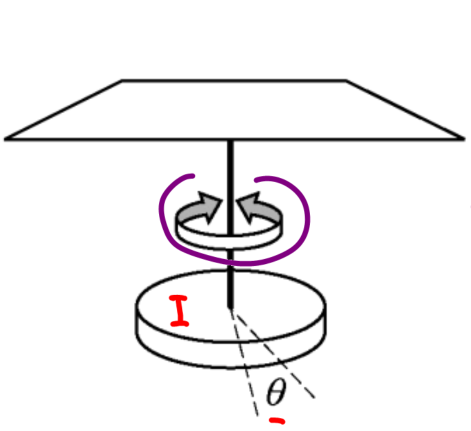
$\frac{d^2x}{dt^2} = \frac{-k}{m}x$

$\tau_{net} = I\alpha$

$-\beta\theta = I\alpha$

$\alpha = -\frac{\beta}{I}\theta$

$\boxed{\frac{d^2\theta}{dt^2} = \frac{-\beta}{I}\theta}$



$I\frac{d^2\theta}{dt^2} = -\beta\theta$ ✓

$\frac{d^2\theta}{dt^2} + \frac{\beta}{I}\theta = 0$ ✓

2. The torsion pendulum shown above consists of a disk of rotational inertia I suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle θ, the twisted rod exerts a restoring torque t that is proportional to the angular displacement: $\tau = -\beta\theta$, where β is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.

   (a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement θ of the torsion pendulum as a function of time t.

   (b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.

To determine the torsion constant β of the rod, disks of different, known values of rotational inertia I are attached to the rod, and the data below are obtained from the resulting oscillations.

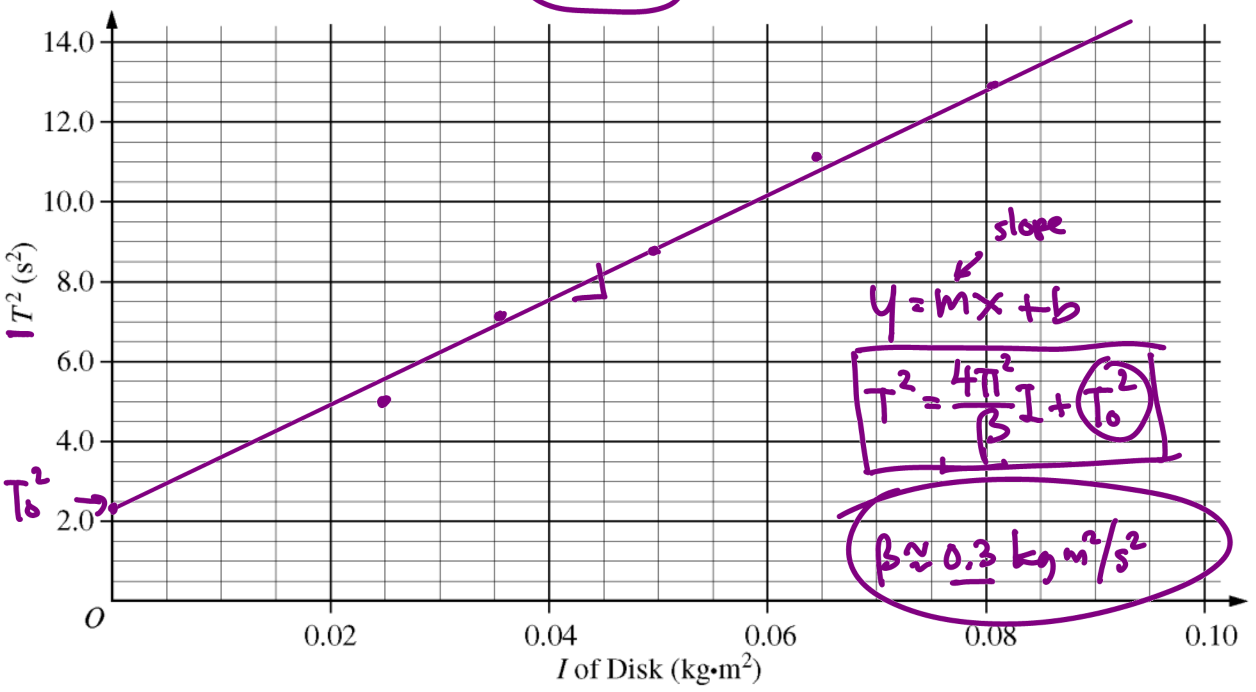
10T

Rotational Inertia I of Disk (kg · m^2)	Average Time for Ten Oscillations (s)	Period T (s)	T^2 (s^2)
0.025	22.4	2.24	5.0
0.036	26.8	2.68	7.2
0.049	29.5	2.95	8.7
0.064	33.3	3.33	11.1
0.081	35.9	3.59	12.9

   (c) On the graph below, plot the data points. Draw a straight line that best represents the data.

$T = 2\pi\sqrt{\frac{I}{\beta}}$

$\underbrace{T^2}_{y} = \underbrace{\frac{4\pi^2}{\beta}}_{m}\underbrace{I}_{x}$



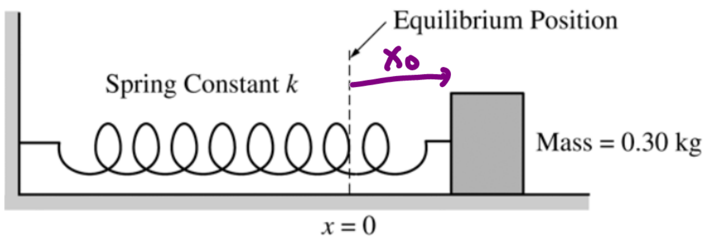
   (d) Determine the equation for your line.

   (e) Calculate the torsion constant β of the rod from your line.

   (f) What is the physical significance of the intercept of your line with the vertical axis?

This is the period when no disk is attached, i.e. the square of the period of oscillation of the flexible rod

b) $\frac{d^2x}{dt^2} = \underbrace{\frac{-k}{m}}_{\omega^2}x$ $\quad \omega = \sqrt{\frac{k}{m}}$ $\longrightarrow$ $\frac{d^2\theta}{dt^2} = \underbrace{\frac{-\beta}{I}}_{\omega^2}\theta$ $\rightarrow \omega = \sqrt{\frac{\beta}{I}}$ $\quad T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{I}{\beta}}}$



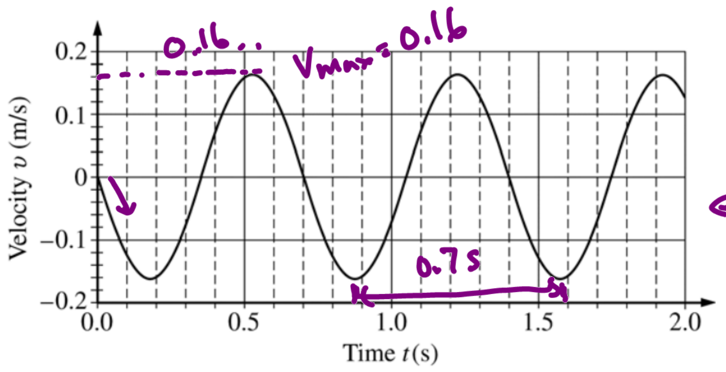
3. Experiment 1. A block of mass 0.30 kg is placed on a frictionless table and is attached to one end of a horizontal spring of spring constant k, as shown above. The other end of the spring is attached to a fixed wall. The block is set into oscillatory motion by stretching the spring and releasing the block from rest at time $t = 0$. A motion detector is used to record the position of the block as it oscillates. The resulting graph of velocity v versus time t is shown below. The positive direction for all quantities is to the right.

a) $V(t) = -V_{max}\sin(\omega t)$

$\boxed{v(t) = -0.16\sin(9.0t)}$

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.7} = 9.0 \text{ rad/s}$



(a) Determine the equation for $v(t)$, including numerical values for all constants.

(b) Given that the equilibrium position is at $x = 0$, determine the equation for $x(t)$, including numerical values for all constants.

(c) Calculate the value of k.

b) $x(t) = \int v(t)\,dt = \int -V_{max}\sin(\omega t)\,dt \rightarrow x(t) = \frac{V_{max}}{\omega}\cos(\omega t) + \cancel{x_0}$

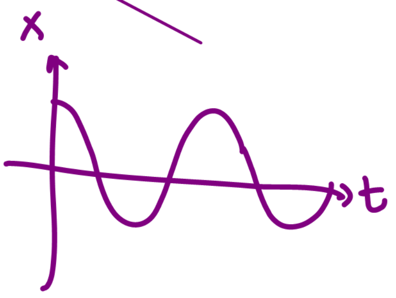
$x(t) = \frac{V_{max}}{\omega}\cos(\omega t) = \frac{0.16}{9.0}\cos(9.0t) \rightarrow \boxed{x(t) = 0.018\cos(9.0t)}$

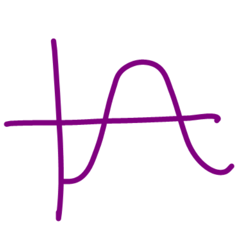
c) $\omega = \sqrt{\frac{k}{m}} \rightarrow \omega^2 = \frac{k}{m} \rightarrow k = \omega^2 m = (9.0)^2(0.3) = \boxed{24.3 \text{ N/m}}$

$x(t) = A\cos(\omega t)$

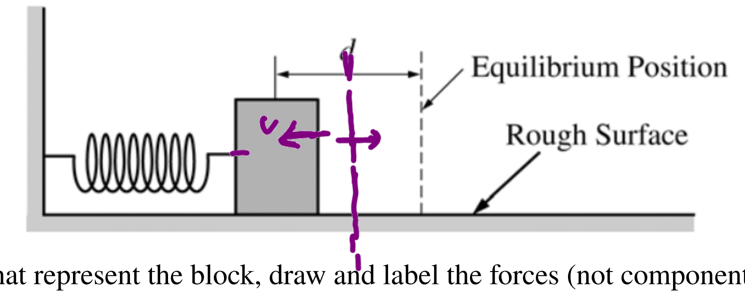
$v(t) = -A\omega\sin(\omega t)$

$a(t) = -A\omega^2\cos(\omega t)$

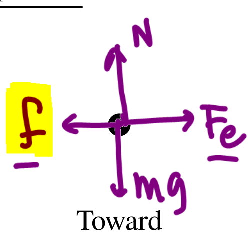




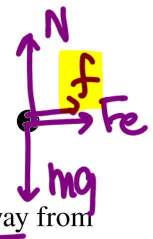
Experiment 2. The block and spring arrangement is now placed on a rough surface, as shown below. The block is displaced so that the spring is compressed a distance d and released from rest.



(d) On the dots below that represent the block, draw and label the forces (not components) that act on the block when the spring is compressed a distance $x = d/2$ and the block is moving in the direction indicated below each dot.

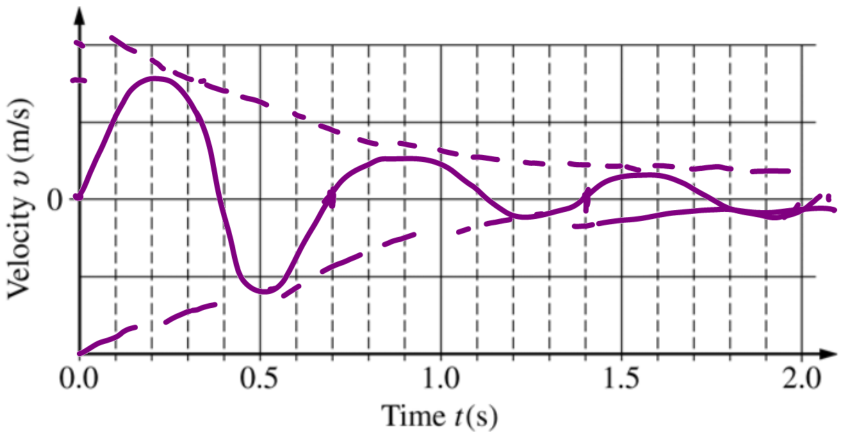


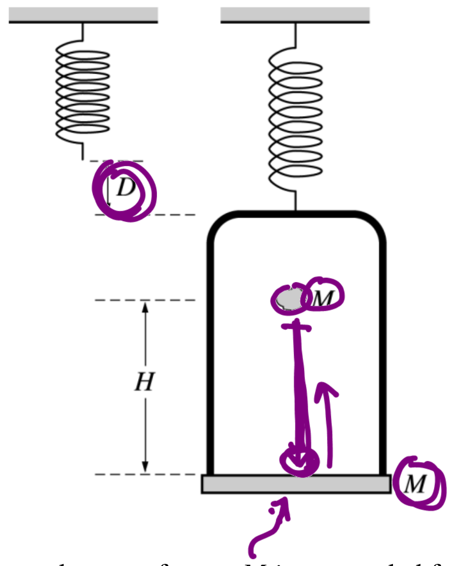
Toward
the equilibrium position



Away from
the equilibrium position

(e) Draw a sketch of v versus t in this case. Assume that there is a negligible change in the period and that the positive direction is still to the right.





$kD = Mg$

$k = \frac{Mg}{D}$

4. An ideal spring is hung from the ceiling and a pan of mass M is suspended from the end of the spring, stretching it a distance D as shown above. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

   (a) Determine the speed of the clay at the instant it hits the pan. → $v^2 = \cancel{v_0^2} + 2gH$ $\boxed{v = \sqrt{2gH}}$

   (b) Determine the speed of the pan just after the clay strikes it.

   (c) Determine the period of the simple harmonic motion that ensues.

   (d) Determine the distance the spring is stretched (from its initial unstretched length) at the moment the speed of the pan is a maximum. ~~Justify your answer.~~ D'

   (e) The clay is now removed from the pan and the pan is returned to equilibrium at the end of the spring. A rubber ball, also of mass M, is dropped from the same height H onto the pan, and after the collision is caught in midair before hitting anything else.

   Indicate below whether the period of the resulting simple harmonic motion of the pan is greater than, less than, or the same as it was in part (c). Justify your answer.

   ____ Greater than ✓ Less than ____ The same as

b) $\cancel{M}v = 2\cancel{M}v'$

$\sqrt{2gH} = 2v'$

$\boxed{v' = \frac{\sqrt{2gH}}{2}} \to \sqrt{\frac{gH}{2}}$

c) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$, $m = 2M$, $k = \frac{Mg}{D}$

$T = 2\pi\sqrt{\frac{2M}{\left(\frac{Mg}{D}\right)}} = \boxed{2\pi\sqrt{\frac{2D}{g}}}$

d) max. speed occurs when $a = 0$ → equilibrium

$kD' = (2M)g$, $k = \frac{Mg}{D}$

$\frac{\cancel{Mg}}{D}D' = 2\cancel{Mg} \to D' = \boxed{2D}$

e) after oscillation started, only M (pan) is oscillating ∴ $T = 2\pi\sqrt{\frac{M}{k}}$ < before when it was $2\pi\sqrt{\frac{2M}{k}}$