Student #: _____ Student Name: _____

AP PHYSICS C CLASS 15: GAUSS'S LAW

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

1. A non-conducting sphere does not have a uniform charge density, but the density ρ varies with the distance r from the center of the sphere according to the equation $\rho = \beta r$ where β is a positive constant. The electric field inside the sphere ($r < R$) at a distance r from the center of the sphere is

   (A) $\dfrac{\beta r^2}{12\epsilon_0}$
   (B) $\dfrac{\beta r^3}{3\epsilon_0}$
   (C) $\dfrac{\beta r}{2\epsilon_0}$
   (D) $\dfrac{\beta r^2}{2\epsilon_0}$
   (E) $\dfrac{\beta r^2}{4\epsilon_0}$

2. The electric potential at the surface of the sphere from the last question is

   (A) $\dfrac{\beta R^3}{12\epsilon_0}$
   (B) $\dfrac{\beta R}{2\epsilon_0}$
   (C) $\dfrac{\beta R^3}{3\epsilon_0}$
   (D) $\dfrac{\beta R^2}{2\epsilon_0}$
   (E) $\dfrac{\beta R^2}{4\epsilon_0}$

3. According to Gauss's law, the net electric flux passing through a closed surface is

   (A) positive if the flux is entering the surface
   (B) negative if the flux is exiting the surface
   (C) positive if the net charge inside the surface is zero
   (D) negative if the net charge inside the surface is zero
   (E) zero if the net charge inside the surface is zero

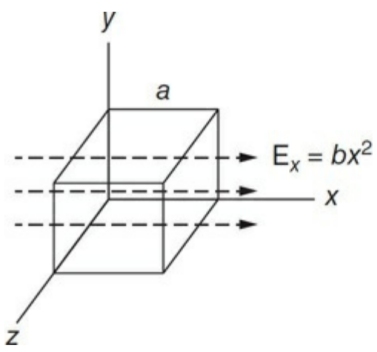
4. According to Gauss's law, which of the following statements is true?

   (A) It is possible to have a nonzero electric field, but zero electric flux.
   (B) It is possible to have a nonzero electric flux, but zero electric field.
   (C) It is possible to have a nonzero electric flux through a closed surface even if the enclosed charge in a surface is zero.
   (D) If a surface is not closed (such as a sheet of paper), the flux through it must be zero.
   (E) It is possible for charges located outside a closed surface to produce a net positive flux through the surface.

5. Gauss's law is most convenient to use when calculating an electric field due to

   (A) charges outside a closed surface
   (B) charges inside a closed surface that has high symmetry
   (C) charges inside a closed surface that has low symmetry
   (D) a potential difference that is negative
   (E) a potential difference that is positive

Question 6-7



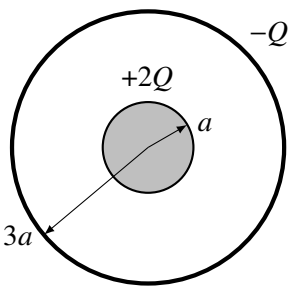
6. A cube has sides of length a. The cube rests so that one side rests on the x-axis as shown. An electric field is established in the x-direction according to the function $E_x = bx^2$, where b is a positive constant. Which of the following statements is true?

   (A) There is a net charge inside the cube.
   (B) There is no net charge inside the cube.
   (C) The flux passing through the cube is negative.
   (D) The flux passing through the cube is zero.
   (E) The flux diminishes while passing through the cube.

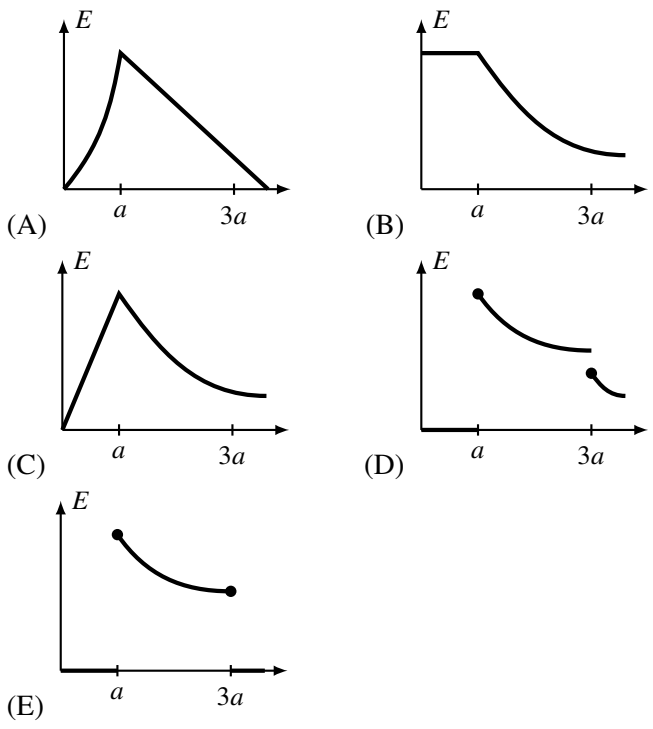
7. The charge inside the cube can be expressed by the equation

   (A) $\epsilon_0 ba$
   (B) $\epsilon_0 ba^2$
   (C) $\epsilon_0 ba^3$
   (D) $\epsilon_0 ba^4$
   (E) $\epsilon_0 b^2 2a^2$

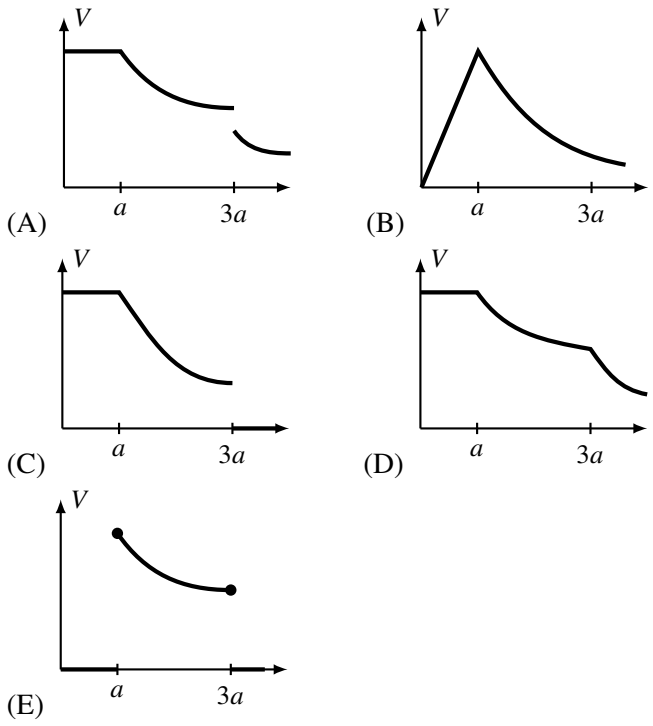
Questions 8–9: A solid conducting sphere of radius a is placed inside a conducting spherical shell of radius $3a$, as shown. A charge $+2Q$ is placed on the inner sphere, and a charge $-Q$ is placed on the outer sphere.



8. Which of the following graphs best represents the electric field $\vec{E}$ as a function of the distance r from the center of the spheres?

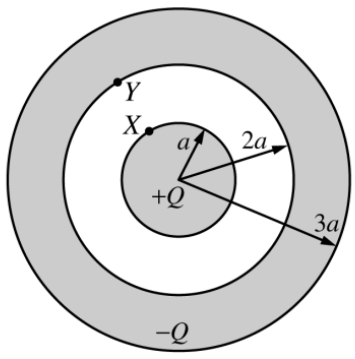


9. Which of the following graphs best represents the electric potential V as a function of the distance r from the center of the spheres?



AP PHYSICS C CLASS 15: GAUSS'S LAW
SECTION II
3 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.



1. In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.
   (a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.
      i. Within the solid sphere ($r < a$)
      ii. Between the solid sphere and the spherical shell ($a < r < 2a$)
      iii. Within the spherical shell ($2a < r < 3a$)
      iv. Outside the spherical shell ($r > 3a$)
   (b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.
   (c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

2. A spherically symmetric charge distribution has net positive charge Q_0 distributed within a radius of R. Its electric potential V as a function of the distance r from the center of the sphere is given by the following

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 R}\left[-2 + 3\left(\frac{r}{R}\right)^2\right] \text{ for } r < R$$

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 r} \text{ for } r > R$$

Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) For the following regions, indicate the direction of the electric field $E(r)$ and derive an expression for its magnitude.

i. $r < R$

_____ Radially inward

_____ Radially outward

ii. $r > R$

_____ Radially inward

_____ Radially outward

(b) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of r.

i. $r < R$

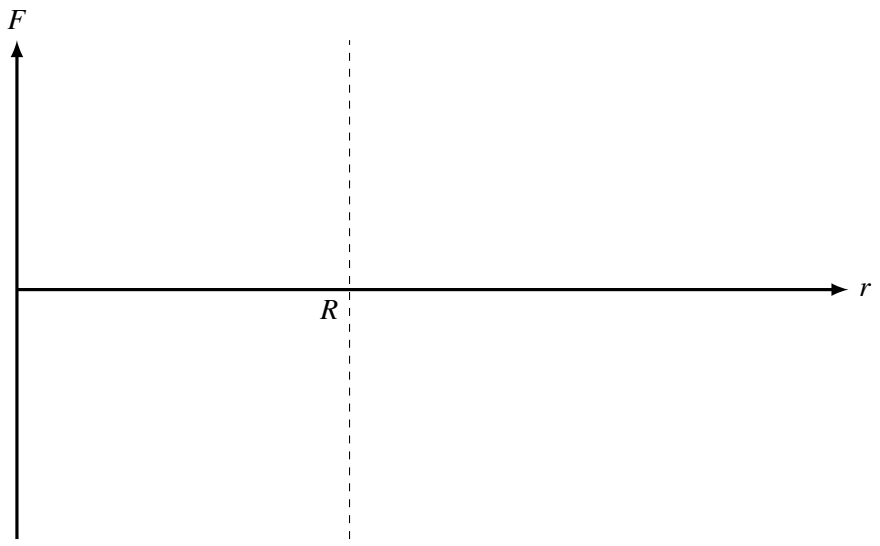
ii. $r > R$

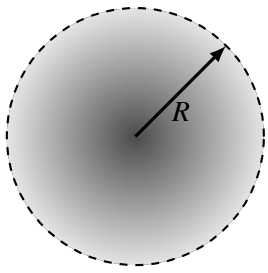
(c) Is there any charge on the surface of the sphere ($r = R$)?

_____ Yes _____ No

If there is, determine the charge. In either case, explain your reasoning.

(d) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions $r < R$ and $r > R$. Assume that a force directed radially outward is positive.





3. A spherical cloud of charge of radius R contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \text{ for } r \leq R \text{ and}$$
$$\rho(r) = 0 \text{ for } r > R$$

where r is the distance from the center of the cloud. Express all algebraic answers in terms of Q, R, and fundamental constants.

(a) Determine the following as a function of r for $r > R$.
   i. The magnitude E of the electric field
   ii. The electric potential V

(b) A proton is placed at point P shown above and released. Describe its motion for a long time after its release.

(c) An electron of charge magnitude e is now placed at point P, which is a distance r from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of r as it strikes the cloud.

(d) Derive an expression for ρ_0.

(e) Determine the magnitude E of the electric field as a function of r for $r \leq R$.