AP PHYSICS 1: ROTATIONAL MOTION

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\text{m/s}^2$ in all problems.

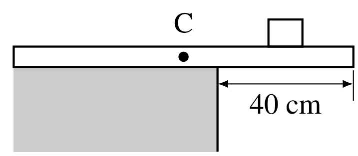
1. What defines a lever arm?
   (A) The distance parallel to the line of action of the force
   (B) The distance parallel to the line of action of the force from the pivot point
   (C) The length of the lever
   (D) The perpendicular distance from the pivot to the line of action of the force
   (E) $F = |F|cos(\theta)x$

2. The most general statement of Newton's first law is:
   (A) $\sum \mathbf{F} = \mathbf{0}$
   (B) $\sum F_x = 0$, $\sum F_y = 0$ and $\sum F_z = 0$
   (C) All the forces or moments must be zero.
   (D) All the forces added together must be zero.
   (E) $\sum F_x = 0$, $\sum F_y = 0$ and $\sum F_z = 0$, $\sum \tau_x = 0$, $\sum \tau_y = 0$ and $\sum \tau_z = 0$,

3. When a skater performs a spin on ice with his arms outstretched, what happens when he brings his arms close to his body?
   (A) His angular acceleration decreases because his moment of inertia was decreased.
   (B) His angular acceleration increases because his moment of inertia was decreased.
   (C) His angular velocity decreases because his moment of inertia was decreased.
   (D) His angular velocity increases because his moment of inertia was decreased.
   (E) His angular displacement increases because his moment of inertia was decreased

4. Linear acceleration is to force as angular acceleration is to
   (A) kinetic energy
   (B) angular velocity
   (C) rotational inertia
   (D) torque
   (E) angular momentum

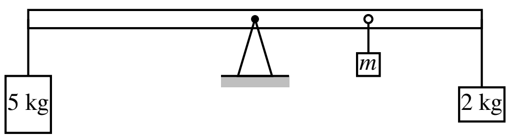
5. A meter stick of mass 0.1 kg rests on a table as shown. A length of 40 cm extends over the edge of the table. How far from the edge of the table could a 0.05 kg mass be placed on the meter stick so that the stick just begins to tip?



   (A) 5 cm
   (B) 10 cm
   (C) 15 cm
   (D) 20 cm
   (E) 30 cm

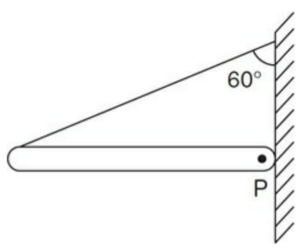
6. The moment of inertia of a particular thin-walled cylinder around its central axis is given by mR^2. What expression best represents the angular momentum of this cylinder if it spins about the central axis at a rate of 12 revolutions per second?
   (A) $24\pi mR^2$
   (B) $12\pi mR^2$
   (C) $12mR^2$
   (D) $\frac{1}{12}\pi mR^2$
   (E) $\frac{1}{12\pi}mR^2$

7. A meter stick is balanced on a fulcrum at its center, as shown. A mass of 5 kg is hung on the left end of the stick, and a mass of 2 kg is hung on the right end. In order to balance the system, a mass m is hung at the 25-cm mark on the right side. What is the value of the mass m?



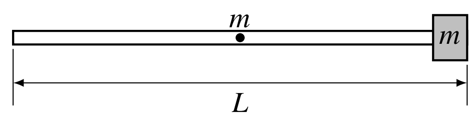
(A) 12 kg
(B) 6 kg
(C) 4 kg
(D) 3 kg
(E) 2 kg

8. A metal bar of constant density and weight W is attached to a pivot on the wall at point P and supported by a rope that makes an angle of 60° with the vertical wall. The reaction force exerted by the pivot on the bar at point P is best represented by which arrow?



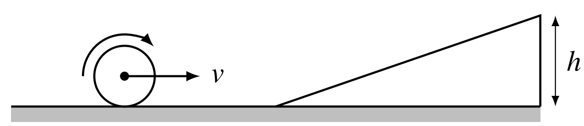
(A) $\nearrow$
(B) $\uparrow$
(C) $\downarrow$
(D) $\nwarrow$
(E) $\searrow$

9. A uniform rod of length L and mass m has a rotational inertia of $\frac{1}{12}mL^2$ about its center. A particle, also of mass m, is attached to one end of the stick. The combined rotational inertia of the stick and particle about the center of the rod is



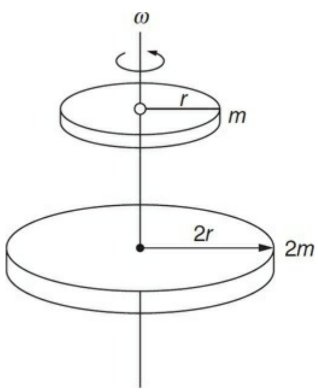
(A) $\dfrac{mL^2}{3}$
(B) $\dfrac{12mL^2}{13}$
(C) $\dfrac{13mL^2}{12}$
(D) $\dfrac{mL^2}{156}$
(E) $\dfrac{13mL^2}{156}$

10. A hoop of radius R and mass m has a rotational inertia of mR^2. The hoop rolls without slipping along a horizontal floor with a constant speed v and then rolls up a long incline. The hoop can roll up the incline to a maximum vertical height of



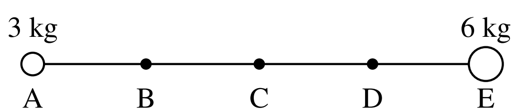
(A) $\dfrac{v^2}{g}$
(B) $\dfrac{2v^2}{g}$
(C) $\dfrac{v^2}{2g}$
(D) $\dfrac{4v^2}{g}$
(E) $\dfrac{v^2}{4g}$

11. Two disks are fixed to a vertical axle that is rotating with a constant angular speed ω. The smaller disk has a mass m and a radius r, and the larger disk has a mass $2m$ and radius $2r$. The general equation for the rotational inertia of a disk of mass M and radius R is $\frac{1}{2}MR^2$. The ratio of the angular momentum of the larger disk to the smaller disk is



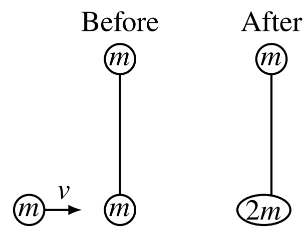
(A) 1 : 4
(B) 4 : 1
(C) 1 : 2
(D) 2 : 1
(E) 8 : 1

12. A light rod has a mass attached at each end. At one end is a 6 kg mass, and at the other end is a 3 kg mass. An axis can be placed at any of the points shown. Through which point should an axis be placed so that the rotational inertia is the greatest about that axis?



(A) A
(B) B
(C) C
(D) D
(E) E

13. Astronauts are conducting an experiment in a negligible gravity environment. Two spheres of mass m are attached to either end of a light rod. As the rod and spheres float motionless in space, an astronaut launches a piece of sticky clay, also of mass m, toward one of the spheres so that the clay strikes and sticks to the sphere perpendicular to the rod. Which of the following statements is true of the motion of the rod, clay, and spheres after the collision?

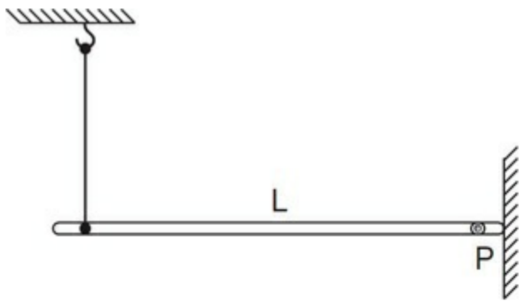


(A) Linear momentum is not conserved, but angular momentum is conserved.
(B) Angular momentum is not conserved, but linear momentum is conserved.
(C) Kinetic energy is conserved, but angular momentum is not conserved.
(D) Kinetic energy is conserved, but linear momentum is not conserved.
(E) Both linear momentum and angular momentum are conserved, but kinetic energy is not conserved.

14. A cylinder has a moment of inertia, I. How much time does it take a torque, τ, to increase its angular speed from ω_1 to ω_2?

(A) $\dfrac{I(\omega_2 - \omega_1)}{\tau}$
(B) $\dfrac{\tau}{I(\omega_2 - \omega_1)}$
(C) $I(\omega_2 - \omega_1)\tau$
(D) $\dfrac{2\tau}{I(\omega_2^2 - \omega_1^2)}$
(E) $\dfrac{1}{2}I(\omega_2^2 - \omega_1^2)\tau$

15. One end of a stick of length L, rotational inertia I, and mass m is pivoted on an axle with negligible friction at point P. The other end is tied to a string and held in a horizontal position. When the string is cut, the stick rotates counterclockwise. The angular speed ω of the stick when it reaches the bottom of its swing is



(A) $\frac{mgL}{I}$
(B) $\sqrt{\frac{mgL}{I}}$
(C) $\sqrt{\frac{2mgL}{I}}$
(D) $\sqrt{\frac{mgL}{2I}}$
(E) $\sqrt{\frac{4mgL}{I}}$

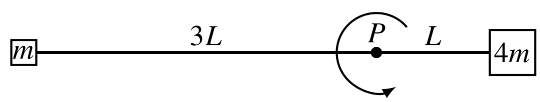
16. A disk is mounted on a fixed axle. The rotational inertia of the disk is I. The angular velocity of the disk is decreased from ω_0 to ω_f during a time Δt due to friction in the axle. The magnitude of the average net torque acting on the wheel is

(A) $\frac{\omega_f - \omega_0}{\Delta t}$
(B) $\frac{(\omega_f - \omega_o)^2}{\Delta t}$
(C) $\frac{I(\omega_f - \omega_o)}{\Delta t}$
(D) $\frac{I(\omega_f - \omega_o)^2}{\Delta t}$
(E) $\frac{I(\omega_f - \omega_o)}{\Delta t^2}$

17. The average power developed by the friction in the axle of the disk from the previous question to bring it to a complete stop is

(A) $\frac{\omega_0}{\Delta t}$
(B) $\frac{(\omega_0)^2}{\Delta t}$
(C) $\frac{I\omega_0}{2\Delta t}$
(D) $\frac{I\omega_0^2}{2\Delta t}$
(E) $\frac{I(\omega_f - \omega_0)}{\Delta t^2}$

18. A light rod of negligible mass is pivoted at point P a distance L from one end as shown. A mass m is attached to the left end of the rod at a distance of $3L$ from the pivot, and another mass $4m$ is attached to the other end a distance L from the pivot. The system begins from rest in the horizontal position. The net torque acting on the system due to gravitational forces is



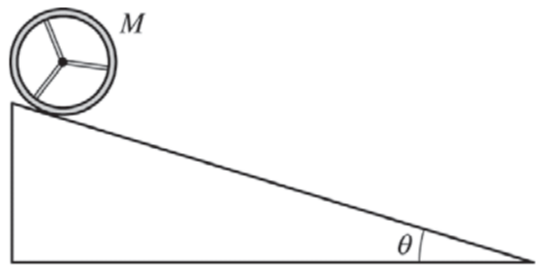
(A) $4mgL$ clockwise
(B) $3mgL$ clockwise
(C) $3mgL$ counterclockwise
(D) mgL counterclockwise
(E) mgL clockwise

19. The angular acceleration of the system when it is released from rest is

(A) zero
(B) $\frac{g}{5L}$
(C) $\frac{g}{4L}$
(D) $\frac{g}{13L}$
(E) $\frac{g}{L}$

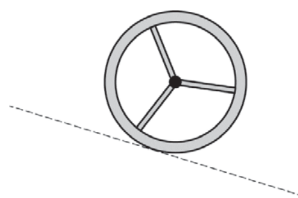
AP PHYSICS 1: ROTATIONAL MOTION
SECTION II
3 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.



1. A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

   (a) i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.

   ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass? Briefly explain your reasoning.

   (b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of M, θ, and physical constants, as appropriate.

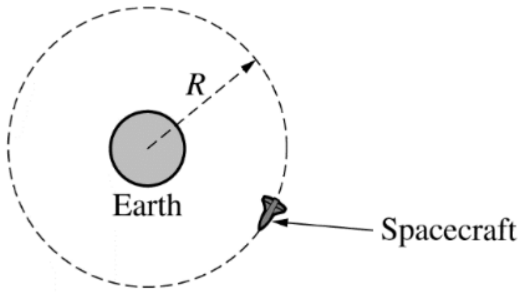
   (c) In a second experiment on the same ramp, a block of ice, also with mass M, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

   i. Which object, if either, reaches the bottom of the ramp with the greatest speed?

   ____ Wheel ____ Block ____ Neither; both reach the bottom with the same speed.

   Briefly explain your answer, reasoning in terms of forces.

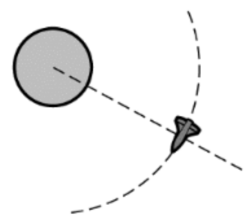
   ii. Briefly explain your answer again, now reasoning in terms of energy.



Note: Figure not drawn to scale.

2. A spacecraft of mass m is in a clockwise circular orbit of radius R around Earth, as shown in the figure above. The mass of Earth is M_E.

   (a) In the figure below, draw and label the forces (not components) that act on the spacecraft. Each force must be represented by a distinct arrow starting on, and pointing away from, the spacecraft.



Note: Figure not drawn to scale.

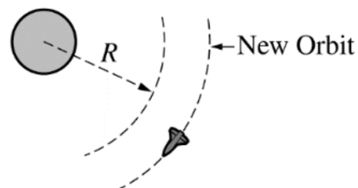
   (b) i. Derive an equation for the orbital period T of the spacecraft in terms of m, M_E, R, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

   ii. A second spacecraft of mass $2m$ is placed in a circular orbit with the same radius R. Is the orbital period of the second spacecraft greater than, less than, or equal to the orbital period of the first spacecraft?

   ____Greater than ____Less than ____Equal to

   Briefly explain your reasoning.

   (c) The first spacecraft is moved into a new circular orbit that has a radius greater than R, as shown in the figure below.

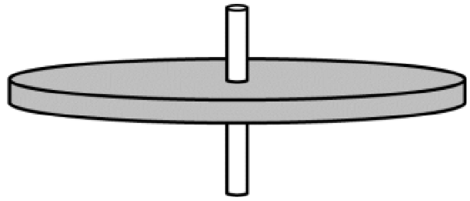


Note: Figure not drawn to scale.

Is the speed of the spacecraft in the new orbit greater than, less than, or equal to the original speed?

____Greater than ____Less than ____Equal to

Briefly explain your reasoning.

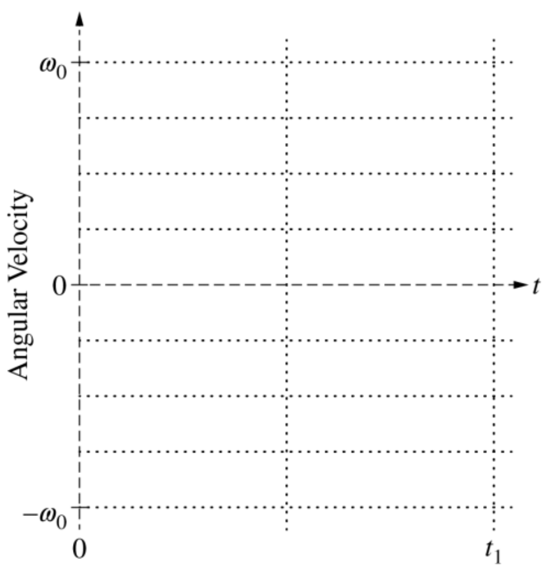


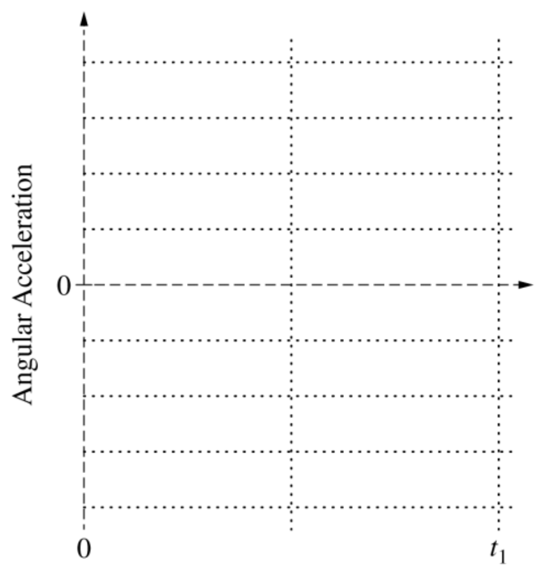
3. The disk shown above spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

   (a) At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity ω_0. The disk later comes to rest at time $t = t_1$.

      i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.

      ii. On the grid at right below, sketch the disk's angular acceleration as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.



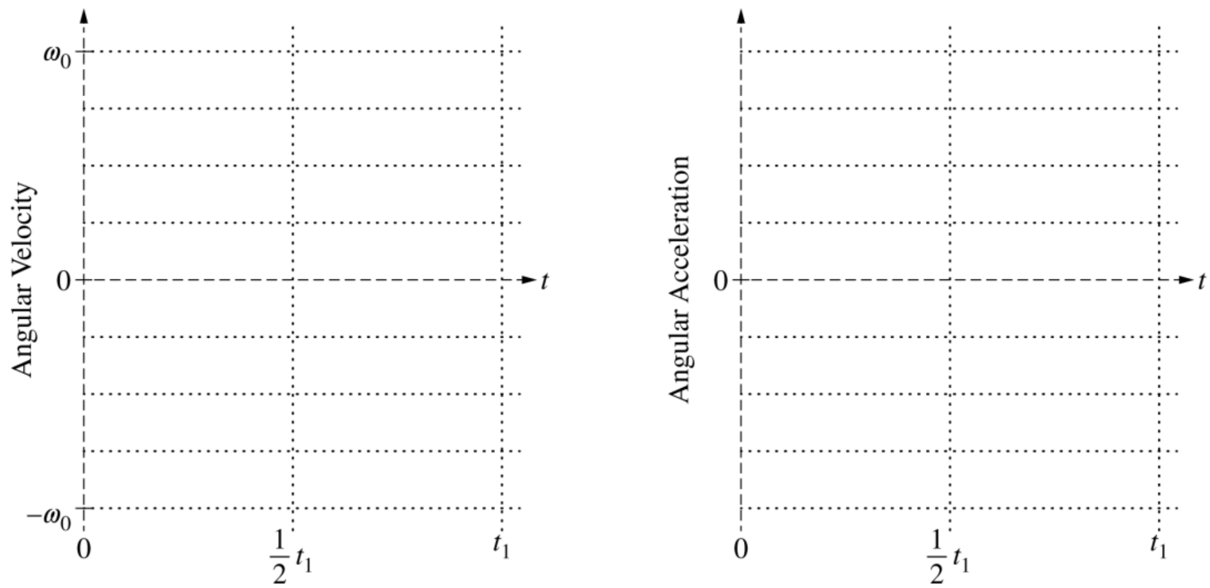


   (b) The magnitude of the frictional torque exerted on the disk is τ_0 . Derive an equation for the rotational inertia I of the disk in terms of τ_0, ω_0, t_1, and physical constants, as appropriate.

(c) In another experiment, the disk again has an initial positive angular velocity ω_0 at time $t = 0$. At $t = \frac{1}{2}t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time from $t = 0$ to $t = t_1$.



(d) The student is trying to mathematically model the magnitude τ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant (C_1 or C_2) with appropriate units.

(1) $\tau = C_1\left(t - \frac{1}{2}t_1\right)$ (for $t > \frac{1}{2}t_1$)

(2) $\tau = \dfrac{C_2}{\left(t + \frac{1}{2}t_1\right)}$ (for $t > \frac{1}{2}t_1$)

Which equation better mathematically models this experiment?

____Equation (1) ____Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.