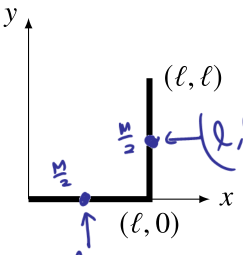
Student #: ____________________________ Student Name: ___________________________________

AP PHYSICS C: MOMENTUM AND CENTER OF MASS

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

1. A uniform rod of length 2ℓ is bent, as shown in the figure below. What are the coordinates of the center of mass of the rod?



$X_{CM} = \frac{\sum x_i m_i}{M} = \frac{\frac{\ell}{2}(\frac{M}{2}) + \ell(\frac{M}{2})}{M} = \frac{\ell}{4} + \frac{\ell}{2} = \frac{3\ell}{4}$

$Y_{CM} = \frac{\sum y_i m_i}{M} = \frac{0(\frac{M}{2}) + \frac{\ell}{2}(\frac{M}{2})}{M} = \frac{\ell}{4}$

(A) $(\ell/4, 3\ell/4)$
(B) $(3\ell/4, \ell/4)$
(C) $(2\ell/3, \ell/3)$
(D) $(2\ell/3, 2\ell/3)$
(E) $(\ell/2, \ell/3)$

B

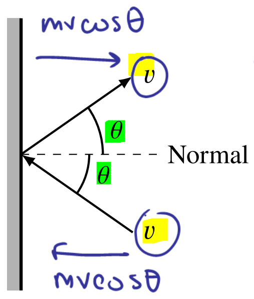
2. Two uniform spheres of mass M and $4M$ are connected by a rod whose mass is negligible, and the distance between the centers of the spheres is d. The $4M$ sphere is then moved a distance of $d/3$ toward the smaller sphere. How far has the center of mass of the entire object moved?

(A) The center of mass has not moved, because both spheres still have their original masses.
(B) $d/15$
(C) $d/5$
(D) $4d/15$
(E) $8d/15$

$\frac{12d}{15} - \frac{8d}{15} = \frac{4d}{15}$

D

3. A rubber ball of mass m strikes a wall with a speed v at an angle θ below the normal line and rebounds from the wall at the same speed and angle above the normal line as shown. The magnitude of the change in momentum of the ball is



$\Delta p = 2mv\cos\theta$ [normal]

(A) mv
(B) $2mv$
(C) $mv\cos\theta$
(D) $2mv\cos\theta$
(E) zero

D

4. Two blocks are connected by a compressed spring and rest on a frictionless surface. The blocks are released from rest and pushed apart by the compressed spring. If one mass is twice the mass of the other, which of the following is the same for both blocks?

(A) magnitude of momentum
(B) acceleration
(C) speed
(D) kinetic energy
(E) potential energy

$\vec{p} = 0$ $\vec{p} = 0$

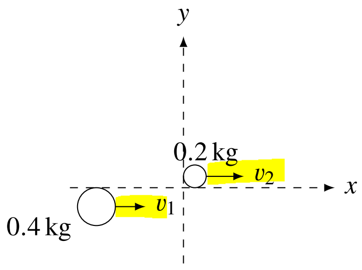
A

5. A known ~~net force F~~ net impulse acts on an unknown mass for a known time Δt. From this information, you could determine the

(A) change in kinetic energy of the object
(B) change in velocity of the object
(C) acceleration of the object
(D) mass of the object
(E) change in momentum of the object

E

6. Two billiard balls are rolling to the right on a table as shown. The 0.4 kg ball is moving faster than the 0.2 kg ball, so it catches up and strikes it from behind at a slight angle. Immediately after the collision, the y-component of the 0.4 kg ball is 2 m/s downward. The y-component of the velocity of the 0.2 kg ball must be



(A) 1 m/s upward
(B) 2 m/s upward
(C) 1 m/s downward
(D) 2 m/s downward
(E) 4 m/s upward

E

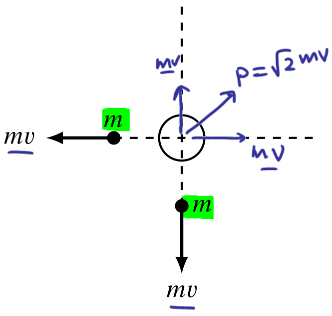
7. A small mass m is moving with a speed v toward a stationary mass M. The speed of the center of mass of the system is

(A) $\left(\frac{m}{m+M}\right)v$
(B) $\left(\frac{m+M}{m}\right)v$
(C) $\left(\frac{m}{M}\right)v$
(D) $\left(1+\frac{m}{M}\right)v$
(E) $\left(1+\frac{M}{3m}\right)v$

$mv = (M+m)v' \to v' = \frac{mv}{(m+M)}$

A

8. An object has a mass $4m$. The object explodes into three pieces of mass m, m, and $2m$. The two pieces of mass m move off at right angles to each other with the same momentum mv, as shown below. The speed of mass $2m$ after the explosion is



$p = \sqrt{2}mv$

$v = \frac{p}{m} = \frac{\sqrt{2}mv}{2m}$

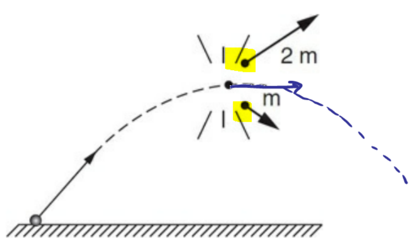
$\frac{\sqrt{2}}{2}v$

(A) $2v$
(B) $\sqrt{2}v$
(C) $\frac{\sqrt{2}}{2}v$
(D) $\frac{\sqrt{2}}{3}v$
(E) $\frac{\sqrt{3}}{2}v$

C

Questions 9–10

A projectile is launched at an angle to the level ground as shown. At the top of the trajectory at point P, the projectile explodes into two pieces of mass $2m$ and m.



9. Which of the following arrows best represents the direction of the velocity of the center of mass of the projectile at point P after the explosion?

D

(A) ←
(B) ↙
(C) ↘
(D) →
(E) ↗

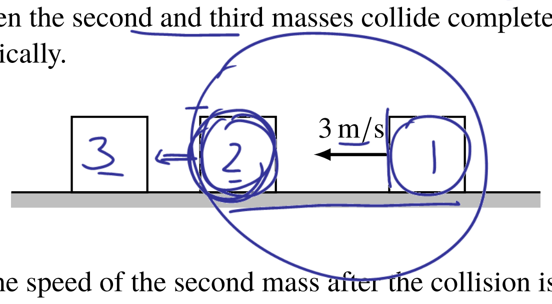
10. Which of the following statements is true of the center of mass of the projectile after the explosion?

A

(A) The center of mass will continue on a parabolic path and land on the ground at the place where it would have landed had it not exploded.
(B) The center of mass will alter its parabolic path and land on the ground farther from where it would have landed had it not exploded.
(C) The center of mass will alter its parabolic path and land on the ground at a shorter distance than it would have landed had it not exploded.
(D) The center of mass will fall straight downward from the point of explosion.
(E) The center of mass will travel straight upward from the point of explosion.

Questions 11–12

Three identical masses can slide freely on a horizontal surface as shown. The first mass moves with a speed of 3.0 m/s toward the second and third masses, which are initially at rest. The first and second mass collide elastically, and then the second and third masses collide completely inelastically.



11. The speed of the second mass after the collision is

C

(A) zero
(B) 1.5 m/s
(C) 3.0 m/s
(D) 6.0 m/s
(E) 9.0 m/s

12. The speed of the second and third masses after they collide inelastically is

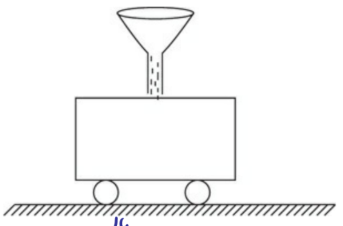
B

(A) zero
(B) 1.5 m/s
(C) 3.0 m/s
(D) 6.0 m/s
(E) 9.0 m/s

13. A mass traveling in the $+x$ direction collides with a mass at rest. Which of the following statements is true?

~~(A)~~ After the collision, the two masses will move with parallel velocities
~~(B)~~ After the collision, the masses will move with anti-parallel velocities
~~(C)~~ After the collision, the masses will both move along the x-axis
~~(D)~~ After the collision, the y-components of the velocities of the two particles will sum to zero.
(E) None of the above

14. A 1000 kg (empty mass) railroad car is rolling without friction on a horizontal track at a speed of 2.0 m/s. Sand is poured into the open top of the car for the time interval from $t = 0$ to $t = 4.0$ s. The mass of the sand poured into the car as a function of time is $m(t) = 60t^2$. The velocity of the car at a time of 4.0 s is most nearly



A

(A) 1 m/s
(B) 2 m/s
(C) 3 m/s
(D) 4 m/s
(E) 5 m/s

$\Delta m = \int_0^4 m(t)\,dt = 60\int_0^4 t^2\,dt$

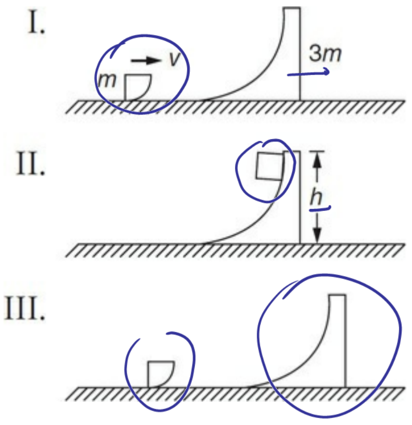
$= \cancel{60}^{20} \frac{t^3}{\cancel{3}}\Big|_0^4$

$= 20t^3\Big|_0^4$

$= 1280$

Questions 15–16

A small block of mass m slides on a horizontal frictionless surface toward a ramp of mass $3m$ which is also free to move on the surface. The small block slides up to a height h on the ramp with no friction (Figure I), then they move together (Figure II), and the small block slides back down the ramp to the horizontal surface (Figure III). Both the block and the ramp continue to slide on the horizontal surface after they separate.



15. Which of the following is true regarding the conservation laws throughout this process?

~~(A)~~ Kinetic energy is conserved from Figure I to Figure II.
(B) Momentum is conserved from Figure I to Figure III.
~~(C)~~ Kinetic energy is conserved from Figure II to Figure III.
~~(D)~~ Potential energy is conserved from Figure I to Figure II.
~~(E)~~ Potential energy is conserved from Figure II to Figure III.

16. Which of the following is a true statement regarding Figure III?

A

(A) The small block is moving to the left and the ramp is moving to the right.
(B) The small block is moving to the right and the ramp is moving to the left.
(C) The small block is moving to the right and the ramp is moving to the right.
(D) The small block is moving to the left and the ramp is moving to the left.
(E) The small block and the large block are moving with the same velocity.

Questions 17–18

A remote controlled stunt car of mass 800 kg initially moving at 10 m/s is crashed into a rail car of mass m that is initially at rest. The cars stick together, and the speed v of both cars after the collision is given by $v' = \frac{6}{t+1}$.

17. By considering the fact that the crash occurs at time $t = 0$, determine the mass m of the rail car.

    (A) 288 kg
    (B) 445 kg
    (C) 533 kg
    (D) 698 kg
    (E) 800 kg

C

at $t=0$ $v = \frac{6}{1} = 6\,m/s^2$

$(800)(10) = (800+m)6$

18. The magnitude of the resisting force acting on the cars as a function of time after the collision is

    (A) $\frac{6m}{t+1}$
    (B) $6m(t+1)$
    (C) $6m(t+1)^2$
    (D) $\frac{6m}{(t+1)^2}$
    (E) $\frac{m(t+1)^2}{6}$

D

$v = \frac{6}{t+1}$

$a = \frac{dv}{dt} = \frac{-6}{(t+1)^2}$

$|F| = ma$

19. A moving object is changing its momentum during a time interval. If a graph of momentum vs. time is plotted, the net force acting on the mass at any time can be determined by finding the

    (A) slope of line tangent to the graph at that time
    (B) area under the graph
    (C) y-intercept of the graph
    (D) x-intercept of the graph
    (E) change in slope of the graph from beginning to end

A

$F = \frac{dp}{dt}$

20. A mass m_1 initially moving at speed v_0 collides with and sticks to a spring attached to a second, initially stationary mass m_2. The two masses continue to move to the right on a frictionless surface as the length of the spring oscillates. At the instant that the spring is maximally extended, the velocity of the first mass is

both m_1 and m_2 move with the same speed!

    (A) v_0
    (B) $m_1^2 v_0/(m_1+m_2)^2$
    (C) $m_2 v_0/m_1$
    (D) $m_1 v_0/m_2$
    (E) $m_1 v_0/(m_1+m_2)$

$m_1 v_0 = (m_1+m_2)\,v$

p

$v = \frac{m_1 v_0}{m_1+m_2}$

E

AP PHYSICS C: MOMENTUM, IMPULSE, COLLISIONS, AND CENTER OF MASS
SECTION II
4 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. A projectile is fired from the edge of a cliff 100 m high with an initial speed of 60 m/s at an angle of elevation of 45°.
   (a) Write equation for $x(t)$, $y(t)$, v_x and v_y. Choose the origin of your coordinate system at the particle's original location.
   (b) Calculate the location and velocity of the particle at time $t = 5$ s.

   Suppose the projectile experiences an internal explosion at time $t = 4$ s with an internal force purely in the y-direction, causing it to break into a 2 kg and a 1 kg fragment.
   (c) If the 2 kg fragment is 77 m above the height of the cliff at $t = 5$ s, what is the y-coordinate of the position of the 1 kg piece?
   (d) If the speed of the 2 kg fragment is 46 m/s and the fragment is falling at $t = 5$ s, what is the y-component of the velocity of the 1 kg fragment?

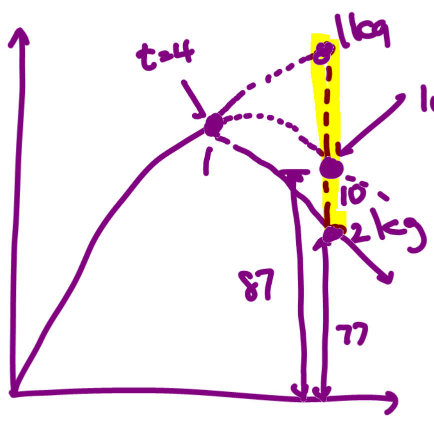
a) $V_x = 60\cos 45 = \boxed{42.4\ \text{m/s}\ \hat{i}}$ $V_y = V_{y0} - gt = \boxed{(42.4 - 10t)\ \text{m/s}\ \hat{j}}$ $V_{y0} = 60\sin 45$

$x(t) = V_x t = \boxed{42.4t\ \text{m}\ \hat{i}}$ $y(t) = \cancel{y_0} + V_{y0}t - \frac{1}{2}gt^2 = \boxed{42.4t - 5t^2\ \text{m}\ \hat{j}}$

b) $x(5) = 212.15$, $y(5) = 87.15$ → $\vec{x}(5) = \boxed{(212\hat{i} + 87\hat{j})\ \text{m}}$

$V_x = 42.4$, $V_y = -7.6$ → $\vec{v}(5) = \boxed{(42.4\hat{i} - 7.6\hat{j})\ \text{m/s}}$

c)



2kg fragment is 10 m below CM
1kg fragment is 20 m above CM
$y(1\text{kg}) = 87 + 20\text{ m} = \boxed{107\text{ m}}$

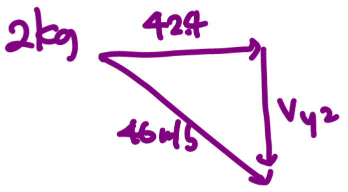
$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$

$87 = \frac{(1)(y_1) + (2)(77)}{1+2}$ → $\boxed{y_1 = 107\text{ m}}$

d) if no explosion, y momentum of the projectile

$P_y = (-7.6)(3) = -22.8\ \text{kg}\cdot\text{m/s}$ (down)

total y momentum of the fragments must be the same



$V_{y2} = \sqrt{46^2 - 42.4^2} = -17.8\ \text{m/s}$

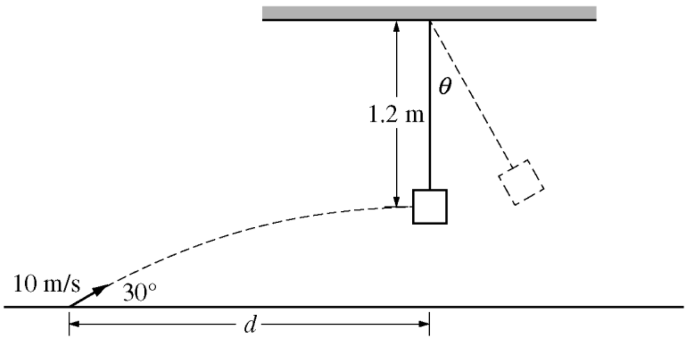
$P_{y2} = m_2V_{y2} = (2)(-17.8) = -35.7\ \text{kg}\cdot\text{m/s}$

$P_{y1} + P_{y2} = P_y$

$P_{y1} - 35.7 = -22.8$

$P_{y1} = 12.9\ \text{kg m/s}$

$\boxed{V_{y1} = 12.9\ \text{m/s}}$ ← up



2. A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

   (a) Determine the speed of the dart just before it strikes the block.

   (b) Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.

   (c) Calculate the speed of the block just after the dart strikes.

   (d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

   (e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.

   (f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

      i. Would the angle θ that the dart and block swing to increase, decrease, or stay the same? Justify your answer.

      ____ Increase ____ Decrease ____ Stay the same

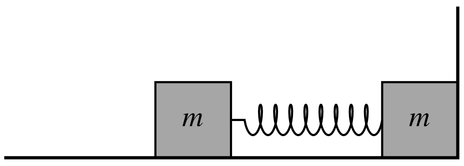
      ii. Would the period of oscillation after the collision increase, decrease, or stay the same? Justify your answer.

      ____ Increase ____ Decrease ____ Stay the same

3. Two masses are connected by a spring (spring constant k) resting on a frictionless horizontal surface as shown. The right mass is initially in contact with a wall. A brief blow to the left block leaves it with an initial velocity v_0 to the right.

   (a) What is the maximum compression of the spring as the left block moves to the right?

   After the spring is maximally compressed, it eventually moves to the left, away from wall. As it moves away from the wall, it continues oscillating.

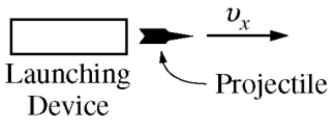


   (b) What is the net momentum of the two masses after they leave the wall?

   (c) What is the total mechanical energy of the oscillating spring system?

   (d) What is the relative velocity of the two masses when the spring is maximally compressed?

   (e) What is the maximum compression of the spring after the two masses have left the wall? Compare the compression to the maximum compression calculated in part (a) and explain any similarities and differences.



4. A projectile is fired horizontally from a launching device, exiting with a speed v_x. While the projectile is in the launching device, the impulse imparted to it is J_p, and the average force on it is F_{avg}. Assume the force becomes zero just as the projectile reaches the end of the launching device. Express your answers to parts (a) and (b) in terms of v_x, J_p, F_{avg}, and fundamental constants, as appropriate.

   (a) Determine an expression for the time required for the projectile to travel the length of the launching device.

   (b) Determine an expression for the mass of the projectile.

   The projectile is fired horizontally into a block of wood that is clamped to a tabletop so that it cannot move. The projectile travels a distance d into the block before it stops. Express all algebraic answers to the following in terms of d and the given quantities previously indicated, as appropriate.

   (c) Derive an expression for the work done in stopping the projectile.

   (d) Derive an expression for the average force F_b exerted on the projectile as it comes to rest in the block.

   Now a new projectile and block are used, identical to the first ones, but the block is not clamped to the table. The projectile is again fired into the block of wood and travels a new distance d_n into the block while the block slides across the table a short distance D. Assume the following: the projectile enters the block with speed v_x, the average force F_b between the projectile and the block has the same value as determined in part (d), the average force of friction between the table and the block is f_T, and the collision is instantaneous so the frictional force is negligible during the collision.

   (e) Derive an expression for d_n in terms of d, D, f_T, and F_b, as appropriate.

   (f) Derive an expression for d_n in terms of d, the mass m of the projectile, and the mass M of the block.