

# Class 15: Gauss's Law

## Advanced Placement Physics C

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Olympiads School

# Gauss's Law

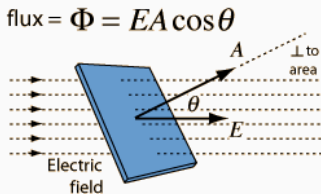
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# Flux

**Flux** is an important concept in many disciplines in physics. The flux of a vector quantity  $\vec{X}$  is the amount of that quantity flowing through a surface. In integral form:

$$\Phi = \int \vec{X} \cdot d\vec{A} \quad \text{or} \quad \Phi = \int (\vec{X} \cdot \hat{n}) dA$$

The direction of the infinitesimal area  $d\vec{A}$  is **outward normal** to the surface.

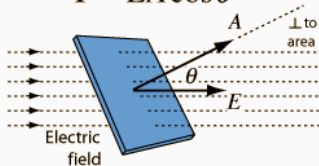


# Flux

$\Phi$  can be something physical, like water, or bananas, or something abstract, like electric field (which is what we are interested in). We can compute a flux as long as there is a vector field i.e.  $\vec{X} = \vec{X}(x, y, z)$ . In the case of **electric flux**, the quantity  $\vec{X}$  is just the electric field, i.e.:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\text{flux} = \Phi = EA \cos \theta$$



# Electric Flux and Gauss's Law

**Gauss's law** tells us that if we have a closed surface (think of the surface of a balloon), the total electric flux is very well defined:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

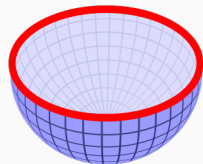
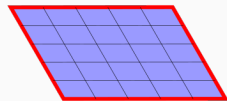
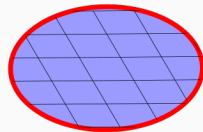
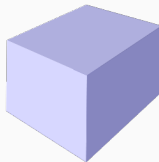
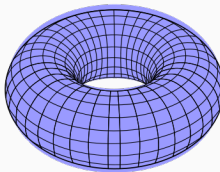
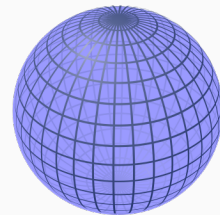
where

- $Q_{\text{encl}}$  is the charge enclosed by the surface
- $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$  is the permittivity of free space

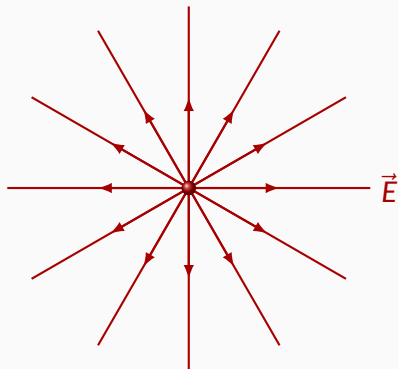
That closed surface is called a **Gaussian surface**

# Closed Surfaces

A **closed surface** is one that does not have a boundary, like the sphere, toroid, and cube on the left.



# Electric Field from a Positive Point Charge



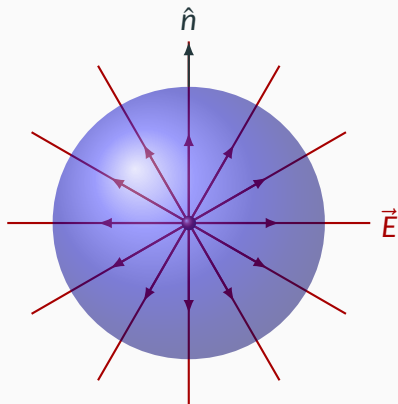
By symmetry, electric field lines must be radially outward from the charge, so the integral reduces to:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = EA = \frac{q}{\epsilon_0}$$

Since area of a sphere is  $A = 4\pi r^2$ , we recover Coulomb's law and the magnitude of the electric field from a point charge:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{kq}{r^2}$$

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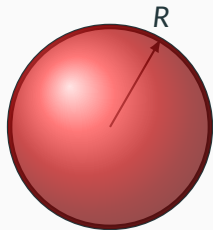
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# Uniformly-Charged Thin Shell

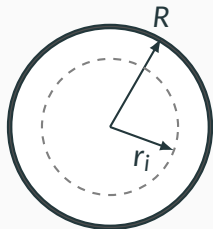
For a uniformly-charged spherical thin shell with radius  $R$  and a total charge of  $Q$ .



# Uniformly-Charged Thin Shell

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Inside the shell ( $r_i < R$ ), there is no enclosed charge, therefore the electric field must be zero:



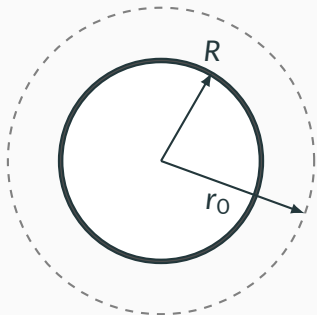
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0} = 0 \quad \rightarrow \quad E = 0$$

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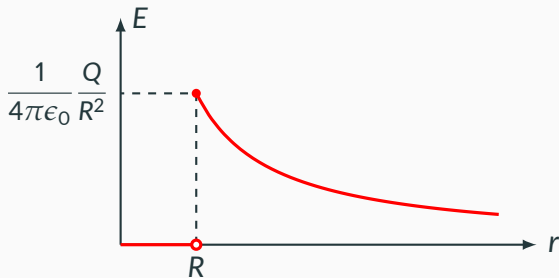


Outside the shell ( $r_o > R$ ), the enclosed charge is  $Q$ , and the electric field is given by the same equation as the point charge:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \quad \rightarrow \quad E = \frac{Q}{\epsilon_0 A} = \frac{1}{4\pi\epsilon_0} \left[ \frac{Q}{r^2} \right]$$

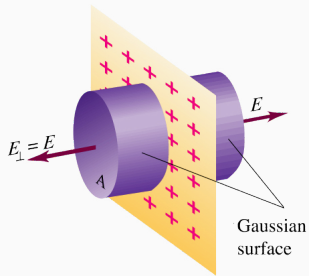
# Uniformly-Charged Thin Shell

The electric field strength  $E$  can be plotted as a function of the distance  $r$  from the center of the shell:



- This graph may be familiar because the graph for the gravitational field strength inside a uniform shell is exactly the same
- Replace  $\epsilon_0$  with  $4\pi G$

# Electric Field Near an Infinite Plane of Charge



- Charge density (charge per unit area)  $\sigma$
- By symmetry,  $\vec{E}$  must be perpendicular to the plane
- Our Gaussian surface is a cylinder shown in the left with an area  $A$ ; the height of the cylinder is unimportant
- Nothing “flows out” of the side of the cylinder, only at the ends
- The total flux is  $\Phi_E = E(2A)$
- The enclosed charge is  $Q_{\text{encl}} = \sigma A$

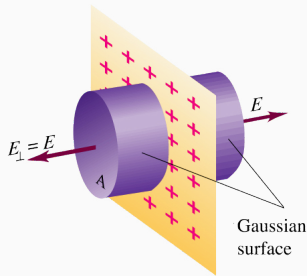
# Electric Field Near an Infinite Plane of Charge

Gauss's law simplifies to:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0} \rightarrow E(2A) = \frac{\sigma A}{\epsilon_0}$$

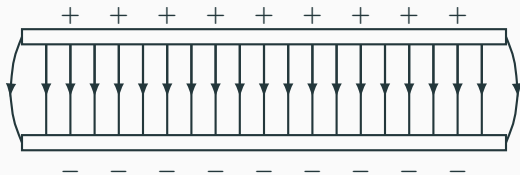
Solving for  $E$ , we get:

$$E = \frac{\sigma}{2\epsilon_0}$$



- $E$  is a constant
- Independent of distance from the plane
- Both sides of the plane are the same

# Electric Field Between Parallel Charged Plates



- Two plates, each producing an electric field pointing in the same direction
- The total electric field is twice the value of *one* infinite plane, pointing from the positively charged plate toward the negatively charged plate

$$E = \frac{\sigma}{\epsilon_0}$$

- $\vec{E}$  outside the plates is very low (close to zero), except for fringe effects at the edges of the plates

# Electric Field and Electric Potential Difference

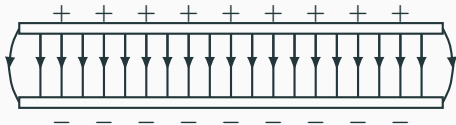
Recall the relationship between electric field ( $\vec{E}$ ) and electric potential difference ( $V$ ):

$$\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$$

This relationship holds regardless of the charge configuration.



# Electric Field and Electric Potential Difference



In the case of two parallel plates, the electric field is uniform, and the relationship simplifies to:

$$E = \frac{\Delta V}{d}$$

Quantity	Symbol	SI Unit
Electric field intensity	$E$	N/C
Potential difference between plates	$\Delta V$	V
Distance between plates	$d$	m