AP PHYSICS 1: HARMONIC MOTION

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

$\omega = \sqrt{\frac{g}{l}} \rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

1. A 0.40-kg mass hangs on a spring with a spring constant of 12 N/m. The system oscillates with a constant amplitude of 12 cm. What is the maximum acceleration of the system?

   D

   $a_{max} = A\omega^2 = \frac{Ak}{m} =$ $\omega = \sqrt{\frac{k}{m}}$

   (A) $0.62\,\mathrm{m/s^2}$
   (B) $1.4\,\mathrm{m/s^2}$
   (C) $1.6\,\mathrm{m/s^2}$
   (D) $3.6\,\mathrm{m/s^2}$
   (E) $9.8\,\mathrm{m/s^2}$

   $a(t) = -A\omega^2\cos(\omega t - \phi)$

2. A mass is attached to a spring and allowed to oscillate vertically. Which of the following would NOT change the period of the oscillation?

   $\omega = \sqrt{\frac{k}{m}}$ $T = 2\pi\sqrt{\frac{m}{k}}$

   A

   (A) Double the mass and double the spring constant
   ~~(B)~~ Double the amplitude of vibration and double the mass
   ~~(C)~~ Double the gravitational field strength and double the mass
   ~~(D)~~ Double the gravitational field strength and double the spring constant
   ~~(E)~~ Double the gravitational field strength and quadruple the mass

3. The Moon is approximately 384 000 km from the Earth. The Moon revolves around the Earth once every 27.3 days. What is the frequency of the Moon's motion?

   $\frac{1}{\text{time}}$

   B

   ~~(A)~~ 14 100 km each day
   (B) 0.0366 revolution each day
   ~~(C)~~ 27.3 revolution each day
   ~~(D)~~ 655 hours for each revolution
   ~~(E)~~ 27.3 days for each revolution

4. A mass is suspended from a spring and allowed to oscillate freely. When the amplitude of vibration is doubled, what happens to frequency of vibration?

   $\omega = \sqrt{\frac{k}{m}}$

   C

   (A) It quadruples.
   (B) It doubles.
   (C) It stays the same.
   (D) It reduces to one-half of what it was.
   (E) It reduces to one-fourth of what it was.

5. A bell is rung when the dangling clapper within it makes contact with the bell. A poorly designed bell has a clapper that swings with the same period as the bell. How can this design be improved?

   ~~(A)~~ Use a clapper with a smaller mass on the end so it is out of period with the bell.
   ~~(B)~~ Use a clapper with a bigger mass on the end so it is out of period with the bell.
   ~~(C)~~ Force the bell to swing with greater amplitude.
   (D) Use a longer clapper so it is out of period with the bell.
   ~~(E)~~ Increase the mass of the bell so it makes better contact with the clapper.

   D

6. Which choice below best explains why a pendulum does not oscillate in zero gravity?

   B

   $mg\sin\theta$

   ~~(A)~~ The pendulum has no mass in zero gravity.
   (B) A pendulum requires gravity to create the restoring force.
   ~~(C)~~ The pendulum is in orbit and considered weightless.
   ~~(D)~~ The pendulum would be too far from the Earth to work properly.
   ~~(E)~~ The pendulum must have an oscillating tension in the string to function properly.

7. A refrigerator compressor that weighs 8 kg is fixed to three separate springs on the refrigerator frame. Each has a spring constant of 0.01 N/m. What is the natural frequency of the system?

   (A) 0.01 cycle/s
   (B) 0.03 cycle/s
   (C) 0.8 cycle/s
   (D) 103 cycles/s
   (E) 0.003 cycle/s

   A

   $k_{eff} = 0.03\,\mathrm{N/m}$

   $\omega = \sqrt{\frac{k}{m}}$

   $f = \frac{\omega}{2\pi} = 0.0097$

8. A pendulum on the surface of the Moon has a period of 1.0 s. If the length of the pendulum is quadrupled, what is the value of the new period?

(A) 0.25 s
(B) 0.50 s
(C) 1.0 s
(D) 2.0 s
(E) 4.0 s

D

$T=2\pi\sqrt{\frac{l}{g}}$

9. A 2.0-m pendulum on a particular planet has a period of 4.6 s. What is the gravitational field strength on that planet?

(A) 1.6 N/kg
(B) 3.7 N/kg
(C) 4.9 N/kg
(D) 9.8 N/kg
(E) 25 N/kg

B

$T=2\pi\sqrt{\frac{l}{g}}$

$\frac{T}{2\pi}=\sqrt{\frac{l}{g}}$ $\quad g=l\left(\frac{2\pi}{T}\right)^2$

$\left(\frac{T}{2\pi}\right)^2=\frac{l}{g}$ $\quad =$

10. The displacement (in centimeters) of the vibrating cone of a large loudspeaker is represented by the equation $\Delta x = 2.0\cos(150t)$, where t is the time in seconds. What distance does the tip of the cone move in half a period?

A

-A 2 2 A

(A) 0.007 cm
(B) 1.0 cm
(C) 2.0 cm
(D) 4.0 cm
(E) 150 cm

D

11. ~~Which choice below best explains why a pendulum does not oscillate in zero gravity?~~

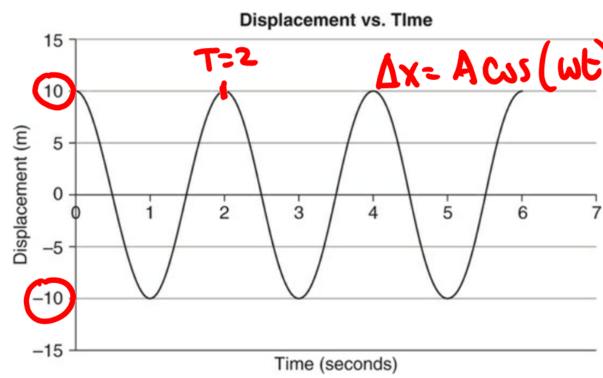
(A) ~~The pendulum has no mass in zero gravity.~~
(B) ~~A pendulum requires gravity to create the restoring force.~~
(C) ~~The pendulum is in orbit and considered weightless.~~
(D) ~~The pendulum would be too far from the Earth to work properly.~~
(E) ~~The pendulum must have an oscillating tension in the string to function properly.~~

12. Some large oil tankers have an antiroll water tank inside the hull that matches the resonant frequency of the ships hull. When ocean waves hit the ship at the resonant frequency, how does the water tank prevent the ship from capsizing in the waves?

(A) The energy of the waves is used by the water in the tank.
(B) The waves enter the tank and are dampened.
(C) The water tank is 180° out of phase with the ship's hull.
(D) The water tank is 90° out of phase with the ship's hull.
(E) The water in the tank is in phase with the ship's hull.

C

13. The graph below shows the displacement versus time for an object. Which equation best describes its displacement in meters?



$\Delta x = A\cos(\omega t)$

(A) $\Delta x = 20\cos(0.5t)$
(B) $\Delta x = 10\cos(2t)$
(C) $\Delta x = 10\cos(\pi t)$
(D) $\Delta x = 20\cos(2t)$
(E) $\Delta x = 20\sin(\pi t)$

$T = 2s$

$f=\frac{1}{2}$ Hz

$\omega=2\pi f = 2\pi\,\frac{1}{2}$

14. The Moon has a gravitational field strength that is approximately one-sixth of the field on the Earth. What is the ratio between the period of a pendulum on the Moon and the period of an identical pendulum on the Earth?

(A) 6
(B) $\sqrt{6}$
(C) $\frac{1}{6}$
(D) $\frac{1}{\sqrt{6}}$
(E) 1

B

$T=2\pi\sqrt{\frac{l}{g}}$

$g \rightarrow \frac{1}{6}g$

15. Which of the following best represent periodic motion?

B

(A) A skydiver who has reached terminal velocity
(B) The Moon in orbit about the Earth
(C) A car driving to each state in the United States
(D) A cart pushed up a frictionless incline plane
(E) A rubber ball bouncing on the floor over 30 seconds.

16. Which of the following significantly affect the period of a simple pendulum?

A

(A) The length of the pendulum
(B) The mass of the pendulum bob
(C) The amplitude of swing
(D) The shape of the mass
(E) The thickness of the string

$T = 2\pi\sqrt{\frac{l}{g}}$

← massless.

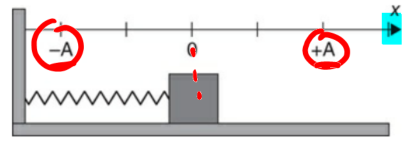
17. A particle oscillates with simple harmonic motion with no damping. Which one of the following statements about the acceleration of the oscillating particle is true?

E

(A) It has a value of $9.8\,\mathrm{m/s^2}$ when the oscillation is vertical.
(B) It is zero when the speed is the minimum.
(C) It is proportional to the frequency.
(D) It is zero throughout the oscillation.
(E) It is zero when the speed is the maximum.

$a = A\omega^2\cos(\omega t)$

Questions 18–21 are based on the figure below of a mass-spring system. Assume the mass is pulled back to position +A and released, and it slides back and forth without friction.



18. When the mass reaches position $-A$, what can be said about its speed?

(A) It is a minimum.
(B) It is a maximum.
(C) It is zero.
(D) It is decreasing.
(E) It is increasing.

C

19. When the mass reaches position 0, what can be said about its speed?

(A) It is a minimum.
(B) It is a maximum.
(C) It is zero.
(D) It is decreasing.
(E) It is increasing.

B

$V_{max} = A\omega$

20. At what position does the mass have the greatest acceleration?

(A) $-A$ → $a = +A\omega^2$
(B) $-A/2$
(C) 0
(D) $+A/2$
(E) $+A$ → $a = -A\omega^2$

21. The mass is released from the $-A$ position at time $t = 0$, and it oscillates with period T, measured in seconds. Which equation best represents the displacement?

(A) $\Delta x = -A\cos\left(\frac{T}{2\pi}t\right)$
(B) $\Delta x = -(A/2)\cos(2\pi T t)$
(C) $\Delta x = -A\cos\left(\frac{2\pi}{T}t\right)$
(D) $\Delta x = (A/2)\cos(Tt)$
(E) $\Delta x = A\cos\left(\frac{2\pi}{T}t\right)$

$T \to f = \frac{1}{T}$

$\downarrow$

$\omega = 2\pi f = \frac{2\pi}{T}$

C

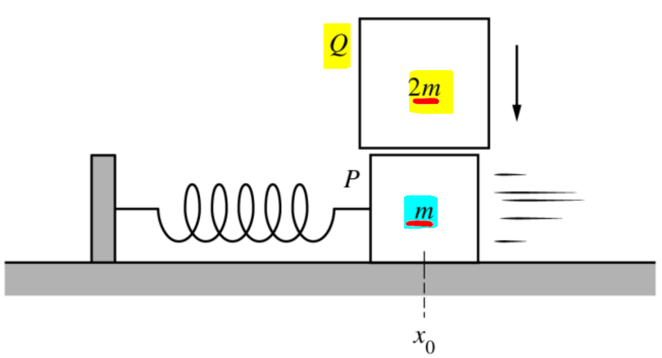
AP PHYSICS 1: SIMPLE HARMONIC MOTION
SECTION II
4 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

$T_P = 2\pi\sqrt{\frac{m}{k}}$

$T_{PQ} = 2\pi\sqrt{\frac{3m}{k}}$

$\frac{T_{PQ}}{T_P} = \frac{\cancel{2\pi}\sqrt{\frac{3m}{\cancel{k}}}}{\cancel{2\pi}\sqrt{\frac{m}{\cancel{k}}}} = \sqrt{3}$



1. Block P of mass m is on a horizontal, frictionless surface and is attached to a spring with spring constant k. The block is oscillating with period T_P and amplitude A_P about the spring's equilibrium position x_0. A second block Q of mass $2m$ is then dropped from rest and lands on block P at the instant it passes through the equilibrium position, as shown above. Block Q immediately sticks to the top of block P, and the two-block system oscillates with period T_{PQ} and amplitude A_{PQ}.

   (a) Determine the numerical value of the ratio T_{PQ}/T_P. $= \sqrt{3}$

   (b) How does the amplitude of oscillation A_{PQ} of the two-block system compare with the original amplitude A_P of block P alone?

   __✓__ $A_{PQ} < A_P$ ____ $A_{PQ} = A_P$ ____ $A_{PQ} > A_P$

   In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, explain your reasoning.

- When Q is dropped on P → inelastic collision
- conservation of momentum → $v_{PQ} = \frac{1}{3}v_P$
- K → U_e $\quad \frac{1}{2}3m\underbrace{\left(\frac{1}{3}v_P\right)^2}_{v_{PQ}} = \frac{1}{2}kA_{PQ}^2 \rightarrow \frac{1}{3}mv_P^2 = kA_{PQ}^2$

$\frac{1}{2}mv_P^2 = \frac{1}{2}kA_P^2 \qquad mv_P^2 = kA_P^2$

$A_{PQ}^2 = \frac{mv_P^2}{3k}$

$A_P^2 = \frac{mv_P^2}{k}$

2. In heavy seas, the bow of a battle ship undergoes a simple harmonic vertical pitching motion with a period of 8.0 s and an amplitude of 2.0 m.

T A

(a) What is the maximum vertical velocity of the battle ship's bow?

(b) What is its maximum acceleration?

(c) An 80 kg sailor is standing on the scale in the bunk room in the bow. What are the maximum and minimum reading on the scale in newtons?

$$v(t) = -A\omega \sin(\omega t - \phi)$$
$$a(t) = -A\omega^2 \cos(\omega t - \phi)$$

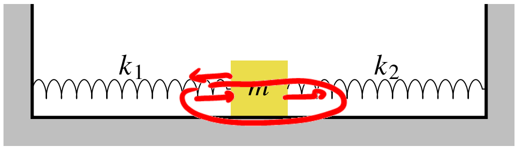
$V_{max} = A\omega = 2\pi A f = \frac{2\pi A}{T} = \frac{(2\pi)(2.0)}{8.0} = \boxed{1.6\ m/s}$

b) ~~max acc~~ $a_{max} = A\omega^2 = 4\pi^2 A f^2 = \frac{4\pi^2 A}{T^2} = \frac{(4\pi^2)(2.0)}{(8.0)^2} = \boxed{1.2\ m/s^2}$

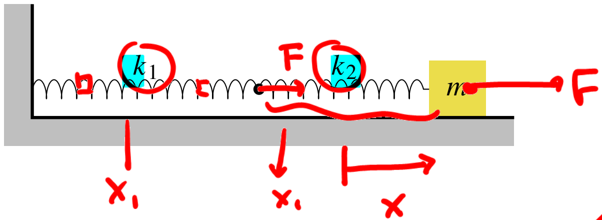
c) ↓a lighter $m(g-a) = 80(9.8 - 1.2) = \boxed{686\ N}$

↑a heavier $m(g+a) = 80(9.81 + 1.2) = \boxed{883\ N}$

3. Show that for the situations in the figures below, the object of mass m oscillates with a frequency of $\omega = \sqrt{\frac{k_{\text{eff}}}{m}}$ where k_{eff} is given by (a) $k_{\text{eff}} = k_1 + k_2$ and (b) $\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2}$. Hint: find the net force on the mass and write $F = -k_{\text{eff}}x$. Note that in (b), the springs stretch by different amounts, the sum of which is x.



(a)



(b)

$x = x_1 + x_2$

stretch of spring 1

stretch of spring 2

$F = k_{eff}\, x$

$F = k_1 x_1 \rightarrow x_1 = \frac{F}{k_1}$

$F = k_2 x_2 \rightarrow x_2 = \frac{F}{k_2}$

$x = x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2}$

$x = F\underbrace{\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}_{k_{eff}^{-1}}$

$\boxed{\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}}$

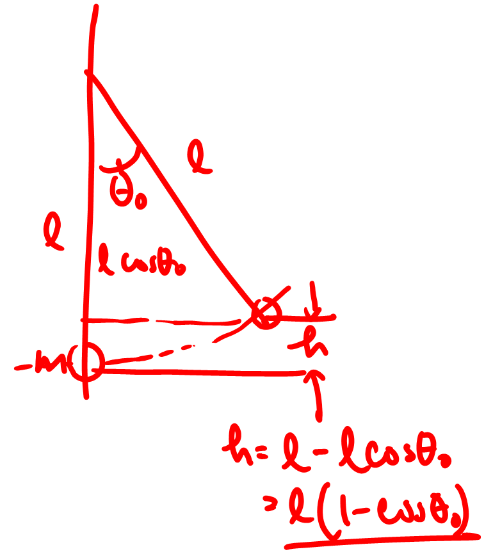
4. A simple pendulum of length L is released from rest from an angle of θ_0.

   (a) Assuming the motion of the pendulum to be simple harmonic motion, find its speed as it passes through $\theta = 0$.

   (b) Using the conservation of energy, find this speed exactly.

   (c) Show that your results for (a) and (b) are the same when θ_0 is small.

   (d) Find the difference in your results for $\theta_0 = 0.20$ rad and $L = 1$ m.



$\theta(t) = \theta_0 \cos(\omega t)$ $\quad \omega = \sqrt{\frac{g}{l}}$

angular velocity

$\Omega(t) = -\omega\theta_0 \sin(\omega t)$

$v = l\Omega = -l\omega\theta_0 \sin(\omega t)$

at bottom, $v = v_{max} = l\omega\theta_0 = l\sqrt{\frac{g}{l}}\theta_0 = \boxed{\sqrt{gl}\,\theta_0}$

$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} \cdots$

b) $mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh} = \boxed{\sqrt{2gl(1-\cos\theta_0)}}$

c) $v \approx \sqrt{2gl\left(\frac{\theta_0^2}{2!}\right)}$ $\quad 1 - \cos\theta = \cancel{1} - \cancel{1} + \frac{\theta^2}{2!} - \frac{\theta^4}{4!}$

$\approx \sqrt{gl\theta_0^2} = \sqrt{gl}\,\theta_0 \leftarrow$ solution for part (a)