

## RIGID CYLINDER ROLLING DOWN A RIGID RAMP WITHOUT SLIPPING

no relative motion between the cylinder and the ramp at the point of contact

Neither  $Mg$  nor  $N$  generates a torque at center of rotation.  
 $\rightarrow$  for the problem to be no-slip, there must be a static friction force

- no motion in  $y$ -direction  $\rightarrow \sum F_y = 0 \rightarrow N = Mg \cos \theta$  (unlikely to be related to  $N$ . (not max))
- there is motion in  $x$ -direction  $\rightarrow \sum F_x = Mg \sin \theta - F_s = ma$  (translational motion in  $x$ -direction)
- (+) rotational motion

$$I = r \cdot F_s = I \alpha$$

torque generated by the static friction

the only force that generates torque

rotational motion

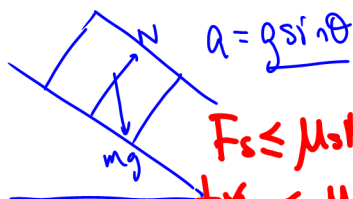
no slip

$$\alpha = \frac{a}{r}$$

(1)  $Mg \sin \theta - F_s = ma$

(2)  $r \cdot F_s = I \alpha$

for a cylinder  $I = \frac{1}{2} Mr^2$



$F_s \leq \mu_s N = \mu_s Mg \cos \theta$

$\frac{1}{2} Ma \leq \mu_s Mg \cos \theta$

$$r \cdot F_s = \left( \frac{1}{2} Mr^2 \right) \left( \frac{a}{r} \right)$$

$$F_s = \frac{1}{2} Ma$$

substitute back into (1)

$$Mg \sin \theta - \frac{1}{2} Ma = Ma$$

$$g \sin \theta = \frac{3}{2} a \rightarrow a = \frac{2}{3} g \sin \theta$$

as long as  $F_s \leq \mu_s N$   
 The cylinder rolls slower than when it slides down without friction