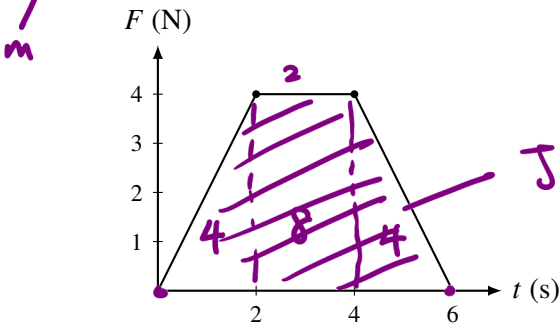


TOPIC 4: MOMENTUM, IMPULSE AND COLLISIONS

1. A toy train car of mass 3 kg rolls to the left at 2 m/s and collides with a 4 kg train car rolling to the right at 1 m/s. The two cars stick together. The velocity of the cars after the collision is
- (A) 2/7 m/s to the left  
(B) 2/7 m/s to the right  
(C) 4/7 m/s to the left  
(D) 4/7 m/s to the right  
(E) 9/7 m/s to the right

$3 \times 2 = 6 \text{ kg m/s}$   
 $4 \times 1 = 4 \text{ kg m/s}$   
 $V = \frac{P}{m} = \frac{2}{7} \text{ m/s [L]}$   
 $2 \text{ kg m/s [L]}$

Questions 2–3. A force acts on a 2.0 kg mass during a time interval as shown in the graph.



$J = \Delta p \rightarrow m \Delta v$

$\Delta v = \frac{\Delta p}{m} = \frac{J}{m}$   
 $= \frac{16}{2} = 8 \text{ m/s}$

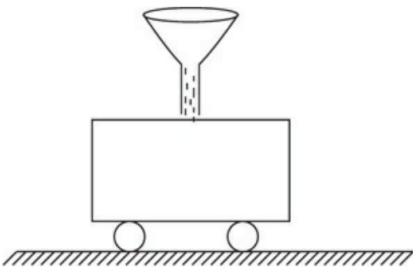
2. The impulse given to the mass from  $t = 0$  to  $t = 6$  s is
- (A)  $4 \text{ N} \cdot \text{s}$   
(B)  $8 \text{ N} \cdot \text{s}$   
(C)  $12 \text{ N} \cdot \text{s}$   
(D)  $16 \text{ N} \cdot \text{s}$   
(E)  $24 \text{ N} \cdot \text{s}$

3. If the initial speed of the mass is 2 m/s at  $t = 0$ , what is its speed at the end of 6 s?
- (A)  $4 \text{ m/s}$   
(B)  $6 \text{ m/s}$   
(C)  $8 \text{ m/s}$   
(D)  $10 \text{ m/s}$   
(E)  $16 \text{ m/s}$

4. Two steel balls, one of mass  $m$  and the other of mass  $2m$ , collide and rebound in a perfectly elastic collision. Which of the following is conserved in this elastic collision?
- (A) velocity only  
(B) momentum only  
(C) momentum and kinetic energy only  
(D) momentum, velocity, and kinetic energy  
(E) kinetic energy only

5. Two blocks are connected by a compressed spring and rest on a frictionless surface. The blocks are released from rest and pushed apart by the compressed spring. If one mass is twice the mass of the other, which of the following is the same for both blocks?
- (A) magnitude of momentum  
(B) acceleration  
(C) speed  
(D) kinetic energy  
(E) potential energy

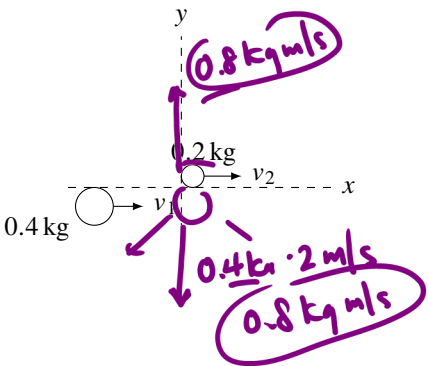
6. A 1000 kg railroad car is rolling without friction on a horizontal track at a speed of 3.0 m/s. Sand is poured into the open top of the car for a time of 5.0 s. The speed of the car after 5.0 s is 1.0 m/s. The mass of the sand added to the car at the end of 5.0 s is



- (A) 500 kg  
(B) 1000 kg  
(C) 2000 kg  
(D) 3000 kg  
(E) 3500 kg

no external force  
 $P_i = P_f$   
 $m_i v_i = m_f v_f$   
 $(1000)(3) = m_f (1)$   
 $m_f = 3000 \text{ kg}$   
1000 kg car  
2000 kg sand

7. Two billiard balls are rolling to the right on a table as shown. The 0.4 kg ball is moving faster than the 0.2 kg ball, so it catches up and strikes it from behind at a slight angle. Immediately after the collision, the y-component of the 0.4 kg ball is 2 m/s downward. The y-component of the velocity of the 0.2 kg ball must be



- (A) 1 m/s upward  
(B) 2 m/s upward  
(C) 1 m/s downward  
(D) 2 m/s downward  
(E) 4 m/s upward

8. A 0.3 kg baseball at rest on a tee is struck by a bat. The ball leaves the bat with a speed of 20 m/s at an angle of  $45^\circ$  above the horizontal. The magnitude of the impulse imparted to the baseball by the bat is most nearly

(A)  $2 \text{ N} \cdot \text{s}$   
 (B)  $6 \text{ N} \cdot \text{s}$   
 (C)  $12 \text{ N} \cdot \text{s}$   
 (D)  $16 \text{ N} \cdot \text{s}$   
 (E)  $20 \text{ N} \cdot \text{s}$

$$\underline{J} = \underline{\Delta p} = m \underline{\Delta v} = (0.3)(20)$$



9. Two ice skaters, a large man and a small woman, are initially at rest and holding each other's hands. They push away horizontally. Afterward, which of the following statements is true?

(A) They have equal and opposite kinetic energies.  
 (B) They have equal and opposite momenta.  
 (C) The large man applies a greater force to the small woman.  
 (D) The small woman applies a greater force to the large man.  
 (E) They recoil with equal and opposite velocities.

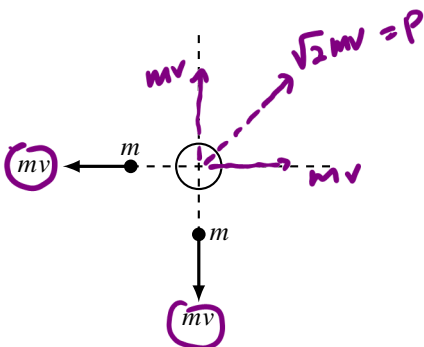
10. A known net force  $F$  acts on an unknown mass for a known time  $\Delta t$ . From this information, you could determine the

(A) change in kinetic energy of the object  
 (B) change in velocity of the object  
 (C) acceleration of the object  
 (D) mass of the object  
 (E) change in momentum of the object

$$\underline{F_{\text{net}} \Delta t} \rightarrow \underline{J_{\text{net}}} \rightarrow \underline{\Delta p}$$

$$\underline{\Delta d}??$$

**Questions 11–12.** An object has a mass  $4m$ . The object explodes into three pieces of mass  $m$ ,  $m$ , and  $2m$ . The two pieces of mass  $m$  move off at right angles to each other with the same momentum  $mv$ , as shown.



$$\Sigma \vec{p} = 0$$

11. The speed of mass  $2m$  after the explosion is

(A)  $2v$   
 (B)  $\sqrt{2}v$   
 (C)  $\frac{\sqrt{2}}{2}v$   
 (D)  $\frac{\sqrt{2}}{3}v$   
 (E)  $\frac{\sqrt{3}}{2}v$

$$v = \frac{P}{m} = \frac{\sqrt{2}mv}{2m} = \frac{\sqrt{2}v}{2}$$

12. The direction of velocity of mass  $2m$  is

(A)  $\rightarrow$   
 (B)  $\swarrow$   
 (C)  $\downarrow$   
 (D)  $\nearrow$   
 (E)  $\uparrow$

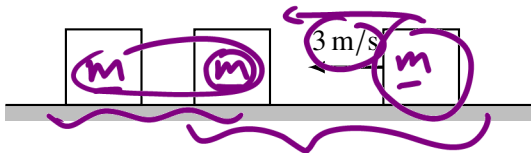
13. A block of mass  $m$  is moving to the right with a speed  $v_0$  on a horizontal surface of negligible friction when it explodes. The explosion causes the block to break into two pieces, each of which moves in the horizontal direction. One piece of mass  $m/4$  moves to the left with a speed of  $2v_0$ . What is the velocity of the other piece?

(A)  $2v_0$  to the right  
 (B)  $v_0$  to the right  
 (C)  $3v_0$  to the right  
 (D)  $12v_0$  to the right  
 (E)  $14v_0$  to the left

$$mv_0 = -\left(\frac{m}{4}\right)2v_0 + \left(\frac{3m}{4}\right)v$$

$$mv_0 = -\frac{mv_0}{2} + \frac{3m}{4}v \quad \frac{2\sqrt{3}v_0}{\sqrt{3}} = \frac{\sqrt{3}v}{\sqrt{3}} \quad v = 2v_0$$

**Questions 14–15.** Three identical masses can slide freely on a horizontal surface as shown. The first mass moves with a speed of 3.0 m/s toward the second and third masses, which are initially at rest. The first and second mass collide elastically, and then the second and third masses collide completely inelastically.



14. The speed of the second mass after the collision is

(A) zero  
 (B) 1.5 m/s  
 (C) 3.0 m/s  
 (D) 6.0 m/s  
 (E) 9.0 m/s

15. The speed of the second and third masses after they collide inelastically is

(A) zero  
 (B) 1.5 m/s  
 (C) 3.0 m/s  
 (D) 6.0 m/s  
 (E) 9.0 m/s

Questions 16–17. A 20 kg boy runs at a speed of 3 m/s and jumps onto a 40 kg sled on frictionless ice that is initially at rest. The boy and the sled then move together for a short time.

16. The speed of the boy and sled after he jumps on it is

- (A) 0.5 m/s
- (B) 0.8 m/s
- (C) 1.0 m/s
- (D) 1.5 m/s
- (E) 2.0 m/s

$$P_i = P_f$$
$$60 = 60v$$
$$v = 1 \text{ m/s}$$

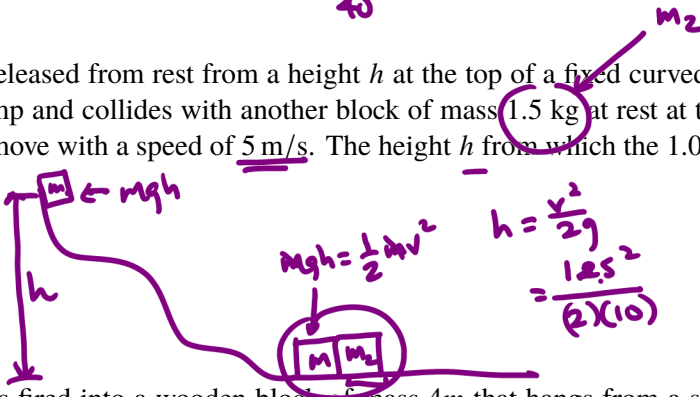
17. While the boy and sled are moving, he jumps off the back of the sled in such a way the boy is at rest, and the sled continues to move forward. The speed of the sled after the boy jumps off is

- (A) 1.5 m/s
- (B) 2.0 m/s
- (C) 3.0 m/s
- (D) 4.5 m/s
- (E) 6.0 m/s

$$60 = 40v$$
$$v = \frac{60}{40} = 1.5$$

18. A 1.0 kg block is released from rest from a height  $h$  at the top of a fixed curved ramp of negligible friction. The block slides down the ramp and collides with another block of mass 1.5 kg at rest at the bottom of the ramp. The two blocks stick together and move with a speed of 5 m/s. The height  $h$  from which the 1.0 kg block began is

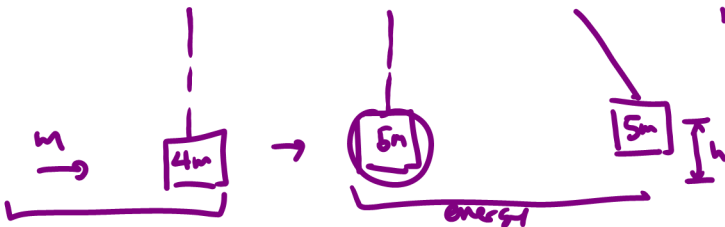
- (A) 0.8 m
- (B) 1.2 m
- (C) 1.8 m
- (D) 2.8 m
- (E) 7.8 m



collision  
$$P_i = P_f$$
$$1.0 v = (1.0 + 1.5)(5)$$
$$v = (2.5)(5) = 12.5 \text{ m/s}$$

19. A dart of mass  $m$  is fired into a wooden block of mass  $4m$  that hangs from a string. The dart and block then rise to a maximum height  $h$ . An expression for the initial speed  $v_0$  of the dart before striking the block is

- (A)  $\sqrt{gh}$
- (B)  $\sqrt{2gh}$
- (C)  $\sqrt{50gh}$
- (D)  $\sqrt{100gh}$
- (E)  $\sqrt{250gh}$



$$\frac{1}{2} (5m) v^2 = (5m) gh$$
$$v = \sqrt{2gh}$$
$$v = \sqrt{2gh}$$
$$v = \sqrt{2gh}$$

20. A mass  $m_1$  initially moving at speed  $v_0$  collides with and sticks to a spring attached to a second, initially stationary mass  $m_2$ . The two masses continue to move to the right on a frictionless surface as the length of the spring oscillates. At the instant that the spring is maximally extended, the velocity of the first mass is

- (A)  $v_0$
- (B)  $m_1^2 v_0 / (m_1 + m_2)^2$
- (C)  $m_2 v_0 / m_1$
- (D)  $m_1 v_0 / m_2$
- (E)  $m_1 v_0 / (m_1 + m_2)$

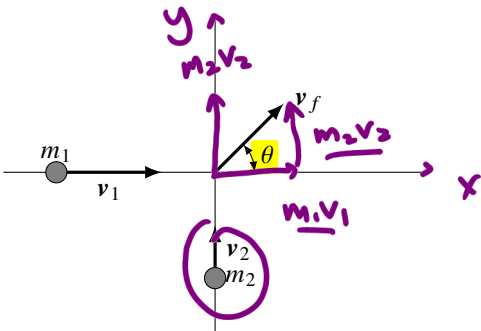
no relative velocity between  $m_1$  and  $m_2$   
both moving with the same speed.



$$m_1 v_0 = (m_1 + m_2) v$$
$$\frac{m_1 v_0}{m_1 + m_2}$$

$$v_0 = 5 \sqrt{2gh}$$
$$v_0 = 5 \sqrt{2gh}$$
$$v_0 = 5 \sqrt{2gh}$$

21. Two masses moving along the coordinates axes as shown collide at the origin and stick to each other. What is the angle  $\theta$  that the final velocity that makes with the  $x$ -axis?



- (A)  $\tan^{-1}(v_2/v_1)$
- (B)  $\tan^{-1}[m_1 v_1 / (m_1 + m_2)]$
- (C)  $\tan^{-1}(m_1 v_2 / m_2 v_1)$
- (D)  $\tan^{-1}(m_2 v_2^2 / m_1 v_1^1)$
- (E)  $\tan^{-1}(m_2 v_2 / m_1 v_1)$

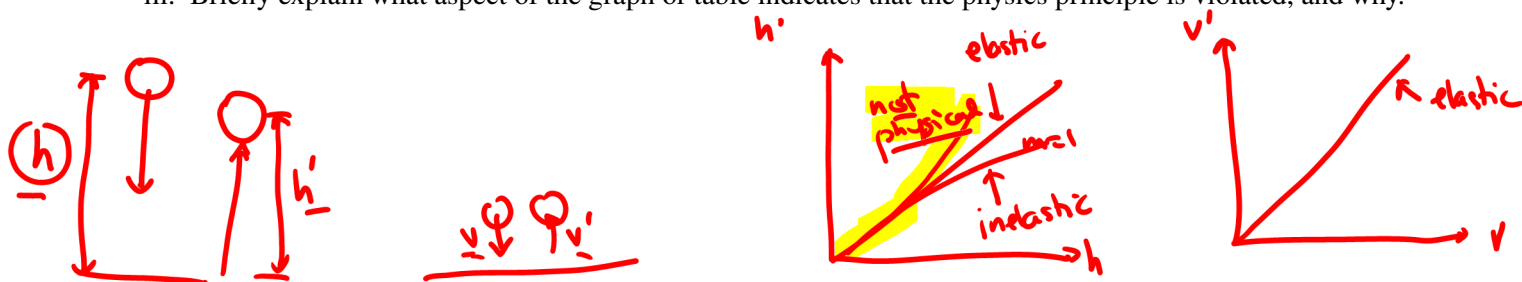
$$\theta = \tan^{-1} \left( \frac{m_2 v_2}{m_1 v_1} \right)$$

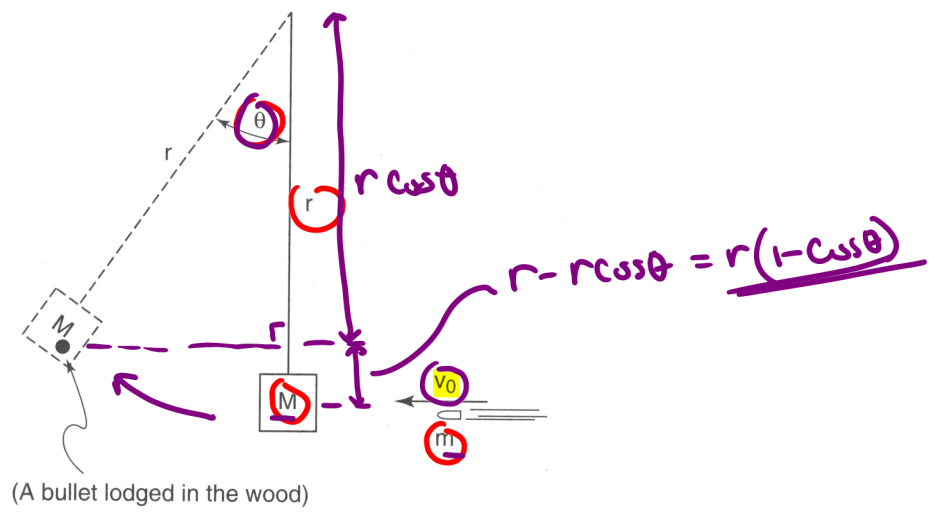
22. A mass traveling in the  $+x$  direction collides with a mass at rest. Which of the following statements is true?

- (A) After the collision, the two masses will move with parallel velocities
- (B) After the collision, the masses will move with anti-parallel velocities
- (C) After the collision, the masses will both move along the  $x$ -axis
- (D) After the collision, the  $y$ -components of the velocities of the two particles will sum to zero.
- (E) None of the above

x momentum

23. A new kind of toy ball is advertised to “bounce perfectly elastically” off hard surfaces. A student suspects, however, that no collision can be perfectly elastic. The student hypothesizes that the collisions are very close to being perfectly elastic for low-speed collisions but that they deviate more and more from being perfectly elastic as the collision speed increases.
- (a) Design an experiment to test the student’s hypothesis about collisions of the ball with a hard surface. The student has equipment that would usually be found in a school physics laboratory.
- What quantities would be measured?
  - What equipment would be used for the measurements, and how would that equipment be used?
  - Describe the procedure to be used to test the student’s hypothesis. Give enough detail so that another student could replicate the experiment.
- (b) Describe how you would represent the data in a graph or table. Explain how that representation would be used to determine whether the data are consistent with the student’s hypothesis.
- (c) A student carries out the experiment and analysis described in parts (a) and (b). The student immediately concludes that something went wrong in the experiment because the graph or table shows behavior that is elastic for low-speed collisions but appears to violate a basic physics principle for high-speed collisions.
- Give an example of a graph or table that indicates nearly elastic behavior for low-speed collisions but appears to violate a basic physics principle for high-speed collisions.
  - State one physics principle that appears to be violated in the graph or table given in part (c)i. Several physics principles might appear to be violated, but you only need to identify one.
  - Briefly explain what aspect of the graph or table indicates that the physics principle is violated, and why.





24. The Ballistic Pendulum. To determine the muzzle speed of a gun, a bullet is shot into a mass  $M$  from a string as shown below, causing  $M$  to swing upward through a maximum angle of  $\theta$ .

(a) What is the speed of  $M$  the instant after the bullet lodges in it?

(b) What is the speed of the bullet before it hits  $M$ ?  $v_0 = ?$

(c) What is the tension in the string at the highest point of the pendulum's swing (when the string makes an angle of  $\theta$  with the vertical as shown)?

a)  $mgh = \frac{1}{2}mv^2$

$$v = \sqrt{2gh}$$

$$\boxed{v = \sqrt{2gr(1 - \cos\theta)}}$$

b)  $mv_0 = (M+m)v$

$$v_0 = \left(\frac{M+m}{m}\right)\sqrt{2gr(1 - \cos\theta)}$$

c)

$v = 0$   
 $a_c = 0$   
 $\underline{F_c = 0.}$

$(M+m)g \cos\theta$

$$\boxed{T = (M+m)g \cos\theta}$$

25. An 800 kg car is traveling along a wet road at a velocity of 25.5 m/s. A 1000.0-kg car is traveling along the same road in the same direction at 34.7 m/s. The two cars collide and lock together. Answer the following questions.

- The two interlocked cars proceed at what velocity after the collision?
- Compare the kinetic energy of the two-car system immediately before the collision and after the collision.
- Discuss any transfer of energy that occurs, and explain whether this is an elastic or inelastic collision.
- If the coefficient of sliding friction between the tires of the cars and the wet pavement is 0.7, calculate the force of friction.
- How long does it take for the two interlocked cars to come to a complete stop on the wet pavement?

a)  $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$   $v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{800(25.5) + (1000)(34.7)}{800 + 1000}$   
 $= 30.61 \rightarrow \boxed{30.6 \text{ m/s}}$

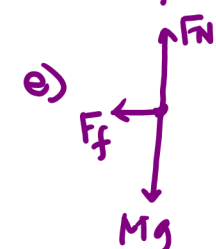
b)  $K_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (800)(25.5)^2 + \frac{1}{2} (1000)(34.7)^2 = 862145 \text{ J}$   
 $= \boxed{8.62 \times 10^5 \text{ J}}$

$K_f = \frac{1}{2} (m_1 + m_2) v^2 = \frac{1}{2} (1800)(30.61)^2 = 843724 \text{ J}$   
 $= \boxed{8.44 \times 10^5 \text{ J}}$

$K_f < K_i$

c) inelastic

d)  $F_f = \mu F_N = \mu (m_1 + m_2) g = (0.7)(1800)(10) = 126000 \rightarrow \boxed{1.26 \times 10^5 \text{ J}}$



$-F_f = Ma$

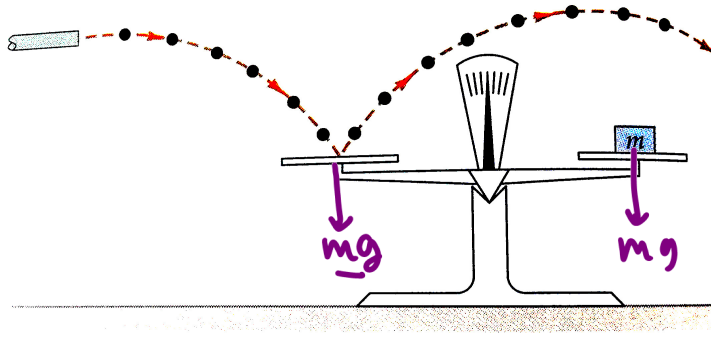
$-\mu Mg = Ma$

$a = -\mu g$

$v = v_0 + at$

$t = \frac{-v_0}{a} = \frac{-v_0}{\mu g} = \frac{30.61}{(0.7)(10)} = \boxed{4.4 \text{ s}}$

$M = m_1 + m_2$

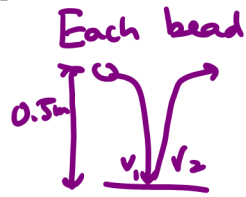


26. A stream of glass beads, each with a mass of 0.5 g, comes out of a horizontal tube at 100 per second. The beads fall a distance of 0.5 m to a balance pan and bounce back to their original height as shown in the figure below. How much mass must be placed in the other pan of the balance to keep the pointer at zero?

$$F = mg = \frac{\Delta p}{\Delta t} = \frac{0.316}{1.0}$$

$$m = \frac{0.316}{10} = 0.0316 \text{ kg.}$$

$$\boxed{m = 31.6 \text{ g}}$$



$$v^2 = 2gd$$

$$v = \sqrt{2gd}$$

$$= \sqrt{2(10)(0.5)} = \sqrt{10}$$

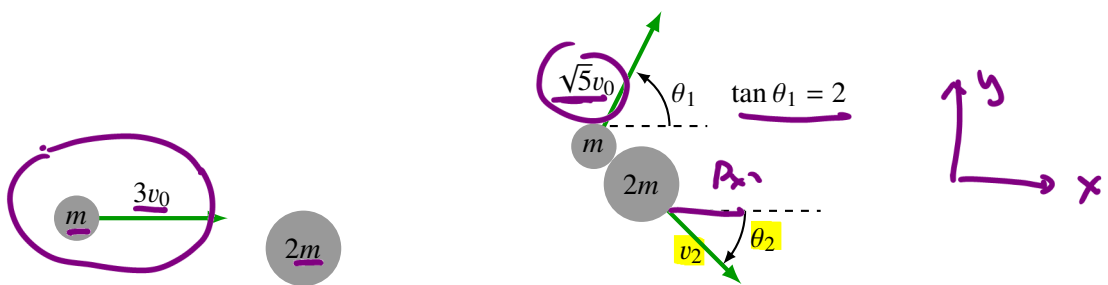
$$p = mv = 0.0005\sqrt{10}$$

$$\Delta p = 2mv = 0.001\sqrt{10} = 0.00316$$

100 beads/sec

$$\Delta p_T = 0.316 \text{ kg m/s.}$$





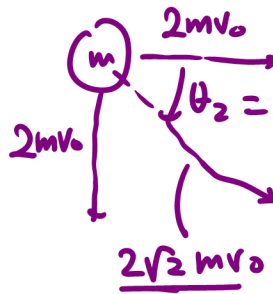
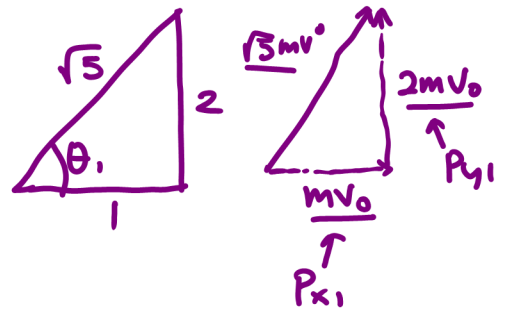
27. The figure above shows the results of a collision of two objects of unequal mass.

- (a) Find the speed of  $v_2$  and angle  $\theta_2$  of the larger mass after the collision.  
 (b) Show that the collision is elastic.

initial  $P_x = 3mv_0$   
 $P_y = 0$ .

$$P_{x1} + P_{x2} = 3mv_0 \rightarrow P_{x2} = 2mv_0$$

$$P_{y1} + P_{y2} = 0 \rightarrow P_{y2} = -2mv_0$$



$$\theta_2 = 45^\circ$$

$$v_2 = \frac{2\sqrt{2}mv_0}{2m}$$

$$v_2 = \sqrt{2}v_0$$

a)  $K_i = \frac{1}{2}(m)(3v_0)^2 = \frac{9}{2}mv_0^2$

$$K_f = \frac{1}{2}m(\sqrt{5}v_0)^2 + \frac{1}{2}(2m)(\sqrt{2}v_0)^2 = \frac{5}{2}mv_0^2 + \frac{4}{2}mv_0^2 = \frac{9}{2}mv_0^2 = K_i \quad \therefore \text{elastic}$$