

# Topic 12: Capacitors

## Advanced Placement Physics C

---

Dr. Timothy Leung

Last Updated: April 12, 2021

Olympiads School

# Parallel-Plate Capacitors

---

# Electric Field and Electric Potential Difference

Recall that the relationship between electrostatic force ( $\mathbf{F}_q$ ) and electric potential energy ( $U_q$ ) can be expressed using definition of mechanical work and the fundamental theorem of calculus:

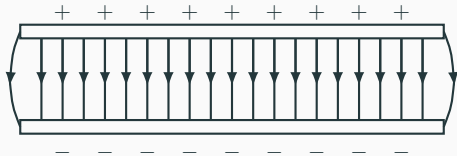
$$\Delta U_q = - \int \mathbf{F}_q \cdot d\mathbf{r} \quad \mathbf{F}_q = -\nabla U_q = -\frac{\partial U_q}{\partial r} \hat{\mathbf{r}}$$

Dividing both sides of the equations by  $q$ , we get the relationship between electric field ( $\mathbf{E}$ ), electric potential ( $V$ ) and electric potential difference ( $\Delta V$ ):

$$\Delta V = - \int \mathbf{E} \cdot d\mathbf{r} \quad \boxed{\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r} \hat{\mathbf{r}}}$$

This relationship holds regardless of the charge configuration.

# Electric Field and Electric Potential Difference



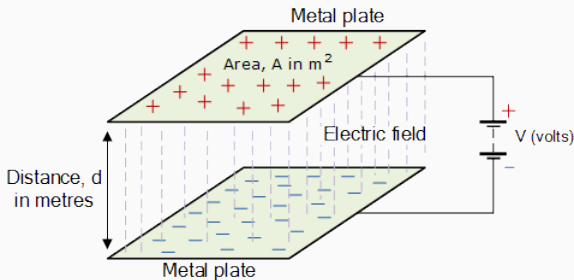
Recall that for two charged parallel plates, the electric field is uniform, and the relationship between electric field and potential difference simplifies to:

$$E = \frac{\Delta V}{d} \quad \text{or} \quad \Delta V = Ed$$

Quantity	Symbol	SI Unit
Electric field intensity	$E$	N/C
Potential difference between plates	$\Delta V$	V
Distance between plates	$d$	m

# Capacitors

**Capacitors** is a device that stores energy in an electric field. The simplest form of a capacitor is a set of closely spaced parallel plates:



When the plates are connected to a battery, the battery transfer charges to the plates until the voltage  $V$  equals the battery terminals. After that, one plate has charge  $+Q$ ; the other has  $-Q$ .

## Parallel-Plate Capacitors

As we have seen already, the (uniform) electric field between two parallel plates is proportional to the charge density  $\sigma$ , which is the charge  $Q$  divided by the area of the plates  $A$ :

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

Substituting this into the relationship between the plate voltage  $V$  and electric field, we find a linear relationship between the charges across the plates and the voltage:

$$\Delta V = Ed = \frac{Qd}{A\epsilon_0} \longrightarrow \boxed{Q = \left[ \frac{A\epsilon_0}{d} \right] \Delta V}$$

## Parallel-Plate Capacitors

Since area  $A$ , distance of separation  $d$  and the vacuum permittivity  $\epsilon_0$  are all constants, the relationship between charge  $Q$  and voltage  $\Delta V$  is *linear*. And the constant is called the **capacitance**  $C$ , defined as:

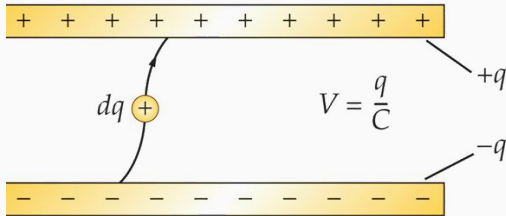
$$C = \frac{Q}{\Delta V}$$

For parallel plates:

$$C = \frac{A\epsilon_0}{d}$$

The unit for capacitance is a **farad** (named after Michael Faraday), where  $1\text{ F} = 1\text{ C/V}$ .

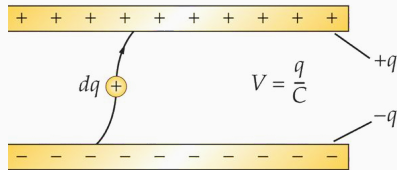
# Storage of Electrical Energy



- When charging up a capacitor, imagine positive charges moving from the negatively charged plate to the positively charged plate
- Initially neither plates are charged, so moving the first charge takes very little work; as the electric field builds, more and more work needs to be done



# Storage of Electrical Energy



In the beginning—when the plates aren't charged—moving an infinitesimal charge  $dq$  across the plates, the infinitesimal work done  $dU$  is very small and related to the capacitance by:

$$dU = Vdq = \frac{q}{C}dq$$

As the electric field begins to form between plates, more and more work is required to move the charges.

## Storage of Electrical Energy

To fully charge the plates, the total work  $U_c$  is the integral:

$$U_c = \int dU = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}$$

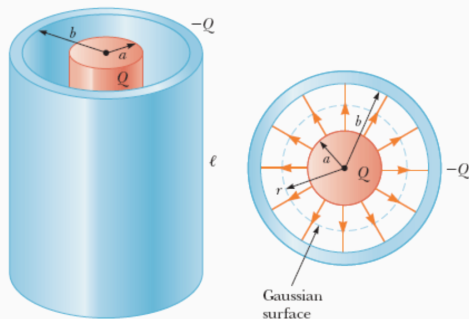
The work done is stored as a potential energy inside the capacitor. There are different ways to express  $U_c$  using definition of capacitance:

$$U_c = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

# Cylindrical Capacitors

---

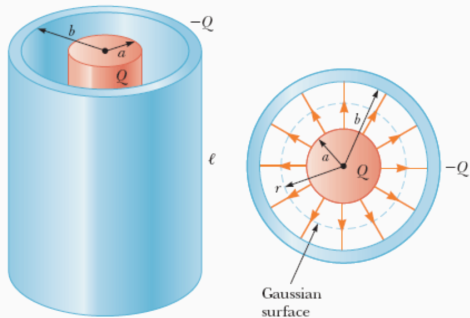
# Cylindrical Capacitors



Not all capacitors are parallel plates. Cylindrical capacitors are also popular.

- The capacitor has length  $\ell$  which is much larger than the radii of the inner & outer cylinders ( $a, b$ )
- Inner cylinder has total charge  $Q$
- Outer cylinder has total charge  $-Q$
- Inside the capacitor, the electric field in the radial direction
- Outside of the capacitor, there is no electric field

## Cylindrical Capacitors: Electric Field



Use Gauss's law to find the electric field between the cylinders, by placing a Gaussian surface of radius  $r$  between the cylinders:

$$\oint \mathbf{E} \cdot d\mathbf{A} = 2\pi r L E = \frac{Q}{\epsilon_0}$$

which gives the expression:

$$E = \frac{Q}{2\pi r L \epsilon_0} \quad \text{or} \quad E = \frac{\lambda}{2\pi r \epsilon_0}$$

where  $\lambda = Q/L$  is the linear charge density

## Cylindrical Capacitors: Voltage Across the Cylinders

Integrating the electric field to get voltage across the plates:

$$\Delta V = - \int_a^b E dr = \frac{Q}{2\pi L \epsilon_0} \int_b^a \frac{1}{r} dr = \frac{Q}{2\pi L \epsilon_0} \ln \left[ \frac{b}{a} \right]$$

Like the parallel plate, the relationship between voltage and charge is still linear, but in this case, the capacitance is defined as:

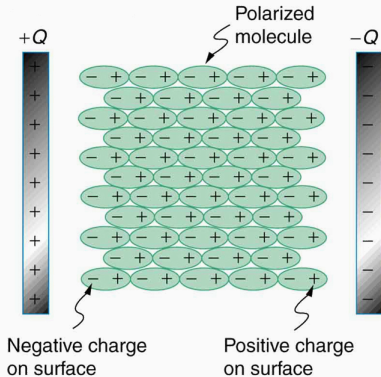
$$C = \frac{Q}{\Delta V} = \frac{2\pi L \epsilon_0}{\ln(b/a)} \longrightarrow \boxed{\frac{C}{L} = \frac{2\pi \epsilon_0}{\ln(b/a)}}$$

The capacitance is generally expressed by  $C/L$  (unit F/m). Like the parallel-plate, the capacitance of the cylindrical capacitor also only depends on the geometry (i.e. the radii  $a$  and  $b$ ) and the permittivity.

# Practical Capacitors

---

# Practical Capacitors



- Capacitors (both parallel-plate and cylindrical) are very common in electric circuits, but the vacuum between the plates is not very effective
- Instead, a non-conducting **dielectric** material is inserted between the plates
- When the plates are charged, the electric field of the plates polarizes the dielectric.
- The polarization produces an electric field that opposes the field from the plates, therefore reduces the effective voltage, and increasing the capacitance



# Dielectric Constant

If electric field without dielectric is  $E_0$ , then  $E$  in the dielectric is reduced by  $\kappa$ , the **dielectric constant**:

$$\kappa = \frac{E_0}{E}$$

The capacitance of the plates with the dielectric is now amplified by the same factor  $\kappa$ :

$$C = \kappa C_0$$

We can also view the dielectric as something that increases the *effective permittivity*:

$$\epsilon = \kappa \epsilon_0$$

# Dielectric Constant

The dielectric constants of commonly used materials are:

Material	$\kappa$
Air	1.000 59
Bakelite	4.9
Pyrex glass	5.6
Neoprene	6.9
Plexiglas	3.4
Polystyrene	2.55
Water (20 °C)	80

## Notes About Storage of Electric Energy

The work done (i.e. the energy stored in the capacitor) is inversely proportional to the capacitance:

$$dU = Vdq = \frac{q}{C}dq$$

- The presence of a dielectric *increases* the capacitance; therefore the work (and potential energy stored) to move the charge  $dq$  *decreases* with the dielectric constant  $\kappa$
- After the capacitor is charged, removing the dielectric material from the capacitor plates will require additional work.

# Capacitors in Electric Circuits

Capacitors are an important part of an electric circuits because it stores energy in the electric field

- Denoted by this symbol (with reference to the parallel-plate capacitor):



- Act like a voltage source
- Unlike a battery, the voltage increases or decreases as the charge across the capacitor plates change.