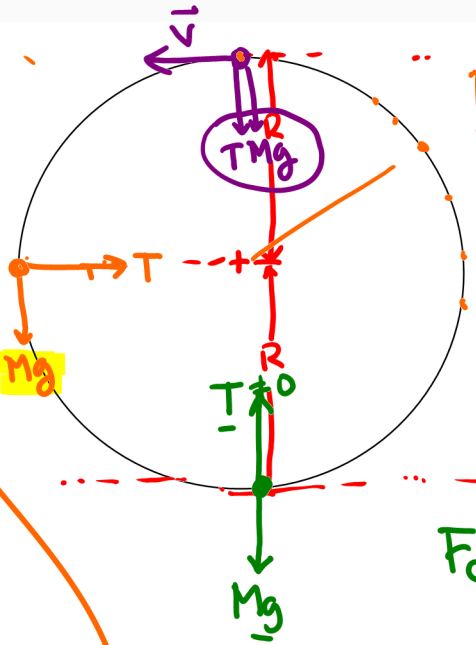
Example 4: You are playing with a yo-yo with a mass M. The length of the string is R. You decide to see how slowly you can swing it in a vertical circle while keeping the string fully extended, even when it is at the top of its swing.

a. Calculate the minimum speed at which you can swing the yo-yo while keeping it on a circular path.

b. Find the tension in the string when the yo-yo is at the side and at the bottom of its swing.



top:

$F_c = T + Mg = \frac{Mv^2}{R}$

at minimum v → $T = 0$

$Mg = \frac{Mv^2}{R}$

$v_{top} = \sqrt{gR}$

Conservation of energy

$\frac{1}{2}mv_i^2 + mg(2R) = \frac{1}{2}mv^2$

$= \frac{1}{2}mgR + 2mgR = \frac{1}{2}mv^2$

$\frac{5}{2}gR = \frac{1}{2}v^2$

$v^2 = 5gR$ at the bottom

$F_c = T - Mg = \frac{Mv^2}{R}$

$T = M\left(g + \frac{v^2}{R}\right)$

$T = M\left(g + \frac{5gR}{R}\right)$

$T = 6Mg$

$v_{bottom} = \sqrt{5gR}$

Conservation of energy

$\frac{1}{2}mv_i^2 + mgR = \frac{1}{2}mv^2$

$\frac{1}{2}gR + gR = \frac{1}{2}v^2$

$\frac{3}{2}gR = \frac{1}{2}v^2$ $v^2 = 3gR$ $v = \sqrt{3gR}$

$F_c = T = \frac{Mv^2}{R}$

$T = \frac{M(3gR)}{R} = 3Mg$

at the side a_c and a_t $Mg = Ma$

$\frac{v^2}{R} = \frac{3gR}{R}$

$= 3g$