Student #: ____________________________ Student Name: ___________________________________

AP PHYSICS C CLASS 16: GAUSS' LAW AND CAPACITORS

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

1. A non-conducting sphere does not have a uniform charge density, but the density ρ varies with the distance r from the center of the sphere according to the equation $\rho = \beta r$ where β is a positive constant. The electric field inside the sphere ($r < R$) at a distance r from the center of the sphere is

   (A) $\dfrac{\beta r^2}{12\epsilon_0}$
   (B) $\dfrac{\beta r^3}{3\epsilon_0}$
   (C) $\dfrac{\beta r}{2\epsilon_0}$
   (D) $\dfrac{\beta r^2}{2\epsilon_0}$
   (E) $\dfrac{\beta r^2}{4\epsilon_0}$

2. The electric potential at the surface of the sphere from the last question is

   (A) $\dfrac{\beta R^3}{12\epsilon_0}$
   (B) $\dfrac{\beta R}{2\epsilon_0}$
   (C) $\dfrac{\beta R^3}{3\epsilon_0}$
   (D) $\dfrac{\beta R^2}{2\epsilon_0}$
   (E) $\dfrac{\beta R^2}{4\epsilon_0}$

3. According to Gauss's law, the net electric flux passing through a closed surface is

   (A) positive if the flux is entering the surface
   (B) negative if the flux is exiting the surface
   (C) positive if the net charge inside the surface is zero
   (D) negative if the net charge inside the surface is zero
   (E) zero if the net charge inside the surface is zero

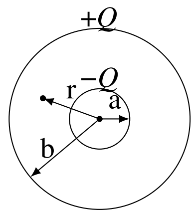
4. According to Gauss's law, which of the following statements is true?

   (A) It is possible to have a nonzero electric field, but zero electric flux.
   (B) It is possible to have a nonzero electric flux, but zero electric field.
   (C) It is possible to have a nonzero electric flux through a closed surface even if the enclosed charge in a surface is zero.
   (D) If a surface is not closed (such as a sheet of paper), the flux through it must be zero.
   (E) It is possible for charges located outside a closed surface to produce a net positive flux through the surface.

5. Gauss's law is most convenient to use when calculating an electric field due to

   (A) charges outside a closed surface
   (B) charges inside a closed surface that has high symmetry
   (C) charges inside a closed surface that has low symmetry
   (D) a potential difference that is negative
   (E) a potential difference that is positive

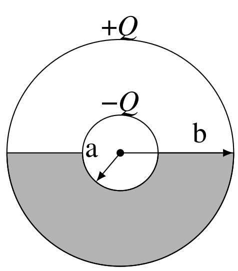
Questions 6–8



The spherical capacitor shown above consists of a conducting shell of radius a inside a larger conducting shell of radius b. A charge $-Q$ is placed on the inner sphere and a charge $+Q$ is placed on the outer shell. The capacitance of the capacitor is C_0.

6. The magnitude of the electric field E at a distance r between the spheres is
   (A) $\dfrac{Q}{4\pi\epsilon_0 r^2}$
   (B) $\dfrac{Q}{4\pi\epsilon_0 r}$
   (C) $\dfrac{Q}{4\pi\epsilon_0 a^2}$
   (D) $\dfrac{Q}{4\pi\epsilon_0 b^2}$
   (E) zero

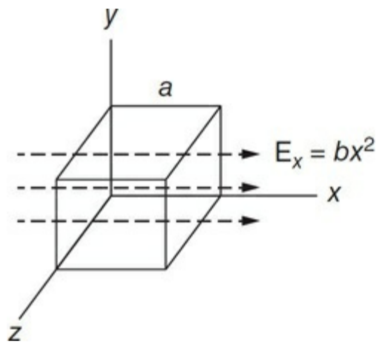
7. The bottom half of the space between the spheres is filled with oil of dielectric constant $\kappa = 3$, creating two capacitors connected to each other. Which of the following is true of the two capacitors?



   (A) They are connected in series.
   (B) They are connected in parallel.
   (C) The total capacitance has not changed.
   (D) The total capacitance of the spheres has decreased.
   (E) The total capacitance is now zero.

8. With the bottom half of the space between the spheres having been filled with oil of dielectric constant $\kappa = 3$, the new capacitance of the spheres is
   (A) zero
   (B) C_0
   (C) $2C_0$
   (D) $3C_0$
   (E) $4C_0$

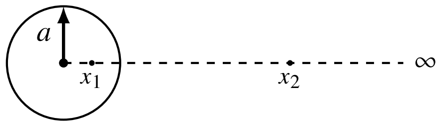
Question 9-10



9. A cube has sides of length a. The cube rests so that one side rests on the x-axis as shown. An electric field is established in the x-direction according to the function $E_x = bx^2$, where b is a positive constant. Which of the following statements is true?
   (A) There is a net charge inside the cube.
   (B) There is no net charge inside the cube.
   (C) The flux passing through the cube is negative.
   (D) The flux passing through the cube is zero.
   (E) The flux diminishes while passing through the cube.

10. The charge inside the cube can be expressed by the equation
   (A) $\epsilon_0 ba$
   (B) $\epsilon_0 ba^2$
   (C) $\epsilon_0 ba^3$
   (D) $\epsilon_0 ba^4$
   (E) $\epsilon_0 b^2 2a^2$

Questions 11–12: A nonconducting spherical charge distribution has a nonuniform positive charge density ρ. The center of the sphere is point O, the radius of the sphere is a. The sphere is centered on the x-axis. A point inside the sphere lies on the x-axis at a distance x_1 from the center of the sphere. Another point, x_2, is outside the sphere on the x-axis.

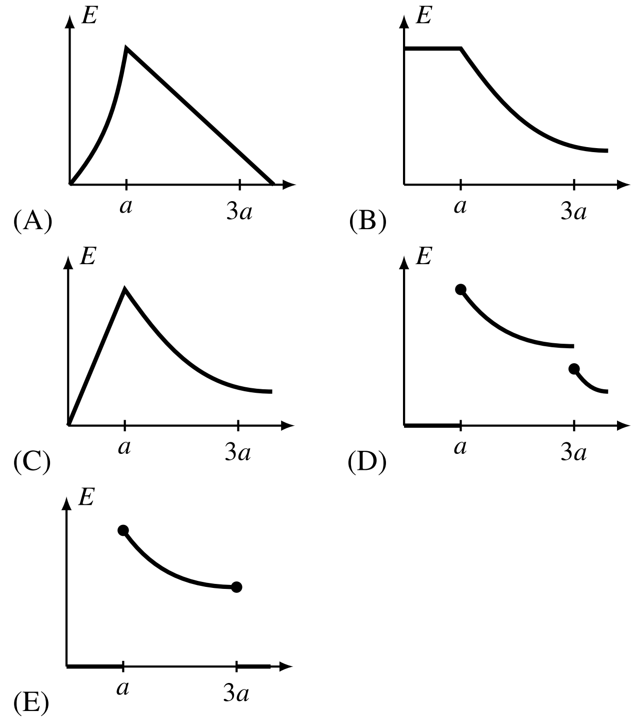


11. The electric field at point x_2 can be determined by
    (A) using Gauss's law to determine the electric field from O to a, then using Gauss's law to determine the electric field from a to x_2, then finding the difference between the two electric fields.
    (B) using Gauss's law to determine the electric field from O to a, then using Gauss's law to determine the electric field from a to x_2, then finding the sum of the two electric fields.
    (C) integrating the electric potential outside the sphere from infinity to a, then integrating the electric potential inside the sphere from a to x_1, then finding the difference between the two potential integrals.
    (D) integrating the electric potential outside the sphere from infinity to a, then integrating the electric potential inside the sphere from a to x_1, then finding the sum of the two potential integrals.
    (E) determining the derivative of the potential function inside and outside the sphere, then finding the difference between the two derivatives.

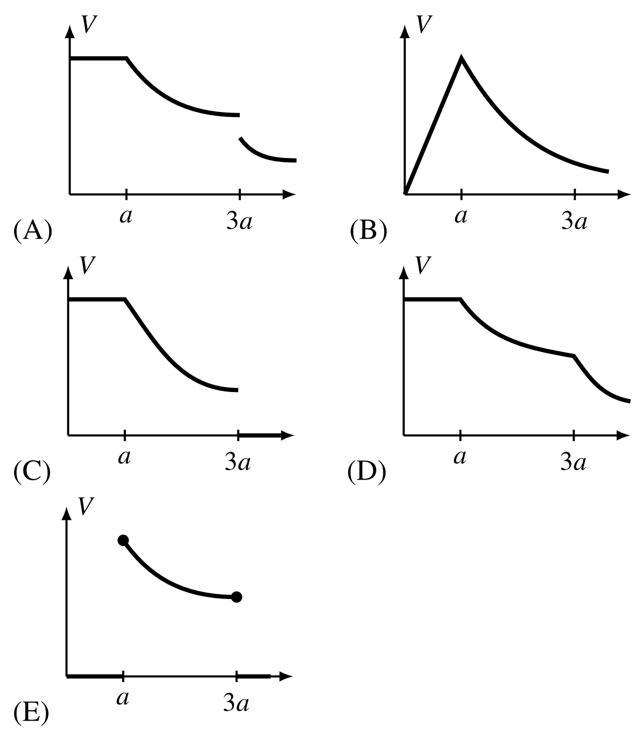
12. The electric potential at point x_1 can be determined by
    (A) determining the derivative of the electric field function inside and outside the sphere, then finding the difference between the two derivatives.
    (B) determining the derivative of the electric field function inside and outside the sphere, then finding the sum of the two derivatives.
    (C) integrating the derivative of the product of the electric field and potential functions, then finding their sum.
    (D) integrating the electric field outside the sphere from infinity to a, then integrating the electric field inside the sphere from a to x_1, then finding the sum of the two potentials.
    (E) integrating the electric field outside the sphere from infinity to a, then integrating the electric field inside the sphere from a to x_1, then finding the difference between the two potentials.

Questions 13–14: A solid conducting sphere of radius a is placed inside a conducting spherical shell of radius $3a$, as shown. A charge $+2Q$ is placed on the inner sphere, and a charge $-Q$ is placed on the outer sphere.

13. Which of the following graphs best represents the electric field $\vec{E}$ as a function of the distance r from the center of the spheres?

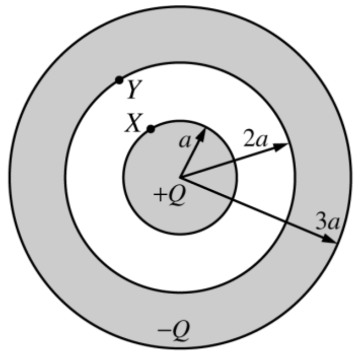


14. Which of the following graphs best represents the electric potential V as a function of the distance r from the center of the spheres?

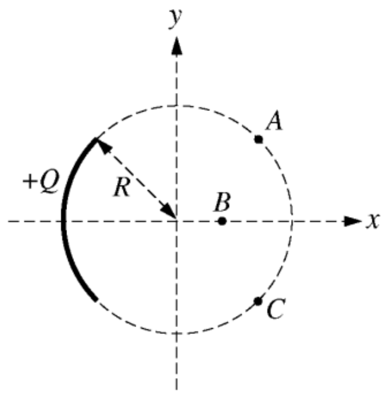


AP PHYSICS C CLASS 16: ELECTROSTATICS & CAPACITORS
SECTION II
4 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.



1. In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.
   (a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.
      i. Within the solid sphere ($r < a$)
      ii. Between the solid sphere and the spherical shell ($a < r < 2a$)
      iii. Within the spherical shell ($2a < r < 3a$)
      iv. Outside the spherical shell ($r > 3a$)
   (b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.
   (c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

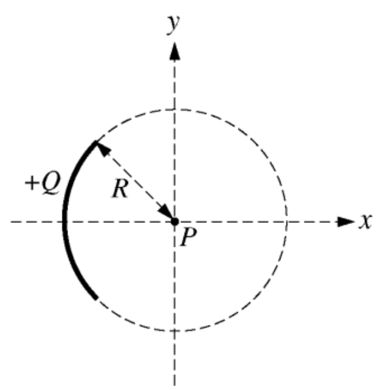


2. A charge $+Q$ is uniformly distributed over a quarter circle of radius R, as shown above. Points A, B, and C are located as shown, with A and C located symmetrically relative to the x-axis. Express all algebraic answers in terms of the given quantities and fundamental constants.

   (a) Rank the magnitude of the electric potential at points A, B, and C from greatest to least, with number 1 being greatest. If two points have the same potential, give them the same ranking. Justify your rankings.

   _____ V_A _____ V_B _____ V_C

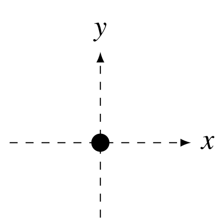
Point P is at the origin, as shown below, and is the center of curvature of the charge distribution.



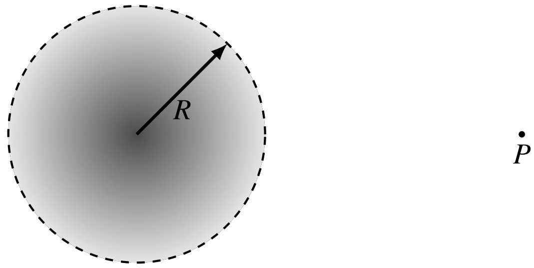
   (b) Determine an expression for the electric potential at point P due to the charge Q.

   (c) A positive point charge q with mass m is placed at point P and released from rest. Derive an expression for the speed of the point charge when it is very far from the origin.

   (d) On the dot representing point P below, indicate the direction of the electric field at point P due to the charge Q.



   (e) Derive an expression for the magnitude of the electric field at point P.



3. A spherical cloud of charge of radius R contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \text{ for } r \leq R \text{ and}$$
$$\rho(r) = 0 \text{ for } r > R$$

where r is the distance from the center of the cloud. Express all algebraic answers in terms of Q, R, and fundamental constants.

(a) Determine the following as a function of r for $r > R$.

i. The magnitude E of the electric field

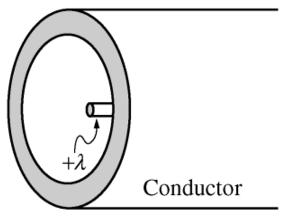
ii. The electric potential V

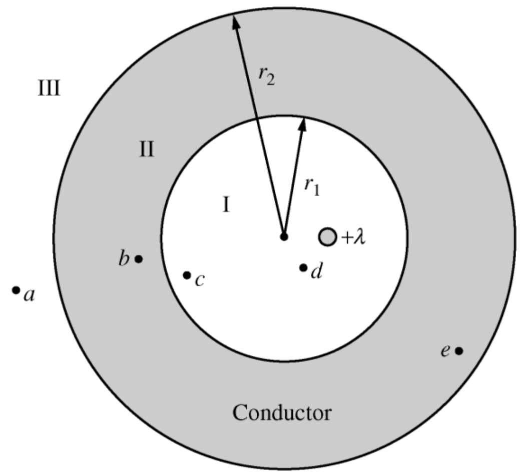
(b) A proton is placed at point P shown above and released. Describe its motion for a long time after its release.

(c) An electron of charge magnitude e is now placed at point P, which is a distance r from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of r as it strikes the cloud.

(d) Derive an expression for ρ_0.

(e) Determine the magnitude E of the electric field as a function of r for $r \leq R$.





Cross Section

4. The figure above left shows a hollow, infinite, cylindrical, uncharged conducting shell of inner radius r_1 and outer radius r_2. An infinite line charge of linear charge density $+\lambda$ is parallel to its axis but off center. An enlarged cross section of the cylindrical shell is shown above right.

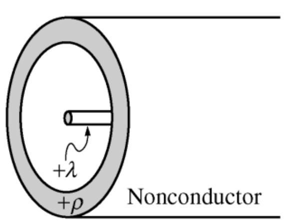
   (a) On the cross section above right,

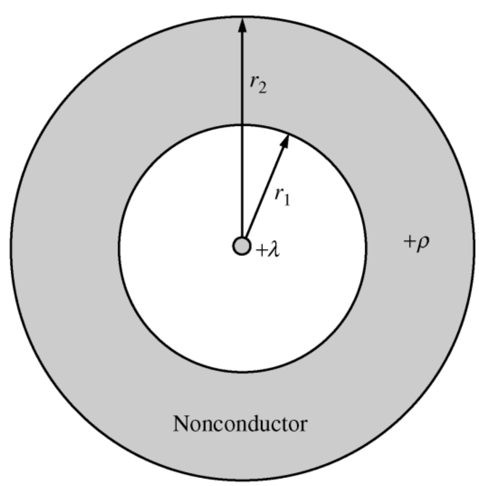
      i. sketch the electric field lines, if any, in each of regions I, II, and III and

      ii. use + and − signs to indicate any charge induced on the conductor.

   (b) In the spaces below, rank the electric potentials at points a, b, c, d, and e from highest to lowest (1 = highest potential). If two points are at the same potential, give them the same number.

   ___ V_a ___ V_b ___ V_c ___ V_d ___ V_e





Cross Section

   (c) The shell is replaced by another cylindrical shell that has the same dimensions but is nonconducting and carries a uniform volume charge density $+\rho$. The infinite line charge, still of charge density $+\lambda$, is located at the center of the shell as shown above. Using Gauss's law, calculate the magnitude of the electric field as a function of the distance r from the center of the shell for each of the following regions. Express your answers in terms of the given quantities and fundamental constants.

      i. $r < r_1$

      ii. $r_1 \leq r \leq r_2$

      iii. $r > r_2$