Student #: ___________ Student Name: ______________________

# AP Physics Class 10 & 11: Electrostatics & Gauss's Law

A 1. Two electric objects experience a repulsive force. What happens to that force if the distance between the objects is doubled?

(A) It decreases to one-fourth its value.
(B) It decreases to one-half its value.
(C) It stays the same.
(D) It doubles.
(E) It quadruples.

B 2. A pith ball is a tiny piece of Styrofoam that is covered with a conductive paint. One pith ball initially has a charge of $6.4 \times 10^{-8}$ C, and it touches an identical, neutral pith ball. After the pith balls are separated, what is the charge on the pith ball that had the initial charge?

(A) $6.4 \times 10^{-8}$ C
(B) $3.2 \times 10^{-8}$ C
(C) 0 C
(D) $-3.2 \times 10^{-8}$ C
(E) $-6.4 \times 10^{-8}$ C

Charge is shared equally between the two balls.

A 3. Glass becomes positively charged when it is rubbed with silk. Which of the following is the best description of whats happening?

(A) Electrons are rubbed off the glass onto the silk.
(B) Electrons are rubbed off the silk onto the glass.
(C) Protons are rubbed off the glass onto the silk.
(D) Protons are rubbed off the silk onto the glass.
(E) Neutrons in the glass have an affinity for positive charge.

B 4. Consider an isolated, neutral system consisting of wool fabric and a rubber rod. If the rubber rod is rubbed with wool to become negatively charged, what can be said about the wool fabric?

(A) It becomes equally negatively charged.
(B) It becomes equally positively charged.
(C) It becomes negatively charged but not equally.
(D) It becomes positively charged but not equally.
(E) In a neutral system, neither object can become charged.

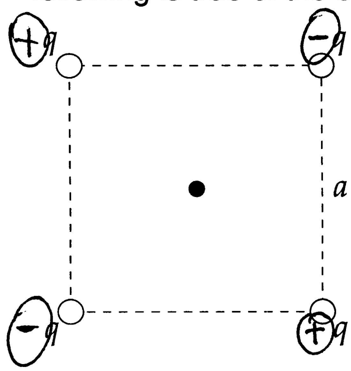
A 5. An electron and a proton are separated by $1.50 \times 10^{-10}$ m. If they are released, which one will accelerate at a greater rate, and what is the magnitude of that acceleration?

(A) The electron; $1.12 \times 10^{22}$ m/s$^2$
(B) The proton; $1.12 \times 10^{22}$ m/s$^2$
(C) The electron; $6.13 \times 10^{18}$ m/s$^2$
(D) The proton; $6.13 \times 10^{18}$ m/s$^2$
(E) They both accelerate at the same rate; $1.02 \times 10^{-8}$ m/s$^2$

$F = ma = \frac{kq_1q_2}{r^2} \rightarrow a = \frac{k}{m}\frac{q^2}{r^2} = \frac{(9.0\times10^9)(1.6\times10^{-19})^2}{(9.1\times10^{-31})(1.5\times10^{-10})^2}$

$\approx 1.12\times10^{22}$ m/s$^2$

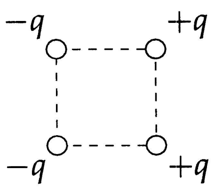
A 6. Four charges are arranged at the corners of a square of side a as shown. Which of the following is true of the electric field and the electric potential at the center of the square?



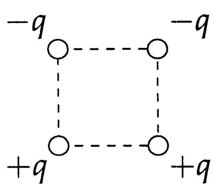

| | Electric Field | Electric Potential |
|---|---|---|
| (a) | zero | zero |
| (b) | $\frac{kQ}{a\sqrt{2}}$ | zero |
| (c) | $\frac{kQ^2}{2a^2}$ | $\frac{kQ}{2a}$ |
| (d) | zero | $\frac{kQ}{\sqrt{2a}}$ |
| (e) | $\frac{kQ^2}{2a}$ | $\frac{kQ}{a\sqrt{2}}$ |

by symmetry.

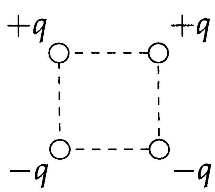
B 7. Which of the following diagrams best represents how you might rearrange the charges so that the electric field at the center would point directly toward the top of the page?



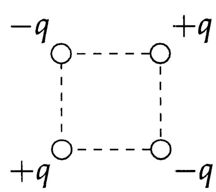
(A)



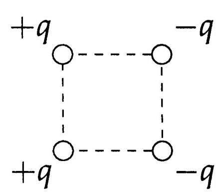
(B)



(C)

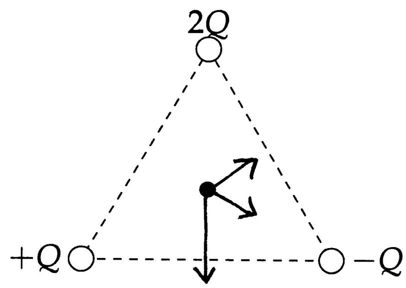


(D)



(E)

C 8. Three charges, $+Q$, $Q$, and $+2Q$, are arranged in an equilateral triangle as shown. Which of the arrows below best represents the direction of the electric field at the center of the triangle?




(A) ↓
(B) ↑
(C) ↘
(D) ↙
(E) ↗

E 9. A nonconducting sphere does not have a uniform charge density, but the density $\rho$ varies with the distance $r$ from the center of the sphere according to the equation $\rho = \beta r$ where $\beta$ is a positive constant. The electric field inside the sphere ($r < R$) at a distance $r$ from the center of the sphere is

(A) $\frac{\beta r^2}{12\epsilon_0}$

(B) $\frac{\beta r^3}{3\epsilon_0}$

(C) $\frac{\beta r}{2\epsilon_0}$

(D) $\frac{\beta r^2}{2\epsilon_0}$

(E) $\frac{\beta r^2}{4\epsilon_0}$

$$Q = \int \rho\, dV = \int \beta r(4\pi r^2)dr = 4\pi\beta \int r^3 dr = \pi\beta r^4$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{\pi\beta r^4}{4\pi r^2 \varepsilon_0} = \frac{\beta r^2}{4\varepsilon_0}$$

A 10. The electric potential at the surface of the sphere from the last question is

(A) $\frac{\beta R^3}{4\epsilon_0}$

(B) $\frac{\beta R}{2\epsilon_0}$

(C) $\frac{\beta R^3}{3\epsilon_0}$

(D) $\frac{\beta R^2}{2\epsilon_0}$

(E) $\frac{\beta R^2}{4\epsilon_0}$

$$V = \int_0^R E\,dr = \int \frac{\beta r^2}{4\varepsilon_0} dr = \frac{\beta R^3}{4\varepsilon_0}$$

E 11. According to Gauss's law, the net electric flux passing through a closed surface is

(A) positive if the flux is entering the surface
(B) negative if the flux is exiting the surface
(C) positive if the net charge inside the surface is zero
(D) negative if the net charge inside the surface is zero
(E) zero if the net charge inside the surface is zero

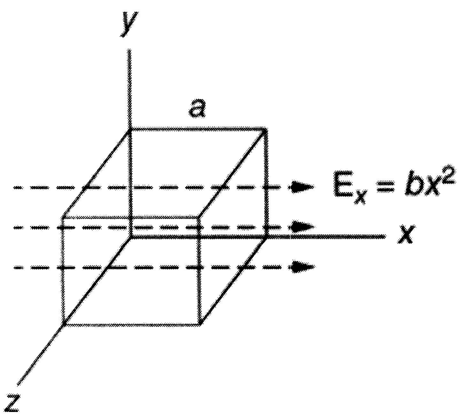
A 12. According to Gauss's law, which of the following statements is true?

(A) It is possible to have a nonzero electric field, but zero electric flux.
(B) It is possible to have a nonzero electric flux, but zero electric field.
(C) It is possible to have a nonzero electric flux through a closed surface even if the enclosed charge in a surface is zero.
(D) If a surface is not closed (such as a sheet of paper), the flux through it must be zero.
(E) It is possible for charges located outside a closed surface to produce a net positive flux through the surface.

B 13. Electric potential

(A) is a vector quantity that depends on the direction of the electric field
(B) is a scalar quantity that depends on the magnitude and sign of charges in the vicinity
(C) is a scalar quantity that depends on the square of the distance from the charges in the vicinity
(D) is a vector quantity that depends on the sign of the charges in the vicinity
(E) is a vector quantity that must point from high to low potential

A 14. A cube has sides of length $a$. The cube rests so that one side rests on the $x$-axis as shown. An electric field is established in the $x$-direction according to the function $E_x = bx^2$, where $b$ is a positive constant. Which of the following statements is true?




(A) There is a net charge inside the cube.
(B) There is no net charge inside the cube.
(C) The flux passing through the cube is negative.
(D) The flux passing through the cube is zero.
(E) The flux diminishes while passing through the cube.

D 15. The charge inside the cube from the previous question can be expressed by the equation

(A) $\epsilon_0 ba$
(B) $\epsilon_0 ba^2$
(C) $\epsilon_0 ba^3$
(D) $\epsilon_0 ba^4$
(E) $\epsilon_0 b^2 2a^2$

$EA = \frac{q}{\varepsilon_0}$ $\quad q = \varepsilon E A = \varepsilon_0 (bx^2)(a^2) = \varepsilon_0 ba^2$

B 16. Gauss's law is most convenient to use when calculating an electric field due to

(A) charges outside a closed surface
(B) charges inside a closed surface that has high symmetry
(C) charges inside a closed surface that has ~~low~~ symmetry
(D) a potential difference that is negative
(E) a potential difference that is positive

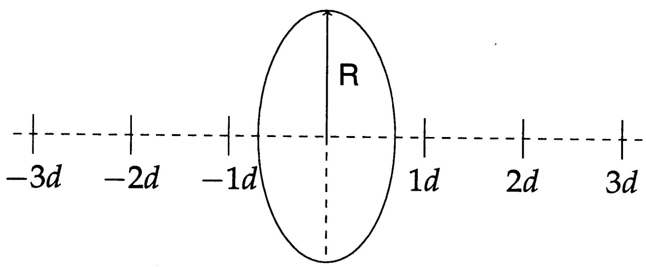
C 17. Which of the following statements is true of electric field and equipotential lines?

(A) The electric field vector always points in the same direction as the equipotential lines.
(B) The electric field always points in the opposite direction of the equipotential lines.
(C) The electric field always points perpendicular to the equipotential lines.
(D) The electric field is always equal to the equipotential lines. (333)
(E) Equipotential lines always form a circle around electric field lines.

E 18. The potential $V$ as a function of distance $r$ for a particular charge distribution is given by the equation $V = ar^{-1}$. The electric field as a function of distance $r$ from the charge distribution is (328)

(A) $1/3\, ar^{-3}$
(B) $2ar^{-1}$
(C) $ar^2$
(D) $a(\ln r)$
(E) $ar^{-2}$

$E = \frac{-\partial V}{\partial r}\ \frac{\partial a}{\partial r\, r} = \frac{a}{r^2} =$




E 19. A positively charged ring of radius R is made of conducting material and has a charge $Q$ distributed uniformly around it. The center of the ring is located at point 0 on the $x$-axis. The potential $V$ at a distance $3d$ from point 0 on the $x$-axis is (326)

(A) $V = \dfrac{kQ}{9d^2}$

(B) $V = \dfrac{kQ}{3d^2}$

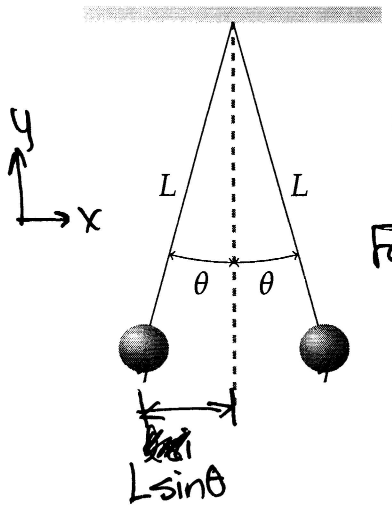
(C) $V = \dfrac{kQ}{R^2 + 9d^2}$

(D) $V = \sqrt{\dfrac{kQ}{R^2 + 9d^2}}$

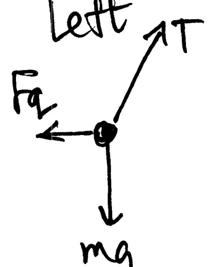
(E) $V = \dfrac{kQ}{\sqrt{R^2 + 9d^2}}$

**Free-Response Questions:**

1. Two identical small spheres of mass $m$ are suspended from a common point by threads of length L. When each sphere carries a charge $q$, each thread makes an angle $\theta$ with the vertical as shown in the figure below. (a) Express charge $q$ in terms of $\theta$, $m$, $L$ and any other relevant constants, and (b) Compute $q$ if $m = 10\,\mathrm{g}$, $L = 50\,\mathrm{cm}$ and $\theta = 10°$.




FBD on the charge on the Left




$\Sigma F_y = T\cos\theta - mg = 0 \rightarrow T = \frac{mg}{\cos\theta}$

$\Sigma F_x = T\sin\theta - \frac{kq^2}{(2L\sin\theta)^2} = 0$

$\frac{mg\sin\theta}{\cos\theta} = \frac{kq^2}{4L^2\sin^2\theta}$

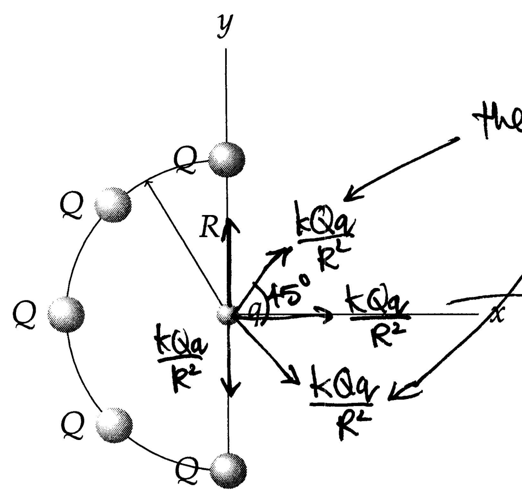
$kq^2 = 4L^2\sin^2\theta\, mg\tan\theta$

$\boxed{q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}}$

plug in values.

$\boxed{\text{(b) } 0.241\,\mu C}$

2. Five equal charges $Q$ are equally spaced on a semicircle or radius $R$ as shown in the figure below. Find the force on a charge $q$ located at the center of the semicircle.




the y-component cancel, x-component is

$\frac{2kQq}{R^2}\cos 45° = \frac{2}{\sqrt2}\frac{kQq}{R^2} = \sqrt2\frac{kQq}{R^2}$

total:

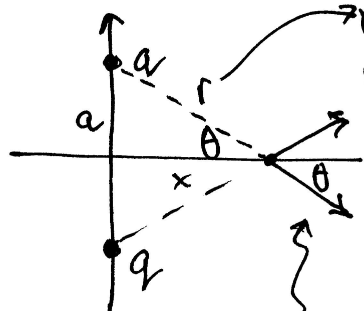
$\boxed{\vec F = (1+\sqrt2)\frac{kQq}{R^2}\hat{i}}$

3. Two positive charges $+q$ are on the $y$ axis at $y = +a$ and $y = -a$.

   (a) Show that the electric field on the $x$ axis is along the $x$ axis with $E_x = 2kqx(x^2 + a^2)^{-3/2}$.
   (b) Show that near the origin, when $x \ll a$, $E_x \approx 2kqx/a^3$.
   (c) Show that for $x \gg a$, $E_x \approx 2kq/x^2$.
   (d) Explain why you should expect the result in (c) even before calculating it.

   A bead of mass $m$ with a negative charge $-q$ slides along a thread that runs along the $x$ axis.

   e ~~(a)~~ Show that for small displacements $x \ll a$, the bead experiences a restoring force that is proportional to $x$ and therefore undergoes simple harmonic motion.
   f ~~(b)~~ Find the period of the motion.




$r^2 = a^2 + x^2$

$r = \sqrt{a^2 + x^2}$

the y-components cancel, leaving only x-component

$E_x = 2\frac{kq}{r^2}\cos\theta$ — but $\cos\theta = \frac{x}{r}$ !

two charges

$= 2\frac{kqx}{r^3}$

$= 2kqxr^{-3}$

(a) $\boxed{E_x = 2kqx\left(\sqrt{x^2+a^2}\right)^{-3}}$

b) near origin $x \ll a \rightarrow r \approx a$

$E_x = 2\frac{kqx}{r^3} \approx \frac{2kqx}{a^3}$

c) for $x \gg a$ (far away) $r \approx x$

$E_x = \frac{2kqx}{r^3} \approx \frac{2kqx}{x^3} \approx \frac{2kq}{x^2}$

(d) we should expect this because far from the charges, it looks like a single $2q$ charge.

(e) for $x \ll a$ the force points toward origin, magnitude

$F_q = -qE_x$ ← use result from (b)

$= -\underbrace{\frac{2kq^2}{a^3}}_{k \text{ constant}} x$

$-k\,x$

(f) period

$T = 2\pi\sqrt{\frac{m}{k}} = \boxed{2\pi\sqrt{\frac{ma^3}{2kq^2}}}$

4. Using Gauss's law, find

   (a) the electric field strength inside and outside of a uniformly charged hollow sphere of radius $R$ and surface charge density $\sigma$ (charge per unit area).

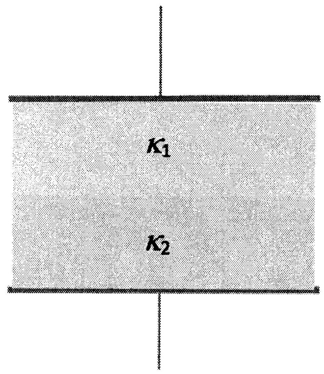
   (b) the electric field inside and outside an infinitely long cyclindrical shell of charge of radius $R$ with charge discibution $\sigma$ (charge per unit area).

   (c) the electric field strength inside and outside a infinitely long solid cylinder of radius $R$ carrying a linear uniform charge density $\rho$ (charge per unit volume).

   Hint: In all cases, think about where to put the Gaussian surface. Take advantage of symmetry.

5. A parallel-plate capacitor has a capacitance $C_0$ and plate separation of $d$. To dielectric slabs of constants $\kappa_1$ and $\kappa_2$, each of thickness $d/2$ and having the same area as the plates, are inserted between the plates as shown in the figure below. When the free charge on the plates are $Q$,

   (a) find the electric field in each of the dielectric
   (b) find the potential difference between the plates
   (c) show that the new capacitance is given by: $C = \dfrac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}C_0$




6. Several point charges produce the equipotential lines shown.

   (a) At which point on the diagram is the magnitude of the electric field greatest? Explain.
   (b) Points C and D are approximately $0.02\,\mathrm{m}$ apart. Point F is halfway between points C and D. What is the electric field at point F?
   (c) A $5.0\,\mu\mathrm{C}$ point charge is moved from point C to point E, then to point D by an external force. Determine the work done by the external force.