Topic 11: Electrostatics

Advanced Placement Physics 1 & 2

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Electrostatic Force

The Charges Are

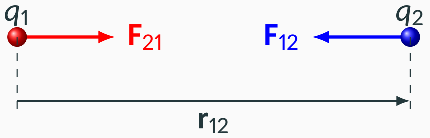
We should already know a bit about charge particles:

- A **proton** carries a **positive** charge
- An **electron** carries a **negative** charge
- A *net charge* of an object means an excess of protons or electrons
- Similar charges are repel; opposite charges attract

We start with electrostatics:

- Charges that are not moving relative to one another

Coulomb’s Law for Electrostatic Force



The **electrostatic force** (or **coulomb force**) is a mutually repulsive/attractive force between all charged objects. The force that charge q_1 exerts on q_2 is given by **Coulomb’s law**:

$$\boxed{\mathbf{F}_{12} = \frac{kq_1q_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12}}$$

Coulomb's Law for Electrostatic Force

$$\boxed{\mathbf{F}_{12} = \frac{kq_1q_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12}}$$

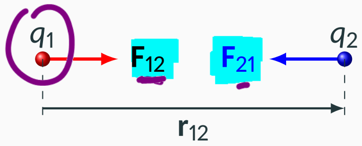
Quantity	Symbol	SI Unit
Electrostatic force	$\mathbf{F}_{12}$	N
Coulomb's constant (electrostatic constant)	k	$\mathrm{N\,m^2/C^2}$
Point charges 1 and 2	q_1, q_2	C
Distance between point charges	$\lvert\mathbf{r}_{12}\rvert$	m
Unit vector of direction between point charges	$\hat{\mathbf{r}}_{12}$	

Coulomb's constant $k = \dfrac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\,\mathrm{N\,m^2/C^2}$ where

$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N\,m^2}$ is called the "permittivity of free space"

Coulomb's Law for Electrostatic Force

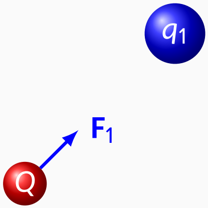
$a = \frac{F}{m}$



- If q_1 exerts an electrostatic force $\mathbf{F}_{12}$ on q_2, then q_2 likewise exerts a force of $\mathbf{F}_{21} = -\mathbf{F}_{12}$ on q_1. The two forces are equal in magnitude and opposite in direction (3rd law of motion).
- q_1 and q_2 are assumed to be *point charges* that do not occupy any space
- The (more familiar) scalar form is often used as well:

$$\boxed{F_q = \frac{k|q_1 q_2|}{r^2}}$$

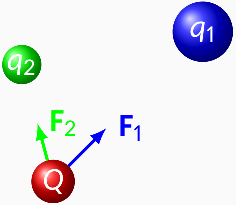
More Than One Charge



For a charge Q that is subjected to the influence of multiple discrete point charges q_i, the total electrostatic force that Q experiences is the vector sum of all the forces $\mathbf{F}_i$:

$$\boxed{\mathbf{F} = \sum_i \mathbf{F}_i = kQ\left(\sum_{i=1}^{N} \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i\right)}$$

More Than One Charge



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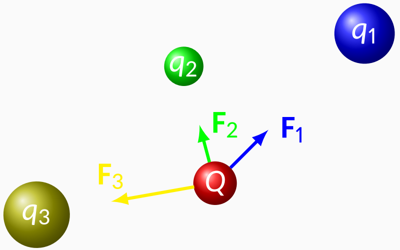

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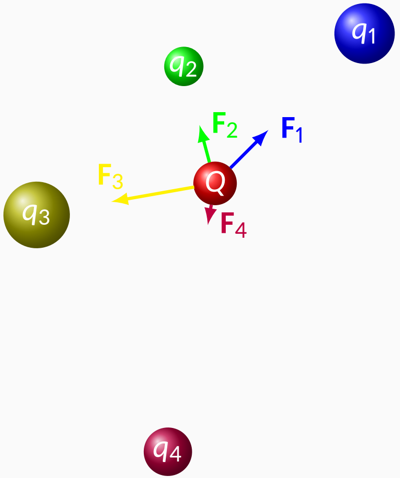

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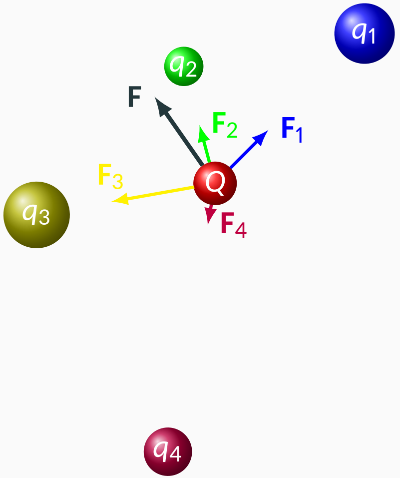

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Electric Field

Electric Field

The expression for **electric field** is obtained by repeating the same procedure as with gravitational field, by groupping the variables in Coulomb's law:

$$F_q = \underbrace{\left[\frac{kq_1}{|\mathbf{r}_{12}|^2} \hat{\mathbf{r}} \right]}_{\mathbf{E}} q_2$$

The electric field **E** created by q_1 is a vector function (called a **vector field**) that shows how it influences other charged particles around it.

Electric Field Near a Point Charge

The electric field a distance r away from a point charge q is given by:

$$\boxed{\mathbf{E}(q, \mathbf{r}) = \frac{kq}{|\mathbf{r}|^2}\hat{\mathbf{r}}}$$

+/-

$$\boxed{E(q,r) = \frac{kq}{r^2}}$$

Quantity	Symbol	SI Unit
Electric field	$\mathbf{E}$	N / C
Coulomb's constant	k	$\mathrm{N\,m^2 / C^2}$
Source charge	q	C
Distance from source charge	$\lvert\mathbf{r}\rvert$	m
Outward unit vector from point source	$\hat{\mathbf{r}}$	

The direction of **E** is radially outward from a positive point charge and radially inward toward a negative charge.

More Than One Charge

When multiple point charges are present, the total electric field at any position **r** is the vector sum of all the fields $\mathbf{E}_i$:

$$\boxed{\mathbf{E} = \sum_i \mathbf{E}_i = k\left(\sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i\right)}$$

Think Electric Field

i.e. putting a charged particle in the electric field.

E iself *doesn't do anything* until another charge interacts with it. And when there is a charge q, the electrostatic force $\mathbf{F}_q$ that the charge experiences is proportional to q and **E**, regardless of how the electric field is generated:

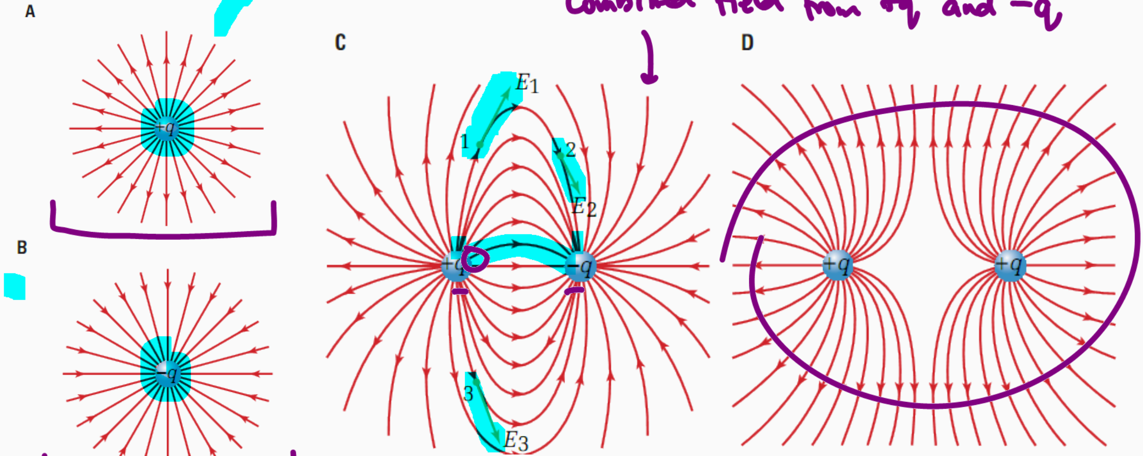
$$\boxed{\mathbf{F}_q = q\mathbf{E}}$$

A positive charge in the electric field experiences a electrostatic force **F** in the same direction as **E**.

Electric Field Lines

4. Fields always start and/or end at a charge
5. Field lines do not touch, and they do not cross

Combined field from +q and -q



1. Field lines tell you the direction of the electric field
2. Field lines do not tell you the magnitude
3. Electric field is tangent to the field lines

$W_q = \Delta K$

Work done by electrostatic force

change in kinetic energy

Electric Potential & Potential Energy

Where did that energy come from?

Electrical Potential Energy

If you know calculus, you can easily find that the work done *by* the electrostatic force is given by this integral:

$$W = \int \mathbf{F}_q \cdot dr = kq_1q_2 \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{kq_1q_2}{r}\Bigg|_{r_1}^{r_2} = -\Delta U_q$$

And if you don't know calculus, all you need to know is the result: Q_q is defined as the **electric potential energy**:

$$\boxed{U_g = -G\frac{m_1m_2}{r}} \qquad \boxed{U_q = \frac{kq_1q_2}{r}}$$

$U_q = 0$ at $r = \infty$

U_q can be $(+)$ or $(-)$, because charges can be either $(+)$ or $(-)$.

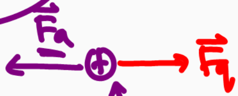
How it Differs from Gravitational Potential Energy

Two positive charges:

$U_q > 0$

Two negative charges:

$U_q > 0$



One positive and one negative charge:

$U_q < 0$

- $U_q > 0$: positive work is done to bring two charges together from $r = \infty$ to r (both charges of the same sign)
- $U_q < 0$: work is done to pull the objects from r to $r = \infty$
- In comparison, gravitational potential U_g is always < 0

When I bring the charges together, Fa does external non-conservative positive work, while Fq does conservative negative work, therefore there is a gain in electric potential energy

Electric Potential

Using gravity as an example: An object at a specific location inside a gravitational field has a gravitational potential energy that is proportional to its mass, i.e.

$$U_g = V_g m$$

$U_g = mgh$

This "constant" V_g is called the **gravitational potential**, which is the *gravitational potential energy per unit mass*. In the trivial case with a uniform **g**:

$$V_g = \frac{U_g}{m} = gh$$

depends on h.

This also applies to the general case of the gravitational potential energy:

$U_g = \frac{G m_1 m}{r}$

$$V_g = \frac{U_g}{m} = -\frac{GM}{r}$$

Electric Potential

This is also true for charged particle q in an electric electric field created by q_s, and the “constant” is called the **electric potential**. For a point charge, it is defined as:

$$\boxed{V = \frac{U_q}{q} = \frac{kq_s}{r}}$$

The unit for electric potential is a *volt*[1] which is *one joule per coulomb*:

$$1\,\mathrm{V} = 1\,\mathrm{J/C}$$

The relationship between V and $\mathbf{E}$ is given by:

$$\boxed{|\mathbf{E}| \approx \frac{\Delta V}{\Delta r}}$$

[1] Named after Italian physicist Alessandro Volta

Potential Difference (Voltage)

The change in electric potential is called the **electric potential difference** or **voltage**:

$$\boxed{\Delta V = \frac{\Delta U_q}{q}}$$

Here, we can relate ΔV to an equation that we knew from Grade 11 Physics, which related to the energy dissipated in a resistor in a circuit ΔU to the voltage drop ΔV:

$$\boxed{\Delta U_q = q\Delta V}$$

Electric potential difference also has the unit *volts* (V)

Getting Those Names Right

Remember that these three scalar quantities, as opposed to electrostatic force $\mathbf{F}_q$ and electric field $\mathbf{E}$ which are vectors

- Electric potential energy:

$$U_q = \frac{kq_1q_2}{r}$$

- Electric potential:

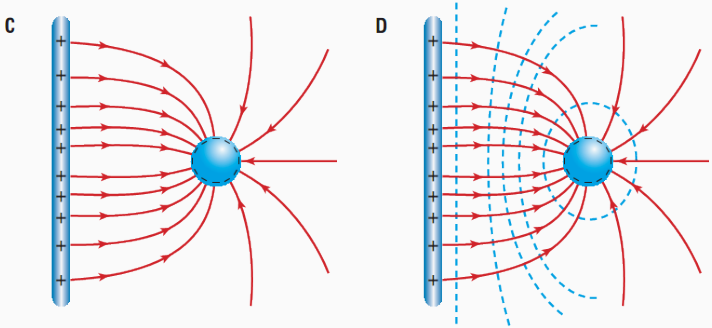
$$V = \frac{kq}{r} = \frac{U_q}{q}$$

point charge

- Electric potential difference (voltage):

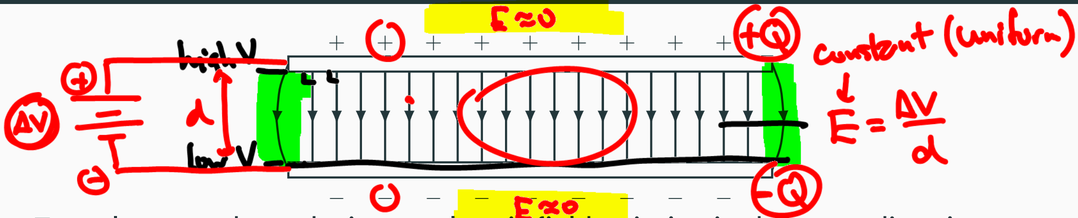
$$\Delta V = \frac{\Delta U_q}{q}$$

Equipotential Lines



The dotted blue lines are called **equipotential lines**. They are always *perpendicular* to the electric field lines. Charges moving in the direction of the equipotential lines have constant electric potential

Electric Field Between Parallel Charged Plates



- Two plates, each producing an electric field pointing in the same direction
- The total electric field is twice the value of *one* infinite plane, pointing from the positively charged plate toward the negatively charged plate

$$E = \frac{\Delta V}{d} = \frac{\sigma}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

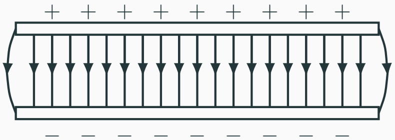
$Q = \sigma A$ ← area of the plate

total charge

surface charge density

- **E outside the plates is very low (close to zero)**, except for fringe effects at the edges of the plates

Electric Field and Electric Potential Difference



In the case of two parallel plates, the electric field is uniform, and the relationship simplifies to:

$$\boxed{E = \frac{\Delta V}{d}}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C
Electric potential difference between plates	ΔV	V
Distance between plates	d	m