Topic 14: Magnetism

AP Physics 2

Dr. Timothy Leung

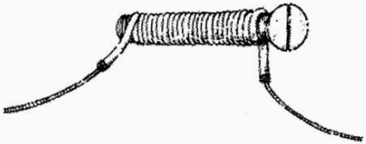
April 15, 2021

Olympiads School

An Electromagnet

A popular experiment in magnetism in elementary schools:

- Wrap a copper wire around an iron nail/screw
- Connect the ends of the wire to a battery
- When the circuit is closed, the nail becomes magnetized, picks up small paper clips
- As soon as the current stops, the nail stops being magnetic

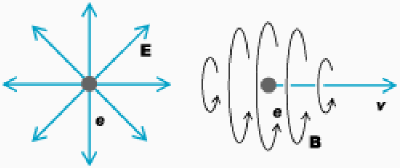


Magnetism

Review of Magnetic Field

- Magnetism is generated by moving charged particles, e.g. a single charge carrier, or an electric current
- It can also be generated by permanent magnets, or Earth
- Magnetism affects other *moving* charged particles
- The vector field is called the **magnetic field**
- Magnetic field has unit **tesla**
- Magnetic field lines have no ends—they always run in a loop

Magnetic Field Generated by a Moving Point Charge



A charged object generates an electric field $\boldsymbol{E}$. When it's moving, it also generates a magnetic field $\boldsymbol{B}$, given by:

$$\boxed{\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}}$$

(The direction of $\boldsymbol{B}$ can be obtained by applying the *right hand rule* if you are not confident with cross products.)

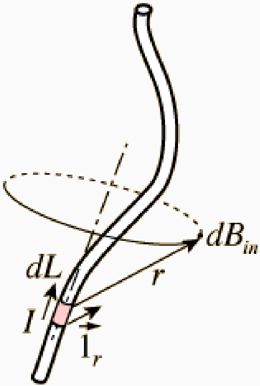
Magnetic Field Generated by a Moving Point Charge

$$\boxed{\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}}$$

Quantity	Symbol	SI Unit
Magnetic field	$\mathbf{B}$	T
Charge	q	C
Velocity of the charge	$\mathbf{v}$	m/s
Distance from the moving charge	r	m
Radial outward unit vector from the charge	$\hat{\mathbf{r}}$	no units
Permeability of free space	μ_0	T · m/A

$\mu_0 = 4\pi \times 10^{-7}$ T · m/A is a universal constant called the **permeability of free space** (or **vacuum permeability**). It measures how easily a space can become magnetized.

Magnetic Generated By a Current

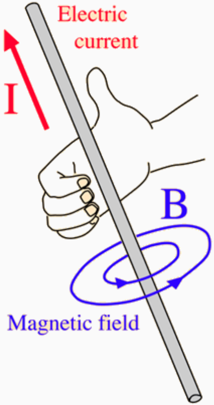


An electric current is really many charges particles moving along a wire; each charge creating its own magnetic field. The total magnetic field in the wire is the integral of the contribution ($d\boldsymbol{B}$) of the current (I) from each infinitesimal sections ($d\boldsymbol{L}$) of the wire, given by the **Biot-Savart law**:

$$\boxed{\mathrm{d}\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\boldsymbol{L} \times \hat{\boldsymbol{r}}}{r^2}}$$

The magnetic field in the diagram goes *into* the page

Magnetic Field Generated By an Infinitely Long Wire



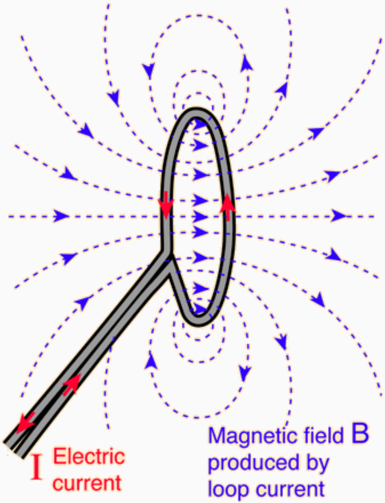
For an *infinitely-long straight* wire, the Biot-Savart law simplifies to:

$$\boxed{\boldsymbol{B} = \frac{\mu_0 (\boldsymbol{I} \times \hat{\boldsymbol{r}})}{2\pi r}} \quad \text{or} \quad \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

The magnitude and direction current "vector" $\boldsymbol{I}$ is straightforward

Quantity	Symbol	SI Unit
Magnetic field	$\boldsymbol{B}$	T
Current	$\boldsymbol{I}$	A
Radial direction from the wire	$\hat{\boldsymbol{r}}$	(no units)
Radial distance from the wire	r	m

Current-Carrying Wire Loop

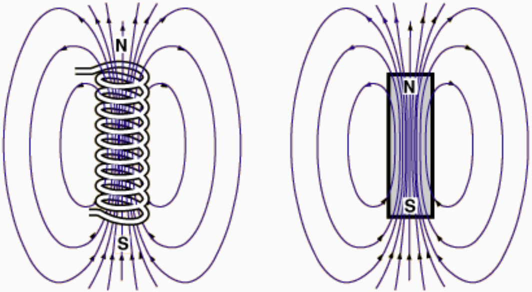


When we shape the current-carrying wire into a loop, we can (again) use the Biot-Savart law to find the magnetic field away from it.

One loop isn’t very interesting (except when you’re integrating Biot-Savart law) but what if we have many loops

Wounding Wires Into a Coil

- A **solenoid** is when you wound a wire into a coil
- You create a magnet very similar to a bar magnet, with an effective north pole and a south pole
- Magnetic field inside the solenoid is uniform
- Magnetic field strength can be increased by the addition of an iron core

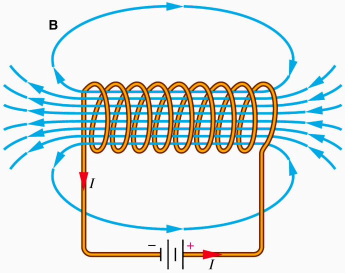


A Practical Solenoid

A practical solenoid usually has hundreds or thousands of turns:

This "air core" coil is used for high school and university experiments. It has approximately 600 turns of copper wire wound around a plastic core.

Magnetic Field Inside a Solenoid



The magnetic field **inside** a solenoid is uniform, with its strength given by:

$$\boxed{B = \frac{\mu N I}{\ell}}$$

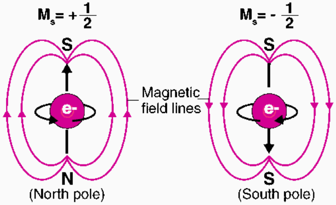
Direction of ***B*** determined by **right hand rule**

Quantity	Symbol	SI Unit
Magnetic field intensity	B	T
Number of coils	N	
Length of the solenoid	ℓ	m
Current	I	A
Effective permeability	μ	$\mathrm{T \cdot m / A}$

Permanent Magnets

Permanent magnets is also based on the motion of charges. This is the "non-technical" version...

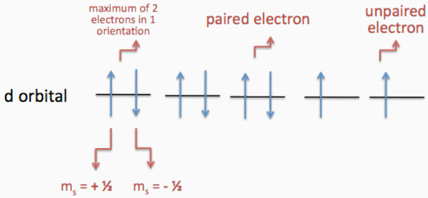
1. Electrons inside an atom *spin*. A spinning electron therefore has an angular momentum, and generates its own tiny magnetic field.



However, in most full "shells", the spin of these electrons are paired, so the magnetic fields cancel each other.

Permanent Magnets

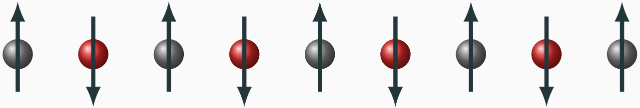
2. The orbits of electrons are not always filled, therefore some atoms do create some (very small) magnetic field.



The atoms that have unpaired electrons are called **paramagnetic** because they are attracted to magnets; atoms that have no unpaired electrons are called **diamagnetic**.

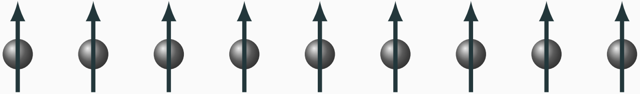
Permanent Magnets

3. While many atoms exhibit paramagnetism, they do not make good magnets, because the atoms are most often arranged in a way where the magnetic fields cancel. This is callled **ferrimagnetism**:

   Ferrimagnetic

   When they do not cancel, then they can become magnets. This is called **ferromagnetism**:

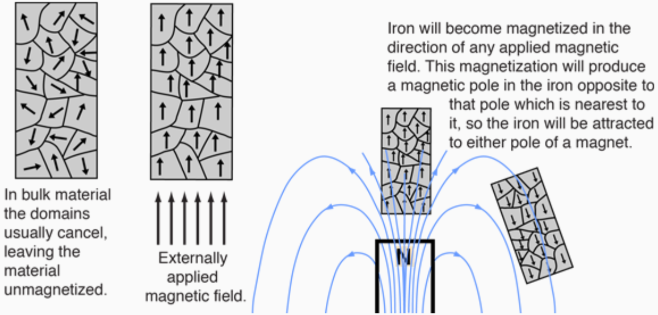
   

   Ferromagnetic

   Transitional elements such as iron, nickel and cobalt, and their alloys will exhibit ferromagnetism.

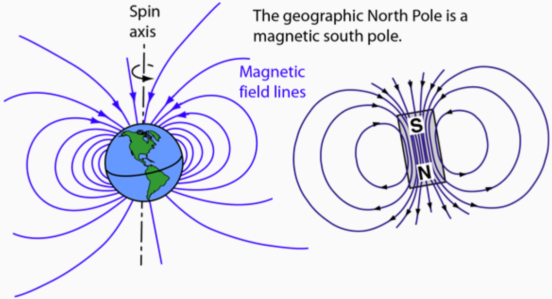
Permanent Magnets

4. The atoms in these ferromagnetic materials are arranged in “domains” wher their magnetic moment is aligned. In the presence of a strong external magnetic field, these domains will line up, creating a magnet.



Earth

Earth is also a “permanent” magnet, with the *magnetic south pole* located near the geographic north pole, and the *magnetic north pole* located near the geographic south pole. The poles are tilted by $\approx 11°$ from the spin axis.



The exact nature of Earth’s magnetic field is not known, although it may be related to “generator effect” from Earth’s rotation, circulating the outer-core fluid around.

Magnetic Force

So What Does the Magnetic Field Do?

Gravitational Field $\boldsymbol{g}$

- Generated by massive objects
- Affects massive objects

Electric Field $\boldsymbol{E}$

- Generated by charged particles
- Affects charged particles

Magnetic Field $\boldsymbol{B}$

- Generated by *moving* charged particles
- Affects moving charged particles

Lorentz Force Law

Since a moving charge or current create both electric and magnetic fields, another moving charge is therefore affected by both $\boldsymbol{E}$ and $\boldsymbol{B}$. The total effect is given by the **Lorentz force law**:

$$\boxed{\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})}$$

$\boldsymbol{F}_q = q\boldsymbol{E}$ is the electrostatic force, and $\boldsymbol{F}_M = q\boldsymbol{v} \times \boldsymbol{B}$ is the magnetic force.

Quantity	Symbol	SI Unit
Total force on the moving charge	$\boldsymbol{F}$	N
Charge	q	C
Velocity of the charge	$\boldsymbol{v}$	m/s
Magnetic field	$\boldsymbol{B}$	T
Electric field	$\boldsymbol{E}$	N/C

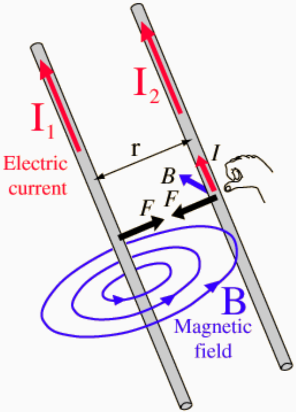
Force on a Current-Carrying Conductor in a Magnetic Field

Likewise, $\boldsymbol{B}$ exerts a force on another current-carrying conductor.

$$\boxed{\boldsymbol{F}_M = \boldsymbol{I}\ell \times \boldsymbol{B}}$$

Quantity	Symbol	SI Unit
Magnetic force on the conductor	$\boldsymbol{F}_M$	N
Electric current in the conductor	$\boldsymbol{I}$	A
Length of the conductor	ℓ	m
Magnetic field	$\boldsymbol{B}$	T

Magnetic Force on Two Current-Carrying Wires



Two parallel current-carrying wires of length L are at a distance r apart. Magnetic field at wire 2 from current I_1 has constant strength along the wire, given by:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

The force of B on I_2 is:

$$F = I_2 L B = \frac{\mu_0 I_1 I_2 L}{2\pi r} \rightarrow \boxed{\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

I_1 also exerts the same force on I_2, pulling the wires toward each other. (We should expect this because of the third law of motion.)

Circular Motion Caused by a Magnetic Field

When a charged particle enters a magnetic field at right angle...

- Magnetic force $\boldsymbol{F}_M$ perpendicular to both velocity $\boldsymbol{v}$ and magnetic field $\boldsymbol{B}$.
- Results in circular motion

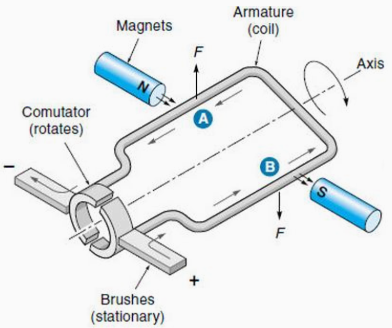
Centripetal force $\boldsymbol{F}_c$ is provided by the magnetic force $\boldsymbol{F}_M$. Equating the two expressions:

$$\frac{mv^2}{r} = qvB$$

We can solve for r get the radius for a charge with a known mass, or solve for mass m of a charged particle based on its radius:

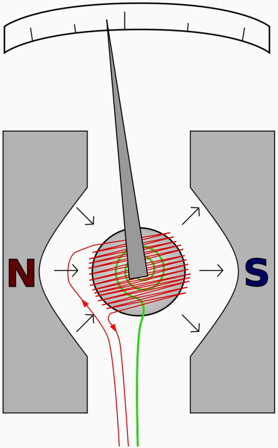
$$r = \frac{mv}{qB} \qquad m = \frac{qrB}{v}$$

DC Motor



- When a current runs through the magnetic field formed by the magnets (called **field magnets** or **stators**), a force is applied to the wire, causing the motor to turn
- The brushes are connected to a DC power source
- The commutators switch current direction every 180° to keep it turning
- The armature will have hundreds or thousands of coils

Galvanometer



Analog **voltmeters** and **ammeters** are based on a **galvanometer** which uses the magnetic force acting on a current.

- Current flows through the coil (red)
- External magnetic field applies a force on the wire
- The coil rotates
- The torque exerted on the coil is balanced by the restoring spring (green)

Faraday’s Law

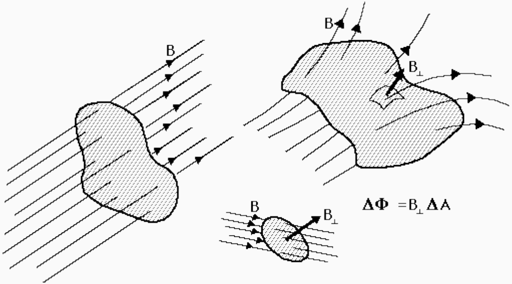
Magnetic Flux

Question: If a current-carrying wire can generate a magnetic field, can a magnetic field affect the current in a wire?

Answer: Yes, sort of...

To understand how to *induce* a current by a magnetic field, we need to look at "fluxes".

Magnetic Flux



The strict mathematical definition (i.e. using calculus) of **magnetic flux** is defined as:

$$\boxed{\Phi_M = \int \boldsymbol{B} \cdot d\boldsymbol{A}}$$

where $\boldsymbol{B}$ is the magnetic field, and $d\boldsymbol{A}$ is the infinitesimal area pointing *outward*. For a uniform magnetic field, this is simplied to (no calculus!):

$$\Phi_M = \boldsymbol{B} \cdot \boldsymbol{A}$$

Magnetic Flux Over a Closed Surface

The unit for magnetic flux is a “weber” (Wb), in honor of German physicist Wilhelm Weber, who invented the electromagnetic telegraph with Carl Gauss:

$$1\,\mathrm{Wb} = 1\,\mathrm{T} \cdot \mathrm{m}^2$$

The magnetic flux over a closed surface is always zero, indicating that magnetic field lines can neither have beginnings nor ends.

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

Changing Magnetic Flux

Changes to magnetic flux can be due to a number of reasons:

1. **Changing magnetic field**...if the magnetic field is created by a time-dependent source (e.g. alternating current)
2. **Changing orientation of magnetic field** because the surface area is moving relative to the magnetic field.
3. **Changing area** the surface area from which the flux is calculated is changing.

When Magnetic Flux is Changing

- When the magnetic flux Φ_M is changing, an electromotive force (*emf*, $\mathcal{E}$) is created in the wire.
- Unlike in a circuit, where the *emf* is concentrated at the terminals of the battery, the induced *emf* is spread across the entire wire.
- Since *emf* is work per unit charge, that means that there is an electric field inside the wire to move the charges.

Faraday’s Law

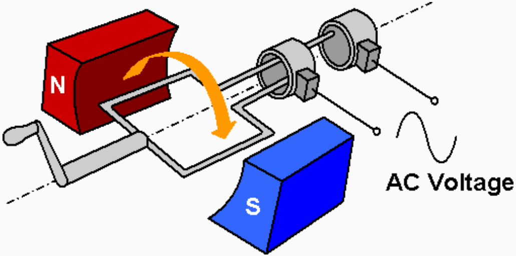
Faraday’s law states that the rate of change of magnetic flux produces an electromotive force:

$$\boxed{\overline{\mathcal{E}} = -\frac{\Delta\Phi_M}{\Delta t}}$$

The negative sign highlighted in red is the result of Lenz’s law, which is related to the conservation energy

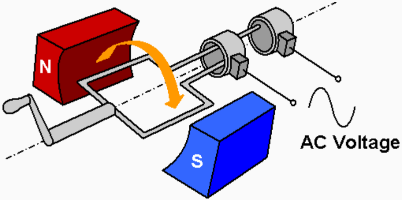
AC Generators

A simple AC (alternating current) generator makes use of the fact that a coil rotating against a fixed magnetic field has a changing flux.



Let's say the permanent magnets produce a uniform magnetic field B, and the coil between them has N turns, and an area A. Now let's say that the coil is rotating with an angular frequency ω.

AC Generators



When the coil is turning at a constant rate ω, the angle between the coil and the magnetic field is:

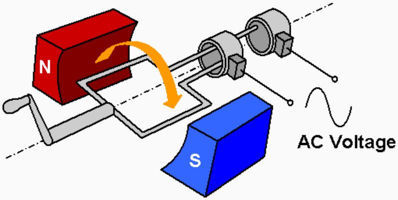
$$\theta = \omega t + \theta_0$$

where θ_0 is the initial angle (not very important coneptually). The magnetic flux through the coil is:

$$\begin{aligned} \Phi_M &= NBA\cos\theta \\ &= NBA\cos(\omega t + \theta_0) \end{aligned}$$

as the coil turns.

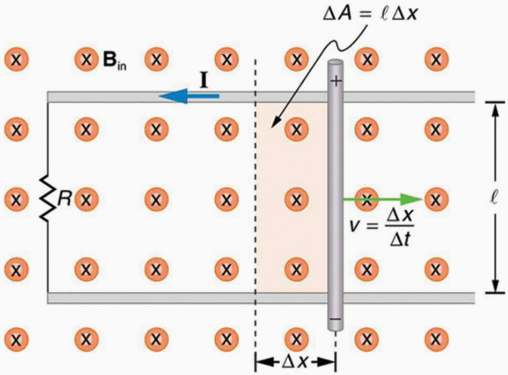
AC Generators



The electromotive force *emf* produced is therefore the rate of change of the magnetic flux:

$$\mathcal{E} = \underbrace{NBA\omega}_{\mathcal{E}_0} \sin(\omega t + \delta)$$

Motional EMF

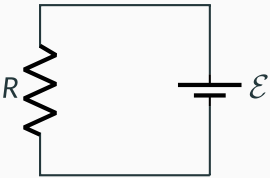


When sliding the rod to the right with speed v, the magnetic flux through the loop (and its rate of change) is:

$$\Phi_M = BA = B\ell x$$
$$\mathcal{E} = B\ell v$$

We can use the Lorentz force law on the charges on the rod to find the direction of the current I.

Motional EMF

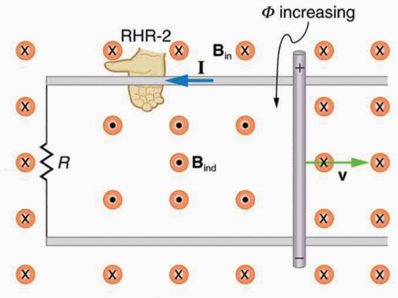


- An equivalent circuit is shown on the left
- The amount of current can be found using Ohm's law: $V = IR$
- Note that the "motional emf" produced is spread over the entire circuit
  - In constrast, in a voltaic cell (or battery), the *emf* is concentrated between the two terminals.

Lenz’s Law

Lenz's Law

Something very interesting happens when the current starts running on the wire.



It produces an "induced magnetic field" out of the page, in the opposite direction as the field that generated the current in the first place.

Lenz's Law

LENZ'S LAW
The induced *emf* and induced current are in such are direction as to oppose the change that produces them

So basically, the conservation of energy