Class 2: Dynamics

Advanced Placement Physics C

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Olympiads School

Dynamics

While **kinematics** describes the motion of any object mathematically, **dynamics** describes *what* causes motion motion to change

Laws of Motion

First Law of Motion

First Law: Every object will remain in its state of rest or uniform motion, until a net external force is applied to it.

- Uniform motion means constant velocity; an object "at rest" is also in uniform motion with $\vec{v} = \vec{0}$
- As long as an object moves in uniform motion, it must be that $\vec{F}_{\text{net}} = \vec{0}$
- Common examples:
    - A hockey puck sliding on very smooth ice has gravity and normal force, but the net force is zero
    - A car traveling on a highway at 100 km / h has many forces acting on it, but the net force is zero
- This is a special case that assumes a constant mass.

Translational Equilibrium

If the net force on an object is zero ($\Sigma\vec{F} = \vec{0}$) then the object is in a *state of translational equilibrium*

- Dynamic equilibrium: the object is moving relative to us
- Static equilibrium: the object is not moving relative to us

Second Law of Motion

Second Law: The acceleration of an object is proportional to, and along the direction of, the net external force.

- Like the first law, this is also a "special case" that assumes a constant mass.
- The first two laws of motion can be summarized in the equation:

$$\boxed{\vec{F}_{\text{net}} = \Sigma\vec{F} = m\vec{a}}$$

- For non-constant mass, net force is the rate of change of momentum $\vec{p}$:

$$\boxed{\vec{F}_{\text{net}} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}}$$

Mass

What is **mass** then? It is

- the property of an object that relates its acceleration to the force applied to it. Literally, it means:

$$m \equiv \frac{F_{\text{net}}}{a}$$

- Intrinsic to the object itself
- This is explicitly referred to as the object's **inertial mass**

Third Law of Motion

Third Law: For every action there is always and opposite and equal reaction; the mutual actions of two objects on each other are always equal, and directed to the other object.

For every action force on an object (B) due to another object (A), there is a reaction force which is equal in magnitude but opposite in direction, on object (A), due to object (B):

$$\boxed{\vec{F}_{\mathrm{AB}} = -\vec{F}_{\mathrm{BA}}}$$

- The action and reaction forces act on different objects!
- Third law is the natural consequence of the first and second law. Action/reaction forces are *internal* forces.

Forces

A **force** is the interaction between the objects.

- When there is interaction, then forces are created
- A “push” or a “pull”

There are two broad categories of forces:

- **Contact forces** act between two objects that are in contact with one another
- **Non-contact forces** act between two objects without them touching each other. They are also called “action-at-a-distance” force

Common Forces

Common Forces

Common forces that we encounter in AP Physics include:

- Weight (gravitational force) $\vec{w}$ (or $\vec{F}_g$, or just $m\vec{g}$)
- Normal force $\vec{N}$
- Friction (static $\vec{f}_s$ and kinetic $\vec{f}_k$)
- Tension $\vec{T}$
- Applied force $\vec{F}_a$
- Spring force $\vec{F}_e$
- Drag $\vec{D}$ (fluid resistance)
- Buoyant force $\vec{D}$ (discussed in fluid mechanics, in AP Physics 2)
- Electrostatic force $\vec{F}_q$ (discussed in E & M)
- Magnetic force $\vec{F}_m$ (discussed E & M)

Gravity

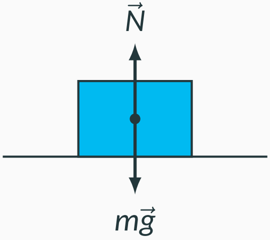
Gravity is the force of attraction between all objects with mass

$$\boxed{\vec{F}_g = m\vec{g}}$$

- Near surface of Earth, use $g = 9.81\,\mathrm{m/s^2}$ (or $g = 10\,\mathrm{m/s^2}$ for your AP exam)
- $\vec{F}_g$ always points *down*
- Based on the law of universal gravitation:

$$\boxed{F_g = \frac{Gm_1m_2}{r^2}} \quad \text{where} \quad G = 6.674 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$$

Normal Force

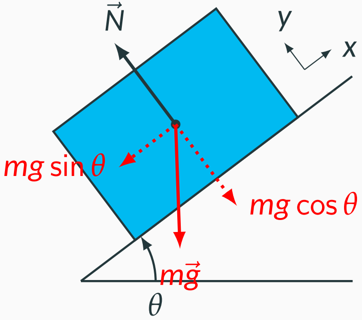


$$m\vec{g} = -\vec{N}$$
(special case)

- A force a surface exerts on another object that it is in contact with
- Always **perpendicular** to the contact surface
- **Special case:** When an object is on a horizontal surface with no additional applied force, the magnitude of the normal force is equal to the magnitude of the weight of the object, i.e. $N = mg$

Normal Force on a Stationary Slope

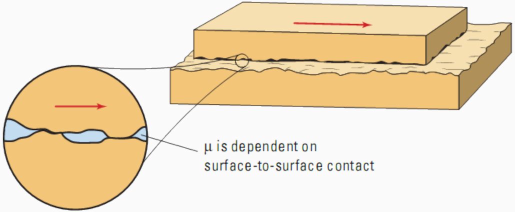
For this case, we define the x-axis to be along the slope, and y-axis to be perpendicular to the slope.



- On a stationary slope: $N = mg\cos\theta$
  - N decreases as ramp angle θ increases
- Weight has a component along the ramp ($mg\sin\theta$) that wants to slide the block down.

Friction

- A force that opposes the sliding of two surface against one another
- Always act in a direction that opposes motion or attempted motion
- Depends on:
    - Normal force N: The force the two surfaces are pressed against each other
    - Coefficient of friction (μ_s and μ_k): Smoothness of the surfaces, which itself depends on
        - The material(s) the surfaces are made of
        - The use of lubricants



Static Friction

Static friction between the two surfaces is when there is no relative motion between them

- Increases with increasing applied force
- Maximum when the object is just about to move

$$\boxed{f_s \leq \mu_s N}$$

Quantity	Symbol	SI Unit
Magnitude of static friction	f_s	N
Static friction coefficient	μ_s	no units
Magnitude of normal force	N	N

Kinetic Friction

Kinetic friction between two surfaces is when they are moving relative to each other. f_k is constant along the path of movement as long as $\vec{N}$ stays constant

$$\boxed{f_k = \mu_k N}$$

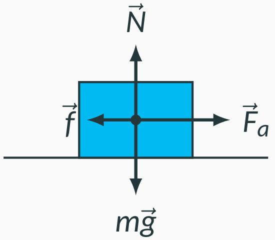
Quantity	Symbol	SI Unit
Magnitude of kinetic friction	f_k	N
Kinetic friction coefficient	μ_k	no units
Magnitude of normal force	N	N

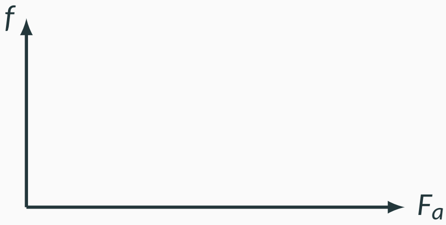
Static and Kinetic Coefficients of Friction

Kinetic friction coefficient is always lower than the static coefficient, otherwise nothing will ever move:

$$\mu_k \leq \mu_s$$

Consider a simple case of a box being pulled along a level floor. The free-body diagram is simple (left). How do the magnitudes of the applied force F_a and friction f compare?





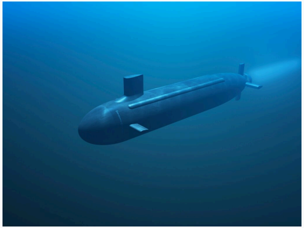
Tires

Most people associate friction as the force that slows down things, but very often friction is what accelerates things. **Example:** the forward acceleration of a car is caused by the static friction between the tires and the road.

Tires also generated a force called *rolling resistance* as it rolls along a road because the weight of the car deforms the tires.

Drag

When an object moves through a fluid (most gases and liquids), it experiences are fluid resistance force called **drag** $\vec{D}$.



Unlike kinetic friction, drag force is highly dependent on the speed of the object relative to the fluid that it is moving in:

$$D = \frac{1}{2}\rho v_\infty^2 C_d A$$

In AP Physics you are *not* asked to know the drag equation. However, you should know that drag depends on speed and is not a constant.

Drag

$$\boxed{D = \frac{1}{2}\rho v_\infty^2 C_d A}$$

Quantity	Symbol	SI Unit
Magnitude of drag force	D	N
Density of the fluid	ρ	$\mathrm{kg/m^3}$
Free-stream velocity	v_∞	$\mathrm{m/s}$
Reference area	A	$\mathrm{m^2}$
Drag coefficient	C_d	(no unit)

Drag coefficient depends on the shape and surface smoothness of the object. For blunt objects ("bluff bodies") A is the frontal area; for streamlined objects A is the planform (top-view) area

Terminal Velocity

When we take drag force into account, we understand that the drag force increases as an object speeds up, and therefore a free-falling object does *not* accelerate infinitely. Instead it reaches a **terminal velocity**.

There is no air resistance just as the object *begins* to fall. Acceleration is due to gravity alone.

Drag increases as v increases. Magnitude of acceleration decreases, but the object continues to gather speed

Terminal velocity is reached when the drag force equals the object's weight. Not net force; no acceleration.

Tension Force

Tension $\vec{T}$ is the force that is the force that is transmitted through objects that can be stretched, e.g. a rope that is being pulled

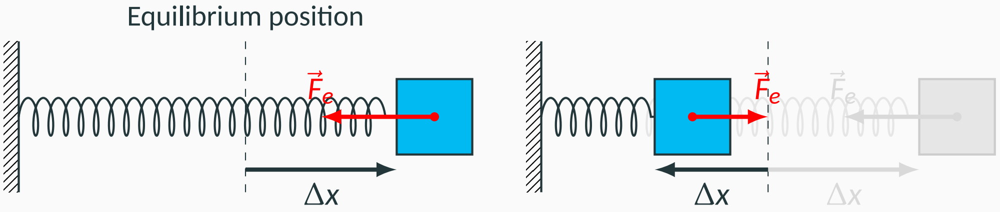
- Examples: ropes, cables, strings, etc.
- Tension force can only be transmitted if the cable is fully extended
- You can't push on a rope
- Can be used with pulleys to change the direction of force

Spring Force & Elastic Potential Energy

The spring force $\vec{F}_e$ is the force that a compressed/stretched spring exerts on the object connected to it. An *ideal* spring obeys Hooke's law:

$$\boxed{\vec{F}_e = -k\vec{x}}$$

The spring force acts in the opposite direction to the spring's displacement, and is proportional to the amount of compression/stretching.



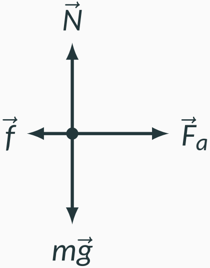
Free-Body Diagrams

Free-Body Diagrams

- Acceleration (if there is going to be any at all) depends on net force $\vec{F}_{\text{net}}$
- Without a vector sum of all the forces, we cannot determine the magnitude, direction of the acceleration, or how acceleration will evolve in time
- We use **free-body diagrams** (FBD) to represent all the forces.
  - Very important in solving any dynamics problems
  - Don't try to save this step, even if the problem does not ask for it
  - Always draw FBD for solving classical mechanics problem

Free-Body Diagrams

For *rectilinear*, or *translational* motion, FBDs are usually drawn by assuming that all forces acting at the center of mass ("CM"), represented by the "big dot". For example:

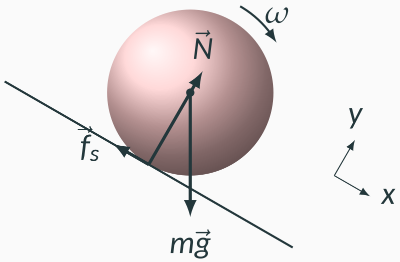


However, for motion where *rotation* is (at least) a possibility, we mist note that:

- Gravitational force $m\vec{g}$ acts at the CM, but
- Normal force $\vec{N}$, friction $\vec{f}$ and applied force $\vec{F}_a$ act at the point of contact

Free-Body Diagrams

In those cases, forces should be drawn where they are applied. For example, a sphere rolling down a ramp should have weight $m\vec{g}$, normal force $\vec{N}$ and static friction $\vec{f}_s$ acting on it:

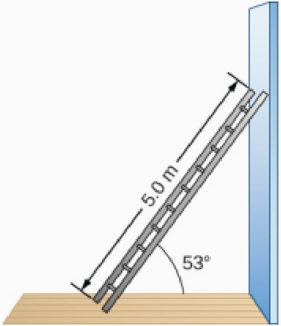


Once the FBD is drawn, decide on the axes to help you solve the motion. One of the axes should line up with the direction of motion. This guarantees that the *other* axis will not have any net force.

Example Problem

A more difficult static problem may involve two surfaces with two different friction coefficients. For example, a ladder leaning on a wall. This problem cannot be solved without first understanding rotational motion, but we can still draw a FBD.

Example: A uniform ladder is 5 m long and weighs 400 N. The ladder rests against a slippery vertical wall, as shown in Figure. The inclination angle between the ladder and the rough floor is 53°. Find the reaction forces from the floor and from the wall on the ladder and the coefficient of static friction μ_s at the interface of the ladder with the floor that prevents the ladder from slipping.



Multi-Body Problems

Applying Third Law on Connected Bodies

- The objects are connected by a cable or a solid linkage with negligible mass
- All objects (usually) have the same acceleration
- Require multiple free-body diagrams

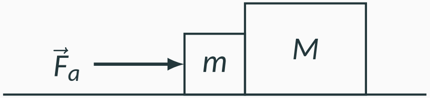
Solving Connected-Bodies Problems

To solve a connected-bodies problem, you can follow these procedures:

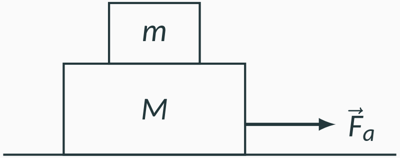
1. Draw a FBD on each of the objects
2. Sum all the forces on all the objects along the direction of motion
    - Direction of motion is usually very obvious
    - All internal forces should cancel and do not figure into the acceleration of the system
3. Compute the acceleration of the entire system using second law of motion
    - Remember that (usually) every object has the same acceleration!
4. Go back to the FBD of each of the objects and compute the unknown forces (usually tension)

Different Types of Connected Bodies

Multiple objects pressed against one another. There may not be friction, but there are definitely action/reaction forces between the blocks.



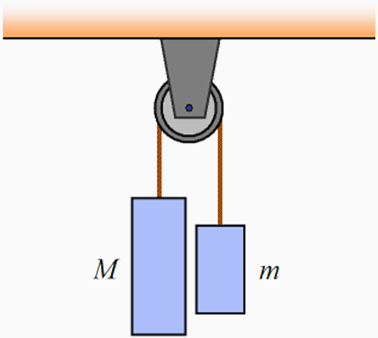
Or multiple objects stacked on top of one another. The contact surface between M and the floor may (or may not) have friction, while the surface between M and m must have a friction coefficient μ.



Pulley Problems

Example Problem: Atwood Machine

An **Atwood machine** is made of two objects connected by a rope that runs over a pulley. The pulley allows the direction of force and direction of motion to change between two objects.



Example: The object on the left has a mass of M and the object on the right has a mass of m.

- What is the acceleration of the masses?
- What is the tension in the rope?

A More Typical Problem

More typically, an Atwood machine problem is one where two objects are sliding on a surface. These surfaces may have (or may not) have friction. In this example, two blocks are connected by a massless string over a frictionless pulley as shown in the diagram.

(a) Determine the acceleration of the blocks.

(b) Calculate the tension in the string.