Topic 16: Wave-Particle Duality

Advanced Placement Physics

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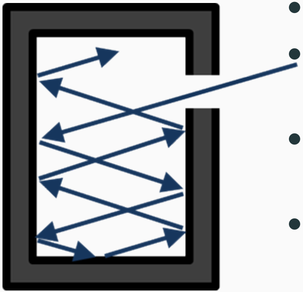
January 11, 2021

Olympiads School

Anyone who is not shocked by quantum theory has not understood it.

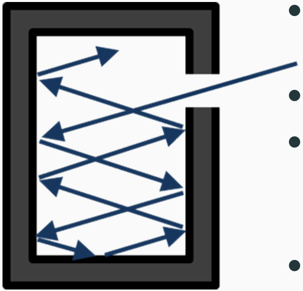
- Niels Bohr

Blackbody Radiation



- The concept was coined by Gustav Kirchhoff in 1860
- An idealized object that absorbs all incident EM radiation, regardless of frequency or angle of incidence
- Think of a box (“cavity”) with a mirror inside, and a hole where light (EM radiation) is allowed in
- Some of the light reflects inside the cavity, and some gets absorbed by the blackbody
- Eventually all the light inside the cavity is absorbed

Blackbody Radiation



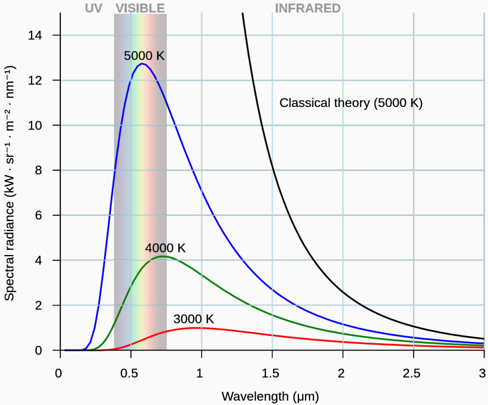
- The object is in thermodynamic equilibrium; all of the absorbed energy is then immediately radiated back as EM radiation
- The spectral distribution depends on temperature
- A blackbody at room temperature appears black, as most of the radiative energy is infrared and cannot be perceived by the human eye
- Thermal radiation spontaneously emitted by many ordinary objects can be approximated as blackbody radiation

Raleigh-Jeans Law and the Ultraviolet Catastrophe

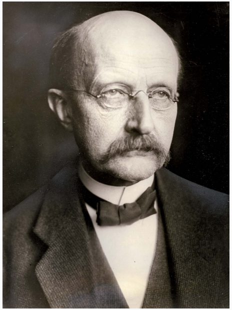
- Based on classical thermodynamics

$$P(\lambda, T) = 8\pi k T \lambda^{-4}$$

  T=temperature, λ=wavelength, k=Boltzmann's constant
- Agrees with experimental results for long wavelengths
- But disagrees violently for short wavelengths:
  - Shorter wavelengths (e.g. ultraviolet waves) seem to have infinite intensity
  - Known as **"Ultraviolet catastrophe"**



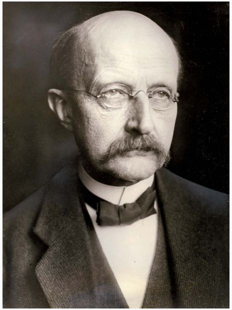
Quantization of Energy



Max Planck

- Made a strange modification in the classical calculations
- Derived a function of $P(\lambda, T)$ that agreed with experimental data for all wavelengths
- First found an empirical function to fit the data
- Then searched for a way to modify the usual calculations

Quantization of Energy



Max Planck

- Argued that the walls of a blackbody are composed of subatomic electric oscillators ("resonators")
  - The nature of these resonators were unknown
  - Billions of resonators vibrating at different frequencies, and therefore
  - Emitting radiation at those frequencies (remember that for a wave, the frequency of disturbance at the source is the frequency of the wave)
  - In classical physics, the resonators can have any value of energy, and change its amplitude continuously
- In order to agree with experiments, Planck discovered that energy emitted the resonator must be *discrete*
  - When energy is emitted from the resonator, it drops to the next lower energy level

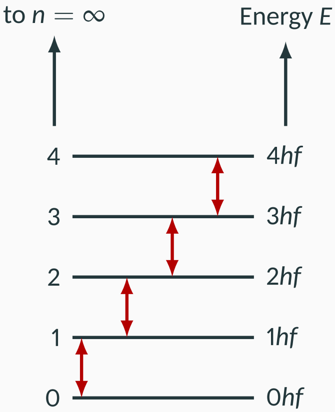
Quantization of Energy

The total energy of *any* harmonic oscillator can only be integral multiples of hf:

$$\boxed{E_{\text{res}} = nhf}$$

Quantity	Symbol	SI Unit
Energy of the resonator	E_{res}	J
Enegy level	n	(no unit)
Planck's constant	h	$\text{J} \cdot \text{s}$
Frequency of resonator	f	Hz

Planck's constant is experimentally determined to be $h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$



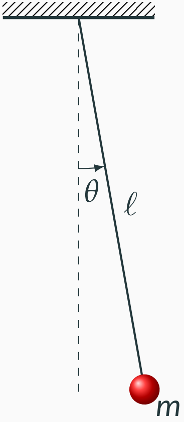
Planck's Law

As for his formula, it's called **Planck's law**:

$$\boxed{P(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}}$$

Classical vs. Quantum Oscillator

This “quantum” behavior exists even for the simple pendulum that we studied in mechanics topics in harmonic motion and circular motion:



The natural frequency for $\ell = 1\,\mathrm{m}$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \approx 0.50\,\mathrm{Hz}$$

Each energy level

$$\Delta E = hf \approx 3.31 \times 10^{-34}\,\mathrm{J}$$

For a pendulum with $m =$100 g and $\theta =$10°:

$$\frac{\Delta E}{E} = \frac{\Delta E}{mg\ell(1 - \cos\theta)} = 2.2 \times 10^{-32}$$

No wonder we can’t observe it in a marcoscopic level!

Maxwell's Equations in a Vacuum

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

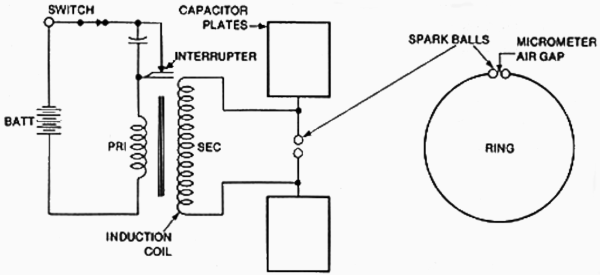
$$\nabla \times \mathbf{B} = \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

Disturbances in **E** and **B** travel as an "electromagnetic wave", with a speed:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299\,792\,458\ \mathrm{m/s}$$

The Spark Gap Experiment

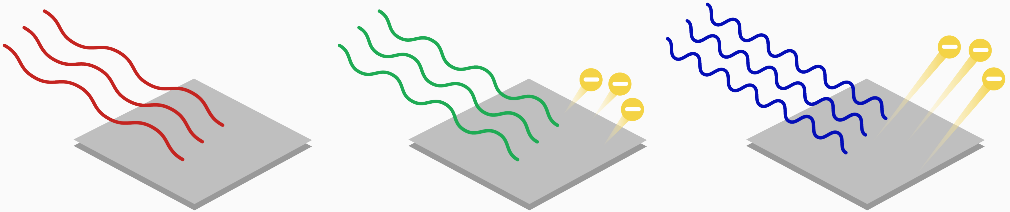
To prove that light is an EM wave, German physicist Heinrich Hertz devised a “spark gap experiment” to generate frequencies in the range of 10^{14} Hz



- Also showed that light has the same wavelengths as predicted by Maxwell’s equations
- Discovered **photoelectric effect** that was caused by ultraviolet radiation
- Physicist who repeated his experiments did not have an explanation

Photoelectric Effect

When electromagnetic waves (e.g. light) hits certain metals, electrons are knocked off the surface.



When observing this **photoelectric effect**, physicists discovered that:

- Increasing intensity of light knocked off more electrons, but doesn't change the maximum kinetic energy of the electrons, but
- Changing the frequency of the light did change K though, although
- Below a certain frequency, *no* electrons were emitted

The Photon: Packets of Energy

In a 1905 paper[1], Einstein argued that light is not continuous wave, but a collection of discrete energy packets (photons), each with energy $E = hf$

$$\boxed{K_{\text{max}} = \begin{cases} hf - \varphi & \text{if } hf > \varphi \\ 0 & \text{otherwise} \end{cases}}$$

Quantity	Symbol	SI Unit
Maximum kinetic energy of	K_{max}	J
Planck's constant	h	$\text{J} \cdot \text{s}$
Frequency of the EM wave	f	Hz
Work function of the metal	φ	J

[1] *On a Heuristic Viewpoint Concerning the Production and Transformation of Light*

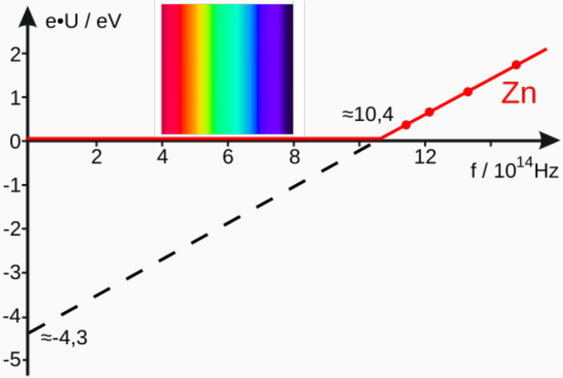
Photon: Particle of Light

In Einstein's view:

- Classical wave theory and laws of electrodynamics cannot explain the photoelectric effect
- When a photon collides with an electron, it is absorbed into the electron entirely, and without delay

Einstein may have been alerted to the fact that the blackbody radiation curve resembles the distribution of energies in a gas

Work Function



Work function the minimum energy required to remove an electron from a solid to a point immediately outside the solid surface. The minimum frequency at which electrons will be ejected is called the **threshold frequency**.

Slope is *h* no matter what metal it is.

Work Functions of Different Materials

The work function φ depends on the metal. They are generally presented in electron volts ($1\,\text{eV} = 1.609 \times 10^{-19}$ J)

Metal	Work function (eV)
aluminum	4.28
calcium	2.87
cesium	2.14
copper	4.65
iron	4.50
lead	4.25
lithium	2.90
nickel	5.15
platinum	5.65
potassium	2.30
tin	4.42
tungsten	4.55
zinc	4.33

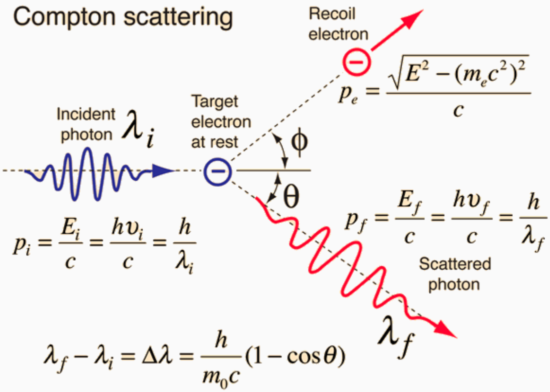
Compton Scattering

American physicist Arthur Holly Compton studied x-ray scattering by free electrons

- Classical theory cannot account for the scattering behaviour
- Frequency shift only depends on scattering angle
- Prediction possible if treating the x-ray as photons with momentum, just like a particle. Compton used Einstein's invariant and applied to a massless particle:

$$\boxed{p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}}$$

Compton Scattering



If we treat the x-ray as a photon with momentum $p = h/\lambda$ then we can use Newton's laws of motion to predict both the recoil electron and scattered x-ray!

Momentum of a Photon

The momentum of a photon is proportional to Planck's constant and inversely proportional to its wavelength.

$$\boxed{p = \frac{h}{\lambda}}$$

Quantity	Symbol	SI Unit
Momentum	p	$\mathrm{kg} \cdot \mathrm{m/s}$
Planck's constant	h	$\mathrm{J} \cdot \mathrm{s}$
Wavelength	λ	m

This is a very odd expression, which treats photon both as a particle (with momentum) and a wave (with a wavelength λ).

Matter Waves

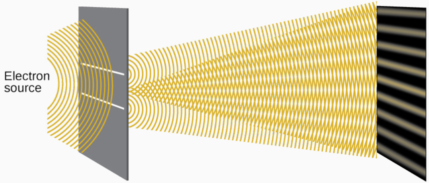
If electromagnetic waves are really particles of energy, then are particles (e.g. electrons) a wave of some sort?

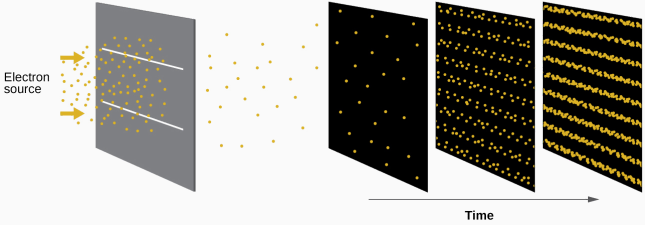
- The De Broglie hypothesis in 1924: a particle can also have a wavelength
- Confirmed accidentally by the Davisson-Germer Experiment in 1927 (beam of electron scattering on nickel crystal surface)

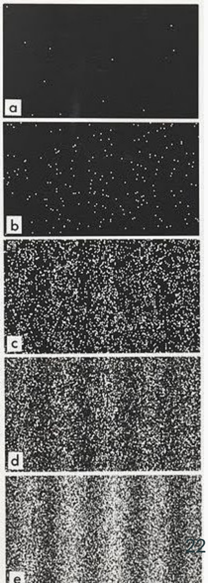
If a particle is also a wave, what *kind* of a wave is it then?

Electron Interference

If I perform a double-slit experiment with a beam of electrons, will I get an interference pattern?







De Broglie Wavelength

If matter is also a wave, then its wavelength can be obtained by solving the momentum equation for λ:

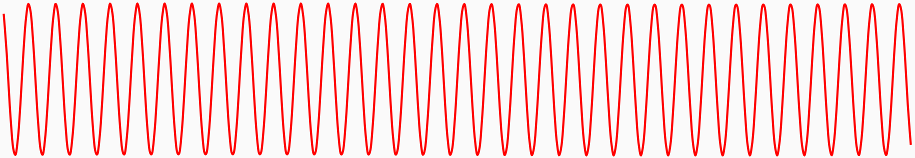
$$p = \frac{h}{\lambda} \quad \rightarrow \quad \lambda = \frac{h}{p} \quad \rightarrow \quad \boxed{\lambda = \frac{h}{mv}}$$

Quantity	**Symbol**	**SI Unit**
Wavelength of a particle	λ	m
Planck's constant	h	$\mathrm{J \cdot s}$
Mass	m	kg
Velocity	v	$\mathrm{m/s}$

Well-Defined Momentum = Poorly-Defined Position

Consider the wave below. At any time t, the wave/particle can be described as the wave function $\Psi(x, t)$.

$$\Psi(x, t) = \cos(\omega t - kx)$$

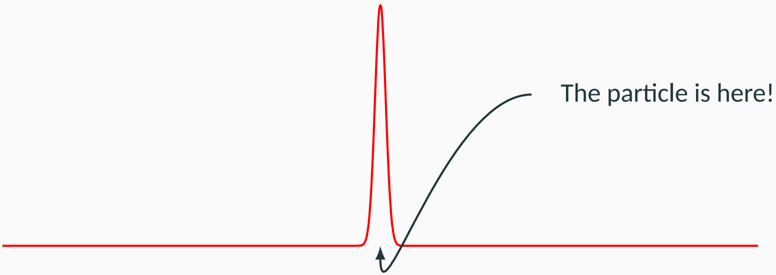


- Ψ has a single wavelength of λ (therefore a single value of momentum p)
- Has no distinguishing features that can tell you the particle's position x

When we have precise knowledge of a particle wave's *momentum*, we have *no* knowledge of *where* it is.

Well-Defined Position = Poorly-Defined Momentum

On the other extreme, consider a particle/wave defined as a delta function:



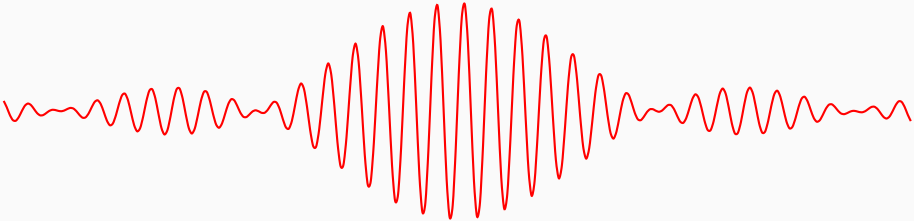
- The particle's position *x* is well-defined
- But its wavelength λ (and therefore momentum *p*) is undefined: it provides no information about the particle's velocity

When we have precise knowledge of a particle wave's *position*, we have *no* knowledge of *where* it is going.

Uncertainty Principle

If a moving particle has small variations/uncertainties in its momentum p (by its wavelength λ), when we add up the different waves together, we begin to see a **packet** forming:

$$\cos(280x - \omega t) + \cos(381x - \omega t) + \cdots + \cos(300x - \omega t) + \cdots + \cos(319x - \omega t) + \cos(320x - \omega t)$$



To gain knowledge about the *location* of a particle, we must give up knowledge about its *momentum*, and vice versa.

Uncertainty Principle

Because of the wave properties of particles, you can never be completely certain of the relationship between an object's momentum p and position x. The more you know about an object's position, the less you know about its momentum, and vice versa.

$$\boxed{\sigma_p \sigma_x \geq \frac{\hbar}{2}}$$

Quantity	Symbol	SI Unit
Uncertainty in momentum	σ_p	$\mathrm{kg} \cdot \mathrm{m}/\mathrm{s}$
Uncertainty in position	σ_x	m
Redued Planck's constant	$\hbar$	$\mathrm{J} \cdot \mathrm{s}$

The **reduced Planck's constant** is just $\hbar = \frac{h}{2\pi}$.

Bohr Atomic Model

The “orbital” model of electrons does not work, because as the electron orbits (accelerates around) the nucleus, it radiates electromagnetic radiation, and therefore lose energy. The orbit will eventually collapse.

Bohr postulated that electron can move in certain “non-radiating” orbits, corresponding to energy levels:

$$\boxed{E_n = -\frac{k^2 e^4 m}{2\hbar^2}\frac{Z^2}{n^2}}$$

From the wave-particle duality perspective, the “orbits” correspond more to a standing wave around the nucleus (a standing wave does not lose energy)

Bohr Atomic Model

$$\boxed{E_n = -\frac{k^2 e^4 m}{2\hbar^2}\frac{Z^2}{n^2}}$$

Quantity	Symbol	SI Unit
Energy at level n	E_n	J
Coulomb's constant	k	$\mathrm{N \cdot m^2 / C^2}$
Elementary charge	e	C
Atomic mass	m	kg
Reduced Planck's constant	$\hbar$	$\mathrm{J \cdot s}$
Atomic number	Z	integer; no units
Energy level	n	integer; no units

Bohr Atomic Model

Successful in describing the behaviour of the hydrogen atom—but fails for heavier atoms—although it still relies on

- Coulomb forces between electrons and protons (classical)
- Centripetal forces (classical)
- Quantization of energy (new physics!)

De Broglie's hypothesis gives us a glimpse of what Bohr is missing

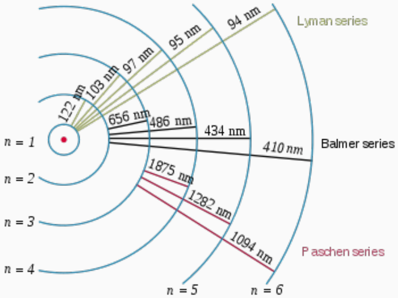
- The "orbits" correspond to a standing wave around the nucleus
- A standing wave does not lose energy

Hydrogen Emission

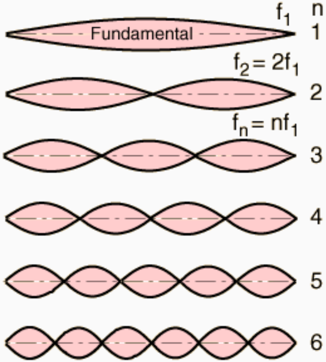
- Lyman series:
  - the EM emissions when the electrons drop from a higher energy state (E_n) to the ground state $n = 1$ (i.e. E_1)
  - The frequency is given by:

$$f = \frac{E_1 - E_n}{h}$$

  - We can apply universal wave equation to get the wavelengths
- Balmer series–dropping to E_2
- Paschen series–dropping to E_3

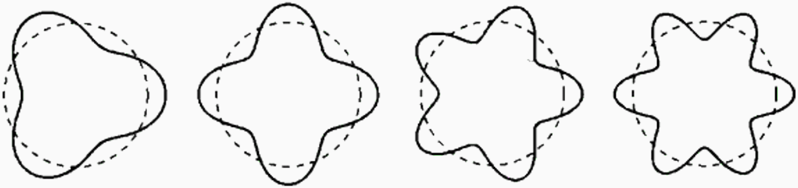


Standing Wave on a String



- We have studied standing waves in Grade 11
- If electron is to be in a “stable orbit” around a nucleus, it has to be in a standing wave pattern
- Otherwise, it will interfere with itself

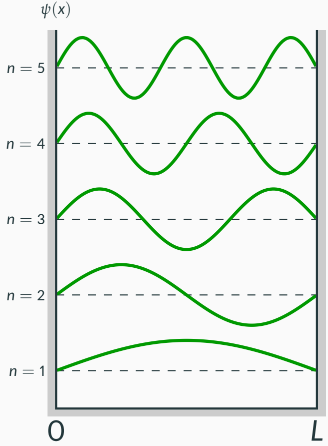
Circular Standing Wave



Electron resonance states $n = 3, 4, 5, 6$

Particle in a Box

A particle in a 1D box (e.g. a billiard ball confined to a pool table and can only roll forward and backward) must behave a standing wave in order to exist.



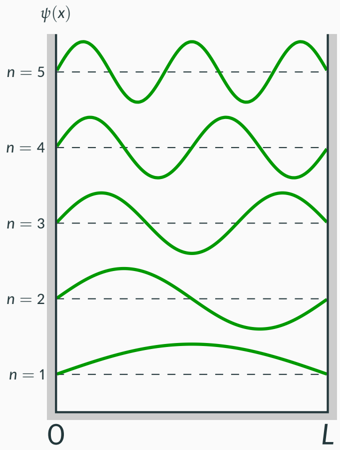
The resonance modes (frequencies where stable standing waves exist) correspond to the wavelengths in the same way that was discussed earlier in the course:

$$\lambda = \frac{2L}{n} \quad n = 1, 2, 3, 4, \ldots$$

and the momentum of the particle is:

$$p = \frac{h}{\lambda} = \frac{nh}{2L}$$

Particle in a Box



Kinetic energy of the particle can be expressed in terms of momentum:

$$K_n = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{n^2h^2}{8mL^2}$$

If a particle is a standing wave, then the kinetic energy of the particle can never be zero (as long as it is confined inside the box), therefore

- It cannot have zero velocity
- The lowest energy level ($n = 1$) is called the **zero-point energy**

Example

Example 3: A 0.150 kg billiard ball is confined to the pool table 1.42 m wide. How long (in seconds) will it take to travel from one side of the table to the other? (Use fundamental mode.)