Student #: ____________________________ Student Name: ___________________________________

AP PHYSICS C: WORK AND ENERGY

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

1. A 1 kg ball is thrown vertically downward from a 50 m high tower with an initial speed of 4 m/s. Just before striking the ground, the speed of the ball is 20 m/s. The energy lost to air friction is most nearly

C

(A) 101 J
(B) 210 J
(C) 308 J
(D) 406 J
(E) 508 J

$E_i = U_g + K = (1)(10)(50) + \frac{1}{2}(1)(4^2)$

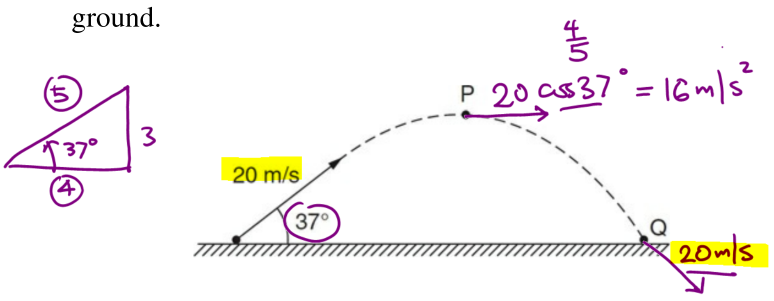
$= 500 + 8 = 508\text{ J}$

$E_f = \cancel{U_g} + K' = \frac{1}{2}mv^2 = \frac{1}{2}(1)(20)^2$

$= 200\text{ J}$

Questions 2–3

A 2 kg projectile is launched with a speed of 20 m/s from horizontal ground at an angle of 37° to the horizontal as shown. Point P is at the top of the path, and point Q is at the end of the path, just before the projectile again reaches the ground.



2. The kinetic energy of the projectile at point P is

C

(A) 108 J
(B) 225 J
(C) 256 J
(D) 400 J
(E) 525 J

$K = \frac{1}{2}mv^2$

3. The kinetic energy of the projectile at point Q is

D

(A) 108 J
(B) 225 J
(C) 256 J
(D) 400 J
(E) 525 J

$\frac{1}{2}mv^2$

4. If a projectile thrown directly upward reaches a maximum height h and spends a total time in the air of T, then returning to the original location, the average power of the gravitational force during the trajectory is

C

(A) $P = 2mgh/T$
(B) $P = -2mgh/T$
(C) 0
(D) $P = mgh/T$
(E) $P = -mgh/T$

$-W$ $+W$

$-mgh$ $+mgh$

$P = \frac{W}{T}$

5. Given that the constant net force on an object and the object's displacement, which of the following quantities can be calculated?

D

m?

(A) the net change in the object's velocity
(B) the net change in the object's mechanical energy
(C) the average acceleration
(D) the net change in the object's kinetic energy
(E) the net change in the object's potential energy

$W_{net} = \Delta K$

6. If the only force acting on an object is given by the equation $F(x) = 2 - 4x$ (where the force is measured in newtons and position in meters), what is the change in the object's kinetic energy as it moves from $x = 2$ to $x = 1$?

A

(A) +4 J
(B) −4 J
(C) +2 J
(D) −2 J
(E) +8 J

$\Delta K = W = \int_2^1 F\,dx = \int_2^1 (2-4x)\,dx$

$= \int_2^1 2\,dx - \int_2^1 4x\,dx = 2x\Big|_2^1 - 2x^2\Big|_2^1 = (2-4)-(2-8) = 4\text{J}$

7. The potential energy of an object varies with the equation $U(x) = 2x^2 + x - 6$, where force is in newtons and displacement is in meters. A force F vs. displacement x graph would yield which of the following?

E

(A) A straight, horizontal line
(B) A parabola
(C) An exponential decay curve
(D) A straight line with a positive slope
(E) A straight line with a negative slope

$\frac{dU}{dx} = 4x+1$

$-\frac{dU}{dx} = -4x-1$

$F(x) = -\frac{dU}{dx} = -4x-1$

8. A particle of mass m moves according to the displacement equation $x = 2t^{5/2}$. The kinetic energy of the particle as a function of time is

C

(A) $10mt^{5/2}$
(B) $10mt^{3/2}$
(C) $\frac{25}{2}mt^3$
(D) $5mt^2$
(E) $2mt^{3/2}$

$v = \frac{dx}{dt} = 2\left(\frac{5}{2}\right)t^{3/2} = 5t^{3/2}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(25t^3\right)$

9. An electron travels in a circle around a hydrogen nucleus at a very high speed. The work done by the electrostatic force acting on the electron after one complete revolution is

A

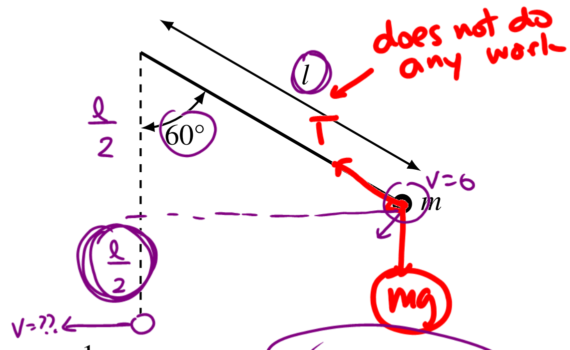
(A) zero
(B) positive
(C) negative
(D) equal to the kinetic energy of the electron
(E) equal to the potential energy of the electron

P $F_c = F_a$ v

10. An object is moved from rest at point P to rest at point Q in a gravitational field. The net work against the gravitational field depends on the $mg\Delta x$

(A) mass of the object and the positions of P and Q
(B) mass of the object only
(C) positions of P and Q only
(D) length moved between points P and Q
(E) coefficient of friction

11. A pendulum bob of mass m is released from rest as shown in the figure below. What is the tension in the string as the pendulum swings through the lowest point of its motion?



D

(A) $T = \frac{1}{2}mg$
(B) $T = mg$
(C) $T = \frac{3}{2}mg$
(D) $T = 2mg$
(E) None of the above

$mg\frac{l}{2} = \frac{1}{2}mv^2$

$mv^2 = mgl \to \frac{mv^2}{l} = mg$

F_c

T

mg

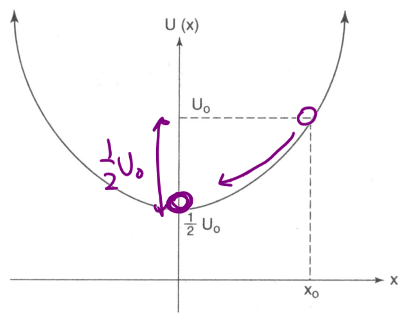
$F_c = T - mg$

$T = F_c + mg = mg + mg$

$= 2mg$

Questions 12–13

Consider the potential energy function shown below.



$\frac{1}{2}U_0 = \frac{1}{2}mv^2$

$v = \sqrt{\frac{U_0}{m}}$

12. Assuming that no non-conservative forces are present, if a particle of mass m is released from position x_0, what is the maximum speed it will achieve?

C

(A) $\sqrt{4U_0/m}$
(B) $\sqrt{2U_0/m}$
(C) $\sqrt{U_0/m}$
(D) $\sqrt{U_0/2m}$
(E) The particle will achieve no maximum speed but instead will continue to accelerate indefinitely.

13. Which of the following is the most accurate description of the system introduced in the previous question?

D

(A) A stable equilibrium
(B) An unstable equilibrium
(C) A neutral equilibrium
(D) A bound system
(E) There is a linear restoring force

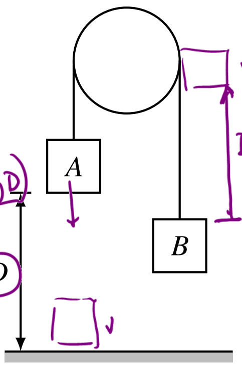
14. Two blocks of mass m_A and m_B are connected by a string that passes over a light pulley. The mass of A is larger than the mass of B. The speed of mass A just before reaching the floor is:

K
$\downarrow$
$\Delta K + \Delta U_g = 0$

$K = -\Delta U_g$

$\frac{1}{2}(m_A+m_B)v^2 = -(m_B g D - m_A g D)$

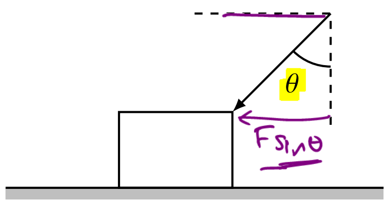
$\frac{1}{2}(m_A+m_B)v^2 = (m_A-m_B)gD$



A

(A) $\sqrt{\frac{2(m_A-m_B)}{m_A+m_B}gD}$
(B) $\sqrt{\frac{2(m_A+m_B)}{m_A-m_B}gD}$
(C) $\sqrt{\frac{2m_A}{m_A+m_B}gD}$
(D) $\sqrt{\frac{2m_B}{m_A+m_B}gD}$
(E) $\sqrt{\frac{2m_A}{m_B}gD}$

15. A force is applied to a block of mass m at a downward angle of θ to the vertical as shown. The block moves with a constant speed across a rough floor for a distance x. The work done by the applied force on the block is



A

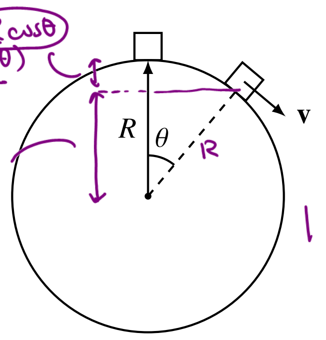
(A) $Fx\sin\theta$
(B) $Fx\cos\theta$
(C) $Fmx\sin\theta$
(D) $Fmx\cos\theta$
(E) zero

Questions 16–17

A small block rests on the top of a smooth sphere of radius R when it is given a light tap so that it just begins sliding on the sphere. When the block reaches the angle θ, it loses contact with the surface of the sphere.

$= R(1-\cos\theta)$ $(R - R\cos\theta)$

$R\cos\theta$



16. The kinetic energy of the block as it leaves the surface of the sphere is

D

$K = -\Delta U_g$
$= mg(R - R\cos\theta)$

(A) mgR
(B) $mgR\cos\theta$
(C) $mgR\sin\theta$
(D) $mg(R - R\cos\theta)$
(E) $mg(R - R\sin\theta)$

17. The speed of the block as it leaves the surface of the sphere is

D

$mg(R - R\cos\theta) = \frac{1}{2}mv^2$
$v = \sqrt{2g(R - R\cos\theta)}$

(A) $\sqrt{2gm}$
(B) $\sqrt{2gRm}$
(C) $\sqrt{2gR\cos\theta}$
(D) $\sqrt{2g(R - R\cos\theta)}$
(E) $\sqrt{2g(R - R\sin\theta)}$

18. A machine can lift large weights according to the power equation $P(t) = 4t^3 + 3t^2 - 2$, where power is in watts and time is in seconds. The energy expended by the machine from $t = 0$ to $t = 10$ s is

D

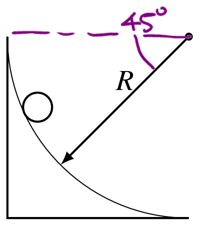
$E = \int_0^{10} P(t)\,dt$

$\int_0^{10} 4t^3 + 3t^2 - 2 = t^4 + t^3 - 2t \Big|_0^{10}$ (10000, 1000, −20)

(A) 1260 J
(B) 3630 J
(C) 9240 J
(D) 10 980 J
(E) 18 150 J

19. A small ball starts from rest and rolls down a quarter-circle ramp of radius R. The speed of the ball at the point halfway down the ramp is most nearly

$R\sin 45°$



D

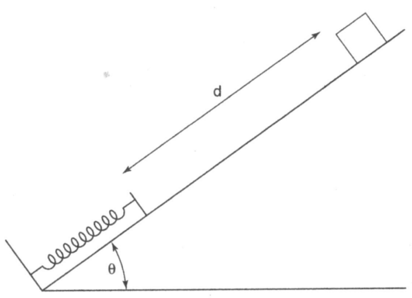
$mgR\sin 45° = \frac{1}{2}mv^2$
$v^2 = 2gR\sin 45°$

(A) gR
(B) $2gR$
(C) $\sqrt{gR\sin 45°}$
(D) $\sqrt{2gR\sin 45°}$
(E) The speed cannot be determined without knowing the mass of the ball.

AP PHYSICS C: WORK AND ENERGY
SECTION II
4 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. A mass m is placed on an incline of angle θ at a distance d from the end of a spring as shown below. The coefficient of kinetic friction between the mass and the plane is μ.



   (a) The mass is released from rest at the position shown. Using Newton's laws, calculate the block's speed when it reaches the spring.

   (b) Using energy conservation, alculate the block's speed when it reaches the spring.

   (c) The spring has spring constant k. At what value x of the compression of the spring does the object reach its maximum speed?

e

2. A 3.0 kg object is moving along the x-axis in a region where its potential energy as a function of x is given as $U(x) = 4.0x^2$, where U is in joules and x is in meters. When the object passes the point $x = -0.50$ m, its velocity is $+2.0$ m/s. All forces acting on the object are conservative.

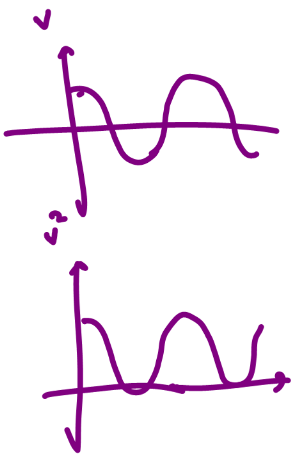
   (a) Calculate the total mechanical energy of the object.

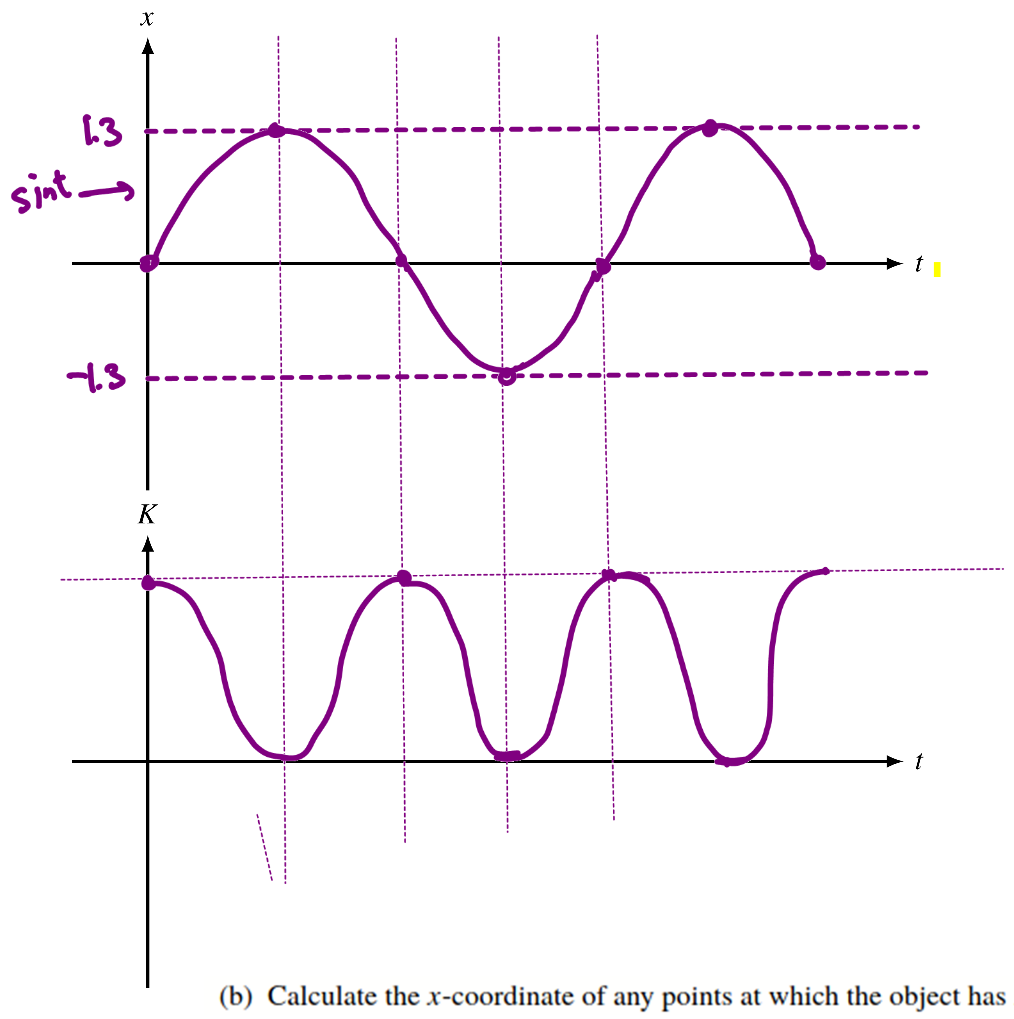
   (b) Calculate the x-coordinate of any points at which the object has zero kinetic energy.

   (c) Calculate the magnitude of the momentum of the object at $x = 0.60$ m.

   (d) Calculate the magnitude of the acceleration of the object as it passes $x = 0.60$ m.

   (e) On the axes below, sketch graphs of the object's position x versus time t and kinetic energy K versus time t. Assume that $x = 0$ at time $t = 0$. The two graphs should cover the same time interval and use the same scale on the horizontal axes.





(b) Calculate the x-coordinate of any points at which the object has zero kinetic energy.

$E = U = 4.0x^2$

$7.0 = 4.0x^2$

$x^2 = \frac{7.0}{4.0} \rightarrow x = \pm\sqrt{\frac{7.0}{4.0}}$

$\boxed{x = \pm 1.3\,m}$

(a) Calculate the total mechanical energy of the object.

$E = K + U = \frac{1}{2}mv^2 + U(x)$

$= \frac{1}{2}(3.0)(2.0)^2 + 4.0(-0.50)^2$

$= 6.0 + 1.0 = \boxed{7.0\,J}$

(c) Calculate the magnitude of the momentum of the object at $x = 0.60$ m.

$K = E - U = 7.0 - (4.0)(0.6)^2 = 5.56\,J$

$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{m^2v^2}{m}\right) = \frac{1}{2}\frac{p^2}{m} \rightarrow p = \sqrt{2mK} = \sqrt{(2)(3.0)(5.56)} = \boxed{5.8\,kg\,m/s}$

$p = mv$

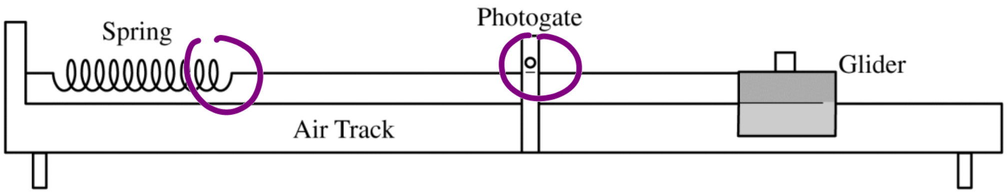
(d) Calculate the magnitude of the acceleration of the object as it passes $x = 0.60$ m.

$\boxed{F = -\frac{dU}{dx}} = -\frac{d}{dx}(4.0x^2) = -8.0x$

$F = -kx$

$a = \frac{F}{m} = \frac{-8.0(0.60)}{3.0} = -1.6\,m/s^2$

$\boxed{|a| = 1.6\,m/s^2}$



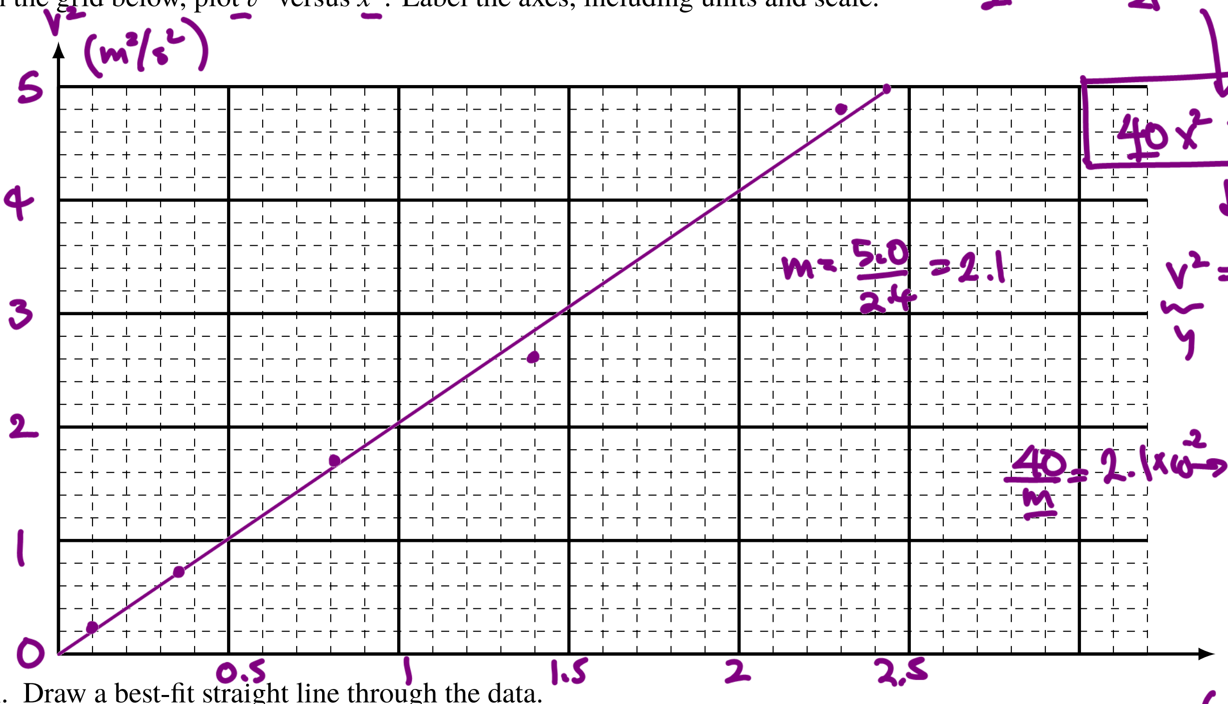
3. The apparatus above is used to study conservation of mechanical energy. A spring of force constant 40 N/m is held horizontal over a horizontal air track, with one end attached to the air track. A light string is attached to the other end of the spring and connects it to a glider of mass m. The glider is pulled to stretch the spring an amount x from equilibrium and then released. Before reaching the photogate, the glider attains its maximum speed and the string becomes slack. The photogate measures the time t that it takes the small block on top of the glider to pass through. Information about the distance x and the speed v of the glider as it passes through the photogate are given below.

Trial #	Extension of the Spring x (m) x (m)	Speed of Glider v (m/s)	Extension Squared x^2 (m^2)	Speed Squared v^2 (m^2/s^2)
1	0.30×10^{-1}	0.47	0.09×10^{-2}	0.22
2	0.60×10^{-1}	0.87	0.36×10^{-2}	0.76
3	0.90×10^{-1}	1.3	0.81×10^{-2}	1.7
4	1.2×10^{-1}	1.6	1.4×10^{-2}	2.6
5	1.5×10^{-1}	2.2	2.3×10^{-2}	4.8

(a) Assuming no energy is lost, write the equation for conservation of mechanical energy that would apply to this situation.

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

(b) On the grid below, plot v^2 versus x^2. Label the axes, including units and scale.



$\boxed{40x^2 = mv^2}$

$v^2 = \frac{40}{m} x^2$

y m x

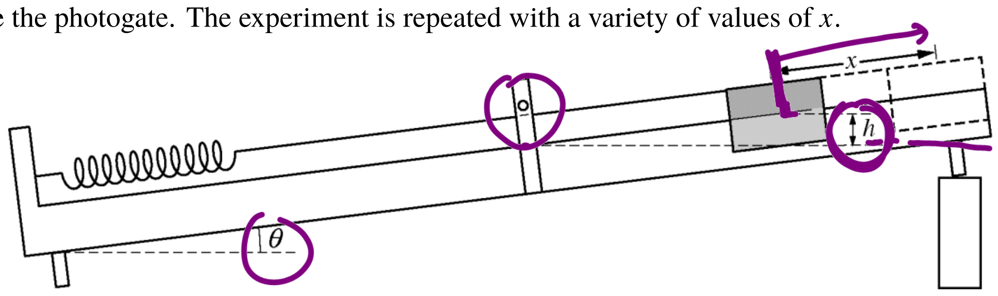
$m = \frac{5.0}{2.4} = 2.1$

$\frac{40}{m} = 2.1 \times 10^{-2} \rightarrow \boxed{m = 0.2\,kg}$

(c) i. Draw a best-fit straight line through the data.

ii. Use the best-fit line to obtain the mass m of the glider. $M \approx 0.20$ kg

(d) The track is now tilted at an angle θ as shown below. When the spring is unstretched, the center of the glider is a height h above the photogate. The experiment is repeated with a variety of values of x.



h=0.

i. Assuming no energy is lost, write the new equation for conservation of mechanical energy that would apply to this situation.

ii. Will the graph of v^2 versus x^2 for this new experiment be a straight line? Justify your answer.

_______ Yes ___✓___ No

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + mgh + mgx\sin\theta \rightarrow \boxed{\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + mg(h + x\sin\theta)}$

quadratic in x.

4. You are to perform an experiment investigating the conservation of mechanical energy involving a transformation from initial gravitational potential energy to translational kinetic energy.

   (a) You are given the equipment listed below, all the supports required to hold the equipment, and a lab table. On the list below, indicate each piece of equipment you would use by checking the line next to each item.

____ Track	____ Meterstick	____ Set of objects of different masses
____ Cart	____ Electronic balance	____ Lightweight low-friction pulley
____ String	____ Stopwatch	

   (b) Outline a procedure for performing the experiment. Include a diagram of your experimental setup. Label the equipment in your diagram. Also include a description of the measurements you would make and a symbol for each measurement.

   (c) Give a detailed account of the calculations of gravitational potential energy and translational kinetic energy both before and after the transformation, in terms of the quantities measured in part (b).

   (d) After your first trial, your calculations show that the energy increased during the experiment. Assuming you made no mathematical errors, give a reasonable explanation for this result.

   (e) On all other trials, your calculations show that the energy decreased during the experiment. Assuming you made no mathematical errors, give a reasonable physical explanation for the fact that the average energy you determined decreased. Include references to conservative and nonconservative forces, as appropriate.