Class 17: Magnetism, Part 1

Advanced Placement Physics C

Dr. Timothy Leung

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Olympiads School

Magnetic Field

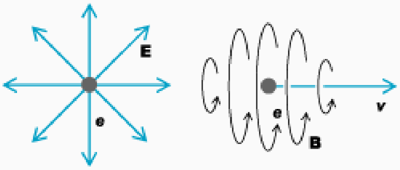
Review of Magnetic Field

- Magnetism is generated by moving charged particles, e.g. a single charge, or an electric current
- It can also be generated by permanent magnets, or Earth

Review of Magnetic Field

- Magnetism affects other *moving* charged particles
- The vector field is called the **magnetic field**
- Magnetic field has unit **tesla**
- Magnetic field lines have no ends—they always run in a loop

Magnetic Field Generated by a Moving Point Charge



A point charge generates an electric field $\vec{E}$. When it's moving, it also generates a magnetic field $\vec{B}$, given by the equation:

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}}$$

The direction of $\vec{B}$ can be obtained by applying the *right hand rule* if you are not confident with cross products.

Reminder on the cross product

Whenever the "right hand rule" is mentioned, or when an equation has "$\sin\theta$" in it, that usually means that the equation involces a cross product in it. Just a reminder on a few properties:

- If $\vec{C} = \vec{A} \times \vec{B}$, then $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.
- The length of the cross product of two vectors is:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

  where θ is the angle between $\vec{A}$ and $\vec{B}$

- Cross products are anti-commutable:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

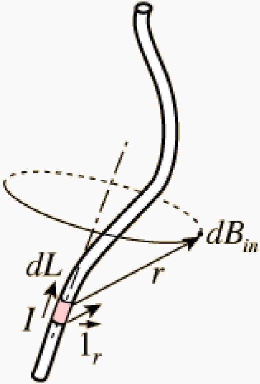
Magnetic Field Generated by a Moving Point Charge

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}}$$

Quantity	Symbol	SI Unit
Magnetic field	$\vec{B}$	T
Charge	q	C
Velocity of the charge	$\vec{v}$	m/s
Distance from the moving charge	r	m
Radial outward unit vector from the charge	$\hat{r}$	no units
Permeability of free space	μ_0	$\mathrm{T} \cdot \mathrm{m}/\mathrm{A}$

Permeability of free space (or **vacuum permeability**) is a constant with a value of $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T} \cdot \mathrm{m}/\mathrm{A}$. It measures how well a space can become magnetized.

Biot-Savart Law

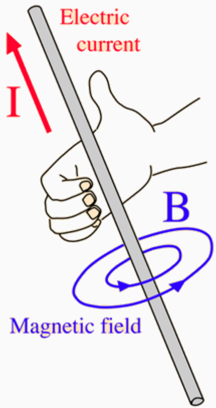


An electric current is really many charges particles moving along a wire; each charge creating its own magnetic field. The total magnetic field in the wire is the integral of the contribution ($\mathrm{d}\vec{B}$) of the current (I) from each infinitesimal sections ($\mathrm{d}\vec{L}$) of the wire, given by the **Biot-Savart law**:

$$\boxed{\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} \frac{I\mathrm{d}\vec{L} \times \hat{r}}{r^2}}$$

The magnetic field in the diagram goes *into* the page

Magnetic Field Generated By an Infinitely Long Wire



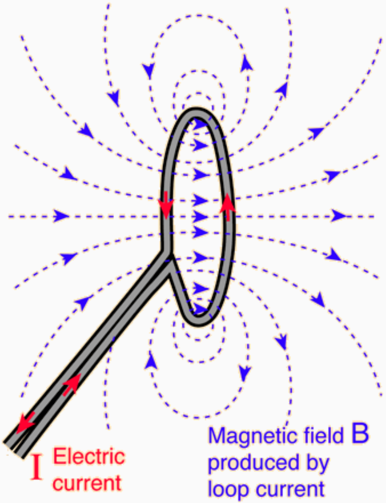
Integrating Biot-Savart law for a point at radial distance r from an *infinitely-long wire* gives a simple expression:

$$\boxed{\vec{B} = \frac{\mu_0(\vec{I} \times \hat{r})}{2\pi r}} \quad \text{or} \quad \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

The magnitude and direction current "vector" $\vec{I}$ is straightforward

Quantity	Symbol	SI Unit
Magnetic field	$\vec{B}$	T
Current	$\vec{I}$	A
Radial direction from the wire	$\hat{r}$	(no units)
Radial distance from the wire	r	m

Current-Carrying Wire Loop

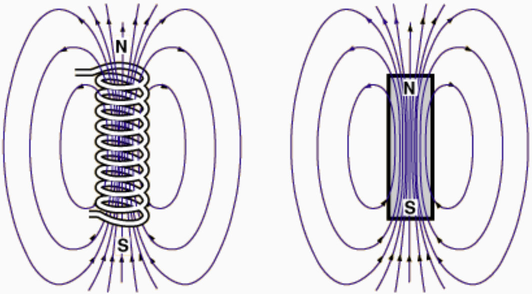


When we shape the current-carrying wire into a loop, we can (again) use the Biot-Savart law to find the magnetic field away from it.

One loop isn't very interesting (except when you're integrating Biot-Savart law) but what if we have many loops

Wounding Wires Into a Coil

- A **solenoid** is when you wound a wire into a coil
- You create a magnet very similar to a bar magnet, with an effective north pole and a south pole
- Magnetic field inside the solenoid is uniform
- Magnetic field strength can be increased by the addition of an iron core

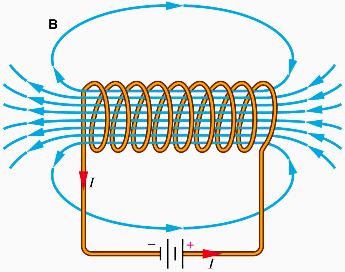


A Practical Solenoid

A practical solenoid usually has hundreds or thousands of turns:

This “air core” coil is used for high school and university experiments. It has approximately 600 turns of copper wire wound around a plastic core.

Magnetic Field Inside a Solenoid



The magnetic field **inside** a solenoid is uniform, with its strength given by:

$$\boxed{B = \frac{\mu N I}{\ell}}$$

Direction of $\vec{B}$ determined by **right hand rule**

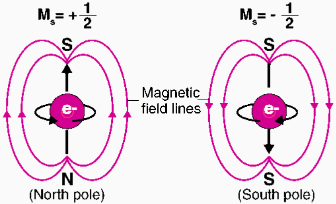
Quantity	Symbol	SI Unit
Magnetic field intensity	B	T
Number of coils	N	
Length of the solenoid	ℓ	m
Current	I	A
Effective permeability	μ	$\mathrm{T \cdot m / A}$

Permanent Magnets

Permanent Magnets

Permanent magnets is also based on the motion of charges. This is the “non-technical” version...

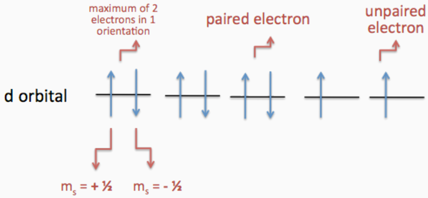
1. Electrons inside an atom *spin*. A spinning electron therefore has an angular momentum, and generates its own tiny magnetic field.



However, in most full “shells”, the spin of these electrons are paired, so the magnetic fields cancel each other.

Permanent Magnets

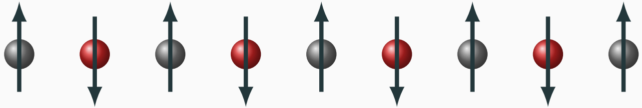
2. The orbits of electrons are not always filled, therefore some atoms do create some (very small) magnetic field.



The atoms that have unpaired electrons are called **paramagnetic** because they are attracted to magnets; atoms that have no unpaired electrons are called **diamagnetic**.

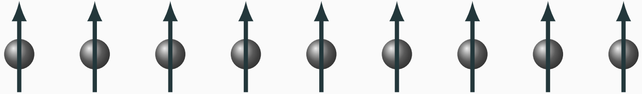
Permanent Magnets

3. While many atoms exhibit paramagnetism, they do not make good magnets, because the atoms are most often arranged in a way where the magnetic fields cancel. This is callled **ferrimagnetism**:

   Ferrimagnetic

   When they do not cancel, then they can become magnets. This is called **ferromagnetism**:

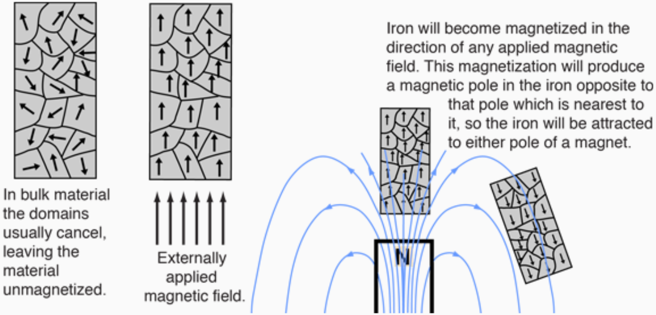
   

   Ferromagnetic

   Transitional elements such as iron, nickel and cobalt, and their alloys will exhibit ferromagnetism.

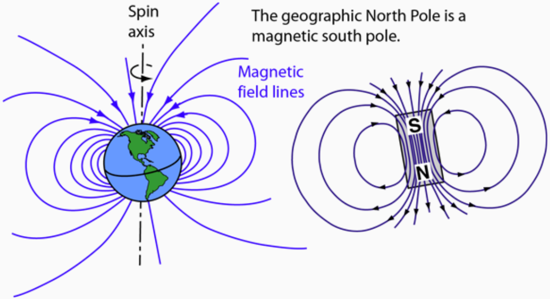
Permanent Magnets

4. The atoms in these ferromagnetic materials are arranged in “domains” wher their magnetic moment is aligned. In the presence of a strong external magnetic field, these domains will line up, creating a magnet.



Earth

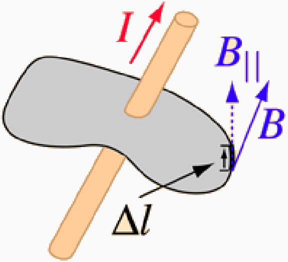
Earth is also a “permanent” magnet, with the *magnetic south pole* located near the geographic north pole, and the *magnetic north pole* located near the geographic south pole. The poles are tilted by $\approx 11°$ from the spin axis.



The exact nature of Earth’s magnetic field is not known, although it may be related to “generator effect” from Earth’s rotation, circulating the outer-core fluid around.

Ampère’s Law

Ampère's Law



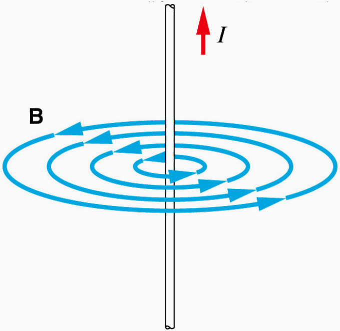
Like Gauss's law is used to calculate electric fields, **Ampère's law** is used to calculate the magnetic field for symmetric configurations:

$$\boxed{\oint_C \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I_C}$$

where

- C is a closed curve around a current ("Amperian loop")
- $\mathrm{d}\vec{\ell}$ is an infinitesimal length along the closed curve
- I_c is the net current that penetrates the area bounded by C

Application of Ampère's Law: Infinitely Long Wire



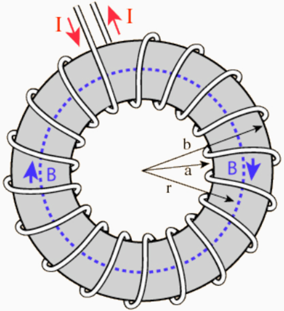
An *infinitely* long wire must generate a magnetic field that only depend on radial distance. We place an Amperian loop as a circle of radius r inside the toroid. Ampère's law reduces to:

$$\oint_C \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I_C \;\rightarrow\; B(2\pi r) = \mu_0 I$$

From this, we get our expression of the magnetic field from an infinitely long wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

Toroid



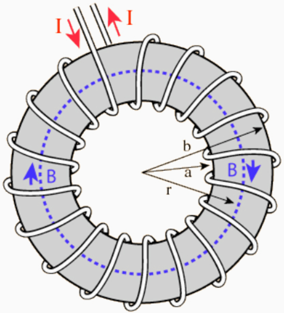
A toroid consists of a current-carrying wire wound around a donut-shaped core

Another application of Ampère's law is the **toroid**. This time, we put our loop at $a < r < b$ inside the toroid. Once again, because of symmetry, Ampère's law reduces to:

$$\oint_C \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I_C$$
$$B(2\pi r) = \mu_0 N I$$
$$B = \frac{\mu_0 N I}{2\pi r}$$

where N is the number of times the wire is wound around the core

Toroid



When the loop is placed at $r < a$, there is no enclosed current, and therefore the magnetic field is zero:

$$B = 0 \quad \text{for} \quad r < a$$

When the loop is placed at $r > b$, the amount of current penetrating the loop is the same in both direction, i.e. $I_c = 0$, and

$$B = 0 \quad \text{for} \quad r > b$$

In fact, magnetic field *only* exists inside the core, between a and b.

Magnetic Force

So What Does the Magnetic Field Do?

Gravitational Field $\vec{g}$

- Generated by massive objects
- Affects massive objects

Electric Field $\vec{E}$

- Generated by charged particles
- Affects charged particles

Magnetic Field $\vec{B}$

- Generated by *moving* charged particles
- Affects moving charged particles

Lorentz Force Law

Since a moving charge or current create both electric and magnetic fields, another moving charge is therefore affected by both $\vec{E}$ and $\vec{B}$. The total effect is given by the **Lorentz force law**:

$$\boxed{\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})}$$

$\vec{F}_q = q\vec{E}$ is the electrostatic force, and $\vec{F}_m = q\vec{v} \times \vec{B}$ is the magnetic force.

Quantity	Symbol	SI Unit
Total force on the moving charge	$\vec{F}$	N
Charge	q	C
Velocity of the charge	$\vec{v}$	m/s
Magnetic field	$\vec{B}$	T
Electric field	$\vec{E}$	N/C

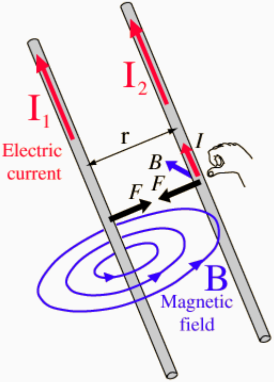
Force on a Current-Carrying Conductor in a Magnetic Field

Likewise, $\vec{B}$ exerts a force on another current-carrying conductor.

$$\boxed{\mathrm{d}F_m = \vec{I}\mathrm{d}\ell \times \vec{B}}$$

Quantity	Symbol	SI Unit
Magnetic force on the conductor	$\vec{F}_m$	N
Electric current in the conductor	$\vec{I}$	A
Length of the conductor	ℓ	m
Magnetic field	$\vec{B}$	T

Magnetic Force on Two Current-Carrying Wires



Two parallel current-carrying wires of length L are at a distance r apart. Magnetic field at wire 2 from current I_1 has constant strength along the wire, given by:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

The force of B on I_2 is:

$$F = I_2 L B = \frac{\mu_0 I_1 I_2 L}{2\pi r} \rightarrow \boxed{\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

I_1 also exerts the same force on I_2, pulling the wires toward each other. (We should expect this because of third law of motion.)

Circular Motion Caused by a Magnetic Field

When a charged particle enters a magnetic field at right angle...

- Magnetic force $\vec{F}_m$ perpendicular to both velocity $\vec{v}$ and magnetic field $\vec{B}$.
- Results in circular motion

Centripetal force $\vec{F}_c$ is provided by the magnetic force $\vec{F}_m$. Equating the two expressions:

$$\frac{mv^2}{r} = qvB$$

We can solve for r get the radius for a charge with a known mass, or solve for mass m of a charged particle based on its radius:

$$r = \frac{mv}{qB} \qquad m = \frac{qrB}{v}$$