Topic 6: Rotational Motion of a Rigid Body

Advanced Placement Physics 1

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Olympiads School

Torque

Equilibrium: First Law of Motion

An object is in **translational equilibrium** is when the net unbalanced force acting it is zero:

$$\mathbf{F}_{\text{net}} = \mathbf{0}$$

Having no net force does *not* mean that the object has no translational motion; it just means that the object's overall *transtational state* is not changing, i.e. the translational momentum **p** is constant. For constant mass, the acceleration of its center of mass is zero.

Equilibrium: First Law of Motion

Likewise, an object is in **rotational equilibrium** when the net torque acting on it is zero:

$$\boldsymbol{\tau}_{\text{net}} = \mathbf{0}$$

Having no net torque does *not* mean that the object has no rotational motion; it just means that the object's overall *rotational state* is not changing, i.e. $\alpha = \mathbf{0}$, or that the **angular momentum L** is constant.

Second Law of Motion

For translational motion, the general form of the second law of motion states that the net force is rate of change of the object's momentum:

$$\bar{\mathbf{F}}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

For objects with constant mass, the second law reduces to the more familiar form:

$$\mathbf{F} = m\mathbf{a}$$

Second Law of Motion for Rotational Motion

Likewise, the second law of motion for rotational motion has a very similar form, but with average torque $\overline{\tau}$ replacing average force $\overline{\mathbf{F}}$, and angular momentum $\mathbf{L}$ replacing linear momentum $\mathbf{p}$:

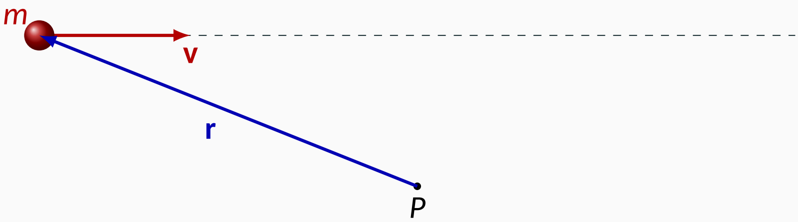
$$\boxed{\overline{\tau} = \frac{\Delta \mathbf{L}}{\Delta t}}$$

For objects with constant momentum of inertia *I* (instead of constant mass *m* in translational motion), the second law reduces to:

$$\tau = I\alpha$$

But there is no rotational motion, is there?

Even when there is no apparent rotational motion, it does not necessarily mean that angular momentum is zero! In this case, mass m travels along a straight path at constant velocity (uniform motion), but the angular momentum around point P is not zero:



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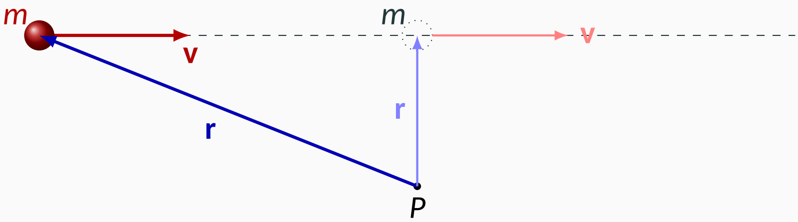

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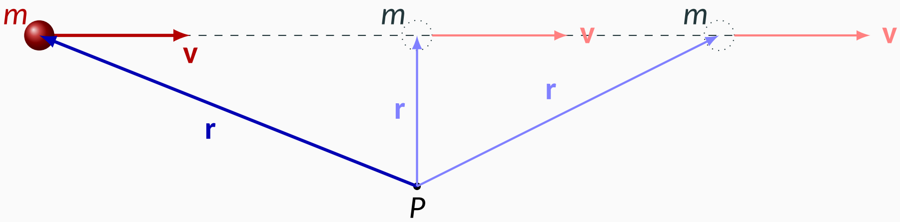

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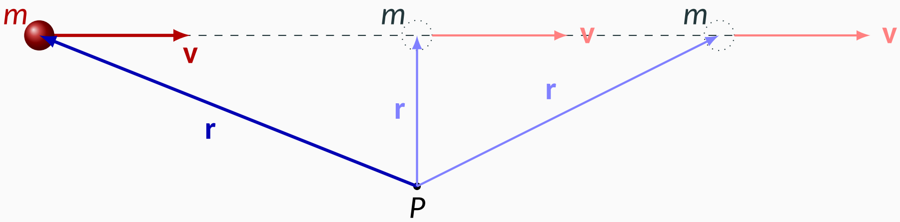

Since there is no force and no torque acting on the object, both the linear momentum ($\mathbf{p} = m\mathbf{v}$) and angular momentum ($\mathbf{L} = \mathbf{r} \times \mathbf{v}$) are constant.