

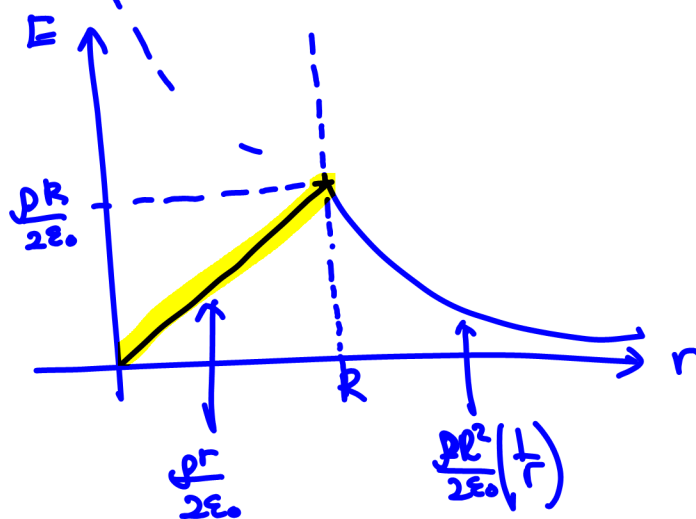
$$2E = \frac{\rho r}{\epsilon_0} \rightarrow \underline{E = \frac{\rho r}{2\epsilon_0} \propto r}$$

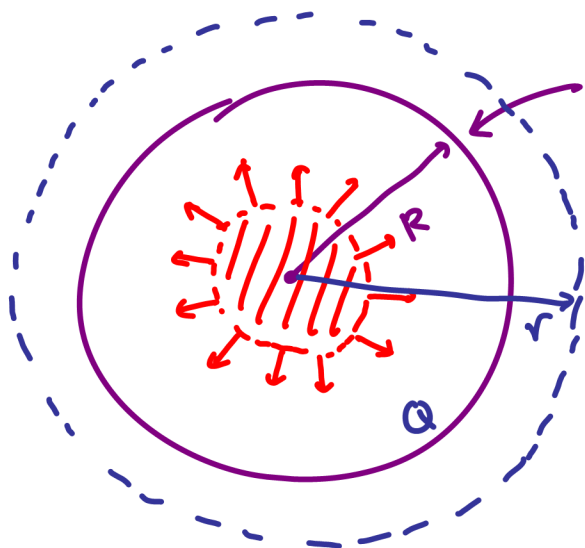
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

constant

$$E \cdot (2\pi r L) = \frac{\pi R^2 L \rho}{\epsilon_0}$$

$$E = \left[\frac{\rho R^2}{2\epsilon_0} \right] \frac{1}{r} \rightarrow E \propto \frac{1}{r}$$





sphere with uniform charge density ρ

total charge

$$Q = \rho V = \frac{4}{3}\pi R^3 \rho \rightarrow \rho = \frac{3Q}{4\pi R^3} \quad \left. \begin{array}{l} \text{positive.} \\ \text{constant.} \end{array} \right\}$$

1. What is the electric field inside the sphere?
2. What is the electric field outside of the sphere?

1. Gaussian surface inside the sphere at radius $r < R$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E(A) = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0 R^3} \rho r^3$$

$$E = \frac{Q}{4\pi \epsilon_0 R^3} r$$

$$E \propto r$$

$$E = \frac{kQ}{R^3} r$$

$$\rho V = \left(\frac{3Q}{4\pi R^3} \right) \left(\frac{4}{3}\pi r^3 \right) = \frac{Q}{R^3} r^3$$

2. Gaussian surface outside of the sphere at radius $r > R$

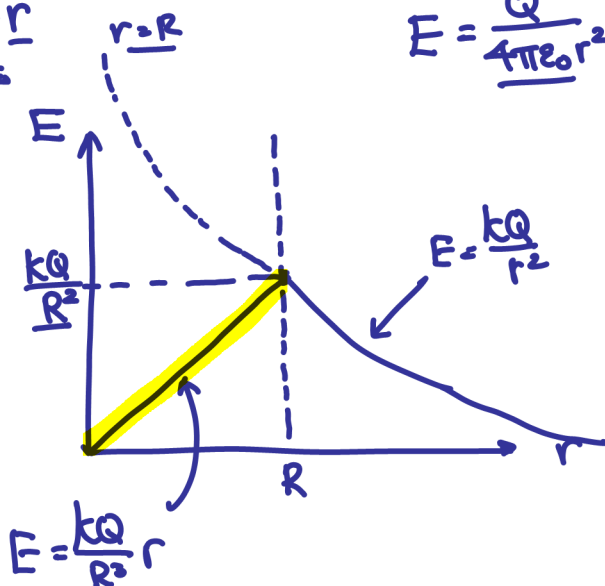
$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

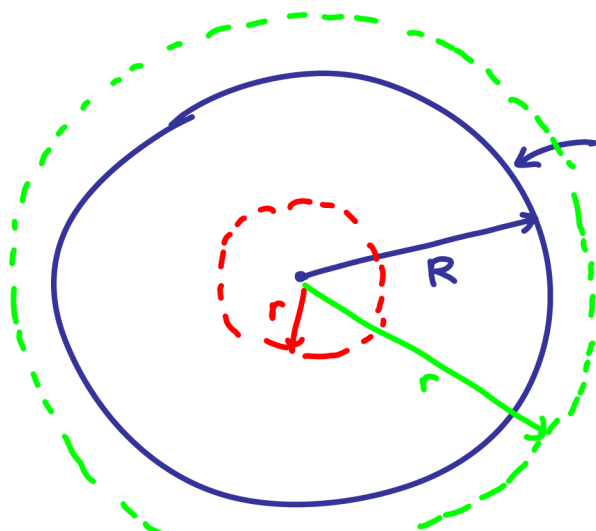
$$E(A) = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi \epsilon_0 r^2} \rightarrow \frac{kQ}{r^2}$$

looks a point charge.





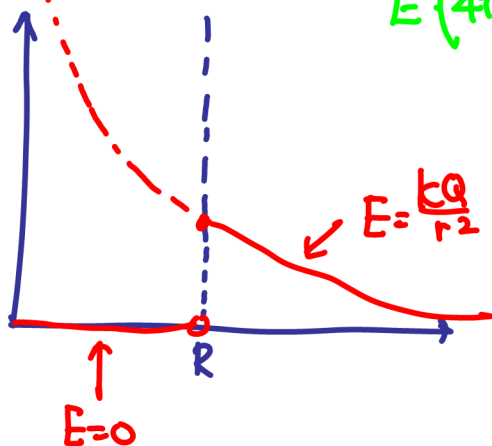
charged spherical shell
total charge $+Q$.

1. What is the electric field inside the sphere?
2. What is the electric field outside the sphere?

1. $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} = 0$
 $\vec{E} = 0$.

2. $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} = \frac{Q}{\epsilon_0}$

$E(4\pi r^2) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$



DOES GAUSS' LAW HAVE AN EQUIVALENT FOR GRAVITY?

Answer: YES!!!!

no one ever writes it this way!
Just for illustration!

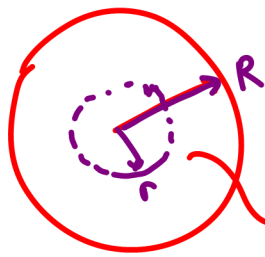
electricity
 $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} = 4\pi k Q_{enc}$

$k = \frac{1}{4\pi\epsilon_0}$

$\frac{1}{\epsilon_0} = 4\pi k$

Gravity inward.
 $\boxed{\oint \vec{g} \cdot d\vec{A} = -4\pi G M_{enc}}$
 Gauss's law for gravity.

A solid sphere (radius R) with charge density that varies with distance r to the center



$$\rho(r) = br^2$$

(+) constant

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \rightarrow \mathbf{E} \cdot (\cancel{4\pi r^2}) = \frac{4\pi b r^3}{5\epsilon_0}$$

$$\mathbf{E} \cdot A = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\boxed{E = \frac{br^3}{5\epsilon_0}}$$

What is the electric field inside the sphere?

aside:

Volume of a sphere

$$V = \int_0^R dv = \int_0^R (4\pi r^2 dr) = \frac{4}{3}\pi R^3$$

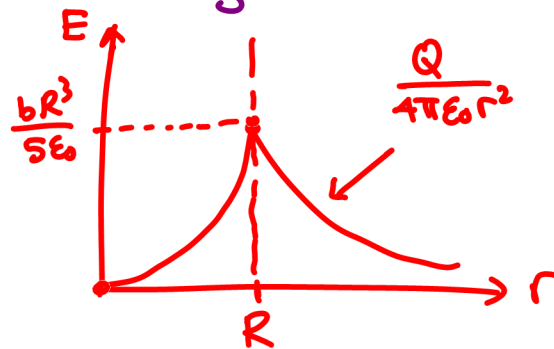
charge:

$$Q = \int dQ = \int \rho dv = \int \rho (4\pi r^2 dr)$$

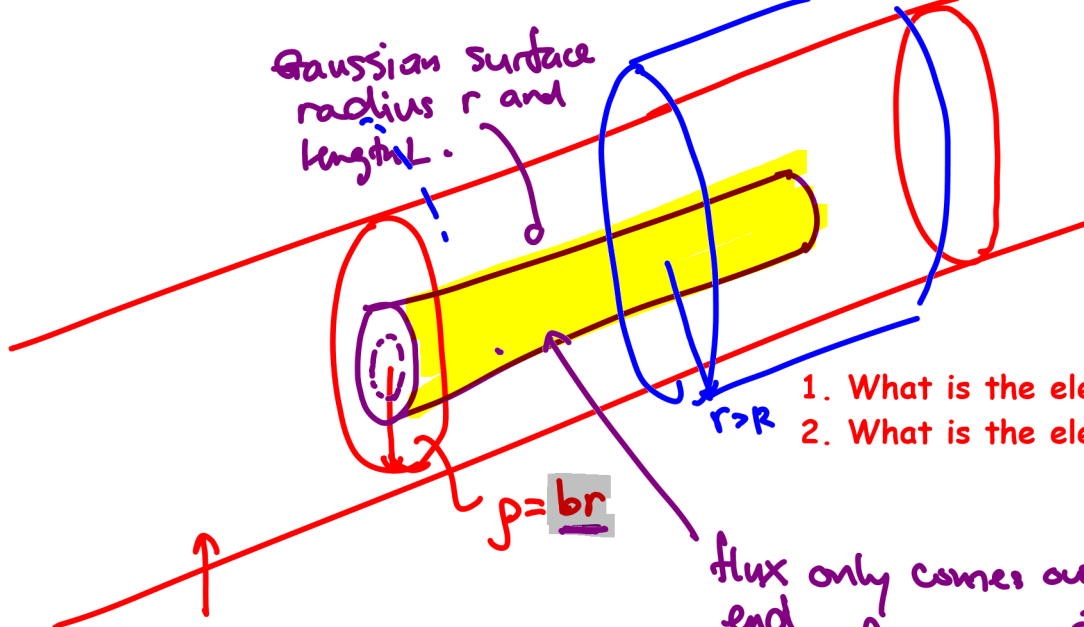
$$Q_{\text{enc}} = \int_0^r dQ = \int_0^r \rho(r) 4\pi r^2 dr$$

$$= \int_0^r (br^2) 4\pi r^2 dr = 4\pi b \int_0^r r^4 dr$$

$$Q_{\text{enc}} = \frac{4\pi b}{5} r^5$$



Gaussian surface
radius r and
length L .



1. What is the electric field inside the rod?
2. What is the electric field outside the rod?

flux only comes out of the side, not the end.

$$\oint E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

$$E \cdot (2\pi r L) = \frac{2\pi b L}{3\epsilon_0} r^3$$

$$E = \frac{br^2}{3\epsilon_0}$$

Infinitely long rod with radius R

$$Q_{enc} = \int_0^r dQ = \int_0^r \rho dV = \int_0^r \rho (2\pi r L dr)$$

$$= \int_0^r (br) (2\pi r L) dr = 2\pi b L \int_0^r r^2 dr$$

$$= \frac{2\pi b L}{3} r^3$$

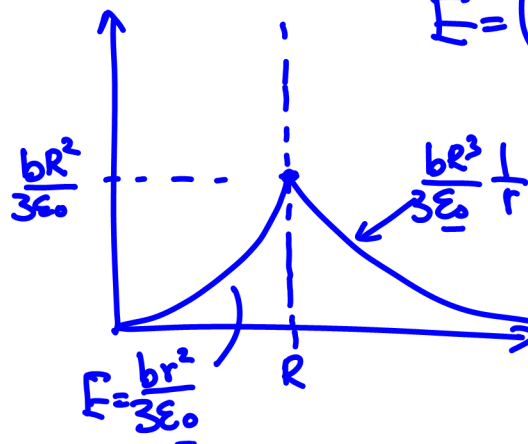
$$Q_{enc} = \int_0^R dQ = \frac{2\pi b L}{3} R^3$$

outside

$$\oint E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

$$E (2\pi r L) = \frac{2\pi b L R^3}{3\epsilon_0}$$

$$E = \left(\frac{b R^3}{3\epsilon_0} \right) \frac{1}{r}$$



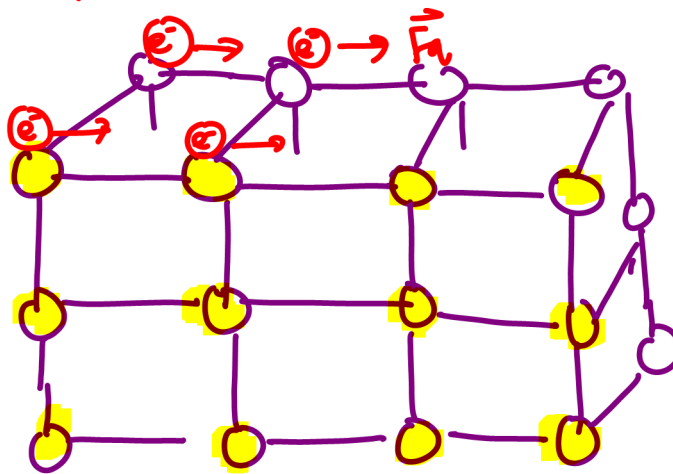
E



In a conductor, each atom has 1 or 2 "free electrons" that can "hop" from atom to atom

The motion of the electrons create electric current

Atoms are organized in a "lattice" structure



+ block of metal

(-) charged block of metal

more or less electrically neutral.
electrons are pushed away from the (+) charge.
 \therefore (+) charge on the edge

excess electrons on this side
 \sim electrically neutral.

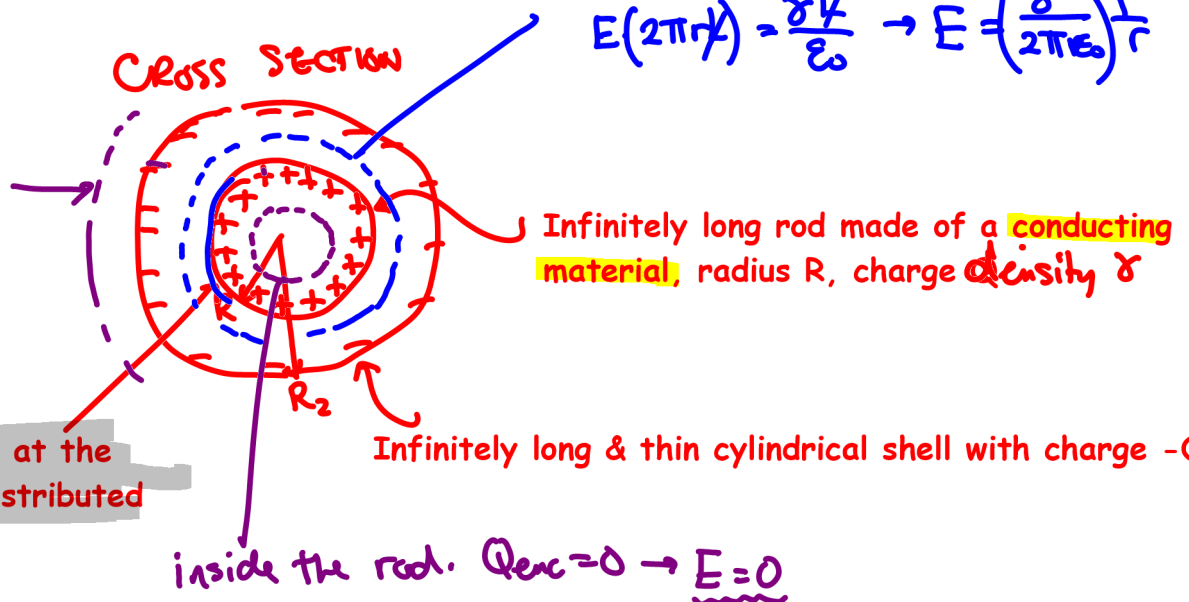
between $R \rightarrow R_2$

$$E(2\pi r l) = \frac{\sigma l}{\epsilon_0} \rightarrow E = \left(\frac{\sigma}{2\pi\epsilon_0}\right) \frac{1}{r}$$

for $r > R_2$

$$Q_{enc} = 0$$

$$\underline{E = 0}$$



Charge on the rod is at the surface, but NOT distributed uniformly inside.

Infinitely long & thin cylindrical shell with charge $-Q$

inside the rod. $Q_{enc} = 0 \rightarrow \underline{E = 0}$