# Topic 17: Nuclear Physics

AP Physics 2

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# Nucleus of the Atom

# The Rutherford-Bohr Model

In the **Rutherford-Bohr atomic model**, an atom consists of

- A dense positively-charged nucleus
- Negatively-charged electrons “orbiting” the nucleus in predefined energy levels
- Most of the atom is empty space

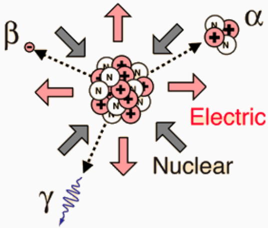
Most of the atom is empty space, although almost all the mass is concentrated at the nucleus

# Nucleus of an Atom

The nucleus—where most of the mass of the atom is concentrated—are made up of **nucleons** which are:

- Positively charged **protons**, and
- Electrically neutral **neutrons**

# Balancing Fundamental Forces




The nucleus of an atom is held together by balancing two fundamental forces:

- **Electromagnetic force**: the repulsive force between protons
  - The force drop off as the square of the distance (inverse square law)
- **Nuclear strong force**: short-distance force holding nucleons together
  - Attractive between nucleons at about $1$ fm ($10^{-15}$ m)
  - Insignificant at distances beyond $2.5$ fm
  - Repulsive at distances below $0.7$ fm

# Atomic Properties

The nucleus of an atom is identified by two numbers:


- **Atomic number** $(Z)$: the number of protons
    - Determines what element it is, and
    - its chemical properties
- **Mass number** $(A)$: number of nucleons
- The number of neutrons $(N)$ is therefore $N = A - Z$

# Isotopes

Carbon has three common **isotopes**, all with the same atomic numbers, but different number of neutrons (called "carbon-12", "carbon-13" and "carbon-14" respectively). The atomic number is sometimes omitted.

$$^{12}C \quad ^{13}C \quad ^{14}C$$

Hydrogen also has three common isotopes (hydrogen, deuterium and tritium)

$$^{1}H \quad ^{2}H \quad ^{3}H$$

On average, there are $2.6$ isotopes for each element.

# Mass of the Nucleus

The **unified atomic mass unit** (u)[1] is defined as $1/12$ of the rest mass of a carbon-12 atom in its nuclear ground state:

$$1\,\mathrm{u} = \frac{1}{12} m(^{12}\mathrm{C}) \approx 1.660\,54 \times 10^{-27}\,\mathrm{kg}$$

The rest mass of the elementary particles can be expressed in this unit:

| **Particle** | **Mass** (u) | **Mass** (kg) |
|---|---|---|
| Proton | $1.007\,276$ | $1.672\,614 \times 10^{-27}$ |
| Neutron | $1.008\,665$ | $1.674\,920 \times 10^{-27}$ |
| Electron | $0.000\,549$ | $9.109\,56 \times 10^{-31}$ |

[1] It replaces the "atomic mass unit" (amu) which is no longer used. The atomic mass value in the periodic table is a weighted average across all isotopes.

# Mass Defect & Nuclear Binding Energy

# Mass-Energy Equivalence

One of the discoveries in special relativity[2] is the fundamental equivalence of mass and energy. While it is based on *relativistic kinematics*, it also applies to *quantum mechanics*, from which nuclear physics is based on.

$$\boxed{E = mc^2}$$

| Quantity | Symbol | SI Unit |
|---|---|---|
| Energy | $E$ | J |
| Rest mass of a particle | $m$ | kg |
| Speed of light | $c$ | m/s |

The speed of light is a universal constant: $c = 2.998 \times 10^8$ m/s.

[2] Einstein, A., “Does the Inertia of a Body Depend Upon Its Energy Content?”, *Annelen der Physik*, 18(13):639-641, 21 November, 1905.

# Mass-Energy Equivalence

Any change in energy is *always* accompanied by a change in mass, and vice versa.

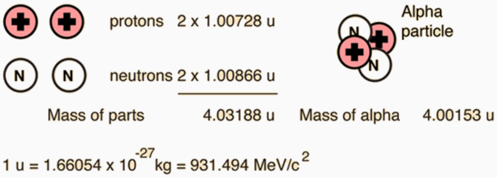
$$E = mc^2 \quad \longrightarrow \quad \Delta E = \Delta mc^2$$

- The concept is difficult to visualize in everyday life. Example: A baseball moving at $160\,\mathrm{km/h}$ is more massive than a baseball travelling at $20\,\mathrm{km/h}$, but the $\Delta m$s is too small to measure.
- But in cases when
  - speeds approaches the speed of light ($v > 0.3c$), or when
  - the masses are small (e.g. electrons, protons, neutrons)

  the difference significant

# Mass Defect

The sum of the masses of the nucleons are always higher than the mass of the nucleus itself. **Example:** the mass of the alpha particle (helium nucleus) is lower than 2 protons plus 2 neutrons:



# Mass Defect

This difference in mass is called the **mass defect** $\Delta m$, which can be calculated with a simple equation:

$$\boxed{\Delta m = [Zm_P + (A - Z)m_N] - m_A}$$

| Quantity | Symbol | SI Unit |
|---|---|---|
| Mass defect | $\Delta m$ | kg |
| Atomic number and mass numbers | $Z$, $A$ | |
| Rest mass of a proton and neutron | $m_P$, $m_N$ | kg |
| Rest mass of the nucleus | $m_A$ | kg |

# Mass Defect

Mass defect exists in all nuclei because the nucleus of an atom is always *in a lower energy state* than the individual nucleons alone

Similar examples:

- The total energy of a planet orbiting the Sun is lower than the plant and the Sun individually
- An electron orbiting the nucleus is at a lower energy state also

# Nuclear Binding Energy

The amount of energy that is equivalent to the mass defect is called the **nuclear binding energy** $E_b$, defined as:

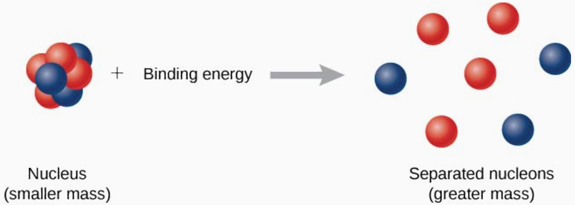
$$\boxed{E_b = (\Delta m)c^2}$$

| **Quantity** | **Symbol** | **SI Unit** |
|---|---|---|
| Nuclear binding energy | $E_b$ | J |
| Mass defect | $\Delta m$ | kg |
| Speed of light | $c$ | m/s |

- The energy required to break up the nucleus into its individual nucleons
- Generally expressed in *electron volts* (eV) where $1\ \text{eV} = 1.602 \times 10^{-19}$ J, rather than in joules

# Nuclear Binding Energy

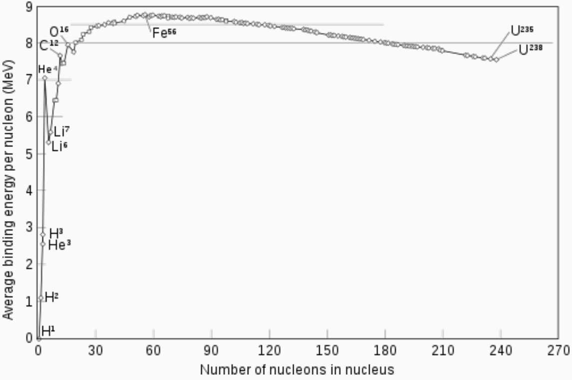
The nuclear binding energy is the amount of work required to separate the nucleons




The higher the binding energy, the more *tightly bound* a nucleus is, therefore more work is required to separate the nucleons

# Nuclear Binding Energy & Stability of the Nucleus

The nuclear binding energy is highest for iron-56 ($E_b = 492.275$ MeV, or $8.7906$ MeV per nucleon)




It means that the nucleus

- requires the most energy to separate the nucleons
- most tightly bound
- most stable

To achieve greater stability in the nucleus

- Heavier atoms can split into lighter nuclei, while
- Lighter atoms can combine into heavier nuclei

# Radioactivity

# Radioactive Decay

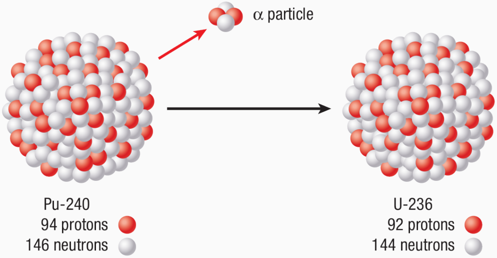
**Radioactivity**[3] is the spontaneously disintegration of a nucleus. There are *three* types of radioactive decay:

- **Alpha decay**, or $\alpha$-decay
- **Beta decay**, or $\beta$-decay
- **Gamma decay**, or $\gamma$-decay

[3] Or **radioactive decay**

# Alpha Decay

In $\alpha$-decay, an **alpha particle** (a helium-4 nucleus, with 2 protons and 2 neutrons), is spontaneously emitted from the nucleus. Example: plutonium-240 nucleus decays into a uranium-236 nucleus, emitting an alpha particle




$$\boxed{{}^{240}_{94}\mathrm{Pu} \longrightarrow {}^{236}_{92}\mathrm{U} + {}^{4}_{2}\mathrm{He}}$$

Note the binding energy in the helium-4 nucleus in previous slides

# General Formula for Alpha Decay

The general formula for an $\alpha$-decay is shown by the equation:

$$\boxed{{}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}}$$

The **parent atom** $(X)$ is the "reactant", and the **daughter atom** $(Y)$ is the "product". This reaction is called a **transmutation**, because a new element is formed.

- The nuclear binding energy of the daughter atom and the alpha particle are *higher* than the parent atom.
- The combined mass of the daughter atom plus the alpha particle is *lower* than the parent atom, meaning that energy is released

# Beta Decays

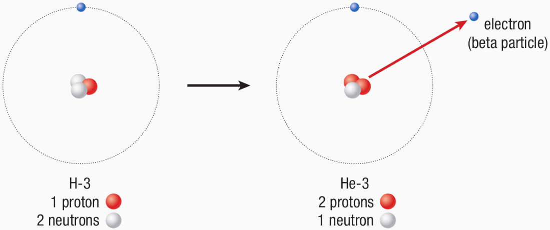
**Beta decays** involve the emission/capture of a **beta particle** (electron or a positron). There are three types of beta decays:

- Beta-negative decay (emission of an electron)
- Beta-positive decay (emission of an positron)
- Electron capture

Beta decays generally occur in smaller (less massive) nuclei

# Beta-Negative Decay

In a **beta-negative decay** ($\beta^-$-decay), a neutron spontaneously decays into a proton and an electron, and the electron is ejected from the nucleus. For example, the $\beta^-$-decay of a tritium atom is:




$$\boxed{{}^{3}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + {}^{0}_{-1}\mathrm{e}}$$

# Beta-Negative Decay

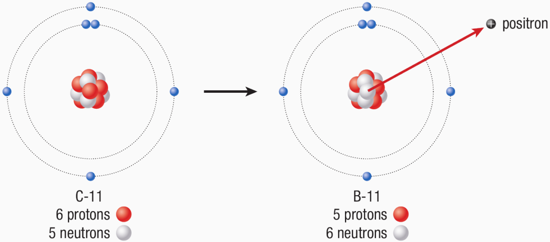
The general form for $\beta^-$-decay is given by:

$$\boxed{{}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\mathrm{e}}$$

The daughter atom has a higher nuclear binding energy than the parent atom. This decay is also a transmutation because a new element is formed.

# Beta-Positive Decay

In **beta-positive decay** ($\beta^+$-decay), a proton spontaneously decays into a neutron and a positron. Example: decay of carbon-11 into boron-11:




$$\boxed{{}^{11}_{6}\mathrm{C} \longrightarrow {}^{11}_{5}\mathrm{B} + {}^{0}_{+1}\mathrm{e}}$$

A positron has the same mass as an electron, but with a *positive* elementary charge. It is not a stable particle.

# Beta-Positive Decay

The general equation for $\beta^+$-decay is given by:

$$\boxed{{}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + {}^{0}_{+1}\mathrm{e}}$$

Similar to a $\beta^-$ decay, the daughter nucleus has a higher nuclear binding energy than the parent nucleus (i.e. lower energy state). This decay is also a transmutation because a new element is formed.

# Electron-Positron Annihilation

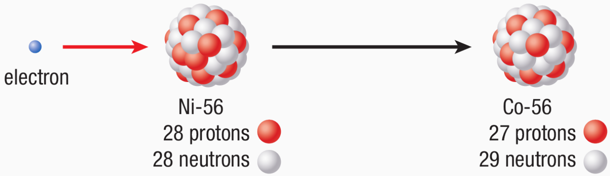
The positron released in the decay will be (almost) immediately *annihilated*[4] by another electron, releasing a pair of gamma particle (photons with no mass), each with an energy of $0.511$ MeV, moving in opposite directions at the speed of light:

$$\boxed{{}^{0}_{+1}\mathrm{e} + {}^{0}_{-1}\mathrm{e} \longrightarrow 2\left({}^{0}_{0}\gamma\right)}$$

[4] It means that the two particles collide, and all of their masses are converted into energy

# Electron Capture

**Electron capture** is a rarer form of beta decay where an electron is absorbed by a nucleus and combines with a proton to form a neutron. For example, a nickel-56 nucleus can capture an electron to form a cobalt-56 nucleus:




$$\boxed{{}^{56}_{28}\mathrm{Ni} + {}^{0}_{-1}\mathrm{e} \longrightarrow {}^{56}_{27}\mathrm{Co}}$$

# Electron Capture

The general equation for electron capture is given by:

$$\boxed{{}^{A}_{Z}\mathrm{X} + {}^{0}_{-1}\mathrm{e} \longrightarrow {}^{\phantom{Z-}A}_{Z-1}\mathrm{Y}}$$

The electron that is absorbed usually comes from the lowest energy shell ($n = 1$, or "K-shell") that is closest to the nucleus, so it is often called *K-capture*.

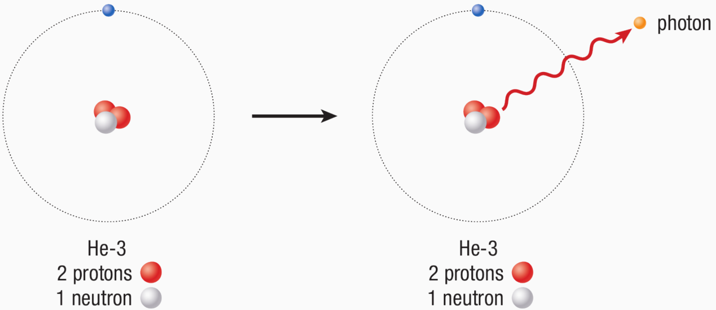
# Gamma Radiation

**Gamma decay** ($\gamma$-decay) occurs after a nuclear reaction (e.g. $\alpha$ or $\beta$ decay)

- The daughter nucleus is in a high-energy (excited) state
- The nucleus spontaneously releases energy in a **gamma particle** to return to a lower (therefore more stable) energy state.
- A gamma particle:
    - is a highly energetic form of electromagnetic radiation that is emitted as a **photon**
    - Has zero mass

# Gamma Radiation

As an example, the $\gamma$-decay of a helium-3 atom is given by:




$$\boxed{{}^{3}_{2}\mathrm{He}^{*} \longrightarrow {}^{3}_{2}\mathrm{He} + {}^{0}_{0}\gamma}$$

# Gamma Radiation

The general equation for $\gamma$-decay is:

$$\boxed{{}^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + {}^{0}_{0}\gamma}$$

- The parent and daughter nuclei are identical
- Only the energy level of the nucleus has changed
    - The asterisk indicates that the parent an excited state
- Notice that the mass number and atomic number of a gamma ray (${}^{0}_{0}\gamma$) are both zero.

# Energies of Radiation

Radioactive particles post danger to living tissues, because

- they can ionize (or strip the electrons from) atoms
- $\alpha$ particles have strongest ionizing ability, but can only travel a relatively short distance before becoming absorbed
- $\beta$ particles and $\gamma$ rays have a greater penetrating range in air and must be shielded against

| Type | Radiation | Charge | Penetrating Ability |
|---|---|---|---|
| $\alpha$-decay | Alpha particle (He-4 nucleus) | $+2$ | Skin or paper |
| $\beta^-$-decay | Beta particle (electron) | $-1$ | thin sheet of aluminum |
| $\beta^+$-decay | Beta particle (positron) | $+1$ | thin sheet of aluminum |
| $e^-$ capture | None | N/A | N/A |
| $\gamma$-decay | Gamma particle (photon) | $0$ | Few centimetres of lead |

# Half-Life

While the radioactive decay of a single atom is *random* (like all quantum mechanics, radioactivity is driven by probability), when there are a large number of atoms, the overall rate of decay is very predictable.

$$\boxed{N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{\tau}}}$$

| Quantity | Symbol | SI Unit |
| --- | --- | --- |
| Amount of radioactive material | $N(t)$ | kg |
| Initial sample amount | $N_0$ | kg |
| Time | $t$ | s |
| Half-life | $\tau$ | s |

**Half-life:** the time it requires for a radioactive material to decay to half of its original amount.

# Half-Life of Radioactive Isotopes

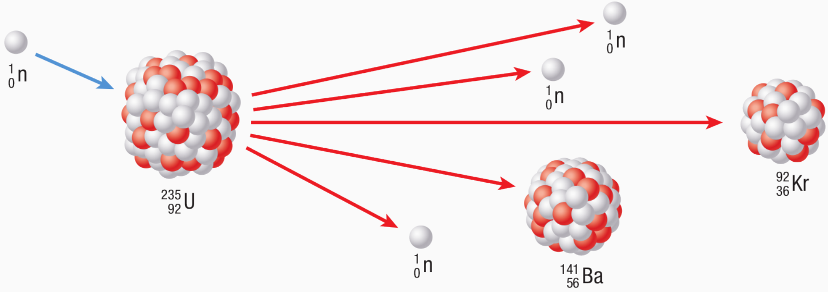
The half-life of radioactive substance can vary from a fraction of a second to billions of years.

| **Substance** | **Half-Life** ($t_0$) |
|---|---|
| Polonium-215 | $0.0018$ s |
| Bismuth-212 | $60.5$ s |
| Sodium-24 | $15$ h |
| Iodine-131 | $8.07$ d |
| Cobalt-60 | $5.26$ yr |
| Radium-226 | $1600$ yr |
| Uranium-238 | $4.5 \times 10^9$ yr |

# Nuclear Fission

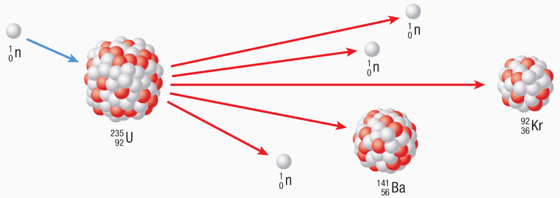
# Nuclear Fission

The release of energy in a nuclear reaction primarily comes from **nuclear fission**, where a heavier atomic is split into lighter atoms. For example, the fission reaction of uranium-235 splitting into krypton-92 and barium-141 atoms:




$$\boxed{^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{92}_{36}Kr + ^{141}_{56}Ba + 3\left(^{1}_{0}n\right) + \text{energy}}$$

# Nuclear Fission




- Fission reaction begins with the uranium nucleus capturing a neutron
- The addition of hte neutron deforms the nucleus into a double-loped “drop” shape
- The attractive nuclear strong force can no longer hold the two fragments together, and they electrically repel each other away
- In splitting the atom, several neutrons are released from the nucleus
- The neutrons may be captured by other uranium-235 atoms, causing further reaction

# Nuclear Fission

- By probability, a nuclear fission is usually *binary* (two daughter nuclei in the process)
- Nuclear fuels can be
  - **Fissile:** the capture of any neutron will be sufficicient to cause fission, e.g. $^{235}$U or $^{239}$Pu
  - **Fissionable:** requires additional energy from a fast-moving neutron to cause fission to begin, e.g. $^{238}$U
- Fission products usually cluster masses of $95 \pm 15$ u and $135 \pm 15$ u

# Example Problem

**Example:** What is the energy yield of the following fission reaction?

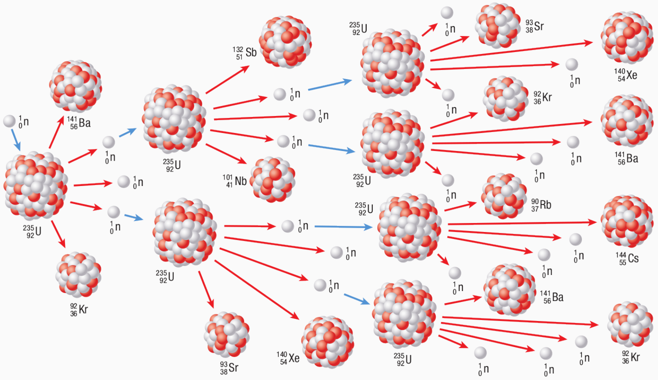
$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{140}_{55}\mathrm{Cs} + ^{93}_{37}\mathrm{Rb} + 3\left(^{1}_{0}\mathrm{n}\right)$$

$$\begin{aligned} m(\text{U-235}) &= 235.044\,\text{u} \\ m(\text{Cs-140}) &= 139.909\,\text{u} \\ m(\text{Rb-93}) &= 92.922\,\text{u} \\ m(\text{n}) &= 1.009\,\text{u} \end{aligned}$$

# Chain Reaction

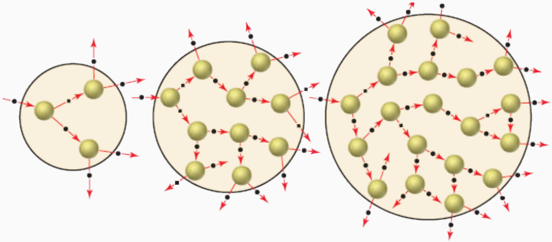
A **chain reaction** is a series of reactions that can repeat over several cycles

- Reactions occur without requiring any material being added to the system



# Critical Mass

The amount of fuel required to sustain a chain reaction is called **critical mass**



**Subcritical mass**
Rate of neutron loss
$>$ rate of neutron
creation by fission

**Critical mass**
Rate of neutron loss
$=$ rate of neutron
creation by fission

**Supercritical mass**
Rate of neutron loss
$<$ rate of neutron
creation by fission

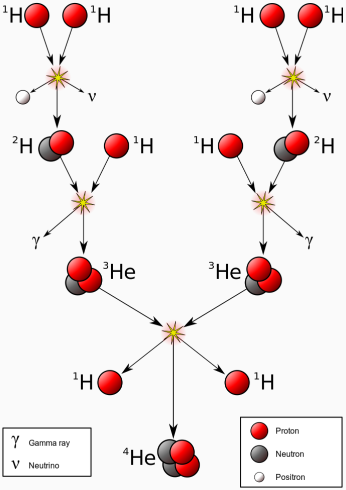
# Nuclear Fusion

# Proton-Proton Reaction

The simplest fusion reaction occurs under high temperature about $4 \times 10^7$ K is the proton-proton chain. In most the basic form, it is written as:

$$4\left({}_1^1\text{H}\right) \longrightarrow {}_2^4\text{He} + 2\left({}_{+1}^{\;0}\text{e}\right) + \text{energy}$$

In each p-p reaction, $26.732$ MeV is released. The exact mechanism for the p-p chain is shown in the right.



# CNO Cycle

A fusion reaction that occurs in even higher temperatures (between 1.5 to $1.7 \times 10^7$ K) is the carbon-nitrogen-oxygen (CNO) cycle. The total energy released in one cycle is $26.73$ MeV.

- The core temperature of the sun is about $1.56 \times 10^7$ K, so only 1.7% of helium-4 nuclei produced in the Sun are from this process