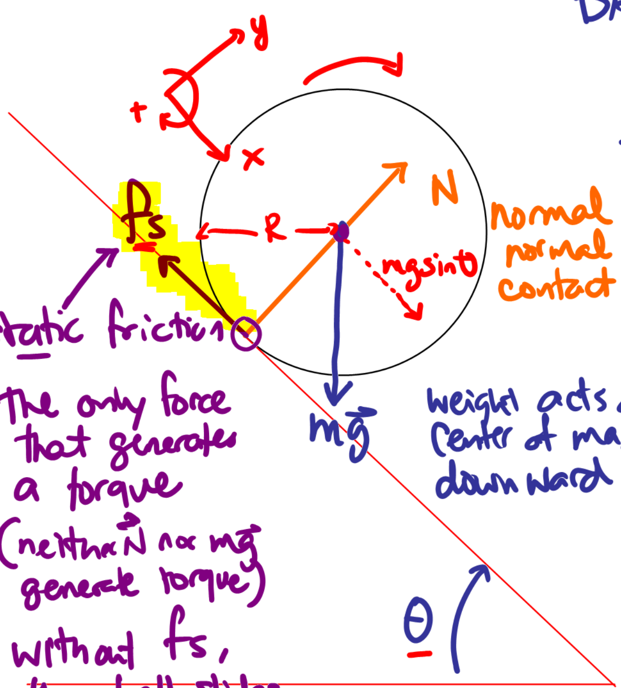
BALL ROLLING DOWN AN INCLINE WITHOUT SLIPPING

- assume rigid body → no deformation



normal force acts normal to the contact point.

static friction
- The only force that generates a torque (neither N nor $m\vec{g}$ generate torque)
- without f_s, the ball slides down instead.

grade 11 problem!!

weight acts at the center of mass. downward

$$\sum F_x = mg\sin\theta - f_s = ma$$

$$\sum F_y = N - mg\cos\theta = 0 \leftarrow \text{this may not have any effect}$$

$$\sum \tau = f_s \cdot R = I\alpha$$

depends on what is rolling down. eq for a solid sphere $I = \frac{2}{5}mR^2$

- if the ball does not slip. then $\begin{cases} a = \alpha R \\ v = \omega R \end{cases}$

- torque

$$f_s \cdot \cancel{R} = \left(\frac{2}{5}mR^{\cancel{2}}\right)\left(\frac{a}{\cancel{R}}\right)$$

$$f_s = \frac{2}{5}ma$$

do not yet know whether this exceed $f_{s,max} = \mu N$

now we can relate friction to translational motion.

put this in the F_x equation

$$mg\sin\theta - \frac{2}{5}ma = ma$$

$$\cancel{m}g\sin\theta = \frac{7}{5}\cancel{m}a \rightarrow \boxed{a = \frac{5}{7}g\sin\theta}$$

→ Sliding case without friction $a = g\sin\theta$

→ rotating without slipping $a = \frac{5}{7}g\sin\theta \leftarrow$ smaller value

need a sanity check: is $\frac{2}{5}ma > f_{s,max}$??

$$\frac{2}{\cancel{5}}\cancel{m}\left(\frac{\cancel{5}}{7}g\sin\theta\right) > \mu \cancel{mg}\cos\theta$$

$$\frac{2}{7}\sin\theta > \mu\cos\theta$$

if this is satisfied, then kinetic friction. the ball slip.