Solutions to Mechanics Mock Exam

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1. **(D)** (A), (B) and (C) are all straight lines which means no acceleration. (E) opens downwards which means negative acceleration.

2. **(D)** Displacement depends only on the initial and final position.

3. **(B)** Velocity is defined as the rate of change of position (a vector) $\dfrac{d\boldsymbol{x}}{dt}$ while acceleration is defined as the rate of change of of velocity $\dfrac{d\boldsymbol{v}}{dt}$ which is also a vector.

4. **(E)** Both projectiles have the same range, but the one at 60° goes higher and therefore stays in the air longer.

5. **(D)** $t = 2\,\text{s}$, there is no external net force, so there is no acceleration.

6. **(A)** By definition, equilibrium means no net force (II) acting on the object, and by the second law of motion, this means no acceleration (I). However, (III) and (IV) are not necessarily true because equilibrium does not imply whether it is stationary or in motion. (It depends on the frame of reference of the observer.)

7. **(A)** The initial velocity can be broken down in to the vertical ($v_0 \sin\theta$) and horizontal ($v_0 \cos\theta$) components. The horizontal component while vertical acceleration is constant due to gravity, g. At P, there is no vertical velocity.

8. **(D)** At Q, horizontal velocity and acceleration is the same as in the previous question, because they are constant. Since Q is at the same height as where the rock is thrown, the vertical speed is the same, at $v_0 \sin\theta$.

9. **(D)** This is a conservation of momentum problem:

$$m_c v_c = (m_c + m_s)v' \quad \longrightarrow \quad v' = \frac{m_c v_c}{m_c + m_s} = \frac{20 \times 4.0}{20 + 5.0} = 3.2\,\text{m/s}$$

10. **(A)** None of the statement is true. At the toy reaches the highest point, it has zero velocity. We can use the conservation of momentum to see that if the 0.02 kg piece moves east, the 0.08 kg piece moves north, therefore (I) is not true. The same conservation of momentum will also show that the 0.08 kg piece will have a slower horizontal speed than the 0.02 kg piece, therefore it lands closer to the launch point, therefore (II) is not true. Finally, neither piece has any vertical velocity when the toy splits, therefore they both reach the ground at the same time, therefore (II) is not true.

11. **(A)** Before the student jumps off the platform, there is no motion, afterwards both are moving, so kinetic energy is not conserved (i.e. the students does positive work). This is analogous to the "explosion problem" in linear momentum conservation, which there is a net kinetic energy after the explosion. Linear momentum is also not conserved because the center of mass of the platform is not moving after the student jumps off (i.e. the axle of the platform exerts an external force as the student jumps off), but angular momentum is conserved because the student does not exert a torque (nor does the external force from the axle).

12. **(D)** The relationship between period of the simple pendulum and its length is related by:

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \longrightarrow \quad T^2 = \frac{4\pi^2}{g}L$$

To get a straight graph, plot $\boxed{T^2 \text{ vs. } L}$.

13. **(B)** When a planet orbits the sun, there is a non-zero angular momentu. Since gravity is a central force, in that it does not generate any torque about the sun, the angular momentum is constant.

14. **(D)** Two satellites orbiting the same planet must have the same constant when using Kepler's third law of planetary motion, i.e.

$$\frac{T_x^2}{R_x^3} = \frac{T_Y^2}{R_Y^3} \quad \longrightarrow \quad \frac{T^2}{R^3} = \frac{(8T)^2}{R_Y^3} \quad \longrightarrow \quad R_Y^3 = 64R^3 \quad \longrightarrow \quad R_Y = \boxed{4R}$$

15. **(D)**

16. **(E)** The acceleration of the object is given by the second law of motion:

$$a = \frac{F}{m} = \frac{-3}{6}t = \frac{-1}{2}t$$

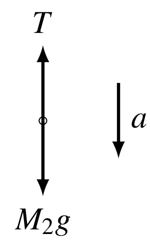
Integrating in time to get the velocity, and then substituting the initial velocity $v_0 = 4\,\mathrm{m/s}$:

$$v = \frac{-1}{2}\int t\,dt = \frac{-1}{4}t^2 + v_0 = \frac{-1}{4}t^2 + 4$$

Setting $v = 0$ and gives $\boxed{t = 4\,\mathrm{s}}$.

17. **(E)** The velocity function is quadratic in time, which means that the position function is cubic. There is an initial positive position, which means that the y-intercept is not zero. (E) is the only graph that fits. There is no calculations needed.

18. **(B)** This can be solved by a simple free body diagram of M_2:



Using the second law of motion:

$$\sum F = ma \quad \longrightarrow \quad M_2 g - T = M_2(0.6g) \quad \longrightarrow \quad T = \boxed{0.4 M_2 g}$$

19. **(D)** The weight $M_2 g$ is the only force that causes motion, so

$$\sum F = ma \quad \longrightarrow \quad M_2 g = (M_2 + M_1)(0.6g) \quad \longrightarrow \quad 0.4 M_2 g = 0.6 M_1 g \quad \longrightarrow \quad \frac{M_2}{M_1} = \boxed{1.5}$$

The most common mistake is to reverse the ratio.

20. **(E)** We can do a quick energy conservation equation:

$$\cancel{K} + \cancel{U_{g1}} + \cancel{U_{g2}} + \cancel{U_e} + \underbrace{W_f}_{<0} = \cancel{K'} + \cancel{U'_{g1}} + \underbrace{U'_{g2}}_{<0} + U'_e$$

Since friction does negative non-conservative work on the system, the gain in elastic potential energy must be less than the loss in gravitational potential energy.

21. **(C)** After the block stops, the forces on Block 1 must be balanced.

22. **(C)** The area under the force vs. time graph is the impuslse, which is also the change in momentum. In this case, the area is in the form of a triangle, with an area of

$$\Delta p = I = \boxed{\frac{1}{2} F t}$$

23. **(E)** This is a simple center of mass calculation. If the distance between the moon and the planet is r_A, then the CM would be located at $\frac{m}{50m} r_A = \frac{1}{50} r_A$ from the planet, or $\boxed{\frac{49}{50} r_A}$ from the moon.

24. **(B)** The angular momentum of the moon about the planet is constant because gravity does not generate a torque about the planet. Therefore

$$L = r_A m v_0 = r_B m (5 v_0) \quad \longrightarrow \quad r_B = \boxed{\frac{1}{5} r_A}$$

25. **(D)** There is no need to do calculations here, but it is worthwhile to see how angular acceleration changes as the bar rotates. The torque generated by the weight is

$$\tau = mg\sin\theta = I\alpha$$

This means that angular acceleration α increases as the bar rotates, reaching *maximum value* at $\theta = 90°$. While θ is not linear with t, (D) is only one graph that shows both an increasing $\alpha(y)$ as well as a one with a *maximum* at the end (i.e. zero slope).

26. **(E)** The angular velocity function can be obtained simply by integrating the angular acceleration function. . . except we don't have to do that! Since we know that angular acceleration is maximum at the end, the slope of the angular velocity graph must also have the steepest positive slope. Graph (E) is the only one that fits that.

27. **(C)** As the stone falls, it is subjected to a constant gravitational force mg, which exerts a constant torque about point P. Therefore the angular momentum about P increases linearly.

28. **(C)** The table shows that the force is related to the spring displacement by $F = x^3$. Integrating this from 0 to 3 gives the potential energy of:

$$U = \int_0^3 F\,dx = \int_0^3 x^3\,dx = \frac{1}{4}x^4\Big|_0^3 = \frac{81}{4}$$

29. **(B)** In such a spring, the velocity function will be given by

$$v(t) = A\omega\cos(\omega t)$$

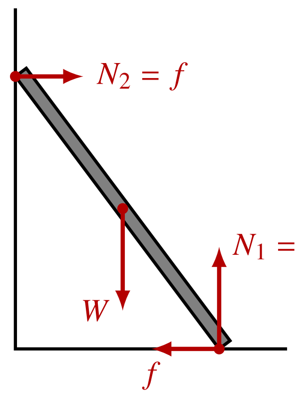
If you have forgotten, you can differentiate the position function $x(t) = A\cos(\sin t)$. It does not matter if you use $\cos(\omega t)$ or $\sin(\omega t)$. The kinetic energy is given by:

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}A^2\underbrace{m\omega^2}_{=m\left(\sqrt{\frac{k}{m}}\right)^2=k}\cos^2(2\omega t) = \boxed{\frac{1}{2}kA^2\cos^2(\omega t)}$$

30. **(C)** The work done by the external force can be found by integrating the force by displacment dx:

$$W = \int_D^0 F\,dx = \int_D^0 (-ax + bx^2)\,dx = \left(\frac{-1}{2}ax^2 + \frac{1}{3}bx^3\right)\Bigg|_D^0 = \boxed{\frac{1}{2}aD^2 - \frac{1}{3}bD^3}$$

31. **(A)** The forces acting on the ladder are:



The normal force exerted by the floor is equaled to the weight of the ladder, i.e. $N_1 = W$, since they are the only vertical forces.

32. **(E)** In order for the ladder to be static, the sum of all the torques must be zero. The net torque can be calculated *anywhere*, but for this problem it is easiest to calculate it at the center of mass. The normal force at the wall must be equal to static friction, $N_2 = f$, because these are the only two horizontal forces. The torques are:

$$\sum \tau = -f\frac{L}{2}\sin\theta - f\frac{L}{2}\sin\theta + W\frac{L}{2}\cos\theta = 0$$
$$W\cos\theta = 2f\sin\theta$$
$$f = \frac{W}{2}\frac{\cos\theta}{\sin\theta} = \boxed{\frac{W}{2}\cot\theta}$$

33. **(E)** When the spring is cut in half, it would take twice as much force to stretch it by the same distance, therefore the spring constant increases by a factor of 2, i.e. $\boxed{2k}$.

34. **(C)** This planet has half the radius of earth, $r' = \frac{1}{2}r$, but the weight of the astronaut is also twice that of Earth, i.e. $g' = 2g$. Using the definition of gravitational field, we find the mass of the planet is half that of Earth:

$$g' = \frac{GM}{r'^2} \quad\longrightarrow\quad (2g) = \frac{GM'}{(\frac{1}{2}r)^2} \quad\longrightarrow\quad 2g = \frac{4GM'}{r^2} \quad\longrightarrow\quad M' = \frac{gr^2}{2G} = \frac{1}{2}M$$

The escape speed of any small object from a much larger planet is given by

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

In this case, decreasing the mass by half, while decreasing the radius by half does not change the escape velocity.

35. **(E)** The gravitational force on m is linearly proportional to the distance from the center $F_g = \frac{GMm}{R^3}r$, therefore the kinetic energy at the center is given by:

$$W = K = \int F_g dr = \frac{GMm}{R^3}\int_0^R r\,dr = \frac{GMm}{2R^3}r^2\Big|_0^R = \boxed{\frac{GMm}{2R}}$$

Which is (III). However, given that the gravitational field at the surface $g = \frac{GM}{R^2}$, the expression for kinetic energy can be written as

$$K = \frac{GMm}{2R} = \frac{GMm}{R^2}\frac{R}{2} = \boxed{\frac{1}{2}mgR}$$