Topic 12: Capacitors

Advanced Placement Physics 2

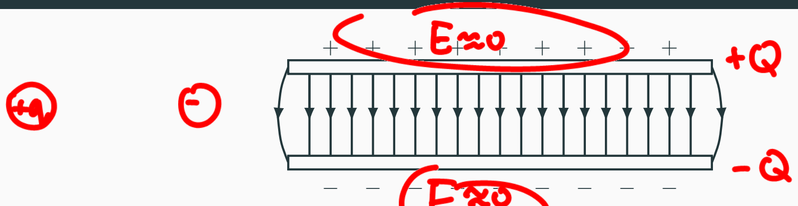
Dr. Timothy Leung

Summer 2021

Olympiads School

Capacitors

Electric Field and Electric Potential Difference



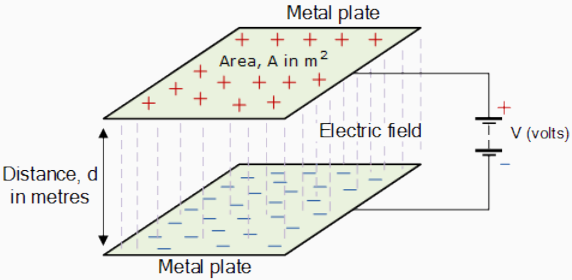
Recall that the electric field between two (infinitely) large parallel plates is uniform, and the relationship between electric field and voltage is given by:

$$\boxed{E = \frac{V}{d}}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C
Electric potential difference between plates	V	V
Distance between plates	d	m

Capacitors

Capacitors is a device that stores energy in an electric field. The simplest form of a capacitor is a set of closely spaced parallel plates:



When the plates are connected to a battery, the battery transfer charges to the plates until the voltage V equals the battery terminals. After that, one plate has charge $+Q$; the other has $-Q$.

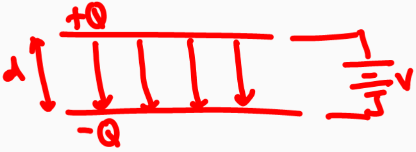
Parallel-Plate Capacitors

As we have seen already, the (uniform) electric field between two parallel plates is proportional to the charge density σ, which is the charge Q divided by the area of the plates A:

$\frac{Q}{A}$

$E = \underbrace{\frac{V}{d}}\ = \frac{\sigma}{\epsilon_0} = \underbrace{\frac{Q}{A\epsilon_0}}$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$



Substituting this into the relationship between the plate voltage V and electric field, we find a relationship between the charges across the plates and the voltage:

$$V = Ed = \frac{Qd}{A\epsilon_0} \longrightarrow \boxed{Q = \left[\frac{A\epsilon_0}{d}\right] V}$$

capacitance

Parallel-Plate Capacitors

Since area A, distance of separation d and the vacuum permittivity ϵ_0 are all constants, the relationship between charge Q and voltage V is *linear*. And the constant is called the **capacitance** C, defined as:

$$\boxed{C = \frac{Q}{V}}$$

$Q = CV$

For parallel plates:

$$\boxed{C = \frac{A\epsilon_0}{d}}$$

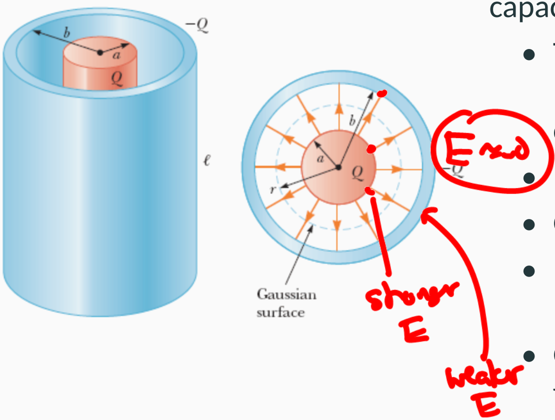
only depends on geometry

$A \gg d$

The unit for capacitance is a **farad** (named after Michael Faraday), where $1\,\mathrm{F} = 1\,\mathrm{C}/\mathrm{V}$.

Cylindrical Capacitors

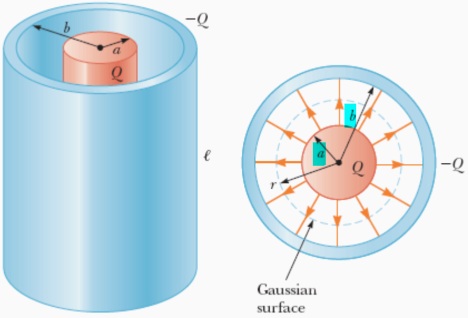
Cylindrical Capacitors



Not all capacitors are parallel plates. Cylindrical capacitors are also popular.

- The capacitor has length ℓ which is much larger than the radii of the inner & outer cylinders (a, b)
- Inner cylinder has total charge Q
- Outer cylinder has total charge $-Q$
- Inside the capacitor, the electric field in the radial direction
- Outside of the capacitor, there is no electric field

Cylindrical Capacitors: Electric Field



Using a bit of calculus, we can also see that, like the parallel plate, the relationship between voltage and charge is still linear. In this case, the capacitance is defined as:

$$\boxed{C = \frac{Q}{V} = \frac{2\pi \ell \epsilon_0}{\ln(b/a)}}$$

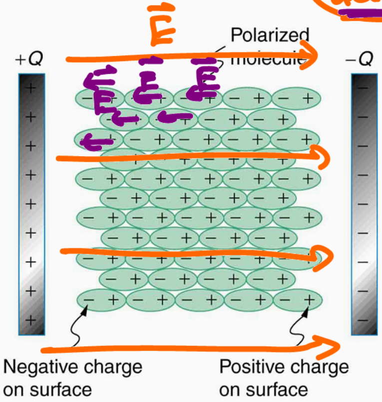
The capacitance is generally expressed in terms of C/ℓ.Capacitance depends only on the geometry (i.e. the raidii *a* and *b*) and the permittiviy.

Practical Capacitors

between capacitor plates.

decrease $\vec{E}$ → decrease voltage for the same Q → increase C

$V = E \cdot d$



- Parallel-plate capacitors are very common in electric circuits, but the vacuum between the plates is not very effective

$\uparrow C = \frac{Q}{V \downarrow}$ ← same

- Instead, a non-conducting **dielectric** material is inserted between the plates
- When the plates are charged, the electric field of the plates polarizes the dielectric.
- The polarization produces an electric field that opposes the field from the plates, therefore reduces the effective voltage, and increasing the capacitance

when dielectric material inserted between the plates, they will create an opposite electric field

$E_{total} = \overbrace{E_{plates} - E_{dielectric}}^{\text{reduced } \vec{E}}$

Dielectric Constant

If electric field without dielectric is E_0, then E in the dielectric is reduced by κ, the **dielectric constant**:

vacuum between plates.

$$\boxed{\kappa = \frac{E_0}{E}}$$

← high

← low

$E_0 = V d$

$E = \frac{V}{\kappa} d$

The capacitance of the plates with the dielectric is now amplified by the same factor κ:

$$\boxed{C = \kappa C_0}$$

$C_0 = \frac{Q}{V}$ $C = \frac{Q}{\left(\frac{V}{\kappa}\right)} = \kappa \frac{Q}{V}$

We can also view the dielectric as something that increases the *effective permittivity*:

$$\boxed{\epsilon = \kappa \epsilon_0}$$

higher.

Permittivity: the material's ability to resist the formation of an electric field; therefore if effective permittivity is higher, then electric field is weaker

Dielectric Constant

The dielectric constants of commonly used materials are:

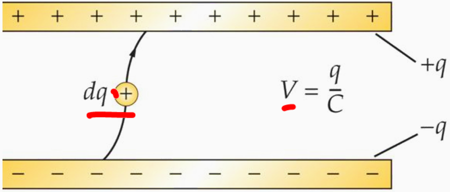
Material	κ
Air	1.000 59
Bakelite	4.9
Pyrex glass	5.6
Neoprene	6.9
Plexiglas	3.4
Polystyrene	2.55
Water (20 °C)	80

— N_2, O_2 not polarized.

Storage of Electrical Energy

When charging up a capacitor, imagine positive charges moving from the $(-)$ plate to the $(+)$ plate.

C



$W = \int V\,dq \rightarrow$

$= \int \frac{q\,dq}{C} \rightarrow \frac{1}{2}\frac{Q^2}{C}$

Initially neither plates are charged, so moving the first charge takes very little work; as the electric field builds, more work needs to be done. The total work done is the potential energy inside the capacitor:

$V = \frac{Q}{C}$

$$\boxed{U_c = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2}$$

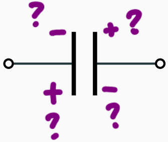
Notes About Storage of Electric Energy

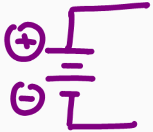
- The presence of a dielectric *increases* the capacitance; therefore the work (and potential energy stored) to move a charge *decreases* with the dielectric constant κ
- After the capacitor is charged, removing the dielectric material from the capacitor plates will require additional work.

Capacitors in Electric Circuits

Capacitors are an important part of an electric circuits because it stores energy in the electric field

- Denoted by this symbol (with reference to the parallel-plate capacitor):





- Act like a voltage source
- Unlike a battery, the voltage increases or decreases as the charge across the capacitor plates change.