#### Class 15: Gauss's Law

Advanced Placement Physics C

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Olympiads School

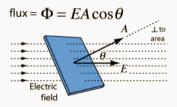
Gauss's Law

#### Flux

**Flux** is an important concept in many disciplines in physics. The flux of a vector quantity  $\vec{X}$  is the amount of that quantity flowing through a surface. In integral form:

$$\Phi = \int \vec{X} \cdot d\vec{A}$$
 or  $\Phi = \int (\vec{X} \cdot \hat{n}) dA$ 

The direction of the infinitesimal area  $d\vec{A}$  is **outward normal** to the surface.

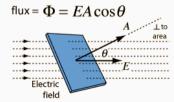


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#### Flux

 $\Phi$  can be something physical, like water, or bananas, or something abstract, like electric field (which is what we are interested in). We can compute a flux as long as there is a vector field i.e.  $\vec{X} = \vec{X}(x, y, z)$ . In the case of **electric flux**, the quantity  $\vec{X}$  is just the electric field, i.e.:

$$\Phi_{\it E} = \int \vec{E} \cdot {\sf d}\vec{\sf A}$$



#### **Electric Flux and Gauss's Law**

**Gauss's law** tells us that if we have a closed surface (think of the surface of a balloon), the total electric flux is very well defined:

$$\Phi_{ extsf{E}} = \oint ec{ extsf{E}} \cdot ext{d} ec{ extsf{A}} = rac{Q_{ ext{encl}}}{\epsilon_0}$$

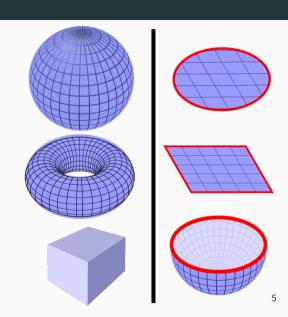
#### where

- Q<sub>encl</sub> is the charge enclosed by the surface
- $\epsilon_0 = 8.85 \times 10^{-12} \, \text{C}^2/\text{N} \cdot \text{m}^2$  is the permittivity of free space

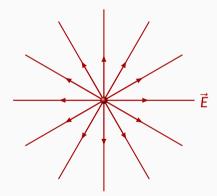
That closed surface is called a Gaussian surface

#### **Closed Surfaces**

A **closed surface** is one that does not have a boundary, like the sphere, toroid, and cube on the left.



# **Electric Field from a Positive Point Charge**



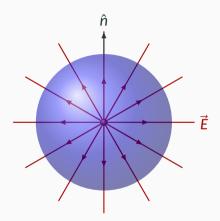
By symmetry, electric field lines must be radially outward from the charge, so the integral reduces to:

$$\Phi_{\mathrm{E}} = \oint \vec{\mathrm{E}} \cdot \mathrm{d} \vec{\mathrm{A}} = \mathrm{E} \mathrm{A} = rac{q}{\epsilon_{\mathrm{O}}}$$

Since area of a sphere is  $A=4\pi r^2$ , we recover Coulomb's law and the magnitude of the electric field from a point charge:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{kq}{r^2}$$

# **Electric Field from a Positive Point Charge**



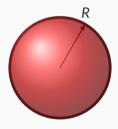
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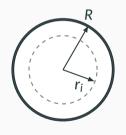
$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{kq}{r^2}$$

For a uniformly-charged spherical thin shell with radius R and a total charge of Q.



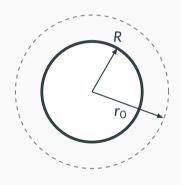
For a uniformly-charged spherical thin shell with radius R and a total charge of Q.

Inside the shell ( $r_i < R$ ), there is no enclosed charge, therefore the electric field must be zero:



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0} = 0 \quad o \quad E = 0$$

For a uniformly-charged spherical thin shell with radius R and a total charge of Q.



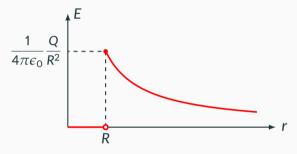
Inside the shell ( $r_i < R$ ), there is no enclosed charge, therefore the electric field must be zero:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0} = 0 \quad o \quad E = 0$$

Outside the shell  $(r_0 > R)$ , the enclosed charge is Q, and the electric field is given by the same equation as the point charge:

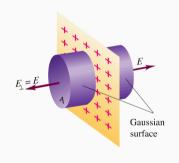
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \rightarrow E = \frac{Q}{\epsilon_0 A} = \frac{1}{4\pi\epsilon_0} \left[ \frac{Q}{r^2} \right]$$

The electric field strength E can be plotted as a function of the distance r from the center of the shell:



- This graph may be familiar because the graph for the gravitational field strength inside a uniform shell is exactly the same
- Replace  $\epsilon_0$  with  $4\pi G$

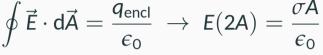
# Electric Field Near an Infinite Plane of Charge



- Charge density (charge per unit area)  $\sigma$
- By symmetry,  $\vec{E}$  must be perpendicular to the plane
- Our Gaussian surface is a cylinder shown in the left with an area A; the height of the cylinder is unimportant
- Nothing "flows out" of the side of the cylinder, only at the ends
- The total flux is  $\Phi_E = E(2A)$
- The enclosed charge is  $Q_{encl} = \sigma A$

# Electric Field Near an Infinite Plane of Charge

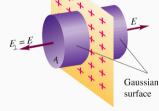




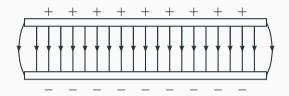
Solving for *E*, we get:

$$E = \frac{\sigma}{2\epsilon_0}$$

- E is a constant
- Independent of distance from the plane
- Both sides of the plane are the same



### **Electric Field Between Parallel Charged Plates**



- Two plates, each producing an electric field pointing in the same direction
- The total electric field is twice the value of *one* infinite plane, pointing from the positively charged plate toward the negatively charged plate

$$E = \frac{\sigma}{\epsilon_0}$$

•  $\vec{E}$  outside the plates is very low (close to zero), except for fringe effects at the edges of the plates

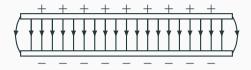
#### **Electric Field and Electric Potential Difference**

Recall the relationship between electric field  $(\vec{E})$  and electric potential difference (V):

$$\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$$

This relationship holds regardless of the charge configuration.

#### **Electric Field and Electric Potential Difference**



In the case of two parallel plates, the electric field is uniform, and the relationship simplifies to:

$$E = \frac{\Delta V}{d}$$

Quantity	Symbol	SI Unit
Electric field intensity	Ε	N/C
Potential difference between plates	$\Delta V$	V
Distance between plates	d	m