Topic 2: Dynamics

Advanced Placement Physics C

Dr. Timothy Leung

August 24, 2020

Olympiads School

Dynamics

While we use **kinematics** to describe the motion of any object mathematically, we use **dynamics** to describe *what* causes motion (or more precisely, *what causes motion to change?*)

- Newton's three laws of motion

Laws of Motion

First Law of Motion

Every body persists in its state of being at rest or moving uniformly straight forward, except insofar as it is compelled to change its state by force impress'd.[1]

- "Moving uniformly" means *uniform motion* with constant velocity
  - An object "at rest" is also in uniform motion with $\mathbf{v} = \mathbf{0}$
- As long as an object moves in uniform motion, it must be that $\mathbf{F}_{\text{net}} = \mathbf{0}$
- Common examples:
  - A hockey puck sliding on very smooth ice has gravity and normal force, but the net force is zero
  - A car traveling on a highway at 100 km/h has many forces acting on it, but the net force is zero

---

[1] Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

Second Law of Motion

The alteration of motion is ever proportional to the motive force impress'd; and is made in the direction of the right line in which that force is impress'd.[2]

The *alteration of motion* means *acceleration*. The first two laws of motion can be summarized in the equation:

$$\boxed{\mathbf{F}_{\text{net}} = \Sigma\mathbf{F} = m\mathbf{a}}$$

This equation is a "special case" that assumes a constant mass. If mass is not constant, the net force is equal to the rate of change of momentum $\mathbf{p}$.

$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt}$$

[2] Lex II: Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.

Mass

What is **mass** then? It is

- the property of an object that relates its acceleration to the force applied to it
- Intrinsic to the object itself (“how much of it”)
- This is explicitly referred to as the object’s **inertial mass**

Third Law of Motion

To every action there is always opposed and equal reaction: the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.[3]

For every action force on an object (B) due to another object (A), there is a reaction force which is equal in magnitude but opposite in direction, on object (A), due to object (B):

$$\boxed{\mathbf{F}_{\text{A on B}} = -\mathbf{F}_{\text{B on A}}}$$

- The action and reaction forces act on different objects!
- Third law is the natural consequence of the first and second law. Action/reaction forces are *internal* forces.

[3] Lex III: Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi.

Forces

A **force** is the interaction between the objects.

- When there is interaction, then forces are created
- A “push” or a “pull”

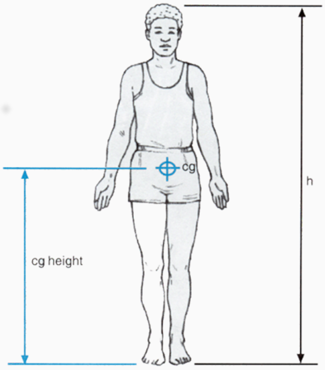
There are two broad categories of forces:

- **Contact forces** act between two objects that are in contact with one another
- **Non-contact forces** act between two objects without them touching each other. They are also called “action-at-a-distance” force

Center of Mass

Newton considered all forces acting at a single point of an object called the center of mass (“CM”)

- The center of mass is also called the center of gravity (“CG”), if the entire object is inside a uniform gravitational field
- If the density of an object is constant, then the CM/CG is also the geometric center (centroid) of the object
- In a later topic, we will study how to compute the centers of mass of different objects



Static & Dynamic Equilibrium

If the net force on an object is zero ($\Sigma \mathbf{F} = \mathbf{0}$) then the object is in a *state of equilibrium*

- Dynamic equilibrium: the object is moving relative to us
- Static equilibrium: the object is not moving relative to us

Common Forces

Common Forces

Common everyday forces that we encounter in Physics 12 include:

- Weight (gravitational force) $\mathbf{w}$ (or $\mathbf{F}_G$)
- Normal force $\mathbf{N}$
- Friction (static $\mathbf{f}_s$ and kinetic $\mathbf{f}_k$)
- Tension $\mathbf{T}$
- Applied force $\mathbf{F}_a$
- Spring force $\mathbf{F}_e$
- Drag $\mathbf{D}$ (fluid resistance, then again in fluid mechanics)
- Buoyant force $\mathbf{B}$ (discussed in fluid mechanics)
- Electrostatic force $\mathbf{F}_E$ (discussed in E & M exam)
- Magnetic force $\mathbf{F}_M$ (discussed E & M exam)

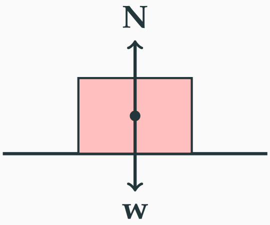
Gravity

Gravity is the force of attraction between all objects with mass

$$\boxed{\mathbf{w} = m\mathbf{g}}$$

- Near surface of Earth, use $g = 9.81\,\mathrm{m/s^2}$ (or $g = 10\,\mathrm{m/s^2}$ for your AP exam)
- You may be asked to find the value of g on some "unknown planet".
- $\mathbf{w}$ always points *down* (the direction of $\mathbf{w}$ is how down is defined)

Normal Force

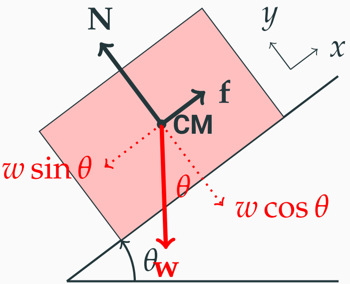


$$\mathbf{w} = m\mathbf{g} = -\mathbf{N}$$

- A force a surface exerts on another object that it is in contact with
- Always **perpendicular** to the contact surface
- **Special case:** When an object is on a horizontal surface with no additional applied force, the magnitude of the normal force is equal to the magnitude of the weight of the object, i.e. $N = w$

Normal Force on a Slope

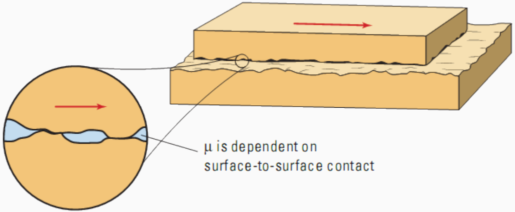
For this case, we label the x-axis to be along the slope, and y-axis to be perpendicular to the slope.



- If on a slope: $N = w\cos\theta$
  - N decreases as ramp angle θ increases
- w has a component along the ramp $w\sin\theta$ that wants to slide the block down.
- Friction force f opposes the motion
  - Be careful: if the block is moving *up* the ramp with an applied force, then f will point *down* the ramp

Friction

- A force that opposes the sliding of two surface against one another
- Always act in a direction that opposes motion or attempted motion
- Depends on:
  - Normal force N: The force the two surfaces are pressed against each other
  - Coefficient of friction (μ_s and μ_k): Smoothness of the surfaces, which itself depends on
    - The material(s) the surfaces are made of
    - The use of lubricants



Static Friction

Static friction between the two surfaces is when there is no relative motion between them

- Increases with increasing applied force
- Maximum when the object is just about to move

$$\boxed{f_s \leq \mu_s N}$$

Quantity	Symbol	SI Unit
Magnitude of static friction	f_s	N
Static friction coefficient	μ_s	no units
Magnitude of normal force	N	N

Kinetic Friction

Kinetic friction between two surfaces is when they are moving relative to each other. f_k is constant along the path of movement as long as $\mathbf{N}$ stays constant

$$\boxed{f_k = \mu_k N}$$

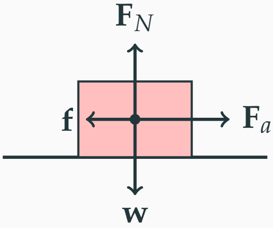
Quantity	Symbol	SI Unit
Magnitude of kinetic friction	f_k	N
Kinetic friction coefficient	μ_k	no units
Magnitude of normal force	N	N

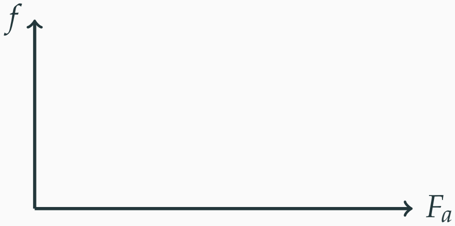
Static and Kinetic Coefficients of Friction

Kinetic friction coefficient is always lower than the static coefficient, otherwise nothing will ever move:

$$\mu_k \leq \mu_s$$

Consider a simple case of a box being pulled along a level floor. The free-body diagram is simple (left). How do the magnitudes of the applied force F_a and friction f compare?





Drag

Drag (or **fluid resistance**) is the force opposing the motion of an object moving in a fluid, with magnitude:

$$D = \frac{1}{2}\rho V_\infty^2 A_{\text{ref}} C_d$$

Quantity	Symbol	SI Unit
Magnitude of drag force	D	N
Density of the fluid	ρ	kg/m^3
Free-stream fluid velocity	V_∞	m/s
Reference area	A_{ref}	m^2
Drag coefficient	C_d	(no unit)

Drag coefficient depends on the shape and surface smoothness of the object. For blunt objects A_{ref} is the frontal area; for streamlined objects A_{ref} is the planform (top-view) area

Drag

In AP Physics you are *not* asked to know the drag equation. However, you should know that drag (air resistance) depends on the motion of the object and is not a constant.

Terminal Velocity

When we take drag force into account, we understand that the drag force increases as an object speeds up, and therefore a free-falling object does *not* accelerate infinitely. Instead it reaches a **terminal velocity**.

There is no air resistance just as the object *begins* to fall. Acceleration is due to gravity alone.

Drag increases as v increases. Magnitude of acceleration decreases, but the object continues to gather speed

Terminal velocity is reached when the drag force equals the object's weight. Not net force; no acceleration.

Tension in a Cable

Tension is the force exerted on and by a cable, rope, or string.

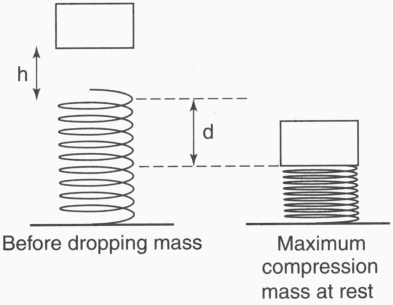
- You can't push on a rope
- Assume the cable/rope/string to be mass less
- Force can change direction when used with pulleys

Spring Force

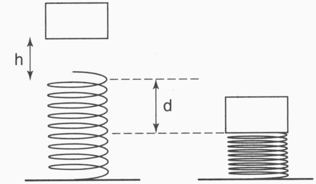
The spring force F_e is the force a compressed or stretched spring exerts onto objects connected to it. It obeys Hooke's Law:

$$\mathbf{F}_e = -k\mathbf{x}$$

where $\mathbf{x}$ is the relative displacement of the ends of the spring.

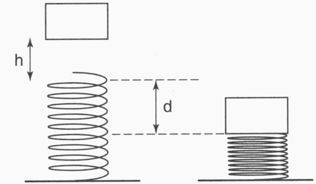


Spring Force



- As the object falls onto the spring, the spring begins to compress
- As the spring compresses, the spring force (pointing up!) increases linearly (Hooke's law)
- At some point, the spring force balances the weight of the block
  - At this point, the *acceleration* is zero
  - But the velocity continues to be downward
- The spring continues to compress until velocity is zero

Spring Force



- Solving this problem using dynamics is difficult, because:
  - Spring force scales linearly with *displacement*, but
  - Net force scales linearly with *acceleration* (2nd time derivative of displacement)
- Note that the block continues to *increase* velocity even after it starts to compress the spring.
- Acceleration is zero only after the spring has compressed some amount

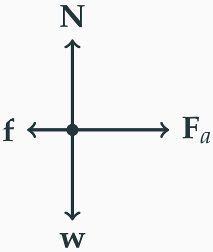
Free-Body Diagrams

Free-Body Diagrams

- Acceleration (if there is going to be any at all) depends on net force $\mathbf{F}_{\mathrm{net}}$
- Without a vector sum of all the forces, we cannot determine the magnitude, direction of the acceleration, or how acceleration will evolve in time
- We use **free-body diagrams** (FBD) to represent all the forces.
  - Very important in solving any dynamics problems
  - Don't try to save this step, even if the problem does not ask for it
  - Always draw FBD for solving classical mechanics problem

Free-Body Diagrams

In Grade 11/12 Physics, for rectilinear motion (no *rotational motion*), FBDs are usually drawn by assuming that all forces acting at the CM, represented by the "big dot". For example:

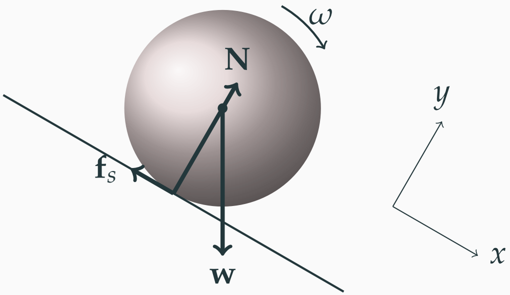


We can still use this method for rectilinear motion. However, this is not entirely correct. We notice that:

- Gravitational force $\mathbf{w}$ acts at the CM, but
- Normal force $\mathbf{N}$, friction $\mathbf{f}$ and applied force $\mathbf{F}_a$ act at the point of contact

Free-Body Diagrams

Instead, forces should be drawn at the point where they are applied. In the example below, a sphere rolling down a ramp should have weight $\mathbf{w}$, normal force $\mathbf{N}$ and static friction $\mathbf{f}_s$ acting on it:

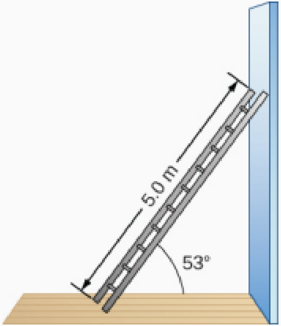


Once the FBD is drawn, decide on the axes to help you solve the motion. One of the axes should line up with the direction of motion. This guarantees that the *other* axis will not have any net force

Example Problem

A more difficult static problem may involve two surfaces with two different friction coefficients. For example, a ladder leaning on a wall. This problem cannot be solved without first understanding rotational motion, but we can still draw a FBD.

Example: A uniform ladder is 5.0 m long and weighs 400 N. The ladder rests against a slippery vertical wall, as shown in Figure. The inclination angle between the ladder and the rough floor is 53°. Find the reaction forces from the floor and from the wall on the ladder and the coefficient of static friction μ_s at the interface of the ladder with the floor that prevents the ladder from slipping.



Multi-Body Problems

Applying Newton's Third Law on Connected Bodies

- The objects are connected by a cable or a solid linkage with negligible mass
- All objects (usually) have the same acceleration
- Require multiple free-body diagrams

Solving Connected-Bodies Problems

To solve a connected-bodies problem, you can follow these procedures:

1. Draw a FBD on each of the objects
2. Sum all the forces on all the objects along the direction of motion
   - Direction of motion is usually very obvious
   - All internal forces should cancel and do not figure into the acceleration of the system
3. Compute the acceleration of the entire system using Newton’s second law
   - Remember that (usually) every object has the same acceleration!
4. Go back to the FBD of each of the objects and compute the unknown forces (usually tension)

Connected Bodies: Example

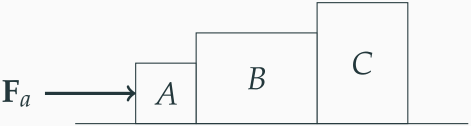
Example: A tractor-trailer pulling two trailers starts from rest and accelerates with an acceleration a on a straight, level road. The mass of the truck (T) is 5450 kg, the mass of the first trailer (A) is 31 500 kg, and the mass of the second trailer (B) is 19 600 kg.

1. What magnitude of force must the truck generate in order to accelerate the entire vehicle?
2. What magnitude of force must each of the trailer hitches withstand while the vehicle is accelerating?

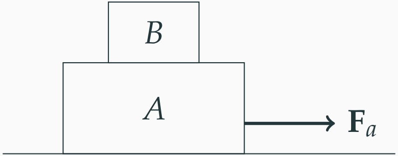
Assume that frictional forces are negligible in comparison with the forces needed to accelerate the large masses.

Different Types of Connected Bodies

Multiple objects pressed against one another. There may not be friction, but there are definitely action/reaction forces between the blocks.



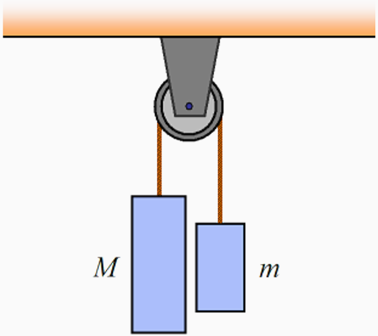
Or multiple objects stacked on top of one another. The contact surface between A and the floor may (or may not) have friction, while the surface between A and B must have a friction coefficient μ.



Pulley Problems

Example Problem: Atwood Machine

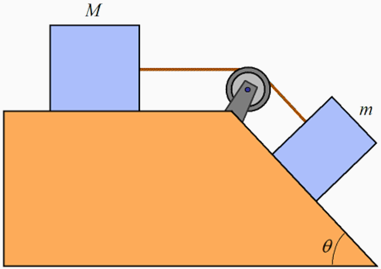
An **Atwood machine** is made of two objects connected by a rope that runs over a pulley. The pulley allows the direction of force and direction of motion to change between two objects.



Example: The object on the left has a mass of M and the object on the right has a mass of m.

- What is the acceleration of the masses?
- What is the tension in the rope?

A Slightly More Difficult Problem



Two blocks of mass m and M are connected via pulley with a configuration as shown on the left. The coefficient of static friction is μ_s, between blocks and surface. What is the maximum mass m so that no sliding occurs?