Student #: ______________________ Student Name: ______________________________

AP PHYSICS C: UNIVERSAL GRAVITATION

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

1. Two satellites of equal mass orbit a planet. Satellite B orbits at twice the orbital radius of Satellite A. Which of the following statements is true?
   (A) The gravitational force on Satellite A is four times less than that on Satellite B.
   (B) The gravitational force on Satellite A is two times less than that on Satellite B.
   (C) The gravitational force on the satellites is equal.
   (D) The gravitational force on Satellite A is two times greater than that on Satellite B.
   (E) The gravitational force on Satellite A is four times greater than that on Satellite B.

2. A 70 kg astronaut floats at a distance of 10 m from a 50 000 kg spacecraft. What is the force of attraction between the astronaut and spacecraft?
   (A) 2.4×10^{-6} N
   (B) 2.4×10^{-5} N
   (C) Zero; there is no gravity in space.
   (D) 2.4×10^{5} N
   (E) 2.4×10^{6} N

3. The centripetal acceleration on 1000 kg car in a turn is $1 \times 10^5\,\mathrm{m/s^2}$. The radius of the turn is 10 m. What is the car's speed?
   (A) 1×10^1 m/s
   (B) 1×10^2 m/s
   (C) 1×10^3 m/s
   (D) 1×10^4 m/s
   (E) 1×10^5 m/s

4. The Earth is at an average distance of 1 AU from the Sun and has an orbital period of 1 year. Jupiter orbits the Sun at approximately 5 AU. About how long is the orbital period of Jupiter?
   (A) 1 year
   (B) 2 years
   (C) 5 years
   (D) 11 years
   (E) 125 years

5. A proposed "space elevator" can lift a 1000 kg payload to an orbit of 150 km above the Earth's surface. The radius of the Earth is 6.4×10^6 m, and the Earth's mass is 6.0×10^{24} kg. What is the gravitational potential energy of the payload when it reaches orbit?
   (A) 1.0×10^3 J
   (B) 2.7×10^6 J
   (C) 6.1×10^{10} J
   (D) 2.7×10^{12} J
   (E) 1.0×10^{15} J

6. A satellite orbits the Earth at a distance of 200 km. If the mass of the Earth is 6.0×10^{24} kg and the Earth's radius is 6.4×10^6 m, what is the satellite's speed?
   (A) 1.0×10^3 m/s
   (B) 3.5×10^3 m/s
   (C) 7.8×10^3 m/s
   (D) 5×10^6 m/s
   (E) 6.1×10^7 m/s

7. Mars orbits the Sun at a distance of 2.3×10^{11} m. The mass of the Sun is 2.0×10^{30} kg, and the mass of Mars is 6.4×10^{23} kg. Approximately what is the gravitational force that the Sun exerts on Mars?
   (A) 1.6×10^{20} N
   (B) 1.6×10^{21} N
   (C) 3.7×10^{21} N
   (D) 3.7×10^{32} N
   (E) 3.7×10^{42} N

8. The mass of a planet is 1/4 that of Earth and its radius is half of Earth's radius. The acceleration due to gravity on this planet is most nearly
   (A) $2\,\mathrm{m/s^2}$
   (B) $4\,\mathrm{m/s^2}$
   (C) $5\,\mathrm{m/s^2}$
   (D) $10\,\mathrm{m/s^2}$
   (E) $20\,\mathrm{m/s^2}$

9. When climbing from sea level to the top of Mount Everest, a hiker changes elevation by 8848 m. By what percentage will the gravitational field of the Earth change during the climb? (The Earth's mass is 6.0×10^{24} kg, and its radius is 6.4×10^6 m.)
   (A) It will increase by approximately 0.3 %.
   (B) It will decrease by approximately 0.3 %.
   (C) It will increase by approximately 12 %.
   (D) It will decrease by approximately 12 %.
   (E) The gravitational field strength will not change.

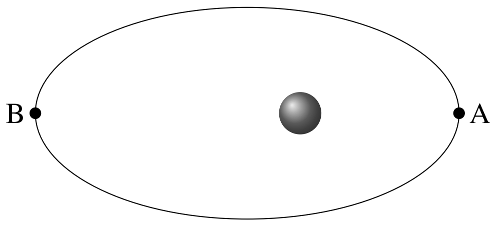
10. Four planets, A through D, orbit the same star. The relative masses and distances from the star for each planet are shown in the table. For example, Planet A has twice the mass of Planet B, and Planet D has three times the orbital radius of Planet A. Which planet has the highest gravitational attraction to the star?

Planet	Relative mass	Relative distance
A	$2m$	r
B	m	$0.1r$
C	$0.5m$	$2r$
D	$4m$	$3r$

   (A) Planet A
   (B) Planet B
   (C) Planet C
   (D) Planet D
   (E) All have the same gravitational attraction to the star.

11. A satellite orbits the Earth at a distance that is four times the radius of the Earth. If the acceleration due to gravity near the surface of the Earth is g, the acceleration of the satellite is most nearly
   (A) zero
   (B) $\frac{g}{2}$
   (C) $\frac{g}{4}$
   (D) $\frac{g}{8}$
   (E) $\frac{g}{16}$

12. A satellite orbits the Earth in an elliptical orbit, with point A being close to the Earth and point B farther away. As the satellite moves from point A to point B, which of the following is true of the angular momentum and kinetic energy of the satellite?



	Angular momentum	Kinetic energy
(A)	Increases	Remains constant
(B)	Remains constant	Increases
(C)	Decreases	Remains constant
(D)	Remains constant	Decreases
(E)	Remains constant	Remains constant

13. Two planets of mass M and $9M$ are in the same solar system. The radius of the planet of mass M is R. In order for the acceleration due to gravity to be the same for each planet, the radius of the planet of mass $9M$ would have to be

(A) $\frac{R}{2}$

(B) R

(C) $2R$

(D) $3R$

(E) $9R$

14. A satellite is in a stable circular orbit around the Earth at a radius R and speed v. At what radius would the satellite travel in a stable orbit with a speed $2v$?

(A) $\frac{R}{4}$

(B) $\frac{R}{2}$

(C) R

(D) $2R$

(E) $4R$

15. Two masses exert a gravitational force F on each other. If one of the masses is doubled, and the distance between the masses is tripled, the new force between them is

(A) $6F$

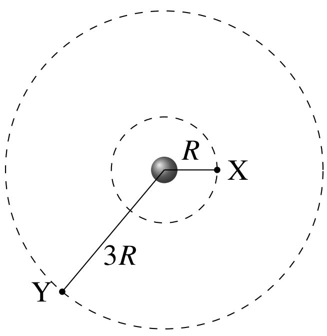
(B) $\frac{2F}{3}$

(C) $\frac{2F}{9}$

(D) $\frac{3F}{2}$

(E) $\frac{4F}{9}$

16. Two planets, X and Y, orbit a star. Planet X orbits at a radius R, and Planet Y orbits at a radius $3R$. Which of the following best represents the relationship between the acceleration a_X of Planet X and the acceleration a_Y of Planet Y?



(A) $a_X = 9a_Y$
(B) $9a_X = a_Y$
(C) $a_X = 3a_Y$
(D) $3a_X = a_Y$
(E) $a_X = a_Y$

17. The Earth and the moon apply a gravitational force to each other. Which of the following statements is true?

(A) The Earth applies a greater force on the moon than the moon exerts on the Earth.
(B) The Earth applies a smaller force on the moon than the moon exerts on the Earth.
(C) The Earth applies a force on the moon, but the moon does not exert a force on the Earth.
(D) The Earth does not apply a force on the moon, but the moon exerts a force on the Earth.
(E) The force the Earth applies to the moon is equal and opposite to the force the moon applies to the Earth.

18. A planet orbits at a radius R around a star of mass M. The period of orbit of the planet is

(A) $\sqrt{\frac{4\pi^2 R^2}{GM}}$

(B) $\frac{4\pi^2 R^3}{GM}$

(C) $\sqrt{\frac{4\pi^2 R^3}{GM}}$

(D) $\sqrt{\frac{4\pi^2 R}{GM}}$

(E) $\frac{GM}{4\pi^2 R}$

19. A moon orbits a large planet in an elliptical orbit, with its closest approach at a distance a, and its farthest distance b. The speed of the moon at point b is v. The speed at point a is

(A) $\frac{av}{b}$

(B) $\frac{bv}{a}$

(C) $\frac{(a+b)v}{b}$

(D) $\frac{(b-a)v}{b}$

(E) $\frac{2bv}{a}$

20. A satellite orbits the Earth in an elliptical orbit. Which of the following statements is true?

(A) The angular velocity of the satellite increases as it travels farther from the Earth.
(B) The acceleration of the satellite increases as it travels closer to the Earth.
(C) The angular momentum of the satellite increases as it travels closer to the Earth.
(D) The potential energy of the satellite is equal to its kinetic energy at all points in the orbit.
(E) The speed of the satellite must remain constant for it to remain in orbit around the Earth.

21. If a planet has twice the radius of Earth and half of Earth's density, what is the acceleration due to gravity on the surface of the planet (in terms of the gravitational acceleration g on the surface of Earth)?

(A) $4g$

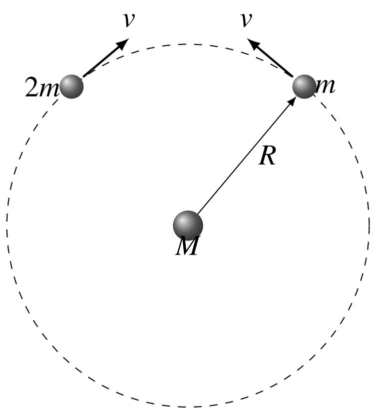
(B) $2g$

(C) g

(D) $\frac{g}{2}$

(E) $\frac{g}{4}$

22. Two moons of mass m and $2m$ orbit a planet of mass M at the same radius R and speed v toward each other, as shown. The moons collide and stick together without destroying either moon. The total momentum of the moons after the collision is



(A) mv
(B) $2mv$
(C) $3mv$
(D) $6mv$
(E) zero

23. The velocity of the two masses after the collision above is

(A) v counterclockwise
(B) $v/2$ counterclockwise
(C) $v/2$ clockwise
(D) $v/3$ counterclockwise
(E) $v/3$ clockwise

24. A satellite of mass m travels in an elliptical orbit around a planet of mass M. The satellite has a speed v when it is closest to the planet at a distance r. Work is done by the engines of the satellite to change its orbit to a circular orbit when it is at this distance r. Which of the following statements is true of the transition from an elliptical orbit to a circular orbit?

(A) The work done by the satellite engines to change the orbit is equal to the change in kinetic energy of the satellite.
(B) The work done by the satellite engines to change the orbit is equal to the change in potential energy of the satellite.
(C) The work done by the satellite engines to change the orbit is equal to the change in angular momentum of the satellite.
(D) The work done by the satellite engines to change the orbit is equal to the change in speed of the satellite.
(E) The work done by the satellite engines to change the orbit is equal to the change in orbital radius of the satellite.

25. A satellite of mass m orbits the Earth with a potential energy U and a kinetic energy K. Which of the following statements would have to be true for the satellite to escape the Earth's gravity completely?

(A) The kinetic energy of the satellite would have to be equal to the potential energy between the Earth and the satellite.
(B) The potential energy between the Earth and the satellite would have to be greater than the kinetic energy of the satellite.
(C) The total energy of the satellite would have to be greater than the kinetic energy of the satellite.
(D) The kinetic energy of the satellite would have to be greater than the potential energy of the satellite.
(E) The total energy of the satellite would have to be equal to the potential energy of the satellite.

AP PHYSICS C: UNIVERSAL GRAVITATION
SECTION II
6 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 $\times 10^{10}$ s^2	R^3 $\times 10^{25}$ m^3
8.14×10^4	1.85×10^8	0.66	0.63
1.18×10^5	2.38×10^8	1.39	1.35
1.63×10^5	2.95×10^8	2.66	2.57
2.37×10^5	3.77×10^8	5.62	5.30

6.63×10^9

(a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

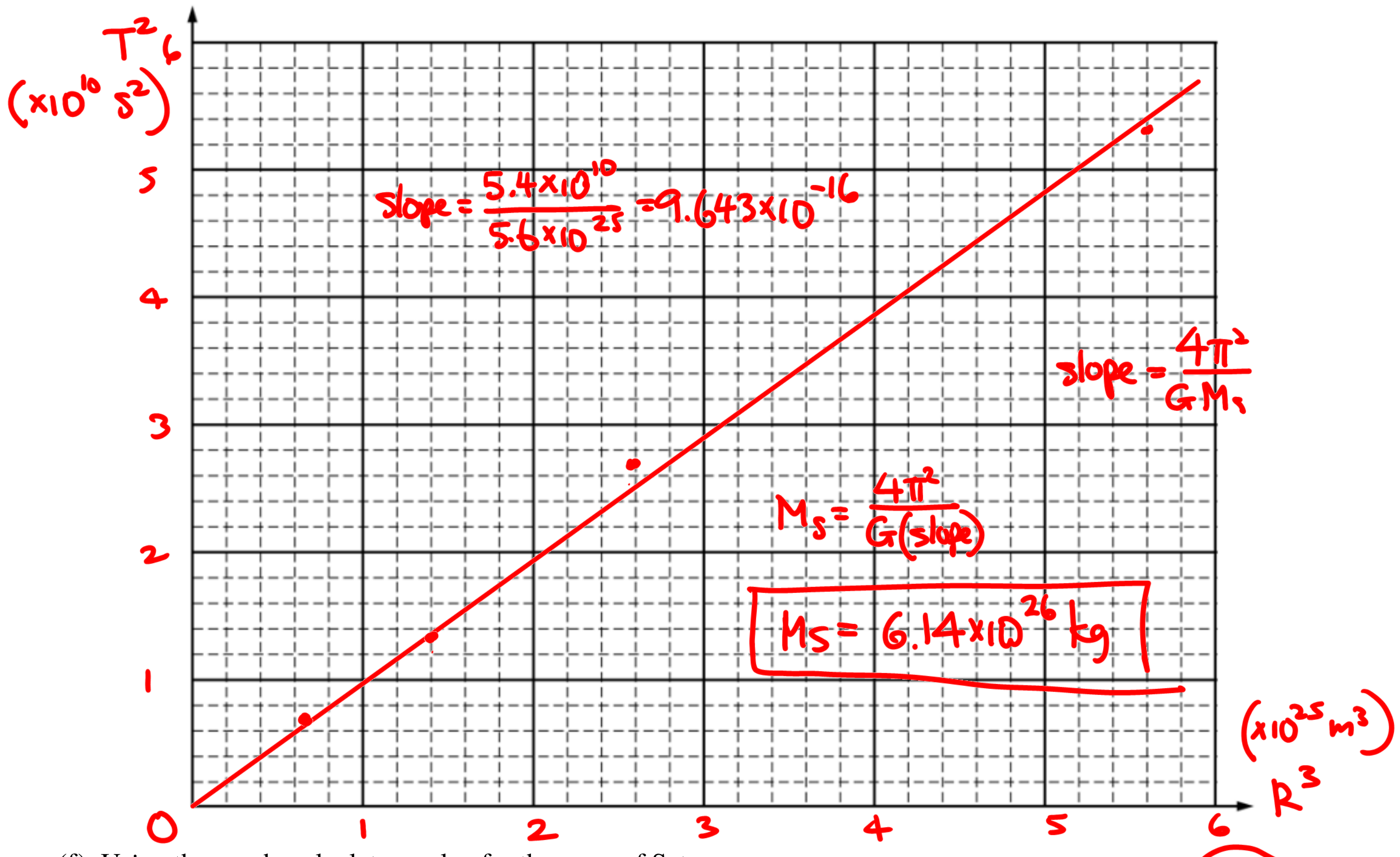
(b) Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

(c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

$y = mx$

(d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

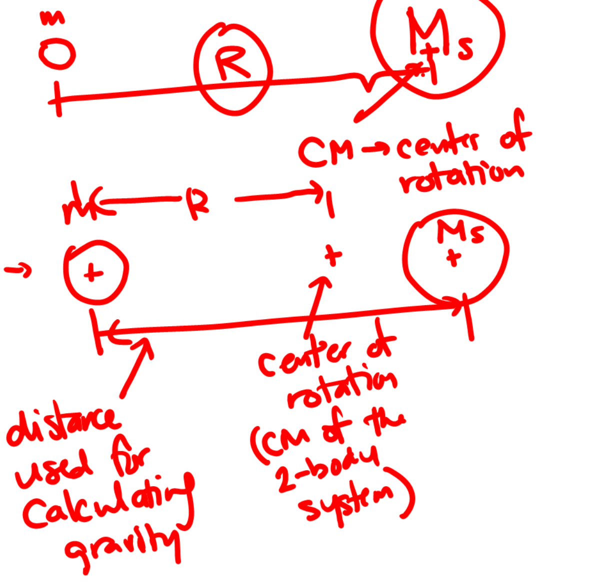
(e) Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



(f) Using the graph, calculate a value for the mass of Saturn.

a) $\boxed{F_g = \frac{GM_s m}{R^2}}$ (assumes $M_s > m$)

where m is the mass of the moon



b) $F_g = F_c$ $\quad v = \frac{2\pi R}{T}$

$\frac{GM_s m}{R^2} = \frac{mv^2}{R}$

$\frac{GM_s}{R} = v^2 = \frac{4\pi^2 R^2}{T^2}$

$\boxed{T^2 = \frac{4\pi^2}{GM_s} R^3}$

$\boxed{T = \sqrt{\frac{4\pi^2}{GM_s} R^3}}$

d) vertical axis (dependent variable) → T^2

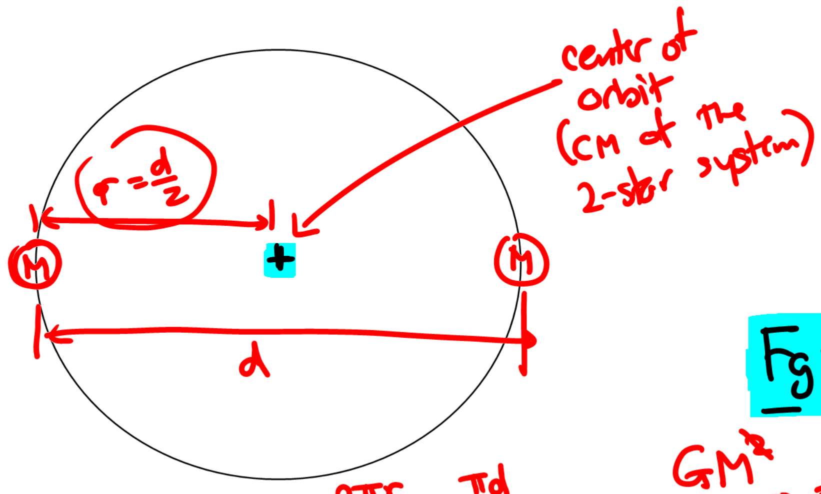
horizontal axis (independent variable) → R^3

slope → constant $\frac{4\pi^2}{GM_s}$

2. Two stars of equal mass M are orbiting each other in a circular path. Show that the orbital period is given by:

$$T^2 = \frac{2\pi^2 d^3}{GM}$$

where d is the distance between the stars.



$$\boxed{F_g = F_c}$$

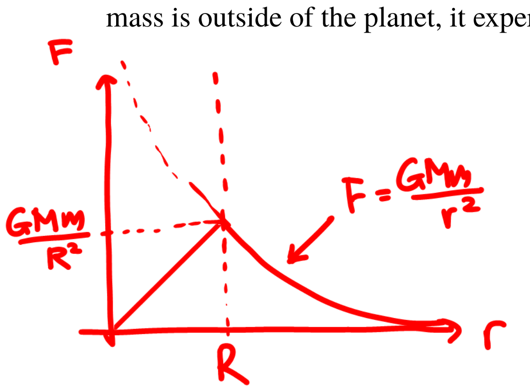
$$\frac{GM^2}{d^2} = \frac{2Mv^2}{d}$$

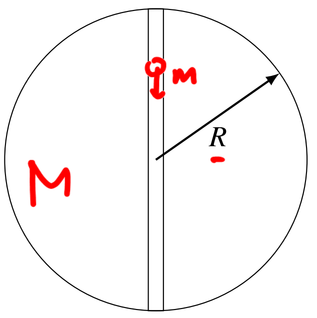
$$v = \frac{2\pi r}{T} \rightarrow \frac{\pi d}{T}$$

$$\frac{GM}{d} = 2v^2 = 2\,\frac{\pi^2 d^2}{T^2} \longrightarrow \boxed{T^2 = \frac{2\pi^2 d^3}{GM}}$$

3. A planet of mass M, radius R, and uniform density has a small tunnel drilled through the center of the planet, as shown below. When the mass is inside the tunnel, it experiences a force of $F = (GmM/R^3)r$, whereas when the mass is outside of the planet, it experiences a gravitational force of $F = GmM/r^2$.

k





$F_g = kr$

(a) Setting the potential energy of the mass to be zero at the planet's center, calculate the mass's potential energy as a function of distance from the center of the planet $U(r)$, for values $r < R$. Sketch this potential function.

(b) If the mass is dropped from R from the center of the planet, how long will it take until it returns to its original position?

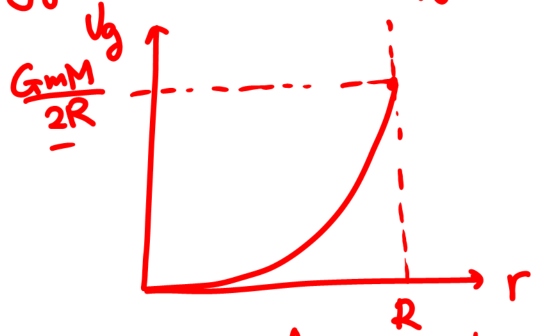
T

(c) If the mass is dropped from $R/2$ from the center of the planet, will it require more, or less, or the same amount of time to return to its original position compared to if it was dropped from R?

(d) If the mass is dropped from $2R$ from the center of the planet, will it require more, or less, or the same amount of time to return to its original position compared to if it was dropped from R?

a) $\Delta U_g = \int_0^r F_g(r)\,dr = \int_0^r kr\,dr = \frac{1}{2}kr^2 \Big|_0^r \rightarrow U_g = \frac{1}{2}kr^2$

$U_g = \left(\frac{GmM}{2R^3}\right) r^2$



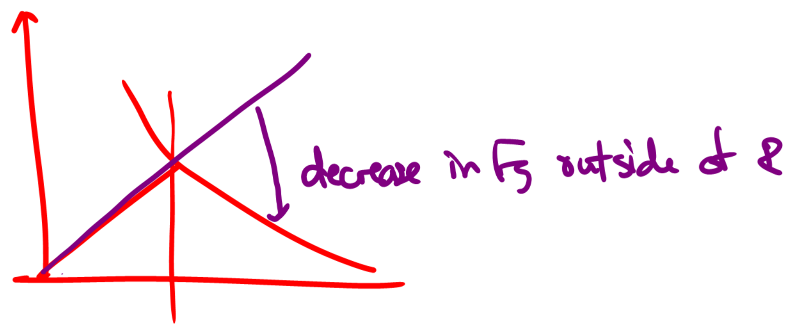
$-\frac{GMm}{R}$

b) - gravitational force $\rightarrow$ like a spring force $\rightarrow$ harmonic motion

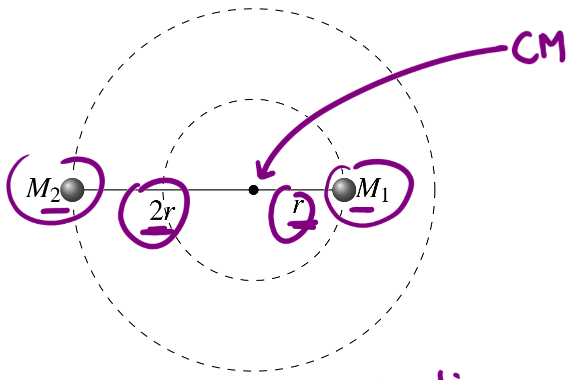
$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{GM}{R^3}}$ $\quad T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{R^3}{GM}}}$

c) Same amount (T not affected by "amplitude")

d) more time $\rightarrow$ "restoring force" decreases when $r > R$



4. Two stars of unequal mass orbit each other about their common center of mass as shown. The star of mass M_1 orbits in a circle of radius r, and the star of mass M_2 orbits in a circle of radius $2r$.



(a) Determine the ratio of masses M_1/M_2.

centripetal acceleration.

(b) Determine the ratio of the acceleration a_1 of M_1 to the acceleration a_2 of M_2.

(c) Determine the ratio of the period T_1 of M_1 to the period T_2 of M_2.

c) $\boxed{T_1 = T_2}$ for a stable orbit.

a) $\frac{M_1}{M_2} = \frac{2r}{r} = \boxed{2}$

$v^2 = \left(\frac{2\pi(2r)}{T}\right)^2 = \frac{16\pi^2}{T^2}$

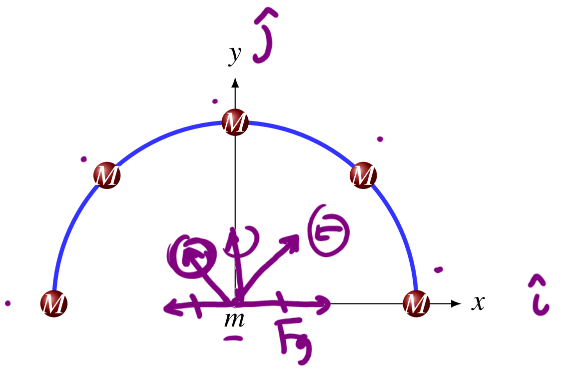
c) $a_2 = \frac{v^2}{2r}$

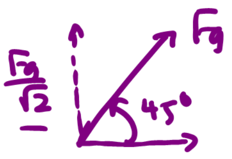
$a_2 = \frac{8\pi^2 r}{T^2}$

$a_1 = \frac{v_1^2}{r} = \frac{2\pi^2 r}{T^2}$

$\frac{a_1}{a_2} = \frac{\left(\frac{4\pi^2 r}{T^2}\right)}{\frac{8\pi^2 \cancel{r}}{\cancel{T^2}}} = \boxed{\frac{1}{2}}$

5. Five equal masses M are equally spaced on the arc of a semicircle of radius R as shown in the figure below. A mass m is located at the center of curvature of the arc. If M is 3 kg, m is 21 kg, nad R is 10 cm, what is the force on m due to the five masses?





$$F_g = \frac{GMm}{R^2}$$

$$\vec{F}_G = ??$$

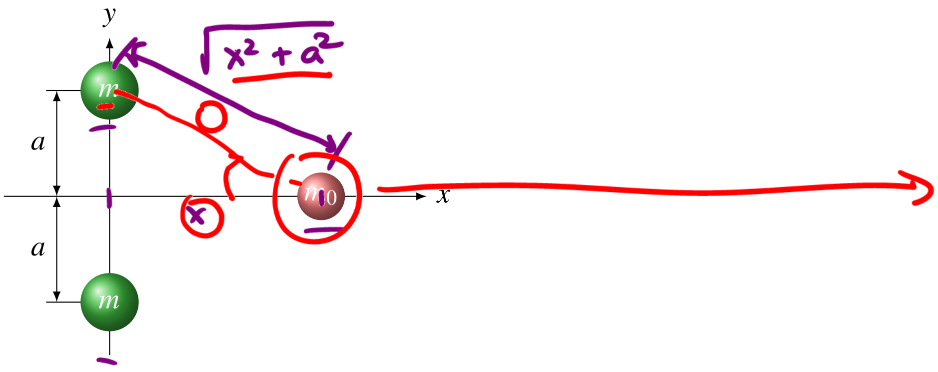
$$-\sum F_x = 0$$

$$\sum F_y = F_g + \frac{F_g}{\sqrt{2}} + \frac{F_g}{\sqrt{2}} = \frac{GMm}{R^2}(1+\sqrt{2})$$

$$\boxed{\vec{F}_g = (1+\sqrt{2})\frac{GMm}{R^2}\,\hat{j}}$$

+y-direction

6. Two point particles of mass m are on the y axis at $y = a$ and $y = -a$, as shown in the figure below.



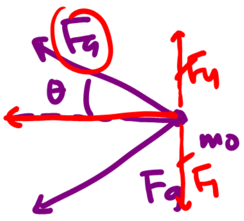
(a) Derive the expression for the gravitational force exerted by these two particles on a third particle of mass m_0 located on the x axis at a distance x away from the origin.

(b) What is the gravitational field $\mathbf{g}$ on the x-axis due to the two particles?

(c) Show that g_x (the x component of $\mathbf{g}$) due to the two particles on the y axis is approximately $-\dfrac{2Gm}{x^2}$ when x is much greater than a.

(d) Show that the maximum value of $|g_x|$ occurs at the point $x = \dfrac{\pm a}{\sqrt{2}}$.

a)



$$\vec{F_g} = 2F_g\cos\theta = -2\underbrace{\left(\frac{Gmm_0}{x^2+a^2}\right)}_{F_g}\frac{x}{\sqrt{x^2+a^2}}\,\hat{i}$$

$$\boxed{\vec{F_g} = \frac{-2Gmm_0\,x}{(x^2+a^2)^{3/2}}\,\hat{i}}$$

b) $$\vec{g} = \frac{\vec{F_g}}{m_0} = \boxed{\frac{-2Gmx}{(x^2+a^2)^{3/2}}\,\hat{i}}$$

c) if $x >> a$ then

$\therefore x^2 >> a^2$ $\quad\therefore x^2 + a^2 \approx x^2$

$$\vec{g} \approx \frac{-2Gmx}{(x^2)^{3/2}} \approx \boxed{-\frac{2Gm}{x^2}\,\hat{i}}$$

$$g_x = \frac{2Gmx}{(x^2+a^2)^{3/2}} \qquad \frac{dg_x}{dx} = 0$$