Topic 12: Capacitors

Advanced Placement Physics C

Dr. Timothy Leung

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Olympiads School
Toronto, ON, Canada

Capacitors

Electric Field and Electric Potential Difference

Recall that the relationship between electrostatic force ($\mathbf{F}_q$) and electric potential energy (U_q) can be expressed using definition of mechanical work and the fundamental theorem of calculus:

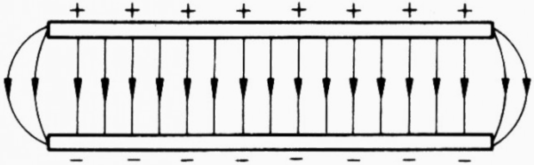
$$\Delta U_q = -\int \mathbf{F}_q \cdot d\mathbf{r} \qquad \mathbf{F}_q = -\nabla U_q = -\frac{\partial U_q}{\partial r}\hat{\mathbf{r}}$$

Dividing both sides of the equations by q, we get the relationship between electric field ($\mathbf{E}$), electric potential (V) and electric potential difference (ΔV):

$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{r} \qquad \boxed{\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{\mathbf{r}}}$$

This relationship holds regardless of the charge configuration.

Electric Field and Electric Potential Difference



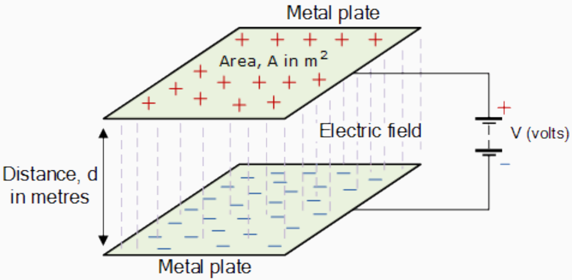
In the case of two parallel plates (as we have worked out using Gauss's law), the electric field is uniform, and the relationship simplifies to:

$$\boxed{E = \frac{\Delta V}{d}} \quad \text{or} \quad \boxed{\Delta V = Ed}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C
Electric potential difference between plates	ΔV	V
Distance between plates	d	m

Capacitors

Capacitors is a device that stores energy in a circuit. The simplest form of a capacitor is a set of closely spaced parallel plates:



When the plates are connected to a battery, the battery transfer charges to the plates until the voltage V equals the battery terminals. After that, one plate has charge $+Q$; the other has $-Q$.

Parallel-Plate Capacitors

As we have seen already, the (uniform) electric field between two parallel plates is proportional to the charge density σ, which is the charge Q divided by the area of the plates A:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

Substituting this into the relationship between the plate voltage V and electric field, we find a relationship between the charges across the plates and the voltage:

$$V = Ed = \frac{Qd}{A\epsilon_0} \quad \longrightarrow \quad \boxed{Q = \left[\frac{A\epsilon_0}{d}\right] V}$$

Parallel-Plate Capacitors

Since area A, distance of separation d and the vacuum permittivity ϵ_0 are all constants, the relationship between charge Q and voltage V is *linear*. And the constant is called the **capacitance** C, defined as:

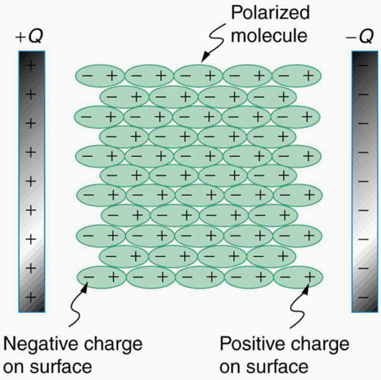
$$\boxed{C = \frac{Q}{V}}$$

For parallel plates:

$$C = \frac{A\epsilon_0}{d}$$

The unit for capacitance is a **farad** (named after Michael Faraday), where $1\,\mathrm{F} = 1\,\mathrm{C}/\mathrm{V}$.

Practical Capacitors



- Parallel-plate capacitors are very common in electric circuits, but the vacuum between the plates is not very effective
- Instead, a non-conducting **dielectric** material is inserted between the plates
- When the plates are charged, the electric field of the plates polarizes the dielectric.
- The polarization produces an electric field that opposes the field from the plates, therefore reduces the effective voltage, and increasing the capacitance

Dielectric Constant

If electric field without dielectric is E_0, then E in the dielectric is reduced by κ, the **dielectric constant**:

$$\boxed{\kappa = \frac{E_0}{E}}$$

The capacitance of the plates with the dielectric is now amplified by the same factor κ:

$$\boxed{C = \kappa C_0}$$

We can also view the dielectric as something that increases the *effective permittivity*:

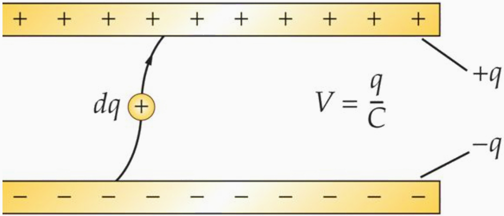
$$\boxed{\epsilon = \kappa \epsilon_0}$$

Dielectric Constant

The dielectric constants of commonly used materials are:

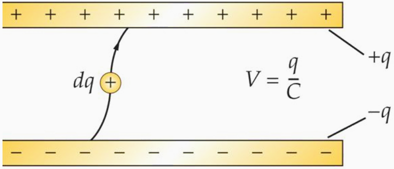
Material	κ
Air	1.000 59
Bakelite	4.9
Pyrex glass	5.6
Neoprene	6.9
Plexiglas	3.4
Polystyrene	2.55
Water (20 °C)	80

Storage of Electrical Energy



- When charging up a capacitor, imagine positive charges moving from the negatively charged plate to the positively charged plate
- Initially neither plates are charged, so moving the first charge takes very little work; as the electric field builds, more and more work needs to be done

Storage of Electrical Energy



In the beginning—when the plates aren't charged—moving an infinitesimal charge *dq* across the plates, the infinitesimal work done *dU* is related to the capacitance:

$$dU = Vdq = \frac{q}{C}dq$$

As the electric field begins to form between plates, more and more work is required to move the charges.

Storage of Electrical Energy

To fully charge the plates, the total work U_c is the integral:

$$U_c = \int dU = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$

The work done is stored as a potential energy inside the capacitor. There are different ways to express U_c using definition of capacitance:

$$\boxed{U_c = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2}$$

Notes About Storage of Electric Energy

- The work done (i.e. the energy stored in the capacitor) is inversely proportional to the capacitance:

$$dU = Vdq = \frac{q}{C}dq$$

- The presence of a dielectric *increases* the capacitance; therefore the work (and potential energy stored) to move the charge *dq decreases* with the dielectric constant κ
- After the capacitor is charged, removing the dielectric material from the capacitor plates will require additional work.