Topic 17: Nuclear Physics

AP Physics 2

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Nucleus of the Atom

The Rutherford-Bohr Model

In the **Rutherford-Bohr atomic model**, an atom consists of

- A dense positively-charged nucleus
- Negatively-charged electrons “orbiting” the nucleus in predefined energy levels
- Most of the atom is empty space

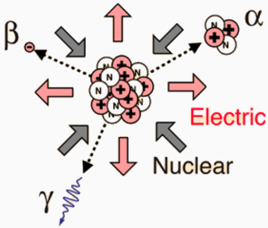
Most of the atom is empty space, although almost all the mass is concentrated at the nucleus

Nucleus of an Atom

The nucleus—where most of the mass of the atom is concentrated—are made up of **nucleons** which are:

- Positively charged **protons**, and
- Electrically neutral **neutrons**

Balancing Fundamental Forces



The nucleus of an atom is held together by balancing two fundamental forces:

- **Electromagnetic force**: the repulsive force between protons
  - The force drop off as the square of the distance (inverse square law)
- **Nuclear strong force**: short-distance force holding nucleons together
  - Attractive between nucleons at about 1 fm (10^{-15} m)
  - Insignificant at distances beyond 2.5 fm
  - Repulsive at distances below 0.7 fm

Atomic Properties

The nucleus of an atom is identified by two numbers:

- **Atomic number** (Z): the number of protons
  - Determines what element it is, and
  - its chemical properties
- **Mass number** (A): number of nucleons
- The number of neutrons (N) is therefore $N = A - Z$

Isotopes

Carbon has three common **isotopes**, all with the same atomic numbers, but different number of neutrons (called "carbon-12", "carbon-13" and "carbon-14" respectively). The atomic number is sometimes omitted.

$$^{12}\mathrm{C} \quad ^{13}\mathrm{C} \quad ^{14}\mathrm{C}$$

Hydrogen also has three common isotopes (hydrogen, deuterium and tritium)

$$^{1}\mathrm{H} \quad ^{2}\mathrm{H} \quad ^{3}\mathrm{H}$$

On average, there are 2.6 isotopes for each element.

Mass of the Nucleus

The **unified atomic mass unit** (u)[1] is defined as $1/12$ of the rest mass of a carbon-12 atom in its nuclear ground state:

$$1\,\mathrm{u} = \frac{1}{12} m(^{12}\mathrm{C}) \approx 1.660\,54 \times 10^{-27}\,\mathrm{kg}$$

The rest mass of the elementary particles can be expressed in this unit:

Particle	**Mass** (u)	**Mass** (kg)
Proton	$1.007\,276$	$1.672\,614 \times 10^{-27}$
Neutron	$1.008\,665$	$1.674\,920 \times 10^{-27}$
Electron	$0.000\,549$	$9.109\,56 \times 10^{-31}$

[1] It replaces the "atomic mass unit" (amu) which is no longer used. The atomic mass value in the periodic table is a weighted average across all isotopes.

Mass Defect & Nuclear Binding Energy

Mass-Energy Equivalence

One of the discoveries in special relativity[2] is the fundamental equivalence of mass and energy. While it is based on *relativistic kinematics*, it also applies to *quantum mechanics*, from which nuclear physics is based on.

$$\boxed{E = mc^2}$$

Quantity	Symbol	SI Unit
Energy	E	J
Rest mass of a particle	m	kg
Speed of light	c	m/s

The speed of light is a universal constant: $c = 2.998 \times 10^8$ m/s.

[2] Einstein, A., "Does the Inertia of a Body Depend Upon Its Energy Content?", *Annelen der Physik*, 18(13):639-641, 21 November, 1905.

Mass-Energy Equivalence

Any change in energy is *always* accompanied by a change in mass, and vice versa.

$$E = mc^2 \quad \longrightarrow \quad \Delta E = \Delta mc^2$$

- The concept is difficult to visualize in everyday life. Example: A baseball moving at $160\,\mathrm{km/h}$ is more massive than a baseball travelling at $20\,\mathrm{km/h}$, but the Δms is too small to measure.
- But in cases when
  - speeds approaches the speed of light ($v > 0.3c$), or when
  - the masses are small (e.g. electrons, protons, neutrons)

  the difference significant

Mass Defect

The sum of the masses of the nucleons are always higher than the mass of the nucleus itself. **Example:** the mass of the alpha particle (helium nucleus) is lower than 2 protons plus 2 neutrons:

$$1\ u = 1.66054 \times 10^{-27}\ kg = 931.494\ MeV/c^2$$

Mass Defect

This difference in mass is called the **mass defect** Δm, which can be calculated with a simple equation:

$$\boxed{\Delta m = [Zm_P + (A - Z)m_N] - m_A}$$

Quantity	Symbol	SI Unit
Mass defect	Δm	kg
Atomic number and mass numbers	Z, A	
Rest mass of a proton and neutron	m_P, m_N	kg
Rest mass of the nucleus	m_A	kg

Mass Defect

Mass defect exists in all nuclei because the nucleus of an atom is always *in a lower energy state* than the individual nucleons alone

Similar examples:

- The total energy of a planet orbiting the Sun is lower than the plant and the Sun individually
- An electron orbiting the nucleus is at a lower energy state also

Nuclear Binding Energy

The amount of energy that is equivalent to the mass defect is called the **nuclear binding energy** E_b, defined as:

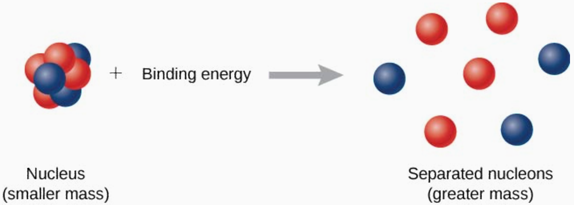
$$\boxed{E_b = (\Delta m)c^2}$$

Quantity	Symbol	SI Unit
Nuclear binding energy	E_b	J
Mass defect	Δm	kg
Speed of light	c	m/s

- The energy required to break up the nucleus into its individual nucleons
- Generally expressed in *electron volts* (eV) where $1\ \text{eV} = 1.602 \times 10^{-19}$ J, rather than in joules

Nuclear Binding Energy

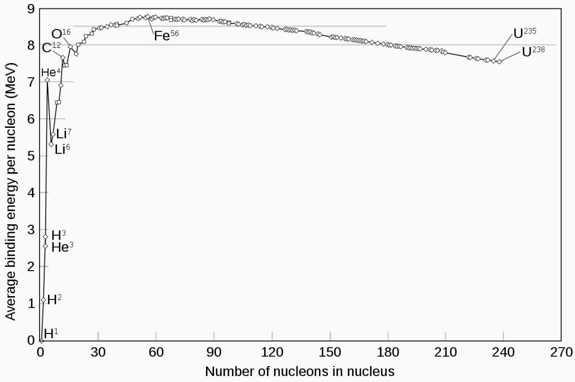
The nuclear binding energy is the amount of work required to separate the nucleons



The higher the binding energy, the more *tightly bound* a nucleus is, therefore more work is required to separate the nucleons

Nuclear Binding Energy & Stability of the Nucleus

The nuclear binding energy is highest for iron-56 ($E_b = 492.275$ MeV, or 8.7906 MeV per nucleon)



It means that the nucleus

- requires the most energy to separate the nucleons
- most tightly bound
- most stable

To achieve greater stability in the nucleus

- Heavier atoms can split into lighter nuclei, while
- Lighter atoms can combine into heavier nuclei

Radioactivity

Radioactive Decay

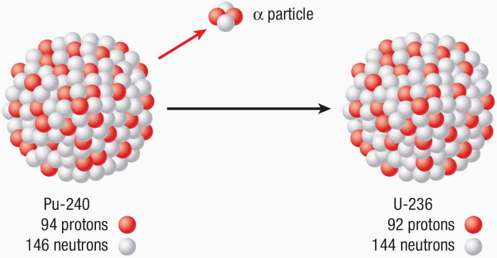
Radioactivity[3] is the spontaneously disintegration of a nucleus. There are *three* types of radioactive decay:

- **Alpha decay**, or α-decay
- **Beta decay**, or β-decay
- **Gamma decay**, or γ-decay

[3] Or **radioactive decay**

Alpha Decay

In α-decay, an **alpha particle** (a helium-4 nucleus, with 2 protons and 2 neutrons), is spontaneously emitted from the nucleus. Example: plutonium-240 nucleus decays into a uranium-236 nucleus, emitting an alpha particle



$$\boxed{{}^{240}_{94}\mathrm{Pu} \longrightarrow {}^{236}_{92}\mathrm{U} + {}^{4}_{2}\mathrm{He}}$$

Note the binding energy in the helium-4 nucleus in previous slides

General Formula for Alpha Decay

The general formula for an α-decay is shown by the equation:

$$\boxed{{}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}}$$

The **parent atom** (X) is the "reactant", and the **daughter atom** (Y) is the "product". This reaction is called a **transmutation**, because a new element is formed.

- The nuclear binding energy of the daughter atom and the alpha particle are *higher* than the parent atom.
- The combined mass of the daughter atom plus the alpha particle is *lower* than the parent atom, meaning that energy is released

Beta Decays

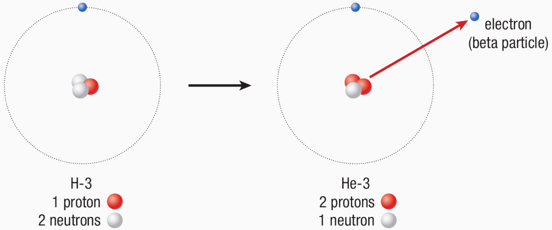
Beta decays involve the emission/capture of a **beta particle** (electron or a positron). There are three types of beta decays:

- Beta-negative decay (emission of an electron)
- Beta-positive decay (emission of an positron)
- Electron capture

Beta decays generally occur in smaller (less massive) nuclei

Beta-Negative Decay

In a **beta-negative decay** (β^--decay), a neutron spontaneously decays into a proton and an electron, and the electron is ejected from the nucleus. For example, the β^--decay of a tritium atom is:



$$\boxed{{}^{3}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + {}^{0}_{-1}\mathrm{e}}$$

Beta-Negative Decay

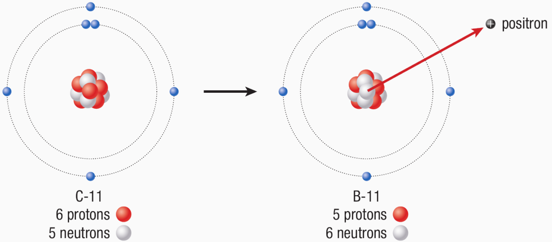
The general form for β^--decay is given by:

$$\boxed{{}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\mathrm{e}}$$

The daughter atom has a higher nuclear binding energy than the parent atom. This decay is also a transmutation because a new element is formed.

Beta-Positive Decay

In **beta-positive decay** (β^+-decay), a proton spontaneously decays into a neutron and a positron. Example: decay of carbon-11 into boron-11:



$$\boxed{{}^{11}_{6}\mathrm{C} \longrightarrow {}^{11}_{5}\mathrm{B} + {}^{0}_{+1}\mathrm{e}}$$

A positron has the same mass as an electron, but with a *positive* elementary charge. It is not a stable particle.

Beta-Positive Decay

The general equation for β^+-decay is given by:

$$\boxed{{}^{A}_{Z}\mathrm{X} \longrightarrow {}^{\;\;A}_{Z-1}\mathrm{Y} + {}^{\;\;0}_{+1}\mathrm{e}}$$

Similar to a β^- decay, the daughter nucleus has a higher nuclear binding energy than the parent nucleus (i.e. lower energy state). This decay is also a transmutation because a new element is formed.

Electron-Positron Annihilation

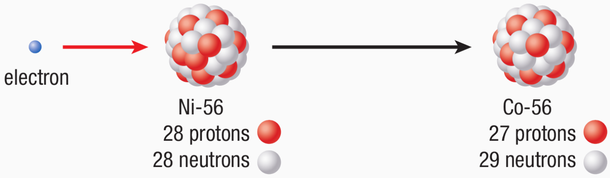
The positron released in the decay will be (almost) immediately *annihilated*[4] by another electron, releasing a pair of gamma particle (photons with no mass), each with an energy of 0.511 MeV, moving in opposite directions at the speed of light:

$$\boxed{{}^{0}_{+1}\mathrm{e} + {}^{0}_{-1}\mathrm{e} \longrightarrow 2\left({}^{0}_{0}\gamma\right)}$$

[4] It means that the two particles collide, and all of their masses are converted into energy

Electron Capture

Electron capture is a rarer form of beta decay where an electron is absorbed by a nucleus and combines with a proton to form a neutron. For example, a nickel-56 nucleus can capture an electron to form a cobalt-56 nucleus:



$$\boxed{{}^{56}_{28}\mathrm{Ni} + {}^{0}_{-1}\mathrm{e} \longrightarrow {}^{56}_{27}\mathrm{Co}}$$

Electron Capture

The general equation for electron capture is given by:

$$\boxed{{}^{A}_{Z}\mathrm{X} + {}^{\;\,0}_{-1}\mathrm{e} \longrightarrow {}^{\quad\;\, A}_{Z-1}\mathrm{Y}}$$

The electron that is absorbed usually comes from the lowest energy shell ($n = 1$, or "K-shell") that is closest to the nucleus, so it is often called *K-capture*.

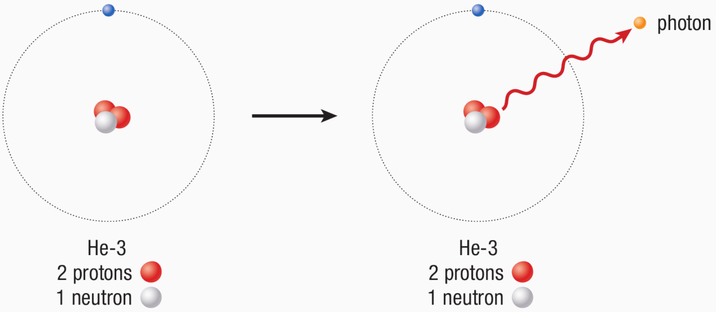
Gamma Radiation

Gamma decay (γ-decay) occurs after a nuclear reaction (e.g. α or β decay)

- The daughter nucleus is in a high-energy (excited) state
- The nucleus spontaneously releases energy in a **gamma particle** to return to a lower (therefore more stable) energy state.
- A gamma particle:
    - is a highly energetic form of electromagnetic radiation that is emitted as a **photon**
    - Has zero mass

Gamma Radiation

As an example, the γ-decay of a helium-3 atom is given by:



$$\boxed{{}^3_2\mathrm{He}^* \longrightarrow {}^3_2\mathrm{He} + {}^0_0\gamma}$$

Gamma Radiation

The general equation for γ-decay is:

$$\boxed{{}^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + {}^{0}_{0}\gamma}$$

- The parent and daughter nuclei are identical
- Only the energy level of the nucleus has changed
    - The asterisk indicates that the parent an excited state
- Notice that the mass number and atomic number of a gamma ray (${}^{0}_{0}\gamma$) are both zero.

Energies of Radiation

Radioactive particles post danger to living tissues, because

- they can ionize (or strip the electrons from) atoms
- α particles have strongest ionizing ability, but can only travel a relatively short distance before becoming absorbed
- β particles and γ rays have a greater penetrating range in air and must be shielded against

Type	Radiation	Charge	Penetrating Ability
α-decay	Alpha particle (He-4 nucleus)	$+2$	Skin or paper
β^--decay	Beta particle (electron)	-1	thin sheet of aluminum
β^+-decay	Beta particle (positron)	$+1$	thin sheet of aluminum
e^- capture	None	N/A	N/A
γ-decay	Gamma particle (photon)	0	Few centimetres of lead

Half-Life

While the radioactive decay of a single atom is *random* (like all quantum mechanics, radioactivity is driven by probability), when there are a large number of atoms, the overall rate of decay is very predictable.

$$\boxed{N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{\tau}}}$$

Quantity	Symbol	SI Unit
Amount of radioactive material	$N(t)$	kg
Initial sample amount	N_0	kg
Time	t	s
Half-life	τ	s

Half-life: the time it requires for a radioactive material to decay to half of its original amount.

Half-Life of Radioactive Isotopes

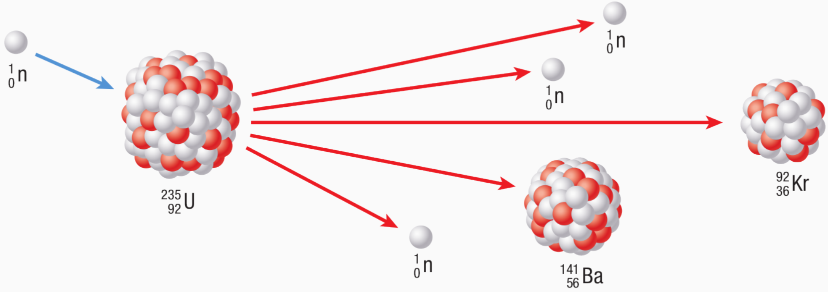
The half-life of radioactive substance can vary from a fraction of a second to billions of years.

Substance	**Half-Life** (t_0)
Polonium-215	0.0018 s
Bismuth-212	60.5 s
Sodium-24	15 h
Iodine-131	8.07 d
Cobalt-60	5.26 yr
Radium-226	1600 yr
Uranium-238	4.5×10^9 yr

Nuclear Fission

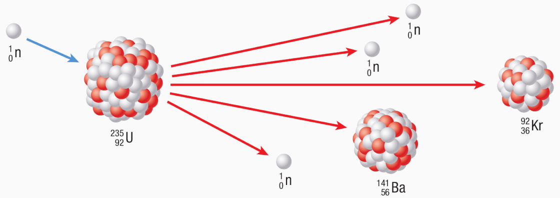
Nuclear Fission

The release of energy in a nuclear reaction primarily comes from **nuclear fission**, where a heavier atomic is split into lighter atoms. For example, the fission reaction of uranium-235 splitting into krypton-92 and barium-141 atoms:



$$\boxed{{}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{92}_{36}\mathrm{Kr} + {}^{141}_{56}\mathrm{Ba} + 3\left({}^{1}_{0}\mathrm{n}\right) + \text{energy}}$$

Nuclear Fission



- Fission reaction begins with the uranium nucleus capturing a neutron
- The addition of hte neutron deforms the nucleus into a double-loped “drop” shape
- The attractive nuclear strong force can no longer hold the two fragments together, and they electrically repel each other away
- In splitting the atom, several neutrons are released from the nucleus
- The neutrons may be captured by other uranium-235 atoms, causing further reaction

Nuclear Fission

- By probability, a nuclear fission is usually *binary* (two daughter nuclei in the process)
- Nuclear fuels can be
  - **Fissile:** the capture of any neutron will be sufficicient to cause fission, e.g. ^{235}U or ^{239}Pu
  - **Fissionable:** requires additional energy from a fast-moving neutron to cause fission to begin, e.g. ^{238}U
- Fission products usually cluster masses of 95 ± 15 u and 135 ± 15 u

Example Problem

Example: What is the energy yield of the following fission reaction?

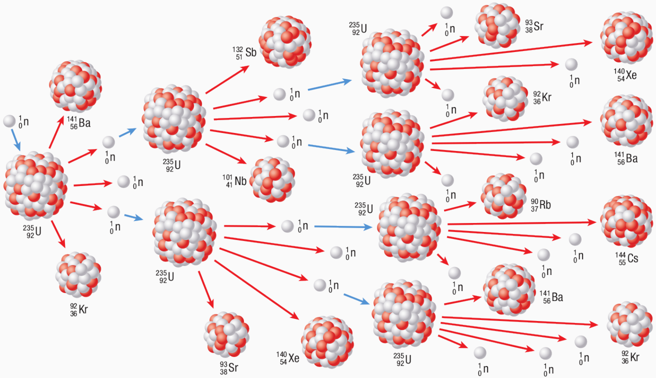
$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{140}_{55}\mathrm{Cs} + ^{93}_{37}\mathrm{Rb} + 3\left(^{1}_{0}\mathrm{n}\right)$$

$$\begin{aligned} m(\text{U-235}) &= 235.044\,\text{u} \\ m(\text{Cs-140}) &= 139.909\,\text{u} \\ m(\text{Rb-93}) &= 92.922\,\text{u} \\ m(\text{n}) &= 1.009\,\text{u} \end{aligned}$$

Chain Reaction

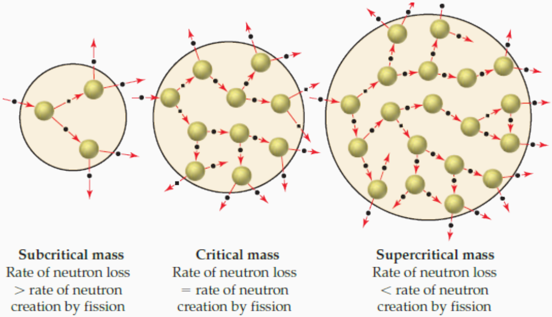
A **chain reaction** is a series of reactions that can repeat over several cycles

- Reactions occur without requiring any material being added to the system



Critical Mass

The amount of fuel required to sustain a chain reaction is called **critical mass**



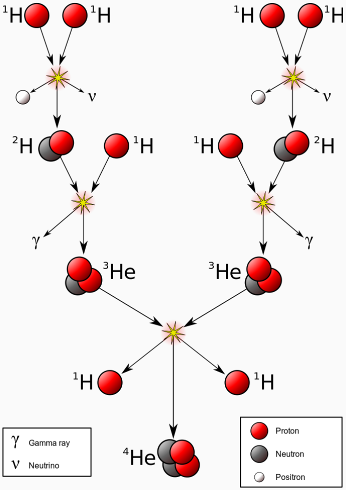
Nuclear Fusion

Proton-Proton Reaction

The simplest fusion reaction occurs under high temperature about 4×10^7 K is the proton-proton chain. In most the basic form, it is written as:

$$4\,({}^1_1\mathrm{H}) \longrightarrow {}^4_2\mathrm{He} + 2\,({}^{\;\,0}_{+1}\mathrm{e}) + \text{energy}$$

In each p-p reaction, 26.732 MeV is released. The exact mechanism for the p-p chain is shown in the right.



CNO Cycle

A fusion reaction that occurs in even higher temperatures (between 1.5 to 1.7×10^7 K) is the carbon-nitrogen-oxygen (CNO) cycle. The total energy released in one cycle is 26.73 MeV.

- The core temperature of the sun is about 1.56×10^7 K, so only 1.7% of helium-4 nuclei produced in the Sun are from this process