Student #: ____________________________ Student Name: ___________________________________

TOPIC 5: CIRCULAR MOTION AND GRAVITY

1. A 1000 kg car experiences a centripetal force of 1.8×10^5 N while making a turn. The car is moving at a constant speed of 30 m/s. What is the radius of the turn?

E

$F_c = ma_c = \frac{mv^2}{r}$ $\quad r = \frac{mv^2}{F_c} = \frac{(1000)(30)^2}{1.8\times10^5} = 5m$

   (A) 0.2 m
   (B) 1 m
   (C) 2 m
   (D) 4 m
   (E) 5 m

2. A record player has four coins at different distances from the center of rotation. Coin A is 1 cm away, Coin B is 2 cm away, Coin C is 4 cm away, and Coin D is 8 cm away. If the player is spinning 45 rotations/min, what coin has the greatest tangential velocity?

f

D

$v = \omega R$, $\omega = 2\pi f$

   (A) Coin A
   (B) Coin B
   (C) Coin C
   (D) Coin D
   (E) All the coins have equal tangential velocities.

3. Friction allows a car to make a turn at a speed of 10 miles per hour. By what factor will the friction have to change to allow the driver to make the same turn at twice the speed?

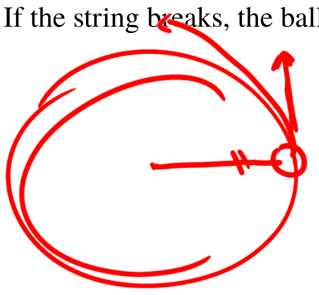
A

$F_f = F_c = \frac{mv^2}{r}$

   (A) Four times the friction
   (B) Twice the friction
   (C) The same amount of friction
   (D) One-half the friction
   (E) One-fourth the friction

4. A tetherball is whirled in a horizontal circle above your head. If the string breaks, the ball will follow what type of path if it is observed from above?

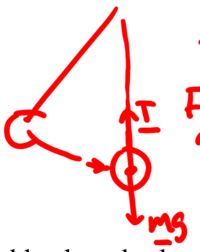
E



   (A) Straight outward from the center
   (B) Straight toward the center
   (C) An expanding spiral
   (D) A curved path that gradually approaches a straight line
   (E) Tangent to the original circular path

5. A pendulum bob is attached to a string that is tied to the ceiling, and the bob is pulled back and released. As the bob moves through the bottom of the swing, how does the magnitude of the tension force from the string compare to the gravitational force on the bob?

B



$\neq 0$
$F_c = T - mg$

   (A) The tension force is less than the gravitational force.
   (B) The tension force is greater than the gravitational force.
   (C) The tension force is equal to the gravitational force.
   (D) The mass of the ball is needed in order to compare these forces.
   (E) The release height of the ball is needed in order to compare these forces.

6. A pendulum bob is attached to a string that is tied to the ceiling, and the bob is pulled back and released from different heights. As the bob moves through the bottom of the swing, how is its centripetal acceleration related to its speed?

C

$a_c = \frac{v^2}{r}$

   (A) The centripetal acceleration is directly proportional to the speed of the pendulum.
   (B) The centripetal acceleration is inversely proportional to the speed of the pendulum.
   (C) The centripetal acceleration is directly proportional to the square of the speed of the pendulum.
   (D) The centripetal acceleration is inversely proportional to the square of the speed of the pendulum.
   (E) There is no relationship between the centripetal acceleration and the speed.

7. Two satellites of equal mass orbit a planet. Satellite B orbits at twice the orbital radius of Satellite A. Which of the following statements is true?

E

$F_g = \frac{Gm_1 m_2}{r^2}$

   (A) The gravitational force on Satellite A is four times less than that on Satellite B.
   (B) The gravitational force on Satellite A is two times less than that on Satellite B.
   (C) The gravitational force on the satellites is equal.
   (D) The gravitational force on Satellite A is two times greater than that on Satellite B.
   (E) The gravitational force on Satellite A is four times greater than that on Satellite B.

1.5×10^5

8. A proposed "space elevator" can lift a 1000 kg payload to an orbit of 150 km above the Earth's surface. The radius of the Earth is 6.4×10^6 m, and the Earth's mass is 6.0×10^{24} kg. What is the gravitational potential energy of the payload when it reaches orbit?

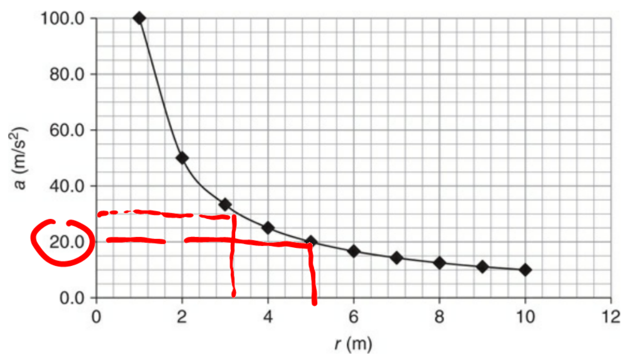
C

$U_g = -\frac{Gm_1 m_2}{r} = -\frac{(6.67\times10^{-11})(1000)(6.0\times10^{24})}{6.55\times10^6} =$

$r = 6.4\times10^6 + 1.5\times10^5 = 6.55\times10^6$

   (A) -1.0×10^3 J
   (B) -2.7×10^6 J
   (C) -6.1×10^{10} J
   (D) -2.7×10^{12} J
   (E) -1.0×10^{15} J

Questions 9–10 are based on the following graph:



9. Engineers have designed a centrifuge for studying the effects of high gravity environments on plants and animals. This graph shows the results of the relationship between the radius and the centripetal acceleration. If the scientists want to simulate a "3-G environment," then what should the radius of the centrifuge be?

a

$3g = 30\,m/s^2$

C

(A) 1 m
(B) 2 m
(C) 3 m
(D) 5 m
(E) 10 m

10. If an astronaut with a mass of 70 kg were placed in that centrifuge with a radius of 5 m, what would be the centripetal force acting on him?

$70 \times 20 = 1400\,N$

C

(A) 300 N
(B) 700 N
(C) 1400 N
(D) 2100 N
(E) 2400 N

2.0×10^5 M

11. A satellite orbits the Earth at a distance of 200 km. If the mass of the Earth is 6.0×10^{24} kg and the Earth's radius is 6.4×10^6 m, what is the satellite's speed?

C

(A) 1.0×10^3 m/s
(B) 3.5×10^3 m/s
(C) 7.8×10^3 m/s
(D) 5×10^6 m/s
(E) 6.1×10^7 m/s

$F_c = F_g$

$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$

$v = \sqrt{\frac{GM}{r}} = 7787\,m/s$

$r = 6.6\times10^6$

8.8 km

12. When climbing from sea level to the top of Mount Everest, a hiker changes elevation by 8848 m. By what percentage will the gravitational field of the Earth change during the climb? (The Earth's mass is 6.0×10^{24} kg, and its radius is 6.4×10^6 m.)

(A) It will increase by approximately 0.3 %.
(B) It will decrease by approximately 0.3 %.
(C) It will increase by approximately 12 %.
(D) It will decrease by approximately 12 %.
(E) The gravitational field strength will not change.

13. Four planets, A through D, orbit the same star. The relative masses and distances from the star for each planet are shown in the table. For example, Planet A has twice the mass of Planet B, and Planet D has three times the orbital radius of Planet A. Which planet has the highest gravitational attraction to the star?

A: $F_g = \frac{GM(2m)}{r^2} = \frac{2GMm}{r^2}$

Planet	Relative mass	Relative distance
A	$2m$	r
B	m	$0.1r$
C	$0.5m$	$2r$
D	$4m$	$3r$

B: $\frac{GMm}{(0.1r)^2} = \frac{100GMm}{r^2}$

$\frac{GM(4m)}{(3r)^2} = \frac{4}{9}\frac{GMm}{r^2}$

C: $\frac{GM(0.5m)}{(2r)^2} = \frac{GMm}{8r^2}$

B

(A) Planet A
(B) Planet B
(C) Planet C
(D) Planet D
(E) All have the same gravitational attraction to the star.

14. A satellite orbits the Earth at a distance that is four times the radius of the Earth. If the acceleration due to gravity near the surface of the Earth is g, the acceleration of the satellite is most nearly

E

$g_E = \frac{GM}{r_E^2}$

$g' = \frac{GM}{(4r_E)^2} = \frac{1}{16}g_E$

(A) zero
(B) $g/2$
(C) $g/4$
(D) $g/8$
(E) $g/16$

15. The mass of a planet is 1/4 that of Earth and its radius is half of Earth's radius. The acceleration due to gravity on this planet is most nearly

E

$g_E = 10\,m/s^2$

$g' = \frac{G(\frac{1}{4}m)}{(\frac{1}{2}r)^2} = \frac{G(\cancel{\frac{1}{4}}m)}{(\cancel{\frac{1}{4}}r^2)} = \frac{Gm}{r^2} = g$

(A) $2\,m/s^2$
(B) $4\,m/s^2$
(C) $5\,m/s^2$
(D) $10\,m/s^2$
(E) $20\,m/s^2$

16. Two planets of mass M and $9M$ are in the same solar system. The radius of the planet of mass M is R. In order for the acceleration due to gravity to be the same for each planet, the radius of the planet of mass $9M$ would have to be

D

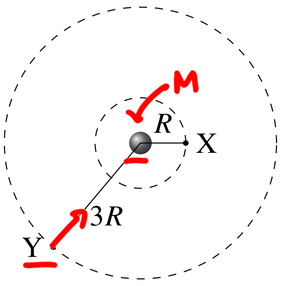
$g = \frac{\cancel{GM}}{R^2} = \frac{\cancel{G}(9\cancel{M})}{r^2} \rightarrow 9R^2 = r^2 \quad r = 3R$

(A) $R/2$
(B) R
(C) $2R$
(D) $3R$
(E) $9R$

17. Two planets, X and Y, orbit a star. Planet X orbits at a radius R, and Planet Y orbits at a radius $3R$. Which of the following best represents the relationship between the acceleration a_X of Planet X and the acceleration a_Y of Planet Y?

$F_g = \frac{G m_1 m_2}{r^2} = ma$

$g = \frac{GM}{r^2}$



A

(A) $a_X = 9a_Y$
(B) $9a_X = a_Y$
(C) $a_X = 3a_Y$
(D) $3a_X = a_Y$
(E) $a_X = a_Y$

18. A satellite is in a stable circular orbit around the Earth at a radius R and speed v. At what radius would the satellite travel in a stable orbit with a speed $2v$?

A

$F_c = F_g$

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$v^2 = \frac{GM}{R} \rightarrow R = \frac{GM}{v^2}$

$R' = \frac{GM}{(2v)^2} = \frac{GM}{4v^2} = \frac{1}{4}R$

(A) $R/4$
(B) $R/2$
(C) R
(D) $2R$
(E) $4R$

19. The Earth and the moon apply a gravitational force to each other. Which of the following statements is true?

E

(A) The Earth applies a greater force on the moon than the moon exerts on the Earth.
(B) The Earth applies a smaller force on the moon than the moon exerts on the Earth.
(C) The Earth applies a force on the moon, but the moon does not exert a force on the Earth.
(D) The Earth does not apply a force on the moon, but the moon exerts a force on the Earth.
(E) The force the Earth applies to the moon is equal and opposite to the force the moon applies to the Earth.

3rd law of motion

20. A planet orbits at a radius R around a star of mass M. The period of orbit of the planet is

C

$F_g = F_c$

$v^2 = \frac{GM}{R}$

$T = \frac{2\pi R}{v} \rightarrow v = \frac{2\pi R}{T} \rightarrow v^2 = \frac{4\pi^2 R^2}{T^2}$

$\frac{GM}{R} = \frac{4\pi^2 R^2}{T^2} \rightarrow T^2 = \frac{4\pi^2 R^3}{GM}$

$\frac{T^2}{R^3} = \frac{4\pi^2}{GM}$

$T = \sqrt{\frac{4\pi^2 R^3}{GM}}$

3rd law of planetary motion

(A) $\sqrt{\frac{4\pi^2 R^2}{GM}}$
(B) $\frac{4\pi^2 R^3}{GM}$
(C) $\sqrt{\frac{4\pi^2 R^3}{GM}}$
(D) $\sqrt{\frac{4\pi^2 R}{GM}}$
(E) $\frac{GM}{4\pi^2 R}$

21. If a planet has twice the radius of Earth and half of Earth's density, what is the acceleration due to gravity on the surface of the planet (in terms of the gravitational acceleration g on the surface of Earth)?

C

$M = \frac{4}{3}\pi r^3 \rho$

$M' = \frac{4}{3}\pi (2r)^3 \left(\frac{1}{2}\rho\right) = 4M$

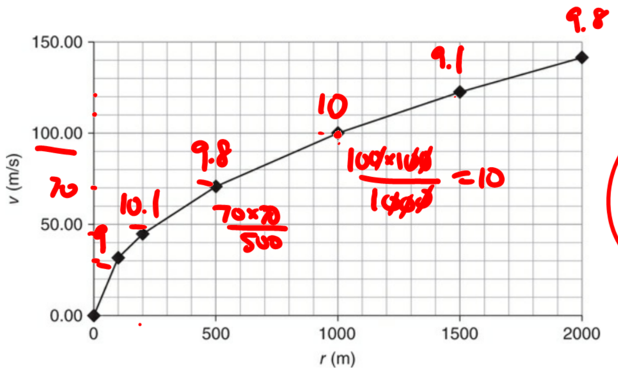
$g = \frac{GM}{r^2}$

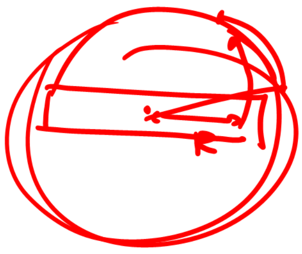
$g' = \frac{G(4M)}{(2r)^2} = g$

(A) $4g$
(B) $2g$
(C) g
(D) $g/2$
(E) $g/4$

22. The graph below depicts the tangential velocities of several circular space stations with different radii. All the stations are spinning. Which of the following statements is true?

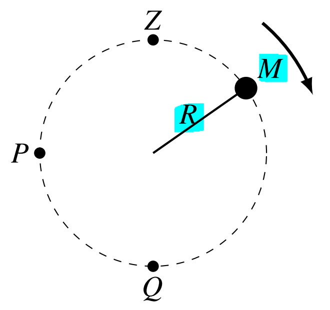
$a = \frac{v^2}{r}$





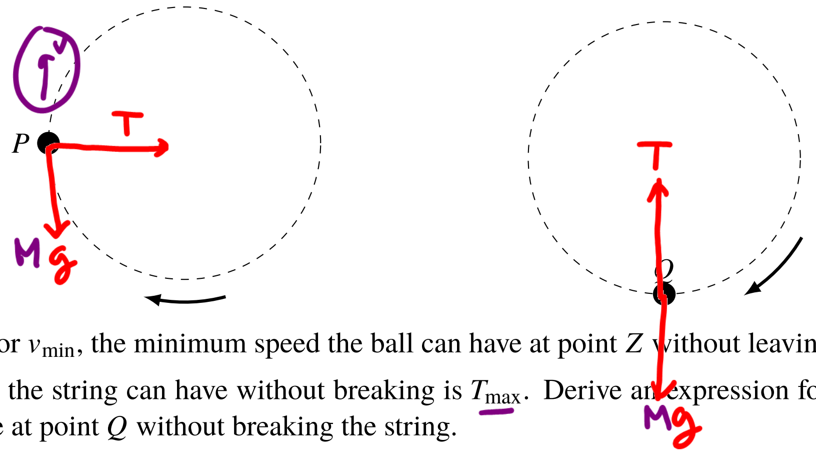
D

(A) The centripetal accelerations of the three shorter radii space stations are greater than $10\,\mathrm{m/s^2}$; those of the larger ones are less than $10\,\mathrm{m/s^2}$.
(B) The centripetal accelerations of the three shorter radii space stations are greater than $5\,\mathrm{m/s^2}$; those of the larger ones are less than $5\,\mathrm{m/s^2}$.
(C) The centripetal accelerations of all the stations are all nearly $5\,\mathrm{m/s^2}$.
(D) The centripetal accelerations of all the stations are all nearly $10\,\mathrm{m/s^2}$.
(E) The centripetal accelerations of the three shorter radii space stations are less than $10\,\mathrm{m/s^2}$; those of the larger ones are greater than $10\,\mathrm{m/s^2}$.



23. A ball of mass M is attached to a string of length R and negligible mass. The ball moves clockwise in a vertical circle, as shown above. When the ball is at point P, the string is horizontal. Point Q is at the bottom of the circle and point Z is at the top of the circle. Air resistance is negligible. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) On the figures below, draw and label all the forces exerted on the ball when it is at points P and Q, respectively.

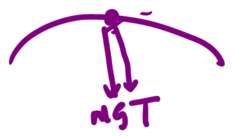


(b) Derive an expression for v_{min}, the minimum speed the ball can have at point Z without leaving the circular path.

(c) The maximum tension the string can have without breaking is T_{max}. Derive an expression for v_{max}, the maximum speed the ball can have at point Q without breaking the string.

(d) Suppose that the string breaks at the instant the ball is at point P. Describe the motion of the ball immediately after the string breaks.

b)



$F_c = Mg + T = Ma_c = \frac{Mv^2}{R}$ $\quad Mg + T = \frac{Mv^2}{R}$ $\quad$ min v → min T = 0

$Mg = \frac{Mv^2}{R}$

$\boxed{v_{min} = \sqrt{gR}}$

c) max. tension occurs at the botton

$F_c = Ma_c$ $\quad$ max T → max v.

$T - Mg = \frac{Mv^2}{R}$ → $T_{max} - Mg = \frac{Mv_{max}^2}{R}$ $\quad \frac{RT_{max}}{M} - Rg = v_{max}^2$

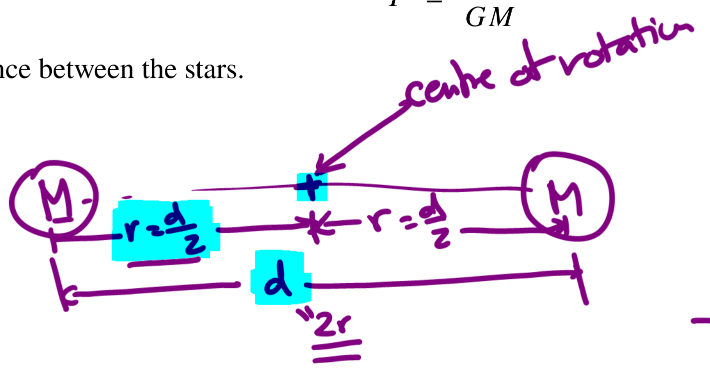
$$\boxed{v_{max} = \sqrt{\frac{RT_{max}}{M} - Rg}}$$

d) at P the $\boxed{\text{ball travel straight up}}$

24. Two stars of equal mass M are orbiting each other in a circular path. Show that the orbital period is given by:

$$T^2 = \frac{2\pi^2 d^3}{GM}$$

where d is the distance between the stars.



$$T = \frac{2\pi r}{v} = \frac{\pi d}{v}$$

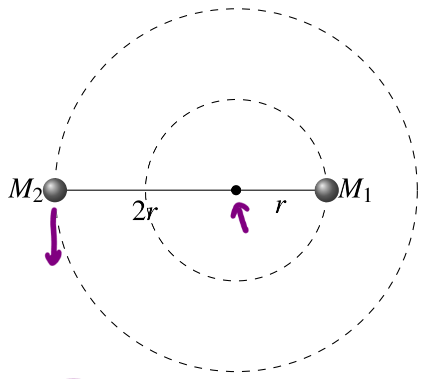
$$v = \frac{\pi d}{T} \qquad v^2 = \frac{\pi^2 d^2}{T^2}$$

$$F_g = F_c$$

$$\frac{GM^{\cancel{2}}}{d^{\cancel{2}}} = \frac{Mv^2}{\left(\frac{d}{2}\right)} = \frac{2\cancel{M}v^2}{\cancel{d}} \rightarrow 2v^2 = \frac{GM}{d}$$

$$\frac{2\pi^2 d^2}{T^2} = \frac{GM}{d} \rightarrow \boxed{T^2 = \frac{2\pi^2 d^3}{GM}}$$

25. Two stars of unequal mass orbit each other about their common center of mass as shown. The star of mass M_1 orbits in a circle of radius r, and the star of mass M_2 orbits in a circle of radius $2r$.



$M_2 = \frac{M_1}{2}$

$M_1 = 2M_2$

(a) Determine the ratio of masses M_1/M_2. $=2$

(b) Determine the ratio of the acceleration a_1 of M_1 to the acceleration a_2 of M_2.

(c) Determine the ratio of the period T_1 of M_1 to the period T_2 of M_2.

c) $\boxed{T_1 = T_2}$ for a stable orbit

$V_2 = \frac{2\pi r_2}{T}$ $-2r$ $\quad V_1 = \frac{2\pi (r)}{T}$

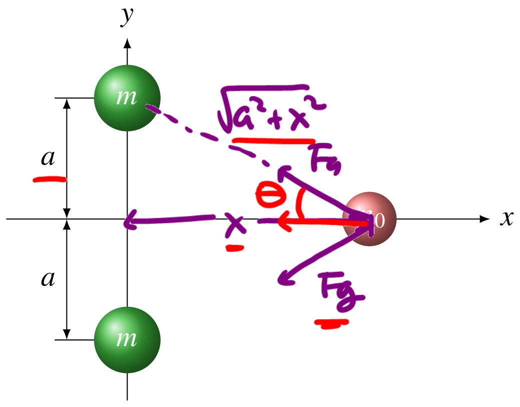
$V_2 = \frac{4\pi r}{T}$ $\quad V_1 = \frac{2\pi r}{T}$

$V_2 = 2V_1$ $\quad V_1 = \frac{V_2}{2}$

b) $a_1 = a_c = \frac{M_1 v_1^2}{2r}$ $\quad a_2 = \frac{M_2 v_2^2}{r}$

$$\frac{a_1}{a_2} = \frac{\left(\frac{2M_2 v_1^2}{2r}\right)}{\frac{M_2 (2v_1)^2}{r}} = \frac{\frac{\cancel{M_2 v_1^2}}{\cancel{r}}}{\frac{4\cancel{M_2 v_1^2}}{\cancel{r}}} = \boxed{\frac{1}{4}}$$

26. Two point particles of mass m are on the y axis at $y = a$ and $y = -a$, as shown in the figure below.



$F_g = \frac{Gmm_0}{a^2+x^2}$

$F_{gx} = F_g \cos\theta$

$= F_g \frac{x}{\sqrt{a^2+x^2}}$

(a) Derive the expression for the gravitational force exerted by these two particles on a third particle of mass m_0 located on the x axis at a distance x away from the origin.

(b) What is the gravitational field $\boldsymbol{g}$ on the x-axis due to the two particles?

(c) Show that g_x (the x component of $\boldsymbol{g}$) due to the two particles on the y axis is approximately $-\frac{2Gm}{x^2}$ when x is much greater than a.

m

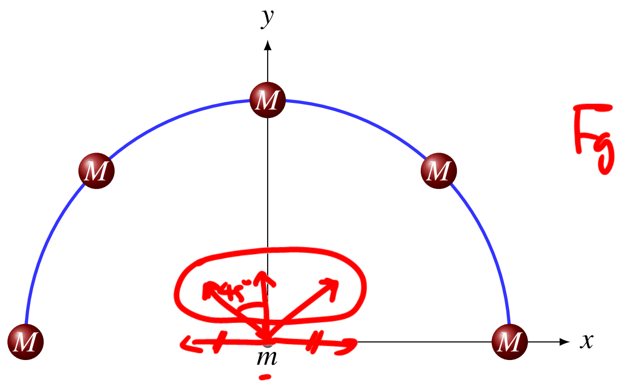
(d) (optional) Show that the maximum value of $|g_x|$ occurs at the point $x = \frac{\pm a}{\sqrt{2}}$.

a) $F_{gx} = \frac{Gmm_0}{a^2+x^2}\left(\frac{x}{\sqrt{a^2+x^2}}\right) \rightarrow \vec{F}_{net} = 2\vec{F}_{gx} = \boxed{\frac{-2Gmm_0 x}{(a^2+x^2)^{3/2}}\hat{i}}$

b) $g = \frac{\vec{F}_g}{m_0} = \boxed{-\frac{2Gmx}{(a^2+x^2)^{3/2}}\hat{i}}$

c) when $x \gg a \rightarrow a^2+x^2 \approx x^2$ $\quad g = -\frac{2GMx}{\underbrace{(a^2+x^2)}_{x^2}{}^{3/2}} = \frac{-2Gmx}{x^3} = \boxed{-\frac{2Gm}{x^2}}$

d) $\frac{dg}{dx} = 0 \rightarrow x = \_\_\_$



$F_g = \frac{GMm}{R^2}$

27. Five equal masses M are equally spaced on the arc of a semicircle of radius R as shown in the figure below. A mass m is located at the center of curvature of the arc. If M is 3 kg, m is 21 kg, and R is 10 cm, what is the force on m due to the five masses?

$$\vec{F} = \left(F_g + \frac{F_5}{\sqrt{2}} + \frac{F_5}{\sqrt{2}}\right)\hat{j} = F_5\left(1 + \underbrace{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}_{\frac{2}{\sqrt{2}}}\right)\hat{j} = F_5(1+\sqrt{2})\hat{j}$$

$$\vec{F} = \boxed{\frac{GMm}{R^2}(1+\sqrt{2})\hat{j}}$$