Topic 3: Work and Energy

Advanced Placement Physics 1

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Olympiads School

Files for You to Download

These are the slides and homework questions for this week.

- `PhysAP1-03-workEnergy.pdf`—Slides on work and energy
- `PhysAP1-04-momentum.pdf`—Slides on momentum, impulse and general collisions

Please download/print the PDF file before class. There is no advantage to copying notes that are already printed out for you. Instead, focus on details that aren't necessarily on the slides. If you want to print the slides, we recommend that you print 4 slides per page to save paper.

Work and Energy

We start with some definition at are (unfortunately) not very useful:

- **Energy** is the ability to do work.
- **Work** is the mechanism in which energy is transformed.

Luckily, we can also use equations to define these concepts.

Work

Mechanical Work

Mechanical work is performed when a force $\mathbf{F}$ displaces an object by $\Delta\mathbf{x}$. If the force is *constant*, then the work done is given by the dot product between the force and displacement vectors:

$$\boxed{W = \mathbf{F} \cdot \Delta\mathbf{x}} \quad \text{or} \quad \boxed{W = |\mathbf{F}||\Delta\mathbf{x}|\cos\theta}$$

where θ is the angle between the two vectors. When force and displacement vectors are expressed by the IJK notation, work is:

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

From the definition of a dot product

Mechanical Work

Mechanical work is a scalar quantity:

$$\boxed{W = \mathbf{F} \cdot \Delta\mathbf{x}}$$

- No work done if no displacement ($\Delta\mathbf{x} = \mathbf{0}$)
- No work done when $\mathbf{F} \perp \Delta\mathbf{x}$, when (i.e. $\mathbf{F} \cdot \Delta\mathbf{x} = 0$; the force did not cause the displacement)
- Work can be positive or negative depending on the dot product (or in the scalar case, $\cos\theta$)

Definition of Mechanical Work

Work done by a force

- The work done by *one specific force*
- Example: A boy pushes a cart forward. The “work done by the boy” is the work done by the applied force.

Work done on an object

- There may be more than one force acting on an object
- The *sum* of all the work done on the object by each force
- The work done by the net force
- Also called the **net work** W_{net}

Kinetic Energy

Kinetic Energy

When a constant net force accelerates an object from v_0 to v_1, work is being done (can be positive or negative). Combining a kinematic equation with the second law of motion:

$$v_1^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{F_{\text{net}}}{m}\Delta x$$

Re-arranging terms and solving for $F_{\text{net}}\Delta x$ (the work done by F_{net}), we get a familiar expression:

$$F_{\text{net}}\Delta x = \frac{1}{2}m(v_1^2 - v_0^2) = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2$$

Kinetic Energy

The change in the "quantity of motion" caused by the work done is defined as the **translational kinetic energy** K:

$$\boxed{K = \frac{1}{2}mv^2}$$

Later in the course we will discuss *rotational* kinetic energy. We arrive at this result using calculus even when F_{net} is not constant.

Work and Kinetic Energy

The **work-energy theorem** states that:

$$\boxed{W_{\text{net}} = \Delta K}$$

We don't have to prove this theorem because kinetic energy is defined to fit this theorem.

- ΔK can be positive or negative depending on whether the work done is positive or negative
- When multiple forces acting on an object, each force can add/remove kinetic energy from an object

Potential Energy

Gravitational Force & Gravitational Potential Energy

The gravitational force (weight) of an object is defined as:

$$\mathbf{w} = m\mathbf{g}$$

Near Earth's surface, $\mathbf{g} = -g\hat{\boldsymbol{k}}$ can be considered to be constant. Using the definition of mechanical work, the work done to move an object from height h_0 to h_1 is therefore:

$$W = \mathbf{w} \cdot \Delta\mathbf{h} = -mg\hat{\boldsymbol{k}} \cdot \Delta h\hat{\boldsymbol{k}} = -\left[mgh_1 - mgh_0\right]$$

From this, we define the **gravitational potential energy** U_g:

$$U_g = mgh$$

Spring Force & Elastic Potential Energy

The spring force $\mathbf{F}_e$ is the force that a compressed/stretched spring exerts on the object connected to it. It obeys Hooke's law:

$$\mathbf{F}_e = -k\mathbf{x}$$

We can find the work done to displace a spring using some calculus that is not shown here:

$$W = \mathbf{F}_e \cdot \Delta\mathbf{x} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_0^2$$

From this we define the **elastic potential energy** U_e:

$$U_e = \frac{1}{2}kx^2$$

Conservative Forces

Gravitational force, spring force, electrostatic force (studied in Grade 12 Physics) are called **conservative forces**

- The work done by these forces relate to a change of another quantity called *potential energy*
- Since the potential energy is evaluated at the end points, the work done by a conservative force is *path independent*

Conservative Forces

With a little bit of calculus, a constant conservative forces can be related to the change in potential energy by the expression in one-dimension:

$$\boxed{F = -\frac{\Delta U}{\Delta x}}$$

Pay attention to the negative sign. Students often forget it.

Work and Potential Energy

With conservative forces, *in addition* to changing kinetic energy, work equals to the change in *something*, and we define that as *potential energy*. Therefore:

$$W_{\text{cons.}} = -\Delta U$$

- ΔU can be positive/negative depending on the direction of the conservative force
- Positive work *decreases* potential energy, while
- Negative work *increases* potential energy

Conservation of Mechanical Energy

Positive work done by conservative forces on an object does two things:

1. *Decrease* its potential energy, while
2. *Increase* its kinetic energy by the same amount

In other words:

$$W_{\text{cons.}} = -\Delta U = \Delta K$$

This means that total mechanical energy E must be conserved when only conservative forces are doing work, or that the total *change* in mechanical energy is zero:

$$\boxed{\Delta E = \Delta K + \Delta U = 0}$$

That's why those forces are called conservative forces!

Conservation of Energy

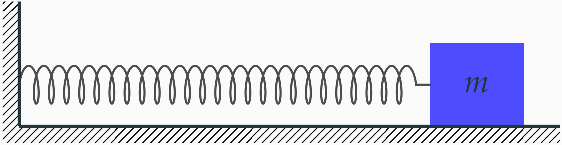
Example: Horizontal Spring-Mass System

Total mechanical energy is conserved when there are only conservative forces.

- Assuming that there is no friction in any part of the system
- The isolated system consists of the mass and the spring

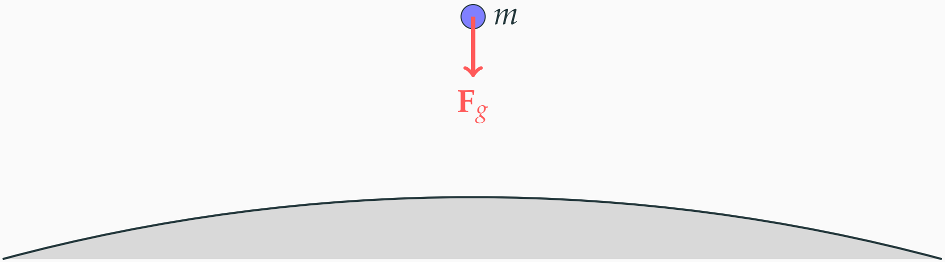
Energies:

- Kinetic energy of the mass
- Elastic potential energy stored in the spring



Example: Gravity

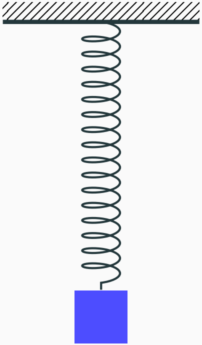
Gravity is another example, as an object falls towards Earth's surface, the only force acting on it is its weight, which is conservative:



Assuming no friction, the total mechanical energy consists of:

- Kinetic energy of the mass
- Gravitational potential energy of the mass

Example: Vertical spring-mass system



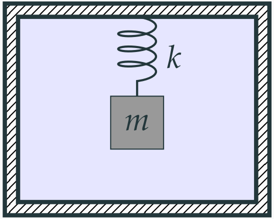
In a vertical spring-mass system, the work done on the mass are the spring force and gravity (both are conservative). Therefore the energy stored in the system are:

- Kinetic energy of the mass
- Gravitational potential energy of the mass
- Elastic potential energy stored in the spring

The total mechanical energy of the system is conserved if there is no friction

What if there is friction?

Energy is always conserved as long as your system is defined properly



- The system consists of a mass, a spring, Earth and all the air particles inside the box
- As the mass vibrates, friction with air slows it down
- While the mass loses energy, the temperature of the air rises due to friction
- Energies:
  - Kinetic and gravitational potential energies of the mass
  - Elastic potential energy stored in the spring
  - Kinetic energy of the vibration of the air molecules
- Total energy is conserved even as the mass stops moving

Conservation of Energy

If *only* conservative forces do work, then the total mechanical energy (sum of all the kinetic and potential $K + U$ energies) is conserved:

$$\boxed{\sum K_i + \sum U_i = \sum K_i' + \sum U_i'}$$

(The summation notation is used because there can be multiple particles, each with its kinetic energy, and multiple potential energies being stored). This is consistent with the equation shown in the previous slide:

$$\boxed{\Delta E = \Delta K + \Delta U = 0}$$

Conservation of Energy

When non-conservative forces are also doing work, the work done by those forces W_{nc} must be taken into account as well:

$$\boxed{\sum K_i + \sum U_i + \sum W_{\mathrm{nc}} = \sum K_i' + \sum U_i'}$$

Expressed in terms the total mechanical energy (like in the previous slides), the conservation of energy equation becomes:

$$\boxed{\Delta E = \sum \Delta K_i + \sum \Delta U_i = W_{\mathrm{net}}}$$

We do not have to distinguish between conservative and non-conservative work in this equation (only non-conservative work contributes to ΔE).

Non-Conservative Force

Work done by these non-conservative forces are *usually* negative because they oppose the direction of motion

- Drag (fluid resistance)
- Kinetic friction

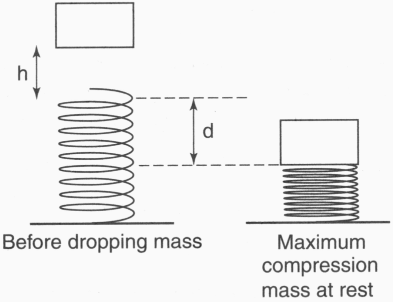
The work done by these non-conservative forces may be positive or negative, depending on the problem

- Applied force
- Tension force
- Normal force

Note that the work-kinetic energy theorem still applies when non-conservative forces are present

Example

Example 2: A mass m is dropped from a height of h above the equilibrium position of a spring. Set up the equation that determines the spring's compression d when the object is instantaneously at rest.



Power & Efficiency

Power

Power is the *rate* at which work is done, i.e. the rate at which energy is being transformed. The average power $\overline{P}$ is work done over a finite time interval δt. The unit for power is a **watt** W:

$$\boxed{\overline{P} = \frac{W}{\Delta t}}$$

Quantity	Symbol	SI Unit
Average power	$\overline{P}$	W
Work done	W	J
Time interval	Δt	s

In engineering, power is often more critical than the actual amount of work done.

Power

If a constant force is used to push an object at a constant velocity, the power produced by the force is:

$$P = \frac{W}{\Delta t} = \frac{\mathbf{F} \cdot \Delta \mathbf{x}}{\Delta t} = \mathbf{F} \cdot \frac{\Delta \mathbf{x}}{\Delta t} \quad \rightarrow \quad \boxed{P = \mathbf{F} \cdot \mathbf{v}}$$

Application: aerodynamics

- When an object moves through air, the applied force must overcome air resistance (drag force), which is proportional with v^2
- Therefore "aerodynamic power" must scale with v^3 (i.e. doubling your speed requires $2^3 = 8$ times more power)
- Important when aerodynamic forces dominate

Efficiency

Efficiency is the ratio of useful energy or work output to the total energy or work input

$$\boxed{\eta = \frac{E_o}{E_i} \times 100\,\%} \qquad \boxed{\eta = \frac{W_o}{W_i} \times 100\,\%}$$

Quantity	Symbol	SI Unit
Useful output energy	E_o	J
Input energy	E_i	J
Useful output work	W_o	J
Input work	W_i	J
Efficiency	η	no units

Efficiency is always $0 \leq \eta \leq 100\,\%$