Student #: ____________________ Student Name: ____________________

AP PHYSICS C CLASS 11: UNIVERSAL GRAVITATION

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\text{m/s}^2$ in all problems.

1. A 70 kg astronaut floats at a distance of 10 m from a 50 000 kg spacecraft. What is the force of attraction between the astronaut and spacecraft?
   (A) 2.4×10^{-6} N
   (B) 2.4×10^{-5} N
   (C) Zero; there is no gravity in space.
   (D) 2.4×10^{5} N
   (E) 2.4×10^{6} N

2. A proposed "space elevator" can lift a 1000 kg payload to an orbit of 150 km above the Earth's surface. The radius of the Earth is 6.4×10^6 m, and the Earth's mass is 6×10^{24} kg. What is the gravitational potential energy of the payload when it reaches orbit?
   (A) -1.0×10^{3} J
   (B) -2.7×10^{6} J
   (C) -6.1×10^{10} J
   (D) -2.7×10^{12} J
   (E) -1.0×10^{15} J

3. The mass of a planet is $1/4$ that of Earth and its radius is half of Earth's radius. The acceleration due to gravity on this planet is most nearly
   (A) $2\,\text{m/s}^2$
   (B) $4\,\text{m/s}^2$
   (C) $5\,\text{m/s}^2$
   (D) $10\,\text{m/s}^2$
   (E) $20\,\text{m/s}^2$

4. Four planets, A through D, orbit the same star. The relative masses and distances from the star for each planet are shown in the table. For example, Planet A has twice the mass of Planet B, and Planet D has three times the orbital radius of Planet A. Which planet has the highest gravitational attraction to the star?

Planet	Relative mass	Relative distance
A	$2m$	r
B	m	$0.1r$
C	$0.5m$	$2r$
D	$4m$	$3r$

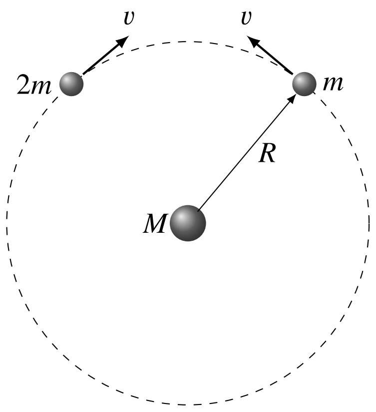
   (A) Planet A
   (B) Planet B
   (C) Planet C
   (D) Planet D
   (E) All have the same gravitational attraction to the star.

5. A satellite orbits the Earth at a distance that is four times the radius of the Earth. If the acceleration due to gravity near the surface of the Earth is g, the acceleration of the satellite is most nearly
   (A) zero
   (B) $\frac{g}{2}$
   (C) $\frac{g}{4}$
   (D) $\frac{g}{8}$
   (E) $\frac{g}{16}$

6. Two planets of mass M and $9M$ are in the same solar system. The radius of the planet of mass M is R. In order for the acceleration due to gravity to be the same for each planet, the radius of the planet of mass $9M$ would have to be
   (A) $\frac{R}{2}$
   (B) R
   (C) $2R$
   (D) $3R$
   (E) $9R$

Questions 7–8

Two moons of mass m and $2m$ orbit a planet of mass M at the same radius R and speed v toward each other, as shown. The moons collide and stick together without destroying either moon.



7. The total momentum of the moons after the collision is
   (A) mv
   (B) $2mv$
   (C) $3mv$
   (D) $6mv$
   (E) zero

8. The velocity of the two masses after the collision above is
   (A) v counterclockwise
   (B) $v/2$ counterclockwise
   (C) $v/2$ clockwise
   (D) $v/3$ counterclockwise
   (E) $v/3$ clockwise

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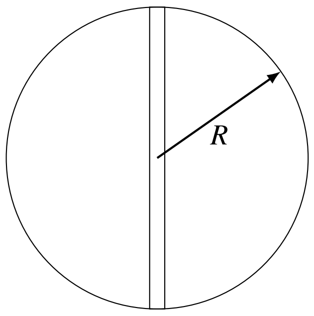
SECTION II
3 Questions

Directions: Answer all questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. A planet of mass M, radius R, and uniform density has a small tunnel drilled through the center of the planet, as shown below. When the mass is inside the tunnel, it experiences a force of $F = (GmM/R^3)r$, whereas when the mass is outside of the planet, it experiences a gravitational force of $F = GmM/r^2$.



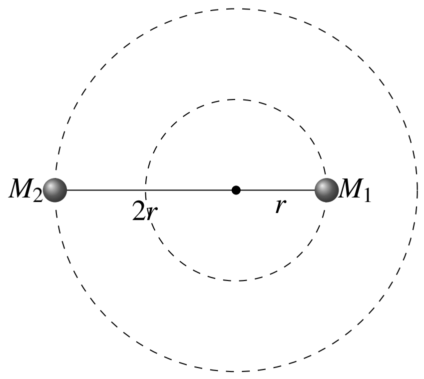
   (a) Setting the potential energy of the mass to be zero at the planet's center, calculate the mass's potential energy as a function of distance from the center of the planet $U(r)$, for values $r < R$. Sketch this potential function.

   (b) If the mass is dropped from R from the center of the planet, how long will it take until it returns to its original position?

   (c) If the mass is dropped from $R/2$ from the center of the planet, will it require more, or less, or the same amount of time to return to its original position compared to if it was dropped from R?

   (d) If the mass is dropped from $2R$ from the center of the planet, will it require more, or less, or the same amount of time to return to its original position compared to if it was dropped from R?

2. Two stars of unequal mass orbit each other about their common center of mass as shown. The star of mass M_1 orbits in a circle of radius r, and the star of mass M_2 orbits in a circle of radius $2r$.



(a) Determine the ratio of masses M_1/M_2.

(b) Determine the ratio of the acceleration a_1 of M_1 to the acceleration a_2 of M_2.

(c) Determine the ratio of the period T_1 of M_1 to the period T_2 of M_2.

3. An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

(a) If the radius of the planned orbit is R, use Newton's laws to show each of the following.

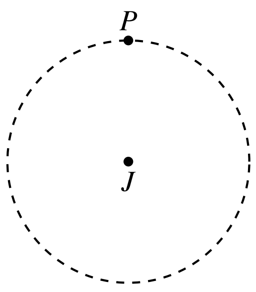
i. The orbital speed of the planned satellite is given by $v = \sqrt{\dfrac{GM_J}{R}}$.

ii. The period of the orbit is given by $T = \sqrt{\dfrac{4\pi^2 R^3}{GM_J}}$.

(b) The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

(c) Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly faster than the correct speed for a circular orbit of that radius.



ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly slower than the correct speed for a circular orbit of that radius.