Exercise sheet #2

Course: *Machine Learning in Physics (PHYS3151)* – Professor: *Dr. Ziyang Meng*
Due date: *Mar.7th, 2022*

1. **Gradient descent method and steepest descent method**
   Please read and follow the example of *GradientDescent&ConjugateGradient.ipynb*. Given $Q = \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix}$, and $b = \begin{pmatrix} 3 & -1 \end{pmatrix}$, and $c = 5$, please draw a 3D plot of the cost function $J(\theta) = \frac{1}{2}(\theta^T Q \theta - b\theta + c)$, and draw a contour plot. Take an initial guess of (θ_0, θ_1), and use gradient descent and steepest gradient methods to find the minimum and plot the trace of the optimization processes. Compare the performance of two methods.

2. **Conjugate gradient method**
   Use the conjugate gradient method to solve the equation

$$Q\theta = b$$

   with $Q = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. Please set your intial guess as $\theta_0 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$.
   (a). Write down u_k, α_k and θ_k step by step and check that:

   - $r_k \cdot r_{k+1} = 0$, where the residual vector $r_k = b - Q\theta_k$;

   - $u_k Q u_{k+1} = 0$, where $\{u_k\}$ are the conjugate vectors of matrix Q;

   - conjugate gradient method converges after 2 steps.

   (b). Please draw a contour plot to show the process above. (Using the code in *GradientDescent&ConjugateGradient.ipynb*)