THE UNIVERSITY OF HONGKONG
DEPARTMENT OF PHYSICS

Assignment 3

Course: *Computational Physics (PHYS4150/8150)* – Professor: *Dr. Ziyang Meng*
Tutor: *Mr. Menghan Song, Mr. Ting-Tung Wang*
Due date: *Nov. 7th, 2022*

1. Stationary Schrödinger Equation
   Use the Numerov matrix method to solve the stationary Schrödinger equation with potential $V = |x|^3$. Similar with the case of quantum harmonic oscillator, we set the term $\gamma^2 = 2m\omega/\hbar = 2$ and $N = 2(4\pi + \frac{x_t}{dx})$ (we set as $N = 224$) with $x_t = 10$ and $dx = 0.1$. Therefore, the range of x is from -11.15 to 11.15. Please numerically solve the equation and:

   (a) Plot the numerical probability density $P_n(x) = |\psi_n(x)|^2$ for $n = 1, \ldots, 5$ and see whether it is the same as the following:

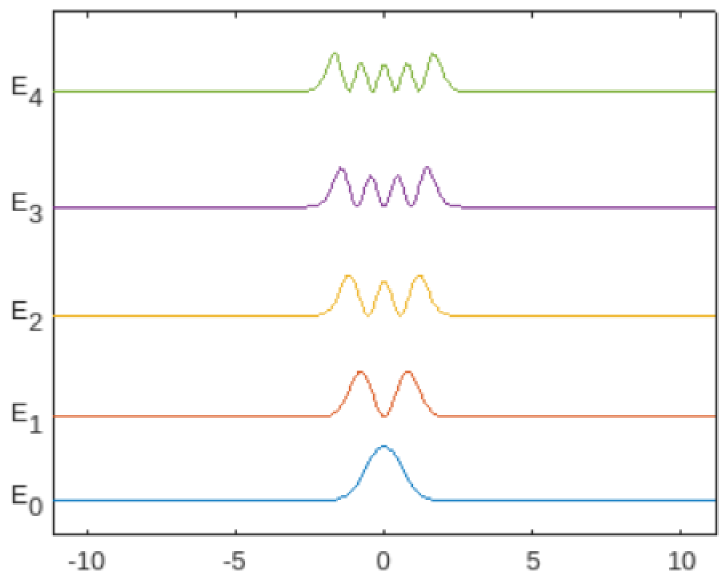

Figure 1: 1(a)

   (b) Show the five lowest eigen-energies.

2. (a) Time-dependent Schrödinger Equation Please use the Crank-Nicolson Scheme to numerically solve the time-dependent Schrödinger equation. Using the same scheme as in the lecture: $\hbar = m = 1$, $x_0 = -10$, $L = 40$, $\sigma_0 = 1$, $k_0 = 7$, $N = 401$, $\tau = 0.03$, $h = 0.1$, (the range of x is -20 to 20) and the initial

condition is $\Psi(t=0,x) = \frac{1}{\sqrt{\sigma_0\sqrt{2\pi}}} e^{ik_0 x} e^{-\frac{1}{2}\frac{(x-x_0)^2}{2\sigma_0^2}}$. The difference is we add a sinusoidal potential barrier $V(x) = \frac{k_0^2}{2} cos(\frac{\pi x}{6})$ at $-3 < x < 3$. Please use an animation to show the probability density $P(t,x) = |\Psi_n(t,x)|^2$ for t from 0 to 15. (Attached below is a screenshot at $t = 150\tau$)

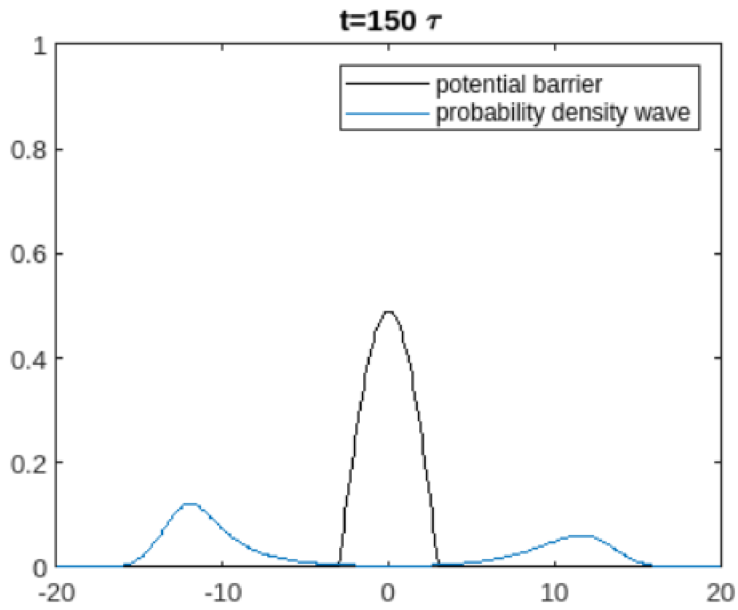

Figure 2: 2

(b) Show that the probability density $P(t,x)$ is conserved during time evolution.