THE UNIVERSITY OF HONGKONG
DEPARTMENT OF PHYSICS

Assignment 1

Course: *Computational Physics (PHYS4150/8150)*
Professor: *Dr. Ziyang Meng*
Tutor: *Mr. Menghan Song, Mr. Ting-Tung Wang*
Due date: *Sept. 29th, 2022*

1. Hope you all enjoyed the mid-autumn festival a few days ago. The moon conveys people's wish and carries people's sentiment. We have learned that the moon moves in an ellipse around the Earth. The gravitational force between the moon and the Earth is $\overrightarrow{F} = -\frac{GMm}{r^3}\overrightarrow{r}$, and the equation of motion is: $\frac{d^2\overrightarrow{r}}{dt^2} = -\frac{\mu}{r^3}\overrightarrow{r}$, where $\mu = G(M+m)$. Here, take the moon as an example. We set the initial condition of the moon in a two-dimensional space as: $x = a(1-e), y = 0, v_x = 0, v_y = \sqrt{\mu(1+e)/[a(1-e)]}$. We consider the unit of t as 1 day, length unit as 3.84748×10^8 m and the mass unit as 5.972×10^{24} kg. So the semi-major axis $a = 1$, mass of the Earth $M = 1$, mass of the moon $m = 0.0123$, the eccentricity $e = 0.0549$, and thus $\mu = 0.05288$.

Figure 1: Sik Sik Yuen at Wong Tai Sin Temple

Please use Euler method, Velocity Verlet method, and 4-th order Runge-Kutta method to computationally solve the equation of motion for the moon.

(a) For each method, please use 3 τs (Euler: 0.05, 0.1, 0.5; Verlet: 0.1, 1, 2 Runge-Kutta: 0.5, 1, 3) and compare the results respectively.

(b) Please compare the results from different methods.

Complete code and necessary figures with analysis are required.

2. Kepler's second law states that the moon moves in its ellipse so that the line between it and the Earth placed at a focus sweeps out equal areas in equal times. Please use the numerical results from the 4-th Runge-Kutta method ($\tau = 0.5$) to prove this law with time duration equals to 1 day, as specified in Fig 2. Complete code and necessary explanation are required.

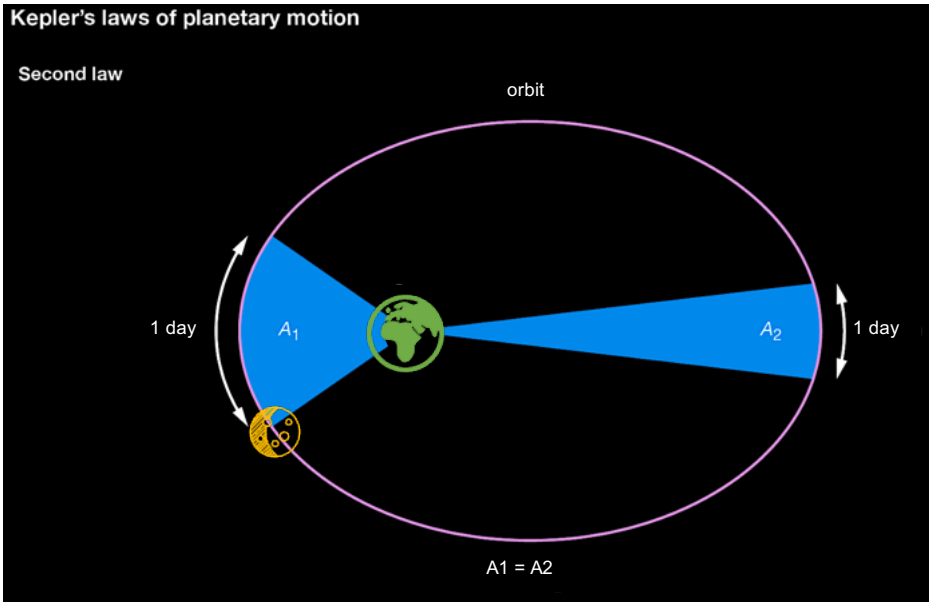

Figure 2: schematic diagram of Kepler's second law