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Endogenous Technological Change in a General Equilibrium Climate-Economy Model

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α	Capital cost shares in production
β	Discount factor
γ	Regional damage parameter
ζ	Carbon content of the exhaustible resource
η_i	Success probability of research in sector $i \in \mathbb{I}, \in [0, 1]$
$\eta_{i,j}$	Individual success probability in sector $i \in \mathbb{I}$ and machine line $j \in \mathbb{J}$
ϑ	Optimal R&D subsidy
ι	Quality ladder step size
κ	Share parameter in CES production function
Λ	Social Cost of Carbon
λ	Technology parameter relating R&D expenditure to success probability
ν	Factor cost shares for energy and the exhaustible resource
Π_i	Aggregated profits in R&D in sector $i \in \mathbb{I}$
$\pi_{i,j}$	Individual profits in R&D in sector $i \in \mathbb{I}$ and machine line $j \in \mathbb{J}$
ρ	Elasticity of substitution between energy inputs
σ	Intertemporal elasticity of substitution
τ	Efficient emissions tax
υ	Global resource price
ϕ	Share of emissions decaying at a geometric rate
ϕ_0	Share of immediately absorbed CO_2 emissions
ϕ_L	Share of permanent CO_2 emissions
χ_i	Share in the nominal energy mix of sector $i \in \mathbb{I}$
ψ	Marginal costs in machine production
ω	Technology parameter representing the cost of R&D
B	Length of policy intervention
C	Aggregate consumption
c	Extraction cost for the exhaustible resource
D	Climate damages
E	Energy aggregate used in final goods production
E_i	Energy production in sector $i \in \mathbb{I}$
g	Growth rate on the balanced growth path
g^N	Population growth rate

H_i	Aggregated R&D expenditures in sector $i \in \mathbb{I}$
$h_{i,j}$	Individual R&D expenditures in sector $i \in \mathbb{I}$ and machine line $j \in \mathbb{J}$
$\mathbb{I}_0$	Set of production sectors: $\mathbb{I} \coloneqq \{0, 1, 2\}$
$\mathbb{I}$	Set of energy production sectors: $\mathbb{I} \coloneqq \{1, 2\}$
i	Subindex for production sectors $i \in \{0, 1, 2\}$
$\mathbb{J}$	Set of sector-specific machine lines $J \coloneqq [0, 1]$
j_i	Machine line in sector $i \in \mathbb{I}, j \in \mathbb{J}$
K	Capital used for final goods production
$\mathbb{L}$	Set of regions
ℓ	Region index $\in \mathbb{L}$
$M_{i,j}$	Machine demand in sector $i \in \mathbb{I}$ and machine line $j \in \mathbb{J}$
N_i	Labor in sector $i \in \mathbb{I}_0$
p_i	Energy price in sector $i \in \mathbb{I}$
$p_{i,j}$	Price of machines in sector $i \in \mathbb{I}$ and machine line $j \in \mathbb{J}$
Q_i	Quality index in sector $i \in \mathbb{I}$
$Q_{i,j}$	Machine quality in sector $i \in \mathbb{I}$ and machine line $j \in \mathbb{J}$
R	Resource stock
r	Interest rate
S_1	Permanent atmospheric CO_2
S_2	Non-permanent atmospheric CO_2
s	Mass of scientists in sector $i \in \mathbb{I}, \in [0, 1]$
t	Index of discrete time
w	Regional price of labor or wage level
X	Exhaustible resource
Y	Final output
Z	Global carbon dioxide emissions

List of Abbreviations

CO_2	Carbon dioxide
CES	Constant elasticities of substitution
DICE	Dynamic Integrated Climate-Economy Model
EU	European Union
GDP	Gross domestic product
GHG	Greenhouse gas
GtC	Gigatonnes of carbon
IAM	Integrated Assessment Model
IPCC	Intergovernmental Panel on Climate Change
NASA	National Aeronautics and Space Administration
NOECD	Non-OECD countries
OECD	Organization for Economic Co-operation and Development
R&D	Research and Development
RICE	Regional Integrated Climate-Economy Model
SCC	Social Cost of Carbon

1 Introduction

1.1 Motivation

Human activities, especially the increasing use of fossil fuels, have unequivocally led to a rise in the global mean temperature above the pre-industrial level. Unsustainable energy sources, e.g., coal, gas, and oil, deforestation, and a change in agricultural and industrial practices are responsible for the increase in greenhouse gas emissions (GHG), specifically carbon dioxide and methane. The repercussions of climate change consequently manifest in heat waves, wildfires, acidification of the oceans, and melting glaciers which pose a threat to nature, biodiversity, public health, food and water security, and infrastructure (IPCC, 2022).

Avoiding severe damage to health, infrastructure, and living requires international coordination and investments in, e.g., renewable energies which emphasizes the economic importance of addressing global warming. The interdependencies between economic activities, emissions trajectories, and climate variables can be analyzed within Integrated Assessment Models (IAM) that synthesize interdisciplinary knowledge by combining economics, climate sciences, and policy analysis in a single framework.

In this context, it is of special interest how new technologies can offer a sustainable solution to the increase in emissions and which policies are needed to implement relevant changes. Empirical evidence indicates that directed technical change can be used to combat climate change by redirecting research and technological innovations to renewable energies. Moscona & Sastry (2022) show that the redirection of technology towards sustainable alternatives decreased damages to U.S. agriculture caused by extreme temperatures since 1960 by 20%. Furthermore, Aghion *et al.* (2016) find strong evidence for *path-dependence* in the auto industry, suggesting that firms that are especially engaged with innovation of clean technologies are more likely to still direct their research to clean technologies in the future. Conversely, firms that have primarily been focusing on innovating non-sustainable technologies are more likely to keep their focus on these. This raises the question of which policy measures are necessary and optimal to overcome path dependencies and redirect technologies to sustainable alternatives. Moreover, this study seeks to further understand whether policy intervention can reconcile economic growth with environmental sustainability.

This study addresses these questions by combining the theoretical framework of Acemoglu *et al.* (2012), who analyze the determinants of directed technological change and its policy implications, with the model by Hillebrand & Hillebrand (2019), who developed a dynamic general equilibrium model with heterogeneous world regions. Hillebrand & Hillebrand (2019) include a comprehensive description of the production process in their IAM framework by explicitly modeling different energy sectors and a resource extraction stage next to the production of a final good. This allows us to study how climate policies affect the transition from dirty to clean energies in greater detail. However, their framework only considers exogenous technological growth whereas Acemoglu *et al.*

(2012) endogenize technical change and its direction based on profit incentives, however, with a less comprehensive model structure. Therefore, I combine the concept of directed technological change with the IAM framework by Hillebrand & Hillebrand (2019) to obtain a more exhaustive and complete description of the interplay between economic sectors and the world climate.

Furthermore, I extend the research process in Acemoglu *et al.* (2012) by explicitly modeling a micro-founded R&D sector. Thereby, we can gain greater insights into firms‘ investment decisions, and it allows me to add the reasonable assumption that funds are required to finance research, which in turn affects the success of the research project. This further endogenizes the research stage and the process of technological change and emphasizes the interdependencies between the research sector and the rest of the economy. This connection is missing in Acemoglu *et al.* (2012) as they assume the outcome of research to be exogenous.

1.2 Literature review

By incorporating directed technological change in an Integrated Assessment Model, I combine two large and growing strands of economic literature. The evolution of the IAM literature started with the pioneering work of Nordhaus (1977) who developed the Dynamic Integrated Climate-Economy Model (DICE) which was later updated to a multi-region framework called Regional Dynamic Integrated Climate-Economy Model (RICE) (Nordhaus & Yang, 1996) and further refined in Nordhaus & Boyer (2000). However, these models lack explicit modeling of markets, price formations, and endogenous economic reactions to changes in the climate model (Hassler *et al.*, 2016).

Consequently, Golosov *et al.* (2014) set up a more transparent IAM based on a modern macroeconomic framework that uses the neoclassical growth model, allowing for micro-founded market mechanisms and an analysis of a decentralized equilibrium. Prior contributions to this class of models are Kelly & Kolstad (1999) who focus on uncertainty and learning of economic agents and Leach (2009) who introduce overlapping generations into an IAM framework. Hassler & Krusell (2012) extend the model framework to a multi-region model taking into account the heterogeneity of regions, e.g., due to differences in their resource stocks, climate damages, and their initial capital endowment. The model by Golosov *et al.* (2014) has been further refined and extended in various directions by e.g., Rezai & van der Ploeg (2015), Gerlagh & Liski (2018), Hassler *et al.* (2019) and Barrage (2020).

Based on Golosov *et al.* (2014) and its various extensions, Hillebrand & Hillebrand (2019) developed a multi-region dynamic general equilibrium model featuring international trade between regions and emphasize the heterogeneity of world regions. Their model contains an environmental constraint in the form of a climate externality that originates from the use of an exhaustible resource causing GHG emissions. Furthermore, they propose an international transfer scheme that incentivizes each region to implement an optimal emissions tax. This study extends their existing framework by incorporating an R&D sector that improves the quality and productivity levels at the energy stage over time. In addition to the climate externality, the introduction of an R&D sector necessitates to correct for an R&D externality and requires policy measures beyond a carbon tax, e.g., in the form of a clean research subsidy.

The theoretical framework for the R&D sector is based on the endogenous growth literature. Most of the early literature dealing with environmental policies assumes that economic growth is exogenous, including the DICE and RICE framework. One of the seminal papers on endogenous growth by Romer (1990) emphasizes the role of knowledge accumulation and technological progress as key drivers of economic growth. Technological progress can help to decompose economic growth and pollution growth by decreasing the pollution intensity of production inputs, developing renewable alternatives, and increasing factor productivity for clean inputs (Burghaus, 2013). The concept of endogenous technological change asserts that investments in R&D, human capital, and innovation in general create new knowledge and technologies leading to sustainable economic growth. Consequently, there have been various models of technological change that assess how environmental policies shape economic growth, among others, e.g., Bovenberg & Smulders (1996), Jonathan Kohler *et al.* (2006), Wing (2006), and La Torre & Marsiglio (2010).

The R&D framework in this study assumes that technological progress is based on Schumpeterian growth as in Aghion & Howitt (1992) and Grossman & Helpman (1991) who assume improvements in the quality or productivity of products accompanied by property rights protection and profit incentives in R&D. Although these models allow us to study structural and institutional determinants of endogenous technological progress, they do not take into account that technological progress might be biased towards particular production factors. Therefore, Acemoglu (2002) set up a theoretical framework that allows us to analyze how market forces like the size of a market or factor prices affect the direction of technological change. Acemoglu *et al.* (2012), incorporate the concept of directed technological change in a climate-economy model with environmental constraints. They find that if energy inputs are sufficiently substitutable, sustainable growth can be achieved using both carbon taxes and a temporary research subsidy. The model has been extended and refined several times, e.g., to multi-region frameworks as in Acemoglu *et al.* (2014) or van den Bijgaart (2017) or by considering carbon capture and storage technologies as in Hou *et al.* (2020). In contrast, Amigues & Durmaz (2019) consider research activities for either resource-saving or abatement-augmenting technologies instead of focusing on the trade-off between two energy inputs.

In contrast to models dealing with directed technical change, e.g., Acemoglu *et al.* (2012), Hemous (2013), and van den Bijgaart (2017), this study further endogenizes the research process by linking the research outcome to R&D expenditure. Furthermore, by introducing directed technical change to the model by Hillebrand & Hillebrand (2019) this model offers a more comprehensive setting to analyze policy alternatives. Energy is explicitly modeled as a production input and the energy production stage is decoupled from final goods production, which allows us to include labor and capital as additional production factors for final output and separate output growth and the transition from dirty to clean energies. Instead of a simple difference equation describing the environmental quality as in Acemoglu *et al.* (2012), the climate model features a more detailed carbon cycle. The combustion of an exhaustible resource emits CO_2 which increases the atmospheric CO_2 concentration, which in turn increases the global mean surface temperature, causing climate damages.

The literature about whether a reconciliation of economic growth and environmental sustainability is feasible can roughly be subdivided into three positions. The most optimistic position by Nordhaus (2008) suggests that only limited and gradual intervention is necessary to combat the effects of global warming and to allow economic long-run growth. Stern (2009) takes a more pessimistic position, arguing that policy intervention has to be immediate and permanent despite their significant economic costs. Lastly, the Greenpeace answer, as labeled by Acemoglu *et al.* (2012), requires all economic growth to come to an end to save the planet. Note, however, that the aforementioned positions are based on exogenous growth models.

In their model of endogenous technical change, Acemoglu *et al.* (2012) take the position that immediate but only temporary intervention is necessary given a sufficiently high elasticity of substitution between energy inputs. They suggest that already sub-optimal policies just using carbon taxes or research subsidies are sufficient to redirect technological change. In a quantitative general equilibrium analysis, Fried (2018) finds that carbon taxes can induce innovation in green technologies and increase the share of green energies in the energy mix. This study, in turn, highlights that the sole use of a Pigouvian carbon tax is not capable of redirecting innovation and may ultimately even fail to stay within the 2°C-target set by the Paris accord on climate change. To avoid excessively high carbon taxes, I also introduce a clean research subsidy.

The remainder of this thesis is structured as follows. Section 2 introduces the general equilibrium climate model framework. The general characterization of the decentralized equilibrium and the determinants of directed technological change are analyzed in Section 3. Section 4 then provides optimal climate policies that implement the optimal solution. A numerical exercise is presented in Section 5. Mathematical proofs and details regarding the numerical algorithm and computational methods can be found in Appendix A and B, respectively.

2 The Model

2.1 World economy

The world economy consists of $L \geq 1$ regions, indexed by $\ell \in \mathbb{L} := \{1, ..., L\}$. Each region represents a union of countries or a single country that can make autonomous policy decisions. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The production process can be decomposed into a final good stage, an energy stage, a resource stage, and an R&D stage. In each region $\ell \in \mathbb{L}$ there are $I = 3$ production sectors identified by the index $i \in \mathbb{I}_0 := \{0, 1, 2\}$, a resource extraction sector and a research sector.

The sector $i = 0$ is the *final sector* that produces a homogenous final good that can be consumed or invested in capital and R&D. The final good serves as the numeraire, i.e., all prices, returns, and costs are measured in terms of the final good. The second stage comprises a dirty ($i = 1$) and a clean ($i = 2$) *energy sector* that provide energy goods and services to the final sector based on either exhaustible or renewable resources. The set of energy producing sectors is given by $\mathbb{I} := \{1, 2\}$. The *resource sector* represents the third stage that extracts the available stock of an exhaustible resource. In the undermost stage, the *R&D sector*, scientists direct their research to sector-specific machines that complement either the clean or the dirty energy sector.

2.2 Production sectors

2.2.1 Final good

Sector $i = 0$ in region $\ell \in \mathbb{L}$ produces the final good Y_t^ℓ using labor $N_{i,t}^\ell$, capital K_t^ℓ, and energy goods and services E_t^ℓ. The production technology is

$$Y_t^\ell = (1 - D_t^\ell) Q_{0,t}^\ell F_0(K_t^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}) = (1 - D_t^\ell) Q_{0,t}^\ell (K_t^\ell)^{\alpha_0} (N_{0,t}^\ell)^{1-\alpha_0-\nu_0} (E_t^\ell)^{\nu_0} \quad (1)$$

where $D_t^\ell \in [0, 1[$ denotes climate damages that are specified further below. Here, $Q_{0,t}^\ell > 0$ is an exogenous productivity parameter that may vary over time and across regions. Energy goods and services E_t^ℓ are a composite of the energy goods produced in the second stage which are aggregated as

$$E_t^\ell = [\kappa_1 (E_{1,t}^\ell)^\rho + \kappa_2 (E_{2,t}^\ell)^\rho]^{\frac{1}{\rho}} \quad \text{where} \quad \sum_{i=1}^{2} \kappa_i = 1. \quad (2)$$

The elasticity of substitution between the two energy goods is given by $1/(1 - \rho)$. κ_i describes the relative importance of the specific energy goods within the CES-aggregate. Given the climate damages D_t^ℓ, the wages for labor w_t^ℓ, energy prices $p_{i,t}^\ell$, and the rental price for capital r_t, final good producers maximize profits

$$\max_{(K_t^\ell, N_{0,t}^\ell, (E_t^\ell)_{i\in\mathbb{I}})\in\mathbb{R}_+^{2+I}} \left\{ (1 - D_t^\ell) Q_{0,t}^\ell F_0(K_t^\ell, N_{0,t}^\ell, (E_t^\ell)_{i\in\mathbb{I}}) - w_t^\ell N_{0,t}^\ell - r_t K_t^\ell - \sum_{i\in\mathbb{I}} p_{i,t}^\ell E_{i,t}^\ell \right\} \quad (3)$$

The maximization problem solves for the standard first order conditions

$$\frac{\alpha_0 Y_t^\ell}{K_t^\ell} = r_t, \quad \frac{(1 - \alpha_0 - \nu_0) Y_t^\ell}{N_{0,t}^\ell} = w_t^\ell, \quad \text{and} \quad \frac{\nu_0 Y_t^\ell}{E_{i,t}^\ell} \kappa_i \left(\frac{E_{i,t}^\ell}{E_t^\ell} \right)^\rho = p_{i,t}^\ell \quad \text{for } i = 1, 2. \quad (4)$$

2.2.2 Energy stage

Each region $\ell \in \mathbb{L}$ has two energy sectors $i \in \mathbb{I} := \{1, 2\}$ that produce energy $E^{\ell}_{i,t}$, using labor $N^{\ell}_{i,t}$ and a continuum of sector-specific machines (intermediates) $M^{\ell}_{i,j,t}$, where $M^{\ell}_{i,j,t}$ denotes the machine demand from machine line $j \in \mathbb{J}$ in sector $i \in \mathbb{I}$ at time t. The set of machine lines will be further specified below. The quality or productivity level of machine line $j \in \mathbb{J}$ in sector $i \in \mathbb{I}$ is denoted by $Q^{\ell}_{i,j,t}$ and is determined endogenously through innovation efforts in the R&D sector. Sector-specific machines depreciate fully after use in one model period.[1] The dirty energy sector additionally uses an exhaustible resource, e.g., coal, oil, or gas as input for production.

Dirty energy sector
The production technology for the dirty energy sector $i = 1$ takes the form

$$E^{\ell}_{1,t} = F_1(N^{\ell}_{1,t}, M^{\ell}_{1,j,t}, X^{\ell}_t, Q^{\ell}_{1,j,t}) = (X^{\ell}_t)^{\nu_1}(N^{\ell}_{1,t})^{1-\alpha_1-\nu_1} \int_0^1 (Q^{\ell}_{1,j,t})^{1-\alpha_1}(M^{\ell}_{1,j,t})^{\alpha_1} dj \tag{5}$$

where X^{ℓ}_t denotes the exhaustible resource. Since the combustion of exhaustible resources generates greenhouse gas emissions proportional to the input of the exhaustible resource, this sector represents the production stage where emissions are potentially generated. Emissions caused by the usage of the exhaustible resource are given by

$$Z^{\ell}_t = \zeta X^{\ell}_t. \tag{6}$$

The parameter ζ describes the physically determined carbon content of X^{ℓ}_t. Furthermore, I assume that every region $\ell \in \mathbb{L}$ imposes a per-unit tax $\tau^{\ell}_t \geq 0$ (which may be zero) on emissions in period $t \geq 0$. The energy producers take the carbon tax, wages for labor, rental prices for machines $p_{i,j,t}$, and the world price of the exhaustible resource υ_t as given and maximize profits. The decision problem in period $t \geq 0$ reads

$$\max_{(N^{\ell}_{1,t}, M^{\ell}_{1,j,t}, X^{\ell}_t) \in \mathbb{R}^3_+} \left\{ p^{\ell}_{1,t} F_1(N^{\ell}_{1,t}, M^{\ell}_{1,j,t}, X^{\ell}_t, Q^{\ell}_{1,j,t}) - w^{\ell}_t N^{\ell}_{1,t} - \int_0^1 p_{1,j,t} M^{\ell}_{1,j,t} dj - (\upsilon_t + \tau_t \zeta) X^{\ell}_t \right\}. \tag{7}$$

The first order conditions w.r.t. $N^{\ell}_{1,t}$ and X^{ℓ}_t take the form

$$\frac{(1-\alpha_1-\nu_1)p^{\ell}_{1,t}E^{\ell}_{1,t}}{N^{\ell}_{1,t}} = w^{\ell}_t, \quad \text{and} \quad \frac{\nu_1 p^{\ell}_{1,t}E^{\ell}_{1,t}}{X^{\ell}_t} = \upsilon_t + \zeta\tau_t. \tag{8}$$

The first order condition w.r.t. $M^{\ell}_{1,j,t}$ solves for the demand for machines in the dirty sector which depends on their rental price $p_{i,j,t}$. The iso-elastic demand function takes the form

$$M^{\ell}_{1,j,t} = \left(\frac{\alpha_1 p^{\ell}_{1,t}}{p^{\ell}_{1,j,t}}\right)^{\frac{1}{1-\alpha_1}} (X^{\ell}_t)^{\frac{\nu_1}{1-\alpha_1}} (N^{\ell}_{1,t})^{\frac{1-\alpha_1-\nu_1}{1-\alpha_1}} Q^{\ell}_{1,j,t}. \tag{9}$$

[1] As stated in Acemoglu (2002), the rate of depreciation of sector-specific machines does not affect the balanced growth path equilibrium, but only affects the transitional dynamics.

Renewable energy sector
Sector $i = 2$ produces clean energy goods and services based on renewable sources like wind, solar, and water which do not cause any emissions. The production technology in the clean sector is

$$E_{2,t}^{\ell} = F_2(N_{2,t}^{\ell}, M_{2,j,t}^{\ell}, Q_{2,j,t}^{\ell}) = (N_{2,t}^{\ell})^{1-\alpha_2} \int_0^1 (Q_{2,j,t}^{\ell})^{1-\alpha_2} (M_{2,j,t}^{\ell})^{\alpha_2} dj. \tag{10}$$

The firms in the clean sector also take factor prices as given and maximize profits

$$\max_{(N_{2,t}^{\ell}, M_{2,j,t}^{\ell}) \in \mathbb{R}_+^2} \left\{ p_{2,t}^{\ell} F_2(N_{2,t}^{\ell}, M_{2,j,t}^{\ell}, Q_{2,j,t}^{\ell}) - w_t^{\ell} N_{2,t}^{\ell} - \int_0^1 p_{2,j,t} M_{2,j,t}^{\ell} dj \right\} \tag{11}$$

The first order condition for profit maximization in period $t \geq 0$ w.r.t to labor is given by

$$\frac{(1-\alpha_2) p_{2,t}^{\ell} E_{2,t}^{\ell}}{N_{2,t}^{\ell}} = w_t^{\ell}. \tag{12}$$

Analogous to the dirty energy sector the first order condition w.r.t. to $M_{2,j,t}^{\ell}$ yields the demand function for machines in sector $i = 2$:

$$M_{2,j,t}^{\ell} = \left(\frac{\alpha_2 p_{2,t}^{\ell}}{p_{2,j,t}^{\ell}} \right)^{\frac{1}{1-\alpha_2}} N_{2,t}^{\ell} Q_{2,j,t}^{\ell}. \tag{13}$$

2.2.3 Resource extraction

Exhaustible resources are solely used by firms in sector $i = 1$. In each region $\ell \in \mathbb{L}$ a single firm extracts resources like coal, oil, or gas from their regional resource stock denoted by $R_0^{\ell} \geq 0$. The amount extracted and supplied to the world resource market by the firm is denoted by $X_t^{\ell,s}$ (to abstract from the amount X_t^{ℓ} demanded by the energy sectors $i = 1$). Resource firms take constant extraction costs $c \geq 0$, the initial resource stock $R_0^{\ell} \geq 0$, and global market prices $(\upsilon_t)_{t \geq 0}$ as given. Profits made in the resource sectors in period $t \geq 0$ are discounted by the factor $q_t := \prod_{s=1}^{t} r_s^{-1}$ where $q_0 = 1$. Given the discount factor, the firms aim to maximize their discounted future stream of profits. The decision problem in the resource sector then reads

$$\max_{(X_{i,t}^{\ell,s})_{t \geq 0}} \left\{ \sum_{t=0}^{\infty} q_t (\upsilon_t - c) X_{i,t}^{\ell,s} \,\middle|\, \sum_{t=0}^{\infty} X_t^{\ell,s} \leq R_0^{\ell},\ X_t^{\ell,s} \geq 0 \ \ \forall t \geq 0 \right\}. \tag{14}$$

In the following, I assume $R_0^{\ell} > 0$. According to Hillebrand & Hillebrand (2019), this assumption implies that there exists an optimal interior extraction sequence $(X_t^{\ell,s})_{t \geq 0}$ if the resource price satisfies $\upsilon_0 \geq 0$ and the Hotelling rule

$$\upsilon_t = c + r_t(\upsilon_{t-1} - c) \ \ \forall t > 0. \tag{15}$$

Only if $\upsilon_t = c$ for all $t \geq 0$ it may be optimal not to exhaust the entire resource stock.

Hillebrand & Hillebrand (2019) show that in either case the Hotelling rule given by (15) allows that profits in the resource sectors can be written as

$$\Pi_x^\ell = (\upsilon_0 - c)R_0^\ell. \tag{16}$$

2.3 Research sector

2.3.1 Innovation and the quality ladder

In the undermost stage, research and development firms employ scientists that aim to improve the quality of already existing sector-specific machines $M_{i,j,t}^\ell$. If scientists are successful at improving the quality $Q_{i,j,t}^\ell$ of a machine, the new machine type is supplied to the energy producing sectors. On machine lines without a researcher or on machine lines where scientists were not successful, machines are produced with the existing but lower quality levels.

In this thesis, I assume that scientists focus on creating new machine types that exceed the quality of existing machines such that innovations are vertically related to the existing products. Product improvements occur along a quality ladder for each machine line $j \in \mathbb{J}$. However, only the leading-edge quality of each machine line is produced and supplied to the energy sector. The increase in average machine quality therefore creates one source of economic growth in this model. The set of machines is defined as follows:

DEFINITION 1. Let $\mathbb{J}_i$ denote the set of sector-specific machines supplied to the energy sector $i \in \mathbb{I}$. Since I abstract from expanding product varieties, the variety of machine types $\mathbb{J}_i$ is fixed throughout time and therefore normalized to unity, i.e., $\mathbb{J}_i \coloneqq [0,1]$ for $i = \{1,2\}$. Each $j \in \mathbb{J}_i$ corresponds to a different type of machine.

Therefore, the energy sectors both use a continuum of sector-specific machines normalized to unity. Machines of type $j_1(j_2)$ cannot be used for production in sector $i = 2(i = 1)$. In this setting of Schumpeterian endogenous growth, new inventions tend to replace older products with lower quality levels. This process is known as "creative destruction" (Barro & Sala-i Martin, 2004). Products on different machine lines j_i in sector $i \in \mathbb{I}$ are imperfect substitutes, but machines of different qualities within a machine line j_i are assumed to be perfect substitutes. Thus, only the highest-quality machine types are used for energy production, as they are most productive. Successful scientists enhance the quality level of a machine line by one rung on the quality ladder. Feasible rungs are equidistantly distributed across the quality ladder. The law of motion of a machine's quality level along the quality ladder is described by the following assumption.

ASSUMPTION 1. Let $Q_{i,j,t}^\ell$ be the quality level of machine type $j \in \mathbb{J}_i$ in sector $i \in \mathbb{I}$ for period $t \geq 0$ and region $\ell \in \mathbb{L}$. Successful improvement of a machine type increases the quality of a machine by the factor $(1 + \iota)$ such that

$$Q_{i,j,t}^\ell = (1 + \iota)Q_{i,j,t-1}^\ell \quad \forall j_i \in \mathbb{J}_i, \quad \iota > 1. \tag{17}$$

The innovation step size is exogenously given for both sectors $i \in \mathbb{I}$ and therefore treated as a parameter by research and development firms.[2] The timing of quality improvements for specific machines, however, is stochastic.

2.3.2 Scientists

Each region $\ell \in \mathbb{L}$ is equipped with a unit mass of scientists s^ℓ that are bounded to stay within their original region. At the beginning of each period, each scientist decides whether to direct their research to machines used in the clean or the dirty energy sector. Then, each scientist is randomly allocated to one machine within this sector and a machine is allocated to at most one scientist. Hence, machine lines may be unoccupied but never occupied by more than one scientist. If a scientist is successful at improving a machine, which happens with probability $\eta^\ell_{i,j,t}$ in sector $i \in \{1, 2\}$ and on machine line $j \in \mathbb{J}_i$, the machine quality increases according to the law of motion given by (17). Successful scientists receive a one-period patent for their innovation, allowing them to exclusively produce and supply the newly invented machine type to the energy sector. The enforcement of patents does not extend across regional borders in this model, i.e., scientists can only sell their products to domestic energy producers. Moreover, machines are not traded between regions either, implying that the scientists' decision is solely driven by domestic machine demand. If research is unfruitful or a machine is not occupied with a scientist, no patent is granted, but scientists competitively supply the machines with their current quality level.

This framework implies that future innovations build on existing quality or productivity levels, integrating the "building on the shoulder of giants" effect into the model. The fact that improvements in one sector make further improvements in the same sector even more profitable or productive describes *"path- or state- dependence"* as labeled by Acemoglu (2002). However, note that advances in the machine quality of one sector $i \in \mathbb{I}$ do not affect the quality of machines in the other sector.[3]

After scientists have decided which sector $i \in \{1, 2\}$ to direct their research to, they have to secure funding, since research is costly, and they do not have the necessary capital at their disposal. Thus, consumers, who own the initial capital stock K_0, lend out the capital needed for research at the international interest rate r_t.[4] This ensures that consumers can not exploit any arbitrage opportunities by investing into the research sector rather than lending capital to the final good sector. Scientists borrow $h^\ell_{i,j,t}$ units of the final good after they have been allocated to a machine line j_i. Given the current state of the machines, they attempt to increase their quality levels. Then, they either

[2]This model of the R&D sector closely follows Barro & Sala-i Martin (2004) and Hillebrand (2016). Alternatively, the rungs on the quality ladder could be distributed proportionally to some parameter as, e.g., in Aghion & Howitt (1992), or the size of the quality improvement may be linked to R&D spending.

[3]See Acemoglu (2002) for a model that considers "limited state dependence" and allows spillovers between two sectors.

[4]See Hillebrand (2016) for a setting where banks set the interest rate by taking into account the default rate of the research project.

exclusively supply their new product as monopolists or produce machines competitively on lines where innovation failed. Independent of the energy sector $i \in \mathbb{I}$ and whether a patent was obtained or not, machines are produced at a unit cost of ψ. Without loss of generality, I normalize $\psi \equiv 1$, i.e., unit costs equal one unit of the final good.

It follows that scientists face a two-stage decision process. First, they undergo a strategic phase where they decide whether to engage in R&D or not. If they do, they must decide on the amount of R&D expenditures $h^{\ell}_{i,j,t}$. A scientist will conduct research if the expected return is at least as high as the research and production costs. In the second stage, successful researchers determine the optimal monopoly price at which they rent out their newly invented machine to the energy producers. The scientists‘ decision problem is solved backwards. I start by determining the monopoly price for machines and subsequently derive the optimal amount of individual R&D expenditure.

2.3.3 Optimal scientist behavior

Second decision stage: Optimal prices

Researchers take the demand for sector-specific machines denoted by (13) for sector $i = 2$ and (9) for sector $i = 1$ as given. A successful scientist, who has obtained a patent, maximizes profits by choosing the machine price $p^{\ell}_{i,j,t}$. Demand $M^{\ell}_{i,j,t}(p^{\ell}_{i,j,t})$ is a decreasing function of the price chosen by the monopolist. The second stage decision problem takes the form

$$\max_{p^{\ell}_{i,j,t} \in \mathbb{R}_+} \left\{ (p^{\ell}_{i,j,t} - \psi) M^{\ell}_{i,j,t}(p^{\ell}_{i,j,t}) - r_t h^{\ell}_{i,j,t} \right\}. \tag{18}$$

The first order condition with respect to $(p^{\ell}_{i,j,t})$ solves for the optimal monopoly price of machine type j_i:

$$p^{\ell}_{i,j,t} = \frac{\psi}{\alpha_i}. \tag{19}$$

Therefore, the monopoly price is a constant markup over marginal costs and is constant across sectors $i \in \mathbb{I}$ and machine lines $j \in \mathbb{J}_i$.[5] Recall that the unit cost of machine production is normalized to $\psi = 1$, so that (19) can be written as $p^{\ell}_{i,j,t} = 1/\alpha_i$. I implicitly assume that machine producers have the necessary resources to finance the unit cost ψ and do not need external funding like for the R&D expenditures. The monopoly profit for a successful scientist then reads

$$\pi^{\ell}_{i,j,t} = \overline{\pi}^{\ell}_{i,t} Q^{\ell}_{i,j,t} - r_t h^{\ell}_{i,j,t}, \tag{20}$$

where

$$\overline{\pi}^{\ell}_{1,t} = (1 - \alpha_1)\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} (p^{\ell}_{1,t})^{\frac{1}{1-\alpha_1}} (X^{\ell}_t)^{\frac{\nu_1}{1-\alpha_1}} (N^{\ell}_{1,t})^{\frac{1-\alpha_1-\nu_1}{1-\alpha_1}} \tag{21}$$

[5] Although the monopoly price is constant across sectors and machine lines, I keep the indices i and j in the expression to distinguish the monopoly price $p^{\ell}_{i,j,t}$ from the prices for energy goods and services $p^{\ell}_{i,t}$.

and

$$\overline{\pi}^{\ell}_{2,t} = (1-\alpha_2)\alpha_2^{\frac{1+\alpha_2}{1-\alpha_2}}(p^{\ell}_{2,t})^{\frac{1}{1-\alpha_2}}N^{\ell}_{2,t}. \tag{22}$$

First decision stage: Optimal R&D expenditures

During the first decision stage, scientists decide whether to engage in research and if they do, they must decide on the amount of spending on R&D. Again, scientists aim to maximize profits, this time by choosing R&D expenditures $h^{\ell}_{i,j,t}$. In contrast to the second decision stage, the research outcome is not yet certain. Therefore, the individual return on R&D depends on the individual success probability $\eta^{\ell}_{i,j,t}$ and is therefore stochastic. To obtain deterministic profit streams, I follow the model in Barro & Sala-i Martin (2004, chap. 7) and assume that scientists form research syndicates that are large enough to diversify the idiosyncratic risk they are exposed to. While the outcome of each individual research project is stochastic, the outcome of joint projects carried out by research syndicates is deterministic using a law of large numbers reasoning, since realizations on individual machine lines are independent of each other (see, e.g., Acemoglu, 2009, chap. 12). Hence, research syndicates convert individual success probabilities $\eta^{\ell}_{i,j,t}$ into average sectoral success probabilities $\eta^{\ell}_{i,t}$.[6] The average profit for a scientist in sector $i \in \mathbb{I}$ then becomes $\eta^{\ell}_{i,t}\pi^{\ell}_{i,j,t}$.

Since I assume that the innovation step size ι on the quality ladder is fixed, the amount of R&D expenditure does not affect the magnitude of innovation. Instead, the probability of research success $\eta^{\ell}_{i,j,t}$ depends on R&D spending $h^{\ell}_{i,j,t}$.[7] Larger R&D outlays clearly increase the probability of success. Furthermore, I assume that the marginal effect of $h^{\ell}_{i,j,t}$ on $\eta^{\ell}_{i,j,t}$ diminishes with increasing R&D spending $h^{\ell}_{i,j,t}$. Alternatively, the marginal effect on the probability of success could be increasing, or the relationship could be proportional (see, e.g., Barro & Sala-i Martin, 2004, chap. 7). Moreover, $\eta^{\ell}_{i,j,t}$ could also depend on the current position on the quality ladder. Higher existing quality levels could make subsequent discoveries easier, and therefore less costly due to the "standing on the shoulder of giants" effect or learning effects. On the other hand, new inventions could become increasingly difficult the higher the current quality level. Then, $\eta^{\ell}_{i,j,t}$ would depend negatively on $Q^{\ell}_{i,j,t}$. In this thesis, I assume the latter case. The relationship between the probability of success, R&D outlays, and the current quality ladder position is defined as

$$\eta^{\ell}_{i,j,t} = (h^{\ell}_{i,j,t})^{\lambda}\Xi^{\ell}_{i,j,t} \tag{23}$$

where $\lambda \in]0,1[$ and

[6] Typically, the law of large numbers applies to the average of a countable sequence of random variables, whereas this model features a continuum of scientists and machines. In this context, Uhlig (1996) shows how to obtain a law of large numbers for a continuum of random variables.

[7] In contrast to Barro & Sala-i Martin (2004), the success probability for research depends on individual R&D outlays. Alternatively, $\eta^{\ell}_{i,j,t}$ could be a function of total R&D expenditure.

$$\Xi^{\ell}_{i,j,t} := \frac{1}{\omega}(Q^{\ell}_{i,j,t})^{-\lambda}. \tag{24}$$

The function $\Xi^{\ell}_{i,j,t}$ captures the diminishing effect of the current quality ladder position. Thus, machine lines with higher existing quality levels require more R&D spending. The parameter $\omega > 0$ represents the cost of doing research. Setting the technology parameter λ on the interval $]0,1[$ makes the success probability a strictly concave function of R&D expenditure and thus incorporates the diminishing marginal effect.[8] Scientists take energy prices $p^{\ell}_{i,t}$, the monopoly price determined in the second decision stage $p^{\ell}_{i,j,t}$, the interest rate r_t, factor inputs X^{ℓ}_t and $(N^{\ell}_{i,t})_{i\in\mathbb{I}}$, and the relationship between R&D spending and the success probability in (23) as given. Scientists choose R&D expenditures to maximize expected profits

$$\max_{h^{\ell}_{i,j,t}\in\mathbb{R}_+} \left\{ \eta^{\ell}_{i,j,t}(\overline{\pi}^{\ell}_{i,j,t}Q^{\ell}_{i,j,t} - r_t h^{\ell}_{i,j,t}) \right\}. \tag{25}$$

Plugging in equation (23) and solving the first order condition reads

$$\lambda(\overline{\pi}^{\ell}_{i,j,t}Q^{\ell}_{i,j,t} - r_t h^{\ell}_{i,j,t}) = r_t h^{\ell}_{i,j,t}. \tag{26}$$

Solving for $h^{\ell}_{i,j,t}$ yields

$$h^{\ell}_{i,j,t} = \frac{\lambda}{1+\lambda}\frac{\overline{\pi}^{\ell}_{i,j,t}}{r_t}Q^{\ell}_{i,j,t}. \tag{27}$$

Equation (27) denotes the optimal amount of R&D spending. Intuitively, higher profits from the production of machines set incentives to increase $h^{\ell}_{i,j,t}$. As outlined before, the expression also shows that a higher level of $Q^{\ell}_{i,j,t}$ demands higher R&D outlays, since it becomes increasingly more difficult to innovate. Furthermore, higher values of the technology parameter λ require larger R&D outlays due to the increasing marginal return of research. If λ were set to unity, the relationship between R&D and the success probability would be linear. Lastly, higher rental costs for capital r_t naturally decrease the demand for R&D. Given optimal R&D in (27) the profit for a successful scientist researching on machine line j can be written as

$$\pi^{\ell}_{i,j,t} = \frac{1}{1+\lambda}\overline{\pi}^{\ell}_{i,t}Q^{\ell}_{i,j,t}. \tag{28}$$

Using optimal R&D given by (27), the equation for the individual success probabilities given by (23) allows me to derive the sectoral success probabilities which are independent of the current quality level and machine line $j \in \mathbb{J}$. The sectoral success probability then reads

$$\eta^{\ell}_{i,j,t} = \eta^{\ell}_{i,t} = \omega^{-1}\left(\frac{\lambda}{1+\lambda}\frac{\overline{\pi}^{\ell}_{i,t}}{r_t}\right)^{\lambda}. \tag{29}$$

This expression shows that the sectoral success probability for research negatively de-

[8]The expression for the success probability is not a probability in a strict sense since it is not numerically bounded between 0 and 1. However, I choose the parameters in a way that ensures that the values of the success probabilities are contained within this interval.

pends on the research cost parameter ω and the interest rate r_t. On the other hand, the probability increases with greater returns $\overline{\pi}^{\ell}_{i,t}$ because larger profits incentivize scientists to invest more in R&D, which in turn increases the probability of research success. Note that $\overline{\pi}^{\ell}_{i,t}$ still contains the endogenous price for energy goods and services $p^{\ell}_{i,t}$.

2.3.4 Aggregation of microeconomic variables

So far, I have only considered the microeconomic decision process in the research sector, i.e., the decision problems of individual scientists. Therefore, I define an aggregate quality index and compute aggregate R&D spending to implement the micro-based R&D model into the macroeconomic framework of this study. Based on Acemoglu *et al.* (2012), the average quality index in sector $i \in \mathbb{I}$ for region $\ell \in \mathbb{L}$ is defined as

$$Q^{\ell}_{i,t} \equiv \int_0^1 Q^{\ell}_{i,j,t} dj. \tag{30}$$

Although the individual quality levels of each machine line follow a stochastic process, the law of motion for the aggregate quality index $Q^{\ell}_{i,t}$ is deterministic. This holds true since the individual realizations of quality improvements are independent of each other and I can apply a law of large numbers type of reasoning (Hillebrand, 2016).[9] The quality index $Q^{\ell}_{i,t}$ is still specific to the energy sectors $i \in \mathbb{I}$ and affects the quality or productivity of energy goods production. Thus, technological change can still be directed or biased towards either the clean or the dirty energy producing sector.

Let now $s^{\ell}_1 := (1 - s^{\ell})$ be the number of scientists allocated to sector $i = 1$ and let $s^{\ell}_2 := s^{\ell}$ be the number of scientists allocated to sector $i = 2$. Note that quality improvements only occur in a sector $i \in \mathbb{I}$ if scientists are present in this sector. Using the aforementioned law of large numbers and the sectoral success probability derived in (29), the law of motion for the aggregate quality index takes the form of a first-order difference equation

$$Q^{\ell}_{i,t} = (1 + \iota \eta^{\ell}_{i,t} s^{\ell}_{i,t}) Q^{\ell}_{i,t-1}. \tag{31}$$

Using the quality index $Q^{\ell}_{i,t}$ in (27) and accounting for the number of scientists $s^{\ell}_{i,t}$ in sector $i \in \mathbb{I}$ yields the aggregate sectoral R&D expenditures

$$H^{\ell}_{i,t} = \frac{\lambda}{1+\lambda} \frac{\overline{\pi}^{\ell}_{i,t}}{r_t} s^{\ell}_{i,t} Q^{\ell}_{i,t}. \tag{32}$$

This expression shows that R&D resources are only devoted to sector $i \in \mathbb{I}$ if $0 < s^{\ell}_{i,t} \leq 1$, i.e., scientists must have decided to dedicate their research to this very sector. In general, it is possible that scientists are present in both sectors simultaneously. Total R&D expenditure in period t and region $\ell \in \mathbb{L}$ is finally given by

[9] Again, see Uhlig (1996) on how to obtain a law of large numbers for a continuum of random variables normalized to unity.

$$H_t^\ell = \sum_{i=1}^{2} H_{i,t}^\ell. \tag{33}$$

Lastly, using the quality index (30), the amount of scientists present and the success probability in sector $i \in \mathbb{I}$, aggregated sectoral profits from R&D can be written as

$$\Pi_{i,t}^\ell = \eta_{i,t}^\ell \frac{1}{1+\lambda} \overline{\pi}_{i,t}^\ell s_{i,t}^\ell Q_{i,t}^\ell. \tag{34}$$

2.4 Climate model

Emissions of CO_2 are generated when the exhaustible resource that may represent, e.g., coal, oil, and gas is burned for energy production. In this model, this happens during the energy production process in sector $i = 1$. Energy production in sector $i = 2$ does not cause any emissions since it is based on renewable energy sources. Furthermore, this model abstracts from emissions that occur at the resource extraction stage. The carbon content ζ of the exhaustible resource determines the amount of CO_2 emitted to the atmosphere. Given regional emissions in (6), worldwide CO_2 emissions in period $t \geq 0$ can be written as

$$Z_t = \sum_{\ell \in \mathbb{L}} \zeta X_t^\ell. \tag{35}$$

For the upcoming analysis, I will utilize the climate model from Golosov *et al.* (2014), which was also used by Hillebrand & Hillebrand (2019). The climate state in period $t \geq 0$ is assumed to comprise a permanent ($S_{1,t}$) and a non-permanent ($S_{2,t}$) component of CO_2 in the atmosphere. Given the sequence of global emissions $\{Z\}_{t\geq 0}$ in (35), the atmospheric CO_2 concentration evolves as

$$S_{1,t} = S_{1,t-1} + \phi_L Z_t \tag{36a}$$
$$S_{2,t} = (1-\phi) S_{2,t-1} + (1-\phi_L)\phi_0 Z_t. \tag{36b}$$

In this law of motion for the climate state $\phi_L \in [0,1[$ denotes the share of emissions that become permanent CO_2 in the atmosphere. Parameter ϕ_0 represents the share out of the remaining emissions that becomes non-permanent CO_2 and decays at the constant rate $\phi \in]0,1[$. The remaining share of emissions $(1-\phi_0)$ instantly leaves the atmosphere, e.g., into the biosphere and the surface of the oceans (Golosov *et al.*, 2014). Hence, the total concentration of CO_2 in the atmosphere is given by $S_t = S_{1,t} + S_{2,t}$. Let $\overline{S} > 0$ denote the pre-industrial CO_2 concentration. Climate damages in region $\ell \in \mathbb{L}$ are given by a function of the atmospheric CO_2 concentration

$$D_t^\ell = D^\ell(S_t) := 1 - \exp\{-\gamma^\ell (S_t - \overline{S})\}, \;\; \gamma^\ell > 0. \tag{37}$$

Since climate damages might be of different magnitudes around the world, regional differences may enter the model via the damage parameter γ^ℓ.

2.5 Consumption sector

The consumption sector in each region $\ell \in \mathbb{L}$ is represented by a single representative household consumer. This consumer supplies labor and capital to the production and research sectors. Furthermore, the consumer is entitled to receive all profits and transfer payments from the government. Note that government transfers may be negative, e.g., in case of a lump-sum tax on the representative consumer. The linear homogeneity of the production functions in sectors $i \in \{0, 1, 2\}$ implies that profits in the final goods sector and the energy sectors are zero. Thus, profits may only occur in the resource stage or in the R&D sector. Therefore, taking profits in the resource sector given by (16) and the discounted sum of profit streams in the R&D sector using equation (34) yields the total profit income of the consumer in region $\ell \in \mathbb{L}$

$$\Pi^\ell = \Pi_x^\ell + \sum_{t=0}^{\infty} \sum_{i=1}^{2} q_t \Pi_{i,t}^\ell. \tag{38}$$

The household chooses a non-negative consumption sequence $(C_t^\ell)_{t \geq 0}$ given discount factor $0 < \beta < 1$ and the utility function

$$U((C_t^\ell)_{t \geq 0}) = \sum_{t=0}^{\infty} \beta^t u(C_t^\ell), \tag{39}$$

where

$$u(C) = \frac{C^{1-\sigma} - 1}{1 - \sigma}, \quad \text{if} \quad \sigma > 0, \sigma \neq 1 \quad \text{or} \quad u(C) = \log(C), \quad \text{if} \quad \sigma = 1. \tag{40}$$

The household is endowed with the initial capital stock K_0^ℓ in $t = 0$ and supplies labor $N_t^{\ell,s}$ to the production processes, which is exogenous in this model. Lifetime labor income is defined as $W^\ell := \sum_{t=0}^{\infty} q_t w_t^\ell N_t^{\ell,s}$, transfer payments are denoted by T^ℓ and profit income Π^ℓ is given by (38). The representative consumers in all world regions $\ell \in \mathbb{L}$ maximize their consumption path $(C_t^\ell)_{t \geq 0}$ given their budget constraint

$$\sum_{t=0}^{\infty} q_t C_t^\ell \leq r_0 K_0^\ell + W^\ell + \Pi^\ell + T^\ell. \tag{41}$$

Hillebrand & Hillebrand (2019) show that the optimal consumption path in region $\ell \in \mathbb{L}$ is given by a constant share of total world consumption. The path of aggregate consumption $\overline{C}_t := \sum_{\ell \in \mathbb{L}} C_t^\ell$ then evolves according to the Euler equation

$$\overline{C}_{t+1} = \overline{C}_t (\beta r_{t+1})^{\frac{1}{\sigma}} \tag{42}$$

and satisfies the transversality condition,

$$\lim_{T \to \infty} \beta^T \overline{C}_T^{-\sigma} \overline{K}_{T+1} = 0 \tag{43}$$

where $\overline{K}_t$ denotes the aggregated stock of world capital in period t.

2.6 Market clearing

Trade between regions $\ell \in \mathbb{L}$ occurs on global markets for the exhaustible resource, capital, and the final consumption good. International trade is assumed to be possible without any tariffs and transaction or transportation costs. Consumers can freely lend or borrow capital on the international capital market since there is no sign restriction on the individual level of households' capital holding but only the transversality condition for the aggregate capital stock in (43). Sector-specific machines and energy products cannot be traded across regional borders.

Labor supply is immobile across regions and can only be supplied to domestic production. Thus, market clearing for the domestic labor market in region $\ell \in \mathbb{L}$ and period $t \geq 0$ takes the form

$$\sum_{i=0}^{2} N_{i,t}^{\ell} \stackrel{!}{=} N_t^{\ell,s}. \tag{44}$$

Scientists are not a member of the representative household and therefore do not consume the final good, own capital or supply labor to the production process in sector $i \in \mathbb{I}_0$. The mass of scientists in each region $\ell \in \mathbb{L}$, which is normalized to unity, only serves the purpose to improve the quality of machines. They have to direct their research either to the clean or dirty energy production sector in their home region such that market clearing in region $\ell \in \mathbb{L}$ and period $t \geq 0$ requires that

$$s_{1,t}^{\ell} + s_{2,t}^{\ell} = 1. \tag{45}$$

Domestic energy production has to coincide with domestic energy demand in equilibrium, since excess production cannot be traded abroad. Market clearing in sector $i \in \mathbb{I}$ in region $\ell \in \mathbb{L}$ and period $t \geq 0$ requires

$$E_{i,t}^{\ell} \stackrel{!}{=} E_{i,t}^{\ell,s}. \tag{46}$$

Sector-specific machines used in the energy production process are also only traded domestically. Here, scientists form the supply side for machines and energy firms the demand side. Market clearing requires for each machine $M_{i,j,t}^{\ell}$ that

$$M_{i,j,t}^{\ell} \stackrel{!}{=} M_{i,j,t}^{\ell,s} \quad j_i \in [0,1], i \in \mathbb{I} \text{ and } \forall t \geq 0. \tag{47}$$

Let $\overline{K}_t > 0$ denote the aggregate world capital stock in period t. The market clearing condition for the international capital market takes the form

$$\sum_{\ell \in \mathbb{L}} K_t^{\ell} \stackrel{!}{=} \overline{K}_t \quad \forall t \geq 0. \tag{48}$$

In the resource stage, market clearing requires that $\sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell,s} \stackrel{!}{=} \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell}$. Furthermore, the exhaustible resource used in production must fulfill the world resource constraint

$$\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}} X_t^\ell \leq R_0, \tag{49}$$

where R_0 is the initial global stock of the exhaustible resource. Hillebrand & Hillebrand (2019) point out that due to the application of the Hotelling rule given by (15), the exact extraction date of the resource is indeterminate. Lastly, market clearing for the final good accounts for consumption, R&D expenditure and spendings on machines and resources. It describes the evolution of the global capital stock and takes the form

$$\overline{K}_{t+1} = \sum_{\ell\in L} Y_t^\ell - \overline{C}_t - c\sum_{\ell\in L} X_t^\ell - \psi\sum_{i=1}^{2}\sum_{\ell\in\mathbb{L}}\left(\int_0^1 M_{i,j,t}^\ell dj\right) - \sum_{\ell\in L} H_t^\ell. \tag{50}$$

3 General equilibrium analysis

3.1 Definition and characterization

Definition of the equilibrium

In this section, I define the *laissez-faire* equilibrium, which corresponds to the decentralized solution of the model without any policy intervention. First, I characterize the equilibrium for given quality levels $Q_{i,j,t}^\ell$ and then determine the direction of technical change endogenously based on market forces. Section 4 then adapts the equilibrium definition and introduces climate policies.

To simplify notation, I adopt the following vector notation for equilibrium variables in each period $t \geq 0$:

$$\begin{array}{lll}
\mathbf{Q_t} := (Q_{i,j,t}^\ell)_{(\ell,i,j)\in\mathbb{L}\times\mathbb{I}\times[0,1]} & \mathbf{N_t^s} := (N_t^{\ell,s})_{\ell\in\mathbb{L}} & \mathbf{H_t} := (H_{i,t}^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}} \\
\mathbf{Y_t} := (Y_t^\ell)_{\ell\in\mathbb{L}} & \mathbf{E_t} := (E_{i,t}^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}} & \mathbf{X_t} := (X_t^\ell)_{\ell\in\mathbb{L}} \\
\mathbf{N_t} := (N_{i,t}^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} & \mathbf{K_t} := (K_t^\ell)_{\ell\in\mathbb{L}} & \mathbf{S_t} := (S_{t,1}, S_{t,2}) \\
\mathbf{M_t} := (M_{i,j,t}^\ell)_{(\ell,i,j)\in\mathbb{L}\times\mathbb{I}\times[0,1]} & \mathbf{s_t} := (s_{i,t}^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}} & \mathbf{p_t} := (p_{i,t}^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}} \\
\mathbf{\tilde{p}_t} := (p_{i,j,t}^\ell)_{(\ell,i,j)\in\mathbb{L}\times\mathbb{I}\times[0,1]} & \mathbf{w_t} := (w_t^\ell)_{\ell\in\mathbb{L}} &
\end{array} \tag{51}$$

DEFINITION 2. An aggregate equilibrium $\xi_t(\mathbf{A},\mathbf{P})$ of the economy is defined for each $t \geq 0$ and consists of an allocation

$$\mathbf{A} = (\overline{C}_t, \mathbf{Y_t}, \mathbf{Q_t}, \mathbf{N_t}, \mathbf{M_t}, \mathbf{E_t}, \mathbf{H_t}, \mathbf{X_t}, \mathbf{S_t}, \overline{K_t}, \mathbf{s_t})_{t\geq 0} \tag{52}$$

and a price vector

$$\mathbf{P} = (r_t, \mathbf{w_t}, \mathbf{p_t}, \mathbf{\tilde{p}_t}, \upsilon_t)_{t\geq 0} \tag{53}$$

such that

i. The allocation is in conformity with the production technologies (1), (5), and (10), and the market clearing conditions and resource constraints (44), (45), (46), (47), (48), (49), and (50).

ii. Producers and scientists behave optimally by maximizing profits and thus equations (4), (8), (12), (19), (27), and (28) hold for $\forall t \geq 0$. Profits are given by (16) for the resource sector and by (34) for the R&D sector while the resource price evolves according to equation (15).

iii. The representative household behaves optimally given its household income according to the Euler equation (42) and the transversality condition (43) for all $t \geq 0$.

iv. The average quality index for sector-specific machines evolves according to its law of motion (30).

v. The climate state evolves according to (36). Emissions follow equation (6) and climate damages are specified by (37).

The equilibrium is considered an aggregate equilibrium because it specifies aggregate world consumption, but not its distribution across regions. Recall that on the microeconomic level, the outcome of R&D decisions is stochastic since it depends on the success probability $\eta^{\ell}_{i,j,t}$. Consequently, profits and R&D investments are stochastic too. However, by applying a law of large number type of reasoning, the uncertainty disappears for aggregated macroeconomic variables. This holds true since the individual outcomes of the random events in the R&D sector are stochastically independent. Therefore, the sequences of variables is fully deterministic as well as the equilibrium analysis (Hillebrand, 2016).

Economic growth enters the model in two ways: exogenous labor-augmenting productivity growth and endogenous directed technological change. Most studies in the directed technical change literature assume growth to solely originate from improvements or innovation in an intermediate sector due to profit incentives in the R&D sector.[10] In Acemoglu *et al.* (2012), for example, intermediate goods are the only inputs for final output production and directly enter the CES production function for the final good. In my integrated assessment model framework the clean and dirty energy inputs also enter a CES-aggregate, which is, however, subordinate to the final output stage that is characterized by a Cobb Douglas production function. Hence, growth as it enters the CES function at the energy stage does not affect final output as strong as in Acemoglu *et al.* (2012) because energy only enters the production function with a factor cost share of ν_0. Therefore, only focusing on endogenous directed technical change in this model would require unrealistically large step sizes for innovations on the quality ladder to match empirical estimates of output growth.[11] In a framework that disaggregates the production process on as many levels as in Hillebrand & Hillebrand (2019), I find it useful to consider two sources of growth. Thus, labor productivity growth complements the endogenous directed technological change mechanism.

[10]See, e.g., Acemoglu *et al.* (2012, 2014), Hemous (2013), and van den Bijgaart (2017).

[11]For example, Acemoglu *et al.* (2012), Golosov *et al.* (2014), and Hillebrand & Hillebrand (2019) aim to match an output growth between 1.5% and 2% in their numerical simulations.

Perfect competition in the absence of innovation
In order to determine the direction of endogenous technological change in this model, it is useful to first compute the equilibrium values for renewable energy, dirty energy, and the exhaustible resource as in Acemoglu *et al.* (2012). Recall that if a scientist is unsuccessful or a machine line has not been occupied with a scientist, no monopoly rights are assigned to anyone. Therefore, in the absence of innovation, sector-specific machines are supplied to the energy sector under perfect competition. The equilibrium price for machines produced under perfect competition naturally equals marginal costs, i.e., $p^{\ell}_{i,j,t} = \psi = 1$. Hence, no profits are made on these machine lines. The presence of two different equilibrium prices requires different demand functions for monopolistically and competitively produced machines. The demand for machines that are produced monopolistically takes the form

$$M^{\ell,m}_{1,j,t} = (\alpha_1^2 p^{\ell}_{1,j,t})^{\frac{1}{1-\alpha_1}} (X^{\ell}_t)^{\frac{\nu_1}{1-\alpha_1}} (N^{\ell}_{1,t})^{\frac{1-\alpha_1-\nu_1}{1-\alpha_1}} Q^{\ell}_{1,j,t} \text{ and } M^{\ell,m}_{2,j,t} = (\alpha_2^2 p^{\ell}_{2,t})^{\frac{1}{1-\alpha_2}} N^{\ell}_{2,t} Q^{\ell}_{2,j,t} \quad (54)$$

for sector $i = 1$ and sector $i = 2$, respectively. For those machines produced competitively and priced at $\psi \equiv 1$ the demand functions for sectors $i \in \{1, 2\}$ write

$$M^{\ell,c}_{1,j,t} = (\alpha_1 p^{\ell}_{1,j,t})^{\frac{1}{1-\alpha_1}} (X^{\ell}_t)^{\frac{\nu_1}{1-\alpha_1}} (N^{\ell}_{1,t})^{\frac{1-\alpha_1-\nu_1}{1-\alpha_1}} Q^{\ell}_{1,j,t} \text{ and } M^{\ell,c}_{2,j,t} = (\alpha_2 p^{\ell}_{2,t})^{\frac{1}{1-\alpha_2}} N^{\ell}_{2,t} Q^{\ell}_{2,j,t} \quad (55)$$

The aggregate number of machines produced monopolistically is given by the number of successful scientists $\eta^{\ell}_{i,t} s_{i,t}$ in sector $i \in \mathbb{I}$. Conversely, the number of machines produced competitively is given by $(1 - \eta^{\ell}_{i,t} s_{i,t})$. This allows me to write the equilibrium production of clean energy as

$$\begin{aligned} E^{\ell}_{2,t} &= N^{1-\alpha_2}_{2,t} \int_0^1 Q^{1-\alpha_2}_2 (\eta^{\ell}_{2,t} s^{\ell}_{2,t} (M^{\ell,m}_{2,j,t})^{\alpha_2} + (1 - \eta^{\ell}_{2,t} s^{\ell}_{2,t})(M^{\ell,c}_{2,j,t})^{\alpha_2} dj \\ &= (\alpha_2 p^{\ell}_{2,t})^{\frac{\alpha_2}{1-\alpha_2}} \tilde{Q}^{\ell}_{2,t} N^{\ell}_{2,t} \end{aligned} \quad (56)$$

where

$$\tilde{Q}^{\ell}_{2,t} = (\eta^{\ell}_{2,t} s^{\ell}_{2,t} (\alpha_2^{\frac{\alpha_2}{1-\alpha_2}} - 1) + 1) Q^{\ell}_{2,t} \quad (57)$$

which denotes the corrected quality index, accounting for the differences in the market structure for machines. Similarly, the equilibrium demand for dirty energy goods takes the form

$$E^{\ell}_{1,t} = \alpha_1^{\frac{\alpha_1}{1-\alpha_1-\nu_1}} \left(\frac{\nu_1 \tilde{Q}^{\ell}_{1,t}}{\upsilon_t + \tau_t \zeta} \right)^{\frac{\nu_1}{1-\alpha_1-\nu_1}} (p^{\ell}_{1,t})^{\frac{\alpha_1+\nu_1}{1-\alpha_1-\nu_1}} N^{\ell}_{1,t} \tilde{Q}^{\ell}_{1,t} \quad (58)$$

where

$$\tilde{Q}^{\ell}_{1,t} = (\eta^{\ell}_{1,t} s^{\ell}_{1,t} (\alpha_1^{\frac{\alpha_1}{1-\alpha_1}} - 1) + 1) Q^{\ell}_{1,t}. \quad (59)$$

Eventually, after plugging in the equilibrium values for dirty and clean energy production in equation (8), the first order condition of the profit maximization problem in sector $i = 1$ with respect to X_t^ℓ determines the equilibrium value of the exhaustible resource as

$$X_t^\ell = (p_{1,t}^\ell)^{\frac{1}{1-\alpha_1-\nu_1}} N_{1,t}^\ell \alpha_1^{\frac{\alpha_1}{1-\alpha_1-\nu_1}} \left(\frac{\nu_1 \tilde{Q}_{1,t}^\ell}{\upsilon_t + \tau_t^\ell \zeta} \right)^{\frac{1-\alpha_1}{1-\alpha_1-\nu_1}}. \tag{60}$$

3.2 Determinants of directed technological change

The direction of innovation for sector-specific machines directly depends on the relative profitability of research in sectors $i \in \mathbb{I}$. Following Acemoglu *et al.* (2012), I start off considering the individual profits of a scientist in machine line $j \in \mathbb{J}$ as given in (28). Taking into account the success probability of research $\eta_{i,t}^\ell$, the law of motion for machine quality (17) and the definition of the quality index (30), the expected profit for a scientist in sector $i \in \mathbb{I}$ at $t \geq 0$ can be written as

$$\Pi_{i,t}^\ell = \frac{1}{1+\lambda} \eta_{i,t}^\ell (1+\iota) \overline{\pi}_{i,t}^\ell Q_{i,t-1}^\ell. \tag{61}$$

The ratio of sectoral profits allows us to point out the essential market forces shaping the direction of technological change. Thus, the profit ratio describing the relative advantage of innovating in sector $i = 2$ relative to $i = 1$ is given by

$$\frac{\Pi_{2,t}^\ell}{\Pi_{1,t}^\ell} = \underbrace{\frac{\eta_{2,t}^\ell}{\eta_{1,t}^\ell}}_{\substack{\text{Risk}\\ \text{effect}}} \frac{(1-\alpha_2)\alpha_2^{\frac{1+\alpha_2}{1-\alpha_2}}}{(1-\alpha_1)\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} \alpha_1^{\frac{\alpha_1 \nu_1}{(1-\alpha_1-\nu_1)(1-\alpha_1)}}} \underbrace{\frac{(p_{2,t}^\ell)^{\frac{1}{1-\alpha_2}}}{(p_{1,t}^\ell)^{\frac{1}{1-\alpha_1-\nu_1}}}}_{\text{Price effect}} \underbrace{\frac{N_{2,t}^\ell}{N_{1,t}^\ell}}_{\substack{\text{Market}\\ \text{size}\\ \text{effect}}} \underbrace{\frac{1}{(\frac{\nu_1 \tilde{Q}_{1,t}^\ell}{\upsilon_t + \tau_t \zeta})^{\frac{\nu_1}{1-\alpha_1-\nu_1}}}}_{\substack{\text{Resource price}\\ \text{effect}}} \underbrace{\frac{Q_{2,t-1}^\ell}{Q_{1,t-1}^\ell}}_{\substack{\text{Productivity}\\ \text{effect}}}. \tag{62}$$

THEOREM 1. The direction of technological change depends on five market forces that affect the profitability in sector $i \in \mathbb{I}$: The productivity effect, the market size effect, the price effect, the risk effect, and the resource price effect.

- The *productivity effect*: As the ratio of sectoral quality or productivity levels denoted by $Q_{2,t-1}^\ell / Q_{1,t-1}^\ell$ increases, the profit ratio $\Pi_{2,t}^\ell / \Pi_{1,t}^\ell$ increases as well. Thus, the productivity effect moves research in the direction of the more advanced sector resulting from the law of motion for the quality index given in (31) resembling the "standing on the shoulders of giants" effect (Acemoglu *et al.*, 2012).

- The *market size effect*: Greater relative employment $N_{2,t}^\ell / N_{1,t}^\ell$ increases the profit ratio $\Pi_{2,t}^\ell / \Pi_{1,t}^\ell$. Hence, the market size effect pushes researchers into the sector with larger employment, which is thus also the sector with a larger market for sector-specific machines.

- The *price effect*: The relative profitability of research increases in relative energy prices $p_{2,t}^\ell / p_{1,t}^\ell$. Thus, the price effect encourages research in the sector with higher energy prices. Naturally, this is the sector that is relatively more backward, since relatively scarce factors are relatively more expensive (Acemoglu *et al.*, 2012). Hence, the price effect directs research towards the sector that uses more scarce factors and works in the opposite direction compared to the market size effect.

- The *risk effect*: The risk effect as labeled by Hillebrand (2016) is denoted by the relative success probabilities of research $\eta^{\ell}_{2,t}/\eta^{\ell}_{1,t}$. The profit ratio increases in the risk effect, indicating that scientists are attracted to the sector with higher success probabilities.

- The *resource price effect*: The relative profitability of research increases in the gross resource price $\upsilon_t + \tau_t\zeta$. An increase in the global resource price υ_t, an increase in the carbon tax τ_t, or an increase in both encourages research in the clean sector. This result indicates that government intervention is able to affect the relative profitability of research by setting a carbon tax τ_t sufficiently high.

The essential market forces that determine the direction of technological change in Theorem 1 resemble the three market forces in Acemoglu *et al.* (2012), the risk effect from Hillebrand (2016) and the resource price effect. In contrast to Hillebrand (2016), the risk effect does not stem from the explicit consideration of financial intermediation, but simply from the fact that R&D requires funding that is available from the consumers at rate r_t. Although Acemoglu *et al.* (2012) consider an economy that employs an exhaustible resource they do not explicitly label the resource price effect. By pointing out the resource price effect this study emphasizes the role of carbon taxation as an important determinant of directed technical change.

ASSUMPTION 2. For the rest of this analysis, I assume that $\alpha_2 \equiv \alpha_1+\nu_1$ and $\kappa_1 = \kappa_2$ which is also implicitly assumed by Acemoglu *et al.* (2012). Furthermore, I assume from now on that it holds true for the elasticity of substitution that $1/(1-\rho) > 1$ which is equivalent to $0 < \rho < 1$.[12] This assumption contributes to a simpler and more tractable equilibrium analysis.

From Theorem 1 follows that the direction of technical change is determined by the net-effect of the five aforementioned market forces. To determine the strength of these forces and their net-effect, it is useful to derive some equilibrium properties of the model. First, consider the ratio of equilibrium energy demand schedules determined in (56) and (58) under Assumption 2:

$$\frac{E^{\ell}_{2,t}}{E^{\ell}_{1,t}} = \frac{\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}}{\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}} \left(\frac{p^{\ell}_{2,t}}{p^{\ell}_{1,t}}\right)^{\frac{\alpha_2}{1-\alpha_2}} \frac{\tilde{Q}^{\ell}_{2,t}}{\tilde{Q}^{\ell}_{1,t}} \frac{N^{\ell}_{2,t}}{N^{\ell}_{1,t}} \left(\frac{\nu_1 \tilde{Q}^{\ell}_{1,t}}{\upsilon_t + \tau_t\zeta}\right)^{-\frac{\nu_1}{1-\alpha_2}}. \tag{63}$$

As in Acemoglu *et al.* (2012) and Golosov *et al.* (2014), the energy price ratio can be written as

[12] In the IAM literature it is common to assume an elasticity of substitution for clean and dirty energy above unity. However, there is no consensus about how far above unity it should be (see, e.g., Acemoglu *et al.*, 2012; Golosov *et al.*, 2014; Rezai & van der Ploeg, 2015; Hassler *et al.*, 2019).

$$\left(\frac{E^{\ell}_{2,t}}{E^{\ell}_{1,t}}\right)^{\rho-1} = \frac{p^{\ell}_{2,t}}{p^{\ell}_{1,t}}, \tag{64}$$

which follows directly from the necessary first-order conditions of the final good given by equation (4). In general, the relative price of energy goods decreases in the supply of factor $E^{\ell}_{2,t}$ relative to factor $E^{\ell}_{1,t}$. The elasticity of substitution determines the strength of the price change responding to a change in relative energy supply. The relative price given by (64) can now be used to eliminate the term $E^{\ell}_{2,t}/E^{\ell}_{1,t}$ in (63) and eventually solving for $p^{\ell}_{2,t}/p^{\ell}_{1,t}$ yields

$$\frac{p^{\ell}_{2,t}}{p^{\ell}_{1,t}} = \left(\frac{\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}}{\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}}\frac{\tilde{Q}^{\ell}_{2,t}}{\tilde{Q}^{\ell}_{1,t}}\frac{N^{\ell}_{2,t}}{N^{\ell}_{1,t}}\left(\frac{\nu_1\tilde{Q}^{\ell}_{1,t}}{\upsilon_t+\tau_t\zeta}\right)^{-\frac{\nu_1}{1-\alpha_2}}\right)^{\frac{1-\alpha_2}{\varphi-1}} \tag{65}$$

where $\varphi = \frac{(\rho-1)-\alpha_2(\rho-1)+(1-\alpha_2)}{\rho-1} = \frac{\rho(1-\alpha_2)}{\rho-1}$. The relative energy price is thus an increasing function of relative labor supply, relative productivity, and the resource price effect. Next, take the ratio of success probabilities as described in (23) and plug in profits and the equilibrium demand for the exhaustible resource given by (20) and (60), respectively:

$$\frac{\eta^{\ell}_{2,t}}{\eta^{\ell}_{1,t}} = \left(\frac{\overline{\pi}^{\ell}_{2,t}}{\overline{\pi}^{\ell}_{1,t}}\right)^{\lambda} = \left(\frac{(1-\alpha_2)\alpha_2^{\frac{1+\alpha_2}{1-\alpha_2}}}{(1-\alpha_1)\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}+\frac{\alpha_1\nu_1}{(1-\alpha_2)(1-\alpha_1)}}}\left(\frac{p^{\ell}_{2,t}}{p^{\ell}_{1,t}}\right)^{\frac{1}{1-\alpha_2}}\frac{N^{\ell}_{2,t}}{N^{\ell}_{1,t}}\left(\frac{\nu_1\tilde{Q}^{\ell}_{1,t}}{\upsilon_t+\tau_t\zeta}\right)^{-\frac{\nu_1}{1-\alpha_2}}\right)^{\lambda}. \tag{66}$$

Eliminating relative energy prices by using (65) yields

$$\frac{\eta^{\ell}_{2,t}}{\eta^{\ell}_{1,t}} = \mu^{\lambda}\left(\left(\frac{\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}}{\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}}\right)^{\frac{1}{\varphi}}\frac{N^{\ell}_{2,t}}{N^{\ell}_{1,t}}\left(\frac{\nu_1\tilde{Q}^{\ell}_{1,t}}{\upsilon_t+\tau_t\zeta}\right)^{-\frac{\nu_1}{1-\alpha_2}}\left(\frac{\tilde{Q}^{\ell}_{2,t}}{\tilde{Q}^{\ell}_{1,t}}\right)^{\frac{1}{\varphi}}\right)^{\frac{\lambda\varphi}{\varphi-1}}, \tag{67}$$

where

$$\mu = \frac{(1-\alpha_2)\alpha_2^{\frac{1+\alpha_2}{1-\alpha_2}}}{(1-\alpha_1)\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}+\frac{\alpha_1\nu_1}{(1-\alpha_2)(1-\alpha_1)}}}. \tag{68}$$

Examining equation (67) indicates that the value of $\varphi = \frac{\rho(1-\alpha_2)}{\rho-1}$ is crucial to determine the response of the risk effect to an increase in relative employment $N^{\ell}_{2,t}/N^{\ell}_{1,t}$. If $0 < \rho < 1 \Leftrightarrow \varphi < 0$, as in Assumption 2, then the two energy inputs are considered gross substitutes and the risk effect increases with an increase in the relative employment in sectors $i \in \mathbb{I}$ since it holds that $\lambda > 0$. Hence, under Assumption 2 the risk effect and the market size effect push research in the same direction. Note that equation (67) contains the corrected quality index $\tilde{Q}^{\ell}_{i,t}$, which in turn contains the success probability $\eta^{\ell}_{i,t}$. However, because $\eta^{\ell}_{i,t}$ takes on values on the interval $[0, 1]$, the correction term as defined in (57) and (59) is strictly positive. The presence, of the corrected quality index, hence, does not change the result.

Eliminating the energy price ratio allows writing the relative expected profits from research in the clean sector as

$$\frac{\Pi^{\ell}_{2,t}}{\Pi^{\ell}_{1,t}} = \frac{\eta^{\ell}_{2,t}}{\eta^{\ell}_{1,t}} \times \mu \times \left(\left(\frac{\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}}{\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}} \right)^{\frac{1}{\varphi}} \frac{N^{\ell}_{2,t}}{N^{\ell}_{1,t}} \left(\frac{\nu_1 \tilde{Q}^{\ell}_{1,t}}{\upsilon_t + \tau_t \zeta} \right)^{-\frac{\nu_1}{1-\alpha_2}} \frac{\tilde{Q}^{\ell}_{2,t}}{\tilde{Q}^{\ell}_{1,t}} \right)^{\frac{\varphi}{\varphi-1}}. \tag{69}$$

The relationship in equation (69) allows analyzing the size of the market size effect compared to the price effect as described in Theorem 1. Ignore the risk effect for the moment, i.e., assume it is set to unity. If relative employment $N^{\ell}_{2,t}/N^{\ell}_{1,t}$ increases, employment in sector $i = 2$ becomes relatively greater leading to a larger market for sector-specific machines, increasing the relative profitability of research in sector $i = 2$. This describes the market size effect. On the other hand, an increase in relative employment also leads to a smaller labor force in sector $i = 1$ translating to relatively higher prices. In turn, the price increase positively affects relative profitability, which can be described as the price effect. Hence, the price effect and the market size effect work in different directions.

Whether the market size effect or the price effect dominates crucially depends on the size of $\varphi = \frac{\rho(1-\alpha_2)}{\rho-1}$.[13] If $\varphi < 0$, i.e., the two energy goods are gross substitutes, an increase in the relative employment $N^{\ell}_{2,t}/N^{\ell}_{1,t}$ increases the relative profitability of undertaking research in the clean sector. Consequently, the market size effect dominates. Conversely, if $0 < \varphi < 1$, which is the case if $\rho < 0$, the profit ratio decreases in the size of relative employment and the price effect dominates the market size effect. Thus, under Assumption 2 the market size effect dominates in this model. Now, consider that the risk effect additionally affects the relative profitability of research. Inserting the ratio of success probabilities as given by (67) into (69) gives

$$\frac{\Pi^{\ell}_{2,t}}{\Pi^{\ell}_{1,t}} = \mu^{1+\lambda} \times \left(\frac{\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}}{\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}} \right)^{\frac{1+\lambda}{\varphi-1}} \times \left(\frac{\tilde{Q}^{\ell}_{2,t}}{\tilde{Q}^{\ell}_{1,t}} \right)^{\frac{\lambda+\varphi}{\varphi-1}} \times \left(\frac{N^{\ell}_{2,t}}{N^{\ell}_{1,t}} \left(\frac{\nu_1 \tilde{Q}^{\ell}_{1,t}}{\upsilon_t + \tau_t \zeta} \right)^{-\frac{\nu_1}{1-\alpha_2}} \right)^{\frac{(1+\lambda)\varphi}{\varphi-1}}. \tag{70}$$

Even after the risk effect has been taken into account, the profit ratio responds to an increase in relative employment in the same way as described above. From Assumption 2 follows that $(1 + \lambda)\varphi/(\varphi - 1) > 0$, hence, relative profitability increases if $N^{\ell}_{2,t}/N^{\ell}_{1,t}$ increases. Consequently, the market size effect dominates under this model setup. Altogether, these results replicate the finding in Acemoglu (2002) that the elasticity of substitution plays a crucial role in determining the direction of technical change and shows that this result still holds under the modified setup including a micro-founded R&D sector, a fossil resource, and carbon taxes.

[13] Recall that by definition of the CES production function if ρ approaches 1, the two energy inputs are perfect substitutes. If ρ approaches zero in the limit, the production function becomes Cobb-Douglas and if ρ approaches negative infinity, the two energy goods are perfect complements. Since for the CES-production function $\rho \leq 1$, it holds that $\varphi < 1$.

3.3 Scientists under the laissez-faire equilibrium

This section analyzes the equilibrium scientist allocation under the laissez-faire scenario, i.e., a decentralized equilibrium without policy intervention. Depending on the relative profitability of research, scientists decide to direct their research to one of the two energy sectors. It is possible that scientists conduct research in both sectors simultaneously. However, note that although the presence of scientists in both sectors is feasible, the analysis of corner solutions is important. Corner solution describe equilibria where all scientists undertake research in one of the two sectors exclusively. When modeling path-dependencies and future climate policies that seek to direct research solely to one sector it is important to consider corner solutions. Rewriting the profit ratio given by (62) establishes the following

LEMMA 1. Under the laissez-faire equilibrium, the profit ratio (62) can be written as

$$\frac{\Pi^{\ell}_{2,t}}{\Pi^{\ell}_{1,t}} = \frac{\eta^{\ell}_{2,t}}{\eta^{\ell}_{1,t}}\Gamma\left(\frac{\nu_1}{\upsilon_t+\tau^{\ell}_t\zeta}\right)^{\frac{\nu_1\varphi}{1-\alpha_2}} \frac{\left((\eta^{\ell}_{1,t}s^{\ell}_{1,t}(\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}-1)+1)(1+\iota\eta^{\ell}_{1,t}s^{\ell}_{1,t})\right)^{\varphi_1+1}}{\left((\eta^{\ell}_{2,t}s^{\ell}_{2,t}(\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}-1)+1)(1+\iota\eta^{\ell}_{2,t}s^{\ell}_{2,t})\right)^{\varphi+1}} \frac{(Q^{\ell}_{2,t-1})^{-\varphi}}{(Q^{\ell}_{1,t-1})^{-\varphi_1}} \tag{71}$$

where

$$\Gamma = \frac{(1-\alpha_2)^{-(\varphi-1)}}{1-\alpha_1}\frac{\alpha_2^{\frac{1-\alpha_2\varphi}{1-\alpha_2}}}{\alpha_1^{-\frac{\alpha_1\varphi(1-\alpha_1)-(1-\alpha_2)}{(1-\alpha_1)(1-\alpha_2)}}},\quad \varphi=\frac{\rho(1-\alpha_2)}{\rho-1}\quad\text{and}\quad \varphi_1=\frac{\rho(1-\alpha_1)}{\rho-1}. \tag{72}$$

PROOF:

See Appendix A.

The next Proposition directly follows from Lemma 1. Details about the equilibrium allocation of scientists can also be found in Appendix A.

PROPOSITION 1. Under the laissez-faire equilibrium, innovation at time $t\geq 0$ in region $\ell\in\mathbb{L}$ occurs in the clean sector only if

$$(Q^{\ell}_{2,t-1})^{\varphi} < \Gamma_1\left((\eta^{\ell}_{2,t}(\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}-1)+1)(1+\iota\eta^{\ell}_{2,t})\right)^{-(\varphi+1)}(Q^{\ell}_{1,t-1})^{\varphi_1},$$

and in the dirty sector only if

$$(Q^{\ell}_{1,t-1})^{-\varphi_1} > \Gamma_1\left((\eta^{\ell}_{1,t}(\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}-1)+1)(1+\iota\eta^{\ell}_{1,t})\right)^{\varphi_1+1}(Q^{\ell}_{2,t-1})^{-\varphi},$$

and in both sectors if

$$\frac{(Q^{\ell}_{2,t-1})^{\varphi}}{(Q^{\ell}_{1,t-1})^{\varphi_1}} = \Gamma_1 \frac{\left((\eta^{\ell}_{1,t}s^{\ell}_{1,t}(\alpha_1^{\frac{\alpha_1}{1-\alpha_2}} - 1) + 1)(1 + \iota\eta^{\ell}_{1,t}s^{\ell}_{1,t})\right)^{\varphi_1+1}}{\left((\eta^{\ell}_{2,t}s^{\ell}_{2,t}(\alpha_2^{\frac{\alpha_2}{1-\alpha_2}} - 1) + 1)(1 + \iota\eta^{\ell}_{2,t}s^{\ell}_{2,t})\right)^{\varphi+1}},$$

where I define $\Gamma_1 = \frac{\eta^{\ell}_{2,t}}{\eta^{\ell}_{1,t}}\Gamma(\frac{\nu_1}{\upsilon_t+\tau^{\ell}_t\zeta})^{\frac{\nu_1\varphi}{1-\alpha_2}}$ for simplicity and better readability.

PROOF:

See Appendix A.

Proposition 1 comprises three scenarios. If $\Pi^{\ell}_{2,t}/\Pi^{\ell}_{1,t} > 1$, innovation in the clean sector is relatively more profitable and therefore, all scientists direct their research to the clean sector, i.e., $s^{\ell}_{2,t} = 1$ and $s^{\ell}_{1,t} = 0$. Contrarily, if $\Pi^{\ell}_{2,t}/\Pi^{\ell}_{1,t} < 1$, then all scientists will work on machine lines in the dirty sector since it is more profitable. Lastly, if $\Pi^{\ell}_{2,t}/\Pi^{\ell}_{1,t} = 1$ scientists are indifferent between sectors because they are equally profitable, hence, scientists may be employed in both sectors simultaneously. Note, however, that the existence of scientists in both sectors at the same time only constitutes a knife-edge solution. Recall from Assumption 2 that $\rho \in (0, 1)$ which is equivalent to $\varphi < 0$ and $\varphi_1 < 0$. Thus, like the findings in Acemoglu *et al.* (2012), innovation favors the sector that is relatively more advanced.

Moreover, just like Acemoglu *et al.* (2012) I assume that sector $i = 1$ is initially sufficiently more advanced in machine quality and productivity than the clean sector. Hence, under *laissez-faire*, scientists start to direct their research to machines complementing the dirty energy sector:

ASSUMPTION 3.

$$\frac{(Q^{\ell}_{2,-1})^{-\varphi}}{(Q^{\ell}_{1,-1})^{-\varphi_1}} < \min \begin{pmatrix} \Gamma_1^{-1}\left((\eta^{\ell}_{1,1}(\alpha_1^{\frac{\alpha_1}{1-\alpha_2}} - 1) + 1)(1 + \iota\eta^{\ell}_{1,1})\right)^{-(\varphi_1+1)} \\ \Gamma_1^{-1}\left((\eta^{\ell}_{2,1}(\alpha_2^{\frac{\alpha_2}{1-\alpha_2}} - 1) + 1)(1 + \iota\eta^{\ell}_{2,1})\right)^{\varphi+1} \end{pmatrix} \tag{73}$$

This assumption implies built-in path-dependence, i.e., decisions of earlier periods still constrain the path of future research. Historical decisions in terms of technology are considered to have long-lasting impacts on the path of the economy and the environment. It may, therefore, be difficult to overcome and change the established economic situation, that mostly relies on fossil fuels, and promote the transition to renewable alternatives. Under Assumption 3, scientists start to improve the quality of machines in the dirty sector due to higher profit incentives. This widens the quality gap between the clean and the dirty sector. The increasing productivity gap, again, favors the dirty sector for research in the following periods. This path-dependence can lead to lock-in effects, where past decisions in research and policy set the course for the future in this model.

From Lemma 1 follows that in equilibrium the direction of technological change is essentially characterized by the gross resource price $\upsilon_t + \tau_t\zeta$, the aggregate quality index, and sectoral success probabilities. One of the key differences between the model by Acemoglu *et al.* (2012) and this study is that the probability of success for innovation is determined exogenously in the former and endogenously in this model. Hence, $\eta^{\ell}_{i,t}$ is not a constant, but may vary over time. However, $\eta^{\ell}_{i,t}$ is not designed to exhibit continuous growth but represents a probability that shall not exceed unity, whereas the gross resource price $\upsilon_t + \tau_t\zeta$ and the aggregate quality index may grow continuously over time. Suppose that $\upsilon_0 > c$, then according to the Hotelling rule (15), υ_t grows at rate $r_t = \beta(g-1)^{\sigma}$ given by the Euler equation in (42), where $g \geq 0$ is the consumption growth rate on a balanced growth path. In this section, I focus on the laissez-faire equilibrium, hence, $\tau_t = 0$. The possibility that the tax rate is positive and grows is discussed in Section 4. Given Assumption 3, only the quality level in sector $i = 1$ increases during the first period, since all scientists direct their research to the dirty sector. Following the law of motion given by (31), $Q^{\ell}_{1,t}$ grows at rate $1 + \iota\eta^{\ell}_{1,t}$ in the initial period $t = 0$. This implies for the following period, that the initial gap between the quality levels of both sectors widens further, increasing profit incentives in sector $i = 1$. If the productivity in sector $i = 1$ grows faster than the resource price and variations in the risk effect do not offset this development, then innovation will henceforth only occur in the dirty energy sector due to the built-in path-dependence. Additionally, consider the case where $\upsilon_0 = c$, then $\upsilon_t = c$ for all $t \geq 0$ and the resource price is constant. Then, only the quality index experiences continuous growth over time and research will only occur in the dirty energy sector.

4 Optimal climate policies

4.1 Efficient carbon tax

This section analyses optimal climate policies in order to implement an equilibrium allocation which is socially optimal. The socially optimal allocation can be determined as the solution of a planning problem where a social planner maximizes consumption of a world representative consumer. This maximization problem is subject to feasibility constraints given by production technologies, the resource stock, the climate model, and market clearing conditions. In contrast to the laissez-faire equilibrium, the social planner takes into account that resource prices do not reflect the social cost of CO_2 emissions. Furthermore, I show that the social planner introduces a subsidy for the use of sector-specific machines to offset monopoly distortions and an R&D subsidy to direct scientists to the sector with the highest social gain. To describe different climate policies in this model, I establish the following

DEFINITION 3. A climate policy is given by a non-negative sequence of emission taxes $\tau = (\tau_t)_{t\geq 0}$ to be paid per unit of CO_2 emitted, and proportional, region-specific profit subsidies for clean research with subsidy rate $\vartheta^{\ell} = (\vartheta^{\ell}_t)_{t\geq 0}$.

First, I characterize the properties of the emissions tax and focus on a scenario where the social planner only corrects the monopoly distortions and implements the carbon emissions tax. The properties and effects of a research subsidy are then analyzed in section 4.2. The carbon emissions tax is paid by dirty energy producers because carbon is assumed to be emitted at the energy production stage. All regions $\ell \in \mathbb{L}$ are assumed to cooperate and impose the same emissions tax τ_t.[14] The tax revenue is then distributed by the governments to the consumers in each region via lump-sum transfers. Eyckmans & Tulkens (2003) study game theoretical approaches to find cooperative and Pareto-improving climate policies. They suggest international transfer schemes that distribute tax revenues across borders to incentivize countries to cooperate. Hillebrand & Hillebrand (2019) later adapted and refined this concept for their IAM framework. Contrarily, this study does not consider a transfer scheme that allows tax revenues to be transferred across regions. Instead, I assume that transfers received by the consumers have to balance with tax revenues on the regional level.

An equilibrium allocation as in Definition 2 that is characterized by a certain climate policy will be written as $\mathbf{A}(\tau, \vartheta)$. In contrast to Hillebrand & Hillebrand (2019), the transfer policy is not part of this notation anymore, since transfers are limited to the national level. The laissez-faire equilibrium studied in section 3.1 without policy intervention therefore takes the form $\mathbf{A}^{\mathbf{LF}} := \mathbf{A}(0, 0)$. In this scenario, the market fails to fully reflect the social costs for the use of the exhaustible resource, since its effects at the climate stage and regional climate damages D_t^ℓ are disregarded. This denotes the *climate externality*. Internalizing this externality requires a carbon tax that can be determined by solving the social planning problem. The aggregate planning problem where the social planner maximizes utility of a world representative household reads

$$\max_{\overline{\mathbf{A}}} \left\{ \sum_{t=0}^{\infty} \beta^t u(\overline{C}_t) \;\middle|\; \overline{\mathbf{A}} = (\overline{C}_t, \mathbf{Y_t}, \mathbf{Q_t}, \mathbf{N_t}, \mathbf{M_t}, \mathbf{E_t}, \mathbf{H_t}, \mathbf{X_t}, \mathbf{S_t}, \overline{K_t}, \mathbf{s_t})_{t \geq 0} \right\}. \tag{74}$$

The planning problem is solved using the standard Lagrangian approach. Appendix A formally develops the Lagrangian function. An allocation that is implemented through the sequence of efficient emission taxes $(\tau_t^{\text{eff}})_{t \geq 0}$ will be referred to as efficient allocation $\mathbf{A}^{\mathbf{eff}} := \mathbf{A}(\tau_t^{\text{eff}}, 0)$. Hillebrand & Hillebrand (2019) show that the efficient carbon tax is determined by the Pigouvian solution

$$\tau_t^{\text{eff}} = \sum_{n=0}^{\infty} \beta^n \left(\frac{\overline{C}_{t+n}}{\overline{C}_t} \right)^{-\sigma} (\phi_L + (1 - \phi_L)\phi_0(1 - \phi)^n) \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_{t+n}^\ell. \tag{75}$$

The efficient tax determines the efficient allocation $\mathbf{A}^{\text{eff}}$ and fully internalizes the climate externality. If the efficient solution follows a balanced growth path on which consumption and output grow at a constant and identical rate $g \geq 0$, then the efficient tax is determined by a constant share of world output and takes the form

[14]See Hillebrand & Hillebrand (2023) for a study that considers region specific emission taxes and additionally Hambel *et al.* (2021) for studies with non-cooperative climate policies.

$$\tau_t^{\text{eff}} = \overline{\tau}_t^{\text{eff}} \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_t^\ell, \quad \overline{\tau}_t^{\text{eff}} := \frac{\phi_L}{1-\beta(1+g)^{1-\sigma}} + \phi_0 \frac{1-\phi_L}{1-\beta(1+g)^{1-\sigma}(1-\phi)}. \tag{76}$$

Although this study constitutes a modified version of the model in Hillebrand & Hillebrand (2019), the efficient tax formula given by (76) still holds true because the specifications of the final good production function (1), the climate model (36), the damage function (37), and the consumer preferences (39) remain unchanged. Appendix A formally shows that the results from Hillebrand & Hillebrand (2019) can be transferred to this study.[15]

As shown in Theorem 1, the efficient carbon tax is a determinant of the direction of endogenous technical change. As apparent in Lemma 1, the carbon tax increases the gross resource price $\upsilon_t + \tau_t\zeta$ and further discourages innovation in the dirty sector given that $\rho \in (0, 1)$. If τ_t is set sufficiently high, the government can influence the direction of technological change solely through the carbon tax. However, the socially optimal tax rate τ_t^{eff} might be too low to redirect research. In this case, additional policy instruments are required. Since taxes are determined by a constant share of output, the tax rate τ_t^{eff} also grows at rate g if the economy is on a balanced growth path.

4.2 R&D subsidies

In contrast to the laissez-faire equilibrium and the efficient allocation featuring a carbon tax, the socially optimal allocation has to correct another market failure; the *knowledge externality*. The knowledge externality exists because scientists do not take into account how their research outcomes affect future economic developments. Therefore, assume that the social planner introduces a subsidy to redirect research towards the sector with the highest social gain. Intuitively, the emissions tax internalizes the current climate externality by decreasing production of dirty energy, whereas the R&D subsidy takes into account the future effects of current research decisions (Acemoglu *et al.*, 2012). As pointed out in the previous section, the emissions tax discourages investments in the dirty energy sector and promotes clean research. However, if the efficient tax is not sufficiently high to overcome the initial path-dependence induced by Assumption 3, the productivity gap between $Q_{2,t}^\ell$ and $Q_{1,t}^\ell$ will further increase in favor of dirty energy production. The absence of a research subsidy typically requires higher carbon taxes, which can also be shown in the numerical simulations. Therefore, to base future production on more advanced clean technologies, subsidies for clean research should be implemented additionally to emission taxes. This result is backed up by Varga *et al.* (2022), who find that the costs of transitioning to a net-zero emission economy can be greatly reduced in the long run if the proceeds from carbon taxes are used to subsidize clean energy.[16] The socially optimal solution additionally corrects for the standard static monopoly distor-

[15] For details, see Hillebrand & Hillebrand (2019) Appendix A.7.

[16] Alternatively, the efficient allocation of scientists could also be achieved via profit taxes on the dirty energy sector (Acemoglu *et al.*, 2012)

tion, encouraging higher use of sector-specific machines as shown in Appendix A (see, e.g., Acemoglu, 2009; Acemoglu *et al.*, 2012).

Suppose the governments in regions $\ell \in \mathbb{L}$ subsidize scientists working on machine lines in the clean sector via a proportional profit subsidy ϑ_t^ℓ beginning in $t = 0$. The subsidy is financed via a lump-sum tax on the representative consumers. In contrast to the efficient tax for all regions $\ell \in \mathbb{L}$, the optimal subsidy does not have to be equal across regions. Since initial quality levels for machines may differ across world regions, the subsidy to redirect innovation can vary too. Otherwise, a region could be subject to subsidies that exceed the necessary effort to redirect innovation and, therefore, distort the consumption path.

The optimal allocation that corrects for all three market failures and includes the efficient tax τ_t^{eff} and optimal research subsidies ϑ_t^ℓ, is denoted by $\mathbf{A}^{\text{opt}} := \mathbf{A}(\tau_t^{\text{eff}}, \vartheta_t^\ell)$. Under the optimal allocation, the social planner allocates scientists to the sector with the highest social gain. Hence, whenever the ratio

$$\frac{\eta_{2,t}^\ell \iota(1+\iota\eta_{2,t}^\ell s_{2,t}^\ell)^{-1}(1-\alpha_2)\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}\sum_{n\geq t}(\hat{p}_{2,n}^\ell)^{\frac{1}{1-\alpha_2}}N_{2,n}^\ell Q_{2,n}^\ell}{\eta_{1,t}^\ell \iota(1+\iota\eta_{1,t}^\ell s_{1,t}^\ell)^{-1}(1-\alpha_1)\alpha_1^{\frac{\alpha_1}{1-\alpha_1}}\sum_{n\geq t}(\hat{p}_{1,n}^\ell)^{\frac{1}{1-\alpha_1}}(N_{1,n}^\ell)^{\frac{1-\alpha_1-\nu_1}{1-\alpha_1}}(X_t^\ell)^{\frac{\nu_1}{1-\alpha_1}}Q_{1,n}^\ell} \tag{77}$$

is greater than unity, scientists are allocated towards the clean sector. The allocation, thus, does not depend on the private return to research anymore.

PROOF:

See Appendix A.

Since profits are subsidized, that are the result of the scientists‘ decision process, the decision problem for optimal R&D expenditure and optimal monopoly pricing itself remains unchanged. However, correcting for the monopoly distortion leads to a higher utilization of sector-specific machines and, hence, to a different demand function, changing the relative profitability of research in sectors $i \in \{1, 2\}$.

LEMMA 2. Under both, the optimal and efficient allocation the profit ratio takes the form

$$\frac{\Pi_{2,t}^\ell}{\Pi_{1,t}^\ell} = \frac{\eta_{2,t}^\ell}{\eta_{1,t}^\ell}\Gamma_{\text{opt}}\left(\frac{\nu_1}{\upsilon_t + \tau_t^\ell \zeta}\right)^{\frac{\nu_1\varphi}{1-\alpha_2}}\frac{(1+\iota\eta_{1,t}^\ell(1-s_{2,t}^\ell))^{\varphi_1+1}}{(1+\iota\eta_{2,t}^\ell s_{2,t}^\ell)^{\varphi+1}}\frac{(Q_{2,t-1}^\ell)^{-\varphi}}{(Q_{1,t-1}^\ell)^{-\varphi_1}}. \tag{78}$$

PROOF:

See Appendix A.

As in Acemoglu *et al.* (2012), a sufficiently high subsidy rate ϑ_t^ℓ redirects all innovation effort immediately to the clean sector due to its direct effect on relative profitability. Given the results in Lemma 2, the following proposition determines the necessary subsidy to redirect innovation.

PROPOSITION 2. If $1+\varphi > 0$ and $1+\varphi_1 > 0$ or equivalently $\rho < \frac{1}{2-\alpha_2}$ and $\rho < \frac{1}{2-\alpha_1}$, the subsidy that is necessary to redirect innovation to the clean sector is

$$\hat{\vartheta}_t^\ell > \vartheta_t^\ell \equiv \frac{\eta_{1,t}^\ell}{\eta_{2,t}^\ell}\Gamma_{\text{opt}}^{-1}\left(\frac{\nu_1}{\upsilon_t + \tau_t^\ell \zeta}\right)^{-\frac{\nu_1\varphi}{1-\alpha_2}}(1+\iota\eta_{2,t}^\ell)^{\varphi+1}\frac{(Q_{2,t-1}^\ell)^\varphi}{(Q_{1,t-1}^\ell)^{\varphi_1}} - 1. \tag{79}$$

If $\rho > \frac{1}{2-\alpha_2}$ and $\rho > \frac{1}{2-\alpha_1}$, then the optimal subsidy is

$$\hat{\vartheta}_t^\ell > \vartheta_t^\ell \equiv \frac{\eta_{1,t}^\ell}{\eta_{2,t}^\ell}\Gamma_{\text{opt}}^{-1}\left(\frac{\nu_1}{\upsilon_t + \tau_t^\ell \zeta}\right)^{-\frac{\nu_1\varphi}{1-\alpha_2}}(1+\iota\eta_{1,t}^\ell)^{-\varphi_1-1}\frac{(Q_{2,t-1}^\ell)^\varphi}{(Q_{1,t-1}^\ell)^{\varphi_1}} - 1 \tag{80}$$

where $\hat{\vartheta}_t^\ell = \vartheta_t^\ell + \epsilon$ and ϵ is an arbitrarily small positive number to ensure that the subsidy is large enough to redirect innovation.

PROOF:

See Appendix A.

The subsidy rate determined in Proposition 2 positively depends on machine quality $Q_{1,t}^\ell$ in sector $i = 1$. Intuitively, the larger the gap between the two productivity levels, the larger the necessary subsidy to redirect research. Furthermore, the gross resource price $\upsilon_t + \tau_t\zeta$ negatively affects the subsidy rate. This indicates that the existence of emission taxes τ_t already mitigates the required subsidy. If, however, the efficient carbon tax is not sufficiently high, it still requires the R&D subsidy to redirect innovation to the clean sector.

The optimal subsidy ensures that the ratio $Q_{2,t}^\ell/Q_{1,t}^\ell$ grows at rate $\iota\eta_{2,t}^\ell$ as long as it is implemented. Given that $0 < \rho < 1$, it is sufficient to maintain the subsidy only temporarily for B periods. After B periods, the productivity ratio $Q_{2,t}^\ell/Q_{1,t}^\ell$ has become sufficiently large to fulfill the condition in Proposition 1 to direct research to the clean sector without subsidy. For initially larger gaps between the quality levels, the subsidy has to be larger to compensate the initially greater relative profitability of sector $i =$ 1. The length of the subsidy can be measured as the number of periods after which innovation occurs in the clean sector without the help of a subsidy. The subsidy on clean research is at work for B periods where B is the smallest integer such that

$$\frac{Q_{2t+B-1}^\varphi}{Q_{1t+B-1}^{\varphi_1}} < \Gamma_{\text{opt}}(1+\iota\eta_2)^{-\varphi-1}, \tag{81}$$

if $\rho < \frac{1}{2-\alpha_2}$ and $\rho < \frac{1}{2-\alpha_1}$, or

$$\frac{Q_{2t+B-1}^\varphi}{Q_{1t+B-1}^{\varphi_1}} < \Gamma_{\text{opt}}(1+\iota\eta_1)^{\varphi_1+1}, \tag{82}$$

if $\rho > \frac{1}{2-\alpha_2}$ and $\rho > \frac{1}{2-\alpha_1}$.

The subsidy is already implemented in the first period. While Proposition 2 quantifies the size of the R&D subsidy and equations (81) and (82) quantify the length of the intervention, it might be insightful to measure the climate policy in terms of adjustment costs. Due to the built-in path-dependence in favor of the dirty energy sector, the less advanced clean sector has to catch up to close the initial gap, causing output to grow slower initially. I assume that the government intervenes already in the first period $t = 0$. The cost of adjustment is determined by the number of periods it takes the economy under policy intervention to achieve the same level of GDP growth as it would have achieved within one period under laissez-faire. Formally, this condition is fulfilled in period t as soon as the following equation holds:

$$\frac{Y_1^{\mathrm{LF}} - Y_0^{\mathrm{LF}}}{Y_0^{\mathrm{LF}}} = \frac{Y_t^{\mathrm{opt}} - Y_{t-1}^{\mathrm{opt}}}{Y_{t-1}^{\mathrm{opt}}}. \tag{83}$$

The adjustment costs, the policy duration and the size of the subsidy are used to evaluate the optimal climate policy $\mathbf{A}^{\mathrm{opt}}(\tau_t^{\mathrm{eff}}, \vartheta_t^{\ell})$ in the numerical simulation in Section 5.

5 Numerical simulation

5.1 Calibration

This section performs a numerical exercise of the developed model and analyzes the different model outcomes for the laissez-faire scenario, the efficient equilibrium with taxes and the optimal solution with complementary R&D subsidies. The world economy consists of two regions $\ell \in \mathbb{L}$ that will represent OECD countries denoted by $\ell = 1$ and the rest of the world denoted by $\ell = 2$, which will be called NOECD countries. As in Golosov *et al.* (2014) and Hillebrand & Hillebrand (2019), one model period comprises ten years. The initial period ends in $t = 2020$ and will be referred to as the baseline period.

Main calibration targets

The initial world population is set to unity and population is distributed according to empirical population shares. The current population share of OECD countries is 18%, thus, $\overline{N}_t^1 = 0.18$ and $\overline{N}_t^2 = 0.82$ (World Bank, 2020b). Exogenous labor-augmenting growth increases labor supply $N_t^{\ell,s}$ at a constant growth rate of $g^N = 0.16$ per period, which implies an annual growth rate of 1.5%. This factor is in line with values used in Hillebrand & Hillebrand (2019) and Golosov *et al.* (2014). Furthermore, OECD countries make up 62% of world GDP (World Bank, 2020a) and are assumed to own 68.5% of the world's capital stock as in Hillebrand & Hillebrand (2019). Productivity levels in the final sector are not affected by the mechanism of endogenous technical change. Thus, Q_0^1 and Q_0^2 are exogenous and constant throughout time but may differ across regions $\ell \in \mathbb{L}$ and capture differences in productivity. Relative productivity is chosen to match empirical GPD shares. The absolute values for productivity induce a value for world output of about 800 trillion U.S.-\$ which roughly matches its empirical counterpart for the baseline period as presented in World Bank's World Development Indicators (2020a).

Production and resource sectors
In the final sector I set the factor shares for capital α_0 and for energy ν_0 as in Golosov *et al.* (2014) such that the labor share is $1-\alpha_0-\nu_0 = 0.66$. There are two energy sectors, one dirty energy sector ($i = 1$) where energy is produced using an exhaustible resource that emits CO_2 and a clean energy sector ($i = 2$) where energy is produced based on renewable resources, e.g., water, wind, solar, and nuclear power. The factor cost shares for sector-specific machines in sector $i = 2$ is chosen similarly to the values in Lennox & Witajewski-Baltvilks (2017) and Hemous (2013) and set to $\alpha_2 = 0.5$. In the dirty sector, the factor share for the exhaustible resource is set to $\nu_1 = 0.2$, which implies $\alpha_1 = 0.3$ to satisfy Assumption 2. The elasticity of substitution $(1/1-\rho)$ is set to 2, hence, $\rho = 0.5$, which is in line with the IAM literature.[17] As in Golosov *et al.* (2014), I choose $\kappa = 0.5$ to imply a relative price of unity between the two energy goods.

In contrast to the productivity parameters in the final sector, the quality levels at the energy stage are determined via the endogenous mechanism of directed technical change and may evolve over time. The initial values for $Q^{\ell}_{1,-1}$ and $Q^{\ell}_{2,-1}$ are chosen to obtain a share of 22% from clean energy in the energy mix of OECD countries and 14% in NOECD countries, as in Hillebrand & Hillebrand (2019). The initial quality levels shown in Table 1 satisfy the assumption that the clean sector is sufficiently backward relative to the dirty sector such that technical change occurs in the dirty sector in the first model period.

In this model, there is only a single exhaustible resource that comprises all fossil fuels, e.g., coal, oil, and gas. All resources are assumed to be abundant, such that, $R_0 = \infty$ as in Golosov *et al.* (2014). This implies by (15) that $\upsilon_t = \upsilon_0 = c$ for all $t \geq 0$, i.e., the resource price is constant and equals extraction costs. Both, the value for extraction costs c and the value for the carbon content of the exhaustible resource ζ are taken from Hillebrand & Hillebrand (2019) as they also combine all fossil fuels in a single resource.

Climate model and damage function
The parameter values describing the carbon cycle and climate damages are mostly taken from Golosov *et al.* (2014) because the climate model in this study is identical. The damage parameter is kept equal for both regions $\ell \in \{1, 2\}$ at $\gamma_1 = \gamma_2 = 0.000053$. The initial values for the climate state $\mathbf{S_{-1}} = (S_{1,-1}, S_{2,-1})$ are chosen to match the empirical observations of 875.1 GtC atmospheric CO_2 concentration in 2020 as observed from the Earth System Research Laboratories (2023). The pre-industrial level of CO_2 in the atmosphere is set to $\overline{S} = 581$. Furthermore, I follow Golosov *et al.* (2014) to map the atmospheric CO_2 concentration to the global mean temperature using

$$T_t = 3 \log\left(\frac{S_t}{\overline{S}}\right) / \log 2. \tag{84}$$

R&D sector
As in Hillebrand (2016), the parameters describing the cost of research and the share

[17] Acemoglu *et al.* (2012) and Rezai & van der Ploeg (2015) assume even higher values or perfect substitutability, respectively.

of profit devoted to R&D are set to $\omega = 1.6$ and $\lambda = 0.06$ to ensure that the success probability of research takes on values on the interval $[0, 1]$.

To determine the constant step size ι on the quality ladder, I use the empirical growth rate of energy productivity, defined as the ratio between world GDP and total primary energy supply. Data for world GDP is again taken from World Bank (2020b). Using data for energy supply from OECD (2022), the average growth rate of energy productivity in the baseline period is 0.3121, which corresponds to an annual growth rate of about 2.75%. Considering an average success probability of 0.29 obtained in the numerical simulation, the step size parameter is set to $\iota = 1.076$. This implies annual productivity jumps of about 2.75%.

Consumer sector

As in Hillebrand & Hillebrand (2019) I choose $\sigma = 1$ which implies a logarithmic utility function. The annual discount factor is $\beta = 0.985$ so that the decadal discount factor is set to $\beta = 0.985^{10}$, which is again taken from Golosov *et al.* (2014). The initial capital stock that is owned by the consumer is set to $\overline{K}_0 = 0.18$ to ensure a stationary capital-to-labor ratio.

Simulation parameters			
Final sector			
$\alpha_0 = 0.3$	$\nu_0 = 0.04$	$\kappa = 0.5$	$\rho = 0.5$
$Q_0^1 = 4.02$	$Q_0^2 = 0.66$		
Energy and resource sector			
$\alpha_1 = 0.3$	$\nu_1 = 0.2$	$\alpha_2 = 0.5$	$c = 0.000071$
Climate parameters			
$\zeta = 0.5835$	$\overline{S} = 581$	$\phi_L = 0.2$	$\phi_0 = 0.393$
$\phi = 0.0228$	$\gamma_1 = 0.000053$	$\gamma_2 = 0.000053$	
Consumption sector			
$\beta = 0.985^{10}$	$\sigma = 1$	$g = 0.16$	
R&D sector			
$\iota = 1.076$	$\lambda = 0.06$	$\omega = 1.6$	$\psi = 1$
Initial values			
$Q_{1,-1}^1 = 4.2$	$Q_{2,-1}^1 = 15$	$Q_{1,-1}^2 = 12$	$Q_{2,-1}^2 = 25$
$S_{1,-1} = 744$	$S_{2,-1} = 112$	$R_0 = \infty$	$\overline{K}_0 = 0.18$

Table 1: Parameter set used for numerical simulation.

5.2 Equilibrium dynamics

Using the parametrization from Table 1, this section presents the simulation results for key variables under the laissez-faire scenario, the efficient emissions tax and the optimal solution which combines the efficient tax and green research subsidies.

5.2.1 Final output and growth

The trajectory for GDP growth in both OECD and NOECD countries is shown in Figure 1. In the baseline period, output for both regions combined is predicted to be 803 trillion U.S.-\$ under laissez-faire and 797 trillion U.S.-\$ under the optimal solution, which roughly matches the calibration target of 800 trillion U.S.-\$. Under all policy scenarios, OECD countries produce about 75.1% of the entire world output. Compared to the laissez-faire scenario, the implementation of an optimal subsidy and emission taxes reduces GDP in OECD countries by about -0.69% and by about -0.87% in NOECD countries. In $t = 2100$ GDP continues to be lower compared to laissez-faire by about -0.47% in OECD countries and by about -0.56% in NOECD countries. Only in 2190 output starts to be higher under the optimal subsidy than under laissez-faire in both OECD and NOECD countries. After 200 years in $t = 2220$, output in OECD countries is about 9.79% higher than in the case without policy intervention and about 8.31% higher in NOECD countries.

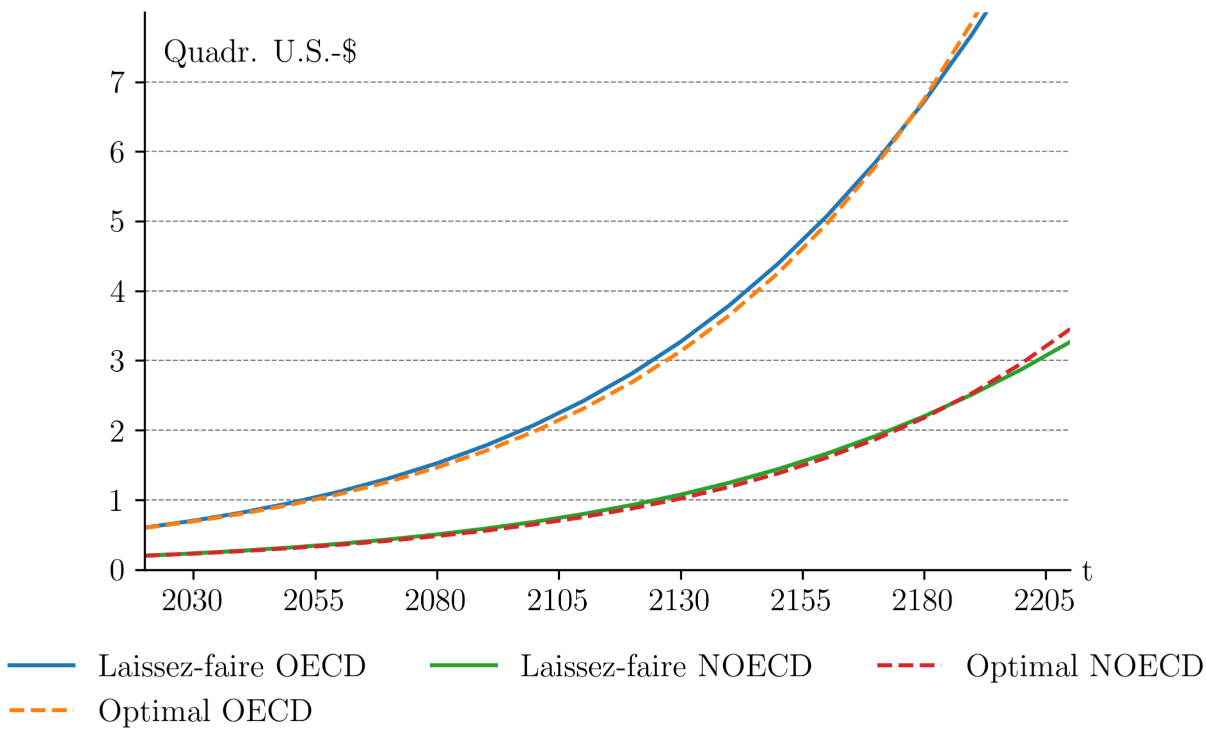


Figure 1: GDP in OECD and NOECD countries.

Furthermore, under the optimal solution with emission taxes and subsidies, the economy converges towards a balanced growth path. Output and consumption grow at an approximately identical and constant rate of $g = 16.5\%$ which corresponds to an annual rate of 1.54%. This growth rate accommodates the exogenous labor-augmenting growth at $g^N = 0.16$, the endogenous growth given by improvements in machine quality, and the productivity-diminishing effect caused by climate damages. In contrast, without any climate policy, output growth is increasingly harmed due to the presence of the climate externality and becomes less than 1.43% per year after $t = 2170$ and less than 1.3% per year after $t = 2210$. Similar to the findings in Hillebrand & Hillebrand (2019) and Acemoglu *et al.* (2012), these results suggest that climate policies come at an initial cost, which, however, becomes increasingly insignificant over time whereas the absence of government intervention leads to dramatic climate damages.

The initially lower level of output under the optimal solution compared to the laissez-faire case partially arises from the initial gap between machine qualities in the energy sectors. Productivity in the clean sector has to catch up to the productivity level in the dirty sector, causing adjustment costs. Figure 2 depicts these costs as the percentage point deviation of regional output growth from one period of laissez-faire growth, as defined in (83). The results suggest that under the optimal allocation $\mathbf{A}^{\text{opt}}(\tau_t^{\text{eff}}, \vartheta_t^{\ell})$, the economy initially grows slower given the negative deviation from laissez-faire. Moreover, it takes NOECD countries significantly longer to reach output growth higher than in one period of laissez-faire than OECD countries. While OECD countries achieve this level of growth after 70 years, it takes NOECD countries 40 years longer. This might be due to NOECD countries being relatively less advanced if it comes to clean energy infrastructure, which naturally requires a longer adjustment period.

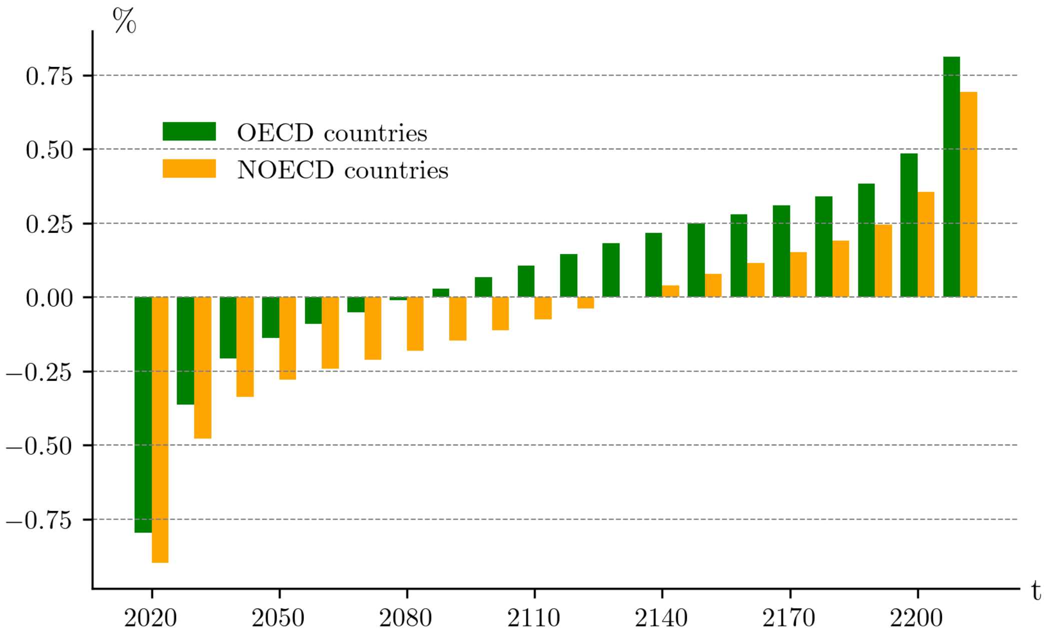
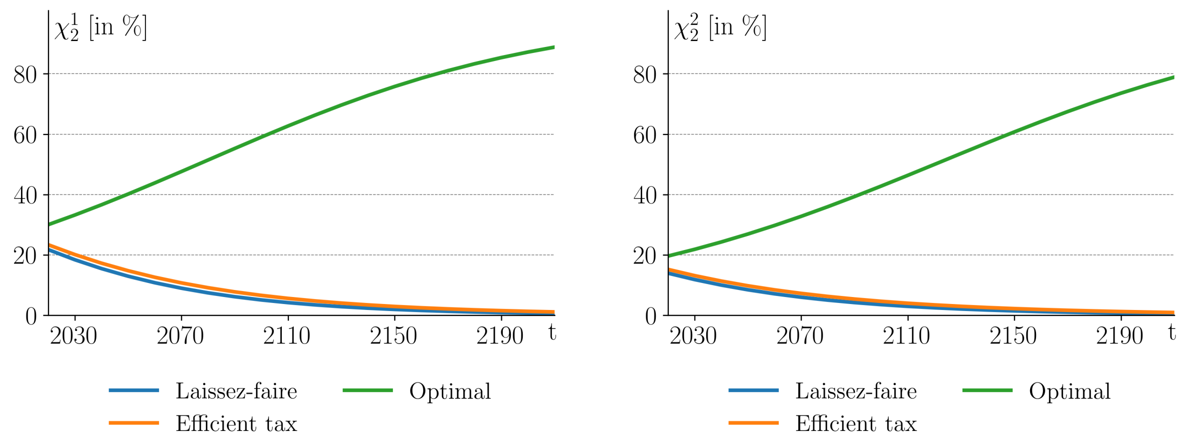


Figure 2: Adjustment costs: deviation of output growth compared to laissez-faire.

5.2.2 Energy stage

The production structure at the energy stage which was adapted from Hillebrand & Hillebrand (2019) allows analyzing the transition from fossil fuel based energies, e.g., coal, oil, and gas towards renewable alternatives. To measure this transition, I define the energy mix as $\chi_{i,t}^{\ell} := p_{i,t}^{\ell} E_{i,t}^{\ell} / \sum_{j \in \mathbb{I}} p_{j,t}^{\ell} E_{j,t}^{\ell}$ for each $i \in \mathbb{I}$. The energy mix $\chi_{i,t}^{\ell}$ measures the share that the clean or the dirty energy sector, respectively, contribute to total energy production used at the final output stage. Figure 3 shows the development of the share of clean energies used in total production over time. In the initial period $t = 0$, the clean energy share closely matches the calibration targets of 22% in OECD countries and 14% in NOECD countries. Without any policy intervention, the share of clean energy asymptotically decreases towards zero, indicating that only fossil fuel based energy inputs will be used in the future.

Although the carbon tax has a negative effect on research incentives in the dirty sector, no scientist is doing research in the clean sector even under the efficient tax policy. In contrast to Hillebrand & Hillebrand (2019), this model has to deal with more market failures than just the climate externality. The explicit modeling of directed technical change additionally introduces the knowledge externality. The efficient carbon tax, however, is only designed to internalize the climate externality. Hence, the efficient carbon tax is not sufficiently high to redirect research in the first place. Due to the initial path-dependence, the productivity gap between machines in the clean and the dirty sector further increases over time.



(a) OECD countries

(b) NOECD countries

Figure 3: Share of clean energy used in final goods production.

The optimal solution including subsidies leads to an immediate increase in the use of clean inputs to a share of 30% in OECD countries and 20% in NOECD countries in the initial period. After 100 years, clean energies already make up 66% of the energy mix in OECD countries and 50% in NOECD countries. The immediate use of the R&D subsidy incentivizes all scientists to innovate in the clean energy sector. However, compared to the development of dirty energy under laissez-faire, it takes clean energies much longer to reach a share of 90%. For OECD countries, our model predicts that this level will be reached after 210 years. In NOECD countries, clean energies only make up 81% of the energy mix after 210 years. This relatively slow adjustment towards clean energies can be explained by the initial state of the energy mix and the built-in path-dependence. First, the initial gap between productivity levels has to be closed. It takes time for machine quality in the clean sector to catch up to the more advanced machines in the dirty sector. Second, the initial share of clean energies in the energy mix is relatively low compared to dirty energies. Thus, countries with a less advanced infrastructure for clean energy production face longer adjustment periods.

5.2.3 Measures of climate policies

The climate policies employed in this model can be measured by the size of intervention, i.e., the amount of tax or subsidy required, or by the length of intervention. Figure 4 shows the trajectory for the global efficient emissions tax τ_t^{eff} under all three policy scenarios. Naturally, without any policy intervention, the tax is zero. In $t = 0$ the tax in both scenarios that employ the tax is about 39\$/tCO_2. This value is in the range of estimates for optimal emission taxes reported, e.g., in Nordhaus (1977), Hillebrand & Hillebrand (2019) and Golosov *et al.* (2014). Since the efficient tax is proportional to world GDP, the tax evolves along the same balanced growth path and grows steadily. The tax under the optimal solution grows slower initially due to adjustment costs that reduce world GDP. Intuitively, the carbon tax does not have to be as high if there is an additional subsidy that redirects research towards the clean sector because renewable energies become more productive due to the subsidy.

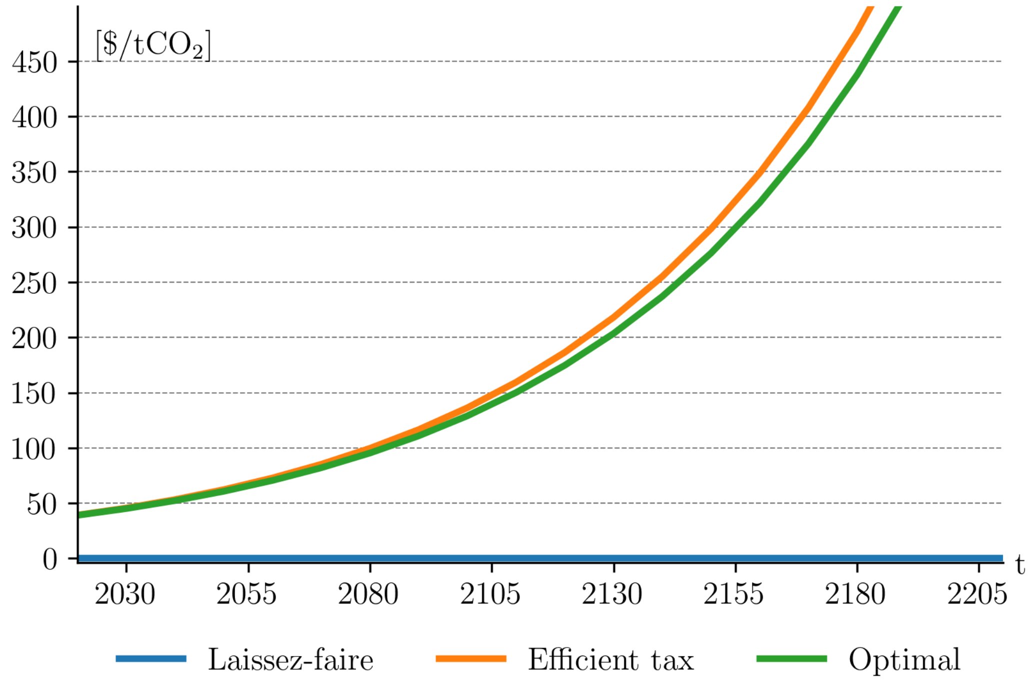


Figure 4: Efficient emissions tax τ_t^{eff}.

While the adjustment costs of the climate policy are already visualized in Figure 2, the size and length of the R&D subsidy can be analyzed in Figure 5. The graph shows the subsidy that is necessary to redirect research to the clean sector in percent of regional GDP. Recall that the subsidy is implemented in the very first period of the simulation already. The results suggest that NOECD countries face higher investments in R&D subsidies with 1.5% of regional GDP than OECD countries with 0.3%. Intuitively, since

NOECD are assumed to have a weaker clean infrastructure, it takes higher investments to close the productivity gap between the clean and the dirty sector. The subsidy expressed in percent of regional GDP first increases before it eventually declines towards zero in both regions. Formally, the subsidy as a share of regional GDP is given by the term $(\vartheta_t^\ell \Pi_{2,t}^\ell)/Y_t^\ell$. In contrast, the subsidy rate ϑ_t^ℓ, which is not shown in Figure 5, declines steadily. This makes sense because the productivity gap decreases with every improvement in the clean sector requiring less subsidies. The subsidy in percentage of regional GDP also has to account for GDP and profit growth. The world GDP grows along the balanced growth path, however, profits obtained through research in the clean sector grow relatively faster. First, this is due to the higher growth rate of the quality level in the clean sector, which is defined through the step size ι and the probability of research $\eta_{2,t}^\ell$. Second, the share of consumers offering their labor to the clean sector increases with the growing market. Both, labor and machine quality positively affect profit and therefore, profit growth exceeds GDP growth. On the other hand, the subsidy rate gradually declines, since the productivity gap closes asymptotically. These factors cause the concavity of the research subsidy when measured in percentage of GDP.

Ultimately, the subsidy is no longer needed in either region. The number of periods B that it takes for the optimal subsidy to become zero serves as a measure for the length of intervention. In OECD countries, the subsidy is required for $B = 14$ periods, whereas in NOECD countries the subsidy has to be at work for $B = 18$ periods. Thereafter, the clean sector is sufficiently advanced in order to offer profit incentives large enough for scientists to engage in research without an additional subsidy.

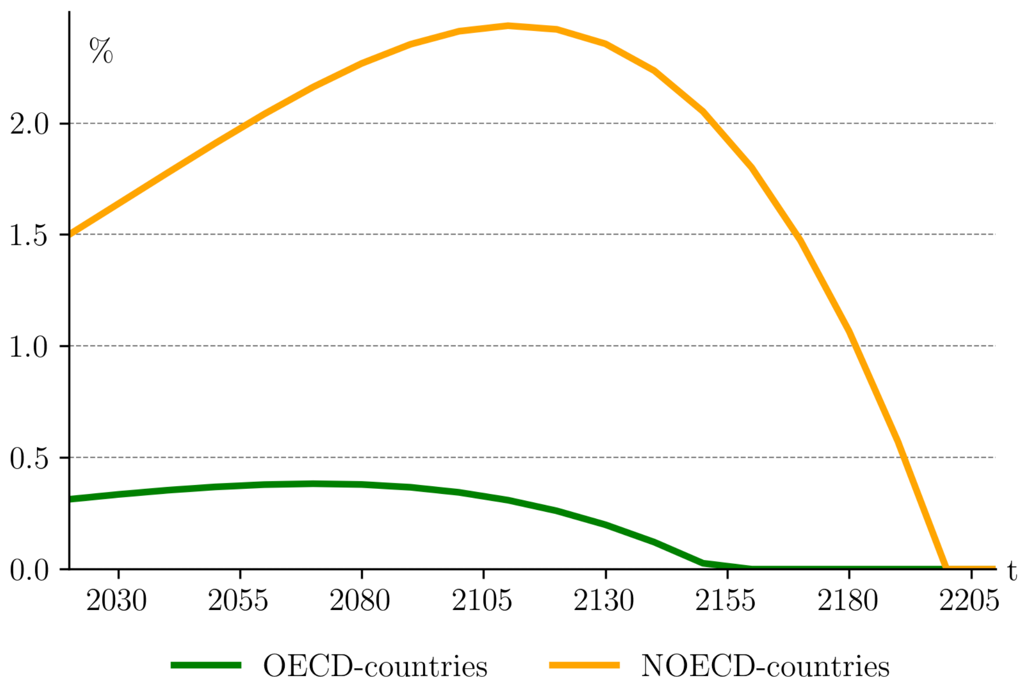


Figure 5: Optimal subsidy for R&D in % of regional GDP.

5.2.4 Climate stage

To quantify the manifestations of climate change, I employ two different measures. The first one are climate damages measured as percentage loss of potential output. Following Hillebrand & Hillebrand (2019), potential output is defined as $Y_t^{\ell,\text{pot}} := Y_t^\ell/(1-D_t^\ell)$, from which directly follows that D_t^ℓ can be interpreted as percentage loss of regional potential output $Y_t^{\ell,\text{pot}}$. Since for this parameter setup, climate damages are equal across regions $\ell \in \mathbb{L}$ by setting $\gamma_1 = \gamma_2$, it holds that $D_t^\ell \equiv D_t$ can be interpreted as global climate damages. Figure 6 shows the evolution of climate damages as defined before. Under the optimal solution, damages can be contained below 2% of world GDP throughout the simulation. If the climate policy only features the efficient tax, damages exceed the 2%-mark by $t = 2110$ and increase up to 2.92% after 200 years. If the government does not intervene at all, climate damages increase exponentially, reaching 5% of world GDP in $t = 2130$ and up to 13.6% in $t = 2200$.

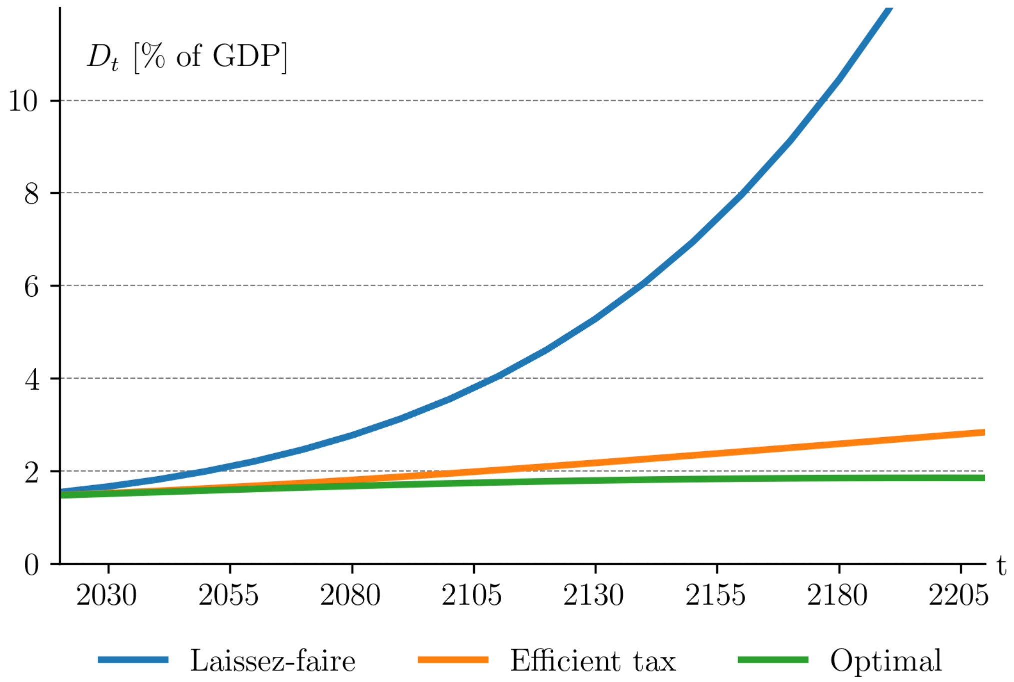


Figure 6: Global climate damages as percentage losses of world GDP.

The second measure to evaluate the climate state is the global mean surface temperature. Data from the NASA Earth observatory (2022) shows that earth was approximately 1.11 °C warmer in 2022 compared to the pre-industrial level. This value is assigned to my baseline period. Therefore, to stay within the two-degree target set by the Paris accord on climate change in 2015 temperature shall not increase by more than 0.89 °C relative to the baseline period. The increase in temperature is computed by (84) which describes a logarithmic relationship between global mean surface temperature and the atmospheric CO_2 concentration.

Figure 7 depicts the evolution of the global mean surface temperature under all three climate policies. Under the optimal policy, the temperature will have increased by 0.23 °C in $t = 2100$ compared to the baseline period. Thereafter, temperature continues to increase only slightly and eventually decreases after 22 model periods. This result suggests that under the optimal solution, the global mean temperature can be contained within roughly 1.45 °C above the pre-industrial level. Without the optimal subsidy and only using the efficient tax, the temperature will have already increased by 0.43°C in $t = 2100$ compared to the baseline period. Ultimately, this policy scenario even exceeds the 2 °C-mark by $t = 2180$.

The results are again dramatically different under the laissez-faire scenario. By 2100 temperature will have increased by 1.59 °C compared to the baseline period without any intervention, which corresponds to a 2.7 °C increase relative to the average of the late 19th century. Within 200 years, the global mean temperature increases by 6.8 °C relative to the baseline period. The simulation predicts that the laissez-faire policy fails the 2 °C-target by 2080. These results are in line with the predictions in Hillebrand & Hillebrand (2019) which is, however, not surprising as I make use of the same climate model and a similar model structure. An important difference, however, is that the efficient tax alone is not able to fulfill the 2 °C-target in the long-run. Due to the knowledge externality in the research sector and the built-in path-dependence, it takes more than just an emissions tax to redirect research to the clean sector and contain the increase in global mean temperature within 2 °C above the pre-industrial level.

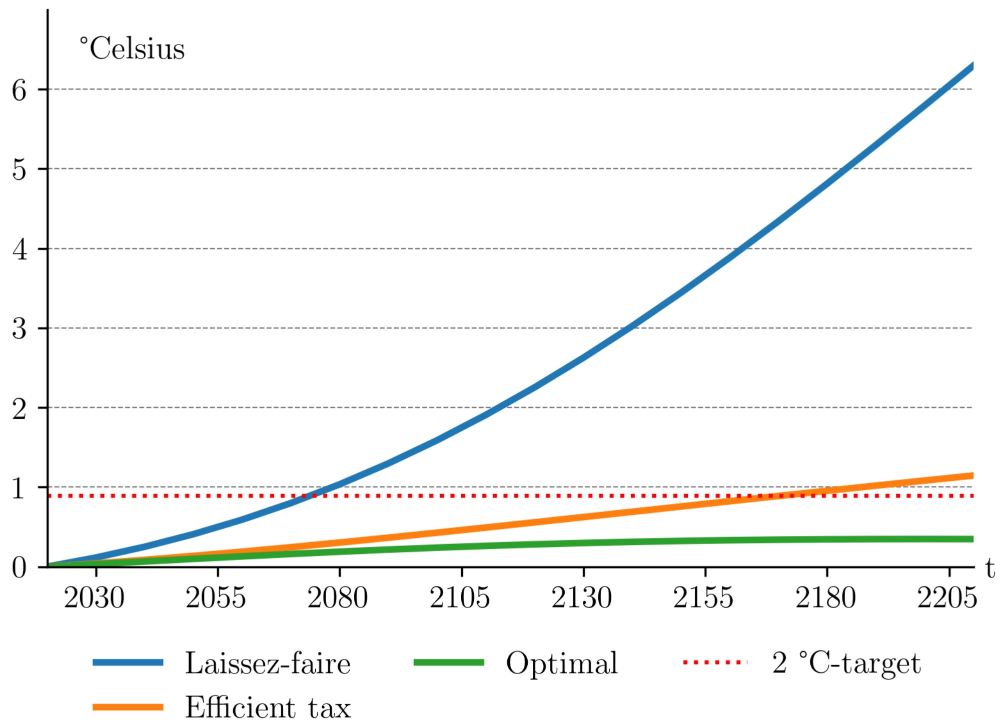


Figure 7: Global mean surface temperature relative to 2020.

6 Conclusions

This thesis formulates a general equilibrium climate-economy model with endogenous technical change and environmental constraints. By combining directed technological change and an IAM framework with each other, this thesis offers a comprehensive framework to model the interaction between climate change, endogenous technological change and policy intervention. The environmental constraint is an externality that originates from the emission of carbon at the energy stage due to the use of an exhaustible resource. A second externality arises from the circumstance that scientists base their research decisions on profit incentives and do not take into account the future consequences of their innovations.
Recent literature in this field has either assumed exogenous technological change, e.g., Golosov *et al.* (2014), Hassler & Krusell (2012), and Hillebrand & Hillebrand (2019, 2023), or based their endogenous growth model on a simple two-sector model, e.g., Acemoglu *et al.* (2012, 2014), van den Bijgaart (2017), or Lennox & Witajewski-Baltvilks (2017). This thesis brings together a comprehensive IAM framework with an extensive energy supply side featuring an exhaustible resource. The research process is based on an endogenous and micro-founded model, where research requires funding. Acemoglu *et al.* (2012) and van den Bijgaart (2017), e.g., simply assume exogenous success probabilities and do not require funds for research. Furthermore, growth enters the model via endogenous technological change and exogenous labor-augmenting growth. This is useful for this model framework because the effect of technical progress in energy on output is small, since the energy share in GDP is relatively small.

The direction of technical change in this model is determined by five market forces: the market size effect, the price effect, the productivity effect, the risk effect and the resource price effect. The first three of these resemble the essential market forces found in Acemoglu *et al.* (2012). In addition, I highlight the role of the resource price effect, as it shows that government intervention can affect the direction of research by increasing the gross resource price via an emissions tax. Furthermore, the size of the elasticity of substitution between the two energy inputs is crucial to determine the net-effect of the five market forces mentioned above. Thus, the parameter ρ is critical for the long-run properties for both, the laissez-faire scenario and also for the optimal climate policy as it affects the size and length of the optimal R&D subsidy. Due to the assumption of path-dependence, innovation starts in the dirty energy sector, leading to an increasing productivity gap between the dirty and clean energy sector. In a decentralized equilibrium without policy intervention, this ultimately leads to increasing emissions, catastrophic climate damages and temperatures above the 2 °C-target.

In section 5 of this thesis, I analyze optimal climate policies in a simple quantitative study where the outcome of the laissez-faire scenario is compared to an economy under the Pigouvian tax and a third scenario that additionally subsidizes R&D in the clean sector. Without any policy intervention, climate damages measured in percent of global

GDP increase exponentially and global mean temperature will exceed the 2 °C-target by 2080. Using the efficient carbon tax, extensively mitigates climate damages and the increase in global temperature, but does not achieve a transition from dirty to clean energies. Redirecting technical change towards the clean sector only using a carbon tax is feasible, but would require excessively high tax rates. Only if the government intervenes using both, the efficient tax and R&D subsidies, technological change can be directed towards renewable energies without excessive taxation and a dramatic increase in temperature and climate damages can be avoided.

The quantitative results support the findings in Hillebrand & Hillebrand (2019) that the implementation of the optimal climate policy comes at initial costs that are, however, negligible in the future. The adjustment phase for aggregate output growth to reach the same level as under laissez-faire in one period are estimated 70 years for OECD countries and 110 years for NOECD countries. While the efficient carbon tax is assumed to be implemented permanently, a temporary R&D subsidy is sufficient to redirect research. Productivity levels in the green sector will eventually catch up to the initial level of the dirty sector. In general, NOECD countries face higher adjustment costs and longer investments in the R&D subsidy, as their infrastructure of renewable energies is relatively more backwards compared to OECD countries.

In conclusion, to answer the initial questions of this thesis, it takes Pigouvian emissions taxes and R&D subsidies to overcome the initial path-dependence and redirect technologies toward the green sector. Furthermore, as long as the elasticity of substitution is sufficiently high, economic growth can be reconciled with environmental sustainability if policy intervention is immediate and uses both taxes and subsidies. These results resemble the position taken by Acemoglu *et al.* (2012) which is not surprising as this thesis builds on their framework of directed technical change.

Altogether, this model framework can be used for theoretical and quantitative analysis of international climate policies and economic growth. Moreover, I think that this framework is amendable to several fruitful extensions. It would be straightforward to extend the number of regions in this study to further elaborate on international policy coordination. Since this thesis shows that relatively less developed countries face higher adjustment costs, it would be interesting to adapt the idea of transfer payments as in Hillebrand & Hillebrand (2019) to set incentives to participate in internationally coordinated climate policies. Considering the side of technological growth, it could be interesting to allow international trade in sector-specific machines and model various ways of how technology diffuses across countries.

A Mathematical Appendix

A.1 Proof of Lemma 1

The relative price between the two energy goods can be expressed as

$$\left(\frac{E^{\ell}_{2,t}}{E^{\ell}_{1,t}}\right)^{\rho-1} = \frac{p^{\ell}_{2,t}}{p^{\ell}_{1,t}} \tag{A.1}$$

if $\kappa_1 = \kappa_2$ holds for the CES-function describing the energy aggregate used in final output production as in (2) which also holds true in Acemoglu *et al.* (2012) and Golosov *et al.* (2014).

Combining equilibrium demand for machines $M^{\ell}_{i,j,t}$ in sector $i \in \mathbb{I}$ as given in (13) and (9) with the necessary first order conditions for sectors $i \in \mathbb{I}$ with respect to $N^{\ell}_{i,t}$ as in (12) and (8) and equating these, allows writing the relative price for energy goods as

$$\frac{(p^{\ell}_{2,t})^{\frac{1}{1-\alpha_2}}}{(p^{\ell}_{1,t})^{\frac{1}{1-\alpha_1-\nu_1}}} = \frac{\tilde{Q}^{\ell}_{1,t}\alpha_1^{\frac{\alpha_1}{1-\alpha_1-\nu_1}}\left(\frac{\nu_1\tilde{Q}^{\ell}_{1,t}}{\upsilon+\tau^{\ell}_t\zeta}\right)^{\frac{\nu_1}{1-\alpha_1-\nu_1}}}{(1-\alpha_2)\tilde{Q}^{\ell}_{2,t}\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}}. \tag{A.2}$$

Now, note that Assumption 2 holds true. Taking the ratio of the equilibrium demands for energy goods (56) and (58) and plugging in (A.1) into the price ratio and solving for the labor ratio $N^{\ell}_{2,t}/N^{\ell}_{1,t}$ yields

$$\frac{N^{\ell}_{2,t}}{N^{\ell}_{1,t}} = \left(\frac{\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}\tilde{Q}^{\ell}_{2,t}}{\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}\left(\frac{\nu_1\tilde{Q}^{\ell}_{1,t}}{\upsilon_t+\tau^{\ell}_t\zeta}\right)^{\frac{\nu_1}{1-\alpha_2}}\tilde{Q}^{\ell}_{1,t}}\right)^{-1}\left(\frac{p^{\ell}_{2,t}}{p^{\ell}_{1,t}}\right)^{\frac{-\alpha_2(\rho-1)+(1-\alpha_2)}{(1-\alpha_2)(\rho-1)}}. \tag{A.3}$$

Given that Assumption 2 holds, plugging in the price ratio (A.2) into the labor ratio (A.3) gives

$$\frac{N^{\ell}_{2,t}}{N^{\ell}_{1,t}} = \left(\frac{\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}\left(\frac{\nu_1\tilde{Q}^{\ell}_{1,t}}{\upsilon_t+\tau^{\ell}_t\zeta}\right)^{\frac{\nu_1}{1-\alpha_2}}\tilde{Q}^{\ell}_{1,t}}{\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}\tilde{Q}^{\ell}_{2,t}}\right)\left(\frac{\tilde{Q}^{\ell}_{1,t}\alpha_1^{\frac{\alpha_1}{1-\alpha_2}}\left(\frac{\nu_1\tilde{Q}^{\ell}_{1,t}}{\upsilon_t+\tau^{\ell}_t\zeta}\right)^{\frac{\nu_1}{1-\alpha_2}}}{(1-\alpha_2)\tilde{Q}^{\ell}_{2,t}\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}}\right)^{\frac{-\alpha_2(\rho-1)+(1-\alpha_2)}{\rho-1}}. \tag{A.4}$$

Recall that $\varphi = \frac{(\rho-1)-\alpha_2(\rho-1)+(1-\alpha_2)}{\rho-1} = \frac{\rho(1-\alpha_2)}{\rho-1}$. Finally, combining (A.2) and (A.3) with (62) yields (71) in Lemma 1. ■

A.2 Proof of Proposition 1

This section follows Acemoglu *et al.* (2012) to first characterize the equilibrium allocation of scientists and then provide proof for Proposition 1 in this study. For $s \in [0,1]$ the profit ratio $\Pi^{\ell}_{2,t}/\Pi^{\ell}_{1,t}$ can be rewritten as a function of the scientists directing their research to the clean sector, $f(s^{\ell}_t)$, where s^{ℓ}_t represents scientists in sector $i = 2$.

$$f(s_t^\ell) \equiv \Gamma_1 \frac{\left((\eta_{1,t}^\ell (1-s_t^\ell)(\alpha_1^{\frac{\alpha_1}{1-\alpha_2}} - 1) + 1)(1 + \iota \eta_{1,t}^\ell (1-s_t^\ell)) \right)^{\varphi_1 + 1}}{\left((\eta_{2,t}^\ell s_t^\ell (\alpha_2^{\frac{\alpha_2}{1-\alpha_2}} - 1) + 1)(1 + \iota \eta_{2,t}^\ell s_t^\ell) \right)^{\varphi + 1}} \frac{(Q_{2,t-1}^\ell)^{-\varphi}}{(Q_{1,t-1}^\ell)^{-\varphi_1}}. \qquad (A.5)$$

Recall that $\Gamma_1 = \frac{\eta_2}{\eta_1} \Gamma (\frac{\nu_1}{\upsilon_0 + \tau\zeta})^{\frac{\nu_1 \varphi}{1-\alpha_2}}$. Clearly, if all scientists direct their research to sector $i = 2$ and the profit ratio exceeds unity, i.e., $f(1) > 1$, then $s_t^\ell = 1$ is an equilibrium. Similarly, if $f(0) < 1$, then $s_t^\ell = 0$ is an equilibrium; and lastly, if $f(s_t^{\ell *}) = 1$ for some $s_t^{\ell *} \in [0, 1]$, then $s_t^{\ell *}$ is an equilibrium. Depending on φ and φ_1 which depend on the elasticity of substitution, mainly three cases characterize the equilibrium:

- If $1 + \varphi > 0$ and $1 + \varphi_1 > 0$ (or equivalently, $\rho < \frac{1}{2-\alpha_2}$ and $\rho < \frac{1}{2-\alpha_1}$), then $f(s_t^\ell)$ is strictly decreasing in s_t^ℓ. It follows that if $f(1) > 1$, then $s_t^\ell = 1$ is the unique equilibrium which is a corner solution, i.e., the equilibrium solution to the scientist allocation problem lies at the extreme of the feasible set which is $s_t^\ell \in [0, 1]$. If $f(0) < 1$, then $s_t^\ell = 0$ is the unique equilibrium, again, a corner solution. If $f(0) > 1 > f(1)$, then there exists a unique $s_t^{\ell *} \in (0, 1)$ such that $f(s_t^{\ell *}) = 1$, which allows a unique interior equilibrium.
- If $1 + \varphi < 0$ and $1 + \varphi_1 < 0$ (or equivalently, $\rho > \frac{1}{2-\alpha_2}$ and $\rho > \frac{1}{2-\alpha_1}$), then $f(s_t^\ell)$ is strictly increasing in s_t^ℓ. Thus, if $1 < f(0) < f(1)$, then $s_t^\ell = 1$ is the unique corner solution; if $f(0) < f(1) < 1$, then $s_t^\ell = 0$ is the unique equilibrium and all scientists direct research towards sector $i = 1$; and lastly, if $f(0) < 1 < f(1)$, then there are three equilibria, the two corner solutions where $s_t^\ell = 1$ and $s_t^\ell = 0$ and the interior solution where $s_t^\ell = s_t^{\ell *} \in (0, 1)$ such that $f(s_t^{\ell *}) = 1$.
- If $1 + \varphi = 0$ and $1 + \varphi_1 = 0$, then $f(s_t^\ell) \equiv f$ is a constant and does not depend on s_t^ℓ anymore. It follows that if $f > 1$, $s_t^\ell = 1$ is the unique equilibrium; if $f < 1$, then $s_t^\ell = 0$ is the unique equilibrium.

Note that more combinations of the terms $1 + \varphi$ and $1 + \varphi_1$ exist, and therefore the equilibrium allocation of scientists is characterized by more than the aforementioned three cases. If, e.g., $1 + \varphi < 0$ and $1 + \varphi_1 > 0$ or vice versa $1 + \varphi > 0$ and $1 + \varphi_1 < 0$ it cannot clearly be determined whether $f(s_t^\ell)$ increases or decreases in s_t^ℓ. Therefore, I focus on the aforementioned three cases in this study, as they form the essential equilibrium allocation for scientists.

This equilibrium allocation of scientists implies the results in Proposition 1.■

A.3 Computing the socially optimal allocation

Lagrangian approach

In order to compute the socially optimal solution and determine the efficient carbon tax, the optimal R&D subsidy, and the subsidy for machine use to correct for the

monopoly distortion, I adapt the infinite dimensional standard Lagrangian approach used in Golosov *et al.* (2014) and Hillebrand & Hillebrand (2019). A social planner maximizes consumer utility given the equilibrium conditions described in Definition 2. For brevity, I again use the vector notation introduced in (51). Let the Lagrange multipliers be denoted by
$\lambda_t := (\lambda_{0,t}, \lambda_{K,t}, \lambda_{S_1,t}, \lambda_{S_2,t}, (((\lambda^\ell_{Q_{i,j,t}}, \lambda^\ell_{h_{i,j,t}}, \lambda^\ell_{M_{i,j,t}})_{j\in[0,1]}, \lambda^\ell_{i,t}, \lambda^\ell_{s_{i,t}})_{i\in\mathbb{I}}, \lambda^\ell_{N,t})_{\ell\in\mathbb{L}})$ for each $t \geq 0$ and $\mu = (\mu_i)_{i\in\mathbb{I}}$ and the Lagrangian function

$$\mathcal{L}\Big((\overline{C}_t, K_{t+1}, \mathbf{K_t}, \mathbf{N_t}, \mathbf{E_t}, \mathbf{S_t}, \mathbf{X_t}, \mathbf{H_t}, \mathbf{s_t}, \mathbf{M_t}, \mathbf{Q_t})_{t\geq 0}, (\lambda_t)_{t\geq 0}, \mu\Big) := \sum_{t=0}^{\infty} \beta^t u(\overline{C}_t) \tag{A.6a}$$

$$+\sum_{t=0}^{\infty} \lambda_{0,t}\Bigg(\sum_{\ell\in\mathbb{L}}(1-D^\ell(S_t))Q^\ell_{0,t}F_0(K^\ell_{0,t}, N^\ell_{0,t}, \mathbf{E_t}^\ell) - C_t - K_{t+1} - \sum_{\ell\in\mathbb{L}} cX^\ell_t \tag{A.6b}$$

$$-\sum_{i=1}^{2}\sum_{\ell\in\mathbb{L}}\Big(\int_0^1 M^\ell_{i,j,t}dj\Big) - \sum_{\ell\in L} H^\ell_t\Bigg) \tag{A.6c}$$

$$+\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}} \lambda^\ell_{i,t}\Big(F_i(M^\ell_{i,j,t}, N^\ell_{i,t}, X^\ell_t, Q^\ell_{i,j,t}) - E^\ell_{i,t}\Big) \tag{A.6d}$$

$$+\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}} \lambda^\ell_{N,t}\Big(N^{\ell,s}_t - \sum_{i\in\mathbb{I}} N^\ell_{i,t}\Big) + \sum_{t=0}^{\infty}\lambda_{K,t}\Big(K_t - \sum_{\ell\in\mathbb{L}} K^\ell_t\Big) \tag{A.6e}$$

$$+\sum_{t=0}^{\infty}\lambda_{S_1,t}\Big(S_{1,t} - S_{1,t-1} - \phi_L\sum_{\ell\in\mathbb{L}}\zeta X^\ell_t\Big) \tag{A.6f}$$

$$+\sum_{t=0}^{\infty}\lambda_{S_2,t}\Big(S_{2,t} - (1-\phi)S_{2,t-1} - (1-\phi_L)\phi_0\sum_{\ell\in\mathbb{L}}\zeta X^\ell_t\Big) \tag{A.6g}$$

$$+\mu\Big(R_0 - \sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}} X^\ell_t\Big) \tag{A.6h}$$

$$+\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}}\int_0^1 \lambda^\ell_{Q_{i,j,t}}dj\Big((1+\iota\eta^\ell_{i,t}s^\ell_{i,t})Q^\ell_{i,t-1} - Q^\ell_{i,t}\Big) \tag{A.6i}$$

$$+\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}}\int_0^1 \lambda^\ell_{H_{i,j,t}}dj\Big(\eta_{i,t} - (h^\ell_{i,j,t})^\lambda \Xi^\ell_{i,j,t}\Big) \tag{A.6j}$$

$$+\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}\lambda^\ell_{s_t}\Big(s^\ell_{1,t} + s^\ell_{2,t} - 1\Big) \tag{A.6k}$$

$$+\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}}\int_0^1 \lambda^\ell_{M_{i,j,t}}dj\Big(M^\ell_{i,t} - \int_0^1 M^\ell_{i,j,t}dj\Big). \tag{A.6l}$$

Optimal carbon tax
As in Hillebrand & Hillebrand (2019), the Lagrange multiplier for consumption $\lambda_{0,t}$ can be interpreted as the shadow price of consumption in period t and all other prices can be measured in time t consumption. The shadow price of energy can hence be written

as

$$\hat{p}^{\ell}_{i,t} \coloneqq \frac{\lambda^{\ell}_{i,t}}{\lambda_{0,t}} = (1 - D_t)^{\ell} Q^{\ell}_{0,t} \partial_{E_i} F_0(K^{\ell}_t, N^{\ell}_{0,t}, \mathbf{E_t}). \tag{A.7}$$

Since the production technology for the final good and the climate system are identical to the specifications in Hillebrand & Hillebrand (2019) the Lagrange multipliers $\lambda_{0,t}, \lambda_{S_1,t}, \lambda_{S_2,t}$ and $\lambda^{\ell}_{i,t}$ are identical too. Therefore, the efficient carbon tax that internalizes the climate externality can be adopted without modifications as

$$\Lambda_t \coloneqq \phi_L \frac{\lambda_{S_1,t}}{\lambda_{0,t}} + (1 - \phi_L)\phi_0 \frac{\lambda_{S_2,t}}{\lambda_{0,t}}. \tag{A.8}$$

Monopoly distortions
To formally show the knowledge externality and how monopoly prices are distorted, I follow Hillebrand (2016). The first order condition of the Lagrangian with respect to sectoral machine investments is given by

$$\frac{\partial \mathcal{L}}{\partial M^{\ell}_{i,t}} = -\lambda_{0,t} + \lambda^{\ell}_{M_{i,j,t}} = 0 \Leftrightarrow \frac{\lambda^{\ell}_{M_{i,j,t}}}{\lambda_{0,t}} = 1 \quad \forall t,\ \forall j \in [0,1],\ i \in \mathbb{I}. \tag{A.9}$$

Thus, the socially optimal allocation implies that the shadow price of sector specific machines is equal to unity, and hence identical to marginal production costs ψ, in both energy sectors. Recall, that in the laissez-faire equilibrium machine producers set prices equal to ψ/α_i which is a mark-up over marginal costs. Thus, the socially optimal allocation can be interpreted as involving a subsidy for the use of sector-specific machines while machine producers still obtain monopoly profits if successful in innovation. Using (A.9) in the first order condition of the Lagrangian with respect to $M^{\ell}_{i,j,t}$ yields the social optimum for machine production

$$M^{\ell *}_{1,j,t} = (\alpha_1 \hat{p}^{\ell}_1)^{\frac{1}{1-\alpha_1}} (X^{\ell}_t)^{\frac{\nu_1}{1-\alpha_1}} (N^{\ell}_{1,t})^{\frac{1-\alpha_1-\nu_1}{1-\alpha_1}} Q^{\ell}_{1,j,t} \tag{A.10}$$

for sector $i = 1$ and for sector $i = 2$:

$$M^{\ell *}_{2,j,t} = (\alpha_2 \hat{p}^{\ell}_2)^{\frac{1}{1-\alpha_2}} N^{\ell}_{2,t} Q^{\ell}_{2,j,t}. \tag{A.11}$$

Compared to the laissez-faire equilibrium, machines are used more intensively leading to a higher production of energy goods, given the quality index, labor allocation and prices.

Knowledge externality
Since scientists do not internalize the effects their research has on the average machine quality level in the future, the socially optimal solution also has to correct for a knowledge externality. The shadow value of a unit increase in machine quality in sector $i \in \mathbb{I}$ can be found by taking the first order condition of the Lagrangian with respect to $Q^{\ell}_{i,j,t}$ in relation to the shadow price of consumption $\lambda_{0,t}$ and using the shadow price for energy production $\hat{p}^{\ell}_{i,t}$:

$$\hat{\lambda}^{\ell}_{Q_{i,t}} = \hat{p}^{\ell}_{i,t} \partial_{Q^{\ell}_{i,j,t}} F_i(M^{\ell}_{i,j,t}, Q^{\ell}_{i,j,t}, N^{\ell}_{i,t}, X^{\ell}_t) + (1 + \iota \eta^{\ell}_{i,t} s^{\ell}_{i,t}) \hat{\lambda}^{\ell}_{Q_{i,t+1}} \tag{A.12}$$

where $\hat{\lambda}^{\ell}_{Q_{i,t+1}} := \lambda^{\ell}_{Q_{i,j,t}}/\lambda_{0,t}$. The shadow value of a unit increase in machine quality is thus equal to the sum of the marginal contribution to utility in period t and its own shadow value in the following period $t+1$ times $(1+\iota\eta^{\ell}_{i,t}s^{\ell}_{i,t})$. This last term captures the knowledge externality (Acemoglu *et al.*, 2012). In contrast to the laissez-faire equilibrium, the social planner allocates scientists to the sector that yields larger social gains from innovation. The social value of conducting research is measured by the term $\iota\eta^{\ell}_{i,t}\hat{\lambda}^{\ell}_{Q_{i,t}}Q^{\ell}_{i,t}$ for sector $i \in \mathbb{I}$. Using the law of motion for the quality index given by (31), the social value of research can be written as

$$\iota\eta^{\ell}_{i,t}\hat{\lambda}^{\ell}_{Q_{i,t}}Q^{\ell}_{i,t} = \iota\eta^{\ell}_{i,t}\hat{\lambda}^{\ell}_{Q_{i,t}}\frac{Q^{\ell}_{i,t+1}}{1+\iota\eta^{\ell}_{i,t}s^{\ell}_{i,t}}. \tag{A.13}$$

Inserting (A.12) and taking the ratio of variables relating to sector $i=2$ and $i=1$ yields

$$\frac{\eta^{\ell}_{2,t}\iota(1+\iota\eta^{\ell}_{2,t}s^{\ell}_{2,t})^{-1}(1-\alpha_2)\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}\sum_{n\geq t}(\hat{p}^{\ell}_{2,n})^{\frac{1}{1-\alpha_2}}N^{\ell}_{2,n}Q^{\ell}_{2,n}}{\eta^{\ell}_{1,t}\iota(1+\iota\eta^{\ell}_{1,t}s^{\ell}_{1,t})^{-1}(1-\alpha_1)\alpha_1^{\frac{\alpha_1}{1-\alpha_1}}\sum_{n\geq t}(\hat{p}^{\ell}_{1,n})^{\frac{1}{1-\alpha_1}}(N^{\ell}_{1,n})^{\frac{1-\alpha_1-\nu_1}{1-\alpha_1}}(X^{\ell}_t)^{\frac{\nu_1}{1-\alpha_1}}Q^{\ell}_{1,n}}. \tag{A.14}$$

The social planner will allocate scientists to the clean sector whenever this ratio is greater than unity because then the social gain from green innovation is higher. The social planner implements the optimal allocation via a green research subsidy.

A.4 Proof of Lemma 2

After introducing a subsidy for machine users to remove monopoly distortions, machine demand in sectors $i \in 1,2$ is given by (A.10) and (A.11). In contrast to the laissez-faire equilibrium, there is no correction term for the quality level, as determined in (57) and (59), anymore. Intuitively, due to the monopoly subsidy, machine users pay the same, independent of whether machines are produced monopolistically or competitively and thus the demand function does not change because of the different market structures. Hence, energy production under the socially optimal allocation is given by

$$E^{\ell}_{1,t} = \alpha_1^{\frac{\alpha_1}{1-\alpha_1-\nu_1}}\left(\frac{\nu_1 Q^{\ell}_{1,t}}{\upsilon_t+\tau^{\ell}_t\zeta}\right)^{\frac{\nu_1}{1-\alpha_1-\nu_1}}(\hat{p}^{\ell}_{1,t})^{\frac{\alpha_1+\nu_1}{1-\alpha_1-\nu_1}}N^{\ell}_{1,t}Q^{\ell}_{1,t}. \tag{A.15}$$

for the dirty sector and for the clean sector by

$$E^{\ell}_{2,t} = (\alpha_2\hat{p}^{\ell}_{2,t})^{\frac{\alpha_2}{1-\alpha_2}}Q^{\ell}_{2,t}N^{\ell}_{2,t} \tag{A.16}$$

Following the structure of the Proof of Lemma 1 and using (A.15) and (A.16) as energy demand function this time, we obtain relative energy prices $\hat{p}^{\ell}_{2,t}/\hat{p}^{\ell}_{1,t}$ and the labor ratio for the socially optimal allocation.

The scientists‘ profit maximization problem from (18) essentially remains the same, except for the fact that the machine demand functions become (A.10) and (A.11) as mentioned before. Scientists still only obtain profits on machine lines where they were successful and therefore have received monopoly rights. Using (A.10), (A.11), and optimal R&D expenditures as given by (27) in the profit maximization problem, gives expected profits for sectors $i \in \{1, 2\}$ under the social optimum

$$\Pi^{\ell}_{2,t} = \frac{1}{1+\lambda}\eta^{\ell}_{2,t}(\alpha_2^{-1} - 1)(\alpha_2 \hat{p}^{\ell}_{2,t})^{\frac{1}{1-\alpha_2}} N^{\ell}_{2,t} Q^{\ell}_{2,t} \tag{A.17a}$$

$$\Pi^{\ell}_{1,t} = \frac{1}{1+\lambda}\eta^{\ell}_{1,t}(\alpha_1^{-1} - 1)\alpha_1^{\frac{(1-\alpha_1-\nu_1)+\alpha_1\nu_1}{(1-\alpha_1-\nu_1)(1-\alpha_1)}} (\hat{p}^{\ell}_{1,t})^{\frac{1}{1-\alpha_1-\nu_1}} N^{\ell}_{1,t} \left(\frac{\nu_1 Q^{\ell}_{1,t}}{\upsilon_t + \tau^{\ell}_t \zeta}\right)^{\frac{\nu_1}{1-\alpha_1-\nu_1}} Q^{\ell}_{1,t} \tag{A.17b}$$

Next, take the profit ratio $\Pi^{\ell}_{2,t}/\Pi^{\ell}_{1,t}$ and plug in the price and labor ratio, which are essentially the same as for the laissez-faire equilibrium except for that there is no need to correct the quality level due to the monopoly subsidy. The profit ratio under the optimal allocation can then be written as

$$\frac{\Pi^{\ell}_{2,t}}{\Pi^{\ell}_{1,t}} = \frac{\eta^{\ell}_{2,t}}{\eta^{\ell}_{1,t}} \Gamma_{\text{opt}} \left(\frac{\nu_1}{\upsilon_t + \tau^{\ell}_t \zeta}\right)^{\frac{\nu_1 \varphi}{1-\alpha_2}} \frac{(1 + \iota \eta^{\ell}_{1,t}(1 - s^{\ell}_{2,t}))^{\varphi_1 + 1}}{(1 + \iota \eta^{\ell}_{2,t} s^{\ell}_{2,t})^{\varphi+1}} \frac{(Q^{\ell}_{2,t-1})^{-\varphi}}{(Q^{\ell}_{1,t-1})^{-\varphi_1}} \tag{A.18}$$

where

$$\Gamma_{\text{opt}} = \frac{(\alpha_2^{-1} - 1)\alpha_2^{\frac{1-\alpha_2-\alpha_2\varphi}{1-\alpha_2}} (1-\alpha_2)^{-\varphi}}{(\alpha_1^{-1} - 1)\alpha_1^{\frac{(1-\alpha_1-\nu_1)+\alpha_1\nu_1-\alpha_1(1-\alpha_1)-\alpha_1\varphi(1-\alpha_1}{(1-\alpha_1-\nu_1)(1-\alpha_1)}}}. \blacksquare \tag{A.19}$$

A.5 Proof of Proposition 2

After the introduction of the R&D subsidy, the equilibrium ratio of expected profits in the clean and dirty sector takes the form

$$\frac{\Pi^{\ell}_{2,t}}{\Pi^{\ell}_{1,t}} = (1 + \vartheta^{\ell}_t)\frac{\eta^{\ell}_{2,t}}{\eta^{\ell}_{1,t}} \Gamma_{\text{opt}} \left(\frac{\nu_1}{\upsilon_t + \tau^{\ell}_t \zeta}\right)^{\frac{\nu_1 \varphi}{1-\alpha_2}} \frac{(1 + \iota \eta^{\ell}_{1,t}(1 - s^{\ell}_{2,t}))^{\varphi_1 + 1}}{(1 + \iota \eta^{\ell}_{2,t} s^{\ell}_{2,t})^{\varphi+1}} \frac{(Q^{\ell}_{2,t-1})^{-\varphi}}{(Q^{\ell}_{1,t-1})^{-\varphi_1}}. \tag{A.20}$$

If the optimal solution suggests that $s^{\ell}_{2,t} = 1$, the subsidy allocates all scientists to the clean sector. This is the case when $\Pi^{\ell}_{2,t}(1 + \vartheta^{\ell}_t) > \Pi^{\ell}_{1,t}$. Rearranging yields

$$\vartheta^{\ell}_t \geq \hat{\vartheta}^{\ell}_t \equiv \frac{\eta^{\ell}_{1,t}}{\eta^{\ell}_{2,t}} \Gamma^{-1}_{\text{opt}} \left(\frac{\nu_1}{\upsilon_t + \tau^{\ell}_t \zeta}\right)^{-\frac{\nu_1 \varphi}{1-\alpha_2}} \frac{(1 + \iota \eta^{\ell}_{1,t} s^{\ell}_{1,t})^{-(\varphi_1+1)}}{(1 + \iota \eta^{\ell}_{2,t} s^{\ell}_{2,t})^{-(\varphi+1)}} \frac{(Q^{\ell}_{2,t-1})^{\varphi}}{(Q^{\ell}_{1,t-1})^{\varphi_1}} - 1 \tag{A.21}$$

which constitutes the lower bound for the subsidy. The more scientist $s^{\ell}_{2,t}$ are engaged in the clean sector, the lower the necessary subsidy if $1 + \varphi < 0$ and $1 + \varphi_1 < 0$. Setting $s^{\ell}_{2,t} = 0$, thus, yields the necessary subsidy for this case. If $1 + \varphi > 0$ and $1 + \varphi_1 > 0$, then analogously using the corner solution $s^{\ell}_{1,t} = 0$ gives the subsidy presented in Proposition 2. This proves the proposition.$\blacksquare$

B Computational Details

B.1 Recursive equilibrium structure

The algorithm for the numerical simulation in this study closely follows the computational algorithm developed by Hillebrand & Hillebrand (2019, 2023) with slight modifications to accommodate the extended model structure. The algorithm aims to solve the equilibrium of the economy ξ_t as defined in Definition 2 by recursively iterating the model forward given ξ_{t-1} and exogenous variables. To elucidate this concept, deviate from the idea that the equilibrium vector consists of an allocation **A** and a Price vector **P** and segment the equilibrium vector as $\xi_t = (\xi_t^1, \xi_t^2)$ where

$$\xi_t^1 := (\mathbf{Y_t}, \mathbf{E_t}, \mathbf{K_t}, \mathbf{X_t}, \mathbf{M_t}, \mathbf{N_t}, \mathbf{Q_t}, \mathbf{s_t}, \tau_t, r_t, \mathbf{w_t}, \mathbf{p_t}, \upsilon_t) \in \Xi^1 \tag{B.1a}$$

$$\xi_t^2 := (\overline{C}_t, \overline{K}_{t+1}, \mathbf{S_t}, \mathbf{H_t}) \in \Xi^2 \tag{B.1b}$$

where

$$\Xi^1 := \mathbb{R}_{++}^L \times \mathbb{R}_{++}^{2L} \times \mathbb{R}_{++}^L \times \mathbb{R}_{++}^L \times \mathbb{R}_{++}^{2L} \times \mathbb{R}_{++}^{3L} \times \mathbb{R}_{++}^{2L} \times [0,1]^{2L} \times \mathbb{R}_{+} \times \mathbb{R}_{++} \times \mathbb{R}_{++}^L \times \mathbb{R}_{++}^{2L} \times [c, \infty[\tag{B.2a}$$

$$\Xi^2 := \mathbb{R}_{++} \times \mathbb{R}_{++} \times \mathbb{R}_{++}^2 \times \mathbb{R}_{+}^{2L}. \tag{B.2b}$$

In addition, the pre-determined variables for periods $t \geq 0$ and exogenous variables from ξ_t^2 are collected in the vector

$$\theta_t := (\mathbf{N_t^s}, \upsilon_{t-1}, \mathbf{S_{t-1}}, \overline{C}_{t-1}, \overline{K}_t, \mathbf{Q_{t-1}}) \in \Theta := \mathbb{R}_{++}^L \times [c, \infty[\times \mathbb{R}_{++}^2 \times \mathbb{R}_{++} \times \mathbb{R}_{++} \times \mathbb{R}_{++}^{2L}. \tag{B.3}$$

Given the pre-determined and exogenous variables in $\theta_t \in \Theta$, the main step is to simultaneously solve the $M := 17L + 2$ dimensional vector ξ_t^1 by simultaneously solving the $L + L + L$ production equations (1), (5), and (10), the $L + 1 + L$ market clearing conditions (44), (48), and (45), the $3L + 2L + L + L + L$ optimality conditions (4), (8), (12), (9), and (13), the $2L + 2L$ equations for the success probability of research and the law of motion for the quality index (29) and (31), and lastly the $1 + 1$ equations describing the Hotelling rule (15) and the optimal tax formula (76). These conditions constitute a system of $3L + 2L + 1 + 8L + 4L + 2 = M$ non-linear equations that can potentially be solved uniquely and determine the M-dimensional vector ξ_t^1. These M conditions can be solved for given $\theta \in \Theta$ and for the function $\Phi : \Xi^1 \times \Theta \to \mathbb{R}^M$ if and only if $\Phi(\xi_t^1, \theta_t) = 0$. This problem computes the equilibrium production allocation. The algorithm to solve this problem is also adopted and modified from Hillebrand & Hillebrand (2019) and presented in section B.3.

If a solution ξ_t^1 has been found, given the pre-determined variables θ_t, the components of ξ_t^2 can be computed using equations (42), (50), (36), (27) and (32). This defines a mapping $\Psi : \Xi^1 \times \Theta \to \Xi^2$ such that $\xi_t^2 = \Psi(\xi_t^1, \theta_t)$.

B.2 Numerical algorithm

This section briefly describes the numerical algorithm from Hillebrand & Hillebrand (2019) which guesses initial values for consumption and resource prices and adjusts them until a stable solution has been found.

Step 1: Initialization for $t = 0$:

(a) Choose candidate values for consumption $\overline{C}_{t-1} > 0$ and resource price $\upsilon_{t-1} \in [c, \infty[$. If $R_0 = \infty$, set $\upsilon_{t-1} = c$, otherwise $\upsilon_{t-1} > c$.

(b) Use the values together with the given parameters $\mathbf{S_{-1}} = (S_{1,-1}, S_{2,-1})$ and $\overline{K}_0 > 0$ to determine the endogenous part of θ_0. Set $t = 0$.

Step 2: Iteration for $0 \leq t \leq t^{\max}$:

(a) Set up θ_t as in (B.3) using N_t^s and relevant endogenous variables from $t-1$.

(b) Compute ξ_t^1 by solving the problem $\Phi(\xi_t^1, \theta_t) = 0$ as outlined above.

(c) Compute $\xi_t^2 = \Psi(\xi_t^1, \theta_t)$ as outlined above and check the following conditions:

   i. If $\overline{K}_{t+1} < 0$, return to **Step 1** and decrease $\overline{C}_{-1}$.

   ii. If $\overline{C}_t < \overline{C}^{\text{crit}}$, return to step one and increase $\overline{C}_{-1}$.

   iii. Otherwise, increase t by 1.

Step 3: Verification of the resource constraint in $t = t^{\max}$:

(a) If $R_0 < \infty$, compute $R_{t^{\max}+1} := R_0 - \sum_{t=0}^{t^{\max}} \sum_{\ell \in \mathbb{L}} X_t^{\ell}$:

   i. If $R_{t^{\max}+1} < 0$, return to **Step 1** and increase υ_{-1}.

   ii. If $R_{t^{\max}+1} > R^{\text{crit}}$, return to **Step 1** and decrease υ_{-1}.

(b) If $0 < R_{t^{\max}+1} < R^{\text{crit}}$ complete the iteration.

Step 2(c) in this algorithm requires the specification of a (typically time-dependent) lower bound $\overline{C}_t^{\text{crit}}$ for consumption in period t. The condition $\overline{C}_t > \overline{C}_t^{\text{crit}}$ for all t serves to avoid consumption to implode and converge to zero. This is the case when initial consumption $\overline{C}_{-1}$ is chosen too small. Conversely, if $\overline{C}_{-1}$ is chosen too large, the exploding consumption path is too large compared to output. In this case, the condition $\overline{K}_{t+1} > 0$ for all t will eventually be violated. Excluding both cases determines a unique initial value $\overline{C}_{-1}$ for which the equilibrium dynamics are well-defined and satisfy the transversality condition (43).

B.3 Computing the equilibrium production allocation

This section develops an argument how the equilibrium production allocation ξ_t^1 can be computed, which then solves $\Phi(\xi_t^1, \theta_t) = 0$ in Step 2(b). Again, this argument is based on Hillebrand & Hillebrand (2023) and modified for the extended model structure in this paper. The production allocation problem can be solved as a fixed point problem given vector θ_t determining labor supply $N_t^{\ell,s}$, the previous resource price v_{t-1}, the previous climate state $\mathbf{S_{t-1}}$, and aggregate capital $\overline{K}_t$. Recall from Section 5 that I define the nominal energy mix as $\chi_{i,t}^\ell := p_{i,t}^\ell E_{i,t}^\ell / \sum_{j\in\mathbb{I}} p_{j,t}^\ell E_{j,t}^\ell$ for each $i \in \mathbb{I}$. This notation will be used to present the optimality conditions in a more accessible way. The argument is structured into three steps:

I. Fix arbitrary values for outputs $(\tilde{Y}_t^\ell)_{\ell\in\mathbb{L}}$, energy inputs $(\tilde{E}_{i,t}^\ell)_{(i,\ell)\in\mathbb{I}\times\mathbb{L}}$, and the energy aggregate $(\tilde{E}_t^\ell)_{\ell\in\mathbb{L}}$. In contrast to Hillebrand & Hillebrand (2019), I include energy inputs $(\tilde{E}_{i,t}^\ell)_{(i,\ell)\in\mathbb{I}\times\mathbb{L}}$, and the energy aggregate $(\tilde{E}_t^\ell)_{\ell\in\mathbb{L}}$ as fix point variables instead of the nominal energy mix since the computation of the equilibrium demand for machines requires sector-specific energy inputs. The factor allocation

$$A_t^f := ((N_{i,t}^\ell)_{i\in\mathbb{I}_0}, K_t^\ell, X_t^\ell)_{\ell\in\mathbb{L}} \tag{B.4}$$

consistent with the equilibrium conditions in (4), (8), and (12) can be determined as follows:

(a) For each $\ell \in \mathbb{L}$, determine the regional labor allocation $(N_{i,t}^\ell)_{i\in\mathbb{I}_0}$ by solving

$$\frac{1-\alpha_0-\nu_0}{N_{0,t}^\ell} = \frac{\nu_1(1-\alpha_1-\nu_1)}{N_{1,t}^\ell}\tilde{\chi}_{1,t}^\ell = \frac{\nu_2(1-\alpha_2-\nu_2)}{N_{2,t}^\ell}\tilde{\chi}_{2,t}^\ell \tag{B.5a}$$

$$N_t^{\ell,s} = \sum_{i=0}^{2} N_{i,t}^\ell \tag{B.5b}$$

which leads to the following solution:

$$N_{i,t}^\ell = \frac{n_{i,t}^\ell}{\sum_{j=0}^2 n_{j,t}^\ell} N_t^{\ell,s}, \quad i \in \mathbb{I}_0 \tag{B.6}$$

where

$$n_{0,t}^\ell = 1-\alpha_0-\nu_0 \tag{B.7a}$$
$$n_{1,t}^\ell = \nu_0(1-\alpha_1-\nu_1)\tilde{\chi}_{1,t}^\ell \tag{B.7b}$$
$$n_{2,t}^\ell = \nu_0(1-\alpha_2)\tilde{\chi}_{2,t}^\ell. \tag{B.7c}$$

(b) Since capital is only used as an input for final goods production, the international capital allocation for region $\ell \in \mathbb{L}$ can be determined as

$$K_t^\ell = \frac{\tilde{Y}_t^\ell}{\sum_{\ell\in\mathbb{L}} \tilde{Y}_t^\ell} \overline{K}_t. \tag{B.8}$$

(c) The resource allocation X_t^ℓ for each region $\ell \in \mathbb{L}$ is obtained by

$$X_t^\ell = \frac{\nu_0 \nu_1 \tilde{Y}_t^\ell \chi_{1,t}^\ell}{c + \frac{\alpha_0 \tilde{Y}_t^\ell}{K_t^\ell}(\upsilon_{t-1} - c) + \zeta \tau_t \sum_{h\in\mathbb{L}} \gamma^h Y_t^h}. \tag{B.9}$$

The determination of the factor allocation A_t^f from initial guesses for output, energy inputs, and the energy aggregate define a first mapping

$$G : ((\tilde{Y}_t^\ell), (\tilde{E}_{i,t}^\ell)_{i\in\mathbb{I}}, (\tilde{E}_t^\ell))_{\ell\in\mathbb{L}} \to A_t^f = ((N_{i,t}^\ell)_{i\in\mathbb{I}_0}, K_t^\ell, X_t^\ell)_{\ell\in\mathbb{L}}. \tag{B.10}$$

II. Using the initial guesses for output, energy inputs and the energy aggregate from step I, the energy price can be determined by the optimality condition in (4). The factor allocation A_t^f from (B.10) and energy prices $p_{i,t}^\ell$ can now be used to recursively determine the following variables:

(a) Emissions Z_t and climate variables $S_{1,t}, S_{2,t}$, and S_t using (6) and (36)

(b) Interest rate r_t and the sectoral success probabilities of research $\eta_{i,t}^\ell$ by using (4) and (29)

(c) The optimal R&D subsidy rate ϑ_t^ℓ and thereafter the optimal scientist allocation $s_{i,t}^\ell$ using Proposition 1 and Proposition 2

(d) Quality index $Q_{i,t}^\ell$, given the optimal scientist allocation, following the law of motion in (31)

(e) Aggregated sectoral machine demand $M_{i,t}^\ell$ using (13) and (9)

(f) Final output Y_t^ℓ, energy inputs $E_{i,t}^\ell$, and the energy aggregate E_t^ℓ using (1), (5), (10), and (2).

This second step defines a second mapping

$$H : A_t^f = ((N_{i,t}^\ell)_{i\in\mathbb{I}_0}, K_t^\ell, X_t^\ell)_{\ell\in\mathbb{L}} \to ((Y_t^\ell), (E_{i,t}^\ell)_{i\in\mathbb{I}}, (E_t^\ell))_{\ell\in\mathbb{L}} \tag{B.11}$$

updating output, energy inputs and the energy aggregate from the factor allocation and pre-determined variables.

III. The composition of mappings (B.10) and (B.11)

$$F := H \circ G : ((\tilde{Y}_t^\ell), (\tilde{E}_{i,t}^\ell)_{i\in\mathbb{I}}, (\tilde{E}_t^\ell))_{\ell\in\mathbb{L}} \to ((Y_t^\ell), (E_{i,t}^\ell)_{i\in\mathbb{I}}, (E_t^\ell))_{\ell\in\mathbb{L}} \tag{B.12}$$

maps the initial guesses $((\tilde{Y}_t^\ell), (\tilde{E}_{i,t}^\ell)_{i\in\mathbb{I}}, (\tilde{E}_t^\ell))_{\ell\in\mathbb{L}}$ to a new value $((Y_t^\ell), (E_{i,t}^\ell)_{i\in\mathbb{I}}, (E_t^\ell))_{\ell\in\mathbb{L}}$. The new variables are then used as new guesses in the first step. For period $t \geq 0$

iterate through Steps I-III until the initial guesses in Step I converge to the updated values $((Y_t^\ell), (E_{i,t}^\ell)_{i\in\mathbb{I}}, (E_t^\ell))_{\ell\in\mathbb{L}}$ in Step III. It follows that if convergence is achieved, the equilibrium solution $((Y_t^{\ell*}), (E_{i,t}^{\ell*})_{i\in\mathbb{I}}, (E_t^{\ell*}))_{\ell\in\mathbb{L}}$ is a fix point of F.

Once the fix point $((Y_t^{\ell*}), (E_{i,t}^{\ell*})_{i\in\mathbb{I}}, (E_t^{\ell*}))_{\ell\in\mathbb{L}}$ is found, the equilibrium factor allocation follows from $A_t^{f*} = G((Y_t^{\ell*}, (E_{i,t}^{\ell*})_{i\in\mathbb{I}})_{\ell\in\mathbb{L}})$. The remaining variable w_t can be computed using A_t^{f*} and (4). This completes the vector ξ_t^1.

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Declaration of Academic Integrity

I hereby confirm that this thesis, entitled *Endogenous Technological Change in a General Equilibrium Climate-Economy Model* is solely my own work and that I have used no sources or aids other than the ones stated. All passages in my thesis for which other sources, including electronic media, have been used, be it direct quotes or content references, have been acknowledged as such and the sources cited. I am aware that plagiarism is considered an act of deception which can result in sanction in accordance with the examination regulations.

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