Supplemental Appendix to Approaches to Statistical Efficiency when comparing the embedded adaptive interventions in a SMART

Timothy Lycurgus, Daniel Almirall & Amy Kilbourne

Appendices

A Common Primary Aims

Consider the following marginal structural mean model, i.e. , $\mu^{(a_1,a_{2NR})}(\boldsymbol{\gamma})$ for a prototypical SMART as follows:

$$\mu^{(a_1,a_{2NR})}(\boldsymbol{\gamma}) = \gamma_0 + \gamma_1 a_1 + \gamma_2 a_{2NR} + \gamma_3 a_1 a_{2NR},$$

where $a_1, a_{2NR} \in \{-1, 1\}$. This corresponds to the marginal structural mean model found in Equation 1 in the manuscript.

Table 1 shows each of the most common primary aims in a prototypical SMART. The first is the main effect for the first-stage intervention. The second common primary aim is the main effect of the second-stage intervention among the non-responders. The final six are comparisons of embedded adaptive interventions. For each of these primary aims, Table 1 provides the causal effect along with a regression formulation for that causal effect using the marginal structural mean model found above.

Primary Aim	**Causal Effect**	**Regression Formulation**
Main effect of 1st Stage	$\mathbb{E}[Y(1, A_{2NR}) - Y(-1, A_{2NR})]$	$2\gamma_1$
Main effect of 2nd Stage among NR	$\mathbb{E}[Y(A_1, 1) - Y(A_1, -1) \mid R(A_1) = 0)]$	$2\gamma_2 \times (1 - R)^{-1}$
AI Comparison: $(1, 1)$ vs. $(1, -1)$	$\mathbb{E}[Y(1, 1) - Y(1, -1)]$	$2(\gamma_2 + \gamma_3)$
AI Comparison: $(1, 1)$ vs. $(-1, 1)$	$\mathbb{E}[Y(1, 1) - Y(-1, 1)]$	$2(\gamma_1 + \gamma_3)$
AI Comparison: $(1, 1)$ vs. $(-1, -1)$	$\mathbb{E}[Y(1, 1) - Y(-1, -1)]$	$2(\gamma_1 + \gamma_2)$
AI Comparison: $(1, -1)$ vs. $(-1, 1)$	$\mathbb{E}[Y(1, -1) - Y(-1, 1)]$	$2(\gamma_1 - \gamma_2)$
AI Comparison: $(1, -1)$ vs. $(-1, -1)$	$\mathbb{E}[Y(1, -1) - Y(-1, -1)]$	$2(\gamma_1 - \gamma_3)$
AI Comparison: $(-1, 1)$ vs. $(-1, -1)$	$\mathbb{E}[Y(-1, 1) - Y(-1, -1)]$	$2(\gamma_2 - \gamma_3)$

Table 1: Each of the primary aims for a prototypical SMART along with their causal effects and the corresponding regression-based formulation for these causal effects using the marginal structural mean model found above.

B Variance-Covariance Matrix for ASIC Simulations

This is the variance-covariance matrix for our baseline covariates. The covariates are:

- Number of students >500.
- Percentage of students on free/reduced price lunch.
- Rural versus urban school.
- Total school professional level of education.
- Total school professional tenure.
- Delivered pre-randomization or none.

All six variables are grand-mean centered.

$$\begin{pmatrix} 3.78 & 0.05 & 0.06 & -0.18 & -0.02 & -2.33 & 0.36 \\ 0.05 & 0.23 & -0.04 & -0.10 & 0.03 & 0.50 & 0.04 \\ -0.06 & -0.04 & 0.23 & 0.00 & -0.03 & 0.11 & -0.02 \\ -0.18 & -0.10 & 0.00 & 0.25 & -0.03 & -0.36 & -0.04 \\ -0.02 & 0.03 & -0.02 & -0.03 & 0.07 & -0.19 & 0.01 \\ -1.33 & 0.50 & 0.11 & -0.36 & -0.19 & 35.51 & 0.07 \\ 0.36 & 0.04 & -0.03 & -0.04 & 0.01 & 0.07 & 0.25 \end{pmatrix}$$