```
--- Recursively divide data based on two

distant points (found in linear time using the Fastmap
heuristic [Fa95]). Then find and print the attribute range
that best distinguishes these halves. Recurse on each half.

(which is sort of like PDDP [Bo98] but faster; and we
offers a human-readable description for each division).

To find those ranges, this code uses a variant of the ChiMerge
discretizer (but we select on entropy and size,
not the Chi statistic)

To avoid spurious outliers, this code separates using '-furthest=.9';
i.e. the 90% furthest points.

To avoid long runtimes, this code only searches at most '-keep=512'
randomly selected examples to find those furtherst points.

To suport multi-objective optimization, this code reads cav files
whose headers may contain markers for "minimize this" or "maximize
that" (see the 'lessp, morep' functions).

To support explanation, optionally, at each level of recursion,
this code reports what ranges can best distinguish sibling clusters
Cl,C2. The discretizer is inspired by the ChiMerge algorithm:
numerics are divided into, say, 16 bins. Then, while we can find
adjacent bins with the similar distributions in Cl,C2, then

(a) merge then (b) look for other merges.

local help = [[
   15 == a little LUA learning library (c) 2022, Tim Menzies, BSD 2-clause license.
USAGE:
lua 15.lua [OPTIONS]
 OPTIONS:
                                                    F Cohen's delta = .35
N data file = ../etc/data/auto93.csv
stack dump on assert fails false
F far = .9
S format string = $5.2f
P max kept items = 512
P distance coefficient = 2
P set seed = 10019
S start up action (or 'all') = nothing
show help = false
F recurse until rows^want = .5
          -cohen -c
-data -d
-Dump -D
-furthest -f
-Format -F
-keep -k
                                         -p
-s
-t
-h
-w
          -p
-seed
-todo
-help
-want
 KEY: N=fileName F=float P=posint S=string
   -- ## Definitions
  -- Cache current names (used at end to find rogue variables) local b4={}; for k,_ in pairs(\_ENV) do b4[k]=k end
   -- Derine locals.
local any asserts, big, cli, distance2Heaven
local fails, firsts, fmt, goalp, ignorep, klassp
local lessp, map, main, many, max, merge, min, morep, new, nump, o, oo, per, pop, push
local r, rows, rnd, rnds, slots, sort, sum, thing, things, file2things, unpack
  -- Define parameter settings.
-- Update parameter defaults from command line. Allow for some shorthand:
-- e.g. _-k N_ ⇒ 'keep=N';
-- and _-booleanFlag_ ⇒ 'booleanFlag=not default').
  local the={} help:gsub("\n([-]([^%s]+))[%s]+(-[^%s]+)[^\n]*%s([^%s]+)",
       -- ### Define headers for row1 of csv files
           Columns to ignore
 -- Columns to ignore function ignorep(x) return x:find":$" end
-- Symbolic class columns.
function klassp(x) return not nump(x) and x:find"!$" end
-- Goal columns to minimize
function lessp(x) return nump(x) and x:find"-$" end
 - Goal columns to maximize function more(x) - Numeric columns to maximize return nump(x) and x:find"-$" end return nump(x) and x:find"+$" end return nump(x) and x:find"-$" end return x:find"^[A-Z]" end return x:find"^[A-Z]" end
                                                    clumns
(x) return morep(x) or lessp(x) or klassp(x) end
```

function goalp(x)

```
-- ## Misc Utils
105
107 -- Strings
108 fmt = string.format
110
     big = math.huge
max = math.max
min = math.min
r = math.random
111
112
113
114
115
116
117
     function rnds(t,f) return map(t, function(x) return rnd(x,f) end) end function rnd(x,f) return fmt(type(x)=="number" and (x~=x//1 and f or the.Format) or "%s",x) end
     pop = table.remove
unpack = table.unpack
function any(t)
function firsts(a,b)
     -- Meta function map(t,f, u) u={}; for k,v in pairs(t) do push(u,f(v)) end; return u end function sum(t,f, n) n=0; for _,v in pairs(t) do n=n+f(v) end; return n end function slots(t, u)
          d='[for k,v in pairs(t) do k=tostring(k);if k:sub(1,1)~="_" then push(u,k) end end
return sort(u) end
     -- Print tables, recursively function oo(t) print(o(t)) end function o(t) if type(t)-="table" then return tostring(t) end local key=function(k) return fmt(":%% %s",k,o(t[k])) end local u = #t>0 and map(t,o) or map(slots(t),key) return '{'..table.concat(u,"").."}" end
     -- Coerce strings to things function thing(x) x = x:match"^%s*(.-)%s*$" if x=="false" then return false end return tonumber(x) or x end
      function things(x,sep, t) t=\{\}; \ for y \ in \ x: gmatch (sep or"([\land]+)") \ do \ push(t,thing(y)) \ end return t \ end
      function file2things(file, x)
file = io.input(file)
return function()
x=io.read(); if x then return things(x) else io.close(file) end end end
     -- Multi-objectives. Normalized, scored via distance to heaven.
function distance2Heaven(t,heaven, num,d)
for n,txt in pairs(heaven) do
num = Num(at,txt)
for _, z in pairs(t) do num:add(z,ys[n]) end
for _,z in pairs(t) do z,ys[n] = num:distance2heaven(z,ys[n]) end d = function(one) return (sum(one,ys)/#one,ys)^5 end
return sort(t, function(a,b) return d(a) < d(b) end) end
     -- While we can find similar adjacent ranges, then merge them. function merge (b4, j,n,now,a,b,merged) j,n,now = 0,\#b4,{} while j < \#b4 do
              iile j < #b4 do
j = j+1
a, b = b4[j], b4[j+1]
if b then
merged = a:merge(b)
if merged then a, j = merged, j+1 end end</pre>
          push (now, a)

j = j+1 end

return #now == #b4 and b4 or merge (now) end
          - Objects
     function new(k,t) k.__index=k; k.__tostring=o; return setmetatable(t,k) end
```

```
-- ## COLS
-- ## COLS
-- Factory. Turns list of column names into NUMs, SYMs, or SKIPs
function COLS.new(k,row, i,createl)
createl = function(at,txt, col)
if ignorep(txt) then return SKIP:new(at,txt) end
col = (nump(txt) and i.y or i.x, col)
if klassp(txt) then i.klass = col end
return col
end
---
i= new(k,{all={},x={},y={},names=row})
for at,txt in ipairs(row) do push(i.all, createl(at,txt)) end
return i end
 function COLS.add(i,t)
  for _,col in pairs(i.all) do col:add( t[col.at] ) end
  return t end
                   -- NUM: summarizes a stream of numbers
function NUM.new(k,n,s)
return new(k,(n=0,at=n or 0,txt=s or*",has=SOME:new(),ok=false,
w=lessp(s or "") and -1 or 1, lo=big, hi=-big) end
function NUM.add(i,x)
if x -= "?" then
i.n = i.n + 1
if i.has:add(x) then i.ok=false end
i.lo,i.hi = min(x,i.lo), max(x,i.hi); end end
function NUM.dist(i,x,y)
if     x==""" and y=="?" then return 1
elseif x=="?" then y=i:norm(y); x=y<0.5 and 1 or 0
elseif y=="" then x=i:norm(x); y=x<0.5 and 1 or 0
else     x,y = i:norm(x), i:norm(y) end
return math.abs(x-y) end</pre>
 function NUM.distance2heaven(x, w)
  return ((i.w>0 and 1 or 0) - i:norm(x))^2 end
 function NUM.mid(i) return per(i:sorted(), .5) end
 function NUM.norm(i,x)
  return math.abs(i.hi-i.lo)<1E-9 and 0 or (x-i.lo)/(i.hi - i.lo) end</pre>
 function NUM.sorted(i)
  if i.ok==false then table.sort(i.has.all); i.ok=true end
  return i.has.all end
 -- ROWS: manages 'rows', summarized in 'cols' (columns).

function ROWS.new(k,inits, i)

i = new(k,frows={},cols=nil})

if type(inits)=="table" then for t in inits do i:add(t) end end

if type(inits)=="string" then for t in file2things(inits) do i:add(t) end end

return i end
function ROWS.add(i,t)
    if i.cols then push(i.rows,i.cols:add(t)) else i.cols=COLS:new(t) end end
 function ROWS.clone(i, j) j= ROWS:new(); j:add(i.cols.names);return j end
function ROWS.dist(i,row1,row2,     d,fun)
   function fun(col) return col:dist(row1[col.at], row2[col.at])^the.p end
   return (sum(i.cols.x, fun)/ #i.cols.x)^(1/the.p) end
 function ROWS.furthest(i,row1,rows, fun)
  function fun(row2) return (i:dist(row1,row2), row2) end
  return unpack(per(sort(map(rows,fun),firsts), the.furthest)) end
 function ROWS.half(i, top)
  local some, top,c,x,y,tmp,mid,lefts,rights,_
  some= many(i.rows, the.keep)
    some= many(i.rows, the.keep)
top = top or i
    _,x = top:furthest(any(some), some)
    c,y = top:furthest(x, some)
tmp = sort(map(i.rows,function(r) return top:fastmap(r,x,y,c) end),firsts)
mid = #i.rows/2
lefts, rights = i:clone(), i:clone()
for at,row in pairs(tmp) do (at <=mid and lefts or rights):add(row[2]) end
return lefts,rights,x,y,c, tmp[mid] end</pre>
 function ROWS.mid(i,cols)
      return map(cols or i.cols.all, function(col) return col:mid() end) end
function ROWS.fastmap(i, r,x,y,c, a,b) 
 a,b = i : dist(r,x), i : dist(r,y); return \{(a^2 + c^2 - b^2)/(2*c), r\} end
                   -- SKIP: summarizes things we want to ignore (so does nothing) function SKIP.new(k,n,s) return new(k,\{n=0,at=at\ or\ 0,txt=s\ or""\}) end function SKIP.add(i,x) return x end function SKIP.mid(i) return "?" end
 -- SOME: keeps a random sample on the arriving data function SOME.new(k,keep) return new(k,{n=0,all={}}, keep=keep or the.keep}) end function SOME.add(i,x)
i.n = i.n+1
if #i.all < i.keep then push(i.all,x) ; return i.all
elseif r() < i.keep/i.n then i.all[r(#i.all)]=x; return i.all end end
             [__]
 -- SYM: summarizes a stream of symbols function SYM.new(k,n,s) return new(k,{n=0,at=n or 0,txt=s or"",has={},most=0}) end
function SYM.add(i,x,inc)
   if x == "?" then
   inc = inc or 1
   i.n = i.n + inc
   i.has[x] = inc + (i.has[x] or 0)
   if i.has[x] > i.most then i.most,i.mode=i.has[x],x end end end
 function SYM.dist(i,x,y) return(x==^n?" and y==^n?" and 1) or(x==y and 0 or 1) end function SYM.mid(i) return i.mode end
```

```
function SYM.div(i, p)
return sum(i.has,function(k) p=-i.has[k]/i.n;return -p*math.log(p,2) end) end
              k = SYM:new(i.at,itxt)
for x,n in pairs(i.has) do k:add(x,n) end
for x,n in pairs(j.has) do k:add(x,n) end
ei, ej, ejk= i:div(), j:div(), k:div()
if i.n=0 or j.n=0 or .99*ek <= (i.n*ei + j.n*ej)/k.n then
return k end end
          function SYM.merge(i,j,
 329
                                 -- CLUSTER: recursively divides data by clustering towards two distant points function CLUSTER.new(k,egs,top)
local i,want,left,right
i = new(k, (here=egs))
top = top or egs
want = (#top.rows) 'the.want
if #egs.rows >= 2*want then
left, right, i.x, i.y, i.c, i.mid = egs:half(top)
if #left.rows < #egs.rows then
i.left = CLUSTER:new(left, top)
i.right= CLUSTER:new(right, top) end end
return i end
          function CLUSTER.show(i,pre, here)
              nnction CLUSTER.show(1,pre, neig)
pre = pre or ""
here=""
here=""
if not i.left and not i.right then here= o(i.here:mid(i.here.cols.y)) end
print(fmt("%6s:%-30s%",#i.here.rows, pre, here))
for _,kid in pairs(i.left, i.right) do
    if kid then kid:show(pre .. "[.") end end end
                               -- SPAN: keeps a random sample on the arriving data function SPAN.new(k, col, lo, hi, has) return new(k, (col=col,lo=lo,hi=hi or lo,has=has or SYM:new())) end
          function SPAN.add(i,x,y,n) i.lo,i.hi=min(x,i.lo),max(x,i.hi); i.has:add(y,n) end
function SPAN.merge(i,j)
local has = i.has:merge(j.has)
if now then return SPAN:new(i.col, i.lo, j.hi, has) end end
          function SPAN.select(i,row, x)
    x = row[i.col.at]
    return (x=="?") or (i.lo==i.hi and x==i.lo) or (i.lo <= x and x < i.hi) end</pre>
          function SPAN.score(i) return {i.has.n/i.col.n, i.has:div()} end
                                        EXPLHIN
          -- ### EXPLAIN:
function EXPLAIN.new(k,egs,top)
local i,top,want,left,right,spans,best,yes,no
i = new(k,(here = egs))
               i = new(k, {here = egs})
top = top or egs
want = (#top.rows)^the.want
if #top.rows >= 2*want then
left,right = egs:half(top)
                    left,right = egs:hair(top)
spans = {}
for n,col in pairs(i.cols.x) do
    for _,s in pairs(col:spans(f.cols.x(n])) do
        push(spans, {ys=s:score(),it=s}) end end
    best = distance2beaven(spans,("+","-"))[1]
yes,no = egs:clone(), egs:clone()
for _,row in pairs(egs.rows) do
    (best:selects(row) and yes or no):add(row) end -- divide data in two
    if #yes.rows<#egs.rows then -- make kids if kid size different to parent siz</pre>
                           if #yes.rows>=want then i.yes=EXPLAIN:new(yes,top) end
if #no.rows >=want then i.no =EXPLAIN:new(no, top) end end end
                return i end
          function EXPLAIN.show(i,pre)
               inction EXPLAIN.show(i,pre)
pre = pre or ""
if not pre then
tmp = i.here:mid(i.here.y)
print(fmt("%s:%-30s %s", $i.here.rows, pre, o(i.here:mid(i.here.cols.y))))
for _,pair in pairs{{true,i.yes},{false,i.no}} do
status,kid = unpack(pair)
k:shpw(pre .. "|..") end end end
                               function SYM.spans(i, j)
  local xys,all,one,last,xys,x,c n = {},{}
  for x,n in pairs(i.has) do push(xys, {x, "this",n}) end
  for x,n in pairs(j.has) do push(xys, {x, "that",n}) end
  for _,tmp in ipairs(sort(xys,firsts)) do
   x,c,n = unpack(tmp)
  if x ~= last then
                last = x
  one = push(all, Span(i,x,x)) end
  one:add(x,y,n) end
return all end
         return all end
function NUM.spans(i, j)
local xys,all,lo,hi,gap,xys,one,x,c,n = {},{}
lo,hi = min(i.lo, j.lo), max(i.hi,j.hi)
gap = (hi - lo) / (6/thc.cohen)
for x,n in pairs(i.has) do push(xys, {x,"this",l}) end
for x,n in pairs(j.has) do push(xys, {x,"thit",l}) end
one = Span:new(i,lo,lo)
all = {one}
for _,tmp in ipairs(sort(xys,first)) do
    x,c,n = unpack(tmp)
    if one.hi - one.lo > gap then one = push(all, Span(i, one.hi, x)) end
    oneradd(x,y) end
all = merge(all)
all[1    ].lo = -big
all[#all].hi = big
return all end
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                                skarkuns
       fails=0
function asserts(test, msg)
print(test and "PASS:"or "FAIL:",msg or "")
if not test then
fails=fails+1
if the dump then assert(test,msg) end end end
        function EGS.nothing() return true end
function EGS.the() oo(the) end
function EGS.snan() print(r()) end
function EGS.snan(s,t)
s=SOME:new(100)
for i=1,100000 do s:add(i) end
asserts(100=#s.all,"lengh")
for j, x in pairs(sort(s.all)) do
--if (j % 10)==0 then print("") end
fmt("%6s",x) end end
         function EGS.clone( r,s)
              mction EGS.clone( r,s)
r = ROWS:new(the data)
s = r:clone()
for _row in pairs(r.rows) do s:add(row) end
asserts(r.cols.x[1].lo==s.cols.x[1].lo, "clone.lo")
asserts(r.cols.x[1].hi==s.cols.x[1].hi, "clone.hi")
end
        function EGS.data( r)
  r = ROWS:new(the.data)
  asserts(r.cols.x[1].hi == 8, "data.columns") end
        function EGS.dist( r,rows,n)
    r = ROWS.new(the.data)
    rows = r.rows
    n = NUM:new()
    for _,row in pairs(rows) do n:add(r:dist(row, rows[1])) end
    oo(rnds(n:sorted()))
    --oo(r.cols.x[2]:sorted())
    o(r.cols.x[2]:sorted()) end
        function EGS.many( t)
t={}; for j=1,1000 do push(t,j) end
--print(oo(many(t, 10))) end
oo(many(t, 10)) end
        function EGS.half( r,c,row1,row2)
local lefts,rights,x,y,x
r = ROWS.new(the.data)
r:mid(r.cols.y)
lefts,rights,x,y,c = r:half()
lefts.mid(lefts.cols.y)
rights:mid(rights.cols.y)
asserts(199==fights.rows, "leftrows")
asserts(199==fights.rows, "rightrows")
asserts(true, "half") end
        --CLUSTER:new(r):show() er
CLUSTER:new(r):show() end
       -- start-up
if arg[0] == "I5.lua" then
if the .help then print(help) else
  local b4={}; for k,v in pairs(the) do b4{k}=v end
  for _, todo in pairs(the.todo=="all" and slots(EGS) or {the.todo}) do
    for k,v in pairs(b4) do the[k]=v end
    math.randomseed(the.seed)
    if type(EGS[todo])=="function" then EGS[todo]() end end
end
              end
for k,v in pairs(_ENV) do if not b4[k] then print("?",k,type(v)) end end
os.exit(fails)
              return {CLUSTER=CLUSTER, COLS=COLS, NUM=NUM, ROWS=ROWS, SKIP=SKIP, SOME=SOME, SYM=SYM,the=the,oo=oo,o=o}
        end
-- git rid of SOME for rows
-- nss = NUM | SYM | SKIP
-- cols = all:[nss]+, x:[nss]*, y:[nss]*, klass;col?
-- ROWS = cols:COLS, rows:SOME
        -- [Ah91]: Aha, D.W., Kibler, D. & Albert, M.K. Instance-based learning algori thms. Mach Learn 6, 37âM-^@M-^S66 (1991). https://doi.org/10.1007/BF00153759
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