```
\<sub>\L\</sub>\
                                                                            /\<u>\</u>'',
Recursively divide data based on two

distant points (found in linear time using the Fastmap

heuristic [Fa95]). Then find and print the attribute range

that best distinguishes these halves. Recurse on each half.

(which is sort of like PDDP [Bo98] but faster; and we

offers a human-readable description for each division)

To find those ranges, this code uses a variant of the ChiMerge

discretizer (but we select on entropy and size,

not the Chi statistic]

To avoid spurious outliers, this code separates using `furthest=.9`;

i.e. the 90% furthest points.

To avoid long runtimes, this code only searches at most `heep=512`

randomly selected examples to find those furtherst points.

To suport multi-objective optimization, this code reads csv files

whose headers may contain markers for "minimize this" or "maximize

that" (see the `lessp, morep` functions).

To support explanation, optionally, at each level of recursion,

this code reports what ranges can best distinguish sibling clusters

Cl,C2. The discretizer is inspired by the ChiMerge algorithm:

numerics are divided into, say, 16 bins. Then, while we can find

adjacent bins with the similar distributions in Cl,C2, then

(a) merge then (b) look for other merges.

local help = [[

15 == a little lab of lots of LUA learning algorithms.
   15 == a little lab of lots of LUA learning algorithms. (c) 2022, Tim Menzies, BSD 2-clause license.
 USAGE:
lua 15.lua [OPTIONS]
 OPTIONS:
                                                         F Cohen's delta = .35
N data file = etc/data/auto93.csv
stack dump on assert fails = false
F far = .9
S format string = $5.2f
P max kept items = 512
P distance coefficient = 2
P set seed = 10019
S start up action (or 'all') = nothing
show help = false
F recurse until rows^want = .5
           -data
             -Dump -D
-furthest -f
-Format -F
-keep -k
   KEY: N=fileName F=float P=posint S=string
  -- ## Definitions
  -- ### Cache current names (used at end to find rogue variables) local b4={}; for k,_ in pairs(_ENV) do b4[k]=k end
  local any.asserts,big,cli,csv,fails,firsts,fmt,goalp,ignorep,klassp
local lessp,map,main,many,max,merge,min,morep,new,nump,o,oo,per,pop,push
local r,rows,rnd,rnds,slots,sort,sum,thing,things,unpack
  -- ### Define classes
local CLUSTER, COLS, EGS, EXPLAIN, NUM, ROWS = {},{},{},{},{},{},d}
local SKIP, SOME, SPAN, SYM = {},{},{},{}
  -- ### Define parameter settings.
-- Update parameter defaults from command line. Allow for some shorthand:
             e.g. _-k N_ ⇒ 'keep=N'; and _-booleanFlag_ ⇒ 'booleanFlag=not default').
 -- and _-booleanFlag_ ⇒ 'booleanFlag=not default').
local the={}
help:gsub("\n [-\[(\)^{\%}\])\[\%]\]*\[\]\]/\n\]*\%\[(\]^{\%}\]*\, function(key, flag1,x)
for n, flag2 in ipairs (arg) do
    if flag1==flag2 or "-". key =="flag2"then
        x = x=="false" and true or x=="true" and "false" or arg[n+1] end end
if x=="false" then the [key]=false elseif x=="true" then the [key]=true else
the[key] = tonumber(x) or x end end )
  -- ### Define headers for row1 of csv files
-gmotep(x) return x:find":$" end
-- Symbolic classes
function klassp(x) return not nump(x) and x:find"!$" end
-- Goals to minimize
function lessp(x) return nump(x) and x:find"-$" end
-- Goals to maximize
function morep(x) return nump(x) and x:find"-$" end
-- Numeric columns
function
 -- Columns to ignore
function ignorep(x) return x:find":$" end
-- Symbolic classes
                                                           x) return nump(x) and x:find"+$" end
mns

return x:find"^[A-Z]" end
 -- Numeric columns
function nump(x)
-- Dependent attri
function goalp(x)
```

return morep(x) or lessp(x) or klassp(x) end

```
-- ## Misc Utils
103
    -- ### Strings
fmt = string.format
105
106
107
108
109
110
    big = math.huge
max = math.max
min = math.min
r = math.random
111
112
113
114
115
     function rnds(t,f) return map(t, function(x) return rnd(x,f) end) end
    function rnd(x, f)

return fmt(type(x) == "number" and (x = x / / 1 and f or the Format) or "%s", x) end
     pop = table.remove
unpack = table.unpack
function any(t)
function firsts(a,b)
     - ### Meta
     Tunction nmap(t,f, u) u={}; for k,v in pairs(t) do push(u,f(v)) end; return u end function sum(t,f, n) n=0; for _,v in pairs(t) do n=n+f(v) end; return n end function slots(t, u)
         d='[for k,v in pairs(t) do k=tostring(k);if k:sub(1,1)~="_" then push(u,k) end end
return sort(u) end
     -- ### Print tables, recursively function oo(t) print(o(t)) end function o(t) = "lable" then return tostring(t) end local key=function(k) return fmt(":%% %s",k,o(t[k])) end local u = #t>0 and map(t,o) or map(slots(t),key) return '{'..table.concat(u,"").."}" end
     -- ### Coerce strings to things
function csv(file, x)
file = io.input(file)
return function()
x=io.read(); if x then return things(x) else io.close(file) end end end
     function things(x,sep, t)
         t={}
for y in x:gmatch(sep or"([^,]+)") do push(t,thing(y)) end
return t end
     -- ### Misc stuff
     -- Multi-objectives. Normalized, scored via distance to heaven. function distance2Heaven(t,heaven, num,d)
       runction distance/reaven(t, neaven, num, a)
for n,txt in pairs(heaven) do
   num = Num(at,txt)
for _, z in pairs(t) do num:add(z.ys[n]) end
  for _, z in pairs(t) do z.ys[n] = num:distance2heaven(z.ys[n]) end end
  d = function(one) return (sum(one.ys)/#one.ys)^5 end
return sort(t, function(a,b) return d(a) < d(b) end) end</pre>
     return SUTI(T, runction(a,b) return d(a) < d(b) end) end
-- While we can find similar adjacent ranges, then merge them.
function merge (b4,
   j,n,now = 0, #b4,{}
   while j < #b4 do
   j = j+1
   a, b = f(j), b4[j+1]
   if b then
       merged = a:merge(b)
       if merged then a, j = merged, j+1 end end
   push (now,a)
   j = j+1 end</pre>
         j = j+1 end
return #now == #b4 and b4 or merge(now) end
function new(k,t) k.__index=k; k.__tostring=o; return setmetatable(t,k) end
```

```
function COLS.add(i,t)
  for _,col in pairs(i.all) do col:add( t[col.at] ) end
  return t end
                 -- NUM: summarizes a stream of numbers
function NUM.new(k,n,s)
return new(k,(n=0,at=n or 0,txt=s or*",has=SOME:new(),ok=false,
w=lessp(s or "") and -1 or 1, lo=big, hi=-big) end
function NUM.add(i,x)
if x -= "?" then
i.n = i.n + 1
if i.has:add(x) then i.ok=false end
i.lo,i.hi = min(x,i.lo), max(x,i.hi); end end
function NUM.dist(i,x,y)
if     x==""" and y=="?" then return 1
elseif x=="?" then y=i:norm(y); x=y<0.5 and 1 or 0
elseif y=="" then x=i:norm(x); y=x<0.5 and 1 or 0
else     x,y = i:norm(x), i:norm(y) end
return math.abs(x-y) end</pre>
 function NUM.distance2heaven(x, w)
  return ((i.w>0 and 1 or 0) - i:norm(x))^2 end
 function NUM.mid(i) return per(i:sorted(), .5) end
 function NUM.norm(i,x)
  return math.abs(i.hi-i.lo)<1E-9 and 0 or (x-i.lo)/(i.hi - i.lo) end</pre>
 function NUM.sorted(i)
  if i.ok==false then table.sort(i.has.all); i.ok=true end
  return i.has.all end
 -- ROWS: manages 'rows', summarized in 'cols' (columns).

function ROWS.new(k,inits, i)
   i = new(k,[rows={},cols=ni])

if type(inits)=="string" then for t in csv(inits) do i:add(t) end end
if type(inits)=="table" then for t in inits do i:add(t) end end
return i end
 function ROWS.add(i,t)
    if i.cols then push(i.rows,i.cols:add(t)) else i.cols=COLS:new(t) end end
function ROWS.clone(i, j) j= ROWS:new(); j:add(i.cols.names);return j end
function ROWS.dist(i,row1,row2,     d,fun)
   function fun(col) return col:dist(row1[col.at], row2[col.at])^the.p end
   return (sum(i.cols.x, fun)/ #i.cols.x)^(1/the.p) end
function ROWS.furthest(i,row1,rows, fun)
  function fun(row2) return {i:dist(row1,row2), row2} end
  return unpack(per(sort(map(rows,fun),firsts), the.furthest)) end
 function ROWS.half(i, top)
  local some, top,c,x,y,tmp,mid,lefts,rights,_
  some= many(i.rows, the.keep)
    some= many(i.rows, the.keep)
top = top or i
    _,x = top:furthest(any(some), some)
    c,y = top:furthest(x, some)
tmp = sort(map(i.rows,function(r) return top:fastmap(r,x,y,c) end),firsts)
mid = #i.rows/2
lefts, rights = i:clone(), i:clone()
for at,row in pairs(tmp) do (at <=mid and lefts or rights):add(row[2]) end
return lefts,rights,x,y,c, tmp[mid] end</pre>
 function ROWS.mid(i,cols)
      return map(cols or i.cols.all, function(col) return col:mid() end) end
function ROWS.fastmap(i, r,x,y,c, a,b) 
 a,b = i : dist(r,x), i : dist(r,y); return \{(a^2 + c^2 - b^2)/(2*c), r\} end
                  -- SKIP: summarizes things we want to ignore (so does nothing) function SKIP.new(k,n,s) return new(k,\{n=0,at=at\ or\ 0,txt=s\ or""\}) end function SKIP.add(i,x) return x end function SKIP.mid(i) return "?" end
                  -- SOME: keeps a random sample on the arriving data function SOME.new(k,keep) return new(k,{n=0,all={}}, keep=keep or the.keep}) end function SOME.add(i,x)
i.n = i.n+1
if #i.all < i.keep then push(i.all,x) ; return i.all
elseif r() < i.keep/i.n then i.all[r(#i.all)]=x; return i.all end end
           [__]
 -- SYM: summarizes a stream of symbols function SYM.new(k,n,s) return new(k,{n=0,at=n or 0,txt=s or"",has={},most=0}) end
function SYM.add(i,x,inc)
   if x == "?" then
   inc = inc or 1
   i.n = i.n + inc
   i.has[x] = inc + (i.has[x] or 0)
   if i.has[x] > i.most then i.most,i.mode=i.has[x],x end end end
 function SYM.dist(i,x,y) return(x==^n?" and y==^n?" and 1) or(x==y and 0 or 1) end function SYM.mid(i) return i.mode end
```

```
1322 function SYM.div(i, p)
1333 return sum(i.has,function(k) p=-i.has[k]/i.n;return -p*math.log(p,2) end) end
              k = SYM:new(i.at,itxt)
for x,n in pairs(i.has) do k:add(x,n) end
for x,n in pairs(j.has) do k:add(x,n) end
ei, ej, ejk= i:div(), j:div(), k:div()
if i.n=0 or j.n=0 or .99*ek <= (i.n*ei + j.n*ej)/k.n then
return k end end
          function SYM.merge(i,j,
                                 -- CLUSTER: recursively divides data by clustering towards two distant points function CLUSTER.new(k,egs,top)
local i,want,left,right
i = new(k, {here=egs})
top = top or egs
want = (#top.rows) 'the.want
if #egs.rows >= 2*want then
left, right, i.x, i.y, i.c, i.mid = egs:half(top)
if #left.rows < #egs.rows then
i.left = CLUSTER:new(left, top)
i.right= CLUSTER:new(right, top) end end
return i end
 336
          function CLUSTER.show(i,pre, here)
              nnction CLUSTER.show(1,pre, neig)
pre = pre or ""
here=""
here=""
if not i.left and not i.right then here= o(i.here:mid(i.here.cols.y)) end
print(fmt("%6s:%-30s%",#i.here.rows, pre, here))
for _,kid in pairs(i.left, i.right) do
    if kid then kid:show(pre .. "[.") end end end
                               -- SPAN: keeps a random sample on the arriving data function SPAN.new(k, col, lo, hi, has) return new(k, (col=col,lo=lo,hi=hi or lo,has=has or SYM:new())) end
          function SPAN.add(i,x,y,n) i.lo,i.hi=min(x,i.lo),max(x,i.hi); i.has:add(y,n) end
function SPAN.merge(i,j)
local has = i.has:merge(j.has)
if now then return SPAN:new(i.col, i.lo, j.hi, has) end end
          function SPAN.select(i,row, x)
    x = row[i.col.at]
    return (x=="?") or (i.lo==i.hi and x==i.lo) or (i.lo <= x and x < i.hi) end</pre>
          function SPAN.score(i) return {i.has.n/i.col.n, i.has:div()} end
                                        EXPLHIN
          -- ### EXPLAIN:
function EXPLAIN.new(k,egs,top)
local i,top,want,left,right,spans,best,yes,no
i = new(k,(here = egs))
               i = new(k, {here = egs})
top = top or egs
want = (#top.rows)^the.want
if #top.rows >= 2*want then
left,right = egs:half(top)
                    left,right = egs:hair(top)
spans = {}
for n,col in pairs(i.cols.x) do
    for _,s in pairs(col:spans(f.cols.x(n])) do
        push(spans, {ys=s:score(),it=s}) end end
    best = distance2beaven(spans,("+","-"))[1]
yes,no = egs:clone(), egs:clone()
for _,row in pairs(egs.rows) do
    (best:selects(row) and yes or no):add(row) end -- divide data in two
    if #yes.rows<#egs.rows then -- make kids if kid size different to parent siz</pre>
                           if #yes.rows>=want then i.yes=EXPLAIN:new(yes,top) end
if #no.rows >=want then i.no =EXPLAIN:new(no, top) end end end
                return i end
          function EXPLAIN.show(i,pre)
               inction EXPLAIN.show(i,pre)
pre = pre or ""
if not pre then
tmp = i.here:mid(i.here.y)
print(fmt("%s:%-30s %s", $i.here.rows, pre, o(i.here:mid(i.here.cols.y))))
for _,pair in pairs{{true,i.yes},{false,i.no}} do
status,kid = unpack(pair)
k:shpw(pre .. "|..") end end end
                               function SYM.spans(i, j)
  local xys,all,one,last,xys,x,c n = {},{}
  for x,n in pairs(i.has) do push(xys, {x, "this",n}) end
  for x,n in pairs(j.has) do push(xys, {x, "that",n}) end
  for _,tmp in ipairs(sort(xys,firsts)) do
    x,c,n = unpack(tmp)
  if x = last then
                last = x
  one = push(all, Span(i,x,x)) end
  one:add(x,y,n) end
return all end
         return all end
function NUM.spans(i, j)
local xys,all,lo,hi,gap,xys,one,x,c,n = {},{}
lo,hi = min(i.lo, j.lo), max(i.hi,j.hi)
gap = (hi - lo) / (6/thc.cohen)
for x,n in pairs(i.has) do push(xys, {x,"this",l}) end
for x,n in pairs(j.has) do push(xys, {x,"thit",l}) end
one = Span:new(i,lo,lo)
all = {one}
for _,tmp in ipairs(sort(xys,first)) do
    x,c,n = unpack(tmp)
    if one.hi - one.lo > gap then one = push(all, Span(i, one.hi, x)) end
    oneradd(x,y) end
all = merge(all)
all[1    ].lo = -big
all[#all].hi = big
return all end
```

```
439
440
441
442
443
444
445
446
447
448
449
                                    ekenek una
        fails=0
function asserts(test, msg)
print(test and "PASS:"or "FAIL:",msg or "")
if not test then
fails=fails+1
if the.dump then assert(test,msg) end end end

"bing() return true end
         function EGS.nothing() return true and
function EGS.the() oo(the) end
function EGS.snad() print(r()) end
function EGS.some(s,t)
s=SOME:new(100)
for i=1,100000 do s:add(i) end
for j,x in pairs(sort(s.all)) do
--if (j % 10)==0 then print("") end
--io.write(fmt("%65",x)) end end
fmt("%6s",x) end end
         function EGS.clone( r,s)

r = ROWS:new(the.data)

s = r:clone()

for __row in pairs(r.rows) do s:add(row) end

asserts(r.cols.x[1].lo==s.cols.x[1].lo, "clone.lo")

asserts(r.cols.x[1].hi==s.cols.x[1].hi, "clone.li")

end
          function EGS.data( r)
  r = ROWS:new(the.data)
  asserts(r.cols.x[1].hi == 8, "data.columns") end
         function EGS.dist( r,rows,n)
    r = ROWS.new(the.data)
    rows = r.rows
    n = NOM:new()
    for _,row in pairs(rows) do n:add(r:dist(row, rows[1])) end
    --oo(r.cols.x[2]:sorted()) end
    o(r.cols.x[2]:sorted()) end
          function EGS.many(     t)
t={}; for j=1,100 do push(t,j) end
--print(oo(many(t, 10))) end
o(many(t, 10)) end
          function EGS.far(    r,c,row1,row2)
    r = ROWS:new(the.data)
    row1 = r.rows[1]
    c,row2 = r:far(r.rows[1], r.rows) end
    --print(c,"\n",o(row1),"\n", o(row2)) end
         function EGS.half( r,c,row1,row2)
local lefts,rights,x,y,x
r = ROWS:new(the.data)
r:mid(r.cols.y)
lefts,rights,x,y,c = r:half()
lefts:mid(lefts.cols.y)
rights:mid(rights.cols.y)
asserts(true,"half") end
          function EGS.cluster(r)
  r = ROWS:new(the.data)
  --CLUSTER:new(r):show() end
  CLUSTER:new(r) end
       -- start-up

if arg[0] == "sllua" then

if the help then print(help:gsub("\nNOTES:*$","")) else

local b4={}; for k,v in pairs(the) do b4(k)=v end

for _,todo in pairs(the todo=="all" and slots(EGS) or {the.todo}) do

for k,v in pairs(b4) do the[k]=v end

math.randomseed(the.seed)

if type(EGS[todo])=="function" then EGS[todo]() end end

end
                end for k,v in pairs(_ENV) do if not b4[k] then print("?",k,type(v)) end end os.exit(fails)
         return {CLUSTER=CLUSTER, COLS=COLS, NUM=NUM, ROWS=ROWS, SKIP=SKIP, SOME=SOME, SYM=SYM, the=the,oo=oo,o=o}
         end
-- git rid of SOME for rows
-- nss = NUM | SYM | SKIP
-- COLS = all: [nss]+, x:[nss]*, y:[nss]*, klass;col?
-- ROWS = cols:COLS, rows:SOME
          -- [Ah91]: Aha, D.W., Kibler, D. & Albert, M.K. Instance-based learning algorithms. Mach Learn 6, 37âM-^@M-^S66 (1991). https://doi.org/10.1007/BF00153759
```