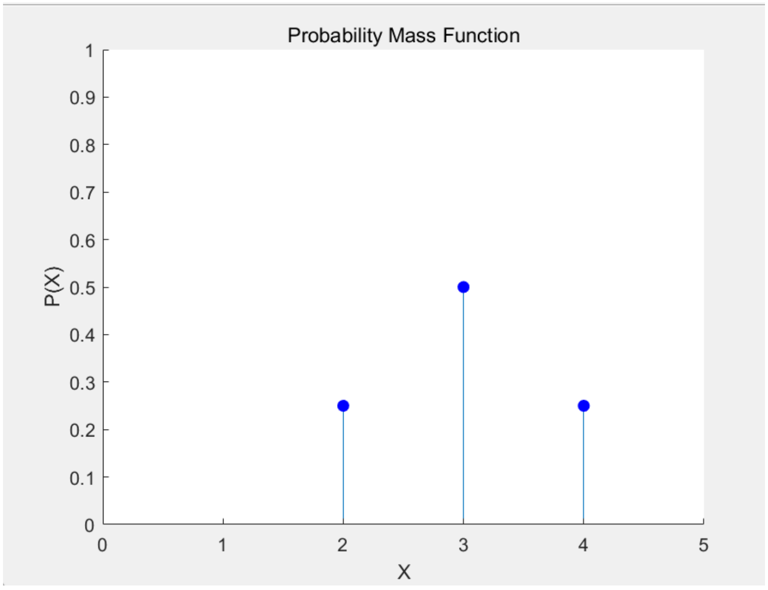
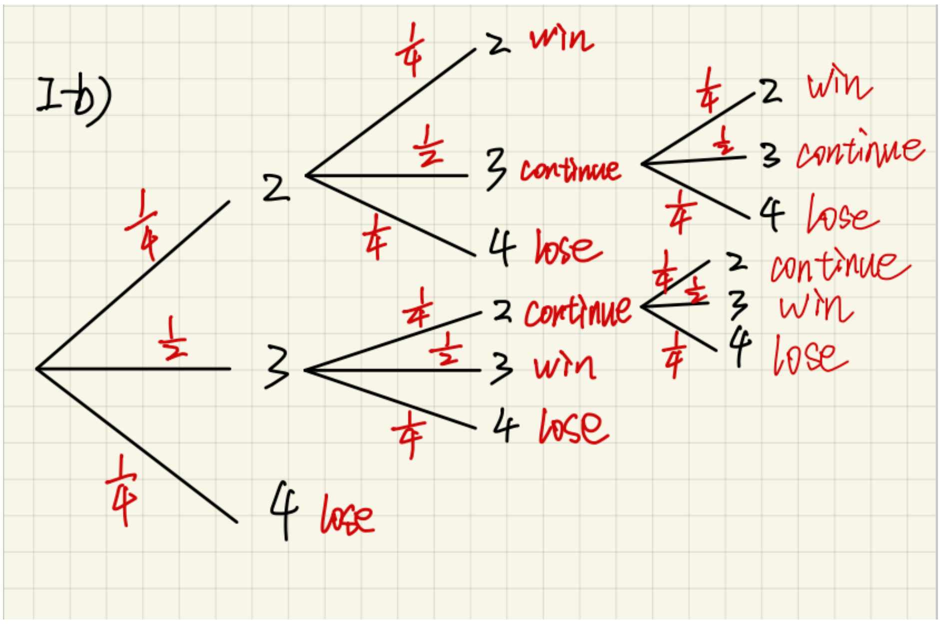
Workshop1

Part I:

I-a)



I-b)



I-c)

The sample space S for this game:

$S = \{4, 22, 24, 33, 34, 232, 234, 323,$
$324, \cdots\}$

Because,

When $T(1) = 4$, the player loses immediately.

When $T(1) = 2, T(2) = T(3) = \cdots = T(N) = 3$, the game is continuing. Only when $T(N+1) = 2$, the player wins. For the same reason, when $T(1) = 3, T(2) = T(3) = \cdots = T(N) = 2$ and $T(N+1) = 3$, the player wins.

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I-d)

According to the tree diagram in I-b), the possible outcomes are “win” and “lose”.

I-e)

Let the probability of incident A is “win” and the probability of incident B is “lose”.

$$P(A) = \frac{1}{4} \times \sum_{n=0}^{\infty} \frac{1}{2^n} \times \frac{1}{4} + \frac{1}{2} \times \sum_{n=0}^{\infty} \frac{1}{4^n} \times \frac{1}{2} = \frac{11}{24}$$

$$P(B) = \frac{1}{4} + \frac{1}{4} \times \sum_{n=0}^{\infty} \frac{1}{2^n} \times \frac{1}{4} + \frac{1}{2} \times \sum_{n=0}^{\infty} \frac{1}{4^n} \times \frac{1}{4} = \frac{13}{24}$$

$$P(A) + P(B) = 1$$

I-f)

Let the probability of incident C is “never finish”.

$$P(C) = \lim_{n\to\infty} \left(\frac{1}{2} \times \frac{1}{4^n} + \frac{1}{4} \times \frac{1}{2^n}\right) = 0$$

I-g)

Let the probability of incident A is “win”.

$$P(A) = \frac{1}{4} \times \sum_{n=0}^{\infty} \frac{1}{2^n} \times \frac{1}{4} + \frac{1}{2} \times \sum_{n=0}^{\infty} \frac{1}{4^n} \times \frac{1}{2} = \frac{11}{24}$$

I-h)

Code:

```
%part1_h
function win_game = part1_h()

    diceRoll1 = randi([1,2]);
    diceRoll2 = randi([1,2]);
    sum = diceRoll1 + diceRoll2;
    win_game = 0;

    if sum ~= 4
        old_sum = sum;
        while true
            diceRoll1 = randi([1,2]);
            diceRoll2 = randi([1,2]);
            new_sum = diceRoll1 + diceRoll2;
            if new_sum == 4
                break;
            elseif new_sum == old_sum
                win_game = 1;
                break;
            end
        end
    end

    if win_game == 1
        fprintf('win\n');
    else
        fprintf('lose\n');
    end
end
```

Result:

```
>> win_game_h = part1_h();
lose
>>
```

I-i)

Code:

```
%part1_i
function win_game = part1_i()
    n = 10;
    win_times = 0;

    for i = 1:n
        if playRound()
            win_times = win_times + 1;
        end
    end

    win_fraction = win_times / n;
    fprintf('Player won %d times in %d games\n', win_times, n);
    fprintf('The fraction of player winning is %.2f\n', win_fraction);

    win_game = win_times;
end

function win = playRound()
    diceRoll1 = randi([1,2]);
    diceRoll2 = randi([1,2]);
    sum = diceRoll1 + diceRoll2;
    win = 0;

    if sum ~= 4
        old_sum = sum;
        while true
            diceRoll1 = randi([1,2]);
            diceRoll2 = randi([1,2]);
            new_sum = diceRoll1 + diceRoll2;
            if new_sum == 4
                break;
            elseif new_sum == old_sum
                win = 1;
                break;
            end
        end
    end
end
```

Result:

```
>> win_game_i = part1_i();
Player won 5 times in 10 games
The fraction of player winning is 0.50
>>
```

I-j)

Change the n in I-i) from 10 to 50000.

Code:

```
%part1_j
function win_game = part1_j()
    n = 50000;
    win_times = 0;

    for i = 1:n
        if playRound()
            win_times = win_times + 1;
        end
    end

    win_fraction = win_times / n;
    fprintf('Player won %d times in %d games\n', win_times, n);
    fprintf('The fraction of player winning is %.2f\n', win_fraction);

    win_game = win_times;
end

function win = playRound()
    diceRoll1 = randi([1,2]);
    diceRoll2 = randi([1,2]);
    sum = diceRoll1 + diceRoll2;
    win = 0;

    if sum ~= 4
        old_sum = sum;
        while true
            diceRoll1 = randi([1,2]);
            diceRoll2 = randi([1,2]);
            new_sum = diceRoll1 + diceRoll2;
            if new_sum == 4
                break;
            elseif new_sum == old_sum
                win = 1;
                break;
            end
        end
    end
end
```

Result:

```
>> win_game_j = part1_j();
Player won 22869 times in 50000 games
The fraction of player winning is 0.46
>>
```

I-k)

Let the probability of incident A is “win” and the probability of incident B is “lose”.

From I-j), we have:

$$P(A) = \frac{22869}{50000} = 45.74\%$$

$$P(B) = \frac{27131}{50000} = 54.25\%$$

From I-i), we have:

$$P(A) = \frac{5}{10} = 50.00\%$$

$$P(B) = \frac{5}{10} = 50.00\%$$

But in I-e), we have:

$$P(A) = \frac{11}{24} = 45.83\%$$

$$P(B) = \frac{5}{10} = 54.17\%$$

The outcomes in I-e) and I-k) are very closed to theoretical calculate results.

I-l)

Repeat I-h) under new condition.

Code:

```
%part1_l_1
function win_game = part1_l_1()

    diceRoll1 = randi([1,6]);
    diceRoll2 = randi([1,6]);
    sum = diceRoll1 + diceRoll2;
    win_game = 0;

    if sum ~= 12
        old_sum = sum;
        while true
            diceRoll1 = randi([1,6]);
            diceRoll2 = randi([1,6]);
            new_sum = diceRoll1 + diceRoll2;
            if new_sum == 12
                break;
            elseif new_sum == old_sum
                win_game = 1;
                break;
            end
        end
    end

    if win_game == 1
        fprintf('win\n');
    else
        fprintf('lose\n');
    end
end
```

Result:

```
>> win_game_l_1 = part1_l_1();
win
>>
```

Repeat I-i) under new condition.

Code:

```
%part1_1_2
function win_fraction = part1_1_2()
    n = 10;
    win_times = 0;

    for i = 1:n
        win_game = dicegame();
        win_times = win_times + win_game;
    end

    win_fraction = win_times / n;
    fprintf('Player won %d times in %d games\n', win_times, n);
    fprintf('The fraction of players winning is %.2f\n', win_fraction);
end

function win_game = dicegame()
    diceRoll1 = randi([1,6]);
    diceRoll2 = randi([1,6]);
    sum = diceRoll1 + diceRoll2;
    win_game = 0;

    if sum ~= 12
        old_sum = sum;
        while true
            diceRoll1 = randi([1,6]);
            diceRoll2 = randi([1,6]);
            new_sum = diceRoll1 + diceRoll2;
            if new_sum == 12
                break;
            elseif new_sum == old_sum
                win_game = 1;
                break;
            end
        end
    end
end
```

Result:

```
>> win_game_1_2 = part1_1_2();
Player won 7 times in 10 games
The fraction of players winning is 0.70
fx >>
```

Repeat I-j) under new condition.

Code:

```
%part1_1_3
function win_fraction = part1_1_3()
    n = 50000;
    win_times = 0;

    for i = 1:n
        win_game = dicegame();
        win_times = win_times + win_game;
    end

    win_fraction = win_times / n;
    fprintf('Player won %d times in %d games\n', win_times, n);
    fprintf('The fraction of players winning is %.2f\n', win_fraction);
end

function win_game = dicegame()
    diceRoll1 = randi([1,6]);
    diceRoll2 = randi([1,6]);
    sum = diceRoll1 + diceRoll2;
    win_game = 0;

    if sum ~= 12
        old_sum = sum;
        while true
            diceRoll1 = randi([1,6]);
            diceRoll2 = randi([1,6]);
            new_sum = diceRoll1 + diceRoll2;
            if new_sum == 12
                break;
            elseif new_sum == old_sum
                win_game = 1;
                break;
            end
        end
    end
end
```

Result:

```
>> win_game_1_3 = part1_1_3();
Player won 38171 times in 50000 games
The fraction of players winning is 0.76
>>
```

Let the probability of incident A is “win” and the probability of incident B is “lose”.

So $P(A)$ is about 76% and $P(B)$ is about 24%.

Part II:

II-a)

Code:

```
%part2_a
function win_game = part2_a()
    n = 1;
    player_stick = 0;
    player_switch = 0;

    for i = 1 : n
        disp(['Time: ', num2str(i)]);
        car = randi([1,3],1);
        choose = randi([1,3],1);
        doors = [1 2 3];
        doors([car, choose]) = 0;
        host = doors(doors ~= 0);
        open = host(randi(length(host)));

        if car == choose
            disp('The player won by sticking her choice.');
            player_stick = player_stick + 1;
        else
            disp('The player lost by sticking her choice.');
        end

        change = setdiff([1, 2, 3], [choose, open]);
        if car == change
            disp('The player won by switching her choice.');
            player_switch = player_switch + 1;
        else
            disp('The player lost by switching her choice.');
        end
    end

    disp(' ');
    disp(['The probability of winning the car if the player sticks: ', num2str(player_stick/n)]);
    disp(['The fraction of times that the player wins when sticking: ', num2str(player_stick)]);
    disp(['The probability of winning the car if the player switches: ', num2str(player_switch/n)]);
    disp(['The fraction of times that the player wins when switching: ', num2str(player_switch)]);

    win_game = 0;
end
```

Result:

```
>> win_game_a = part2_a();
Time: 1
The player lost by sticking her choice.
The player won by switching her choice.

The probability of winning the car if the player sticks: 0
The fraction of times that the player wins when sticking: 0
The probability of winning the car if the player switches: 1
The fraction of times that the player wins when switching: 1
>>
```

II-b)

Repeat II-a) under new conditions.

Result:

```
>> win_game_b = part2_b();
Time: 1
The player lost by sticking her choice.
The player won by switching her choice.
Time: 2
The player won by sticking her choice.
The player lost by switching her choice.
Time: 3
The player lost by sticking her choice.
The player won by switching her choice.
Time: 4
The player won by sticking her choice.
The player lost by switching her choice.
Time: 5
The player lost by sticking her choice.
The player won by switching her choice.
Time: 6
The player won by sticking her choice.
The player lost by switching her choice.
Time: 7
The player lost by sticking her choice.
The player won by switching her choice.
Time: 8
The player lost by sticking her choice.
The player won by switching her choice.
Time: 9
The player won by sticking her choice.
The player lost by switching her choice.
Time: 10
The player lost by sticking her choice.
The player won by switching her choice.

The probability of winning the car if the player sticks: 0.4
The fraction of times that the player wins when sticking: 4
The probability of winning the car if the player switches: 0.6
The fraction of times that the player wins when switching: 6
>>
```

II-c)

Repeat II-a) under new conditions.

Result:

```
The probability of winning the car if the player sticks: 0.33506
The fraction of times that the player wins when sticking: 16753
The probability of winning the car if the player switches: 0.66494
The fraction of times that the player wins when switching: 33247
>>
```

II-d)

Let the probability of incident A is “choose car firstly and switch” and the probability of incident B is “choose goat firstly and switch”.

$$P(A) = 0$$

Because the player can't choose the car after she switches.

$$P(B) = \frac{2}{3}$$

Because the player has the probability to choose goats, which is $\frac{2}{3}$ firstly. The host shows another goat and she is absolutely choosing the car after she switches.

So $P = P(A) + P(A) = \frac{2}{3} = 0.667$

II-e)

In I-b) the probability of winning after switching is 0.6, and in I-c), which is 0.66494. Obviously, I-c) is very closed to theoretical calculate result with more simulation times.

II-f)

She should switch. If she sticks, the probability of winning is 0.333. But the probability is 0.667 if she switches.