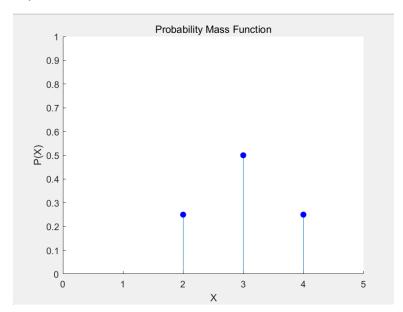
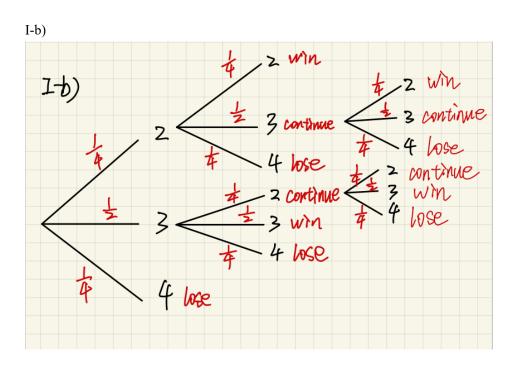
# Workshop1

Part I:

I-a)





I-c) The sample space S for this game:

Because,

When T(1) = 4, the player loses immediately.

When T(1) = 2,  $T(2) = T(3) = \cdots = T(N) = 3$ , the game is continuing. Only when T(N + 1) = 2, the player wins. For the same reason, when T(1) = 3,  $T(2) = T(3) = \cdots = T(N) = 2$  and T(N + 1) = 3, the player wins.

When T(1) = 2,  $T(2) = T(3) = \cdots = T(N) = 3$ , the game is continuing. Only when T(N + 1) = 4, the player loses. For the same reason, when T(1) = 3,  $T(2) = T(3) = \cdots = T(N) = 2$  and T(N + 1) = 4, the player loses.

I-d)

According to the tree diagram in I-b), the possible outcomes are "win" and "lose".

I-e)

Let the probability of incident A is "win" and the probability of incident B is "lose".

$$P(A) = \frac{1}{4} \times \sum_{n=0}^{\infty} \frac{1}{2^n} \times \frac{1}{4} + \frac{1}{2} \times \sum_{n=0}^{\infty} \frac{1}{4^n} \times \frac{1}{2} = \frac{11}{24}$$

$$P(B) = \frac{1}{4} + \frac{1}{4} \times \sum_{n=0}^{\infty} \frac{1}{2^n} \times \frac{1}{4} + \frac{1}{2} \times \sum_{n=0}^{\infty} \frac{1}{4^n} \times \frac{1}{4} = \frac{13}{24}$$

$$P(A) + P(B) = 1$$

I-f)

Let the probability of incident C is "never finish".

$$P(C) = \lim_{n \to \infty} \left( \frac{1}{2} \times \frac{1}{4^n} + \frac{1}{4} \times \frac{1}{2^n} \right) = 0$$

I-g)

Let the probability of incident A is "win".

$$P(A) = \frac{1}{4} \times \sum_{n=0}^{\infty} \frac{1}{2^n} \times \frac{1}{4} + \frac{1}{2} \times \sum_{n=0}^{\infty} \frac{1}{4^n} \times \frac{1}{2} = \frac{11}{24}$$

I-h)

```
1
       %part1_h
 2 🖃
       function win_game = part1_h()
 3
 4
           diceRoll1 = randi([1,2]);
           diceRoll2 = randi([1,2]);
 5
           sum = diceRoll1 + diceRoll2;
 6
 7
           win_game = 0;
 8
 9
           if sum \sim=4
10
               old_sum = sum;
11 🚊
               while true
12
                   diceRoll1 = randi([1,2]);
                    diceRoll2 = randi([1,2]);
13
14
                   new_sum = diceRoll1 + diceRoll2;
15
                    if new_sum == 4
16
                        break;
17
                    elseif new_sum == old_sum
18
                        win_game = 1;
19
                        break;
20
                    end
               end
21
22
           end
23
24
           if win_game == 1
25
               fprintf('win\n');
26
27
               fprintf('lose\n');
28
           end
29 L
       end
```

```
>> win_game_h = partl_h();
lose
fx >>
```

I-i)

```
1
       %part1 i
2 🖃
        function win_game = part1_i()
 3
           n = 10;
 4
           win_times = 0;
 5
           for i = 1:n
 6 🖨
               if playRound()
 7
8
                    win_times = win_times + 1;
9
10
           end
11
           win_fraction = win_times / n;
12
13
           fprintf('Player won %d times in %d games\n', win_times, n);
14
           fprintf('The fraction of player winning is %.2f\n', win_fraction);
15
16
           win_game = win_times;
17
       end
18
19 🖃
       function win = playRound()
20
           diceRoll1 = randi([1,2]);
21
           diceRoll2 = randi([1,2]);
22
           sum = diceRoll1 + diceRoll2;
23
           win = 0;
24
25
           if sum \sim=4
26
               old_sum = sum;
               while true
27 🗀
28
                    diceRoll1 = randi([1,2]);
29
                    diceRoll2 = randi([1,2]);
30
                    new_sum = diceRoll1 + diceRoll2;
31
                    if new_sum == 4
32
                       break;
33
                    elseif new_sum == old_sum
34
                       win = 1;
35
                        break;
36
                    end
37
               end
38
           end
39
       end
```

```
>> win_game_i = partl_i();
Player won 5 times in 10 games
The fraction of player winning is 0.50

fx >>
```

I-j)
Change the n in I-i) from 10 to 50000.
Code:

```
1
        %part1_j
 2 📮
        function win_game = part1_j()
 3
            n = 50000;
 4
            win_times = 0;
 5
            for i = 1:n
 6 🖨
                if playRound()
 7
 8
                    win_times = win_times + 1;
 9
10
            end
11
            win_fraction = win_times / n;
12
            fprintf('Player won %d times in %d games\n', win_times, n);
13
14
            fprintf('The fraction of player winning is %.2f\n', win_fraction);
15
16
            win_game = win_times;
17
        end
18
19 🖃
        function win = playRound()
            diceRoll1 = randi([1,2]);
20
            diceRoll2 = randi([1,2]);
21
            sum = diceRoll1 + diceRoll2;
22
23
            win = 0;
24
25
            if sum \sim=4
                old_sum = sum;
26
                while true
27 😑
                    diceRoll1 = randi([1,2]);
28
29
                    diceRoll2 = randi([1,2]);
30
                    new_sum = diceRoll1 + diceRoll2;
31
                    if new_sum == 4
32
                        break;
33
                    elseif new_sum == old_sum
                        win = 1;
34
35
                        break;
36
                    end
37
                end
38
            end
39
        end
Result:
```

```
>> win_game_j = partl_j();
Player won 22869 times in 50000 games
The fraction of player winning is 0.46

fx >>
```

I-k)

Let the probability of incident A is "win" and the probability of incident B is "lose". From I-j), we have:

$$P(A) = \frac{22869}{50000} = 45.74\%$$

$$P(B) = \frac{27131}{50000} = 54.25\%$$

From I-i), we have:

$$P(A) = \frac{5}{10} = 50.00\%$$

$$P(B) = \frac{5}{10} = 50.00\%$$

But in I-e), we have:

$$P(A) = \frac{11}{24} = 45.83\%$$

$$P(B) = \frac{5}{10} = 54.17\%$$

The outcomes in I-e) and I-k) are very closed to theoretical calculate results.

I-l)
Repeat I-h) under new condition.

Code:

```
1
       %part1_1_1
2 🖃
       function win_game = part1_1_1()
3
 4
           diceRoll1 = randi([1,6]);
 5
           diceRoll2 = randi([1,6]);
 6
           sum = diceRoll1 + diceRoll2;
 7
           win_game = 0;
 8
9
           if sum \sim= 12
10
               old_sum = sum;
11 📥
                while true
12
                    diceRoll1 = randi([1,6]);
                    diceRoll2 = randi([1,6]);
13
14
                    new_sum = diceRoll1 + diceRoll2;
15
                    if new_sum == 12
16
                        break;
17
                    elseif new_sum == old_sum
18
                        win_game = 1;
19
                        break;
20
                    end
                end
21
22
           end
23
24
            if win game == 1
25
                fprintf('win\n');
26
27
                fprintf('lose\n');
28
           end
29
       end
```

Result:

```
>> win_game_1_1 = part1_1_1();
win
fx >>
```

Repeat I-i) under new condition.

```
1
       %part1_1_2
2 🖃
       function win_fraction = part1_1_2()
3
           n = 10;
 4
           win_times = 0;
 5
 6 🗀
           for i = 1:n
 7
               win_game = dicegame();
 8
               win_times = win_times + win_game;
 9
           end
10
11
           win_fraction = win_times / n;
12
           fprintf('Player won %d times in %d games\n', win_times, n);
13
           fprintf('The fraction of players winning is %.2f\n', win_fraction);
14
15
       function win_game = dicegame()
16 🖃
17
           diceRoll1 = randi([1,6]);
           diceRoll2 = randi([1,6]);
18
           sum = diceRoll1 + diceRoll2;
19
20
           win_game = 0;
21
22
           if sum ~= 12
23
               old_sum = sum;
24 崫
               while true
25
                   diceRoll1 = randi([1,6]);
                   diceRoll2 = randi([1,6]);
26
                   new_sum = diceRoll1 + diceRoll2;
27
28
                    if new_sum == 12
29
                        break;
30
                    elseif new_sum == old_sum
31
                        win_game = 1;
32
                        break;
33
                    end
34
                end
35
           end
36
       end
37
```

```
>> win_game_1_2 = part1_1_2();
Player won 7 times in 10 games
The fraction of players winning is 0.70

fx >>
```

Repeat I-j) under new condition.

```
1
       %part1_1_3
2 🖃
       function win_fraction = part1_1_3()
3
           n = 50000;
4
           win_times = 0;
5
6 🗕
           for i = 1:n
7
               win_game = dicegame();
8
               win_times = win_times + win_game;
9
10
11
           win_fraction = win_times / n;
           fprintf('Player won %d times in %d games\n', win_times, n);
12
13
           fprintf('The fraction of players winning is %.2f\n', win_fraction);
14
       end
15
16 🗐
       function win_game = dicegame()
17
           diceRoll1 = randi([1,6]);
18
           diceRoll2 = randi([1,6]);
           sum = diceRoll1 + diceRoll2;
19
20
           win_game = 0;
21
           if sum ~= 12
22
23
               old_sum = sum;
24 🗀
               while true
25
                   diceRoll1 = randi([1,6]);
26
                   diceRoll2 = randi([1,6]);
27
                   new_sum = diceRoll1 + diceRoll2;
28
                    if new_sum == 12
29
                        break;
30
                    elseif new_sum == old_sum
31
                        win_game = 1;
32
                        break;
33
                    end
34
                end
35
           end
36
       end
37
38
```

```
>> win_game_1_3 = partl_1_3();
Player won 38171 times in 50000 games
The fraction of players winning is 0.76

fx >>
```

Let the probability of incident A is "win" and the probability of incident B is "lose". So P(A) is about 76% and P(B) is about 24%.

## Part II:

II-a)

Code:

```
%part2_a
        function win_game = part2_a()
3
           n = 1;
 4
            player_stick = 0;
 5
            player_switch = 0;
 6
            for i = 1 : n
 7
               disp(['Time: ', num2str(i)]);
car = randi([1,3],1);
 8
 9
10
                choose = randi([1,3],1);
                doors = [1 2 3];
11
12
                doors([car, choose]) = 0;
13
                host = doors(doors ~= 0);
14
                open = host(randi(length(host)));
15
                if car == choose
16
17
                    disp('The player won by sticking her choice.');
18
                    player_stick = player_stick + 1;
19
20
                   disp('The player lost by sticking her choice.');
21
22
23
                change = setdiff([1, 2, 3], [choose, open]);
24
                if car == change
25
                    disp('The player won by switching her choice.');
26
                    player_switch = player_switch + 1;
27
                else
                   disp('The player lost by switching her choice.');
28
               end
29
30
            end
31
            disp(' ');
32
           disp(['The probability of winning the car if the player sticks: ', num2str(player_stick/n)]);
disp(['The fraction of times that the player wins when sticking: ', num2str(player_stick)]);
disp(['The probability of winning the car if the player switches: ', num2str(player_switch/n)]);
disp(['The fraction of times that the player wins when switching: ', num2str(player_switch)]);
33
34
35
36
37
38
            win_game = 0;
39
Result:
     >> win_game_a = part2_a();
     Time: 1
     The player lost by sticking her choice.
     The player won by switching her choice.
     The probability of winning the car if the player sticks: 0
     The fraction of times that the player wins when sticking: 0
     The probability of winning the car if the player switches: 1
     The fraction of times that the player wins when switching: 1
 fx >>
```

II-b)

Repeat II-a) under new conditions.

Result:

```
>> win game b = part2 b();
  Time: 1
  The player lost by sticking her choice.
  The player won by switching her choice.
  Time: 2
  The player won by sticking her choice.
  The player lost by switching her choice.
  The player lost by sticking her choice.
  The player won by switching her choice.
  The player won by sticking her choice.
  The player lost by switching her choice.
  Time: 5
  The player lost by sticking her choice.
  The player won by switching her choice.
  Time: 6
  The player won by sticking her choice.
  The player lost by switching her choice.
  Time: 7
  The player lost by sticking her choice.
  The player won by switching her choice.
  Time: 8
  The player lost by sticking her choice.
  The player won by switching her choice.
  Time: 9
  The player won by sticking her choice.
  The player lost by switching her choice.
  Time: 10
  The player lost by sticking her choice.
  The player won by switching her choice.
  The probability of winning the car if the player sticks: 0.4
  The fraction of times that the player wins when sticking: 4
  The probability of winning the car if the player switches: 0.6
  The fraction of times that the player wins when switching: 6
f_{\underline{x}} >>
```

# II-c)

Repeat II-a) under new conditions.

#### Result

```
The probability of winning the car if the player sticks: 0.33506
The fraction of times that the player wins when sticking: 16753
The probability of winning the car if the player switches: 0.66494
The fraction of times that the player wins when switching: 33247

Ex. >>
```

### II-d)

Let the probability of incident A is "choose car firstly and switch" and the probability of incident B is "choose goat firstly and switch".

$$P(A) = 0$$

Because the player can't choose the car after she switches.

$$P(B) = \frac{2}{3}$$

Because the player has the probability to choose goats, which is  $\frac{2}{3}$  firstly. The host shows another goat and she is absolutely choosing the car after she switches.

So 
$$P = P(A) + P(A) = \frac{2}{3} = 0.667$$

II-e)

In I-b) the probability of winning after switching is 0.6, and in I-c), which is 0.66494. Obviously, I-c) is very closed to theoretical calculate result with more simulation times.

II-f)

She should switch. If she sticks, the probability of winning is 0.333. But the probability is 0.667 if she switches.