Introduction to Programming using R

Organizational Matters

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Timetable:

- Monday - Thursday, 9am - 4pm; Friday, 9am - 2pm
- Morning: Lectures and presenation of solutions
- Lunchbreak: 12-1pm
- Afternoon: Supervised learning

Preliminaries

Before starting the class make sure to install the required software. We will be using the programming language R [R Core Team, 2019] and the development environment RStudio [RStudio Team, 2015]. We recommend first installing R and then RStudio (desktop).[1]

[1] In case you have problems with the installation process press here.

- R: `https://cran.rstudio.com`
- RStudio: `https://rstudio.com/products/rstudio/download`

1 *Introduction*

"[...] computers and mathematics are like beer and potato chips: two fine tastes that are best enjoyed together. Mathematics provides the foundations of our models and of the algorithms we use to solve them. Computers are the engines that run these algorithms." – [Stachurski, 2009]

In this section we will present the very basics of R. We will go through some arithmetic, variables, special numerical objects, comments and data types.[2]

[2] All statements are typed into the R console and the results are displayed after `[1]`.

1.1 *Arithmetic*

```
1 + 1
```

```
[1] 2
```

```
1 - 1
```

```
[1] 0
```

Decimals:[3]

[3] Note that the decimal mark is denoted by a dot (.) and not a comma (,).

```
2.5 * 4
```

```
[1] 10
```

```
1 / 3
```

```
[1] 0.3333333
```

```
2 ^ 2 ^ 3
```

```
[1] 256
```

```
2 ** 2 ** 3
```

```
[1] 256
```

Parentheses:[4]

[4] If in doubt use parentheses to ensure that R will compute the correct expression.

```
(2 ** 2) ** 3
```

```
[1] 64
```

1.2 Variables

```
x <- 10 + 5
x
```

```
[1] 15
```

```
x <- x + x
x
```

```
[1] 30
```

```
x ** 2
```

```
[1] 900
```

```
y <- x
y
```

```
[1] 30
```

1.3 Special Numerical Objects

Constants, infinity and NaNs (Not a Number):

```
pi
```

```
[1] 3.141593
```

```
1/0
```

```
[1] Inf
```

```
0/0
```

```
[1] NaN
```

1.4 Data Types

- `numeric`: `x <- 1.25`
- `integer`: `x <- 1L`
- `character`: `x <- "this_works"`[5]
- `logical`: `x <- TRUE`, `y <- FALSE`
- `complex`: `x <- 1 + 2i`

[5] This data type is also known as a *String*.

2 Data Structures

In this section we consider the most common data structures used in R. This includes

- `list`
- `(atomic) vector`
- `matrix`
- `data.frame`

2.1 Lists

Lists are objects which can contain different objects of different data types.[6] [7]

```
x <- list(1, 1.25, "this works?")
x
```

```
[[1]]
[1] 1

[[2]]
[1] 1.25

[[3]]
[1] "this works?"
```

[6] `list` is a (built-in) function which takes as argument multiple objects and combines them to a list. See section function.`print` is a (built-in) function which prints its argument on the console.

[7] `length` is a (built-in) function which returns the length of its argument.

```
x <- list(x, 1 + 2i)
x
```

```
[[1]]
[[1]][[1]]
[1] 1

[[1]][[2]]
[1] 1.25

[[1]][[3]]
[1] "this works?"


[[2]]
[1] 1+2i
```

```
length(x)
```

```
[1] 2
```

2.2 *Vectors*

Vectors are similar to lists in that they can contain multiple objects, however, any vector can contain only objects of one data type.[8]

[8] `c` is a (built-in) function which takes as argument multiple objects of the same data type and combines them to a vector. `c` stands for `combine`.

```
x <- c(1, 2, 3)
x
```

```
[1] 1 2 3
```

```
x <- c("this", "actually", "works")
x
```

```
[1] "this"     "actually" "works"
```

```
x <- c("wait?", 1)
x
```

```
[1] "wait?" "1"
```

```
length(x)
```

```
[1] 2
```

2.3 *Indexing of Lists and Vectors*

We access elements of lists and vectors via their index. If `x` is a list (or vector) we get the `i`-th element as `x[i]`.[9] Note that for a list (or vector) of length `n` we can of course only ask for elements `1` to `n`, otherwise R returns `NA` which stands for Not Available. If we supply a vector of indices we can access more than one element, i.e. `x[c(1, 3, 5)]` will return the first, third and fifth element of `x`.
Examples:[10]

[9] This is slightly different to many other programming languages which start indexing at `0` instead of `1`.

[10] Note the difference between `y[1]` and `y[[1]]` for lists.

```
x <- c(2, 4, 6, 8, 10)
x[1]
```

```
[1] 2
```

```
x[2]
```

```
[1] 4
```

```
x[6]
```

```
[1] NA
```

```
x[6] <- 0
x
```

```
[1]  2  4  6  8 10  0
```

```
x[c(1, 3, 5)]
```

```
[1]  2  6 10
```

```
x[3] <- 100
x
```

```
[1]   2   4 100   8  10   0
```

```
x[c(2, 4)] <- -100
x
```

```
[1]    2 -100  100 -100   10    0
```

```
x[-1]
```

```
[1] -100  100 -100   10    0
```

```
x[-c(1, 2)]
```

```
[1]  100 -100   10    0
```

```
y <- list(1, 1.25, "this works?")
y[1]
```

```
[[1]]
[1] 1
```

```
y[[1]]
```

```
[1] 1
```

Useful commands:[11] [12]

[11] `seq` is a (built-in) function which produces sequences from a specified number to another; with the extra argument `by` one can specify the increment; with the extra argument `length.out` one can specify the desired length of the sequence.

[12] A quick way to create sequences which increment by one is by using the syntax `a:b` to create the sequence (vector) `c(a, a + 1, ..., b - 1, b)`.

```
x <- 1:10
x
```

```
 [1]  1  2  3  4  5  6  7  8  9 10
```

```
x <- seq(from=1, to=10, by=2)
x
```

```
[1] 1 3 5 7 9
```

```
x <- seq(from=0, to=1, length.out=20)
x
```

```
 [1] 0.00000000 0.05263158 0.10526316 0.15789474 0.21052632 0.26315789
 [7] 0.31578947 0.36842105 0.42105263 0.47368421 0.52631579 0.57894737
[13] 0.63157895 0.68421053 0.73684211 0.78947368 0.84210526 0.89473684
[19] 0.94736842 1.00000000
```

2.4 Calculating with Vectors

```
x <- 1:10
x
```

```
 [1]  1  2  3  4  5  6  7  8  9 10
```

```
y <- -4:5
y
```

```
 [1] -4 -3 -2 -1  0  1  2  3  4  5
```

```
x + y
```

```
 [1] -3 -1  1  3  5  7  9 11 13 15
```

```
x * y
```

```
 [1] -4 -6 -6 -4  0  6 14 24 36 50
```

```
x ** y
```

```
 [1] 1.000000e+00 1.250000e-01 1.111111e-01 2.500000e-01 1.000000e+00
 [6] 6.000000e+00 4.900000e+01 5.120000e+02 6.561000e+03 1.000000e+05
```

```
2 * x
```

```
 [1]  2  4  6  8 10 12 14 16 18 20
```

```
10 + x
```

```
 [1] 11 12 13 14 15 16 17 18 19 20
```

```
x ** 2
```

```
 [1]   1   4   9  16  25  36  49  64  81 100
```

Recycling:

```
x <- 1:4
y <- c(1, 5, 10)
x + y
```

```
Warning in x + y: longer object length is not a multiple of shorter object
length
```

```
[1]  2  7 13  5
```

(Some) useful functions:[13]

[13] `sum` is a (built-in) function which sums all elements of its argument (also works on matrices). `mean` computes the mean of its argument, `sd` the (unbiased) standard deviation, `var` the variance and `cumsum` the cumulative sum.

```
x <- -5:5
x
```

```
 [1] -5 -4 -3 -2 -1  0  1  2  3  4  5
```

```
sum(x)
```

```
[1] 0
```

```
mean(x)
```

```
[1] 0
```

```
sd(x)
```

```
[1] 3.316625
```

```
var(x)
```

```
[1] 11
```

```
cumsum(x)
```

```
 [1]  -5  -9 -12 -14 -15 -15 -14 -12  -9  -5   0
```

2.5 *Matrices*

Matrices represent two dimensional arrays which works similar to vectors in that matrices can only contain objects of a single data type. To create a matrix we need to know how many rows and columns it should have and what data it should contain.[14]

[14] Note that `1:x` is equivalent to `c(1,2,...,x)`. Also note `matrix(data, rows, cols)` is not equal to `matrix(data, cols, rows)`; if you do not know which argument comes when, simply ask R for help: `?matrix`. (This works for any `R` function, just type `?function_name`).

```
data <- 1:9
rows <- 3
cols <- 3

x <- matrix(data, rows, cols)
x
```

```
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
```

```
y <- matrix(data, rows, cols, byrow=TRUE)
y
```

```
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
```

```
dim(x)
```

```
[1] 3 3
```

```
nrow(x)
```

```
[1] 3
```

```
ncol(x)
```

```
[1] 3
```

2.6 *Combining Vectors and Matrices*

When computing different intermediate results it is often useful to combine them to get an end result.[15]

[15] The (built-in) function `rbind` takes two matrices (or data frames) as input and stacks them on top of each other (on the rows). Similarly, `cbind`, stacks the two arrays next to each other (on the columns).

```
x <- matrix(1:9, 3)
x
```

```
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
```

```
y <- matrix(1:6, 2)
y
```

```
     [,1] [,2] [,3]
[1,]    1    3    5
[2,]    2    4    6
```

```
z <- rbind(x, y)
z
```

```
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
[4,]    1    3    5
[5,]    2    4    6
```

```
x <- cbind(1:3, 4:6)
x
```

```
     [,1] [,2]
[1,]    1    4
[2,]    2    5
[3,]    3    6
```

(Some) useful functions:[16]

[16] `rowSums` computes the sum of each row of a matrix (or data frame) and returns the resulting vector. `colSums` , `rowMean` and `colMeans` work analogously.

```
m <- matrix(1:9, 3)
m
```

```
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
```

```
rowSums(m)
```

```
[1] 12 15 18
```

```
colSums(m)
```

```
[1]  6 15 24
```

```
rowMeans(m)
```

```
[1] 4 5 6
```

In more general settings we might wish to apply an arbitrary function to the rows or columns of a matrix. We can do this with the function `apply`.[17] Example:

[17] `apply(X, MARGIN, FUN)`, `X` = matrix of interest, `MARGIN` = 1 to apply the function over the rows and 2 to apply the function over the columns, `FUN` = the function of interest.

```
m <- matrix(1:9, 3)
apply(m, 1, sd)
```

```
[1] 3 3 3
```

```
apply(m, 2, sd)
```

```
[1] 1 1 1
```

2.7 Matrix algebra[18]

```
X <- matrix(1:9, 3)
y <- -1:1

X * y
```

```
     [,1] [,2] [,3]
[1,]   -1   -4   -7
[2,]    0    0    0
[3,]    3    6    9
```

```
X %*% y
```

```
     [,1]
[1,]    6
[2,]    6
[3,]    6
```

```
X * X
```

```
     [,1] [,2] [,3]
[1,]    1   16   49
[2,]    4   25   64
[3,]    9   36   81
```

```
X %*% X
```

```
     [,1] [,2] [,3]
[1,]   30   66  102
[2,]   36   81  126
[3,]   42   96  150
```

(Some) useful functions:[19]

```
t(X)
```

```
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
```

[18] Note the difference between `X * y` and `X %*% y`; the first multiples the `i`th element of `y` onto the `i`th row of `X` and the second computes the regular matrix product known from linear algebar.

[19] `t` computes the inverse of its argument, which can be either a matrix or a data frame. `diag` returns the diagonal entries of its argument. `solve` can be used to either solve a linear system of equations or compute the inverse of its argument: if we provide `solve` with one matrix (or data frame) then it returns the inverse; if we supply a matrix (or data frame) and a vector, `solve` returns the solution to the system of linear equations. That is, $\text{solve}(X) = X^{-1}$ and $\text{solve}(X, y) = b$ with $Xb = y$ (if the system has a solution).

```
diag(X)
```

```
[1] 1 5 9
```

```
A <- matrix(c(1, 10, -2, 3), 2)
A
```

```
     [,1] [,2]
[1,]    1   -2
[2,]   10    3
```

```
solve(A)
```

```
           [,1]       [,2]
[1,]  0.1304348 0.08695652
[2,] -0.4347826 0.04347826
```

```
b <- c(-1, 1)
solve(A, b)
```

```
[1] -0.04347826  0.47826087
```

2.8 Data Frames

Data frames represent data sets. The difference to matrices is that different columns can have different data types. Note that there are many different ways of creating a data frame.

```
x <- c("Micheal", "Sofia", "Jonah")
y <- c(1.0, 1.3, 3.0)
z <- c("a", "b", "c")

df <- data.frame(name=x, grades=y, type=z)
df
```

```
     name grades type
1 Micheal    1.0    a
2   Sofia    1.3    b
3   Jonah    3.0    c
```

```
m <- matrix(1:9, 3)
df <- as.data.frame(m)
df
```

```
  V1 V2 V3
1  1  4  7
2  2  5  8
3  3  6  9
```

The iris data set:[20]

[20] The function `head` returns the first few rows of a data frame, which can be useful to have a quick glance at a data frame. `str` returns the internal structure of its argument and is particularly useful for data frames to output the key information in a data set.

```
head(iris)
```

```
  Sepal.Length Sepal.Width Petal.Length Petal.Width Species
1          5.1         3.5          1.4         0.2  setosa
2          4.9         3.0          1.4         0.2  setosa
3          4.7         3.2          1.3         0.2  setosa
4          4.6         3.1          1.5         0.2  setosa
5          5.0         3.6          1.4         0.2  setosa
6          5.4         3.9          1.7         0.4  setosa
```

```
str(iris)
```

```
'data.frame':   150 obs. of  5 variables:
 $ Sepal.Length: num  5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...
 $ Sepal.Width : num  3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...
 $ Petal.Length: num  1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...
 $ Petal.Width : num  0.2 0.2 0.2 0.2 0.2 0.4 0.3 0.2 0.2 0.1 ...
 $ Species     : Factor w/ 3 levels "setosa","versicolor",..: 1 1 1 1 1 1 1 1 1 1 ...
```

2.9 Indexing of Matrices and Data Frames

Matrices and data frames constitute two dimensional objects, this means we can ask for submatrices, columns, rows or individual elements.

```
m <- matrix(1:9, 3)
m
```

```
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
```

```
m[1, 1]
```

```
[1] 1
```

```
m[1, ]
```

```
[1] 1 4 7
```

```
m[, 1]
```

```
[1] 1 2 3
```

```
m[c(1, 2), c(2, 3)]
```

```
     [,1] [,2]
[1,]    4    7
[2,]    5    8
```

When dealing with data frames we can also access the columns by their respective names.[21]

[21] To access a column of a data frame by name use `df$column_name`. Note the different results of `df[["grade"]]` and `df["grade"]`.

```
df <- data.frame(name=c("Thomas", "Susan"), grade=c(1, 2))
df
```

```
    name grade
1 Thomas     1
2  Susan     2
```

```
df$name
```

```
[1] Thomas Susan
Levels: Susan Thomas
```

```
df[["grade"]]
```

```
[1] 1 2
```

```
df["grade"]
```

```
  grade
1     1
2     2
```

3 *Digression on asserting data types*

When building programs which handle data and objects that are unknown while developing the code it is often necessary to check of what type they are. Say we get an object (a variable) `x` from somewhere and we need to evaluate if its a number or data frame; and if it is a number, is it an integer or a real number. For these questions are supplies the many functions of the type `is.vector`, `is.matrix`, `is.integer`. We will not list them all and only provide a small example.

```
x <- 1:10
is.integer(x)
```

```
[1] TRUE
```

```
is.numeric(x)
```

```
[1] TRUE
```

```
is.vector(x)
```

```
[1] TRUE
```

```
is.data.frame(x)
```

```
[1] FALSE
```

```
is.function(x)
```

```
[1] FALSE
```

4 *Logical Operators*

IN THIS SECTION we consider logical operators which form the direct equivalent to logical operators in mathematics. We first note that we can induce boolean values by comparison via relations (`<`, `>`, `<=`, `>=`) or (in)equalities (`==`, `!=`). On boolean values we may use logical operators as *and* (`&`), *or* (`|`), but also quantifier as $\exists$ (`any`) and $\forall$ (`all`).

```
x <- 1:10
```

```
x < 5
```

```
 [1]  TRUE  TRUE  TRUE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE
```

```
sum(x < 5)
```

```
[1] 4
```

```
x[x < 5]
```

```
[1] 1 2 3 4
```

```
x[!(x < 5)]
```

```
[1]  5  6  7  8  9 10
```

```
x[(x < 3) | (x > 7)]
```

```
[1]  1  2  8  9 10
```

```
x[(x < 8) & (x > 3)]
```

```
[1] 4 5 6 7
```

```
any(x < 8)
```

```
[1] TRUE
```

```r
all(x < 8)
```

```
[1] FALSE
```

Often we are interested in the (indices of the) elements of a vector (matrix) that fulfill a certain condition.

```r
x <- c(3, 2, -100, 400)
which(x > 100)
```

```
[1] 4
```

5 *Conditional Expressions*

In many scenarios our decisions depend on the specific state of the situation. For example, *if* it rains we will take the umbrella with us. Or a little more complex. *If* it rains we will take the umbrella, otherwise, *if* we fixed the flat bike tires already we will go by bike. (We illustrate the a fictional conditional decision tree on the blackboard.) This brings us to conditional expressions.

5.1 *if*

```r
x <- 10
if (x < 10) {
  print("x is smaller than 10.")
}
```

5.2 *else*

```r
x <- 10
if (x < 10) {
  print("x is smaller than 10.")
} else{
  print("x is *not* smaller than 10.")
}
```

```
[1] "x is *not* smaller than 10."
```

5.3 *else if*

```r
x <- 10
if (x < 10) {
  print("x is smaller than 10.")
} else if (x > 0) {
  print("x is between 0 and 10.")
} else {
  print("x is either smaller than 0 or bigger than 10.")
}
```

```
[1] "x is between 0 and 10."
```

5.4 *Short digression into* `User Input`

Sometimes we want to write programs which work in many different scenarios that can be specified by the user of the program.[22] [23]

[22] Note that the (built-in) function `readline` reads input from the user in the `R` console and stores it as a string.

[23] The function `as.integer` takes as argument an `R` object and tries to *coerce* the input to an `integer` if possible; For example the string `"1"` can be coerced to a `1` but the string `"text"` cannot.

```
cat("Please choose which type of regression should be run:\n")
x <- readline(prompt="Linear regression (1); Polynomial regression (2): ")
x <- as.integer(x)

if (x == 1) {
  print("Okay lets do linear regression!")
} else if (x == 2) {
  print("Oh no I hate polynomial regression :(")
} else {
  print("There were only two options what did you do?")
}
```

6 *Control Flow Statements*

When working on nearly any project we often find ourselves repeating simple tasks over and over again. If this happens with tasks that cannot be managed on a computer we hire research assistants; however, if it can be done on a computer there are cheaper ways.

6.1 *For Loops*

Let's say we want to create a list with 10 entries and the `i`th entry is a matrix of dimension `ixi` filled with numbers 1 to i^2. This can be achieved very easily with a `for` loop.

```
matrices <- list()
for (i in 1:10) {
  imatrix <- matrix(1:(i ** 2), nrow=i)

  matrices[[i]] <- imatrix
}
matrices[[5]]
```

```
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    6   11   16   21
[2,]    2    7   12   17   22
[3,]    3    8   13   18   23
[4,]    4    9   14   19   24
[5,]    5   10   15   20   25
```

6.2 *Short digression into Monte Carlo simulation*

Say we have two uniform random variables on $[0,1]$, i.e. $X, Y \sim \mathcal{U}[0,1]$. And say we want to estimate $\mathbb{P}(X+Y \in [0.75, 1.25])$ without doing any analytical mathematics. One solution to problems of this kind are so called *Monte Carlo estimates*, in which we *simulate* (in this case) two uniform random variables for many many times and each time we simply check if the sum of the realizations fulfills the statement. The frequency of times when the statement was fulfilled then approximates the probability.[24] [25]

[24] In the latter chapters we will consider the function `runif` in more detail; for here only note that `runif(1, 0, 1)` evaluates to a realization of a uniform random variable on $[0,1]$.

[25] Note the use of `#` which tells `R` to ignore the following statement; These are called comments and should be used to clarify ones code.

```
count <- 0
nsim <- 10000
for (i in 1:nsim) {
  x <- runif(1, min=0, max=1)
  y <- runif(1, min=0, max=1)

  z <- x + y
  if (z >= 0.75 && z <= 1.25) {
    count <- count + 1
  }

}
count / nsim # analytical solution = 7/6 = 0.4375
```

```
[1] 0.442
```

6.3 *While Loops*

For loops are very useful if we know exactly how many times we need to execute some statement. If we do not know the number of repetitions before starting the loop we can use `while` loops.
Cherry picking results:

Once we introduced linear models and ordinary least squares regression we will show a simple example on how to cherry pick your data such that you can claim statistical significance even if there is none.
Example:

```
userinput <- NULL
while(is.null(userinput)) {
  input <- readline("Type in a number between 0 and 10. \n")
  input <- as.integer(input)

  if (is.numeric(input)) {
    if (input >= 0 && input <= 10) {
      userinput <- input
    }
  }
}
userinput
```

7 Functions

FUNCTIONS ARE arguably the most important building block when writing large programs. We have already seen the use of many (built-in) functions. Functions, in general, allow us to use a piece of code multiple times in a program without repeating all of the code at every instance.

7.1 A normal example

Say we need to compute the value of a normal density with mean `mu` and standard deviation `sigma` at some point `x`. In case we need to compute this value for many different means, variances or points we can save on time (and erros) by implementing a function once.[26] [27]

[26] Note the use of special (built-in) mathematical functions `sqrt` and `exp`, which compute the square root and exponential of their arguments, respectively.

[27] The (built-in) function `curve` displays the graph of a (mathematical) function.

```
normaldensity <- function(x, mu, sigma) {
  constant <- 1 / sqrt(2 * pi * sigma ** 2)
  exponential <- exp(- (x - mu) ** 2 / (2 * sigma ** 2))

  return(constant * exponential)
}

normaldensity(x=0, mu=0, sigma=1)
```

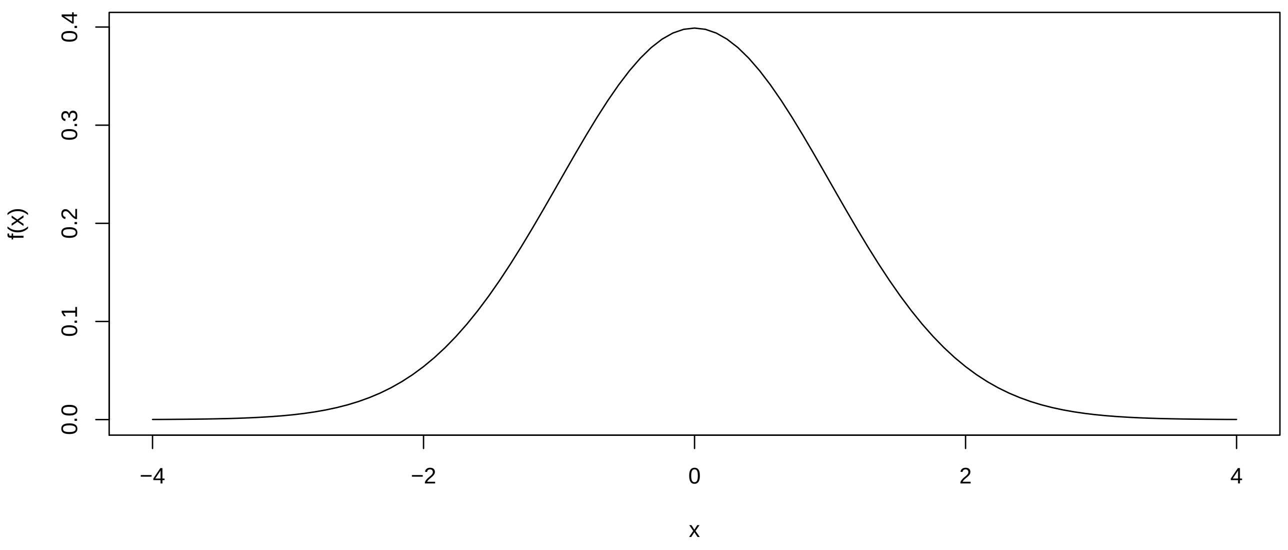
```
[1] 0.3989423
```

```
normaldensity(x=1, mu=0, sigma=1)
```

```
[1] 0.2419707
```

```
standardnormaldensity <- function(x) {
  normaldensity(x, mu=0, sigma=1)
}

curve(standardnormaldensity, from=-4, to=4, xlab="x", ylab="f(x)")
```



7.2 *Recursive functions*

The Fibonacci sequence is defined by the following (recursive) function

$$f(n) = \begin{cases} 0, \, n = 0 \\ 1, \, n = 1 \\ f(n-1) + f(n-2), \, n > 1 \,. \end{cases}$$

We can implement this function easily using an `R` function.[28]

```
fibonacci <- function(n) {
  if (n == 0) {
    return(0)
  } else if (n == 1) {
    return(1)
  } else {
    return(fibonacci(n - 1) + fibonacci(n - 2))
  }
}

n <- 1:10
fib <- sapply(n, fibonacci)
rbind(n, fib)
```

```
    [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
n      1    2    3    4    5    6    7    8    9    10
fib    1    1    2    3    5    8   13   21   34    55
```

[28] The (built-in) function `sapply` applies a function to each element of its first argument, which is typically a `list` or a `vector`.

References

R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2019. URL `https://www.R-project.org/`.

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