CSC 212: Data Structures and Abstractions

Hash Tables (part 1)

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THINK BIG WE DO℠

Data Structure	Worst-case			Average-case			Ordered?
	insert at	delete	search	insert at	delete	search	
sequential (unordered)	O(n)	O(n)	O(n)	O(n)	O(n)	O(n)	No
sequential (ordered) binary search	O(n)	O(n)	O(log n)	O(n)	O(n)	O(log n)	Yes
BST	O(n)	O(n)	O(n)	O(log n)	O(log n)	O(log n)	Yes
2-3-4	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	Yes
Red-Black	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	Yes

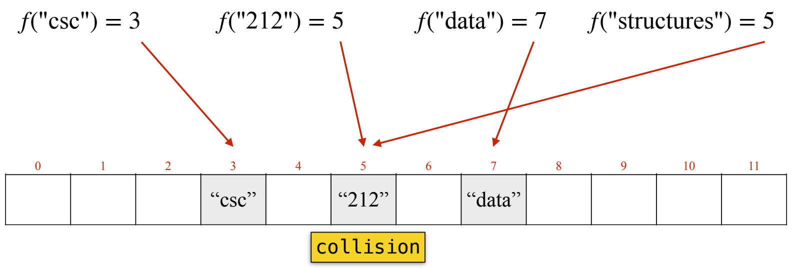
Can we do better?

Random access memory

- Random Access Memory (RAM)
  - ✓ fundamental principle in computer science
  - ✓ enables constant-time access to any memory location, given its address — foundation for efficient array operations
  - ✓ arrays exploit RAM by mapping indices directly to memory addresses
- Arrays in C++
  - ✓ contiguous memory allocation
  - ✓ homogeneous elements (same data type)
  - ✓ fixed size (determined at creation, cannot be modified)
  - ✓ zero-based indexing
  - ✓ $O(1)$ random access

Hash tables

- A hash table is a data structure that implements an associative array
  - ✓ stores keys (**set**), or key-value pairs (**map**)
  - ✓ uses a **hash** function to compute an array index from a key
  - ✓ provides average-case $O(1)$ for search, insertion, deletion



Hash function

Hash function

- A hash function maps an input key to an integer value
  - ✓ hash value is then mapped to a valid array index using modulo
- Essential properties
  - ✓ **deterministic**: same key must always produce the same hash value
  - ✓ **uniform distribution**: hash values should be spread evenly
  - ✓ **efficient to compute**: execute in $O(1)$ time

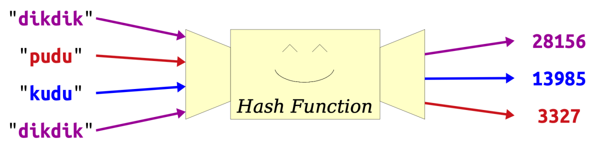


Image credit: CS106B @ Stanford

Hash functions

- Space efficiency
  - ✓ supporting all possible keys directly would require enormous arrays => use a hash function to map keys to a much smaller array of size m (the **capacity** of the array)

```
// if hash() returns non-negatives
index = hash(key) % capacity

// if hash() returns any integer
index = abs(hash(key) % capacity)
```

The **load factor** α in a hash table is the ratio of N, the number elements, to M, the total capacity

Practice

- Which of the following tables is a better choice?
  - ✓ what is the load factor α?

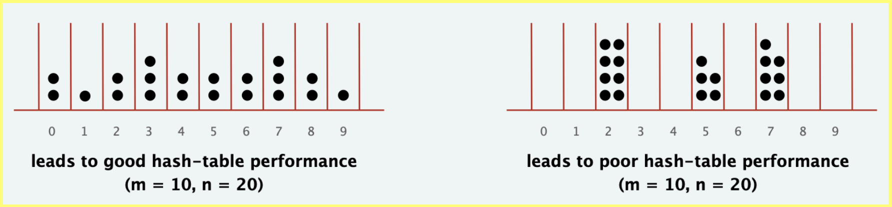

0 1 2 3 4 5 6 7 8 9
leads to good hash-table performance (m = 10, n = 20)
0 1 2 3 4 5 6 7 8 9
leads to poor hash-table performance (m = 10, n = 20)

Image credit: COS 226 @ Princeton

Designing hash functions

- Design principles
  - ✓ good hash functions satisfy: determinism, efficiency, and uniform distribution
- Hash functions on different data types
  - ✓ **integers**: may use the integer value as the hash value, or may apply transformations to break patterns
  - ✓ **floats**: convert to binary and treat as an integer, or manipulate the bits (e.g. XOR the mantissa and exponent)
  - ✓ **strings**: use the polynomial rolling hash ($31x + y$ rule) or other variants
  - ✓ **compound objects**: combine hash values of individual fields, may use the $31x + y$ rule or others

Designing hash functions

- Importance of a hash function
  - ✓ determines hash table's storage capacity
  - ✓ directly impacts collision probability
  - ✓ influences overall data structure performance
- Size selection strategies — mapping hashes into $[0, M-1]$
  - ✓ **M is prime**: safe default, helps distributing keys more uniformly, minimizing collisions
    - hash % M
  - ✓ **M is a power of two**: faster, enables fast modulo operation via bitwise AND
    - hash & $(M-1)$

Collisions

- Definition
  - ✓ occurs when two distinct keys hash to the same index in the table
    - inevitable consequence when $\alpha > 1$
- Resolution strategies:
  - ✓ **separate chaining**: each table slot contains a linked list (or other secondary structure) of all elements hashing to that index
    - insertion $O(1)$, search/delete $O(1+\alpha)$
  - ✓ **open addressing**: all elements stored directly in the table, collisions trigger probe sequences
    - open addressing is more space-efficient than chaining, but it can be slower

Hash functions (beyond hash tables)

- Storing passwords
  - ✓ never store plaintext passwords — use cryptographic hash functions
- File verification
  - ✓ checksum mechanism — detect file corruption and provide data integrity validation
  - ✓ verification process: generate and publish file hash, client recomputes hash of downloaded file, compare computed and published hashes
- Cryptographic hash functions
  - ✓ one-way property: computationally infeasible to reverse the hash, find input producing specific hash, generate collision
  - ✓ MD5, SHA-1, SHA-256, SHA-512, SHA3-512, …

Separate chaining

Separate chaining

- Collision resolution mechanism
  - ✓ utilizes a linked list at each hash table index (array of linked lists)
  - ✓ guarantees $O(1)$ average-case and $O(n)$ worst-case search times
  - ✓ supports dynamic memory allocation
  - ✓ maintains key uniqueness constraint
- Core operations (assume a hash function h)
  - ✓ **insert**: add a new key (or key/value pair) to the linked list at $h(key)$
    - insert at front for faster operations, no need to keep the keys on each list in sorted order
  - ✓ **search**: search the linked list at $h(key)$
  - ✓ **delete**: remove the key (or key/value pair) from linked list at $h(key)$
- Alternative collision resolution
  - ✓ replace linked list with a self-balancing tree (e.g. Red-Black, AVL), guarantees $O(\log n)$ worst-case search

Practice

- Perform the following operations
  - ✓ insert(L, 11), delete(D), insert(M, 12), delete(E), search(C)
    - assume: insertions occur at front of the lists, hash(L)=3, hash(M)=0

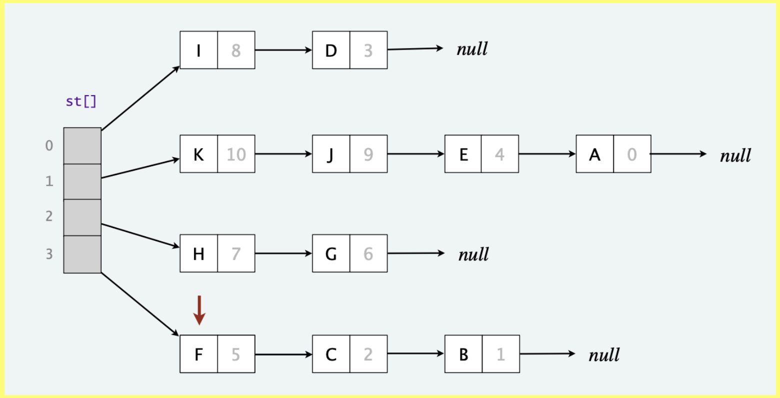
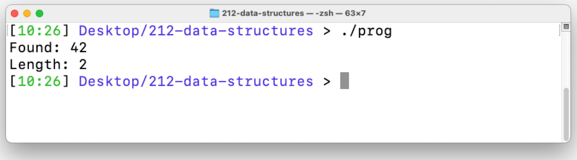


Image credit: COS 226 @ Princeton

```cpp
#include <string>
#include <iostream>
#include <list>
#include <vector>

const size_t tableSize = 101;
typedef std::string Key;
typedef int Value;
typedef std::pair<std::string, int> HashEntry;
typedef std::list<HashEntry> HashChain;
typedef std::vector<HashChain> HashTable;


size_t hash(const Key& k) {
    size_t hashValue = 0;
    for (char ch : k) {
        hashValue = hashValue * 31 + ch;
    }
    return hashValue;
}
```

```cpp
void insert(HashTable& table, const Key& key, const Value& value) {
    size_t index = hash(key) % tableSize;
    table[index].push_front({key, value});
}

Value* search(HashTable& table, const Key& key) {
    size_t index = hash(key) % tableSize;
    for (auto& entry : table[index]) {
        if (entry.first == key)
            return &entry.second;
    }
    return nullptr;
}

bool remove(HashTable& table, const Key& key) {
    size_t index = hash(key) % tableSize;
    auto& list = table[index];
    for (auto it = list.begin(); it != list.end(); ++it) {
        if (it->first == key) {
            list.erase(it);
            return true;
        }
    }
    return false;
}
```

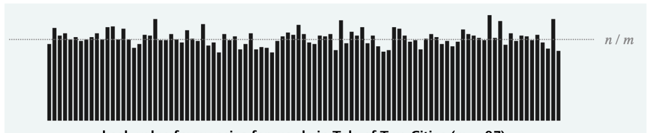
```cpp
int main() {
    HashTable table(tableSize);
    insert(table, "example", 42);
    insert(table, "test", 84);
    insert(table, "csc", 126);
    insert(table, "212", 168);
    Value* val = search(table, "example");
    if (val) {
        std::cout << "Found: " << *val << std::endl;
    }
    remove(table, "example");
    remove(table, "test");
    size_t count = 0;
    for (const auto& chain : table) {
        count += chain.size();
    }
    std::cout << "Length: " << count << std::endl;
    return 0;
}
```



```
[10:26] Desktop/212-data-structures > ./prog
Found: 42
Length: 2
[10:26] Desktop/212-data-structures >
```

Analysis

- Uniform hashing assumption
  - ✓ assume the hash function is a good one, and all keys are uniformly distributed



hash value frequencies for words in Tale of Two Cities (m = 97)

- Load factor (α)
  - ✓ the ratio of the number of keys (n) to the number of slots (m)

$$\alpha = \frac{n}{m}$$

- Time complexity
  - ✓ insert
    - $O(c)$ for all cases, if inserting at front of the list
  - ✓ search and delete
    - average case is $O(c + \alpha)$, where c is the cost of the hash function
    - worst case is $O(c + n)$, all the keys hash to the same index

Considerations

- Choices for α
  - ✓ too small, results in excessive table size with inefficient space utilization
  - ✓ too large, results in insufficient table size, increasing collision probability and degrading lookup performance
- Typical values
  - ✓ between 0.5 and 1.0 often provide a reasonable balance of space efficiency and lookup performance
  - ✓ higher load factors (>1.0) remain functional but with degraded performance
  - ✓ for performance-critical applications, conduct benchmarks with representative data sets to determine the optimal load factor

Data Structure	**Worst-case**			**Average-case**			**Ordered?**
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Red-Black	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	O(log n)	Yes
Hash table (separate chaining)	O(1)	O(n)	O(n)	O(1)*	O(1)*	O(1)*	No

(*) assumes uniform hashing and appropriate load factor