Your Name: _____

1. (10 points) Write a function $T(n)$ that counts the number of multiplications performed by the following function foo on an input of size $n \geq 1$. You do not need to find a closed form for $T(n)$.

```
int foo(int n) {
    int result = 1;
    for (int i = 1; i <= n; ++i)
        for (int j = 1; j <= i; ++j)
            result = result * i * j;
    return result;
}
```

2. (10 points) Which of the following asymptotic descriptions of $n^2 + 3n \lg n$ are correct?
   ○ $\mathcal{O}(n^2)$
   ○ $\mathcal{O}(n \lg n)$
   ○ $\mathcal{O}(n)$
   ○ $\Omega(n^2)$
   ○ $\Omega(n \lg n)$
   ○ $\Omega(n)$
   ○ $\Theta(n^2)$
   ○ $\Theta(n \lg n)$
   ○ $\Theta(n)$

3. (10 points) What is the growth rate of the following function?

$$2n + 3 \lg n + 5$$

○ Constant

○ Logarithmic

○ Linear

○ Linearithmic

○ Quadratic

○ Cubic

○ Exponential

4. (10 points) (*) Suppose algorithm A always runs in $\Theta(n)$ time and algorithm B always runs in $\Theta(\lg n)$ time. Which algorithm should you prefer for small values of n? Which algorithm should you prefer for large values of n? Justify your answer.

5. (10 points) Suppose v is a grow-by-doubling dynamic array with size 0 and capacity 1. What is the capacity of v after 5 calls to push_back?

6. (10 points) What is the output of the following program?

```
queue<int> q;
q.push(0);
q.push(1);
q.push(2);
q.pop();
cout << q.front() << '-';
q.pop();
cout << q.front();
```

7. (10 points) (*) Give a Θ-bound on the time complexity of the following program. Justify your answer.

```cpp
int baz(const vector<int>& v) {
    stack<int> s;
    int result = 0;
    for (int i = 0; i < v.size(); i++) {
        while (!s.empty() && v[i] >= v[s.top()])
            s.pop();
        int x = i + 1;
        if (!s.empty())
            x = i - s.top();
        result = max(result, x);
        s.push(i);
    }
    return result;
}
```

8. (10 points) What are the contents of v after this program executes?

```
vector<int> v{0, 1, 4, 2, 3}; // min-heap
pop_heap(v.begin(), v.end());
v.pop_back();
v.push_back(1)
push_heap(v.begin(), v.end())
```


9. (10 points) (**) Give an input of size n such that heapsort sorts it in linear time. Justify your answer.

10. (10 points) (*) You are a software engineer working on a cloud computing platform. Users submit computational jobs to the system, and the system executes them on a server. A **job** is a unit of work, like running a simulation or processing a dataset. Jobs arrive in the order that customers submit them. Some jobs are marked as **urgent**, and should be executed as soon as possible.

    For example, suppose jobs are represented by letters, and the system has two servers:

    1. Job A (not urgent) arrives, Job B (not urgent) and Job C (not urgent) arrive.
        - Server 1 executes A.
        - Server 2 executes B.

    2. Job D (urgent), Job E (not urgent) and Job F (urgent) arrive.
        - Server 1 executes D.
        - Server 2 executes F.

    3. No new jobs arrive.
        - Server 1 executes C.
        - Server 2 executes E.

    What abstract data type best models the system's waiting area for jobs before they are sent to a server? Here, best means efficiently solves the problem. Justify your answer.