CSC 212: Data Structures and Abstractions

Graphs

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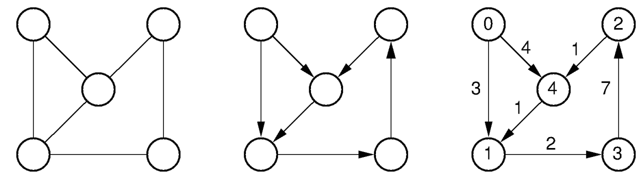
THINK BIG WE DO℠

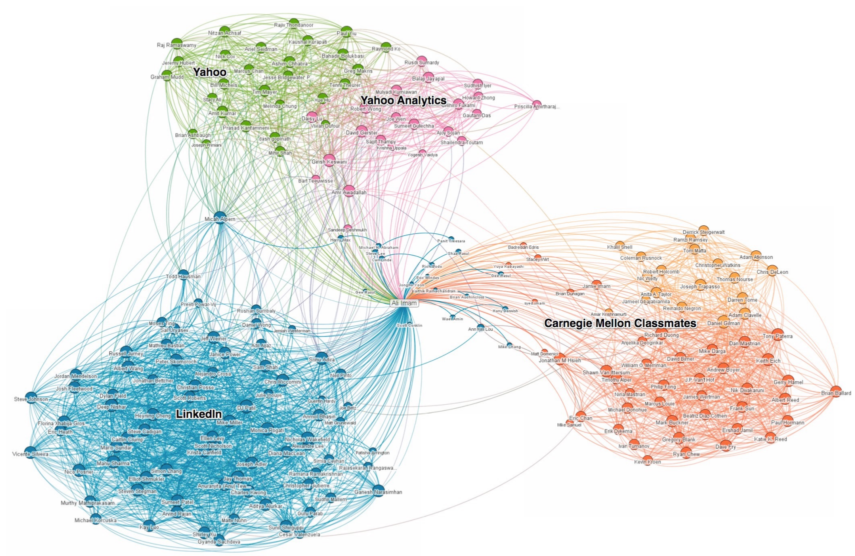
What is a graph?

A *graph* G is an ordered pair $G = (V, E)$, comprising a finite set of *nodes* V and a finite set of *edges* E

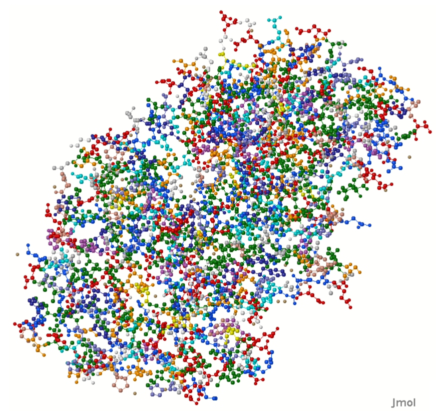


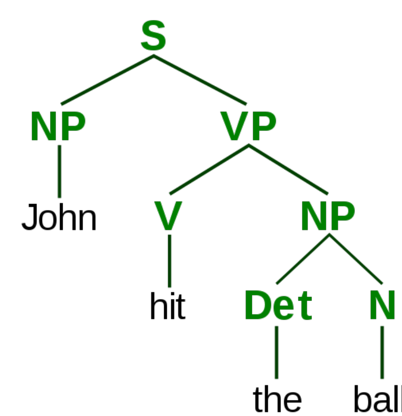
- Edges
  - ✓ can be **undirected** (u, v) or **directed** $(u \rightarrow v)$
  - ✓ may have additional attributes: weights or labels
- Why study graphs?
  - ✓ broadly useful abstraction with efficient algorithms
  - ✓ real-world applications

Graphs everywhere

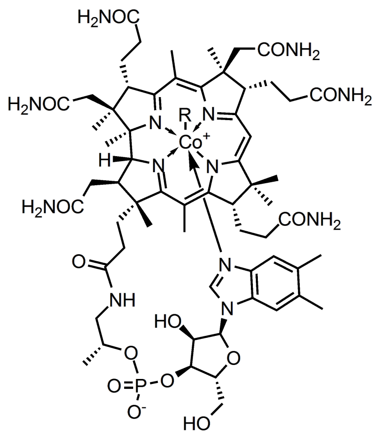


Graphs everywhere

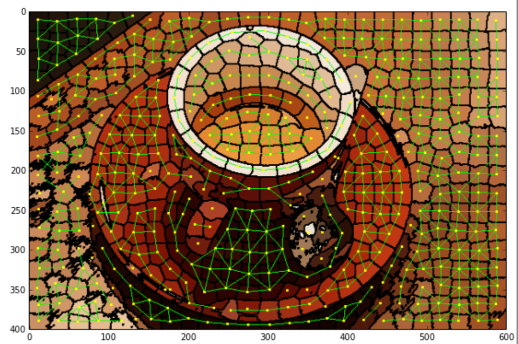




Graphs everywhere



R = 5'-deoxyadenosyl, Me, OH, CN

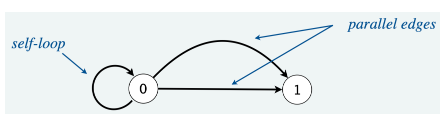


Applications

- Graphs model relationships
  - ✓ networks, dependencies, maps, molecules, social graphs, etc.
- Real-world graphs
  - ✓ social networks
    - vertices = users, edges = friendships/follows
  - ✓ road networks
    - vertices = intersections, edges = streets
  - ✓ course prerequisites
    - vertices = courses, edges = directed dependencies
  - ✓ computer networks
    - vertices = routers, edges = links

Types of graphs

- Based on edges
  - ✓ **undirected** — all edges have no orientation
  - ✓ **directed (digraphs)** — all edges point from one vertex to another
- Based on edge attributes
  - ✓ **weighted** — edges store weights
  - ✓ **unweighted** — edges do not carry information
- Based on structure
  - ✓ **simple** — no self-loops, no parallel edges
  - ✓ **multigraph** — parallel edges allowed
  - ✓ **pseudograph** — self-loops allowed



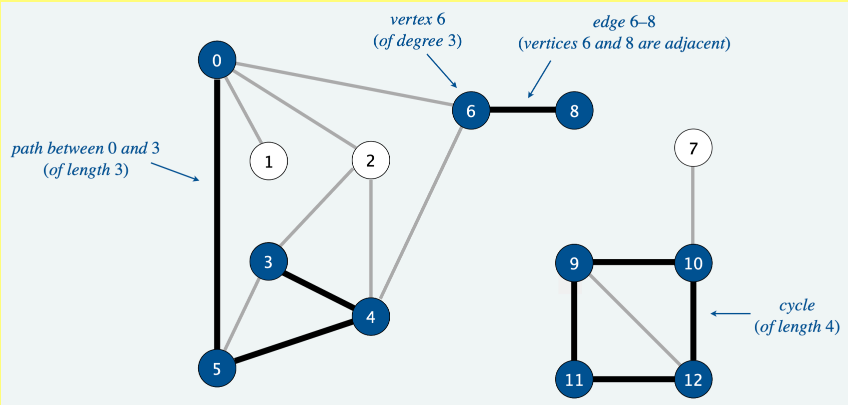
- Based on Density
  - ✓ **sparse** — $|E| \approx |V|$
  - ✓ **dense** — $|E| \approx |V|^2$

Basic terminology

Term	Meaning
Vertex	A node in the graph
Edge	A connection between two vertices
Adjacent	Two vertices connected by an edge
Neighbors	Adjacent vertices
Incident	An edge that touches a vertex
Degree	Number of incident edges
In-degree / Out-degree	Directed version of degree
Path	Sequence of vertices connected by edges (no repeated edges, undirected or directed)
Cycle	Path (with >= 1 edge) that starts and ends at the same vertex (undirected or directed)
Connected vertices	There is a path between them
Connected graph	Every vertex reachable (undirected graphs)

Practice

- Given this undirected graph
  - ✓ list all possible cycles



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Practice

- Given this directed graph
  - ✓ identify all cycles

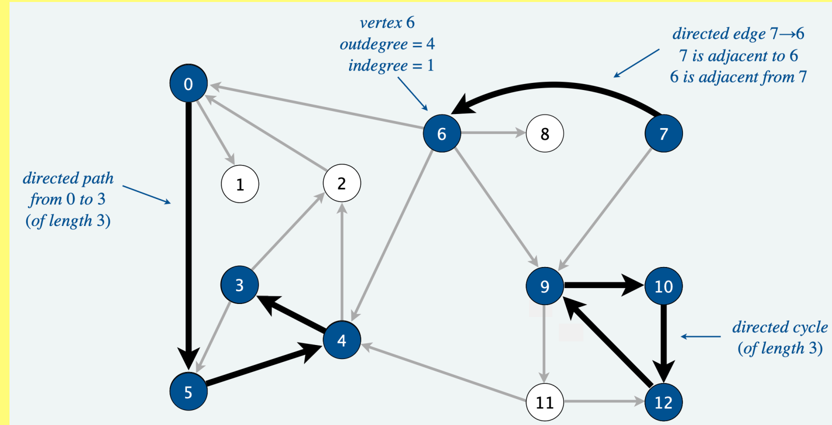


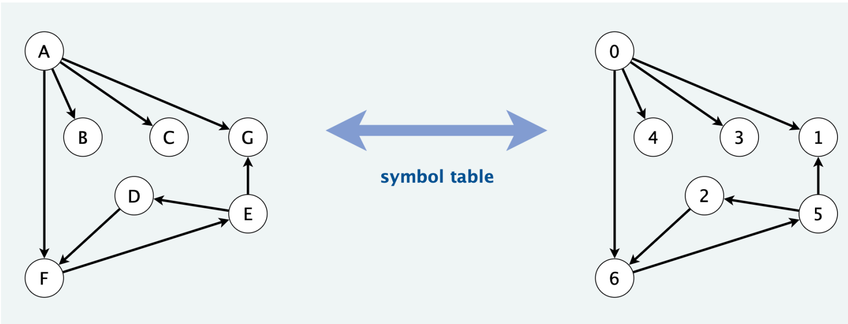
Image credit: COS 226 @ Princeton

Graph representation

Vertex representation

- Representing vertices by labels
  - ✓ human-readable identifiers
  - ✓ examples: "A", "v3", "Paris"
  - ✓ implementation: often mapped to integers for efficiency
- Representing vertices by integer values
  - ✓ vertices numbered $0, 1, \ldots, n-1$
  - ✓ fast array-based storage and efficient graph representation
  - ✓ standard in algorithm textbooks and implementations
- Representing vertices by objects
  - ✓ each vertex stores label/ID and additional attributes (metadata)
  - ✓ optional metadata (color, state, coordinates, other attributes)
- Vertices must be consistently indexed or labeled
  - ✓ all graph data structures rely on a clear method of labeling or indexing vertices

Vertex representation



Symbol tables (maps) can be used to convert between names and integers

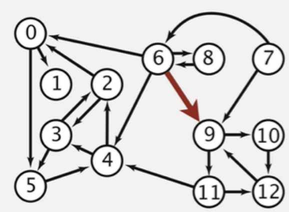
Graph representations

- Why do we need data structures for graphs?
  - ✓ efficiency of storage and operations depends on representation
  - ✓ sparse graphs and dense graphs require different structures
  - ✓ some algorithms prefer matrix access, others list traversal
- Representations
  - ✓ adjacency matrix
  - ✓ adjacency list
  - ✓ edge list

Adjacency matrix

- Definition
  - ✓ $n \times n$ matrix where $n = |V|$ and element at (row u column v) is 1 if edge exists connecting vertices u and v, 0 otherwise
  - ✓ for weighted graphs, replace 1 by the weight w and 0 by a special value that indicates the edge does not exist
- Space Complexity
  - ✓ $\Theta(V^2)$ regardless of actual number of edges
- When is it useful?
  - ✓ representing dense graphs
  - ✓ fast $O(1)$ adjacency queries

Example



to

from	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	0	0	0	1	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	1	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	1	0	0	0	0	0	0	0
4	0	0	1	1	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0	0	0	0
6	1	0	0	0	1	0	0	0	1	1	0	0	0
7	0	0	0	0	0	0	1	0	0	1	0	0	0
8	0	0	0	0	0	0	1	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	0
10	0	0	0	0	0	0	0	0	0	0	0	0	1
11	0	0	0	0	1	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	0	0	1	0	0	0

Note: parallel edges disallowed

Adjacency list

- Definition
  - list (or dictionary) where each vertex stores all its adjacent vertices
  - for weighted graphs, add the weight information to each element
    - $u : (v, w_1), (x, w_2)$
- Space Complexity
  - $\Theta(V + E)$ ideal for sparse graphs
- Advantages
  - efficient traversal
  - compact storage

Preferred in practice.
Real-world graphs tend to be sparse (not dense).

Example

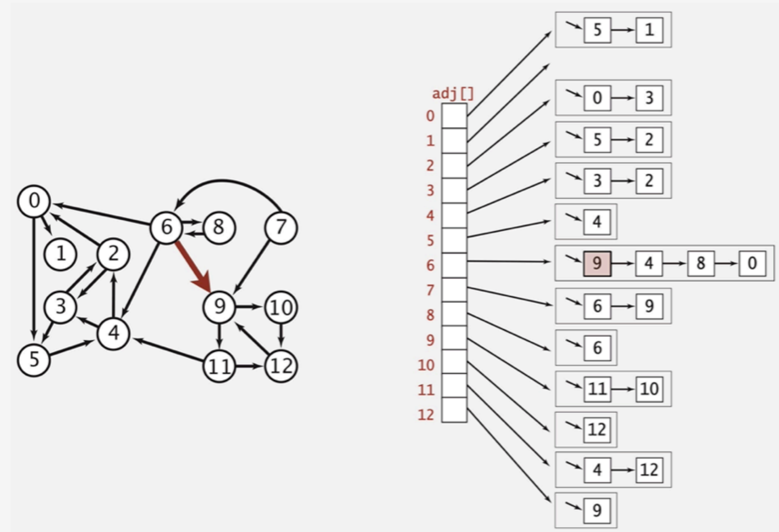


Image credit: COS 226 @ Princeton

Edge list

- Definition
  - a simple list of all edges: $[(u_1, v_1), (u_2, v_2), \ldots, (u_n, v_n)]$
  - for weighted graphs, add the weight information to each element
    - (u, v, w)
- Space Complexity
  - $\Theta(E)$
- Advantages
  - very compact
  - good for input parsing
- Cons
  - slow adjacency checks

Practice

- Given an undirected graph:
  - $V = \{0,1,2,3\}, \quad E = \{(0,1), (0,2), (1,2), (2,3)\}$
  - draw the corresponding representations: adjacency matrix, adjacency list, edge list

Practice

- Given an directed graph:
  - $V = \{0,1,2,3\}, \quad E = \{(0,1), (0,2), (1,2), (2,3)\}$
  - draw the corresponding representations: adjacency matrix, adjacency list, edge list

Practice

- Draw an undirected graph with 5 vertices where each vertex has degree >= 2
  - draw the corresponding representations: adjacency matrix, adjacency list, edge list
  - identify all cycles
  - is the graph connected?

Practice

- For each of the 3 representations, indicate the computational cost of:
  - checking if two vertices are adjacent
  - iterating all of the neighbors of a vertex u