PROJECT NAME: ME 564

PROJECT NUMBER: HW #4

BY: Tim Yoder

SHEET NUMBER: 1 OF 3

$\boxed{4-1}$

$$\frac{d}{dx}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sigma(y-x) \\ x(\rho - z) - y \\ xy - \beta z \end{bmatrix} \qquad \begin{aligned} \sigma &= 10 \\ \rho &= 28 \\ \beta &= 8/3 \end{aligned}$$

$FP_1 \quad (0, 0, 0)$

$FP_2 \quad (\sqrt{\beta(\rho-1)}, \sqrt{\beta(\rho-1)}, \rho-1)$

$FP_3 \quad (-\sqrt{\beta(\rho-1)}, -\sqrt{\beta(\rho-1)}, \rho-1)$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix} = \begin{bmatrix} -\sigma & \sigma & 0 \\ \rho - z & -1 & -x \\ y & x & -\beta \end{bmatrix}$$

$FP_1 : (0, 0, 0)$

$$A = \begin{bmatrix} -10 & -10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -8/3 \end{bmatrix}$$

Using Matlab

$\lambda_1 = -22.827 \qquad V_1 = (-0.6148, 0.7887, 0)$

$\lambda_2 = 11.827 \qquad V_2 = (-0.4165, -0.9091, 0)$

$\lambda_3 = -2.667 \qquad V_3 = (0, 0, 1)$

x and y are independent of z.

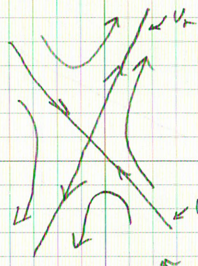
λ_1 and λ_2 are associated with x, y.

$\text{Im}\,\lambda_1, \lambda_2 = 0$ $\Rightarrow$ saddle point in x y

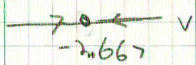
$\text{Re}\,\lambda_1 < 0, \text{Re}\,\lambda_2 > 0$

λ_3 is an in $\mathbb{R}_2$ z direction

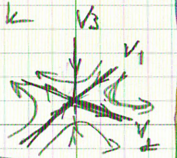
$\text{Re}\,\lambda_3 \leq 0, \text{Im}\,\lambda_3 = 0 \Rightarrow$ stable sink in Z direction



xy plane saddle



z. direction sink



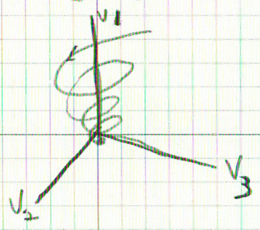
$FP_2 : (6\sqrt{2}, 6\sqrt{2}, 27)$

$$A_2 = \begin{bmatrix} -10 & 10 & 0 \\ 1 & -1 & -6\sqrt{2} \\ 6\sqrt{2} & -6\sqrt{2} & -8/3 \end{bmatrix}$$

$\lambda_1 = -13.85 \qquad V_1 = (0.8557, -0.3298, -0.3988)$

$\lambda_2 = 0.094 + 10.2i \qquad V_2 = (-0.2661 - 0.295i, 0.0321 - 0.569i, -0.7192)$

$\lambda_3 = 0.094 - 10.1945i \qquad V_3 = (-0.2661 + 0.295i, 0.0321 + 0.569i, -0.7192)$



$\text{Max Re}\,\lambda_1, \lambda_2, \lambda_3 > 0$

$\lambda_2, \lambda_3 \rightarrow$ plane of unstable spiral

$\lambda_1 \rightarrow$ stable line of solutions

$FP_3 = (-6\sqrt{2}, -6\sqrt{2}, 27)$

$\lambda_1 = -13.85 \qquad v_1 = (0.8557, -0.3298, 0.3988)$

$\lambda_2 = 0.094 + 10.1945i \qquad V_2 = (0.2661 - 0.295i, 0.032 - 0.569i, 0.719)$

$\lambda_3 = 0.094 - 10.1945i \qquad V_3 = (0.2661 + 0.295i, 0.0321 + 0.569i, 0.719)$

$$A_3 = \begin{bmatrix} -10 & 10 & 0 \\ 1 & -1 & 6\sqrt{2} \\ -6\sqrt{2} & -6\sqrt{2} & -8/3 \end{bmatrix}$$

FP_3 is also a spiral saddle.

4-2 Refer to Attached Matlab Plots

ρ	λ_1	λ_2	λ_3	Stability
5				
10				
15				
20				
25				
30				
35				
40				
45				
50				

I don't feel like copying all of this busy work down by hand.

Refer to attach Matlab output.

Yes the plotted results agree w/ linearization Results.

4-3

$$-4\left[f(t-\Delta t) = f(t) - f'(t)\Delta t + \frac{f''(t)}{2!}\Delta t^2 - \frac{f'''(t)}{3!}\Delta t^3 + \mathcal{O}(t^4)\right]$$

$$f(t-2\Delta t) = f(t) - 2f'(t)\Delta t + \frac{f''(t)}{2!}4\Delta t^2 - \frac{f'''(t)}{3!}8\Delta t^3 + \mathcal{O}(t^4)$$

$$3\left[f(t) = f(t)\right]$$

$$\boxed{\frac{df}{dt} \approx \frac{3f(t) - 4f(t-\Delta t) + f(t-2\Delta t)}{2\Delta t} = \frac{3f(t) - 4[f(t) - f'(t)\Delta t + \mathcal{O}(\Delta t^2)] + [f(t) - 2f'(t)\Delta t + \mathcal{O}(\Delta t^2)]}{2\Delta t} = \frac{2f'(t)\cancel{\Delta t}}{\cancel{2\Delta t}} + \mathcal{O}(\Delta t^2)}$$

```
fp1 = [0, 0, 0];
```

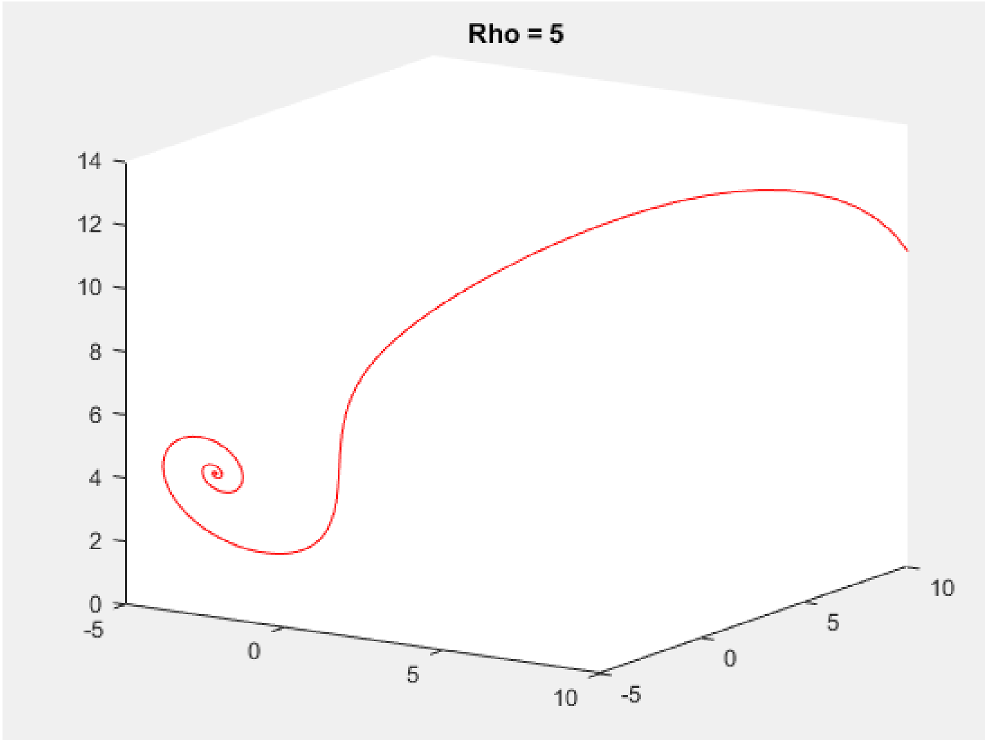
"Rho"	5.0000	10.0000	15.0000	20.0000	25.0000	30.0000	35.0000	40.0000	45.0000	50.0000
"lambda 1"	-13.8815	-16.4659	-18.5480	-20.3408	-21.9393	-23.3955	-24.7419	-26.0000	-27.1852	-28.3090
"lambda 2"	2.8815	-2.6667	7.5480	9.3408	10.9393	12.3955	13.7419	15.0000	16.1852	17.3090
"lambda 3"	-2.6667	5.4659	-2.6667	-2.6667	-2.6667	-2.6667	-2.6667	-2.6667	-2.6667	-2.6667
	Unstable	Unstable	Unstable	Unstable	Unstable	Unstable	Unstable	Unstable	Unstable	Unstable

```
fp2 = [sqrt(beta*(rho-1)), sqrt(beta*(rho-1)), (rho-1)];
```

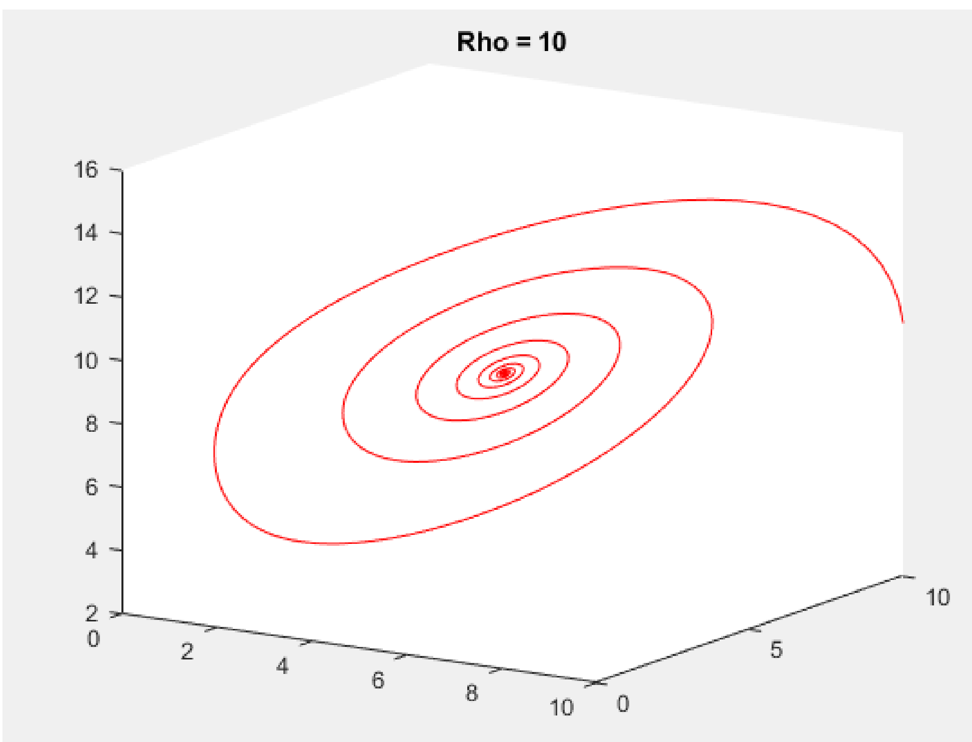
"Rho"	5.0000 + 0.0000i	10.0000 + 0.0000i	15.0000 + 0.0000i	20.0000 + 0.0000i	25.0000 + 0.0000i	30.0000 + 0.0000i	35.0000 + 0.0000i	40.0000 + 0.0000i	45.0000 + 0.0000i	50.0000 + 0.0000i
"lambda 1"	-11.8092 + 0.0000i	-12.4757 + 0.0000i	-12.9663 + 0.0000i	-13.3571 + 0.0000i	-13.6825 + 0.0000i	-13.9614 + 0.0000i	-14.2053 + 0.0000i	-14.4219 + 0.0000i	-14.6165 + 0.0000i	-14.7929 + 0.0000i
"lambda 2"	-0.9287 + 4.1476i	-0.5955 + 6.1742i	-0.3502 + 7.5804i	-0.1548 + 8.7087i	0.0079 + 9.6721i	0.1474 +10.5243i	0.2693 +11.2951i	0.3776 +12.0034i	0.4749 +12.6619i	0.5631 +13.2794i
"lambda 3"	-0.9287 - 4.1476i	-0.5955 - 6.1742i	-0.3502 - 7.5804i	-0.1548 - 8.7087i	0.0079 - 9.6721i	0.1474 -10.5243i	0.2693 -11.2951i	0.3776 -12.0034i	0.4749 -12.6619i	0.5631 -13.2794i
	Stable	Stable	Stable	Stable	Unstable	Unstable	Unstable	Unstable	Unstable	Unstable

```
fp3 = [-sqrt(beta*(rho-1)), -sqrt(beta*(rho-1)), (rho-1);];
```

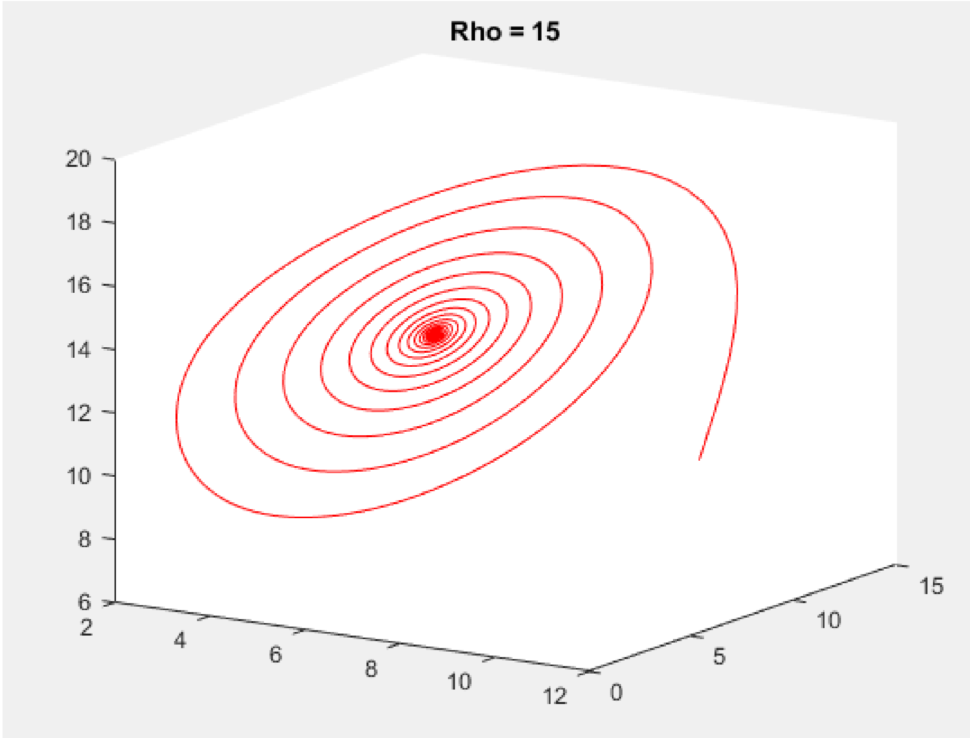
"Rho"	5.0000 + 0.0000i	10.0000 + 0.0000i	15.0000 + 0.0000i	20.0000 + 0.0000i	25.0000 + 0.0000i	30.0000 + 0.0000i	35.0000 + 0.0000i	40.0000 + 0.0000i	45.0000 + 0.0000i	50.0000 + 0.0000i
"lambda 1"	-11.8092 + 0.0000i	-12.4757 + 0.0000i	-12.9663 + 0.0000i	-13.3571 + 0.0000i	-13.6825 + 0.0000i	-13.9614 + 0.0000i	-14.2053 + 0.0000i	-14.4219 + 0.0000i	-14.6165 + 0.0000i	-14.7929 + 0.0000i
"lambda 2"	-0.9287 + 4.1476i	-0.5955 + 6.1742i	-0.3502 + 7.5804i	-0.1548 + 8.7087i	0.0079 + 9.6721i	0.1474 +10.5243i	0.2693 +11.2951i	0.3776 +12.0034i	0.4749 +12.6619i	0.5631 +13.2794i
"lambda 3"	-0.9287 - 4.1476i	-0.5955 - 6.1742i	-0.3502 - 7.5804i	-0.1548 - 8.7087i	0.0079 - 9.6721i	0.1474 -10.5243i	0.2693 -11.2951i	0.3776 -12.0034i	0.4749 -12.6619i	0.5631 -13.2794i
	Stable	Stable	Stable	Stable	Unstable	Unstable	Unstable	Unstable	Unstable	Unstable



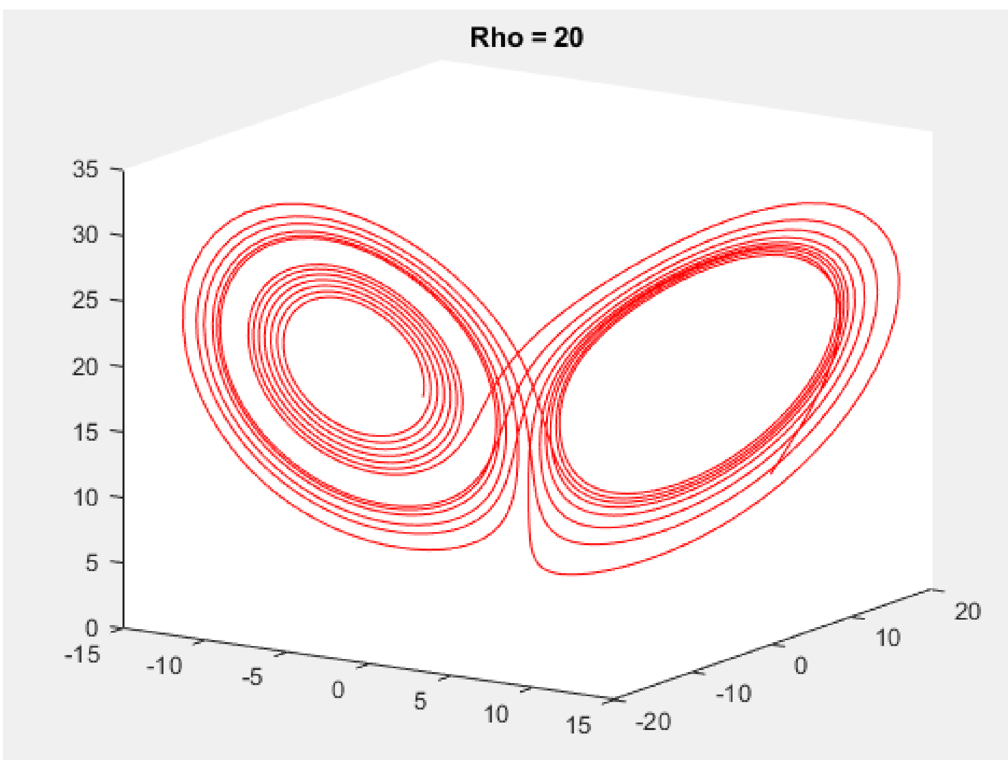
Rho = 5
14
12
10
8
6
4
2
0
-5
0
5
10
-5
0
5
10



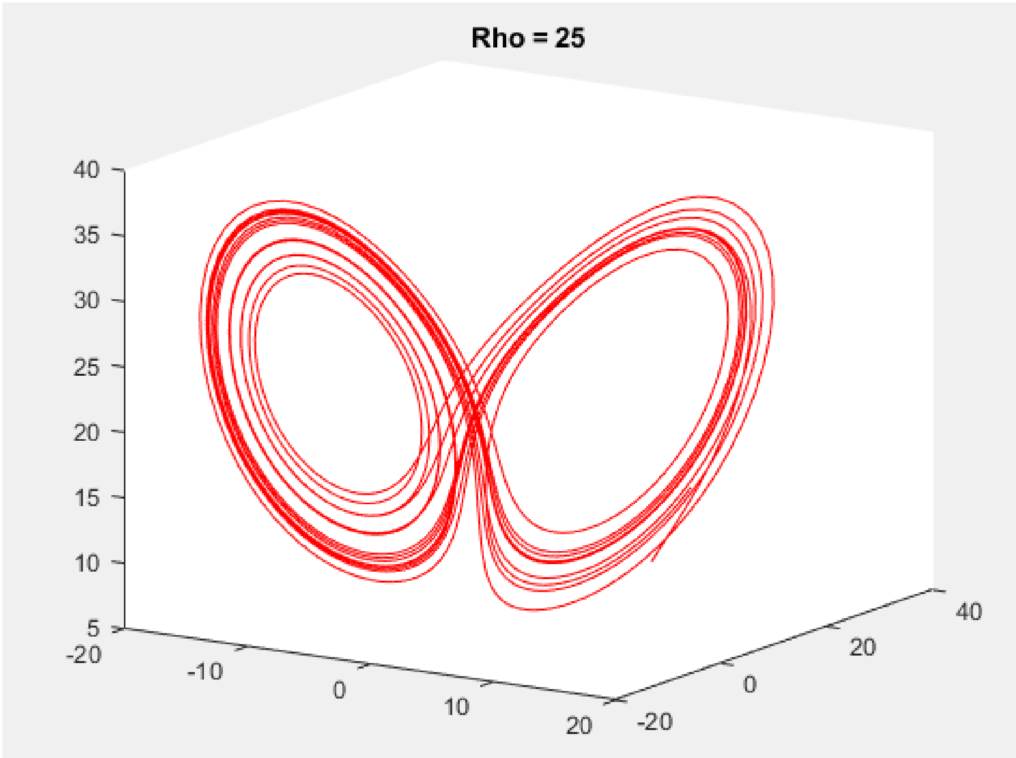
Rho = 10
16
14
12
10
8
6
4
2
0
2
4
6
8
10
0
5
10

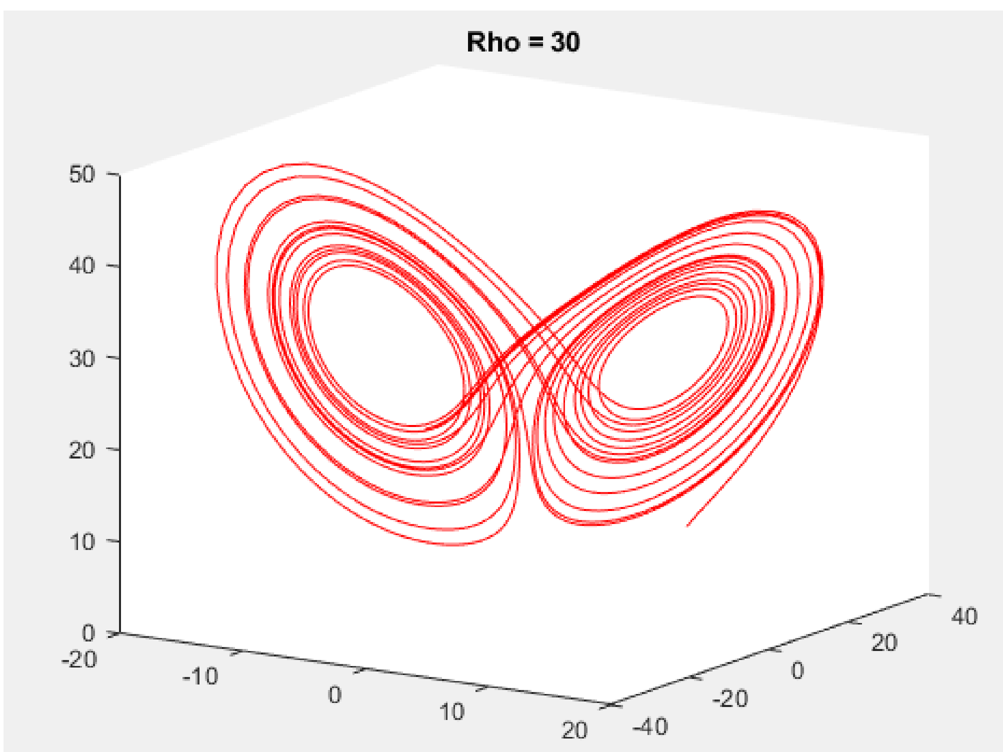


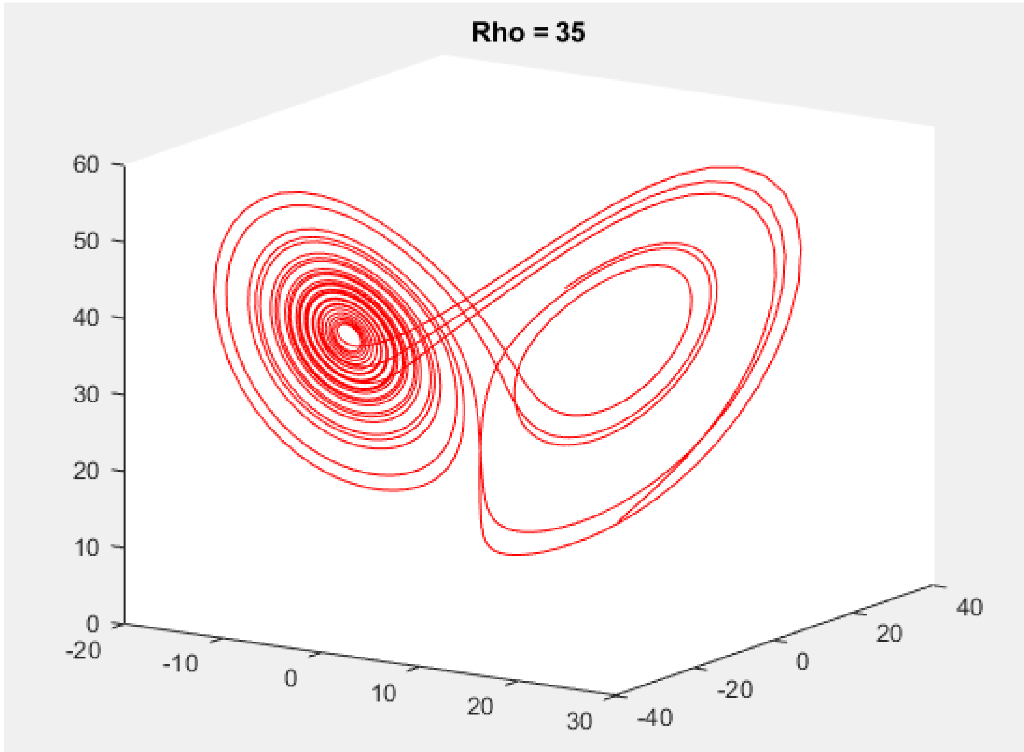
Rho = 15
20
18
16
14
12
10
8
6
2
4
6
8
10
12
0
5
10
15

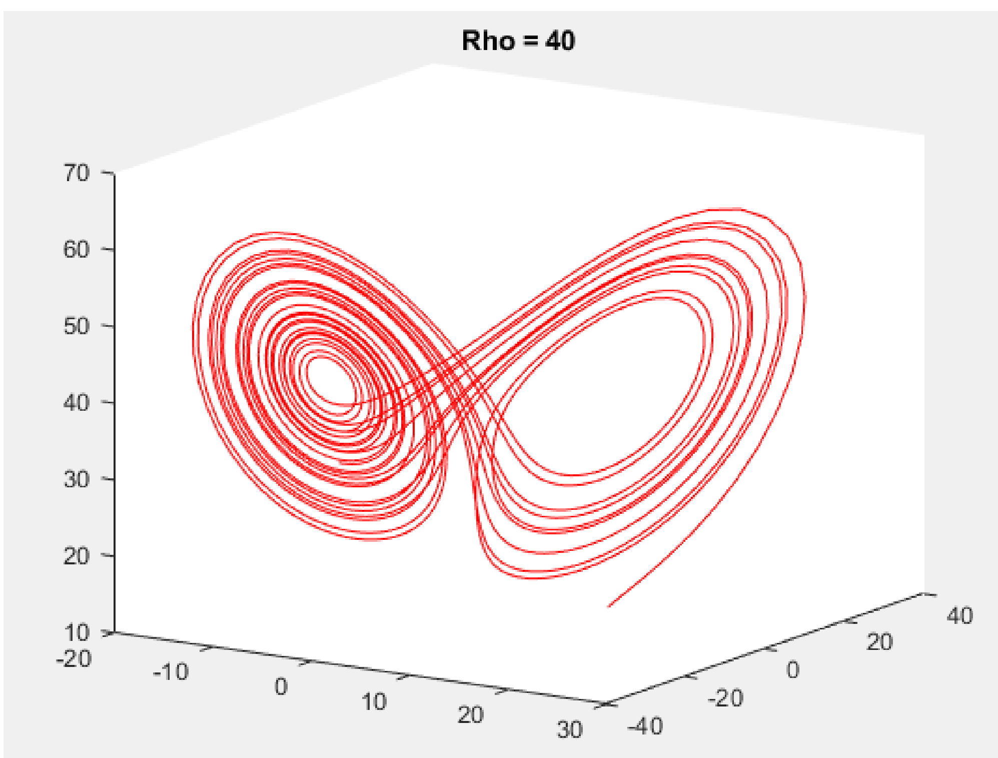


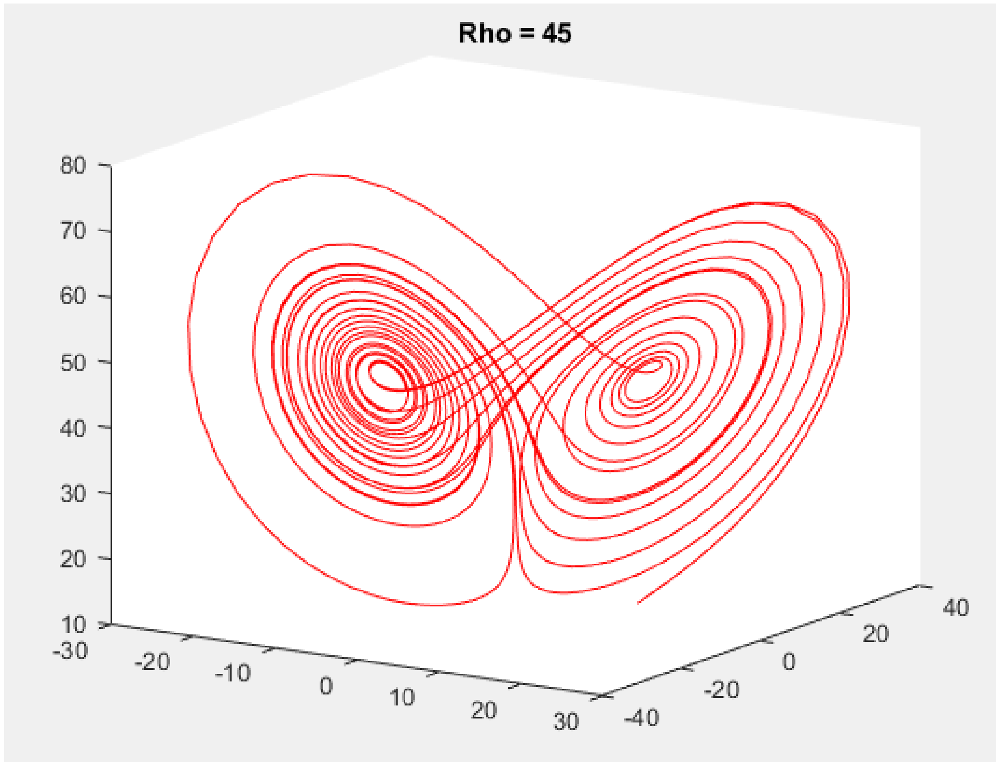
Rho = 20
35
30
25
20
15
10
5
0
-15
-10
-5
0
5
10
15
-20
-10
0
10
20

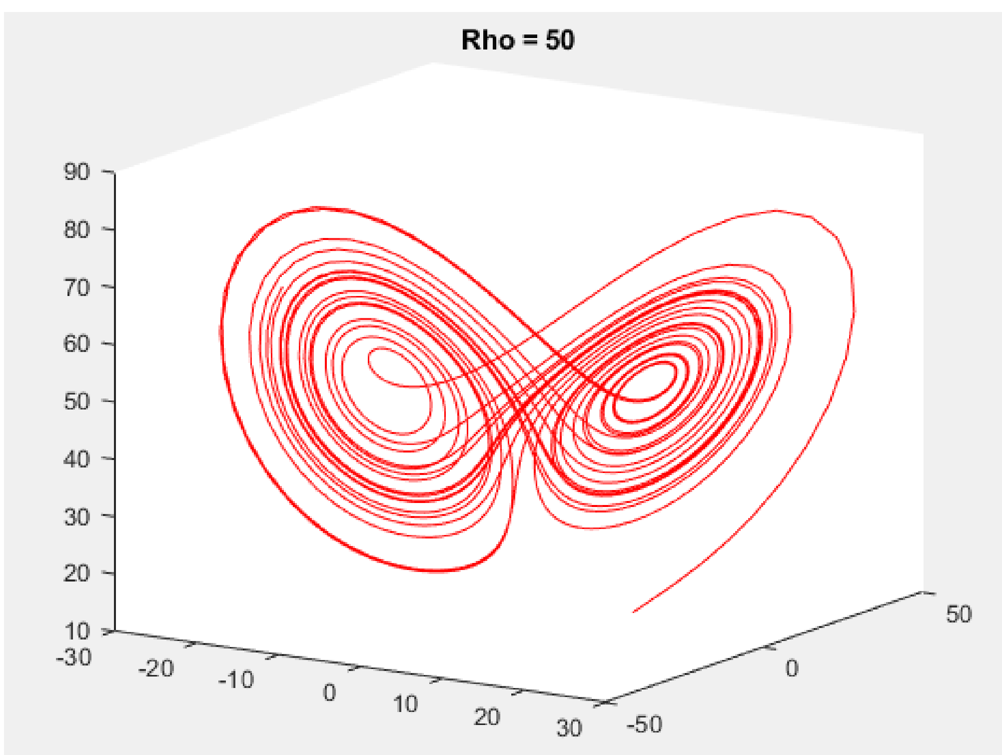

Rho = 25
40
35
30
25
20
15
10
5
-20
-10
0
10
20
-20
0
20
40


Rho = 30
50
40
30
20
10
0
-20
-10
0
10
20
-40
-20
0
20
40


Rho = 35
60
50
40
30
20
10
0
-20
-10
0
10
20
30
-40
-20
0
20
40


Rho = 40
70
60
50
40
30
20
10
-20
-10
0
10
20
30
-40
-20
0
20
40


Rho = 45
80
70
60
50
40
30
20
10
-30
-20
-10
0
10
20
30
-40
-20
0
20
40


Rho = 50
90
80
70
60
50
40
30
20
10
-30
-20
-10
0
10
20
30
-50
0
50

BALLINGER

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PROJECT NAME: ME 564

PROJECT NUMBER: HW #4

BY: Tim Yaler

4-4 $\quad \ddot{y} + \varepsilon(y^2-1)\dot{y} + y = 0$

a) $\dot{y} = z$

$\dot{z} = \ddot{y} \qquad \dot{z} + \varepsilon(y^2-1)z + y = 0$

$\dot{z} = \varepsilon(1-y^2)z - y$

$$\boxed{\frac{d}{dt}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} z \\ \varepsilon(1-y^2)z - y \end{bmatrix}}$$

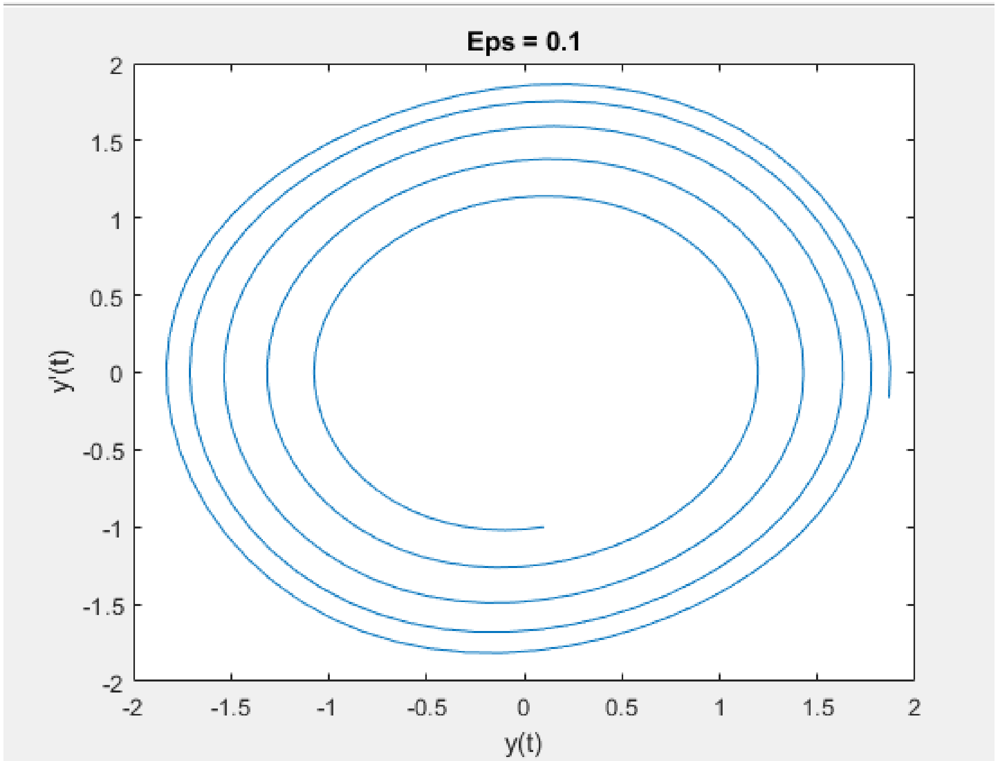
$\varepsilon - \varepsilon y^2 z - y$

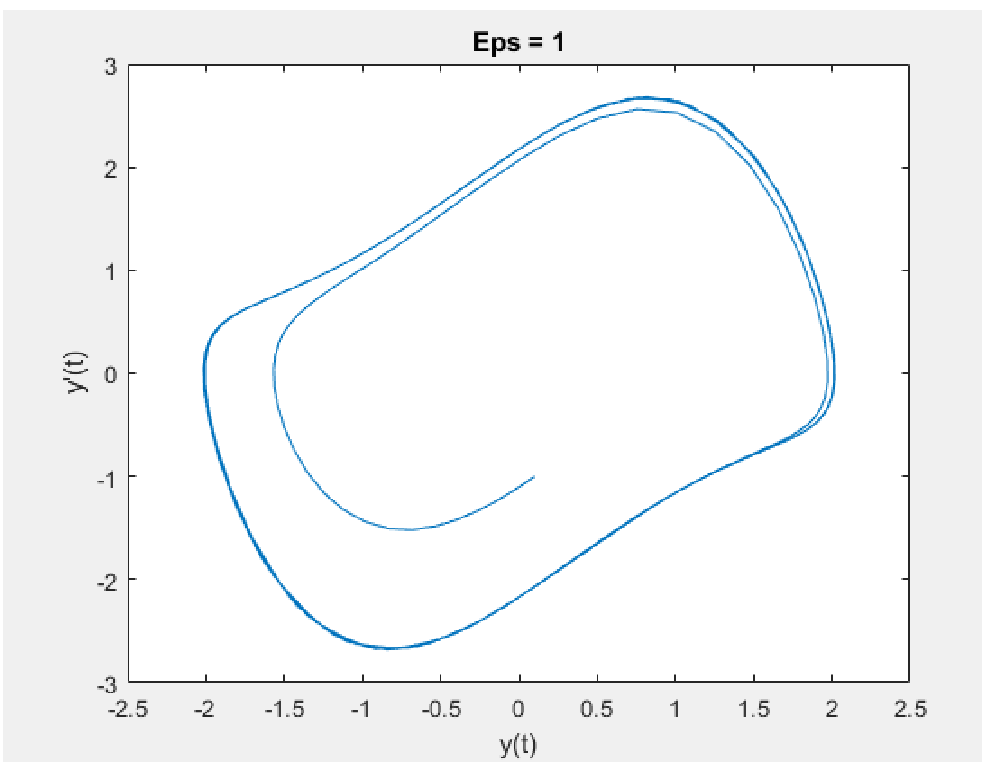
b) FP $= 0, 0$

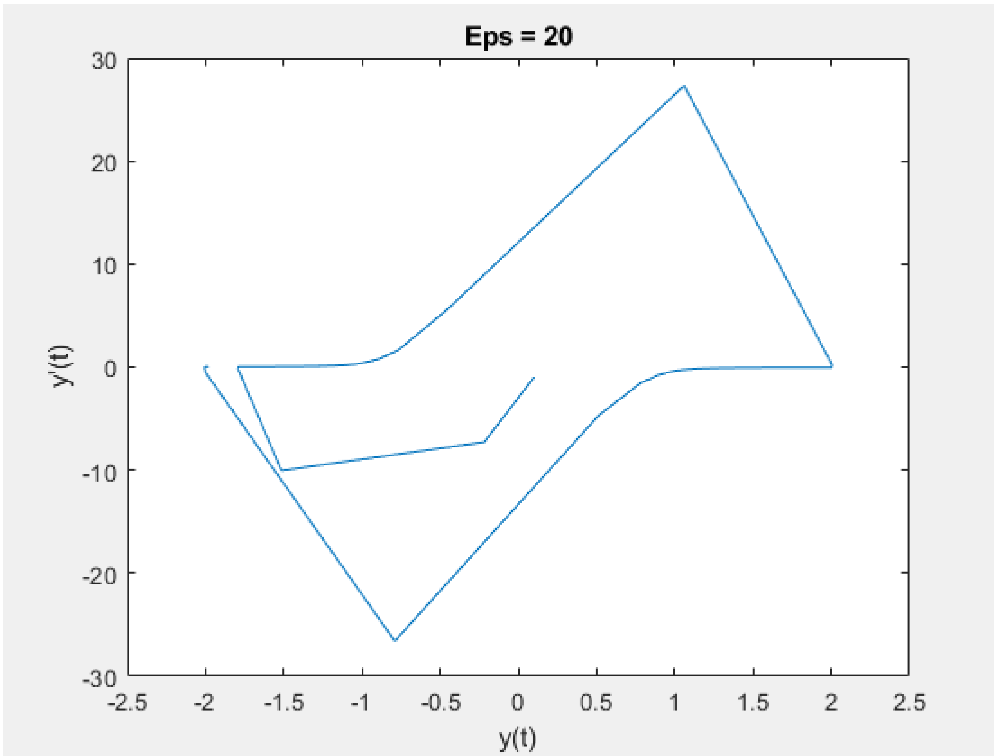
$$J = \begin{bmatrix} 0 & 1 \\ -2\varepsilon y z - 1 & -y^2 \end{bmatrix}\Bigg|_{(0,0)} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$\lambda = \pm i \Rightarrow$ Neutrally stable center

See Attached Matlab plots.


Eps = 0.1
y'(t)
y(t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Eps = 1
y'(t)
y(t)
3
2
1
0
-1
-2
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5



Eps = 20
30
20
10
0
-10
-20
-30
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
y'(t)
y(t)