BALLINGER

833 CHESTNUT ST,
SUITE 1400
PHILADELPHIA, PA 19107

T 215 446 0900
F 215 446 0901
BALLINGER.COM

PROJECT NAME ME 564
PROJECT NUMBER HW #2

BY Tim Yoder
SHEET NUMBER 1 OF 5

a) substitute $e^{\lambda t}$
b) $v = \dot{x}$
A into eqn
c) IC
D) long term behavior

$\ddot{x} - 10\dot{x} + 21 = 0$

1) $\ddot{x} + 10\dot{x} + 21x = 0$ I.C. $x(0) = 2$ $\dot{x}(0) = -10$

a)

$x = e^{\lambda t}$

$\dot{x} = \lambda e^{\lambda t}$

$\ddot{x} = \lambda^2 e^{\lambda t}$

$(\lambda^2 + 10\lambda + 21)e^{\lambda t} = 0$

$(\lambda + 3)(\lambda + 7) = 0$

$\lambda = -3, -7$

$$\boxed{x(t) = c_1 e^{-3t} + c_2 e^{-7t}}$$

b) $v = \dot{x}$

$\dot{v} = \ddot{x}$

$\ddot{x} = -21x - 10\dot{x}$

$\dot{v} = -21x - 10v$

$\dot{x} = v$

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -21 & -10 \end{bmatrix}}_{A}\begin{bmatrix} x \\ v \end{bmatrix}$$

$$(A - \lambda I) = \begin{bmatrix} -\lambda & 1 \\ -21 & -10-\lambda \end{bmatrix}$$

$\det(A - \lambda I) = 0$

$-\lambda(-10-\lambda) + 21 = 0$

$(\lambda^2 + 10\lambda + 21) = 0$

$(\lambda + 3)(\lambda + 7) = 0$

$\lambda = -3, -7$

c) $x(0) = c_1 + c_2 = 2 \Rightarrow c_1 = 2 - c_2$

$\dot{x}(0) = -3c_1 - 7c_2 = -10$

$-3(2 - c_2) - 7c_2 = -10$

$-6 - 4c_2 = -10$

$-4c_2 = -4$

$c_2 = 1 \rightarrow c_1 = 1$

$$\boxed{x(t) = e^{-3t} + e^{-7t}}$$

$(A - \lambda I)x = 0$

$\lambda = -3$: $(A + 3I)x = 0$ $3x_1 + x_2 = 0$ $x_2 = -3x_1$

$$\begin{bmatrix} 3 & 1 \\ -21 & -7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\boxed{\lambda = -3 \quad \begin{bmatrix} 1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix} \leftarrow \text{eig vector}}$$

d) The long term behavior of the system is a stable solution (i.e. 0) both eigen values are negative which results in a stable solution. if the equation was $\ddot{x} - 10\dot{x} + 21 = 0$ the eigen values would both be positive ($\lambda = 3, 7$) and the solution would be unstable.

$\lambda = -7$ $7x_1 + x_2 = 0$ $x_2 = -7x_1$

$$\begin{bmatrix} 7 & 1 \\ -21 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$

$$\boxed{\lambda = -7 \quad \begin{bmatrix} 1/\sqrt{50} \\ -7/\sqrt{50} \end{bmatrix} = \begin{bmatrix} 1/5\sqrt{2} \\ -7/5\sqrt{2} \end{bmatrix}}$$

eigenvector

$$D = \begin{bmatrix} -3 & 0 \\ 0 & -7 \end{bmatrix} \qquad T = \begin{bmatrix} .3162 & -.1414 \\ -.9487 & .9899 \end{bmatrix}$$

→ see Next Page

$$T = \begin{bmatrix} 0.3162 & -0.1414 \\ -0.9487 & 0.9899 \end{bmatrix} \qquad T^{-1} = \begin{bmatrix} 5.534 & 0.7906 \\ 5.3033 & 1.7678 \end{bmatrix}$$

$$\begin{matrix} x(t) \\ v(t) \end{matrix} = T \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} T^{-1} \begin{bmatrix} x(0) \\ v(0) \end{bmatrix}$$

$$\begin{bmatrix} x(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} 0.3162 & -0.1414 \\ -0.9487 & 0.9899 \end{bmatrix} \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-7t} \end{bmatrix} \begin{bmatrix} 5.534 & 0.7906 \\ 5.3033 & 1.7678 \end{bmatrix} \begin{bmatrix} 2 \\ -10 \end{bmatrix}$$

$$\begin{matrix} x(t) \\ v(t) \end{matrix} = \begin{bmatrix} 0.3162 & -0.1414 \\ -0.9487 & 0.9899 \end{bmatrix} \begin{bmatrix} 5.534e^{-3t} & 0.7906e^{-3t} \\ 5.3003e^{-7t} & 1.7678e^{-7t} \end{bmatrix} \begin{bmatrix} 2 \\ -10 \end{bmatrix}$$

1(c) continued — using easier eigen vectors [non-unitized]

2 of 5

$$T = \begin{bmatrix} 1 & 1 \\ -3 & -7 \end{bmatrix} \qquad T^{-1} = \begin{bmatrix} 1.75 & .25 \\ -.75 & -.25 \end{bmatrix}$$

$$\begin{bmatrix} x(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -7 \end{bmatrix} \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-7t} \end{bmatrix} \begin{bmatrix} 1.75 & .25 \\ -.75 & -.25 \end{bmatrix} \begin{bmatrix} 2 \\ -10 \end{bmatrix}$$

$$\begin{matrix} x(t) \\ v(t) \end{matrix} = \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} 1.75e^{-3t} & .25e^{-3t} \\ -.75e^{-7t} & -.25e^{-7t} \end{bmatrix} \begin{bmatrix} 2 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} 1.75e^{-3t} - .75e^{-7t} & .25e^{-3t} - .25e^{-7t} \\ -5.25e^{-3t} + 5.25e^{-7t} & -.75e^{-3t} + 1.75e^{-7t} \end{bmatrix} \begin{bmatrix} 2 \\ -10 \end{bmatrix}$$

$$x(t) = 3.5e^{-3t} - 1.5e^{-7t} - 2.5e^{-3t} + 2.5e^{-7t}$$

$$\boxed{x(t) = e^{-3t} + e^{-7t}}$$

See previous page for Part (d)

2)

a) $\dddot{x} + 2\ddot{x} - \dot{x} - 2x = 0$

$\dot{x} = y$

$\ddot{x} = \dot{y} = z$

$\dddot{x} = \ddot{y} = \dot{z}$

$\dot{z} = -2z + y + 2x$

$$\boxed{\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}}$$

$$\boxed{\begin{aligned} \lambda_1 &= -2 \\ \lambda_2 &= -1 \\ \lambda_3 &= 1 \end{aligned}}$$

See Matlab code attached

long term behavior of $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an unstable solution i.e. $x(t) \to \infty$

long term behavior of $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is a stable solution, $x(t) \to 0$

These results do agree. There is a positive eigenvalue so any I.C. that is not exactly on a stable manifold with blow up. $[1 \;\; -1 \;\; 1]$ is an eigen vector $[\lambda = -1]$ and therefore the solution is stable. See attached matlab plot and code.

b) $x^{(4)} + 5\dddot{x} + 5\ddot{x} - 5\dot{x} - 6x = 0$

$\dot{x} = m$

$\ddot{x} = \dot{m} = n$

$\dddot{x} = \ddot{m} = \dot{n} = p$

$x^{(4)} = \dddot{m} = \ddot{n} = \dot{p}$

$x^4 = -5\dddot{x} - 5\ddot{x} + 5\dot{x} + 6x = 0$

$\dot{p} = -5p - 5n + 5m + 6x$

$$\boxed{\begin{gathered}\frac{d}{dt}\begin{bmatrix} x \\ m \\ n \\ p \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6 & 5 & -5 & -5 \end{bmatrix}\begin{bmatrix} x \\ m \\ n \\ p \end{bmatrix} \\ \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -2 \\ -1 \end{bmatrix}\end{gathered}}$$

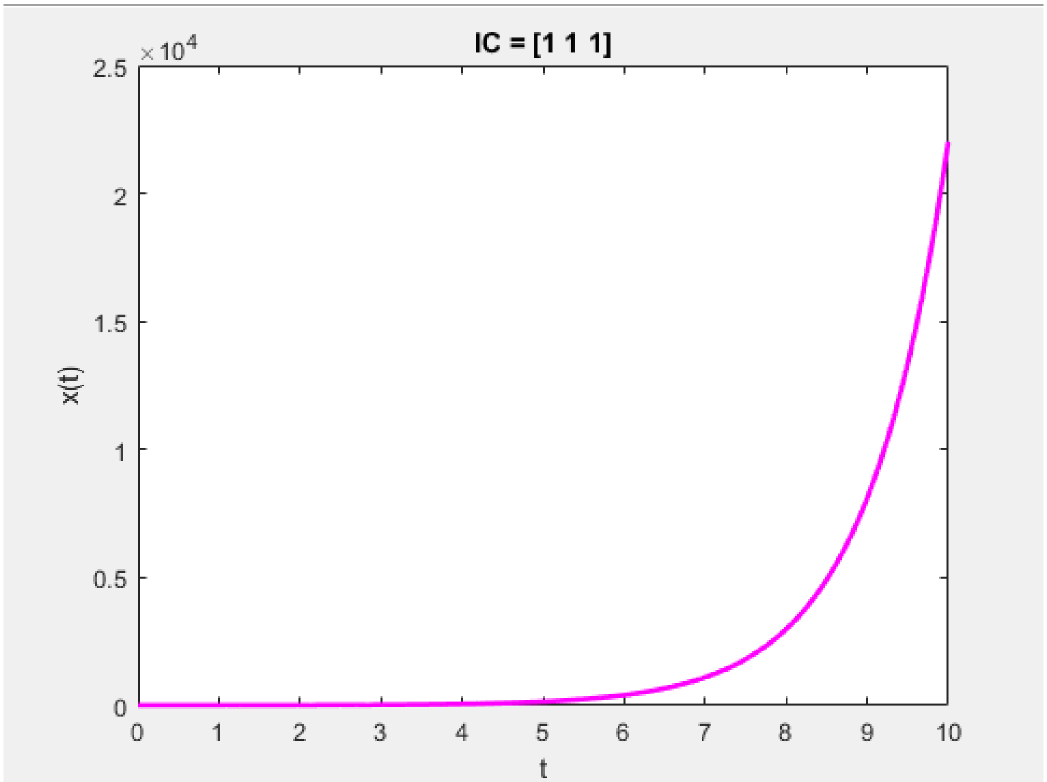
long term behavior of $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \\ \dddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is unstable. $x(t) \to \infty$

longer term behavior of $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \\ \dddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ eigen vector of $\lambda = -1$ is stable $x(t) \to 0$

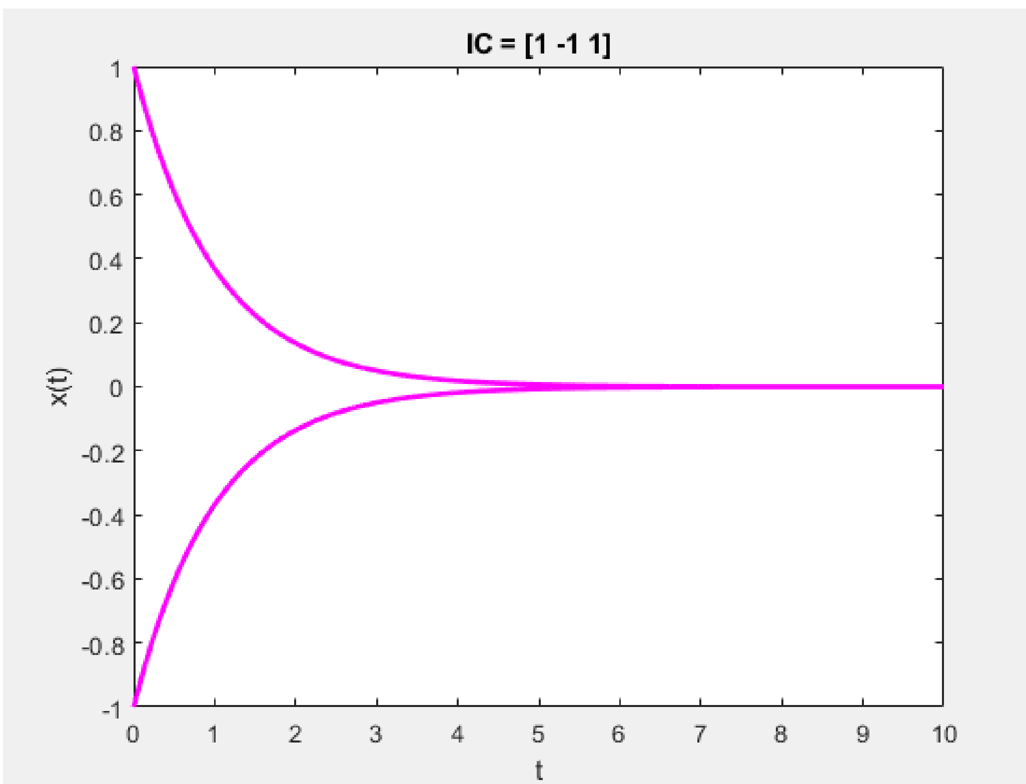
These results also agree for same reasons listed above.

See Attached Matlab Plots

2A

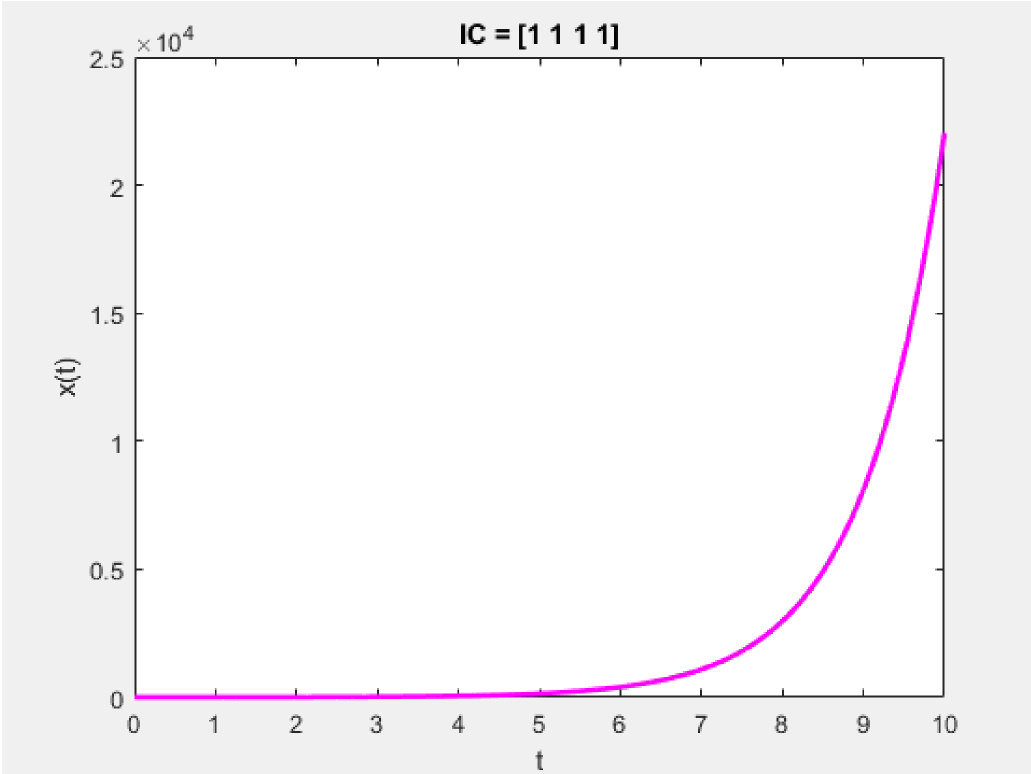


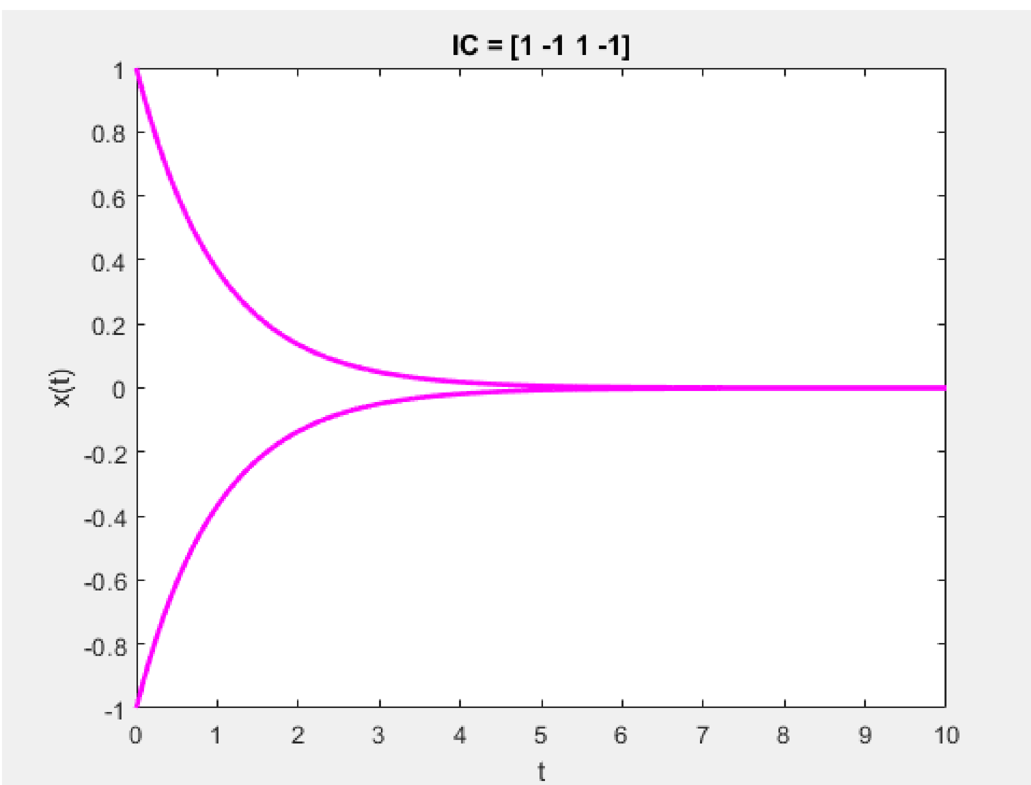
IC = [1 1 1]
$\times 10^4$
2.5
2
1.5
1
0.5
0
x(t)
0 1 2 3 4 5 6 7 8 9 10
t



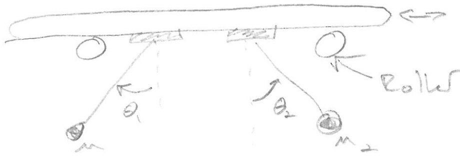
IC = [1 -1 1]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
x(t)
0 1 2 3 4 5 6 7 8 9 10
t

2B


IC = [1 1 1 1]
×10^4
2.5
2
1.5
1
0.5
0
x(t)
0 1 2 3 4 5 6 7 8 9 10
t


IC = [1 -1 1 -1]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
x(t)
0 1 2 3 4 5 6 7 8 9 10
t

2-3



$$\ddot{\theta}_1 = -\omega_1^2\theta_1 + \varepsilon(\theta_2-\theta_1)$$
$$\ddot{\theta}_2 = -\omega_2^2\theta_2 + \varepsilon(\theta_1-\theta_2)$$

ω = constant

$\dot{\theta} = \Omega$ ← angular Velocity

$\theta \Rightarrow$ angular position

a)

$$\dddot{\theta}_1 = -\omega_1^2\dot{\theta}_1 + \varepsilon(\dot{\theta}_2-\dot{\theta}_1)$$
$$\theta_1^{(4)} = -\omega_1^2\ddot{\theta}_1 + \varepsilon(\ddot{\theta}_2-\ddot{\theta}_1)$$

$$\ddot{\theta}_1 = -\omega_1^2\theta_1 + \varepsilon(\theta_2-\theta_1)$$
$$\varepsilon(\theta_2-\theta_1) = \ddot{\theta}_1 + \omega_1^2\theta_1$$
$$\underbrace{\theta_2 = \frac{\ddot{\theta}_1 + \omega_1^2\theta_1}{\varepsilon} + \theta_1}$$

$$\theta_1^{(4)} = -\omega_1^2\ddot{\theta}_1 + \varepsilon\ddot{\theta}_2 - \varepsilon\ddot{\theta}_1$$
$$\theta_1^{(4)} = -\omega_1^2\ddot{\theta}_1 - \varepsilon\ddot{\theta}_1 + \varepsilon\left[-\omega_2^2\theta_2 + \varepsilon\theta_1 - \varepsilon\theta_2\right]$$
$$\theta_1^{(4)} = -\omega_1^2\ddot{\theta}_1 - \varepsilon\ddot{\theta}_1 + \varepsilon^2\theta_1 - \varepsilon(\omega_2^2+\varepsilon)\theta_2$$
$$\theta_1^{(4)} = -\omega_1^2\ddot{\theta}_1 - \varepsilon\ddot{\theta}_1 + \varepsilon^2\theta_1 - \varepsilon(\omega_2^2+\varepsilon)\left(\frac{\ddot{\theta}_1 + \omega_1^2\theta_1}{\varepsilon} + \theta_1\right)$$
$$\theta_1^{(4)} = -\omega_1^2\ddot{\theta}_1 - \varepsilon\ddot{\theta}_1 + \varepsilon^2\theta_1 - \omega_2^2\ddot{\theta}_1 - \varepsilon\ddot{\theta}_1 - \omega_1^2\omega_2^2\theta_1 - \varepsilon\omega_1^2\theta_1 - \varepsilon(\omega_2^2+\varepsilon)\theta_1$$
$$= \ddot{\theta}_1(-\omega_1^2 - \varepsilon - \varepsilon - \omega_2^2) + \theta_1(\cancel{\varepsilon^2} - \omega_1^2\omega_2^2 - \varepsilon\omega_1^2 - \varepsilon\omega_2^2 - \cancel{\varepsilon^2})$$
$$\boxed{\theta_1^{(4)} = -\ddot{\theta}_1(2\varepsilon + \omega_1^2 + \omega_2^2) - \theta_1(\varepsilon\omega_1^2 + \varepsilon\omega_2^2 + \omega_1^2\omega_2^2)}$$

b) $\varepsilon(\theta_1-\theta_2) = \ddot{\theta}_2 + \omega_2^2\theta_2$

$$\underbrace{\theta_1 = \frac{\ddot{\theta}_2}{\varepsilon} + \frac{\omega_2^2}{\varepsilon}\theta_2 + \theta_2}$$
$$\theta_2^{(4)} = -\omega_2^2\ddot{\theta}_2 - \varepsilon\ddot{\theta}_2 + \varepsilon\ddot{\theta}_1$$
$$\theta_2^{(4)} = -\omega_2^2\ddot{\theta}_2 - \varepsilon\ddot{\theta}_2 + \varepsilon\left[-\omega_1^2\theta_1 + \varepsilon\theta_2 - \varepsilon\theta_1\right]$$
$$\theta_2^{(4)} = -\omega_2^2\ddot{\theta}_2 - \varepsilon\ddot{\theta}_2 + \varepsilon^2\theta_2 - \varepsilon(\omega_1^2+\varepsilon)\theta_1$$
$$\theta_2^{(4)} = -\omega_2^2\ddot{\theta}_2 - \varepsilon\ddot{\theta}_2 + \varepsilon^2\theta_2 - \varepsilon(\omega_1^2+\varepsilon)\left[\frac{\ddot{\theta}_2}{\varepsilon} + \frac{\omega_2^2}{\varepsilon}\theta_2 + \theta_2\right]$$
$$\theta_2^{(4)} = -\omega_2^2\ddot{\theta}_2 - \varepsilon\ddot{\theta} + \varepsilon^2\theta_2 - \omega_1^2\ddot{\theta}_2 - \varepsilon\ddot{\theta}_2 - \omega_1^2\omega_2^2\theta_2 - \varepsilon\omega_2^2\theta_2 - \varepsilon(\omega_1^2+\varepsilon)\theta_2$$
$$\theta_2^{(4)} = -\ddot{\theta}_2(\omega_2^2+\omega_1^2+\varepsilon+\varepsilon) + \theta_2(\cancel{\varepsilon^2} - \omega_1^2\omega_2^2 - \varepsilon\omega_2^2 - \varepsilon\omega_1^2 - \cancel{\varepsilon^2})$$
$$\boxed{\theta_2^{(4)} = -\ddot{\theta}_2(2\varepsilon + \omega_2^2 + \omega_1^2) - \theta_2(\varepsilon\omega_1^2 + \varepsilon\omega_2^2 + \omega_1^2\omega_2^2)}$$

c)

$\dot{\theta}_1 = \Omega_1 \qquad \dot{\Omega}_1 = -\omega_1^2\theta_1 + \varepsilon(\theta_2-\theta_1) \qquad \ddot{\theta}_1 = \dot{\Omega}_1$

$\dot{\theta}_2 = \Omega_2 \qquad \dot{\Omega}_2 = -\omega_2^2\theta_2 + \varepsilon(\theta_1-\theta_2) \qquad \ddot{\theta}_2 = \dot{\Omega}_2$

$$\boxed{\frac{d}{dt}\begin{bmatrix}\theta_1\\ \theta_2\\ \Omega_1\\ \Omega_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\omega_1^2-\varepsilon & \varepsilon & 0 & 0\\ \varepsilon & -\omega_2^2-\varepsilon & 0 & 0\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ \Omega_1\\ \Omega_2\end{bmatrix}}$$

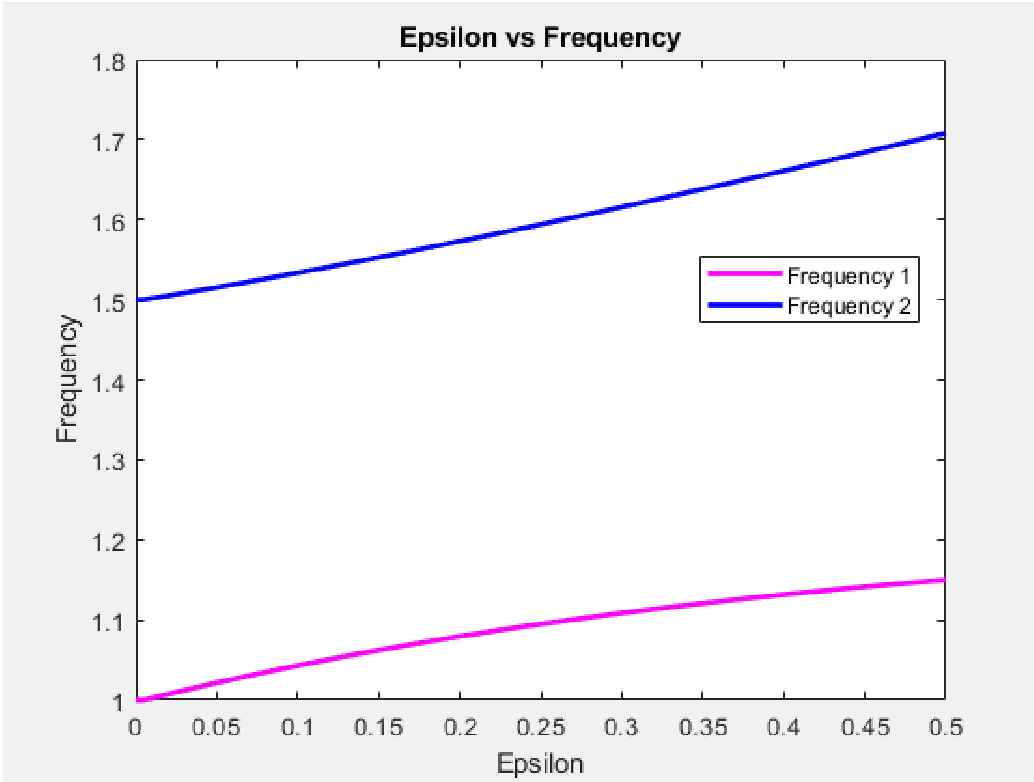
d) See Matlab

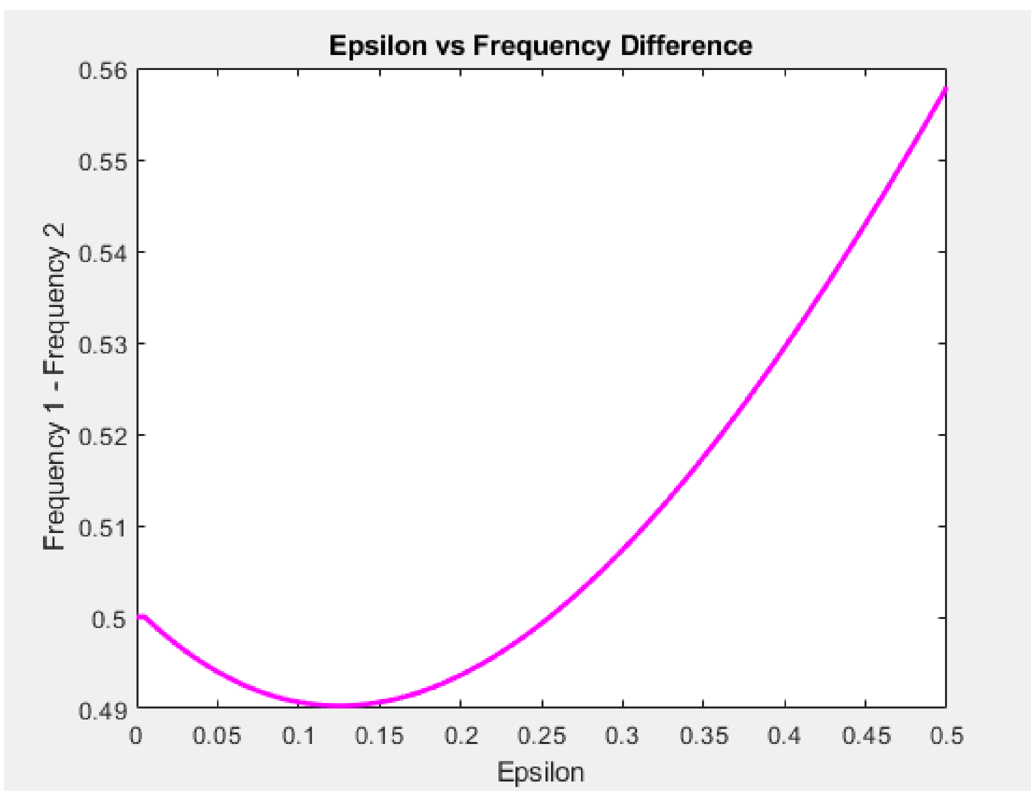
They will never be equal. The eigen values are always positive for the values of ω_1 and ω_2

$$D = \begin{bmatrix}\lambda_1 & & & \\ & \lambda_2 & 0 & \\ & & \lambda_3 & \\ 0 & & & \lambda_4\end{bmatrix}$$

↑ frequencies

e) They are never equal. see plots and codes


Epsilon vs Frequency
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
Frequency
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Epsilon
Frequency 1
Frequency 2


Epsilon vs Frequency Difference
0.56
0.55
0.54
0.53
0.52
0.51
0.5
0.49
Frequency 1 - Frequency 2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Epsilon

PROJECT NAME: ME 564

PROJECT NUMBER: HW #2

BY: Tim Yoder

SHEET NUMBER 5 OF 5

2-4 $\quad x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2 - a^2)y = 0 \qquad a = 1$

$$y(x) = c_0 + c_1 x + c_2 x^2 + \dots = \sum_{n=0}^{\infty} c_n x^n$$

$$y'(x) = c_1 + 2c_2 x + 3c_3 x^2 + \dots = \sum_{n=0}^{\infty} n c_n x^{n-1}$$

$$y''(x) = 2c_2 + (2\cdot 3)c_3 x + (2\cdot 3\cdot 4)c_4 x^2 = \sum_{n=0}^{\infty} n(n-1)c_n x^{n-2}$$

$$x^2\left(\sum_{n=0}^{\infty} n(n-1)c_n x^{n-2}\right) + x\left(\sum_{n=0}^{\infty} n c_n x^{n-1}\right) + (x^2-1)\left(\sum_{n=0}^{\infty} c_n x^n\right) = 0$$

$$\left(\sum_{n=0}^{\infty} n(n-1)c_n x^n\right) + \left(\sum_{n=0}^{\infty} n c_n x^n\right) + \left(\sum_{n=0}^{\infty} c_n x^{n+2}\right) - \left(\sum_{n=0}^{\infty} c_n x^n\right) = 0$$

$$= \sum_{n=2}^{\infty} n(n-1)c_n x^n + \cancel{c_1 x} + \sum_{n=2}^{\infty} n c_n x^n + \sum_{n=2}^{\infty} c_{n-2} x^n - c_0 - \cancel{c_1 x} - \sum_{n=2}^{\infty} c_n x^n = 0$$

$$= \sum_{n=2}^{\infty}\left[n(n-1)c_n + n c_n + c_{n-2} - c_n\right]x^n - c_0 = 0$$

$$= \sum_{n=2}^{\infty}\left[n^2 c_n \underbrace{\cancel{- n c_n + n c_n}}_{0} + c_{n-2} - c_n\right]x^n - c_0 = 0$$

$$= \sum_{n=2}^{\infty}\left[n^2 c_n + c_{n-2} - c_n\right]x^n - c_0 = 0$$

x^0: $\quad -c_0 = 0$

x^1: $\quad 0 = 0$

x^2: $\quad 2^2 c_2 + \cancel{c_0}^{\,0} - c_2 = 0$

$\quad (2^2 - 1)c_2 = 0 \Rightarrow c_2 = 0$

x^3: $\quad 3^2 c_3 + c_1 - c_3 = 0$

$\quad (3^2 - 1)c_3 = -c_1$

$\quad c_3 = \left(\dfrac{-1}{3^2-1}\right)c_1$

8

x^4: $\quad 4^2 c_4 + \cancel{c_2}^{\,0} - c_4 = 0$

$\quad (4^2-1)c_4 = 0 \Rightarrow c_4 = 0$

x^5: $\quad 5^2 c_5 + c_3 - c_5 = 0$

$\quad (5^2-1)c_5 = -c_3$

$$c_5 = \frac{-1}{(5^2-1)}c_3 = \left(\frac{-1}{3^2-1}\right)\left(\frac{-1}{5^2-1}\right)c_1 = \frac{(-1)^2}{(3+1)(3-1)(5+1)(5-1)}c_1$$

8 $\quad$ 24

x^{2k}: $\quad \boxed{c_{2k} = 0}$

x^{2k-1}: $\quad \boxed{c_{2k-1} = \dfrac{2(-1)^{k+1}}{(2k-2)!!\,(2k)!!}c_1}$

$(3^2-1)(5^2-1)(7^2-1)$

$(3+1)(3-1)(5+1)(5-1)(7+1)(7-1) + \dots$

$2\cdot 4\cdot 4\cdot 6\cdot 6\cdot 8 + \dots$

$\underbrace{(2\cdot 4\cdot 6\cdot 8 + \dots)(4\cdot 6\cdot 8\cdots)}$

$(7-1)!! \quad \dfrac{(7+1)!!}{2}$

$\dfrac{(n-1)!!\,(n+1)!!}{2}$

```
clear all; close all; clc

%% 2a
b = [0 1 0;
    0 0 1;
    2 1 -2;];
[v d]= eig(b);

y0A= [1; 1; 1];
y0B = [1; -1; 1];
y0C= [-1; -1; -1];

tspan = 0:.01:10;

[t,yA] = ode45(@(t,y)b*y,tspan,y0A);
[t,yB] = ode45(@(t,y)b*y,tspan,y0B);
[t,yC] = ode45(@(t,y)b*y,tspan,y0C);

figure(1)
plot(t,yA,'m','linewidth',2)
title('IC = [1 1 1]')
ylabel('x(t)')
xlabel('t')

figure(2)
plot(t,yB,'m','linewidth',2)
title('IC = [1 -1 1]')
ylabel('x(t)')
xlabel('t')

figure(3)
plot(t,yB,'m','linewidth',2)
title('IC = [-1 -1 -1] - starting on other eigvec')
ylabel('x(t)')
xlabel('t')

%% 2b

c = [0 1 0 0;
    0 0 1 0;
    0 0 0 1;
    6 5 -5 -5;];
[vc dc]= eig(c)

y0D= [1; 1; 1; 1;];
y0E = [1; -1; 1; -1;];

tspan = 0:.01:10;

[t,yD] = ode45(@(t,y)c*y,tspan,y0D);
[t,yE] = ode45(@(t,y)c*y,tspan,y0E);
```

```
figure(4)
plot(t,yD,'m','linewidth',2)
title('IC = [1 1 1 1]')
ylabel('x(t)')
xlabel('t')

figure(5)
plot(t,yE,'m','linewidth',2)
title('IC = [1 -1 1 -1]')
ylabel('x(t)')
xlabel('t')

%% 3

e = 0:.005:.5;

w1 = 1;
w2 = 1.5;

ep = 0;

A = [0 0 1 0;
    0 0 0 1;
    -w1^2-ep ep 0 0;
    ep -w2^2-ep 0 0;];

eig3 = eig(A);

for i = 1:size(e,2)-1;
    eig_out = eig(A);
    ep = ep + .005;
    A = [0 0 1 0;
        0 0 0 1;
        -w1^2-ep ep 0 0;
        ep -w2^2-ep 0 0;];
    eig3 = [eig3 eig_out];
end


freq1 = imag(eig3(1,:));
freq2 = imag(eig3(3,:));


figure
plot(e,freq1, 'm', 'linewidth',2)
hold on
plot(e,freq2, 'b', 'linewidth',2)
title('Epsilon vs Frequency')
xlabel('Epsilon')
ylabel('Frequency')
```

```
legend('Frequency 1', 'Frequency 2', 'location', 'best')

figure
plot(e,abs(freq1-freq2), 'm','linewidth',2)
title('Epsilon vs Frequency Difference')
xlabel('Epsilon')
ylabel('Frequency 1 - Frequency 2')
```