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ME 564
PROJECT NAME
HW #5
PROJECT NUMBER

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BY
1 2
SHEET NUMBER OF

5-1 $\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} x$ $\lambda_1 = -2$, $\lambda_2 = -4$ linearly independent eigenvectors can be investigated separately.

a) F.E. unstable if $|1 + \lambda \Delta t| > 1$

$|1 - 2\Delta t| > 1$

$|1 - 4\Delta t| > 1$

$-1 > 1 - 2\Delta t > 1$

$\Delta t < 0$, $\boxed{\Delta t > 1/2}$

$\Delta t > 1$, $t < 0$

$\therefore$ unstable if time step exceeds

b) B.E unstable if $|(1 - \Delta t \lambda)^{-1}| < 1$

$\lambda = -4$

$|(1 + 4\Delta t)^{-1}| > 1$

$-1 > \frac{1}{1+4\Delta t} > 1$

$-1 > \frac{1}{1+4\Delta t}$

$-1 - 4\Delta t > 1$

$-4\Delta t > 1$

$\Delta t < -\frac{1}{4}$

$1 > 1 + 4\Delta t$

$0 > 4\Delta t$

$\Delta t < 0$

$\lambda = -2$: $|(1 + 2\Delta t)^{-1}| > 1$

$-1 > \frac{1}{1+2\Delta t} > 1$

$-1 > \frac{1}{1+2\Delta t}$

$-2\Delta t - 1 > 1$

$-2\Delta t > 2$

$\Delta t < -1$

$1 > 1 + 2\Delta t$

$0 > 2\Delta t$

$\Delta t < 0$

$\therefore$ B.E. is stable for all positive Δt

5-2

1) $\langle x, y \rangle = \langle y, x \rangle$

2) $\langle ax, y \rangle = a\langle x, y \rangle \quad a \in \mathbb{R}$

3) $\langle x, x \rangle \geq 0$ with equality if $x = 0$

$[0, 1]$ Bounded interval

$$\langle f, g \rangle = \int_0^1 f(x) g(x)\, dx$$

a)

1) $\int_0^1 f(x) g(x)\, dx = \int_0^1 g(x) f(x)\, dx$ ✓

2) $\int_0^1 a f(x) g(x)\, dx = a \int_0^1 f(x) g(x)\, dx$ ✓

3) $\int_0^1 f(x) f(x)\, dx = 0 \Rightarrow f(x) = 0$ ✓

b) $\int_0^1 \cos(\pi m x) \cos(\pi n x)\, dx = 0$ $\quad \cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$$\frac{1}{2}\int_0^1 \cos(\pi x(m-n))\, dx + \frac{1}{2}\int_0^1 \cos(\pi x(m+n))\, dx = 0$$

$$\frac{1}{2}\int_0^1 \cos(\pi x(m-n))\, dx = -\frac{1}{2}\int_0^1 \cos(\pi x(m+n))\, dx$$

$$\frac{1}{2\pi(m-n)} \sin(\pi x(m-n))\Big|_0^1 = -\frac{1}{2\pi(m+n)} \sin(\pi x(m+n))\Big|_0^1$$

$$\frac{1}{m-n}\left[\sin[\pi(m-n)] - \sin(0)\right] = -\frac{1}{m+n}\left[\sin[\pi(m+n)] - \sin 0\right]$$

($\sin(0)$ and $\sin 0$ are 0)

If $m \neq n$

$$\frac{\sin(\pi(m-n))}{m-n} = -\frac{\sin(\pi(m+n))}{m+n}$$

(both sides are 0) ✓

for any integers $m \neq n$

If $m = n$

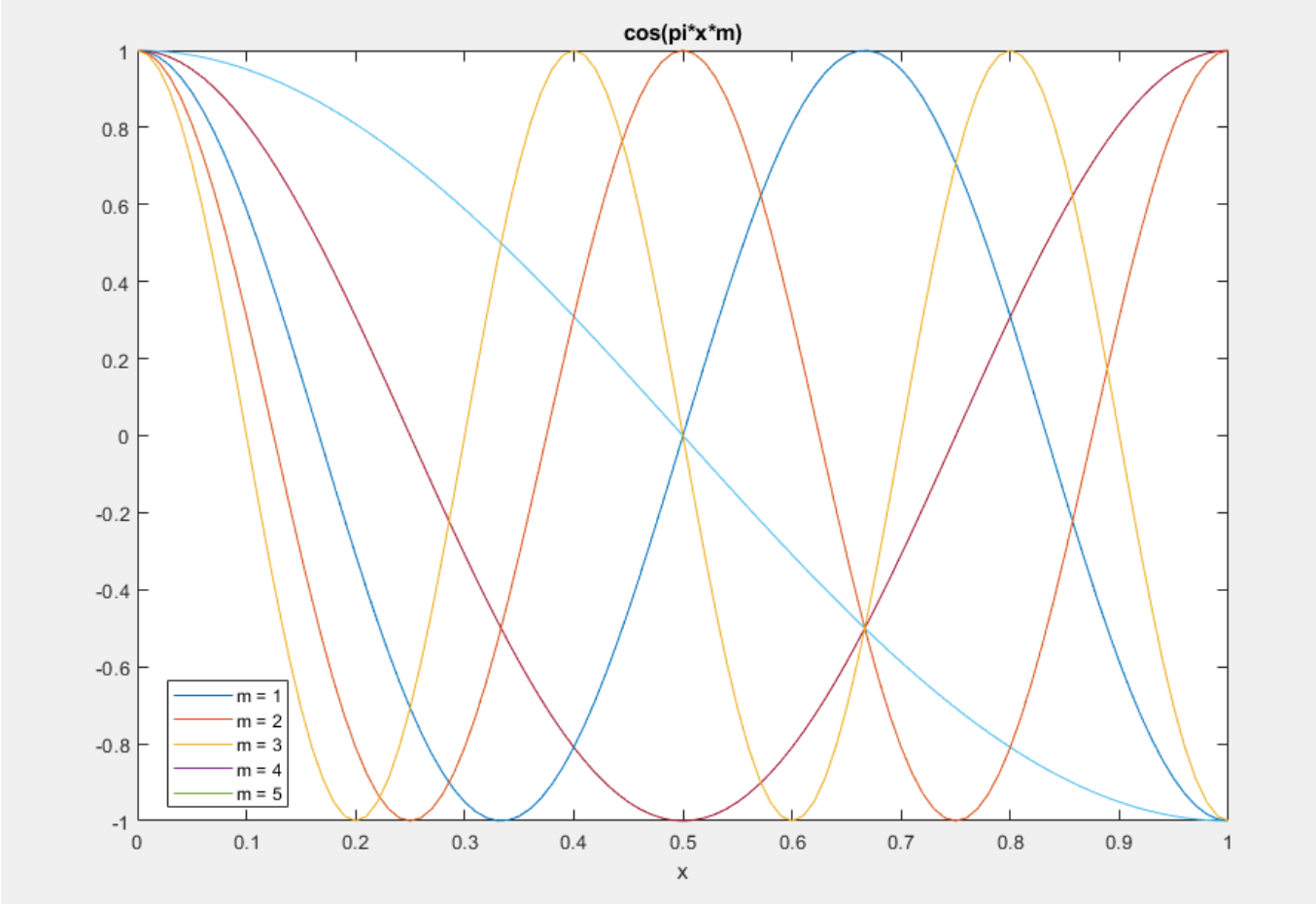
$$\frac{1}{2}\int_0^1 \cos(0)\, dx = \frac{1}{2}$$

$$+\frac{1}{2}\int_0^1 \cos(2\pi x m)\, dx = \frac{1}{4\pi m}\sin(2\pi m)$$

$$\frac{1}{2} + \frac{1}{4\pi m}\sin(2\pi m) \neq 0$$

for any non-negative m.

c) See Matlab output


cos(pi*x*m)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
m = 1
m = 2
m = 3
m = 4
m = 5

```
dt = 0.01;
x = 0:dt:1;

for i = 1:5
    fun = cos(i*pi()*x);
    plot(x,fun)
    legend('m = 1','m = 2', 'm = 3', 'm = 4', 'm = 5', 'location', 'best')
    xlabel('x')
    title('cos(pi*x*m)')
    hold on
end

%%

test = [1, 4;
    2, 6;
    3, 15;];

out = zeros(3,1);

for i = 1:3

v = cos(pi()*x*test(i,1));
w = cos(pi()*x*test(i,2));
Y = v.*w;

out(i) = trapz(Y);

end
```

```
out =

   1.0e-13 *

    0.0133
   -0.1188
   -0.1865
```

Zero to machine precision