Project 1 Report
Water Tower
CIVL/ECPE 259: Sensor Networks
Fall 2016
September 28, 2016
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Section 1: Introduction

Project 1 was constructed as an introduction to various foundational skills that will be applied deeper as CIVL/ECPE 259 progresses. Some of these skills include the fundamental understanding and equation sets of vibrations in structures, working knowledge of microcontrollers as a tool to acquire data, and the utilization of system modeling software that can simulate different phenomena in particular systems. The vehicle to incorporate these various skills was the the modeling, building, analyzing, and testing of a single degree of freedom water tower system.

The process used to implement the water tower project was to first develop a theoretical model of the system for predictions of system behavior, then simulate the model using the RISA software, acquire experimental acceleration data of the system in motion, and finally, analyze the resultant data to make a decision on how the system changed after liquid had been removed from the water tower. The predictive model results, simulated model results, and measured model results are presented in the next 3 sections followed by a conclusive summary of the overall results of the project.

Section 2: Theoretical System

The following equations characterized the theoretical model of the single degree of freedom water tower system. Each were used in the calculations of the parameters outlined in Table 2.1. Equation 2.1 is the most fundamental equation of this project. The equation of motion relates the mass acceleration, damping coefficient velocity, and the spring constant position to be the sum of the forces adding to zero. This follows Newton's Second Law of Motion where the sum of all forces must equal zero. Equation 2.1 results in the most practical equation utilized in the project, Equation 2.2. The exact position of the water tower system in oscillation can be found during any time after an initial displacement or velocity gives energy to the structure by using Equation 2.2. The natural frequency in Equation 2.3 represents the vibrational response of the system to an initial energy input without any damping effect present in the structure. To get a better understanding of how the structure behaves including a damping effect like, possibly friction, Equation 2.5 is used. The damping ratio in Equation 2.4 is found by experimentation after acquiring the logarithmic decrement using Equation 2.7. The logarithmic decrement can also be found experimentally by looking at two different peaks, u_i and u_{i+j}, in the oscillation plot of the moving system. This collection of equations allowed the theoretical and experimental calculations to be established.

$$mu'' + cu' + ku = 0$$

Equation 2.1: Equation of Motion

$$u(t) = e^{-\zeta\omega_n t}[u_o \mathrm{Cos}(\omega_D t) + \frac{u'_o + \zeta\omega_n u_o}{\omega_D}\mathrm{Sin}(\omega_D t)]$$

Equation 2.2: Solution to Equation of Motion for Initial Conditions

$$\omega_n = \sqrt{\frac{k}{m}}$$

Equation 2.3: Natural Frequency

$$\zeta = \frac{\delta}{2\pi}$$

Equation 2.4: Damping Ratio

$$\omega_D = \omega_n \sqrt{1 - \zeta^2}$$

Equation 2.5: Damped Natural Frequency

$$T_D = \frac{2\pi}{\omega_D}$$

Equation 2.6: Damped Natural Period

$$\delta = \frac{1}{j} ln(\frac{u_i}{u_{i+j}})$$

Equation 2.7 Logarithmic Decrement

Table 2.1 presents the parameters and values used in the predictive modeling of the system. The parameter, m, is the water bottle mass of the top of the water tower that was measured on a digital scale. The spring constant, k, was determined experimentally for the yardstick used . Damping ratio, ζ, was calculated by Equation 2.4 to be 0.0102. Since ζ < 1, the system as a whole is underdamped. This means that the amplitude of the damped frequency decreases logarithmically as time increases and contains overshoot. From intuition, the behavior of an underdamped system should accurately represent an oscillating water tower. The natural frequency, ω_n, and damped natural frequency, ω_D, where found simply by using Equations 2.3 and 2.5 respectively.

Parameter	**Value**
m - mass	0.24 kg
k - Stiffness	46.3 N/m
ζ - Damping Ratio	0.0102
T_n - Period	0.523 s
ω_D - Damped Natural Frequency	12.007 rad/s
ω_n - Natural Frequency	12.008 rad/s

Table 2.1: Provided Constants

Section 3: RISA System Simulations

RISA (Rapid Interactive Structural Analysis) was a secondary method to analyze the vibration characteristics of our simple structure. By inputting the material properties of the yardstick used to construct the "water tower", and applying a load which simulated the mass of the attached water bottle, RISA was then capable of running an analysis analogous to FEA in order to determine the vibrational modes the structure will encounter, as well as the relative displacement the structure will endure while going through those vibration modes. Table 3.1 describes the three primary modes of vibration which will exist in the structure. For this analysis, we are most interested in mode 1 because it poses the greatest threat to the structure's integrity. Mode 1 is the iconic mode which is seen during an earthquake when a skyscraper slowly sways back and forth.

Mode	**Frequency (Hz)**	**Period (s)**	**Mode Shape**
1	2.347	0.426	Rotation about Z-axis
2	13.278	0.075	Bowing along X-axis
3	648.104	0.002	Elongation in Y-axis

Table 3.1: RISA Simulation Frequency Response

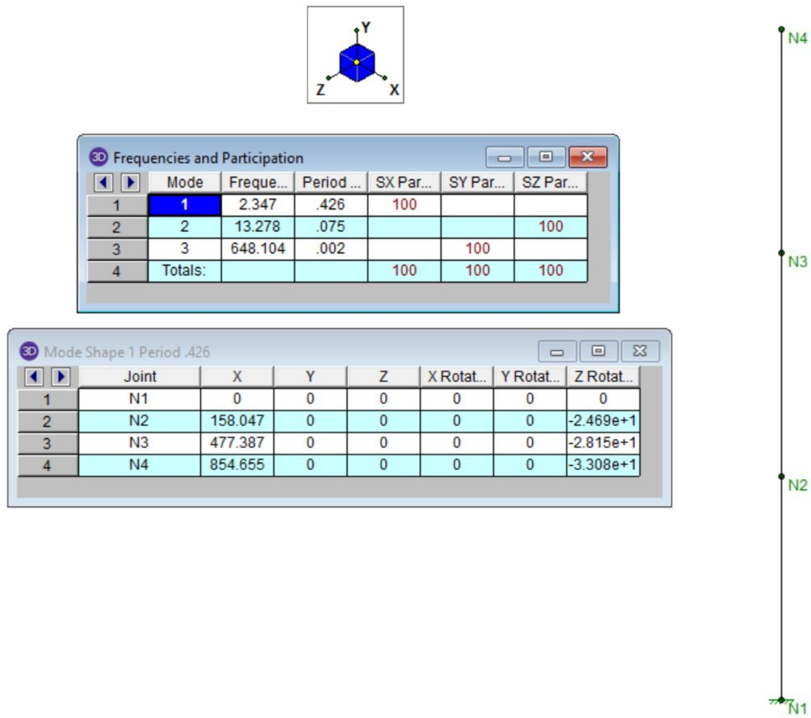
Table 3.2 shows the relative motion of the structure during the first mode of vibration. Although these values are unitless, it describes how much displacement exists in one node relative to another. In this case, when node 4 (or N4 in the table) has moved 854.7 mm, node 2 is displaced 158 mm. (In this example it is assumed the units are in millimeters, but this could be nanometers, or inches. These values describe the ratio of how much one node moves relative to the movement of another node.)

Joint	**Vibration Magnitude along X-axis**
N1	0
N2	158.0
N3	477.4
N4	854.7

Table 3.2: RISA Simulation Mode 1 Response Relative Magnitudes

Figure 3.1 demonstrates how the "water tower" is modeled in RISA. Four nodes are specified on the structure (yardstick), and a force is applied at N4 to simulate the mass of the water bottle. Each node is a location where the software will simulate how the structure will respond at that specific location.

Notice N1 is a fixed point. All other points will move relative to N1. This is also why zero magnitude of displacement exists at N1.



Frequencies and Participation

	Mode	Freque...	Period ...	SX Par...	SY Par...	SZ Par...
1	1	2.347	.426	100		
2	2	13.278	.075			100
3	3	648.104	.002		100	
4	Totals:			100	100	100

Mode Shape 1 Period .426

	Joint	X	Y	Z	X Rotat...	Y Rotat...	Z Rotat...
1	N1	0	0	0	0	0	0
2	N2	158.047	0	0	0	0	-2.469e+1
3	N3	477.387	0	0	0	0	-2.815e+1
4	N4	854.655	0	0	0	0	-3.308e+1

Figure 3.1: RISA Simulation Screenshot

Section 4: Experimental System

To experimentally determine the vibrational response of the system, an ADXL335 3-axis accelerometer is used to measure the acceleration of the water bottle at the top of the tower. From the acceleration data and known stiffness (k) of the structure, the mass of the water bottle can be calculated.

Figure 4.1 displays the 1-D acceleration of the system, as well as the local maxima of each vibration cycle. At this point, Matlab is used to numerically calculate the period of the response, Equation 2.7 to determine the damping ratio, then Equation 2.3 is used to calculate the “mass”. The mass is not exactly the same as the water bottle, but it turns out that due to the approximate linearity of the system, as water is removed from the water bottle, the delta of the calculated mass is about one-to-one.

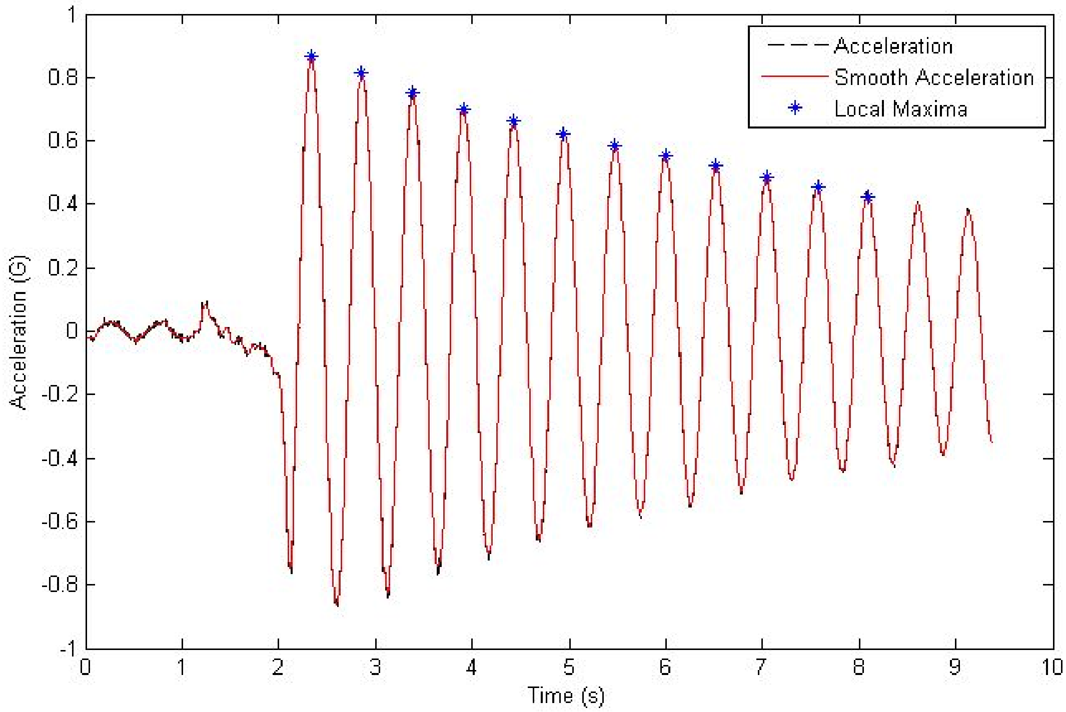


Figure 4.1: Acceleration Response of Full Water Tower

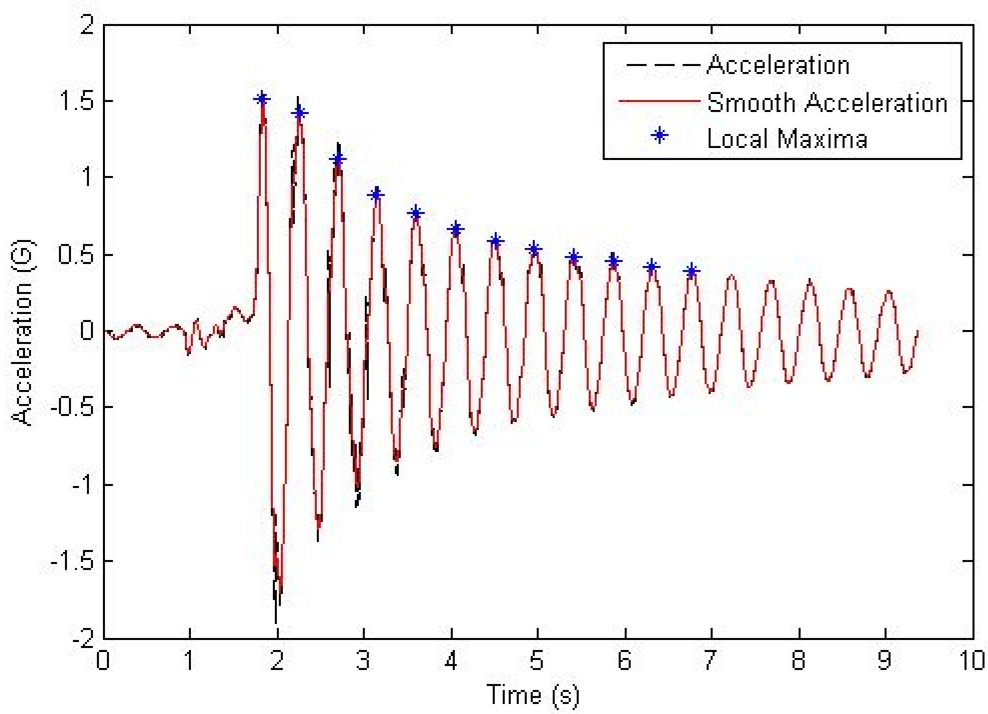


Figure 4.2: Acceleration Response of Partially Empty Water Tower

Table 4.1 shows how the parameters of the system change after removing an unknown volume of water from the water tower. While measuring the vibration of the "undamaged" system, the mass was measured to be 0.32 kg. After removing water from the tower, the mass was reduced to 0.24 kg.

To clarify, the water bottle was known to be 0.24 kg, but the experiment showed that the mass of the water was reduced by 0.08 kg, which means the actual mass of the water bottle was reduced to **0.16 kg.**

To determine the actual mass of the water bottle, it was measured on a scale four separate times to account for potential error in the scale. Twice it measured 0.16 kg, and twice it measured 0.14 kg. These values were averaged, and 0.15 kg was accepted to be the actual mass of the water bottle.

As a side note, because the scale proved to be not repeatable, there is clearly a significant amount of error in the mass measurement itself. The standard deviation of the scale was 0.012. It seems reasonable to argue the estimated weight of the water was within the error of the scale, thus, with the measurement limitations, the estimate is accurate.

System	Experimental Mass (kg)	Natural Frequency (rad/s)	Natural Period (s)	Damping Ratio
Full	0.32	12.0	0.52	0.010
Partially Empty	0.24	17.0	0.37	0.010

Table 4.1: Experimental Calculated Parameters

Experimentally Determined New Mass	0.16 kg
Actual New Mass (According to Scale)	0.15 kg
Water Bottle Experimental Measurement Error	6.7 %
Scale Accuracy (STD)	± 0.012

Table 4.2: Error of Calculated Mass

Theoretically, if a double integration of the acceleration is taken, the position of the system should be the result. Unfortunately, this method of determining the position results in substantial error (as seen in Figure 4.4).

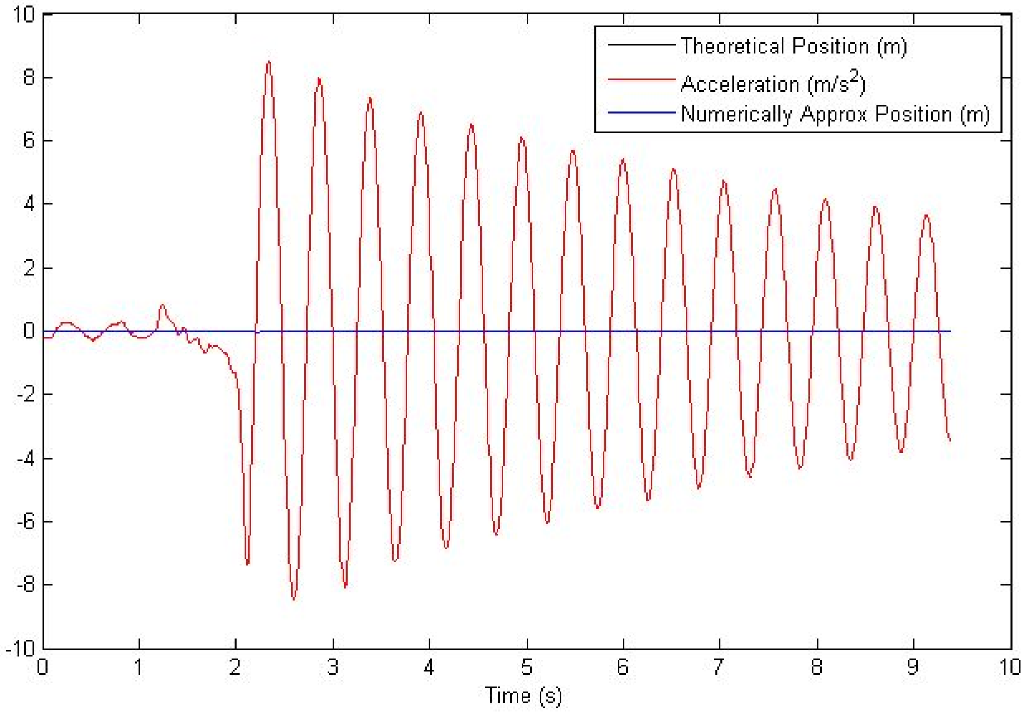


Figure 4.3: Numerical Double Integration to Derive Position

In order to transform this erroneous position graph (figure 4.4) into a plot that properly represents the displacement of the water tower (figure 4.5), a moving average of the graph (Figure 4.4) is used to properly shift the plot in the y-axis. After that shift, the plot is still slightly shifted down in y-direction (negative displacement), so a total mean of the data is used to shift the data in positive Y. The resulting graph is compared to the theoretical model in Figure 4.5. There is still about a 25% discrepancy between the theoretical model and the model based on the double integration of the accelerometer data, but most of this error may be due to the initial displacement of the water tower. In the theoretical model, a 1" (0.0254 m) initial displacement is applied. In the experimental model, an attempt of applying the same initial displacement was made, but substantial error existed due to the imprecision of our hand pulling the water back to a displacement of 1" .

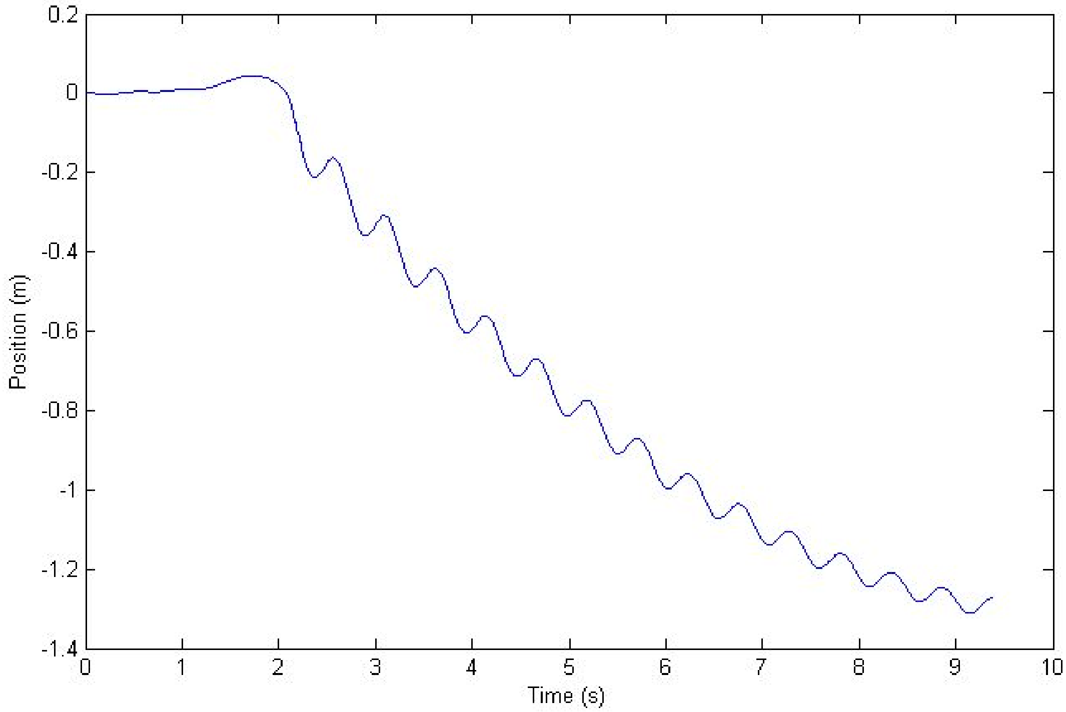


Figure 4.4: Erroneous Numerically Calculated Position

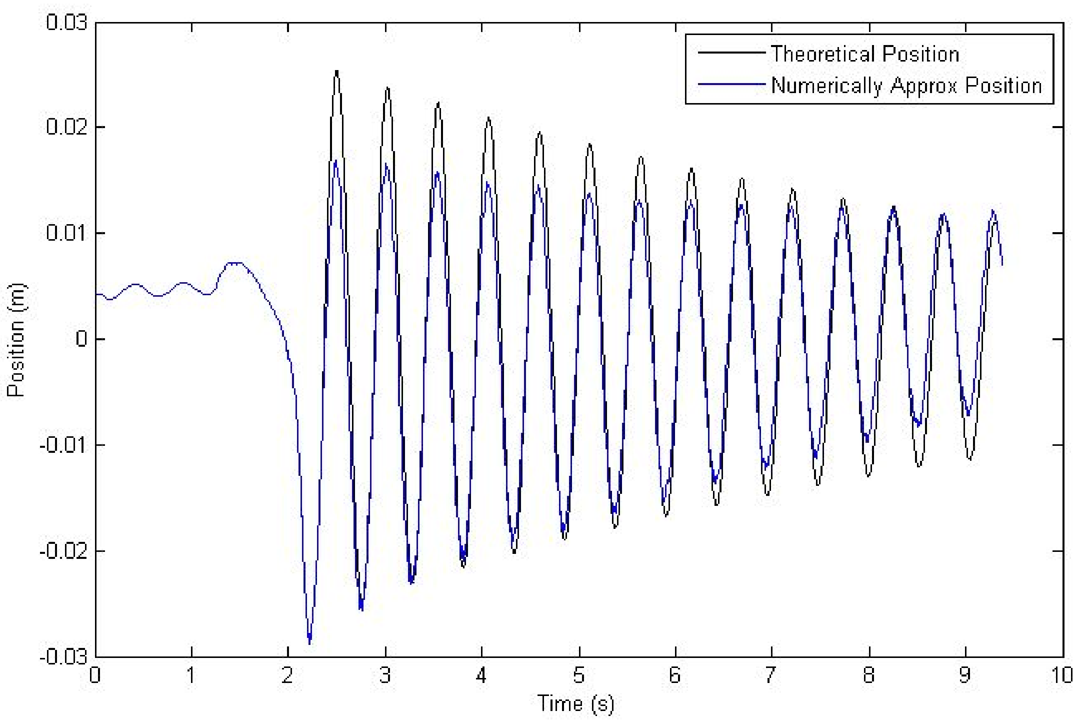


Figure 4.5: Compare Theoretical Position to Numerically Calculated Position

Section 5: Model Comparison

Significant discrepancies exist between RISA and the experimental models. In the experimental measurement, the natural frequency was 1.91 Hz while the RISA model calculated a 2.35 Hz frequency (and the period varied accordingly). These variations can be attributed to both the imperfect RISA model, and error in the accelerometer. Although the accelerometer tracks the general shape of the vibration well, enough noise exists to potentially throw off the ability to determine the position of each local maxima in the raw data. If this experiment was done again, a fourier transform would have effectively removed this noise, and allowed us to focus only on the first vibration mode, effectively removing the noise. The other issue was that the RISA model assumed the water tower was perfectly fixed to the ground, however, that was not the case. The water tower was attached to the yardstick with glue and tape, and the yardstick was mounted to a table with clamps. Any vibration in the table would propagate through the yardstick and generate unexpected vibration modes in the tower, creating error. There is another vibration mode caused by the glue interface between the yardstick and water bottle which would also cause unexpected vibration. (the glue would expand and contract). All these variables can account for the discrepancies between these two models.

Model	**Frequency (Hz)**	**Period (s)**
RISA	2.35	0.43
Experimental	1.91	0.52
Theoretical (Parameters are determined from experiment)	1.91	0.52

Table 5.1: Compare RISA to Experimental Response

Because many of the parameters input in the theoretical model (such as natural frequency, period, and damping ratio) are based on the results of the experimental model, there are not many differences between the two. The only major difference is that the numerically calculated displacement is slightly shifted down in the y-axis. (see figure 4.4 and 4.5) This is because any slight error in the accelerometer will accumulate through time as we integrate (and even moreso after integrating twice), thus the entire displacement plot shift away from position = 0 as time increases (figure 4.4). Because we physically know the position oscillates about zero, the data was shifted using a moving average to account for this shift. (figure 4.5). Because to extent the data was numerically "fudged" so the data matched the result of a known system, it is suspected the shift in the amplitude is due to the numerical analysis of the data.

Section 6: Conclusion

Because the experimental system successfully measured the change of mass of the water bottle within the error of the scale used, we would claim the experimental model successfully analyzed the change of mass of the water tower. This success is due to the simplicity of the model. This water tower was effectively a one dimensional system, with few causes of noise and error. As more dimensions of motion are induced in a system, the analysis becomes exponentially more difficult to analyze. This analysis lacked that complexity.

The primary assumption of this experiment was that the accelerometer could accurately measure the motion of the system, and the sensor's capabilities surpassed all expectations. At a sampling rate of 0.007 seconds/measurement, the sensor was fully capable of determining the how the acceleration changed through time, enabling the calculations of the critical vibrational characteristics: natural frequency and damping ratio.

Overall, from the experimental setup used to collect data, or the RISA simulation, to the theoretical modeling, the experiment proved to be a success.