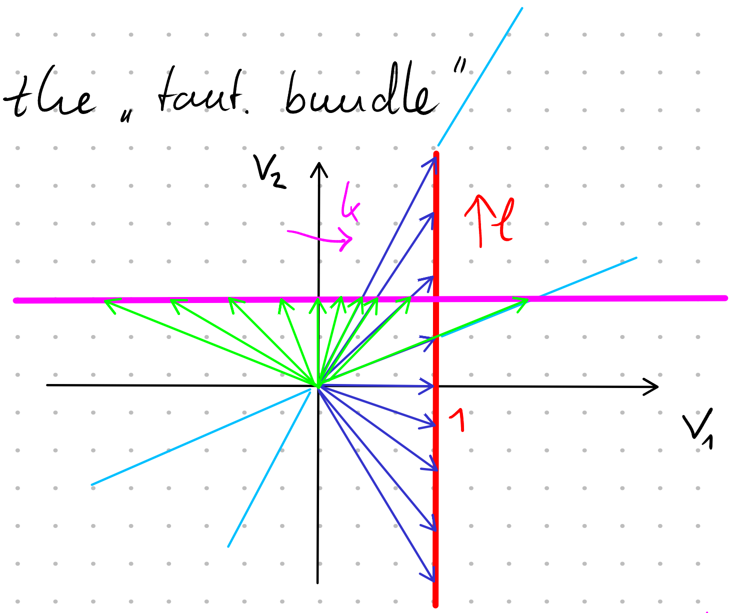
What vector bundles are there on $\mathbb{P}^1$?

$\mathcal{O}_{\mathbb{P}^1}(-1)$ the „taut. bundle"



„blue vs. green arrow" $\hat{=}$ transition fct.

$$\underbrace{\begin{pmatrix} 1 \\ \ell \end{pmatrix}}_{\vec{e}_1} = \underbrace{\ell}_{\in \mathcal{O}(U_1 \cap U_2)} \cdot \begin{pmatrix} k \\ 1 \end{pmatrix} =: \vec{e}_2$$

$k = \frac{1}{\ell} \quad \ell = \frac{1}{k}$

What is $\mathcal{O}(-d)$?

$$\mathcal{O}(-d) := \underbrace{\mathcal{O}(-1) \otimes \mathcal{O}(-1) \otimes \dots \otimes \mathcal{O}(-1)}_{d \text{ times}}$$

this is trivialized ...

... on U_1 by $\quad \vec{e}_1 \otimes \vec{e}_1 \otimes \dots \otimes \vec{e}_1$

... on U_2 by $\quad \vec{e}_2 \otimes \vec{e}_2 \otimes \dots \otimes \vec{e}_2$

with an induced transition

$$\vec{e}_1 \otimes \vec{e}_1 \otimes \dots \otimes \vec{e}_1 = \ell \cdot \vec{e}_2 \otimes \ell \cdot \vec{e}_2 \otimes \dots \otimes \ell \cdot \vec{e}_2$$

$$= (\ell)^d \cdot \vec{e}_2 \otimes \vec{e}_2 \otimes \dots \otimes \vec{e}_2$$

Upshot: $\mathcal{O}(-d)$ has $(\ell)^d$ as transition-functions

Similar: $\mathcal{O}(d)$ has $\underbrace{(\ell)^{-d}}_{= k^d}$ as transition-functions

Looking at the end of Guille's computation we see that we found $\mathcal{O}(3)$!