CCSing - Introduction to Chern classes on sing spaces

$E \in \mathrm{Vect}_n^{\mathbb{C}}(N)$, $E \mapsto c_k(E) \in H^k(N)$

$f: M \to N$, $f^*E \mapsto f^* c_k(E) \in H^k(M)$

$\in \mathrm{Vect}_n^{\mathbb{C}}(M)$ i.e. $c_k(f^*E)$

$= f^*(c_k E)$

1. Chern classes on smooth manifold

Let M be a smooth mfd

Chern class c_n = natural transformation $\left(\mathrm{Vect}_n^{\mathbb{C}} \longrightarrow H^*(\cdot\,;\mathbb{Z})\right)$

s.t. (i) $\forall E \in \mathrm{Vect}_n^{\mathbb{C}}(M)$: $c_0(E) = 1$

(ii) $c = \sum c_i$: $c(E \oplus F) = c(E) \cup c(F)$ — Whitney Sum Formula

(iii) $c(\underbrace{\mathcal{O}_{\mathbb{C}P^k}(-1)}_{\text{tautological bundle}}) = 1 - [\mathbb{C}P^{k-1}] \cap [\mathbb{C}P^k]$ — Normalization

Chern class of M: $c_k(TM)$

2. Chern classes on singular varieties X

Constructible functions: $\alpha = \sum_i \underbrace{n_i}_{\text{integers}} \underbrace{1\!\!1_{V_i}}_{\text{subvariety}}$

$F(X)$

Ex: $\mathcal{F}_\bullet$ = constr. complex of sheaves

$\Rightarrow$ $p \mapsto \chi(\mathcal{F}_\bullet)(p) = \sum_i (-1)^i \dim H^i(\mathcal{F}_\bullet)_p \in F(X)$

$f: X \to Y$

Pushforward: $\boxed{f_*(1\!\!1_V)(p) = \chi(f^{-1}(p) \cap V)}$

top. Euler char.

F = covariant functor

Deligne-Grothendieck-Conjecture: $f: X \to Y \rightsquigarrow f_*: F(X) \to F(Y)$

$\exists$ nat. transformation of covariant functors $: c_{CSM}: F \longrightarrow H_*(\cdot, \mathbb{Z})$ s.t. $c_{CSM}(f_*(\alpha)) = f_*(c_{CSM}(\alpha))$

on V = smooth: $1\!\!1 \longmapsto c(V) \cap [V] \in H_0(V; \mathbb{Z})$ $\quad c_{CSM}(\alpha + \beta) = c_{CSM}(\alpha) + c_{CSM}(\beta)$

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<u>Thm:</u> There is one unique nat. transformation like this.

<u>Proof:</u> Two Ingredients: Mather-class $V \subseteq X$. $c_M(V) \in H_*(X)$

$A(X) \longrightarrow H_*(X)$

local Euler obstruction $\to$ identifies constr functions with algebraic cycles = formal sums of subvarieties

$F(X) \cong A(X)$

Technique: Graph construction is used to prove the important properties of: $F \dashrightarrow H_*(\cdot\,; \mathbb{Z})$

Date/time:

			Yes	No	Maybe
Mo	2:15	CET	6	/	/
Mo	4:15	CET	6	/	/
Is another day better ?			/	6	/

Speakers (prelim. list)

- Guille after 28th of Nov. (or after Christmas)
- Timo (1st Talk)
- Matthias (3rd Talk ?)
- Markus (Talk 6, concrete examples for derived pushforward)
- Alex (maybe, after December)
- Abraham (Talk 4 (?) or sth. around Talk 6)