Monopolistically Skewed Business Cycles*

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September 27, 2025

Abstract

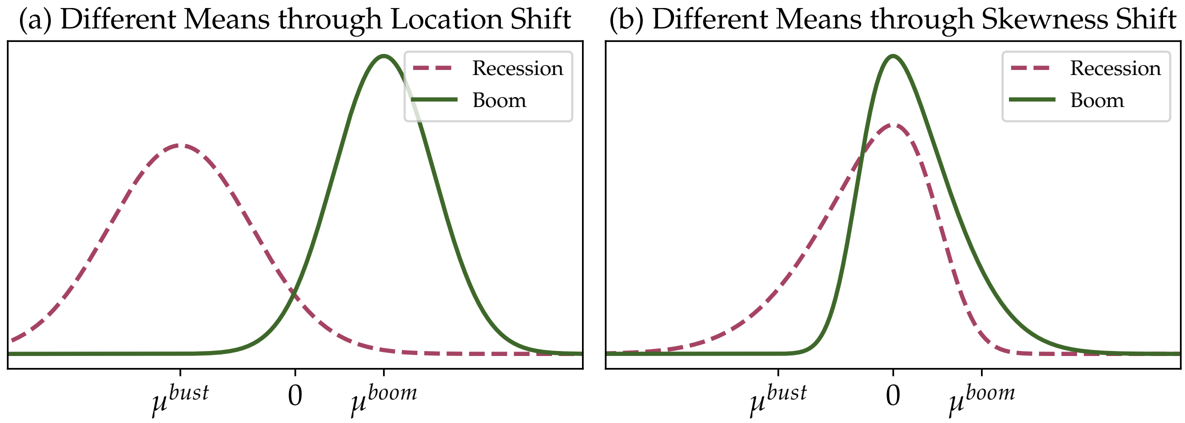
We show that the cross-sectional distribution of firm growth rates changes shape over the business cycle: it is right-skewed in booms, left-skewed in recessions, and these swings are more pronounced for larger firms. We call this the size gradient of skewness. We show that one way of explaining this pattern is through a parsimonious demand-side framework in which market power maps symmetric shocks into skewed growth outcomes. Stronger market power implies more concave responses and thus greater skewness. Countercyclical variance — or equivalently heterogeneous exposures to aggregate impulses — then generates the procyclical, size-dependent skewness we observe. Consistent with this mechanism, impulse responses to aggregate shocks show that growth and skewness move in tandem, with the skewness response concentrated among large firms. The results imply that large firms amplify cyclical asymmetry through market power, and that outcome-based policies risk responding to distributional patterns that reflect propagation rather than the shocks themselves.

Keywords: business cycle, skewness, market power
JEL Codes: D21, E32, L11

*The views expressed in this paper are those of the authors and should not be attributed to De Nederlandsche Bank. This paper builds on *Granular Responses to Aggregate Shocks: An Explanation of Skewness over the Business Cycle* by Niklas Schmitz and supersedes it. We thank Guido Ascari, Igli Bajo, Anmol Bhandari, Leonard Bocquet, Paul Bouscasse, Vasco Carvalho, Elisa Faraglia, Lukas Freund, Maren Froemel, Xavier Gabaix, Chryssi Giannitsarou, Fatih Guvenen, Sean Holly, Felix Kübler, Hanbaek Lee, Glenn Magerman, Shane Mahen, Leonardo Melosi, Mathis Momm, Charles Parry, Alba Patozi, Margit Reischer, Petr Sedlacek, Adam Hal Spencer, and Philipp Sternal for their valuable feedback and participants at the Cambridge Macro PhD Workshop and seminars at De Nederlandsche Bank for helpful comments. We also thank Alba Patozi for generously sharing data with us.

1 Introduction

Figure 1: Distributional Shift Location vs. Skewness



Note: The figure shows unskewed but scaled and mode-shifted distributions on the left, versus skewed and scaled, but unshifted distributions on the right. We think of the left distributions as canonical *input* distributions to economic models and the right ones as *output* distributions of economic observables.

Firm growth rate distributions change in shape over the business cycle: their tails thicken or thin out asymmetrically (cf. Salgado et al. (2025); Ilut et al. (2018)). In contrast, classical assumptions about aggregate fluctuations do not affect the shape. They concern cyclical movement of the mean and, perhaps, the variance of *input* shocks, i.e., shock which hit production cost of or the demand for firms' products. Figure 1 illustrates this contrast over the cycle: classical input shock distributions in panel (a) versus right- and left-skewed growth outcomes in panel (b).[1] In this paper, we establish additional descriptive evidence about the cyclical shifts of growth distributions, and how these shifts vary for the very largest firms. We then reconcile the tension between in- and output distributions. We propose a transmission mechanism which links parsimonious input distributions to output distributions that change shape over the business cycle. Hence, this paper gives one answer to a central question in macroeconomics: *How do unobservable shocks to firms translate into observable outcomes of growth?*

To make progress on this question, we show empirically that firm size is a key dimension determining how the skewness of firm growth changes over the cycle. We argue that heterogeneity by market power and, by extension, firm size is evidence of a demand-side mechanism at play. Market power, it turns out, is the ghost in the machine of our transmission model.

Our paper makes four contributions to answer the original question. First, we doc-

[1]Right-skewed (left-skewed) means the distribution has a positive (negative) skewness index. In comparison, a distribution that is 'more left skewed' than another has a lower, generally more negative skewness index.

ument three stylized facts using Compustat data (1983–2021): *(i)* firm growth rates are procyclically skewed: negatively in recessions and positively in expansions; *(ii)* the procyclicality of skewness is strongly size dependent, with larger firms exhibiting more pronounced swings from negative to positive skewness over the cycle. We call this fact the *size gradient of skewness*. Finally, *(iii)*, cross-sectional variance is a strong predictor of cross-sectional skewness in all size groups. Facts *(ii)* and *(iii)* are new to the literature. Fact *(ii)* is particularly surprising given Crouzet and Mehrotra (2020)'s documentation that large firms have less cyclical outcomes in levels with smaller variance. The size gradient persists even after controlling for industry composition and differential exposure to aggregate shocks, consistent with structural differences linked to market power rather than sectoral reallocation.

Second, motivated by these stylized facts, we develop a theory that maps symmetric cost shocks into skewed growth outcomes through a market power mechanism.[2] We parametrize market power as demand curvature. A strong version of Marshall's Second Law of Demand ensures that symmetric input shocks generate skewness in outcomes, while a slightly stronger condition — inverse demand has an increasing super-elasticity — implies that skewness rises systematically with market power. Intuitively, firms with greater market power face more concave first-order conditions, so output falls more in response to cost increases than it rises in response to cost declines. This prediction has a transparent empirical counterpart: increasing pass-through rates. If firms pass on a larger share of cost shocks at higher cost levels, then the concavity of responses, and hence skewness, increases with market power. Together, these properties deliver *monotone skewness*: a skewness index that is nil for price takers and increasingly negative as market power rises. They provide a parsimonious explanation for the size gradient of skewness observed in the data.

Third, while our mechanism explains why market power generates skewness and its size gradient, it does not yet account for the systematic variation of skewness over the business cycle. Our third contribution is therefore to show that when the variance of input shocks is countercyclical, like in Figure 1(a), the framework produces procyclical, size-dependent skewness in firm growth rates. We furthermore show that countercyclical variance need not be taken as a primitive. An equivalent interpretation arises from an aggregate impulse interacting with heterogeneous firm exposures. Large negative shocks then generate recessions by lowering mean growth and simultaneously raising the cross-sectional variance of shocks across firms. Conversely, small positive shocks generate expansions with tighter cross-sectional shock distributions. In this way, the scaling and shifting of the input distributions can be understood as

[2] Closely related work highlights two approaches: one treats skewed shocks as primitive (e.g. Salgado et al., 2025; Kamepalli et al., 2025), the other emphasizes propagation from symmetric shocks, for instance through hiring-firing asymmetry (Ilut et al., 2018) or network complementarities (Dew-Becker et al., 2021). We follow the latter, focusing on a demand-side channel.

the result of an aggregate impulse to which firms are exposed heterogeneously.

Fourth, we use our framework to derive and test empirical predictions. It predicts that *(i)* a single adverse aggregate shock lowers the level of growth and induces negative skewness in the cross-section, and *(ii)* this skewness response is particularly strong among firms with greater market power, and hence among large firms. Both claims are borne out in the data. Using impulse-response methods and a range of identified aggregate shocks, we show that aggregate growth and cross-sectional skewness move in tandem, and the decline in skewness is concentrated on large firms. A complementary factor decomposition confirms that most of the cyclical variation in skewness can be traced to an aggregate component with heterogeneous firm exposures. These results reinforce the interpretation that procyclical skewness does not require exotic shock distributions, but emerges from the interaction of aggregate impulses with heterogeneity in market power.

Our findings have several implications. For aggregate fluctuations, the size gradient suggests that large firms amplify business cycle asymmetry through their market power, not despite it.[3] For empirical work, our results caution against assuming shape-preserving shock propagation and highlight the importance of studying distributional changes beyond mean and variance. For policy, interventions targeting large firms may have disproportionate effects on the shape, not just the level, of aggregate outcomes. More broadly, policy makers who design insurance or compensation schemes based on the observed distribution of outcomes should be cautious. Our evidence shows that the cross-sectional skewness reflects the endogenous influence of market power, rather than directly capturing the underlying primitives that such policies aim to address.

Since the empirical size gradient is central to our results, it is important to be clear about how we interpret the relation between firm size and market power. We assume that larger firms tend to have greater market power. This view is consistent with standard theories in the literature (e.g. Atkeson and Burstein, 2008; Melitz and Ottaviano, 2008; Edmond et al., 2015; Parenti, 2018; Boar and Midrigan, 2024), as well as empirical evidence linking firm size to market power and markups (e.g. De Loecker and Warzynski, 2012; Autor et al., 2020). We use this assumption parsimoniously: not to propose a new theory of market power, but to interpret the size dependence of skewness implied by our mechanism and to organize the empirical facts.

Finally, it is worth clarifying the scope of our analysis and how it relates to our contribution. Our analysis focuses on annual firm growth rates in the cross-section and on fluctuations around trend, not on long-run growth. In a similar vein, the cycli-

[3]The connection between market structure and macroeconomic fluctuations has long been emphasized; see, for example, Hall (1986), who discusses how concentration and market power can shape aggregate dynamics.

cal movement of mean growth rates is the very definition of the business cycle and therefore not our object of study. Instead, we ask how the higher-order properties of the growth distribution such as variance, skewness, and their dependence on firm size, systematically evolve over the cycle. In doing so, we complement existing work that emphasizes structural change, network structures, or skewed idiosyncratic shocks (e.g. Ilut et al., 2018; Dew-Becker, 2023; Salgado et al., 2025), without dismissing their relevance. In particular, our key contribution is to explain the differential pattern: why skewness rises systematically with firm size.

So, according to our evidence, the size gradient arises from skewed responses driven by market power, not from increasingly skewed shocks hitting larger firms. The gradient we document is pronounced in Compustat, a universe of publicly traded firms with significant concentration, but may be weaker in datasets with more small firms or less concentrated markets. In settings with smaller firms, such as those studied by Bloom et al. (2018), idiosyncratic shocks are likely to play a more important role in generating skewness. Our evidence is thus complementary, not contrary, to these studies: idiosyncratically skewed shocks can coexist with the market power mechanism we emphasize.

Literature

Our paper connects to strands of literature studying the cross-section of firms over the business cycle focusing on the distribution of shocks, propagation mechanisms, and the distribution of outcomes. We also build on recent work in granular macroeconomics and firm heterogeneity.

Shock distributions One view is that cyclical asymmetries originate in the shocks themselves. Salgado et al. (2025) document that firm growth distributions are procyclically skewed (left-tailed in recessions and right-tailed in booms) using U.S. and international micro data, and interpret this through "skewness shocks" that directly shift higher moments of disturbances. Analyzing firm-level outcomes from over 40 countries, they document a consistent and robust relationship between aggregate output growth and the skewness of firm outcomes, such as sales growth, value added, and employment. Kamepalli et al. (2025) build a framework where skewness can arise either from non-Gaussian shocks or from endogenous propagation, nesting both sources within a production-network environment. These contributions motivate our stylized fact that cross-sectional skewness of firm growth rates is strongly procyclical, while our approach shows that symmetric shocks combined with demand-side propagation suffice to reproduce this fact.[4]

Propagation mechanisms A second view is that symmetric shocks can be transformed into asymmetric outcomes through endogenous firm responses. Bloom (2009) shows that uncertainty shocks — modeled as increases in second-moment volatility — induce firms to pause hiring and investment, generating sharp recessions followed by rebounds. This supports our finding that variance and skewness co-move across the business cycle. Similarly, Ilut et al. (2018) demonstrate that U.S. manufacturing firms employ concave employment responses to aggregate shocks. They are "slow to hire, quick to fire," which produces negative skewness and countercyclical volatility even when shocks are symmetric. Dew-Becker et al. (2021) develop a nonlinear production-network model where input complementarities generate left-skewed aggregate fluctuations together with countercyclical dispersion. Bloom et al. (2018) extend the uncertainty-shock perspective in a DSGE framework with heterogeneous firms, showing how volatility shocks generate asymmetric business cycle dynamics. We introduce a demand-side mechanism, which generates skewed growth rates through firms' endogenous responses to idiosyncratic shocks to their marginal costs (or, equivalently, to their demand curves). Doing so, we add to this strand of literature.

Outcome distributions A third strand takes the outcome distribution itself as the primary object of study. Dew-Becker (2024) measures option-implied skewness and shows that cross-sectional ('micro') skewness is procyclical while skewness in the time series of aggregate outcomes ('macro skewness') is largely acyclical, helping distinguish mechanisms that operate at the firm versus aggregate level. Crouzet and Mehrotra (2020) study the dynamics of large and small firms, showing that large firms are less cyclical in levels and variances. We extend their insights to higher moments, documenting that while large firms are dampened in levels, they amplify cyclical movements in skewness. This contrast highlights the novelty of our stylized fact on the size gradient of skewness.

Granular and network origins Our work also relates to granular and network-based approaches to business cycles. Carvalho and Grassi (2019) show that firm-level disturbances alone can generate aggregate volatility, persistence, and time-varying higher moments, providing a micro foundation for nontrivial aggregate dynamics. Acemoglu et al. (2017) develop a theory of macroeconomic tail risks from micro shocks, while

[4]Skewness has been treated not only in work focusing on macroeconomics and firm dynamics. Especially the household income literature (Guvenen et al., 2014, 2022; Busch et al., 2022) has brought much early attention to heterogeneity and higher moments of distributions of economic outcomes.

Acemoglu et al. (2012) formalize how network structure shapes aggregate volatility and amplification. Using French data, Di Giovanni et al. (2014) demonstrate empirically that firm-specific shocks contribute substantially to aggregate fluctuations, comparable to sectoral disturbances. This research aligns with our perspective of decomposing an input distribution into heterogeneous firm exposures to a common aggregate impulse, which suffices to generate the observed recession–expansion asymmetries in skewness.

Taken together, the literature shows that skewness can arise from skewed shocks, nonlinear propagation, or structural heterogeneity. Our contribution is to show how far one can go with a simple starting point: symmetric shocks, a demand-side propagation channel, and countercyclical variance interacting with heterogeneous firm exposures. This parsimonious framework reproduces the skewed, size-dependent outcome distributions that characterize firm growth over the business cycle.

Plan for the paper

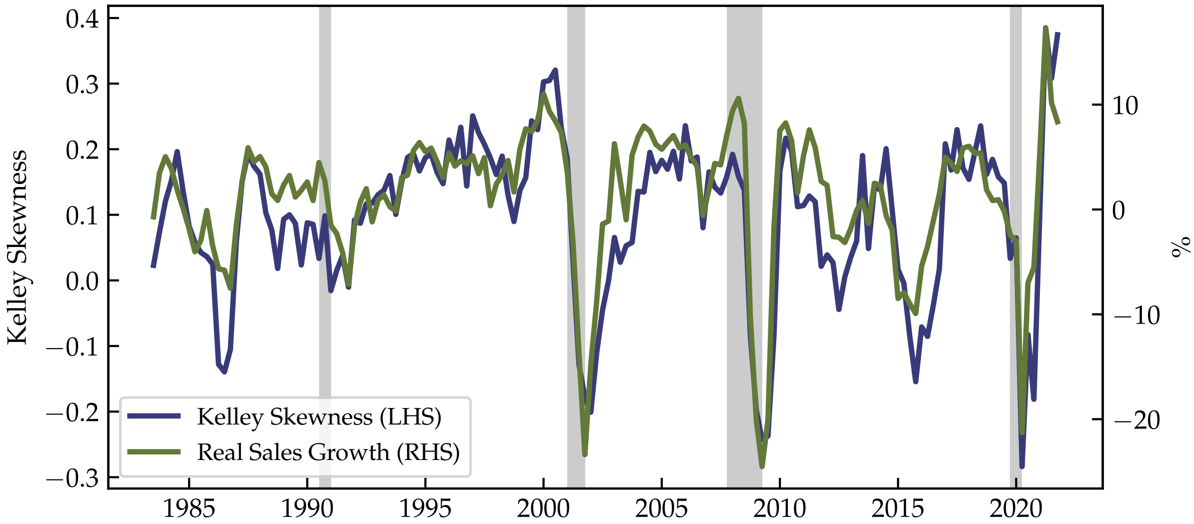
The rest of the paper is structured as follows. Section 2 describes the key stylized facts around procyclical skewness. Section 3 describes our simple theoretical framework linking the size gradient of skewness to market power. Section 4 gives an overview of the data and further empirical analysis that confirm our theoretical predictions. Finally, Section 5 concludes the paper.

2 Stylized Facts

In this section we present the three main stylized facts which motivate our theory. All of these facts relate to the business cycle properties of firm-level skewness and thus abstract from any long-term, secular phenomena.

Stylized Fact 1: Skewness is procyclical A well-documented empirical regularity is that the skewness of the firm growth distribution is procyclical: it rises in booms and falls in recessions. The pattern reflects an asymmetry in firm dynamics. During downturns, a small subset of firms suffer large negative growth rates, while positive growth remains more compressed. As a result, sales growth distributions are negatively skewed in recessions and more symmetric or right-skewed in expansions. This finding is robust across settings, countries, and measurement approaches, as shown by Dew-Becker et al. (2021), Ilut et al. (2018), Salgado et al. (2025), and Kamepalli et al. (2025).

Figure 2: Skewness and mean of sales growth in Compustat



Note: YoY sales growth is computed as $\log(s_{i,t}) - \log(s_{i,t})$ where $s_{i,t}$ denotes Compustat item `saleq`, deflated by the GDP deflator. Skewness is measured by Kelley Skewness and NBER recessions are shaded in gray.

We confirm this fact using quarterly Compustat data on publicly listed U.S. firms. Figure 2 plots the Kelley skewness of year-over-year sales growth against aggregate real sales growth.[5] Periods of stronger aggregate growth coincide with more positive skewness, consistent with the procyclical pattern. The result holds across alternative skewness measures and data samples, as shown in Figure A.5 (cf. appendix).

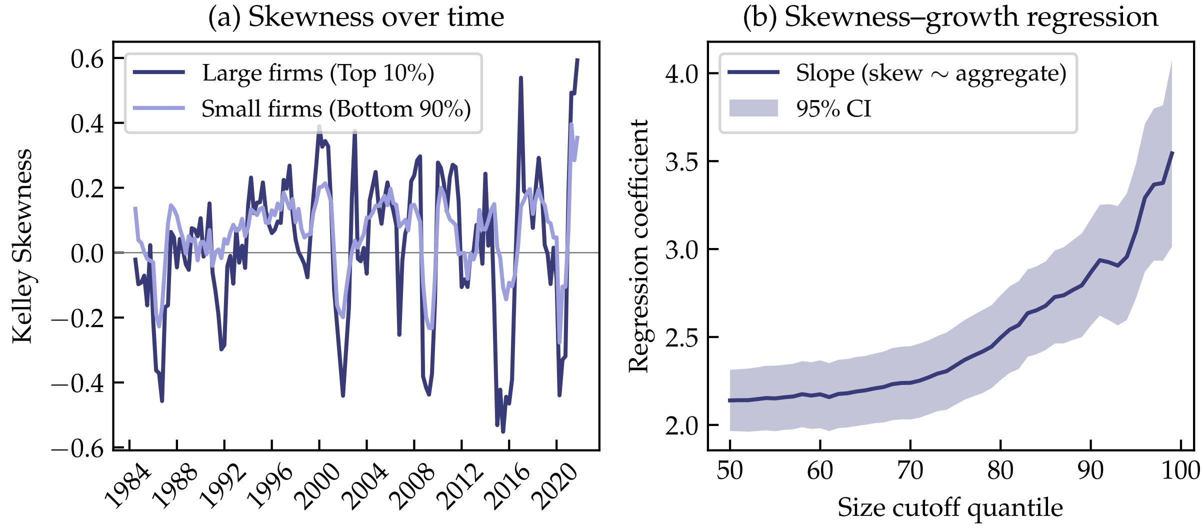
Stylized Fact 2: Procyclical skewness increases with firm size Having re-established the procyclical nature of aggregate skewness, we now turn to the core of our empirical analysis: how this relationship varies across firm size. Panel (a) of Figure 3 plots the evolution of Kelley skewness over time for large and small firms in the Compustat dataset.[6] The figure demonstrates that skewness for large firms is considerably more procyclical in amplitude than for smaller firms. This heightened procyclicality is particularly evident during recessions, when large-firm skewness exhibits both deep declines and sharp recoveries.

Crucially, this result does not hinge on a specific size cutoff. Panel (b) shows the estimated sensitivity of Kelley skewness with respect to aggregate growth for increasing

[5] Kelley skewness is defined as $\mathrm{KSK}[X] = \frac{X_{0.9}+X_{0.1}-2X_{0.5}}{X_{0.9}-X_{0.1}}$, where X_r denotes the r-th quantile.

[6] Firm size is measured as the rolling average of real sales over the previous three years, with large firms defined as those above the 90th percentile of this distribution. For ease of writing, we refer to 'small' and 'large' firms. These terms should be interpreted within the confines of the size distribution that Compustat allows to study, acknowledging that 'small' firms in Compustat are significantly larger on average than small firms in a representative sample. Additionally, since the smallest firms in Compustat are often startups and may differ from typical small firms across a range of features, we abstain from directly comparing the largest firms to the smallest firms in the data. Instead, we generally focus on comparing the top of the size distribution to the rest of the distribution.

Figure 3: Size-dependent skewness



Note: Size groups are defined based on average real sales over previous three years. The standard deviation of Kelley skewness for large firms is about 0.23 — more than twice the corresponding value of 0.11 for small firms.

size thresholds for large firms. It is clear that the sensitivity of skewness to aggregate sales growth increases systematically with firm size. The estimated sensitivity rises steadily across the distribution, becoming especially pronounced above the 70th percentile. This pattern demonstrates that the size–skewness relationship is a pervasive feature of the data, not an artifact of an arbitrary split. Taken together, these results show that the procyclical swings in skewness increase with firm size.

Stylized Fact 3: Countercyclical variance amplifies large-firm skewness To better understand the drivers of skewness, we examine its relationship with the dispersion of sales growth rates. Specifically, we estimate

$$\Delta\gamma_{g,t} = \alpha + \beta\,\Delta\sigma_{g,t} + u_{g,t}, \tag{1}$$

where $\Delta\gamma_{g,t}$ denotes the change in skewness for group g at time t, and $\Delta\sigma_{g,t}$ captures the change in the standard deviation of sales growth in the same group. Importantly, the standard deviation of growth rates is not a pure measure of exogenous shock variance, since it also reflects firms' endogenous responses. Nevertheless, it provides a useful summary of how volatile growth outcomes are across firms at a given point in time. If greater volatility is systematically associated with more negative skewness, this points to an important role for uncertainty in shaping asymmetries.

The results confirm this intuition. Across the sample, increases in dispersion are positively correlated with declines in skewness. Crucially, the relationship is much stronger for large firms: the estimated β is substantially larger than for small firms. In

Table 1: Regression of Changes in Skewness on Changes in Standard Deviation by Firm Size

	$\Delta\text{Skewness}_t = \alpha + \beta\Delta\text{Std Dev}_t + \varepsilon_t$		
	All Firms	Large Firms (Top 10%)	Small Firms (Bottom 90%)
β (Coefficient)	-2.15***	-3.23***	-1.81***
	(0.51)	(0.64)	(0.52)
t-statistic	-4.18	-5.02	-3.50
R^2	0.171	0.300	0.139
Observations	146	146	146

Note: This table reports results from regressions of year-on-year changes in cross-sectional skewness on year-on-year changes in cross-sectional standard deviation of real sales growth. Large firms are defined as those above the 90th percentile of average firm size within each quarter. Standard errors (shown in parentheses) are computed using the Newey-West HAC estimator with automatic lag selection. Sample period: 1983–2021. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

other words, when volatility rises, skewness becomes disproportionately more negative among the largest firms.

3 Theory

The stylized facts reveal a systematic relationship between firm size and the cyclical properties of growth rate skewness. To explain these patterns, we develop a theoretical framework linking market power to asymmetric firm responses. Our theory rests upon two key mechanisms: first, optimizing firms pass symmetric cost shocks through to output in an asymmetric fashion (with negative skew) if and only if they possess market power; and second, a countercyclical shock variance amplifies this cross-sectional skewness, generating procyclical skewness patterns that increase in magnitude with the degree of market power. The intuition is straightforward. When facing cost shocks, firms with market power adjust quantities along their downward-sloping demand curves. The concavity of their first-order conditions creates asymmetric responses: positive cost shocks reduce output more than negative shocks increase it. This asymmetry becomes more pronounced when shock variance rises (typically in recessions), generating the size-dependent procyclical skewness patterns observed in the data.

3.1 Set-Up and Firm Problem

We model firm production with a convex cost function $c(q) = q^{\eta} e^{\epsilon}$ with $\eta > 1$, where q is its production output. The term e^{ϵ} is a stochastic cost shifter, where ϵ is drawn from a symmetric input distribution with zero mean and finite variance, and observed by the firm at time of production. Because we want to focus on the strategic output adjustment of firms, a simple, isoelastic cost function serves as a technological constraint, while the inverse demand function is kept as general as possible.[7] Furthermore, our empirical sample contains firms that are large by global standards and have a very low exit rate ($< 1\%$), firm exit and sample attrition are unlikely to be driving results. Therefore, we abstract from firm exit decisions in the theory, too.

Monopolist Let $p(q)$ be the inverse demand function that the monopolistic firm is facing. Assumption 1 on p ensures that the firm's profit-maximizing output is unique and that the problem is well-behaved.

Assumption 1. *The inverse demand function p satisfies: $p : \mathbb{R}_+ \to \mathbb{R}_+$ (non-negative domain and range), $p \in \mathcal{C}^3(\mathbb{R}_+)$ (differentiability), $p' < 0$ (decreasing in quantity) and $\frac{\partial^2}{\partial q^2} \ln p \leq 0$ (log-concavity). By convention, $\mathbb{R}_+ = [0, \infty)$. Furthermore, assume that marginal revenue, $\mathfrak{mr}(r) \equiv \frac{\partial}{\partial q} qp(q)$ satisfies $\mathfrak{mr}(a) > c$ for some $0 < a < \infty$.*

The monopolist's problem is

$$\max_{q \geq 0} qp(q) - c(q) \tag{2}$$

Note that eq. (2) depends on the cost shock only through c. The reader may make a mental note for the remainder of this paper that all subsequent results will also hold for shocks to the scale of the inverse demand function. Such shocks generate a monopolist's problem that is isomorphic to the one at hand. The problem features an interior solution $q^\star > 0$ if and only if $q^\star$ satisfies the first order condition. Lemma 1 characterizes the solution. Here, we define the elasticity operator $\mathcal{E}$ as $\mathcal{E}f(x) \equiv \frac{f'(x)}{f(x)}x$ for any differentiable function, f, that is either strictly positive or strictly negative.

Lemma 1 (Solution of Firm Problem). *The solution $q^\star$ of the monopolist's problem is unique, interior ($q^\star > 0$) and implicitly given as the solution to the first order condition*

$$c'(q) = \underbrace{p(q)\,(1 + \mathcal{E}p(q))}_{\equiv \mathfrak{mr}(q) \text{ (marginal revenue)}} . \tag{3}$$

Marginal revenue, $\mathfrak{mr}$, is defined on some open interval $D \subset [0, a]$, on which $\mathcal{E}p \in (0, -1)$ holds, too. The markup is given by $\mu(q) \equiv (1 + \mathcal{E}p(q))^{-1}$.

The optimality condition in Lemma 1 is a condition in the style of Lerner (1934). Denote log-quantities with a hat-accent, e.g., $\ln q = \widehat{q}$. The optimality condition can be

[7] We note, however, that all our results would apply to a setting in which all firms face marginal cost constant in q, subject to a capacity constraint for price taking firms. The capacity constraint ensures that $q = \infty$ is not a possible outcome.

rewritten in terms of marginal revenue, $\mathfrak{mr}$, and as a function of log output, $\widehat{q}$:

$$\ln \circ \mathfrak{mr}(e^{\widehat{q}}) - [\ln \eta + (\eta - 1)\widehat{q}] = \epsilon \tag{4}$$

Using the conditions cited in Assumption 1, it is easy to show that the function $\ln \circ \mathfrak{mr}$ and the whole left hand side of eq. (4) are decreasing in $\widehat{q}$.

We now examine our first shape restriction on inverse demand, p. Marshall's second law of demand (MSLD) says that the absolute elasticity of demand increases with price, so $|(\mathcal{E}p)'| > 0$, or, equivalently, that low-cost firms set higher mark-ups. Marshall's second law has been subject to much empirical and theoretical scrutiny. It is prevalent in the trade literature and key insights of seminal papers like Krugman (1979) rest on it. In more recent theoretical work by Matsuyama and Ushchev (2022), the second law has been shown to be instrumental for rationalizing incomplete pass-through and strategic complementarities in pricing, which happen when firms reduce their mark-ups in response to higher competitive pressures. Melitz (2018) strengthens the second law to MSLD′, which we refer to as *the Strong Second Law*: the absolute elasticity of *marginal revenue* decreases as output decreases, so $|(\mathcal{E}\mathfrak{mr})'| > 0$. MSLD is equivalent to the average elasticity of marginal revenues being increasing, i.e., $\int_0^q |\mathcal{E}\mathfrak{mr}(q)| \, \mathrm{d}q > 0$. MSLD′ means that this also holds at the margin, making it a just slightly stronger concept. For reference, we define MSLD and MSLD′ in the following two properties:

Property 1 (MSLD). *We say that Marshall's Second Law of Demand holds if for all* $q \in D$, $|\frac{\partial}{\partial q}\mathcal{E}p(q)| > 0$. *We say it only holds weakly, if the inequality is weak.*

Property 2 (MSLD′). *We say that Marshall's Strong Second Law of Demand holds if for all* $q \in D$, *it holds that* $|\frac{\partial}{\partial q}\mathcal{E}\mathfrak{mr}(q)| > 0$. *We say it only holds weakly, if the inequality is weak.*

Price Taker A natural benchmark for the monopolist is the behavior of a price-taking firm.[8] A price taker accepts the market price, denoted by $\bar{p}$, and chooses output q_{pt}, which satisfies $\epsilon = \ln \bar{p} - \ln \eta - (\eta - 1)\widehat{q_{\mathrm{pt}}}$. Hence, $\widehat{q_{\mathrm{pt}}}$ is a linear function of ϵ. Going forward, we use pt-subscripts to refer to variables calculated in the price-taker equilibrium.

Alternative Cost Functions We assume an iso-elastic demand function in order to focus on the effects of competition on growth rate skewness. While cost functions may be a driver, too, it is harder to rationalize a dependence on skewness on firm size using cost functions alone. We discuss avenues to create cross-sectional skewness driven by the cost structure of firms in more detail in Section B.

[8]Note that a price taking firm does not necessarily operate in a perfectly competitive market.

3.2 Skewness Measure and Growth Rates

We now formalize how we measure skewness in the distribution of firm output responses to shocks. Let the steady state output q_0 be the output level corresponding to $\epsilon = 0$. *Growth rates relative to the steady state* are log-differences of output to steady state output: $\widehat{q} - \widehat{q}_0$. Consequently, the distributions of growth rates and $\widehat{q}$ are the same up to a constant shift. It is important to note that $\widehat{q} - \widehat{q}_0$ is not a time-series growth rate. In Section 3.3, we exclusively refer to growth rates relative to the steady state. In Section 3.4, we link our theory back to time-series growth rates, which we use to derive our stylized facts and the empirical part of this paper.

Following Groeneveld and Meeden (1984), we define the skewness of a random variable X with quantiles X_r, $r \in (0,1)$, as:

$$skew[X] := \frac{X_r + X_{1-r} - 2X_{0.5}}{X_{1-r} - X_r} \in (-1,1). \tag{5}$$

Setting $r = 0.1$ defines the classic 'Kelley skewness' measure. One can create negative skew with a symmetrically distributed, zero-mean, random variable by transforming it using some concave, increasing function. This negative skewness is exacerbated when additional concave transformations are applied or when the standard deviation of the original variable increases. This claim is proven in Lemma 2.

Lemma 2 (Skewness of Transformed RVs)**.** *Let Z be a random variable, continuously and symmetrically distributed about its mean $\mathbb{E}[Z]$. Let $X = \sigma_X Z$ with $0 < \sigma_X < \infty$. Suppose g is concave and increasing over the support of X. Then:*

1. *It holds that $skew[g(X)] \leq 0$, and strictly if g is strictly concave.*
2. *If h is another concave, increasing transformation, then $skew[h(g(X))] \leq skew[g(X)]$, strictly if h is strictly concave.*
3. *Skewness decreases for larger σ_X:*

$$\frac{\partial}{\partial \sigma_X} skew[g(X)] \leq 0,$$

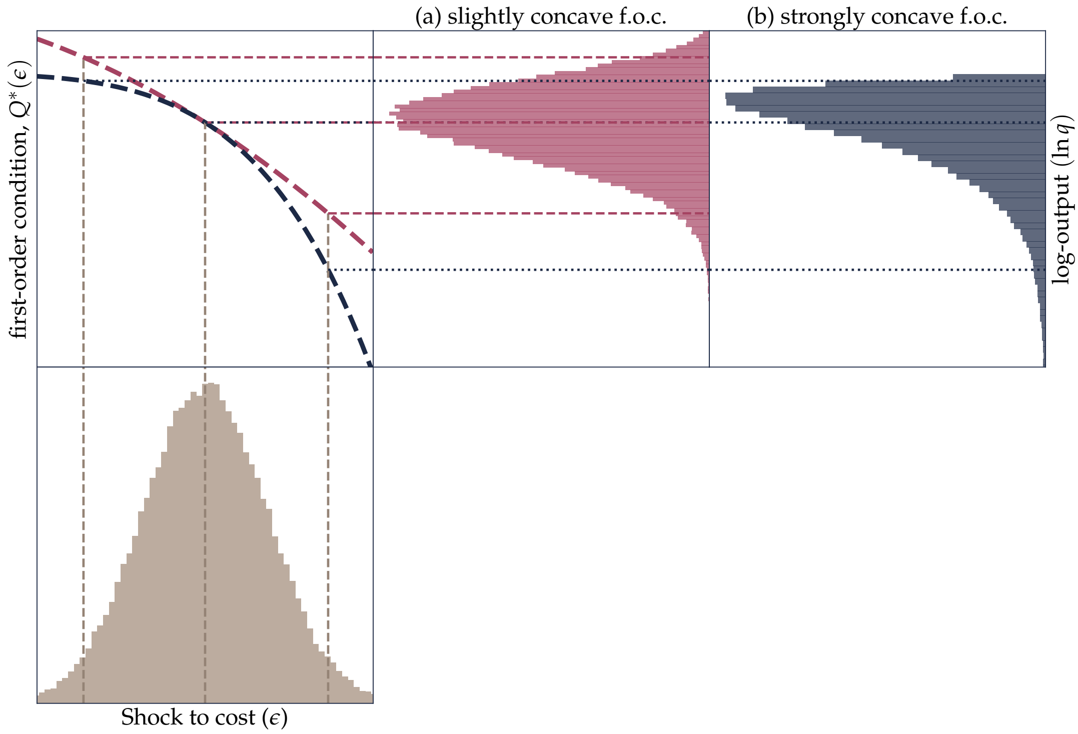
which also holds strictly if g is strictly concave.

Proof. We provide a self-contained proof using a second degree approximation in the appendix. The results, however, are also corollaries of the notion of '$c-$comparability' of Groeneveld and Meeden (1984). ■

3.3 Skewed Responses to Shocks

The general insights about skewness from concave transformations apply directly to our model of firm behavior with shocks. We now examine how the endogenous response of firm output to cost shocks can generate negatively skewed growth rates in the cross section. Figure 4 illustrates our strategy. In the lower left panel, it shows a symmetric shock distribution, which is, by a first order condition in the upper left panel, mapped to growth rates relative to the steady state. There are two first order conditions shown: The red f.o.c. maps to panel (a), yielding a slightly left-skewed distribution of growth rates. The blue f.o.c. maps to panel (b), which displays a distribution that is much more strongly left skewed, as indicated by its quantile lines. The reason for these differences in distribution is the higher degree of concavity of the blue f.o.c. relative to the red one. This is a direct illustration of points 1. and 2. of Lemma 2. In this section, we devise conditions on the inverse demand such that *(i)* first order conditions are concave mappings in the sense of this figure and *(ii)* the concavity increases in the degree of market power associated with the demand curve.

Figure 4: Concave first-order conditions and skewness in $\log(q/q_0)$



3.3.1 Monopolist v. Price Taker

The monopolistic firm adjusts its quantity because a shift in its unit cost implies a different profit maximizing output. In that sense, it responds endogenously to shocks to c. We write these endogenous responses in log-quantity, $\widehat{q}$, as a function of the shock, $\widehat{q} = Q^\star(\epsilon)$. Lemma 2 immediately tells us that growth rates are negatively skewed if $Q^\star$ is concave. Strict concavity of $Q^\star$ is determined by properties of the inverse demand function, p, as stated in Lemma 3. In it, we characterize sufficient conditions for negatively skewed growth rates in terms of elasticities.

Lemma 3 (Concavity of Real Growth Rates)**.** *Consider eq. (4) and let* $H(\widehat{q}) := (\ln \circ \mathfrak{mr})(e^{\widehat{q}}) + [\eta - (\eta - 1)\widehat{q}]$ *denote the left hand side of the FOC. Then,* $Q^\star = H^{-1}$*. Then:*

$Q^\star$ is a concave function of ϵ

$\Leftrightarrow$ *Log marginal revenues are log-log concave, i.e.* $(\ln \circ \mathfrak{mr})(e^{\widehat{q}})$ *is concave in* $\widehat{q}$*. If concavity holds strictly, then* $Q^\star$ *is strictly concave.*

$\Leftrightarrow$ *The term*

$$\mathcal{E}\mathfrak{mr}(q) = \mathcal{E}p + \frac{\mathcal{E}p}{\mathcal{E}p + 1}\mathcal{E}^2 p \tag{6}$$

is decreasing in q, where $\mathcal{E}^2 = \mathcal{E} \circ \mathcal{E}$ is the elasticity of the elasticity function ('superelasticity').

$\Leftrightarrow$ *MLSD' holds.*

Note that Lemma 3 can be relaxed slightly: conditions do not need to hold globally, but only on the support of $Q^\star(\epsilon)$. While similar local relaxations can be made to all results in this paper, we omit these additional remarks.

We are now ready to state the first main result of this section, which formalizes a key comparative prediction. Proposition 1, shows that, under concavity of H, sales growth rates of a firm with market power are negatively skewed, in contrast to the symmetric responses of a price taker.

Proposition 1. *Let $\widehat{q}$ be log-output of a monopolistic firm, and suppose the conditions of Lemma 3 hold strictly. Then,*

$$skew[\widehat{q}] < skew[\widehat{q}_{\mathrm{pt}}] = 0.$$

Proof. This is a direct consequence of Lemmas 2 and 3. ■

3.3.2 Parametrized Market Power

To explore how the degree of market power shapes the skewness of firm responses more systematically, we next introduce a simple and flexible parameterization of the inverse demand function that allows us to vary market power continuously. Specifically, write the inverse demand function as $p(q) = p^*(q)^\alpha \bar{p}^{1-\alpha}$ for some inverse demand function p^*, some fixed price $\bar{p}$ and parameter $\alpha \in (0, 1]$. For low values of α, the firm faces a highly elastic demand and has little influence over the price. For α close to 1, the firm faces an elasticity that is lower and determined through p^*. The elasticity of inverse demand is now $\mathcal{E}p = \alpha\mathcal{E}p^*$. The firm's first order condition now depends on α explicitly:

$$\alpha \ln \circ\, p^*(e^{\widehat{q}}) + (1-\alpha)\ln \bar{p} + \ln\left(1 + \alpha\, \mathcal{E}p^*(e^{\widehat{q}})\right) + [\ln \eta - (\eta - 1)\widehat{q}] = \epsilon. \tag{7}$$

We can generalize Proposition 1 under slightly stronger conditions to guarantee that skewness is monotone in α. Monotonicity of skewness in market power is the key property which generates the size gradient of skewness described in the stylized facts. Hence, we define it formally in Property 3.

Property 3 (Monotone Skewness). *Let q_α be the output produced by a firm with market power $\alpha \in [0, 1]$. We say Monotone Skewness holds if skewness is monotone in market power. Specifically, we require skew$[q_\alpha] \leq 0$ is decreasing in α, with skew$[q_1]$ equaling monopolist and skew$[q_0]$ equaling price taker output, respectively. Furthermore, skew$[q_0] = 0$.*

Monotone skewness is a global property in the sense that it concerns all degrees of market power. To devise characteristics on p^* which imply monotone skewness, we first establish a condition which allows ranking the skewness of output of a firm of a single, given degree of market power α against other firms. To this end, index the markup of this firm by α, $\mu_\alpha(q) = (1 + \alpha\mathcal{E}p^*(q))^{-1}$. We call the following auxiliary property *monotone markup-to-price elasticities*:

Property 4 (Monotone Markup-to-Price Elasticities (MMPE$_\alpha$)). *The ratio of markup elasticity to price elasticity is strictly increasing:*

$$\frac{\partial}{\partial q}\frac{\mathcal{E}\mu_\alpha(q)}{\mathcal{E}p^*(q)} > 0.$$

MMPE$_\alpha$ states that given some degree of α, markups change more strongly than the price elasticity, if quantities are perturbed. This derivative describes the nonlinearities in the first order condition of the firm, as one traces out the demand curve. It turns out that the condition is sufficient to guarantee monotone skewness: Higher degrees of market power correspond to firms with more negatively skewed growth rates. Before discussing details, consider CES demand as a sanity check, for which growth rates are generally unskewed. Under CES, both — the markup and the price — are constant, hence the derivative evaluates to 0. This is not surprising, since then the first-order condition of the firm is linear in $\widehat{q}$ and growth rates are symmetrically distributed. Thus, MMPE$_\alpha$ is indeed not satisfied for CES demands.

If and only if MMPE$_\alpha$ holds, the inverse markup μ_α is concave relative to the inverse demand function in the log-log space[9]. Relative concavity can be intuitively

[9] A function $f(x)$ in log-log space is given by the mapping $\widehat{x} \mapsto \ln f(\exp(\widehat{x}))$. When taking logs of the FOC and rewriting it in terms of $\widehat{q}$, we are operating in the log-log space. If a function, f is an

understood as the notion that some function is *more concave* than another. If f is concave relative to g, then plotting f on the y- and g on the x-axis produces a concave curve. To denote that the log-inverse mark-up is concave relative to the log-inverse demand function, we write

$$\frac{1}{\alpha}\ln\big(\underbrace{1+\alpha\mathcal{E}p^*(\cdot)}_{1/\mu_\alpha}\big) \prec \ln\left(p^*(\cdot)\right). \tag{8}$$

Under this condition, the inverse mark-up becomes the chief contributor of concavity to the first order condition. Since the concavity of the inverse markup is increasing in market power, α, a higher market power causes more concave responses to shocks and thus stronger left-skew. This informal reasoning is correct and carries over to the following proposition:

Proposition 2. *Consider two firms which have the same long-run output, $\widehat{q}_1 = \widehat{q}_0 = \widehat{q}$, but different degrees of market power, $1 \geq \alpha_1 > \alpha_0 \geq 0$, and otherwise face the same demand curves, p^*, up to differences in $\bar{p}$[10]. If MMPE$_{\alpha_0}$ or MMPE$_{\alpha_1}$ holds, then skew$(\widehat{q}_1) <$ skew$(\widehat{q}_0) \leq 0$.*

Proof (intuition). Fix two levels of market power, $\alpha_1 > \alpha_0 \geq 0$. One can show that the entire LHS of the first order condition of the high-market power firm is a concave transformation of the FOC of the lower market power firm. One concludes that the inverse of the FOC is also more concave for the high market power firm, leading to stronger left-skew in growth rates. (For details, see appendix.) ■

Proposition 2 states that at whatever degree α of market power MMPE$_\alpha$ holds, we can rank the skewness created by all other degrees of market power against it. It affirms that growth rates of any other firm that chooses the same output are more left-skewed if and only if that other firm has more market power. Clearly, if the property holds for any α, then monotone skewness is implied. This is an unwieldy demand to make. It turns out, however, that the relation in eq. (8) yields a sufficient condition on p^* which guarantees monotone skewness without reference to α. Note that the LHS of eq. (8) becomes *more concave* if α increases. Hence, MMPE$_\alpha$ automatically holds for all α, if eq. (8) holds in the limit as $\alpha \to 0$:[11]

$$\mathcal{E}p^*(\cdot) \prec \ln\left(p^*(\cdot)\right). \tag{9}$$

increasing, concave transformation, h, of another function, g, i.e., if $f = h \circ g$, then f is concave relative to g. We use it extensively in the proofs of the theorems in the appendix. Note that relative concavity is invariant to linear transformations.

[10] Note that $\bar{p}$ has to be different for both firms in order to equalize their output and facilitate the local comparison.

[11] Here, one notes the convergence of the LHS to the exponential function.

This condition is equivalent to the superelasticity of p^*, $\mathcal{E}^2 p^*$, being increasing. This feature is defined in Property 5 and shortened to *ISID*:

Property 5 (ISID). *An inverse demand function, p, (locally) satisfies an increasing superelasticity of inverse demand (ISID) if $\mathcal{E}^2 p(q) \equiv \mathcal{E}(\mathcal{E}p)(q)$ is (locally) strictly increasing in q.*

ISID turns out to be a key property to generate monotone skewness. It not only implies MMPE$_\alpha$ but it also *implies MSLD′* for any given α, and thus can be seen as a further strengthening of Marshall's second law. Yet, despite its centrality for characterizing the curvature of demand, the superelasticity is an object about which it is hard to form a prior. Á priori, we have no strong intuition about whether $\mathcal{E}^2 p^*$ should be constant, increasing or decreasing in q. To make the concept of ISID less vacuous, we note that it relates on a deep level to a more interpretable metric: *pass-through rates*.

The *pass through*, $\tau(\epsilon)$, is defined as the share of a cost increase that is passed on to customers in equilibrium. Formally, τ equals one minus the elasticity of the markup with respect to the cost shifter $\bar{c} \equiv e^{\epsilon}$, i.e. $\tau(\bar{c}) = 1 - \frac{d \log \mu}{d \log \bar{c}}$. We formalize the feature that the pass-through is increasing in Property 6:

Property 6 (Increasing Pass-Through (IPT)). *An inverse demand function p features increasing pass-through if $\frac{\partial}{\partial \bar{c}}\tau(\bar{c}) \geq 0$.*

The pass through simplifies an empirical treatment of ISID and monotone skewness significantly. Indeed, one only needs to assume the (weak) second law of demand, MSLD, and increasing pass-through rates, IPT, to conclude that ISID holds. In turn, MSLD′ and monotone skew both hold. This chain of implications is part of the main result of this section. Since both MSLD and IPT are properties which are empirically identifiable and supported by evidence, we have arrived at a set of properties that are sufficiently strong to provide the sharp theoretical prediction of monotone skewness, while maintaining interpretability. We thus summarize our insight in the main proposition of this section.

Proposition 3 (Implications of Properties). *The following implications hold:*

$$ISID \implies (MSLD' \wedge MMPE_\alpha \, \forall \alpha \in [0,1]) \implies Monotone\ Skewness \tag{10}$$

$$IPT \wedge MSLD \implies ISID \tag{11}$$

Equation (10) of Proposition 3 states that under ISID the model predicts a market power gradient of skewness for growth rates relative to the steady state. Equation (11) ensures that increasing pass-through rates and a Marshall's second law suffice. Next, we connect this gradient to time series growth rates, which are the relevant metric of our stylized facts and empirical evaluation. A brief discussion of empirical evidence for ISID is relegated to the end of the theory section.

3.4 Pro-Cyclical Skewness

In the first part of this section, we explain pro-cyclical skewness in time-series growth rates with counter-cyclical fluctuations in the cross-sectional shock variance. Thereby, we tacitly assume ISID. Subsequently, we show how firms' heterogeneous exposures to the same aggregate shock can drive counter-cyclical fluctuations in variance. This completes our theory of pro-cyclical skewness: Besides an aggregate shock, no exogenous variation is required to generate counter-cyclical variances and a channel of pro-cyclical skewness, which monotonously depends on market power.

To illuminate the intuition behind the first part and illustrate Lemma 2(3), consider Figure 5. The narrow distribution of ϵ in the bottom left panel maps to a narrow distribution of growth rates in the top right panel. Importantly, by inspection of the quantile lines of the growth rates, the resulting distribution is left-skewed, but not extremely so. In contrast, the wide distribution of shocks maps to an extremely left-skewed distribution of growth rates. If one identifies times of wide distributions as times of recession and times of narrow shock distributions as expansions, the leap to pro-cyclical skewness is small. Note, however, that the wide and the narrow distribution are both centered. In our theory, we are intentionally silent about the location of shocks and mean growth rates in general. Because we have a theory of *centered* business cycle moments, we neglect shifts in location altogether.

3.4.1 Countercyclical Variance Drives Procyclical Skewness

It is de facto consensus that expansions are relatively smooth with a low latent cross-sectional shock variance $\underline{\sigma} \equiv \sqrt{\mathbb{V}(\epsilon)}$, and that the variance increases to some $\overline{\sigma} > \underline{\sigma}$ in recessions. Suppose that $\sigma_t = \underline{\sigma}$ for even ('booms'), and $\sigma_t = \overline{\sigma}$ for odd t ('busts') in our model. For this section, assume that the ISID sufficient condition of Section 3.3 is satisfied, i.e. $\mathcal{E}^2 p^*$ is strictly increasing. Then, the countercyclical process immediately implies procyclical skewness of growth rates relative to the steady state.

Corollary 1. *Let $(\ldots, \sigma_{t-1}, \sigma_t, \sigma_{t+1}, \ldots) = (\ldots, \underline{\sigma}, \overline{\sigma}, \underline{\sigma} \ldots)$ be an alternating sequence of the cross-sectional standard deviation of ϵ_t corresponding to booms and busts, respectively. Denote the skewness of the monopolist's growth rates relative to the steady state by $\mathbb{S}_t = skew[\widehat{q}_t - \widehat{q}_0]$. Accordingly, denote the skewness of the price taking firm by $\mathbb{S}_t^{\mathrm{pt}}$. Then*

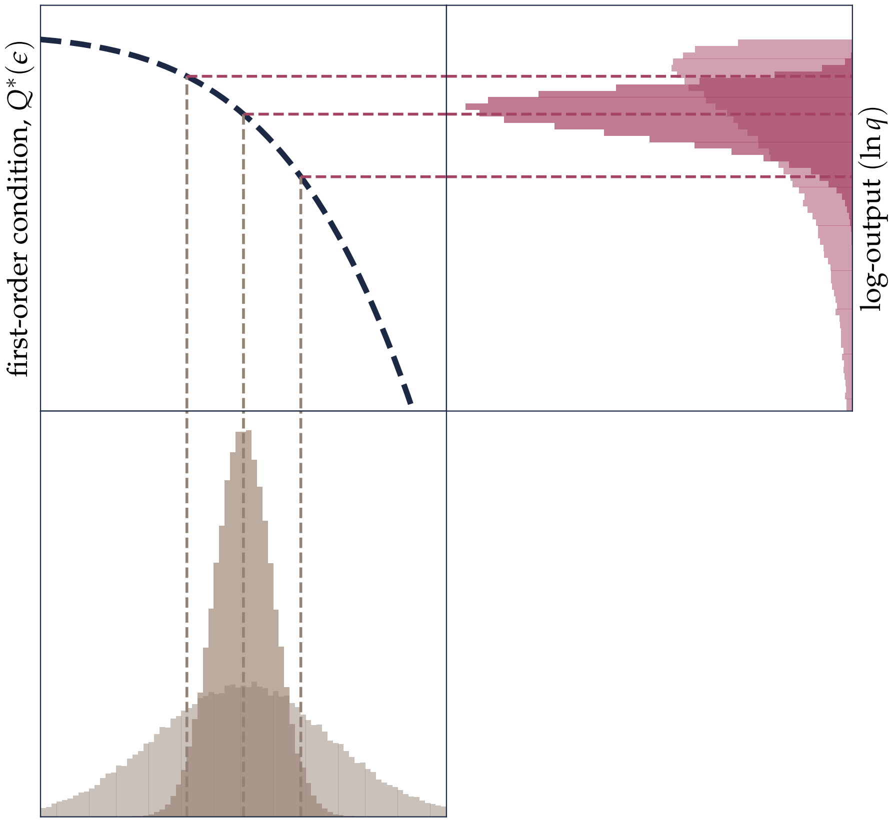
$$(\ldots\mathbb{S}_{t-1}, \mathbb{S}_t, \mathbb{S}_{t+1}\ldots) = (\ldots, \underline{\mathbb{S}}, \overline{\mathbb{S}}, \underline{\mathbb{S}}\ldots)$$

with $\mathbb{S}_t = \overline{\mathbb{S}} < \underline{\mathbb{S}} < 0$ if t corresponds to a bust, and

$$\mathbb{S}_t^{\mathrm{pt}} = 0 \quad \textit{for all } t.$$

Additionally, the amplitude of the skewness sequence of the monopolist is strictly increasing in

Figure 5: Shock Variance and Skewness in $\log q/q_0$



market power, i.e.

$$\frac{\partial}{\partial \alpha}|\mathbb{S}_t| > 0 \quad \textit{for all } t.$$

Proof. This is a direct consequence of Lemma 2, combined with Propositions 1 and 2. ■

The conditions of Corollary 1 describe a world visualized in Figure 5, in which increased shock variances amplify left-skewness. Additionally, skewness is exacerbated by market power in the sense described in the preceding section.

Up to this point, we have derived skewness properties of $\widehat{q} - \widehat{q}_0$, which is growth relative to the steady state. We now turn to *time-series growth rates*, $r_t \equiv \widehat{q}_t - \widehat{q}_{t-1}$. Conceptually, the limit case of $\underline{\sigma} \to 0$ gives us, without any effort of calculation, the skewness pattern we see in the data. In this extreme case, output is either hit by a random shock in a bust, or equals steady state output in a boom.[12] Then either the economy is in a bust and $r_t = \widehat{q} - \widehat{q}_0$, so $skew[r_t] = \overline{\mathbb{S}} > 0$. Or it is in a boom and $r_t = \widehat{q}_0 - \widehat{q}$, and thus $skew[r_t] = \underline{\mathbb{S}} = -\overline{\mathbb{S}}$. In effect, skewness alternates between positive and negative values as the economy passes through boom-bust cycles. The amplitude

[12]Note that the shocks are allowed to have a non-zero mean, since the location of a random variable is irrelevant to its skewness.

of this alternating sequence depends on the considered firms' market power. This fully rationalizes the stylized facts.

To conclude this section, we provide Proposition 4, which characterizes the skewness cycle for $\sigma \in \{\underline{\sigma}, \overline{\sigma}\}$ with $0 < \underline{\sigma} < \overline{\sigma}$. It ties the theory presented above to the stylized facts which we elaborate on further in the next section.

Proposition 4 (Procyclical Skewness in $\ln q_t/q_{t-1}$, Countercyclical σ and Market Power). *Define $(\ldots, \sigma_{t-1}, \sigma_t, \sigma_{t+1}, \ldots) = (\ldots, \underline{\sigma}, \overline{\sigma}, \underline{\sigma} \ldots)$ like above. Denote the skewness of the monopolist's time series growth rates by $\mathbb{S}_t^\alpha = skew[\widehat{q}_t - \widehat{q}_{t-1}]$, where $\alpha \in (0, 1]$ parameterizes market power. Then*

$$(\ldots \mathbb{S}_{t-1}^\alpha, \mathbb{S}_t^\alpha, \mathbb{S}_{t+1}^\alpha \ldots) = (\ldots, \underline{\mathbb{S}^\alpha}, \overline{\mathbb{S}^\alpha}, \underline{\mathbb{S}^\alpha} \ldots)$$

with $\mathbb{S}_t = \overline{\mathbb{S}^\alpha} < 0$ if t corresponds to a bust and $\mathbb{S}_t = \underline{\mathbb{S}^\alpha} > 0$ otherwise. Additionally, $\overline{\mathbb{S}^\alpha}$ is increasing and $\underline{\mathbb{S}^\alpha}$ is decreasing in α, with limit 0 for $\alpha = 0$.

3.4.2 Heterogeneous Business Cycle Exposure Drives Countercyclical Variance

Up to now, we have taken the existence of i.i.d. symmetric shocks, $\epsilon_{i,t}$, as given. However, why should $\epsilon_{i,t}$ have a time-varying variance at all? We provide a simple theoretical explanation, why $\mathbb{V}(\epsilon_{i,t} \mid \text{recession}) > \mathbb{V}(\epsilon_{i,t} \mid \text{expansion})$, which immediately ties the changes in variance to the mean of the shock distribution. To this end, suppose that $u_{l,t}$ is one of $l = 1, ..., L$ *aggregate shocks* or *factors*, which are drawn from some conditional distribution $P_l(a \mid u_{l,t-k},\ k \in K \subseteq \mathbb{N})$, say. Let there be a unit measure of firms, $i \in [0, 1]$ and let each firm's shock be related to the aggregate factor through a constant marginal effect $\dfrac{\partial \epsilon_{i,t}}{\partial u_{l,t}} = \widetilde{\lambda}_{i,l}$. Hence, we can write

$$\epsilon_{i,t} = e_{i,t} + \sum_{l=1}^{L} \widetilde{\lambda}_{i,l} u_{l,t} = e_{i,t} + \sum_{l=1}^{L} \sum_{i=1}^{N} \lambda_{i,l} u_{l,t} + \sum_{l=1}^{L} u_{l,t} \bar{\lambda}_l = e_{i,t} + \boldsymbol{\lambda}_i^T \boldsymbol{u}_t + \bar{\boldsymbol{\lambda}}^T \boldsymbol{u}_t \tag{12}$$

where $e_{i,t}$ is an idiosyncratic shock assumed i.i.d., and $\bar{\lambda}_l = \int_0^1 \lambda_{i,l}\, \mathrm{d}i$ and $\lambda_{i,l} = (\widetilde{\lambda}_{i,l} - \bar{\lambda}_l)$ are the (centered) individual exposures or factor loadings. W.l.o.g. exposures are normalized to have unit variance, and $\bar{\boldsymbol{\lambda}}_l \geq 0$[13]. Exposures are of reduced form for our purpose but may, for example, be the result of a production network. We must assume that the histogram of each set of factor loadings, $\{\lambda_{i,l}\}_{i \in [0,1]}$, is symmetric in order to generate the symmetric location-scale type input distributions of Figure 1 (a).

Consider now the cross-sectional variance of $\epsilon_{i,t}$ at a given point in time, and ex-

[13]The latter assumption is also w.l.o.g. and ensures that positive shocks corresponding to an increase in cost and hence have a recession interpretation.

amine what happens if $u_{t,l}^2$ is large relative to all other factors:

$$\mathbb{V}_t(\epsilon_{i,t}) = \mathbb{V}(e_{i,t} + \boldsymbol{\lambda}_i^T \boldsymbol{u}_t + \bar{\boldsymbol{\lambda}}^T \boldsymbol{u}_t \mid \boldsymbol{u}_t) \propto u_{t,l}^2. \tag{13}$$

Hence, large shocks drive up the variance of $\epsilon_{i,t}$ in the cross section and lead to a left-skewed distribution of $\widehat{q}_t$ among the subset of firms with market power. In the special case of $L = 1$ drop the subscript, and consider a sequence of aggregate shocks $\{u_t\}_t$. Suppose the sequence is such that it oscillates between $u_t \approx 0$ and $u_t \gg 0$. Recall that large shocks correspond to recessions because they drive up firms' cost. Additionally, they increase cross-sectional variance. Note that any deterministic growth can be subsumed in a positive mean of the process of $e_{i,t}$, which we do not specify any further. We then have the following Corollary:

Corollary 2. *Assume the shock-exposure structure of this section with $L = 1$, and let $\{u_t^2\}_t$ be the oscillating (countercyclical) sequence of shocks just described. Consider the cross-section of growth rates $\{r_{i,t}\}_{i\in[0,1]}$ over time. Then, the resulting sequence of cross-sectional skewness measures, $\{\mathbb{S}_t\}_t$ is (procyclically) oscillating, with* $\operatorname{sgn} \mathbb{S}_t = -\operatorname{sgn} \mathbb{S}_{t-1}$.

3.5 Discussion

3.5.1 Is ISID Exotic?

An (at least locally) increasing superelasticity is no skeleton in the closet of our model. For example, the family of *linear inverse demand* functions have an increasing superelasticity, or, as a more complex example, the constant pass-through demand family (CoPaTh) of demand systems by Matsuyama and Ushchev (2020) satisfies the property. In general, since ISID is implied by a weakly increasing pass-through plus MSLD, it suffices to examine the evidence for IPT[14]. Recently, Baqaee et al. (2024) have, using a non-parametric calibration of Matsuyama and Ushchev (2022)'s H.S.A. demand system, provided evidence in support of IPT (and, additionally, on MSLD). Other recent empirical work also comes down in favor of IPT (cf. Berman et al. (2012) and Amiti et al. (2019)).

Yet, ISID is not a property that has historically received too much attention in emprirical work as a demand-side restriction. To make this point, note that the widely used Kimball (1995) aggregator in the parametrization of Klenow and Willis (2016) has a globally constant superelasticity. Its special case of CES has a superelasticity of zero. Analogously, the homothetic translog aggregator by Feenstra (2003), which is popular in the trade literature, violates IPT (see also Matsuyama and Ushchev (2022)).

[14] Matsuyama and Ushchev (2022) refer to IPT even as "Marshall's 3rd Law of Demand", implying that IPT is a natural restriction on the demand function.

CES turns out to be a special case also when it comes to skewness. A CES-style demand function $p^*(q) = Aq^{-\tau}$ for $A > 0, \tau \in (0,1)$, implies a constant inverse demand elasticity. Therefore, the first order condition is linear in $\widehat{q}$ and no degree of market power induces skewness. Note here that the irrelevance of market power for skewness is obvious: In this special case, our formalization of market power implies that p is also a CES inverse demand function. Another insight is that CES inverse demand is indeed the *only inverse demand function* for which growth rates are globally unskewed. This is formalized in Proposition 5. [15]

Proposition 5 (the Special Case of CES Demand)**.** *The only inverse demand function which satisfies the regularity conditions (Assumption 1) for which growth rates are globally unskewed is of CES type, i.e.* $p^*(q) = Aq^{-\tau}$ *with* $A > 0, \tau \in (0,1)$.

Proof. Note first that any twice continuously differentiable, function $f : \mathbb{R} \supset D \to \mathbb{R}$ is either locally s-convex or s-concave in some point x_0, unless it is affine. Therefore, any inverse demand function which guarantees globally unskewed growth rates must be such that $\ln \circ \mathfrak{mr} \circ e^x$ is linear in x, which implies that $\mathfrak{mr}(q) = e^{A+B\ln q} = \bar{A}q^B$. Since marginal revenue is $p(q) + p'(q)q$, we are given the ODE: $p(q) + qp'(q) = Aq^B$. This is a linear first-order ODE. Noting that $p(q) + qp'(q) = \frac{\partial}{\partial q} qp(q)$, we can integrate both sides and obtain $p(q) = \frac{\bar{A}}{B+1}q^B + \frac{C}{q}$ for $B \neq -1$. Suppose the integrating constant, C, is zero. Then we must have $B \in (-1,0)$ because $B = \mathcal{E}p$. This is indeed the only possible case, since otherwise p is not log-concave around 0. (See appendix for details.) ■

3.5.2 Why Growth Rates?

In principle, we could benchmark our theory against log-real sales, $\widehat{q}_t$, directly. Why not calculate the cross sectional distribution of $\widehat{q}_t$ at any point in time and assess its skewness? There are three problems. First, the distribution of $\widehat{q}_t$ is essentially the size distribution. Trying to keep market power fixed by fixing size bins trivializes the distribution of $\widehat{q}_t$. Second, if one used generously wide size bins instead, the size distribution itself, which is heavily right-skewed in general, would obscure any contribution to cross-sectional left-skewness in $\widehat{q}_t$ caused by shocks. Third, any shock requires a comparison of the 'shocked' state to a baseline, and it is natural to make the baseline either the state of the previous period, or some kind of long-run steady state. The former describes a time series growth rate, $\widehat{q}_t - \widehat{q}_{t-1}$, whereas the latter describes

[15]We note that it is possible to construct pathological examples of demand functions which have a *decreasing* superelasticity of demand. In unreported results, we found a parameter configuration in a quartic demand function, for which the first order condition turns convex and yields right-skewed growth relative to the steady state. This feature, however, turned out to be very delicate, and required much tinkering with parameters, as well as a configuration, in which prices do not drop to 0 as q grows large.

growth relative to the steady state, $\widehat{q}_t - \widehat{q}_0$. While we have used the latter as a theoretical device above, in the presence of idiosyncratic growth trends, measurement of $\widehat{q}_0$ becomes elusive.

3.5.3 Other Sources of Skewed Growth Rates

We note that we do not preclude the existence of other sources of skewness in growth rates. By offering a theory which is able to explain the differential pattern between skewness of small v. large firms over the business cycle, we allow that other sources of skewness may well play a role in shaping outcomes. For example, the input distributions of shocks may be systematically skewed (Salgado et al., 2025). Alternatively, the cost function may be log-convex, which can be an additional contributor to left-skewness of $\widehat{q} - \widehat{q}_0$. We outline two mechanisms for this 'supply-side skewness' in Appendix B: Capacity adjustments and customer acquisitions. Yet, these complications do not offer a natural reason for why skewness may differ with market power.

3.5.4 Implications for the Real World

Our formal results on procyclical skewness in growth rates are likely to hold with more generality 'in the wild'. Suppose, for example, that (i) expansions are smooth via a negative trend in $e_{i,t}$, and (ii) recessions are abrupt positive impulses in $u_t \uparrow$, which decay in an AR(1) fashion. Therefore, from an expansion to a recession, one observes a sudden increase in variance, leading to sudden negative skew in growth rates. From there, a decay of u_t causes the shock variance to decrease steadily, leading to a sequence of cross-sections that features an initially positive but slowly vanishing skew in growth rates.

Yet, Figure 2 suggests that skewness does not decay to zero in times of expansion but rather stays positive. We advise to be careful when interpreting this fact within our model. The theory implies a mechanism in which symmetric input distributions yield skewed outcomes. Fixing the mechanism while changing the input distribution will generally lead to similar dynamics of the skewness index over the cycle, however, may affect its level. Besides such concerns about the level, we can think of at least two intuitive reasons for why the level of skewness may not return to nil after a crisis. The first is that the decay of the impact of aggregate shocks is asynchronous across firms. If the impact of the shock lasts longer for some firms, they will recover later. Therefore, their contribution to a positive skewness index during times of expansion may materialize substantially later in time. Second, the distribution of growth through technological innovation (which aggregates into heterogeneous, secular growth trends) may simply be right-skewed in the cross-section. In fact, any model in which only some firms innovate while others keep their current technology implies a right-skewed distribution of technological growth.[16]

4 Empirical Evidence

Having established the theoretical foundations linking market power to skewed firm responses and procyclical skewness, we now provide empirical evidence testing these predictions. Our theory generates three testable hypotheses: (1) aggregate shocks should induce negatively skewed growth rate distributions in the cross-section of firms, (2) this skewness response should be stronger for larger firms with greater market power, and (3) the comovement between aggregate growth and cross-sectional skewness should be driven primarily by aggregate factors rather than idiosyncratic shocks. To test these predictions, we use quarterly Compustat data on US public firms spanning multiple business cycles. The empirical strategy proceeds in three steps: first, we document the data and measurement approach; second, we estimate impulse responses to identified aggregate shocks to test whether shocks generate the predicted skewness patterns; and third, we decompose growth rate fluctuations to assess whether aggregate or idiosyncratic factors drive cross-sectional skewness.

4.1 Data

Our analysis uses data on US public firms from Compustat. Compustat is the benchmark firm-level data set for the United States, providing detailed balance sheet information at the quarterly frequency over a long sample period of over 35 years. The long sample period enables us to cover multiple recessions and draw general conclusions about skewness facts in the US business cycle. Estimating impulse responses to aggregate shocks at the firm level also requires a sufficiently long time series for each firm. Let $q_{i,t}$ be firm i's real sales in quarter t. Real sales our key measure of firm size and output. Year-on-year real sales growth is $g_{i,t} = \ln\left(q_{i,t}/q_{i,t-4}\right)$. The business cycle indicator is aggregate real sales growth, constructed as the size-weighted average of existing firms' growth rates:[17]

$$g_t = \frac{\sum_i g_{i,t} q_{i,t-4}}{\sum_i q_{i,t-4}}.$$

This definition of aggregate sales growth only considers firms that exist in both t and $t-4$ and therefore abstracts from entry and exit dynamics, which could affect the comovement of aggregate growth and micro skewness but are not the focus of this study.

[16] This idea is, for example, compatible with recent work on *innovation bursts* by Berlingieri et al. (2025).

[17] Note that we treat production and sales as equal, hence we are ignoring inventories.

The main skewness measure is the *Kelley skewness*, which we obtain from setting $r = 0.1$ in eq. (5), and applying it to the empirical CDF of a given set of growth rates, G_t.[18] Kelley skewness compares the distance of the 90% quantile of the time-t distribution of firm growth rates ($[G_t]_{0.9}$) from the median ($[G_t]_{0.5}$) to the distance of the median from the 10% quantile, rescaled by the overall 90-10-spread of the distribution. If the 90% quantile is further above the median than the 10% quantile is below the median, the distribution is right-skewed and Kelley skewness is positive. Kelley skewness allows for an easy decomposition of skewness movements into changes in upper and lower parts of the distribution and is more robust to outliers than the third moment.

Details on the sample construction are contained in Appendix C. Besides Compustat, we use data from CRSP for stock prices and Worldscope Fundamentals because of its good coverage of the date of incorporation. All variable definitions are listed in the appendix. The data cleaning filters out roughly half of the observations from the raw Compustat files. Since estimating firm-level impulse responses requires a sufficiently long time series for each firm, we focus on firms that have at least 40 consecutive observations for sales growth. This reduces the sample size further, see Figure A.1 in Appendix C. Despite the smaller sample size, the time series of cross-sectional skewness are very similar before and after data cleaning, see Figure A.2. Appendix D confirms that cross-sectional skewness is strongly procyclical in Compustat data, for a variety of skewness measures and data cleaning procedures.

Table 2 compares the full Compustat sample against the cleaned version of firm growth streaks. For comparison, the table also reports summary statistics from the Quarterly Financial Reports (QFR), which have been used by Crouzet and Mehrotra (2020) to construct a representative sample of US firms in certain sectors. For example, the QFR can be used to construct a sample accurately reflecting the firm size distribution of US manufacturing firms, including private firms. Relative to this representative sample of manufacturing firms, the average firm in the Compustat data (which is not limited to manufacturing firms) is considerably larger, both in terms of assets (USD 3.99bn vs USD 43mln) and sales (USD 399mln vs USD 11mln). The sales growth distribution in the QFR sample is more dispersed and more symmetric than in the Compustat sample with a mean growth rate closer to zero. Compared to the QFR, leverage and short-term debt are higher in raw Compustat data but lower in the cleaned data. The number of observations in the cleaned data is roughly half of the number of ob-

[18] Explicitly,

$$skew\,(G_t) = \frac{\left([G_t]_{0.9} - [G_t]_{0.5}\right) - \left([G_t]_{0.5} - [G_t]_{0.1}\right)}{[G_t]_{0.9} - [G_t]_{0.1}} \in (-1, 1), \tag{14}$$

where $G_t := \{g_{i,t}\}_{i=1,\ldots,n_t}$ is the set of firm growth rates at time t.

Table 2: Summary Statistics for Compustat Data

	Full Compustat	Cleaned Sample	QFR
Assets (mln. USD)	3,941	1,873	43.2
Sales (mln. USD)	396.3	411.7	10.8
Sales Growth (%)	7.2	7.6	0.63
$Q(\text{Sales Growth})_{0.25}$ (%)	-7.8	-7.6	-25.3
$Q(\text{Sales Growth})_{0.75}$ (%)	20.7	21.2	26.6
Net Leverage (%)	26.9	12.4	20.0
Short-term debt (%)	75.8	8.8	33.0
Obs./quarter	6,338	4,844	6,122
Unique firms	22,097	17,388	–
ρ(Sales Gr., GDP Gr.)	0.64	0.55	–
ρ(Sales Gr., Skew)	0.85	0.88	–

Note: Statistics for QFR are for the manufacturing subset of Crouzet and Mehrotra (2020) from 1977Q3–2014Q1 and directly taken from tables 1 and 3 of their paper; the values are unweighted averages across size bins. The Compustat statistics are for 1983Q3–2014Q1. The reported values for assets and sales are in 2009 USD. Values from Crouzet and Mehrotra (2020) are deflated using the price index for value added in manufacturing. Compustat values are deflated using the GDP deflator since the data covers multiple industries. The full Compustat sample is the raw Compustat data but removes all firm-quarter observations with non-positive assets.

servations per quarter in the QFR. The number of unique firms falls from $22,397$ to $5,061$. Importantly, although the data cleaning affects multiple firm characteristics on average, the correlation between aggregate sales growth and GDP growth is similar for both Compustat samples (0.56 vs 0.69). The correlation between skewness and aggregate sales growth, which is the key object of study in this chapter, is virtually identical for both samples (0.85 vs 0.84).

In the Compustat sample, all firms are large compared to the universe of US firms. Therefore, there is little movement at the extensive margin, and any bias in cross-sectional skewness due to firm exit should be negligible. Hence, we implicitly condition on firm survival in our results, and abstract from entry/exit dynamics as much as possible, which are not focus of this work.

Despite the underrepresentation of small firms, sales concentration in the sample is still high. The largest 10% of firms account for 70% of sales on average, and the top 30% account for over 90% of sales. For comparison, the largest 1% of firms in the QFR sample of Crouzet and Mehrotra (2020) represent ca. 75% of total sales.

To support our theoretical model with empirical evidence, we derive the following econometric specification that is guided by our theoretical model. Within our framework, we can express firm level growth rates as

$$g_{i,t} = \ln \frac{q_{i,t}}{q_{i,t-1}} = \gamma + f(\alpha_i, \epsilon_{i,t}) - f(\alpha_i, \epsilon_{i,t-1}) + w_{i,t} \tag{15}$$

where γ is an aggregate growth trend[19] and α_i is firm-specific market power (which we assume to be monotonic in firm size) and $w_{i,t}$ is a zero-mean i.i.d. disturbance. We use eq. (12) of the previous section in place of $\epsilon_{i,t}$, whereby we assume that $e_{i,t}$ is fully absorbed by the secular growth trend, γ and $w_{i,t}$. Relative to the no-disturbance, steady-state baseline, one can write

$$g_{i,t} = \ln \frac{q_{i,t}}{q_{i,t-1}} = \gamma + f(\alpha_i, \boldsymbol{\lambda}_i^T \boldsymbol{u}_t) - f(\alpha_i, 0) + w_{i,t}. \tag{16}$$

From here, we derive two simple hypotheses: (i) The presence of $f(\alpha_i, \boldsymbol{\lambda}_i^T \boldsymbol{u}_t)$ should induce negative skewness in the cross-section of growth rates upon impact of a shock. (ii) The impact on left-skewness, when indexed by α, is increasing in the market power of the considered cross-section. We examine these two hypotheses in the next section (Section 4.2) using a simple impulse-response framework and a battery of off-the-shelf aggregate shocks.

Even though we document a new channel of skewness propagation in the business cycle, we are not claiming that this mechanism is exclusive. In fact, skewed idiosyncratic shocks may still play a role for the cross-section of growth rates. To examine this thought, consider the case where market power plays no role in the transmission. This implies $\alpha = 0$ and f is a linear function in the shock, and we can write (up to an affine transformation)

$$g_{i,t} = \ln \frac{q_{i,t}}{q_{i,t-1}} = \gamma + \widetilde{\boldsymbol{\lambda}}_i^T \boldsymbol{u}_t + w_{i,t}. \tag{17}$$

One can estimate this equation with a simple decomposition using principal component analysis (PCA). This allows analyzing skewness properties of an aggregate factor $a_{i,t} = \widetilde{\boldsymbol{\lambda}}_i^T \boldsymbol{u}_t$ vis-à-vis those of the idiosyncratic component, $w_{i,t}$: Maintaining the assumption of symmetrically distributed $\boldsymbol{\lambda}_i$ would imply that skewness of growth rates at a given time t exclusively lives in the idiosyncratic component of this equation, $w_{i,t}$. Reintroducing market power would reintroduce skewness into $a_{i,t}$ by skewing the distribution of exposures. Thus, a central question we can apply this PCA to is about the importance of our mechanism *a priori*: whether skewness occurs in the aggregate component or in the individual component. A large relative contribution to skewness by the aggregate factor reinforces the importance of our mechanism, directly linking aggregate shocks in levels to cyclicality properties of higher business cycle moments. It further adds evidence to a growing literature documenting heterogeneous responses of firms to aggregate shocks; for monetary policy shocks, for example, see Ottonello and Winberry (2020) or Cloyne et al. (2023). As the ultimate analysis of this paper, we

[19]One can motivate the trend from the growth trends brought up in Section 3.4.2, which turn into an aggregate trend in outcomes if demand systems are homothetic.

Table 3: Local projection specifications

Shock	Reference	Controls (lagged)	Sample period
Monetary	Bu et al. (2021)	Real GDP, GDP deflator, Shadow Rate, EBP	1994Q1 – 2019Q4
Oil	Baumeister and Hamilton (2019)	Real GDP, GDP deflator, Oil price	1983Q3 – 2019Q4
Credit	Gilchrist and Zakrajšek (2012)	Real GDP, GDP deflator, EBP	1983Q3 – 2019Q4
Uncertainty	Ludvigson et al. (2021)	Real GDP, GDP deflator, VXO	1983Q1 – 2015Q4
Sentiment	Lagerborg et al. (2023)	ICE, real GDP, uncertainty, Real stock prices	1983Q3 – 2019Q4
TFP	Ben Zeev and Khan (2015)	Real GDP per capita, real stock prices per capita, labor productivity	1983Q3 – 2012Q1

Note: All specifications include lags of the dependent variable and the shock series as controls and are estimated with two lags. 'ICE' is the University of Michigan Index of Consumer Expectations. Uncertainty is measured as the 12-month Jurado et al. (2015) uncertainty index.

discuss the PCA and its results in Section 4.3.

4.2 Aggregate Shocks Cause Growth-Skewness Correlation

With an impulse-response framework, we show that aggregate shocks move level and skewness of growth rates in lock-step, on the aggregate and for large firms, but much less so for the small firms of our sample. We estimate impulse responses of skewness and growth to monetary, oil, credit, uncertainty, sentiment, and TFP shocks. These shocks are different in nature and timing, constructed using varying identification schemes and sample periods. We find that all shocks induce a close co-movement pattern between skewness and growth that is at least as strong as measured in the raw data. We estimate the impulse responses of skewness and sales growth using local projections (Jordà, 2005):

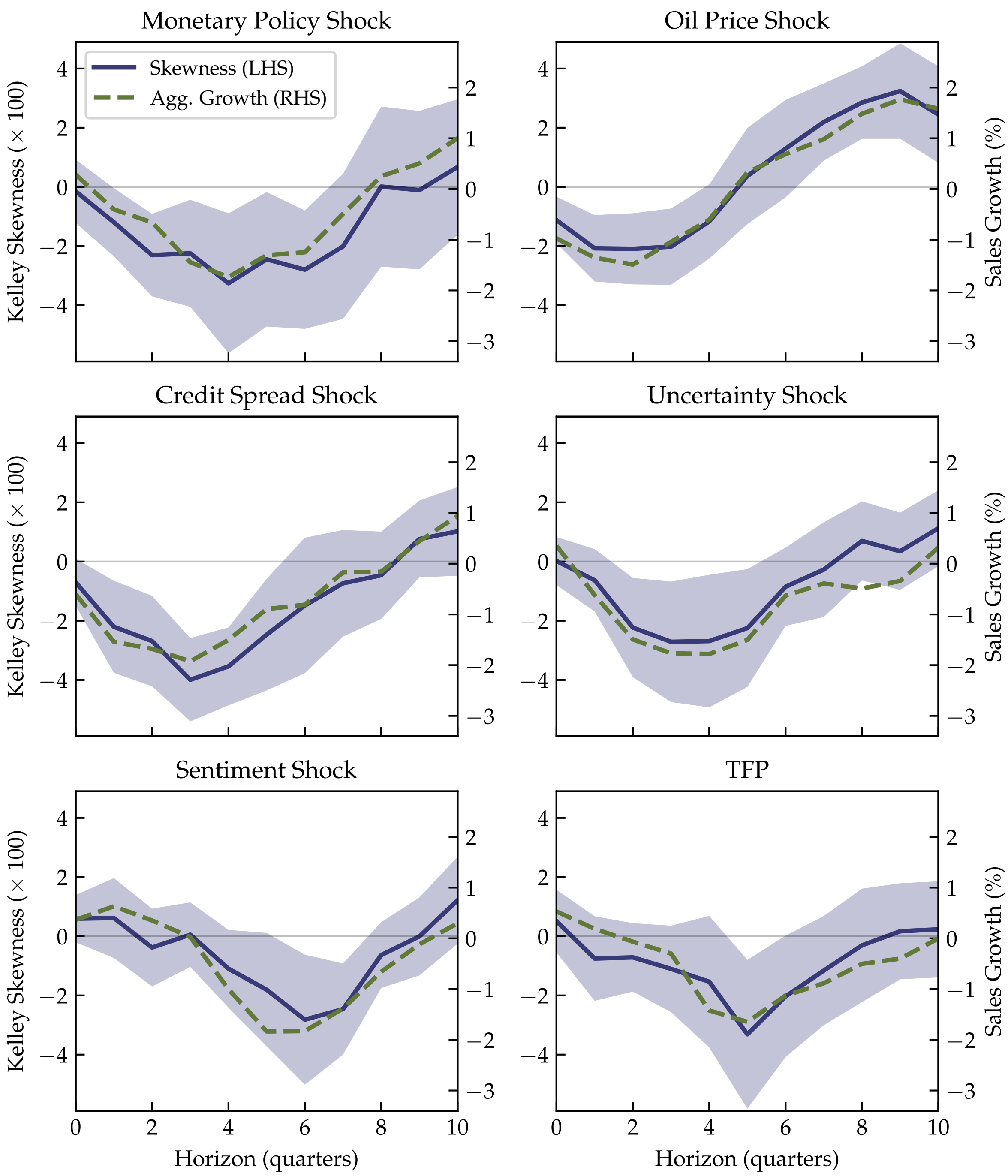
$$y_{t+h} = \alpha_h + \beta_h \text{shock}_t + \sum_{\ell=1}^{L} \gamma'_{\ell,t} \text{controls}_{t-\ell} + e_{t+h} \tag{18}$$

for $h = 0, ..., 11$ quarters using up to $L = 2$ lags. The β_h coefficients give the impulse response of interest. The variable y is either cross-sectional skewness or aggregate sales growth. The shock series and controls are taken off-the-shelf from existing work. Table 3 summarizes the regression specifications across the different shocks. Appendix D.2 covers robustness checks and contains details on the variable definitions as well as data sources. We also describe each shock series in detail in Appendix D.2.

4.2.1 IRF Results

Figure 6 shows the impulse response estimates for the six different shocks. All aggregate shocks are associated with a subsequent decline in cross-sectional skewness (blue lines; left axis). Following an adverse one standard deviation shock, the skewness index declines by between 0.02 and 0.06 points. The decline is strongest for the credit

Figure 6: Comovement of growth and skew after aggregate shocks



Note: The 90% confidence bands are based on Newey-West standard errors. Shock magnitudes are normalized to be one standard deviation. The signs of the sentiment and the TFP shock are reversed to be contractionary.

shock and weakest for the monetary shock. The peak effect occurs 4 to 6 quarters after impact and is statistically significant across all shocks. The effects on skewness are not long-lived and die out after at most 10 quarters. The response of aggregate sales growth (black dashed lines; right axis) to the aggregate shocks looks very similar to the responses of skewness. The correlations of the impulse responses for a given shock range between 0.89 and 0.98. Aggregate shocks therefore appear capable of 1) inducing significant movements in skewness and 2) generating strong co-movement between sales growth and skewness.

These findings confirm the insights of Figure 2. Cross-sectional skewness moves closely with aggregate growth across many US recessions (including the Covid recession), suggesting the high correlation is a robust business cycle fact that does not only pertain to certain types of recessions. It is therefore encouraging to see that different types of shocks, all of which are considered potentially important drivers of the US business cycle, induce the procyclical skewness pattern.

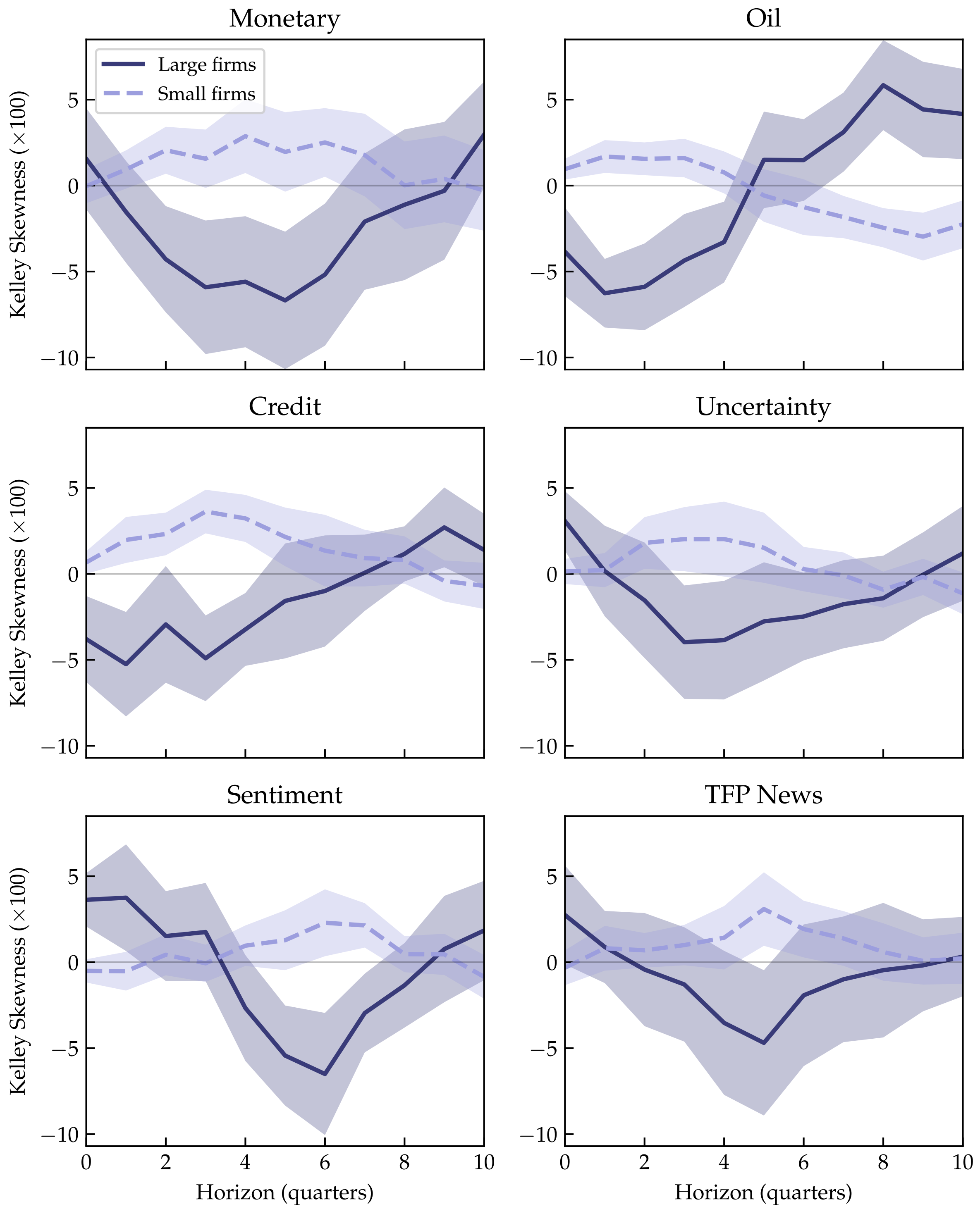
Splitting up the linear projections in equation (18) into the top-10% and bottom-90% firm size bins also confirms the evidence conveyed in Figure 3. The skewness of large firms' growth rates drops considerably more than for the smaller firms with striking regularity. This result is displayed in Figure 7. Small firms experience almost no effect on skewness, whereas the skewness index of large firms drops by -0.04 to -0.06. Through the lens of our model, this is evidence of price taking and price setting behavior among small and large firms, respectively.

In Appendix D.1, we corroborate the results of this section using a bottom-up approach to impulse responses. Our findings are highly robust to this alternative approach.

4.3 Aggregate Factors Explain Skewness Fluctuations

We decompose sales growth rates and thereby cross-sectional skewness into an *aggregate* and an *idiosyncratic* component using eq. (17). We define as the aggregate component the contribution of the business cycle to individual growth rates, and the idiosyncratic component as any residual random fluctuation. The evidence from the previous literature regarding the importance of both components to cross-sectional skewness is mixed: Ilut et al. (2018) find no significant skewness in establishment-level TFP shocks, while Salgado et al. (2025) argue for strong procyclical skewness in TFP shocks computed using various methods. Neither approach allows for clear conclusions about the relative importance of idiosyncratic shocks: Even if TFP shocks are not skewed as in Ilut et al. (2018), there may be other idiosyncratic shocks with a skewed distribution that drive skewness in sales growth rates; even if TFP shocks are skewed as in Salgado et al. (2025), their contribution to sales growth rates may be minute because the shocks

Figure 7: Skewness response of small and large firms



Note: The 90% confidence bands are based on Newey-West standard errors. Shock magnitudes are normalized to be one standard deviation. The signs of the sentiment and the TFP shock are reversed to be contractionary.

are small[20]. Focusing on skewness in a particular idiosyncratic shock can therefore not provide conclusive evidence about whether skewed idiosyncratic shocks cause cross-sectional skewness unless the shock is both skewed and explains a significant share of variation in sales growth rates.

We decompose sales growth rates into aggregate and idiosyncratic components using eq. (17). The same approach is used in Herskovic et al. (2016) to extract the idiosyncratic component of sales growth rates. We use the two components to study their impact on cross-sectional skewness. The results obtained this way are conservative in the sense that the idiosyncratic component may still contain aggregate fluctuations that firms could respond to in a nonlinear fashion. However, the idiosyncratic component is certain to capture all firm-specific sources of variation[21]. If skewness in idiosyncratic shocks affects skewness in sales growth rates, the idiosyncratic component must explain a significant share of the skewness in growth rates. The estimates from this approach therefore provide an upper bound for the importance of skewness in the idiosyncratic component in explaining skewness in growth rates.

Table 4: Common vs idiosyncratic drivers of skewness

No. Factors:	1	4	8
Correlations with skewness:			
$\rho\left(skew_u, skew_g\right)$	0.89	0.76	0.72
$\rho\left(skew_a, skew_g\right)$	0.68	0.72	0.81
Decomposition of variation in skewness:			
R_u^2	0.21	0.26	0.32
R_a^2	0.79	0.74	0.68
Fit of aggregate component:			
R_i^2 q(0.25)	0.01	0.09	0.16
R_i^2 q(0.5)	0.06	0.20	0.30
R_i^2 q(0.75)	0.18	0.36	0.47
Observations	398,316		

Note: Each column refers to a decomposition using a different number of principal components. The decomposition uses the weighted PCA algorithm of Delchambre (2015) with zero weights for missing values and unit weights for all other observations. The first two rows measure the correlation of 90% Kelley skewness in sales growth rates with the skewness in the idiosyncratic components ($skew_u$) or the aggregate components ($skew_a$). The following two rows decompose the variation in Kelley skewness into the contributions by skewness in the idiosyncratic part and skewness in the aggregate part. The last three rows show the 25, 50, and 75% quantile of the distribution across R^2 from firm-level time series regressions of the sales growth rate onto the aggregate component. The number of observations refers to the actual firm-quarter observations.

[20]Panel regressions in Salgado et al. (2025) confirm this intuition. The skewness in TFP shocks explains virtually none of the variation in firm-level sales, employment, or investment growth as observed from the R^2 values of zero reported in Table 2 of their paper.

[21]This is true except under a network perspective in which idiosyncratic shocks may cause comovement across firms that is perceived as aggregate fluctuations by the PCA algorithm. See Foerster et al. (2011) for a discussion of this point.

4.3.1 Factor Decomposition Results

Table 4 shows the results. Skewness across the aggregate components $a_{i,t}$ correlates closely with skewness in growth rates even with only one factor included in the decomposition (row 1). The comovement between skewness in the idiosyncratic components and in the growth rates decreases with the number of factors, though it remains sizeable even for the case of eight factors (row 2).

Correlations can be deceiving because comovement patterns may be strong while magnitudes of variation differ. To analyze which component explains most of the variation in Kelley skewness, we decompose the numerator of the skewness measure. The numerator is the component representing asymmetries in the distribution, while the denominator is solely a scaling factor ensuring Kelley skewness always lies between -1 and 1.

Let the numerator of the Kelley skewness expression be $\eta\left(X\right) \equiv [X]_{0.9} - 2[X]_{0.5} + [X]_{0.1}$, where $[X]_r$ indicates the r-quantile of $X := \{x_i\}_{i=1,\ldots,N}$. The decomposition of demeaned growth rates $\bar{g}_{i,t} := g_{i,t} - \gamma_i$ is then

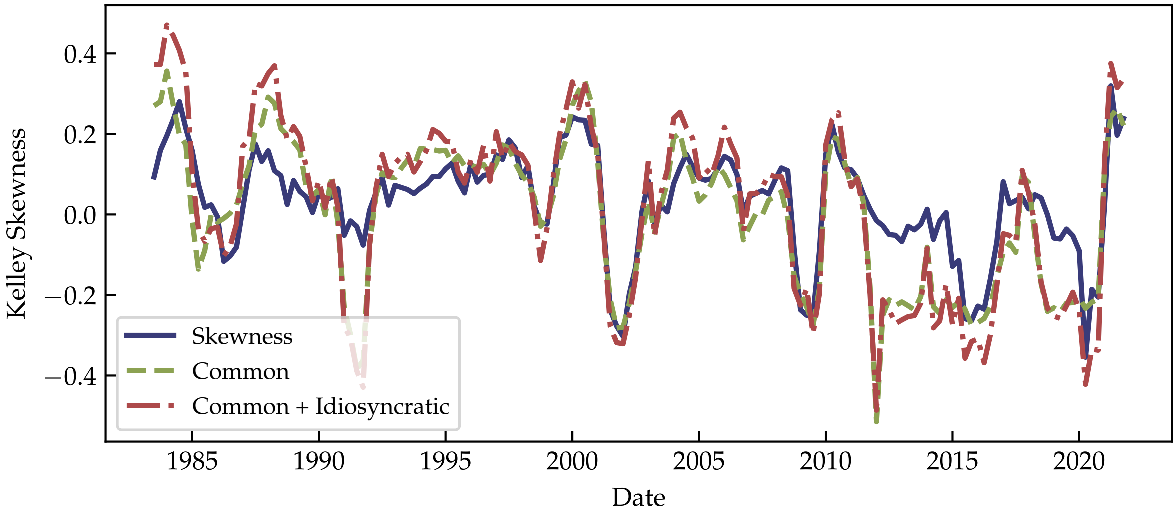
$$\frac{\eta\left(\bar{g}_t\right)}{[\bar{g}_t]_{0.9} - [\bar{g}_t]_{0.1}} = \frac{\eta\left(a_t\right)}{[\bar{g}_t]_{0.9} - [\bar{g}_t]_{0.1}} + \frac{\eta\left(\varepsilon_t\right)}{[\bar{g}_t]_{0.9} - [\bar{g}_t]_{0.1}} + \Delta_{a_t} + \Delta_{\varepsilon_t}, \tag{19}$$

where a_t and ε_t refer to the distributions of the aggregate and the idiosyncratic component. Because the ordering of firms within these three distributions may change relative to the ordering of sales growth rates, the decomposition is not exact. The difference is captured by approximation errors

$$\Delta_a = \left([\widetilde{a}_t]_{0.9} - [a_t]_{0.9} + 2\left([\widetilde{a}_t]_{0.5} - [a_t]_{0.5}\right) + [\widetilde{a}_t]_{0.1} - [a_t]_{0.1}\right) / \left([\bar{g}_t]_{0.9} - [\bar{g}_t]_{0.1}\right) \tag{20}$$

with $[\widetilde{a}_t]_r$ denoting the aggregate component of the r-quantile of the growth rate distribution g_t, and by Δ_ε, which is defined analogously to Δ_a. Given these objects, we can compute partial contributions to explained variance in growth rate skewness. Of the skewness that is unexplained by the approximation error, the idiosyncratic component explains only 25% (R^2_ε). The remaining 75% of unexplained variation are attributed to skewness in the common factors (R^2_a). This decomposition result is broadly stable across the number of aggregate factors used. Because the skewness of the different components is not orthogonal, the explained variance attributed to each component depends on the ordering of the variables. The results presented here order the idiosyncratic component first to give conservative results for the aggregate component. Flipping the ordering indicates a contribution between 92% and 96% for the aggregate component (result not shown). To stress the importance of aggregate factors in driving cross-sectional skewness, Figure 8 shows that skewness in the idiosyncratic component adds little information beyond the procyclical pattern present in skewness

Figure 8: Skewness in common vs idiosyncratic component



Note: The blue line is the skewness in demeaned growth rates. The red line shows the contribution of skewness in the aggregate component ($\eta(a_t)$ in equation (19)) to skewness in demeaned sales growth rates. The green line adds the contribution of skewness in the idiosyncratic component ($\eta(\varepsilon_t)$ in equation (19)) to the green line.

of the common component. The figure also demonstrates that the approximation error of eq. (19) is small since the common and idiosyncratic contributions (orange line) closely track the skewness measure (blue line), apart from deviations in the early 1990s and mid-2010s.

The weak contribution of the idiosyncratic component is not due to a small size of that component. For most firms, the idiosyncratic component remains large after removing the aggregate factors. The last three rows of Table 4 show the 25%, 50%, and 75% quantiles of the distribution of R^2 values from firm-level time series regressions of the demeaned sales growth rate onto the aggregate factors. Even when including eight factors, the aggregate component explains no more than 30% of time series variation for half the firms (R_i^2 q(0.5)), and explains more than 47% of variation for only 25% of firms (R_i^2 q(0.75)). To emphasize, the first column of Table 4 shows that one aggregate factor explains 79% of the variation in skewness (R_a^2) even though it only explains 6% of firm-level sales growth variation on average.

5 Conclusion

This paper has documented new evidence on the shape of firm growth distributions over the business cycle. Using Compustat data, we confirmed that skewness is procyclical, becoming negative in recessions and positive in booms. We then showed that this pattern is strongly size dependent: large firms display much larger swings in skewness than smaller firms. Finally, we established that countercyclical variance amplifies these effects, with increases in volatility leading to disproportionately negative skewness among the largest firms. Taken together, these findings point to a systematic size gradient of skewness in the cross-section of firms.

Since this size gradient cannot be explained by theories in which firms are fundamentally homogeneous, we set out to develop a simple framework to rationalize all empirical observations jointly. When firms possess market power, Marshall's Second Law of Demand implies that symmetric shocks translate into concave output adjustments. This mechanism generates systematically skewed growth responses, with the effect increasing in the degree of market power and in the variance of shocks. The model rationalizes both the procyclicality of skewness and its dependence on firm size, and it matches the empirical impulse responses we estimate from the data. In this sense, skewness is not a primitive property of the shock distribution but an endogenous outcome of firm behavior under imperfect competition.

The results carry important policy implications. Skewness means that observed outcomes systematically differ from the underlying incidence of shocks. If policymakers allocate support or compensation in proportion to realized outcomes, they risk mistaking endogenous asymmetries for differences in exposure. Large firms in particular may appear to be disproportionately hit during downturns, when in fact their outcomes reflect amplified responses due to market power. Effective stabilization or redistribution policies must therefore look beyond average growth and volatility, and take into account how market structure shapes the distribution of outcomes across firms. Ignoring these mechanisms risks systematic misallocation of resources and a reinforcement of downside risks in aggregate fluctuations.

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Online Appendix

Monopolistically Skewed Business Cycles

A Proofs

A.1 Theory

Proof of Lemma 1. Marginal revenue is $\mathfrak{mr}(q) = p'(q)q + p(q)$. Any solution to the monopolist's problem must live within a non-empty, open interval, D, on which $\mathfrak{mr}$ is strictly positive. Thus, $p'(q)q + p(q) > 0 \Leftrightarrow \mathcal{E}p(q) > -1$. By log-concavity of p, we have $p''(q)p(q) - p'(q)^2 \leq 0$, thus, on D,

$$\begin{aligned}
\mathfrak{mr}'(q) &= 2p'(q) + p''(q)q \\
&= p'(q)(2 + qp''(q)/p'(q)) \\
&\leq p'(q)(2 + qp'(q)/p(q)) \\
&= p'(q)(2 + \mathcal{E}p(q)) < 0,
\end{aligned}$$

because $p'(q) < 0$. Since p is decreasing to 0, $p' \overset{q\to\infty}{\longrightarrow} 0$ and some $q^\star$ exists for which $\mathfrak{mr}(q^\star) = c'(q)$ (recall that c' is increasing). ■

Proof of Lemma 2. We prove each claim individually.

1. Note that X is unskewed because Z is symmetric about its mean. Write

$$\begin{aligned}
skew[g(X)] &= \frac{[g(X_\varepsilon) + g(X_{1-\varepsilon})]/2 - g(X_{0.5})}{(g(X_{1-\varepsilon}) - g(X_\varepsilon)/2)} \\
&\leq \frac{g\left(\left(X_\varepsilon + X_{1-\varepsilon}\right)/2\right) - g(X_{0.5})}{(g(X_{1-\varepsilon}) - g(X_\varepsilon))/2} \\
&= \frac{g(X_{0.5}) - g(X_{0.5})}{(g(X_{1-\varepsilon}) - g(X_\varepsilon))/2} = 0
\end{aligned}$$

   by Jensen's inequality and unskewedness of X.

2. Let $Y = g(X)$, then Y has negative skew. Consider a second-degree Taylor expansion of h about the median of Y

$$h^*(y) = h(Y_{0.5}) + h'(Y_{0.5})(y - Y_{0.5}) + \frac{h''(Y_{0.5})}{2}(y - Y_{0.5})^2$$

and rewrite the skewness coefficient:

$$
\begin{aligned}
skew[h(Y)] &\approx \frac{\left(\frac{Y_{1-\varepsilon}+Y_\varepsilon}{2} - Y_{0.5}\right)\left(1+\frac{h''}{2h'}\frac{(Y_{1-\varepsilon}-Y_{0.5})^2+(Y_\varepsilon-Y_{0.5})^2}{(Y_{1-\varepsilon}-Y_{0.5})+(Y_\varepsilon-Y_{0.5})}\right)}{\left(\frac{Y_{1-\varepsilon}-Y_\varepsilon}{2}\right)\left(1+\frac{h''}{2h'}[(Y_{1-\varepsilon}-Y_{0.5})+(Y_\varepsilon-Y_{0.5})]\right)} \\
&= skew[Y]\underbrace{\frac{\left(1+\frac{h''}{2h'}\frac{(Y_{1-\varepsilon}-Y_{0.5})^2+(Y_\varepsilon-Y_{0.5})^2}{(Y_{1-\varepsilon}-Y_{0.5})+(Y_\varepsilon-Y_{0.5})}\right)}{\left(1+\frac{h''}{2h'}[(Y_{1-\varepsilon}-Y_{0.5})+(Y_\varepsilon-Y_{0.5})]\right)}}_{=:\Delta},
\end{aligned}
$$

where one notes that $\Delta > 1$ since $h'' < 0 < h'$. Hence $skew[h(Y)] \approx \Delta \cdot skew[Y] < skew[Y] < 0$.

3. Finally, consider some positive number a and the mapping $Z \mapsto g(a \cdot Z)$. Take a second order Taylor expansion over the median of Z,

$$
g^*(a \cdot x) = g(0) + ag'(0)(x - Z_{0.5}) + \frac{a^2 g''(0)}{2}(x - Z_{0.5})^2,
$$

substituting into the skewness measure yields

$$
\begin{aligned}
skew[g(aZ)] &\approx \frac{ag'\overbrace{(Z_{1-\varepsilon}+Z_\varepsilon-2Z_{0.5})}^{=0}+\frac{a^2g''}{2}((Z_\varepsilon-Z_{0.5})^2+(Z_{1-\varepsilon}-Z_{0.5})^2)}{ag'(Z_{1-\varepsilon}-Z_\varepsilon)+\underbrace{\frac{a^2g''}{2}[(Z_{1-\varepsilon}-Z_{0.5})^2-(Z_\varepsilon-Z_{0.5})^2])}_{=0}} \\
&= \frac{\frac{a^2g''}{2}((Z_\varepsilon-Z_{0.5})^2+(Z_{1-\varepsilon}-Z_{0.5})^2)}{ag'(Z_{1-\varepsilon}-Z_\varepsilon)} \\
&= \frac{\frac{a^2g''}{2}\left(\frac{Z_{1-\varepsilon}-Z_\varepsilon}{2}\right)^2}{ag'(Z_{1-\varepsilon}-Z_\varepsilon)} \\
&= a\frac{g''}{g'}\left(Z_{1-\varepsilon}-Z_\varepsilon\right)/8.
\end{aligned}
$$

Therefore, $skew[g(aZ)] \approx a\frac{g''}{g'}\left(Z_{1-\varepsilon}-Z_\varepsilon\right)/8$. Setting $a = 1$ yields the skewness for the transformed variable $g(Z)$, and an explicit formula to part (1) of the Lemma. Additionally, increasing a yields more negative skewness for a concave, increasing transformation ($g'' < 0 < g'$) and more positive skew for a convex increasing transformation.

■

Proof of Lemma 3. Define $\widetilde{p}(\widehat{q}) := p(\exp\widehat{q})$, then $\ln(\widetilde{p}(\widehat{q}) + \widetilde{p}'(\widehat{q})) = \ln \circ \mathfrak{mr}(e^{\widehat{q}})$. Note first that H is (strictly) concave if and only if $\ln \circ \mathfrak{mr}(e^{\widehat{q}})$ is strictly concave, since the marginal cost part of the equation is linear. The two assertions on the list as proved as follows:

1. This condition also implies that H is (strictly) concave. By eq. (4), $H(\widehat{q}) = \epsilon$ and thus $Q^\star = (\ln \circ H)^{-1}$. Since marginal revenue is strictly decreasing on D, so is H. Suppose now that H is (strictly) concave. This implies that H^{-1} is (strictly) concave and decreasing. (This fact is treated in exercise 3.3 of Boyd and Vandenberghe, 2004.)

2. Rewrite concavity of $(\ln \circ \mathfrak{mr})(e^{\widehat{q}})$ using the following identities:

$$\mathcal{E}p = \frac{\partial \ln p(q)}{\partial \ln q} = \frac{\partial \ln \widetilde{p}(\widehat{q})}{\partial \widehat{q}} = \frac{\widetilde{p}'(\widehat{q})}{\widetilde{p}(\widehat{q})} \tag{A.1}$$

$$\mathcal{E}^2 p(e^{\widehat{q}}) = \frac{\partial \mathcal{E}p}{\partial q}\frac{q}{\mathcal{E}p} = \frac{\frac{\partial \mathcal{E}p}{\partial \ln q}}{\mathcal{E}p} = \frac{\frac{\widetilde{p}''(\widehat{q})}{\widetilde{p}(\widehat{q})} - \frac{\widetilde{p}'(\widehat{q})^2}{\widetilde{p}(\widehat{q})^2}}{\mathcal{E}p} = \frac{\widetilde{p}''(\widehat{q})}{\widetilde{p}'(\widehat{q})} - \mathcal{E}p(e^{\widehat{q}}) \tag{A.2}$$

   Note that the condition is equivalent to the following chain of expressions being decreasing in $\widehat{q}$:

$$\frac{\widetilde{p}'(\widehat{q}) + \widetilde{p}''(\widehat{q})}{\widetilde{p}(\widehat{q}) + \widetilde{p}'(\widehat{q})} = \frac{1 + \mathcal{E}p(e^{\widehat{q}}) + \mathcal{E}^2 p(e^{\widehat{q}})}{1 + 1/\mathcal{E}p(e^{\widehat{q}})} = \mathcal{E}p(e^{\widehat{q}}) + \frac{\mathcal{E}p(e^{\widehat{q}})}{\mathcal{E}p(e^{\widehat{q}}) + 1}\mathcal{E}^2 p(e^{\widehat{q}}) \tag{A.3}$$

   which is decreasing in $\widehat{q}$ if and only if it is decreasing in $q = e^{\widehat{q}}$.

3. We write the elasticity of marginal revenue as

$$\begin{aligned}\frac{\partial \ln[p(q)(1+\mathcal{E}p(q))]}{\partial \ln q} &= \mathcal{E}p(q) + \frac{\partial \ln(1+\mathcal{E}p(q))}{\partial \ln q} \\ &= \mathcal{E}p(q) + \frac{\frac{\partial \mathcal{E}p(e^{\ln q})}{\partial \ln q}}{1+\mathcal{E}p(q)} \\ &= \mathcal{E}p(e^{\widehat{q}}) + \frac{\mathcal{E}p(e^{\widehat{q}}) \cdot \mathcal{E}^2 p(e^{\widehat{q}})}{1+\mathcal{E}p(e^{\widehat{q}})}\end{aligned}$$

Finally, the two statements with explicit dependency on α are proven by inspection. ∎

Proof of Proposition 2. See section A.2 "Monotone Skewness". ∎

Proof of Proposition 5. Continuing the second part of the proof in the main text: Suppose, $C \neq 0$. Then

$$\frac{d}{dq}\ln(p(q)) = \frac{\frac{A}{B+1}Bq^{B-1} - \frac{C}{q^2}}{\frac{A}{B+1}q^B + \frac{C}{q}},$$

and

$$\frac{d^2}{dq^2}\ln(p(q)) = \frac{\left(\frac{A}{B+1}B(B-1)q^{B-2} + \frac{2C}{q^3}\right)\left(\frac{A}{B+1}q^B + \frac{C}{q}\right) - \left(\frac{A}{B+1}Bq^{B-1} - \frac{C}{q^2}\right)^2}{\left(\frac{A}{B+1}q^B + \frac{C}{q}\right)^2}$$

must be < 0 by log-concavity. As $q \downarrow 0$, the dominating term is C/q^4, and thus $C < 0$ is necessary to ensure log-concavity. But if $C < 0$, then $\left(\frac{A}{B+1}q^B + \frac{C}{q}\right) < 0$ for small enough q. But this implies

$$\left(\frac{A}{B+1}B(B-1)q^B + \frac{3C}{q}\right) > \frac{A}{B+1}Bq^B > q$$

for all small q, which cannot hold since the LHS tends to $-\infty$ as $q \downarrow 0$. Thus, $C = 0$ is the only solution. ■

Proof of Proposition 4. Take a concave function f with $f' < 0$ and a random variable Z that is symmetrically distributed, and let $a > 1$ be a constant. We want to calculate the skewness of $X - Y$ where $X = f(Z)$, $Y = f(aZ)$, which is

$$skew[X-Y] = \frac{(X-Y)_r + (X-Y)_{1-r} - 2(X-Y)_{0.5}}{(X-Y)_{1-r} - (X-Y)_r} = \frac{(X-Y)_r + (X-Y)_{1-r}}{(X-Y)_{1-r} - (X-Y)_r}.$$

In the proposition, f is the first order condition, $Z = \epsilon\underline{\sigma}$ and $a = \overline{\sigma}/\underline{\sigma}$. This is the case in which t is a time of boom. We want to show that $skew[X-Y] > 0$. Because f is concave, we have $f(Z) - f(aZ)$ is increasing in Z, so

$$(X-Y)_r = f(Z_r) - f(aZ_r) \;<\; 0 \;<\; (f(Z_{1-r}) - f(aZ_{1-r})) = (X-Y)_{1-r}.$$

By concavity of f and since $(Z_r - aZ_r) = (aZ_{1-r} - Z_{1-r})$, the sum $(X-Y)_{1-r} + (X-Y)_r$ is positive, which we needed to show. Clearly, at time of recession, the skewness flips sign.

To compute the effect of lower market power during a boom, one computes the skewness for a less concave first order condition $\varphi \circ f$, where φ is convex. The proof rewrites the skewness in terms of the fraction $F = -(X-Y)_r/(X-Y)_{1-r}$ and shows that $skew[X-Y]$ decreases in F and that the presence of φ increases F. Details are left to the reader. ■

A.2 Monotone Skewness

A.2.1 Proof of Proposition 2

We develop a series of Lemmas to eventually prove that skewness is monotone in market power, α. We apply the notion of *relative convexity* due to Palmer (2003).

Definition A.1. *Consider two strictly monotone functions f, g. f is convex relative to a g if there exists a convex, strictly increasing transformation s such that $s \circ f = g$. We write $f \succ g$. Conversely, f is concave relative to g if there exists a strictly increasing, concave function s such that $s \circ f = g$. We write $f \prec g$.*

Note that the inverse of a strictly increasing, convex function exists and is strictly increasing and concave, and vice versa. Hence, $f \prec g$ if and only if $g \succ f$. The following is a criterion for relative concavity for twice differentiable functions.

Lemma A.4 (Relative Concavity of Twice Differentiable Functions)**.** *If $f, g \in \mathcal{C}^2$ then the following are equivalent:*

$$g \prec f \iff \frac{g''}{|g'|} < \frac{f''}{|f'|}.$$

Proof. See Palmer (2003). ∎

We now relate MMPE$_\alpha$ to this notion of relative concavity:

Lemma A.5 (MMPE$_\alpha$ implies relative concavity, $\phi_\alpha \prec \ln p^*$)**.** *As before, denote marginal profits in terms of log sales, $\widehat{q}$, by H, but add a subscript for market power. Write*

$$\begin{aligned} H_\alpha(\widehat{q}) &= \overbrace{\alpha \ln(p^*(e^{\widehat{q}}))}^{\text{log inverse demand}} + \overbrace{\ln\left(1/\mu(e^{\widehat{q}})\right)}^{\text{log inverse markup, } \equiv \phi_\alpha(\widehat{q})} + \overbrace{\ln \eta - (\eta - 1)\widehat{q}}^{\text{log marginal cost, } c'(e^{\widehat{q}})} \\ &= \alpha \ln(\widetilde{p}^*(\widehat{q})) + \phi_\alpha(\widehat{q}) + c'(e^{\widehat{q}}). \end{aligned}$$

Suppose, ϕ_α and $\ln p^*$ are both concave and decreasing. Then:

$$\phi_\alpha \prec \ln p^*(\exp(\cdot)) \iff MMPE_\alpha.$$

Proof of Lemma A.5. The proof does not depend on the value of α, only that at this value MMPE$_\alpha$ holds. Hence, suppress α subscripts and the $*$ superscript in the following equations. Note that $f \prec g$ iff $f \circ \exp \prec g \circ \exp$. Then:

$$\begin{aligned}
& \ln\left(1+\mathcal{E}p(e^{\widehat{q}})\right) \prec \ln\left(p(e^{\widehat{q}})\right) \\
\Leftrightarrow \quad & \ln(1+\mathcal{E}p(q)) \prec \ln(p(q)) \\
\Leftrightarrow \quad & \frac{\frac{\partial^2}{\partial q^2}\ln(1+\mathcal{E}p(q))}{-\frac{\partial}{\partial q}\ln(1+\mathcal{E}p(q))} < \frac{\frac{\partial^2}{\partial q^2}\ln(p(q))}{-\frac{\partial}{\partial q}\ln(p(q))} \\
\Leftrightarrow \quad & \frac{\frac{\partial^2}{\partial q^2}\ln\left(\frac{1}{\mu(q)}\right)}{-\frac{\partial}{\partial q}\ln\left(\frac{1}{\mu(q)}\right)} < \frac{\frac{\partial^2}{\partial q^2}\ln(p(q))}{-\frac{\partial}{\partial q}\ln(p(q))} \\
\Leftrightarrow \quad & \frac{\frac{\partial^2}{\partial q^2}\ln(\mu(q))}{\frac{\partial}{\partial q}\ln(\mu(q))} > \frac{\frac{\partial^2}{\partial q^2}\ln(p(q))}{\frac{\partial}{\partial q}\ln(p(q))}
\end{aligned}$$

Note that $\frac{f''(x)}{f'(x)} = \frac{\partial}{\partial x}\ln(f'(x))$ and with $f = \ln \circ \mu$, we have $f'(x) = \frac{\mu'(x)}{\mu(x)}$. Since the same goes with $f = \ln \circ p$, we obtain:

$$\begin{aligned}
\Leftrightarrow \quad & \frac{\partial}{\partial q}\ln\left(\frac{\mu'(q)}{\mu(q)}\right) > \frac{\partial}{\partial q}\ln\left(\frac{p'(q)}{p(q)}\right) \\
\Leftrightarrow \quad & \frac{\partial}{\partial q}\left(\ln\left(\frac{\mu'(q)}{\mu(q)}\right) + \ln q - \ln\left(\frac{p'(q)}{p(q)}\right) - \ln q\right) > 0 \\
\Leftrightarrow \quad & \frac{\partial}{\partial q}\left(\ln(\mathcal{E}\mu(q)) - \ln(\mathcal{E}p(q))\right) > 0 \\
\Leftrightarrow \quad & \ln\left(\frac{\mathcal{E}\mu(q)}{\mathcal{E}p(q)}\right) \text{ is increasing} \\
\Leftrightarrow \quad & \frac{\mathcal{E}\mu(q)}{\mathcal{E}p(q)} \text{ is increasing}
\end{aligned}$$

■

The next Lemma on relative concavity is new and applies directly to our setting.

Lemma A.6. *Let $f, g, h \in \mathcal{C}^2$, $X \subseteq \mathbb{R}$, $f : X \to \mathbb{R}$, $g : X \to D \subset \mathbb{R}$ and $h : D \to \mathbb{R}$. Let $a > 0$, $g \prec f \prec \mathrm{Id}$, and $f' < 0, g' < 0$. Then*

1. *multiply-and-transform induces concavity:*

$$(h \prec \mathrm{Id} \wedge h' > a) \implies af + h \circ g \prec f + g.$$

2. *multiply-and-transform induces convexity:*

$$(h \succ \mathrm{Id} \wedge h' < a) \implies af + h \circ g \succ f + g.$$

Proof. We only proof the first implication, because the proof of the second follows the same logic with flipped sign. Using the criterion of Lemma A.4, we need to verify that

$$\frac{af''(x) + h'(g(x))g''(x) + h''(g(x))g'(x)^2}{-[af'(x) + h'(g(x))g'(x)]} < \frac{f''(x) + g''(x)}{-[f'(x) + g'(x)]}.$$

Because $\frac{h''(g(x))g'(x)^2}{-[f'(x)+h'(g(x))h'(x)]} < 0$ by concavity of h, it suffices to check that

$$\frac{af''(x) + h'(g(x))g''(x)}{-[af'(x) + h'(g(x))g'(x)]} < \frac{f''(x) + g''(x)}{-[f'(x) + g'(x)]}.$$

Rearranging, we obtain

$$\begin{aligned}
&(af'' + h'g'')(f' + g') > (f'' + g'')(af' + h'g') \\
\iff\ & \cancel{af''f'} + af''g' + h'g''f' + \cancel{h'g''g'} > \cancel{f''af'} + f''g'h' + g''af' + \cancel{g''h'g'} \\
\iff\ & h'(g''f' - f''g') > a(g''f' - f''g')
\end{aligned}$$

But then note that $g''f' - f''g' > 0 \iff g \prec f$, hence the last line is equivalent to $h' > a$, which is true by supposition. ■

Lemma A.7. *Let $f, g, h \in C^2$ and $f \prec g \prec \mathrm{Id}$ with $f'' < g''$ as well as $f' < 0$, $g' < 0$. Let $v(x) := bx + c$ with $b < 0$, $c \in \mathbb{R}$. Then*

$$f + v \prec g + v$$

Proof. Evaluate the differentiability criterion directly:

$$\begin{aligned}
\frac{f''}{|f'+b|} = -\frac{f''}{f'+b} &< -\frac{g''}{g'+b} = \frac{g''}{|g'+b|} \\
\iff f''(g'+b) &> g''(f'+b) \\
\iff f''g' - g''f' &> (g'' - f'')b.
\end{aligned}$$

Then we have $f''g' - g''f' > 0$ by $f \prec g$ and $(g'' - f'') > 0$, too. Hence

$$\iff \frac{f''g - g''f}{g'' - f''} > b,$$

which is true as $b < 0$. ■

We are now ready to prove skewness monotonicity in α. Denote the marginal revenue in terms of $\widehat{q}$ by

$$\begin{aligned}
H_\alpha(\widehat{q}) &= \alpha \ln(\widetilde{p}^*(\widehat{q})) + \ln\left(1 + \alpha \frac{\widetilde{p}^{*\prime}(\widehat{q}))}{\widetilde{p}^*(\widehat{q}))}\right) + \ln \eta - (\eta - 1)\widehat{q} \\
&= \alpha \ln(\widetilde{p}^*(\widehat{q})) + \phi_\alpha(\widehat{q}) + \ln \eta - (\eta - 1)\widehat{q}.
\end{aligned}$$

Note that $\phi_\alpha < 0$ because $\frac{\widetilde{p}^{*\prime}(\widehat{q})}{p^*(\widehat{q})} = \mathcal{E}p(e^{\widehat{q}}) \in (-1, 0)$. Consider two degrees of market power $\alpha_1 > \alpha_0$ and set the subscripts of H and ϕ to 0 and 1, respectively. First, assume MMPE$_{\alpha_0}$, so the relative concavity condition holds for the low market power firm. Then, we have

$$\begin{aligned} \phi_1 &= h(\phi_0(x)) \quad \text{with} \\ h(y) &:= \ln\left(1 + \frac{\alpha_1}{\alpha_0} e^y\right), \quad \forall\, y \in (-\infty, 0). \end{aligned}$$

The function h is strictly concave and increasing with $h'(y) > \frac{\alpha_1}{\alpha_0}$. This follows from

$$h'(y) = \frac{e^y}{\frac{\alpha_0}{\alpha_1} - 1 + e^y} > 0,$$
$$h''(y) = \frac{\alpha_0/\alpha_1 - 1}{\left(\frac{\alpha_0}{\alpha_1} - 1 + e^y\right)^2} < 0,$$

and plugging in. We use Lemma A.6 part 1 with the h we just found as well as $g = \phi_0$, $f = \ln \circ\, \widetilde{p}^*$ and $a = \frac{\alpha_1}{\alpha_0}$. Therefore,

$$\frac{\alpha_1}{\alpha_0} \cdot \alpha_0(\ln \circ\, p^*) + h \circ \phi_0 \prec \alpha_0(\ln \circ\, p^*) + \phi_0.$$

That is, the high market power (α_1) firm features a more concave LHS of its first order condition. Speaking loosely, its marginal revenues are more concave.

Second, we can follow the same steps under the assumption of MMPE$_{\alpha_1}$. Now, h satisfies the conditions of Lemma A.6 part 2. We arrive at the conclusion that, again, the high-market power firm has a more concave marginal revenue in log-log space.

We finally need to add the linear terms coming from the marginal cost back in. Here, note that second derivative of the left expression is necessarily smaller than that of the right. Thus, Lemma A.7 is applicable with $v(x) = -(\eta - 1)x + \ln \eta$ and therefore

$$\begin{aligned} H_1(\widehat{q}) &= \alpha_1(\ln \circ\, p^*)(\widehat{q}) + \phi_1(\widehat{q}) + \ln \eta - (\eta - 1)\widehat{q} \\ &\prec \alpha_0(\ln \circ\, p^*)(\widehat{q}) + \phi_0(\widehat{q}) + \ln \eta - (\eta - 1)\widehat{q} = H_0(\widehat{q}). \end{aligned}$$

Under MMPE$_\alpha$ we thus conclude that concavity of H_α increases in market power. Proposition 2 asserts that this translates into more left-skewed growth rates for higher market power.

Proof of Proposition 2 (final bits). We need to show that the inverse, H_1^{-1}, of the high-market power firm is more concave than H_0^{-1}. The relative ordering of skewness then follows from Lemma 2 (2). Note that by supposition, $H_0 = \phi \circ H_1$ for a strictly increasing, convex function ϕ. We collect the first and second derivatives:

$$H_0'(\widehat{q}) = \phi'(H_1(\widehat{q}))H_1'(\widehat{q})$$
$$H_0''(\widehat{q}) = \phi''(H_1(\widehat{q}))H_1'(\widehat{q})^2 + \phi'(H_1(\widehat{q}))H_1''(\widehat{q}).$$

Note that for any invertible, twice differentiable function f holds $(f^{-1})'(y) = 1/f'(x)$ and $(f^{-1})''(y) = -f''(x)/f'(x)^3$ for $y = f(x)$. We apply this insight with $\epsilon = H_0(\widehat{q}) = H_1(\widehat{q})$:

$$\begin{aligned}
(H_0^{-1})''(\epsilon) &= -\frac{\phi''(H_1(\widehat{q}))H_1'(\widehat{q})^2 + \phi'(H_1(\widehat{q}))H_1''(\widehat{q})}{(\phi'(H_1(\widehat{q}))H_1'(\widehat{q}))^3} \\
&= -\frac{\phi''(\epsilon)}{\phi'(\epsilon)^3 H_1'(\widehat{q})} - \frac{H_1''(\widehat{q})}{\phi'(\epsilon)^2 H_0'(\widehat{q})^3} \\
(H_0^{-1})'(\epsilon) &= \frac{1}{\phi'(H_1(H_0^{-1}(\widehat{q})))H_1'(H_0^{-1}(\epsilon))} \\
&= \frac{1}{\phi'(\epsilon)H_1'(\widehat{q})}.
\end{aligned}$$

Therefore,

$$\frac{(H_0^{-1})''(\epsilon)}{|(H_0^{-1})'(\epsilon)|} = -\frac{(H_0^{-1})''(\epsilon)}{(H_0^{-1})'(\epsilon)} = \phi''(\epsilon) - \frac{(H_1^{-1})''(\epsilon)}{(H_1^{-1})'(\epsilon)} = \phi''(\epsilon) + \frac{(H_1^{-1})''(\epsilon)}{|(H_1^{-1})'(\epsilon)|}$$

and since $\phi'' > 0$, we have $H_1^{-1} \prec H_0^{-1}$. Therefore, H_1^{-1} is more concave and there exists a concave, strictly increasing transformation mapping H_0^{-1} to H_1^{-1}. ■

A.2.2 Proof of Proposition 3, Implication (10)

For this section, note that MSLD′, MMPE$_\alpha$ and relative concavity are strict inequalities/relations. As argued in Lemma A.5, MMPE$_\alpha$ is equivalent to the technical condition

$$\frac{1}{\alpha}\ln\left(1 + \alpha\mathcal{E}p^*(\cdot)\right) \prec \ln\left(p^*(\cdot)\right). \tag{A.4}$$

Because concavity of the LHS is increasing in α (this was shown in Lemma A.5), one only needs the relation to hold strictly at the limit $\alpha \to 0$, i.e.

$$\mathcal{E}p^*(\cdot) \prec \ln\left(p^*(\cdot)\right), \tag{A.5}$$

which states that the elasticity of the inverse demand function is more concave than its logarithm. We have thus proven:

Lemma A.8. *For any $\alpha \in (0,1]$: $\mathcal{E}p^*(\cdot) \prec \ln\left(p^*(\cdot)\right) \implies \ln\left(1 + \alpha\mathcal{E}p^*(\cdot)\right) \prec \ln\left(p^*(\cdot)\right)$ (that is, MMPE$_\alpha$).*

Then note the following Lemma:

Lemma A.9. *$\mathcal{E}p^*(\cdot) \prec \ln(p^*(\cdot)) \iff \mathcal{E}^2 p^*$ is strictly increasing.*

Proof. Following the steps similar to the proof of Lemma A.5, we obtain

$$\begin{aligned}
& \mathcal{E}p^*(\cdot) \prec \ln p^*(\cdot) \\
\iff & -\frac{(\mathcal{E}p^*)''}{(\mathcal{E}p^*)'} < -\frac{(\ln p^*)''}{(\ln p^*)'} \\
\iff & -\frac{\partial}{\partial q}\ln[-(\mathcal{E}p^*)'] < -\frac{\partial}{\partial q}\ln[-(\ln p^*)'] \\
\iff & \frac{\partial}{\partial q}\ln[-(\mathcal{E}p^*)'] > \frac{\partial}{\partial q}\ln[-(\ln p^*)'] \\
\iff & \frac{\partial}{\partial q}(\ln[-(\mathcal{E}p^*)'] - \ln[-(\ln p^*)']) > 0 \\
\iff & \frac{\partial}{\partial q}\left(\ln\frac{-(\mathcal{E}p^*)'}{-(\ln p^*(q))'}\right) > 0 \\
\iff & \ln\frac{(\mathcal{E}p^*)'}{(\ln p^*(q))'} \quad \text{is increasing} \\
\iff & \frac{(\mathcal{E}p^*)'}{(p^*)'(q)/p^*(q)} \quad \text{is increasing} \\
\iff & \frac{(\mathcal{E}p^*)'}{q(p^*)'(q)/p^*(q)}q \quad \text{is increasing} \\
\iff & \frac{(\mathcal{E}p^*)'}{\mathcal{E}p^*}q \quad \text{is increasing} \\
\iff & \mathcal{E}^2 p^* \quad \text{is increasing}
\end{aligned}$$

■

We now tie the proof together. We follow the approach of Mrázová and Neary (2017), who express local properties of (inverse) demand functions in terms of three unit-free statistics:

$$\varepsilon(x) \equiv \mathcal{E}p^*(x) = \frac{xp^{*\prime}(x)}{p(x)}, \quad \rho(x) \equiv \frac{xp^{*\prime\prime}(x)}{p^{*\prime}(x)}, \quad \chi(x) \equiv \frac{xp^{*\prime\prime\prime}(x)}{p^{*\prime\prime}(x)}.$$

The second quantity, ρ, measures the curvature of demand, and must be strictly larger than -2 by log-concavity of p^*. The third, χ, is also known as the *coefficient of relative temperance* of Eeckhoudt et al. (1995). We then have the following lemma, which implies (10).

Lemma A.10. *(1) If MSLD′ holds strictly for all $\alpha \in (0,1]$, then*

$$(\rho\chi - \rho^2 + \rho\varepsilon) \geq 0.$$

(2) An increasing superelasticity, $\mathcal{E}^2 p^*$, implies that MMPE$_\alpha$ holds for all $\alpha \in (0,1]$. The former is equivalent to

$$(\rho\chi - \rho^2 + \rho\varepsilon) + (\varepsilon^2 - \varepsilon - 2) > 0.$$

(3) An increasing superelasticity, $\mathcal{E}^2 p^*$, implies MSLD′ for any $\alpha \in (0,1]$.

Proof. Part 1: First, note that marginal revenue is now given by $\mathfrak{mr}(q) = p^*(q)^\alpha(1 + \alpha\mathcal{E}p^*(q))$. We calculate the elasticity, which is

$$\alpha\frac{p^{*\prime}}{p^*}q + \alpha\frac{\frac{\partial}{\partial \ln q}\mathcal{E}p^*}{1+\alpha\mathcal{E}p^*} = \alpha\frac{p^{*\prime}}{p^*}q + \alpha\frac{1 + \frac{p^{*\prime\prime}}{p^{*\prime}}q - \frac{p^{*\prime}}{p^*}q}{1+\alpha\frac{p^{*\prime}}{p^*}q} \tag{A.6}$$

where we have used the identities

$$\frac{\partial}{\partial \ln q}\mathcal{E}p^* = \frac{\frac{\partial}{\partial q}\mathcal{E}p^*}{\mathcal{E}p^*}q \quad \text{and} \quad \frac{\partial}{\partial q}\mathcal{E}p^* = \frac{p^{*\prime}}{p^{*\prime}} + \frac{p^{*\prime\prime}}{p^{*\prime}} - \left(\frac{p^{*\prime}}{p^*}\right)^2 q. \tag{A.7}$$

MSLD′ states that the first derivative of this expression must be positive. We take $\frac{\partial}{\partial q}$. Doing so, we apply the definitions of ϵ, ρ, χ and use

$$\frac{\partial}{\partial q}\varepsilon = \frac{\partial}{\partial q}\frac{p^{*\prime}}{p^*}q = \frac{1}{q}(\rho\varepsilon + \varepsilon - \varepsilon^2), \quad \frac{\partial}{\partial q}\rho = \frac{\partial}{\partial q}\frac{p^{*\prime\prime}}{p^{*\prime}}q = \frac{1}{q}(\rho\chi + \rho - \rho^2). \tag{A.8}$$

Doing so yields, for all $\alpha \in (0,1]$

$$\begin{aligned}
&\alpha\frac{1}{q}(\rho\varepsilon + \varepsilon - \varepsilon^2) + \\
&\alpha\frac{(1+\alpha\varepsilon)\Big[-\frac{1}{q}(\rho\varepsilon+\varepsilon-\varepsilon^2) + \frac{1}{q}(\rho\chi+\rho-\rho^2)\Big] - \alpha\frac{1}{q}(\rho\varepsilon+\varepsilon-\varepsilon^2)(1+\rho-\varepsilon)}{(1+\alpha\varepsilon)^2} > 0 \\
\iff\ &(\rho\varepsilon+\varepsilon-\varepsilon^2) + \frac{(1+\alpha\varepsilon)\Big[-(\rho\varepsilon+\varepsilon-\varepsilon^2) + (\rho\chi+\rho-\rho^2)\Big] - \alpha\varepsilon(1+\rho-\varepsilon)^2}{(1+\alpha\varepsilon)^2} > 0 \\
\iff\ &(\rho\varepsilon+\varepsilon-\varepsilon^2) + \frac{\Big[-(\rho\varepsilon+\varepsilon-\varepsilon^2) + (\rho\chi+\rho-\rho^2)\Big]}{(1+\alpha\varepsilon)} + \frac{-\alpha\varepsilon(1+\rho-\varepsilon)^2}{(1+\alpha\varepsilon)^2} > 0
\end{aligned} \tag{A.9}$$

which implies, as we take $\alpha \to 0$

$$(\rho\chi + \rho - \rho^2) \geq 0 \tag{A.10}$$

Part 2: By Lemmas A.8 and A.9, we only need to check the second part (equivalence of eq. (9) to the inequality). To this end, note that for any thrice differentiable function

f, we have

$$
\begin{aligned}
(\mathcal{E}f)'(x) &= x\frac{f''}{f} + \frac{f'}{f} - x\left(\frac{f'}{f}\right)^2 \\
&= (\rho_f(x) + 1 - \varepsilon_f(x))\varepsilon_f(x)/x \\
(\mathcal{E}f)''(x) &= \frac{f''}{f} + \frac{f'''f - f''f'}{f^2}x + \frac{f''f - (f')^2}{f^2} - \left(\left(\frac{f'}{f}\right)^2 + 2x\frac{f'}{f}\frac{f''f - (f')^2}{f^2}\right) \\
&= \underbrace{\rho_f(x)\varepsilon_f(x)/x^2}_{I} + \underbrace{\chi_f(x)\rho_f(x)\varepsilon_f(x) - (\rho_f(x)\varepsilon_f(x)^2)/x^2}_{II} \\
&\quad + \underbrace{(\rho_f(x)\varepsilon_f(x) - \varepsilon_f(x)^2)/x^2}_{III} - \underbrace{(\varepsilon_f(x)^2 + 2\varepsilon_f(x)^2(\rho_f(x) - \varepsilon_f(x)))}_{IV} \\
&= \frac{\varepsilon_f(x)}{x^2}(\rho_f(x)(\chi_f(x) + \varepsilon_f(x) + 2) - 2\varepsilon_f(x)(1 + \varepsilon_f(x)))
\end{aligned}
$$

Moreover,

$$
\frac{\frac{\partial^2 \ln f(x)}{\partial x^2}}{\frac{\partial \ln f(x)}{\partial x}} = \frac{1}{\chi_f(x)}(\rho_f(x) - \varepsilon_f(x)).
$$

Therefore, the condition can be written as (note that $p^{*\prime} < 0$, hence the inequality is flipped and suppressing subscripts and arguments for brevity)

$$
\begin{aligned}
\frac{(\ln p^*)''}{(\ln p^*)'} &< \frac{\varepsilon''}{\varepsilon'} \\
\iff \rho - \varepsilon &< \frac{\rho(\chi + \varepsilon + 1) - 2(\varepsilon + 1)}{\rho + 1 + \varepsilon} \\
\iff \rho\chi - \rho^2 + \rho\varepsilon + \varepsilon^2 - \varepsilon - 2 &\geq 0
\end{aligned}
$$

Part 3: Part (2) tells us that the inequality (A.9), which is equivalent to MSLD′ for a given α, is implied if MMPE$_\alpha$ holds for all α. This follows from the fact that $\varepsilon^2 - \varepsilon - 2 < 0$. Additionally, in eq. (A.9), note that part $*$ is positive, so the inequality is implied if

$$
(\rho\varepsilon + \varepsilon - \varepsilon^2) + \frac{\left[-(\rho\varepsilon + \varepsilon - \varepsilon^2) + (\rho\chi + \rho - \rho^2)\right]}{(1 + \alpha\varepsilon)} > 0.
$$

This, in turn, holds true if $-(\rho\varepsilon + \varepsilon - \varepsilon^2) + (\rho\chi + \rho - \rho^2) > 0$ which holds iff $2/\varepsilon \leq \rho$. But this has to be the case, since $2/\varepsilon < -2 < \rho$. Therefore: ($\mathcal{E}^2 p$ increasing) $\implies$ (MMPE$_\alpha$ $\forall\alpha \in (0,1]$) and (MMPE$_\alpha$ $\forall\alpha \in (0,1]$) $\implies$ (MSLD′ $\forall\alpha \in (0,1]$). ■

A.2.3 Proof of Proposition 3, Implication (11)

Let inverse demand be $p : \mathbb{R}_+ \to \mathbb{R}_+$, twice continuously differentiable with $p'(q) < 0$. Costs are isoelastic and convex: $\bar{c}\, q^\eta$. Define the *(negative) inverse-demand elasticity* $\varepsilon(q) \equiv \frac{q\, p'(q)}{p(q)}$ (< 0), the *curvature* $\rho(q) \equiv \frac{q\, p''(q)}{p'(q)}$, and the *superelasticity* $\psi(q) \equiv \frac{q}{\varepsilon(q)}\, \varepsilon'(q)$. (Equivalently, $\psi(q) = \dfrac{d \ln |\varepsilon(q)|}{d \ln q}$.) A useful identity obtained by direct differentiation is

$$\psi(q) = 1 - \varepsilon(q) + \rho(q). \tag{A.11}$$

Let $q^\star(\bar{c})$ solve the monopolist's problem $\max_{q \geq 0}\ p(q)\, q - \bar{c}\, q^\eta$ and let $p^\star(\bar{c}) = p\big(q^\star(\bar{c})\big)$. Note that $q^\star(\bar{c})$ is strictly decreasing in $\bar{c}$ (so $\dfrac{dq^\star}{d\bar{c}} < 0$) (MONO). We want to prove the claim: Suppose,

- (MSLD) $\varepsilon'(q) < 0$ (Marshall's Second Law in this notation) (hence $\psi(q) \geq 0$) and
- (PT↑) the pass-through w.r.t. the cost shifter $\bar{c}$, $\tau(\bar{c}) \equiv \frac{dp^\star(\bar{c})}{d\bar{c}}$, is strictly increasing in $\bar{c}$.

Then $\psi'(q) > 0$ for all relevant q.

Proof: (1) Optimality and reduced-form $\tau(q)$: The first-order condition is

$$p(q) + q\, p'(q) - \eta\, \bar{c}\, q^{\eta-1} = 0. \tag{A.12}$$

By the Implicit Function Theorem and $p^\star(\bar{c}) = p(q^\star(\bar{c}))$,

$$\tau(\bar{c}) = p'(q)\, \frac{dq^\star}{d\bar{c}} = \frac{\eta\, p'(q)\, q^{\eta-1}}{2p'(q) + q\, p''(q) - \eta(\eta-1)\bar{c}\, q^{\eta-2}}, \quad q = q^\star(\bar{c}).$$

Use (A.12) to eliminate $\bar{c}$ and substitute $q\, p'(q) = \varepsilon(q)\, p(q)$ and $q\, p''(q) = \rho(q)\, p'(q)$ to express τ purely as a function of q (and primitives):

$$\tau(q) = \frac{\eta\, \varepsilon(q)\, q^{\eta-1}}{\varepsilon(q)\big(\varepsilon(q) + \psi(q) + 2 - \eta\big) - (\eta - 1)}. \tag{A.13}$$

(Identity (A.11) was used to eliminate ρ.)

(2) From (PT ↑) *and (MONO) to $\tau'(q) < 0$:* Since $\dfrac{dq^\star}{d\bar{c}} < 0$, the assumption $\dfrac{d\tau}{d\bar{c}} > 0$ implies, by the chain rule,

$$\frac{d\tau(q)}{dq} < 0 \qquad \text{along } q = q^\star(\bar{c}). \tag{A.14}$$

(3) Differentiate $\tau(q)$ and sign it: Write (A.13) as $\tau(q) = \dfrac{N(q)}{D(q)}$ with

$$N(q) = \eta\, \varepsilon(q)\, q^{\eta-1}, \qquad D(q) = \varepsilon(q)\big(\varepsilon(q) + \psi(q) + 2 - \eta\big) - (\eta - 1).$$

Using $\varepsilon'(q) = \dfrac{\psi(q)\, \varepsilon(q)}{q}$,

$$N'(q) = \eta\, q^{\eta-2}\, \varepsilon(q)\big(\psi(q) + \eta - 1\big),$$

$$D'(q) = \varepsilon'(q)\big(2\varepsilon(q) + \psi(q) + 2 - \eta\big) + \varepsilon(q)\psi'(q) = \varepsilon(q)\left[\frac{\psi(q)}{q}\big(2\varepsilon(q) + \psi(q) + 2 - \eta\big) + \psi'(q)\right].$$

Hence

$$\frac{d\tau(q)}{dq} = \frac{N'(q)D(q) - N(q)D'(q)}{D(q)^2} = \frac{\eta\, q^{\eta-2}\, \varepsilon(q)}{D(q)^2}$$
$$\times \Big\{ \big(\psi(q) + \eta - 1\big)D(q) - \varepsilon(q)\Big[\psi(q)\big(2\varepsilon(q) + \psi(q) + 2 - \eta\big) + q\, \psi'(q)\Big]\Big\}.$$

Because $D(q)^2 > 0$ and $\varepsilon(q) < 0$, the inequality (A.14) is equivalent to

$$\big(\psi(q) + \eta - 1\big)D(q) - \varepsilon(q)\Big[\psi(q)\big(2\varepsilon(q) + \psi(q) + 2 - \eta\big) + q\, \psi'(q)\Big] \;>\; 0. \tag{A.15}$$

Substitute $D(q) = \varepsilon(q)\big(\varepsilon(q) + \psi(q) + 2 - \eta\big) - (\eta - 1)$ and simplify; after canceling like terms one obtains the linear inequality in $q\, \psi'(q)$:

$$q\, \psi'(q) \;>\; \frac{\eta - 1}{-\varepsilon(q)}\big(\psi(q) + \eta - 1\big) \;+\; (\eta - 1)\big(1 - \varepsilon(q) + \psi(q) - (\eta - 1)\big) \;+\; \psi(q)\big(-\varepsilon(q)\big) \tag{A.16}$$

$$= \underbrace{\frac{\eta - 1}{-\varepsilon(q)}\psi(q)}_{\geq 0} + (\eta - 1)^2 \underbrace{\left(\frac{1}{-\varepsilon(q)} - 1\right)}_{\geq 0} + (\eta - 1)\underbrace{\big(1 - \varepsilon(q)\psi(q)\big)}_{\geq 0} + \underbrace{\psi(q)(-\varepsilon(q))}_{>0}. \tag{A.17}$$

The bracketed terms on the right-hand side are nonnegative because $\eta \geq 1$, $\varepsilon(q) < 0$, and by (MSLD) we have $\psi(q) = \dfrac{q}{\varepsilon(q)}\varepsilon'(q) \geq 0$. Therefore the right-hand side of (A.16) is strictly positive, which yields

$$q\, \psi'(q) > 0 \quad \Longrightarrow \quad \psi'(q) > 0.$$

Conclusion: Under (MSLD), (Mono), and (PT $\uparrow$), the superelasticity $\psi(q)$ is strictly increasing in q. $\square$

B Supply Side Skewness

Up until now, we have focused on (negative) skew driven by the shape of the demand curve, neglecting contributions of the cost function to skewness. We discuss two types of cost functions which can contribute to skewed growth rates. Either approach rests on the assumption that adjustment of some status quo (or the 'current state') is costly in one direction and cheap in the other. This leads to a cost function with a kink located at previous-period output, which immediately implies local log-convexity of the cost function and locally left-skewed values of $\widehat{q}$.

We start from a price taking firm's static profit maximization problem (suppressing price taker subscripts),

$$\max_q q\,[\bar{p} - c(q)], \qquad c(q) = e^{\epsilon}\Psi(q).$$

Here, $\bar{p}$ is the given output price, $\Psi(q)$ is the baseline cost function, and the exponential term e^{ϵ} is a multiplicative cost shifter. The first-order condition equates marginal revenue and marginal cost. Differentiating, we obtain

$$0 = \bar{p} - e^{\epsilon}\big(\Psi(q) + q\Psi'(q)\big) \equiv \bar{p} - e^{\epsilon}\mathfrak{mc}(q),$$

where $\mathfrak{mc}(q)$ is the slope of total cost. Taking logs yields $\ln \mathfrak{mc}(q) = \log \bar{p} - \epsilon$. This condition allows us to study how equilibrium output responds to the cost shifter. Differentiating the FOC, one can show that

$$\frac{d\widehat{q}}{d\epsilon} = -\frac{1}{\mathcal{E}\mathfrak{mc}(q)}, \qquad \frac{d^2\widehat{q}}{d\epsilon^2} = \frac{1}{(\mathcal{E}\mathfrak{mc}(q))^2}\,(\mathcal{E}\mathfrak{mc})'(q)\,\frac{dq}{d\epsilon}.$$

Since $dq/d\epsilon < 0$ (higher costs reduce output), concavity of $\widehat{q}$ in ϵ requires that $(\mathcal{E}\mathfrak{mc})'(q) \geq 0$, i.e. that the elasticity of marginal cost is increasing in quantity. Put differently: if $\mathfrak{mc}(q)$ is log-convex, then $\widehat{q}$ is locally concave in the cost shifter ϵ. Now consider how different frictions shape $\Psi(q)$, and thereby $\mathfrak{mc}(q)$.

Capacity adjustments Work on asymmetric capital adjustment costs goes back to investment Q-theory. More recent seminal work which finds empirical evidence for convex adjustment costs includes Cooper and Haltiwanger (2006). Meanwhile, Christiano et al. (2005) delivered an important milestone for the inclusion of such cost into modern quantitative macroeconomic models. To build a simple model of this class, suppose there is a baseline cost $\psi_0(q)$, but producing beyond installed capacity $\bar{q}$ requires increasingly costly effort. We assume $\bar{q} = q_0$, so capacity is set to the steady-state level of output. A reduced-form way to capture this structure is

$$\Psi_{\text{cap}}(q) = \psi_0(q) + \chi\,[q - \bar{q}]_+^{\nu}, \qquad \nu > 1,\ \chi > 0.$$

Once output exceeds capacity, the derivative $\mathfrak{mc}'(q)$ rises steeply (jumps, in fact), and thus the elasticity $\mathcal{E}\mathfrak{mc}(q)$ is increasing. This satisfies the log-convexity condition, ensuring that $\ln q$ is concave in ϵ locally around $\ln q_0$. Intuitively, expansions beyond capacity are disproportionately costly, so output falls quickly in response to cost shocks but rises only sluggishly.

Customer acquisition The approach of embedding a customer base into the firm problem was pioneered by Gourio and Rudanko (2014). Roldan-Blanco and Gilbukh (2021) formalize a customer base through a matching model between customers and firms. Through this approach, they capture rich business cycle dynamics. More recently, Ignaszak and Sedlácek (2025) use customer bases as a device to analyze the tension between productivity and profitability, and explain how the latter may become uncoupled from the former.

Suppose, sales depend on a pre-existing base b plus new customers from acquisition effort a, with $q \leq b + H(a)$. Acquisition costs are convex, $S(a)$, while customer recruitment is concave, $H(a)$. Assume that the firm has built a customer base for steady state output in the past, hence $b = q_0$. Embedding this into the per-unit cost gives

$$\Psi_{\text{sell}}(q) = \psi_0(q) + \frac{S(a^*(q))}{q},$$

where $a^*(q)$ is the optimal acquisition choice. By the envelope theorem, the shadow value of an extra unit of demand, $\lambda(q)$, enters marginal cost, and rises with q because $S'' > 0$ and $H'' < 0$. Thus, as output expands beyond the base b, $\mathfrak{mc}(q)$ grows increasingly steep, again making $\mathcal{E}\mathfrak{mc}(q)$ increasing. This ensures local concavity of $\log q$ in ϵ. The economic interpretation is that when demand is weak, the firm lets its base erode (no acquisition), whereas when demand is strong, acquiring new customers is costly and slow, so output cannot expand proportionally.

In both cases, the critical feature is that the marginal cost function becomes increasingly steep as output rises. This makes the mapping from cost shocks ϵ to equilibrium quantities concave in logs, and thereby generates the negative skewness in firm-level growth rates. Existence of log-convexities in firms' cost functions can explain some pattern in Figure 3 (b). For a small size cutoff quantile, the plotted regression coefficients taper off and become constant at about 2.2. This suggests that growth rate skewness is still procyclical, even if market power, α, becomes small. Part of the reason may be that shocks are inherently skewed, but a more structural explanation is existence of log-convex cost functions even for price takers.

Yet, cost functions are unable to generate size dependence of skewness. To this observation, a demand-side oriented explanation caters nicely. Second, the approaches discussed below—capacity adjustments and customer acquisition cost—each only guarantee local skewness: growth rates relative to the steady state are skewed locally around long run output, q_0. That means, if the shock is not mean zero but, say, has a positive support such that *every* firm i in the cross-section adjusts to some $q_i < q_0$, then the negative skewness in growth rates is no longer guaranteed. In contrast, invoking ISID as a demand curve property immediately guarantees global left-skewness of growth rates, $\ln q/q_0$.

C Data Preparation

We start from the entire Compustat database at the quarterly frequency. After the download, the data has 1,928,055 quarter-firm observations and covers the period 1961Q1 - 2022Q3. The date is defined using the item `datacqtr`, not the fiscal quarter. The unique firm identifier is `gvkey`. We drop firms that are not incorporated (variable `fic`) or headquartered (`loc`) in the United States. We remove any companies with an SIC code above 9000, which includes non-operating establishments. We drop any observations with negative nominal sales (`saleq`) and remove all duplicates of the firm-quarter identifier (`gvkey` and `datacqtr`).

Nominal sales are deflated with the GDP price deflator (USAGDPDEFQISMEI on FRED) to obtain real sales $s_{i,t}$ of firm i in quarter t. If a firm shows a missing value of real sales in a period that is surrounded by non-missing sales observations, we fill the missing value via linear imputation. If two missing values are adjacent, no imputation is performed. Real sales growth is the year-on-year growth rate of quarterly real sales: $g_{i,t} = \ln(s_{i,t}) - \ln(s_{i,t-4})$.[22] Aggregate real sales growth is

$$g_t = \frac{\sum_i s_{i,t-4} g_{i,t}}{\sum_i s_{i,t-4}} \tag{A.18}$$

This way of computing aggregate sales ensures that only growth rates of firms are considered that experience non-missing sales in both quarters. It is not biased by the entry of new firms or exit of dying firms.

We construct several variables for firm characteristics, following Ottonello and Winberry (2020), Crouzet and Mehrotra (2020), and Cloyne et al. (2023). Leverage is the ratio of total debt (sum of items `dlcq` and `dlttq`) to total assets (`atq`). Net leverage is the ratio of total debt minus net current assets (`actq`) to total assets. Liquid assets ("liquidity ") is the ratio of cash and short-term investments (`cheq`) to total assets.

This yields the *full sample*. The full sample of non-missing sales growth rate observations has 1,146,214 firm-quarter observations and covers the period 1962Q1 – 2022Q3. Additional steps yield the *cleaned sample*, which aims to remove sales growth rate outliers:

1. Remove any firm-quarter observations with negative current assets (`actq`), total assets (`atq`), or liquid assets.
2. Remove the observation if net current assets relative to total assets falls outside of $[-10, 10]$.
3. Remove observations with leverage above 10 or below zero.

[22] In unreported results, we confirm that all main results are robust to using growth rates defined as $g_{i,t} = \frac{s_{i,t} - s_{i,t-4}}{s_{i,t-4}}$ instead of using log differences.

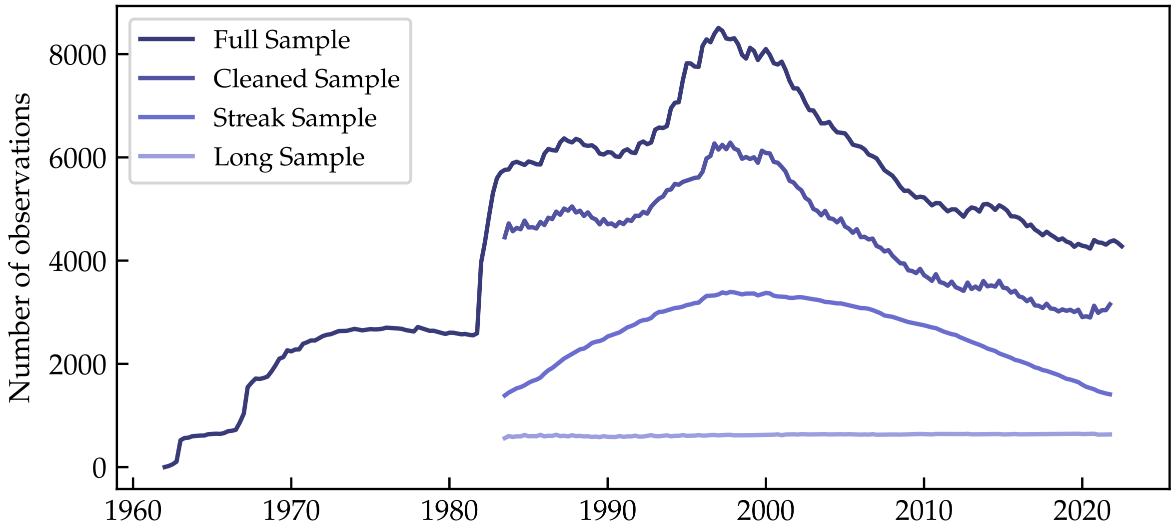
4. Remove any observations with percentage sales growth rate $(s_{i,t} - s_{i,t-4}) / s_{i,t-4}$ below -1 (We do not apply this filter to log growth rates $g_{i,t}$).
5. Remove any observations where the ratio of sales to total assets is in the top 0.1% of observations. This is to clean any sales growth observations that may be due to mistakes in the data.
6. To further account for growth rate outliers, we remove the top and bottom 1% of growth rate observations in each quarter.
7. Since data on acquisitions is only available from 1983Q3 onwards, we remove all earlier observations.
8. We remove any observations after 2021Q4 since Compustat may face a reporting lag such that 2022 values may have been disproportionately missing at the time of data collection.

The resulting sample covers the period from 1983Q3 until 2021Q4 and has 699,440 firm-quarter observations. We merge this sample with information on stock prices (variable `PCLOSE`) and the first date of trading (`BEGDAT`) from CRSP using the `PERMCO-GVKEY` linking table. We also merge the sample with information on dates of incorporation from Worldscope Fundamentals (variable `INCORPDAT`) using the CUSIP identifier. This allows us to construct firm age as the minimum across 1) the date of the first observation in Compustat, 2) the first date of trading from CRSP, and 3) the date of incorporation as indicated in Worldscope Fundamentals. This approach makes use of the well-populated and accurate information in Worldscope while avoiding negative firm ages, as discussed in Cloyne et al. (2023). To obtain analyst forecast errors, we merge with I/B/E/S based on the `PERMNO-GVKEY` link.

To be able to work with within-firm time series variation in some parts of our analysis, we perform a final step of cleaning to yield the *streak sample*. As in Ottonello and Winberry (2020), we only keep growth rate streaks of 40 consecutive quarters, and remove all other observations. This yields a sample of 5,332 unique growth streaks for 5,061 unique companies. 271 companies have two sales growth rate streaks in the data. The sample period is 1983Q3 until 2021Q4 and there are 396,722 firm-quarter observations. To approximate a balanced panel, the *long* sample only consider firms within the clean sample that have observations before 1985Q1 and after 2021Q1. This leaves 661 unique firms.

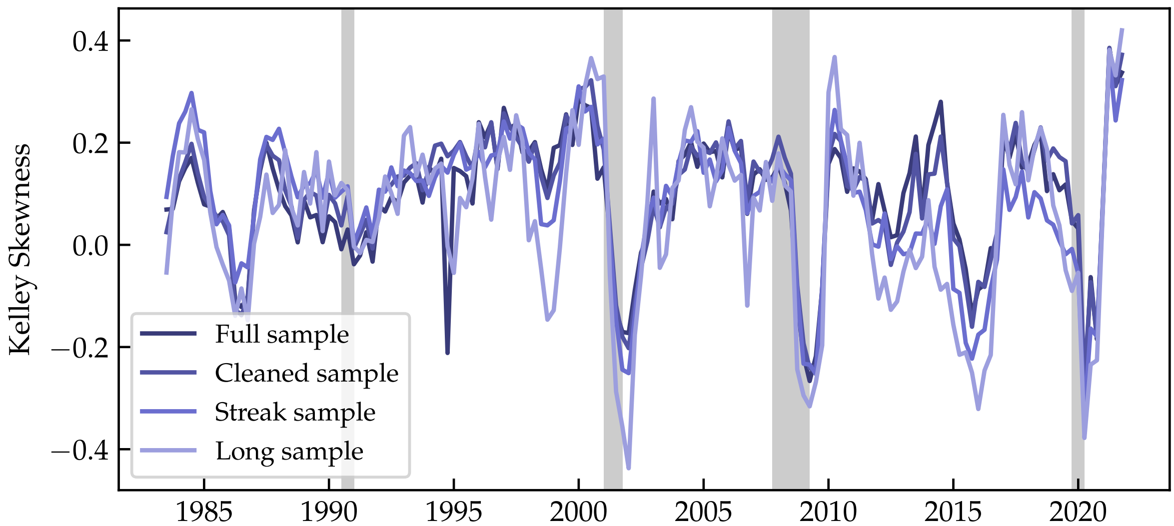
Figure A.1 shows the number of firm-level observations per quarter for the different samples. Despite differences in the number of observations, the skewness pattern across samples is very similar, see Figure A.2. The business cycle pattern of cross-sectional skewness is especially similar between the *cleaned* and the *streak* sample, which are used in the main analysis.

Figure A.1: Number of sales growth observations per quarter



Note: The full sample of growth rates covers 1962Q1-2022Q3. The other samples cover 1983Q3-2021Q4.

Figure A.2: Kelley skewness for different samples



Note: Skewness is computed using 90% Kelley skewness. The sample period is 1983Q3-2021Q4.

Table A.1: Firm-level local projections – Summary statistics

	Monetary	Oil	Credit	Uncertainty	Sentiment	TFP News
# Streaks	4,120	5,332	2,813	5,115	5,125	4,829
# Firms	4,017	5,061	2,813	4,893	4,902	4,651
Avg. # Obs.	64	70	84	66	66	63
Avg. R^2	0.33	0.29	0.25	0.26	0.32	0.31
Sign. IRFs (%)	77	80	83	78	82	80
$Q_{0.1}^{\text{IRF}}$ (%)	-5.4	-14.3	-2.1	-5.0	-4.5	-5.4
$Q_{0.5}^{\text{IRF}}$ (%)	-0.16	-0.14	-0.33	-0.30	-0.33	-0.39
$Q_{0.9}^{\text{IRF}}$ (%)	5.0	13.5	1.2	4.1	3.8	4.7

Note: The number of streaks can be larger than the number of unique firms. The average number of time series observations is measured for impact effect regressions and rounded to the nearest integer. The adjusted R^2 values are averaged across horizons and firms. The share of significant IRFs is the relative frequency of statistically significant IRFs for the peak of the impulse response estimates, measured using 90% confidence intervals based on Newey–West standard errors. Quantiles across firm-level IRFs are averaged across horizons. IRFs for the credit shock are only estimated for firms existing during the Great Financial Crisis.

D Additional empirical results

D.1 Skewness across firm-level (bottom-up) impulse responses

In the main text we have used measures of aggregate growth and cross-sectional skewness as inputs to the local projections to study their impulse responses. Instead, this section estimates impulse responses of firm-level sales growth rates to aggregate shocks and then constructs the response of cross-sectional skewness and aggregate sales growth bottom-up from the distribution of firm-level IRFs. The local projection specification at the firm level is

$$g_{i,t+h} = \alpha_{i,h} + \beta_{i,h}\text{shock}_t + \sum_{\ell=1}^{L} \gamma'_{i,\ell,t}\text{controls}_{i,t-\ell} + e_{i,t+h}, \tag{A.19}$$

where $\beta_{i,h}$ is the response of firm i's year-on-year sales growth rate at horizon h to a shock at horizon 0. All firm-level regressions control for lagged values of the shock and lagged GDP, as well as sales growth at the firm and the 2-digit NAICS level. In addition, we include shock-specific controls: shadow rate and leverage (monetary shock), GDP deflator (oil supply), excess bond premium (credit shock), Jurado et al. (2015) financial uncertainty (uncertainty), ICE consumer sentiment, macroeconomic uncertainty, and S&P500 stock prices (sentiment), and GDP per capita, labor productivity, and S&P500 stock prices per capita (TFP news). All controls are included with two lags. The only exception is a contemporaneous control for GDP growth in the credit shock regression, mirroring the specification in Gilchrist and Zakrajšek (2012).[23]

[23] See Table A.2 for variable definitions.

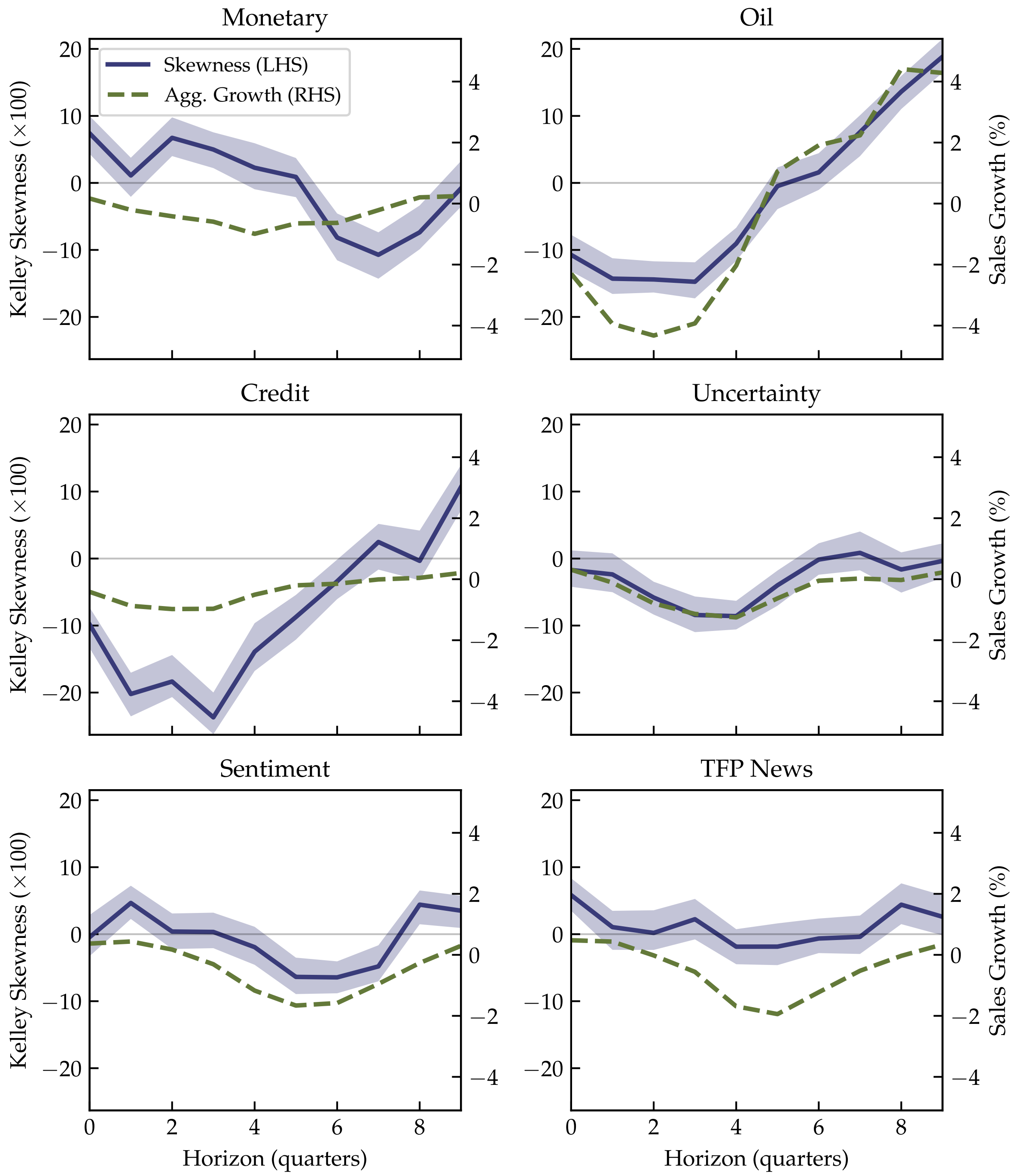
The summary statistics for the distribution of firm-level impulse response estimates are reported in Table A.1. Varying sample periods across shocks and missing values for firm-specific controls (in particular leverage for the monetary shock) imply differences in sample sizes. The number of unique streaks is above 4,000 for all shocks except credit. The sample for the credit shock is smaller since we only consider streaks covering the Great Financial Crisis, which turns out to be crucial to identify the effects of credit shocks using the Gilchrist and Zakrajšek (2012) specification. The number of streaks can be larger than the number of unique firms in the sample since some firms can have multiple streaks in the data, although this does not happen frequently. The average time series is roughly 70 quarters long. The firm-level regressions have average R^2 values of at least 25% and over 77% of impulse response estimates have statistically significant peak effects for each shock. The distribution of IRF estimates is widely dispersed with negative (unweighted) mean estimates, reflecting the contractionary nature of the shocks, and large heterogeneity in terms of firm responses. These distributions look very similar when only considering IRFs with significant peak effects (results not shown).

Based on the firm-level IRF estimates $\widehat{\beta}_{i,h}$ from equation (A.19), we construct the response of skewness and aggregate sales growth from the bottom up. The aggregate sales growth IRF is the size-weighted average of the firm IRFs: $\widehat{\beta}_h^{agg} = \sum_i \omega_{\beta,i}\widehat{\beta}_{i,h}$, where $\omega_{\beta,i}$ is the average real sales of firm i divided by the sum of average real sales across firms. The response of cross-sectional skewness is estimated from the cross section of firm IRFs: $\widehat{\beta}_h^{ksk} = ksk\left(\widehat{\mathcal{B}}_h\right)$, where $\widehat{\mathcal{B}}_h = \{\widehat{\beta}_{i,h}\}_{i=1,\ldots,N}$ is the set of firm IRF estimates at horizon h. Testing for procyclical skewness in this exercise is significantly harder than when using a top-down skewness index since individual firm IRFs are much more volatile than aggregate sales and the only source of procyclical skewness in response to a properly identified aggregate shock are heterogeneous responses across firms.

The results are shown in Figure A.3, where shaded areas are 90% confidence intervals based on a simple bootstrap with 2000 replications.[24] Following a contractionary aggregate shock, cross-sectional skewness (solid blue) and aggregate sales growth

[24]The bootstrap procedure resamples from the set of impulse responses with replacement. This takes the IRF estimates as given and does not consider the sampling uncertainty surrounding these estimates, thus understating the width of the confidence intervals (Pagan, 1984). Using a parametric bootstrap to estimate bottom-up statistics based on a distribution of firm-level IRFs across firms and bootstrap samples would correct the confidence intervals but is infeasible because of the computational burden involved. A proper estimation requires to run 1000 bootstrap replications for 5000 firms for 6 shocks, resulting in 30 million regressions. In addition, the parametric bootstrap would reduce the effective sample size even further due to the lag structure of the data-generating process, which is undesirable given the already short time series samples for some firms.

Figure A.3: Comovement of cross-sectional growth and skew after aggregate shocks



Note: The figure shows the response of aggregate sales growth ($\widehat{\beta}_h^{agg}$; green dashed line) and cross-sectional skewness ($\widehat{\beta}_h^{ksk}$; blue solid line) to different aggregate shocks. The blue shaded areas are 90% confidence bands for the skewness response, based on a bootstrap with 2000 replications. Shock magnitudes are normalized to be one standard deviation. The signs of the sentiment and the TFP shock are reversed to be contractionary.

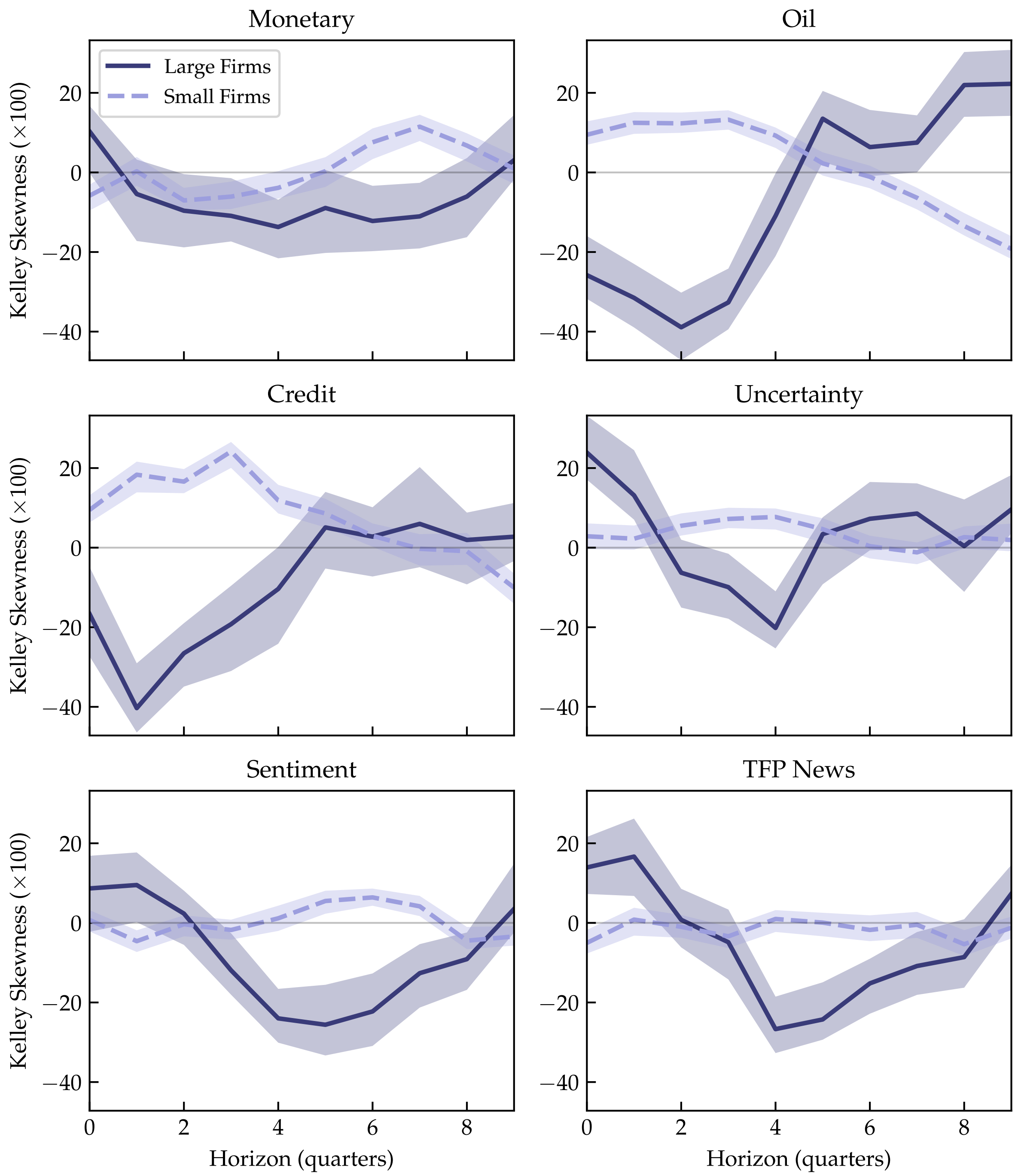
(dashed black) show a closely correlated decline. This is especially true for the oil, credit and uncertainty shocks. The correlations of skewness and growth following a sentiment or TFP shock are also close but the evidence for a negative skewness response is less clear. The monetary shock leads to a severe contraction in skewness but only after eight quarters, with a positive resonse on impact. Sales growth declines earlier and is recovering while skewness bottoms out. Without putting too much emphasis on any individual impulse response estimate, the findings across the six different shocks confirm that 1) cross-sectional skewness declines following contractionary aggregate shocks and 2) aggregate sales growth and cross-sectional skewness are strongly correlated following aggregate shocks.

How does the response of large firms differ from the response of small firms? We split the sample into two size groups (largest firms versus the rest) to study the impulse response of skewness across large vs small firms and compute their contribution to aggregate sales growth. Figure A.4 shows the IRFs for the largest 10% of firms (defined by average real sales) and the IRFs for the bottom 90% of firms. The black dotted line shows the contribution of large firms to the impulse response of aggregate sales growth, and the black dashed line shows the contribution of small firms. By construction, the sum of the two lines equals the impulse response of aggregate sales growth shown in Figure A.3. The red (blue) line shows the impulse response of skewness across large (small) firms. The shaded areas are 90% confidence intervals.

The bottom-up skewness response of large firms is significantly negative across shocks and in line with the impulse responses for the skewness index (Figure 6). The response of the largest firms is more skewed than the response of the rest of the firms. The differences in skewness can be large. For example, the minimum of the skewness IRF in response to a one standard deviation sentiment shock is around -0.2 for the largest firms but only -0.04 for the smaller firms. In response to an oil shock, large firms' skewness declines by over 0.3 points, while smaller firms' skewness falls by 0.1 points at most. The differences are also large for the monetary and the TFP shock and less pronounced for the credit and the uncertainty shock. In any case, the response across large firms is *not less* skewed than the response of small firms.

Summarizing the results of this section so far, aggregate shocks induce procyclical skewness through heterogeneous responses across firms. Aggregate shocks explain most of the business cycle variation of cross-sectional skewness. The set of the largest firms in the US economy also shows skewed responses to aggregate shocks, suggesting that some large firms respond strongly to those shocks.

Figure A.4: Large vs small firms: Cross-sectional skewness responses



Note: The figure shows the responses of cross-sectional skewness ($\widehat{\beta}_h^{ksk}$; blue solid line for large and light blue dashed for small) to different aggregate shocks, split by large and small firms. Large firms are the top 10% of the sales distribution, which averages real sales over time for each firm. Small firms are all other firms. The shaded areas are 90% confidence bands for the skewness responses, based on a bootstrap with 2000 replications. Shock magnitudes are normalized to be one standard deviation. The signs of the sentiment and the TFP shock are reversed to be contractionary

D.2 Cross-Sectional Skewness v. Aggregate Shocks

D.2.1 Shock Series for IRFs

Monetary shock We use the Bu et al. (2021) shock series, which are constructed to bridge periods of conventional and unconventional monetary policy. This is useful because the skewness series only starts in the mid-1980s while unconventional monetary policy became an important policy tool from 2008 onwards. Being restricted to a 1985-2008 sample period would make identification difficult, especially with quarterly data. The shock is estimated with Fama-MacBeth regressions using changes in interest rates at different maturities around FOMC announcements such that the second-stage coefficient estimates are the monetary shock series. In our local projection specification, we include lags of real GDP and the GDP deflator (both as detrended log levels) as well as the Wu and Xia (2016) shadow rate and the excess bond premium as controls. The excess bond premium captures financial conditions and is a useful control for the predictable component of the business cycle. We also include lags of the dependent variable and the shock as controls.

Oil supply shock The oil supply shocks are identified following Baumeister and Hamilton (2019), who use carefully selected priors for demand and supply elasticities in the oil market (among priors for other coefficients) in a Bayesian VAR. Their identification scheme allows them to relax some identifying assumptions previously imposed in the literature, for example that oil supply does not respond on impact to shocks to the oil price. Under the new identification strategy, the authors find oil supply shocks to be a more important determinant of historical oil price movements than found in the previous literature. The shock series we use is the median of the posterior distribution. We add lags of the shock, GDP, the GDP deflator, the crude petroleum producer price index, and the dependent variable as controls.

Credit shock. The credit shock uses innovations in the excess bond premium (EBP) following Gilchrist and Zakrajšek (2012). The EBP is constructed from corporate bond spreads to proxy investor risk appetite and is orthogonal to the risk of corporate default. Gilchrist and Zakrajšek (2012) use a recursive identification strategy in a VAR to study the effect of EBP innovations on macroeconomic variables. They assume that indicators of economic activity do not respond to EBP shocks within the same quarter while financial variables can respond immediately. We replicate their VAR, extract the shock, and use it in a local projection controlling for lags of the shock and the dependent variable, GDP, the GDP deflator, and the EBP.

Uncertainty shock The identification of the uncertainty shock follows Ludvigson et al. (2021), who use restrictions on the time series of the structural shocks to jointly identify financial uncertainty, macroeconomic uncertainty, and output shocks. Given the VAR residuals, the authors randomly draw many candidates for the time series of the structural shocks and only retain those that satisfy restrictions motivated from economic theory and narratives of historical events. For example, financial uncertainty should be high in October 1987 ('Black Monday') and September 2008 (Lehman collapse).[25] The remaining shocks series can be used for set identification of the impulse responses. The authors find that financial uncertainty shocks are a source of business cycle fluctuations, while macroeconomic uncertainty is more likely to be an endogenous response to output shocks. To obtain a single shock series for the financial uncertainty shock, we use the 'maxG' solution, which jointly maximizes the inequalities associated with a subset of the constraints. The controls are lags of the shock, GDP, the GDP deflator, VXO, and the dependent variable.

Sentiment shock While the previous shocks are related to economic fundamentals or financial conditions, business cycles may also be affected by fluctuations in consumer sentiment that are unrelated to economic conditions. Lagerborg et al. (2023) show that exogenous changes in consumer confidence can be recessionary. Their identification strategy relies on mass shootings in the United States, which are widely reported in the media and are shown to be predictors of downturns in sentiment. The authors show that the number of fatalities in mass shooting events can be viewed as exogenous to the state of the economy and used as a valid instrument to identify the effect of consumer confidence shocks on the business cycle. The authors estimate impulse responses in a proxy SVAR, and we extract the shock series from this system using the authors' replication codes. Similar to Lagerborg et al. (2023), we include lags of the shock, the University of Michigan Index of Consumer Expectations, real GDP, the Jurado et al. (2015) 12-month macroeconomic uncertainty index, real stock prices, and the dependent variable in the local projections.

TFP news shock News about future productivity can explain a significant share of business cycle variation, as shown in Beaudry and Portier (2006). We use shocks following the identification strategy of Ben Zeev and Khan (2015), who impose medium-run restrictions to identify news about investment-specific technology. Their shock is chosen to maximize the explained variance in (the inverse of) the relative price of investment in the medium term, while being orthogonal to both current TFP and the

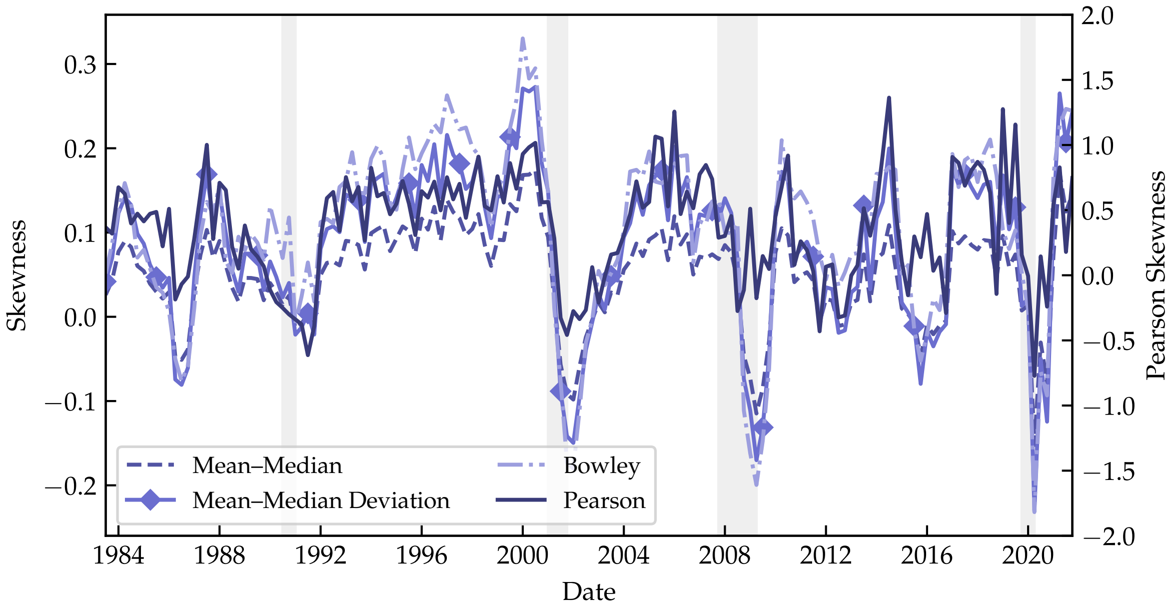
[25]The idea behind the identification scheme is similar to the classic sign restrictions, except that the restrictions are directly imposed on the time series of the structural shocks as opposed to the shape or magnitude of the impulse response estimates.

current relative price of investment. The authors find TFP news to account for a significant share of business cycle fluctuations. The impulse responses are estimated similar to the local projections of Ramey (2016), controlling for lags of the shock, real GDP per capita, real stock prices per capita, labor productivity, and the dependent variable.

D.2.2 Local projection specifications and robustness.

Table A.2 describes the construction of all data entering the local projections, including the shocks. We use existing data or the authors' replication codes for all shock series. For the baseline specifications, cross-sectional skewness and aggregate sales growth are computed using the streak sample as described in Appendix C. We confirm that the results for the impulse responses of cross-sectional skewness to aggregate shocks (Figure 6) are robust to several robustness checks: Using four lags instead of two, including lagged values of aggregate sales growth as controls, or using the cleaned sample instead of the streak sample to compute the skewness index. Results are not shown here to conserve space, but all alternative specifications yield very similar results, sometimes so close that the different impulse responses are indistinguishable from each other because they agree up to the third decimal.

Figure A.5: Comparison of different skewness measures.



Note. The first three measures are plotted on the LHS axis. The Pearson Skewness measure is plotted on the RHS axis.

Table A.2: Data for local projections

Variable	Transformation from raw data	Data source
Real GDP	Log level	FRED (GDPC1)
GDP Deflator	Log level	FRED (GDPDEF)
Real oil price	Quarterly average of monthly data, deflated	FRED (WPU0561, GDPDEF)
GDP per capita	Real GDP ('rgdp') per population ('civpop')	Ramey (2016) TFP data
Labor productivity	Real GDP ('rgdp') per hours worked ('tothours')	Ramey (2016) TFP data
Shadow rate	Quarterly average of monthly data	Atlanta Fed*
Stock prices	Shiller stock prices divided by GDP deflator	Ramey (2016), FRED
Stock prices per capita	Stock prices per population ('civpop')	Ramey (2016)
VXO	Quarterly average of daily data	FRED (VXOCLS)
Uncertainty Index	Log level of Jurado et al. (2015) index	Lagerborg et al. (2023)
Consumer Expectations	Log level	Lagerborg et al. (2023)
Monetary shock	Quarterly sum of monthly data	Fed Board**
Oil shock	Quarterly average of monthly data	Baumeister***
Credit shock	Quarterly average of monthly data	Favara et al. (2016)†
Uncertainty shock	Quarterly average of monthly 'maxG' shock	Ludvigson et al. (2021)
Sentiment shock	Quarterly average of monthly data	Lagerborg et al. (2023)‡
TFP News Shock	Level of Ben Zeev and Khan (2015) shock	Ramey (2016) TFP data
Cross-sectional skewness	Own construction based on streak sample	–
Sales growth	Own construction based on streak sample	–

Note:
(*) Shadow rate: https://www.atlantafed.org/cqer/research/wu-xia-shadow-federal-funds-rate.
(**) Bu et al. (2021) monetary shocks: https://www.federalreserve.gov/econres/feds/a-unified-measure-of-fed-monetary-policy-shocks.htm.
(***) Baumeister and Hamilton (2019) shocks: https://sites.google.com/site/cjsbaumeister/datasets?authuser=0.
(†) Credit shock from eight-variable VAR of Gilchrist and Zakrajšek (2012), 1973Q1–2019Q4.
(‡) Sentiment shock from Lagerborg et al. (2023) proxy SVAR, 1965:1–2018:11. Instrument is number of fatalities ($\leq$7), excluding 2017 Las Vegas shooting.

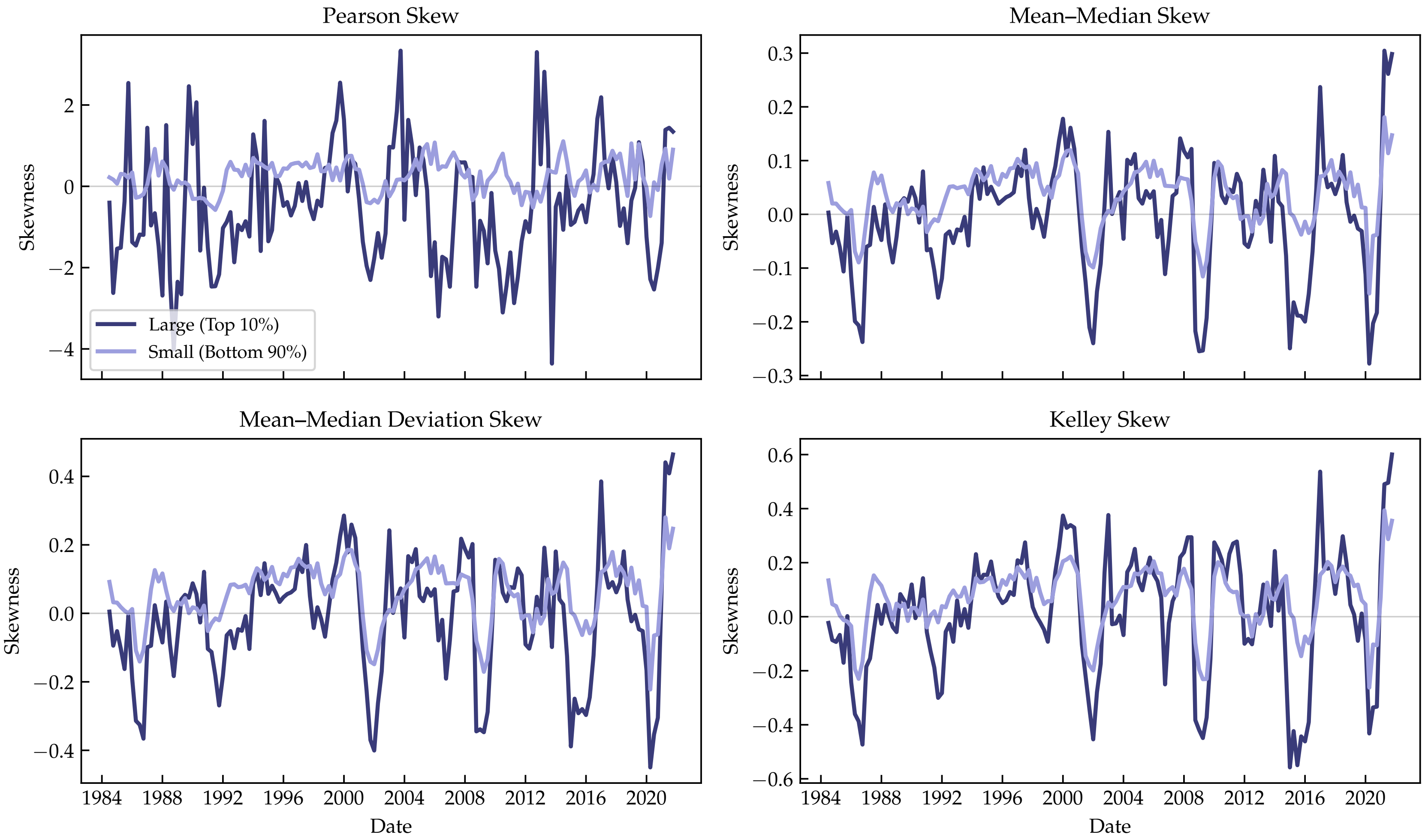


Figure 6: Skewness by firm size across different measures.