```
In[55]:= (* The following Calculations are done by
           using Jaakko's d and e^2 vectors parametrizations ,
         which was proven is lower energy in  equlibrium  state.
                The parametrization  is eα^2 = -cosθ ẑ -sinθ ŷ
                    dα = cosθ ŷ -sinθ ẑ *)
         (*** Ξ33,α=3,λ=3 ***)
         Clear["context`*"];
         Clear[Ξ33, c1, c2, gD, γ, χv, α, λ, V, V1, V2, V3, Kij, K1, K1ij, K2, K2ij,
             K3, K3ij, Qβj, Q1βj, Q2βj, Δp, Δv1, Δv2, X, θ, x1, x2, x3 , b, a, d, Va, Vb,
             Vd, Vj, Vβ, Xa, Xb, Xd, Λij33, Λij331, Λij332, Λij333, Λij334, Λij335, Pai1,
             Pai2, Pai3, Paj1, Paj2, Paj3, Pai, Paj, PaiVb, PajVb, KijΛijαλ, KaΛaαλ,
             KbaΛbaαλ, RjλαβQβj, VdVbϵjλζϵαβγ, VdVbϵjλζϵαβγQβj, LeviCivitą ϵijk];
         α =
           3;
         λ = 3;
         KaΛaαλ = 0;
         KbaΛbaαλ = 0;
         RbλαQb = 0;
         RdbλαQdb = 0;
         VVbϵλϵαQb = 0;
         VdVbϵλϵαQbd = 0;
         V1 = {1, 0, 0};
         V2 = {0, -Sin[θ[x1, x2, x3]], -Cos[θ[x1, x2, x3]]};
         V3 = {0, Cos[θ[x1, x2, x3]], -Sin[θ[x1, x2, x3]]};
         (*************************************************)
         (*V tensor for d e^1 e^2*)
         V = {V1, V2, V3};
         (*TreeForm [V]*)(*Level[V,{2}][[1]]*)
         (*************************************************)
         (*X tensor for X̄^1 X̄^2 X̄^3*)
         X = {{Δv1, 0, 0}, {0, Δv2, 0}, {0, 0, Δp}};
         (*X_i^a tensor symbol  Row is a index and colum  is spatial index *)
         (*************************************************)
         (**  Sumover  ba **)
         (*****************)
         (*K tensor K_ij^ba*)
         Do[Do[
             Xb = Level[X, {1}][[b]]; Xa = Level[X, {1}][[a]];
             K1ij = K1 Array[KroneckerDelta, {3, 3}] (Tr[({Xb})ᵀ.{Xa}]);
             K2ij = K2 (({Xb})ᵀ.{Xa});
             K3ij = K3 (({Xa})ᵀ.{Xb});
             Kij = K1ij + K2ij + K3ij;
             Print[Style["Kij=", Red, 12], Kij // MatrixForm ,
               "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
             (*****************************************************)
             (* Λ_ij33^ba *)
             Vb = Level[V, {1}][[b]]; Va = Level[V, {1}][[a]];
```

```
Pai = Grad[δs[x1, x2, x3], {x1, x2, x3}];
Paj = Grad[δs[x1, x2, x3], {x1, x2, x3}];
PaiPaj = Grad[Grad[δs[x1, x2, x3], {x1, x2, x3}], {x1, x2, x3}];
(********** the symbol  Λijxx(1..5) means  the 1st to 5th terms  of Λijxx,
  where xx is αλ *********)
Λij331 = (Grad[Grad[Vb[[λ]], {x1, x2, x3}], {x1, x2, x3}]) Va[[α]];
Λij331 = Λij111δs[x1, x2, x3];
(**********)
(*Λij112=
    (FullSimplify [Vb.Va,Assumptions →{{x1,x2,x3,θ}∈Reals}]-Vb[[1]] Va[[1]])
      (({Pai})ᵀ.{Paj});*)
Λij332 = (FullSimplify [Vb.Va, Assumptions →{{x1, x2, x3, θ} ∈ Reals}]
          KroneckerDelta[α, λ] - Vb[[α]] Va[[λ]]) (PaiPaj);
(*Λij332//MatrixForm *)
(**********)
PaiVb = (Grad[Vb, {x1, x2, x3}])ᵀ;
Λij333 = {(FullSimplify [(PaiVb).(Va), Assumptions →{{x1, x2, x3, θ} ∈ Reals}]
              KroneckerDelta[α, λ] -
            ((Grad[Vb[[α]], {x1, x2, x3}]) (Va[[λ]])))}ᵀ.{Paj};
(***********)
PajVb = (Grad[Vb, {x1, x2, x3}])ᵀ;
Λij334 =
  ({Pai})ᵀ.{(FullSimplify [(PajVb).(Va), Assumptions →{{x1, x2, x3, θ} ∈ Reals}]
            KroneckerDelta[α, λ] - ((Grad[Vb[[α]], {x1, x2, x3}]) (Va[[λ]])))};
(***********)
(*b=1;a=1;Vb=Level[V,{2}][[b]];Va=Level[V,{2}][[a]];*)
Λij335 = -(Grad[Grad[Vb[[α]], {x1, x2, x3}], {x1, x2, x3}]) Va[[λ]];
Λij335 = Λij115δs[x1, x2, x3];
(***********)
Λij33 = Λij331+Λij332+Λij333+Λij334+Λij335;
FullSimplify [Λij33, Assumptions →{{{x1, x2, x3, θ, K1, K2, K3} ∈ Reals}}];
(************************************************)
KijΛijαλ= FullSimplify [Tr[Kij.(Λij33ᵀ)],
    Assumptions →{{x1, x2, x3, θ, K1, K2, K3} ∈ Reals}];
Print[Style["KijΛijαλFull=", Red, 12], KijΛijαλ// TraditionalForm ,
  "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
Print[Style["KijΛijαλ=", Red, 12],
  (KijΛijαλ/. {δs^(0,0,1)[x1, x2, x3] → 0, θ^(0,0,1)[x1, x2, x3] → 0,
          δs^(0,0,2)[x1, x2, x3] → 0, δs^(1,0,1)[x1, x2, x3] → 0}) // TraditionalForm ,
  "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
KaΛaαλ = KaΛaαλ + FullSimplify [KijΛijαλ/. {δs^(0,0,1)[x1, x2, x3] → 0,
          θ^(0,0,1)[x1, x2, x3] → 0, δs^(0,0,2)[x1, x2, x3] → 0, δs^(1,0,1)[x1, x2, x3] → 0},
      Assumptions →{{x1, x2, x3, θ} ∈ Reals}];
 , {a, 3}];
KbaΛbaαλ = KbaΛbaαλ + KaΛaαλ; KaΛaαλ = 0;, {b, 3}];
```

```
KbaΛbaαλ =
  FullSimplify[KbaΛbaαλ, Assumptions → {{x1, x2, x3, θ} ∈ Reals}] // TraditionalForm
(*********************************************)
(*********************************************)
LeviCivita[i_, j_, k_] := Module[{test1 = (i == j || j == k || i == k), test2 =
      ((i == 1 && j == 2 && k == 3) || (i == 2 && j == 3 && k == 1) || (i == 3 && j == 1 && k == 2))},
    If[test1, ϵijk = 0;, If[test2, ϵijk = 1;, ϵijk = -1;];]; ϵijk];
Do[Do[(* Q tensor Qβj^bd *)
    Xb = Level[X, {1}][[b]]; Xd = Level[X, {1}][[d]];
    Q1βj = ({Xb})ᵀ.{Xd};
    Q2βj = ({Xd})ᵀ.{Xb};
    Qβj = (Q1βj + Q2βj);
    Print[Style["Qβj=", Red, 12], Qβj // MatrixForm ,
     ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
    (*********************************************)
    (* R^db_jλαβ *)
    Vd = Level[V, {1}][[d]]; Vb = Level[V, {1}][[b]];
    Rjλαβ = (({Vd})ᵀ.{Vb}) KroneckerDelta[λ, α] -
       (({Vd})ᵀ (Vb[[α]])).{Table[KroneckerDelta[λ, β], {β, 1, 3, 1}]};
    (*Print["Rjλαβ=",Rjλαβ//MatrixForm ,",","d=","d",",","b=",b];*)
    RjλαβQβj = FullSimplify [Tr[Rjλαβ.Qβj], Assumptions →
        {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
    Print[Style["RjλαβQβj=", Red, 12], RjλαβQβj // TraditionalForm ,
     ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
    RbλαQb = RbλαQb + RjλαβQβj;
    (*********************************************************)
    (* V^d_ζ V^b_γ ϵ_jλζ ϵ_αβγ Q^bd_βj *)
    (****************************)
    Vj = Table[Sum [Vd[[ζ]] LeviCivita[j, λ, ζ], {ζ, 3}], {j, 3}];
    Vβ = Table[Sum [Vb[[γ]] LeviCivita[α, β, γ], {γ, 3}], {β, 3}];
    VdVbϵjλζϵαβγ = ({Vj})ᵀ.{Vβ};
    VdVbϵjλζϵαβγQβj = FullSimplify [Tr[VdVbϵjλζϵαβγ.Qβj], Assumptions →
        {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
    Print[Style["VdVbϵjλζϵαβγQβj=", Red, 12], VdVbϵjλζϵαβγQβj // TraditionalForm ,
     ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
    VVbϵλϵαQb = VVbϵλϵαQb + VdVbϵjλζϵαβγQβj;, {b, 3}];
  RdbλαQdb = RdbλαQdb + RbλαQb;
  RbλαQb = 0;
  VdVbϵλϵαQbd = VdVbϵλϵαQbd + VVbϵλϵαQb;
  VVbϵλϵαQb = 0;, {d, 3}];
(*********** Show results ***********)
RdbλαQdb = FullSimplify [RdbλαQdb,
    Assumptions → {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
VdVbϵλϵαQbd = FullSimplify [VdVbϵλϵαQbd,
    Assumptions → {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
Print[Style["RdbλαQdb=", Red, 12], RdbλαQdb // TraditionalForm , ",",
  Style["α=", Red, 12], α, ",", Style["λ=", Red, 12], λ];
```

```
Print[Style["VdVbϵλϵαQbd=", Red, 12], VdVbϵλϵαQbd // TraditionalForm ,
   ",", Style["α=", Red, 12], α, ",", Style["λ=", Red, 12], λ];
(***********************************)
(***** Ξ11= γ²/χv KbaΛbaαλ + 6 gD γ2/(5 χv) RdbλαQdb + 6 gD γ²/(5 χv) VdVbϵλϵαQbd ******)
(** c1=γ²/χv c2=6 gD γ2/(5 χv) **)
Print[Style[" Ξ33 in Jaakko's parametrizetion : ", Red, 18]]
Ξ33 = (c1 KbaΛbaαλ + c2 RdbλαQdb + c2 VdVbϵλϵαQbd);
Ξ33 // TraditionalForm
```

$\text{Kij}=\begin{pmatrix} K1\,\Delta v1^2+K2\,\Delta v1^2+K3\,\Delta v1^2 & 0 & 0 \\ 0 & K1\,\Delta v1^2 & 0 \\ 0 & 0 & K1\,\Delta v1^2 \end{pmatrix}$, b=1, a=1

KijΛijαλFull=$\Delta v1^2 \left((K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)+K1\left(\delta s^{(0,0,2)}(x1,x2,x3)+\delta s^{(0,2,0)}(x1,x2,x3)\right)\right)$, b=1, a=1

KijΛijαλ=$\Delta v1^2\left((K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)+K1\,\delta s^{(0,2,0)}(x1,x2,x3)\right)$, b=1, a=1

$\text{Kij}=\begin{pmatrix} 0 & K2\,\Delta v1\,\Delta v2 & 0 \\ K3\,\Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, b=1, a=2

KijΛijαλFull=0, b=1, a=2

KijΛijαλ=0, b=1, a=2

$\text{Kij}=\begin{pmatrix} 0 & 0 & K2\,\Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ K3\,\Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}$, b=1, a=3

KijΛijαλFull=0, b=1, a=3

KijΛijαλ=0, b=1, a=3

$\text{Kij}=\begin{pmatrix} 0 & K3\,\Delta v1\,\Delta v2 & 0 \\ K2\,\Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, b=2, a=1

KijΛijαλFull=0, b=2, a=1

KijΛijαλ=0, b=2, a=1

$\text{Kij}=\begin{pmatrix} K1\,\Delta v2^2 & 0 & 0 \\ 0 & K1\,\Delta v2^2+K2\,\Delta v2^2+K3\,\Delta v2^2 & 0 \\ 0 & 0 & K1\,\Delta v2^2 \end{pmatrix}$, b=2, a=2

KijΛijαλFull=$\Delta v2^2 \sin(\theta(x1,x2,x3)) \Big(2\cos(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)+\sin(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(0,0,2)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\right)\Big)$, b=2, a=2

KijΛijαλ=$\Delta v2^2 \sin(\theta(x1,x2,x3)) \Big(2\cos(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)+\sin(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\right)\Big)$, b=2, a=2

$\text{Kij}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K2\,\Delta p\,\Delta v2 \\ 0 & K3\,\Delta p\,\Delta v2 & 0 \end{pmatrix}$, b=2, a=3

$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=\Delta p\,\Delta v2\,(-(K2+K3))\cos(\theta(x1,x2,x3))\left(\left(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)\right)\cos(\theta(x1,x2,x3))+\delta s^{(0,1,1)}(x1,x2,x3)\sin(\theta(x1,x2,x3))\right)$, b=2, a=3

$\text{Kij}\Lambda\text{ij}\alpha\lambda=\Delta p\,\Delta v2\,(-(K2+K3))\,\delta s^{(0,1,1)}(x1,x2,x3)\sin(\theta(x1,x2,x3))\cos(\theta(x1,x2,x3))$, b=2, a=3

$\text{Kij}=\begin{pmatrix}0 & 0 & K3\,\Delta p\,\Delta v1\\ 0 & 0 & 0\\ K2\,\Delta p\,\Delta v1 & 0 & 0\end{pmatrix}$, b=3, a=1

$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=0$, b=3, a=1

$\text{Kij}\Lambda\text{ij}\alpha\lambda=0$, b=3, a=1

$\text{Kij}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & K3\,\Delta p\,\Delta v2\\ 0 & K2\,\Delta p\,\Delta v2 & 0\end{pmatrix}$, b=3, a=2

$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=\Delta p\,\Delta v2\,(K2+K3)\sin(\theta(x1,x2,x3))\left(\left(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)\right)\sin(\theta(x1,x2,x3))-\delta s^{(0,1,1)}(x1,x2,x3)\cos(\theta(x1,x2,x3))\right)$, b=3, a=2

$\text{Kij}\Lambda\text{ij}\alpha\lambda=\Delta p\,\Delta v2\,(-(K2+K3))\,\delta s^{(0,1,1)}(x1,x2,x3)\sin(\theta(x1,x2,x3))\cos(\theta(x1,x2,x3))$, b=3, a=2

$\text{Kij}=\begin{pmatrix}K1\,\Delta p^2 & 0 & 0\\ 0 & K1\,\Delta p^2 & 0\\ 0 & 0 & K1\,\Delta p^2+K2\,\Delta p^2+K3\,\Delta p^2\end{pmatrix}$, b=3, a=3

$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=\Delta p^2\cos(\theta(x1,x2,x3))\left(\cos(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,0,2)}(x1,x2,x3)+K1\left(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\right)\right)-2\sin(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+K1\left(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)\right)\right)$, b=3, a=3

$\text{Kij}\Lambda\text{ij}\alpha\lambda=\Delta p^2\cos(\theta(x1,x2,x3))\left(K1\left(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\right)\cos(\theta(x1,x2,x3))-2\,K1\left(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)\sin(\theta(x1,x2,x3))\right)$, b=3, a=3

Out[63]//TraditionalForm=

$$\Delta v1^2\left((K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)+K1\,\delta s^{(0,2,0)}(x1,x2,x3)\right)+\Delta v2^2\sin(\theta(x1,x2,x3))\left(2\cos(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)+\sin(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\right)\right)+\Delta p^2\cos(\theta(x1,x2,x3))\left(K1\left(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\right)\cos(\theta(x1,x2,x3))-2\,K1\left(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)\sin(\theta(x1,x2,x3))\right)-\Delta p\,\Delta v2\,(K2+K3)\,\delta s^{(0,1,1)}(x1,x2,x3)\sin(2\,\theta(x1,x2,x3))$$

$\text{Q}\beta\text{j}=\begin{pmatrix}2\,\Delta v1^2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$, d=1, b=1

$\text{Rj}\lambda\alpha\beta\text{Q}\beta\text{j}=2\,\Delta v1^2$, d=1, b=1

$\text{VdVb}\epsilon\text{j}\lambda\zeta\epsilon\alpha\beta\gamma\text{Q}\beta\text{j}=0$, d=1, b=1

$\text{Q}\beta\text{j}=\begin{pmatrix}0 & \Delta v1\,\Delta v2 & 0\\ \Delta v1\,\Delta v2 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$, d=1, b=2

$\text{Rj}\lambda\alpha\beta\text{Q}\beta\text{j}=-\Delta v1\,\Delta v2\sin(\theta(x1,x2,x3))$, d=1, b=2

$\text{VdVb}\epsilon\text{j}\lambda\zeta\epsilon\alpha\beta\gamma\text{Q}\beta\text{j}=-\Delta v1\,\Delta v2\sin(\theta(x1,x2,x3))$, d=1, b=2

$Q\beta j = \begin{pmatrix} 0 & 0 & \Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ \Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}, d=1, b=3$

$Rj\lambda\alpha\beta Q\beta j = 0, d=1, b=3$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=1, b=3$

$Q\beta j = \begin{pmatrix} 0 & \Delta v1\,\Delta v2 & 0 \\ \Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, d=2, b=1$

$Rj\lambda\alpha\beta Q\beta j = -\Delta v1\,\Delta v2\,\sin(\theta(x1, x2, x3)), d=2, b=1$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = -\Delta v1\,\Delta v2\,\sin(\theta(x1, x2, x3)), d=2, b=1$

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2\,\Delta v2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, d=2, b=2$

$Rj\lambda\alpha\beta Q\beta j = 2\,\Delta v2^2\,\sin^2(\theta(x1, x2, x3)), d=2, b=2$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=2, b=2$

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta p\,\Delta v2 \\ 0 & \Delta p\,\Delta v2 & 0 \end{pmatrix}, d=2, b=3$

$Rj\lambda\alpha\beta Q\beta j = -\Delta p\,\Delta v2\,\cos^2(\theta(x1, x2, x3)), d=2, b=3$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=2, b=3$

$Q\beta j = \begin{pmatrix} 0 & 0 & \Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ \Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}, d=3, b=1$

$Rj\lambda\alpha\beta Q\beta j = -\Delta p\,\Delta v1\,\sin(\theta(x1, x2, x3)), d=3, b=1$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=3, b=1$

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta p\,\Delta v2 \\ 0 & \Delta p\,\Delta v2 & 0 \end{pmatrix}, d=3, b=2$

$Rj\lambda\alpha\beta Q\beta j = \Delta p\,\Delta v2\,\sin^2(\theta(x1, x2, x3)), d=3, b=2$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=3, b=2$

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2\,\Delta p^2 \end{pmatrix}, d=3, b=3$

$Rj\lambda\alpha\beta Q\beta j = 0, d=3, b=3$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=3, b=3$

$Rdb\lambda\alpha Qdb = -\Delta v1\,(\Delta p + 2\,\Delta v2)\,\sin(\theta(x1, x2, x3)) - \Delta v2\,(\Delta p + \Delta v2)\,\cos(2\,\theta(x1, x2, x3)) + 2\,\Delta v1^2 + \Delta v2^2, \alpha=3, \lambda=3$

$VdVb\epsilon\lambda\epsilon\alpha Qbd = -2\,\Delta v1\,\Delta v2\,\sin(\theta(x1, x2, x3)), \alpha=3, \lambda=3$

## $\Xi_{33}$ in Jaakko's parametrizetion :

Out[71]//TraditionalForm=

$$c1\left(\cos(\theta(x1,x2,x3))\left(K1\cos(\theta(x1,x2,x3))\left(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\right)-2\,K1\sin(\theta(x1,x2,x3))\left(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)\right)\Delta p^2-(K2+K3)\,\Delta v2\sin(2\,\theta(x1,x2,x3))\,\delta s^{(0,1,1)}(x1,x2,x3)\,\Delta p+\Delta v1^2\left(K1\,\delta s^{(0,2,0)}(x1,x2,x3)+(K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)\right)+\Delta v2^2\sin(\theta(x1,x2,x3))\left(2\cos(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)+\sin(\theta(x1,x2,x3))\left((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\right)\right)\right)+$$

$$c2\left(-\Delta v1\,(\Delta p+2\,\Delta v2)\sin(\theta(x1,x2,x3))-\Delta v2\,(\Delta p+\Delta v2)\cos(2\,\theta(x1,x2,x3))+2\,\Delta v1^2+\Delta v2^2\right)-$$

2

c2

Δv1

Δv2

sin (θ(x1, x2, x3))

In[72]:=
```
(* The following Calculations are done by
  using Jaakko's d and e² vectors parametrizations ,
which was proven is lower energy in  equlibrium  state.
      The parametrization  is eα² = -cosθ ẑ -sinθ ŷ
          dα = cosθ ŷ -sinθ ẑ *)
(*** Ξ22,α=2,λ=2 ***)
Clear["context`*"];
Clear[Ξ22, c1, c2, gD, γ, χv, α, λ, V, V1, V2, V3, Kij, K1, K1ij, K2, K2ij,
   K3, K3ij, Qβj, Q1βj, Q2βj, Δp, Δv1, Δv2, X, Θ, x1, x2, x3 , b, a, d, Va, Vb,
   Vd, Vj, Vβ, Xa, Xb, Xd, Λij33, Λij331, Λij332, Λij333, Λij334, Λij335, Pai1,
   Pai2, Pai3, Paj1, Paj2, Paj3, Pai, Paj, PaiVb, PajVb, KijΛijαλ, KaΛaαλ,
   KbaΛbaαλ, RjλαβQβj, VdVbϵjλζϵαβγ, VdVbϵjλζϵαβγQβj, LeviCivita, ϵijk];
α =
  2;
λ = 2;
KaΛaαλ = 0;
KbaΛbaαλ = 0;
RbλαQb = 0;
RdbλαQdb = 0;
VVbϵλϵαQb = 0;
VdVbϵλϵαQbd = 0;
V1 = {1, 0, 0};
V2 = {0, -Sin[θ[x1, x2, x3]], -Cos[θ[x1, x2, x3]]};
V3 = {0, Cos[θ[x1, x2, x3]], -Sin[θ[x1, x2, x3]]};
(***************************************************)
(*V tensor for d e¹ e²*)
V = {V1, V2, V3};
(*TreeForm [V]*)(*Level[V,{2}][[1]]*)
(***************************************************)
(*X tensor for X̄¹ X̄² X̄³*)
```

```
X = {{Δv1, 0, 0}, {0, Δv2, 0}, {0, 0, Δp}};
(*X_i^a tensor symbol  Row is a index and colum  is spatial index *)
(************************************************)
(**  Sumover  ba **)
(*****************)
(*K tensor K_ij^ba*)
Do[Do[
    Xb = Level[X, {1}][[b]]; Xa = Level[X, {1}][[a]];
    K1ij = K1 Array[KroneckerDelta, {3, 3}] (Tr[({Xb})ᵀ.{Xa}]);
    K2ij = K2 (({Xb})ᵀ.{Xa});
    K3ij = K3 (({Xa})ᵀ.{Xb});
    Kij = K1ij + K2ij + K3ij;
    Print[Style["Kij=", Red, 12], Kij // MatrixForm ,
      "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    (*******************************************************)
    (* Λ_ij22^ba *)
    Vb = Level[V, {1}][[b]]; Va = Level[V, {1}][[a]];
    Pai = Grad[δs[x1, x2, x3], {x1, x2, x3}];
    Paj = Grad[δs[x1, x2, x3], {x1, x2, x3}];
    PaiPaj = Grad[Grad[δs[x1, x2, x3], {x1, x2, x3}], {x1, x2, x3}];
    (********** the symbol  Λijxx(1..5) means  the 1st to 5th terms  of Λijxx,
      where xx is αλ *********)
    Λij331 = (Grad[Grad[Vb[[λ]], {x1, x2, x3}], {x1, x2, x3}]) Va[[α]];
    Λij331 = Λij111δs[x1, x2, x3];
    (**********)
    (*Λij112=
        (FullSimplify [Vb.Va,Assumptions →{{x1,x2,x3,θ}∈Reals}]-Vb[[1]] Va[[1]])
          (({Pai})ᵀ.{Paj});*)
    Λij332 = (FullSimplify [Vb.Va, Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
              KroneckerDelta[α, λ] - Vb[[α]] Va[[λ]]) (PaiPaj);
    (*Λij332//MatrixForm *)
    (**********)
    PaiVb = (Grad[Vb, {x1, x2, x3}])ᵀ;
    Λij333 = {(FullSimplify [(PaiVb).(Va), Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
                  KroneckerDelta[α, λ] -
                ((Grad[Vb[[α]], {x1, x2, x3}]) (Va[[λ]])))}ᵀ.{Paj};
    (***********)
    PajVb = (Grad[Vb, {x1, x2, x3}])ᵀ;
    Λij334 =
      ({Pai})ᵀ.{(FullSimplify [(PajVb).(Va), Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
                KroneckerDelta[α, λ] - ((Grad[Vb[[α]], {x1, x2, x3}]) (Va[[λ]])))};
    (***********)
    (*b=1;a=1;Vb=Level[V,{2}][[b]];Va=Level[V,{2}][[a]];*)
    Λij335 = -(Grad[Grad[Vb[[α]], {x1, x2, x3}], {x1, x2, x3}]) Va[[λ]];
    Λij335 = Λij115δs[x1, x2, x3];
    (***********)
    Λij33 = Λij331 + Λij332 + Λij333 + Λij334 + Λij335;
```

```
    FullSimplify [Λij33, Assumptions → {{{x1, x2, x3, θ, K1, K2, K3} ∈ Reals}}];
    (**********************************************)
    KijΛijαλ= FullSimplify [Tr[Kij.(Λij33ᵀ)],
        Assumptions → {{x1, x2, x3, θ, K1, K2, K3} ∈ Reals}];
    Print[Style["KijΛijαλFull=", Red, 12], KijΛijαλ// TraditionalForm ,
      ","Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    Print[Style["KijΛijαλ=", Red, 12],
      (KijΛijαλ/. {δs^(0,0,1)[x1, x2, x3] → 0, θ^(0,0,1)[x1, x2, x3] → 0,
             δs^(0,0,2)[x1, x2, x3] → 0, δs^(1,0,1)[x1, x2, x3] → 0}) // TraditionalForm ,
      ","Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    KaΛaαλ = KaΛaαλ + FullSimplify [KijΛijαλ/. {δs^(0,0,1)[x1, x2, x3] → 0,
             θ^(0,0,1)[x1, x2, x3] → 0, δs^(0,0,2)[x1, x2, x3] → 0, δs^(1,0,1)[x1, x2, x3] → 0},
          Assumptions → {{x1, x2, x3, θ} ∈ Reals}];
    , {a, 3}];
  KbaΛbaαλ = KbaΛbaαλ + KaΛaαλ; KaΛaαλ = 0;, {b, 3}];
KbaΛbaαλ =
  FullSimplify [KbaΛbaαλ, Assumptions → {{x1, x2, x3, θ} ∈ Reals}] // TraditionalForm
(**********************************************)
(**********************************************)
LeviCivita[i_, j_, k_] := Module[{test1 = (i == j || j == k || i == k), test2 =
          ((i == 1 && j == 2 && k == 3) || (i == 2 && j == 3 && k == 1) || (i == 3 && j == 1 && k == 2))},
        If[test1, ϵijk= 0;, If[test2, ϵijk= 1;, ϵijk= -1;];]; ϵijk];
Do[Do[(* Q tensor Q_βj^bd*)
      Xb = Level[X, {1}][[b]]; Xd = Level[X, {1}][[d]];
      Q1βj = ({Xb})ᵀ.{Xd};
      Q2βj = ({Xd})ᵀ.{Xb};
      Qβj = (Q1βj + Q2βj);
      Print[Style["Qβj=", Red, 12], Qβj // MatrixForm ,
        ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
      (**********************************************)
      (* R^db_jλαβ *)
      Vd = Level[V, {1}][[d]]; Vb = Level[V, {1}][[b]];
      Rjλαβ = (({Vd})ᵀ.{Vb}) KroneckerDelta[λ, α] -
          (({Vd})ᵀ (Vb[[α]])).{Table[KroneckerDelta[λ, β], {β, 1, 3, 1}]};
      (*Print["Rjλαβ=",Rjλαβ//MatrixForm ,",",",","d=","d",",",",","b=",b];*)
      RjλαβQβj = FullSimplify [Tr[Rjλαβ.Qβj], Assumptions →
            {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
      Print[Style["RjλαβQβj=", Red, 12], RjλαβQβj // TraditionalForm ,
        ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
      RbλαQb = RbλαQb + RjλαβQβj;
      (******************************************************)
      (* V^d_ζ V^b_γ ϵ_jλζ ϵ_αβγ Q^bd_βj *)
      (***************************)
      Vj = Table[Sum [Vd[[ξ]] LeviCivita[j, λ, ξ], {ξ, 3}], {j, 3}];
      Vβ = Table[Sum [Vb[[γ]] LeviCivita[α, β, γ], {γ, 3}], {β, 3}];
```

```
        VdVbejλζεαβγ = ({Vj})ᵀ.{Vβ};
        VdVbejλζεαβγQβj = FullSimplify[Tr[VdVbejλζεαβγ.Qβj], Assumptions →
              {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
        Print[Style["VdVbejλζεαβγQβj=", Red, 12], VdVbejλζεαβγQβj // TraditionalForm,
          ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
        VVbeλεαQb = VVbeλεαQb + VdVbejλζεαβγQβj;, {b, 3}];
     RdbλαQdb = RdbλαQdb + RbλαQb;
     RbλαQb = 0;
     VdVbeλεαQbd = VdVbeλεαQbd + VVbeλεαQb;
     VVbeλεαQb = 0;, {d, 3}];
(*********** Show results ***********)
RdbλαQdb = FullSimplify[RdbλαQdb,
        Assumptions →{{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
VdVbeλεαQbd = FullSimplify[VdVbeλεαQbd,
        Assumptions →{{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
Print[Style["RdbλαQdb=", Red, 12], RdbλαQdb // TraditionalForm, ",",
     Style["α=", Red, 12], α, ",", Style["λ=", Red, 12], λ];
Print[Style["VdVbeλεαQbd=", Red, 12], VdVbeλεαQbd // TraditionalForm,
     ",", Style["α=", Red, 12], α, ",", Style["λ=", Red, 12], λ];
(*************************************)
```

(***** $\Xi$11= $\frac{\gamma^2}{\chi_v}$KbaΛbaαλ + $\frac{6\ gD\ \gamma2}{5\ \chi_v}$RdbλαQdb + $\frac{6\ gD\ \gamma^2}{5\ \chi_v}$VdVbeλεαQbd ******)

(** c1=$\frac{\gamma^2}{\chi_v}$ c2=$\frac{6\ gD\ \gamma2}{5\ \chi_v}$ **)

```
Print[Style[" Ξ22 in Jaakko's parametrizetion  : ", Red, 18]]
Ξ22 = (c1 KbaΛbaαλ + c2 RdbλαQdb + c2 VdVbeλεαQbd);
Ξ22 // TraditionalForm
```

Kij= $\begin{pmatrix} K1\,\Delta v1^2 + K2\,\Delta v1^2 + K3\,\Delta v1^2 & 0 & 0 \\ 0 & K1\,\Delta v1^2 & 0 \\ 0 & 0 & K1\,\Delta v1^2 \end{pmatrix}$, b=1, a=1

KijΛijαλFull= $\Delta v1^2 \left((K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3) + K1\left(\delta s^{(0,0,2)}(x1,x2,x3)+\delta s^{(0,2,0)}(x1,x2,x3)\right)\right)$, b=1, a=1

KijΛijαλ= $\Delta v1^2 \left((K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3) + K1\,\delta s^{(0,2,0)}(x1,x2,x3)\right)$, b=1, a=1

Kij= $\begin{pmatrix} 0 & K2\,\Delta v1\,\Delta v2 & 0 \\ K3\,\Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, b=1, a=2

KijΛijαλFull=0, b=1, a=2

KijΛijαλ=0, b=1, a=2

Kij= $\begin{pmatrix} 0 & 0 & K2\,\Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ K3\,\Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}$, b=1, a=3

KijΛijαλFull=0, b=1, a=3

KijΛijαλ=0, b=1, a=3

Kij= $\begin{pmatrix} 0 & K3\,\Delta v1\,\Delta v2 & 0 \\ K2\,\Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, b=2, a=1

KijΛijαλFull=0, b=2, a=1

KijΛijαλ=0, b=2, a=1

$$\text{Kij}=\begin{pmatrix} \text{K1}\,\Delta\text{v2}^2 & 0 & 0 \\ 0 & \text{K1}\,\Delta\text{v2}^2+\text{K2}\,\Delta\text{v2}^2+\text{K3}\,\Delta\text{v2}^2 & 0 \\ 0 & 0 & \text{K1}\,\Delta\text{v2}^2 \end{pmatrix},\ \text{b}=2,\ \text{a}=2$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=\Delta\text{v2}^2\cos(\theta(\text{x1},\text{x2},\text{x3}))\Big(\cos(\theta(\text{x1},\text{x2},\text{x3}))\big((\text{K1}+\text{K2}+\text{K3})\,\delta s^{(0,2,0)}(\text{x1},\text{x2},\text{x3})+\text{K1}\,\delta s^{(0,0,2)}(\text{x1},\text{x2},\text{x3})+\text{K1}\,\delta s^{(2,0,0)}(\text{x1},\text{x2},\text{x3})\big)-2\sin(\theta(\text{x1},\text{x2},\text{x3}))\big((\text{K1}+\text{K2}+\text{K3})\,\delta s^{(0,1,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,1,0)}(\text{x1},\text{x2},\text{x3})+\text{K1}\,\delta s^{(0,0,1)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,0,1)}(\text{x1},\text{x2},\text{x3})+\text{K1}\,\delta s^{(1,0,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(1,0,0)}(\text{x1},\text{x2},\text{x3})\big)\Big),\ \text{b}=2,\ \text{a}=2$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda=\Delta\text{v2}^2\cos(\theta(\text{x1},\text{x2},\text{x3}))\Big(\cos(\theta(\text{x1},\text{x2},\text{x3}))\big((\text{K1}+\text{K2}+\text{K3})\,\delta s^{(0,2,0)}(\text{x1},\text{x2},\text{x3})+\text{K1}\,\delta s^{(2,0,0)}(\text{x1},\text{x2},\text{x3})\big)-2\sin(\theta(\text{x1},\text{x2},\text{x3}))\big((\text{K1}+\text{K2}+\text{K3})\,\delta s^{(0,1,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,1,0)}(\text{x1},\text{x2},\text{x3})+\text{K1}\,\delta s^{(1,0,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(1,0,0)}(\text{x1},\text{x2},\text{x3})\big)\Big),\ \text{b}=2,\ \text{a}=2$$

$$\text{Kij}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \text{K2}\,\Delta\text{p}\,\Delta\text{v2} \\ 0 & \text{K3}\,\Delta\text{p}\,\Delta\text{v2} & 0 \end{pmatrix},\ \text{b}=2,\ \text{a}=3$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=\Delta\text{p}\,\Delta\text{v2}\,(-(\text{K2}+\text{K3}))\sin(\theta(\text{x1},\text{x2},\text{x3}))\Big(\big(\delta s^{(0,1,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,0,1)}(\text{x1},\text{x2},\text{x3})+\delta s^{(0,0,1)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,1,0)}(\text{x1},\text{x2},\text{x3})\big)\sin(\theta(\text{x1},\text{x2},\text{x3}))-\delta s^{(0,1,1)}(\text{x1},\text{x2},\text{x3})\cos(\theta(\text{x1},\text{x2},\text{x3}))\Big),\ \text{b}=2,\ \text{a}=3$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda=\Delta\text{p}\,\Delta\text{v2}\,(\text{K2}+\text{K3})\,\delta s^{(0,1,1)}(\text{x1},\text{x2},\text{x3})\sin(\theta(\text{x1},\text{x2},\text{x3}))\cos(\theta(\text{x1},\text{x2},\text{x3})),\ \text{b}=2,\ \text{a}=3$$

$$\text{Kij}=\begin{pmatrix} 0 & 0 & \text{K3}\,\Delta\text{p}\,\Delta\text{v1} \\ 0 & 0 & 0 \\ \text{K2}\,\Delta\text{p}\,\Delta\text{v1} & 0 & 0 \end{pmatrix},\ \text{b}=3,\ \text{a}=1$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=0,\ \text{b}=3,\ \text{a}=1$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda=0,\ \text{b}=3,\ \text{a}=1$$

$$\text{Kij}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \text{K3}\,\Delta\text{p}\,\Delta\text{v2} \\ 0 & \text{K2}\,\Delta\text{p}\,\Delta\text{v2} & 0 \end{pmatrix},\ \text{b}=3,\ \text{a}=2$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=\Delta\text{p}\,\Delta\text{v2}\,(\text{K2}+\text{K3})\cos(\theta(\text{x1},\text{x2},\text{x3}))\Big(\big(\delta s^{(0,1,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,0,1)}(\text{x1},\text{x2},\text{x3})+\delta s^{(0,0,1)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,1,0)}(\text{x1},\text{x2},\text{x3})\big)\cos(\theta(\text{x1},\text{x2},\text{x3}))+\delta s^{(0,1,1)}(\text{x1},\text{x2},\text{x3})\sin(\theta(\text{x1},\text{x2},\text{x3}))\Big),\ \text{b}=3,\ \text{a}=2$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda=\Delta\text{p}\,\Delta\text{v2}\,(\text{K2}+\text{K3})\,\delta s^{(0,1,1)}(\text{x1},\text{x2},\text{x3})\sin(\theta(\text{x1},\text{x2},\text{x3}))\cos(\theta(\text{x1},\text{x2},\text{x3})),\ \text{b}=3,\ \text{a}=2$$

$$\text{Kij}=\begin{pmatrix} \text{K1}\,\Delta\text{p}^2 & 0 & 0 \\ 0 & \text{K1}\,\Delta\text{p}^2 & 0 \\ 0 & 0 & \text{K1}\,\Delta\text{p}^2+\text{K2}\,\Delta\text{p}^2+\text{K3}\,\Delta\text{p}^2 \end{pmatrix},\ \text{b}=3,\ \text{a}=3$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=\Delta\text{p}^2\sin(\theta(\text{x1},\text{x2},\text{x3}))\Big(2\cos(\theta(\text{x1},\text{x2},\text{x3}))\big((\text{K1}+\text{K2}+\text{K3})\,\delta s^{(0,0,1)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,0,1)}(\text{x1},\text{x2},\text{x3})+\text{K1}\big(\delta s^{(0,1,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,1,0)}(\text{x1},\text{x2},\text{x3})+\delta s^{(1,0,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(1,0,0)}(\text{x1},\text{x2},\text{x3})\big)\big)+\sin(\theta(\text{x1},\text{x2},\text{x3}))\big((\text{K1}+\text{K2}+\text{K3})\,\delta s^{(0,0,2)}(\text{x1},\text{x2},\text{x3})+\text{K1}\big(\delta s^{(0,2,0)}(\text{x1},\text{x2},\text{x3})+\delta s^{(2,0,0)}(\text{x1},\text{x2},\text{x3})\big)\big)\Big),\ \text{b}=3,\ \text{a}=3$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda=\Delta\text{p}^2\sin(\theta(\text{x1},\text{x2},\text{x3}))\Big(2\,\text{K1}\big(\delta s^{(0,1,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(0,1,0)}(\text{x1},\text{x2},\text{x3})+\delta s^{(1,0,0)}(\text{x1},\text{x2},\text{x3})\,\theta^{(1,0,0)}(\text{x1},\text{x2},\text{x3})\big)\cos(\theta(\text{x1},\text{x2},\text{x3}))+\text{K1}\big(\delta s^{(0,2,0)}(\text{x1},\text{x2},\text{x3})+\delta s^{(2,0,0)}(\text{x1},\text{x2},\text{x3})\big)\sin(\theta(\text{x1},\text{x2},\text{x3}))\Big),\ \text{b}=3,\ \text{a}=3$$

Out[80]//TraditionalForm=

$$\Delta v1^2 \left((K1+K2+K3)\, \delta s^{(2,0,0)}(x1, x2, x3) + K1\, \delta s^{(0,2,0)}(x1, x2, x3)\right) + \Delta v2^2 \cos(\theta(x1, x2, x3)) \left(\cos(\theta(x1, x2, x3)) \left((K1+K2+K3)\, \delta s^{(0,2,0)}(x1, x2, x3) + K1\, \delta s^{(2,0,0)}(x1, x2, x3)\right) - 2 \sin(\theta(x1, x2, x3)) \left((K1+K2+K3)\, \delta s^{(0,1,0)}(x1, x2, x3)\, \theta^{(0,1,0)}(x1, x2, x3) + K1\, \delta s^{(1,0,0)}(x1, x2, x3)\, \theta^{(1,0,0)}(x1, x2, x3)\right)\right) + \Delta p^2 \sin(\theta(x1, x2, x3)) \left(2\, K1 \left(\delta s^{(0,1,0)}(x1, x2, x3)\, \theta^{(0,1,0)}(x1, x2, x3) + \delta s^{(1,0,0)}(x1, x2, x3)\, \theta^{(1,0,0)}(x1, x2, x3)\right) \cos(\theta(x1, x2, x3)) + K1 \left(\delta s^{(0,2,0)}(x1, x2, x3) + \delta s^{(2,0,0)}(x1, x2, x3)\right) \sin(\theta(x1, x2, x3))\right) + \Delta p\, \Delta v2\, (K2+K3)\, \delta s^{(0,1,1)}(x1, x2, x3) \sin(2\, \theta(x1, x2, x3))$$

$Q\beta j = \begin{pmatrix} 2\Delta v1^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, d=1, b=1

$Rj\lambda\alpha\beta Q\beta j = 2\Delta v1^2$, d=1, b=1

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=1, b=1

$Q\beta j = \begin{pmatrix} 0 & \Delta v1\,\Delta v2 & 0 \\ \Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, d=1, b=2

$Rj\lambda\alpha\beta Q\beta j = 0$, d=1, b=2

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=1, b=2

$Q\beta j = \begin{pmatrix} 0 & 0 & \Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ \Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}$, d=1, b=3

$Rj\lambda\alpha\beta Q\beta j = -\Delta p\,\Delta v1 \sin(\theta(x1, x2, x3))$, d=1, b=3

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = -\Delta p\,\Delta v1 \sin(\theta(x1, x2, x3))$, d=1, b=3

$Q\beta j = \begin{pmatrix} 0 & \Delta v1\,\Delta v2 & 0 \\ \Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, d=2, b=1

$Rj\lambda\alpha\beta Q\beta j = -\Delta v1\,\Delta v2 \sin(\theta(x1, x2, x3))$, d=2, b=1

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=2, b=1

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2\Delta v2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, d=2, b=2

$Rj\lambda\alpha\beta Q\beta j = 0$, d=2, b=2

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=2, b=2

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta p\,\Delta v2 \\ 0 & \Delta p\,\Delta v2 & 0 \end{pmatrix}$, d=2, b=3

$Rj\lambda\alpha\beta Q\beta j = \Delta p\,\Delta v2 \sin^2(\theta(x1, x2, x3))$, d=2, b=3

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=2, b=3

$Q\beta j = \begin{pmatrix} 0 & 0 & \Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ \Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}$, d=3, b=1

$Rj\lambda\alpha\beta Q\beta j = -\Delta p\,\Delta v1 \sin(\theta(x1, x2, x3))$, d=3, b=1

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = -\Delta p\,\Delta v1 \sin(\theta(x1, x2, x3))$, d=3, b=1

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta p\,\Delta v2 \\ 0 & \Delta p\,\Delta v2 & 0 \end{pmatrix}$, d=3, b=2

RjλαβQβj = $-\Delta p\,\Delta v2\cos^2(\theta(x1, x2, x3))$, d = 3, b = 2

VdVbϵjλζϵαβγQβj = 0, d = 3, b = 2

Qβj = $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2\,\Delta p^2 \end{pmatrix}$, d = 3, b = 3

RjλαβQβj = $2\,\Delta p^2 \sin^2(\theta(x1, x2, x3))$, d = 3, b = 3

VdVbϵjλζϵαβγQβj = 0, d = 3, b = 3

RdbλαQdb =
$-\Delta v1\,(2\,\Delta p + \Delta v2)\sin(\theta(x1, x2, x3)) - \Delta p\,(\Delta p + \Delta v2)\cos(2\,\theta(x1, x2, x3)) + \Delta p^2 + 2\,\Delta v1^2$, α = 2, λ = 2

VdVbϵλϵαQbd = $-2\,\Delta p\,\Delta v1\sin(\theta(x1, x2, x3))$, α = 2, λ = 2

## Ξ22 in Jaakko's parametrizetion :

Out[88]//TraditionalForm=

$$c1\Big(\sin(\theta(x1,x2,x3))\Big(2\,K1\cos(\theta(x1,x2,x3))\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)+K1\sin(\theta(x1,x2,x3))\Big(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big)\Delta p^2+(K2+K3)\,\Delta v2\sin(2\,\theta(x1,x2,x3))\,\delta s^{(0,1,1)}(x1,x2,x3)\,\Delta p+\Delta v1^2\Big(K1\,\delta s^{(0,2,0)}(x1,x2,x3)+(K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)+\Delta v2^2\cos(\theta(x1,x2,x3))\Big(\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)-2\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)\Big)\Big)+$$

$$c2\left(-\Delta v1\,(2\,\Delta p+\Delta v2)\sin(\theta(x1,x2,x3))-\Delta p\,(\Delta p+\Delta v2)\cos(2\,\theta(x1,x2,x3))+\Delta p^2+2\,\Delta v1^2\right)-$$

2
c2
Δp
Δv1
sin(θ(x1, x2, x3))

In[106]:=
```
(* The following Calculations are done by
  using Jaakko's d and e² vectors parametrizations ,
which was proven is lower energy in  equlibrium state.
       The parametrization  is eα² = -cosθ ẑ -sinθ ŷ
           dα = cosθ ŷ -sinθ ẑ *)
(*** Ξ32,α=3,λ=2 ***)
Clear["context`*"];
Clear[Ξ32, c1, c2, gD, γ, χv, α, λ, V, V1, V2, V3, Kij, K1, K1ij, K2, K2ij,
    K3, K3ij, Qβj, Q1βj, Q2βj, Δp, Δv1, Δv2, X, θ, x1, x2, x3 , b, a, d, Va, Vb,
    Vd, Vj, Vβ, Xa, Xb, Xd, Λij33, Λij331, Λij332, Λij333, Λij334, Λij335, Pai1,
    Pai2, Pai3, Paj1, Paj2, Paj3, Pai, Paj, PaiVb, PajVb, KijΛijαλ, KaΛaαλ,
    KbaΛbaαλ, RjλαβQβj, VdVbϵjλζϵαβγ, VdVbϵjλζϵαβγQβj, LeviCivita, ϵijk];
α =
  3;
λ = 2;
KaΛaαλ = 0;
KbaΛbaαλ = 0;
```

```
RbλαQb = 0;
RdbλαQdb = 0;
VVbϵλϵαQb = 0;
VdVbϵλϵαQbd = 0;
V1 = {1, 0, 0};
V2 = {0, -Sin[θ[x1, x2, x3]], -Cos[θ[x1, x2, x3]]};
V3 = {0, Cos[θ[x1, x2, x3]], -Sin[θ[x1, x2, x3]]};
(**************************************************)
(*V tensor for d e^1 e^2*)
V = {V1, V2, V3};
(*TreeForm [V]*)(*Level[V,{2}][[1]]*)
(**************************************************)
(*X tensor for X̄^1 X̄^2 X̄^3*)
X = {{Δv1, 0, 0}, {0, Δv2, 0}, {0, 0, Δp}};
(*X_i^a tensor symbol  Row is a index and colum  is spatial index *)
(**************************************************)
(**  Sumover  ba **)
(*****************)
(*K tensor K_ij^ba*)
Do[Do[
    Xb = Level[X, {1}][[b]]; Xa = Level[X, {1}][[a]];
    K1ij = K1 Array[KroneckerDelta, {3, 3}] (Tr[({Xb})ᵀ.{Xa}]);
    K2ij = K2 (({Xb})ᵀ.{Xa});
    K3ij = K3 (({Xa})ᵀ.{Xb});
    Kij = K1ij + K2ij + K3ij;
    Print[Style["Kij=", Red, 12], Kij // MatrixForm ,
      "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    (*******************************************************)
    (* Λ_ij33^ba *)
    Vb = Level[V, {1}][[b]]; Va = Level[V, {1}][[a]];
    Pai = Grad[δs[x1, x2, x3], {x1, x2, x3}];
    Paj = Grad[δs[x1, x2, x3], {x1, x2, x3}];
    PaiPaj = Grad[Grad[δs[x1, x2, x3], {x1, x2, x3}], {x1, x2, x3}];
    (********** the symbol  Λijxx(1..5) means  the 1st to 5th terms  of Λijxx,
      where xx is αλ *********)
    Λij331= (Grad[Grad[Vb[[λ]], {x1, x2, x3}], {x1, x2, x3}]) Va[[α]];
    Λij331= Λij111δs[x1, x2, x3];
    (**********)
    (*Λij112=
        (FullSimplify [Vb.Va,Assumptions →{{x1,x2,x3,θ}∈Reals}]-Vb[[1]] Va[[1]])
          (({Pai})ᵀ.{Paj});*)
    Λij332= (FullSimplify [Vb.Va, Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
              KroneckerDelta[α, λ] - Vb[[α]] Va[[λ]]) (PaiPaj);
    (*Λij332//MatrixForm *)
    (**********)
    PaiVb= (Grad[Vb, {x1, x2, x3}])ᵀ;
    Λij333= {(FullSimplify [(PaiVb).(Va), Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
```

```
                  KroneckerDelta[α, λ] -
                 ((Grad[Vb[[α]], {x1, x2, x3}]) (Va[[λ]])))}ᵀ.{Paj};
    (***********)
    PajVb = (Grad[Vb, {x1, x2, x3}])ᵀ;
    Λij334 =
      ({Pai})ᵀ.{(FullSimplify [(PajVb).(Va), Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
                 KroneckerDelta[α, λ] - ((Grad[Vb[[α]], {x1, x2, x3}]) (Va[[λ]])))};
    (***********)
    (*b=1;a=1;Vb=Level[V,{2}][[b]];Va=Level[V,{2}][[a]];*)
    Λij335 = -(Grad[Grad[Vb[[α]], {x1, x2, x3}], {x1, x2, x3}]) Va[[λ]];
    Λij335 = Λij115δs[x1, x2, x3];
    (***********)
    Λij33 = Λij331+Λij332+Λij333+Λij334+Λij335;
    FullSimplify [Λij33, Assumptions → {{{x1, x2, x3, θ, K1, K2, K3} ∈ Reals}}];
    (***********************************************)
    KijΛijαλ= FullSimplify [Tr[Kij.(Λij33ᵀ)],
        Assumptions → {{x1, x2, x3, θ, K1, K2, K3} ∈ Reals}];
    Print[Style["KijΛijαλFull=", Red, 12], KijΛijαλ // TraditionalForm ,
      "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    Print[Style["KijΛijαλ=", Red, 12],
      (KijΛijαλ /. {δs^(0,0,1)[x1, x2, x3] → 0, θ^(0,0,1)[x1, x2, x3] → 0,
              δs^(0,0,2)[x1, x2, x3] → 0, δs^(1,0,1)[x1, x2, x3] → 0}) // TraditionalForm ,
      "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    KaΛaαλ = KaΛaαλ + FullSimplify [KijΛijαλ /. {δs^(0,0,1)[x1, x2, x3] → 0,
              θ^(0,0,1)[x1, x2, x3] → 0, δs^(0,0,2)[x1, x2, x3] → 0, δs^(1,0,1)[x1, x2, x3] → 0},
          Assumptions → {{x1, x2, x3, θ} ∈ Reals}];
    , {a, 3}];
  KbaΛbaαλ = KbaΛbaαλ + KaΛaαλ; KaΛaαλ = 0;, {b, 3}];
KbaΛbaαλ =
  FullSimplify [KbaΛbaαλ, Assumptions → {{x1, x2, x3, θ} ∈ Reals}] // TraditionalForm
(***********************************************)
(***********************************************)
LeviCivita[i_, j_, k_] := Module[{test1 = (i == j || j == k || i == k), test2 =
          ((i == 1 && j == 2 && k == 3) || (i == 2 && j == 3 && k == 1) || (i == 3 && j == 1 && k == 2))},
        If[test1, ϵijk = 0;, If[test2, ϵijk = 1;, ϵijk = -1;];]; ϵijk];
Do[Do[(* Q tensor Q_βj^bd *)
      Xb = Level[X, {1}][[b]]; Xd = Level[X, {1}][[d]];
      Q1βj = ({Xb})ᵀ.{Xd};
      Q2βj = ({Xd})ᵀ.{Xb};
      Qβj = (Q1βj + Q2βj);
      Print[Style["Qβj=", Red, 12], Qβj // MatrixForm ,
        ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
      (***********************************************)
      (* R^db_jλαβ *)
      Vd = Level[V, {1}][[d]]; Vb = Level[V, {1}][[b]];
```

```
        Rjλαβ = (({Vd})ᵀ.{Vb}) KroneckerDelta[λ, α] -
             (({Vd})ᵀ (Vb[[α]])).{Table[KroneckerDelta[λ, β], {β, 1, 3, 1}]};
        (*Print["Rjλαβ=",Rjλαβ//MatrixForm ,",",",","d=","d",",",",","b=",b];*)
        RjλαβQβj = FullSimplify [Tr[Rjλαβ.Qβj], Assumptions →
               {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
        Print[Style["RjλαβQβj=", Red, 12], RjλαβQβj // TraditionalForm ,
          ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
        RbλαQb = RbλαQb + RjλαβQβj;
        (*********************************************************)
        (* Vdζ Vbγ ϵjλζ ϵαβγ Qbdβj *)
        (*****************************)
        Vj = Table[Sum [Vd[[ξ]] LeviCivita[j, λ, ξ], {ξ, 3}], {j, 3}];
        Vβ = Table[Sum [Vb[[γ]] LeviCivita[α, β, γ], {γ, 3}], {β, 3}];
        VdVbϵjλξϵαβγ = ({Vj})ᵀ.{Vβ};
        VdVbϵjλξϵαβγQβj = FullSimplify [Tr[VdVbϵjλξϵαβγ.Qβj], Assumptions →
               {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
        Print[Style["VdVbϵjλξϵαβγQβj=", Red, 12], VdVbϵjλξϵαβγQβj // TraditionalForm ,
          ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
        VVbϵλϵαQb = VVbϵλϵαQb + VdVbϵjλξϵαβγQβj;, {b, 3}];
     RdbλαQdb = RdbλαQdb + RbλαQb;
     RbλαQb = 0;
     VdVbϵλϵαQbd = VdVbϵλϵαQbd + VVbϵλϵαQb;
     VVbϵλϵαQb = 0;, {d, 3}];
(*********** Show results ***********)
RdbλαQdb = FullSimplify [RdbλαQdb,
        Assumptions →{{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
VdVbϵλϵαQbd = FullSimplify [VdVbϵλϵαQbd,
        Assumptions →{{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
Print[Style["RdbλαQdb=", Red, 12], RdbλαQdb // TraditionalForm , ",",
     Style["α=", Red, 12], α, ",", Style["λ=", Red, 12], λ];
Print[Style["VdVbϵλϵαQbd=", Red, 12], VdVbϵλϵαQbd // TraditionalForm ,
     ",", Style["α=", Red, 12], α, ",", Style["λ=", Red, 12], λ];
(*************************************)
(***** Ξ11= γ²/χv KbaΛbaαλ + 6 gD γ2/(5 χv) RdbλαQdb + 6 gD γ²/(5 χv) VdVbϵλϵαQbd ******)
(** c1=γ²/χv c2=6 gD γ2/(5 χv) **)
Print[Style[" Ξ32 in Jaakko's parametrizetion : ", Red, 18]]
Ξ32 = (c1 KbaΛbaαλ + c2 RdbλαQdb + c2 VdVbϵλϵαQbd);
Ξ32 // TraditionalForm
```

$$\text{Kij}=\begin{pmatrix} K1\,\Delta v1^2+K2\,\Delta v1^2+K3\,\Delta v1^2 & 0 & 0 \\ 0 & K1\,\Delta v1^2 & 0 \\ 0 & 0 & K1\,\Delta v1^2 \end{pmatrix}, \text{b}=1, \text{a}=1$$

KijΛijαλFull=0, b=1, a=1

KijΛijαλ=0, b=1, a=1

$$\text{Kij}=\begin{pmatrix} 0 & K2\,\Delta v1\,\Delta v2 & 0 \\ K3\,\Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{b}=1, \text{a}=2$$

KijΛijαλFull=0, b=1, a=2

KijΛijαλ=0, b=1, a=2

$$Kij=\begin{pmatrix} 0 & 0 & K2\,\Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ K3\,\Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}, b=1, a=3$$

KijΛijαλFull=0, b=1, a=3

KijΛijαλ=0, b=1, a=3

$$Kij=\begin{pmatrix} 0 & K3\,\Delta v1\,\Delta v2 & 0 \\ K2\,\Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, b=2, a=1$$

KijΛijαλFull=0, b=2, a=1

KijΛijαλ=0, b=2, a=1

$$Kij=\begin{pmatrix} K1\,\Delta v2^2 & 0 & 0 \\ 0 & K1\,\Delta v2^2+K2\,\Delta v2^2+K3\,\Delta v2^2 & 0 \\ 0 & 0 & K1\,\Delta v2^2 \end{pmatrix}, b=2, a=2$$

$$KijΛijαλFull=\Delta v2^2 \sin(\theta(x1,x2,x3)) \Big(2\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)-\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(0,0,2)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big), b=2, a=2$$

$$KijΛijαλ=\Delta v2^2 \sin(\theta(x1,x2,x3)) \Big(2\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)-\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big), b=2, a=2$$

$$Kij=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K2\,\Delta p\,\Delta v2 \\ 0 & K3\,\Delta p\,\Delta v2 & 0 \end{pmatrix}, b=2, a=3$$

$$KijΛijαλFull=\Delta p\,\Delta v2\,(-(K2+K3))\cos(\theta(x1,x2,x3))\Big(\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)\Big)\sin(\theta(x1,x2,x3))-\delta s^{(0,1,1)}(x1,x2,x3)\cos(\theta(x1,x2,x3))\Big), b=2, a=3$$

$$KijΛijαλ=\Delta p\,\Delta v2\,(K2+K3)\,\delta s^{(0,1,1)}(x1,x2,x3)\cos^2(\theta(x1,x2,x3)), b=2, a=3$$

$$Kij=\begin{pmatrix} 0 & 0 & K3\,\Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ K2\,\Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}, b=3, a=1$$

KijΛijαλFull=0, b=3, a=1

KijΛijαλ=0, b=3, a=1

$$Kij=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K3\,\Delta p\,\Delta v2 \\ 0 & K2\,\Delta p\,\Delta v2 & 0 \end{pmatrix}, b=3, a=2$$

$$KijΛijαλFull=\Delta p\,\Delta v2\,(-(K2+K3))\sin(\theta(x1,x2,x3))\Big(\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)\Big)\cos(\theta(x1,x2,x3))+\delta s^{(0,1,1)}(x1,x2,x3)\sin(\theta(x1,x2,x3))\Big), b=3, a=2$$

$$KijΛijαλ=\Delta p\,\Delta v2\,(-(K2+K3))\,\delta s^{(0,1,1)}(x1,x2,x3)\sin^2(\theta(x1,x2,x3)), b=3, a=2$$

$$Kij=\begin{pmatrix} K1\,\Delta p^2 & 0 & 0 \\ 0 & K1\,\Delta p^2 & 0 \\ 0 & 0 & K1\,\Delta p^2+K2\,\Delta p^2+K3\,\Delta p^2 \end{pmatrix}, b=3, a=3$$

$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full}=\Delta p^2 \cos(\theta(x1,x2,x3)) \Big(2\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3) + K1\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3) + \delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)\Big) + \sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,0,2)}(x1,x2,x3) + K1\Big(\delta s^{(0,2,0)}(x1,x2,x3) + \delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big)\Big), b=3, a=3$

$\text{Kij}\Lambda\text{ij}\alpha\lambda=\Delta p^2 \cos(\theta(x1,x2,x3)) \Big(2\,K1\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3) + \delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)\cos(\theta(x1,x2,x3)) + K1\Big(\delta s^{(0,2,0)}(x1,x2,x3) + \delta s^{(2,0,0)}(x1,x2,x3)\Big)\sin(\theta(x1,x2,x3))\Big), b=3, a=3$

Out[114]//TraditionalForm=

$$2\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)\left(\Delta v2^2\,(K1+K2+K3)\sin^2(\theta(x1,x2,x3)) + \Delta p^2\,K1\cos^2(\theta(x1,x2,x3))\right) + \frac{1}{2}\sin(2\,\theta(x1,x2,x3))\left(\delta s^{(0,2,0)}(x1,x2,x3)\left(\Delta p^2\,K1 - \Delta v2^2\,(K1+K2+K3)\right) + K1\,(\Delta p-\Delta v2)\,(\Delta p+\Delta v2)\,\delta s^{(2,0,0)}(x1,x2,x3)\right) + \cos(2\,\theta(x1,x2,x3))\left(K1\,(\Delta p-\Delta v2)\,(\Delta p+\Delta v2)\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3) + \Delta p\,\Delta v2\,(K2+K3)\,\delta s^{(0,1,1)}(x1,x2,x3)\right) + K1\left(\Delta p^2+\Delta v2^2\right)\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)$$

$Q\beta j=\begin{pmatrix} 2\Delta v1^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, d=1, b=1$

$Rj\lambda\alpha\beta Q\beta j=0, d=1, b=1$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j=0, d=1, b=1$

$Q\beta j=\begin{pmatrix} 0 & \Delta v1\,\Delta v2 & 0 \\ \Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, d=1, b=2$

$Rj\lambda\alpha\beta Q\beta j=\Delta v1\,\Delta v2\cos(\theta(x1,x2,x3)), d=1, b=2$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j=0, d=1, b=2$

$Q\beta j=\begin{pmatrix} 0 & 0 & \Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ \Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}, d=1, b=3$

$Rj\lambda\alpha\beta Q\beta j=0, d=1, b=3$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j=-\Delta p\,\Delta v1\cos(\theta(x1,x2,x3)), d=1, b=3$

$Q\beta j=\begin{pmatrix} 0 & \Delta v1\,\Delta v2 & 0 \\ \Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, d=2, b=1$

$Rj\lambda\alpha\beta Q\beta j=0, d=2, b=1$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j=\Delta v1\,\Delta v2\cos(\theta(x1,x2,x3)), d=2, b=1$

$Q\beta j=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2\Delta v2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, d=2, b=2$

$Rj\lambda\alpha\beta Q\beta j=-\Delta v2^2\sin(2\,\theta(x1,x2,x3)), d=2, b=2$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j=0, d=2, b=2$

$Q\beta j=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta p\,\Delta v2 \\ 0 & \Delta p\,\Delta v2 & 0 \end{pmatrix}, d=2, b=3$

$Rj\lambda\alpha\beta Q\beta j = -\frac{1}{2}\Delta p\,\Delta v2\,\sin(2\,\theta(x1, x2, x3))$, d=2, b=3

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=2, b=3

$Q\beta j = \begin{pmatrix} 0 & 0 & \Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ \Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}$, d=3, b=1

$Rj\lambda\alpha\beta Q\beta j = 0$, d=3, b=1

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=3, b=1

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta p\,\Delta v2 \\ 0 & \Delta p\,\Delta v2 & 0 \end{pmatrix}$, d=3, b=2

$Rj\lambda\alpha\beta Q\beta j = -\frac{1}{2}\Delta p\,\Delta v2\,\sin(2\,\theta(x1, x2, x3))$, d=3, b=2

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=3, b=2

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2\,\Delta p^2 \end{pmatrix}$, d=3, b=3

$Rj\lambda\alpha\beta Q\beta j = 0$, d=3, b=3

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0$, d=3, b=3

$Rdb\lambda\alpha Qdb = \Delta v1\,\Delta v2\,\cos(\theta(x1, x2, x3)) - \Delta v2\,(\Delta p + \Delta v2)\,\sin(2\,\theta(x1, x2, x3))$, $\alpha$=3, $\lambda$=2

$VdVb\epsilon\lambda\epsilon\alpha Qbd = \Delta v1\,(\Delta v2 - \Delta p)\,\cos(\theta(x1, x2, x3))$, $\alpha$=3, $\lambda$=2

$\Xi_{32}$ in Jaakko's parametrizetion :

Out[122]//TraditionalForm=

$$c1\Big(2\left(K1\,\Delta p^2\cos^2(\theta(x1,x2,x3)) + (K1+K2+K3)\,\Delta v2^2\sin^2(\theta(x1,x2,x3))\right)\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3) + K1\left(\Delta p^2+\Delta v2^2\right)\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3) + \cos(2\,\theta(x1,x2,x3))\Big((K2+K3)\,\Delta p\,\Delta v2\,\delta s^{(0,1,1)}(x1,x2,x3) + K1\,(\Delta p-\Delta v2)\,(\Delta p+\Delta v2)\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big) + \frac{1}{2}\sin(2\,\theta(x1,x2,x3))\Big(\left(K1\,\Delta p^2-(K1+K2+K3)\,\Delta v2^2\right)\delta s^{(0,2,0)}(x1,x2,x3) + K1\,(\Delta p-\Delta v2)\,(\Delta p+\Delta v2)\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big) + c2\,\Delta v1\,(\Delta v2-\Delta p)\cos(\theta(x1,x2,x3)) + c2\,(\Delta v1\,\Delta v2\cos(\theta(x1,x2,x3)) - \Delta v2\,(\Delta p+\Delta v2)\sin(2\,\theta(x1,x2,x3)))$$

```
(* The following Calculations are done by
  using Jaakko's d and e² vectors parametrizations ,
which was proven is lower energy in  equlibrium  state.
       The parametrization  is eα² = -cosθ ẑ -sinθ ŷ
           dα = cosθ ŷ -sinθ ẑ *)
(*** Ξ23,α=2,λ=3 ***)
Clear["context`*"];
Clear[Ξ23, c1, c2, gD, γ, χv, α, λ, V, V1, V2, V3, Kij, K1, K1ij, K2, K2ij,
    K3, K3ij, Qβj, Q1βj, Q2βj, Δp, Δv1, Δv2, X, θ, x1, x2, x3 , b, a, d, Va, Vb,
    Vd, Vj, Vβ, Xa, Xb, Xd, Λij33, Λij331, Λij332, Λij333, Λij334, Λij335, Pai1,
    Pai2, Pai3, Paj1, Paj2, Paj3, Pai, Paj, PaiVb, PajVb, KijΛijαλ, KaΛaαλ,
    KbaΛbaαλ, RjλαβQβj, VdVbϵjλζϵαβγ, VdVbϵjλζϵαβγQβj, LeviCivitą ϵijk];
```

```
α =
  2;
λ = 3;
KaΛaαλ = 0;
KbaΛbaαλ = 0;
RbλαQb = 0;
RdbλαQdb = 0;
VVbϵλϵαQb = 0;
VdVbϵλϵαQbd = 0;
V1 = {1, 0, 0};
V2 = {0, -Sin[θ[x1, x2, x3]], -Cos[θ[x1, x2, x3]]};
V3 = {0, Cos[θ[x1, x2, x3]], -Sin[θ[x1, x2, x3]]};
(***************************************************)
(*V tensor for d e^1 e^2*)
V = {V1, V2, V3};
(*TreeForm [V]*)(*Level[V,{2}][[1]]*)
(***************************************************)
(*X tensor for X̄^1 X̄^2 X̄^3*)
X = {{Δv1, 0, 0}, {0, Δv2, 0}, {0, 0, Δp}};
(*X_i^a tensor symbol  Row is a index and colum  is spatial index *)
(***************************************************)
(**  Sumover  ba **)
(*****************)
(*K tensor K_ij^ba*)
Do[Do[
    Xb = Level[X, {1}][[b]]; Xa = Level[X, {1}][[a]];
    K1ij = K1 Array[KroneckerDelta, {3, 3}] (Tr[({Xb})ᵀ.{Xa}]);
    K2ij = K2 (({Xb})ᵀ.{Xa});
    K3ij = K3 (({Xa})ᵀ.{Xb});
    Kij = K1ij + K2ij + K3ij;
    Print[Style["Kij=", Red, 12], Kij // MatrixForm ,
      "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    (*******************************************************)
    (* Λ_ij33^ba *)
    Vb = Level[V, {1}][[b]]; Va = Level[V, {1}][[a]];
    Pai = Grad[δs[x1, x2, x3], {x1, x2, x3}];
    Paj = Grad[δs[x1, x2, x3], {x1, x2, x3}];
    PaiPaj = Grad[Grad[δs[x1, x2, x3], {x1, x2, x3}], {x1, x2, x3}];
    (********** the symbol  Λijxx(1..5) means  the 1st to 5th terms  of Λijxx,
      where xx is αλ *********)
    Λij331 = (Grad[Grad[Vb[[λ]], {x1, x2, x3}], {x1, x2, x3}]) Va[[α]];
    Λij331 = Λij111δs[x1, x2, x3];
    (**********)
    (*Λij112=
        (FullSimplify [Vb.Va,Assumptions →{{x1,x2,x3,θ}ϵReals}]-Vb[[1]] Va[[1]])
          (({Pai})ᵀ.{Paj});*)
    Λij332 = (FullSimplify [Vb.Va, Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
```

```
            KroneckerDelta[α, λ] - Vb[[α]] Va[[λ]]) (PaiPaj);
    (*Λij332//MatrixForm *)
    (**********)
    PaiVb= (Grad[Vb, {x1, x2, x3}])ᵀ;
    Λij333= {(FullSimplify [(PaiVb).(Va), Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
                KroneckerDelta[α, λ] -
              ((Grad[Vb[[α]], {x1, x2, x3}]) (Va[[λ]])))}ᵀ.{Paj};
    (***********)
    PajVb = (Grad[Vb, {x1, x2, x3}])ᵀ;
    Λij334=
      ({Pai})ᵀ.{(FullSimplify [(PajVb).(Va), Assumptions → {{x1, x2, x3, θ} ∈ Reals}]
              KroneckerDelta[α, λ] - ((Grad[Vb[[α]], {x1, x2, x3}]) (Va[[λ]])))};
    (***********)
    (*b=1;a=1;Vb=Level[V,{2}][[b]];Va=Level[V,{2}][[a]];*)
    Λij335= -(Grad[Grad[Vb[[α]], {x1, x2, x3}], {x1, x2, x3}]) Va[[λ]];
    Λij335= Λij115δs[x1, x2, x3];
    (***********)
    Λij33= Λij331+Λij332+Λij333+Λij334+Λij335;
    FullSimplify [Λij33, Assumptions → {{{x1, x2, x3, θ, K1, K2, K3} ∈ Reals}}];
    (***********************************************)
    KijΛijαλ= FullSimplify [Tr[Kij.(Λij33ᵀ)],
        Assumptions → {{x1, x2, x3, θ, K1, K2, K3} ∈ Reals}];
    Print[Style["KijΛijαλFull=", Red, 12], KijΛijαλ// TraditionalForm ,
      "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    Print[Style["KijΛijαλ=", Red, 12],
      (KijΛijαλ/. {δs^(0,0,1)[x1, x2, x3] → 0, θ^(0,0,1)[x1, x2, x3] → 0,
            δs^(0,0,2)[x1, x2, x3] → 0, δs^(1,0,1)[x1, x2, x3] → 0}) // TraditionalForm ,
      "," Style["b=", Red, 12], b, ",", Style["a=", Red, 12], a];
    KaΛaαλ = KaΛaαλ + FullSimplify [KijΛijαλ/. {δs^(0,0,1)[x1, x2, x3] → 0,
            θ^(0,0,1)[x1, x2, x3] → 0, δs^(0,0,2)[x1, x2, x3] → 0, δs^(1,0,1)[x1, x2, x3] → 0},
         Assumptions → {{x1, x2, x3, θ} ∈ Reals}];
    , {a, 3}];
  KbaΛbaαλ = KbaΛbaαλ + KaΛaαλ; KaΛaαλ = 0;, {b, 3}];
KbaΛbaαλ =
  FullSimplify [KbaΛbaαλ, Assumptions → {{x1, x2, x3, θ} ∈ Reals}] // TraditionalForm
(***********************************************)
(***********************************************)
LeviCivita[i_, j_, k_] := Module[{test1 = (i == j || j == k || i == k), test2 =
        ((i == 1 && j == 2 && k == 3) || (i == 2 && j == 3 && k == 1) || (i == 3 && j == 1 && k == 2))},
      If[test1, ϵijk= 0;, If[test2, ϵijk= 1;, ϵijk= -1;];]; ϵijk];
Do[Do[(* Q tensor Qβj^bd*)
      Xb = Level[X, {1}][[b]]; Xd = Level[X, {1}][[d]];
      Q1βj = ({Xb})ᵀ.{Xd};
      Q2βj = ({Xd})ᵀ.{Xb};
      Qβj = (Q1βj + Q2βj);
```

```
        Print[Style["Qβj=", Red, 12], Qβj // MatrixForm ,
          ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
        (*************************************************)
        (* R^db_jλαβ *)
        Vd = Level[V, {1}][[d]]; Vb = Level[V, {1}][[b]];
        Rjλαβ = (({Vd})ᵀ.{Vb}) KroneckerDelta[λ, α] -
            (({Vd})ᵀ (Vb[[α]])).{Table[KroneckerDelta[λ, β], {β, 1, 3, 1}]};
        (*Print["Rjλαβ=",Rjλαβ//MatrixForm ,",","d=","d",",","b=",b];*)
        RjλαβQβj = FullSimplify [Tr[Rjλαβ.Qβj], Assumptions →
              {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
        Print[Style["RjλαβQβj=", Red, 12], RjλαβQβj // TraditionalForm ,
          ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
        RbλαQb = RbλαQb + RjλαβQβj;
        (**********************************************************)
        (* V^d_ζ V^b_γ ε_jλζ ε_αβγ Q^bd_βj *)
        (******************************)
        Vj = Table[Sum [Vd[[ξ]] LeviCivita[j, λ, ξ], {ξ, 3}], {j, 3}];
        Vβ = Table[Sum [Vb[[γ]] LeviCivita[α, β, γ], {γ, 3}], {β, 3}];
        VdVbεjλξεαβγ = ({Vj})ᵀ.{Vβ};
        VdVbεjλξεαβγQβj = FullSimplify [Tr[VdVbεjλξεαβγ.Qβj], Assumptions →
              {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
        Print[Style["VdVbεjλξεαβγQβj=", Red, 12], VdVbεjλξεαβγQβj // TraditionalForm ,
          ",", Style["d=", Red, 12], d, ",", Style["b=", Red, 12], b];
        VVbελεαQb = VVbελεαQb + VdVbεjλξεαβγQβj;, {b, 3}];
      RdbλαQdb = RdbλαQdb + RbλαQb;
      RbλαQb = 0;
      VdVbελεαQbd = VdVbελεαQbd + VVbελεαQb;
      VVbελεαQb = 0;, {d, 3}];
(*********** Show results ***********)
RdbλαQdb = FullSimplify [RdbλαQdb,
        Assumptions →{{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
VdVbελεαQbd = FullSimplify [VdVbελεαQbd,
        Assumptions →{{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals && Δp > 0 && Δv1 > 0 && Δv2 > 0}];
Print[Style["RdbλαQdb=", Red, 12], RdbλαQdb // TraditionalForm , ",",
    Style["α=", Red, 12], α, ",", Style["λ=", Red, 12], λ];
Print[Style["VdVbελεαQbd=", Red, 12], VdVbελεαQbd // TraditionalForm ,
    ",", Style["α=", Red, 12], α, ",", Style["λ=", Red, 12], λ];
(*************************************)
(***** Ξ11= γ²/χv KbaΛbaαλ + (6 gD γ2)/(5 χv) RdbλαQdb + (6 gD γ²)/(5 χv) VdVbελεαQbd ******)
(** c1=γ²/χv c2=(6 gD γ2)/(5 χv) **)
Print[Style[" Ξ23 in Jaakko's parametrizetion : ", Red, 18]]
Ξ23 = (c1 KbaΛbaαλ + c2 RdbλαQdb + c2 VdVbελεαQbd);
Ξ23 // TraditionalForm
```

Kij= $\begin{pmatrix} K1\,\Delta v1^2 + K2\,\Delta v1^2 + K3\,\Delta v1^2 & 0 & 0 \\ 0 & K1\,\Delta v1^2 & 0 \\ 0 & 0 & K1\,\Delta v1^2 \end{pmatrix}$, b=1, a=1

KijΛijαλFull=0, b=1, a=1

KijΛijαλ=0, b=1, a=1

$$Kij=\begin{pmatrix} 0 & K2\,\Delta v1\,\Delta v2 & 0 \\ K3\,\Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, b=1, a=2$$

KijΛijαλFull=0, b=1, a=2

KijΛijαλ=0, b=1, a=2

$$Kij=\begin{pmatrix} 0 & 0 & K2\,\Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ K3\,\Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}, b=1, a=3$$

KijΛijαλFull=0, b=1, a=3

KijΛijαλ=0, b=1, a=3

$$Kij=\begin{pmatrix} 0 & K3\,\Delta v1\,\Delta v2 & 0 \\ K2\,\Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, b=2, a=1$$

KijΛijαλFull=0, b=2, a=1

KijΛijαλ=0, b=2, a=1

$$Kij=\begin{pmatrix} K1\,\Delta v2^2 & 0 & 0 \\ 0 & K1\,\Delta v2^2+K2\,\Delta v2^2+K3\,\Delta v2^2 & 0 \\ 0 & 0 & K1\,\Delta v2^2 \end{pmatrix}, b=2, a=2$$

$$KijΛijαλFull=-\Delta v2^2\cos(\theta(x1,x2,x3))\Big(2\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)+\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(0,0,2)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big), b=2, a=2$$

$$KijΛijαλ=-\Delta v2^2\cos(\theta(x1,x2,x3))\Big(2\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)+\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big), b=2, a=2$$

$$Kij=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K2\,\Delta p\,\Delta v2 \\ 0 & K3\,\Delta p\,\Delta v2 & 0 \end{pmatrix}, b=2, a=3$$

$$KijΛijαλFull=\Delta p\,\Delta v2\,(-(K2+K3))\sin(\theta(x1,x2,x3))\Big(\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)\Big)\cos(\theta(x1,x2,x3))+\delta s^{(0,1,1)}(x1,x2,x3)\sin(\theta(x1,x2,x3))\Big), b=2, a=3$$

$$KijΛijαλ=\Delta p\,\Delta v2\,(-(K2+K3))\,\delta s^{(0,1,1)}(x1,x2,x3)\sin^2(\theta(x1,x2,x3)), b=2, a=3$$

$$Kij=\begin{pmatrix} 0 & 0 & K3\,\Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ K2\,\Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}, b=3, a=1$$

KijΛijαλFull=0, b=3, a=1

KijΛijαλ=0, b=3, a=1

$$Kij=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & K3\,\Delta p\,\Delta v2 \\ 0 & K2\,\Delta p\,\Delta v2 & 0 \end{pmatrix}, b=3, a=2$$

$$KijΛijαλFull=\Delta p\,\Delta v2\,(-(K2+K3))\cos(\theta(x1,x2,x3))\Big(\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3)+\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)\Big)\sin(\theta(x1,x2,x3))-\delta s^{(0,1,1)}(x1,x2,x3)\cos(\theta(x1,x2,x3))\Big), b=3, a=2$$

$\text{Kij}\Lambda\text{ij}\alpha\lambda = \Delta p\,\Delta v2\,(K2+K3)\,\delta s^{(0,1,1)}(x1,x2,x3)\cos^2(\theta(x1,x2,x3)),\ b=3,\ a=2$

$$\text{Kij} = \begin{pmatrix} K1\,\Delta p^2 & 0 & 0 \\ 0 & K1\,\Delta p^2 & 0 \\ 0 & 0 & K1\,\Delta p^2 + K2\,\Delta p^2 + K3\,\Delta p^2 \end{pmatrix},\ b=3,\ a=3$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda\text{Full} = \Delta p^2 \sin(\theta(x1,x2,x3)) \Big(\cos(\theta(x1,x2,x3)) \Big((K1+K2+K3)\,\delta s^{(0,0,2)}(x1,x2,x3) + K1\left(\delta s^{(0,2,0)}(x1,x2,x3) + \delta s^{(2,0,0)}(x1,x2,x3)\right)\Big) - 2\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,0,1)}(x1,x2,x3)\,\theta^{(0,0,1)}(x1,x2,x3) + K1\left(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3) + \delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)\Big)\Big),\ b=3,\ a=3$$

$$\text{Kij}\Lambda\text{ij}\alpha\lambda = \Delta p^2 \sin(\theta(x1,x2,x3)) \Big(K1\left(\delta s^{(0,2,0)}(x1,x2,x3) + \delta s^{(2,0,0)}(x1,x2,x3)\right)\cos(\theta(x1,x2,x3)) - 2\,K1\left(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3) + \delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\right)\sin(\theta(x1,x2,x3))\Big),\ b=3,\ a=3$$

Out[131]//TraditionalForm=

$$-2\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)\left(\Delta v2^2\,(K1+K2+K3)\cos^2(\theta(x1,x2,x3)) + \Delta p^2\,K1\sin^2(\theta(x1,x2,x3))\right) + \frac{1}{2}\sin(2\,\theta(x1,x2,x3))\Big(\delta s^{(0,2,0)}(x1,x2,x3)\left(\Delta p^2\,K1 - \Delta v2^2\,(K1+K2+K3)\right) + K1\,(\Delta p - \Delta v2)\,(\Delta p + \Delta v2)\,\delta s^{(2,0,0)}(x1,x2,x3)\Big) + \cos(2\,\theta(x1,x2,x3))\Big(K1\,(\Delta p - \Delta v2)\,(\Delta p + \Delta v2)\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3) + \Delta p\,\Delta v2\,(K2+K3)\,\delta s^{(0,1,1)}(x1,x2,x3)\Big) - K1\left(\Delta p^2 + \Delta v2^2\right)\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)$$

$$Q\beta j = \begin{pmatrix} 2\,\Delta v1^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ d=1,\ b=1$$

$\text{Rj}\lambda\alpha\beta Q\beta j = 0,\ d=1,\ b=1$

$\text{VdVb}\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0,\ d=1,\ b=1$

$$Q\beta j = \begin{pmatrix} 0 & \Delta v1\,\Delta v2 & 0 \\ \Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ d=1,\ b=2$$

$\text{Rj}\lambda\alpha\beta Q\beta j = 0,\ d=1,\ b=2$

$\text{VdVb}\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = \Delta v1\,\Delta v2\cos(\theta(x1,x2,x3)),\ d=1,\ b=2$

$$Q\beta j = \begin{pmatrix} 0 & 0 & \Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ \Delta p\,\Delta v1 & 0 & 0 \end{pmatrix},\ d=1,\ b=3$$

$\text{Rj}\lambda\alpha\beta Q\beta j = -\Delta p\,\Delta v1\cos(\theta(x1,x2,x3)),\ d=1,\ b=3$

$\text{VdVb}\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0,\ d=1,\ b=3$

$$Q\beta j = \begin{pmatrix} 0 & \Delta v1\,\Delta v2 & 0 \\ \Delta v1\,\Delta v2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ d=2,\ b=1$$

$\text{Rj}\lambda\alpha\beta Q\beta j = 0,\ d=2,\ b=1$

$\text{VdVb}\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0,\ d=2,\ b=1$

$$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2\,\Delta v2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ d=2,\ b=2$$

$\text{Rj}\lambda\alpha\beta Q\beta j = 0,\ d=2,\ b=2$

$\text{VdVb}\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0,\ d=2,\ b=2$

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta p\,\Delta v2 \\ 0 & \Delta p\,\Delta v2 & 0 \end{pmatrix}, d=2, b=3$

$Rj\lambda\alpha\beta Q\beta j = \Delta p\,\Delta v2\,\sin(\theta(x1, x2, x3))\cos(\theta(x1, x2, x3)), d=2, b=3$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=2, b=3$

$Q\beta j = \begin{pmatrix} 0 & 0 & \Delta p\,\Delta v1 \\ 0 & 0 & 0 \\ \Delta p\,\Delta v1 & 0 & 0 \end{pmatrix}, d=3, b=1$

$Rj\lambda\alpha\beta Q\beta j = 0, d=3, b=1$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = -\Delta p\,\Delta v1\cos(\theta(x1, x2, x3)), d=3, b=1$

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta p\,\Delta v2 \\ 0 & \Delta p\,\Delta v2 & 0 \end{pmatrix}, d=3, b=2$

$Rj\lambda\alpha\beta Q\beta j = \Delta p\,\Delta v2\,\sin(\theta(x1, x2, x3))\cos(\theta(x1, x2, x3)), d=3, b=2$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=3, b=2$

$Q\beta j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2\,\Delta p^2 \end{pmatrix}, d=3, b=3$

$Rj\lambda\alpha\beta Q\beta j = \Delta p^2\sin(2\,\theta(x1, x2, x3)), d=3, b=3$

$VdVb\epsilon j\lambda\zeta\epsilon\alpha\beta\gamma Q\beta j = 0, d=3, b=3$

$Rdb\lambda\alpha Qdb = \Delta p\cos(\theta(x1, x2, x3))\,(2\,(\Delta p + \Delta v2)\sin(\theta(x1, x2, x3)) - \Delta v1), \alpha=2, \lambda=3$

$VdVb\epsilon\lambda\epsilon\alpha Qbd = \Delta v1\,(\Delta v2 - \Delta p)\cos(\theta(x1, x2, x3)), \alpha=2, \lambda=3$

## $\Xi_{23}$ in Jaakko's parametrizetion :

Out[139]//TraditionalForm=

$$
\begin{aligned}
&c1\Big(-2\left((K1 + K2 + K3)\,\Delta v2^2\cos^2(\theta(x1, x2, x3)) + K1\,\Delta p^2\sin^2(\theta(x1, x2, x3))\right)\delta s^{(0,1,0)}(x1, x2, x3)\,\theta^{(0,1,0)}(x1, x2, x3) - \\
&\quad K1\left(\Delta p^2 + \Delta v2^2\right)\delta s^{(1,0,0)}(x1, x2, x3)\,\theta^{(1,0,0)}(x1, x2, x3) + \\
&\quad \cos(2\,\theta(x1, x2, x3))\Big((K2 + K3)\,\Delta p\,\Delta v2\,\delta s^{(0,1,1)}(x1, x2, x3) + \\
&\qquad K1\,(\Delta p - \Delta v2)\,(\Delta p + \Delta v2)\,\delta s^{(1,0,0)}(x1, x2, x3)\,\theta^{(1,0,0)}(x1, x2, x3)\Big) + \\
&\quad \frac{1}{2}\sin(2\,\theta(x1, x2, x3))\Big(\left(K1\,\Delta p^2 - (K1 + K2 + K3)\,\Delta v2^2\right)\delta s^{(0,2,0)}(x1, x2, x3) + \\
&\qquad K1\,(\Delta p - \Delta v2)\,(\Delta p + \Delta v2)\,\delta s^{(2,0,0)}(x1, x2, x3)\Big)\Big) + \\
&c2\,\Delta v1\,(\Delta v2 - \Delta p)\cos(\theta(x1, x2, x3)) + c2\,\Delta p\cos(\theta(x1, x2, x3)) \\
&\quad (2\,(\Delta p + \Delta v2)\sin(\theta(x1, x2, x3)) - \Delta v1)
\end{aligned}
$$

In[144]:=
```
Print[Style["Ξ33+Ξ22=", Red, 18]]
FullSimplify[Ξ33 + Ξ22,
  Assumptions → Assumptions → {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals &&
      Δp > 0 && Δv1 > 0 && Δv2 > 0}] // TraditionalForm
```

Ξ33+Ξ22=

Out[145]//TraditionalForm=

$$
\begin{aligned}
&c1\Big(\Big(\sin(\theta(x1,x2,x3))\Big(2\,K1\cos(\theta(x1,x2,x3))\\
&\quad\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)+\\
&\quad K1\sin(\theta(x1,x2,x3))\Big(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big)\Delta p^2+\\
&\quad(K2+K3)\,\Delta v2\sin(2\,\theta(x1,x2,x3))\,\delta s^{(0,1,1)}(x1,x2,x3)\,\Delta p+\\
&\quad\Delta v1^2\Big(K1\,\delta s^{(0,2,0)}(x1,x2,x3)+(K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)+\Delta v2^2\cos(\theta(x1,x2,x3))\\
&\quad\Big(\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)-\\
&\quad 2\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\\
&\quad K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)\Big)\Big)+\\
&\Big(\cos(\theta(x1,x2,x3))\Big(K1\cos(\theta(x1,x2,x3))\Big(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\Big)-\\
&\quad 2\,K1\sin(\theta(x1,x2,x3))\\
&\quad\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)\Big)\Delta p^2-\\
&\quad(K2+K3)\,\Delta v2\sin(2\,\theta(x1,x2,x3))\,\delta s^{(0,1,1)}(x1,x2,x3)\,\Delta p+\\
&\quad\Delta v1^2\Big(K1\,\delta s^{(0,2,0)}(x1,x2,x3)+(K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)+\\
&\quad\Delta v2^2\sin(\theta(x1,x2,x3))\Big(2\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\\
&\quad\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)+\\
&\quad\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big)\Big)\Big)+\\
&c2\Big(-5\,\Delta v1\,(\Delta p+\Delta v2)\sin(\theta(x1,x2,x3))-(\Delta p+\Delta v2)^2\cos(2\,\theta(x1,x2,x3))+\\
&\quad\Delta p^2+\\
&\quad 4\,\Delta v1^2+\\
&\quad\Delta v2^2\Big)
\end{aligned}
$$

In[142]:=
```
Print[Style["Ξ32-Ξ23=", Red, 18]]
FullSimplify [Ξ33 + Ξ22,
    Assumptions → Assumptions → {{x1, x2, x3, θ, Δp, Δv1, Δv2} ∈ Reals &&
        Δp > 0 && Δv1 > 0 && Δv2 > 0}] // TraditionalForm
```

Ξ32-Ξ23=

Out[143]//TraditionalForm=

$$c1\Big(\Big(\sin(\theta(x1,x2,x3))\Big(2\,K1\cos(\theta(x1,x2,x3))\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)+K1\sin(\theta(x1,x2,x3))\Big(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big)\Delta p^2+(K2+K3)\,\Delta v2\sin(2\,\theta(x1,x2,x3))\,\delta s^{(0,1,1)}(x1,x2,x3)\,\Delta p+\Delta v1^2\Big(K1\,\delta s^{(0,2,0)}(x1,x2,x3)+(K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)+\Delta v2^2\cos(\theta(x1,x2,x3))\Big(\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)-2\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)\Big)\Big)+\Big(\cos(\theta(x1,x2,x3))\Big(K1\cos(\theta(x1,x2,x3))\Big(\delta s^{(0,2,0)}(x1,x2,x3)+\delta s^{(2,0,0)}(x1,x2,x3)\Big)-2\,K1\sin(\theta(x1,x2,x3))\Big(\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)\Big)\Delta p^2-(K2+K3)\,\Delta v2\sin(2\,\theta(x1,x2,x3))\,\delta s^{(0,1,1)}(x1,x2,x3)\,\Delta p+\Delta v1^2\Big(K1\,\delta s^{(0,2,0)}(x1,x2,x3)+(K1+K2+K3)\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)+\Delta v2^2\sin(\theta(x1,x2,x3))\Big(2\cos(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,1,0)}(x1,x2,x3)\,\theta^{(0,1,0)}(x1,x2,x3)+K1\,\delta s^{(1,0,0)}(x1,x2,x3)\,\theta^{(1,0,0)}(x1,x2,x3)\Big)+\sin(\theta(x1,x2,x3))\Big((K1+K2+K3)\,\delta s^{(0,2,0)}(x1,x2,x3)+K1\,\delta s^{(2,0,0)}(x1,x2,x3)\Big)\Big)\Big)\Big)+c2\Big(-5\,\Delta v1\,(\Delta p+\Delta v2)\sin(\theta(x1,x2,x3))-(\Delta p+\Delta v2)^2\cos(2\,\theta(x1,x2,x3))+\Delta p^2+4\,\Delta v1^2+\Delta v2^2\Big)$$