# CPSC 316 HOMEWORK ASSIGNMENT: ANALYSIS TOOLS

In all cases, "lg" refers to base-2 logarithms, "ln" refers to natural (base-*e*) logs and "log" refers to base-10 logs. Write out your answers legibly. Work that isn't organized or readable will not be accepted for grade.

1. For each expression, select a big-oh notation from this table.

| | | | |
|---|---|---|---|
| $O(1)$ | constant | $O(n^2)$ | quadratic |
| $O(\lg \lg n)$ | log log | $O(n^3)$ | cubic |
| $O(\lg n)$ | logarithmic | $O(n^m)$ | polynomial |
| $O(n)$ | linear | $O(m^n)$, $m \geq 2$ | exponential |
| $O(n \lg n)$ | n log n | $O(n!)$ | factorial |

   a. $6n+1$
   b. $3n^2+2n \lg n$
   c. $2 + 4 + 6 + \ldots + 2n$
   d. $(n+1)(n+3) / (n+2)$

2. Show that
   a. $\lfloor x + ½ \rfloor \in O(x)$
   b. $\log(x^2+1) \in O(\lg x)$
   c. $\log x \in O(\lg x)$

3. Let $f,g,h : \mathbb{Z}^+ \to \mathbb{R}$ be functions. Prove each of the following.

   a. If $f \in O(g)$ and $g \in O(h)$ prove that $f \in O(h)$.
   b. If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where $a_n,\ldots,a_0$ are positive real numbers and $a_n \neq 0$, then $f(x) \in O(x^n)$.
   c. If $f(x) = 1^k + 2^k + \ldots + x^k$, where $k$ is a positive integer, then $f(x) \in O(x^{k+1})$.

4. In each of the following pseudo code program segments, select a big-oh notation from the above table for the number of times the statement "*x*++" is executed.

a.
```
for i←1 to 2n do
    x++
```

b.
```
for i←1 to 2n do
    for j←1 to n do
        x++
```

c.
```
for i←1 to n do
    for j←1 to n² do
        x++
```

d.
```
for i←1 to n do
    for j←i to n do
        x++
```

e.
```
for i←1 to n do
    for j←1 to n do
        for k←1 to i do
            x++
```

f.
```
i←n
while i>0 do
    x++
    i ← ⌊i / 2⌋
```

g.
```
for i←1 to n do
    j←n
    while j>0 do
        x++
        j ← ⌊j / 2⌋
```