Gramatici regulate. Automate finite

March 26, 2019

Outline

Gramatici regulate 2. Expresii regulate

Analiza lexicala cu LEX

Regular expression - expresii regulate

Expresiile regulate descriu limbajele regulate

Fie V un vocabular si simbolurile $E, \varepsilon, +, *, (,) \notin V$.
Un string ρ peste $V \cup \{E, \varepsilon, +, *, (,)\}$ este o expresie regulata peste V daca

1. ρ este un simbol peste V sau unul dintre simbolurile E, ε, sau
2. ρ este de forma $(X + Y)$, (XY), $(X)^*$, unde X si Y sunt expresii regulate.

Descriere expresii regulate

- $L(E) = \emptyset$ este limbajul empty
- $L(\varepsilon) = \{\varepsilon\}$ este limbajul format din stringul empty
- $L(v), v \in V$ descrie limbajul $\{v\}$
- $L(\ (X+Y)\ ) = \{w | w \in X \ sau \ w \in Y\}$
- $L(\ (XY)\ ) = \{\chi\gamma | \chi \in X \ si \ \gamma \in Y\}$
- operatorul * inchidere (Kleene closure):

$$X^* = \varepsilon + X + XX + XXX + ...$$

expresii regulate 2

- Parantezele se pot omite
- $*$ este operator unar, cu prioritate mai mare decat oricare operator binar
- $+$ are prioritate mai mica decat concatenarea

$W + XY^*$ este echivalent cu $(W \; + \; (X \; (Y^*) \;) \;)$

Exemple expresii regulate

- $L(01) = \{01\}$

Exemple expresii regulate

- $L(01) = \{01\}$
- $L(01 + 0) = \{01, 0\}$ - in lex |

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- $L(01) = \{01\}$
- $L(01 + 0) = \{01, 0\}$ - in lex |
- $L(0(1 + 0)) = \{01, 00\}$

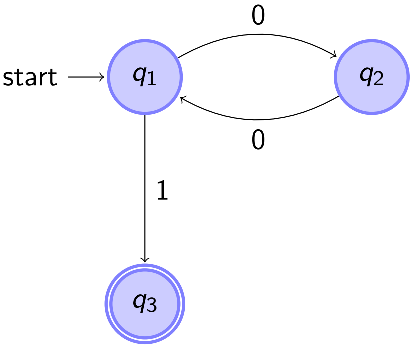
Exemple expresii regulate

- $L(01) = \{01\}$
- $L(01 + 0) = \{01, 0\}$ - in lex |
- $L(0(1 + 0)) = \{01, 00\}$
- $L(0^*) = \{\varepsilon, 0, 00, 000, ...\}$

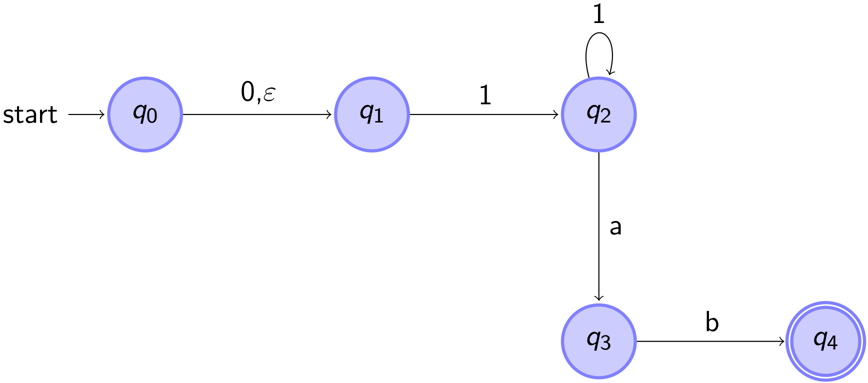
Exemple expresii regulate

- $L(01) = \{01\}$
- $L(01 + 0) = \{01, 0\}$ - in lex |
- $L(0(1 + 0)) = \{01, 00\}$
- $L(0^*) = \{\varepsilon, 0, 00, 000, ...\}$
- $L((0 + 10)^*(\varepsilon + 1))$ Toate stringurile de 0 si 1 fara doua consecutive 1

Exemplu $(00)^*1$



Exemplu $0?1^+ab$



Operatori aditionali: ? $+$

Nu permit definirea unor limbaje aditionale, dar permit exprimarea mai usoara a expresiilor regulate

- Operatorul optional: ?
  Daca R este o expresie regulata, $R? = \varepsilon + R$
- Operatorul $+$
  Daca R este o expresie regulata $R^+ = RR^*$:

$$L(R^+) = L(R) \cup L(RR) \cup L(RRR) \cup \ldots$$

Proprietati algebrice ale expresiilor regulate

$X + Y = Y + X$	*comutativitate*
$(X + Y) + Z = Z + (Y + Z)$	*asociativitate*
$X(YZ) = (XY)Z$	
$X(Y + Z) = XY + XZ$	*distributivitate*
$(X + Y)Z = XZ + YZ$	
$X + \emptyset = \emptyset + X = X$	*identitate*
$X\varepsilon = \varepsilon X$	
$X\emptyset = \emptyset X = X$	*zero*
$X + X = X$	*idempotenta*
$(X^*)^* = X^*$	
$X^* = \varepsilon + XX^*$	
$X^* = X + X^*$	
$\varepsilon^* = \varepsilon$	
$\emptyset^* = \varepsilon$	

Echivalenta expresii regulate - automate finite

Fie R o expresie regulata care descrie un subset $S \subseteq T^*$.
Exista un automat finit determinist $A = (T, Q, P, q_0, F)$ a.i. $L(A) = S$.

Construire automat

1. $R = l(l + d)^*$
2. $R' = 1(2 + 3)^*$ - o noua expresie in care se inlocuiesc elementele lui T din R cu simboluri distincte
   Aparitii multiple ale aceluiasi element - simboluri diferite
3. $R' = 01(2 + 3)^*$ - se adauga un prefix (un simbol distinct)
   Daca $R = E$ atunci R' este doar simbolul de start
4. starile automatului corespund submultimilor setului de simboluri.
5. Se inspecteaza pe rand starile lui lui Q si daca e necesar, se adauga stari noi:
   $pentru\ \forall q \in Q\ si\ \forall t \in T,\ fie\ q'$ corespondentul setului de simboluri din R':
   - care inlocuiesc pe t si
   - urmeaza unui simbol din setul corespunzator lui q

   Daca setul corespounzator lui q' nu e vid, se adauga $qt \rightarrow q'$ la P si se include q' in Q.
6. setul F de starile finale = toate starile care includ un simbol final posibil al lui R'

$l(l+d)^*$. $R' = 01(2+3)^*$, l is $1, 2$, d is 3

	l	d	
q_0	q_1		$\{0\}$
q_1			$\{1\}$

q_0 l: $\{1\ ,\ 2\}$, dar numai 1 urmeaza lui 0   deci $q_0 l \rightarrow q_1$
q_0 d: $\{3\}$, dar nu urmeaza lui 0   deci $q_1 d$ nu face parte dint

$l(l+d)^*$. $R' = 01(2+3)^*$, l is $1, 2$, d is 3

	l	d	
q_0	q_1		$\{0\}$
q_1	q_2		$\{1\}$
q_2			$\{2\}$

q_0 l: $\{1\ ,\ 2\}$, dar numai 1 urmeaza lui 0   deci $q_0 l \rightarrow q_1$

q_0 d: $\{3\}$, dar nu urmeaza lui 0   deci $q_1 d$ nu face parte dint

q_1 l: $\{1,\ 2\}$, dar numai 2 urmeaza lui 1   deci $q_1 l \rightarrow q_2$

$l(l+d)^*$. $R' = 01(2+3)^*$, l *is* $1, 2$, d *is* 3

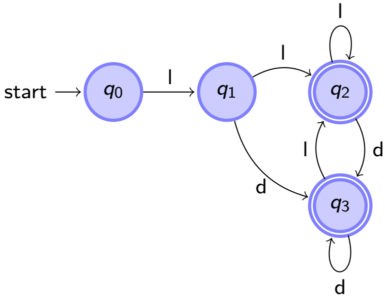
	l	d	
q_0	q_1		$\{0\}$
q_1	q_2	q_3	$\{1\}$
q_2	q_2	q_3	$\{2\}$
q_3	q_2	q_3	$\{3\}$

q_0 l: $\{1 , 2\}$, dar numai 1 urmeaza lui 0 deci $q_0 l \rightarrow q_1$
q_0 d: $\{3\}$, dar nu urmeaza lui 0 deci $q_1 d$ nu face parte din
q_1 l: $\{1, 2\}$, dar numai 2 urmeaza lui 1 deci $q_1 l \rightarrow q_2$
q_1 d: $\{3\}$ si 3 urmeaza lui 1 deci $q_1 d \rightarrow q_3$
q_2 l: $\{1, 2\}$, dar numai 2 urmeaza lui 2 deci $q_2 l \rightarrow q_2$
q_2 d: $\{3\}$, si 3 urmeaza lui 2 deci $q_2 d \rightarrow q_3$
q_3 l: $\{1, 2\}$, dar numai 2 urmeaza lui 3 - deci $q_3 l \rightarrow q_2$
q_3 d: $\{3\}$, si 3 urmeaza lui 3 - deci $q_3 d \rightarrow q_3$

Stari finale: q_2 si q_3

$l(l+d)^*$. $R' = 01(2+3)^*$, l is $1, 2$, d is 3

	l	d	
q_0	q_1		$\{0\}$
q_1	q_2	q_3	$\{1\}$
q_2	q_2	q_3	$\{2\}$
q_3	q_2	q_3	$\{3\}$



q_0 l: $\{1\ ,\ 2\}$, dar numai 1 urmeaza lui 0   deci $q_0 l \rightarrow q_1$

q_0 d: $\{3\}$, dar nu urmeaza lui 0   deci $q_1 d$ nu face parte din

q_1 l: $\{1,\ 2\}$, dar numai 2 urmeaza lui 1   deci $q_1 l \rightarrow q_2$

q_1 d: $\{3\}$ si 3 urmeaza lui 1   deci $q_1 d \rightarrow q_3$

q_2 l: $\{1,\ 2\}$, dar numai 2 urmeaza lui 2   deci $q_2 l \rightarrow q_2$

q_2 d: $\{3\}$, si 3 urmeaza lui 2   deci $q_2 d \rightarrow q_3$

q_3 l: $\{1,\ 2\}$, dar numai 2 urmeaza lui 3 -   deci $q_3 l \rightarrow q_2$

q_3 d: $\{3\}$, si 3 urmeaza lui 3 -   deci $q_3 d \rightarrow q_3$

Stari finale: q_2 si q_3

Conversie expresie regulata - automat finit determinist - continuare

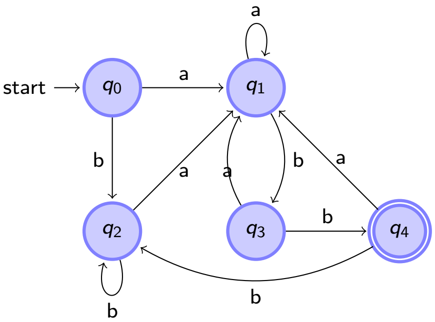
$(a+b)*abb$ becomes $0(1+2)^{*}345$

	a	b		a	follow	b	follow
q_0	q_1	q_2	0	1,3	1,3	2,4,5	2
q_1	q_1	q_3	1,3	1,3	1,3	2,4,5	2,4
q_2	q_1	q_2	2	1,3	1,3	2,4,5	2
q_3	q_1	q_4	2,4	1,3	1,3	2,4,5	2,5
q_4	q_1	q_2	2,5	1,3	1,3	2,4,5	2

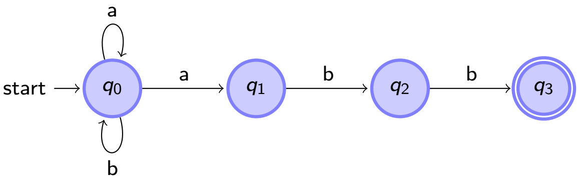
use jflab: minimize DFA + convert to grammar (alg DFA2RegularGrammar)

DFA NFA: (a+b)*abb

DFA

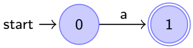


NFA

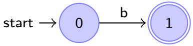


Alternativa

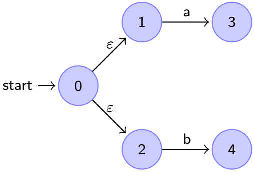
1. a



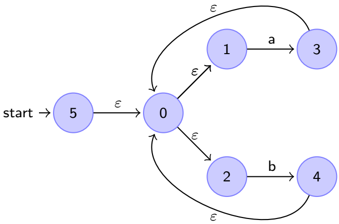
2. b



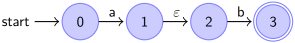
3. $a + b$



4. $(a + b)^*$



5. ab



LEX - analiza lexicala

Caractere operator

$$'' \backslash \; [\;] \; \wedge \; - \; ?\,. \; * \; + \,|\, (\;) \; \$ \; / \; \{\}\% <>$$

Folosirea lor drept caractere text: precedate de $\backslash$ sau intre $''$.

$$xyz'' + +''$$

$$''xyz + +''$$

$$xyz \backslash + \backslash +$$

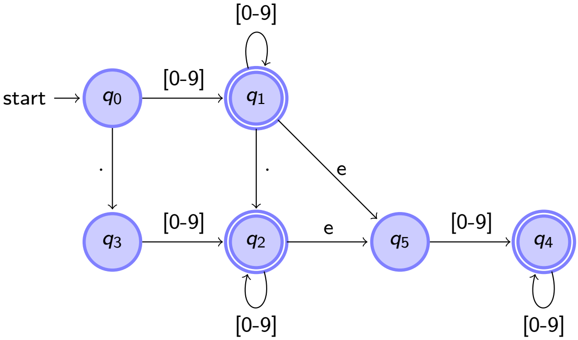
Expresii regulate in LEX

1. Clase de caractere $[a-z0-9<>\_]$, $[-0-9]$, $[^\wedge abc]$
2. Caracter arbitrar .
3. Element optional $ab?c$
4. Repetitii $a*$, $a+$, $[a-z]+$
5. Alternare $ab|cd$ $a(b|d)$
6. Doar la inceput/final de linie $^\wedge abc$, $abc\$$
7. Context ab/cd
8. Operator $\{\}$: $a\{1,5\}$, $a\{2,\}$

Analizor lexical controlat prin automat finit 3.6.2

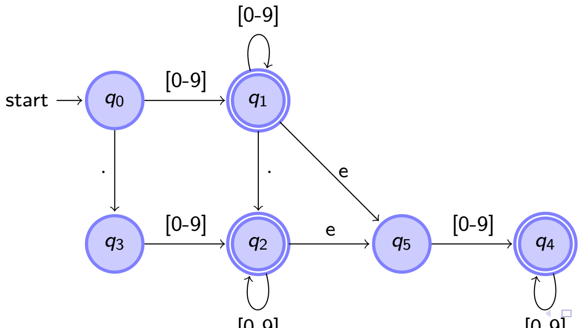
Fie:

```
D   [0-9]
%%
{D}+                                        return ICON;
({D}*|{D}*\.{D}+|{D}+\.{D}*)(e{D}+)?       return FCON;
```



Tabel de tranzitie

Stare curenta	caracter examinat inainte (lookahead) caracter de intrare			actiune la acceptare
	.	0-9	e	
0	3	1	-	-
1	2	1	5	return ICON;
2	-	2	5	return FCON;
3	-	2	-	-
4	-	4	-	return FCON;
5	-	4	-	-



Algoritmul utilizat de LEX (greedy) 3.6.3

```
stare_curenta = 0;
stare_acceptoare_vazuta_anterior = nimic_vazut;
if (caracter lookahead este end_of_input)
   return 0;
while(caracter lookahead nu este end_of_input){
  if (exista tranzitie din starea curenta cu caracterul
       lookahead curent){
     stare_curenta = acea stare;
     avanseaza in intrare;
     if (starea curenta este o stare acceptoare){
        memoreaza pozitia curenta in intrare
        si actiunea asociata starii curente;
     }  }
  else{
    if (nu a fost vazuta nicio stare acceptoare){
       exista o eroare:
          descarca lexemul curent si caracterul de intrare
              curent;
          stare_curenta = 0;
       }
    else {
      salveaza intrarea in pozitia in care se afla cand a
      vazut ultima stare acceptoare; realizeaza actiunea
      asociate acelei stari acceptoare;
      }
 }
```

Exemplu: $stare\_urmatoare = array[stare\_curenta][intrare]$

Stare curenta	caracter examinat inainte (lookahead) caracter de intrare			actiune la acceptare
	.	0-9	e	
0	3	1	-	-
1	2	1	5	return ICON;
2	-	2	5	return FCON;
3	-	2	-	-
4	-	4	-	return FCON;
5	-	4	-	-

Stare	Intrare	Ultima stare acceptoare	actiune	pozitie in intrare
0	1.2e4			
1	.2e4	1	ICON	.
2	2e4	2	FCON	2
2	e4	2	FCON	e
5	4			
4		4	FCON	

Exemplu: $stare\_urmatoare = array[stare\_curenta][intrare]$

Stare curenta	caracter examinat inainte (lookahead) caracter de intrare			actiune la acceptare
	.	0-9	e	
0	3	1	-	-
1	2	1	5	return ICON;
2	-	2	5	return FCON;
3	-	2	-	-
4	-	4	-	return FCON;
5	-	4	-	-

Stare	Intrare	Ultima stare acceptoare	actiune	pozitie in intrare
0	1.2e			
1	.2e	1	ICON	.
2	2e	2	FCON	2
2	e	2	FCON	e
5				
	e			

why? Concluzii

- Assuming that the language has been described by a grammar, we are interested in techniques for automatically generating a **recognizer from that grammar**. There are two reasons for this requirement:
  - It provides a guarantee that the language recognized by the compiler is identical to that defined by the grammar.
  - It simplies the task of the compiler writer.

Expresiile regulate = un mod algebric de a descrie limbajele
Descriu limbajele regulate

$a^n b^n$

NU e un limbaj regulat: nu exista niciun automat finit care sa-l aiba ca limbaj

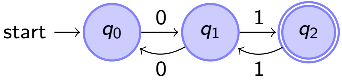
Dar limbajul *aaabbb*?

Rezumat

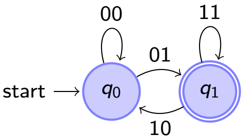
Gramatici regulate 2. Expresii regulate

Analiza lexicala cu LEX

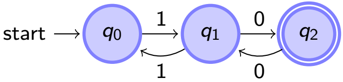
Care automate definesc acelasi limbaj?



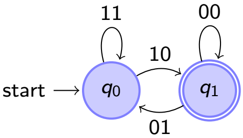
a)



b)



c)



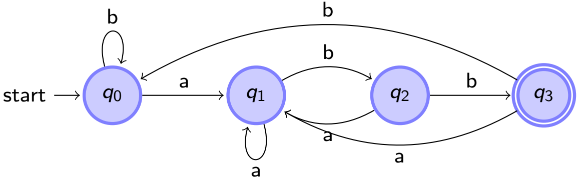
d)

- a si d
- b si c
- c si d
- a si c

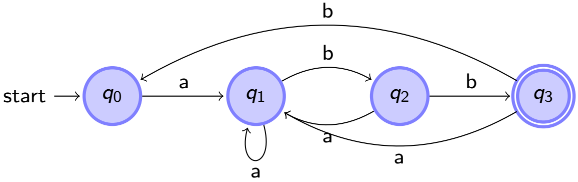
Kahoot

1. $L((a+b)(a+b)) = \{aa, ab, ba, bb\}$
2. $L(\ (a+b)^*\ )$ $=\{w|w$ are acelasi numar de a si $b\}$
3. Operatorul ? defineste un nou limbaj
4. Pentru orice expresie regulata exista un automat finit determinist
5. Expresiile regulate sunt o notatie pentru gramatici regulate.

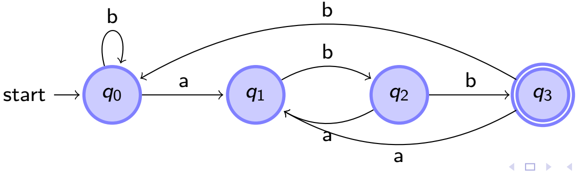
$(a+b)^*abb$


b
b
b
b
start
q_0
a
q_1
q_2
q_3
a
a
a

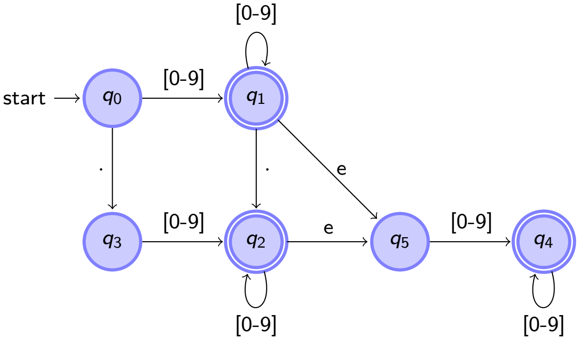
a)


b
b
b
start
q_0
a
q_1
q_2
q_3
a
a
a

b)


b
b
b
b
start
q_0
a
q_1
q_2
q_3
a
a

c)



Stare	Intrare	Ultima stare acceptoare	actiune	pozitie in intrare
0	.1e2			