Parsare ascendenta (Bottom-up) 1.

Parsare ascendenta

- necesita luarea unei decizii dupa analiza sirului derivat dintr-o productie
- mai multa informatie pt decizie $\rightarrow$
  - clasa larga de gramatici
  - pret: cresterea complexitati procedurii de analiza si a automatului rezultat

Automat stiva - analiza ascendenta

Fie $G = (T, N, P, Z)$ o CFG si automatul stiva $A = (T, \{q\}, R, q, \{q\}, V, \varepsilon)$ cu $V = T \cup N$ si R:
(alfabet, stari, productii, stare initiala, stari finale, alfabet stiva, continut initial stiva)

$$\{x_1 x_2 ... x_n q \rightarrow Xq | X \rightarrow x_1 x_2 ... x_n \in P, n \geq 0, X \in N, X_i \in V\} \cup$$
$$\{qt \rightarrow tq | t \in T\} \cup$$
$$\{Zq \rightarrow q\}$$

Automatul accepta un sir din L(G) lucrand inapoi printr-o derivare cea mai din dreapta a sirului.

Comparatie automat stiva in analiza descendenta vs ascendenta pt $G = (T, N, P, Z)$ o CFG

- descedenta - cursuri anterioare
  $A = (T, \{q\}, R, q, \{q\}, V, Z)$ cu $V = T \cup N$ si R:
  $\{tqt \to q | t \in T\} \cup$
  $\{Xq \to x_n....x_1 q | X \to x_1 x_2 ... x_n \in P, n \geq 0, X \in N, X_i \in V\}$

- ascendenta
  $A = (T, \{q\}, R, q, \{q\}, V, \varepsilon)$ cu $V = T \cup N$ si R:

  $\{x_1 x_2 ... x_n q \to Xq | X \to x_1 x_2 ... x_n \in P, n \geq 0, X \in N, X_i \in V\} \cup$

  $\{qt \to tq | t \in T\} \cup$

  $\{Zq \to q\}$

Exemplu

Fie $G_1 = (T, N, E, P)$

- $T = \{+, *, (,), i\}$, $N = \{E, T, F\}$
- cu productiile P
  - $(1, 2) E \to T | E + T$
  - $(3, 4) T \to F | T * F$
  - $(5, 6) F \to i | (E)$

Automatul stiva:

- $T = \{+, *, (,), i\}$, $Q = \{q\}$, $q_0 = q, F = \{q\}, S = \{+, -, *, (,), i, E, T, F\}, s_0 = E$
- cu productiile R
  1. $Tq \to Eq, E + Tq \to Eq,$
  2. $Fq \to Tq, T * Fq \to Tq,$
  3. $iq \to Fq, (E)q \to Fq,$
  4. $q+ \to +q, q* \to *q, q(\to (q, q) \to q), qi \to iq\}$
  5. $Eq \to q\}$

Derivarea gasita: i+i*i

stiva	stare	intrare	derivarea cea mai din dreapta
	q	$i + i * i$	i+i*i
i	q	$+i * i$	
F	q	$+i * i$	F+i*i
T	q	$+i * i$	T+i*i
E	q	$+i * i$	E+i*i
E+	q	$i * i$	
E+i	q	$*i$	
E+F	q	$*i$	E+F*i
E+T	q	$*i$	E+T*i
E+T*	q	i	
E+T*i	q	i	
E+T*F	q		
E+T	q		E+T*F
E	q		E+T
	q		E

Observatii

- coloana din dreapta este inversul derivarii celei mai din dreapta

- ascendenta - traseaza derivarea de jos la simbolul de start
- stiva contine la fiecare pas un sir din care se poate deriva portiunea de sir deja citita
- informatia semnificativa: perechea (ρ, σ), unde
  - $\rho \in V^*$ - continutul stivei,
  - $\sigma \in T^*$ - restul sirului de la intrare

LL	LR
Does a leftmost derivation.	Does a rightmost derivation in reverse.
Starts with the root nonterminal on the stack.	The last nonterminal on the stack is the root nonterminal.
Ends when the stack is empty.	Starts with an empty stack.
Uses the stack for designating what is still to be expected.	Uses the stack for designating what is already seen.
Builds the parse tree top-down.	Builds the parse tree bottom-up.
Continuously pops a nonterminal off the stack, and pushes the corresponding right hand side.	Tries to recognize a right hand side on the stack, pops it, and pushes the corresponding nonterminal.
Expands the non-terminals.	Reduces the non-terminals.
Reads the terminals when it pops one off the stack.	Reads the terminals while it pushes them on the stack.
Pre-order traversal of the parse tree.	Post-order traversal of the parse tree.

Clase de echivalenta pentru perechile (ρ, σ)

Pentru $p \in 1..n,$ fie $X_p \to \chi_p$ productia a $p-a$ a gramaticii independente de context $G = (T, N, P, Z)$. Clasele de reducere $R_j, j \in 0, ..n$ sunt definite de

$$R_0 = \{(\rho, \sigma) | \rho = \mu\gamma, \sigma = \nu\omega \text{ a.i. } Z \Rightarrow^R \mu Y \omega, Y \Rightarrow^{R'} \gamma\nu, \nu \neq \varepsilon\}$$
$$R_p = \{(\rho, \sigma) | \rho = \mu\chi_p, Z \Rightarrow^R \mu X_p \sigma, X_p \Rightarrow \chi_p\}$$

unde $Y \Rightarrow^{R'} \alpha$ este $Y \Rightarrow^R \alpha$ si ultimul pas din derivare nu ia forma $Y_1\alpha \Rightarrow \alpha$

Clase de reducere - continuare

- clasele de reducere - perechile de siruri care ar putea sa apara in timpul analizei ascendente a unei propozitii din $L(G)$ de catre automatul stiva
- clasa de reducere careia ii apartine o pereche caracterizeaza tranzitia efectuata de catre automat cand acea pereche apare ca o configuratie
  1. $(\rho, \sigma) \in R_0$ - fraza simpla χ nu e complet in stiva; se aplica $qt \to tq$ cu $t = 1 : \sigma$ tranzitie de deplasare
  2. $(\rho, \sigma) \in R_p, p \in 1..n$ - fraza simpla χ_p e complet in stiva; se aplica $\chi_p q \to X_p q$ tranzitie de reducere
     Obs: pt $p = 1$ tranzitia $Zq \to q$ si automatul se opreste
  3. $(\rho, \sigma) \notin R_j, j \in 0..n$. nu mai sunt posibile alte tranzitii; sirul de intrare nu apartine $L(G)$

Clase stiva k

- Pentru un $k \geq 0$, multimile $R_{j,k}, k \in 0..n$ se numesc clase stiva k al gramaticii G daca

$$R_{j,k} = \{(\rho, \tau) | \exists (\rho, \sigma) \in R_j, \tau = k : \sigma\}$$

- Daca clasele stiva k sunt mutual disjuncte atuncti automatul stiva este determinist chiar si cand examinarea inainte este limitata la k simboluri

Gramatica LR(k)

O gramatica independenta de context $G = (T, N, P, Z)$ este $LR(k)$ pentru un $k \geq 0$ dat daca pentru derivari arbitrare

$$Z \Rightarrow^R \mu X \omega \Rightarrow \mu \chi \omega \quad \mu \in V^*, \omega \in T^*, X \rightarrow \chi \in P$$

$$Z \Rightarrow^R \mu' Y \omega' \Rightarrow \mu' \gamma \omega' \quad \mu' \in V^*, \omega' \in T^*, Y \rightarrow \gamma \in P$$

$(|\mu\chi| + k) : \mu\chi\omega = (|\mu'\chi| + k) : \mu'\gamma\omega'$ implica
$\mu = \mu', X = Y, \chi = \gamma$

LR(k)

Automatul

- baleiaza sirul de intrare de la stanga la dreapta (Left to right)
- traversand inversa celei mai din dreapta derivari (Right)
- fara sa examinze mai mult de k simboluri de intrare intr-un pas

Teorema

O gramatica independenta de context este LR(k) daca si numai daca clasele sale stiva k sunt mutual disjuncte.

Verificarea proprietatii LR(k) prin intersectarea claselor stiva

- clasele stiva k contin o infinitate de perechi (ρ, τ), $\#\tau$ fiind finit datorita limitei de lungime, insa lungimea stivei nefiind limitata, $\#\rho$ este infinitate
- pentru fiecare clasa stiva k $R_{j,k}$ se poate preciza o gramatica regulata G_j a.i.

$$L(G_j) = \{(\rho\&\tau) | (\rho, \tau) \in R_{j,k}\}$$

- exista algoritmi pt a determina daca doua limbaje regulate sunt distincte

Situatii si inchidere nonterminal

Gramaticile regulate care genereaza clase stiva k:
Simbolurile nonterminale:

$$W = \{[X \to \mu.\nu;\omega] | X \to \mu\nu \in P, \omega \in FOLLOW_k(X)\}$$

Gramatici care genereaza clasele stiva k, fara a fi regulate

$$G'_j = (V \cup \{\&, \#\}, W, P' \cup P'' \cup P_j, [Z \rightarrow .S; \#])$$

$$P' = \{[X \rightarrow \mu.\nu\gamma;\omega] \rightarrow \nu[X \rightarrow \mu\nu.\gamma;\omega] \qquad |\nu \in V\}$$
$$P'' = \{[X \rightarrow \mu.Y\gamma;\omega] \rightarrow [Y \rightarrow .\beta;\tau] \qquad |Y \rightarrow \beta \in P, \tau \in EFF_k(\gamma\omega)\}$$
$$P_0 = \{[X \rightarrow \mu.\nu;\omega] \rightarrow \&\tau] \qquad |\nu \neq \varepsilon, \tau \in EFF_k(\nu\omega)\}$$
$$P_p = \{[X_p \rightarrow \chi_p.;\omega] \rightarrow \&\omega\} \qquad p \in 1..n$$

Care productii sunt permise in gramatica regulata?
Lungimile $\&\tau$, $\&\omega$ sunt finite datorita lui k;

Gramatici care genereaza clasele stiva k, fara a fi regulate

$$G'_j = (V \cup \{\&, \#\}, W, P' \cup P'' \cup P_j, [Z \to .S; \#])$$

$$P' = \{[X \to \mu.\nu\gamma; \omega] \to \nu[X \to \mu\nu.\gamma; \omega] \qquad |\nu \in V\}$$
$$P'' = \{[X \to \mu.Y\gamma; \omega] \to [Y \to .\beta; \tau] \qquad |Y \to \beta \in P, \tau \in EFF_k(\gamma\omega)\}$$
$$P_0 = \{[X \to \mu.\nu; \omega] \to \&\tau] \qquad |\nu \neq \varepsilon, \tau \in EFF_k(\nu\omega)\}$$
$$P_p = \{[X_p \to \chi_p.; \omega] \to \&\omega\} \qquad p \in 1..n$$

Care productii sunt permise in gramatica regulata?
P' si P_j unde $j \in 0..n$
Lungimile $\&\tau$, $\&\omega$ sunt finite datorita lui k; sunt considerate simboluri terminale

Inchiderea nonterminalului

o gramatica se poate rescrie a.i. sa nu contina productii precum cele din P''

- Inchiderea unui nonterminal

$$H(X) = \{X\} \cup \{Y | Y_l \to Y \in P, Y_l \in H(X)\}$$

Algoritm de rescriere a gramaticii

1. se selecteaza un $X \in N$ pentru care $H(X) \neq \{X\}$.
2. $P = P - \{X \rightarrow Y | Y \in N\}$
3. $P = P \cup \{X \rightarrow \beta | Y \rightarrow \beta \in P, Y \in H(X), \beta \notin N\}$
   Alg se termina cand nu se mai poate face nicio selectie la pasul 1

din G'_j rezulta G_j. Sirurile β sunt toate de forma $\nu[..], \&\tau$ sau $\&\omega$: deci gramatica regulata

Teorema

Pentru orice gramatica G de tipul LR(k) exista un automat stiva determinist A a.i. L(A) = L(G).

Constructia automatului se bazeaza pe gramaticile G_j:

- automatul genereaza clasele stiva k
- si le verifica fata de inversa celei mai din dreapta derivari a sirului
- in functie de clasa stiva k particulara, automatul
  - stivuieste simbolul de intrare, sau
  - reduce un numar de simboluri stivuite la un nonterminal

..continuare construire automat LR

- alg de construire genereaza treptat situatiile necesare si utilizeaza operatia de inchidere pentru evitarea productiilor din P''.
- o stare - o multime de situatii:
  - fiecare situatie dintr-o stare poate fi utilizata pentru derivarea clasei stiva k curente
- o alta formulare a inchiderii direct in functie de o multime de situatii M:

$$
\begin{aligned}
H(M) = M \cup \{[Y \rightarrow .\beta;\tau] \mid \; & \exists[X \rightarrow \mu.Y\gamma;\omega] \in H(M), \\
& Y \rightarrow \beta \in P, \\
& \tau \in FIRST_k(\gamma\omega)\}
\end{aligned}
$$

Algoritm LR(k)-determinare Q si R :

1. $Q = \{q_0\}$ si $R = \emptyset$ cu $q_0 = H([Z \to .S; \#])$
2. pt orice $q \in Q$ se efectueaza pasii 3-5 pt fiecare $\nu \in V$
3. fie $basis(q, \nu) = \{[X \to \mu\nu.\gamma; \omega] | [X \to \mu.\nu\gamma; \omega] \in q\}$
4. daca $basis(q, \nu) \neq \emptyset$ atunci $next(q, \nu) = H(basis(q, \nu))$. Se include $q' = next(q, \nu)$ in Q
5. daca $basis(q, \nu) \neq \emptyset$ si $\nu \in T$ se actualizeaza
   $R = R \cup \begin{cases} \{q\nu \to qq'\}, & k \leq 1 \\ \{q\nu\tau \to qq'\tau | [X \to \mu.\nu\gamma; \omega] \in q, \tau \in FIRST_{k-1}(\gamma\omega)\}, & k > 1 \end{cases}$
6. daca toate elementele lui Q au fost tratate se executa pasul 7 pt fiecare $q \in Q$ si alg se termina; altfel se continua pasul 2
7. pentru fiecare $[X \to \chi.; \omega] \in q$, unde $\chi = x_1..x_n$ se face

$$\begin{aligned} R = R \cup \{q_1..q_n q\omega \to q_1 q'\omega | [X \to .\chi; \omega] \in q_1, \\ q_{i+1} = next(q_i, x_i)(i \in 1..n-1), \\ q = next(q_n, x_n), \\ q' = next(q_1, X)\} \end{aligned}$$

Exemplu LR(k) cu k=2

$$
\begin{aligned}
T &= \{a, b, c\}, N = \{Z.X, Y\} \\
P &= \{(1) Z \to X, \\
&\qquad (2,3) X \to Y | bYa, \\
&\qquad (4,5) Y \to c | ca\}
\end{aligned}
$$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X; \#])$ $[Z \to .X; \#]$, $[X \to .Y; \#]$, $[X \to .bYa; \#]$, $[Y \to .c; \#]$, $[Y \to .ca; \#]$	$[Z \to X.; \#]$	$[X \to Y.; \#]$	-	$[X \to b.Ya; \#]$	$[Y \to c.; \#]$ $[Y \to c.a; \#]$
$next(q_0, \ldots)$	q_1	q_2		q_3	q_4
R				$q_0 bc \to q_0 q_3 c$ $FIRST_1(Ya\#)$	$q_0 c\# \to q_0 q_4 \#$ $FIRST_1(\#)$ $q_0 ca \to q_0 q_4 a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
q_0 $H([Z \rightarrow .X;\#])$ $[Z \rightarrow .X;\#]$, $[X \rightarrow .Y;\#]$, $[X \rightarrow .bYa;\#]$, $[Y \rightarrow .c;\#]$, $[Y \rightarrow .ca;\#]$	$[Z \rightarrow X.;\#]$	$[X \rightarrow Y.;\#]$	-	$[X \rightarrow b.Ya;\#]$	$[Y \rightarrow c.;\#]$ $[Y \rightarrow c.a;\#]$
$next(q_0, \ldots)$	q_1	q_2		q_3	q_4
R				$q_0 bc \rightarrow q_0 q_3 c$ $FIRST_1(Ya\#)$	$q_0 c\# \rightarrow q_0 q_4 \#$ $FIRST_1(\#)$ $q_0 ca \rightarrow q_0 q_4 a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X;\#])$ $[Z \to .X;\#]$, $[X \to .Y;\#]$, $[X \to .bYa;\#]$, $[Y \to .c;\#]$, $[Y \to .ca;\#]$	$[Z \to X.;\#]$	$[X \to Y.;\#]$	-	$[X \to b.Ya;\#]$	$[Y \to c.;\#]$ $[Y \to c.a;\#]$
$next(q_0,\ldots)$	q_1	q_2		q_3	q_4
R				$q_0bc \to q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \to q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \to q_0q_4a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X;\#])$ $[Z \to .X;\#]$, $[X \to .Y;\#]$, $[X \to .bYa;\#]$, $[Y \to .c;\#]$, $[Y \to .ca;\#]$	$[Z \to X.;\#]$	$[X \to Y.;\#]$	-	$[X \to b.Ya;\#]$	$[Y \to c.;\#]$ $[Y \to c.a;\#]$
$next(q_0, \ldots)$	q_1	q_2		q_3	q_4
R				$q_0 bc \to q_0 q_3 c$ $FIRST_1(Ya\#)$	$q_0 c\# \to q_0 q_4 \#$ $FIRST_1(\#)$ $q_0 ca \to q_0 q_4 a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X; \#])$ $[Z \to .X; \#]$, $[X \to .Y; \#]$, $[X \to .bYa; \#]$, $[Y \to .c; \#]$, $[Y \to .ca; \#]$	$[Z \to X.; \#]$	$[X \to Y.; \#]$	-	$[X \to b.Ya; \#]$	$[Y \to c.; \#]$ $[Y \to c.a; \#]$
$next(q_0, \ldots)$	q_1	q_2		q_3	q_4
R				$q_0bc \to q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \to q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \to q_0q_4a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
q_0 $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#]$, $[X \rightarrow .Y; \#]$, $[X \rightarrow .bYa; \#]$, $[Y \rightarrow .c; \#]$, $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$ $[Y \rightarrow c.a; \#]$
$next(q_0, \ldots)$	q_1	q_2		q_3	q_4
R				$q_0 bc \rightarrow q_0 q_3 c$ $FIRST_1(Ya\#)$	$q_0 c\# \rightarrow q_0 q_4 \#$ $FIRST_1(\#)$ $q_0 ca \rightarrow q_0 q_4 a$ $FIRST_1(a\#)$
q_1 $H([Z \rightarrow X.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X; \#])$ $[Z \to .X; \#]$, $[X \to .Y; \#]$, $[X \to .bYa; \#]$, $[Y \to .c; \#]$, $[Y \to .ca; \#]$	$[Z \to X.; \#]$	$[X \to Y.; \#]$	-	$[X \to b.Ya; \#]$	$[Y \to c.; \#]$ $[Y \to c.a; \#]$
$next(q_0, \ldots)$	q_1	q_2		q_3	q_4
R				$q_0 bc \to q_0 q_3 c$ $FIRST_1(Ya\#)$	$q_0 c\# \to q_0 q_4 \#$ $FIRST_1(\#)$ $q_0 ca \to q_0 q_4 a$ $FIRST_1(a\#)$
q_1 $H([Z \to X.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
q_2 $H([X \to Y.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X; \#])$ $[Z \to .X; \#]$, $[X \to .Y; \#]$, $[X \to .bYa; \#]$, $[Y \to .c; \#]$, $[Y \to .ca; \#]$	$[Z \to X.; \#]$	$[X \to Y.; \#]$	-	$[X \to b.Ya; \#]$	$[Y \to c.; \#]$ $[Y \to c.a; \#]$
$next(q_0, \ldots)$	q_1	q_2		q_3	q_4
R				$q_0 bc \to q_0 q_3 c$ $FIRST_1(Ya\#)$	$q_0 c\# \to q_0 q_4 \#$ $FIRST_1(\#)$ $q_0 ca \to q_0 q_4 a$ $FIRST_1(a\#)$
q_1 $H([Z \to X.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
q_2 $H([X \to Y.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$q_3 : H([B \to b.Ya; \#])$ $[B \to b.Ya; \#]$ $[Y \to .c; a\#]$ $[Y \to .ca; a\#]$	$\emptyset$	$[B \to bY.a.; \#]$	$\emptyset$	$\emptyset$	$[Y \to c.; a\#]$ $[Y \to c.a; a\#]$
$next(q_3, \ldots)$		q_5			q_6
R					$q_3 ca \to q_3 q_6 a$

Stare	X	Y	a	b	c
q_4 $H([Y \to c.; \#],)$ $[Y \to c.a; \#])$	$\emptyset$	$\emptyset$	$[Y \to ca.; \#]$	$\emptyset$	$\emptyset$
$next(q_5 \ldots)$			q_7		
R			$q_4 a\# \to q_4 q_7 \#$		
q_5 $H([X \to bY.a; \#],)$	$\emptyset$	$\emptyset$	$[X \to bYa.; \#]$	$\emptyset$	$\emptyset$
$next(q_5, \ldots)$			q_8		
R			$q_5 a\# \to q_5 q_8 \#$		
q_6 $H([Y \to c.; a\#],)$ $[Y \to c.a; a\#]$	$\emptyset$	$\emptyset$	$[Y \to ca.; a\#]$	$\emptyset$	$\emptyset$
$next(q_6, \ldots)$			q_9		
R			$q_6 aa \to q_6 q_9 a$		
q_7 $H([Y \to ca.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
q_8 $H([X \to bYa.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
q_9 $H([Y \to ca.; a\#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

pas 7. Pentru fiecare $[X \to \chi.;\omega] \in q$, unde $\chi = x_1..x_n$ se face

$$
\begin{aligned}
R = R \cup \{q_1..q_n q\omega \to q_1 q'\omega | [X \to .\chi;\omega] \in q_1,\\
q_{i+1} = next(q_i, x_i)(i \in 1..n-1),\\
q = next(q_n, x_n),\\
q' = next(q_1, X)\}
\end{aligned}
$$

Pt q_2: $H([X \to Y.;\#])$

$$
\begin{array}{rl}
q_1 & [X \to .Y;\#] \in ?\\
q & ? = next(?, Y)\\
q' & ? = next(?, X)
\end{array}
$$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X;\#])$ $[Z \to .X;\#]$, $[X \to .Y;\#]$, $[X \to .bYa;\#]$, $[Y \to .c;\#]$, $[Y \to .ca;\#]$	$[Z \to X.;\#]$	$[X \to Y.;\#]$	-	$[X \to b.Ya;\#]$	$[Y \to c.;\#]$ $[Y \to c.a;\#$
$next(q_0, ...)$	q_1	q_2		q_3	q_4

pas 7. Pentru fiecare $[X \to \chi.;\omega] \in q$, unde $\chi = x_1..x_n$ se face

$$\begin{aligned} R = R \cup \{q_1..q_n q\omega \to q_1 q'\omega | [X \to .\chi;\omega] \in q_1, \\ q_{i+1} = next(q_i, x_i)(i \in 1..n-1), \\ q = next(q_n, x_n), \\ q' = next(q_1, X)\} \end{aligned}$$

Pt q_2: $H([X \to Y.;\#])$

q_1 $[X \to .Y;\#] \in q_0$

q $q_2 = next(q_0, Y)$

q' $q_2 = next(q_0, X)$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X;\#])$ $[Z \to .X;\#]$, $[X \to .Y;\#]$, $[X \to .bYa;\#]$, $[Y \to .c;\#]$, $[Y \to .ca;\#]$	$[Z \to X.;\#]$	$[X \to Y.;\#]$	-	$[X \to b.Ya;\#]$	$[Y \to c.;\#]$ $[Y \to c.a;\#$
$next(q_0, ...)$	q_1	q_2		q_3	q_4

pas 7. Pentru fiecare $[X \to \chi.;\omega] \in q$, unde $\chi = x_1..x_n$ se face

$$\begin{aligned} R = R \cup \{q_1..q_n q\omega \to q_1 q'\omega | [X \to .\chi;\omega] \in q_1, \\ q_{i+1} = next(q_i, x_i)(i \in 1..n-1), \\ q = next(q_n, x_n), \\ q' = next(q_1, X)\} \end{aligned}$$

Pt q_2: $H([X \to Y.;\#])$

q_1 $[X \to .Y;\#] \in q_0$

q $q_2 = next(q_0, Y)$

q' $q_2 = next(q_0, X)$

$q_0 q_2 \# \to q_0 q_1 \#$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X;\#])$ $[Z \to .X;\#]$, $[X \to .Y;\#]$, $[X \to .bYa;\#]$, $[Y \to .c;\#]$, $[Y \to .ca;\#]$	$[Z \to X.;\#]$	$[X \to Y.;\#]$	-	$[X \to b.Ya;\#]$	$[Y \to c.;\#]$ $[Y \to c.a;\#$
$next(q_0, ...)$	q_1	q_2		q_3	q_4

pas 7. Pentru fiecare $[X \to \chi.;\omega] \in q$, unde $\chi = x_1..x_n$ se face

$$\begin{aligned} R = R \cup \{q_1..q_n q\omega \to q_1 q'\omega | [X \to .\chi;\omega] \in q_1, \\ q_{i+1} = next(q_i, x_i)(i \in 1..n-1), \\ q = next(q_n, x_n), \\ q' = next(q_1, X)\} \end{aligned}$$

Pt q_4: $H([Y \to c.;\#],[Y \to c.a;\#])$

$$\begin{array}{ll} q_1 & [Y \to .c;\#] \in q_0 \\ q & q_4 = next(q_0, c) \\ q' & q_2 = next(q_0, Y) \end{array}$$

Stare	X	Y	a	b	c
q_0 $H([Z \to .X;\#])$ $[Z \to .X;\#]$, $[X \to .Y;\#]$, $[X \to .bYa;\#]$, $[Y \to .c;\#]$, $[Y \to .ca;\#]$	$[Z \to X.;\#]$	$[X \to Y.;\#]$	-	$[X \to b.Ya;\#]$	$[Y \to c.;\#]$ $[Y \to c.a;\#$
$next(q_0, ...)$	q_1	q_2		q_3	q_4

pas 7. Pentru fiecare $[X \to \chi.;\omega] \in q$, unde $\chi = x_1..x_n$ se face

$$R = R \cup \{q_1..q_n q\omega \to q_1 q'\omega | [X \to .\chi;\omega] \in q_1,$$
$$q_{i+1} = next(q_i, x_i)(i \in 1..n-1),$$
$$q = next(q_n, x_n),$$
$$q' = next(q_1, X)\}$$

Pt q_4: $H([Y \to c.;\#],[Y \to c.a;\#])$

q_1 $[Y \to .c;\#] \in q_0$

q $q_4 = next(q_0, c)$

q' $q_2 = next(q_0, Y)$ $\quad q_0 q_4 \# \to q_0 q_2 \#$

Stare	X	Y	a	b	c
q_0	$[Z \to X.;\#]$	$[X \to Y.;\#]$	-	$[X \to b.Ya;\#]$	$[Y \to c.;\#]$
$H([Z \to .X;\#])$					
$[Z \to .X;\#]$,					$[Y \to c.a;\#$
$[X \to .Y;\#]$,					
$[X \to .bYa;\#]$,					
$[Y \to .c;\#]$,					
$[Y \to .ca;\#]$					
$next(q_0, ...)$	q_1	q_2		q_3	q_4

Pt q_6: $H([Y \to c.; a\#], [Y \to c.a; a\#])$

q_1 $[Y \to .c; a\#] \in q_3$

q $q_6 = next(q_3, c)$

q' $q_5 = next(q_3, Y)$ $\qquad q_3 q_6 a\# \to q_3 q_5 a\#$

Pt q_7: $H([Y \to ca.; \#])$

q_1 $[Y \to .ca; \#] \in q_0$

q_2 $q_4 = next(q_0, c)$

q $q_7 = next(q_4, a)$

q' $q_2 = next(q_0, Y)$ $\qquad q_0 q_4 q_7\# \to q_0 q_2\#$

Pt q_8: $H([X \to bYa.; \#]$

$$
\begin{array}{rl}
q_1 & [X \to .bYa; \#] \in q_0 \\
q_2 & q_3 = next(q_0, b) \\
q_3 & q_5 = next(q_3, Y) \\
q & q_8 = next(q_5, a) \\
q' & q_1 = next(q_0, X)
\end{array}
$$

$q_0 q_3 q_5 q_8 \# \to q_0 q_1 \#$

Pt q_9: $H([Y \to ca.; a\#])$

$$
\begin{array}{rl}
q_1 & [Y \to .ca; a\#] \in q_3 \\
q_2 & q_6 = next(q_3, c) \\
q & q_9 = next(q_6, a) \\
q' & q_5 = next(q_3, Y)
\end{array}
$$

$q_3 q_6 q_9 a\# \to q_3 q_5 a\#$

$$
\begin{array}{llll}
q_0: & [Z \to \bullet X; \#] & q_4: & [Y \to c \bullet ; \#] \\
 & [X \to \bullet Y; \#] & & [Y \to c \bullet a; \#] \\
 & [X \to \bullet bY a; \#] & & \\
 & [Y \to \bullet c; \#] & q_5: & [X \to bY \bullet a; \#] \\
 & [Y \to \bullet ca; \#] & q_6: & [Y \to c\bullet; a\#] \\
q_1: & [Z \to X\bullet; \#] & & [Y \to c \bullet a; a\#] \\
q_2: & [X \to Y\bullet; \#] & q_7: & [Y \to ca\bullet; \#] \\
q_3: & [X \to b \bullet Y a; \#] & q_8: & [X \to bY a\bullet; \#] \\
 & [Y \to \bullet c; a\#] & q_9: & [Y \to ca\bullet; a\#] \\
 & [Y \to \bullet ca; a\#] & &
\end{array}
$$

$$
\begin{aligned}
R = \{ & q_0bc \to q_0q_3c, \\
& q_0c\# \to q_0q_4\#, \\
& q_0ca \to q_0q_4a, \\
& q_3ca \to q_3q_6a, \\
& q_4a\# \to q_4q_7\#, \\
& q_5a\# \to q_5q_8\#, \\
& q_6aa \to q_6q_9a, \\
& q_0q_2\# \to q_0q_1\#, \\
& q_0q_4\# \to q_0q_2\#, \\
& q_3q_6a\# \to q_3q_5a\#, \\
& q_0q_4q_7\# \to q_0q_2\#, \\
& q_0q_3q_5q_8\# \to q_0q_1\#, \\
& q_3q_6q_9a\# \to q_3q_5a\#\}
\end{aligned}
$$

$$
\begin{array}{lll}
q_0: & [Z \to \bullet X; \#] \\
 & [X \to \bullet Y; \#] \\
 & [X \to \bullet bYa; \#] \\
 & [Y \to \bullet c; \#] \\
 & [Y \to \bullet ca; \#] \\
q_1: & [Z \to X\bullet; \#] \\
q_2: & [X \to Y\bullet; \#] \\
q_3: & [X \to b \bullet Ya; \#] \\
 & [Y \to \bullet c; a\#] \\
 & [Y \to \bullet ca; a\#]
\end{array}
\qquad
\begin{array}{lll}
q_4: & [Y \to c\bullet; \#] \\
 & [Y \to c \bullet a; \#] \\
 & \\
q_5: & [X \to bY \bullet a; \#] \\
q_6: & [Y \to c\bullet; a\#] \\
 & [Y \to c \bullet a; a\#] \\
q_7: & [Y \to ca\bullet; \#] \\
q_8: & [X \to bYa\bullet; \#] \\
q_9: & [Y \to ca\bullet; a\#]
\end{array}
$$

$$
\begin{aligned}
R = \{ & q_0bc \to q_0q_3c, \\
& q_0c\# \to q_0q_4\#, \\
& q_0ca \to q_0q_4a, \\
& q_3ca \to q_3q_6a, \\
& q_4a\# \to q_4q_7\#, \\
& q_5a\# \to q_5q_8\#, \\
& q_6aa \to q_6q_9a, \\
& q_0q_2\# \to q_0q_1\#, \\
& q_0q_4\# \to q_0q_2\#, \\
& q_3q_6a\# \to q_3q_5a\#, \\
& q_0q_4q_7\# \to q_0q_2\#, \\
& q_0q_3q_5q_8\# \to q_0q_1\#, \\
& q_3q_6q_9a\# \to q_3q_5a\#\}
\end{aligned}
$$

Cu $k = 1$: aceleasi stari; $k = 0$ ar fuziona q_4, q_6, respectiv q_7, q_9. Dar: un singur simbol inainte nu face distinctie intre tranzitiile de deplasare (shift) si reducere (reduce) din starea 6.

$Z \Rightarrow X \rightarrow bYa \Rightarrow bcaa$

Stiva	Stare	intrare	$\Rightarrow^R$	*tranzitie*
#	q_0	bcaa#	bcaa	1
$\#q_0$	q_3	caa#		3
$\#q_0q_3$	q_6	aa#		6
$\#q_0q_3q_6$	q_9	a#	bYa	12
$\#q_0q_3$	q_5	a#		5
$\#q_0q_3q_5$	q_8	#	X	11
$\#q_0$	q_1	#	Z	