Principles of Finance

Options in corporate finance

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Options

- Options are financial securities (called derivatives) whose price depends on the price of another asset
- A *call option* gives the buyer the right, but not the obligation
  - To purchase a specific asset
  - For a prespecified price
  - At or until a specific date
- A *put option* gives the buyer the right, but not the obligation
  - To sell a specific asset
  - For a prespecified price
  - At or until a specific date

Options

- Options are everywhere in corporate finance
  - *Executive stock options*: Right to buy the stock of the firm at a given (fixed) price in the future
  - *Real options and intangibles*: Successful R&D gives the firm the right to develop a new product or drug
  - *Real options and tangibles*: Firms have the right to increase output of successful products and reduce output for unsuccessful products
  - *Options attached to securities*: A convertible bond gives the owner the right to exchange his/her bond for stocks in the firm

Option Terminology

- Call, put
- Buyer, writer
- Premium
- Strike/exercise price
- Expiration/maturity
- European, American
- In/out of/at the money

Payoff at maturity: Call

□ Consider a European call with maturity T, exercise price K, written on an underlying asset with price S_t at date t

□ At the maturity date

- If $S_T \leq K$, the option is not exercised
- If $S_T > K$, the option is exercised; its cash flow is $S_T - K$

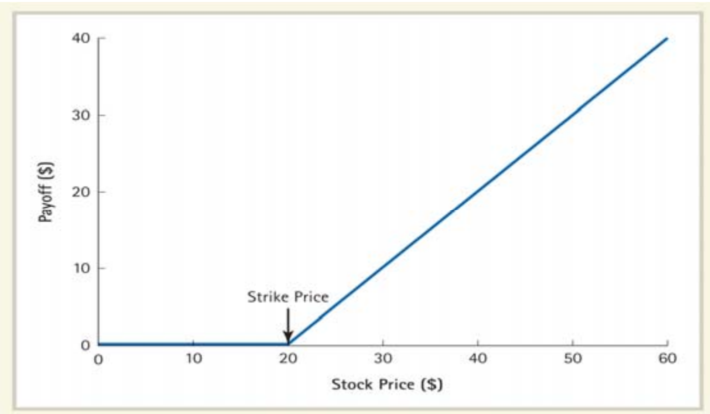
□ The payoff from a long position is

$$(S_T - K)^+ = max\{S_T - K, 0\} \geq 0$$

□ The premium is the present value of this payoff

Graphical representation

□ Payoff of a call option with a strike price of $20 at expiration



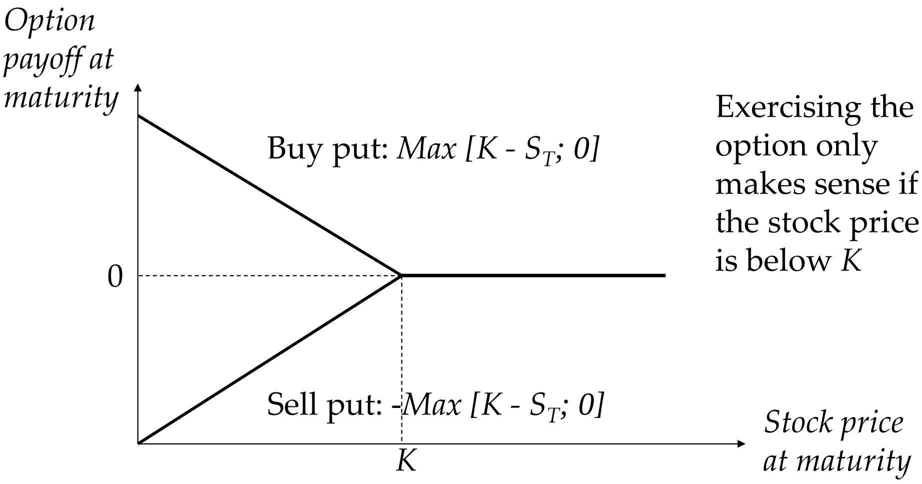
□ Although payouts on a long position in an option contract are never negative, the profit from purchasing an option and holding it to expiration could be negative because the payout at expiration might be less than the initial cost of the option

Payoff at maturity: Put

- Consider a European put with maturity T, exercise price K, written on an underlying asset with price S_t at date t
- At the maturity date
  - If $S_T \geq K$, the option is not exercised
  - If $S_T < K$, the option is exercised; its cash flow is $K - S_T$
- The payoff from a long position is

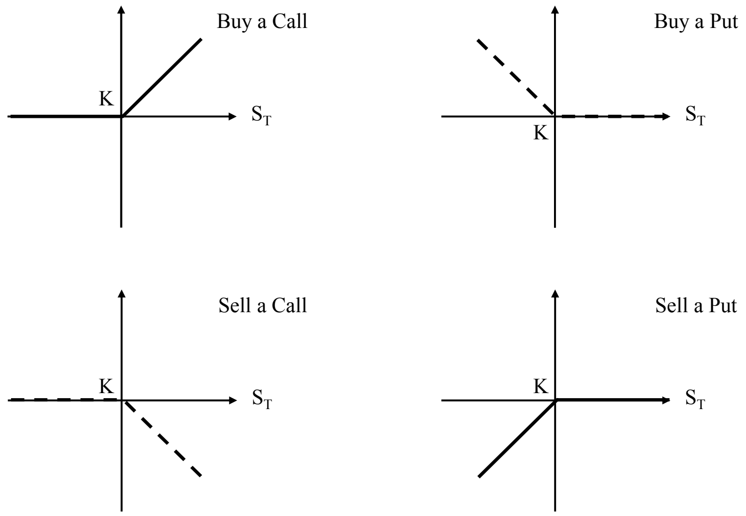
$$(K - S_T)^+ = max\{K - S_T, 0\} \geq 0$$

Put options



Exercising the option only makes sense if the stock price is below K

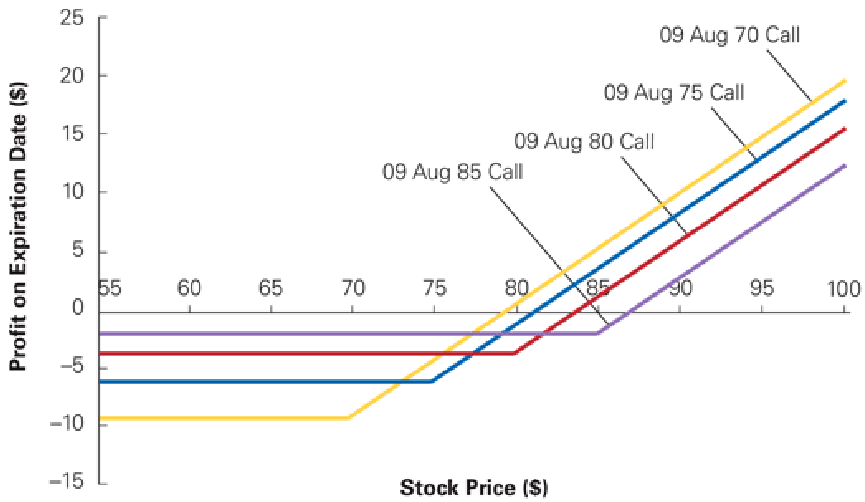
Graphical representation



Profits from holding an option to expiration

□ Although payouts on a long position in an option contract are never negative, the profit from purchasing an option and holding it to expiration could be negative because the payout at expiration might be less than the initial cost of the option

Profits from holding an option to expiration



Portfolio insurance

□ The basic idea of portfolio insurance is to use options to change the support of the distribution of the value of a portfolio

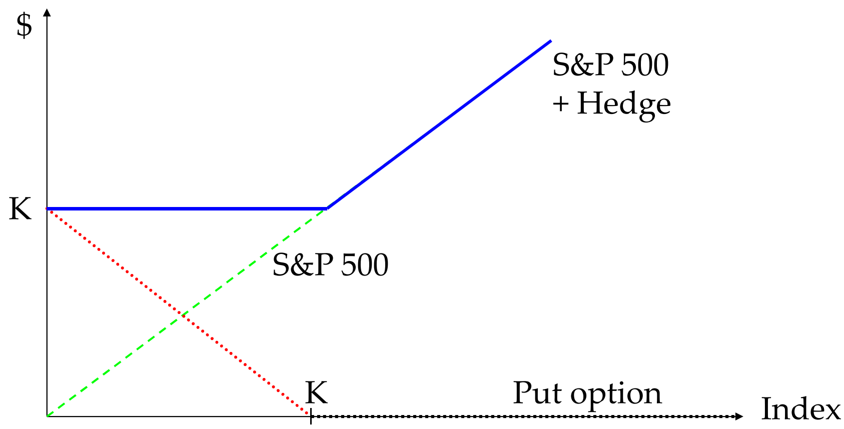
→ One should use put options to eliminate the possibility of bad outcomes

□ When put options are not available, the investor can manufacture them by creating a portfolio of stocks and risk-free bonds. The duplication of a put option requires a

- Long position in a risk-free loan
- Short position in the underlying asset (the index in this particular case)

Portfolio insurance

- A mutual fund holds $100 million in S&P stocks
- Can buy put options to ensure minimum value



Assumptions and notations

- Assumptions
  - Agents can lend an borrow freely at the risk-free rate r
  - There are no arbitrage opportunities
- Notations
  - S_t: Spot price of the underlying asset at time t
  - T: Maturity date of the options;
  - K: Exercise price
  - D_t: PV at time t of the dividends paid between t and T
  - c_t and p_t: Prices of European call and put options at time t
  - C_t and P_t: Prices of American call and put options at time t

Call-Put parity

- Consider two portfolios
  - A: One European call + PV strike price: $c_t + K(1+r)^{-(T-t)}$
  - B: One European put + underlying asset: $p_t + S_t$
- At expiration, they pay off
  - A: $max[S_T - K;0] + K = max[S_T, K]$
  - B: $max[K - S_T;0] + S_T = max[S_T, K]$
- By absence of arbitrage opportunities

$$c_t - p_t = S_t - K(1+r)^{-(T-t)}$$

Call-Put parity

Problem

You are an options dealer who deals in non-publicly traded options. One of your clients wants to purchase a one-year European call option on HAL Computer Systems stock with a strike price of $20. Another dealer is willing to write a one-year European put option on HAL stock with a strike price of $20, and sell you the put option for a price of $1.50 per share. If HAL pays no dividends and is currently trading for $18 per share, and if the risk-free interest rate is 6%, what is the lowest price you can charge for the option and guarantee yourself a profit?

Solution

Using put-call parity, we can replicate the payoff of the one-year call option with a strike price of $20 by holding the following portfolio: Buy the one-year put option with a strike price of $20 from the dealer, buy the stock, and sell a one-year risk-free zero-coupon bond with a face value of $20. With this combination, we have the following final payoff depending on the final price of HAL stock in one year, S_1:

	Final HAL Stock Price	
	$S_1 < \$20$	$S_1 > \$20$
Buy Put Option	$20 - S_1$	0
Buy Stock	S_1	S_1
Sell Bond	-20	-20
Portfolio	0	$S_1 - 20$
Sell Call Option	0	$-(S_1 - 20)$
Total Payoff	0	0

Note that the final payoff of the portfolio of the three securities matches the payoff of a call option. Therefore, we can sell the call option to our client and have future payoff of zero no matter what happens. Doing so is worthwhile as long as we can sell the call option for more than the cost of the portfolio, which is

$$P + S - PV(K) = \$1.50 + \$18 - \$20/1.06 = \$0.632$$

Early exercise

- *European options* can only be exercised at expiration
- *American options* can be exercised at any time before expiration
- Thus American options are typically **worth more** than European options
- One exception is call options **on non-dividend paying** stocks since it is never optimal to exercise these options before maturity
  - No income is sacrificed
  - Payment of the strike price is delayed
  - Holding the call provides insurance against the stock price falling below the strike price

Early exercise: No dividends

□ The call-put parity for European options is

$$c_t - p_t = S_t - K(1+r)^{-(T-t)}$$

□ This implies that the value of the call option alive

$$c_t = S_t - K + [K - K(1+r)^{-(T-t)}] + p_t$$

□ The term in square brackets is positive. So is the value of the put option

- Thus, **it is never optimal to exercise the call option early**

Early exercise: No dividends

□ The call-put parity for European options is

$$c_t - p_t = S_t - K(1+r)^{-(T-t)}$$

□ This implies that the value of the put option alive satisfies

$$p_t = K - S_t + [K(1+r)^{-(T-t)} - K] + c_t$$

□ The term in square brackets is negative.

- Thus, it **may be optimal to exercise the put option** early

Early exercise: Dividends

□ The call-put parity for European options on dividend paying stocks is

$$c_t - p_t = S_t - K(1+r)^{-(T-t)} - D_t$$

□ This implies that the value of the call option alive

$$c_t = S_t - K + [K - K(1+r)^{-(T-t)}] + p_t - D_t$$

□ When dividends are large enough, it may be optimal to exercise early an American call option

Option pricing

□ At maturity, the value of the option only depends on the value of the underlying asset S and the strike price K

□ Prior to maturity the value of a call depends on

- the current price of the underlying asset: S_0
- the exercise price of the option: K
- the time to maturity of the option: T
- the risk free interest rate: $r > 0$
- the volatility of the underlying asset price: σ
- the payout ratio : δ

Option pricing

□ Derivatives are often priced using replicating portfolios

□ A replicating portfolio is an investment portfolio that does not use derivatives but produces the same payoff at maturity than the derivative

□ Pricing derivatives by arbitrage

- Find the replicating portfolio
- By absence of arbitrage opportunities, the value of the derivative must be the same as the value of the replicating portfolio

Option pricing

□ Example: consider an economy with two states of the world and two securities:

- A risk-free bond B with price $B_0 = 1$ and promised interest rate 2,5%
- A risky stock with current price $S_0 = 100$.
- The relevant prices tomorrow are 1 for the bond and either $S_1 = 125$ or $S_1 = 80$ for the stock

□ Consider an option to buy the stock for 102.5. What is the current price of the option?

The Binomial model

- We have

$$S_0 = 100 \quad \begin{cases} u \times S_0 = 1.25 \times S_0 = 125 \\ d \times S_0 = 0.80 \times S_0 = 80 \end{cases}$$

and the tree for the call option is

$$c_0 = ? \quad \begin{cases} c_u = max[u \times S_0 \text{ - } K;0] = 22.5 \\ c_d = max[d \times S_0 \text{ - } K;0] = 0 \end{cases}$$

The Binomial model

- The duplicating portfolio evolves according to the process

$$V = \Delta S_0 + B \quad \begin{cases} V_u = \Delta u S_0 + B(1+r) \\ V_d = \Delta d S_0 + B(1+r) \end{cases}$$

- The replication implies the following two equations

$$(1)\ \Delta u S_0 + B(1+r) = c_u = 22.5$$

$$(2)\ \Delta d S_0 + B(1+r) = c_d = 0$$

The Binomial model

□ Solving these equations yields

$$\Delta = \frac{c_u - c_d}{S_0(u-d)} = \frac{22{,}5}{100(1{,}25 - 0{,}8)} = 0{,}5$$

□ The solution for B is given by

$$B = \frac{1}{1+r}\frac{uc_d - dc_u}{u-d} = \frac{1}{1{,}025}\frac{-0{,}8 * 22{,}5}{1{,}25 - 0{,}80} = -39{,}02$$

□ The value of the option is

$$V = \Delta S_0 + B = 0.5*100 - 39.02 = 10.98$$

The Binomial model

□ Using these equations we get

$$c_0 = \frac{c_u - c_d}{S_0(u-d)} S_0 + \frac{1}{1+r}\frac{uc_d - dc_u}{u-d}$$

□ After simplifications, we get

$$c_0 = \frac{1}{1+r}\left[\frac{1+r-d}{u-d} c_u + \frac{u-(1+r)}{u-d} c_d\right]$$

The Binomial model

□ We can now define

$$p = \frac{1 + r - d}{u - d}$$

□ We then have the option price as

$$c_0 = \frac{1}{1 + r}[pc_u + (1 - p)c_d] = \frac{1}{1 + r}E_p[c_1]$$

which is the present value of the option cash flows using the *risk neutral probabilities* and the *risk free rate* of discount

The Binomial model

□ Why do we use the term *risk neutral probabilities* to describe these weights?

The Binomial model

- Example: Pricing of a call option maturing next period

  $S_0 = 100$

  $K = 102.5$

  $u = 1.25$

  $d = 0.8$

  $r = 2.5\%$

- We then have the option price as

$$c_0 = \frac{1}{1{,}025}\left[\frac{1{,}025 - 0{,}8}{1{,}25 - 0{,}8}22{,}5 + \frac{1{,}25 - 1{,}025}{1{,}25 - 0{,}8}0\right] = 10{,}98$$

The Binomial model

- The model can be extended to consider 2-period options
- The dynamics of the stock price are then given by

$$\begin{array}{ccccc} & & & & u^2 \times S_0 = (1.25)^2 \times 100 \\ & & u \times S_0 = 1.25 \times S_0 & & \\ S_0 = 100 & & & & d \times u \times S_0 = 0.8 \times 1.25 \times 100 \\ & & d \times S_0 = 0.80 \times S_0 & & \\ & & & & d^2 \times S_0 = (0.8)^2 \times 100 \end{array}$$

The Binomial model

- The dynamics of the option price are then given by

$c_0 = ?$

$c_u = ?$

$c_d = ?$

$c_{uu} = max[u^2 \times S_0 \text{-} K;\ 0] = 53.75$

$c_{du} = max[d \times u \times S_0 - K;\ 0] = 0$

$c_{dd} = max[d^2 \times S_0 \text{ - } K;\ 0] = 0$

- The model needs to be solved *recursively*, starting from the last date

The Binomial model

- First step: Starting from the up node we have

$$c_u = \frac{1}{1+r}[pc_{uu} + (1-p)c_{ud}] = \frac{0{,}5 * 53{,}75 + 0{,}5 * 0}{1{,}025} = 26{,}22$$

- Second step: Starting from the down node we have

$$c_d = \frac{1}{1+r}[pc_{du} + (1-p)c_{dd}] = \frac{0{,}5 * 0 + 0{,}5 * 0}{1{,}025} = 0$$

- We then have as before

$$c_0 = \frac{1}{1{,}025}[0{,}5 * 26{,}22 + 0{,}5 * 0] = 12{,}79$$

The Binomial model

□ This expression for the call price is similar to one in which we directly discount the various cash flows from period 2 to the initial date

$$c_0 = \frac{1}{(1+r)^2}\left[p^2 c_{uu} + 2p(1-p)c_{ud} + (1-p)^2 c_{dd}\right]$$

□ In our numerical example, we have

$$c_0 = \frac{1}{(1.025)^2}\left[(0.5)^2 53.75 + 2\times 0.5\times 0.5\times 0 + (0.5)^2 \times 0\right] = 12.79$$

What happens to the duplicating portfolio?

Pricing a European put

□ The same binomial representation applies for the put option

$$p_0 = ? \quad \begin{cases} p_u = max[K - u \times S_0;\ 0] = 0 \\ p_d = max[K - d \times S_0; 0] = 22.5 \end{cases}$$

□ The replication implies the following two equations

$$(1)\ \Delta' u S_0 + B'(1+r) = p_u = 0$$

$$(2)\ \Delta' d S_0 + B'(1+r) = p_d = 22.5$$

Pricing a European put

□ Solving these equations yields

$$\Delta' = \frac{p_u - p_d}{S_0(u-d)} = \frac{-22{,}5}{100(1{,}25 - 0{,}8)} = -0{,}5$$

□ The solution for B' is given by

$$B' = \frac{1}{1+r}\frac{up_d - dp_u}{u-d} = \frac{1}{1{,}025}\frac{1{,}25 * 22{,}5}{1{,}25 - 0{,}80} = 60{,}98$$

□ The value of the put option is then

$$p_0 = \Delta' S_0 + B' = -0.5 \times 100 + 60.98 = 10.98$$

Generalizing the model

- The binomial model can be generalized to consider the case in which the number n of nodes is arbitrarily large
- In this model we denote by $S_{n,j}$ the value of the underlying asset after n periods in the tree if the asset value has gone up j times
- This value is given by

$$S_{n,j} = u^j \times d^{n-j} \times S_0$$

and the value of the call option at this node is then given by

$$c_{n,j} = \frac{1}{1+r}\left[p c_{n+1,j+1} + (1-p) c_{n+1,j}\right]$$

Generalizing the model

- This option value is also equal to

$$c_{n,j} = \frac{1}{(1+r)^2}\left[p^2 c_{n+2,j+2} + 2p(1-p) c_{n+2,j+1} + (1-p)^2 c_{n+2,j}\right]$$

and hence we can express the option price at time $t = 0$ as

$$\boxed{c_0 = \frac{1}{(1+r)^n}\sum_{j:0}^{n} \frac{n!}{j!\,(n-j)!} p^j (1-p)^{n-j} \max(u^j d^{n-j} S_0 - K; 0)}$$

where $n! = 1 \times 2 \times 3 \times \ldots \times (n\text{-}1) \times n$

Generalizing the model

□ This option value can be simplified by using the cumulative density function of the binomial distribution i.e. $\Phi(x;n,p)=\Pr(\text{Bin}(n,p) \geq x)$ where

$$\Phi(x;n,p) = \sum_{j=x}^{n} \frac{n!}{j!(n-j)!} p^j (1-p)^{n-j}$$

□ We can then define the minimum number of up moves such that the option ends in the money as:

$$a = \inf\{k: S_0 u^k d^{(n-k)} \geq K\}$$

Generalizing the model

□ This option value can be rewritten as

$$\boxed{c_0 = S_0 \Phi(a;n,q) - \frac{K}{(1+r)^n} \Phi(a;n,p)}$$

where

$$q = \frac{pu}{1+r}$$

As we will see later, this formula is very close to the Black and Scholes formula for the pricing of European options.

What happens when the option is American?

- The main difference between American and European options is that the former can be exercised at any point in time
- Thus, at each point in time you need to take the maximum between waiting and exercising the option
- Again, the model needs to be solved *recursively*, starting from the last date to capture in a correct way the value of waiting

Pricing an American call

- The dynamics of the option price are then given by

$$
\begin{array}{ccccl}
 & & & & C_{uu} = max[u^2 \times S_0 - K;\ 0] = 53.75 \\
 & & C_u = ? & & \\
C_0 = ? & & & & C_{du} = max[d \times u \times S_0 - K;\ 0] = 0 \\
 & & C_d = ? & & \\
 & & & & C_{dd} = max[d^2 \times S_0 - K;\ 0] = 0
\end{array}
$$

- The model needs to be solved *recursively*, starting from the last date since at the last date the value of waiting is zero

Pricing an American call

□ In the up node, the value of the American call is

$$C_u = max\{exercise; wait\}$$

or

$$C_u = max\left\{uS_0 - K; \frac{1}{1+r}[pC_{uu} + (1-p)C_{ud}]\right\}$$

□ Using the numerical example we have

$$C_u = max\left\{125 - 100; \frac{1}{1{,}025}[0{,}5 * 53{,}75 + 0{,}5 * 0]\right\} = 26{,}22$$

Pricing an American call

□ In the up node the value of waiting exceeds the cash flow obtained by exercising the option

→ It is optimal to wait; the option value is 26.22

□ Similarly, it is optimal to wait in period 0. As a result the option value is given by

$$C_0 = \frac{1}{1+r}[pC_u + (1-p)C_d] = \frac{0{,}5 * 26{,}22 + 0{,}5 * 0}{1{,}025} = 12{,}79$$

□ How does this price compare with that of the European option? Is this result surprising?

Pricing an American put

- The dynamics of the option price are given by

$$
\begin{array}{ccccc}
 & & & & P_{uu} = max[K\text{-}u^2 \times S_0;\ 0] = 0 \\
 & & P_u = ? & & \\
P_0 = ? & & & & P_{du} = max[K\text{-}d \times u \times S_0;\ 0] = 2.5 \\
 & & P_d = ? & & \\
 & & & & P_{dd} = max[K\text{-}d^2 \times S_0;\ 0] = 38.5
\end{array}
$$

- What is the price of the American put?

Pricing an American put

Parameter values

□ How do you get the values of u and d?

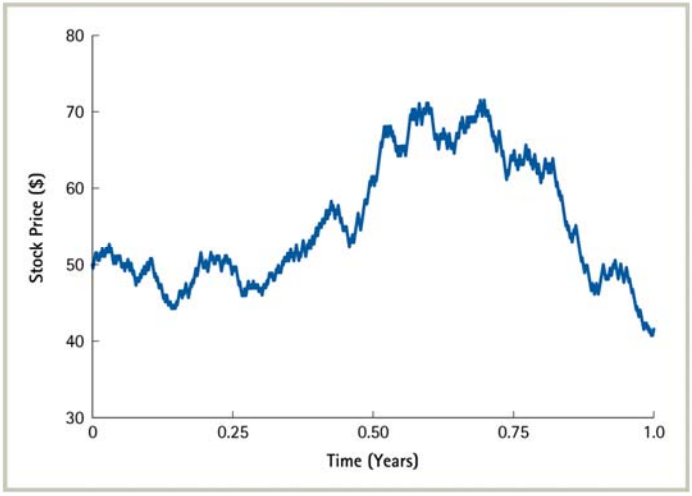
$$u = e^{\sigma\sqrt{T/n}} \quad \text{and} \quad d = e^{-\sigma\sqrt{T/n}}$$

where

- σ is the volatility of the underlying asset measured in years
- T is the maturity of the project in years
- n is the number of nodes

Black and Scholes

□ By decreasing the length of each period, and increasing the number of periods in the stock price tree, a realistic model for the stock price can be constructed



Black and Scholes

□ The Black and Scholes formula gives the price of a **European** option prior to maturity

□ It expresses the value of a call option as a function of

- The current price of the underlying asset: S_0
- The exercise price of the option: K
- The time to maturity of the option: T
- The risk free interest rate: r
- the standard deviation of the return on the underlying asset: σ

□ ***The model is set in continuous time***

Black and Scholes

□ The price of the option is given by the expected payoff of the option under the risk-neutral probability measure discounted at the risk-free rate

$$c_0 = E_Q[e^{-rT} max(S_T - K; 0)]$$

with

$$dS_t = r\, S_t\, dt + \sigma\, S_t\, dB_t, \qquad S_0 > 0$$

and where Q is the risk-neutral pricing measure and $B = (B_t)_{t \geq 0}$ is a standard Brownian motion.

Black and Scholes

□ When stock prices are governed by the above SDE, the formula for the pricing of *European options* is

$$\boxed{c_0 = S_0 N(d_1) - K e^{-rT} N(d_1 - \sigma\sqrt{T})}$$

with

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + rT}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

□ Does it make sense?

Application

□ Assume that

- Underlying stock price is S_0 = *$100*
- The exercise price is K = *$65*
- The maturity of the option is T = *3 years*
- The volatility of returns is σ = *40%*
- No dividend is expected
- The risk-free rate is r = *3%*

□ What is the value of a call option?

Application

□ Computation of d_1

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + rT}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} = \frac{ln\left(\frac{100}{65}\right) + 0{,}03 * 3}{0{,}4\sqrt{3}} = 1{,}098$$

□ The value of the call option is then

$$c_0 = 100 * N(1{,}098) - 65e^{-0{,}03*3}N\left(1{,}098 - 0{,}4\sqrt{3}\right) = 47{,}34$$

The put option

□ The call-put parity is given by

$$c_t = p_t + S_t - Ke^{-r(T-t)}$$

□ Using this relation and the pricing formula for European call options, we can write the value of the European put option in the Black and Scholes model as

$$p_0 = Ke^{-rT}N\left(-d_1 + \sigma\sqrt{T}\right) - S_0N(-d_1)$$

where d_1 is defined as above.

Application

- Assume that
  - Underlying stock price is S_0 = *$100*
  - The exercise price is K = *$65*
  - The maturity of the option is T = *3 years*
  - The volatility of returns is σ = *40%*
  - No dividend is expected
  - The risk-free rate is r = *3%*
- What is the value of a put option?

Estimating volatility

- Volatility can be estimated from historical data

Month	Price	Monthly return
0	20	
1	21	0.05
2	23	0.09
3	17	-0.30
4	16	-0.06
5	18	0.12
6	22	0.20

$R_t = Ln(S_t/S_{t-1}) = Ln(21/20)$

Monthly return using continuous compounding

- The estimate of volatility is:
  - For the monthly volatility: 0.18
  - For the annual volatility: 0.62

Estimating volatility

□ Volatility can be also estimated from options prices

- Find the value of the volatility parameter such that

$$c_{observed} = c_{Black\text{-}Scholes}(\sigma)$$

- Ex: $c_{observed} = 33.376$ and $S = 100$, $K = 77$, $r = 0.0954$, and $T = 1$

Trial with $\sigma = 0.2$: $c_{Black\text{-}Scholes}(\sigma) = 30.254$

Trial with $\sigma = 0.3$: $c_{Black\text{-}Scholes}(\sigma) = 31.435$

Trial with $\boldsymbol{\sigma = 0.4}$: $\boldsymbol{c_{Black\text{-}Scholes}(\sigma) = 33.376}$

□ In practice one uses the Solver in Excel to determine this implied volatility

Using Options to Value Debt and Equity in a Levered Firm

□ Consider a firm that has assets in place with value $V = (V_t)_{t\geq 0}$ that evolves according to the stochastic differential equation

$$dV_t = r\, V_t\, dt + \sigma V_t\, dB_t, \qquad V_0 > 0$$

□ For simplicity, assume that the firm produces no cash flows before some fixed horizon T

□ In this specification

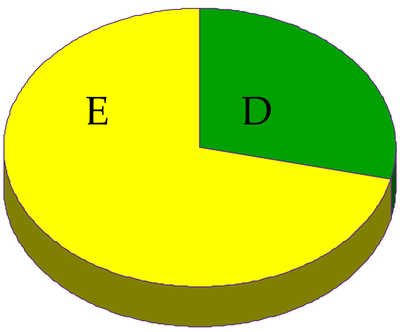
- r is the risk-free interest rate
- σ is the constant volatility of returns on the firm's assets
- B is a standard Brownian motion

Valuing Debt and Equity of a Levered Firm

- Assume that the M/M assumptions hold (no contracting costs, no taxes, and fixed investment policy)
- The firm has a simple capital structure with common stock and a single debt issue outstanding
- The debt contract
  - is a 0-coupon bond
  - has maturity date T
  - promises a single payment F at maturity

Valuing Debt and Equity of a Levered Firm

- Corporate securities are contingent claims: Their value depends on the value of the firm's assets



$$V = E + D$$

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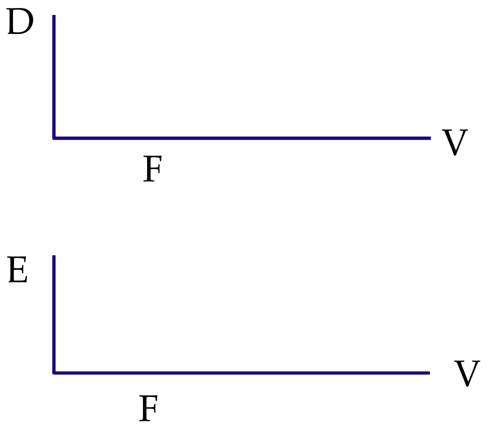
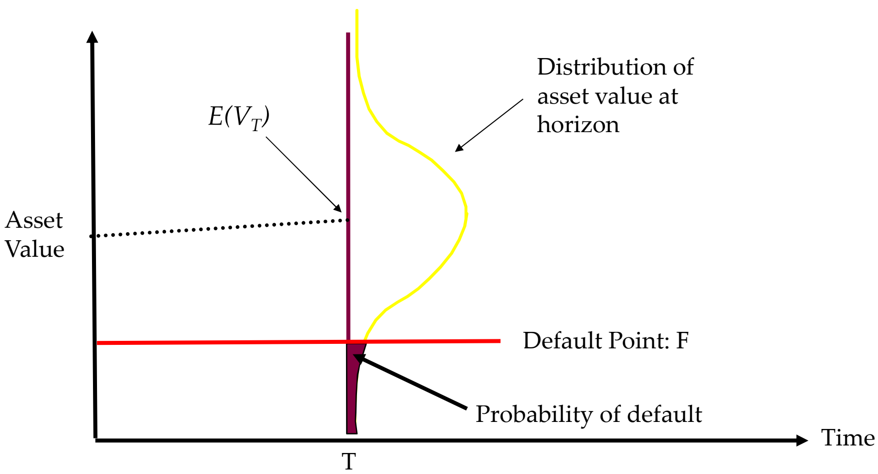


Graphical Representation



Valuing Debt and Equity of a Levered Firm

- Using the Black and Scholes to price equity and debt
- Equity is like a call option on the firm's assets with exercise price given by the promised payment to debtholders
- Mapping
  - c becomes E
  - S becomes V
  - K becomes F

Valuing Debt and Equity of a Levered Firm

- The value of equity in a levered firm is equivalent to a call option on the firm's assets

$$\boxed{E_0 = V_0 N(d_1) - F e^{-rT} N(d_1 - \sigma\sqrt{T})}$$

with

$$d_1 = \frac{ln\left(\frac{V_0}{F}\right) + rT}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

Application

- Assume that
  - Firm value is $V = \$100$
  - The face value of corporate debt is $F = \$80$
  - The maturity of corporate debt is T=*1 year*
  - The volatility of returns is $\sigma = 30\%$
  - No dividend is expected
  - The risk-free rate is $r = 3\%$
- The value of equity is:

Corporate debt

- Credit risk exists as long as the probability of default (V_T<F) is greater than zero
- This implies that at time 0 we have

$$D < Fe^{-rT}$$

- This also implies that the yield to maturity on corporate debt, Y_T, is higher than the risk free rate r

$$\boxed{Y_T = r + \textit{risk premium}}$$

Corporate debt

□ The value of the firm's assets is equal to the value of its liabilities or

$$V = E + D$$

□ This implies that the value of corporate debt satisfies

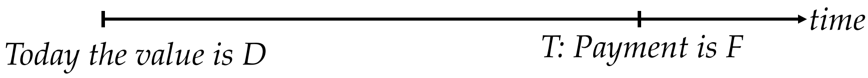
$$D = V - E$$

or

$$D = \underbrace{V\,[1 - N(d_1)]}_{1} + \underbrace{F\,e^{-rT}\,N(d_1 - \sigma\sqrt{T})}_{2}$$

Yields and spreads

□ What is the yield on corporate debt?



□ The market value of corporate debt is D; its face value is F; its time to maturity is *T*. Therefore the yield solves

$$D\,e^{YT} = F$$

□ What is the credit spread on corporate debt?

$$CS = Y - r$$

Yields and spreads

□ The default/credit spread can be computed exactly as a function of:

- the leverage ratio
- the volatility of returns on the underlying asset
- the debt maturity T

□ Using the above numerical example we find

- A credit spread equal to $CS = 0{,}83\%$
- A value for equity of $E = 25{,}28$
- A value for corporate debt of $D = 74{,}72$