Ecole Polytechnique Fédérale de Lausanne
Swiss Finance Institute @ EPFL

# Principles of *Finance*

Erwan Morellec

# Objective of the course

□ This course provides an introduction to financial valuation and value enhancement through financial decision making

□ Main objectives

*1- Understand standard financial valuations models*

*2- Understand the trade-off between risk and return and develop an ability to make portfolio decisions*

*3- Develop an ability to analyze and evaluate firms and investment projects*

*4- Understand the determinants of financing decisions*

*5- Understand derivatives markets and their benefits and costs*

□ Most results will be derived using a heuristic approach.

# Organization of the course

- This course consists of
  - Classroom sessions
  - Exercise sessions, which start on week 3
- I will be away on Thursday October 18 and Friday October 19. The lecture will be given on Monday October 8 and Tuesday October 9 from 10:15am until 12pm
  - **See the syllabus for details**
- Grading will be based on assignments and a final exam in the following way
  - The final exam grade: 50%
  - The assignments grades: 50%

# Evaluation

- **Assignments**
  - Due on the date indicated on the schedule (see Moodle website)
  - Group assignments: done in groups of 3 students
  - Remain with the same group through the entire course
- The final exam will take place during the exam session
  - This is a **closed book exam**
  - I will give you the necessary formulas
  - You can bring your own sheet of formulas (one-sided A4)

# Outline

□ The recommended book for the course is:

Jonathan Berk and Peter DeMarzo, **Corporate Finance**, Pearson, $1^{st}$, $2^{nd}$, $3^{rd}$, or $4^{th}$ Edition

□ Reading assignments follow the sequence indicated in the syllabus of the course

□ There is a **moodle website** for this course where you can find all the documents (handouts, assignments, sample exams)

□ If you get lost with the terminology, you can get useful information on the following web site:

http://people.duke.edu/~charvey/Classes/wpg/glossary.htm

# Outline of the course

□ The course will cover the following topics

- Discounting and valuation (*Handout 1*)
- Risk and return (*Handout 2*)
- Fundamentals of capital budgeting (*Handout 3*)
- Financing and valuation (*Handout 4*)
- Options in corporate finance (*Handout 5*)

# Introduction to finance

*Discounting and valuation*

Erwan Morellec

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

# Definitions

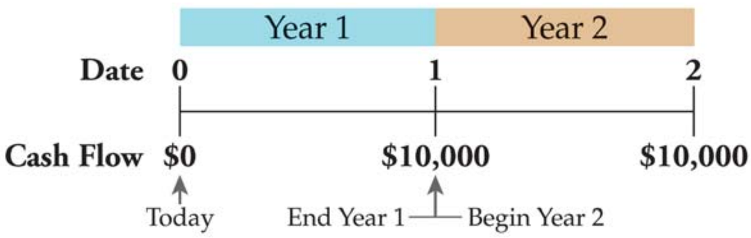
- **Cash flows** are numbers attached to dates in time, with date $t=0$ generally referring to the current date
- For example,
  - The date 0 purchase of a factory of \$10 million is a cash flow to the purchaser of -\$10 million
  - The sale of that same factory at date 1 for \$15 million represents a cash flow to the seller of +\$15 million.
  - The difference is a profit or capital gain

# Definitions

□ A timeline is a linear representation of the timing of potential cash flows

□ Drawing a timeline of the cash flows will help you visualize the financial problem

□ *Example*: Assume that you made a loan to a friend. You will be repaid in two payments, one at the end of each year over the next two years




# Projects

□ A **<u>project</u>** is anything that generates a series of cash flows

□ For example,

- You start a bicycle repair shop. Cost Sfr. 5000 today. Expected earnings Sfr. 1500 per month starting next month for 24 months
- A company decides to sell purple pen and expects to earn $10,000 per year for 10 years beginning next year after an initial investment of $20,000.

# Firms

□ A **firm** or company invests in various projects (for example Apple has invested in the Imac, Iphone, Iwatch, …)

□ These projects generate expenses and produce revenues. Revenues in excess of expenses go to new investments or are paid to **the firm's investors**

- Investors in the firm: Debtholders (lend money to the firm) and equity holders (also called shareholders or stockholders; invest in the firm by buying its shares)

□ **Equity** (actions): Share of ownership in a corporation. Firms obtain equity capital either internally by earning money and retaining it or externally by issuing new equity

□ **Debt** (dette): Financial contract containing a promise to pay a cash flow stream to the holder of the contract

# Different types of firms

□ Sole proprietorship (Raison individuelle)

- Business is owned and run by one person
- Typically has few, if any, employees
- Easy to create but unlimited personal liability

□ Partnership (Société en nom collectif)

- Similar to a sole proprietorship, but more than one owner
- All partners are personally liable for all of the firm's debts. A lender can require any partner to repay all of the firm's outstanding debts
- The partnership ends with the death or withdrawal of any single partner

## Different types of firms

- Limited partnership (Société en commandite) has two types of owners
- General partners
  - Have the same rights and liability as partners in a "regular" partnership
  - Typically run the firm on a day-to-day basis
- Limited partners
  - Have limited liability and cannot lose more than their initial investment
  - Have no management authority and cannot legally be involved in the managerial decision making for the business

## Different types of firms

- Limited Liability Company (SARL and SA)
- A *legal entity* separate from its owners
  - Has many of the legal powers individuals have such as the ability to enter into contracts, own assets, and borrow money
  - The firm is solely responsible for its own obligations. Its owners are not liable for any obligation the firm enters into
- Ownership
  - Represented by shares of stock
  - Sum of all ownership value is called equity
  - There is no limit to the number of shareholders, and thus the amount of funds a company can raise by selling stock

# Financing of firms

□ The initial capital (equity) that is required to start a business is usually provided by the entrepreneur and the immediate family

□ Often, a private company must seek outside sources that can provide additional (equity) capital for growth

- **Angel Investors**: Individual Investors who buy equity in small private firms (difficult to find)
- **Venture Capital Firm**: Limited partnership that specializes in raising money to invest in the private equity of young firms
- **Private Equity Firms**: Like VC but invests in the equity of existing privately held firms rather than start-up companies

□ It is important to understand how the infusion of outside capital will affect the control of the company (Facebook)

# Financing of firms

□ Institutional investors, such as pension funds, insurance companies, endowments, and foundations, are active investors in private companies

- Institutional investors may invest directly in private firms or they may invest indirectly by becoming limited partners in venture capital firms

□ Corporate investors

- A corporation that invests in private companies
- Although most other types of investors in private firms are primarily interested in the financial returns of their investments, corporate investors might invest for corporate strategic objectives, in addition to the financial returns

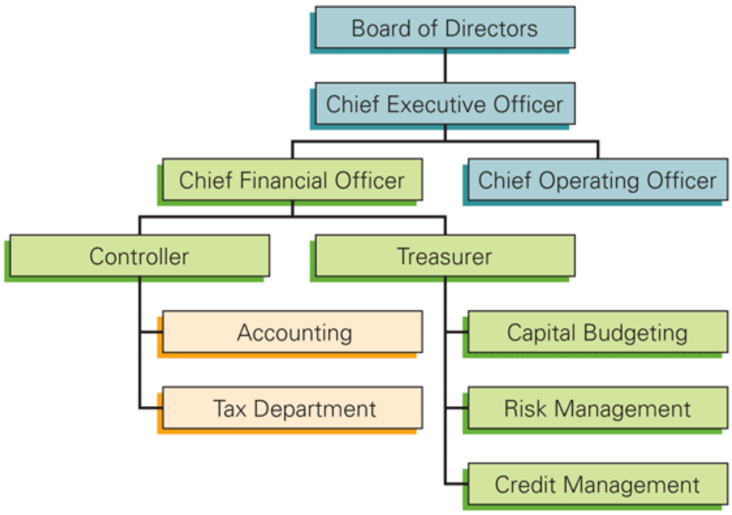
## Financing of firms

- Public company
  - Stock is traded by the public on a stock exchange
- Advantages of going public
  - **Greater liquidity**: PE investors get the ability to diversify
  - **Better access to capital**: Public companies have access to much larger amounts of capital through the public markets
- Disadvantages of going public
  - The equity holders become more widely dispersed. This makes it difficult to monitor management
  - The firm must satisfy all of the requirements (e.g. filings) of public companies

## Ownership versus Control of Corporations

- In a corporation (e.g. Nestlé), ownership and direct control are typically separate
- Board of Directors
  - Elected by shareholders
  - Have ultimate decision-making authority
- Chief Executive Officer (CEO)
  - Board typically delegates day-to-day decision making to CEO

# Organizational chart of a corporation




# Goal of the firm

- Goal of the firm
  - Shareholders (who own the firm) will agree that they are better off if management makes decisions that maximize the value of their shares
- The firm and society
  - Often, a corporation's decisions that increase the value of the firm's equity benefit society as a whole
  - As long as nobody else is made worse off by a firm's decisions, increasing the value of the firm's equity is good for society
  - It becomes a problem when increasing the value of the firm's equity comes at the expense of others

## The balance sheet

□ The projects in which firms invest generate expenses and produce revenues. Revenues in excess of expenses go to either new investments or to **the firm's investors**

- Equity holders
- Debt holders

□ One way firms **evaluate** their performance and **communicate** this information to investors is through their **financial statements**

- Balance sheet
- Income statement
- Statement of cash flows

## The balance sheet

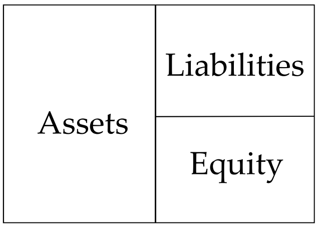
□ Presents a "snapshot" of the investing and financing activities of a firm at a point in time

□ Is based on the simple equation

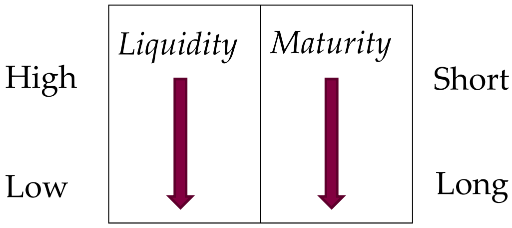
| **Assets** | | **Liabilities** | | **Stockholders' Equity** |
|---|---|---|---|---|
| Economic resources with probable future benefits | = | Economic obligations to outsiders, e.g., creditors | + | Owners' residual interest |

# The balance sheet

- Simple representation




- Simple representation




# The balance sheet

**GLOBAL CONGLOMERATE CORPORATION**

Consolidated Balance Sheet
Year Ended December 31 (in $ million)

| Assets | 2015 | 2014 | Liabilities and Stockholders' Equity | 2015 | 2014 |
|---|---|---|---|---|---|
| **Current Assets** | | | **Current Liabilities** | | |
| Cash | 21.2 | 19.5 | Accounts payable | 29.2 | 24.5 |
| Accounts receivable | 18.5 | 13.2 | Notes payable/short-term debt | 3.5 | 3.2 |
| Inventories | 15.3 | 14.3 | Current maturities of long-term debt | 13.3 | 12.3 |
| Other current assets | 2.0 | 1.0 | Other current liabilities | 2.0 | 4.0 |
| Total current assets | 57.0 | 48.0 | Total current liabilities | 48.0 | 44.0 |
| **Long-Term Assets** | | | **Long-Term Liabilities** | | |
| Land | 22.2 | 20.7 | Long-term debt | 99.9 | 76.3 |
| Buildings | 36.5 | 30.5 | Capital lease obligations | — | — |
| Equipment | 39.7 | 33.2 | Total debt | 99.9 | 76.3 |
| Less accumulated depreciation | (18.7) | (17.5) | Deferred taxes | 7.6 | 7.4 |
| Net property, plant, and equipment | 79.7 | 66.9 | Other long-term liabilities | — | — |
| Goodwill and intangible assets | 20.0 | 20.0 | Total long-term liabilities | 107.5 | 83.7 |
| Other long-term assets | 21.0 | 14.0 | **Total Liabilities** | 155.5 | 127.7 |
| Total long-term assets | 120.7 | 100.9 | **Stockholders' Equity** | 22.2 | 21.2 |
| **Total Assets** | 177.7 | 148.9 | **Total Liabilities and Stockholders' Equity** | 177.7 | 148.9 |

## The balance sheet

- The asset side of the balance sheet comprises current and long-term assets
- **Current assets** include
  - **Cash and other marketable securities**, which are short-term investments that can be easily converted into cash
  - **Accounts receivable**, which are amounts owed to the firm by costumers who have purchased goods on credit
  - **Inventories**, which are composed of raw materials as well as work-in-progress and finished goods
  - Other current assets (catch-all category)

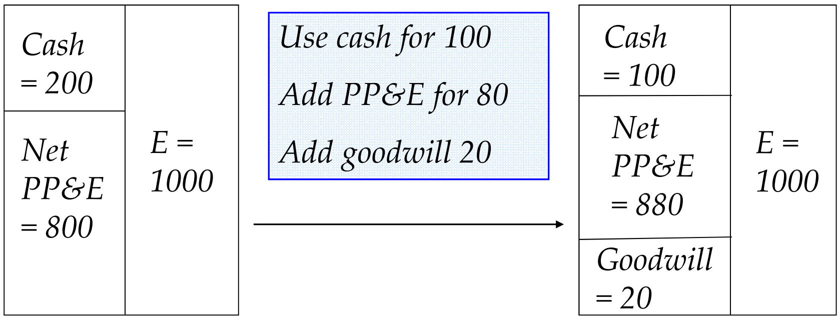
## The balance sheet

- **Long-term assets** include
  - **Net property plant and equipment (Net PP&E)**, which includes assets such as real estate, machines, or factories that produce tangible benefits <u>for more than one year</u>. Because equipment tends to become obsolete over time, the value recorded is reduced each year by an amount called **depreciation**. Depreciation is not an actual expense that the firm pays.
    The **book value** of an asset is equal to its acquisition cost minus accumulated depreciation
  - **Goodwill**, which is the difference between the purchase price of an asset/company and its book value
  - Other long-term assets (e.g. patents, trademarks)

# The balance sheet

□ **Goodwill** example

‣ Suppose you want to acquire a firm with market value 100 and to pay with cash. The book value of the firm's assets is 80. What happens to your balance sheet.




# The balance sheet

□ The liability side comprises current and long-term liabilities

□ **Current liabilities** include

‣ **Accounts payable**, the amounts owed to suppliers for products or services purchased with credit

‣ **Notes payable, short-term debt, and current maturities of long-term debt**, which are all repayments of debt that will occur within the next year

‣ Items such as salary or taxes that are owed but have not been paid yet

□ The difference between current assets and current labilities is the firm's **net working capital**, the capital available in the short term to run the business

## The balance sheet

□ **Long-term liabilities** are liabilities that extend beyond one year. They include

- **Long-term debt**
- **Capital leases**, which are long-term lease contracts that obligate the firm to make regular lease payments in exchange for the use of an asset
- **Deferred taxes**, which are taxes that are owed but have not yet been paid

□ The difference between the firm's assets and liabilities is **stockholders' equity**; it is also called the **book value** of equity

## The balance sheet

□ Ideally, the balance sheet should provide an accurate assessment of the true value of the firm's equity

□ This is unlikely as

- Many assets on the balance sheet are valued based on their **historical cost**
- Many of the firm's most valuable assets, such as its reputation or the expertise of its employees, are not on the balance sheet

□ Because of these limitations, the **book value** of equity often differs substantially from its **market value**, defined as the amount investors are willing to pay for the equity

# The balance sheet

□ The total market value of the firm's equity is defined as

$$Market\ value\ of\ equity$$
$$= Shares\ Outstanding \times Market\ price\ per\ share$$

□ It is often referred to as the firm's **market capitalization**

□ Using the **book and market values** of equity, we can also define the market-to-book ratio

$$Market - to - book\ ratio = \frac{Market\ value\ of\ equity}{Book\ value\ of\ equity}$$

which substantially exceeds 1 for most successful firms

# The income statement

□ The **income statement** lists the firm's revenues and expenses over a period of time

□ The last line of the income statement shows the firm's **net income**, which is a measure of its profitability during the period

□ The income statement is sometimes called a profit and loss, or «P&L», statement and the net income is also referred to as the firm's earnings

□ This course

- Use the income statement to evaluate firms

# The income statement

GLOBAL CONGLOMERATE CORPORATION

Income Statement
Year Ended December 31 (in $ million)

| | 2015 | 2014 |
|---|---|---|
| Total sales | 186.7 | 176.1 |
| Cost of sales | (153.4) | (147.3) |
| **Gross Profit** | 33.3 | 28.8 |
| Selling, general, and administrative expenses | (13.5) | (13.0) |
| Research and development | (8.2) | (7.6) |
| Depreciation and amortization | (1.2) | (1.1) |
| **Operating Income** | 10.4 | 7.1 |
| Other income | — | — |
| **Earnings Before Interest and Taxes (EBIT)** | 10.4 | 7.1 |
| Interest income (expense) | (7.7) | (4.6) |
| **Pretax Income** | 2.7 | 2.5 |
| Taxes | (0.7) | (0.6) |
| **Net Income** | 2.0 | 1.9 |
| Earnings per share: | \$0.556 | \$0.528 |
| Diluted earnings per share: | \$0.526 | \$0.500 |

# The income statement

□ Net income represents the total earnings of the firm's equity holders. It is often reported on a per share basis as the firm's **earnings per share** (**EPS**)

□ We compute EPS by dividing net income by the number of shares

$$\text{EPS} = \frac{\text{Net Income}}{\text{Shares Outstanding}} = \frac{\$2.0 \text{ million}}{3.6 \text{ million shares}} = \$0.556 \text{ per share}$$

□ Used in valuation models relying on multiples

- **Example**: Suppose that in the pharmaceutical industry firms are worth on average 10 times their EPS. With an EPS of $0.83, a firm should have a share price of $8.3

# Statement of cash flows

□ Net Income typically does **NOT** equal the amount of **cash** the firm has earned

□ Need to make adjustments for

- **Cash from operating activities**, e.g. add depreciation and amortization and deduct the increase in net working capital (deduct increase in accounts receivables, …)
- **Cash from investing activities**: Deduct investment in PP&E or the purchase of securities and add the sale of assets and securities
- **Cash from financing activities**: Deduct dividend payments and add new financing

# Statement of cash flows

GLOBAL CONGLOMERATE CORPORATION

Statement of Cash Flows
Year Ended December 31 (in $ million)

| | 2015 | 2014 |
|---|---|---|
| Operating activities | | |
| Net income | 2.0 | 1.9 |
| Depreciation and amortization | 1.2 | 1.1 |
| Other non-cash items | (2.8) | (1.0) |
| Cash effect of changes in | | |
| Accounts receivable | (5.3) | (0.3) |
| Accounts payable | 4.7 | (0.5) |
| Inventory | (1.0) | (1.0) |
| **Cash from operating activities** | **(1.2)** | **0.2** |
| Investment activities | | |
| Capital expenditures | (14.0) | (4.0) |
| Acquisitions and other investing activity | (7.0) | (2.0) |
| **Cash from investing activities** | **(21.0)** | **(6.0)** |
| Financing activities | | |
| Dividends paid | (1.0) | (1.0) |
| Sale (or purchase) of stock | — | — |
| Increase in borrowing | 24.9 | 5.5 |
| **Cash from financing activities** | **23.9** | **4.5** |
| **Change in cash and cash equivalents** | **1.7** | **(1.3)** |

## First principles

□ We have said before that revenues in excess of expenses go to either **new investments** or to **the firm's investors**

□ Corporations create value for their shareholders by making good real investment decisions

□ To do so, corporations must first be able to identify the costs and benefits of their decisions

□ In addition, to determine which projects they should invest in, corporations need rules. These rules should allow firms to compare one project to another one a fair basis

□ Similarly, investors may choose to invest in different securities (shares of Nestlé, Apple, UBS, …). They also need rules to guide them when making these choices

## The time value of money

□ \$100 today is worth more than \$100 one year from today because money that you have today can be invested and will start earning interest immediately

□ The difference in value between money today and money in the future is due to **the time value of money**

□ The rate at which we can exchange money today for money in the future is determined by the current **interest rate**

- Risk-free interest rate $r_f$: The interest rate at which money can be borrowed or lent **without risk**

□ *\$1* today can be invested at the rate $r_f$ to generate $\$(1+r_f)$ in one year. As a result, \$*1* in one year is worth $\$1/(1+r_f)$ today

## The time value of money

□ Consider an investment opportunity with the following certain cash flows

- Cost: \$100,000 today
- Benefit: \$105,000 in one year

□ Suppose the current annual interest rate is 7%. By investing or borrowing at this rate, we can exchange \$1.07 in one year for each \$1 today

## The time value of money

□ If the interest rate is 7%, then we can express our cost in one year as

$$Cost = \$100'000 \times 1{,}07 = \$107'000$$

□ Both costs and benefits are now in terms of "dollars in one year," so we can compare them and compute the investment's net value

$$\$105'000 - \$107'000 = -\$2'000$$

□ In other words, we could earn \$2000 more in one year by putting our \$100,000 in the bank rather than making this investment. **We should reject the investment**

# The time value of money

□ Instead of computing values in one year, we can determine **the value of the investment today**

□ Consider the benefit of $105,000 in one year. What is the equivalent amount in terms of dollars today?

$$Benefit = \$105'000 \times \frac{1}{1{,}07} = \$98'130$$

□ Both costs and benefits are now in terms of "dollars today," so we can compare them and compute the investment's net value

$$\$98'130 - \$100'000 = -\$1'870$$

□ We should reject the investment. **Our decision is the same** whether we express the value of the investment in terms of dollars in one year or dollars today.

# The time value of money

□ The two results are equivalent but expressed as values at different points in time

□ When we express the value in terms of dollars today, we call it the **present value** (PV) of the investment. If we express it in terms of dollars in the future, we call it the **future value** of the investment

□ We can interpret

$$\frac{1}{1+r} = \frac{1}{1{,}07} = 0{,}93458$$

as the price today of $1 in one year. The amount $\frac{1}{1+r}$ is called the one- year **discount factor**. The risk-free rate is also referred to as the **discount rate** for a risk-free investment

# The time value of money

□ This reasoning extends to multiple periods

□ What is the value of \$100 to be received in one year if the annual interest rate is 4%?

□ What is the value of \$100 to be received in two years if the annual interest rate is 4%?

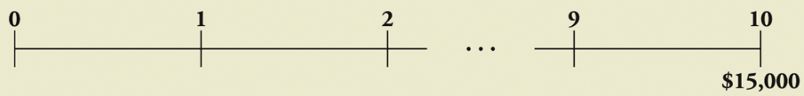
# The time value of money

**Problem**

You are considering investing in a savings bond that will pay \$15,000 in ten years. If the competitive market interest rate is fixed at 6% per year, what is the bond worth today?

**Solution**

The cash flows for this bond are represented by the following timeline:




Thus, the bond is worth \$15,000 in ten years. To determine the value today, we compute the present value:

$$PV = \frac{15{,}000}{1.06^{10}} = \$8375.92 \text{ today}$$

The bond is worth much less today than its final payoff because of the time value of money.

# The time value of money

| | | |
|---|---|---|
| **Rule 1** | Only values at the same point in time can be compared or combined. | |
| **Rule 2** | To move a cash flow forward in time, you must compound it. | Future Value of a Cash Flow $FV_n = C \times (1 + r)^n$ |
| **Rule 3** | To move a cash flow backward in time, you must discount it. | Present Value of a Cash Flow $PV = C \div (1 + r)^n = \dfrac{C}{(1 + r)^n}$ |

# The time value of money

**The Power of Compounding**

**Problem**

Suppose you invest \$1000 in an account paying 10% interest per year. How much will you have in the account in 7 years? in 20 years? in 75 years?

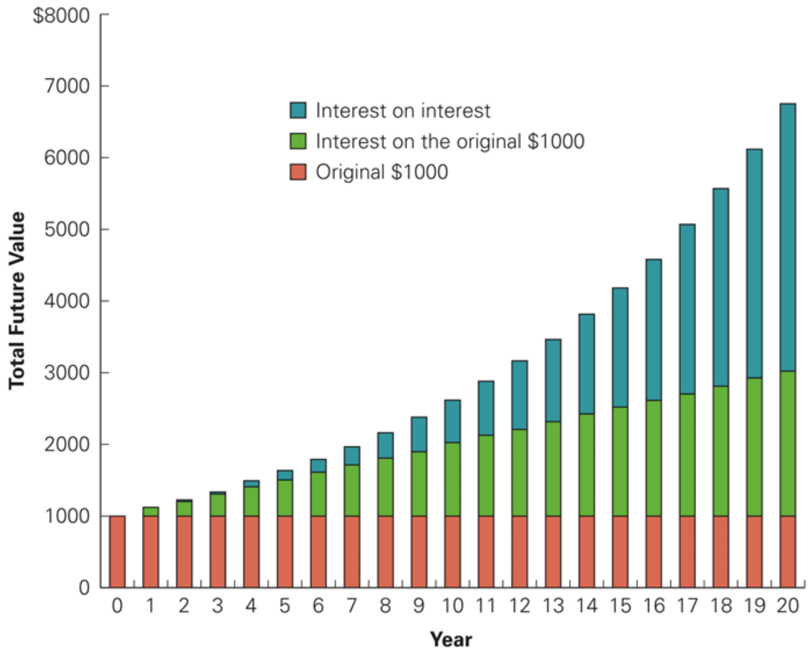
**Solution**

You can apply Eq. 4.1 to calculate the future value in each case:

| | |
|---|---|
| 7 years: | $\$1000 \times (1.10)^7 = \$1948.72$ |
| 20 years: | $\$1000 \times (1.10)^{20} = \$6727.50$ |
| 75 years: | $\$1000 \times (1.10)^{75} = \$1{,}271{,}895.37$ |

Note that at 10% interest, your money will nearly double in 7 years. After 20 years, it will increase almost 7-fold. And if you invest for 75 years, you will be a millionaire!

# The time value of money




# The law of one price

- **Arbitrage**:
  - The practice of buying and selling equivalent goods in different markets to take advantage of a price difference. An **arbitrage opportunity** occurs when it is possible to make a profit without taking any risk or making any investment
- **Normal market**: A market in which there are no arbitrage opportunities
- **2nd Law: <u>The law of one price</u>**
  - If equivalent investment opportunities trade simultaneously in different markets, then they must trade for the same price in both markets

# The law of one price

□ Assume a security promises a risk-free payment of $1000 in one year. If the risk-free interest rate is 5%, what can we conclude about the price of this bond in a normal market?

$$Price = PV(\$1000\ in\ one\ year) = \frac{1000}{1{,}05} = \$952{,}38$$

□ What if the price of the security is **<u>not</u>** $952.38? Assume for example the price is $940

| | Today ($) | In One Year ($) |
|---|---|---|
| Buy the bond | −940.00 | +1000.00 |
| Borrow from the bank | +952.38 | −1000.00 |
| Net cash flow | +12.38 | 0.00 |

The opportunity for arbitrage will force the price of the security to rise until it is equal to $952.38

# The law of one price

□ Unless the price of the security equals the present value of the security's cash flows, an arbitrage opportunity will appear

□ No Arbitrage Price of a Security

$$Price(Security) = PV(All\ cash\ flows\ paid\ by\ security)$$

**Computing the No-Arbitrage Price**

**Problem**

Consider a security that pays its owner $100 today and $100 in one year, without any risk. Suppose the risk-free interest rate is 10%. What is the no-arbitrage price of the security today (before the first $100 is paid)? If the security is trading for $195, what arbitrage opportunity is available?

# The law of one price

**Solution**

We need to compute the present value of the security's cash flows. In this case there are two cash flows: \$100 today, which is already in present value terms, and \$100 in one year. The present value of the second cash flow is

$$\$100 \text{ in one year} \div (1.10 \text{ \$ in one year/\$ today}) = \$90.91 \text{ today}$$

Therefore, the total present value of the cash flows is \$100 + \$90.91 = \$190.91 today, which is the no-arbitrage price of the security.

If the security is trading for \$195, we can exploit its overpricing by selling it for \$195. We can then use \$100 of the sale proceeds to replace the \$100 we would have received from the security today and invest \$90.91 of the sale proceeds at 10% to replace the \$100 we would have received in one year. The remaining \$195 − \$100 − \$90.91 = \$4.09 is an arbitrage profit.

# The law of one price

- The **Law of One Price** also has implications for packages of securities
  - Consider two securities, A and B. Suppose a third security, C, has the same cash flows as A and B combined. In this case, security C is equivalent to a portfolio, or combination, of the securities A and B
  - The law of one price implies

$$P(C) = P(A + B) = P(A) + P(B)$$

# The law of one price

### Valuing an Asset in a Portfolio

**Problem**
Holbrook Holdings is a publicly traded company with only two assets: It owns 60% of Harry's Hotcakes restaurant chain and an ice hockey team. Suppose the market value of Holbrook Holdings is \$160 million, and the market value of the entire Harry's Hotcakes chain (which is also publicly traded) is \$120 million. What is the market value of the hockey team?

**Solution**
We can think of Holbrook as a portfolio consisting of a 60% stake in Harry's Hotcakes and the hockey team. By value additivity, the sum of the value of the stake in Harry's Hotcakes and the hockey team must equal the \$160 million market value of Holbrook. Because the 60% stake in Harry's Hotcakes is worth 60% × \$120 million = \$72 million, the hockey team has a value of \$160 million − \$72 million = \$88 million.

# Value additivity and the law of one price

- Present values (or PVs) obey the principle of **value additivity**

> *The present value of many cash flows is the sum of their individual present values*

- Let $C_0, C_1, C_2, \ldots, C_N$ denote the cash flows at dates $0, 1, 2, \ldots, N$. The present value of this cash flow stream is

$$\boxed{PV = \sum_{n=0}^{N} PV(C_n) = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n}}$$

if for all horizons the discount rate is r. This formula is called the **discounted cash flow (DCF) formula**

⇔ Law of one price

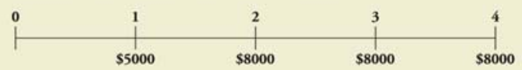
# Value additivity and the law of one price

**Problem**

You have just graduated and need money to buy a new car. Your rich Uncle Henry will lend you the money so long as you agree to pay him back within four years, and you offer to pay him the rate of interest that he would otherwise get by putting his money in a savings account. Based on your earnings and living expenses, you think you will be able to pay him \$5000 in one year, and then \$8000 each year for the next three years. If Uncle Henry would otherwise earn 6% per year on his savings, how much can you borrow from him?

**Solution**

The cash flows you can promise Uncle Henry are as follows:




How much money should Uncle Henry be willing to give you today in return for your promise of these payments? He should be willing to give you an amount that is equivalent to these payments in present value terms. This is the amount of money that it would take him to produce these same cash flows, which we calculate as follows:

$$
\begin{aligned}
PV &= \frac{5000}{1.06} + \frac{8000}{1.06^2} + \frac{8000}{1.06^3} + \frac{8000}{1.06^4} \\
&= 4716.98 + 7119.97 + 6716.95 + 6336.75 \\
&= 24{,}890.65
\end{aligned}
$$

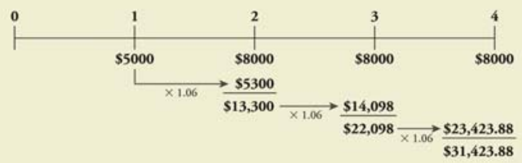
Thus, Uncle Henry should be willing to lend you \$24,890.65 in exchange for your promised payments. This amount is less than the total you will pay him (\$5000 + \$8000 + \$8000 + \$8000 = \$29,000) due to the time value of money.

Let's verify our answer. If your uncle kept his \$24,890.65 in the bank today earning 6% interest, in four years he would have

$$FV = \$24{,}890.65 \times (1.06)^4 = \$31{,}423.87 \text{ in 4 years}$$

Now suppose that Uncle Henry gives you the money, and then deposits your payments to him in the bank each year. How much will he have four years from now?

We need to compute the future value of the annual deposits. One way to do so is to compute the bank balance each year:




We get the same answer both ways (within a penny, which is because of rounding).

# Net Present Value (NPV)

□ The **net present value** of a project is the difference between the project's present value and the cost of implementing the project

$$\boxed{NPV = PV(\text{Benefits}) - PV(\text{Costs})}$$

□ Projects that create value are those whose present values exceed their costs

- They represent situations in which a future cash flow pattern can be produced more cheaply within the firm than by investing in financial assets.
- They are called **positive net present value** investments

# Net Present Value (NPV)

**Net Present Value of an Investment Opportunity**

**Problem**

You have been offered the following investment opportunity: If you invest \$1000 today, you will receive \$500 at the end of each of the next three years. If you could otherwise earn 10% per year on your money, should you undertake the investment opportunity?

**Solution**

As always, we start with a timeline. We denote the upfront investment as a negative cash flow (because it is money we need to spend) and the money we receive as a positive cash flow.


To decide whether we should accept this opportunity, we compute the NPV by computing the present value of the stream:

$$NPV = -1000 + \frac{500}{1.10} + \frac{500}{1.10^2} + \frac{500}{1.10^3} = \$243.43$$

Because the NPV is positive, the benefits exceed the costs and we should make the investment. Indeed, the NPV tells us that taking this opportunity is like getting an extra \$243.43 that you can spend today. To illustrate, suppose you borrow \$1000 to invest in the opportunity and an extra \$243.43 to spend today. How much would you owe on the \$1243.43 loan in three years? At 10% interest, the amount you would owe would be

$$FV = (\$1000 + \$243.43) \times (1.10)^3 = \$1655 \text{ in three years}$$

At the same time, the investment opportunity generates cash flows. If you put these cash flows into a bank account, how much will you have saved three years from now? The future value of the savings is

$$FV = (\$500 \times 1.10^2) + (\$500 \times 1.10) + \$500 = \$1655 \text{ in three years}$$

As you see, you can use your bank savings to repay the loan. Taking the opportunity therefore allows you to spend \$243.43 today at no extra cost.

# Net Present Value (NPV)

- We can also use the NPV decision rule to choose among projects
- To do so, we must compute the NPV of each alternative, and then select the one with the highest NPV. This alternative is the one which will lead to the largest increase in the value of the firm

# Net Present Value (NPV)

## Choosing among Alternative Plans

**Problem**

Suppose you started a Web site hosting business and then decided to return to school. Now that you are back in school, you are considering selling the business within the next year. An investor has offered to buy the business for $200,000 whenever you are ready. If the interest rate is 10%, which of the following three alternatives is the best choice?

1. Sell the business now.
2. Scale back the business and continue running it while you are in school for one more year, and then sell the business (requiring you to spend $30,000 on expenses now, but generating $50,000 in profit at the end of the year).
3. Hire someone to manage the business while you are in school for one more year, and then sell the business (requiring you to spend $50,000 on expenses now, but generating $100,000 in profit at the end of the year).

# Net Present Value (NPV)

- Faced with these three alternatives, the best one is the one with the highest NPV: Hire a manager and sell in one year
- Choosing this alternative is equivalent to receiving $222,727 today

| | Today | In One Year | NPV |
|---|---|---|---|
| Sell Now | $200,000 | 0 | $200,000 |
| Scale Back Operations | −$30,000 | $50,000<br>$200,000 | $-\$30{,}000 + \frac{\$250{,}000}{1.10} = \$197{,}273$ |
| Hire a Manager | −$50,000 | $100,000<br>$200,000 | $-\$50{,}000 + \frac{\$300{,}000}{1.10} = \$222{,}727$ |

# Net Present Value (NPV)

- Regardless of our preferences for cash today versus cash in the future, we should always maximize NPV first
- We can then borrow or lend to shift cash flows through time and find our most preferred pattern of cash flows
- Suppose for example you need \$60'000 today

| | Today | In One Year |
|---|---|---|
| Hire a Manager | −\$50,000 | \$300,000 |
| Borrow | \$110,000 | −\$121,000 |
| Total Cash Flow | \$60,000 | \$179,000 |
| Versus | | |
| Sell Now | \$200,000 | \$0 |
| Invest | −\$140,000 | \$154,000 |
| Total Cash Flow | \$60,000 | \$154,000 |

# Looking for shortcuts

- A perpetuity is a security that gives you a constant payment *C* forever starting next year. Its value is given by
- Dividing both sides by *1+r*, we get
- Subtracting we finally obtain

$$\boxed{PV(C \text{ in perpetuity}) = \frac{C}{r}}$$

# Looking for shortcuts

**Problem**

You want to endow an annual MBA graduation party at your alma mater. You want the event to be a memorable one, so you budget \$30,000 per year forever for the party. If the university earns 8% per year on its investments, and if the first party is in one year's time, how much will you need to donate to endow the party?

**Solution**

The timeline of the cash flows you want to provide is


This is a standard perpetuity of \$30,000 per year. The funding you would need to give the university in perpetuity is the present value of this cash flow stream. From the formula,

$$PV = C / r = \$30{,}000 / 0.08 = \$375{,}000 \text{ today}$$

If you donate \$375,000 today, and if the university invests it at 8% per year forever, then the MBAs will have \$30,000 every year for their graduation party.

# Looking for shortcuts

- A standard **annuity** with payments of C from date *1* to date *N* can be viewed as the difference between
  - A perpetuity with cash flows starting at date *1*
  - A perpetuity with cash flows starting at date *N+1*
- If the discount rate is *r* per period, the PV of this annuity is thus

$$\boxed{PV(\text{annuity of } C \text{ for } N \text{ periods with interest rate } r) = C \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)}$$

# Looking for shortcuts

**Problem**

You are the lucky winner of the $30 million state lottery. You can take your prize money either as (a) 30 payments of $1 million per year (starting today), or (b) $15 million paid today. If the interest rate is 8%, which option should you take?

**Solution**

Option (a) provides $30 million in prize money but paid over time. To evaluate it correctly, we must convert it to a present value. Here is the timeline:

| 0 | 1 | 2 | … | 29 |
|---|---|---|---|---|
| **$1 million** | **$1 million** | **$1 million** | | **$1 million** |

Because the first payment starts today, the last payment will occur in 29 years (for a total of 30 payments). The $1 million at date 0 is already stated in present value terms, but we need to compute the present value of the remaining payments. Fortunately, this case looks like a 29-year annuity of $1 million per year, so we can use the annuity formula:

$$
\begin{aligned}
PV(\text{29-year annuity of \$1 million}) &= \$1 \text{ million} \times \frac{1}{0.08}\left(1 - \frac{1}{1.08^{29}}\right) \\
&= \$1 \text{ million} \times 11.16 \\
&= \$11.16 \text{ million today}
\end{aligned}
$$

Thus, the total present value of the cash flows is $1 million + $11.16 million = $12.16 million. In timeline form:

| 0 | 1 | 2 | … | 29 |
|---|---|---|---|---|
| **$1 million** | **$1 million** | **$1 million** | | **$1 million** |
| **$11.16 million** | | | | |
| **$12.16 million** | | | | |

Option (b), $15 million upfront, is more valuable—even though the total amount of money paid is half that of option (a). The reason for the difference is the time value of money. If you have the $15 million today, you can use $1 million immediately and invest the remaining $14 million at an 8% interest rate. This strategy will give you $14 million × 8% = $1.12 million per year in perpetuity! Alternatively, you can spend $15 million − $11.16 million = $3.84 million today, and invest the remaining $11.16 million, which will still allow you to withdraw $1 million each year for the next 29 years before your account is depleted.

# Looking for shortcuts

□ Similarly, the value of a perpetuity with payments that grow at the rate $g$ when the discount rate is $r$ is

$$\boxed{PV(\text{growing perpetuity}) = \frac{C}{r - g}}$$

□ The value of an annuity with payments that grow at the rate $g$ when the discount rate is $r$ is

$$\boxed{PV = C \times \frac{1}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^N\right)}$$

# Looking for shortcuts

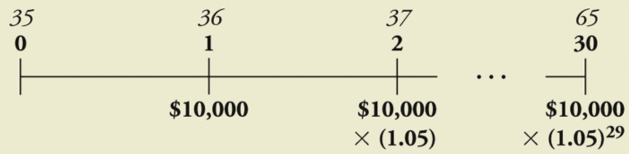
□ Ellen is currently 35-year old and she considers saving $10'000 at the end of this year for her retirement

□ Although $10'000 is the most she can save this year, she expects her salary to increase each year so that she will be able to increase her savings by 5% per year

□ Whis this plan, if she earns 10% per year on her savings, how much will Ellen have saved at age 65?

**Solution**

Her new savings plan is represented by the following timeline:




This example involves a 30-year growing annuity, with a growth rate of 5%, and an initial cash flow of \$10,000. The present value of this growing annuity is given by

$$PV = \$10{,}000 \times \frac{1}{0.10 - 0.05}\left(1 - \left(\frac{1.05}{1.10}\right)^{30}\right)$$

$$= \$10{,}000 \times 15.0463$$

$$= \$150{,}463 \text{ today}$$

Ellen's proposed savings plan is equivalent to having \$150,463 in the bank *today.* To determine the amount she will have at age 65, we need to move this amount forward 30 years:

$$FV = \$150{,}463 \times 1.10^{30}$$

$$= \$2.625 \text{ million in 30 years}$$

Ellen will have saved \$2.625 million at age 65 using the new savings plan. This sum is almost \$1 million more than she had without the additional annual increases in savings.

# Interest rates

□ We have just explored the mechanics of computing present values given a market interest rate

□ To determine the appropriate discount rate from an interest rate, we need to understand how interest rates are quoted

□ Interest rates are often stated as an **effective annual rate** (**EAR**), which indicates the total amount of interest that will be earned at the end of one year

□ **Example**: with an EAR of 5%, a \$100,000 investment grows to

$$\$100{,}000 \times (1+r) = \$100{,}000 \times (1.05) = \$105{,}000$$

□ By raising the interest rate factor (*1+r*) to the appropriate power, we can compute an EAR for longer or shorter time periods

## Interest rates

□ Banks also quote interest rates in terms of an **annual percentage rate** (**APR**), which indicates the amount of **simple interest** earned in one year (i.e. ignoring compounding)

□ The APR with *k* compounding periods is a way of quoting the actual interest earned each compounding period

$$\text{Interest Rate per Compounding Period} = \frac{\text{APR}}{k \text{ periods / year}}$$

□ **Example:** 6% APR with monthly compounding, implies that one earns 0.5% every month

## Interest rates

□ Because it does not include the effect of compounding, the APR quote is **typically less** than the actual amount one will pay or earn

□ Because the **APR** ignores compounding, it **cannot be used as a discount rate**

□ When working with APRs, one must first convert the APR to an EAR before evaluating the PV of a set of cash flows

# Interest rates

□ The EAR corresponding to an APR with $k$ compounding periods satisfies

$$\boxed{1 + EAR = \left(1 + \frac{APR}{k}\right)^k}$$

□ The following table shows the effective annual rate that corresponds to an APR of 6% with varying compounding periods

| Compounding Interval | Effective Annual Rate |
|---|---|
| Annual | $(1 + 0.06 / 1)^1 - 1 = 6\%$ |
| Semiannual | $(1 + 0.06 / 2)^2 - 1 = 6.09\%$ |
| Monthly | $(1 + 0.06 / 12)^{12} - 1 = 6.1678\%$ |
| Daily | $(1 + 0.06 / 365)^{365} - 1 = 6.1831\%$ |

# Continuous compounding

□ In the limit as we increase the number $k$ of compounding periods to infinity, we go to continuous compouding

□ Denote the APR with continuous compounding by $c$

□ We have

$$1 + EAR = e^c$$

or

$$c = ln(1 + EAR)$$

□ With an APR of $c$=6%, we get

$$EAR = e^{0.06} - 1 = 6.1837\%$$

□ As a practical matter, componding more frequently than daily has a negligible impact on the **EAR**

# Interest rates

**Problem**

Ten years ago your firm borrowed $3 million to purchase an office building using a loan with a 7.80% APR and monthly payments for 30 years. How much do you owe on the loan today? How much interest was paid on the loan in the past year?

**Solution**

The first step is to solve for the monthly loan payment. Here is the timeline (in months):


An APR of 7.80% with monthly compounding is equivalent to 7.80% / 12 = 0.65% per month. The monthly payment is then

$$C = \frac{P}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)} = \frac{3{,}000{,}000}{\frac{1}{0.0065}\left(1 - \frac{1}{(1.0065)^{360}}\right)} = \$21{,}596$$

The remaining balance on the loan is the present value of the remaining 20 years, or 240 months, of payments:

$$\text{Balance after 10 years} = \$21{,}596 \times \frac{1}{0.0065}\left(1 - \frac{1}{1.0065^{240}}\right) = \$2{,}620{,}759$$

Thus, after 10 years, you owe $2,620,759 on the loan.

During the past year, your firm made total payments of $21,596 × 12 = $259,152 on the loan. To determine how much of that amount was interest, it is easiest to first determine the amount that was used to repay the principal. Your loan balance one year ago, with 21 years (252 months) remaining, was

$$\text{Balance after 9 years} = \$21{,}596 \times \frac{1}{0.0065}\left(1 - \frac{1}{1.0065^{252}}\right) = \$2{,}673{,}248$$

Therefore, the balance declined by $2,673,248 − $2,620,759 = $52,489 in the past year. Of the total payments made, $52,489 was used to repay the principal and the remaining $259,152 − $52,489 = $206,663 was used to pay interest.

# Interest rates

□ Inflation and Real Versus Nominal Rates:

- **Nominal Interest Rate ($r_n$)**: The rates quoted by financial institutions and used for discounting cash flows

□ Suppose you have \$50’000 in cash and you are interested in buying a car that costs \$50’000

□ If you do not buy the car, you can invest your cash in government bonds that will return **$r_n$=2%** per year

□ If the price of the car increases by

- **Less than 2%** (e.g. to 50’500): Your purchasing power increases, because you can buy more than the car with your investment
- **More than 2%**(e.g. to 52’000): Your purchasing power decreases, because you can no longer buy the car with your investment

# Interest rates

□ Inflation and Real Versus Nominal Rates:

- **Nominal Interest Rate ($r_n$)**: The rates quoted by financial institutions and used for discounting cash flows
- **Real Interest Rate ($r_r$)**: The rate of growth of your purchasing power, after adjusting for inflation

□ We have

$$Growth\ of\ purchasing\ power = \frac{Growth\ of\ money}{Growth\ of\ prices}$$

or

$$1 + r_r = \frac{1 + r_n}{1 + i}$$

so that

$$r_r = \frac{r_n - i}{1 + i} \approx r_n - i$$

# Interest rates

**Calculating the Real Interest Rate**

**Problem**

At the start of 2008, one-year U.S. government bond rates were about 3.3%, while the rate of inflation that year was 0.1%. At the start of 2011, one-year interest rates were about 0.3%, and inflation that year was about 3.0%. What was the real interest rate in 2008 and 2011?

**Solution**

Using Eq. 5.5, the real interest rate in 2008 was $(3.3\% - 0.1\%)/(1.001) = 3.20\%$. In 2011, the real interest rate was $(0.3\% - 3.0\%)/(1.03) = -2.62\%$. Note that the real interest rate was negative in 2011, indicating that interest rates were insufficient to keep up with inflation: Investors in U.S. government bonds were able to buy less at the end of the year than they could have purchased at the start of the year. On the other hand, there was hardly any inflation in 2008, and so the real interest rate earned was only slightly below the nominal interest rate.

# Interest rates

- Interest rates are set in the market in such a way that the **supply of lending matches the demand for borrowing**
- An increase in interest rates will typically reduce the NPV of an investment and the demand for loans
- Consider an project that requires an initial investment of \$10 million and generates a cash flow of \$3 million per year for four years. If the interest rate is 5%, the project has an NPV of

$$NPV = -10 + \frac{3}{1{,}05} + \frac{3}{(1{,}05)^2} + \frac{3}{(1{,}05)^3} + \frac{3}{(1{,}05)^4} = 0{,}638$$

- If the interest rate rises to 9%, the NPV becomes negative and the project is no longer profitable and the demand for loans is 0 for this project

# Interest rates

□ The interest rates that banks offer on investments depends on the horizon, or **term**, of the investment

□ The relation between the investment term and the interest rate is called the **term structure** of interest rates

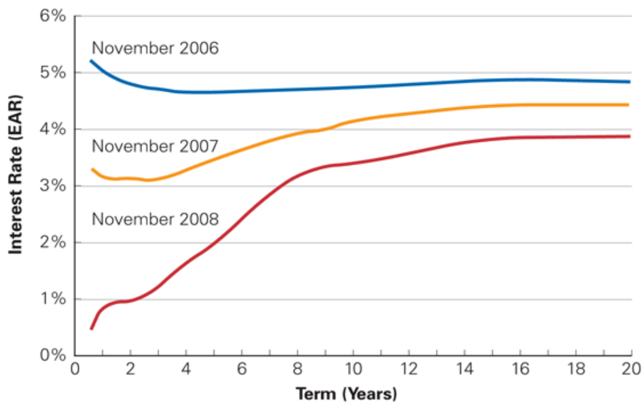
□ We can plot this relation on a graph called the **yield curve**

□ The following shows the interest rate available from investing in *risk-free* U.S. Treasury securities with different investment terms

- In each case, interest rates depend on the horizon

# Interest rates

| Term (years) | Date Nov-06 | Nov-07 | Nov-08 |
|---|---|---|---|
| 0.5 | 5.23% | 3.32% | 0.47% |
| 1 | 4.99% | 3.16% | 0.91% |
| 2 | 4.80% | 3.16% | 0.98% |
| 3 | 4.72% | 3.12% | 1.26% |
| 4 | 4.63% | 3.34% | 1.69% |
| 5 | 4.64% | 3.48% | 2.01% |
| 6 | 4.65% | 3.63% | 2.49% |
| 7 | 4.66% | 3.79% | 2.90% |
| 8 | 4.69% | 3.96% | 3.21% |
| 9 | 4.70% | 4.00% | 3.38% |
| 10 | 4.73% | 4.18% | 3.41% |
| 15 | 4.89% | 4.44% | 3.86% |
| 20 | 4.87% | 4.45% | 3.87% |




□ Because interest rates tend to fall in response to an economic slowdown, an inverted yield curve is often interpreted as a negative forecast for economic growth

# Interest rates

□ All the shortcuts for computing present values we presented above are based on discounting all cash flows at the same rate

□ They cannot be used in situations in which cash flows need to be discounted at different rates

□ When this is the case, the following formula applies

$$PV = \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \cdots + \frac{C_N}{(1 + r_N)^N} = \sum_{n=1}^{N} \frac{C_n}{(1 + r_n)^n}$$

# Interest rates

**Using the Term Structure to Compute Present Values**

**Problem**

Compute the present value in November 2008 of a risk-free five-year annuity of \$1000 per year, given the yield curve for November 2008 in Figure 5.2.

**Solution**

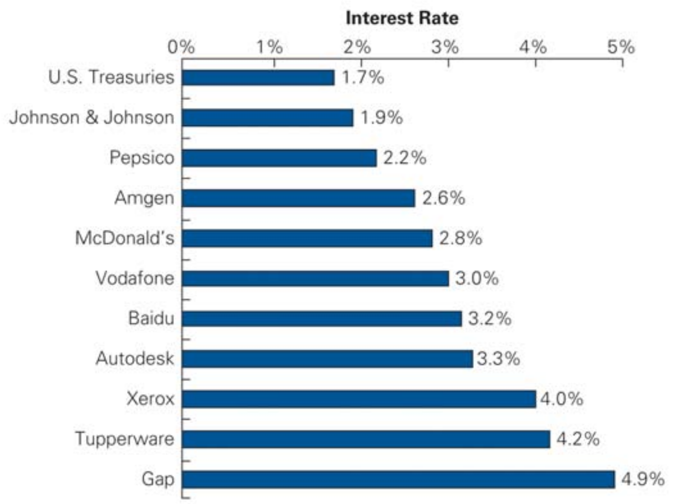
To compute the present value, we discount each cash flow by the corresponding interest rate:

$$PV = \frac{1000}{1.0091} + \frac{1000}{1.0098^2} + \frac{1000}{1.0126^3} + \frac{1000}{1.0169^4} + \frac{1000}{1.0201^5} = \$4775.25$$

Note that we cannot use the annuity formula here because the discount rates differ for each cash flow.

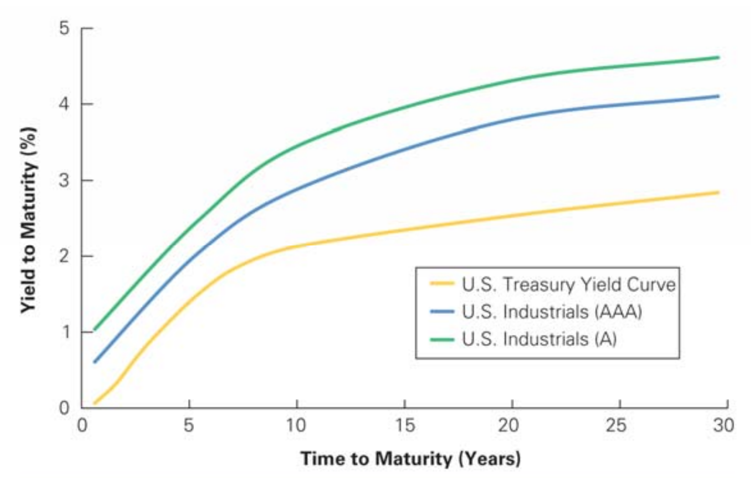
# Interest rates

□ Interest rates typically depend on the term. They also depend on the identity of the borrower. Data for December 2015.




# Interest rates

□ Interest rates typically depend on the term. They also depend on the identity of the borrower. Data August 2015,



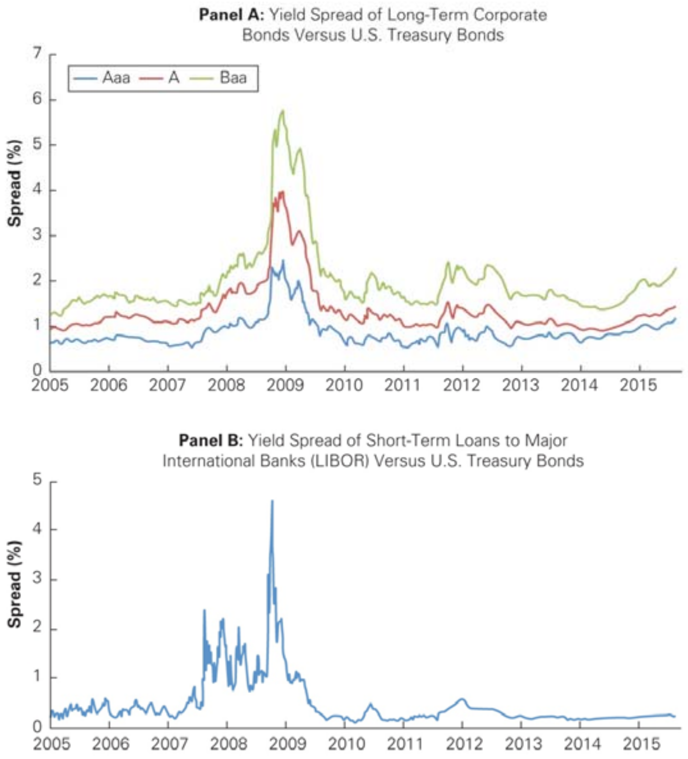
# Credit ratings

- Credit ratings play an important role in helping investors assess the **riskiness** of a bond or an issuer
- The main rating agencies are Moody's Investor Services (or Moody's), Standard and Poors (or S&P), and Fitch
- The following table shows their main rating categories
  - Within each rating category, there are subcategories also called notches

# Credit ratings

- Bonds with a rating equal to or above Baa (Moody's) or BBB (S&P) are considered *investment grade*. Bonds with a lower rating are *speculative grade* or *high-yield* (junk) bonds

| | Moody's | S&P |
|---|---|---|
| Investment Grade | Aaa | AAA |
| | Aa | AA |
| | A | A |
| | Baa | BBB |
| Junk Bonds | Ba | BB |
| | B | B |
| | Caa | CCC |
| | Ca | CC |
| | C | C |




# Credit ratings

- □ Rating agencies use a number of factors to analyze the risk of a company
- □ Rating agencies are concerned with the risks to bondholders
  - ‣ The risk that interest payments will not be made
  - ‣ The risk that the final principal will not be paid
- □ Several factors are used to assess these two risks: financial factors and non-financial factors (product differentiation, geographical diversification, or asset tangibility)
- □ See handouts 4 and 5 for more details

## Valuing bonds

□ A bond is a financial security in which the issuer promises to make interest and principal payments to the holder. It is typically issued by a corporation or a government to raise money

□ The terms of the bond are described in the **bond certificate**, which indicates the amounts and dates of all payments to be made

□ Bonds are primarily characterized by

- **Maturity date** = date of last promised payment
- **Face**, **par**, or **principal** value = promised payment at maturity
- **Coupon** = promised payments prior to maturity

## Valuing bonds

□ The amount of each coupon payment is determined by the **coupon rate** of the bond. This coupon rate is set by the issuer and stated on the bond certificate.

□ By convention, the coupon rate is expressed as an **APR**, so the amount of each coupon, *CPN*, is

$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}}$$

□ For example, a "$1,000 bond with a 10% coupon rate and semi-annual payments" will pay a coupon of $50 every six months.

## Zero-coupon bonds

- The simplest type of bond is a **zero-coupon bond**, a bond that does not make coupon payments (e.g. Treasury Bills)
- The only cash payments that the investor receives is the face value of the bond on the maturity date.
- Zero-coupon bonds always trade at a **discount** (a price lower than the face value), so they are also called **pure discount bonds**
  - The price today is equal to the discounted face value

## Yield to maturity

- The **internal rate of return** (**IRR**) of an investment in zero-coupon bond is the rate of return that investors will earn if they buy the bond at its current price and hold it to maturity
- The IRR of an investment in a bond is called the **yield to maturity** (**YTM**) or just yield
- The YTM for a zero-coupon bond with $n$ periods to maturity, current price $P$, and face value $FV$ is

$$\boxed{P = \frac{FV}{(1 + YTM_n)^n}}$$

- Rearranging the previous expression, we get

$$\boxed{YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1}$$

# Yield to maturity

**Problem**
Suppose the following zero-coupon bonds are trading at the prices shown below per $100 face value. Determine the corresponding yield to maturity for each bond.

| **Maturity** | **1 year** | **2 years** | **3 years** | **4 years** |
|---|---|---|---|---|
| **Price** | $96.62 | $92.45 | $87.63 | $83.06 |

**Solution**
Using Eq. 8.3, we have

$$YTM_1 = (100 / 96.62) - 1 \quad = 3.50\%$$
$$YTM_2 = (100 / 92.45)^{1/2} - 1 = 4.00\%$$
$$YTM_3 = (100 / 87.63)^{1/3} - 1 = 4.50\%$$
$$YTM_4 = (100 / 83.06)^{1/4} - 1 = 4.75\%$$

# Coupon bonds

- Like zero-coupon bonds, **coupon bonds** pay investors their face value at maturity.
  - In addition, these bonds make regular coupon interest payments (e.g Treasury notes and bonds)
- We can also compute the yield to maturity of a coupon bond. The YTM is the interest rate that solves

$$\boxed{P = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N}}$$

# Coupon bonds

□ Using the Law of One Price and the yields of default-free zero-coupon bonds, one can determine the price and yield of any other default-free bond

□ Replicating a three-year $1000 bond that pays 10% annual coupon using three zero-coupon bonds

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Coupon bond: | | $100 | $100 | $1100 |
| 1-year zero: | | $100 | | |
| 2-year zero: | | | $100 | |
| 3-year zero: | | | | $1100 |
| Zero-coupon Bond portfolio: | | $100 | $100 | $1100 |

# Coupon bonds

□ Yields and Prices (per $100 Face Value) for Zero Coupon Bonds

| Maturity | 1 year | 2 years | 3 years | 4 years |
|---|---|---|---|---|
| YTM | 3.50% | 4.00% | 4.50% | 4.75% |
| Price | $96.62 | $92.45 | $87.63 | $83.06 |

□ By the Law of One Price, the three-year coupon bond must trade for a price of $1153

| Zero-Coupon Bond | Face Value Required | Cost |
|---|---|---|
| 1 year | 100 | 96.62 |
| 2 years | 100 | 92.45 |
| 3 years | 1100 | 11 × 87.63 = 963.93 |
| | **Total Cost:** | $1153.00 |

# Coupon bonds

□ Zero-coupon bonds always trade at a discount

□ Coupon bonds may trade at a discount, at **par** (price equal to the face value) or at a **premium** (price greater than the face value)

□ If the bond trades at a **discount**, an investor will earn a return both from receiving the coupons and from receiving a face value that **exceeds** the price paid for the bond

- As a result, its **yield** to maturity will **exceed its coupon rate**

□ Conversely, if a bond trades at a **premium**, its yield to maturity is **less** than its coupon rate

# Coupon bonds

□ The following table summarizes these results

| When the bond price is | We say the bond trades | This occurs when |
|---|---|---|
| greater than the face value | "above par" or "at a premium" | Coupon Rate > Yield to Maturity |
| equal to the face value | "at par" | Coupon Rate = Yield to Maturity |
| less than the face value | "below par" or "at a discount" | Coupon Rate < Yield to Maturity |

# Coupon bonds

□ Suppose you have a bond with face value 100, coupon rate 10%, and one year to maturity. What is the **yield to maturity** on this bond if its price is 90? 100? 110?

# Coupon bonds

**Determining the Discount or Premium of a Coupon Bond**

**Problem**

Consider three 30-year bonds with annual coupon payments. One bond has a 10% coupon rate, one has a 5% coupon rate, and one has a 3% coupon rate. If the yield to maturity of each bond is 5%, what is the price of each bond per $100 face value? Which bond trades at a premium, which trades at a discount, and which trades at par?

## Coupon bonds

**Solution**

We can compute the price of each bond using Eq. 6.5. Therefore, the bond prices are

$$P(10\%\text{ coupon}) = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86 \quad \text{(trades at a premium)}$$

$$P(5\%\text{ coupon}) = 5 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$100.00 \quad \text{(trades at par)}$$

$$P(3\%\text{ coupon}) = 3 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$69.26 \quad \text{(trades at a discount)}$$

## Time and bond prices

**Problem**

Consider a 30-year bond with a 10% coupon rate (annual payments) and a $100 face value. What is the initial price of this bond if it has a 5% yield to maturity? If the yield to maturity is unchanged, what will the price be immediately before and after the first coupon is paid?

**Solution**

We computed the price of this bond with 30 years to maturity in Example 8.5:

$$P = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$$

Now consider the cash flows of this bond in one year, immediately before the first coupon is paid. The bond now has 29 years until it matures, and the timeline is as follows:

| 0 | 1 | 2 | ... | 29 |
|---|---|---|---|---|
| \$10 | \$10 | \$10 | | \$10 + \$100 |

Again, we compute the price by discounting the cash flows by the yield to maturity. Note that there is a cash flow of \$10 at date zero, the coupon that is about to be paid. In this case, it is easiest to treat the first coupon separately and value the remaining cash flows as in Eq. 8.5:

$$P(\text{just before first coupon}) = 10 + 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$185.71$$

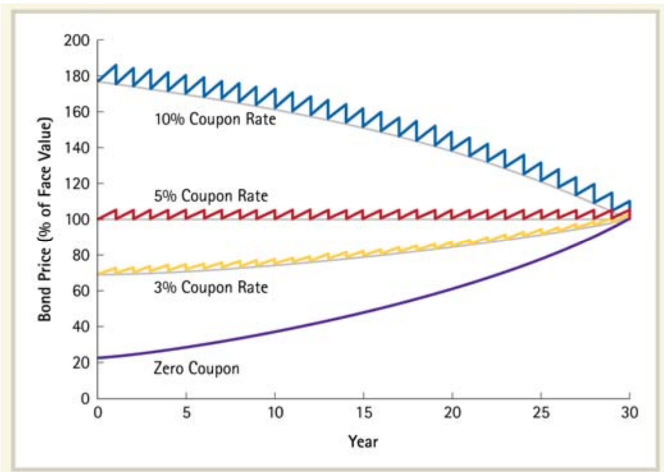
Note that the bond price is higher than it was initially. It will make the same total number of coupon payments, but an investor does not need to wait as long to receive the first one. We could also compute the price by noting that because the yield to maturity remains at 5% for the bond, investors in the bond should earn a return of 5% over the year: \$176.86 × 1.05 = \$185.71.

What happens to the price of the bond just after the first coupon is paid? The timeline is the same as that given earlier, except the new owner of the bond will not receive the coupon at date zero. Thus, just after the coupon is paid, the price of the bond (given the same yield to maturity) will be

$$P(\text{just after first coupon}) = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$175.71$$

The price of the bond will drop by the amount of the coupon (\$10) immediately after the coupon is paid, reflecting the fact that the owner will no longer receive the coupon. In this case, the price is lower than the initial price of the bond. Because there are fewer coupon payments remaining, the premium investors will pay for the bond declines. Still, an investor who buys the bond initially, receives the first coupon, and then sells it, earns a 5% return if the bond's yield does not change: (10 + 175.71) / 176.86 = 1.05.

The graph illustrates the effects of the passage of time on bond prices when the yield remains constant.




The price of a zero-coupon bond rises smoothly.

The price of a coupon bond also rises between coupon payments, but tumbles on the coupon date, reflecting the amount of the coupon payment. For each coupon bond, the gray line shows the trend of the bond price just after each coupon is paid.

# Valuing stocks

□ **Common stock** is a share of ownership in a corporation that **entitles its holders to the firm's profits** after other contractual claims on the firm are satisfied (see *income statement*)

□ The price of a security should be equal to the present value of the expected cash flows an investor will receive from owning it

□ Thus, to value a stock we need to know

- the expected cash flows an investor will receive
- the appropriate discount rate

□ Both of these quantities can be challenging to estimate

# A one-year investor

□ There are two potential sources of cash flows from a stock

- The firm might pay out cash in the form of a dividend
- The investor might generate cash by selling the stock

□ Consider first the case of an investor with a **one-year horizon**

□ Let $Div_1$ be the total dividends paid per share of the stock during the year. At the end of the year the investor will sell his share at the new market price $P_1$

□ These cash flows must be discounted at the equity cost of capital $r_E$. Therefore we have

$$P_0 = \frac{Div_1 + P_1}{1 + r_E}$$

# A one-year investor

□ Rearranging this equation, we can write

$$r_E = \frac{Div_1 + P_1}{P_0} - 1 = \underbrace{\frac{Div_1}{P_0}}_{\text{Dividend Yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital Gain Rate}}$$

□ The right hand side of this equation shows that the **total return** of the stock is given by the sum of

- The stock's **dividend yield**, which is the expected annual dividend of the stock divided by its current price
- The **capital gain rate**, which is the capital gain the investor will earn on the stock divided by the current stock price.

# A one-year investor

**Stock Prices and Returns**

**Problem**

Suppose you expect Walgreen Company (a drugstore chain) to pay dividends of $0.44 per share and trade for $33 per share at the end of the year. If investments with equivalent risk to Walgreen's stock have an expected return of 8.5%, what is the most you would pay today for Walgreen's stock? What dividend yield and capital gain rate would you expect at this price?

# A one-year investor

**Solution**

Using Eq. 9.1, we have

$$P_0 = \frac{Div_1 + P_1}{1 + r_E} = \frac{0.44 + 33.00}{1.085} = \$30.82$$

At this price, Walgreen's dividend yield is $Div_1/P_0 = 0.44/30.82 = 1.43\%$. The expected capital gain is $\$33.00 - \$30.82 = \$2.18$ per share, for a capital gain rate of $2.18/30.82 = 7.07\%$. Therefore, at this price, Walgreen's expected total return is $1.43\% + 7.07\% = 8.5\%$, which is equal to its equity cost of capital.

# A multi-year investor

□ Suppose the investor plans to hold the stock for two years. We then have

$$\boxed{P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2 + P_2}{(1 + r_E)^2}}$$

□ We can extend the horizon by replacing the final stock price with the value that the next holder of the stock would pay

□ Doing this leads to the general **dividend-discount model** of the stock price where the horizon $N$ is arbitrary:

$$\boxed{P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2}{(1 + r_E)^2} + \cdots + \frac{Div_N}{(1 + r_E)^N} + \frac{P_N}{(1 + r_E)^N}}$$

# Valuing stocks

□ Letting $N$ go to infinity, we can write

$$\boxed{P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2}{(1 + r_E)^2} + \frac{Div_3}{(1 + r_E)^3} + \cdots = \sum_{n=1}^{\infty} \frac{Div_n}{(1 + r_E)^n}}$$

□ That is, the price of the stock is equal to the present value of the expected future dividends it will pay

# Valuing stocks

□ The simplest forecast of the firm's dividends states that it will grow at a constant rate $g$ forever. We then have

$$\boxed{P_0 = \frac{Div_1}{r_E - g}}$$

□ In handout 2, we will see how to determine $r_E$

- Risk-free cash flows should be discounted by the return on risk-free securities such as government bonds
- Risky cash flows typically will be discounted at a higher rate to compensate for their risk

□ In handout 3, we will see how to determine $Div_1$ and $g$

# Valuing stocks

**Valuing a Firm with Constant Dividend Growth**

**Problem**

Consolidated Edison, Inc. (Con Edison), is a regulated utility company that services the New York City area. Suppose Con Edison plans to pay $2.36 per share in dividends in the coming year. If its equity cost of capital is 7.5% and dividends are expected to grow by 1.5% per year in the future, estimate the value of Con Edison's stock.

**Solution**

If dividends are expected to grow perpetually at a rate of 1.5% per year, we can use Eq. 9.6 to calculate the price of a share of Con Edison stock:

$$P_0 = \frac{Div_1}{r_E - g} = \frac{\$2.36}{0.075 - 0.015} = \$39.33$$

# Share repurchases

□ In recent years, a number of firms have replaced dividend payouts with share repurchases. In a **share repurchase**, the firm uses excess cash to buy back its own stock

□ When a firm repurchases shares, one should use the **total payout model**, which values all of the firm's equity rather than a single share. We have

$$\boxed{P_0 = \frac{PV(\text{Future Total Dividends and Repurchases})}{\text{Shares Outstanding}_0}}$$

□ We can then apply the same simplifications that we obtained by assuming constant growth to the total payout model

# Share repurchases

- Titan Industries has 217 million shares outstanding and expects earnings at the end of this year of $860 million. Titan plans to pay out 50% of its earnings in total, paying 30% as a dividend and using 20% to repurchase shares.
- If Titan's earnings are expected to grow by 7.5% per year and these payout rates remain constant, determine Titan's share price assuming an equity cost of capital of 10%

**Solution**

Titan will have total payouts this year of 50% × $860 million = $430 million. Based on the equity cost of capital of 10% and an expected earnings growth rate of 7.5%, the present value of Titan's future payouts can be computed as a constant growth perpetuity:

$$PV(\text{Future Total Dividends and Repurchases}) = \frac{\$430 \text{ million}}{0.10 - 0.075} = \$17.2 \text{ billion}$$

This present value represents the total value of Titan's equity (i.e., its market capitalization). To compute the share price, we divide by the current number of shares outstanding:

$$P_0 = \frac{\$17.2 \text{ billion}}{217 \text{ million shares}} = \$79.26 \text{ per share}$$

Using the total payout method, we did not need to know the firm's split between dividends and share repurchases. To compare this method with the dividend-discount model, note that Titan will pay a dividend of 30% × $860 million / (217 million shares) = $1.19 per share, for a dividend yield of 1.19 / 79.26 = 1.50%. From Eq. 9.7, Titan's expected EPS, dividend, and share price growth rate is $g = r_E - Div_1 / P_0 = 8.50\%$. This growth rate exceeds the 7.50% growth rate of earnings because Titan's share count will decline over time due to share repurchases.