ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

# Principles of Finance

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**Assignment 3**

### Instructions

- Assignments should be done in groups of 3 students.
- You should remain with the same group through the entire course.
- Submit on Moodle only one copy of solutions per group.
- For each assignment you can get a maximum of 100 points.
- All assignments turned in late will not be graded (zero points).

### Due date

The due date is indicated on Moodle.

For this assignment, you should hand in two files: First, a separate file (pdf) where you record all your results (tables, graphs, etc.) and interpretations. Second, you should also hand in your Excel file with your calculations. The Excel file we only need if we do not understand something you did. The instructions are very precise on what you should do, what results to report, and what questions to answer.

**Warm-up** You are given the following sample: (15 points)

| Year End | S&P 500 | Dividends Paid |
|---|---|---|
| 2000 | 1330.93 | 16.27 |
| 2001 | 1144.93 | 15.74 |
| 2002 | 899.18 | 16.07 |
| 2003 | 1080.64 | 17.39 |
| 2004 | 1199.21 | 19.44 |
| 2005 | 1262.07 | 22.22 |
| 2006 | 1416.42 | 24.88 |
| 2007 | 1479.22 | 27.73 |
| 2008 | 877.56 | 28.39 |
| 2009 | 1110.38 | 22.41 |
| 2010 | 1241.53 | 22.73 |
| 2011 | 1243.32 | 26.43 |
| 2012 | 1422.29 | 31.25 |
| 2013 | 1807.78 | 34.99 |
| 2014 | 2054.27 | 39.44 |
| 2015 | 2054.08 | 43.39 |
| 2016 | 2246.63 | 45.70 |

Source: http://www.econ.yale.edu/~shiller/data.htm

(a) What is the median dividend yield for the S&P 500 from 2002 to 2015?
(b) What is the volatility of the dividend yield from 2001 to 2016?
(c) What is the median capital gain rate of the S&P 500 from 2003 to 2014?
(d) What is the variance of S&P 500 returns from capital gains from 2001 to 2016?
(e) What is the average total return of the S&P 500 from 2003 to 2016?
(f) What is the variance of S&P 500 total returns from 2001 to 2016?

**Part I (15 points)**

You have just started your new job with a financial planning firm. In addition to studying for your exams, you have been asked to review a portion of a client's stock portfolio to determine the risk/return profiles of 12 stocks in the portfolio. Unfortunately, your small firm cannot afford the expensive databases that would provide all this information with a few simple keystrokes, but that is why they have you. Specifically, you have been asked to determine the monthly average returns and standard deviations for the 12 stocks. The stocks are: Apple (AAPL), Johnson & Johnson (JNJ), J. P. Morgan (JPM), Procter & Gamble (PG), Exxon Mobil (XOM), Pfizer Inc. (PFE), Microsoft Corp. (MSFT), AT & T (T), Citi Group (C), Oracle Corporation (ORCL), General Electric (GE), and Wells Fargo & Co. (WFC).

- Collect price information for each stock from Yahoo! Finance (http://finance.yahoo.com) as follows:
  - Enter the stock symbol. On the page for that stock, click "Historical Data" in the header of the page.
  - Enter the "start date" as Jan 1, 2007 and the "end date" as December 31, 2017. Make sure to access historical data based on "**monthly**" frequency. Download the data to a spreadsheet. Delete all columns except the date and the adjusted close (the first and second to last columns).
  - Keep the Excel file open and go back to the Yahoo! Finance Web page. When you return to the prices page, enter the next stock symbol and hit "Apply" again to get the prices for this stock. Do not change the dates or frequency, but make sure you have the same dates for all the stocks you will download. Again, click "Download to Spreadsheet" and then open the file. Copy the last column, "**Adj. Close**", paste it into the Excel file and change "Adj. Close" to the stock symbol (so that you know which column belongs to which stock). Make sure that the first and last prices are in the same rows as the first stock.
  - Repeat these steps for the remaining ten stocks, pasting each adjusted closing price right next to the other stocks, again making sure that the correct prices on the correct dates all appear on the same rows.
- Convert these prices to monthly returns as the percentage change in the monthly prices. *Hint: Pay attention to the chronological order of the data.*
- Compute the mean monthly returns and standard deviations for the monthly returns of each of the stocks. Convert the monthly statistics to **annual** statistics for easier interpretation. *Hint: The annualized return can be approximated by* $12\bar{r}_{monthly}$. *The annualized standard deviation can be approximated by* $\sqrt{12}\sigma_{monthly}$.
- Add a column in your Excel worksheet with the average returns across stocks for each month. These are the monthly returns to an equally weighted portfolio of the 12 stocks. Compute the mean and standard deviation of monthly returns for the equally weighted portfolio. Double check that the average return on this equally weighted portfolio is equal to the average return of all of the individual stocks. Again, convert these monthly statistics to annual statistics.

**Report** the following results:

1. Report the annual mean return and annualized standard deviation of the 12 stocks and of the equally-weighted portfolio in a table in the separate file.

2. Create a plot with the annual standard deviation on the $x$-axis and the annual return on the $y$-axis and represent the 12 stocks and the portfolio in this plot. Copy the plot into the separate file.

3. How does the portfolio average return and standard deviation compare with those of individual firms? Comment.

### Part II (35 points)

Now you want to rebalance your portfolio with the optimal weights that will provide the best risk and return combination for the 12-stock portfolio. Use the Solver function in Excel to perform this analysis (the time-consuming alternative is to find the optimum weights by trial and error).

- Begin with the equally weighted portfolio analyzed above. Establish the portfolio returns for the stocks in the portfolio using a formula that depends on the portfolio weights. Initially, these weights will all equal 1/12. You would like to allow the portfolio weights to vary, so you will need to list the weights for each stock in separate cells and establish another cell that sums the weights of the stocks. The portfolio returns for each month MUST reference these weights for Excel solver to be of any use.

- Compute the values for the monthly mean return and standard deviation of the portfolio. Convert these values to annual numbers.

- Compute the efficient frontier when short sales are NOT allowed. To activate the Solver function in Excel, click the "Tools" tab, select "Add-Ins...", check "Solver-Add-in" in the pop-up dialog box, and then click "OK". To set the solver parameter:

  - Set the target cell as the cell of interest, making it the cell that computes the (annual) portfolio standard deviation. Minimize this value.
  - Establish the "By Changing Cells" by holding the control key and clicking in each of the 12 cells containing the weights of each stock (all in all you select 12 cells).
  - Add constraints by clicking on the add button next to the "Subject to the constraints" box. One set of constraints will be that the weight of each stock is greater than or equal to zero. A second constraint is that the weights will sum to one.
  - Compute the portfolio with the lowest standard deviation for a given expected return. Start by finding this portfolio with an expected return of 5%. To do this, add a constraint that the (annual) portfolio return equals 0.05.

  - If the parameters are set correctly, you should get a solution when you click "Solve". If there is an error, you will need to double-check the parameters, especially the constraints.

- Record the resulting standard deviation for the "optimally weighted" portfolio with a return of 0.09 in a separate cell on the spreadsheet. Repeat the previous step to solve for the portfolio with the lowest standard deviation for several different choices of expected returns: 0.11, 0.13, 0.15, 0.20, 0.25, 0.30, 0.40. Record these values. Plot the efficient frontier with the constraint of no short sales. To do this, use a plot with portfolio standard deviation on the $x$-axis and the return on the $y$-axis.

- Redo your analysis to allow for short sales by removing the constraint that each portfolio weight is greater than or equal to zero. Use again the solver to calculate the (annual) portfolio standard deviation when the annual portfolio returns are set to 0.09, 0.11, 0.13, 0.15, 0.20, 0.25, 0.30, 0.40. Plot the unconstrained efficient frontier into the same plot as the constrained efficient frontier.

**Report** the following results:

1. Report the values that you used to produce the plots of the constrained and unconstrained efficient frontier in a table in the separate file.

2. Copy the plot into the separate file.

3. What do you observe in the plot? Comment.

### Part III (35 points)

Finally, you would like to use the CAPM to compute expected returns for all 12 stocks in the portfolio. Specifically, you should estimate betas for each stock using ten years of monthly data and an expected return using the historical risk premium of 4.5% and a risk free rate of 2%. You should compute the betas using the S&P 500 as market index.

- Get the returns for the S&P 500 from Yahoo! Finance (symbol ^GSPC). Again, use January 1, 2007 as the start date and December 31, 2017 as the end date to obtain the prices and remember to click "monthly". Download those prices and add the **adjusted** closing price to your spreadsheet. Then create monthly returns for the S&P 500 following the procedure you used for individual stocks.

- For each stock, estimate the following regression:

$$R_{it} = \alpha_i + \beta_i R_{S\&P500,t} + \epsilon_{it}$$

  where $R_{it}$ is the monthly return of stock $i$ at time $t$, $R_{S\&P500,t}$ is the return of the S&P 500 at time $t$, and $\epsilon_{it}$ is a normally distributed error term. To do those regression, you can use any program you like.

**Report** the following results:

1. For the S&P 500 return series, compute the mean, variance, standard deviation, skewness, kurtosis, maximum, and minimum using the adequate Excel functions (use for these questions the monthly simple returns). What do you observe (compare for instance the standard deviation, the maximum and minimum returns of the two series)?

2. In Excel, use the "Data Analysis" package to compute the empirical distribution of the S&P 500. For this purpose, generate bins with 2 percent intervals, i.e., start at 30%, then 28%, 26%, etc. until -28%, and -30%. Use these bins to construct your histogram and empirical cumulative distribution.

3. Using the function NORMDIST as well as the mean and standard deviation computed above, construct a cumulative distribution assuming that returns are normally distributed with mean and standard deviation equal to that of the S&P 500 index. Draw a graph where you include both the empirical cumulative distribution and the cumulative distribution, assuming that returns are normally distributed. What do you observe?

4. Report in one table the following statistics for each stock: The estimate of the constant, the estimate of the slope coefficient (beta), the $t$-statistic of the slope coefficient, the adjusted R-squared, the number of observations you used for the estimation, and the Jensen's Alpha (assume a monthly interest rate of 0.167% to do the Jensen's Alpha computation). Finally, compute and report in the same table the expected return (on an annual basis) for each stock using the beta estimates and the other information provided in this exercise.

5. How do you interpret the slope coefficient (beta)?

6. What does the *adjusted* R-squared mean?

7. What does Jensen's Alpha measure?

8. Assuming that the annual risk-free rate is 2%, does assuming a 4.5% historical risk premium seem reasonable in this case?