MGT-482 Principles of Finance
Assignment 1

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1 Exercise 1

We compute the present value of the benefits:

$$PV(benefits) = \frac{14500}{1.075^2} + \frac{17500}{1.075^5} = 24'737.10 \qquad [\ \$\ ] \quad (1)$$

2 Exercise 2

We compute the net present value of this investment opportunity:

$$\begin{aligned} NPV &= PV(benefits) - PV(costs) \\ &= -3500 + \frac{5000}{1.055^1} + \frac{-2000}{1.055^2} + \frac{-2000}{1.055^3} + \frac{-2000}{1.055^4} + \frac{5000}{1.055^5} \qquad [\ \$\ ] \quad (2) \\ &= -49.56 \end{aligned}$$

Because it is negative, this investment is not worth considering.

3 Exercise 3

Let's compute the net present value of the investment from the Swiss investor perspective:

$$NPV = -350000 + \frac{420000}{1.08 \cdot 1.03^3} = 5888.42 \qquad [\ \text{CHF}\ ] \quad (3)$$

As it is positive, we should invest.

4 Exercise 4

As the price of the security B is not known, we can divide this problem into 3 sub-problems:

$$P(B) = 100 \qquad [\,\$\,] \quad (4)$$

$$P(B) = 100 + \epsilon \qquad [\,\$\,] \quad (5)$$

$$P(B) = 100 - \epsilon \qquad [\,\$\,] \quad (6)$$

with $\epsilon > 0$

When $P(B) = 100$, buying A and B and selling C (or the opposite) results in no arbitrage (cash flows for each year equal zero for the same investment).

The same operations with $P(B) = 100 \pm \epsilon$ would result in cash-flows of zero but with investment of $\pm\epsilon$. Hence there is arbitrage!
Law of one price, no arbitrage:

$$P(C) = P(A + B) = P(A) + P(B) \quad (7)$$

If P(B) $<$ 100, we should buy A and B, sell C.
If P(B) $>$ 100, we should sell A and B, buy C.

5 Exercise 5

Let's compute the net present value of each investment:
A)

$$NPV(A) = -100 + \frac{45}{1.03} + \frac{75}{1.03^2} - \frac{10}{1.03^3} = 5.23 \qquad [\,\$\,] \quad (8)$$

B)

$$NPV(B) = -1000 + \frac{500}{1.03} + \frac{600}{1.03^2} - \frac{70}{1.03^3} = -13.07 \qquad [\,\$\,] \quad (9)$$

Hence we should take investment A but not B.

6 Exercise 6

We need to compute the future value equivalent to the 4 annual payments, in this case we compound:

$$FV = -1200 \cdot 1.065^3 - 1200 \cdot 1.065^2 - 1200 \cdot 1.065 - 1200 = -5288.61 \qquad [\,\$\,] \quad (10)$$

One final payment of 5288.61 is equivalent to the previous case.

7 Exercise 7

There are 17 years remaining, we need to compound to compute the future value:

$$FV_{35yo} = 21000 \cdot 1.05^{17} = 48132.38 \qquad [\,\$\,] \quad (11)$$

To compute how much the parents put at the beginning we need to discount:

$$PV_{0yo} = \frac{21000}{1.05^{18}} = 8725.93 \qquad [\,\$\,] \quad (12)$$

8 Exercise 8

We need to discount an amount which increases each year with the same rate as the interest rate:

$$\sum_{n=1}^{12} 15000 \cdot \frac{1.06^n}{1.06^n} = 12 \cdot 15000 = 180000 \qquad [\ \$\ ] \quad (13)$$

As we can see, interest rate compensates the increase of the tuition fee.

9 Exercise 9

We are computing the present value (today) of all the savings. A saving year n is:

$$Saving(n) = 0.96^{n-1} \cdot 2000 \qquad [\ \$\ ] \quad (14)$$

We can then write the present value as:

$$\begin{aligned} PV &= \sum_{n=1}^{\infty} \frac{Saving(n)}{1.02^n} \\ &= \sum_{n=1}^{\infty} \frac{0.96^{n-1} \cdot 2000}{1.02^n} \\ &= \frac{2000}{1.02} \cdot \sum_{n=0}^{\infty} r^n \\ &= \frac{2000}{1.02} \cdot \frac{1}{1-r} \\ &= 33333.33 \end{aligned} \qquad [\ \$\ ] \quad (15)$$

Where $r = \frac{0.96}{1.02}$

10 Exercise 10

We want the sum of the 25 annual payments to be equal the amount we need to borrow (400000\$). Let's define C the first annual payment.

$$\begin{aligned} 400000 &= \sum_{n=1}^{25} \frac{C}{1.09^n} \\ &= \frac{C}{1.09} \cdot \sum_{n=0}^{24} (\frac{1}{1.09})^n \\ &= \frac{C}{1.09} \cdot \frac{1-(\frac{1}{1.09})^{25}}{1-\frac{1}{1.09}} \end{aligned} \qquad [\ \$\ ] \quad (16)$$

And we solve for C :

$$\begin{aligned} C &= 1.09 \cdot 400000 \cdot \frac{1-\frac{1}{1.09}}{1-(\frac{1}{1.09})^{25}} \\ &= 40722.50 \end{aligned} \qquad [\ \$\ ] \quad (17)$$

Now we want to compare the future value of annual payments of 38000$ compared to the future value borrowed:

$$(400000 - \frac{38000}{1.09} \cdot \frac{1-(\frac{1}{1.09})^{25}}{1-\frac{1}{1.09}}) \cdot 1.09^{25} = 268598.21 \qquad [\ \$\ ] \qquad (18)$$

Above we assumed that the balloon payment includes the last (25th) payment of 38000$

11 Exercise 11

First we compute the present value of the amount we want to have in the future:

$$PV = \frac{2500000}{1.035^{38}} \qquad [\ \$\ ] \qquad (19)$$

Given the present value of annual payment of C during 38 years, we can solve the equation:

$$\begin{aligned} PV &= \sum_{n=0}^{38} C \left(\frac{1.025}{1.035}\right)^n \\ &= C\frac{1-r^{39}}{1-r} \end{aligned} \qquad [\ \$\ ] \qquad (20)$$

With:

$$r = \frac{1.025}{1.035} \qquad (21)$$

Let's solve for C:

$$\begin{aligned} C &= \frac{PV(1-r)}{1-r^{39}} \\ &= 20733.17 \end{aligned} \qquad [\ \$\ ] \qquad (22)$$

In these computations we assumed that we started saving today (n=0) up to and including the 66th birthday (n=38)

12 Exercise 12

$$PV = 250000 \qquad [\ \$\ ] \qquad (23)$$

$$C = 30000 \qquad [\ \$\ ] \qquad (24)$$

$$r = 6\% \qquad (25)$$

find N:

$$PV = \frac{C}{r} \cdot (1 - \frac{1}{(1+r)^N}) \qquad [\ \$\ ] \qquad (26)$$

$$1 - \frac{PV \cdot r}{C} = (1+r)^{-N} \qquad (27)$$

$$\begin{aligned} N &= -\frac{\log(1-\frac{PV \cdot r}{C})}{\log(1+r)} \\ &= 11.896 \end{aligned} \qquad [\ \text{years}\ ] \qquad (28)$$

Hence, after 12 years she would have earned more than 250000$