ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

MGT-482 Principles of Finance
Assignment 8

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1 Exercice 1

The payoff of the call option can be obtained by $max[S_t - K; 0]$:
$c_{uuu} = 37.65625$
$c_{uud} = c_{udu} = c_{uud} = 2.5$
$c_{udd} = c_{dud} = c_{ddu} = 0$
$c_{ddd} = 0$
The risk neutral probability of an upward movement p can be computed:

$$p = \frac{1 + r - d}{u - d} = \frac{5}{9} \tag{1}$$

where $S_0 = \$50, K = \$60, u = 1.25, d = 0.8, r = 5\%$

$$\begin{aligned} c_0 &= \frac{1}{(1+r)^3}[p^3 c_{uuu} + 3p^2(1-p)c_{uud} + 3p(1-p)^2 c_{udd} + (1-p)^3 c_{ddd}] \\ &= \frac{1}{(1.05)^3}[(\frac{5}{9})^3 \cdot 37.65625 + 3(\frac{5}{9})^2(1 - \frac{5}{9}) \cdot 2.5] \\ &= 6.46637 \end{aligned} \tag{2}$$

Hence, the price of the call option should be:

$$c_0 = 6.466 \tag{3}$$

Next, the payoff of the put option can be obtained by $max[K - S_t; 0]$:
$p_{uuu} = 0$
$p_{uud} = p_{udu} = p_{uud} = 0$
$p_{udd} = p_{dud} = p_{ddu} = 20$
$p_{ddd} = 34.4$
where $S_0 = \$50, K = \$60, u = 1.25, d = 0.8, r = 5\%$

$$
\begin{aligned}
p_0 &= \frac{1}{(1+r)^3}[p^3 p_{uuu} + 3p^2(1-p)p_{uud} + 3p(1-p)^2 p_{udd} + (1-p)^3 p_{ddd}] \\
&= \frac{1}{(1.05)^3}[3(\frac{5}{9})(1-\frac{5}{9})^2 \cdot 20 + (1-\frac{5}{9})^3 \cdot 34.4] \\
&= 8.29663
\end{aligned} \tag{4}
$$

In the end, the price of the put option should be:

$$
p_0 = 8.297 \tag{5}
$$

2 Exercice 2

As we sell a put option, at most we can make the price we sold the option, that is the case when the option is not exercised, i.e. IBM share is worth less than \$90. At worst, we lose the difference in the stock's price compared to \$90, with the assumption that a stock price cannot be negative, we can compute the maximum loss:

- price of the European put: p_0 [\$]
- max gain: p_0 [\$]
- max loss: $100 \cdot max(0, 90-0) - p_0 = 9000 - p_0$ [\$]

By selling the put, we play the role of an insurer for the buyer. One can make the analogy with a car insurance, as long as no damage occurs (i.e. IBM share does not go below \$90), we simply earn the insurance price (price of the put). Otherwise, we repair the damage (reimburse the difference up to the \$90 per share).

3 Exercice 3

The payoffs of the buyer of the call option and the seller of the put option (with a strike-price of 100 for both) is shown in Figure 2:

Payoff of the buyer of the call is

$$
max\{0, S_T - k\} \tag{6}
$$

Payoff of the seller of the put is

$$
-max\{0, k - S_T\} \tag{7}
$$

Where S_T is the stock price at time T and k is the exercise price.

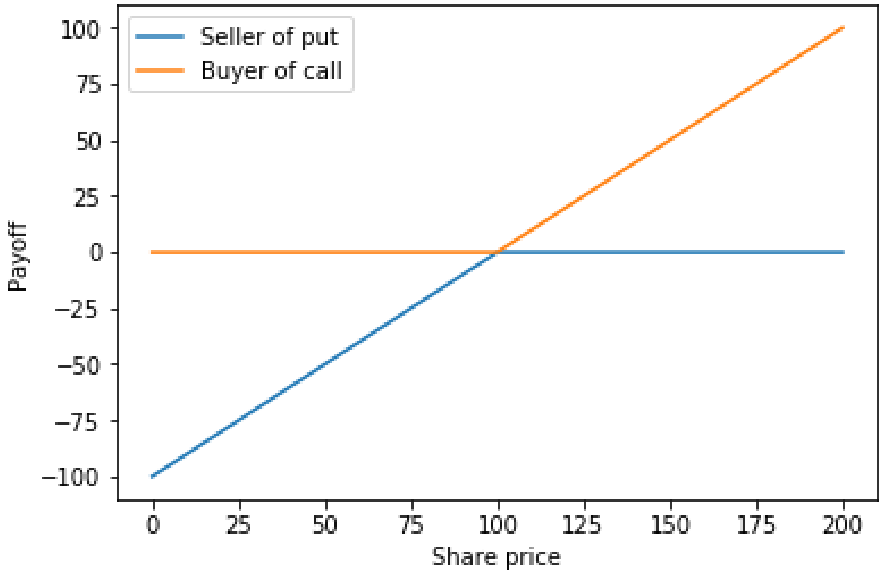

Figure 1: Payoff with strike-price k of 100 and option price of zero.

Indeed both hope that the stock rise but they don't have identical payoff. Simply because as one needs the share price to be more than \$100, no matter how high, the other wants it to be as high as possible. Hence what the speaker says is incorrect.

4 Exercice 4

The payoff of the call option can be obtained by $max[S_t - K; 0]$:
$c_{uu} = 10.5$
$c_{ud} = c_{du} = 0$
$c_{dd} = 0$
The risk neutral probability of an upward movement p can be computed:

$$p = \frac{1 + r - d}{u - d} = 0.6 \tag{8}$$

where $S_0 = \$50, K = \$50, u = 1.1, d = 0.9, r = 2\%$

$$\begin{aligned} V &= \frac{1}{(1+r)^2}[p^2 c_{uu} + 2p(1-p)c_{ud} + (1-p)^2 c_{dd}] \\ &= \frac{1}{(1.02)^2}[0.6^2 \cdot 10.5] \\ &= 3.70588 \end{aligned} \tag{9}$$

Hence the value of the call is 3.706

5 Exercice 5

One can hedge himself by buying a put option contract on IBM shares. If one wants to be fully insured, one can buy a contract for the whole 5000 shares with a strike price of 100\$.

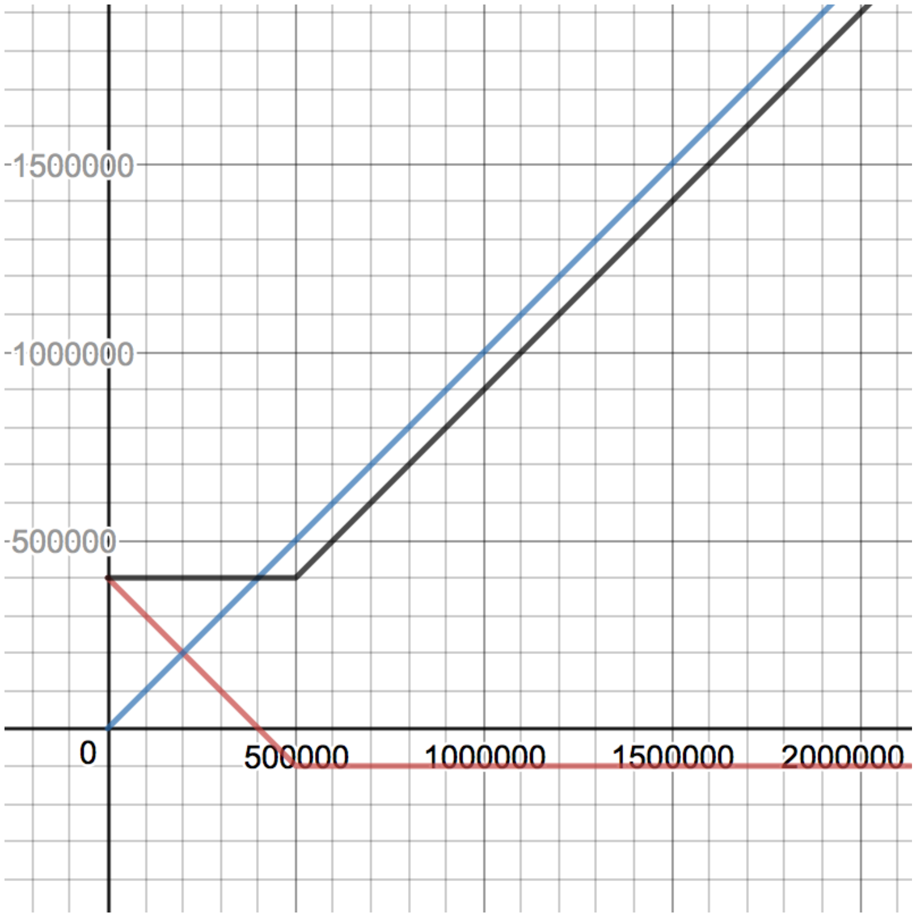

Figure 2: available at https://www.desmos.com/calculator/sgmrpff9yx

- Blue line is the stock price payoff if nothing is done.
- Red line is the put option payoff.
- Black line is the sum of the two. Our "insurance" plan.

6 Exercice 6

a) The payoff of the call option can be obtained by $max[S_t - K; 0]$:
$c_u = 15$
$c_d = 0$
The risk-free interest rate and the risk-neutral probability of an upward movement can be computed by using these formula:

$$p = \frac{1 + r - d}{u - d} \tag{10}$$

$$c_0 = \frac{1}{1 + r}[pc_u + (1 - p)c_d] \tag{11}$$

where $S_0 = \$100, K = \$95, u = 1.1, d = 0.9, c_0 = 10.09615$

Then we got:

$$p = \frac{c_d - c_0 d}{(u-d)c_0 - c_u + c_d} = 0.700 \tag{12}$$

$$r = p(u-d) + d - 1 = 4.00\% \tag{13}$$

b) The payoff of the put option can be obtained by $max[K - S_t; 0]$:
$p_{uu} = 0$
$p_{ud} = p_{du} = 1$
$p_{dd} = 19$
where $S_0 = \$100, K = \$95, u = 1.1, d = 0.9, r = 4\%$

$$\begin{aligned} p_0 &= \frac{1}{(1+r)^2}[p^2 p_{uu} + 2p(1-p)p_{ud} + (1-p)^2 p_{dd}] \\ &= \frac{1}{(1.04)^2}[2 \cdot 0.7 \cdot (1-0.7) \cdot 1 + (1-0.7)^2 \cdot 19] \\ &= 1.969304 \end{aligned} \tag{14}$$

$p_0 = 1.969$

c) The payoff of the option can be obtained by $max[0; min(S_t) - K]$:
$c_{uuu} = c_{uud} = 5$
$c_{udu} = 4$
$c_{udd} = c_{duu} = c_{dud} = c_{ddu} = c_{ddd} = 0$

Then we computed recursively:
$c_{uu} = \frac{1}{1.04}[0.7 \cdot 5 + (1-0.7) \cdot 5] = 4.807692$
$c_{ud} = \frac{1}{1.04}[0.7 \cdot 4] = 2.692307$
$c_{du} = c_{dd} = 0$

$c_u = \frac{1}{1.04}[0.7 \cdot 4.807692 + (1-0.7) \cdot 2.692307] = 4.012573$
$c_d = 0$

$c_0 = \frac{1}{1.04}[0.7 \cdot 4.01257376] = 2.700769$

In the end, we obtained the price of the option:
$c_0 = 2.701$