

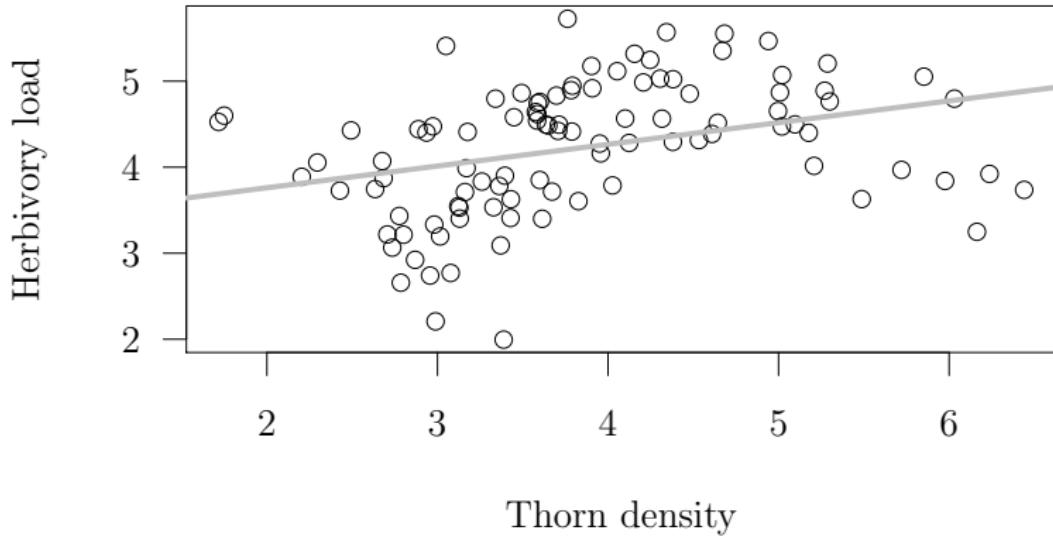
Linear mixed models 2

Random interactions and correlated random effects

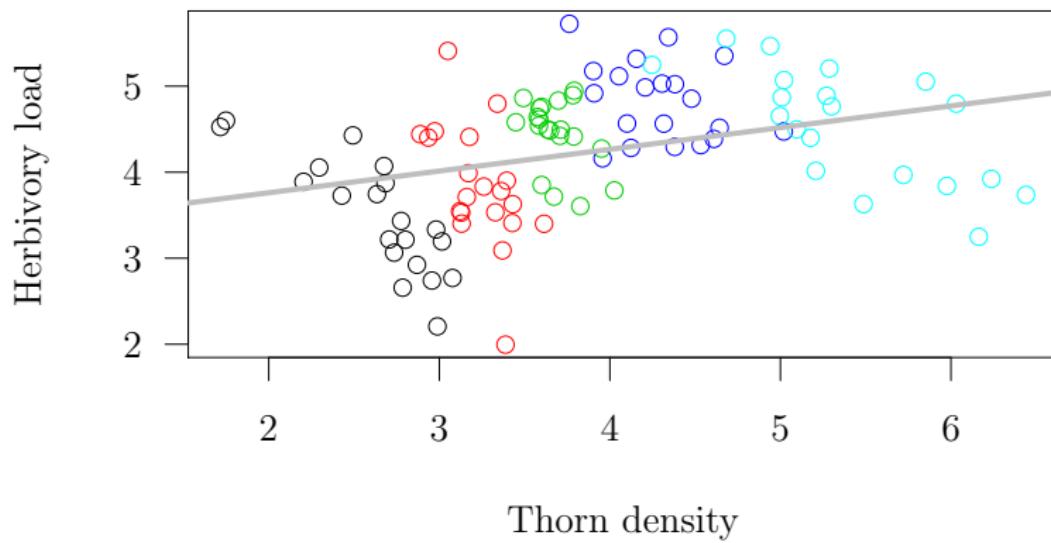
Timothée Bonnet

March 28, 2019

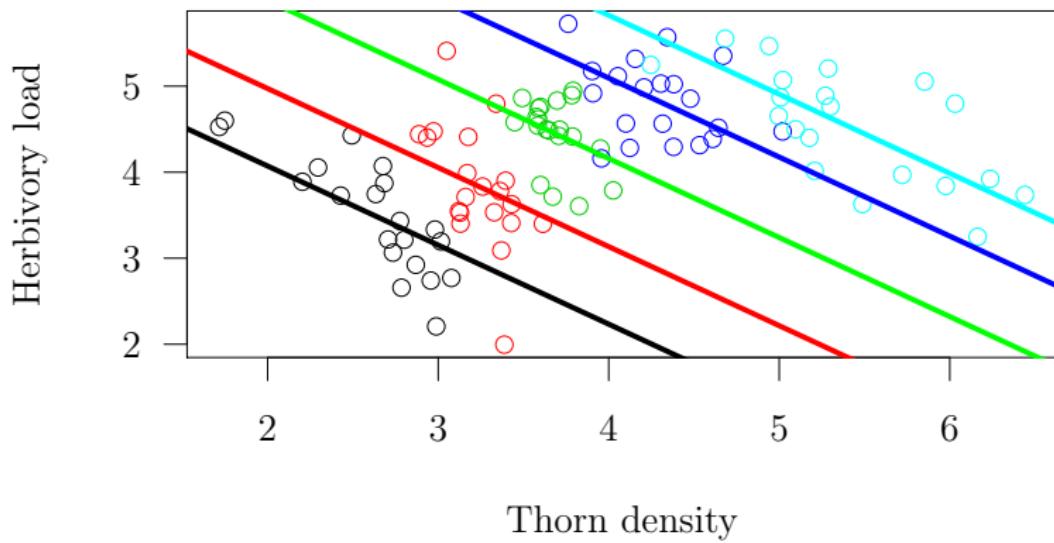
Mixed models reminder



Mixed models reminder

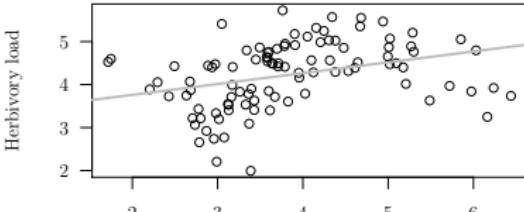


Mixed models reminder

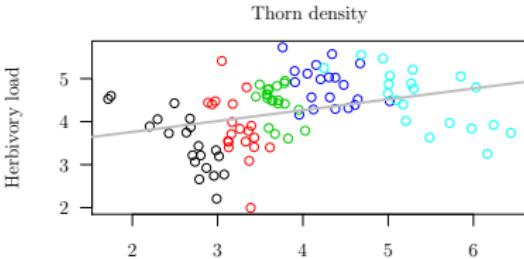


Mixed models reminder

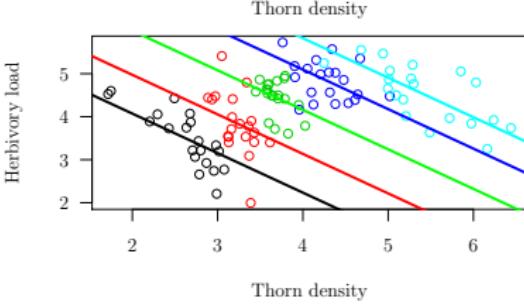
1. First model assumes residuals are independent



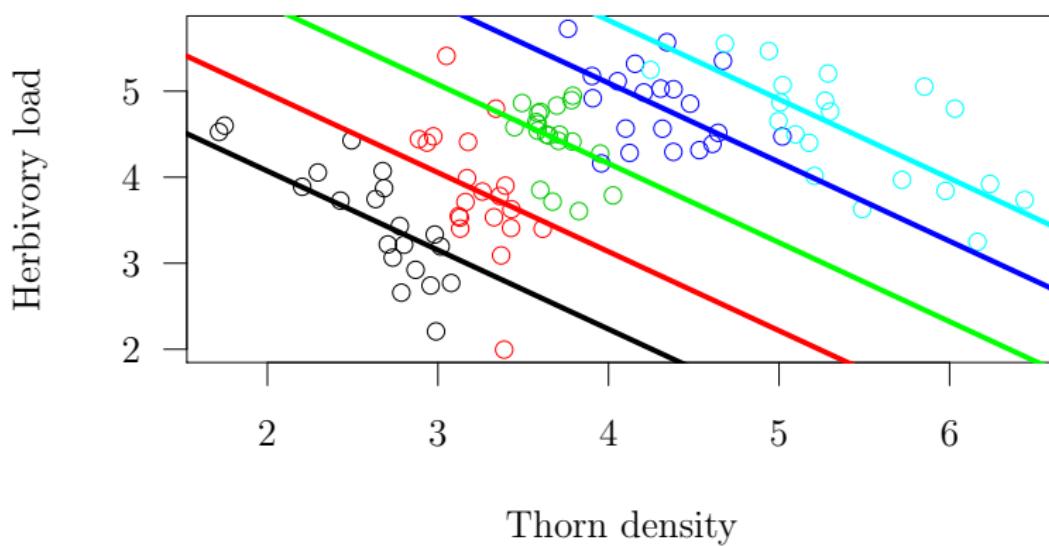
2. But they are not. Data come from five different places



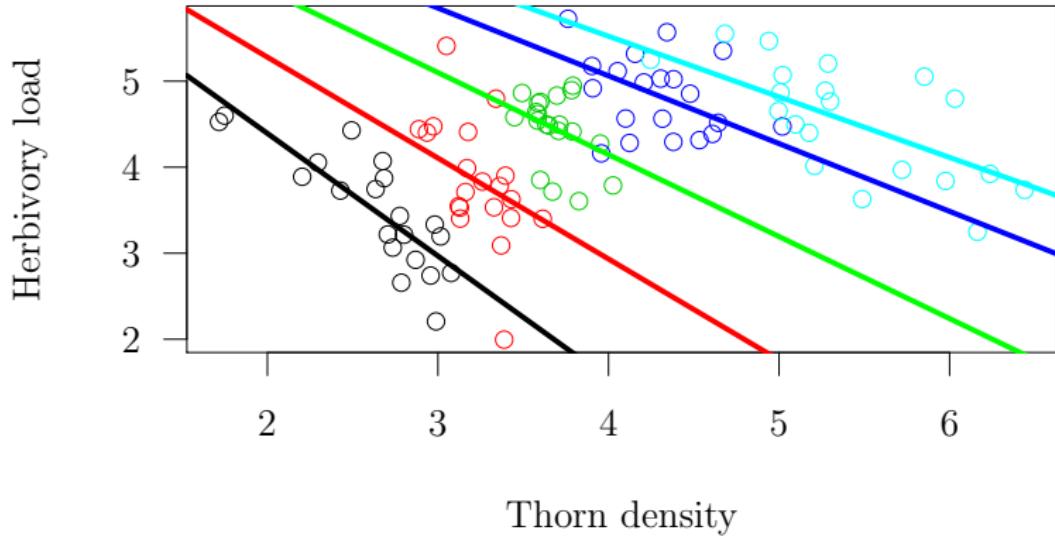
3. Adding random effect “place” gets correct slope. Residuals are now really independent



Limitations: 1) It does not have to be parallel

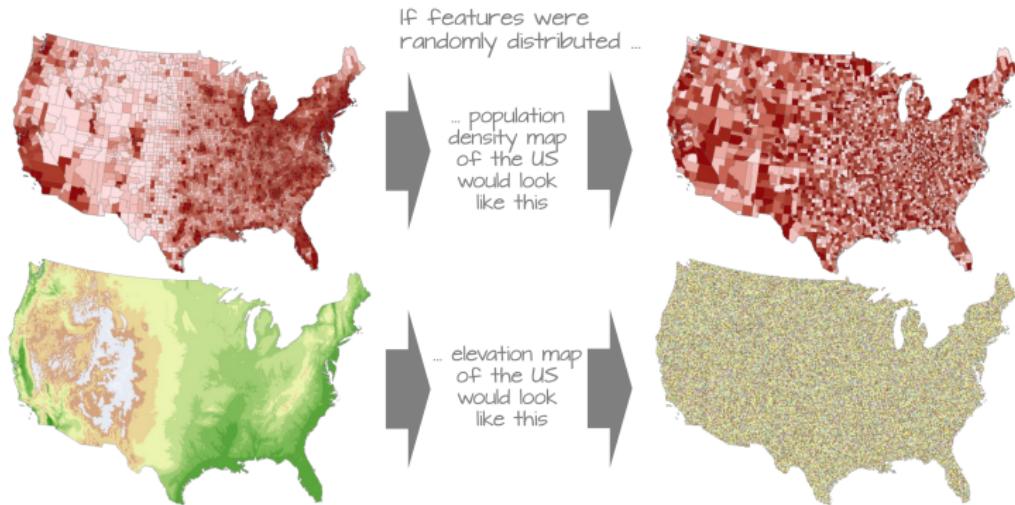


Limitations: 1) It does not have to be parallel



Limitations: 2) It does not have to be independent

Maybe two of the locations are very similar compare to the other ones.
We assume all locations are equally different, maybe wasting information



source: Manuel Gimond <https://mgimond.github.io/Spatial/index.html>

Mixed models today

- ① Random interactions:

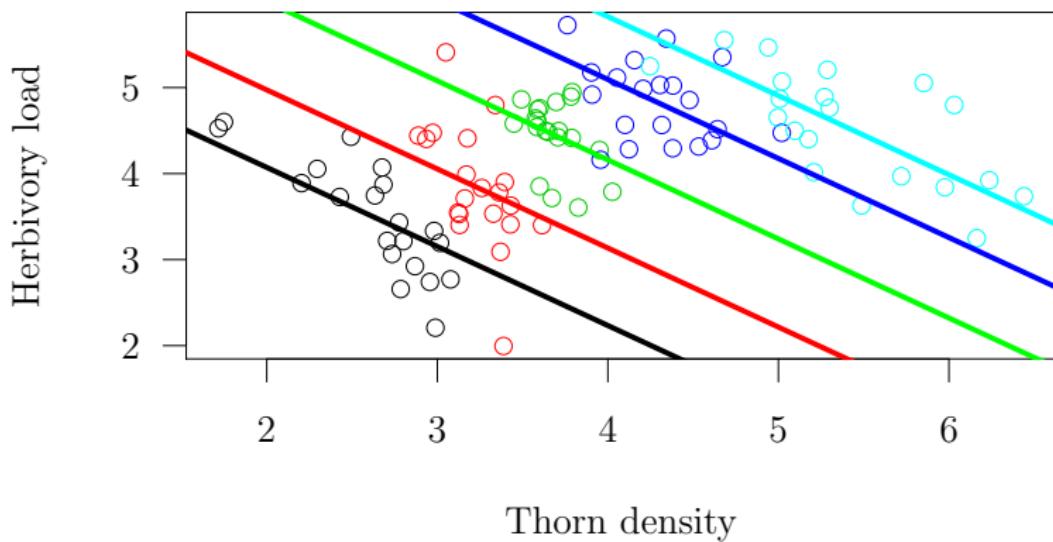
What if a covariate doesn't have the same effect everywhere?

- ② Correlated random effects:

What if we know random effects are not independent?

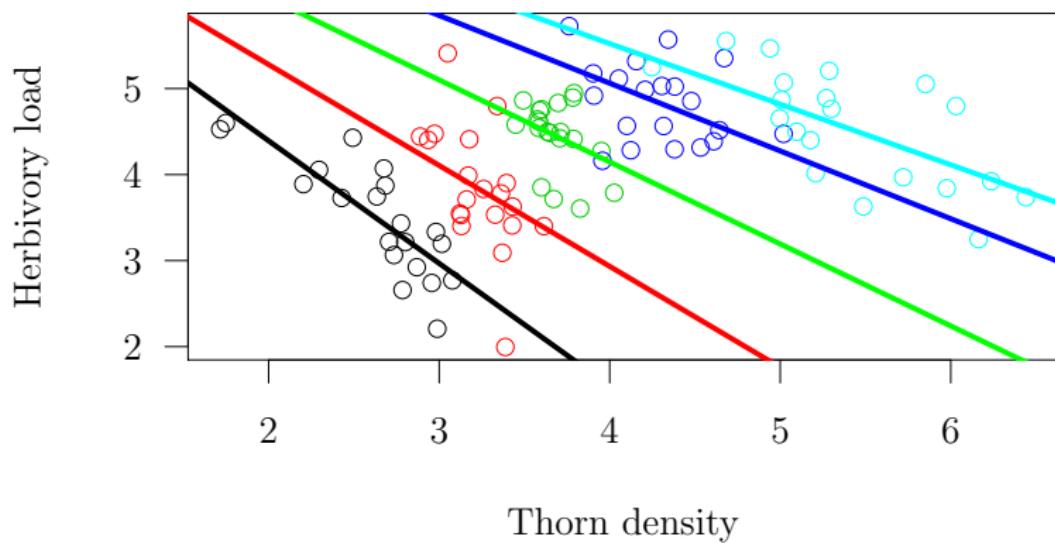
Not only intercept vary!

Assume parallel slopes:



Not only intercept vary!

Allow slopes to vary:



Random slopes and unbalanced data

We saw that random intercept could reveal a hidden pattern

What about random slopes? See exercise 1!

Random slopes and unbalanced data

We saw that random intercept could reveal a hidden pattern

What about random slopes? See exercise 1!

Exercise 2, visualize random slope!

Random slopes and unbalanced data

We saw that random intercept could reveal a hidden pattern

What about random slopes? See exercise 1!

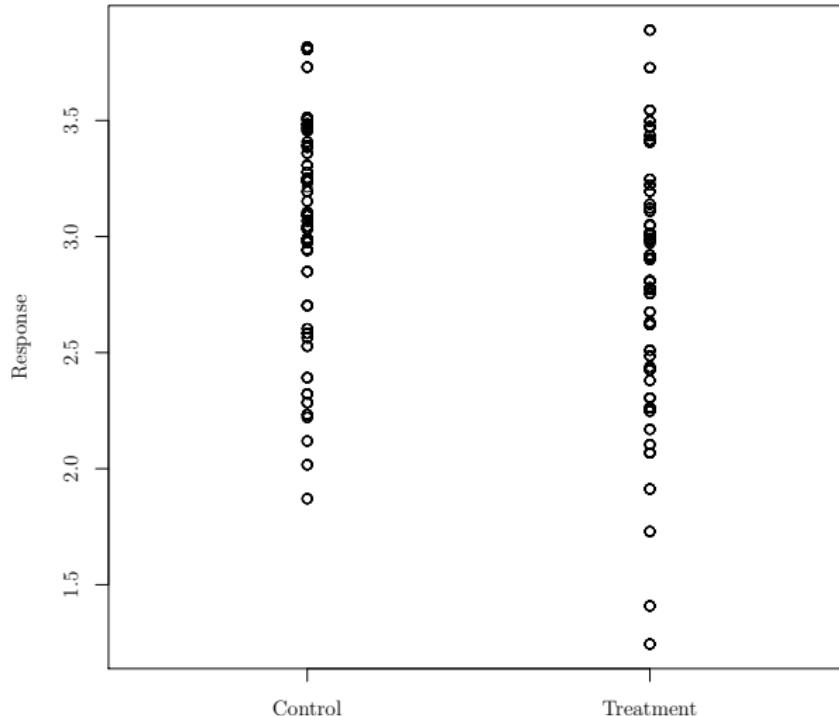
Exercise 2, visualize random slope!

Application: natural selection

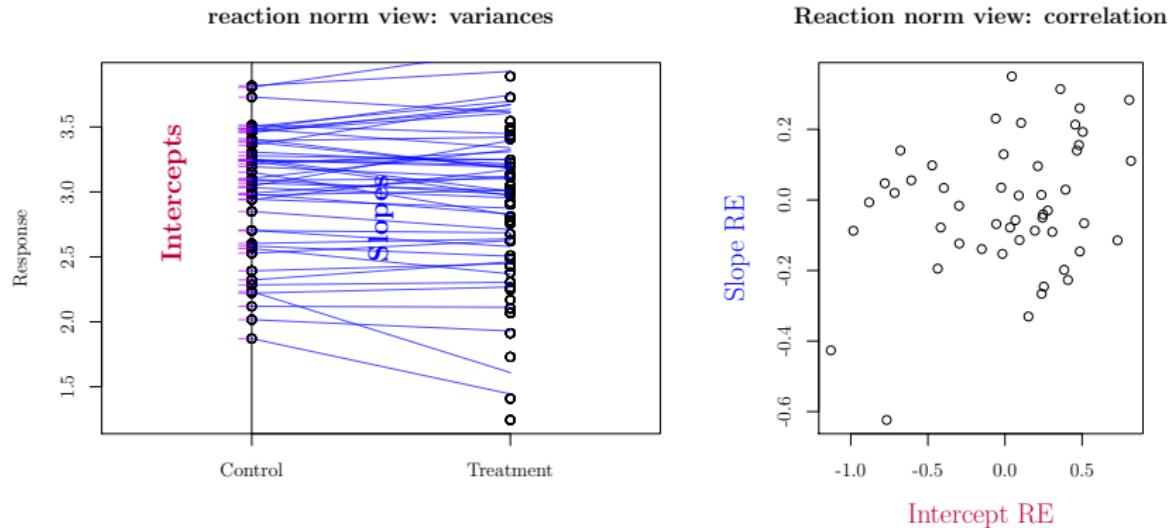
Exercise 3 Does natural selection vary?



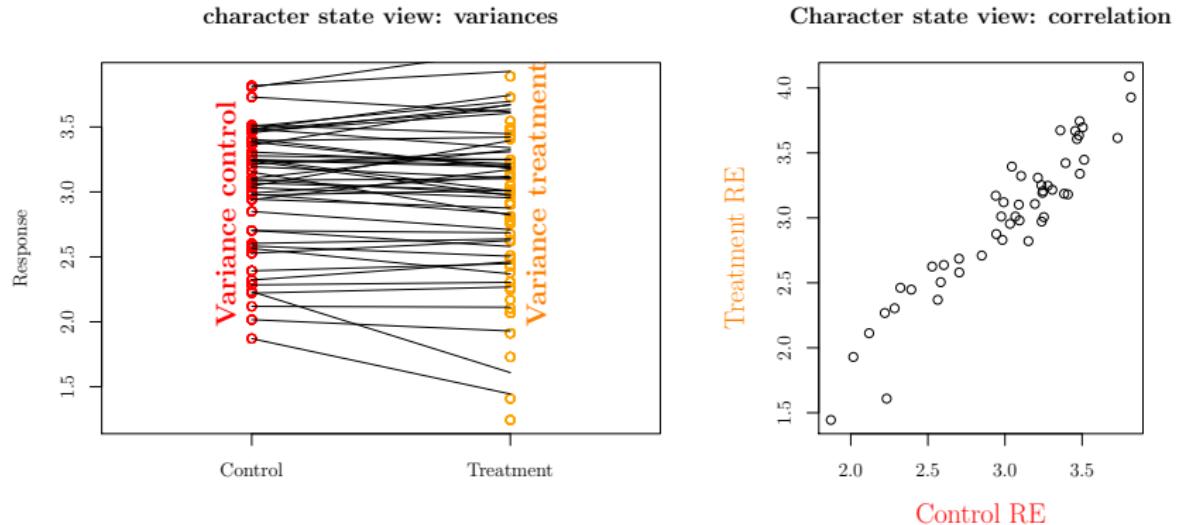
Random interaction with a factor



Random interaction with a factor



Random interaction with a factor



Beetles

Two sides of the random effect term

Left-hand side = what varies according to grouping

The 1 stands for **intercept**

Other things can go to the left hand side.

Random interactions, random regressions, random slopes... e.g.,

$y \sim 1 + x + (1 + x|something)$

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Right-hand side = what groups observations

Nested, crossed et al. on the right hand side of the |: $(1 | something)$

How are data related to each other, what groups them.

When are the “somethings” correlated?

In all models so far:

$\text{Response} = \text{intercept} + \text{covariate} + \text{Random} + \text{error}$

$\text{Random} \sim N(0, V_R \mathbf{G})$

\mathbf{G} is the random effect variance covariance:

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

Each random effect has information only about itself.

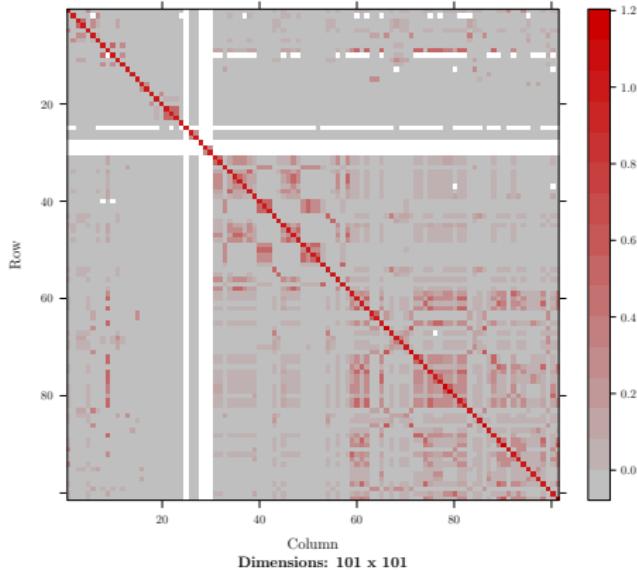
But one random effects can have information about others:

1	0.5	0	0	0	0
0.5	1	0	0.25	0	0
0	0	1	0	0	0
0	0.25	0	1	0	0
0	0	0	0	1	0.9
0	0	0	0	0.9	1

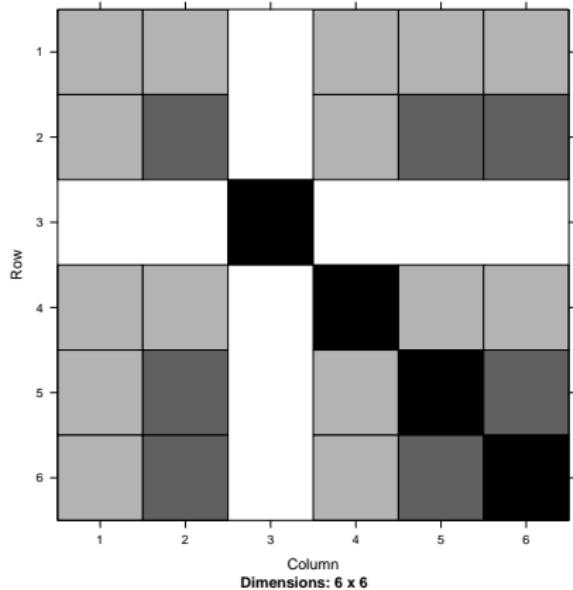
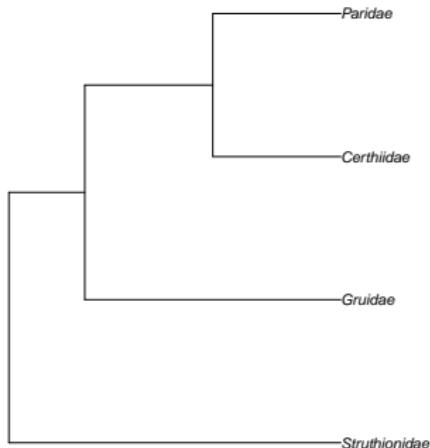
For instance:

- Spatial auto-correlation
- Kin in a population
- Related species in phylogeny

Quantitative genetic demo: section 2.1



Phylogenetic model: section 2.2



Everything you need to know about mixed models

- <http://bbolker.github.io/mixedmodels-misc/glmmFAQ.html>
- Subscribe to mailing-list:
<https://stat.ethz.ch/mailman/listinfo/r-sig-mixed-models>