Statistical Modelling: Beyond Linear Models, Generalized Linear Models

Chapter 5

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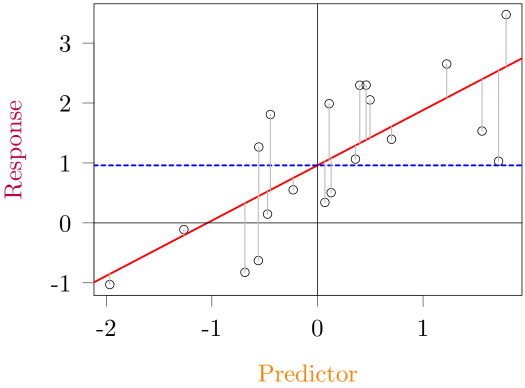
Simple linear models

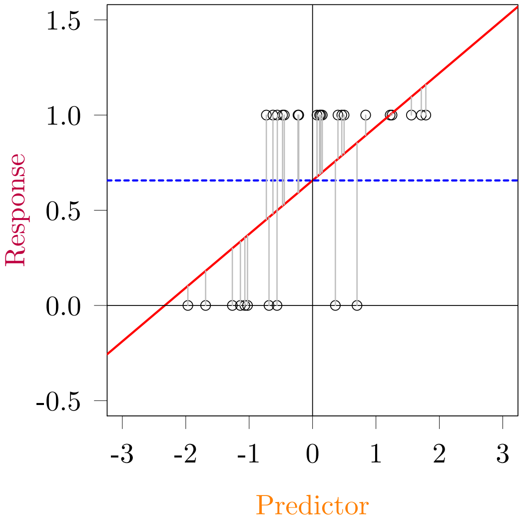
Response = **Intercept** + **Slope** × **Predictor** + **Error**



Linear model basic assumptions

- Predictor not perfectly correlated
  Risk: Model won't run, unstable convergence, or huge SE
- Little error in predictors
  Risk: bias estimates (underestimate with Gaussian error)
- Gaussian error distribution
  Risk: Poor predictions
- Homoscedasticity (constant error variance)
  Risk: Over-optimistic uncertainty, unreliable predictions
- Independence of error
  Risk: Bias and over-optimistic uncertainty

A simple linear model failure: binary data



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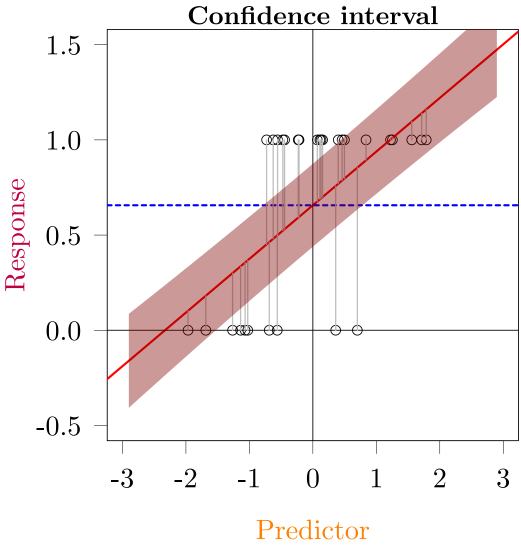
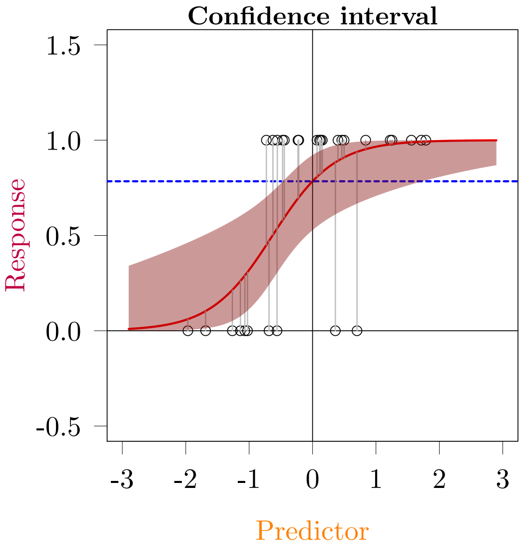


Assumptions violated:

What we want our model to do



That is what a Generalized Linear Model does

Vocabulary warning

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- **Generalized Linear Model (=non-normal errors, and uncertainty dependent on the mean)**

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What a GLM is:

1. A linear function ($y = \mu + \beta x \dots$)
2. A probability distribution (Bernouilli, Binomial, Poisson...)
3. A "link function" to convert between the scale of the linear function ($-\infty$ to $+\infty$) and the scale of the data and the probability distribution (often positive integer: 0, 1, 2, 3...)

A GLM fits a continuous expected response; we observe discrete realizations

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- For a given predicted y, $\exp(y)$ is the odd ratio: probability success / probability failure

What is the Bernouilli distribution?

```
bernouilli_random_sample <- rbinom(n = 10000, size = 1, prob =
hist(bernouilli_random_sample)
mean(bernouilli_random_sample); 0.3
var(bernouilli_random_sample); 0.3*(1-0.3)
```

Logistic regression in R

```r
glm(formula = obs ~ 1 + x, family = "binomial", data=data)
```

Does survival probability depend on size?

Exercise, part 1

1. Load `survivalsize.csv`
2. Plot survival data. What kind of distribution is it?
3. Fit a linear model and a logistic model with intercept only. How to interpret the estimates?

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hints:

1. For a given predicted y, $\exp(y)$ is the odd ratio: probability success / probability failure
2. Back-transformation inverse-logit: $probability = \frac{1}{1+exp(-y)}$

Solutions part 1

```
surv <- read.csv("Data/survival.csv")
plot(surv$survival)
lmsurv <- glm(survival~1, data=surv, family=gaussian)
lregsurv <- glm(survival~1, data=surv, family=binomial)

#linear model prediction:
coefficients(lmsurv)

#logistic reg prediction:
plogis(coefficients(lregsurv))
1/(1+exp(-coefficients(lregsurv)))
exp(coefficients(lregsurv))

#observed mean survival:
mean(surv$survival)
#mean odd-ratio:
mean(surv$survival)/(1-mean(surv$survival))
```

Does survival probability depend on size?

Exercise, part 2

1. Fit a linear regression and a logistic regression of survival on relative size, compare the outputs
2. Check the diagnostic plots for both models. Should you be worried?
3. Extract and visualize a model prediction from both models (use the function predict(), and/or do it by hand to practice link-function back-transformation)

Solutions part 2

```
lmsurvS <- glm(survival~1 + relative_size, data=surv, family=gaussian)
lregsurvS <- glm(survival~1 + relative_size, data=surv, family=binomial)

summary(lmsurvS)
summary(lregsurvS)

plot(lmsurvS)
plot(lregsurvS)

plot(surv$relative_size, surv$survival, ylim=c(-0.2,1.2))
abline(lmsurv, col="red")

plot(surv$relative_size, surv$survival, ylim=c(-0.2,1.2))
datforpred <- data.frame(relative_size=seq(from=-3,to=4, by=0.1))
datforpred$prob <- predict(lregsurvS, newdata = datforpred,
type = "response")
lines(datforpred$relative_size, datforpred$prob, col="red")

ggplot(surv, aes(x = relative_size, y=survival))+geom_point()+
stat_smooth(method = "glm", method.args = list(family = "binomial"))
```

More practice: does survival probability depend on weight? does the relationship depend on sex?

Exercise

1. Load `survivalweight.csv`
2. Plot data
3. Fit a logistic model to address these questions
4. Plot the results

Poisson regression

- Count data
- Poisson distribution
- Link function: logarithm
- Inverse link function: exponential
- Linear function $y = intercept + slope_1 predictor_1 + slope_2 predictor_2 + \ldots$

What is the Poisson distribution?

```
poisson_random_sample <- rpois(n = 10000, lambda = 4)
hist(poisson_random_sample)
mean(poisson_random_sample)
var(poisson_random_sample)
```

Poisson regression in R

```
glm(formula = obs ~1 + x, family = "poisson", data=data)

glm(formula = obs ~1 + x, family = "quasipoisson", data=data)
```

family = "poisson" is dangerous

- A true Poisson distribution has $E(\exp(Y)) = V(\exp(Y))$
- Assumes no unexplained variation in Y
- `glm()` follows this assumption
- In nature, $E(\exp(Y)) < V(\exp(Y))$ most of the time
- SE and p-value to small
- family = "quasipoisson" correct the uncertainty in `glm()`
- other packages never follow the assumption (`MCMCglmm`)