

# Given a helical compression spring in a spring-mass-damper system, what are optimal springs?

Justin Krueger<sup>1</sup>, Alistair Bentley<sup>2</sup>, Tianyu Qiu<sup>3</sup>, Saideep Nannapaneni<sup>4</sup>, Jiahua Jiang<sup>5</sup>, Tim Hodges<sup>6</sup>

Problem Presenters: Jordan Massad<sup>7</sup>, Sean Webb<sup>8</sup>, Faculty Mentors: Ilse Ipsen<sup>9</sup>, Ralph Smith<sup>10</sup>,

## Abstract

- Summarize the results presented in the report, and the contributions of your research.
- Readers should not have to look at the rest of the paper in order to understand the abstract.
- Keep it short and to the point.

## 1 Introduction

The use of mechanical switches in industry is a cornerstone of the modern world. Many mechanical switches can be designed to use a spring. This is useful because depending on the use of the switch, the spring can be optimized to for the function of the spring.

For example, consider an acceleration switch in 1.

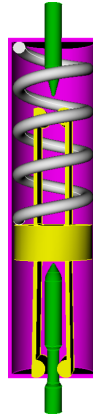


Figure 1: An example of an acceleration switch

This switch is used in high acceleration testing. The use is to only record information at crucial times of the test. For this reason when a certain force is exerted on the switch the two at both ends will connect. When this happens we have a circuit that will enable data collection. It is important that the switch does not open too soon or too late otherwise the data collection will be too large or mean too little. This is just one example of a switch that we must know the best spring to make the performance maximal.

Given that a switch may be used in a myriad of ways, it is important that one can find an optimal spring given the use known. This leads to an optimization problem. Even more this leads to an optimization problem

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<sup>1</sup>Mathematics, Virginia Tech University

<sup>2</sup>Mathematics, Clemson University

<sup>3</sup>Mathematics, University of Delaware

<sup>4</sup>Civil & Environmental Engineering, Vanderbilt University

<sup>5</sup>Mathematics, University of Massachusetts Dartmouth

<sup>6</sup>Mathematics, Colorado State University

<sup>7</sup>Sandia National Laboratory

<sup>8</sup>Sandia National Laboratory

<sup>9</sup>North Carolina State University

<sup>10</sup>North Carolina State University

that requires the flexibility to allow how a spring is determined to be optimal by an objective function and constraints determining the optimization problem to be changed. This does not consider the fabrication constraints that exist during manufacturing, but also needs to be considered as well.

Designing an optimal spring is not a new problem, see [1] [2] [3] [4]. In 2010 at the SAMSI IMSM workshop a team considered the design of an acceleration with enabled uncertainty.

In this paper we will give a description of the springs of interest, Helical Compression Springs. Next, the formulation of the problem, and the approach taken to the problem. Following will be a section on workflow with descriptions of each step in the workflow. Lastly, we will discuss a few case studies given the framework constructed and a summary with future work that can be worked on.

- Describe the problem you are trying to solve, the approach you took, and summarize your contribution and results.
- Review the history of this problem, and existing literature.
- Give an outline of the rest of the paper.

## 2 Helical Compression Springs

Helical springs are a large class of springs sharing the common characteristic of a coiled appearance, see figure 2.

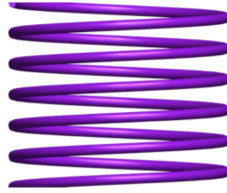
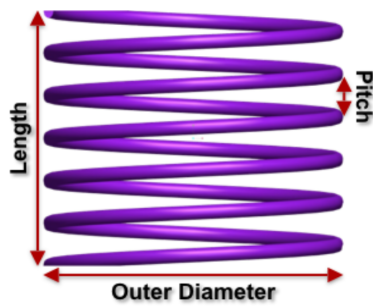
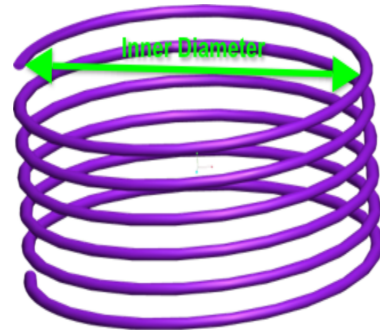


Figure 2: An example of a helical compression spring.

Below is a list of a spring's key design parameters. We have added a few illustrations to keep straight some of the parameters.



(a) Pitch, outer diameter, and length of a spring.



(b) Inner diameter

Figure 3: A few illustrations of a few of the parameters for a helical compression spring.

1. Spring's inner diameter  $d_i$ , illustrated in figure 3b.
2. Spring's outer diameter  $d_o$  illustrated in figure 3a.
3. Spring's wire diameter  $d_w$ .

4. Total number of spring coils  $N_t$ .
5. Active number of spring coils  $N_a$ , active coils are not touching any other coils, and is subject to the spring being closed or open.
6. Pitch  $p$  illustrated in figure 3a.
7. Spring's free length  $L_{free}$ , this is the spring's length without any force applied.
8. Spring's solid length  $L_{solid}$ , this is the spring's length when all coils are compressed together.
9. Spring's open length  $L_{open}$ , spring length at open position, open is beginning state.
10. Spring's open length  $L_{close}$ , spring length at close position, close is the ending state.
11. Spring's open length  $L_{hard}$ , the maximum a spring can compress for the application.
12. Spring's open force  $F_{open}$ , this is the force on the spring in open position.
13. Spring's shear modulus  $G$ , this is determined by the material of the spring.
14. Spring's youngs modulus  $E$ , this is determined by the material of the spring.
15. Spring's poisson ratio  $\nu$ , this is determined by the material of the spring.

In addition to these parameters there are the following attributes that are empirical given some regime we care about.

1. Spring Rate,

$$k = \frac{G}{8N_a} \frac{d_w^4}{(d_i + d_w)^3}$$

2. Spring Index,

$$C = \frac{d_i}{d_w} + 1$$

3. Coil Binding Gap,

$$g = \frac{L_{hard} - L_{solid}(d_w, N_a; ec)}{N_t - 1}$$

- 4.

$$\frac{G(L_{free} - L_{hard})}{4\pi N_a(ec)} \left[ \frac{d_w(4d_i^2 + 9.46d_id_w + 3d_w^2)}{d_i(d_i + d_w)^3} \right] < UTS$$

where UTS is the ultimate torsional stress.

5. Diametral Expansion,

$$d_{expand} = d_w + \sqrt{(d_i + d_w)^2 + \frac{p_{closed}^2 - d_w^2}{\pi^2}}$$

### 3 The Problem

Design an algorithm that optimizes springs with interchangeable objectives and constraints. In addition, attempt to incorporate properties stress relaxation and creep into the available objectives and constraints.

A list of possible constraints/objectives given in minimization form is below

1.  $d_i < d_i^{max}$
2.  $d_i < d_o$
3.  $d_i + 2 * d_w < d_o^{max}$
4.  $d_{expand} - d_0^{max} < 0$
5.  $\frac{G}{8N_a} \frac{d_w^4}{(d_i + d_w)^3} - k_{max} \leq 0$
6.  $\frac{d_i}{d_w} + 1 - C_{max} < 0$
7.  $(L_{free} - L_{open}) \frac{G}{8N_a} \frac{d_w^4}{(d_i + d_w)^3} - F_{open} = 0$
8.  $\frac{L_{hard} - L_{solid}}{N_t - 1} + g_{min} \leq 0$
9.  $\frac{L_{free}}{d_i + d_w} - \pi \sqrt{\frac{2(2\nu + 1)}{\nu + 2}}$
10.  $-UTS + \frac{G(L_{free} - L_{hard})}{4\pi N_a(ec)} \left[ \frac{d_w(4d_i^2 + 9.46d_id_w + 3d_w^2)}{d_i(d_i + d_w)^3} \right] < 0$

In addition to these we could minimize given to any attribute of the spring.

### 4 The Problem

- Give a precise technical description of your problem.
- State and justify all your assumptions.
- Define notation.
- Describe your data, how you collected them, their properties, and whether you did anything to them (removed noise, filled in missing data, applied normalizations).

### 5 The Approach

- Present and justify your approach for solving the problem.
- Explain the advantages of your approach over existing ones.
- Tell a story. Don't just say: "I did this, then I did this, and at last I did this".

### 6 Computational Experiments

Give enough details so that readers can duplicate your experiments.

- Describe the precise purpose of the experiments, and what they are supposed to show.
- Describe and justify your test data, and any assumptions you made to simplify the problem.
- Describe the software you used, and the parameter values you selected.
- For every figure, describe the meaning and units of the coordinate axes, and what is being plotted.
- Describe the conclusions you can draw from your experiments

## 7 Summary and Future Work

- Briefly summarize your contributions, and their possible impact on the field (but don't just repeat the abstract or introduction).
- Identify the limitations of your approach.
- Suggest improvements for future work.
- Outline open problems.

## References

- [1] M. Paredes, M. Sartor, and A. Daidie, “Advanced assistance tool for optimal compression spring design,” *Engineering with Computers*, 2005.
- [2] A. M. N. P. Sastry, B. K. D. Devi, K. H. Reddy, K. M. Reddy, and V. S. Kumar, “Reliability based design optimization of helical compression spring using probabilistic response surface methodology,” *International Conference On Advances in Engineering*, 2012.
- [3] H. Zhao, G. Chen, and J. zhe Zhou, “The robust optimization design for cylindrical helical compression spring,” *Advanced Materials Research*, vol. 433-440, 2012.
- [4] A. M. Wahl, *Mechanical Springs*. Penton Publishing, first ed., 1944.