**Given a helical compression spring in a spring-mass-damper system, what are optimal springs?**

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**Abstract**

- Summarize the results presented in the report, and the contributions of your research.
- Readers should not have to look at the rest of the paper in order to understand the abstract.
- Keep it short and to the point.

# 1 Introduction

The use of mechanical switches in industry is a cornerstone of the modern world. Many mechanical switches can be designed to use a spring. This is useful because depending on the use of the switch, the spring can be optimized to for the function of the spring.

For example, consider an acceleration switch in 1.

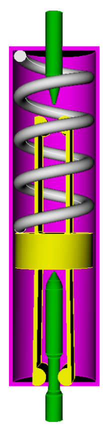

Figure 1: An example of an acceleration switch

**Describe problem, approach, and summarize contribution and results.** This switch is used in high acceleration testing. The use is to only record information at crucial times of the test. For this reason when a certain force is exerted on the switch the pins at both ends will connect. When this happens we have a circuit that will enable data collection. It is important that the switch does not open too soon or too late otherwise the data collection will be too large or mean too little.This is just one example of a switch that we must know the best spring to make the performance maximal. For a more in depth look at this example look at [1]

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Given that a switch may be used in a myriad of ways, it is important that one can find an optimal spring given after the use of the spring is known. This leads to an optimization problem. Even more this leads to an optimization problem that requires the flexibility to allow how a spring is determined to be optimal by an objective function and constraints. This does not consider the fabrication constraints that exist during manufacturing, but also needs to be considered as well.

Given any constraint or objective function it will depend on a set of variables. This dependence will change when a constraint or objective function changes. In optimization we only wish to concern ourselves with the state variables that are allowed to change during optimization. For this reason, we construct a spring object that will hold all attributes of a spring and we can pull the relevant variables when it is determined what they are.

In addition to this spring object, to reduce the uncertainty of the problem we conduct a sensitivity analysis of the objective function subject to the constraints and variables given. This allows the user to reduce the state variables if it is determined that the set of variables given has insensitive variables.

A main contribution of this work is the framework to add constraints and objectives that may be unknown at the point of this paper. In parallel to this we have built a framework that is flexible enough to address the issue of unknown constraints and objectives. **ADD!**

**review history and existing literature** Designing an optimal spring is not a new problem. In 2010 at the SAMSI IMSM workshop a team considered the design of an acceleration with enabled uncertainty. This approach lead to a paper [1]. This is not the only approach to uncertainty, in [3] probabilistic response surface methodology was implemented to investigate the complications of uncertainty in designing a spring. In addition to these types of approaches it is possible to narrow focus to a single parameter, in [4] they focus on the spring stiffness.

Our approach is not limited to an acceleration switch, instead it is but a special case of our approach. In [2] they attempt to design an optimal spring, with the introduction of a sizing tool. All references listed are informed to some extent by [5]. We do not focus on a set of parameters or attempt to fix the uncertainty of the spring design problem initially. We attempt to allow any possible construction of a spring optimization possible.

This does not rule out if a construction is not physically or mathematically invalid. We will dispose of that case when we are unable to find a feasible point for optimization within our feasibility check. In summary, we need to allow any possible construction of a spring design problem, and adjust accordingly.

**outline** In this paper we will give a description of the springs of interest, helical compression springs. Next, the formulation of the problem, and the approach taken to the problem. Following will be a section on workflow with descriptions of each step in the workflow. Lastly, we will discuss a few case studies given the framework constructed and a summary with future work that can be worked on.

- Describe the problem you are trying to solve, the approach you took, and summarize your contribution and results.
- Review the history of this problem, and existing literature.
- Give an outline of the rest of the paper.

# 2 Helical Compression Springs

Helical springs are a large class of springs sharing the common characteristic of a coiled appearance, see figure 2.

Below is a list of a spring's key design parameters. We have added a few illustrations to keep straight some of the parameters.

1. Spring's inner diameter $d_i$, illustrated in figure 3b.
2. Spring's outer diameter $d_o$ illustrated in figure 3a.
3. Spring's wire diameter $d_w$.
4. Total number of spring coils $N_t$.

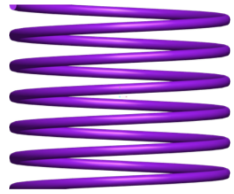
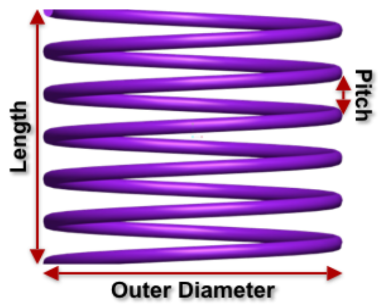
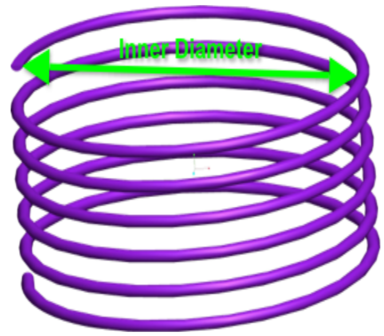


Figure 2: An example of a helical compression spring.




(a) Pitch, outer diameter, and length of a spring.




(b) Inner diameter

Figure 3: A few illustrations of a few of the parameters for a helical compression spring.

5. Active number of spring coils $N_a$, active coils are not touching any other coils, and is subject to the spring being closed or open.
6. Pitch $p$ illustrated in figure 3a.
7. Spring's free length $L_{free}$, this is the spring's length without any force applied.
8. Spring's solid length $L_{solid}$, this is the spring's length when all coils are compressed together.
9. Spring's open length $L_{open}$, spring length at open position, open is beginning state.
10. Spring's open length $L_{close}$, spring length at close position, close is the ending state.
11. Spring's open length $L_{hard}$, the maximum a spring can compress for the application.
12. Spring's open force $F_{open}$, this is the force on the spring in open position.
13. Spring's shear modulus $G$, this is determined by the material of the spring.
14. Spring's youngs modulus $E$, this is determined by the material of the spring.
15. Spring's poisson ratio $\nu$, this is determined by the material of the spring.

In addition to these parameters there are the following attributes that are empirical given some regime we care about.

1. Spring Rate,
$$k = \frac{G}{8N_a} \frac{d_w^4}{(d_i + d_w)^3}$$
2. Spring Index,
$$C = \frac{d_i}{d_w} + 1$$
.

3. Coil Binding Gap,
$$g = \frac{L_{hard} - L_{solid}(d_w, N_a; ec)}{N_t - 1}$$

4. $$\frac{G(L_{free} - L_{hard})}{4\pi N_a(ec)} \left[\frac{d_w(4d_i^2 + 9.46 d_i d_w + 3d_w^2)}{d_i(d_i + d_w)^3}\right] < UTS$$
where UTS is the ultimate torsional stress.

5. Diametral Expansion,
$$d_{expand} = d_w + \sqrt{(d_i + d_w)^2 + \frac{p_{closed}^2 - d_w}{\pi^2}}$$

# 3 The Problem

Design an algorithm that optimizes springs with interchangeable objectives and constraints. In addition, attempt to incorporate properties stress relaxation and creep into the available objectives and constraints.

A list of possible constraints/objectives given in minimization form is below

1. $d_i < d_i^{max}$
2. $d_i < d_o$
3. $d_i + 2 * d_w < d_o^{max}$
4. $d_{expand} - d_0^{max} < 0$
5. $\frac{G}{8N_a} \frac{d_w^4}{(d_i+d_w)^3} - k_{max} \leq 0$
6. $\frac{d_i}{d_w} + 1 - C_{max} < 0$
7. $(L_{free} - L_{open}) \frac{G}{8N_a} \frac{d_w^4}{(d_i+d_w)^3} - F_{open} = 0$
8. $\frac{L_{hard} - L_{solid}}{N_t - 1} + g_{min} \leq 0$
9. $\frac{L_{free}}{d_i + d_w} - \pi \sqrt{\frac{2(2\nu+1)}{\nu+2}}$
10. $-UTS + \frac{G(L_{free}-L_{hard})}{4\pi N_a(ec)} \left[\frac{d_w(4d_i^2+9.46 d_i d_w + 3d_w^2)}{d_i(d_i+d_w)^3}\right] < 0$

**ADD RELAXATION HERE!**

In addition to these we should be able to minimize any attribute of the spring.

In order to simplify the problem we have made some assumptions. First, we have assumed that constraints and objectives are all in terms of the number of total coils. This is to reduce the complexity of knowing if a constraint is dependent on the active number or total number of coils. These two numbers are dependent on the end conditions of the spring, whether it is open or closed at the end. For simplicity we also will consider all springs to be have a closed end condition.

With these assumptions we simplify our constraints for pitch, $p$, solid length, $L_{solid}$, and the diametral expansion, $d_{expand}$.

**JUSTIFY ASSUMPTIONS**

# 4 The Problem

- Give a precise technical description of your problem.
- State and justify all your assumptions.
- Define notation.
- Describe your data, how you collected them, their properties, and whether you did anything to them (removed noise, filled in missing data, applied normalizations).

# 5 The Approach

- Present and justify your approach for solving the problem.
- Explain the advantages of your approach over existing ones.
- Tell a story. Don't just say: "I did this, then I did this, and at last I did this".

# 6 Computational Experiments

Give enough details so that readers can duplicate your experiments.

- Describe the precise purpose of the experiments, and what they are supposed to show.
- Describe and justify your test data, and any assumptions you made to simplify the problem.
- Describe the software you used, and the parameter values you selected.
- For every figure, describe the meaning and units of the coordinate axes, and what is being plotted.
- Describe the conclusions you can draw from your experiments

# 7 Summary and Future Work

- Briefly summarize your contributions, and their possible impact on the field (but don't just repeat the abstract or introduction).
- Identify the limitations of your approach.
- Suggest improvements for future work.
- Outline open problems.

# References


[1] M. R. Brake, J. Mossad, B. Beheshti, J. Davis, R. Smith, K. Chowdhary, and S. Wang, "Uncertainty enable design of an acceleration switch," *Proceedings of ASME 2011 International Mechanical Engineering Congress and Exposition*, 2011.

[2] M. Paredes, M. Sartor, and A. Daidie, "Advanced assistance tool for optimal compression spring design," *Engineering with Computers*, 2005.

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[4] H. Zhao, G. Chen, and J. zhe Zhou, "The robust optimization design for cylindrical helical compression spring," *Advanced Materials Research*, vol. 433-440, 2012.

[5] A. M. Wahl, *Mechanical Springs*. Penton Publishing, first ed., 1944.