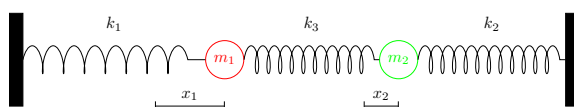


Coupled Harmonic Oscillators

We show how to model and solve the system of ODEs that models the coupled, undamped, spring-mass system.

Consider the coupled system pictured below.



Newton's Second Law says that the sum total force F acting on an object of mass m satisfies $F = [m \cdot a]$, where a is the acceleration. On the other hand, we have **Hooke's Law**: the force F required to stretch a spring x units beyond its natural length is $[k \cdot x]$ where k is some constant depending on the spring. The individual spring constants are in our coupled system are labeled in the image above.

Problem 1 The sum total force acting on m_1 is $F_1 = [-k_1x_1 - k_3(x_2 - x_1)]$

Problem 2 The sum total force acting on m_2 is $F_2 = [-k_2x_2 + k_3(x_2 - x_1)]$

Problem 3 The function $x_1(t)$ satisfies the following differential equation:

$$m_1 x_1'' = -(k_1 - k_3)x_1 - k_3x_2$$

Hint: Remember that acceleration is the second derivative of position