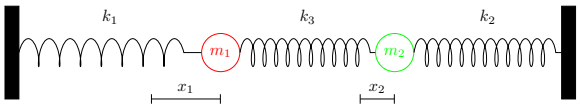
Coupled Harmonic Oscillators

We show how to model and solve the system of ODEs that models the coupled, undamped, spring-mass system.

Consider the coupled system pictured below.



Newton's Second Law says that the sum total force F acting on an object of mass m satisfies $F = \boxed{m \cdot a}$, where a is the acceleration. On the other hand, we have **Hooke's Law**: the force F required to stretch a spring x units beyond its natural length is $\boxed{k \cdot x}$ where k is some constant depending on the spring. The individual spring constants are in our coupled system are labeled in the image above.

Problem 1 *The sum total force acting on m_1 is $F_1 =$* $\boxed{-k_1x_1 - k_3(x_2 - x_1)}$

Problem 2 *The sum total force acting on m_2 is $F_2 =$* $\boxed{-k_2x_2 + k_3(x_2 - x_1)}$

Problem 3 *The function $x_1(t)$ satisfies the following differential equation:*

$$\boxed{m_1x_1'' = -(k_1 - k_3)x_1 - k_3x_2}$$

Hint: *Remember that acceleration is the second derivative of position*

Problem 4 *The function $x_2(t)$ satisfies the following differential equation:*

$$\boxed{m_2x_2'' = -k_3x_1 + (k_3 - k_2)x_2.}$$

Learning outcomes: