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1 Standard approach

This document explores a simple strategy for improving the robustness of randomization tests. Let $T_1, \dots, T_{B-1}$ be the $B-1$ resampled test statistics. Let T^* be the statistic computed on the raw data. Let $\mathrm{rank}(T^*; B-1)$ be the rank of T^* in the set $\{T^*, T_1, \dots, T_B\}$, i.e.

$$\mathrm{rank}(T^*; B) = |\{T \in \{T^*, T_1, \dots, T_B\} : T^* \geq T| = 1 + \sum_{i=1}^{B-1} \mathbb{I}(T^* \geq T_i).$$

Under the null hypothesis, $(T^*, T_1, \dots, T_B)$ is exchangeable, and we have that

$$\mathrm{rank}(T^*; B) = \mathrm{Unif}(\{1, \dots, B\}).$$

We can construct a p-value as

$$p = \mathrm{rank}(T^*; B)/B.$$

2 A simple strategy for improving robustness

Denote $R := \mathrm{rank}(T^*; B)$. Let $f : \{1, \dots, B\} \to \mathbb{R}$ be a function. We consider the transformed random variable $W := f(R)$. Let $\mathcal{X} := \{f(r)\}_{r=1}^{B}$. For $w \in \mathcal{X}$, we have that

$$\mathbb{P}\left(W = w\right) = \mathbb{P}\left(R \in f^{-1}(w)\right) = |f^{-1}(w)|,$$

where $f^{-1}(w) = \{r \in \{1, \dots, B\} : f(r) = w\}$ is the preimage of w. The latter equality follows because R is uniformly distributed on $\{1, \dots, B\}$. If f is injective, then $P(W = w) = 1/B$ for all w, implying that W is uniformly distributed over $\mathcal{X}$.