Introduction to Statistical Programming

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Outline

Introduction

What is the R language?

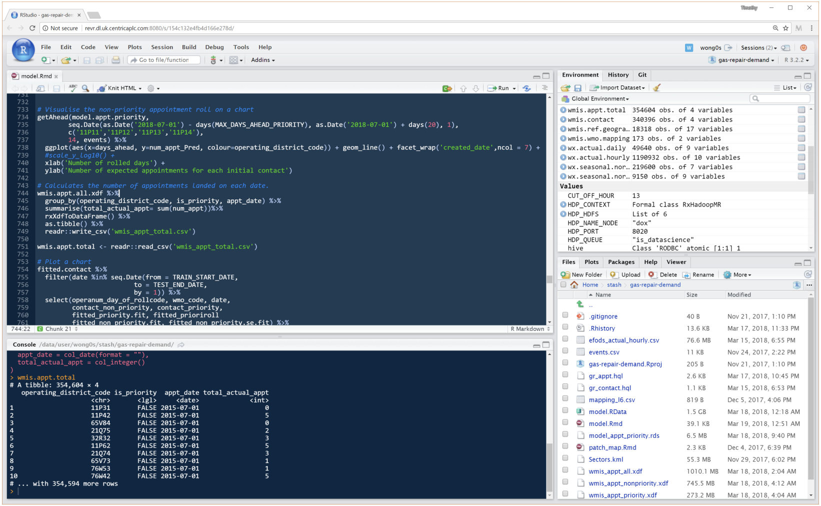
- Offers modern and sophisticated statistical algorithms
- Used by millions of analysts and researchers worldwide
- Has a thriving open-source community
- Enables big data analytics

Easy to Use

- `PROC REG` = `lm()` or `glm()`
- `PROC SQL` = `%>%`
- `PROC SORT` = `arrange()`
- `PROC MEANS` = `mean()`, `sd()`
- `PROC GPLOT` = `plot()`, `ggplot()`, `autoplot()`

RStudio Server Pro

- RStudio is your integrated development environment (IDE)



Packages

- **CRAN** is the Comprehensive R Archive Network.
- User-contributed packages: source code, binaries, documentation.

```r
# Install a new package with all its dependencies
install.packages("ggplot2", dependencies = TRUE)
# Load an installed package
# (both lines are identical)
library(ggplot2)
library("ggplot2")
```

Packages

- **CRAN Task View** is a curated list of packages.
- https://cran.r-project.org/web/views/

```r
library(ctv)
# Install a CRAN Task View
install.views("Econometrics")
# Update a CRAN Task View
update.views("Econometrics")
```

Vectors

- R is a vectorised programming language.
- Vector contains objects of the same data type.

```
# Create a vector of integers one to ten
myVec1 <- 1:10
# Find out the length of vector
length(myVec1)

## [1] 10

# Reverse the vector
rev(myVec1)

##  [1] 10  9  8  7  6  5  4  3  2  1

# Create a custom vector of 10, 15, 20, 25, 30
myVec2 <- c(10, 15, 20, 25, 30)
myVec2

## [1] 10 15 20 25 30

# Create a vector of sequential numbers
myVec3 <- seq(from = -10, to = 10, by = 0.5)
myVec3

##  [1] -10.0  -9.5  -9.0  -8.5  -8.0  -7.5  -7.0  -6.5  -6.0  -5.5  -5.0
## [12]  -4.5  -4.0  -3.5  -3.0  -2.5  -2.0  -1.5  -1.0  -0.5   0.0   0.5
## [23]   1.0   1.5   2.0   2.5   3.0   3.5   4.0   4.5   5.0   5.5   6.0
## [34]   6.5   7.0   7.5   8.0   8.5   9.0   9.5  10.0
```

Subsetting a Vector

- You can subset members from a vector.

```r
# Select the second member of the vector
myVec1[2]

## [1] 2

# Subset a range from the vector
myVec1[2:4]

## [1] 2 3 4

# Subset specified elements
myVec1[c(4,2,3)]

## [1] 4 2 3
```

Vectorised Operations

- Operations in R are vectorised.

```
# Arithmetic operations
myVec1 + 10

##  [1] 11 12 13 14 15 16 17 18 19 20

myVec1 - 10

##  [1] -9 -8 -7 -6 -5 -4 -3 -2 -1  0

myVec1 * 2

##  [1]  2  4  6  8 10 12 14 16 18 20

myVec1 / 2

##  [1] 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0

myVec1 ^ 2

##  [1]   1   4   9  16  25  36  49  64  81 100

log(myVec1)

##  [1] 0.0000000 0.6931472 1.0986123 1.3862944 1.6094379 1.7917595 1.9459101
##  [8] 2.0794415 2.1972246 2.3025851
```

Looping

- Looping can be slow.
- Always try to vectorise your code.

```
# Vectorised operation is fast
system.time({
  myResult <- 1:100000 * 2
})

##    user  system elapsed
##       0       0       0

# Looping is quite slow
system.time({
  myResult <- sapply(1:100000, function(x){ x * 2 })
})

##    user  system elapsed
##    0.08    0.00    0.08

# Appending to vector is much slower
system.time({
  myResult <- c()
  for(i in 1:100000){
    myResult <- c(myResult, i * 2)
  }
})

##    user  system elapsed
##   20.95    0.19   21.17
```

Functions

- Functions are vectorised.

```
# Defines a custom function
myFunc <- function(x) {
  x * 2
}
# Execute the function with one input
myFunc(5)

## [1] 10

# Execute the function with an integer vector
myFunc(1:10)

##  [1]  2  4  6  8 10 12 14 16 18 20
```

Character Vectors

- Vector can also contain character objects.

```
# Vector can contain character objects
myVec4 <- c("Bill", "Mark", "Steve", "Jeff", "Larry")
myVec4

## [1] "Bill"  "Mark"  "Steve" "Jeff"  "Larry"

# Constant character vectors in R
LETTERS

##  [1] "A" "B" "C" "D" "E" "F" "G" "H" "I" "J" "K" "L" "M" "N" "O" "P" "Q"
## [18] "R" "S" "T" "U" "V" "W" "X" "Y" "Z"

letters

##  [1] "a" "b" "c" "d" "e" "f" "g" "h" "i" "j" "k" "l" "m" "n" "o" "p" "q"
## [18] "r" "s" "t" "u" "v" "w" "x" "y" "z"

month.name

##  [1] "January"   "February"  "March"     "April"     "May"
##  [6] "June"      "July"      "August"    "September" "October"
## [11] "November"  "December"

month.abb

##  [1] "Jan" "Feb" "Mar" "Apr" "May" "Jun" "Jul" "Aug" "Sep" "Oct" "Nov"
## [12] "Dec"
```

Vectors - Other Data Types

- Vector can contain objects of any data type.

```
# This is a vector of character objects
myVec5 <- c("2017-07-13",
            "2017-10-11",
            "2017-11-21",
            "2018-01-16",
            "2018-03-27")
# This is a vector of date objects
myVec6 <- as.Date(myVec5)
# Compute the day of week - returns a vector of characters
# Notice that these are all vectorised functions
weekdays(myVec6)

## [1] "Thursday"  "Wednesday" "Tuesday"   "Tuesday"   "Tuesday"
```

List

- List is a generic container for objects of different data types

```
myFavBook <- list(title = "R for Data Science",
                  authors = c("Garrett Grolemund", "Hadley Wickham"),
                  publishDate = as.Date("2016-12-12"),
                  price = 18.17,
                  currency = "USD",
                  edition = 1,
                  isbn = 1491910399)
myFavBook
```

```
## $title
## [1] "R for Data Science"
##
## $authors
## [1] "Garrett Grolemund" "Hadley Wickham"
##
## $publishDate
## [1] "2016-12-12"
##
## $price
## [1] 18.17
##
## $currency
## [1] "USD"
##
## $edition
## [1] 1
##
## $isbn
## [1] 1491910399
```

Subsetting a List

- You can subset a particular member from a list

```
# Select a named member of a list
# Using the dollar sign, followed by name without bracket
myFavBook$title

## [1] "R for Data Science"

# Using double squared brackets with member's name as string
myFavBook[["authors"]]

## [1] "Garrett Grolemund" "Hadley Wickham"

# Select the fourth member in the list
myFavBook[[4]]

## [1] 18.17
```

Special Numbers in R

```
# Pi is constant 3.14159...
pi

## [1] 3.141593

# One divided by zero is infinity
1/0

## [1] Inf

# Negative number divided by zero is negative infinity
-1/0

## [1] -Inf

# Infinity divided by infinity is Not-a-Number (NaN)
Inf/Inf

## [1] NaN

# Not available (NA) plus one is still NA
NA + 1

## [1] NA

# Effects of different special numbers
c(5, 10, 15, NA, 25, 30, NaN, 35, 40, Inf, 50, -Inf, 60) / 5

##  [1]    1    2    3   NA    5    6  NaN    7    8  Inf   10 -Inf   12
```

Data Frame

- Table with rows (observations) and columns (variables).
- Analogous to an Excel workbook.

```
myFavMovies1 <- data.frame(title = c("Dr. No",
                                     "Goldfinger",
                                     "Diamonds are Forever",
                                     "Moonraker",
                                     "The Living Daylights",
                                     "GoldenEye",
                                     "Casino Royale"),
                           year = c(1962, 1964, 1971, 1979,
                                    1987, 1995, 2006),
                           box = c(59.5, 125, 120, 210.3,
                                   191.2, 355, 599),
                           bondActor = c("Sean Connery",
                                         "Sean Connery",
                                         "Sean Connery",
                                         "Roger Moore",
                                         "Timothy Dalton",
                                         "Pierce Brosnan",
                                         "Daniel Craig"))
```

Data Frame

```
myFavMovies1

##                      title year   box      bondActor
## 1                   Dr. No 1962  59.5   Sean Connery
## 2               Goldfinger 1964 125.0   Sean Connery
## 3 Diamonds are Forever 1971 120.0   Sean Connery
## 4                Moonraker 1979 210.3    Roger Moore
## 5 The Living Daylights 1987 191.2 Timothy Dalton
## 6                GoldenEye 1995 355.0 Pierce Brosnan
## 7           Casino Royale 2006 599.0   Daniel Craig
```

Tibble

- Similar to traditional `data frame`.
- `tibble` is the modern standard in R.

```
library(dplyr)
myFavMovies2 <- tibble(title = c("Dr. No",
                                  "Goldfinger",
                                  "Diamonds are Forever",
                                  "Moonraker",
                                  "The Living Daylights",
                                  "GoldenEye",
                                  "Casino Royale"),
                       year = c(1962, 1964, 1971, 1979,
                                1987, 1995, 2006),
                       box = c(59.5, 125, 120, 210.3,
                               191.2, 355, 599),
                       bondActor = c("Sean Connery",
                                     "Sean Connery",
                                     "Sean Connery",
                                     "Roger Moore",
                                     "Timothy Dalton",
                                     "Pierce Brosnan",
                                     "Daniel Craig"))
```

Tibble

```
myFavMovies2

## # A tibble: 7 x 4
##   title                    year   box bondActor
##   <chr>                   <dbl> <dbl> <chr>
## 1 Dr. No                   1962  59.5 Sean Connery
## 2 Goldfinger               1964 125   Sean Connery
## 3 Diamonds are Forever     1971 120   Sean Connery
## 4 Moonraker                1979 210.  Roger Moore
## 5 The Living Daylights     1987 191.  Timothy Dalton
## 6 GoldenEye                1995 355   Pierce Brosnan
## 7 Casino Royale            2006 599   Daniel Craig
```

Subsetting a Tibble

```r
# Get one column by name
myFavMovies2[["title"]]
myFavMovies2$title
# Get a range of columns by position ID
myFavMovies2[, 1:2]
myFavMovies2[1:2]
# Get rows 1 to 3
myFavMovies2[1:3, ]
# Get the "year" variable of row 1-3
myFavMovies2[1:3, "year"]
# Get the "title" and "year" variables of row 4-7
myFavMovies2[4:7, c("title","year")]
```

UK User Communities

- LondonR -
- ManchesterR -
- CaRdiff -
- SheffieldR
- EdinbR -
- CambR -
- NottinghamR
- BirminghamR -
- Oxford RUG -
- Bristol Data Scientists -

International User Communities

- International R User Conference (useR!)
- Enterprise Application of the R Language (EARL)
- European R User Meeting (ERUM)

Introduction to Data Analysis

Regression Models

Simple Regression

Univariate linear regression model

$$\hat{y}_i = \beta_0 + \beta_1 x_i$$

- Analogous to a straight line $y = mx + c$
- Can be chained with M dependent variables (Multivariate)

$$\hat{y}_i = \beta_0 + \sum_{m=1}^{M} \beta_m x_{m,i}$$

- Residual term $\epsilon_i = y_i - \hat{y}_i$ assumed to be Gaussian

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

- Also known as ordinary least squared (OLS) regression

Linear Regression in R

```r
# Build a univariate linear model
# These two lines are equivalent
myModel1 <- lm(mpg ~ wt, mtcars)
myModel1 <- lm(formula = mpg ~ wt, data = mtcars)
# Read the model summary
summary(myModel1)
```

```
##
## Call:
## lm(formula = mpg ~ wt, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.5432 -2.3647 -0.1252  1.4096  6.8727
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.2851     1.8776  19.858  < 2e-16 ***
## wt           -5.3445     0.5591  -9.559 1.29e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.046 on 30 degrees of freedom
## Multiple R-squared:  0.7528,Adjusted R-squared:  0.7446
## F-statistic: 91.38 on 1 and 30 DF,  p-value: 1.294e-10
```

More Linear Regression Models

Multivariate linear model Additional independent variables can be chained using the + symbol. Categorical variables can be encoded as dummy on-the-fly using `factor()`.

```
myModel2 <- lm(mpg ~ wt + hp + qsec + factor(am), mtcars)
```

Polynomial term Model can become more flexible when an independent variable is converted into polynomial terms. Use the `poly()` function.

```
myModel3 <- lm(mpg ~ wt + qsec + factor(am) +
                poly(hp, 3), mtcars)
```

Interaction term Two variables can be combined to create synergy effect. The * symbol is used to combine variables together.

```
myModel4 <- lm(mpg ~ wt * hp + qsec + factor(am), mtcars)
```

Regression Diagnostics

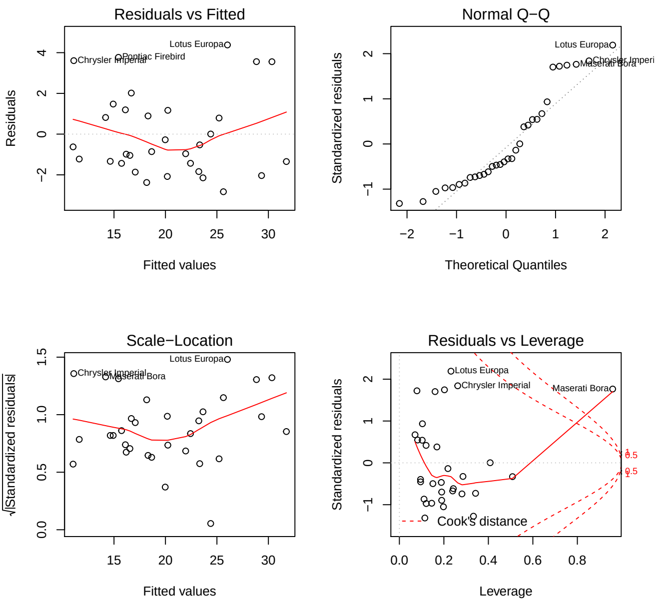
Residuals vs Fitted Checks for non-linear relationship. Look for a near horizontal line.

Normal Quantile-Quantile It aligns model residuals against a theoretical normal distribution. If the residuals spread along a straight diagonal line on the Q-Q plot, it suggests that the residuals are normally distributed.

Scale-Location Checks for homoscedasticity and heteroscedasticity. It is homoscedastic if observations scatter without any observable pattern.

Residual vs Leverage (Cook's Distance) Identifies observations having strong influence to the model.

Diagnostics Plots

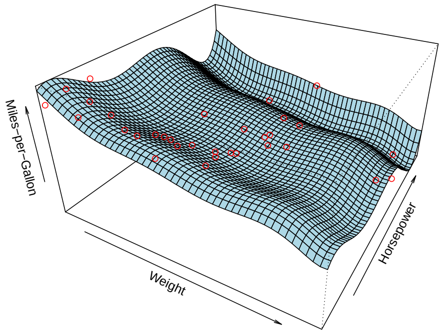


Overfitting

- Flexible models are prone to overfitting.
- Overfitting makes the model less generalisable.
- Solution
  - Use less flexible methods.
  - Impose regularisation.

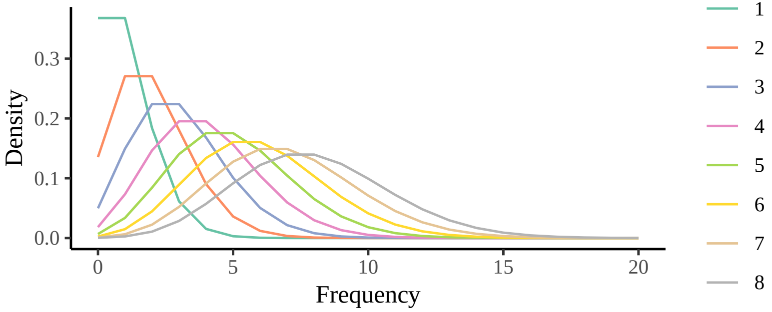
Overfitting: Visual Explaination

$$\hat{y} = \beta_0 + \sum_{j=1}^{8} \beta_{wt_j} x_{wt}^{j} + \sum_{k=1}^{5} \beta_{hp_k} x_{hp}^{k}$$



Poisson Distribution

- Count of distinct events are drawn from Poisson distribution.
  - Always positive.
  - In most cases they are integers.
- e.g. Number of people in a room, number of flights delayed per day... etc.



Testing for Poisson Distribution

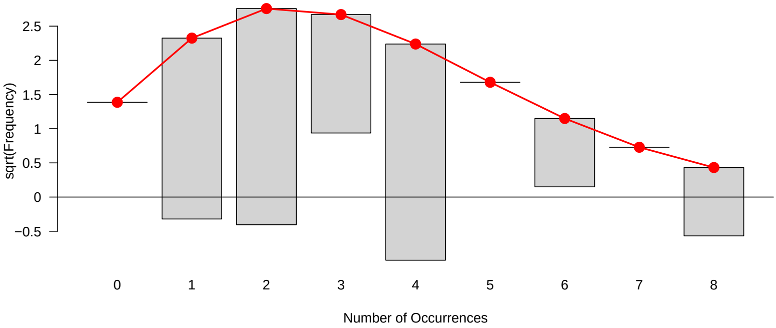
- Chi-square goodness of fit test.
- Fits the data against theoretical Poisson distribution.
- Look for statistical significance.

```
# Performs the Chi-squared goodness-of-fit test.
# It checks whether the variable is drawn from a Poisson distribution.
library(vcd)
gf <- goodfit(mtcars$carb, type= "poisson", method= "ML")
# Checks the statistical p-value of the goodness-of-fit test.
# If p<=0.05 then it is safe to say that the variable is Poisson.
summary(gf)

##
##   Goodness-of-fit test for poisson distribution
##
##                       X^2 df     P(> X^2)
## Likelihood Ratio 20.53973  4 0.0003906369
```

Goodness of Fit Plot

```r
# Plots the observed frequency vs theoretical Poisson distribution.
# The hanging bars should fill the space if it is perfectly Poisson.
plot(gf)
```



Poisson Regression

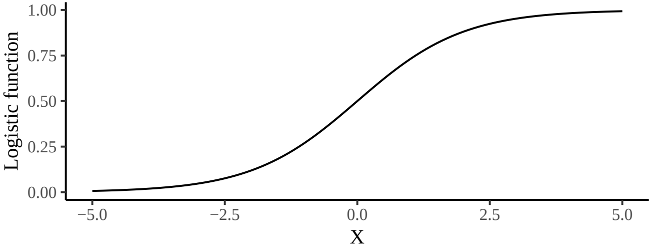
```r
# Build a Poisson model to predict the number of carburetors in a car.
myPoissonModel <- glm(carb ~ hp + wt + factor(am),
                      family="poisson",
                      data=mtcars)
# Read the model summary
summary(myPoissonModel)

##
## Call:
## glm(formula = carb ~ hp + wt + factor(am), family = "poisson",
##     data = mtcars)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -0.91420  -0.48423  -0.07246   0.19252   1.26155
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.418081   0.604211  -0.692   0.4890
## hp           0.004316   0.001880   2.296   0.0217 *
## wt           0.179583   0.191352   0.938   0.3480
## factor(am)1  0.393750   0.324978   1.212   0.2257
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 27.043  on 31  degrees of freedom
## Residual deviance: 10.798  on 28  degrees of freedom
## AIC: 108.16
##
## Number of Fisher Scoring iterations: 4
```

Binomial Distribution

- There are only two possible outcomes $\{Y, \neg Y\}$.
  - Toss a coin (Head or tail)
  - Taking an examination (Pass or fail)
  - Selling a product (Sold or unsold)
- Likelihood of event Y and $\neg Y$ expressed as probablity $P(Y) + P(\neg Y) = 1$.
- Logistic function squeezes real value range X into $(0, 1)$ to express probablity.

$$P(Y) = \frac{1}{1 + e^{-X}}$$



Logistic Regression

- Equation for logistic regression

$$P(Y) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_M x_M)}}$$

- Coefficients $\beta_1, \beta_2, \beta_3, \ldots, \beta_M$ can be converted into odds ratios $OR(x_1), OR(x_2), OR(x_3), \ldots, OR(x_M)$.

$$OR(x_1) = \frac{odds(x_1 + 1)}{odds(x_1)} = \frac{e^{\beta_0 + \beta_1 (x_1 + 1) + \beta_2 x_2 + \ldots + \beta_M x_M}}{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_M x_M}} = e^{\beta_1}$$

- $OR(x_1)$ Represents the change in probability when x_1 increases by 1 unit.

Training a Logistic Regression Model

- Build a logistic regression model to predict the dependent variable `am`. (1=manual; 0=auto)

```
myLogisticModel <- glm(am ~ mpg + hp + disp,
                       family="binomial",
                       data=mtcars)
summary(myLogisticModel)
```

Calculate the odds ratios for this model.

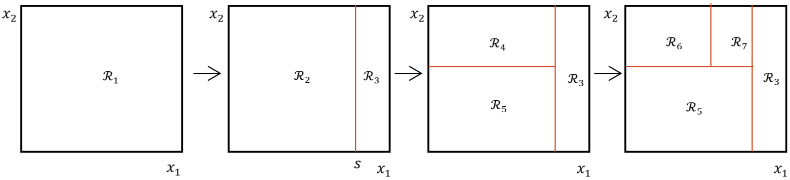
```
# Calculate the odds-ratios by taking the exponential of the coefficients
# You may also calculate the 95% confidence interval of the odds-ratio
exp(cbind(oddsratio = myLogisticModel$coefficients,
          confint(myLogisticModel)))

##                oddsratio        2.5 %      97.5 %
## (Intercept) 2.066683e-15 9.104693e-49  0.02539238
## mpg         3.614582e+00 1.198283e+00 61.43800377
## hp          1.161095e+00 1.049568e+00  1.50974362
## disp        9.366496e-01 8.197068e-01  0.98685201
```

Tree-based Methods

Recursive Partitioning

- Cut off point is denoted as s.
- Divides data into regions (leaves) $\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \ldots$ recursively.
- Works with real values as well as categorical variables.
- Large tree risks overfitting
  - Removes weaker leaves.
  - Regularisation.



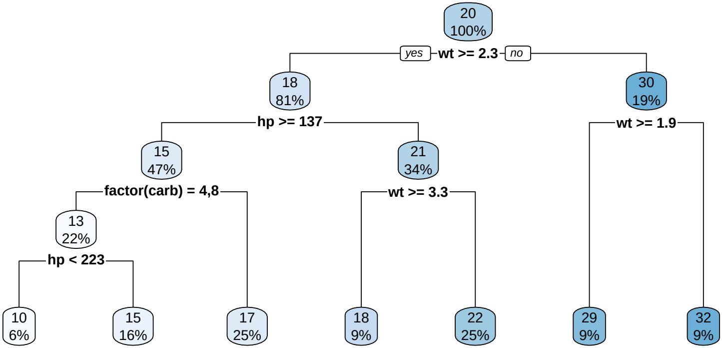
Decision Tree

- Trees can be trained with a formula and optional control parameters.

```r
# Load the rpart package for recursive partitioning
library(rpart)
# Build a decision tree to predict mpg
myTree <- rpart(formula = mpg ~ wt + hp +
                  factor(carb) +
                  factor(am),
                data = mtcars,
                control = rpart.control(minsplit=5))
# Read the detailed summary of the tree
summary(myTree)
```

Decision Tree: Visualisation

```r
# Load the rpart.plot package for tree visualisation
library(rpart.plot)
rpart.plot(myTree)
```

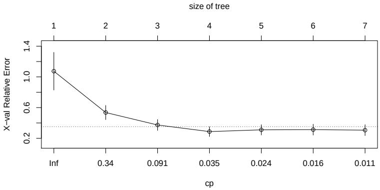


Tree Pruning

```
printcp(myTree)

##
## Regression tree:
## rpart(formula = mpg ~ wt + hp + factor(carb) + factor(am), data = mtcars,
##     control = rpart.control(minsplit = 5))
##
## Variables actually used in tree construction:
## [1] factor(carb) hp           wt
##
## Root node error: 1126/32 = 35.189
##
## n= 32
##
##         CP nsplit rel error  xerror     xstd
## 1 0.652661      0  1.000000 1.07418 0.245782
## 2 0.178618      1  0.347339 0.53593 0.092826
## 3 0.046269      2  0.168721 0.37350 0.071636
## 4 0.026109      3  0.122453 0.28739 0.064465
## 5 0.022593      4  0.096343 0.31045 0.067372
## 6 0.011989      5  0.073751 0.31308 0.071366
## 7 0.010000      6  0.061762 0.30626 0.071398

plotcp(myTree)
```

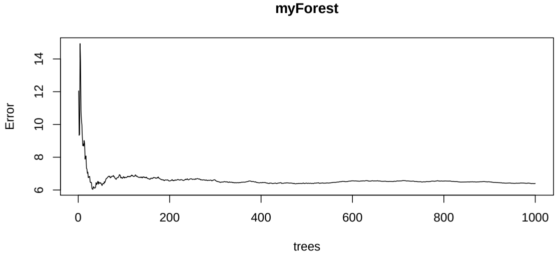


Random Forest

- Consists of many decision trees
  - Randomly selected variables will be used in each split
  - Usually no need to prune them (all trees are allowed to grow big)
- M trees in a random forest produces M predictions
  - Final prediction is calculated as mean value for regression problem
  - Classification problem will use most the common label (majority voting)

Training a Random Forest

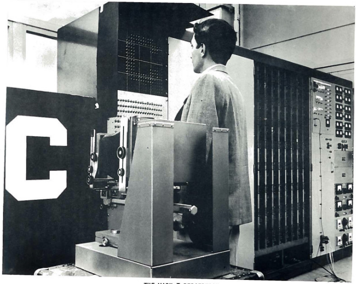
```
library(randomForest)
library(dplyr)
# Build a random forest with 1000 trees
# Each tree has 2 randomly selected variables
# You can change the parameters
myForest <- randomForest(mpg ~ wt + hp + carb + am,
                         ntree = 1000,
                         mtry = 2,
                         data = mtcars %>% mutate(carb = factor(carb),
                                                  am = factor(am)))
# Plot the error as the forest expands
plot(myForest)
```



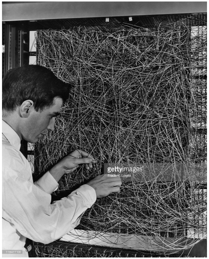
Neural Networks

Artificial Neurons

- Inspired by neurons in biological brain.
- McCulloch and Pitts described neuron as a logical process.
  - Neuron takes several inputs $\{x_1, x_2, x_3, ..., x_M\}$
  - Fires (activates) if the combined weighted input $\sum_{m=1}^{M} w_m x_m$ exceeds threshold.
  - Output can either be fire 1 or not fire 0.
- Rosenblatt's Mark I Perceptron



THE MARK I PERCEPTRON



Modern Neural Networks

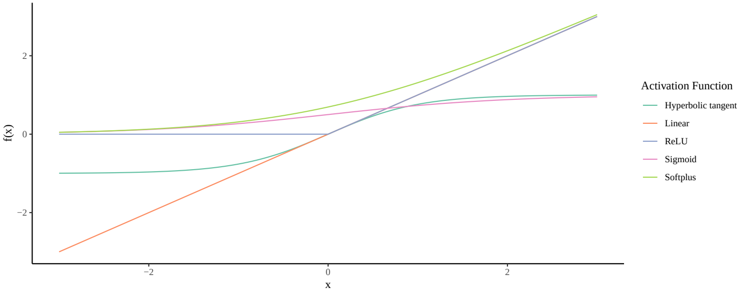
- Neural nets are based on non-linear processing power.
- Trained via gradient descent optimisers (backpropagation).
  - Network initialise randomly.
  - Requires differentiable loss function.
  - Requires strong gradient in order to improve.
  - Model weights improve iteratively.
  - Converge at local minimum.
- Neurons can be stacked as layers.
  - Can either be shallow (1 layer) or deep (many layers).
  - State-of-the art neural networks have highly bespoke topologies.

Activation

- Weighted inputs are combined linearly.

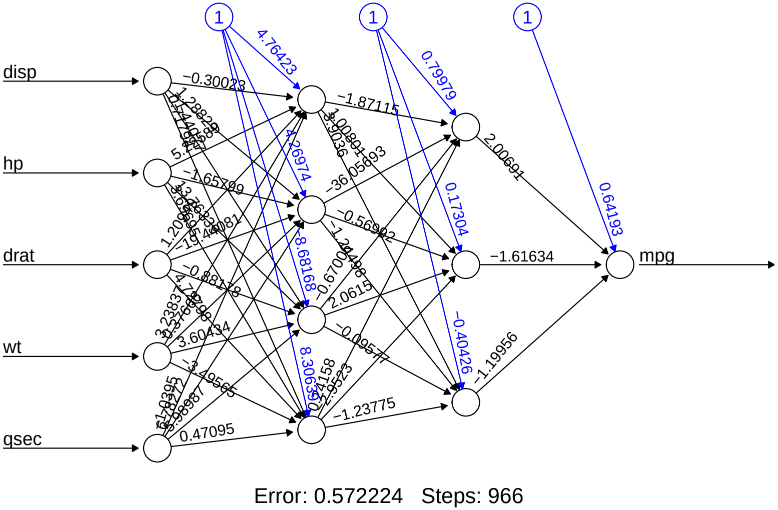
$$X = \sum_{m=1}^{M} w_m x_m$$

- Non-linear activation functions
  - Sigmoid
  - Hyperbolic tangent
  - etc...



Multilayer Perceptron: Topology

- Layers are fully-interconnected.
- Usually having two or more layers.



Time Series Analysis

Time Series Data

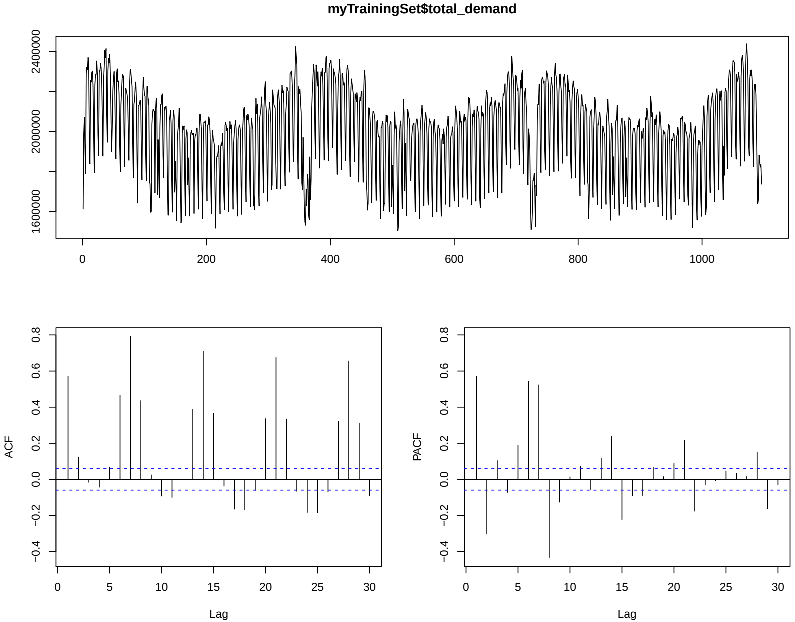
- Observations repeatedly taken at regular interval.
- Explore variable relationship across temporal space.

  Auto-correlation Function (ACF)
  : Measures the correlation of a single variable along the temporal dimension between x_t and x_{t+h}. It shows the correlation of the variable over different lag periods. For most time series variables, correlation is usually strong at lag $h = 1$ and it gradually diminishes as lag period increases. Cyclic pattern in the correlogram suggests possible seasonality which you can analyse further.

  Partial Auto-correlation Function (PACF)
  : Also measures the correlation between different lag periods, but it controls the correlation across the temporal dimsnion so that only the contribution of an individual lag is reflected.

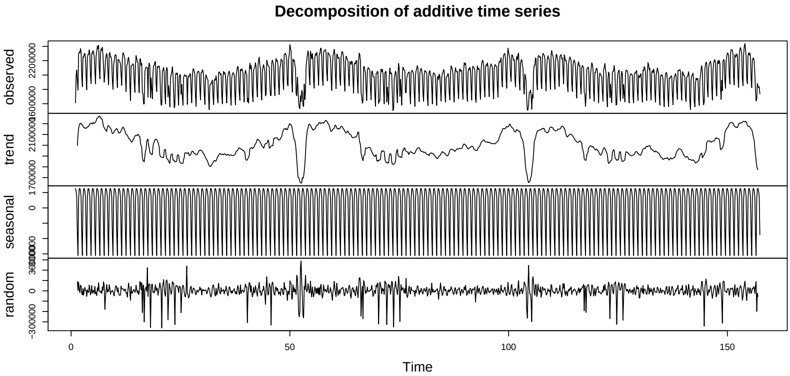
  Cross Correlation Function (CCF)
  : Analyses the temporal correlation between two variables.

ACF and PACF Correlograms



Decomposition

- Time series can be decomposed into:
  - Seasonal component S_t
  - Trend component T_t
  - Residual component ϵ_t
- Additive time series is expressed as $X_t = S_t + T_t + \epsilon_t$
- Multiplicative time series is expressed as $X_t = S_t \times T_t \times \epsilon_t$



Time Series Linear Regression Model

- Decomposed components can be used as independent variable in linear regression:

$$X_t = \beta_0 + \beta_{trend} T_t + \beta_{seasonal} S_t + \sum_{m=1}^{M} (\beta_m x_{mt}) + \epsilon_t$$

- Components can also form interaction terms with other independent variables.

Auto-regressive Moving Average Model (ARMA)

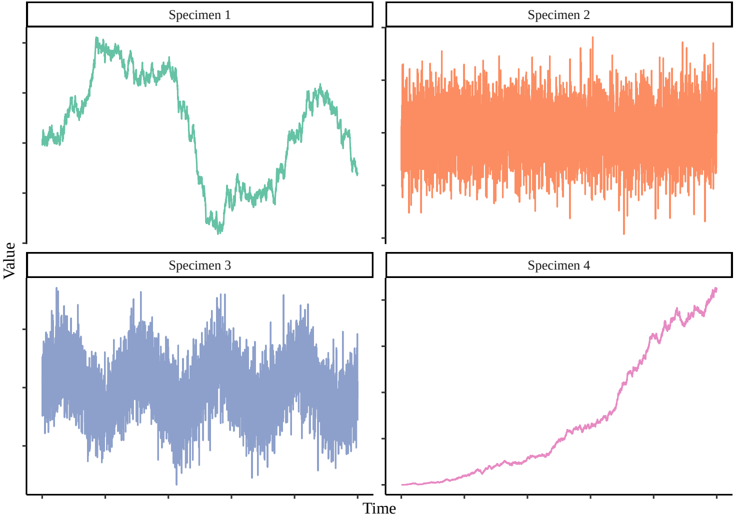
- $ARMA(p, q)$ model

$$\underbrace{X_t}_{\text{Observation}} = \underbrace{\beta_0}_{\text{intercept}} + \underbrace{\sum_{i=i}^{p}(\phi_i X_{t-1})}_{\text{AR(p)}} + \underbrace{\sum_{i=1}^{q}(\theta_i \epsilon_{t-i})}_{\text{MA(q)}} + \underbrace{\epsilon_t}_{\text{residual}}$$

- $ARIMA(p, d, q)$ model
  - Auto-regressive Integrative Moving Average
  - $I(d)$ d^{th} order integration can be added.
    - Integration refers to the difference from previous time step
    - First order differencing $I(1) : X_t^{'} = X_t - X_{t-1}$
    - To satisfy **stationarity** requirement.

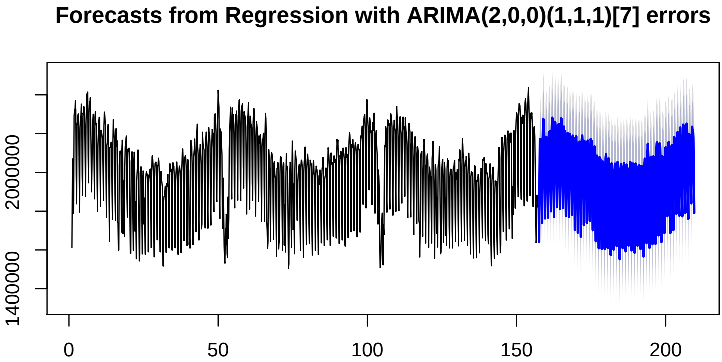
Stationarity

- Equal properties (mean and variance) across time.
  - Only one below is a stationary time series.
  - Which one is it?



ARIMA with Seasonality (SARIMA)

- $ARIMA(p, d, q)(P, D, Q)_m$
  - All parameter values can be automatically identified.
  - Simple models are always preferred
  - Intend to keep $p + q + P + Q$ small.



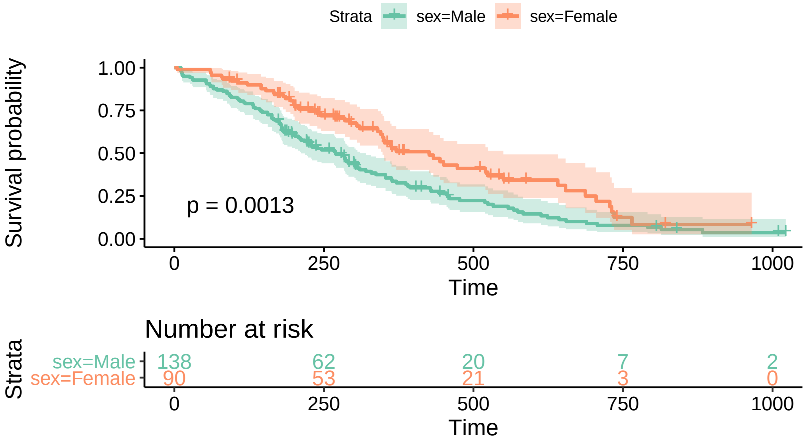
Survival Analysis

Survival Analysis

- Event occuring at irregular intervals.
  - e.g. Patients gets sick, machine failing... etc
- Also known as **time-to-event analysis** or **event history analysis**.

Kaplan-Meier Estimator

- It is used to measure how many subjects survives in a clinical trial since treatment began.
- Categorical variables only.



Cox Proportional Harzard Model

- It is a regression method which can take into account categorical and numeric variables.
- Assumes effects are time-independent (proportional harzard assumption).
- Harzard function h_t is defined as:

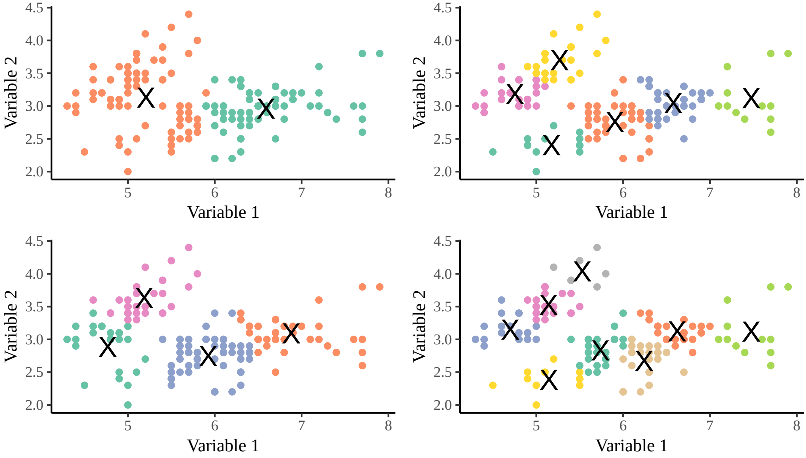
$$h_t = h_{0,t} \times e^{\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_M x_M}$$

```r
# Build a Cox model with predictor variables
myCoxModel1 <- coxph(Surv(time, status==2) ~ factor(sex) + age +
                          ph.ecog + ph.karno +
                          pat.karno +
                          meal.cal + wt.loss, data = lung)
# Read the model summary
summary(myCoxModel1)
```

Unsupervised Learning

K-means Clustering

- Works with unlabelled data.
- Arrange objects into groups (clusters)
- Objective interpretation of results as K is arbitrarily selected.



Agglomerative Hierarchical Clustering

- N objects can form maximum N clusters, each having 1 member object.
- Identify distance between closest cluster pair.
- Merge them.
- Repeat until there are no more clusters left.

Agglomerative Hierarchical Clustering: Example