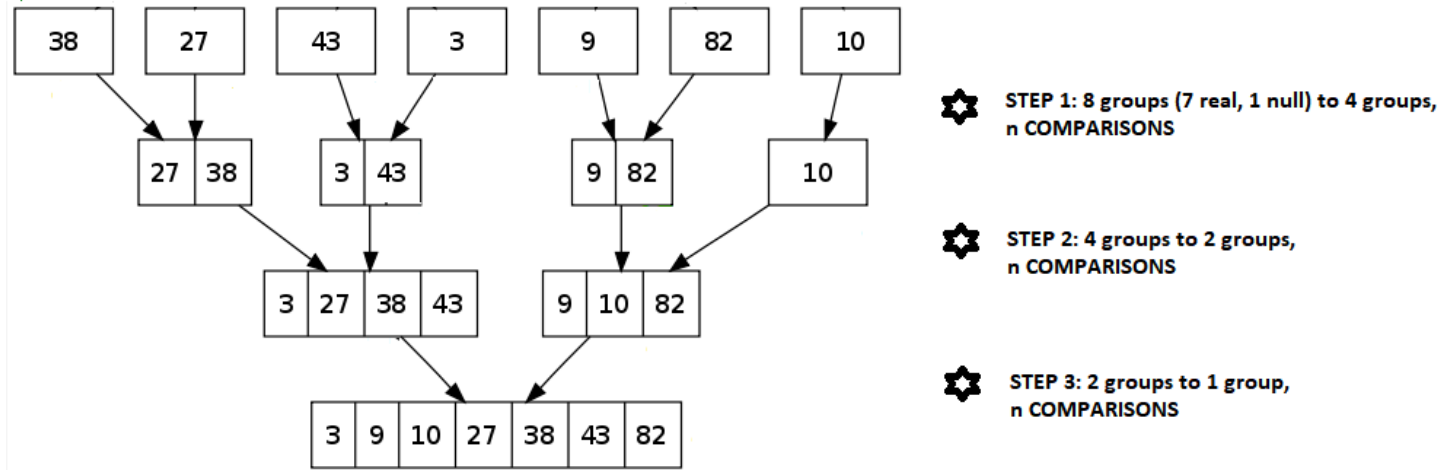


## 1 Merge Sort Complexities

38	27	43	3	9	82	10	$n = 7$
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**TIME COMPLEXITY:** step count \* comparisons at each step  
**SPACE COMPLEXITY:** how much data is stored, at most?  
The dominant factor for space complexity is the data storage.

How many steps are required? If, at each step, we merge two groups together, we are halving our group count at each step. When we arrive at a single group, the algorithm finishes.

Therefore, the group count, starting at  $n$ , diminishes at a rate of  $\log(n)$ .

This rate yields the expected number of steps.

Note:  $\log_2(7) = 2.80735492206 \approx 3$ . Over 7 data points, this algorithm yields 3 steps.

## 2 Inline Merge Sort Algorithm

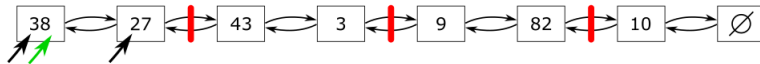
Can we perform merge sort over a single deck of cards? The current algorithm requires two decks. You can use a wooden table to exhibit constant-space additional space for your algorithm. To achieve this in code, use a doubly-linked list (shown below).

↗ COMPARISON POINTER

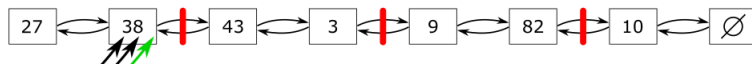
↗ INSERT POINTER



ADD NULLS UNTIL  $n=2^x$ ,  $x = \#steps$ .



UNSORTED

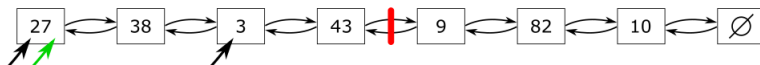


UNSORTED

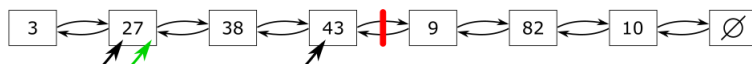
\*Only first group sorting shown.



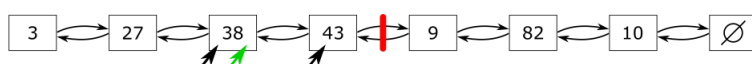
SORTED WITHIN GROUPS, time  $O(n)$ .



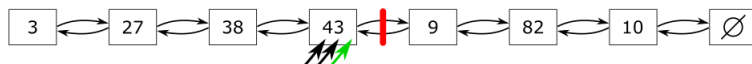
UNSORTED



UNSORTED



UNSORTED

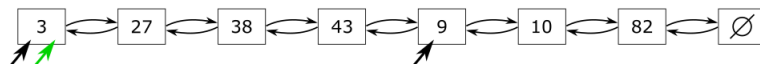


UNSORTED

\*Only first group sorting shown.



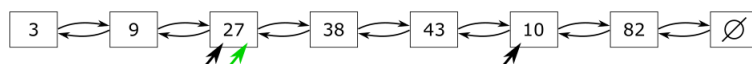
SORTED WITHIN GROUPS, time  $O(n)$ .



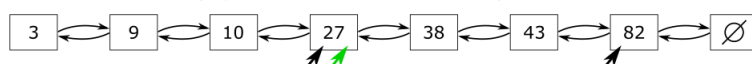
UNSORTED



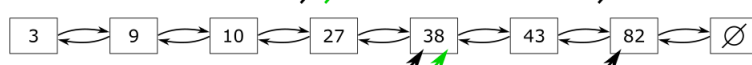
UNSORTED



UNSORTED



UNSORTED



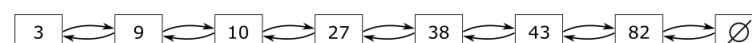
UNSORTED



UNSORTED



UNSORTED



SORTED WITHIN FINAL GROUP, time  $O(n)$ .