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Local influence in elliptical linear regression models

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SUMMARY
Influence diagnostic methods are extended in this paper to elliptical linear models. These include several symmetric multivariate distributions such as the normal, Student *t*-, Cauchy and logistic distributions, among others. For a particular perturbation scheme and for the likelihood displacement the diagnostics agree with those developed for the normal linear regression model by Cook when the coefficients and the scale parameter are treated separately. This result shows the invariance of the diagnostics with respect to the induced model in the elliptical linear family. However, if the coefficients and the scale parameter are treated jointly we have a different diagnostic for each induced model, which makes this approach helpful for selecting the less sensitive model in the elliptical linear family. An example on the salinity of water is given for illustration.

Keywords: Diagnostic; Influence; Likelihood displacement; Multivariate symmetric distributions

1. Introduction

Diagnostic techniques for normal linear regression models have been extensively studied in the statistical literature. See, for example, Belsley *et al.* (1980), Cook and Weisberg (1982) and Chatterjee and Hadi (1988). Several of the diagnostic techniques evaluate the effect of deleting observations on parameter estimates. An alternative approach that assesses the influence of small (local) perturbations from the assumed model on key results is considered in Cook (1986). Additional results on local influence and applications can be found in Beckman *et al.* (1987), Lawrance (1988), Thomas and Cook (1990), Tsai and Wu (1992), Paula (1993) and Kim (1995).

The method of local influence was proposed by Cook (1986, 1987) as a general tool for assessing the effect of local departures from model assumptions. In this paper, the local influence approach is applied to elliptical linear regression models, i.e. when the error vector has an elliptical distribution. The perturbation scheme considered here is the scheme in which the scale parameter is modified to allow convenient perturbations in the model.

In Section 2, along with the notation, the elliptical linear models are defined. The local influence method is reviewed in Section 3. Section 4 deals with the derivation of the diagnostic procedures for the elliptical linear models. An illustrative example is given in the last section.

2. Elliptical linear models

The class of elliptical distributions has been of considerable interest in the recent statistical

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literature. See, for example, Fang *et al.* (1990), Fang and Zhang (1990) and Fang and Anderson (1990). An $n \times 1$ random vector Y has an elliptical distribution with location vector μ and scale positive definite matrix Λ, if its density takes the form

$$f_Y(y) = |\Lambda|^{-1/2}\, g\{(y-\mu)^{\mathrm{T}}\Lambda^{-1}(y-\mu)\}, \tag{1}$$

$y \in \mathscr{R}^n$, where the function g: $\mathscr{R} \rightarrow [0, \infty)$ is such that

$$\int_0^\infty u^{n-1}\, g(u^2)\, \mathrm{d}u < \infty.$$

The function g is typically known as the density generator. For a vector Y distributed according to density (1), we use the notation $Y \sim \mathrm{El}_n(\mu, \Lambda, g)$ or, simply, $Y \sim \mathrm{El}_n(\mu, \Lambda)$. When $\mu = 0$ and $\Lambda = I$, we obtain the spherical family of densities. This class of symmetric distributions includes the normal, Student t-, contaminated normal and logistic (both, univariate and multivariate) distributions, among others, as considered, for example, by Fang *et al.* (1990). Table 1, taken from Fang *et al.* (1990), reports examples of distributions in the elliptical family. The notation c_1, c_2, c_3 and c_4 is used to denote normalizing constants.

Consider now the linear regression model

$$Y = X\beta + \epsilon, \tag{2}$$

where Y is an $n \times 1$ vector of responses, X is a known $n \times p$ matrix of rank p, β is a p-dimensional vector of parameters and ϵ is a p-dimensional error vector with distribution $\mathrm{El}_n(0, \phi I)$, where ϕ is the scale parameter. Thus, it follows that $Y \sim \mathrm{El}_n(X\beta, \phi I)$. This is typically called the elliptical linear regression model. If g is a continuous and decreasing function then the maximum likelihood estimators of β and ϕ are given by (see Fang and Anderson (1990))

$$\begin{aligned} \hat{\beta} &= (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}Y, \\ \hat{\phi} &= Q(\hat{\beta})/u_g, \end{aligned} \tag{3}$$

where $Q(\beta) = (Y - X\beta)^{\mathrm{T}}(Y - X\beta)$ and u_g maximizes the function

$$h(u) = u^{n/2}\, g(u), \qquad u \geqslant 0. \tag{4}$$

Typically, if g in equation (4) is continuous and decreasing then its maximum u_g exists and is finite and positive. Moreover, if g is continuous and differentiable then u_g is the solution to (Fang and Anderson, 1990)

$$g'(u) + \frac{n}{2u}\, g(u) = 0$$

or, equivalently, the solution of the equation

$$\frac{n}{2u} + W_g(u) = 0, \tag{5}$$

TABLE 1
Multivariate elliptical distributions

Distribution	*Notation*	*Generating function*
Normal	$N_n(\mu, \Lambda)$	$g(u) = c_1 \exp(-u/2), \quad u \geqslant 0$
Student t	$t_n(\mu, \Lambda, \nu)$	$g(u) = c_2(1 + u/\nu)^{-(\nu+n)/2}$
Contaminated normal	$\mathrm{CN}_n(\mu, \Lambda, \delta, \tau)$	$g(u) = c_1\{(1-\delta)\exp(-u/2) + \delta\tau^{-n/2}\exp(-u/2\tau)\}$
Cauchy	$C_n(\mu, \Lambda)$	$g(u) = c_3(1 + u)^{-(n+1)/2}$
Logistic	$L_n(\mu, \Lambda)$	$g(u) = c_4 \exp(-u)/\{1 + \exp(-u)\}^2$

where

$$W_g(u) = \frac{\mathrm{d}\{\log g(u)\}}{\mathrm{d}u} = \frac{g'(u)}{g(u)}.$$

It is easy to see that, for the normal and t-distributions, $u_g = n$. However, for the contaminated normal and logistic distributions, u_g must be obtained numerically. For the logistic distribution, for example, equation (5) becomes

$$\frac{n}{2u} = \tanh\left(\frac{u}{2}\right),$$

where $\tanh(\cdot)$ denotes the hyperbolic tangent.

3. Local influence on likelihood displacement

Let $L(\theta)$ denote the log-likelihood function from the postulated model (here $\theta = (\beta^{\mathrm{T}}, \phi)^{\mathrm{T}}$), and let ω be a $q \times 1$ vector of perturbations restricted to some open subset $\omega \in \mathbb{R}^q$. The perturbations are made on the likelihood function, such that it takes the form $L(\theta|\omega)$. Denoting the vector of no perturbation by ω_0, we assume that $L(\theta|\omega_0) = L(\theta)$. To assess the influence of the perturbations on the maximum likelihood estimate $\hat{\theta}$, we may consider the likelihood displacement

$$\mathrm{LD}(\omega) = 2\{L(\hat{\theta}) - L(\hat{\theta}_\omega)\},$$

where $\hat{\theta}_\omega$ denotes the maximum likelihood estimate under the model $L(\theta|\omega)$.

Small perturbations to the model may be important, especially when assessing whether the sample is robust with respect to the induced model. To assess this kind of robustness, Cook (1986) suggested studying the local influence around ω_0. The idea consists of studying the normal curvature of the surface $\alpha(\omega) = (\omega^{\mathrm{T}}, \mathrm{LD}(\omega))^{\mathrm{T}}$ and then taking the direction around ω_0 corresponding to the largest normal curvature.

Cook (1986) showed that the normal curvature in the direction l takes the form

$$C_l(\theta) = 2|l^{\mathrm{T}}\Delta^{\mathrm{T}}(\ddot{L})^{-1}\Delta l|, \tag{6}$$

where $\|l\| = 1$, $-\ddot{L}$ is the observed information matrix for the postulated model ($\omega = \omega_0$) and Δ is the $(p+1) \times q$ matrix with elements

$$\Delta_{ij} = \frac{\partial^2 L(\theta|\omega)}{\partial\theta_i \partial\omega_i},$$

evaluated at $\theta = \hat{\theta}$ and $\omega = \omega_0$, $i = 1, \ldots, p+1$ and $j = 1, \ldots, q$.

Therefore, the maximization of equation (6) is equivalent to finding the largest eigenvalue $C_{\max}$ of the matrix $B = \Delta^{\mathrm{T}}(\ddot{L})\Delta$, and the largest direction around ω_0, denoted by $l_{\max}$, is the corresponding eigenvector. If $C_{\max}$ is much greater than the remaining eigenvalues of B, the index plot for $l_{\max}$ may be helpful in assessing the influence of small perturbations on $\mathrm{LD}(\omega)$. Otherwise, it should be more informative to perform the index plot for the eigenvectors corresponding to the largest eigenvalues.

4. Curvature derivation for elliptical linear models

Firstly, we consider the unperturbed model. The log-likelihood function under the induced model is given by

$$L(\theta) = \log[|\phi I_n|^{-1/2}\, g\{(y - X\beta)^{\mathrm{T}}(\phi I_n)^{-1}(y - X\beta)\}]$$
$$= -\frac{n}{2}\log\phi + \log g(u),$$

where $u = Q(\beta)/\phi$ and $\theta = (\beta^{\mathrm{T}}, \phi)^{\mathrm{T}}$. Then, the first derivatives of $L(\theta)$ with respect to β and ϕ are

$$\frac{\partial L(\theta)}{\partial \beta} = -\frac{2}{\phi} W_g(u) X^{\mathrm{T}}(y - X\beta) \tag{7}$$

and

$$\frac{\partial L(\theta)}{\partial \phi} = -\frac{n}{2\phi} - \frac{1}{\phi^2} W_g(u)\, Q(\beta). \tag{8}$$

From equations (7) and (8) it follows, after some algebraic manipulation, that

$$\frac{\partial^2 L(\theta)}{\partial\beta\partial\beta^{\mathrm{T}}} = \frac{2}{\phi}\left\{ W_g(u)(X^{\mathrm{T}}X) + \frac{2}{\phi} W'_g(u) X^{\mathrm{T}}(y - X\beta)(y - X\beta)^{\mathrm{T}} X \right\},$$
$$\frac{\partial^2 L(\theta)}{\partial\phi\partial\beta^{\mathrm{T}}} = \frac{2}{\phi^3}\{W_g(u) + W'_g(u)\, Q(\beta)\}(y - X\beta)^{\mathrm{T}} X,$$
$$\frac{\partial^2 L(\theta)}{\partial\phi^2} = \frac{n}{2\phi^2} + \frac{Q(\beta)}{\phi^3}\left\{ 2W_g(u) + W'_g(u)\frac{Q(\beta)}{\phi} \right\},$$

where

$$W'_g(u) = \frac{\mathrm{d}W_g(u)}{\mathrm{d}u}.$$

Evaluating these derivatives at $\theta = \hat{\theta}$, given in equations (3), and by noting that $(y - X\hat{\beta})^{\mathrm{T}} X = 0$ and $Q(\hat{\beta})/\hat{\phi} = u_g$, we have

$$\ddot{L} = \begin{pmatrix} \frac{2}{\hat{\phi}} W_g(\hat{u})(X^{\mathrm{T}}X) & 0 \\ 0 & \frac{n}{2\hat{\phi}^2} + \frac{u_g}{\hat{\phi}^2}\{2W_g(\hat{u}) + u_g\, W'_g(\hat{u})\} \end{pmatrix},$$

where $\hat{u} = Q(\hat{\beta})/\hat{\phi} = u_g$.

Table 2 shows $W_g(u)$ and $W'_g(u)$ for the distributions given in Table 1, where

$$f_i(u) = (1 - \delta)\exp(-u/2) + \delta\tau^{-(n/2)-i}\exp(-u/2\tau), \qquad i = 0, 1, 2.$$

Note that for the normal case ($u_g = n$, $W_g(u) = -\frac{1}{2}$ and $W'_g(u) = 0$) the matrix $\ddot{L}$ reduces to

$$\ddot{L} = \begin{pmatrix} -(1/\hat{\phi})(X^{\mathrm{T}}X) & 0 \\ 0 & -n/2\hat{\phi}^2 \end{pmatrix}.$$

Consider now model (2), with the assumption that $\epsilon \sim \mathrm{El}_n\{0,\ \phi\, D^{-1}(\omega)\}$, where $D(\omega) = \mathrm{diag}(\omega_1, \ldots, \omega_n)$ and $D^{-1}(\omega)$ denotes the inverse of $D(\omega)$. Here $q = n$ and ω_i is the weight corresponding to the ith case, $i = 1, \ldots, n$. When $\omega = \omega_0 = 1$, the perturbed model reduces to the postulated model. Under the perturbed model we shall denote $Y \sim \mathrm{El}_n\{X\beta,\ \phi\, D^{-1}(\omega)\}$. Thus, the log-likelihood function is given by

$$L(\theta|\omega) = -\frac{n}{2}\log\phi + \tfrac{1}{2}\log|D(\omega)| + \log g(u_\omega),$$

TABLE 2
$W_g(u)$ and $W'_g(u)$ for some elliptical distributions

Distribution	$W_g(u)$	$W'_g(u)$
Normal	$-\frac{1}{2}$	0
Student t	$-\{(\nu+n)/2\nu\}(1+u/\nu)^{-1}$	$\{(n+\nu)/2\nu^2\}(1+u/\nu)^{-2}$
Cauchy	$-\{(1+n)/2\}(1+u)^{-1}$	$\{(1+n)/2\}(1+u)^{-2}$
Contaminated normal	$-\frac{1}{2}\frac{f_1(u)}{f_0(u)}$	$\frac{1}{4}\left[\frac{f_2(u)}{f_0(u)} - \left\{\frac{f_1(u)}{f_0(u)}\right\}^2\right]$
Logistic	$-\tanh(u/2)$	$-2/\{\exp(u/2)+\exp(-u/2)\}^2$

where $u_\omega = Q_\omega(\beta)/\phi$ and $Q_\omega = (y - X\beta)^{\mathrm{T}} D(\omega)(y - X\beta)$.

Following the same procedure as for the unperturbed case we have

$$\frac{\partial L(\theta|\omega)}{\partial \beta} = -\frac{2}{\phi} W_g(u_\omega)\{X^{\mathrm{T}} D(\omega) y - X^{\mathrm{T}} D(\omega) X\beta\},$$

$$\frac{\partial L(\theta|\omega)}{\partial \phi} = -\frac{n}{2\phi} - \frac{1}{\phi^2} W_g(u_\omega)\, Q_\omega(\beta).$$

From these equations we obtain

$$\frac{\partial^2 L(\theta|\omega)}{\partial \beta \partial \omega^{\mathrm{T}}} = -\frac{2}{\phi}\left\{ W_g(u_\omega) X^{\mathrm{T}} D(\epsilon) + \frac{1}{\phi} W'_g(u_\omega) X^{\mathrm{T}} D(\omega)\epsilon^{\mathrm{T}} D(\epsilon)\right\},$$

$$\frac{\partial^2 L(\theta|\omega)}{\partial \phi \partial \omega^{\mathrm{T}}} = -\frac{1}{\phi^2}\left\{ W_g(u_\omega)\epsilon^{\mathrm{T}} D(\epsilon) + \frac{1}{\phi} W'_g(u_\omega) Q_\omega(\beta)\epsilon^{\mathrm{T}} D(\epsilon)\right\},$$

since $\partial\{X^{\mathrm{T}} D(\omega)\epsilon\}/\partial\omega^{\mathrm{T}} = X^{\mathrm{T}} D(\omega)$ and $\partial u_\omega/\partial\omega^{\mathrm{T}} = (1/\phi)\epsilon^{\mathrm{T}} D(\epsilon)$, where $\epsilon = y - X\beta$. Evaluating the matrix Δ at $(\theta, \omega) = (\hat{\theta}, \omega_0)$ we find

$$\Delta = \begin{pmatrix} -\frac{2}{\hat{\phi}} W_g(\hat{u}) X^{\mathrm{T}} D(e) \\ -\frac{1}{\hat{\phi}^2}\{W_g(\hat{u}) + u_g W'_g(\hat{u})\} e^{\mathrm{T}} D(e) \end{pmatrix},$$

where $e = y - X\hat{\beta}$. For the normal linear case the matrix Δ reduces to

$$\Delta = \begin{pmatrix} X^{\mathrm{T}} D(e)/\hat{\phi} \\ e^{\mathrm{T}} D(e)/2\hat{\phi}^2 \end{pmatrix},$$

which is in agreement with the expression obtained by Cook (1986). Therefore, we may write

$$B = \Delta^{\mathrm{T}} \ddot{L}^{-1} \Delta = B_1 + B_2,$$

where

$$B_1 = \frac{2}{\hat{\phi}} W_g(\hat{u})\, D(e) P\, D(e)$$

and

$$B_2 = \frac{1}{\hat{\phi}^2} \frac{\{W_g(\hat{u}) + u_g\, W'_g(\hat{u})\}^2}{[(n/2) + u_g\{2\, W_g(\hat{u}) + u_g\, W'_g(\hat{u})\}]} D(e)ee^{\mathrm{T}} D(e),$$

with $P = X(X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}$. Also, for the normal linear case, the matrix B reduces to the matrix obtained by Cook (1986), equation (31). Then, the normal curvature in the direction l takes the form

$$C_l(\theta) = 2|l^{\mathrm{T}}(B_1 + B_2)l|.$$

In particular, if we are interested in the vector β, the normal curvature in the direction l yields

$$\begin{aligned} C_l(\beta) &= 2|l^{\mathrm{T}} B_1 l| \\ &= \frac{4}{\hat{\phi}} |W_g(\hat{u}) l^{\mathrm{T}} D(e) P\, D(e) l| \\ &= \frac{4}{\hat{\phi}} |W_g(\hat{u})|\, |l^{\mathrm{T}} D(e) P\, D(e) l|. \end{aligned}$$

Thus, the index plot for $l_{\max}$ of the matrix $D(e)P\,D(e)$ may show how to perturb the scale parameter to obtain larger changes in the regression coefficients.

For a particular coefficient, namely β_1, rearranging the columns of $X = (X_1, X_2)$, such that X_1 and X_2 are matrices with dimensions $n \times 1$ and $n \times (p-1)$ respectively, it follows from expression (33) of Cook (1986) that

$$C_l(\beta_1) = \frac{4|W_g(\hat{u})|\, |l^{\mathrm{T}} D(e) rr^{\mathrm{T}} D(e) l|}{\|r\|^2 \hat{\phi}},$$

where

$$\begin{aligned} r &= (I - P_2)X_1, \\ P_2 &= X_2(X_2^{\mathrm{T}} X_2)^{-1} X_2^{\mathrm{T}} \end{aligned}$$

and $\|a\|$ denotes the norm of the vector **a**. Thus, the maximum curvature occurs in the direction

$$l_{\max} \propto D(e) r.$$

Accordingly, the cases with $|r_i e_i|$ large are locally most influential on the estimate $\hat{\beta}_1$.

Similarly, the normal curvature for the scale parameter ϕ in the direction l is given by

$$\begin{aligned} C_l(\phi) &= 2|l^{\mathrm{T}} B_2 l| \\ &= \frac{2}{\hat{\phi}^2} |C_\omega|\, |l^{\mathrm{T}} D(e) ee^{\mathrm{T}} D(e) l|, \end{aligned}$$

where

$$C_\omega = \frac{\{W_g(\hat{u}) + u_g\, W'_g(\hat{u})\}^2}{n/2 + u_g\{2\, W_g(\hat{u}) + u_g\, W'_g(\hat{u})\}}.$$

Here, for the largest curvature,

$$l_{\max} \propto D(e) e,$$

which means that the observations with large values for e_i^2 are most influential on $\hat{\phi}$.

Therefore, at least for the perturbation scheme defined in Section 4 and for the likelihood displacement, we may conclude that the diagnostics for the elliptical linear models are equivalent to those deduced by Cook (1986) for the normal linear model when β and ϕ are treated separately, i.e. the index plots do not change with the induced model in the elliptical linear family. However, if β and ϕ are treated jointly, the l_{max}-vector may change from one model to another, which suggests a helpful way of discovering those observations that are most locally influential under each model.

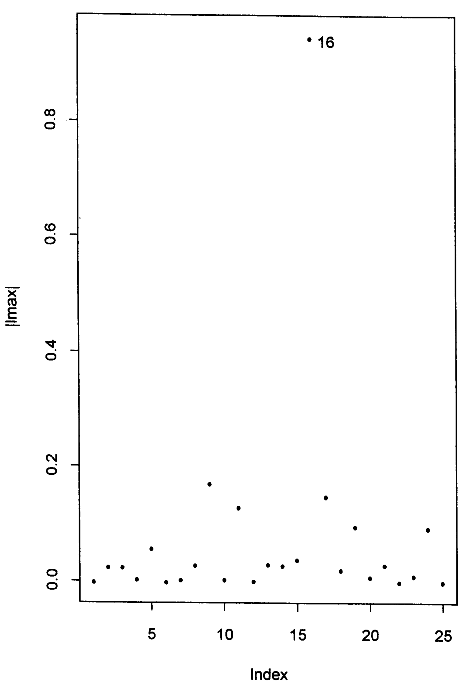
5. Water salinity

To illustrate the methodology described in this paper we consider the data set reported by Ruppert and Carroll (1980) on the salinity of water during the spring in Pamlico Sound, North Carolina. The response Y is biweekly salinity, and the explanatory variables are salinity lagged 2 weeks, x_1, a dummy variable x_2 for the time period and river discharge, x_3. The value of x_{1i} may differ from y_{i-1}, once the data are not a contiguous sequence. This data set has been analysed, for instance, by Atkinson (1985), Carroll and Ruppert (1985) and Davison and Tsai (1992). Atkinson (1985) assumed a normal distribution for the response whereas Davison and Tsai (1992) considered a Student t-distribution with 3 degrees of freedom to allow for the possibility that the data have tails that are longer than for the normal distribution. In both cases, the linear model

$$Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \epsilon_i$$

is assumed, where ϵ_i follows either a normal or a Student t-distribution with 3 degrees of freedom.

Atkinson (1985) and Davison and Tsai (1992) used deletion diagnostic methods to assess the individual influence of the observations on $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)^{\mathrm{T}}$ and $\hat{\phi}$. Atkinson, assuming normally distributed errors, found cases 16 and 5 to be the most influential. Case 5 was shown



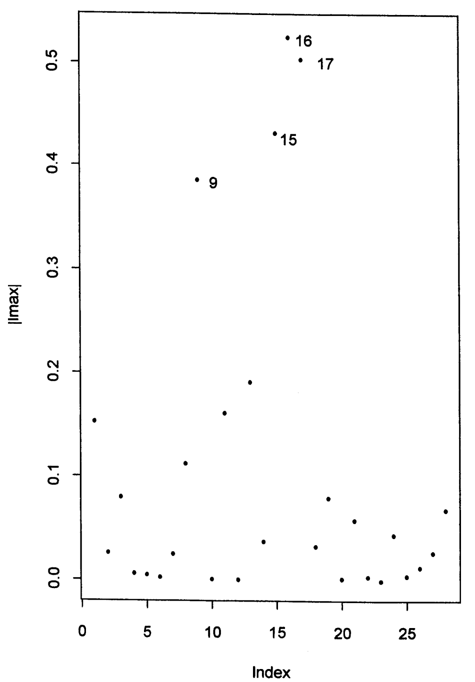
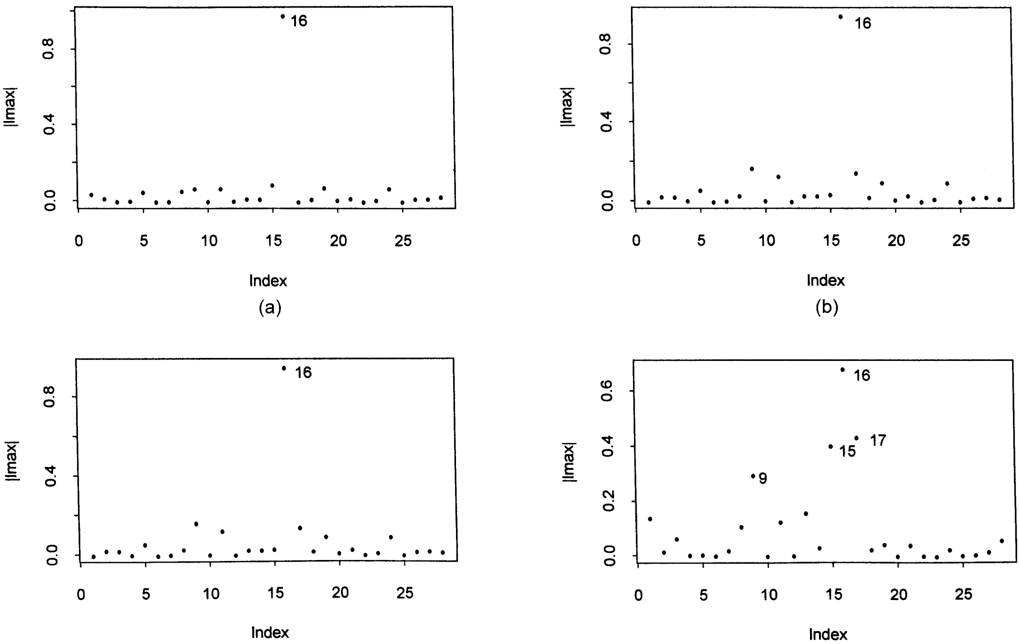


Fig. 1. Index plots of $|l_{max}|$ for (a) $\hat{\beta}$ and (b) $\hat{\phi}$



(a) (b) (c) (d)

Fig. 2. Index plots of $|l_{\max}|$ for $\hat{\theta}$ for (a) the normal, (b) the Cauchy, (c) the Student t- (with 3 degrees of freedom) and (d) the logistic elliptical linear models

to be influential after a correction was made for case 16. In Davison and Tsai's analysis, where a Student t-distribution with 3 degrees of freedom was used, cases 16, 5 and 3 appear the most influential. Fig. 1 presents the index plot of $|l_{\max}|$ for $\hat{\beta}$ and $\hat{\phi}$ separately. We see in Fig. 1(a) outstanding local influence for case 16, whereas in Fig. 1(b) it follows that cases 9, 15, 16 and 17 present the highest local influences. In contrast, when we use the global log-likelihood $L(\theta)$ in Cook's approach instead of the profiles $L(\beta|\phi)$ or $L(\phi|\beta)$, the local influence of the observations on $\hat{\theta}$ is no longer invariant in the elliptical linear family. Fig. 2 illustrates this behaviour. Moreover, Fig. 2 shows that case 16 is the most locally influential for the normal, Cauchy and Student t-models. However, for the logistic model, cases 9, 15 and 17 also appear with a high local influence. Therefore, we may conclude from this example that the index plot of $|l_{\max}|$ for $\hat{\theta}$ may be helpful in selecting the less sensitive model with respect to local perturbations in the elliptical linear family, especially when we are interested in both $\hat{\beta}$ and $\hat{\phi}$.

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