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**Part I**  
**Introduction and Preliminaries**

# 1

## Introduction

### 1.1 *Cash Flow Streams*

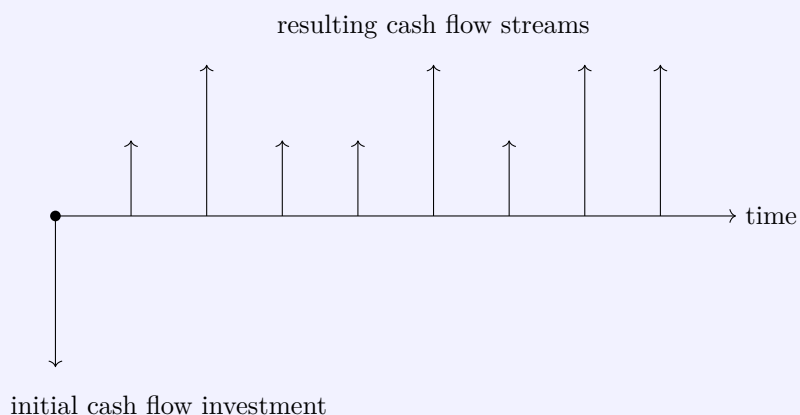
#### Definition of Investments

#### Definition 1.1.1

An **investment** is the **current commitment of resources** in order to achieve later **benefits**. Basically, investing money right now in hopes of earning a return.

Sometimes this amount of money to be obtained later is uncertain.

We can generalise this to say that an investment is defined by the terms of **its resulting cash flow stream**.



We then have the following questions:

- How do I choose a preferred cash flow streams, and how much should I be willing to pay for one?
- Are two streams together worth more to me than the sum of their individual values?
- Given a collection of several streams, what's the most favourable combination of them?

Sometimes the timing and amount of cash flows are not fixed, and can be influenced by the investor. Investment science answers these questions by **determining suitable management strategies to tailor cash flow streams**.

### 1.2 *Investments and the Market*

#### Investment Analysis

#### Definition 1.2.1

The process of examining alternatives and deciding which alternative is the most preferable.

Investment problems are a unique class of decision problems that are carried out within the framework of the financial market, which provides a basis for comparison.

There are several important aspects of the financial market:

- **The Comparison Principle:**

- The comparison principle states that if two alternatives are equivalent, then they are equally preferable.

- **Arbitrage:**

- Arbitrage is the act of taking a risk-free profit on a trade.
- A simple example: In New York, 1 USD = 0.85 EUR. In London, 1 USD = 0.83 EUR. If I had USD, I could buy EUR in New York and convert it back to USD in London to earn 0.02 EUR per USD.

This is a very simple example and almost always, there would be no price discrepancy across exchanges.

- **Dynamics:**

- Dynamics refers to the forces and processes that cause changes within financial systems or markets over time.
- Such changes can be caused by economic events, government policies, technological advancements, and market sentiment.

- **Risk Aversion:**

- Investors are generally risk-averse and will only take on risk if they are compensated for it. So an investor only makes an investment if there is an expected return (that is greater than the risk-free rate, or the interest paid if one was to save the money instead of investing it).

### 1.3 Pricing Coin Tosses

You pay £1, I flip a fair coin.

- If heads, you get £3.
- If tails, you get nothing.
- The value of this game is  $£3 \times \frac{1}{2} - 1 = £0.50$ .

You pay £1, I flip a fair coin.

- If heads, you get £1.
- If tails, you get £1.
- The value of this game is  $£1 \times \frac{1}{2} + £1 \times \frac{1}{2} - 1 = £0$ .

I flip a fair coin twice.

- If at least one flip is heads, you get £9.
- Else, you get nothing.
- Out of the four equally likely outcomes (HH, HT, TH, TT) with probability  $1/4$  each, three outcomes are possible, so the value of this game is  $£9 \times \frac{3}{4} = £6.75$ .

# 2

## Mathematical Preliminaries

### 2.1 Functions

There are certain functions commonly used in finance:

- **Exponential functions:**  $f(x) = ac^{bx}$  where  $a$ ,  $b$ , and  $c$  are constants. Very often  $c$  is  $e = 2.7182818\dots$
- **Logarithmic functions:** the natural logarithm is the function denoted by  $\ln(\cdot)$  which satisfies  $e^{\ln(x)} = x$ .
- **Linear functions:** a function  $f$  of several variables  $x_1, x_2, \dots, x_n$  is linear if it has the form

$$f(x_1, x_2, \dots, x_n) = a_1x_1 + a_2x_2 + \dots + a_nx_n.$$

- **Inverse functions:** a function  $f$  has an inverse function  $g$  if for all  $x$  we have  $g(f(x)) = x$ . Inverse functions are usually denoted by  $f^{-1}$ .

### 2.2 Differential Calculus

One should be familiar with the following concepts:

- Limits
- Derivatives
- Product, Quotient, Chain rules
- Higher order derivatives like  $f, f', f'', \dots$
- Partial Derivatives

$$\frac{\partial f(x_1, x_2, \dots, x_n)}{\partial x_i} = \lim_{\Delta x \rightarrow 0} \frac{f(x_1, x_2, \dots, x_i + \Delta x, \dots, x_n) - f(x_1, x_2, \dots, x_n)}{\Delta x}$$

- Taylor Approximation: approximation of a function  $f$  in a region near point  $x$ .

- $f(x + \Delta x) = f(x) + f'(x)\Delta x + O(\Delta x^2)$
- $f(x + \Delta x) = f(x) + f'(x)\Delta x + \frac{1}{2}f''(x)\Delta x^2 + O(\Delta x^3)$

where  $O(\Delta x^2)$  and  $O(\Delta x^3)$  denote terms of order  $(\Delta x^2)$  and  $(\Delta x^3)$  respectively.

- Taylor Approximation for multivariable functions:

- A function  $f : \mathbb{R}^n \Rightarrow \mathbb{R}$  can be approximated in a region near point  $(x_1, \dots, x_n)$  by partial derivatives.

$$\begin{aligned} f(x_1 + \Delta x_1, x_2 + \Delta x_2, \dots, x_n + \Delta x_n) = & f(x_1, x_2, \dots, x_n) \\ & + \sum_{i=1}^n \frac{\partial f}{\partial x_i} \Delta x_i \\ & + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 f}{\partial x_i \partial x_j} \Delta x_i \Delta x_j \\ & + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n O(\Delta x_i \Delta x_j \Delta x_k) \end{aligned}$$



## 2.3 Optimisation

- **Necessary conditions:** a function  $f$  of a single variable  $x$  is said to have a maximum at point  $x_0$  if  $f(x_0) \geq f(x)$  for all  $x$ . If  $x_0$  is not a boundary point of an interval over which  $f$  is defined, then for  $x_0$  to be a maximum, then  $f'(x_0) = 0$ .

- **Constrained Optimisation with Lagrange Multipliers:**

- Considering maximising multivariable function  $f$  of several variables  $x_1, x_2, \dots, x_n$  required to satisfy constraint  $g(x_1, x_2, \dots, x_n) = 0$ .

$$\max_x f(x_1, x_2, \dots, x_n) \quad \text{subject to} \quad g(x_1, x_2, \dots, x_n) = 0$$

- Introduce a Lagrange multiplier  $\lambda$  to form the Lagrangian function

$$L(x_1, x_2, \dots, x_n, \lambda) = f(x_1, x_2, \dots, x_n) - \lambda g(x_1, x_2, \dots, x_n)$$

According to the Lagrangian Method, we set the partial derivatives of the Lagrangian w.r.t to each variable to zero, leaving us with a system of  $n + 1$  equations for  $n + 1$  unknowns  $x_1, x_2, \dots, x_n, \lambda$ .

$$\frac{\partial L}{\partial x_i} = 0 \quad \text{where } i = 1..n \quad \text{and} \quad \frac{\partial L}{\partial \lambda} = 0$$

- A problem with two constraints involves two Lagrange multipliers  $\mu$  and  $\lambda$ .

$$\max_x f(x_1, x_2, \dots, x_n) \quad \text{subject to} \quad g(x_1, x_2, \dots, x_n) = 0 \quad \text{and} \quad h(x_1, x_2, \dots, x_n) = 0$$

$$L = f(x_1, x_2, \dots, x_n) - \mu g(x_1, x_2, \dots, x_n) - \lambda h(x_1, x_2, \dots, x_n)$$

- More generally, a problem with  $n$  variables and  $m$  constraints is assigned  $m$  Lagrange multipliers, and the function has  $n + m$  arguments, and setting all partial derivatives to zero gives  $n + m$  equations for  $n + m$  unknowns.
- \* Sometimes we have inequality constraints in the form  $g(x_1, x_2, \dots, x_n) \leq 0$ . We have two cases:
  1. If  $g(x_1, x_2, \dots, x_n) < 0$  at the optimum, the constraint is not active and can be dropped, therefore this constraint is non-binding does not require the Lagrange multiplier.
  2. If  $g(x_1, x_2, \dots, x_n) = 0$  at the optimum, the constraint is active and requires the Lagrange multiplier which is non-negative.

## 2.4 Random Variables

- A **discrete random variable**  $x$  has finite possible values  $x_1, x_2, \dots, x_m$  with probabilities  $p_1, p_2, \dots, p_m$ :

$$p_i = \text{prob}(x = x_i) \quad \text{for any } i = 1, 2, \dots, m.$$

The probabilities are nonnegative and sum to unity:

$$\sum_{i=1}^m p_i = 1.$$

- A **continuous random variable**  $x$  is described by a probability density function  $p(\xi)$ :

$$\int_a^b p(\xi) d\xi = \text{prob}(a \leq x \leq b) \quad \text{for any } a < b.$$

The density function is nonnegative and integrates to unity:

$$\int_{-\infty}^{+\infty} p(\xi) d\xi = 1.$$

## 2.5 Probability Distribution

- The **probability distribution** of a random variable  $x$  is the function  $F(\xi)$ :

$$F(\xi) = \text{prob}(x \leq \xi).$$

- It follows that:

$$F(-\infty) = 0, \quad F(+\infty) = 1,$$

$F$  is monotonically increasing.

- If  $x$  is a continuous random variable:

$$F(\xi) = \int_{-\infty}^{\xi} p(\xi') d\xi' \quad \Rightarrow \quad \frac{dF(\xi)}{d\xi} = p(\xi).$$

## 2.6 Dependent Random Variables

- Two **discrete random variables**  $x$  and  $y$  are described by their possible pairs of values  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  and the corresponding probabilities  $p_1, p_2, \dots, p_n$  with the interpretation

$$p_i = \text{prob}(x = x_i \wedge y = y_i).$$

- Two **continuous random variables**  $x$  and  $y$  are described by their joint probability density function  $p(\xi, \eta)$  with the interpretation

$$\int_{a_x}^{b_x} \int_{a_y}^{b_y} p(\xi, \eta) d\eta d\xi = \text{prob}(a_x \leq x \leq b_x \wedge a_y \leq y \leq b_y).$$

- The **joint probability distribution**  $F$  is defined as

$$F(\xi, \eta) = \text{prob}(x \leq \xi, y \leq \eta).$$

- From a joint distribution, the distribution of any of the random variables can easily be recovered:

$$F_x(\xi) = F(\xi, \infty); \quad F_y(\eta) = F(\infty, \eta).$$

- In general,  $n$  random variables are defined by their joint probability distribution defined with respect to  $n$  variables.

## 2.7 Independent Random Variables

- Two **discrete random variables**  $x$  and  $y$  are independent if the possible joint values can be written as  $(x_i, y_j)$  for  $i = 1, 2, \dots, n_x$  and  $j = 1, 2, \dots, n_y$ , while the probability  $p_{ij}$  of outcome  $(x_i, y_j)$  factors into the form

$$p_{ij} = p_{x,i} p_{y,j}.$$

- Two **continuous random variables**  $x$  and  $y$  are independent if the joint density function factors into the form

$$p(\xi, \eta) = p_x(\xi) p_y(\eta).$$

- Example:** The pair of random variables defined as the outcomes on two fair tosses of a die are independent. The probability of obtaining the pair  $(3, 5)$ , say, is

$$\frac{1}{6} \times \frac{1}{6}.$$

- The **expected value** or expectation of a random variable  $x$  is defined as

$$E(x) = \sum_{i=1}^n x_i p_i \text{ if } x \text{ is a discrete r.v.};$$

$$E(x) = \int_{-\infty}^{+\infty} \xi p(\xi) d\xi \text{ if } x \text{ is a continuous r.v.}$$

- The concept of an expectation can be **generalized**. For any function  $f : \mathbb{R} \rightarrow \mathbb{R}$ , we can define

$$E[f(x)] = \sum_{i=1}^n f(x_i) p_i \text{ if } x \text{ is a discrete r.v.};$$

$$E[f(x)] = \int_{-\infty}^{+\infty} f(\xi) p(\xi) d\xi \text{ if } x \text{ is a continuous r.v.}$$

- The **moment** of order  $m$  of any random variable  $x$  is defined as  $E(x^m)$ .

## 2.8 Variance and Standard Deviation

- The **variance** of a r.v.  $x$  is defined as

$$\text{var}(x) = E[(x - E(x))^2].$$

- One easily verifies the identity:

$$\text{var}(x) = E(x^2) - E(x)^2.$$

- Loosely, the **expectation** tells you the 'typical' or 'average' value of a r.v., while the **variance** gives the amount of 'variation' around this value.

- The **standard deviation** of a r.v. is defined as

$$\text{std}(x) = \sqrt{\text{var}(x)}.$$

## 2.9 Generalised Expectation

- The concept of an **expectation** can be further generalised to situations in which there are two dependent random variables  $x$  and  $y$ . For any function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ , we can define

$$E[f(x, y)] = \sum_{i=1}^n f(x_i, y_i) p_i \text{ if } x \text{ and } y \text{ are discrete dependent random variables};$$

$$E[f(x, y)] = \int_{\mathbb{R}^2} f(\xi, \eta) p(\xi, \eta) d\xi d\eta \text{ if } x \text{ and } y \text{ are continuous dependent random variables.}$$

- Expectations of functions of  $n$  random variables are defined analogously. <sup>1</sup> <sup>1</sup> Analogously = simultaneously.

## 2.10 Covariances and Correlations

- The **covariance** of two dependent random variables  $x$  and  $y$  is defined as

$$\text{cov}(x, y) = E[(x - E(x))(y - E(y))].$$

- Note that  $\text{cov}(x, x) = \text{var}(x)$ .

- The **correlation** of  $x$  and  $y$  is defined as

$$\varrho(x, y) = \frac{\text{cov}(x, y)}{\text{std}(x)\text{std}(y)}.$$

- If  $x$  and  $y$  are independent, then

$$\text{cov}(x, y) = E[x - E(x)]E[y - E(y)] = 0 \Rightarrow \rho(x, y) = 0.$$

- By the **Cauchy-Schwartz inequality**, we find

$$|\text{cov}(x, y)| \leq E(|x - E(x)| |y - E(y)|) \leq \sqrt{E[(x - E(x))^2] E[(y - E(y))^2]} = \text{std}(x) \text{std}(y).$$

- The correlation  $\rho(x, y)$  is always between  $-1$  and  $+1$ .
- Two random variables  $x$  and  $y$  are said to be:
  - positively correlated if  $\rho(x, y) > 0$ ;
  - perfectly positively correlated if  $\rho(x, y) = 1$ ;
  - negatively correlated if  $\rho(x, y) < 0$ ;
  - perfectly negatively correlated if  $\rho(x, y) = -1$ ;
  - uncorrelated if  $\rho(x, y) = 0$ .
- A random variable  $x$  is **perfectly positively correlated** with the random variable  $y = ax + b$  for any  $a, b \in \mathbb{R}$  such that  $a > 0$ .
- A random variable  $x$  is **perfectly negatively correlated** with the random variable  $y = ax + b$  for any  $a, b \in \mathbb{R}$  such that  $a < 0$ .
- Note that if  $x$  and  $y$  are **independent**, then they are **uncorrelated**. However, if  $x$  and  $y$  are uncorrelated, then they are not necessarily independent.<sup>2</sup>
- Let  $x$  and  $y$  be two dependent random variables, and let  $\alpha$  and  $\beta$  be real numbers. Then

$$E(\alpha x + \beta y) = \alpha E(x) + \beta E(y),$$

$$\text{var}(\alpha x + \beta y) = \alpha^2 \text{var}(x) + 2\alpha\beta \text{cov}(x, y) + \beta^2 \text{var}(y).$$

- Let  $x_1, x_2, \dots, x_n$  be  $n$  dependent random variables. The **covariance matrix** of these random variables is defined as the  $n \times n$ -matrix  $V$  with entries

$$V_{ij} = \text{cov}(x_i, x_j) \quad \text{for } i, j = 1, \dots, n.$$

- If  $\alpha_1, \alpha_2, \dots, \alpha_n$  are  $n$  real numbers, then

$$E\left(\sum_{i=1}^n \alpha_i x_i\right) = \sum_{i=1}^n \alpha_i E(x_i)$$

and

$$\text{var}\left(\sum_{i=1}^n \alpha_i x_i\right) = \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j V_{ij}.$$

<sup>2</sup> Example: Take  $y = x^2$  and  $x \sim N(0, 1)$ .  $x$  and  $y$  are dependent as they can be described by  $y = x^2$ .

Then,  $E[x] = 0$  by definition and  $E[x^2] = 1$ , then  $E[y] = 0$ .

$E[x^3] = 0$  by symmetry about zero.

So  $\text{cov}(x, y) = E[xy] - E[x]E[y] \Rightarrow E[x^3] - E[x]E[y] = 0$ . So the result shows that  $x$  and  $y$  are uncorrelated, but dependent.

## 2.11 Uniform Random Variables

- A continuous random variable  $x$  with density function

$$p(\xi) = \begin{cases} (\beta - \alpha)^{-1} & \text{for } \alpha \leq \xi \leq \beta, \\ 0 & \text{otherwise,} \end{cases}$$

is said to have a **uniform distribution** over  $[\alpha, \beta]$ .

- $x$  takes only values between  $\alpha$  and  $\beta$  and is equally likely to take any such value.

- The **uniform distribution function** is given by

$$F(x) = \begin{cases} 0 & \text{for } x < \alpha, \\ \frac{x-\alpha}{\beta-\alpha} & \text{for } \alpha \leq x \leq \beta, \\ 1 & \text{for } x > \beta. \end{cases}$$

- $E(x) = \frac{\beta+\alpha}{2}$  and  $\text{var}(x) = \frac{(\beta-\alpha)^2}{12}$ .

## 2.12 Normal Random Variables

- A (continuous) random variable  $x$  is said to be **normal** or **Gaussian** if its probability density function is of the form

$$p(\xi) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(\xi-\mu)^2}.$$

- It follows that  $E(x) = \mu$  and  $\text{var}(x) = \sigma^2$ .
- A normal r.v. is said to be **standard** if  $\mu = 0$  and  $\sigma = 1$ .
- A **standard normal random variable** has density

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2},$$

and the **standard normal distribution**  $N$  is given by

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}\xi^2} d\xi.$$

- There is no analytic expression for  $N(x)$ , but tables of its values are available.
- Let  $x = (x_1, x_2, \dots, x_n)$  be a **vector of  $n$  normal random variables**. We introduce the vector  $\bar{x}$  whose components are the expected values of the components in  $x$ . The covariance matrix  $V$  associated with  $x$  can be written as

$$V = E[(x - \bar{x})(x - \bar{x})^\top].$$

- If the  $n$  variables are **jointly normal**, the density of  $x$  is

$$p(x) = \frac{1}{(2\pi)^{n/2} \det(V)^{1/2}} e^{-\frac{1}{2}(x-\bar{x})^\top V^{-1}(x-\bar{x})}.$$

- If  $n$  jointly normal random variables are **uncorrelated**, then the covariance matrix  $V$  is diagonal  $\Rightarrow$  the joint density function factors into a product of densities for the  $n$  separate variables.
- $\Rightarrow$  If  $n$  jointly normal random variables are uncorrelated, then they are independent.
- **Summation property**: if  $x$  and  $y$  are jointly normal random variables and  $\alpha, \beta \in \mathbb{R}$ , then  $\alpha x + \beta y$  is normal.
- **Generalisation**: if  $x$  is a vector of  $n$  jointly normal r.v.s and  $T$  is an  $m \times n$ -matrix, then  $Tx$  is a vector of  $m$  jointly normal r.v.s.
- To express that  $x$  is a normal r.v. with expected value  $\mu$  and variance  $\sigma^2$  we use the shorthand notation:

$$x \sim \mathcal{N}(\mu, \sigma^2).$$

- To express that  $x$  is a vector of jointly normal r.v.s with expected values  $\bar{x}$  and covariance matrix  $V$  we write:

$$x \sim \mathcal{N}(\bar{x}, V).$$

- Some useful properties of normal r.v.s are:
  - if  $x \sim \mathcal{N}(\mu, \sigma^2)$ , then  $(x - \mu)/\sigma \sim \mathcal{N}(0, 1)$ ;
  - if  $y \sim \mathcal{N}(0, 1)$ , then  $\sigma y + \mu \sim \mathcal{N}(\mu, \sigma^2)$ ;
  - if  $x_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ ,  $x_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$  and  $x_1$  and  $x_2$  are independent, then  $x_1 + x_2 \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ ;

## 2.13 Central Limit Theorem

- Let  $x_1, x_2, x_3, \dots$  be an infinite sequence of **independent, identically distributed (i.i.d.)** random variables, each with expected value  $\mu$  and variance  $\sigma^2$ .
- Define  $S_n = \sum_{i=1}^n x_i$  for  $n = 1, 2, 3, \dots$ . Note that  $E(S_n) = n\mu$  and  $\text{var}(S_n) = n\sigma^2$ .
- The **Central Limit Theorem** says that **for large  $n$**  the random variable  $(S_n - n\mu)/(\sigma\sqrt{n})$  is approximately **standard normally distributed**. In mathematical terms:

$$\text{prob} \left( \frac{S_n - n\mu}{\sigma\sqrt{n}} \leq x \right) \rightarrow N(x) \quad \text{as } n \rightarrow \infty \ (\forall x \in \mathbb{R}).$$

- Real-life systems are **subject to a range of external influences** that can be reasonably **approximated by i.i.d. random variables**.
- Hence, by the **C.L.T.** the **overall effect** can be reasonably modelled by a **single normal random variable** with appropriate mean and variance.
- $\Rightarrow$  Because of the **C.L.T.** normal random variables are ubiquitous in stochastic modelling! <sup>3</sup>

<sup>3</sup> The normal distribution has convenient mathematical properties, such as being completely described by its mean and variance. As a result, normal distributions are often used to approximate more complex distributions in stochastic models.

# 3

## Basic Theory Of Interest

### Further Reading

### Reference 3.0.1

D.G. Luenberger: Investment Science, Chapters 2,4

### 3.1 *Principal and Interest*

#### Interest

#### Example Q 3.1.1

If you invest \$1.00 in a bank account paying 8% interest per year, at the end of the year, you will have \$1.08.

#### Terminology

#### Definition 3.1.1

- **Principal:** The initial investment ( $W$ ).
- **Interest:** The rent paid on investment ( $I$ ).
- **Interest Rate:** Interest rate per unit of currency invested ( $r$ ).
- $\Rightarrow I = W \times r$
- **Initial Wealth (today):**  $W - 0 = W$
- **Terminal Wealth (after one year):**  $W_1 = W(1 + r)$

### 3.2 *Compound Interest*

Consider a situation in which money is invested in a bank account over **several periods**. Assume that the **interest rate** in the  $n$ th year is  $r_n$  for  $n = 1, 2, 3, \dots$ . We obtain the following **account holdings**:

- today:  $W_0 = W$ ;
- after 1 year:  $W_1 = W(1 + r_1)$ ;
- after 2 years:  
$$W_2 = W_1(1 + r_2) = W(1 + r_1)(1 + r_2);$$
- after  $n$  years:

$$W_n = W_{n-1}(1 + r_n) = W \prod_{i=1}^n (1 + r_i).$$

If the **interest rate is constant**, i.e.,  $r_n = r$ , then

$$W_n = W(1 + r)^n \quad \Rightarrow \quad r = \left( \frac{W_n}{W_0} \right)^{1/n} - 1.$$

- The **seven-ten rule**:
  - Money invested at **7%** doubles in about **10 years**;
  - Money invested at **10%** doubles in about **7 years** (Figure 3.1).

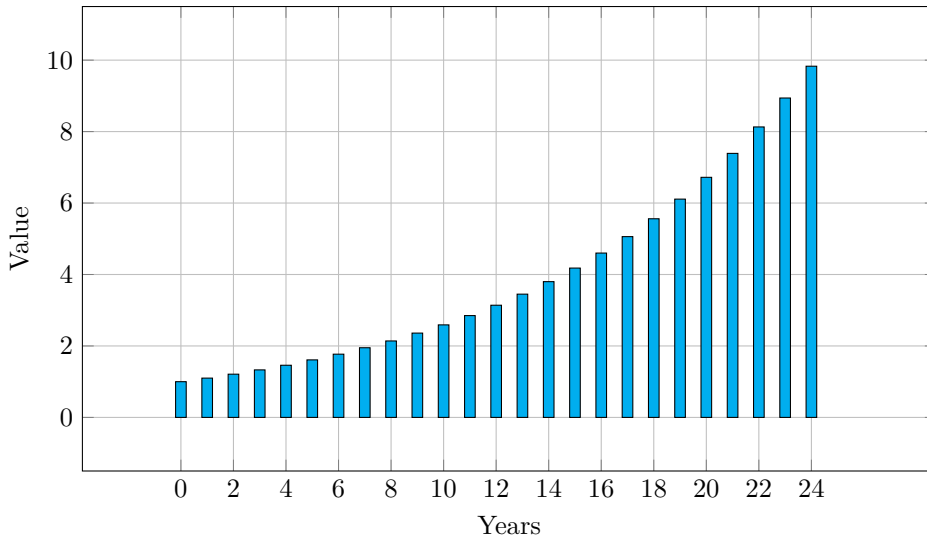


Figure 3.1: Money doubling over time with compound interest.

### 3.3 Compounding at Various Intervals

It is traditional to quote the interest rate on a yearly basis but adjust the proportion of that interest rate over each compounding period. Divide a year into  $m$  equally-spaced compounding periods.

- **Nominal Interest Rate (annual):**  $r$
- **Length of a compounding period :**  $\frac{1}{m}$  years
- **Interest Rate for each of the  $m$  periods:**  $r \frac{1}{m}$
- **Growth of account over  $k$  compounding periods:**  $(1 + \frac{r}{m})^k$
- **Growth of account over 1 year:**  $(1 + \frac{r}{m})^m$
- **The effective interest rate is the number  $r_{\text{eff}}$  such that  $(1 + r_{\text{eff}})^m = (1 + \frac{r}{m})^m$**

### 3.4 Continuous Compounding

Notice that increasing the number of compounding periods  $m$  to infinity gives us the exponential function. If we measure time in years as  $t$ , we have:

$$\lim_{m \rightarrow \infty} \left(1 + \frac{r}{m}\right)^{mt} \rightarrow e^{rt} \quad (3.1)$$

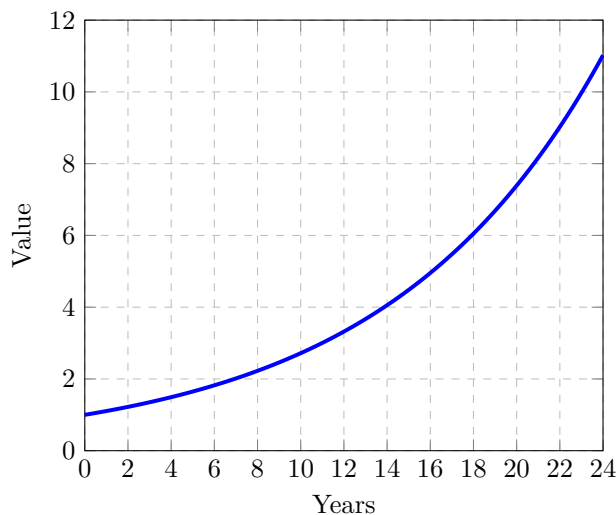


Figure 3.2: Exponential growth. For continuous compounding at 10% the value of \$1 doubles in about 7 years, and grows 8 times in about 20 years.



### 3.5 Time Value of Money

#### Debt

#### Definition 3.5.1

Money has a time value, and is worth more in the present than the future. In other words, money invested/borrowed today leads to increased value/debt in the future as a result of interest.

If I make a deposit in the bank (meaning that I only need it in the future, not the present), it grows over time due to interest compounding.

Similarly, if I borrow from the bank at an interest rate  $r$  (meaning I need money now that I can only pay it back in the future), my debt to the bank increases over time following the the compounding of interest.

#### Present Value

#### Definition 3.5.2

Suppose that the annual interest rate  $r$  is compounded  $m$  times per year.

The final amount attained is denoted as  $A$  which is in the future. This amount can also be equivalently represented as an amount  $d_k A$  today, which is  $A$  discounted to the present, where

$$d_k = \frac{1}{(1 + r/m)^k} < 1$$

$d_k$  is the discount factor, and  $d_k A$  is the present value of  $A$ .

#### The Ideal Bank

#### Definition 3.5.3

An **ideal bank**, for the sake of modelling purposes:

- Applies the same interest rate to deposits and limitations
- Assumes no additional service charges or transaction customizations
- Has a constant interest rate regardless of the size of principal (amount of money loaned or deposited)

In practice, different transactions may have different rates based on their duration. For example, a 2-year certificate of deposit (CD) might offer a higher rate than a 1-year CD.

A **constant ideal bank** has a constant interest rate that is independent of duration.

A certificate of deposit is a promise for the bank to pay interest on money deposited for a fixed period. Banks generally compensate a longer lock-in period with higher interest rates. This is because a longer duration of lock-in period increases the uncertainty regarding future interest rate movements and inflation.

For example, in a volatile period of time in the economy, locking-in a deposit at a fixed rate for 2 years might mean you could lose out on better rates if interest rates increase next year. But also, you could also keep a better rate if interest rates decrease next year.

### 3.6 Future and Present Value of Streams

- Consider a **cash flow stream**  $x_0, x_1, x_2, \dots, x_n$ .
- $x_k$  occurs at the end of period  $k$ .
- We can use a **constant ideal bank** to move all cash flows to the end of period  $n$  or to the **present time**.

**Future Value****Definition 3.6.1**

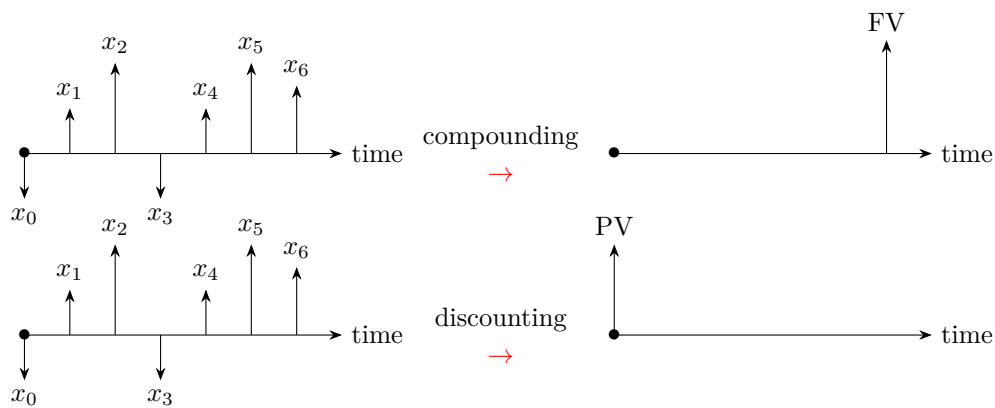
The **future value** of the stream is

$$\text{FV} = \sum_{k=0}^n x_k (1 + r/m)^{n-k} \quad \leftarrow \text{'compounding'}$$

**Present Value****Definition 3.6.2**

The **present value** of the stream is

$$\text{PV} = \sum_{k=0}^n \frac{x_k}{(1 + r/m)^k} \quad \leftarrow \text{'discounting'}$$



### 3.7 Present Value and an Ideal Bank

# 4

## Lookbook

Theorem 4.0.1 Theorem Name

This is the statement of the theorem.

Corollary 4.0.1 Corollary Name

This is the statement of the corollary.

Lemma 4.0.1 Lemma Name

This is the statement of the lemma.

Claim 4.0.1 Claim Name

This is the statement of the claim.

Example 4.0.1 (Example Name)

This is the explanation of the example.

Note 4.0.1 Side Note Box

This is a side note.

This is a block of highlighted text

Definition Title

Definition 4.0.2

This is an example definition.

Extra Title

Non-Examinable 4.0.2

This is an example box with extra information.

Example Title

Example Q 4.0.2

This is an example question.

Answer here

Q1c - 2018

Exam Q 4.0.2

This is an example exam question.

Reference Title

Reference 4.0.2

This is an example reference to source material.

Note 4.0.1 Side Note Box

This is a smaller side note.

This is a block of highlighted text that's smaller.

Small Def Title

Example Text

Definition 4.0.1

Small Title

Example Text

Non-Examinable 4.0.1

Small Title

Example Text

Answer

Example Q 4.0.1

Q1c - 2018

This is an example exam question, smaller.

Exam Q 4.0.1

Reference Title

This is an example reference to source material.

Reference 4.0.1

Intuition Title

Intuition 4.0.2

This is an example of an intuitive explanation.

Intuition Title

This is an example of an intuitive explanation, smaller

Intuition 4.0.1

THE FRONT MATTER of a book refers to all of the material that comes before the main text. The following table from shows a list of material that appears in the front matter of *The Visual Display of Quantitative Information*, *Envisioning Information*, *Visual Explanations*, and *Beautiful Evidence* along with its page number. Page numbers that appear in parentheses refer to folios that do not have a printed page number (but they are still counted in the page number sequence).

Page content	Books			
	<i>VDQI</i>	<i>EI</i>	<i>VE</i>	<i>BE</i>
Blank half title page	(1)	(1)	(1)	(1)
Frontispiece <sup>1</sup>	(2)	(2)	(2)	(2)
Full title page	(3)	(3)	(3)	(3)
Copyright page	(4)	(4)	(4)	(4)
Contents	(5)	(5)	(5)	(5)
Dedication	(6)	(7)	(7)	7
Epigraph	–	–	(8)	–
Introduction	(7)	(9)	(9)	9

<sup>1</sup> The contents of this page vary from book to book. In *VDQI* this page is blank; in *EI* and *VE* this page holds a frontispiece; and in *BE* this page contains three epigraphs.

The design of the front matter in Tufte’s books varies slightly from the traditional design of front matter. First, the pages in front matter are traditionally numbered with lowercase roman numerals (*e.g.*, i, ii, iii, iv, . . . ). Second, the front matter page numbering sequence is usually separate from the main matter page numbering. That is, the page numbers restart at 1 when the main matter begins. In contrast, Tufte has enumerated his pages with arabic numerals that share the same page counting sequence as the main matter.

There are also some variations in design across Tufte’s four books. The page opposite the full title page (labeled “frontispiece” in the above table) has different content in each of the books. In *The Visual Display of Quantitative Information*, this page is blank; in *Envisioning Information* and *Visual Explanations*, this page holds a frontispiece; and in *Beautiful Evidence*, this page contains three epigraphs. The dedication appears on page 6 in *VDQI* (opposite the introduction), and is placed on its own spread in the other books. In *VE*, an epigraph shares the spread with the opening page of the introduction.

None of the page numbers (folios) of the front matter are expressed except in *BE*, where the folios start to appear on the dedication page.

THE FULL TITLE PAGE of each of the books varies slightly in design. In all the books, the author’s name appears at the top of the page, the title it set just above the center line, and the publisher is printed along the bottom margin. Some of the differences are outlined in the following table.

On the side note of...

yeah

Non-Examinable 4.0.3

Feature	<i>VDQI</i>	<i>EI</i>	<i>VE</i>	<i>BE</i>
Author				
Typeface	serif	serif	serif	sans serif
Style	italics	italics	italics	upright, caps
Size	24 pt	20 pt	20 pt	20 pt
Title				
Typeface	serif	serif	serif	sans serif
Style	upright	italics	upright	upright, caps
Size	36 pt	48 pt	48 pt	36 pt
Subtitle				
Typeface	–	–	serif	–
Style	–	–	upright	–
Size	–	–	20 pt	–
Edition				
Typeface	sans serif	–	–	–
Style	upright, caps	–	–	–
Size	14 pt	–	–	–
Publisher				
Typeface	serif	serif	serif	sans serif
Style	italics	italics	italics	upright, caps
Size	14 pt	14 pt	14 pt	14 pt

THE TABLES OF CONTENTS in Tufte’s books give us our first glimpse of the structure of the main matter. *The Visual Display of Quantitative Information* is split into two parts, each containing some number of chapters. His other three books only contain chapters—they’re not broken into parts.

## 4.1 Typefaces

Tufte’s books primarily use two typefaces: Bembo and Gill Sans. Bembo is used for the headings and body text, while Gill Sans is used for the title page and opening epigraphs in *Beautiful Evidence*.

Since neither Bembo nor Gill Sans are available in default L<sup>A</sup>T<sub>E</sub>X installations, the Tufte-L<sup>A</sup>T<sub>E</sub>X document classes default to using Palatino and Helvetica, respectively.

In addition, the Bera Mono typeface is used for monospaced type.

The following font sizes are defined by the Tufte-L<sup>A</sup>T<sub>E</sub>X classes:

L <sup>A</sup> T <sub>E</sub> X size	Font size	Leading	Used for
<code>\tiny</code>	5	6	sidenote numbers
<code>\scriptsize</code>	7	8	–
<code>\footnotesize</code>	8	10	sidenotes, captions
<code>\small</code>	9	12	quote, quotation, and verse environments
<code>\normalsize</code>	10	14	body text
<code>\large</code>	11	15	B-heads
<code>\Large</code>	12	16	A-heads, TOC entries, author, date
<code>\LARGE</code>	14	18	handout title
<code>\huge</code>	20	30	chapter heads
<code>\Huge</code>	24	36	part titles

Table 4.1: A list of L<sup>A</sup>T<sub>E</sub>X font sizes as defined by the Tufte-L<sup>A</sup>T<sub>E</sub>X document classes.

## 4.2 Headings

Tufte’s books include the following heading levels: parts, chapters,<sup>2</sup> sections, sub-sections, and paragraphs. Not defined by default are: sub-subsections and subpara-graphs.

*Paragraph* Paragraph headings (as shown here) are introduced by italicized text and separated from the main paragraph by a bit of space.

<sup>2</sup> Parts and chapters are defined for the `tufte-book` class only.

Heading	Style	Size
Part	roman	24/36×40 pc
Chapter	italic	20/30×40 pc
Section	italic	12/16×26 pc
Subsection	italic	11/15×26 pc
Paragraph	italic	10/14

Table 4.2: Heading styles used in *Beautiful Evidence*.

4.3 Environments

The following characteristics define the various environments:

Environment	Font size	Notes
Body text	10/14×26 pc	
Block quote	9/12×24 pc	Block indent (left and right) by 1 pc
Sidenotes	8/10×12 pc	Sidenote number is set inline, followed by word space
Captions	8/10×12 pc	

Table 4.3: Environment styles used in *Beautiful Evidence*.

Column 1	Column 2	Column 3
Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.	Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.	Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.
Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.	Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.	Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.
Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.	Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.	Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.

Table 4.4: Example table with limited column widths

# On the Use of the `tufte-book` Document Class

The Tufte- $\text{\LaTeX}$  document classes define a style similar to the style Edward Tufte uses in his books and handouts. Tufte’s style is known for its extensive use of sidenotes, tight integration of graphics with text, and well-set typography. This document aims to be at once a demonstration of the features of the Tufte- $\text{\LaTeX}$  document classes and a style guide to their use.

## 5.1 *Page Layout*

### 5.1.1 *Headings*

This style provides A- and B-heads (that is, `\section` and `\subsection`), demonstrated above.

If you need more than two levels of section headings, you’ll have to define them yourself at the moment; there are no pre-defined styles for anything below a `\subsection`. As Bringhurst points out in *The Elements of Typographic Style*,<sup>1</sup> you should “use as many levels of headings as you need: no more, and no fewer.” The Tufte- $\text{\LaTeX}$  classes will emit an error if you try to use `\subsubsection` and smaller headings.

<sup>1</sup> Bringhurst2005

IN HIS LATER BOOKS,<sup>2</sup> Tufte starts each section with a bit of vertical space, a non-indented paragraph, and sets the first few words of the sentence in SMALL CAPS. To accomplish this using this style, use the `\newthought` command:

<sup>2</sup> Tufte2006

```
\newthought{In his later books}, Tufte starts...
```

## 5.2 *Sidenotes*

One of the most prominent and distinctive features of this style is the extensive use of sidenotes. There is a wide margin to provide ample room for sidenotes and small figures. Any `\footnotes` will automatically be converted to sidenotes.<sup>3</sup> If you’d like to place ancillary information in the margin without the sidenote mark (the superscript number), you can use the `\marginnote` command. The specification of the `\sidenote` command is:

<sup>3</sup> This is a sidenote that was entered using the `\footnote` command.

This is a margin note. Notice that there isn’t a number preceding the note, and there is no number in the main text where this note was written.

```
\sidenote[⟨number⟩][⟨offset⟩]{Sidenote text.}
```

Both the `⟨number⟩` and `⟨offset⟩` arguments are optional. If you provide a `⟨number⟩` argument, then that number will be used as the sidenote number. It will change of the number of the current sidenote only and will not affect the numbering sequence of subsequent sidenotes.

Sometimes a sidenote may run over the top of other text or graphics in the margin space. If this happens, you can adjust the vertical position of the sidenote by providing a dimension in the `⟨offset⟩` argument. Some examples of valid dimensions are:

```
1.0in    2.54cm    254mm    6\baselineskip
```

If the dimension is positive it will push the sidenote down the page; if the dimension is negative, it will move the sidenote up the page.

While both the  $\langle number \rangle$  and  $\langle offset \rangle$  arguments are optional, they must be provided in order. To adjust the vertical position of the sidenote while leaving the sidenote number alone, use the following syntax:

```
\sidenote[] [ $\langle offset \rangle$ ] {Sidenote text.}
```

The empty brackets tell the `\sidenote` command to use the default sidenote number.

If you *only* want to change the sidenote number, however, you may completely omit the  $\langle offset \rangle$  argument:

```
\sidenote[ $\langle number \rangle$ ] {Sidenote text.}
```

The `\marginnote` command has a similar *offset* argument:

```
\marginnote[ $\langle offset \rangle$ ] {Margin note text.}
```

### 5.3 References

References are placed alongside their citations as sidenotes, as well. This can be accomplished using the normal `\cite` command.<sup>4</sup>

The complete list of references may also be printed automatically by using the `\bibliography` command. (See the end of this document for an example.) If you do not want to print a bibliography at the end of your document, use the `\nobibliography` command in its place.

To enter multiple citations at one location,<sup>5</sup> you can provide a list of keys separated by commas and the same optional vertical offset argument:

```
\cite{Tufte2006,Tufte1990}.
```

```
\cite[ $\langle offset \rangle$ ] {bibkey1,bibkey2,...}
```

<sup>4</sup> The first paragraph of this document includes a citation.

<sup>5</sup> **Tufte2006, Tufte1990**

### 5.4 Figures and Tables

Images and graphics play an integral role in Tufte’s work. In addition to the standard `figure` and `tabular` environments, this style provides special figure and table environments for full-width floats.

Full page-width figures and tables may be placed in `figure*` or `table*` environments. To place figures or tables in the margin, use the `marginfigure` or `marginfigure` environments as follows (see figure 5.1):

```
\begin{marginfigure}
  \includegraphics{helix}
  \caption{This is a margin figure.}
  \label{fig:marginfig}
\end{marginfigure}
```

The `marginfigure` and `marginfigure` environments accept an optional parameter  $\langle offset \rangle$  that adjusts the vertical position of the figure or table. See the “Sidenotes” section above for examples. The specifications are:

```
\begin{marginfigure}[ $\langle offset \rangle$ ]
  ...
\end{marginfigure}

\begin{marginfigure}[ $\langle offset \rangle$ ]
  ...
\end{marginfigure}
```

Figure ?? is an example of the `figure*` environment and figure ?? is an example of the normal `figure` environment.

Figure 5.1: This is a margin figure. The helix is defined by  $x = \cos(2\pi z)$ ,  $y = \sin(2\pi z)$ , and  $z = [0, 2.7]$ . The figure was drawn using Asymptote (<http://asymptote.sf.net/>).



As with sidenotes and marginnotes, a caption may sometimes require vertical adjustment. The `\caption` command now takes a second optional argument that enables you to do this by providing a dimension  $\langle offset \rangle$ . You may specify the caption in any one of the following forms:

```
\caption{long caption}
\caption[short caption]{long caption}
\caption[][\langle offset \rangle]{long caption}
\caption[short caption][\langle offset \rangle]{long caption}
```

A positive  $\langle offset \rangle$  will push the caption down the page. The short caption, if provided, is what appears in the list of figures/tables, otherwise the “long” caption appears there. Note that although the arguments  $\langle short\ caption \rangle$  and  $\langle offset \rangle$  are both optional, they must be provided in order. Thus, to specify an  $\langle offset \rangle$  without specifying a  $\langle short\ caption \rangle$ , you must include the first set of empty brackets `[]`, which tell `\caption` to use the default “long” caption. As an example, the caption to figure ?? above was given in the form

```
\caption[Hilbert curves...][6pt]{Hilbert curves...}
```

Table 5.1 shows table created with the `booktabs` package. Notice the lack of vertical rules—they serve only to clutter the table’s data.

Margin	Length
Paper width	8 <sup>1</sup> / <sub>2</sub> inches
Paper height	11 inches
Textblock width	6 <sup>1</sup> / <sub>2</sub> inches
Textblock/sidenote gutter	3/ <sub>8</sub> inches
Sidenote width	2 inches

Table 5.1: Here are the dimensions of the various margins used in the Tufte-handout class.

OCCASIONALLY L<sup>A</sup>T<sub>E</sub>X will generate an error message:

```
Error: Too many unprocessed floats
```

L<sup>A</sup>T<sub>E</sub>X tries to place floats in the best position on the page. Until it’s finished composing the page, however, it won’t know where those positions are. If you have a lot of floats on a page (including sidenotes, margin notes, figures, tables, etc.), L<sup>A</sup>T<sub>E</sub>X may run out of “slots” to keep track of them and will generate the above error.

L<sup>A</sup>T<sub>E</sub>X initially allocates 18 slots for storing floats. To work around this limitation, the Tufte-L<sup>A</sup>T<sub>E</sub>X document classes provide a `\morefloats` command that will reserve more slots.

The first time `\morefloats` is called, it allocates an additional 34 slots. The second time `\morefloats` is called, it allocates another 26 slots.

The `\morefloats` command may only be used two times. Calling it a third time will generate an error message. (This is because we can’t safely allocate many more floats or L<sup>A</sup>T<sub>E</sub>X will run out of memory.)

If, after using the `\morefloats` command twice, you continue to get the `Too many unprocessed floats` error, there are a couple things you can do.

The `\FloatBarrier` command will immediately process all the floats before typesetting more material. Since `\FloatBarrier` will start a new paragraph, you should place this command at the beginning or end of a paragraph.

The `\clearpage` command will also process the floats before continuing, but instead of starting a new paragraph, it will start a new page.

You can also try moving your floats around a bit: move a figure or table to the next page or reduce the number of sidenotes. (Each sidenote actually uses *two* slots.)

After the floats have placed, L<sup>A</sup>T<sub>E</sub>X will mark those slots as unused so they are available for the next page to be composed.

## 5.5 Captions

You may notice that the captions are sometimes misaligned. Due to the way L<sup>A</sup>T<sub>E</sub>X’s float mechanism works, we can’t know for sure where it decided to put a float. Therefore, the Tufte-L<sup>A</sup>T<sub>E</sub>X document classes provide commands to override the caption position.

*Vertical alignment* To override the vertical alignment, use the `\setfloatalignment` command inside the float environment. For example:

```
\begin{figure}[btp]
  \includegraphics{sinewave}
  \caption{This is an example of a sine wave.}
  \label{fig:sinewave}
  \setfloatalignment{b}% forces caption to be bottom-aligned
\end{figure}
```

The syntax of the `\setfloatalignment` command is:

```
\setfloatalignment{<pos>}
```

where *<pos>* can be either `b` for bottom-aligned captions, or `t` for top-aligned captions.

*Horizontal alignment* To override the horizontal alignment, use either the `\forceversofloat` or the `\forcerectofloat` command inside of the float environment. For example:

```
\begin{figure}[btp]
  \includegraphics{sinewave}
  \caption{This is an example of a sine wave.}
  \label{fig:sinewave}
  \forceversofloat% forces caption to be set to the left of the float
\end{figure}
```

The `\forceversofloat` command causes the algorithm to assume the float has been placed on a verso page—that is, a page on the left side of a two-page spread. Conversely, the `\forcerectofloat` command causes the algorithm to assume the float has been placed on a recto page—that is, a page on the right side of a two-page spread.

## 5.6 Full-width text blocks

In addition to the new float types, there is a `fullwidth` environment that stretches across the main text block and the sidenotes area.

```
\begin{fullwidth}
  Lorem ipsum dolor sit amet...
\end{fullwidth}
```

*Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.*

## 5.7 *Typography*

### 5.7.1 *Typefaces*

If the Palatino, Helvetica, and Bera Mono typefaces are installed, this style will use them automatically. Otherwise, we'll fall back on the Computer Modern typefaces.

### 5.7.2 *Letterspacing*

This document class includes two new commands and some improvements on existing commands for letterspacing.

When setting strings of ALL CAPS or SMALL CAPS, the letterspacing—that is, the spacing between the letters—should be increased slightly.<sup>6</sup> The `\allcaps` command has proper letterspacing for strings of FULL CAPITAL LETTERS, and the `\smallcaps` command has letterspacing for SMALL CAPITAL LETTERS. These commands will also automatically convert the case of the text to upper- or lowercase, respectively.

<sup>6</sup> Bringhurst2005

The `\textsc` command has also been redefined to include letterspacing. The case of the `\textsc` argument is left as is, however. This allows one to use both uppercase and lowercase letters: THE INITIAL LETTERS OF THE WORDS IN THIS SENTENCE ARE CAPITALIZED.

## 5.8 *Document Class Options*

The `tufte-book` class is based on the L<sup>A</sup>T<sub>E</sub>X `book` document class. Therefore, you can pass any of the typical book options. There are a few options that are specific to the `tufte-book` document class, however.

The `a4paper` option will set the paper size to A4 instead of the default US letter size.

The `sfsidenotes` option will set the sidenotes and title block in a sans serif typeface instead of the default roman.

The `twoside` option will modify the running heads so that the page number is printed on the outside edge (as opposed to always printing the page number on the right-side edge in `oneside` mode).

The `symmetric` option typesets the sidenotes on the outside edge of the page. This is how books are traditionally printed, but is contrary to Tufte's book design which sets the sidenotes on the right side of the page. This option implicitly sets the `twoside` option.

The `justified` option sets `alldocclsoptdef` and `right`). The default is to set the text ragged right. The body text of Tufte's books are set ragged right. This prevents needless hyphenation and makes it easier to read the text in the slightly narrower column.

The `bidirectional` option loads the  `bidi`  package which is used with X<sub>Y</sub>L<sup>A</sup>T<sub>E</sub>X to typeset bi-directional text. Since the  `bidi`  package needs to be loaded before the sidenotes and cite commands are defined, it can't be loaded in the document preamble.

The `debug` option causes the Tufte-L<sup>A</sup>T<sub>E</sub>X classes to output debug information to the log file which is useful in troubleshooting bugs. It will also cause the graphics to be replaced by outlines.

The `nofonts` option prevents the Tufte-L<sup>A</sup>T<sub>E</sub>X classes from automatically loading the Palatino and Helvetica typefaces. You should use this option if you wish to load your own fonts. If you're using X<sub>Y</sub>L<sup>A</sup>T<sub>E</sub>X, this option is implied (*i.e.*, the Palatino and Helvetica fonts aren't loaded if you use X<sub>Y</sub>L<sup>A</sup>T<sub>E</sub>X).

The `nols` option inhibits the letterspacing code. The Tufte-L<sup>A</sup>T<sub>E</sub>X classes try to load the appropriate letterspacing package (either pdfL<sup>A</sup>T<sub>E</sub>X's `letterspace` package

or the `soul` package). If you're using  $\text{\LaTeX}$  with `fontenc`, however, you should configure your own letterspacing.

The `notitlepage` option causes `\maketitle` to generate a title block instead of a title page. The `book` class defaults to a title page and the `handout` class defaults to the title block. There is an analogous `titlepage` option that forces `\maketitle` to generate a full title page instead of the title block.

The `notoc` option suppresses Tufte- $\text{\LaTeX}$ 's custom table of contents (TOC) design.

The current TOC design only shows unnumbered chapter titles; it doesn't show sections or subsections. The `notoc` option will revert to  $\text{\LaTeX}$ 's TOC design.

The `nohyper` option prevents the `hyperref` package from being loaded. The default is to load the `hyperref` package and use the `\title` and `\author` contents as metadata for the generated PDF.

## 6

# Customizing Tufte- $\text{\LaTeX}$

The Tufte- $\text{\LaTeX}$  document classes are designed to closely emulate Tufte’s book design by default. However, each document is different and you may encounter situations where the default settings are insufficient. This chapter explores many of the ways you can adjust the Tufte- $\text{\LaTeX}$  document classes to better fit your needs.

### 6.1 *File Hooks*

If you create many documents using the Tufte- $\text{\LaTeX}$  classes, it’s easier to store your customizations in a separate file instead of copying them into the preamble of each document. The Tufte- $\text{\LaTeX}$  classes provide three file hooks: `tufte-common-local.tex`, `tufte-book-local.tex`, and `tufte-handout-local.tex`.

*tufte-common-local.tex* If this file exists, it will be loaded by all of the Tufte- $\text{\LaTeX}$  document classes just prior to any document-class-specific code. If your customizations or code should be included in both the book and handout classes, use this file hook.

*tufte-book-local.tex* If this file exists, it will be loaded after all of the common and book-specific code has been read. If your customizations apply only to the book class, use this file hook.

*tufte-common-handout.tex* If this file exists, it will be loaded after all of the common and handout-specific code has been read. If your customizations apply only to the handout class, use this file hook.

### 6.2 *Numbered Section Headings*

While Tufte dispenses with numbered headings in his books, if you require them, they can be enabled by changing the value of the `secnumdepth` counter. From the table below, select the heading level at which numbering should stop and set the `secnumdepth` counter to that value. For example, if you want parts and chapters numbered, but don’t want numbering for sections or subsections, use the command:

```
\setcounter{secnumdepth}{0}
```

The default `secnumdepth` for the Tufte- $\text{\LaTeX}$  document classes is `-1`.

Heading level	Value
Part (in <code>tufte-book</code> )	<code>-1</code>
Part (in <code>tufte-handout</code> )	<code>0</code>
Chapter (only in <code>tufte-book</code> )	<code>0</code>
Section	<code>1</code>
Subsection	<code>2</code>
Subsubsection	<code>3</code>
Paragraph	<code>4</code>
Subparagraph	<code>5</code>

Table 6.1: Heading levels used with the `secnumdepth` counter.

### 6.3 Changing the Paper Size

The Tufte-L<sup>A</sup>T<sub>E</sub>X classes currently only provide three paper sizes: A4, B5, and US letter. To specify a different paper size (and/or margins), use the `\geometrysetup` command in the preamble of your document (or one of the file hooks). The full documentation of the `\geometrysetup` command may be found in the `geometry` package documentation.<sup>1</sup>

<sup>1</sup> `pkg-geometry`

### 6.4 Customizing Marginal Material

Marginal material includes sidenotes, citations, margin notes, and captions. Normally, the justification of the marginal material follows the justification of the body text. If you specify the `justified` document class option, all of the margin material will be fully justified as well. If you don't specify the `justified` option, then the marginal material will be set ragged right.

You can set the justification of the marginal material separately from the body text using the following document class options: `sidenote`, `marginnote`, `caption`, `citation`, and `marginals`. Each option refers to its obviously corresponding marginal material type. The `marginals` option simultaneously sets the justification on all four marginal material types.

Each of the document class options takes one of five justification types:

*justified* Fully justifies the text (sets it flush left and right).

*raggedleft* Sets the text ragged left, regardless of which page it falls on.

*raggedright* Sets the text ragged right, regardless of which page it falls on.

*raggedouter* Sets the text ragged left if it falls on the left-hand (verso) page of the spread and otherwise sets it ragged right. This is useful in conjunction with the `symmetric` document class option.

*auto* If the `justified` document class option was specified, then set the text fully justified; otherwise the text is set ragged right. This is the default justification option if one is not explicitly specified.

For example,

```
\documentclass[symmetric,justified,marginals=raggedouter]{tufte-book}
```

will set the body text of the document to be fully justified and all of the margin material (sidenotes, margin notes, captions, and citations) to be flush against the body text with ragged outer edges.

THE FONT AND STYLE of the marginal material may also be modified using the following commands:

```
\setsidenotefont{\font commands}
\setcaptionfont{\font commands}
\setmarginnotefont{\font commands}
\setcitationfont{\font commands}
```

The `\setsidenotefont` sets the font and style for sidenotes, the `\setcaptionfont` for captions, the `\setmarginnotefont` for margin notes, and the `\setcitationfont` for citations. The `\font commands` can contain font size changes (e.g., `\footnotesize`, `\Huge`, etc.), font style changes (e.g., `\sffamily`, `\ttfamily`, `\itshape`, etc.), color changes (e.g., `\color{blue}`), and many other adjustments.

If, for example, you wanted the captions to be set in italic sans serif, you could use:

```
\setcaptionfont{\itshape\sffamily}
```

# 7

## Compatibility Issues

When switching an existing document from one document class to a Tufte- $\text{\LaTeX}$  document class, a few changes to the document may have to be made.

### *7.1 Converting from article to tufte-handout*

The following `article` class options are unsupported: `10pt`, `11pt`, `12pt`, `a5paper`, `b5paper`, `executivepaper`, `legalpaper`, `landscape`, `onecolumn`, and `twocolumn`.  
The following headings are not supported: `\subsubsection` and `\subparagraph`.

### *7.2 Converting from book to tufte-book*

The following `report` class options are unsupported: `10pt`, `11pt`, `12pt`, `a5paper`, `b5paper`, `executivepaper`, `legalpaper`, `landscape`, `onecolumn`, and `twocolumn`.  
The following headings are not supported: `\subsubsection` and `\subparagraph`.



## 8

# Troubleshooting and Support

### 8.1 *Tufte- $\LaTeX$ Website*

The website for the Tufte- $\LaTeX$  packages is located at <http://code.google.com/p/tufte-latex/>. On our website, you'll find links to our SVN repository, mailing lists, bug tracker, and documentation.

### 8.2 *Tufte- $\LaTeX$ Mailing Lists*

There are two mailing lists for the Tufte- $\LaTeX$  project:

*Discussion list* The `tufte-latex` discussion list is for asking questions, getting assistance with problems, and help with troubleshooting. Release announcements are also posted to this list. You can subscribe to the `tufte-latex` discussion list at <http://groups.google.com/group/tufte-latex>.

*Commits list* The `tufte-latex-commits` list is a read-only mailing list. A message is sent to the list any time the Tufte- $\LaTeX$  code has been updated. If you'd like to keep up with the latest code developments, you may subscribe to this list. You can subscribe to the `tufte-latex-commits` mailing list at <http://groups.google.com/group/tufte-latex-commits>.

### 8.3 *Getting Help*

If you've encountered a problem with one of the Tufte- $\LaTeX$  document classes, have a question, or would like to report a bug, please send an email to our mailing list or visit our website.

To help us troubleshoot the problem more quickly, please try to compile your document using the `debug` class option and send the generated `.log` file to the mailing list with a brief description of the problem.

### 8.4 *Errors, Warnings, and Informational Messages*

The following is a list of all of the errors, warnings, and other messages generated by the Tufte- $\LaTeX$  classes and a brief description of their meanings.

Error: `\subparagraph` is undefined by this class.

The `\subparagraph` command is not defined in the Tufte- $\LaTeX$  document classes. If you'd like to use the `\subparagraph` command, you'll need to redefine it yourself. See the "Headings" section on page 23 for a description of the heading styles available in the Tufte- $\LaTeX$  document classes.

Error: `\subsubsection` is undefined by this class.

The `\subsubsection` command is not defined in the Tufte- $\LaTeX$  document classes. If you'd like to use the `\subsubsection` command, you'll need to redefine

it yourself. See the “Headings” section on page 23 for a description of the heading styles available in the Tufte-L<sup>A</sup>T<sub>E</sub>X document classes.

**Error:** You may only call `\morefloats` twice. See the Tufte-L<sup>A</sup>T<sub>E</sub>X documentation for other workarounds.

L<sup>A</sup>T<sub>E</sub>X allocates 18 slots for storing floats. The first time `\morefloats` is called, it allocates an additional 34 slots. The second time `\morefloats` is called, it allocates another 26 slots.

The `\morefloats` command may only be called two times. Calling it a third time will generate this error message. See page 25 for more information.

**Warning:** Option ‘*⟨class option⟩*’ is not supported -- ignoring option.

This warning appears when you’ve tried to use *⟨class option⟩* with a Tufte-L<sup>A</sup>T<sub>E</sub>X document class, but *⟨class option⟩* isn’t supported by the Tufte-L<sup>A</sup>T<sub>E</sub>X document class. In this situation, *⟨class option⟩* is ignored.

**Info:** The ‘*symmetric*’ option implies ‘*twoside*’

You specified the `symmetric` document class option. This option automatically forces the `twoside` option as well. See page 27 for more information on the `symmetric` class option.

## 8.5 *Package Dependencies*

The following is a list of packages that the Tufte-L<sup>A</sup>T<sub>E</sub>X document classes rely upon. Packages marked with an asterisk are optional.

- xifthen
- natbib *and* bibentry
- ifpdf\*
- optparams
- ifxetex\*
- placeins
- hyperref
- mathpazo\*
- geometry
- helvet\*
- ragged2e
- fontenc
- chngpage *or* changepage
- beramono\*
- paralist
- fancyhdr
- textcase
- xcolor
- soul\*
- textcomp
- letterspace\*
- titlesec
- setspace
- titletoc