
Exercise 1 Evaluate the limit, if it exits.

(a) $\lim\limits_{x\to -3} \dfrac{x^2+3x}{x^2-x-12}$

(b) $\lim\limits_{x\to 4} \dfrac{x^2+3x}{x^2-x-12}$

(c) $\lim\limits_{t\to 1} \dfrac{t^4-1}{t^3-1}$

(d) $\lim\limits_{u\to 2} \dfrac{\sqrt{4u+1}-3}{u-2}$

(e) $\lim\limits_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right)$

Exercise 2 If $2x \le g(x) \le x^4 - x^2 + 2$ or all x, evaluate

(a) $\lim_{x\to 1} 2x$

(b) $\lim_{x\to 1} x^4 - x^2 + 2$

(c) $\lim_{x\to 1} g(x) =$ (by _____ Theorem)

Exercise 3 Find the limit, if it exists. If the limit does not exist, explain why.

(a) $\lim\limits_{x\to -6} \dfrac{2x+12}{|x+6|}$

(b) $\lim\limits_{x\to -2} \dfrac{2-|x|}{2+x}$

(c) $\lim\limits_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$

Exercise 4 Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ (x-2)^2 & \text{if } x \geq 1 \end{cases}$$

(a) Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

(b) Does $\lim_{x \to 1} f(x)$ exist?

(c) Sketch the graph of f.

Exercise 5 The figure shows a fixed circle C_1 with equation $(x-1)^2 + y^2 = 1$ and a shrinking circle C_2 with radius r and center the origin. P is the point $(0, r)$, Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the x-axis. What happens to R as C_2 shrinks, that is, as $r \to 0^+$?