

Exercise 1 Draw the graph of a function defined on $[0, 8]$ such that $f(0) = f(8) = 3$ and the function does not satisfy the conclusion of Rolle's Theorem on $[0, 8]$.

Exercise 2 Draw the graph of a function that is continuous on $[0, 8]$ where $f(0) = 1$ and $f(8) = 4$ and that does not satisfy the conclusion of the Mean Value Theorem on $[0, 8]$.

Exercise 3 Verify that the function $f(x) = x^3 - 3x + 2$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-2, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

Exercise 4 Let $f(x) = \tan(x)$. Show that $f(0) = f(\pi)$ but there is no number c in $(0, \pi)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

Exercise 5 Let $f(x) = 2 - |2x - 1|$. Show that there is no value of c such that $f(3) - f(0) = f'(c)(3 - 0)$. Why does this not contradict the Mean Value Theorem?

Exercise 6 Show that the equation $x^3 + e^x = 0$ has 'exactly' one real root.