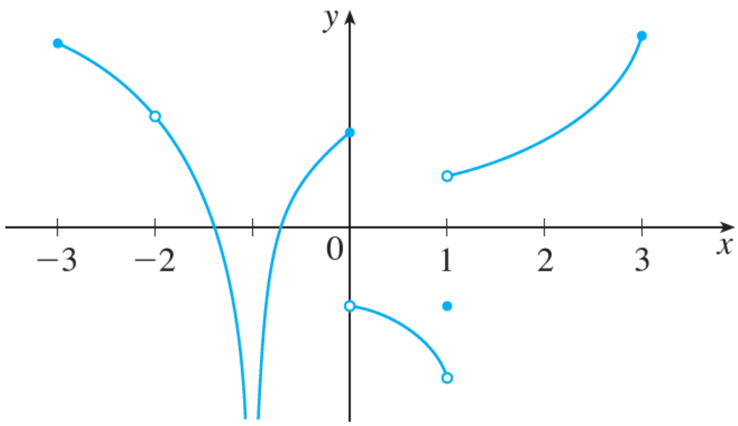
Exercise 1 From the graph of g, determine whether the function g is continuous at a given x-value. Explain. Then state the intervals on which g is continuous.



(a) $x = -3$

(b) $x = -2$

(c) $x = 0$

(d) $x = 1$

(e) $x = 3$

Exercise 2 Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

(*a*) $g(t) = \dfrac{t^2 + 5t}{2t + 1}, \ a = 2$

(*b*) $f(x) = 3x^4 - 5x + \sqrt[3]{x^2 + 4}, \ a = 2$

Exercise 3 Explain why the function

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

is discontinuous at the number $a = 1$. Remove the discontinuity by redefining the function.

Exercise 4 Explain why the function

$$f(x) = \begin{cases} \frac{2x^2 - 5x - 3}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

is discontinuous at the number $a = 3$. Remove the discontinuity by redefining the function.

Exercise 5 Explain, using theorems, why the function is continuous at every number in its domain. State the domain.

(a) $G(x) = \dfrac{x^2+1}{2x^2-x-1}$

(b) $R(t) = \dfrac{e^{\sin(t)}}{2+\cos(\pi t)}$

(c) $B(x) = \dfrac{\tan(x)}{\sqrt{4-x^2}}$

(d) $N(r) = \tan^{-1}(1+e^{-t^2})$

Exercise 6 Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} 2^x & \text{if } x \leq 1 \\ 3-x & \text{if } 1 < x \leq 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$$

Exercise 7 Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$. Find $(f \circ g)(x)$. Is $f \circ g$ continuous everywhere? Explain.

Exercise 8 Use the Intermediate Value Theorem to show that there is a root of the equation $\sin(x) = x^2 - x$ in the interval $(1, 2)$.