
Exercise 1 Find the limit of the function $f(x) = \frac{\sin(x)}{x}$ as x approaches 0.

As it is not easy to draw a graph of the function, we can try numerical calculations to see the limiting values. Use a calculator to complete the tables. Make sure that the calculator is set to RADIAN mode.

x	$f(x)$
-0.1	
-0.01	
-0.001	
-0.0001	

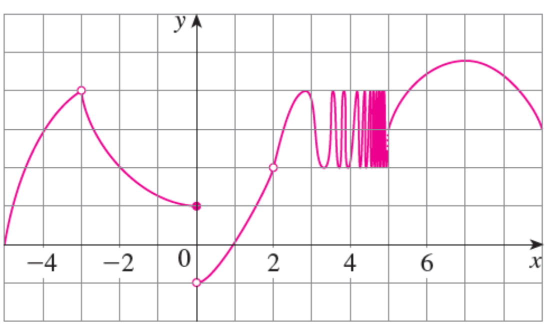
x	$f(x)$
0.1	
0.01	
0.001	
0.0001	

Now make a guess for the limiting value as x approaches 0. Verify graphically by plotting the function $y = \frac{\sin(x)}{x}$ near $x = 0$ on www.desmos.com/calculator.

$$\boxed{\lim_{x\to 0} \frac{\sin(x)}{x} = \qquad}$$

This limit is a kinda important ones to remember for later chapter.

Exercise 2 For the function h whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.



(a) $\lim_{x\to -3^-} h(x)$ (b) $\lim_{x\to -3^+} h(x)$

(c) $\lim_{x\to -3} h(x)$ (d) $h(-3)$

(e) $\lim_{x\to 0^-} h(x)$ (f) $\lim_{x\to 0^+} h(x)$

(g) $\lim_{x\to 0} h(x)$ (h) $h(0)$

(i) $\lim_{x\to 2^-} h(x)$ (j) $\lim_{x\to 2^+} h(x)$ (k) $\lim_{x\to 2} h(x)$

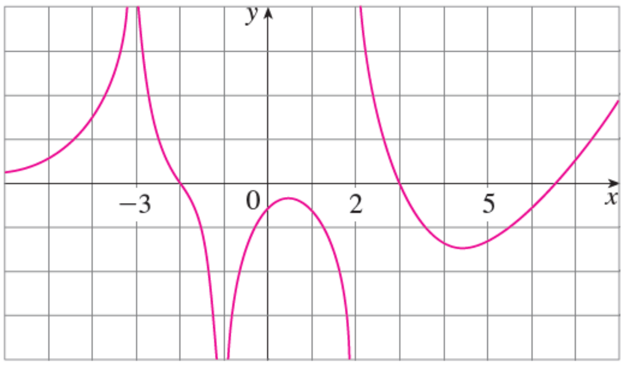
(l) $h(2)$ (m) $\lim_{x\to 5^-} h(x)$ (n) $\lim_{x\to 5^+} h(x)$

Exercise 3 Guess the value of the limit (if it exists) by evaluating the function at the given numbers.

$$\lim_{x\to -3} \frac{x^2 - 3x}{x^2 - 9}$$

$x = -3 \pm 0.5, -3 \pm 0.1, -3 \pm 0.01, -3 \pm 0.001, -3 \pm 0.0001.$

Exercise 4 For the function A whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.



(a) $\lim_{x\to -3^-} A(x)$ (b) $\lim_{x\to -3^+} A(x)$

(c) $\lim_{x\to -3} A(x)$ (d) $\lim_{x\to -1^-} A(x)$

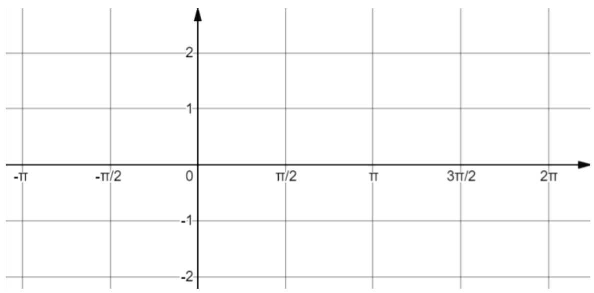
(e) $\lim_{x\to -1^+} A(x)$ (f) $\lim_{x\to -1} A(x)$

(g) $\lim_{x\to 2^-} A(x)$ (h) $\lim_{x\to 2^+} A(x)$

(i) $\lim_{x\to 2} A(x)$

Exercise 5 Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} 1+\sin(x) & \text{if } x < 0 \\ \cos(x) & \text{if } 0 \le x \le \pi \\ \sin(x) & \text{if } x > \pi \end{cases}$$



Exercise 6 Determine the infinite limit.

(a) $\displaystyle\lim_{x\to 5^-} \frac{x+1}{x-5}$

(b) $\displaystyle\lim_{x\to 0^+} \ln(\sin(x))$

(c) $\displaystyle\lim_{x\to \pi^-} \cot(x)$

Exercise 7 Find the vertical asymptotes of the function

$$y = \frac{x^2+1}{3x-2x^2}$$

Exercise 8 In the theory of relativity, the mass of a particle with velocity v is $m = m_0/\sqrt{1-v^2/c^2}$ where m_0 is the mass of the particle at rest and c is the speed of light. What happens as $v \to c^-$?