Exercise 1 Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

(a) $\lim_{x\to 0} \frac{\tan(3x)}{\sin(2x)}$

(b) $\lim_{t\to 0} \frac{8^t - 5^t}{t}$

(c) $\lim_{x\to\infty} x \ln\left(1 - \frac{1}{x}\right)$

(d) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\tan^{-x}(x)}\right)$

(e) $\lim_{x\to\infty} x^{\ln(2)/(1+\ln(x))}$

(f) $\lim_{x\to\infty} \left(\frac{2x-3}{2x+5}\right)^{2x+1}$

Exercise 2 If f'' is continuous, show that

$$\lim_{h\to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x)$$

Exercise 3 Prove that

$$\lim_{x\to\infty} \frac{\ln(x)}{x^p} = 0$$

for any number $p > 0$. This shows that the logarithmic function approaches infinity more slowly than any power of x.