Exercise 1 Show that $(\coth(\mathrm{x}))' = -\operatorname{csch}^2(x)$.

Exercise 2 Prove the identity.

$$\tanh(\ln(x)) = \frac{x^2 - 1}{x^2 + 1}$$

Exercise 3 Find the derivative. Simplify where possible.

(a) $f(x) = e^x \cosh(x)$

(b) $g(x) = \ln(\sinh(x))$

(c) $h(x) = \dfrac{1 + \sinh(x)}{1 - \sinh(x)}$

Exercise 4 The most famous application of the hyperbolic functions is the use of hyperbolic cosine to describe the shape of a hanging wire. It can be proved that if a heavy flexible cable (such as a telephone or power line) is suspended between two points at the same height, then it takes the shape of a curve with equation

$$y = c + a \cosh\left(\frac{x}{a}\right)$$

called a *catenary* (catena means "chain").

Using principles from physics it can be shown that when a cable is hung between two poles, it takes the shape of a curve $y = f(x)$ that satisfies the differential equation

$$\frac{d^2y}{dx^2} = \frac{\rho g}{T}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

where ρ is the linear density of the cable, g is the acceleration due to gravity, T is the tension in the cable at its lowest point, and the coordinate system is chosen appropriately. Verify that the function

$$y = f(x) = \frac{T}{\rho g}\cosh\left(\frac{\rho g x}{T}\right)$$

is a solution of this differential equation. Suggestion: Let $a = \frac{T}{\rho g}$.