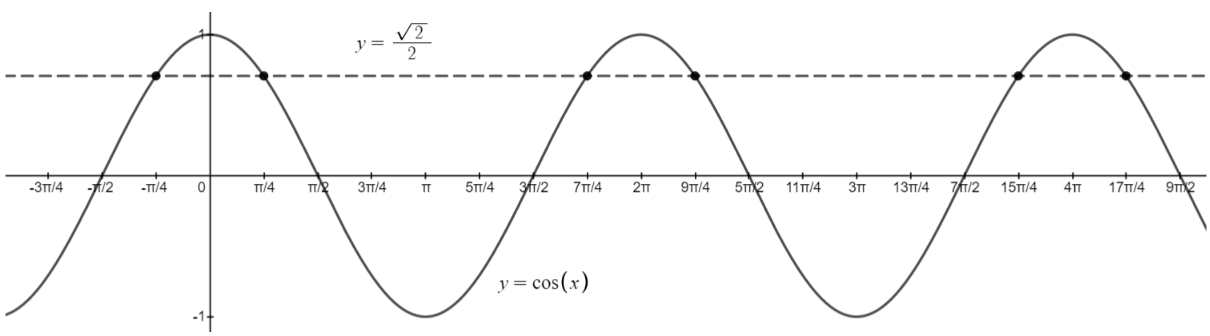
One-to-One Functions

Let $f(x)$ be a function with a domain D and the range R. For $a \in D$, the function value is $f(a) \in R$. It is also denoted by $a \mapsto f(a)$, read "a is **mapped to** $f(a)$" or "the **image** of a under the function f is $f(a)$." In this case, a is called a **preimage** of $f(a)$. For instance, consider the function $f(x) = \cos(x)$ with its (implied) domain $D = (-\infty, \infty)$ and the range $R = [-1, 1]$. For $\frac{\pi}{2} \in D$, the function value is $f(\frac{\pi}{2}) = 0$. Also, we can say that the image of $\frac{\pi}{2}$ under the function f is 0. $\frac{\pi}{2}$ is a preimage of 0. Intuitively, an inverse function, denoted by f^{-1}, should be a function that reverses this process. So we would like $f^{-1}(0) = \frac{\pi}{2}$. In other words, an inverse function should map an image under the function f to its preimage. Since $f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$, we would like $f^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$. However, we have a small problem. Note that $f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$, but also $f(\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$. Then what should $f^{-1}(\frac{\sqrt{2}}{2})$ be? $\frac{\pi}{4}$ or $\frac{7\pi}{4}$? Should we ask an oracle? For f^{-1} to be a 'function', it can only have unique value for a given input value. So we can't say that $f^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$ or $\frac{7\pi}{4}$. In fact, $\frac{\pi}{4}$ and $\frac{7\pi}{4}$ are not the only candidates. $\cos(x)$ has infinitely many input values ($\frac{\pi}{4} + 2\pi k$ and $\frac{7\pi}{4} + 2\pi k$ where $k \in \mathbb{Z}$) whose image is $\frac{\sqrt{2}}{2}$. We say that the function $f(x) = \cos(x)$ is not invertible because of this problem.



The center of the problem with the function $\cos(x)$ is that the function value $\frac{\sqrt{2}}{2}$ has multiple preimages. Graphically, the problem can be spotted if a horizontal line can be drawn such that the line crosses the graph of $y = \cos(x)$ more than once. If that happens, we say that the graph of $y = f(x)$ fails the **Horizontal Line Test** (HLT). On the contrary, if no horizontal line that crosses the graph of $y = f(x)$ more than once can be drawn, then we say that the graph of $y = f(x)$ passes the HLT.

A function f with a domain D is called a **one-to-one function** if it never takes on the same value twice on D; that is, for $x_1, x_2 \in D$,

$$\text{if } x_1 \neq x_2, \text{ then } f(x_1) \neq f(x_2)$$

For a one-to-one function $f(x)$, if input values from the domain are different, then output values should be different. Logically, it is equivalent to say that if $x_1, x_2 \in D$ and $f(x_1) = f(x_2)$, then $x_1 = x_2$.

> A function is one-to-one if and only if no horizontal line intersects its graph more than once, that is, passing the Horizontal Line Test (HLT).

Since $\cos(x)$ fails the HLT, $\cos(x)$ is not a one-to-one function. There are two ways to show that a function is 'not' a one-to-one function: (1) show that the graph of the function fails the HLT; (2) find two different values in a domain whose images are the same value.

Example 1 Show that the function $y = \tan(\pi(x - \frac{1}{2}))$ with its (implied) domain $\mathbb{R} - \mathbb{Z}$ is not a one-to-one function.

$\tan(0) = 0$ and $\tan(\pi) = 0$. The images of 0 and π in the domain are the same. That's it! $\tan(x)$ is not a one-to-one function.

> Let f be a one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by
>
> $$f^{-1}(b) = a \quad \Leftrightarrow \quad f(a) = b$$
>
> for any b in B.

1. You read f^{-1} as "f inverse". Never "f negative one" or "f to the -1 power". Certainly it is not the case that $f^{-1} = \frac{1}{f}$.
2. If (a, b) is a point on the graph of the function $y = f(x)$, then (b, a) must be a point on the graph of $y = f^{-1}(x)$. The inverse function f^{-1} switches an input value and its corresponding output value of the function f.
3. The inverse function f^{-1} is well-defined because f is a one-to-one function. Suppose f is not one-to-one. Then $f(a_1) = b$ and $f(a_2) = b$ for two different input values $a_1 \neq a_2$. Now then what should be the output value of the inverse function f^{-1} for the input value b? Is $f^{-1}(b) = a_1$ or $f^{-1}(b) = a_2$? A function is not well-defined if this happens, i.e. f^{-1} fails the Vertical Line Test.
4. If f^{-1} is the inverse function of the function f, then the function f is again the inverse function of f^{-1}. That is, $(f^{-1})^{-1} = f$.
5. The domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}.

Cancellation Property of Inverse Function

$$f^{-1}(f(x)) = x \text{ for every } x \in A$$

$$f(f^{-1}(x)) = x \text{ for every } x \in B$$

To find the inverse function of a one-to-one function f,

Step 1: Solve the equation $y = f(x)$ for the variable x so that we have $x =$ some expression involving y (but not x).

Step 2: Switch the letters x and y to write as $y = f^{-1}(x)$.

The first step might not be possible because not every equation can be solved for a variable involved. Try to solve $y = x^5 + x^4 + x^3 + x^2 + x$ for the variable x. However, if you are asked to solve for a variable in a class like this, it would be always possible.

Example 2 Consider a function $y = \sqrt[3]{2x-1}$ with the domain $[\frac{1}{2}, \infty)$ and the range $(-\infty, \infty)$. To find the expression for its inverse function, we first solve the equation for the variable x.

$$y = \sqrt[3]{2x-1} \quad \Rightarrow \quad (y)^3 = (\sqrt[3]{2x-1})^3 \quad \Rightarrow \quad y^3 = 2x-1 \quad \Rightarrow \quad \frac{y^3+1}{2} = x$$

Now switch the letters to obtain $y = \frac{x^3+1}{2}$. If $f(x) = \sqrt[3]{2x-1}$, then $f^{-1}(x) = \frac{x^3+1}{2}$. The domain of f^{-1} is $(-\infty, \infty)$, and the range of f^{-1} is $[\frac{1}{2}, \infty)$. Let us check to see if the expression is correct. $f(14) = \sqrt[3]{2(14)-1} = 3$ and $f^{-1}(3) = \frac{(3)^3+1}{2} = 14$. But this only checks for one ordered pair.

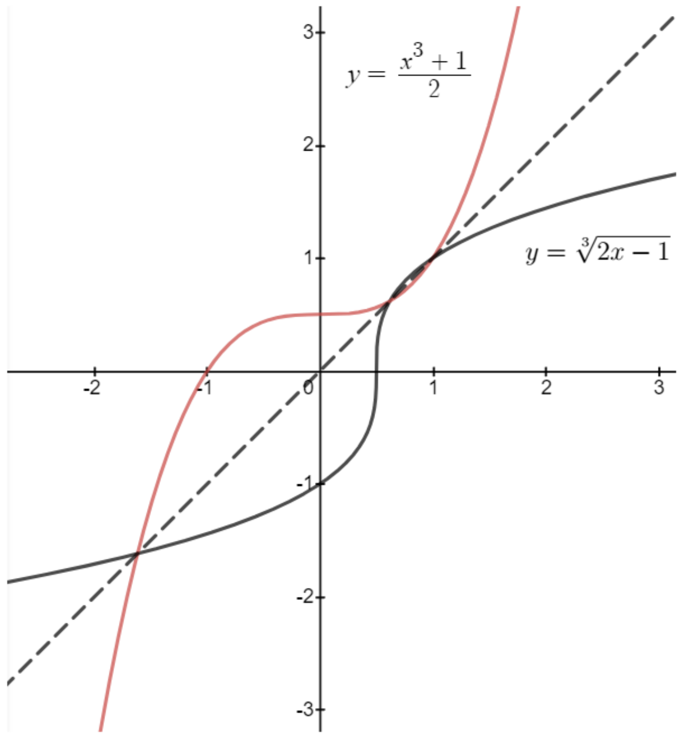
To prove that if an expression in indeed the inverse function, we use the cancellation property to check: Let $g(x) = \frac{x^3+1}{2}$ and $f(x) = \sqrt[3]{2x-1}$. If g and f are inverse functions to each other, it should be the case that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x^3+1}{2}\right) = \sqrt[3]{2\left(\frac{x^3+1}{2}\right) - 1} = \sqrt[3]{x^3} = x$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{2x-1}) = \frac{(\sqrt[3]{2x-1})^3 + 1}{2} = \frac{2x-1+1}{2} = x$$

Hence, f and g are inverse to each other.

The graph of f^{-1} is obtained by reflecting the graph of f about the line $y = x$.



Logarithmic Functions

The exponential function $f(x) = b^x$ we considered in Section 1.4 is either always increasing (if $b > 0$) or always decreasing (if $0 < b < 1$). Its graph must pass the HLT, so the exponential function is invertible. We want to study the inverse functions of the exponential functions.

Consider an exponential function $f(x) = 2^x$. Since $(-1, 0.5)$, $(0, 1)$, and $(1, 2)$ are on the graph of f, we know that the points $(0.5, -1)$, $(1, 0)$, and $(2, 1)$ must be on the graph of the inverse function f^{-1}. The graph of f has a horizontal asymptote $y = 0$. As the inverse function switches x-value and y-value, the graph of the inverse function f^{-1} will have a vertical asymptote $x = 0$ instead. Now we just have to find a graph with which the points $(0.5, -1)$, $(1, 0)$, and $(2, 1)$ would fit. Unfortunately there is no such function known. So we have to invent one! There is no natural way of defining such an inverse function except literally defining the inverse function such that it plays a role of switching input and output of the exponential function $f(x) = b^x$. We name this inverse function **log(arithmic) base b function** and use $\log_b$ to denote. For the exponential function $f(x) = 2^x$, its inverse would be denoted by $f^{-1}(x) = \log_2(x)$.

Q: How is the inverse function $\log_2$ defined?

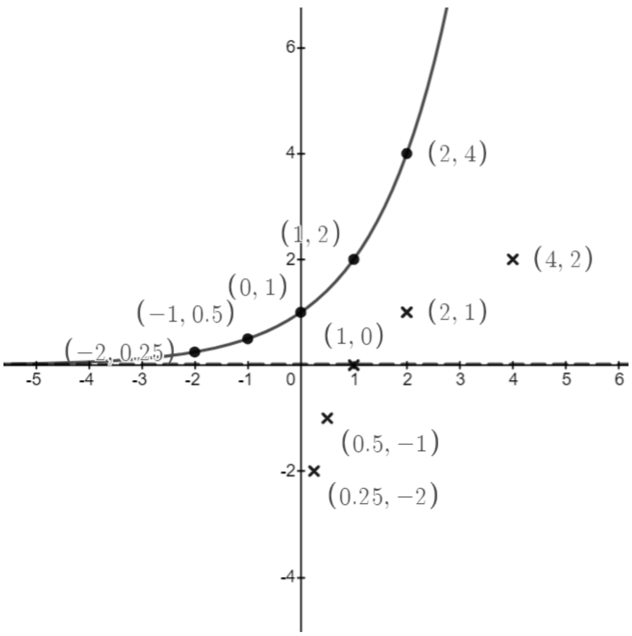
We can easily compute $f(-1) = 2^{-1} = \frac{1}{2}$, $f(0) = 2^0 = 1$, $f(1) = 2^1 = 2$, and $f(2) = 2^2 = 4$.

$f(x) = 2^x$ $\qquad$ $f^{-1}(x) = \log_2(x)$

Input x	Output $f(x)$
-1	$\frac{1}{2}$
0	1
1	2
2	4

$\Leftarrow$ Switch $\Rightarrow$

Input x	Output $f^{-1}(x)$
$\frac{1}{2}$	-1
1	0
2	1
4	2



After switching the input and the output, we can write $f^{-1}(\frac{1}{2}) = \log_2(\frac{1}{2}) = -1$, $f^{-1}(1) = \log_2(1) = 0$, $f^{-1}(2) = \log_2(2) = 1$, and $f^{-1}(4) = \log_2(4) = 2$.

Connecting theses ordered pairs will draw a graph of the inverse function $y = \log_2(x)$.

As x gets closer to 0, the function value of $\log_2(x)$ is becoming very large negative number. Hence, it will create a vertical asymptote $x = 0$ (the y-axis).

$$\log_b(x) = y \quad \Leftrightarrow \quad b^y = x$$

Since b^x and $\log_b(x)$ are inverse functions to each other,

i) $\log_b(b^x) = x$ for every $x \in \mathbb{R}$

ii) $b^{\log_b(x)} = x$ for every $x > 0$

Laws of Logarithms If x and y are positive numbers, then

1. $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$
2. $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$
3. $\log_b(x^r) = r\log_b(x)$ (where r is any real number)

Some properties:

(a) $\log_b(b) = 1$
(b) $\log_b(1) = 0$
(c) $\log_b(x)$ is undefined for $x \leq 0$

Change-of-Base Theorem:

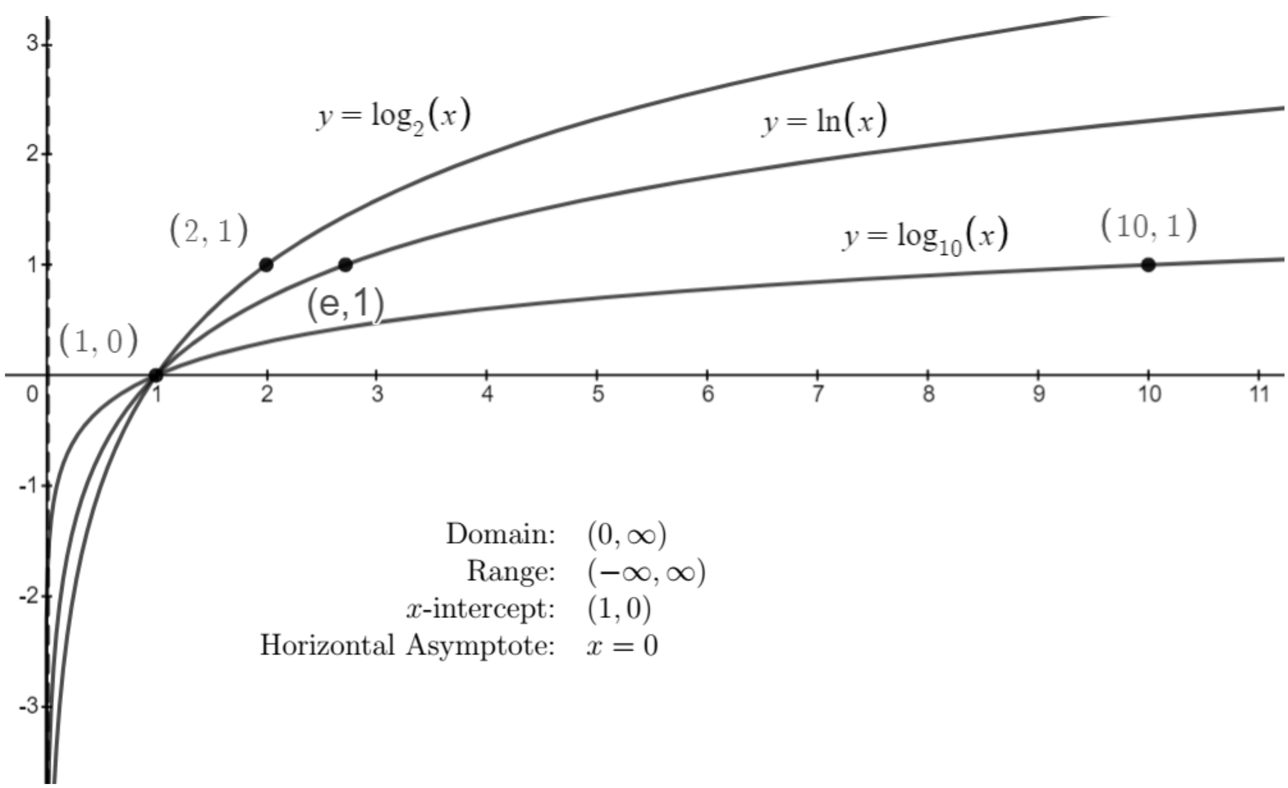
$$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$$

where the new base c is any positive real number such that $c \neq 1$.

If the base $b = 10$, then the logarithmic function $\log_{10}(x)$ is called a **common log** and is denoted with the base 10 omitted, i.e. $\log(x)$.

If the base $b = e \approx 2.71$, then the logarithmic function $\log_e(x)$ is called a **natural log** and is denoted with a special notation $\ln(x)$ (read "el-en").

If the base $b = 2$, then the logarithmic function $\log_2(x)$ is called a **binary log** and is denoted with a special notation $\mathrm{lb}(x)$.



Domain:	$(0, \infty)$
Range:	$(-\infty, \infty)$
x-intercept:	$(1, 0)$
Horizontal Asymptote:	$x = 0$

Example 3 (a) $\log_3(\sqrt{3}) = \frac{1}{2}$ since $3^{1/2} = \sqrt{3}$.

(b) $\log_5(15) = \log_5(5 \cdot 3) = \log_5(5) + \log_5(3) = 1 + \log_5(3)$

(c) $\log_{10}(0.0001) = \log_{10}(\frac{1}{10^4}) = \log_{10}(1) - \log_{10}(10^4) = 0 - 4\log_{10}(10) = -4(1) = -4$

(d) $\log_2(0)$ and $\log_2(-1)$ are undefined.

(e) $\log_1(x)$ does not make sense as 1 is an invalid base.

(f) $\log_{0.1}(0.001) = 3$ since $0.1^3 = 0.001$.

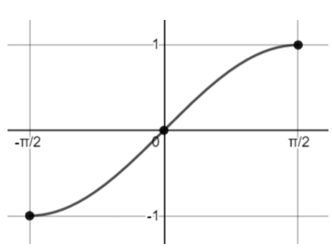
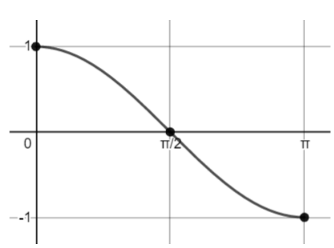
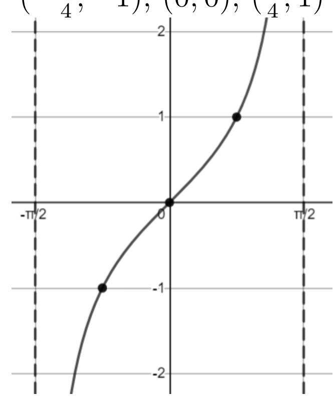
(g) $\log(\text{million}) = 6$ as $10^6 = 1{,}000{,}000$. $\log(0.001) = -3$ as $10^{-3} = 0.001$.

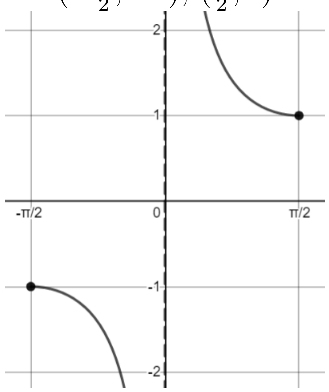
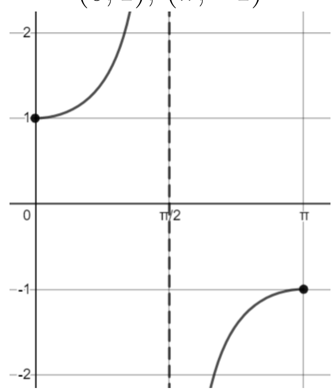
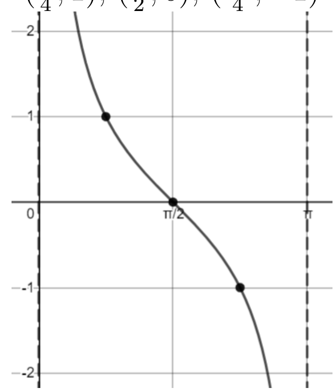
(h) $\ln(2) < 1$ and $\ln(3) > 1$ as $2 < e < 3$. $\ln(e) = 1$ and $\ln(e^5) = 5\ln(e) = 5(1) = 5$.

(i) $\mathrm{lb}(1024) = 10$ since $2^{10} = 1024$. $\mathrm{lb}(4294967296) = 32$ since $2^{32} = 4294967296$.

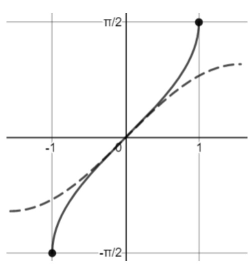
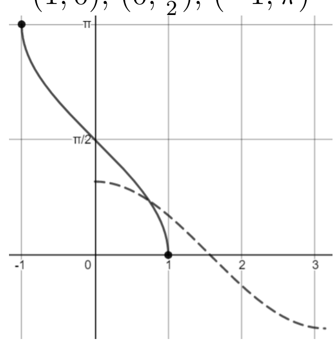
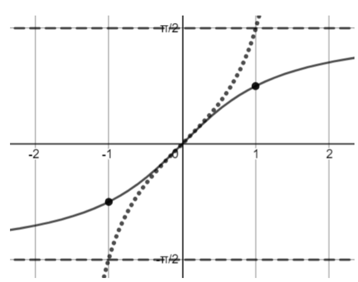
Inverse Trigonometric Functions

If trigonometric functions take angles (in radians or in degrees) as input values and give real numbers as output values, the **inverse trigonometric functions** should do exactly the opposite, i.e. take a real number as its input value and an angle as its output value. Hence, even if we see $\tan^{-1}(\pi)$, we should not interpret the input value π as an angle. $\tan^{-1}(\pi)$ would be an angle θ such that $\tan(\theta) = \pi$. According to a calculator, $\tan^{-1}(\pi) \approx 1.26$ or $72.34°$. All trig functions (six of them) are not one-to-one functions as they miserably fail the horizontal line test. Hence, in order to talk about the inverse functions of trig functions, we need to restrict the domain of each trig function. For each trig function, the domain has been carefully restricted so that the graph passes the horizontal line test (to be one-to-one function graph) and captures all possible trigonometric function values. Here are the restrictions of the domains of trig functions.

Function	$\sin(x)$	$\cos(x)$	$\tan(x)$
Restricted Domain	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[0, \pi]$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
Range	$[-1, 1]$	$[-1, 1]$	$(-\infty, \infty)$
Guide points	$(-\frac{\pi}{2}, -1)$, $(0, 0)$, $(\frac{\pi}{2}, 1)$	$(0, 1)$, $(\frac{\pi}{2}, 0)$, $(\pi, -1)$	$(-\frac{\pi}{4}, -1)$, $(0, 0)$, $(\frac{\pi}{4}, 1)$
			
Vertical Asymptote	None	None	$x = -\frac{\pi}{2}$ $x = \frac{\pi}{2}$

Function	$\csc(x)$	$\sec(x)$	$\cot(x)$
Restricted Domain	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$	$(0, \pi)$
Range	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, \infty)$
Guide points	$(-\frac{\pi}{2}, -1)$, $(\frac{\pi}{2}, 1)$ 	$(0, 1)$, $(\pi, -1)$ 	$(\frac{\pi}{4}, 1)$, $(\frac{\pi}{2}, 0)$, $(\frac{3\pi}{4}, -1)$ 
Vertical Asymptote	$x = 0$	$x = \frac{\pi}{2}$	$x = 0$ $x = \pi$

Now they are all one-to-one functions, so we can define inverse functions of trig functions. The notations are $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$, $\csc^{-1}(x)$, $\sec^{-1}(x)$, and $\cot^{-1}(x)$. Note that the domains and ranges will be switched. The x-coordinates and y-coordinates of guide points will be swapped. The vertical asymptotes will become the horizontal asymptotes. The graphs of the inverse trig functions will be the reflected images of the graphs above over the line $y = x$.

Function	$\sin^{-1}(x)$	$\cos^{-1}(x)$	$\tan^{-1}(x)$
Domain	$[-1, 1]$	$[-1, 1]$	$(-\infty, \infty)$
Range	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[0, \pi]$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
Guide points	$(-1, -\frac{\pi}{2})$, $(0, 0)$, $(1, \frac{\pi}{2})$ 	$(1, 0)$, $(0, \frac{\pi}{2})$, $(-1, \pi)$ 	$(-1, -\frac{\pi}{4})$, $(0, 0)$, $(1, \frac{\pi}{4})$ 
Horizontal Asymptote	None	None	$y = -\frac{\pi}{2}$ $y = \frac{\pi}{2}$

Function	$\csc^{-1}(x)$	$\sec^{-1}(x)$	$\cot^{-1}(x)$
Domain	$(-\infty,-1]\cup[1,\infty)$	$(-\infty,-1]\cup[1,\infty)$	$(-\infty,\infty)$
Range	$[-\frac{\pi}{2},0)\cup(0,\frac{\pi}{2}]$	$[0,\frac{\pi}{2})\cup(\frac{\pi}{2},\pi]$	$(0,\pi)$
Guide points	$(-1,-\frac{\pi}{2})$, $(1,\frac{\pi}{2})$ Graph omitted	$(1,0)$, $(-1,\pi)$ Graph omitted	$(1,\frac{\pi}{4})$, $(0,\frac{\pi}{2})$, $(-1,\frac{3\pi}{4})$ Graph omitted
Horizontal Asymptote	$y=0$	$y=\frac{\pi}{2}$	$y=0$ $y=\pi$

In order to evaluate $\sin^{-1}(x)$, we must have good idea about how $\sin(x)$ is evaluated as $\sin^{-1}(x)$ basically swaps the input and output values of $\sin(x)$. You must remember that the range of $\sin^{-1}(x)$ is $[-\frac{\pi}{2},\frac{\pi}{2}]$, i.e. $-\frac{\pi}{2}\leq\sin^{-1}(x)\leq\frac{\pi}{2}$, due to restriction of the sine graph. For instance, $\sin^{-1}(\frac{1}{2})=\frac{\pi}{6}$ as $\sin(\frac{\pi}{6})=\frac{1}{2}$. How about $\sin^{-1}(-\frac{1}{2})$? A shortcoming would be considering a usual unit circle and answering $\sin^{-1}(-\frac{1}{2})=\frac{7\pi}{6}$ (or $\frac{11\pi}{6}$). But none of these options are in the interval $[-\frac{\pi}{2},\frac{\pi}{2}]$. So we must find the coterminal angle to either $\frac{7\pi}{6}$ or $\frac{11\pi}{6}$ such that it falls in that interval. Only one (and it should be only one) that works would be $-\frac{\pi}{6}$. Thus, $\sin^{-1}(-\frac{1}{2})=-\frac{\pi}{6}$. If $x<0$, then the output value for $\sin^{-1}(x)$ should be an angle coming from Quadrant IV and must be negative. Here are what we should pay attention to:

Function	$\sin^{-1}(x)$	$\cos^{-1}(x)$	$\tan^{-1}(x)$
Range	$[-\frac{\pi}{2},\frac{\pi}{2}]$	$[0,\pi]$	$(-\frac{\pi}{2},\frac{\pi}{2})$
If $x>0$	$\sin^{-1}(x)$ in Q1 & $\sin^{-1}(x)>0$	$\cos^{-1}(x)$ in Q1 & $\cos^{-1}(x)>0$	$\tan^{-1}(x)$ in Q1 & $\tan^{-1}(x)>0$
If $x<0$	$\sin^{-1}(x)$ in Q4 & $\sin^{-1}(x)<0$	$\cos^{-1}(x)$ in Q2 & $\cos^{-1}(x)>0$	$\tan^{-1}(x)$ in Q4 & $\tan^{-1}(x)<0$

How would you evaluate $\csc^{-1}(x)$? Again a shortcoming would be thinking $\csc^{-1}(x)=\frac{1}{\sin^{-1}(x)}$ as $\csc(x)=\frac{1}{\sin(x)}$. But it is a bit more complicated than one can easily guess. $\csc^{-1}(x)$ would be an angle θ such that $\csc(\theta)=x$, i.e. $\frac{1}{\sin(\theta)}=x$, i.e. $\sin(\theta)=\frac{1}{x}$. Hence, $\theta=\sin^{-1}(\frac{1}{x})$ or $\csc^{-1}(x)=\sin^{-1}(\frac{1}{x})$. For example, $\csc^{-1}(2)=\sin^{-1}(\frac{1}{2})=\frac{\pi}{6}$ and $\csc^{-1}(-2)=\sin^{-1}(-\frac{1}{2})=-\frac{\pi}{6}$. Now then can we just understand that $\sec^{-1}(x)=\cos^{-1}(\frac{1}{x})$ and $\cot^{-1}(x)=\tan^{-1}(\frac{1}{x})$? Yes for the former, but absolutely not for the latter. If you see the chart above, the ranges for $\sin^{-1}(x)$ and $\csc^{-1}(x)$ are virtually the same (except at one point $x=0$) and the range for $\cos^{-1}(x)$ and $\sec^{-1}(x)$ are the same (except at $x=\frac{\pi}{2}$). But the ranges of $\tan^{-1}(x)$ and $\cot^{-1}(x)$ are quite different. So adjustment must be made.

$$\boxed{\csc^{-1}(x)=\sin^{-1}\left(\frac{1}{x}\right)} \quad \text{and} \quad \boxed{\sec^{-1}(x)=\cos^{-1}\left(\frac{1}{x}\right)}$$

for $x\in(-\infty,-1]\cup[1,\infty)$ (or $|x|\geq 1$). And

$$\boxed{\cot^{-1}(x)=\begin{cases}\tan^{-1}(\frac{1}{x}) & \text{for } x>0\\ \tan^{-1}(\frac{1}{x})+\pi & \text{for } x<0\\ \frac{\pi}{2} & \text{for } x=0\end{cases}}$$

For example, $\cos^{-1}(\sqrt{3}) = \tan^{-1}(\frac{1}{\sqrt{3}}) = \tan^{-1}(\frac{\sqrt{3}}{3}) = \frac{\pi}{6}$. But $\cot^{-1}(-\sqrt{3}) = \tan^{-1}(\frac{1}{-\sqrt{3}}) = \tan^{-1}(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6}$ would be WRONG. It should be $\cot^{-1}(-\sqrt{3}) = \tan^{-1}(\frac{1}{-\sqrt{3}})+\pi = \tan^{-1}(-\frac{\sqrt{3}}{3})+\pi = -\frac{\pi}{6}+\pi = \frac{5\pi}{6}$. In summary:

Function	$\csc^{-1}(x)$ $= \sin^{-1}(\frac{1}{x})$	$\sec^{-1}(x)$ $= \cos^{-1}(\frac{1}{x})$	$\cot^{-1}(x)$ $= \tan^{-1}(x)$ ($+\pi$ if $x < 0$)
Range	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$	$(0, \pi)$
If $x > 0$	$\csc^{-1}(x)$ in Q1 & $\csc^{-1}(x) > 0$	$\sec^{-1}(x)$ in Q1 & $\sec^{-1}(x) > 0$	$\cot^{-1}(x)$ in Q1 & $\cot^{-1}(x) > 0$
If $x < 0$	$\csc^{-1}(x)$ in Q4 & $\csc^{-1}(x) < 0$	$\sec^{-1}(x)$ in Q2 & $\sec^{-1}(x) > 0$	$\cot^{-1}(x)$ in Q2 & $\cot^{-1}(x) > 0$

$$\sin^{-1}(x) = y \Leftrightarrow \sin(y) = x \quad \text{and} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$
$$\sin^{-1}(\sin(x)) = x \quad \text{for} \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$
$$\sin(\sin^{-1}(x)) = x \quad \text{for} \quad -1 \le x \le 1$$

$$\cos^{-1}(x) = y \Leftrightarrow \cos(y) = x \quad \text{and} \quad 0 \le y \le \pi$$
$$\cos^{-1}(\cos(x)) = x \quad \text{for} \quad 0 \le x \le \pi$$
$$\cos(\cos^{-1}(x)) = x \quad \text{for} \quad -1 \le x \le 1$$

$$\tan^{-1}(x) = y \Leftrightarrow \tan(y) = x \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$
$$\tan^{-1}(\tan(x)) = x \quad \text{for} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$\tan(\tan^{-1}(x)) = x \quad \text{for all } x$$

Assigned Exercises: (p 66) 3 - 11 (odds), 15, 17, 21 - 31 (odds), 35 - 41 (odds), 47, 49, 61 - 67 (odds)