INF1003 Tutorial 4

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Topic: Propositional Logic
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1. Let p, q, and r be the propositions

- p: You have COVID-19;
- q: You miss the final exam;
- r: You pass the course.

Express each of the following compound propositions as an English sentence.

(a) $q \to \neg r$.

Solution. Here q means "you miss the final exam" and $\neg r$ means "you do not pass the course". The implication $q \to \neg r$ is read as: *If you miss the final exam, then you do not pass the course.*

(b) $(p \to \neg r) \vee (q \to \neg r)$.

Solution. $p \to \neg r$ means "If you have COVID-19, then you do not pass the course", and $q \to \neg r$ means "If you miss the final exam, then you do not pass the course". Their disjunction is: *Either, if you have COVID-19 then you do not pass the course, or, if you miss the final exam then you do not pass the course.*

(c) $(p \wedge q) \vee (\neg q \wedge r)$.

Solution. $p \wedge q$ is "you have COVID-19 and you miss the final exam", while $\neg q \wedge r$ is "you do not miss the final exam and you pass the course". Thus the whole proposition says: *Either you have COVID-19 and miss the final exam, or you do not miss the final exam and you pass the course.*

2. Let p, q and r be the propositions

- p: You get an A on the final exam;
- q: You do every exercise in the textbook;
- r: You get an A in this course.

Write the following propositions using p, q, and r, and logical connectives.

(a) You get an A in this course, but you do not do every exercise in the textbook.

Solution. "You get an A in this course" is r and "you do not do every exercise in the textbook" is $\neg q$. The word "but" is logically the same as "and", so the proposition is

$$r \wedge \neg q.$$

(b) You get an A on the final exam, but you do not do every exercise in the textbook; nevertheless, you get an A in this course.

Solution. "You get an A on the final exam" is p, "you do not do every exercise in the textbook" is $\neg q$, and "you get an A in this course" is r. Joining them with "and" gives

$$p \wedge \neg q \wedge r.$$

(c) Getting an A on the final exam and doing every exercise in the textbook is sufficient for getting an A in this course.

Solution. "Getting an A on the final exam and doing every exercise in the textbook" is $p \wedge q$, which is a sufficient condition for r. So the proposition is

$$(p \wedge q) \rightarrow r.$$

(d) You will get an A in this course if and only if you either do every exercise in the textbook or you get an A on the final exam.

Solution. "You will get an A in this course" is r, and "you either do every exercise in the textbook or you get an A on the final exam" is $q \vee p$. "If and only if" is $\leftrightarrow$, so we obtain

$$r \leftrightarrow (q \vee p).$$

3. Write each of these statements in the form "if p, then q" in English.

(a) It is necessary to hike 2 km to get to the top of Bukit Timah Hill.

Solution. Saying that hiking 2 km is *necessary* to get to the top means that reaching the top implies that you have hiked 2 km. In "if p, then q" form:

If you get to the top of Bukit Timah Hill, then you must have hiked 2 km.

(b) If you drive more than 650 km, you will need to buy petrol.

Solution. This is already in the desired form:

If you drive more than 650 km, then you will need to buy petrol.

(c) Xiaoming will go swimming unless the water is too cold.

Solution. "p unless q" is logically equivalent to "if not q, then p". Here, let p be "Xiaoming will go swimming" and let q be "the water is too cold". Then the statement becomes:

If the water is not too cold, then Xiaoming will go swimming.

4. Construct a complete truth table for each of the following compound propositions.

(a) $(p \to q) \leftrightarrow (\neg q \to \neg p)$.

Solution.

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

The final column is always T, so the two implications are logically equivalent.

(b) $(p \oplus q) \wedge (p \oplus \neg q)$.

Solution. Recall that $p \oplus q$ is true exactly when p and q have different truth values.

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

The final column is always F, so the compound proposition is a contradiction.

(c) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$.

Solution.

p	q	r	$p \leftrightarrow q$	$\neg q$	$\neg q \leftrightarrow r$	$(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	F	F	F
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

(d) $\big((p \to q) \to r\big) \to s$.

Solution. Let $A = (p \to q)$ and $B = (A \to r)$. Then the whole formula is

$B \to s$.

p	q	r	s	$p \to q$	$(p \to q) \to r$	$\big((p \to q) \to r\big) \to s$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	T

(e) $(p \wedge r \wedge s) \leftrightarrow (p \vee q)$.

Solution. Here the two sides are $p \wedge r \wedge s$ and $p \vee q$.

p	q	r	s	$p \wedge r \wedge s$	$p \vee q$	$(p \wedge r \wedge s) \leftrightarrow (p \vee q)$
T	T	T	T	T	T	T
T	T	T	F	F	T	F
T	T	F	T	F	T	F
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	F
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	T	F
F	T	F	T	F	T	F
F	T	F	F	F	T	F
F	F	T	T	F	F	T
F	F	T	F	F	F	T
F	F	F	T	F	F	T
F	F	F	F	F	F	T

5. Suppose that

- Smartphone A has 8 GB RAM and 64 GB ROM, and the resolution of its camera is 12 MP;

- Smartphone B has 16 GB RAM and 128 GB ROM, and the resolution of its camera is 5 MP; and
- Smartphone C has 4 GB RAM and 64 GB ROM, and the resolution of its camera is 8 MP.

Determine the truth value of each of the following propositions. Make sure to provide appropriate derivations and explanations.

(a) Smartphone B has the most RAM of these three smartphones.

Solution. RAM values: A has 8 GB, B has 16 GB, C has 4 GB. Since $16 > 8$ and $16 > 4$, B has more RAM than both A and C, so B indeed has the most RAM. The proposition is therefore **true**.

(b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

Solution. ROM values: C has 64 GB, B has 128 GB, so C does *not* have more ROM than B. Camera resolution: C has 8 MP whereas B has 5 MP, so C *does* have a higher resolution camera. The statement is *(more ROM)* $\vee$ *(higher resolution camera)* $= F \vee T = T$. Hence the proposition is **true**.

(c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

Solution. Comparing B to A:

- RAM: $16 > 8$ (B has more RAM) – true.
- ROM: $128 > 64$ (B has more ROM) – true.
- Camera: 5 MP vs 12 MP (B's camera resolution is lower than A's) – false.

The proposition is a conjunction of the three comparisons, so it is

$$T \wedge T \wedge F = F.$$

Thus the proposition is **false**.

(d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

Solution. Antecedent: "B has more RAM and more ROM than C":

$$16 > 4 \quad \text{and} \quad 128 > 64,$$

which is true. Consequent: "B has a higher resolution camera than C":

$$5 > 8$$

is false. So the implication has the form $T \to F$, which is false. Therefore the proposition is **false**.

(e) If Smartphone A has higher resolution camera than Smartphone B and Smartphone C, it has the most RAM of all the three smartphones.

Solution. Antecedent: A's camera resolution is 12 MP, B's is 5 MP, and C's is 8 MP. We have $12 > 5$ and $12 > 8$, so the antecedent is true. Consequent: "A has the most RAM of all three smartphones" would require

$$8 > 16 \quad \text{and} \quad 8 > 4,$$

which is false because B has more RAM than A. Hence the implication again has the form $T \to F$, which is false. The proposition is **false**.

6. Which of the following statement(s) is/are correct? Explain, for each of the statements, whether they are logically equivalent or not.

(a) $p \vee q \equiv q \vee p$.

Solution. Disjunction is commutative: for every choice of truth values of p and q, $p \vee q$ and $q \vee p$ have the same truth value. So the equivalence is **true**.

(b) $p \leftrightarrow q \equiv \neg(p \to q) \wedge (\neg q \to p)$.

Solution. Consider $p = T$ and $q = F$.

$$p \leftrightarrow q = T \leftrightarrow F = F.$$

On the other hand,

$$p \to q = T \to F = F, \quad \neg(p \to q) = T,$$

and

$$\neg q = T, \quad (\neg q \to p) = T \to T = T,$$

so

$$\neg(p \to q) \wedge (\neg q \to p) = T \wedge T = T.$$

The two sides have different truth values for this assignment, so they are *not* logically equivalent. The stated equivalence is therefore **false**.

(c) $p \vee q \equiv \neg p \to q$.

Solution. We transform the implication:

$$\neg p \to q \equiv \neg(\neg p) \vee q \equiv p \vee q.$$

Thus $p \vee q$ and $\neg p \to q$ are logically equivalent, so the equivalence is **true**.

7. Find the converse, contrapositive, and inverse of the following statement:

"When the customer's insurance premium payment does not arrive by the deadline, an email reminder is sent."

Solution. Let

p : The customer's insurance premium payment does not arrive by the deadline, q : An emai

The original statement is the implication $p \to q$.

- **Converse:** $q \to p$. "If an email reminder is sent, then the customer's insurance premium payment does not arrive by the deadline."
- **Contrapositive:** $\neg q \to \neg p$. "If an email reminder is not sent, then the customer's insurance premium payment arrives by the deadline."
- **Inverse:** $\neg p \to \neg q$. "If the customer's insurance premium payment arrives by the deadline, then an email reminder is not sent."

8. Determine whether $\neg q \wedge (p \to q) \to \neg p$ is a tautology.

Solution. We transform the expression using logical equivalences.

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\begin{aligned}
\neg q \wedge (p \to q) \to \neg p &\equiv \neg q \wedge (\neg p \vee q) \to \neg p && \text{since } (p \to q) \equiv \neg p \vee q, \\
&\equiv \big((\neg q \wedge \neg p) \vee (\neg q \wedge q)\big) \to \neg p && \text{distributivity}, \\
&\equiv (\neg q \wedge \neg p) \to \neg p && \text{since } (\neg q \wedge q) \text{ is always false}, \\
&\equiv \neg(\neg q \wedge \neg p) \vee \neg p && \text{because } (A \to B) \equiv \neg A \vee B, \\
&\equiv (q \vee p) \vee \neg p && \text{De Morgan and double negation}, \\
&\equiv q \vee (p \vee \neg p) && \text{associativity and commutativity}, \\
&\equiv q \vee \mathrm{T} && \text{since } (p \vee \neg p) \text{ is a tautology}, \\
&\equiv \mathrm{T}.
\end{aligned}
$$

As the expression simplifies to a statement that is always true, $\neg q \wedge (p \to q) \to \neg p$ is a **tautology**.