Lecture 02: Probability Theory

INF1004 Mathematics II

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1 Types of Probability

Probability is defined as a number in the range $[0, 1]$ that is assigned to an event associated with a random experiment. There are two interpretations of probability theory that guide how such values are assigned:

1.1 Classical Probability

In classical probability, all outcomes in the sample space are assumed to be equally likely. The probability of a random event A occuring is given as

$$P(A) = \frac{\text{Number of different outcomes in A}}{\text{Total number of possible outcomes}}$$

Note: This definition is not very practical for everyday life because it assumes the outcomes are equally likely.

1.2 Empirical Probability

In empirical probability theory, the probability of an event A occurring approximated by the relative frequency of its occurrence over a large number of trials such that

$$P(A) = \frac{\text{Number of times an outcome occurs}}{\text{Total number of observations}}$$

This interpretation relies on collecting data over the years or existing historical data to propose such relative frequencies.

1.3 Definitions

- We define a sample space S as the set of all possible outcomes of a random phenomenon.

- An event A is defined as the set of outcomes under investigation.

- By definition of probability, the following statements must hold to be valid:
$$0 \leq P(A) \leq 1, \ P(S) = 1$$

- If $A_1, A_2, \ldots$ are mutually exclusive events, then

$$P\left(\bigcup_i A_i\right) = \sum_i P(A_i)$$

  .

2 Complementary Events

3   Union of Events

4 Intersection of Events

5 Inclusion-Exclusion Principle

6 Multiplicative Rule of Probability

7 Independent and Mutually Exclusive Events

8 Law of Total Probability

9   Bayes' Theorem