Hierarchical Walk-through

Tim Frasier

Data

- Milk composition and brain characteristics for a range of primates.

clade	kcal.per.g
Strepsirrhine (Strep)	0.49
New World Monkey (NWM)	0.80
Old World Monkey (OWM)	0.73
Ape	0.48




Model

```
model {
  // Definitions
  vector[N] mu;

  // Likelihood
  for (i in 1:N) {
    mu[i] = b1[x[i]];
    y[i] ~ normal(mu[i], sigma[x[i]]);
  }

  // Priors
  for (j in 1:nxLevels) {
    b1[j]    ~ normal(shopMean, shopMeanSD);
    sigma[j] ~ cauchy(1, 1);
  }

  // Hyperpriors
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Stan starts by selecting values, from the prior, for parameters with defined priors (the hyperpriors).

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shopMean    = -0.2784659
shopMeanSD  = 1.215533
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These values are then used to create a prior value for the mean effect of each level in our nominal variable.

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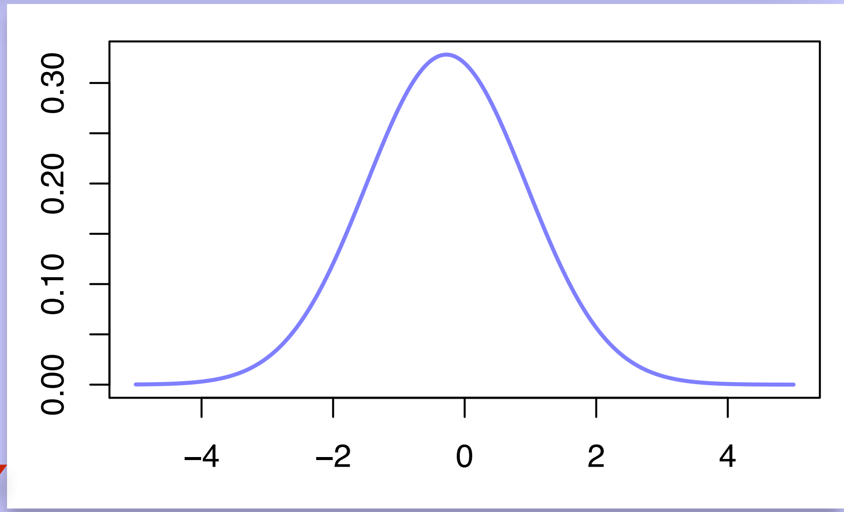


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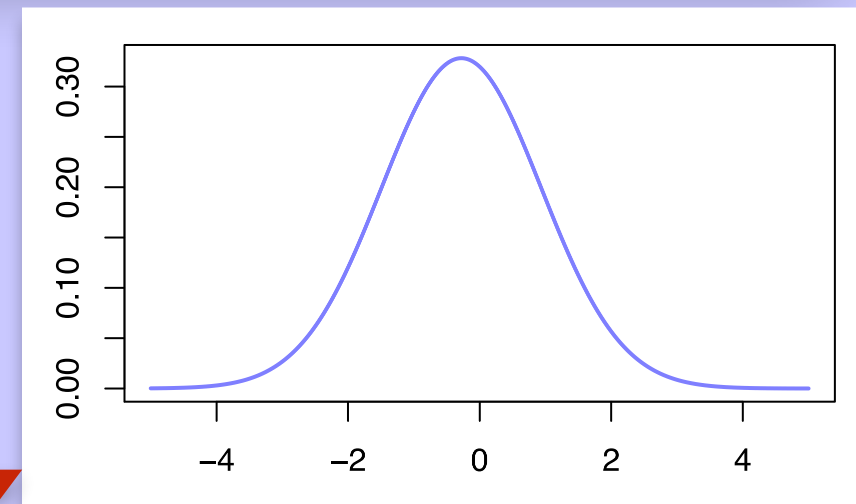


```
b1[Strep] = -0.6652136
b1[NWM]   = -0.786263
b1[OWM]   = -2.408034
b1[Ape]   = -2.666315
```

Model

Sigma values for each level drawn from defined prior.

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  // Definitions
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  // Likelihood
  for (i in 1:N) {
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    y[i] ~ normal(mu[i], sigma[x[i]]);
  }

  // Priors
  for (j in 1:nxLevels) {
    b1[j]     ~ normal(-0.2784659, 1.215533);
    sigma[j] ~ cauchy(1, 1);
  }

  // Hyperpriors
  shopMean    ~ normal(0, 1);
  shopMeanSD ~ normal(1, 1);
}
```

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sigma[Strep] = 1.321518
sigma[NWM]   = 0.1529242
sigma[OWM]   = 0.1786416
sigma[Ape]   = 17.08555
```

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```

Model

Calculate the likelihood of the data given these values for each parameter.

```
model {
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  vector[N] mu;

  // Likelihood
  for (i in 1:N) {
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  }

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  for (j in 1:nxLevels) {
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Model

- Propose new values for the parameters in the same manner
- Calculate the likelihood of the data with these new values
  - If higher, accept new values
  - If not, follow some rules to decide whether or not to accept new values

Model

Will result in posterior probability distributions for the mean and s.d. for the effects of being in each group...

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}
```

```
b1[Strep]
b1[NWM]
b1[OWM]
b1[Ape]
```

```
sigma[Strep]
sigma[NWM]
sigma[OWM]
sigma[Ape]
```

Model

...as well as the mean and s.d. describing the normal distribution from which all effects (across all levels) arose.

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model {
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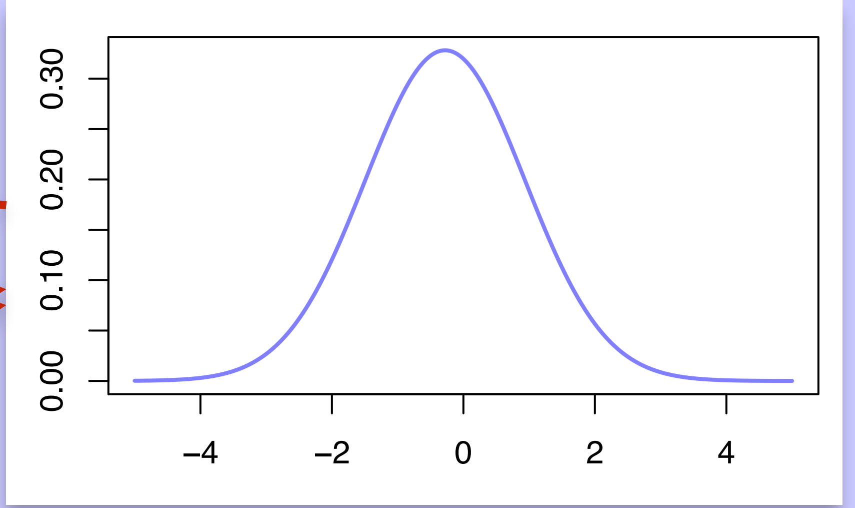

Does that help?