Hierarchical Walk-through

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Data

• Milk composition and brain characteristics for a range of primates.

clade	kcal.per.g
Strepsirrhine (Strep)	0.49
New World Monkey (NWM)	0.80
Old World Monkey (OWM)	0.73
Ape	0.48









```
model {
 // Definitions
  vector[N] mu;
  // Likelihood
  for (i in 1:N) {
    mu[i] = b1[x[i]];
    y[i] ~ normal(mu[i], sigma[x[i]]);
  // Priors
  for (j in 1:nxLevels) {
    b1[j] ~ normal(shopMean, shopMeanSD);
    sigma[j] \sim cauchy(1, 1);
  // Hyperpriors
  shopMean ~ normal(0, 1);
  shopMeanSD ~ normal(1, 1);
```

Stan starts by selecting values, from the prior, for parameters with defined priors (the hyperpriors).

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   y[i] ~ normal(mu[i], sigma[x[i]]);
  // Priors
  for (j in 1:nxLevels) {
    b1[j] ~ normal(shopMean, shopMeanSD);
    sigma[j] \sim cauchy(1, 1);
                                                     shopMean
                                                                  = -0.2784659
  // Hyperpriors
                                                     shopMeanSD
                                                                  = 1.215533
  shopMean
            ~ normal(0, 1);
  shopMeanSD ~ normal(1, 1);
```

These values are then used to create a prior value for the mean effect of each level in our nominal variable.

```
model {
                                                    0.30
  // Definitions
  vector[N] mu;
                                                    0.20
                                                    0.10
  // Likelihood
  for (i in 1:N) {
                                                    0.00
    mu[i] = b1[x[i]];
    y[i] ~ normal(mu[i], sigma[x[i]]);
  // Priors
  for (j in 1:nxLevels) {
    b1[j] ~ normal(-0.2784659, 1.215533);
    sigma[j] ~ cauchy(1, 1);
                                                       shopMean
  // Hyperpriors
                                                       shopMeanSD
  shopMean
            ~ normal(0, 1);
  shopMeanSD ~ normal(1, 1);
```

```
= -0.2784659
= 1.215533
```

These values are then used to create a prior value for the mean effect of each level in our nominal variable.

```
model {
                                                      0.30
  // Definitions
  vector[N] mu;
                                                       0.20
                                                       0.10
  // Likelihood
  for (i in 1:N) {
                                                       0.00
    mu[i] = b1[x[i]];
    y[i] ~ normal(mu[i], sigma[x[i]]);
                                                                           2
  // Priors
  for (j in 1:nxLevels) {
    b1[j] ~ normal(-0.2784659, 1.215533);
sigma[j] ~ cauchy(1, 1);
                                                      b1[Strep] = -0.6652136
  // Hyperpriors
                                                      b1[NWM] = -0.786263
  shopMean
             ~ normal(0, 1);
                                                      b1[OWM] = -2.408034
  shopMeanSD ~ normal(1, 1);
                                                      b1[Ape]
                                                                  = -2.666315
```

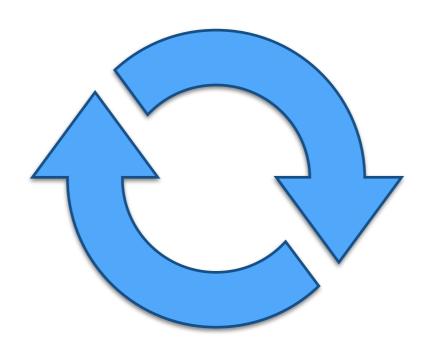
Sigma values for each level drawn from defined prior.

```
model {
 // Definitions
  vector[N] mu;
  // Likelihood
  for (i in 1:N) {
   mu[i] = b1[x[i]];
   y[i] ~ normal(mu[i], sigma[x[i]]);
                                                sigma[Strep] = 1.321518
                                                sigma[NWM] = 0.1529242
                                                sigma[OWM] = 0.1786416
  // Priors
                                                sigma[Ape]
                                                              = 17.08555
  for (j in 1:nxLevels) {
   b1[j] \sim normal(-0.2784659, 1.215533);
   sigma[j] \sim cauchy(1, 1);
                                                b1[Strep] = -0.6652136
  // Hyperpriors
                                                b1[NWM] = -0.786263
  shopMean ~ normal(0, 1);
                                                b1[OWM] = -2.408034
  shopMeanSD ~ normal(1, 1);
                                                b1[Ape] = -2.666315
```

Calculate the likelihood of the data given these values for each parameter.

```
model {
 // Definitions
  vector[N] mu;
  // Likelihood
  for (i in 1:N) {
   mu[i] = b1[x[i]];
   y[i] ~ normal(mu[i], sigma[x[i]]);
                                                sigma[Strep] = 1.321518
                                                sigma[NWM] = 0.1529242
                                                sigma[OWM]
                                                             = 0.1786416
  // Priors
                                                             = 17.08555
                                                sigma[Ape]
  for (j in 1:nxLevels) {
   b1[j] \sim normal(-0.2784659, 1.215533)
   sigma[j] \sim cauchy(1, 1);
                                                b1[Strep] = -0.6652136
                                                b1[NWM]
                                                          = -0.786263
  // Hyperpriors
                                                b1[OWM] = -2.408034
  shopMean ~ normal(0, 1);
                                                          = -2.666315
                                                b1[Ape]
  shopMeanSD ~ normal(1, 1);
```

- Propose new values for the parameters in the same manner
- Calculate the likelihood of the data with these new values
 - If higher, accept new values
 - If not, follow some rules to decide whether or not to accept new values



Will result in posterior probability distributions for the mean and s.d. for the effects of being in each group...

```
model {
  // Definitions
  vector[N] mu;
                                                     b1[Strep]
  // Likelihood
                                                     b1[NWM]
  for (i in 1:N) {
                                                     b1[OWM]
    mu[i] = b1[x[i]];
    y[i] ~ normal(mu[i], sigma[x[i]]);
                                                     b1[Ape]
  // Priors
  for (j in 1:nxLevels) {
                                                     sigma[Strep]
    b1[j] ~ normal(shopMean, shopMeanSD);
                                                     sigma[NWM]
    sigma[j] < cauchy(1, 1);
                                                     sigma[OWM]
                                                     sigma[Ape]
  // Hyperpriors
  shopMean
            ~ normal(0, 1);
  shopMeanSD ~ normal(1, 1);
```

...as well as the mean and s.d. describing the normal distribution from which all effects (across all levels) arose.

```
model {
  // Definitions
  vector[N] mu;
  // Likelihood
  for (i in 1:N) {
    mu[i] = b1[x[i]];
    y[i] ~ normal(mu[i], sigma[x[i]]);
  // Priors
  for (j in 1:nxLevels) {
                                                            0.30
    b1[j] ~ normal(shopMean, shopMeanSD);
                                                            0.20
    sigma[j] \sim cauchy(1, 1);
                                                            0.10
  // Hyperpriors
                                                            0.00
  shopMean ~ normal(0, 1);
                                                                       -2
                                                                                 2
  shopMeanSD ~ normal(1, 1);
```

Does that help?