Fluid Mechanics

Density: $\rho = \frac{M}{V}$; pressure: $p = \frac{\Delta F_{\perp}}{\Delta A}$; force of buoyancy: $F_B = \rho_{fluid} V_{object} g$; mass conservation means:

$$\rho A v = Const.$$

For incompressible fluids: $\rho = Const.$ Bernoulli's equation for any fluid:

$$\frac{p}{\rho} + \frac{1}{2} v^2 + gh = Const.$$

Power flow (with flow $Q = Av$):

$$pQ + \frac{1}{2} \rho v^2 Q + \rho g h Q = Const.$$

Distribution of speeds in a cylindrical pipe due to viscosity:

$$v = \frac{P_1 - P_2}{4\eta l}(R^2 - r^2)$$

where R is the radius of the pipe and r is the radial distance from the center; P_1 is the pressure at one end; l is the length of the pipe and η is the viscosity of the fluid. For laminar flow, the flow rate Q in m^3/s is given by:

$$Q = \frac{\pi R^4}{8\eta l}(P_1 - P_2)$$

The force needed to stretch the surface area is

$$F = 2\gamma l$$

where, the length is l and γ is the surface tension. Pressure across a drop of liquid (δP): consider a spherical water drop of radius R

$$\delta P = \frac{2\gamma}{R}$$

Capillary action: height of liquid inside a tube of radius r and contact angle θ

$$h = \frac{2\gamma \cos\theta}{\rho g r}$$

Thermodynamics

Linear expansion: $\Delta L = \alpha L_0 \Delta T$; volume expansion: $\Delta V = \beta V_0 \Delta T$
Specific Heat: $\Delta Q = mc\Delta T$; in term of moles (n): $\Delta Q = nC\Delta T$; the mass of a mole M then $m = nM$
Heat transfer in change of phase: $\Delta Q = mL$

Heat transfer. Conduction: $H = \frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{L}$; the R factor $R = \frac{L}{k}$.
Radiation: $H = Ae\sigma T^4$ where $\sigma = 5.67\times10^{-8}$; Wien's law: $f_{max} = 5.879\times10^{10} T$

Dependence of entropy on energy: $\frac{1}{T} = \frac{dS}{dE}$. Alternatively: $\Delta S = \int \frac{dQ}{T}$

$T_F = \frac{9}{5} T_C + 32 \qquad T = T_C + 273.15$

Ideal Gas

$C_V = \frac{d}{2} k_B$	$\frac{C_P}{C_V} = \gamma = 1 + \frac{2}{d}$	$v_{rms} = \sqrt{\frac{3k_B T}{m}}$
$PV = Nk_B T$	$\Delta Q = \Delta W + \Delta E$	$E_{th} = \frac{d}{2} N k_B T$
$\Delta W = \int P dV$	$S/k_B = N \ln V + \frac{d}{2} N \ln T + C$	

Isochoric:

$\Delta W = 0$ $\Delta E_{th} = \Delta Q$ $\Delta Q = C_V N \Delta T$ $\Delta E_{th} = N C_V \Delta T$

$\Delta S = N C_V \ln \frac{T_f}{T_i}$ $\frac{T_i}{P_i} = \frac{T}{P}$

Isentropic:

$\Delta S = 0$ $P_i V_i^{\gamma} = P V^{\gamma}$ or $T_i V_i^{\gamma - 1} = T V^{\gamma - 1}$ $\Delta Q = 0$

$\Delta E_{th} = -\Delta W = N C_V \Delta T$ $\Delta W = \frac{1}{\gamma - 1}(PV - P_i V_i)$

Isobaric:

$\Delta Q = C_P N \Delta T$ $C_P = C_V + k_B$ $\frac{T_i}{V_i} = \frac{T}{V}$ $\Delta S = N C_P \ln \frac{T_f}{T_i}$ $\Delta W = P(V_f - V_i)$

Isothermal:

$\Delta E_{th} = 0$ $\Delta Q = \Delta W$ $\Delta W = N k_B T \ln \frac{V_f}{V_i}$

$\Delta S = k_B N \ln \frac{V_f}{V_i}$ $P_i V_i = PV$

Other processes:

Specific heat capacity: $\Delta Q = m c_V \Delta T$ Associate change in entropy: $\Delta S = m c_V \ln \frac{T_f}{T_i}$ (for warming or cooling)

for a change of phase (at a constant temperature): $\Delta S = \frac{\Delta Q}{T}$

so, for melting or vaporization: $\Delta S = \frac{mL}{T}$ and with negative sign for solidification or condensation.

Heat engines and heat pumps:

Working in a cycle at the end we have: $\Delta E_{th} = 0$, $\Delta Q_{Total} = -|\Delta W_{Total}|$

$\Delta S = \Delta S_{device} + \Delta S_{environment} \geq 0$ but the device goes back to original so $\Delta S_{device} = 0$; so $\Delta S_{environment} \geq 0$

$e = \frac{|\Delta W|}{|\Delta Q_{in}|}$ $e = 1 - \frac{|\Delta Q_{out}|}{|\Delta Q_{in}|}$ Carnot cycle: $e_c = 1 - \frac{T_L}{T_H}$ Brayton cycle: $e = 1 - \left(\frac{P_H}{P_L}\right)^{(1/\gamma) - 1}$

Otto cycle $e = 1 - \frac{1}{r^{\gamma - 1}}$

Refrigerators and heat pumps (because the cycle is reverse compared to heat engines):

$|\Delta Q_{out}| > |\Delta Q_{in}|$ $T_H > T_L$ $|\Delta W| = |\Delta Q_{out}| - |\Delta Q_{oin}|$

$C.O.P_{cooling} = \frac{|\Delta Q_{in}|}{|\Delta W|}$ $C.O.P_{heating} = \frac{|\Delta Q_{out}|}{|\Delta W|}$

$C.O.P_{cooling,max} = \frac{T_L}{T_H - T_L}$ $C.O.P_{heating,max} = \frac{T_H}{T_H - T_L}$