Tuneable & Tileable Helmholtz Resonators

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1 Introduction

The following concerns the development of tuneable acoustic resonators/absorbers for the application of 3D printing. This has been developed under Simplectic[1] and authored by Timothy J. Stamm. Simplectic has approved this design to be open source, though retains the intellectual property, and may revoke the open source status at any time for whatever reason[2]. You may make copies, edits and produce goods based off of this design. An honorable mention is appreciated!

The purpose of this project is to give people with sensory issues[3] a way to control their sensory environment while being as inexpensive as possible and easy to construct[4]. As such all designs can be made from cardboard or other readily available (inexpensive/recycleable) material. Simplectic is not responsible for any damages or liabalities resulting from use of the designs (e.g. if you use cardboard...make it fireproof!).

[1]Registered in Utrecht, The Netherlands, KvK88905470

[2]This is mostly to be able to do something about people possible misusing the designs, such as unethical/unseemly economic behavior. Behave!

[3]Such as persons with ASD, ADHD, those who are otherwise highly sensitive (eg misophones)

[4]So that quality noise control is not only a luxury for those who can afford it

2 Hexagonal Helmholtz Resonators

A Helmholtz Resonator is an acoustic system consisting of a body and a neck (think beer bottle). The air in the body acts like a spring, the air in the neck acts like a mass. This system has a resonant frequency, and as such acts like a frequency filter around that frequency (like when you blow over the bottle). Absorption occurs formostly due to viscious forces in the neck section[5] and to a lesser extent by thermal damping in the body[6]. This model is very simplistic (is an approximation), and one quickly falls out of the parameter space in which it is applicable. The reader is referred to [1] for more information on the subject.

2.1 Derivation of basic formulae

The formula used for the frequency of a Helmholtz Resonator is

$$\nu_R = \frac{c}{2\pi}\sqrt{\frac{A_{neck}}{V_{body}L}} \tag{1}$$

Where c the speed of sound in the medium, A_{neck} the area of the neck, V_{body} the volume of the body and $L_{neck} = L_0 + 0.3D$ the effective length of the neck, with hydraulic diameter $D = \frac{4A_{neck}}{P}$ and P the wetted perimiter. This was taken from the wikipedia page [2][7].

The idea here, hardly an original one, is to use a multitude of Helmholtz Resonators to pick out the unwanted frequencies. It has to be easy to use and scalable. Hence a multitude of space-filling tilings (tesselations) were considered [4]. Finally the hexagon was chosen[8]. The body and the neck sections will both be hexagonal. The Hexagon of sides a has as its area

$$A = \frac{3\sqrt{3}}{2}a^2 \tag{2}$$

Writing $V_{body} = A_{body}\mathcal{L}$ we can see that in 1 we get

$$\frac{A_{neck}}{V_{body}} = (\frac{b}{a})^2\mathcal{L}^{-1} \tag{3}$$

Where a and b are the sides of the body and neck hexagons respectively. Next we want the expression for L, starting with[9]

[5] IF the neck length is at least twice as long as the diameter of the hole

[6] IF the body diameter is significantly larger than the body length...up to a point when absorption inverts again

[7] Which is in agreement with the Wiley Engineering Acoustics book[3]

[8] After insistence by the author's dear friend that 'Hexagons are bestagons' (though the author would still like to embed that tesselation in a hyperbolic space or use stereographic projections of E(N) onto the plane to create quasi-crystalline tilings...)

[9] Note that $P = 6b$, the wetted perimeter is simply the circumference here

$$D = \frac{4A}{P} = \frac{3\sqrt{3}}{2}\frac{4b^2}{6b} = \sqrt{3}b \quad (4)$$

Putting it all together gives

$$\nu_R = \frac{c}{2\pi}\frac{b}{a}(\mathcal{L}L)^{-\frac{1}{2}} \quad (5)$$

Where $L = L_{neck} + 0.3\sqrt{3}b$ and all other variables as previously defined. Note that for $L_{neck} \gg b$, $L \to L_{neck}$. In addition we can write $Q = \frac{b}{a}$ so that

$$\nu_R \approx \frac{c}{2\pi}Q(\mathcal{L}L_{neck})^{-\frac{1}{2}} \quad (6)$$

We will stick to this section of the parameter space. This is due to ease of tuning.

2.2 Tuning

We can, in principle, tune a Hexagonal Helmholtz Resonator (Hex-Res) by altering any of the parameters in [5]. The author would like to stress that the model quickly falls out of the parameter space. That is, eq [1] is an approximation of the far more complicated aero-acoustical scenario [1] and as such the formula does not hold arbitrarly[10].

The author recommends to use a reference design with a known (and measured) resonant frequency ν_R (such as given later in this article) and to perform simple alterations. The author recommends, additionally, two tuning procedures:

1) Volume manipulation (body): keep everything else constant and alter the body length $\mathcal{L}$. In Pazhayannur Lakshmanan's thesis [5] it is demonstrated that Helmholtz resonators are most responsive to volume alterations[11]. More from common experience: filling a glass bottle with water raises the frequency

[10]In fact, the smallest gap or hole may significantly alter aborption and resonance frequency and the elestic deformations of the geometry (as well as the types material and thickness of the material) may become dominant when one deviates from the model too much (e.g. reducing the neck side to neck length ratio to $\frac{b}{L} \approx 1$, or having the total volume of air become significantly smaller to the volume of material)

[11]This is rather self-evident from the formulae, but Pazhayannur Lakshmanan demonstrated this physically by making (3D printing) a very similar construction and measuring it. Experimental validation is essential when using approximative formulae! In fact following the thesis designs may be of more interest to some, though the 3D printing files are not so easily found. In the thesis an active noise control mechanism is set up by dynamically adjusting the various lengths of the Helmholtz Resonator bodies.

audibly, noise control concerns what is audible hence the effect is subjectively significant[12]. In any case the formula [5] becomes

$$\nu_R = \mathcal{P}\mathcal{L}^{-\frac{1}{2}} \tag{7}$$

Where $P = \frac{c}{2\pi}Q(L)^{-\frac{1}{2}} = const.$ (because we keep the rest of the parameters constant). Or *the shorter the body, the higher the pitch* and the converse is true as well.

2) Scaling: Introduce a scaling parameter k such that for all Cartesian dimensions $x^i \to kx^i$. Then from [6] we get for the new frequency $\nu_R(k)$

$$\nu_R(k) = \frac{\nu_R}{k} \tag{8}$$

The derivation is trivial. Note that this only holds for $L_{neck} \gg b$

3 The functioning design

The author lives in a city center, surrounded by church bells. The main frequencies were found to be in the range of G#3 to B4[13]. As such the reference pitch chosen became G#3. The parameters settled on are:

$$\nu_{G\#3} = 207.65\text{Hz} \tag{9}$$
$$c = 343\frac{m}{s} \tag{10}$$
$$b = 2\text{mm} \tag{11}$$
$$a = 20\text{mm} \tag{12}$$
$$L_{neck} = 23\text{mm} \tag{13}$$
$$\mathcal{L} = 5.361941056791188\text{mm} \tag{14}$$

Note that additionally

[12] You can, by the way, make your own Helmholtz Resonator by 3D scanning a bottle you have at home (or just measuring the dimensions). Use that same bottle as a tuning device by adding water to it until a target frequency is reached. Mark the water line, and adjust the length of the 3D printed Bottle accordingly. You can also use the bottles themselves, but the downside is: do you really want heavy and bulky glass resonators hanging from your ceiling?

[13] Musical pitches, this was estimated by playing notes on a digital piano. It was additionally found that the most common pitches were in the key of B major. Luck would have it that all additional sounds of noise pollution were at most one octave band separated from this frequency (or: evidently the author's walls/windows do not filter these frequencies effectively). This includes: people talking, ambulances, scooters, etc. Hence G#3 is a good reference frequency.

$$P = 1113.4070604426902 \quad (15)$$
$$Q = 0.1 \quad (16)$$

This was printed using an Ultimaker S2+ Connect with Ultimaker PLA[14] with 10% infill.

Results: The resonant frequency was accurate within a few cents[15] (i.e. spot on), though when printed on its side the hole shape deformed and the shift was about 2 chromatic pitches (25.43 Hz). The Hex-Res was tuneable through scaling and stayed within the simplistic model (with a small error which vanished for major thirds and fifths, suggesting an Eulerian tuning effect).

The wall thickness was set to 1 mm, and 2 mm for the bottom of the body to reduce elastic resonance of the plastic plate.

Note that the speed of sound in PLA is quite high (ranging from 1200-1800 m/s depending on the source). As such it has a very high acoustic reflection coefficient[16] relative to standard air. Meaning that an effective acoustic panel can already be made by simply alternating a sheet of PLA and a gap (a sheet of air). For very thin slices total reflectance can theoretically be achieved with only a small thickness.

The author would, again, like to stress that they have explored the parameter space quite extensively[17]. The above parameters give the most stable design according to current findings. Of course innovation is encouraged, be warned that it is not as straightfoward as it may seem.

The printable files can be found on Simplectic's Thingiverse account (username Simplectic). A link to Simplectic's github will be provided there for other documents.

4 Testing, Discussion & Conclusion

Both scaling and volume manipulation has been tested for the reference design. Both show agreement with the target pitches within a two octave range centered on the reference pitch. For musical scaling use the relation of chromatic pitches

[14] Various colours, no significant effect observed

[15] Measured by ear (noise is subjective so it was decided to measure subjectively) and using a reference electrical piano set to 440.0Hz standard tuning. Note that the author is a hobby musician from a family of professional musicians (precise pitch perception).

[16] $R = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2$ and $n = \frac{c_{ref}}{v}$. Search terms are Fresnel coefficients and refractive index

[17] This project has created a sizeable prototype graveyard

$$v_n = 2^{\frac{n}{12}} v_0 \tag{17}$$
$$\tag{18}$$

This gives as scaling factor:

$$k_n = 2^{-\frac{n}{12}} \tag{19}$$

Note that a reduction in size means a reduction in resonating volume (though an increase in viscious damping). This reduces total absorption (in a linear approximation). Additionally one gets varying tile sizes. Hence it is necessary to add additional units to compensate for the loss. A possible pattern is the Appolonian Gasket. Hence for simple tiling of a surface, volume tuning may be preferable. The usefulness of having such modules is the capability to design one's own meta-material. The original prototype works like a diffuser from the side. One may alter orientations, and sizes, and make new tilings (e.g. fill a larger hexagon with a collection of tiles and use this for tessellation). Note that these transformations are easy to perform on Cura (the most used 3D printed software). It is thus not necessary to know how to use CAD programs (or mathematical analysis) to tune these resonators and so more people can use them.

Lastly: On construction One may print out an entire panel's worth of HexRes units, though this may become rather expensive if one is not using inexpensive filament. Alternatively one may print out a single model for reference and use this to make cavities of the same shape and size in an inexpensive and widely available material (such as cardboard, or plywood). Since a Helmholtz Resonator is the vibration of the air and not the material, the question of which material used hardly matters (as long as it is sufficiently rigid).

One design is: a cover sheet (uncut), some honeycomb sheets stacked onto on another so that each cell has the same dimensions as the body section of reference object. Then a sheet with depth the length of the neck, either drill holes of the neck radius per cell location[18] or only print out the neck sections and hammer those in[19]. If you don't have a 3D printer you can send an order to a local 3D print shop with the files. Otherwise, you can solely copy the dimensions.

Note: Point opening to the noise source! Also do NOT cover the opening! And lastly: if you make this from cardboard, make sure the inner wals of the hexagonal cells are without holes[20]! The cavity will no longer act as a Helmholtz resonator if it is not sufficiently air tight!

[18] Another honeycomb pattern

[19] The neck frequency is very sensitive to alterations in neck profile, so a preformed neck may be preferable

[20] Corrugated cardboard is all tubes when viewed from the side.

References

[1] Mironov, M. & Komkin, Alexander & Bykov, Aleksei. (2017). Sound Absorption by a Helmholtz Resonator. Acoustical Physics. 63. 385-392. 10.1134/S1063771017030071.

[2] `https://en.wikipedia.org/wiki/Helmholtz_resonance`

[3] Engineering Acoustics: Noise and Vibration Control, Malcolm J. Crocker, Jorge P. Arenas, ISBN: 978-1-118-49642-8 January 2021

[4] `https://mathworld.wolfram.com/Tessellation.html`

[5] Active acoustic metamaterial based on Helmholtz resonators to absorb broadband low frequency noise, Pazhayannur Lakshmanan, Sandhya (TU Delft Aerospace Engineering)