Chapter 1 Functions, Limit and Continuity Concept Check

Student name:

Tutorial group:

INSTRUCTIONS

Write your name and tutorial group in the space provided on Page 1.

This exercise must be answered in English.

This exercise consists of FORTY–TWO questions.

Attempt ALL questions in this exercise.

Unless otherwise specified, all working must be clearly shown.

Unless otherwise specified, numerical answer should be EXACT.

Chapter 1 Functions, Limit and Continuity Concept Check

1 Find the domain and range of the function

1. $f(x) = \sqrt{4 - 10x}$
2. $g(x) = \frac{1}{x+1}$
3. $y = 2^x + 1$
4. $A(x) = 6.3 + \ln(x - 1)$
5. $f(x) = \frac{x}{3x-1}$
6. $f(x) = \frac{3x+4}{x^2-x}$
7. $f(t) = \sqrt{2t + 6}$
8. $g(u) = \sqrt{u - 4} + 1.5u$

2 Suppose that a function $f(x)$ has domain $(-5,5)$ and a function $g(x)$ has domain $[0, \infty)$.

1. What is the domain of $f(x) + g(x)$?
2. What is the domain of $f(x)g(x)$?
3. What is the domain of $\frac{f(x)}{g(x)}$?

3 Evaluate the difference quotient for the given function. Simplify your answer.

1. $f(x) = x^2 + 1$ $\frac{f(4+h)-f(4)}{h}$
2. $f(x) = x^3$ $\frac{f(a+h)-f(a)}{h}$
3. $f(x) = 4 + 3x - x^2$ $\frac{f(3+h)-f(3)}{h}$
4. $f(x) = 2x^2 - x$ $\frac{f(t+h)-f(t)}{h}$
5. $f(x) = \frac{1}{x}$ $\frac{f(x)-f(a)}{x-a}$
6. $f(x) = \frac{x+3}{x+1}$ $\frac{f(x)-f(1)}{x-1}$
7. $f(x) = 5^x$ $\frac{f(x+h)-f(x)}{h}$

4 If $p(x) = x^2 - 3x$, evaluate

1. $p(-2)$
2. $p(x - 5)$
3. $\frac{p(a)-p(4)}{a-4}$
4. $\frac{p(x+h)-p(x)}{h}$

5 If $f(x) = 2x^2 - 5x + 1$, evaluate

1. $f(-3)$
2. $f(4) - f(2)$
3. $\frac{f(1+h)-f(1)}{h}$ $(h \neq 0)$

6 If $f(x) = 3x^2 - x + 2$, evaluate

1. $f(2)$
2. $f(-2)$
3. $f(a)$
4. $f(-a)$
5. $f(a+1)$
6. $2f(a)$
7. $f(2a)$
8. $f(a^2)$
9. $[f(a)]^2$
10. $f(a+h)$

7 If $g(t) = 4t - t^2$, evaluate

1. $g(3)$
2. $g(-1)$
3. $g(x)$
4. $g(x-2)$
5. $g(x+h)$

8 Determine whether each of the following functions is even, odd, or neither even nor odd.

1. $f(x) = x^5 + x$
2. $g(x) = 1 - x^4$
3. $h(x) = 2x - x^2$
4. $f(x) = \frac{x}{x^2+1}$
5. $f(x) = \frac{x^2}{x^4+1}$
6. $f(x) = \frac{x}{x+1}$
7. $f(x) = 1 + 3x^3 - x^5$
8. $f(x) = 1 + 3x^2 - x^4$
9. $f(x) = x|x|$
10. $f(x) = 2x^5 - 3x^2 + 2$
11. $f(x) = x^3 - x^7$
12. $f(x) = e^{-x^2}$

9 If the point $(5,3)$ is on the graph of an

1. even function, what other point must also be on the graph?
2. odd function, what other point must also be on the graph?

10 If $f(x) = x^2 - 5x$ and $g(x) = 3x + 12$, write a formula for each of the following functions. What is the corresponding domain and range?

1. $A(x) = f(x) + g(x)$
2. $B(x) = f(x) - g(x)$
3. $C(x) = f(x)g(x)$
4. $D(x) = \frac{f(x)}{g(x)}$

11 If $p(x) = \sqrt{x+1}$ and $q(x) = 2x - 4$, write a formula for each of the following functions. What is the corresponding domain and range?

1. $A(x) = f(x) + g(x)$
2. $B(x) = f(x) - g(x)$
3. $C(x) = f(x)g(x)$
4. $D(x) = \frac{f(x)}{g(x)}$

12 If $f(x) = 3x^2 + 4$ and $g(x) = 2^x - 5$, find each of the following functions.

1. $A(x) = f(x) + g(x)$
2. $B(x) = (f \circ g)(x)$
3. $C(x) = (g \circ f)(x)$

13 Let $f(x) = x^2$ and $g(x) = x - 3$. If $h(x) = (f \circ g)(x)$ and $k(x) = (g \circ f)(x)$, compute $h(5)$ and $k(5)$.

14 Let $f(x) = x^2 + 1$ and $g(t) = 4t - 2$. If $A(t) = (f \circ g)(t)$ and $B(x) = (g \circ f)(x)$, compute $A(3)$ and $B(3)$.

15 Let $f(t) = t^3 + 2$ and $g(x) = 2x + 3$. If $p(x) = (f \circ g)(x)$ and $r(t) = (g \circ f)(t)$, compute $p(-1)$ and $r(-2)$.

16 Let $M(t) = t + \sqrt{t}$ and $N(t) = 3x + 7$. If $C(t) = (M \circ N)(t)$ and $D(t) = (N \circ M)(t)$, compute $C(3)$ and $D(4)$.

17 Let $h(n) = 2 - 5n$ and $p(n) = n^2 - 3$. If $u(n) = (h \circ p)(n)$ and $v(n) = (p \circ h)(n)$, compute $u(2)$ and $v(2)$.

18 Find the functions $p(x) = (f \circ g)(x)$ and $q(x) = (g \circ f)(x)$.

1. $f(x) = x^2 - 1 \qquad g(x) = 2x + 1$
2. $f(x) = 1 - x^3 \qquad g(x) = \frac{1}{x}$
3. $f(x) = x^3 + 2x \qquad g(x) = 1 - \sqrt{x}$
4. $f(x) = 1 - 3x \qquad g(x) = 5x^2 + 3x + 2$
5. $f(x) = x + \frac{1}{x} \qquad g(x) = x + 2$
6. $f(x) = \sqrt{2x + 3} \qquad g(x) = x^2 + 1$

19 Find a formula for $p(x) = f((g \circ h)(x))$.

1. $f(x) = \sqrt{x-1}$ $g(x) = x^2 + 2$ $h(x) = x + 3$
2. $f(x) = 2x - 1$ $g(x) = x^2$ $h(x) = 1 - x$

20 Let f and g be linear functions with equations $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$. If $h(x) = (f \circ g)(x)$, is h also a linear function? If so, what is the slope of its graph?

21 An equation that defines the exponential function with base $a > 0$.

1. Write an equation with the above requirements.
2. What is the domain of this function?
3. If $a \neq 1$, what is the range of this function?

22 Show that $f(t) = \ln(5e^{3t})$ is a linear function.

23 Express the following as a single logarithm.

1. $\ln a + \frac{1}{2}\ln b$
2. $2\ln 4 - \ln 2$
3. $\ln 3 + 2\ln x$
4. $3\ln x - 2\ln 5$
5. $\ln x + a\ln y - b\ln z$

24 Explain

1. why $y = \ln(x^3)$ and $y = 3\ln x$ have the same graph.
2. why $y = \ln(x^2)$ and $y = 2\ln x$ don't have the same graph.

25 what is the inverse function of $f(x) = 3^x$?

26 Find the inverse function to the given function and determine the domain and range of both functions.

1. $y = \frac{x}{5}$
2. $y = 1 - [4x - 7 - (1 - 2x) + 3] - x$
3. $y = \frac{2x+3}{3x+5}$
4. $y = \frac{x-1}{6x+3}$
5. $y = 1 - \frac{1}{2x}$
6. $y = \frac{-x-7}{x+5}$
7. $y = -\frac{9-3x}{9x-3}$
8. $y = \frac{10x-5}{15-10x} + 1$
9. $y = \frac{1-[10-(7-x)+20]-5x}{1+2x-(3-4x)} - 2$
10. $y = \frac{x^3-1}{x^3}$
11. $y = -x^2 - (-x)^2$
12. $y = 2x^{\frac{4}{3}} - 1$
13. $y = x^3 + 3x^2 + 3x + 1$

14. $y = \sqrt{x-1}$

15. $y = \sqrt{(2x+4)^3 - 7}$

16. $y = \left(5 - \sqrt{x+2}\right)^4 + 3$

17. $y = 2^x$

18. $y = \left(\frac{1}{8}\right)^{1-x}$

19. $y = -3 \times 5^x + 6$

20. $y = 1 + \log x$

21. $y = -2\log\left(\frac{x-1}{x+1}\right)^5$

22. $y = \log x - \log 2x + \log 3x$

23. $y = \sin 2x + 1$

24. $y = \left(1 - \cos\frac{x}{2}\right)^2 - 1$

25. $y = 2\tan^2\left(x + \frac{\pi}{2}\right) - 8$

26. $y = \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x}$

27 Evaluate the limit and justify each step by indicating the appropriate Limit Laws.

1. $\lim_{x\to 2}(x^3 + 2x^2 + 1)$
2. $\lim_{x\to 5}(2x^2 - 3x + 4)$
3. $\lim_{t\to -1}(5t^2 - 3t + 2)$
4. $\lim_{v\to 1}\frac{v^2-5}{v}$
5. $\lim_{x\to 4}(3x - 9)^4$

28 Use continuity to evaluate the limit. Round your answer to three decimal places.

1. $\lim_{t\to 1}(3e^t - 4)$
2. $\lim_{x\to 3.5}(2^x + 0.8)$
3. $\lim_{m\to 2}\left(\frac{\ln m}{m+2}\right)$
4. $\lim_{u\to 0.3}\left(\frac{u^2-4u}{3u+5}\right)$

29 Let f by the function defined by $f(x) = \frac{x-1}{x^2-1}$

1. State the domain of f.
2. Find $\lim_{x\to 4} f(x)$.
3. Find $\lim_{x\to 1} f(x)$.

30 Let f be the function defined by $f(x) = \frac{x^2-4}{x-2}$

1. State the domain of f.
2. Find $\lim_{x\to 1} f(x)$.
3. Find $\lim_{x\to 2} f(x)$.

31 Let R be the function defined by $R(x) = \frac{x^2-2x-8}{x^2-16}$

1. State the domain of R.
2. Find $\lim_{x\to 2} R(x)$.
3. Find $\lim_{x\to 4} R(x)$.

32 Let A be the function defined by $A(z) = \frac{2z-6}{z^2-5z+6}$

1. State the domain of A.
2. Find $\lim_{z\to 0} A(z)$.
3. Find $\lim_{z\to 3} A(z)$.

33 Let g be the function defined by $g(t) = \frac{t^2-3t-4}{t+1}$

1. State the domain of g.
2. Find $\lim_{t\to 3} g(t)$.
3. Find $\lim_{t\to -1} g(t)$.

34 Evaluate the limit if exists.

1. $\lim_{r\to 0} \frac{(3+r)^2-9}{r}$
2. $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$
3. $\lim_{x\to 0} \frac{1}{x^2}$
4. $\lim_{t\to 4}(3t-7)$
5. $\lim_{x\to -2}(4x^2+x)$
6. $\lim_{x\to 3} \frac{x^2+5}{x+5}$
7. $\lim_{w\to 5} \frac{3w^2+1}{w}$
8. $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$
9. $\lim_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4}$
10. $\lim_{t\to -3} \frac{t^2-9}{2t^2+7t+3}$
11. $\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4}$
12. $\lim_{h\to 0} \frac{(4+h)^2-16}{h}$
13. $\lim_{x\to -1} \frac{x^2+2x+1}{x^4-1}$
14. $\lim_{x\to -2} \frac{x+2}{x^3+8}$
15. $\lim_{x\to 0} \frac{\sqrt{x^2+b^2}-b}{x^2}$ $(b>0)$
16. $\lim_{x\to 7} \frac{\sqrt{x+2}-3}{x-7}$
17. $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$
18. $\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$
19. $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)$
20. $\lim_{x\to 0} \frac{3}{x^4}$
21. $\lim_{x\to 2} \frac{x}{(x-2)^2}$
22. $\lim_{t\to 0} \frac{e^t}{t}$
23. $\lim_{x\to 0^+} \ln x$
24. $\lim_{x\to 4} \frac{x^2-16}{x-4}$
25. $\lim_{a\to 4} \frac{a^2+2a-24}{a-4}$
26. $\lim_{b\to 0} \frac{(-2+b)^3+8}{b}$
27. $\lim_{t\to 9} \frac{\sqrt{t}-3}{t-9}$
28. $\lim_{h\to 0} \frac{h^2+2xh}{h}$
29. $\lim_{h\to 0} \frac{(x+h)^3+2(x+h)-x^3-2x}{h}$
30. $\lim_{x\to 1}(5x^2-4x+5)$

31. $\lim_{x\to 3}\frac{x^2-9}{x^2+2x-3}$

32. $\lim_{x\to -3}\frac{x^2-9}{x^2+2x-3}$

33. $\lim_{t\to 2}\frac{t^2-4}{t^2+3t-10}$

34. $\lim_{t\to 0} 4e^{-2t}$

35. $\lim_{b\to 1}(\ln b)^2$

36. $\lim_{h\to 0}\frac{(h-1)^3+1}{h}$

37. $\lim_{x\to 1}\left(\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right)$

38. $\lim_{x\to 2}(x^2-4)$

39. $\lim_{x\to 2}\frac{x^3-4x}{2x^2+3x}$

40. $\lim_{x\to -1}\frac{x^3}{(x+1)^2}$

41. $\lim_{x\to -1}\frac{(x+1)^2(x-1)}{x^3+1}$

42. $\lim_{x\to 0}\frac{x^3-2x^2+x}{2x^3+x^2-2x}$

43. $\lim_{x\to 1}\frac{x^2+2x+3}{(x-1)^2}$

44. $\lim_{x\to 0}\frac{x^4-4x^3+x\text{^}2}{x^3+x^2+x}$

45. $\lim_{x\to -1}\frac{x^3+x^2+x+1}{x^4+x^2-2}$

46. $\lim_{x\to 2}\frac{(x+1)^2}{2-x}$

47. $\lim_{x\to 2}\frac{x-2}{x^2-3x+2}$

48. $\lim_{x\to 0}\frac{3x+2x^{-1}}{x+4x^{-1}}$

49. $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-2x}$

50. $\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$

51. $\lim_{x\to 2}\frac{x^2-x-2}{x^2-2x}$

52. $\lim_{x\to 2}\frac{x^2+5}{x^2-3}$

53. $\lim_{x\to 1}\frac{3x^4-4x^3+1}{(x-1)^2}$

54. $\lim_{x\to -2}\frac{3x+6}{x^3+8}$

55. $\lim_{x\to 2}\frac{x+1}{x-1}$

56. $\lim_{x\to -2}\frac{x^3+3x^2+2x}{x^2-x-6}$

57. $\lim_{x\to 1}\frac{x^2-2x+1}{x^3-x}$

58. $\lim_{x\to 4}\frac{x^2+7x-44}{x^2-6x+8}$

59. $\lim_{x\to 1}\frac{x^2-4}{x^2-3x+2}$

60. $\lim_{x\to 1}\frac{x^3-5x+4}{x^3-1}$

61. $\lim_{x\to 2}\frac{x^2-4}{x-2}$

62. $\lim_{x\to 2}\frac{x^2-4}{x^2-3x+2}$

63. $\lim_{x\to 1}\left(\frac{1}{x^2-1}-\frac{2}{x^4-1}\right)$

64. $\lim_{x\to 3}\frac{x-3}{x^2-5x+6}$

65. $\lim_{x\to \infty}\frac{x^2-1}{2x^2+1}$

66. $\lim_{x\to -\infty}\frac{x^3+x^2-4}{2x^3+x+11}$

67. $\lim_{x\to \infty}\frac{3x^2+2x-1}{x^3-x+2}$

68. $\lim_{x\to \infty}\left(\frac{x^3}{x^2+2}-x\right)$

69. $\lim_{x\to \infty}\frac{x^2+3x-4}{3x^2-2x+5}$

70. $\lim_{x\to \infty}\frac{x(x-1)(x-2)}{x^2+6x-9}$

71. $\lim_{x\to \infty}\frac{\sqrt{x^2+9}}{x+3}$

72. $\lim_{x\to \infty}\left(\frac{x^2+x-1}{2x^2-x+1}\right)^3$

73. $\lim_{x\to \infty}\frac{x^2+2x+1}{5x}$

74. $\lim_{x\to -\infty}\frac{x^3+x^4-1}{2x^5+x-x^2}$

75. $\lim_{x\to \infty}\frac{\left(\sqrt{x^2+1}+x\right)^2}{\sqrt[3]{x^6+1}}$

76. $\lim_{x\to -\infty}\frac{x^6+7x^4-40}{1-x-5x^7}$

77. $\lim_{x\to\infty}\frac{(x+1)(x-2)}{3x^2+6x-5}$

78. $\lim_{x\to\infty}\frac{\sqrt{x^2+1}}{x}$

79. $\lim_{x\to\infty}\left(\frac{3x^2+2x+1}{x^2-3x+2}\right)^4$

80. $\lim_{x\to-\infty}\frac{5x^3-x^2+x}{1-x-3x^2}$

81. $\lim_{x\to\infty}\frac{1+x-3x^3}{1+x^2+3x^3}$

82. $\lim_{x\to-\infty}\left(\frac{x^3-8}{x^4+16}\right)^{10}$

83. $\lim_{x\to\infty}\frac{(x+3)(x+4)(x+5)}{x^4+x-11}$

84. $\lim_{x\to-\infty}\frac{8x-2x^5+x^6}{11x+5x^3+3x^5}$

85. $\lim_{x\to\infty}\left(\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}\right)$

86. $\lim_{x\to\infty}\left(x^2-\frac{x^4-1}{x^2-2}\right)$

87. $\lim_{x\to\infty}\frac{(x-1)^{100}(6x+1)^{200}}{(3x+5)^{300}}$

88. $\lim_{x\to\infty}\frac{\sqrt[4]{x^5}+\sqrt[5]{x^3}+\sqrt[6]{x^8}}{\sqrt[3]{x^4+2}}$

89. $\lim_{x\to-\infty}\frac{x^2(2x+1)(3x-2)}{2x^2(5x-8)(x+6)}$

90. $\lim_{x\to\infty}\left(\frac{2x^8+8x^6+6x^4}{4x^8-x^6+12x^4}\right)^5$

91. $\lim_{x\to-\infty}\frac{(2x-3)^{20}(3x+2)^{30}}{(2x+1)^{50}}$

92. $\lim_{x\to0}\frac{\sqrt{1+2x}-1}{3x}$

93. $\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

94. $\lim_{x\to0}\frac{x-\sqrt{x}}{\sqrt{x}}$

95. $\lim_{x\to5}\frac{\sqrt{x-1}-2}{x^2-25}$

96. $\lim_{x\to3}\frac{\sqrt{x+6}-3}{x^3-5x^2+3x+9}$

97. $\lim_{x\to9}\frac{3-\sqrt{x}}{27-\sqrt{x^3}}$

98. $\lim_{x\to0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$

99. $\lim_{x\to1}\frac{x^{2/3}-1}{x^{3/5}-1}$

100. $\lim_{x\to1}\frac{1-\sqrt[n]{x}}{1-\sqrt[m]{x}}$

101. $\lim_{x\to\infty}\left(\sqrt{x-2}-\sqrt{x}\right)$

102. $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

103. $\lim_{x\to-\infty}\left(\sqrt{x^2+x}-x\right)$

104. $\lim_{x\to\infty}\left(\sqrt{x-3}-\sqrt{x}\right)$

105. $\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x-3}-\sqrt{x}\right)$

106. $\lim_{x\to\infty}x\left(\sqrt{x^2+1}-x\right)$

107. $\lim_{x\to-\infty}x\left(\sqrt{x^2+1}-x\right)$

108. $\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$

109. $\lim_{x\to-\infty}\left(\sqrt{x^2+1}-x\right)$

110. $\lim_{x\to\infty}\frac{\sqrt{x+2}-\sqrt{2}}{x}$

111. $\lim_{x\to\infty}\frac{\sqrt{x+5}-\sqrt{5}}{\sqrt{x}-5}$

112. $\lim_{x\to\infty}\frac{\sqrt{x^2+9}-\sqrt{x^2-9}}{6x}$

113. $\lim_{x\to\infty}\frac{\sqrt{x-1}-2x}{x-7}$

114. $\lim_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$

115. $\lim_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^3+x}-x}$

116. $\lim_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^2+1}-x}$

117. $\lim_{x\to\infty}\frac{\sqrt[3]{x}-2\sqrt{x^3}}{\sqrt[4]{x^5}+x\sqrt{x}}$

118. $\lim_{x\to-\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{2\times\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$

119. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$

120. $\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^{3x-4}$

121. $\lim_{x\to\infty}\left(1+\frac{1}{5x}\right)^{2x+6}$

122. $\lim_{x\to\infty}\left(1+\frac{7}{3x}\right)^{x-1}$

123. $\lim_{x\to\infty}\left(1-\frac{1}{3x}\right)^{x}$

124. $\lim_{x\to\infty}\left(1-\frac{5}{x}\right)^{x}$

125. $\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^{x}$

126. $\lim_{x\to\infty}\left(\frac{x+6}{x+5}\right)^{x}$

127. $\lim_{x\to\infty}\left(\frac{x-3}{x}\right)^{\frac{x}{2}}$

128. $\lim_{x\to\infty}\left(\frac{7x+10}{1+7x}\right)^{\frac{x}{3}}$

129. $\lim_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^{x}$

130. $\lim_{x\to\infty}\left(\frac{6+4x}{2+4x}\right)^{3-2x}$

131. $\lim_{x\to\infty}\left(\frac{x+5}{x+4}\right)^{2x-1}$

132. $\lim_{x\to\infty}\left(\frac{2x+5}{2x}\right)^{3x-7}$

133. $\lim_{x\to\infty}\left(\frac{2x+1}{2x-3}\right)^{3x}$

134. $\lim_{x\to\infty}\left(\frac{x-1}{x+3}\right)^{5-4x}$

135. $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^{x}$

136. $\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{2x}$

137. $\lim_{x\to\infty}\left(\frac{2x-5}{2x-2}\right)^{4x^2}$

138. $\lim_{x\to\infty}\left(\frac{3x+6}{3x-1}\right)^{x^2}$

139. $\lim_{x\to\infty}\left(\frac{x^2+2x+2}{x^2+3}\right)^{x}$

140. $\lim_{x\to0}(1+2x)^{\frac{1}{x}}$

141. $\lim_{x\to0}(1+5x)^{\frac{1}{8x}}$

142. $\lim_{x\to\infty}\left[\ln\left(\frac{3x+1}{3x-5}\right)^{2-x}\right]$

143. $\lim_{x\to\infty} x[\ln(x+3)-\ln x]$

144. $\lim_{x\to\infty} x[\ln x-\ln(x+2)]$

145. $\lim_{x\to\infty}(x+1)[\ln(x+1)-\ln x]$

146. $\lim_{x\to0}\frac{\sin 10x}{10x}$

147. $\lim_{x\to0}\frac{\sin 3x}{2x}$

148. $\lim_{x\to0}\frac{\tan 8x}{x}$

149. $\lim_{x\to0}\frac{\sin 3x}{\sin 5x}$

150. $\lim_{x\to0}\frac{\tan 5x}{\sin 4x}$

151. $\lim_{x\to0}\frac{\tan 5x}{\tan 6x}$

152. $\lim_{x\to0}\frac{\tan x}{3x}$

153. $\lim_{x\to0}\frac{1-\cos x}{x}$

154. $\lim_{x\to0}\frac{x}{\sqrt{1-\cos x}}$

155. $\lim_{x\to0}\frac{\sin x}{x^3}$

156. $\lim_{x\to0}\frac{1-\cos 2x}{x\sin x}$

157. $\lim_{x\to0}\frac{\sin^3\left(\frac{x}{2}\right)}{x^3}$

158. $\lim_{x\to0}\frac{\sin 4x+\sin 7x}{\sin 3x}$

159. $\lim_{x\to0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}$

160. $\lim_{x\to0}\frac{\tan x-\sin x}{x^3}$

161. $\lim_{x\to\infty} x\sin\frac{\pi}{x}$

162. $\lim_{x\to-1}\frac{x^3+1}{\sin(x+1)}$

163. $\lim_{x\to0}\frac{|\sin x|}{x}$

164. $\lim_{x\to1}\frac{\tan(x-1)}{\sqrt{x}-1}$

165. $\lim_{x\to0}\frac{\sqrt{\cos x}-1}{\sin^2 x}$

166. $\lim_{x\to0}\frac{\sin 3x+\sin 5x}{\sin 2x}$

167. $\lim_{x\to0}\frac{\cos x-\cos^3 x}{x^2}$

168. $\lim_{x\to 0} x\cot 2x$

169. $\lim_{x\to 0}\sqrt{\frac{1-\cos x}{x^2}}$

170. $\lim_{x\to \pi}\frac{\sqrt{1-\tan x}-\sqrt{1+\tan x}}{\sin 2x}$

171. $\lim_{x\to 0}\frac{\ln x}{\ln(\sin x)}$

172. $\lim_{x\to 0}\frac{\ln(\sin 2x)}{\ln(\sin x)}$

173. $\lim_{x\to 0}\frac{3^x-1}{6^x-1}$

174. $\lim_{x\to \infty} x(2^{\frac{1}{x}}-1)$

175. $\lim_{x\to 0}(e^x+x)^{\frac{1}{x}}$

176. $\lim_{x\to \infty}\frac{e^{\frac{1+x}{1-x}}}{2}$

177. $\lim_{x\to \infty}\frac{e^x-e^{-x}}{2}$

178. $\lim_{x\to -\infty}\frac{e^x+e^{-x}}{e^x-e^{-x}}$

179. $\lim_{x\to 0}\frac{\ln(1+x)}{x}$

180. $\lim_{x\to 0}\frac{\ln(1+3x)}{x}$

181. $\lim_{x\to e}\frac{\ln x-1}{x-e}$

182. $\lim_{x\to -\infty}\left(\ln\frac{x-1}{x+1}\right)$

183. $\lim_{x\to 0}\frac{1}{x}\ln\sqrt{\frac{1+x}{1-x}}$

184. $\lim_{x\to e}\frac{\ln x^x-x}{2-\ln x^2}$

185. $\lim_{x\to 0}\frac{1-3^x}{\sin 3x}$

186. $\lim_{x\to -\infty} 2^x\sin 2\pi x$

187. $\lim_{x\to 0}\frac{\sin 2x}{\sqrt{x+3}-\sqrt{3}}$

188. $\lim_{x\to \infty} x^2\left(1-\cos\frac{1}{x}\right)$

189. $\lim_{x\to 0}(\cos x)^{\frac{1}{x}}$

190. $\lim_{x\to \frac{\pi}{2}}\frac{1-\sin x}{\pi-2x}$

191. $\lim_{x\to \frac{\pi}{4}}\frac{\sin x-\cos x}{\cos 2x}$

192. $\lim_{x\to \infty}\left(\sin\sqrt{x+1}-\sin\sqrt{x}\right)$

193. $\lim_{x\to \frac{\pi}{4}}(\tan x)^{\tan 2x}$

194. $\lim_{x\to 1}\frac{x^3-2x^2+x}{x^2-1}$

195. $\lim_{x\to 3}\frac{x^3-9x}{x^4-3x^3-x+3}$

196. $\lim_{x\to 1}\frac{\sqrt[3]{8x}-2x}{\sqrt[4]{x}-x}$

197. $\lim_{x\to 1}\frac{\sqrt{x+1}-\sqrt{2}}{x^2-1}$

198. $\lim_{x\to 0}\frac{5^x-1}{x}$

199. $\lim_{x\to 1}\frac{3x^3-3}{3^x-3}$

200. $\lim_{x\to 0}\frac{e^x-e^{-x}}{x}$

201. $\lim_{x\to 0}\frac{e^x-1}{x^2}$

202. $\lim_{x\to \frac{\pi}{3}}\frac{1-2\cos x}{\pi-3x}$

203. $\lim_{x\to 0}\frac{\sin x}{e^x-1}$

204. $\lim_{x\to 0}\frac{\tan^{-1}3x}{\sin^{-1}2x}$

205. $\lim_{x\to 0}\frac{e^x-e^{-x}}{2x-\sin x}$

206. $\lim_{x\to 0}\frac{x^3+\pi x}{\sin 3x}$

207. $\lim_{x\to 0}\frac{\ln(1+4x)}{3^x-1}$

208. $\lim_{x\to 0}\frac{\sin 4x}{\ln(1+\sin x)}$

209. $\lim_{x\to 0}\frac{3\ln(1-2x)}{2\tan^{-1}3x}$

210. $\lim_{x\to 0}\frac{\tan^{-1}x+x^2}{2^{3x}-3^{2x}}$

211. $\lim_{x\to \frac{\pi}{2}}\frac{\tan^{-1}\left(x-\frac{\pi}{2}\right)}{\pi-2x}$

212. $\lim_{x\to 0}\frac{e^{3x}-e^{-2x}}{2\sin^{-1}x-\sin x}$

213. $\lim_{x\to 0}\frac{\ln(\cos 3x)}{\tan^{-1}4x}$

214. $\lim_{x\to 0}\frac{(1+x)^2-(1+2x)}{x^2+4x^3}$

215. $\lim_{x\to -1}\frac{\left(x^2+3x+2\right)^2}{x^3-3x-2}$

216. $\lim_{x\to 0}\frac{1-\cos^4 x}{4x^2}$

217. $\lim_{x\to 0}\frac{1-\cos x}{2x\sin x}$

218. $\lim_{x\to 0}\frac{x-\sin x}{e^x-e^{-x}-2x}$

219. $\lim_{x\to 0}\frac{e^{3x}-3x-1}{\sin^2 x}$

220. $\lim_{x\to 0}\frac{x^3}{x-\tan^{-1}x}$

35 Explain what it means to say that

1. $\lim_{x\to 1^-} f(x) = 3$
2. $\lim_{x\to 1^+} f(x) = 7$
3. In this situation is it possible that $\lim_{x\to 1} f(x)$ exist? Explain.

36 Let $g(x) = \begin{cases} -x & if\ x \le -1 \\ 1-x^2 & if\ -1 < x < 1 \\ x-1 & if\ x > 1 \end{cases}$, evaluate each of the following limits if it exists.

1. $\lim_{x\to 1^+} g(x)$
2. $\lim_{x\to 1} g(x)$
3. $\lim_{x\to 0} g(x)$
4. $\lim_{x\to -1^-} g(x)$
5. $\lim_{x\to -1^+} g(x)$
6. $\lim_{x\to -1} g(x)$

37 Let $F(x) = \frac{x^2-1}{|x-1|}$, evaluate

1. $\lim_{x\to 1^+} F(x)$
2. $\lim_{x\to 1^-} F(x)$
3. Does $\lim_{x\to 1} F(x)$ exist? Explain.

38 Recall that $|x| = \begin{cases} x & if\ x \ge 0 \\ -x & if\ x < 0 \end{cases}$, let $f(x) = \frac{|x|}{x}$, find

1. $\lim_{x\to 0^+} f(x)$
2. $\lim_{x\to 0^-} f(x)$
3. Does $\lim_{x\to 0} f(x)$ exist? Explain.

39 Let $f(t) = \begin{cases} t^2 & if\ t < 0 \\ e^t & if\ t \geq 0 \end{cases}$, evaluate each of the following limits if it exists.

1. $\lim_{t \to -1} f(t)$
2. $\lim_{t \to 0^-} f(t)$
3. $\lim_{t \to 0^+} f(t)$
4. $\lim_{t \to 0} f(t)$
5. $\lim_{t \to 2} f(t)$
6. Explain why f is not continuous at $t = 0$.

40 $\frac{x^2+x-6}{x-2} = x + 3$

1. What is wrong with the above equation?
2. In view of part 1, explain why the equation $\lim_{x \to 2} \frac{x^2+x-6}{x-2} = \lim_{x \to 2}(x + 3)$ is correct.

41 For what value of the constant c is the function $f(x) \begin{cases} cx^2 + 2x & if\ x < 2 \\ x^3 - cx & if\ x \geq 2 \end{cases}$

continuous on $(-\infty, \infty)$?

42 If $\lim_{x \to 1} \frac{f(x)-8}{x-1} = 10$, find $\lim_{x \to 1} f(x)$.

END