NUMBER SYSTEM

$\mathbb{C}$ Complex Number: $a + bi$

$\mathbb{R}$ Real Number: The limit of a convergent sequence of rational numbers

$\mathbb{Q}$ Rational Number: Express in form of $\frac{a}{b}$ where $a\epsilon\mathbb{Z}$, $b\epsilon\mathbb{Z}$ and $b \neq 0$

$\mathbb{Z}$ Integer

$\mathbb{N}$ Natural: $\mathbb{N}_0 = \{0,1,2,3,4, \dots\}$

$\mathbb{N}_1 = \{1,2,3,4, \dots\}$

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

ALGEBRA

Arithmetic Operations and Fractions

$a + b = b + a$ $\quad$ $ab = ba$ $\quad$ $a(b + c) = ab + ac$

$(a + b) + c = a + (b + c)$ $\quad$ $(ab)c = a(bc)$ $\quad$ $-(b + c) = -b - c$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

EXERCISE Simplify $\frac{\frac{X}{Y}+1}{1-\frac{Y}{X}}$

Quadratic Formula

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad$ $\Delta = b^2 - 4ac$

$\Delta > 0$, 2 real solutions $\quad$ $\Delta = 0$, 2 equal real solutions

$\Delta < 0$, no real solutions

EXERCISE Solve the equation $5x^2 + 3x - 2 = 1$

Binomial Theorem

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

$(a+b)^2 = a^2 + 2ab + b^2$ $\qquad$ $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

EXERCISE Expand $(x-2)^5$

Expanding and Factoring

$a^2 - b^2 = (a-b)(a+b)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $\qquad$ $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

EXERCISE Simplify $\frac{x^2-16}{x^2-2x-8}$

Completing the square

$$ax^2 + bx + c = a\left[x^2 + \frac{b}{a}x\right] + c$$

$$= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

EXERCISE Rewrite $2x^2 - 12x + 11$ by completing the square

Radicals

$x = \sqrt[n]{a}$ means $x^n = a$ $\qquad$ if n is even, then $a \geq 0$ and $x \geq 0$

$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$ $\qquad$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

EXERCISE Rationalize the numerator in the expression $\frac{\sqrt{x+4}-2}{x}$

Simplify $\sqrt[2]{x^6}$ for $x \epsilon \mathbb{R}^+$

Absolute Value

$|a| = a \text{ if } a \geq 0$ $\quad$ $|a| = -a \text{ if } a < 0$

$\sqrt{a^2} = |a|$

$|ab| = |a||b|$ $\quad$ $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}\ (b \neq 0)$ $\quad$ $|a^n| = |a|^n$

For $a > 0$,

$|x| = a \text{ iff } x = \pm a$ $\quad$ $|x| < a \text{ iff } -a < x < a$ $\quad$ $|x| > a \text{ iff } x > a \text{ or } x < -a$

EXERCISE Solve $|2x - 5| = 3$

Simplify $\sqrt{x^2 y}$

Inequalities

$\forall \{a, b, c, d\} \epsilon \mathbb{R}$

if $a > b$, then $a + c > b + c$

if $a > b$ and $c > d$, then $a + c > b + d$

if $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

if $a > b$ and $c < 0$, then $ac > bc$ and $\frac{a}{c} < \frac{b}{c}$

if $a > b$ and a & b are of the same sign, then $\frac{1}{a} < \frac{1}{b}$

if $a > b > 0$ and $n \in \mathbb{Z}^+$, then $a^n > b^n$ and $\sqrt[n]{a} > \sqrt[n]{b}$

if $ab > 0$, then ($a > 0$ and $b > 0$) or ($a < 0$ and $b < 0$)

if $ab < 0$, then ($a < 0$ and $b > 0$) or ($a > 0$ and $b < 0$)

if $a \neq 0$, then $a^2 > 0$

EXERCISE Solve the inequality $x^2 - 5x + 6 \leq 0$

Exponents

$\forall \{a, b\} \epsilon \mathbb{R}$

$\forall \{m, n\} \epsilon \mathbb{Z}$

When $a = 0$ or $b = 0$, a condition that $m, n \neq 0$

$a^0 = 1$	$a^{-n} = \frac{1}{a^n}$	$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
$a^m a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}\ (a \neq 0)$	$(a^m)^n = a^{mn}$
$(ab)^m = a^m b^m$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\ (b \neq 0)$	

Tom's words,

To multiply two powers of the same number, add the exponents.

To divide two powers of the same number, subtract the exponents.

To raise a power to a new power, multiply the exponents.

To raise a product to a power, raise each factor to the power.

To raise a quotient to a power, raise both numerator and denominator to the power.

EXERCISE Simplify $\frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}$ and $\left(\frac{x}{y}\right)^3 \left(\frac{y^2 x}{z}\right)^4$

Intervals

Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	
$[a, b)$	$\{x \mid a \leq x < b\}$	
$(a, b]$	$\{x \mid a < x \leq b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \geq a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \leq b\}$	
$(-\infty, \infty)$	$\mathbb{R}$	

Remainder Theorem

If a polynomial $p(x)$ is divided by $ax - b$, where $a \neq 0$,

Then the remainder is $p\left(\frac{b}{a}\right)$

Factor Theorem

The polynomial $p(x)$ has a factor $ax - b$, i.e. $p(x)$ is divisible by $ax - b$, where $a \neq 0$,

iff $p\left(\frac{b}{a}\right) = 0$

Rational Root Theorem

Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial with integer coefficients and $a_0, a_n \neq 0$. If $\frac{p}{q}$ is a rational number in the lowest terms and $\frac{p}{q}$ is a root of the equation $p(x) = 0$, then p is a factor of a_0 and q is a factor of a_n

A useful application of this result is to locate the possible rational roots of a polynomial equation in which the coefficients of the polynomial are all integers.

In particular, if $a_n = 1$, then the possible rational roots of $p(x) = 0$ are $\pm$factors of a_0

Tom's STARS

State whether or not the equation is TRUE for ALL values of the variable.

$\sqrt{x^2} = x$

$\frac{16+a}{16} = 1 + \frac{a}{16}$

$\frac{x}{x+y} = \frac{1}{1+y}$

$(x^3)^4 = x^7$

Solve the inequality in terms of intervals

$2x + 7 > 3$

$1 - x \leq 2$

$0 \leq 1 - x \leq 1$

$(x-1)(x-2) > 0$

$x^2 < 3$

$x^3 - x^2 \leq 0$

$x^3 > x$

Rationalize the expression

$\frac{\sqrt{x}-3}{x-9}$

$\frac{x\sqrt{x}-8}{x-4}$

$\frac{2}{3-\sqrt{5}}$

Simplify the expression

$3^{10} \times 9^8$

$\frac{x^9(2x)^4}{x^3}$

$\frac{a^{-3}b^4}{a^{-5}b^5}$

$3^{\frac{-1}{2}}$

$125^{\frac{2}{3}}$

$(2x^2y^4)^{\frac{3}{2}}$

$\frac{1}{(\sqrt{t})^5}$

Prove that $|ab| = |a||b|$

Shows that if $0 < a < b$, then $a^2 < b^2$

GEOMETRY

Distance Formula

The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXERCISE The distance between $(1, -2)$ and $(5,3)$

Lines

For $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ $y - y_1 = m(x - x_1)$ $\qquad$ $y = mx + b$

Slope of a vertical line is undefined

EXERCISE Find an equation of the line through the points $(-2, -9)$ and $(4,6)$

Graph the inequality $x + 2y > 5$

Parallel and Perpendicular Lines

Two nonvertical lines are parallel iff they have the SAME slope

Two lines with slopes m_1 and m_2 are perpendicular iff $m_1 m_2 = -1$, i.e. their slopes are NEGATIVE reciprocals.

EXERCISE Find an equation of the line through the point $(5,2)$ that is parallel to the line $4x + 6y + 5 = 0$

Circle

Equation of circle with centre at (h, k) and radius r

$$(x - h)^2 + (y - k)^2 = r^2$$

Quadratic Function

$$y = ax^2 + bx + c$$

where $a \neq 0$

The parabola,

Intersects the x−axis at 2 distinct points if $\Delta > 0$

Touches the x−axis at one intersection point only if $\Delta = 0$

Does NOT intersect the x−axis if $\Delta < 0$

Apply completing the square to the equality,

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

The parabola is symmetric about the vertical line $x = -\frac{b}{2a}$

If $a > 0$ (resp. $a < 0$), the parabola opens upward (resp. downward) and has the lowest (resp. highest) point at $x = -\frac{b}{2a}$. This point is known as the vertex. Moreover, the corresponding minimum (resp. maximum) value of the quadratic function is $y = \frac{b^2-4ac}{4a}$

Sphere

$V = \frac{4}{3}\pi r^3$ $\qquad$ $A = 4\pi r^2$

Cylinder

$V = \pi r^2 h$

Cone

$V = \frac{1}{3}\pi r^2 h$ $\qquad$ $A = \pi r\sqrt{r^2 + h^2}$

Tom's STARS

Sketch the region in the xy−plane

$\{(x, y)|x < 0\}$

$\{(x, y)|xy < 0\}$

$\{(x, y)||x| \leq 2\}$

$\{(x, y)|0 \leq y \leq 4 \text{ and } x \leq 2\}$

$\{(x, y)|1 + x \leq y \leq 1 - 2x\}$

Show that the lines $2x - y = 4$ and $6x - 2y = 10$ are not parallel and find their point of intersection.