

NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2020/2021 (II)

MA4260 Stochastic Operations Research

Assignment 1

Note. This assignment carries 5% of the grade for this course. Please submit the home-work by 16:00, 11/02/2021 (Thursday).

1. (1 mark, Tutorial 1 Problem 1.) Suppose I arrive at an $M/M/k : FCFS/k + 1/\infty$ queueing system when all servers are busy. What is the probability that I will complete service before at least one of the k customers in service?
2. (1 mark, Tutorial 2 Problem 6.) A small mail order firm Seas Beginnings has one phone line. An average of 60 people per hour call in orders, and it takes an average of 1 minute to handle a call. Time between calls and time to handle calls are exponentially distributed. If the phone line is busy, Seas Beginnings can put up to $c - 1$ people on hold. If $c - 1$ people are on hold, a caller gets a busy signal and calls a competitor (Air End). Seas Beginnings wants only 1% of all callers to get a busy signal. How many people should they be able to put on hold?
3. (1 mark, Tutorial 3 Problem 3.) A small bank is trying to determine how many tellers to employ. The total cost of employing a teller is \$100 per day, and a teller can serve an average of 60 customers per day. An average of 50 customers per day arrive at the bank and both service times and interarrival times are exponential. If the delay (waiting in queue) cost per customer day is \$100, how many tellers should the bank hire?
4. (2 marks, Decision Trees.) You sit on a plot of land that may contain oil. A consultant geologist assesses a 0.25 chance that the land sits on an oil well, in which case the expected revenue is 800. Based on this prospect, a rival company has offered to buy over the land for 90. The cost of drilling is 100. You have two choices (i) sell the land, (ii) drill the land and see what happens.
 - (a) The Maximin Pay-off Criterion is as follows: For each possible decision, identify the minimum pay-off. Then choose the decision that maximizes the minimum pay-off. What is the decision based on the Maximin Pay-off Criterion and what is the pay-off?
 - (b) The Maximum Likelihood Criterion (MLC) selects the optimal decision according to the most likely scenario. What is the decision based on the MLC and what is the pay-off?

(c) The Bayes' Decision Rule uses the best available estimates of the probabilities of each possible states and selects the decision that maximizes the expected pay-off. Based on the current information, what is the optimal Bayes' Decision Rule and what is the pay-off?

You are now offered a third choice of conducting a more detailed geological survey at a cost of 30. The following is known: Given that there is an oil well, the probability of a favourable survey result is 0.6. Given that there is no oil, the probability of an unfavourable survey result is 0.8.

(d) Using Bayes' theorem, compute the probabilities of finding an oil well as well as the probabilities of not finding an oil well, given that the survey results are favourable or not favourable.

(e) Compute the expected payoff corresponding to each action given the survey results.

The Expected Value of Experimentation is define as the expected gain pay-off if you proceed with the experiment.

(f) Using the answers from (e), compute the expected payoff supposing that you proceed with the more detailed geological survey.

(g) Subtract the answer from (c) from the previous answer. This is the Expected Value of Experimentation. Does the gain exceed the cost of the experiment? Should you proceed with experimentation?