NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

2020/2021 (II) MA4260 Stochastic Operations Research Tutorial 2

An asterisk (*) means that you are required to write-up the solution as part of an up-coming graded assignment. We will not discuss these questions during tutorials.

1. Each airline passenger and his or her luggage must be checked to determine whether he or she is carrying weapons onto the airplane. Suppose that at Gotham City Airport, an average of 10 passengers per minute arrive (interarrival times are exponential). To check passengers for weapons, the airport must have a checkpoint consisting of a metal detector and baggege X-ray machine. Whenever a checkpoint is in operation, two employees are required. A checkpoint can check an average of 12 passengers per minute (the time to check a passenger is exponential). Under the assumption that the airport has only one checkpoint, answer the following questions:
   (a) What is the probability that a passenger will have to wait before being checked for weapons?
   (b) On the average, how many passengers are waiting in line to enter the checkpoint?
   (c) On the average, how long will a passenger spend at the checkpoint?
   (d) If arrival and service rates are both doubled, what are the effects on L_q and W?
2. Referring to Question 1, suppose the airline wants to determine how many checkpoints to operate to minimize operating costs and delay costs over a ten-year period. Assume that the cost of delaying a passenger for 1 hour is $10 and that the airport is open everyday for 16 hours per day. It costs $1 million to purchase, staff, and maintain a metal detector and baggage X-ray machine for a ten-year period. Finally, assume that each passenger is equally likely to enter a given checkpoint.
3. Consider an airport where taxis and customers arrive (exponential interarrival times) with respective rates of 1 and 2 per minute. No matter how many other taxis are present, a taxi will wait. If an arriving customer does not find a taxi, the customer immediately leaves. Assume that each taxi takes only one customer.

(a) Model this system as a birth-death process (Hint: Determine what the state of the system is at any given time and draw a rate diagram.)

(b) Find the average number of taxis that are waiting for a customer.

(c) Suppose all customers who use a taxi pay a \$2 fare. During a typical hour, how much revenue will the taxis receive?

4. A small petrol kiosk has only one petrol pump. Cars wanting petrol arrive according to a Poisson process with a mean rate of 15 per hour. However, if the pump already is being used, these potential customers may *balk* (i.e., drive on to another petrol kiosk). In particular, if there are n cars already at the kiosk, the probability that an arriving potential customer will balk is $n/3$ for $n = 1, 2, 3$. The time required to service a car has an exponential distribution with a mean of 4 minutes.

   (a) Does steady-state exist?

   (b) Draw the rate diagram and write down the balance equations.

   (c) Solve these equations to find the steady-state probability distribution of the number of cars at the kiosk.

   (d) Find the expected waiting time (including service) for those cars that use the kiosk.

5. An average of 40 cars per hour (interarrival times are exponentially distributed) are tempted to use the drive-in window at the Hot Dog King Restaurant. If a total of more than or equal to 4 cars are in line (including the car at the window) a car will not enter the line. It takes an average of 4 minutes (exponentially distributed) to serve a car.

   (a) What is the average number of cars waiting for the drive-in window (not including a car at the window)?

   (b) On the average, how many cars will be served per hour?

   (c) I have just jointed the line at the drive-in window. On the average, how long will it be before I have received my food?

6. (*) A small mail order firm Seas Beginnings has one phone line. An average of 60 people per hour call in orders, and it takes an average of 1 minute to handle a call. Time between calls and time to handle calls are exponentially distributed. If the phone line is busy, Seas Beginnings can put up to $c - 1$ people on hold. If $c - 1$ people are on hold, a caller gets a busy signal and calls a competitor (Air End). Seas Beginnings wants only 1% of all callers to get a busy signal. How many people should they be able to put on hold?