NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 2, 2020/2021

MA4264 Game Theory **Tutorial 2**

1. Suppose there are n firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i, and let $Q = q_1 + \cdots + q_n$ denote the aggregate quantity on the market. Let P denote the market-clearing price and assume that inverse demand is given by $P(Q) = a - Q$ (assuming $Q < a$, else $P = 0$). Assume that the total cost of firm i from producing quantity q_i is $C_i(q_i) = cq_i$. That is, there are no fixed costs and the marginal cost is constant at c, where we assume $c < a$. Following Cournot, suppose that the firms choose their quantities simultaneously. What is the Nash equilibrium? What happens as n approaches infinity?

2. Consider the Cournot duopoly model where inverse demand is $P(Q) = a - Q$ but firms have asymmetric marginal costs: c_1 for firm 1 and c_2 for firm 2. What is the Nash equilibrium if $0 < c_i < a/2$ for each firm? What if $c_1 < c_2 < a$ but $2c_2 > a + c_1$?

3. Consider a market of duopoly. The two firms produce the same product. Let q_i be the quantity of the product produced by firm i, $i = 1, 2$. Let the market price be

$$P(q_1, q_2) = \begin{cases} 25 - q_1 - q_2, & \text{if } q_1 + q_2 < 25; \\ 0, & \text{if } q_1 + q_2 \geq 25. \end{cases}$$

Let the cost of producing a unit of the product be $c_1 = 6$ for firm 1 and $c_2 = 5$ for firm 2. Due to the restriction of technology, firm 1 can produce either $q_1 = 5$ or $q_1 = 10$. Firm 2 can produce any quantity $q_2 \geq 0$. Firm i's payoff is its profit $q_i(P(q_1, q_2) - c_i)$.

Find the Nash equilibrium of the game.

4. Two players simultaneously announce their demands: player 1 demands $x \in [0, 1]$ and player 2 demands $y \in [0, 1]$. Suppose that the amount of money available is drawn from the uniform distribution $z \sim U[0, 1]$. The players receive payoffs x and y respectively if $x + y \leq z$ and 0 otherwise. Find all the pure-strategy Nash equilibria of this game.

5. Prove the following statement for a two-player game. If a strategy $s_{kj} \in S_k (k = 1, 2)$ is eliminated by the iterated elimination of strictly dominated strategies, then s_{kj} must be played with zero probability in any mixed strategy Nash equilibrium.

6. Consider the following two-person game.

Player 1 \ Player 2	X	Y
A	$9, 9$	$0, 8$
B	$8, 0$	$7, 7$

   (i) Suppose that Player 1 thinks that Player 2 will play her strategy X with probability y and her strategy Y with probability $1 - y$. For what value of y will Player 1 be indifferent between his two strategies?
   (ii) If y is less than this value what strategy will Player 1 prefer? If y is greater than that value?
   (iii) Graph the best responses of Player 1 to Player 2's mixed strategy.
   (iv) Repeat this analysis with the roles of the players reversed.

7. Consider the following game:

Player 1 \ Player 2	L	M	R
A	$4, 3$	$2, 5$	$2, 0$
B	$6, 2$	$0, 3$	$1, 4$
C	$3, 1$	$1, 0$	$1, 2$
D	$3, 0$	$1, 1$	$3, 3$

   (i) Eliminate strictly dominated strategies.
   (ii) Find all pure-strategy Nash equilibria and write down the corresponding payoffs.
   (iii) Find all mixed-strategy Nash equilibria and write down the corresponding expected payoffs.

End of Tutorial 2