NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

2020/2021 (II) MA4260 Stochastic Operations Research Tutorial 1

An asterisk (*) means that you are required to write-up the solution as part of an up-coming graded assignment. We will not discuss these questions during tutorials.

1. (*) Suppose I arrive at an $M/M/k/FCFS/k+1/\infty$ queueing system when all servers are busy. What is the probability that I will complete service before at least one of the k customers in service?

2. The time between arrivals of buses follows an exponential distribution with a mean of one hour.

   (a) What is the probability that exactly four buses will arrive within the next 2 hours?

   (b) What is the probability that at least two buses will arrive during the next 2 hours?

   (c) What is the probability that no buses will arrive during the next 2 hours?

   (d) A bus has just arrived. What is the probability that the next bus will arrive between 30 and 90 minutes?

3. Suppose a queueing system has two servers and each of them has an exponentially distributed service time with a mean of 2 hours. Suppose the interarrival time is exponentially distributed with a mean of 2 hours. Furthermore, suppose a customer just arrived at 12:00 noon.

   (a) What is the probability that the next arrival will come (i) before 1:00pm? (ii) between 1:00pm and 2:00pm? (iii) after 2:00pm?

   (b) Suppose that no additional customers arrive before 1:00pm. Now, what is the probability that the next arrival will come between 1:00pm and 2:00pm?

   (c) What is the probability that the number of arrivals between 1:00pm and 2:00pm is (i) zero? (ii) one? (iii) two or more?

   (d) Suppose that both servers are serving customers at 1:00pm. What is the probability that neither customers will have service completed (i) before 2:00pm? (ii) before 1:10pm? (iii) before 1:01pm?

4. The time between buses follows the mass function shown in the table below. What is the average length of time one must wait for a bus?

Time between buses		Probability
30	minutes	1/4
1	hour	1/4
2	hour	1/2

5. (i) Show that the chance that an exponentially distributed variable takes on a value below its mean is more than 60% and that the chance that the value will be below half of the mean is almost 40%.

   (ii) When are exponentially distributed interarrival (service) times reasonable/unreasonable for a queuing system?

6. It is known that the interarrival time $T \sim U[0, u]$. Let t_0 denote the time now and suppose that there has not been any arrivals since a_0. Suppose $t_0 - a_0 < u$. What is the distribution of the remaining waiting time?