NATIONAL UNIVERSITY OF SINGAPORE

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MA4264 Game Theory **Tutorial 1**

1. In the following normal-form games, what strategies survive iterated elimination of strictly dominated strategies? What are the pure-strategy Nash equilibria?

	L	C	R
T	$2, 0$	$1, 1$	$4, 2$
M	$3, 4$	$1, 2$	$2, 3$
B	$1, 3$	$0, 2$	$3, 0$

	L	R
U	$1, 3$	$-2, 0$
M	$-2, 0$	$1, 3$
D	$0, 1$	$0, 1$

2. An old lady is looking for help crossing the street. Only one person is needed to help her; more are okay but no better than one. You and I are the two people in the vicinity who can help, each has to choose simultaneously whether to do so. Each of us will get pleasure worth of 3 from her success (no matter who helps her). But each one who goes to help will bear a cost of 1, this being the value of our time taken up in helping. Set this up as a game. Write the payoff table, and find all pure-strategy Nash equilibria.

3. There are three computer companies, each of which can choose to make large (L) or small (S) computers. The choice of company 1 is denoted by S_1 or L_1, and similarly, the choices of companies 2 and 3 are denoted S_i or L_i of $i = 2$ or 3. The following table shows the profit each company would receive according to the choices which the three companies could make. What is the outcome of IESDS and the Nash equilibria of the game?

	S_2S_3	S_2L_3	L_2S_3	L_2L_3
S_1	$-10, -15, 20$	$0, -10, 60$	$0, 10, 10$	$20, 5, 15$
L_1	$5, -5, 0$	$-5, 35, 15$	$-5, 0, 15$	$-20, 10, 10$

4. In the movie, “A Beautiful Mind”, John Nash gets the idea for Nash equilibrium in a student hangout where he is sitting with three buddies. Five women walk in, four brunettes and a stunning blonde. Each of the four buddies starts forward to introduce himself to the blonde. Nash stops them, though, saying, “If we all go for the blonde,

we will all be rejected and none of the brunettes will talk to us afterwards because they will be offended. So let's go for the brunettes." The next thing we see is the four buddies dancing with the four brunettes and the blonde standing alone, looking unhappy.

Assume that if more than one buddy goes after a single woman, they will all be rejected by the woman and end up alone. The payoffs are as follows. Ending up with the blonde has a payoff of 4, ending up with a brunette has a payoff of 1, and ending up alone is 0. The four buddies are players in this noncooperative game.

(i) Is the result in the story a Nash equilibrium?

(ii) Find all pure-strategy Nash equilibria for this game.

(iii) Are the Nash equilibria you find better than what Nash suggested?

5. Players 1 and 2 are bargaining over how to split one dollar. Both players simultaneously name shares they would like to have, s_1 and s_2, where $0 \leq s_1,\ s_2 \leq 1$. If $s_1^2 + s_2^2 \leq 1/2$, then the players receive the shares they named; if $s_1^2 + s_2^2 > 1/2$, then both players receive zero. What are the pure-strategy Nash equilibria of this game? Now we change the payoff rule as follows: If $s_1^2 + s_2^2 < 1/2$, then the players receive the shares they named; if $s_1^2 + s_2^2 \geq 1/2$, then both players receive zero. What are the pure-strategy Nash equilibria of this game?

6. Two firms may compete for a given market of total value, V, by investing a certain amount of effort into the project through advertising, securing outlets, etc. Each firm may allocate a certain amount for this purpose. If firm 1 allocates $x \geq 0$ and firm 2 allocates $y \geq 0$, then the proportion of the market that firm 1 corners is $x/(x+y)$. The firms have different difficulties in allocating these resources. The cost per unit allocation to firm i is c_i, $i = 1, 2$. Thus the profits to the two firms are

$$\pi_1(x, y) = V \cdot \frac{x}{x+y} - c_1 x,$$
$$\pi_2(x, y) = V \cdot \frac{y}{x+y} - c_2 y.$$

If both x and y are zero, the payoffs to both are $V/2$.

Find the equilibrium allocations, and the equilibrium profits to the two firms, as functions of V, c_1 and c_2.

7. Let $n (n \geq 2)$ people play the following game. Simultaneously, each player i announces a number x_i in the set $\{1, \ldots, K\}$. A prize of \$1 is split equally between all the people whose number is closest to $\frac{2}{3} \cdot \frac{x_1 + \cdots + x_n}{n}$. Find all the pure-strategy Nash equilibria.

8. A two-person game is called a zero-sum game (also called a matrix game) if $u_1(s_1, s_2) + u_2(s_1, s_2) = 0$ for all $s_1 \in S_1$ and $s_2 \in S_2$. Show that (s_1^*, s_2^*) is a pure-strategy Nash equilibrium of a two-person zero-sum game if and only if

$$u_1(s_1, s_2^*) \leq u_1(s_1^*, s_2^*) \leq u_1(s_1^*, s_2), \quad \forall s_1 \in S_1,\ s_2 \in S_2.$$

Consider a two-person zero-sum game in strategic form with finitely many strategies for each player (not just two), and assume that player I has two particular pure strategies T and B and that player II has two pure strategies l and r so that both (T, l) and (B, r) are Nash equilibria of the game. Show that there are at least two further pure-strategy Nash equilibria.

Prove that, for each player, the payoffs for the given equilibria are equal.

End of Tutorial 1