MA2213 Cheatsheet 19/20 Sem 1 Midterm

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Lecture 8: Lagrange Interpolation

Solving via linear system.

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix}$$

Solving via basis polynomials. Let $L_k(x) = \frac{(x-x_0)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$ for $k = 0, 1, \cdots, n$. Then $P_n(x) = y_0 L_0(x) + y_1 L_1(x) + \cdots + y_n L_n(x)$.

Error analysis. $x_0, x_1, \cdots, x_n$ are distinct in $[a, b] \implies \forall x \in [a, b], \exists \xi \in (\min\{x, x_0, x_1, \cdots, x_n\}, \max\{x, x_0, x_1, \cdots, x_n\})$ such that

$$f(x) = P_n(x) + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n)$$

Tutorial 3: Lagrange Interpolation

Lecture 9: Divided Differences

Structure of Lagrange polynomial. $P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots + a_n(x-x_0)(x-x_1)\cdots(x-x_{n-1})$ where $a_k = f[x_0, x_1, \cdots, x_k]$ and $f[x_0, x_1, \cdots, x_n] = \frac{f[x_1, x_2, \cdots, x_n] - f[x_0, x_1, \cdots, x_{n-1}]}{x_n - x_0}$. $a_0 = f(x_0)$.

Lecture 10: Cubic Spline Interpolation

Tutorial 4: Divided diff and CSI

Lecture 11: Least Squares Approximation

Lecture 12: Newton-Cotes Formulae

Lecture 13: Composite Numerical Integration

Tutorial 5: LSA and Integration

Miscellaneous

The Gamma function. $\Gamma(x) = \int_0^{+\infty} t^{x-1} \mathrm{e}^{-t} \mathrm{d}t$ and $\Gamma(n) = (n-1)!$ for any positive integer n.