44.

A.

$$x(t) = C_1 e^{2t} + C_2 e^{-\frac{1}{3}t}$$

B.

$$x(t) = C_1 e^{-7t+2it} + C_2 e^{-7t-2it}$$

$$= C_1 \cdot e^{-7t} \cdot (\cos(2t) + i\sin(2t)) + C_2 \cdot e^{-7t}(\cos(2t) - i\sin(2t))$$

$$= e^{-7t}((C_1 + C_2)\cos(2t) + (C_1 - C_2)i\sin(2t))$$

for $C_1 = C_2 = \frac{1}{2}$

$$X_1(t) = e^{-7t}\cos(2t)$$

for $C_1 = \frac{1}{2i}$ $C_2 = -\frac{1}{2i}$

$$X_2(t) = e^{-7t}\sin(2t)$$

45.

A.

$$ma = -kx - Cv$$

$$m \cdot x''(t) + C \cdot x'(t) + k \cdot x(t) = 0$$

B.

$$m \cdot \frac{d^2}{dt^2}(e^{\lambda t}) + C\frac{d}{dt}(e^{\lambda t}) + k e^{\lambda t} = 0$$

$$\Rightarrow e^{\lambda t}(m\lambda^2 + C\lambda + k) = 0$$

$$\Rightarrow e^{\lambda t}(1\lambda^2 + 4\lambda + 29) = 0$$

(use the quadratic formula)

$$\Rightarrow \lambda = -2 \pm 5i$$

$$x_1(t) = c_1 e^{(-2+5i)t} = c_1 e^{-2t}(\cos(5t) + i\sin(5t))$$

$$x_2(t) = c_2 e^{(-2-5i)t} = c_1^{-2t}(\cos(5t) - i\sin(5t))$$

Real solutions would be

$$\frac{1}{2}x_1(t) + \frac{1}{2}x_2(t) = e^{-2t}\cos(5t)$$

$$\frac{1}{2i}x_1(t) - \frac{1}{2i}x_2(t) = e^{-2t}\sin(5t)$$

c.

$$x(t) = c_1 e^{-2t}\cos(5t) + c_2 e^{-2t}\sin(5t)$$

$$x(0) = 0$$

$$\Rightarrow c_1 e^{-2(0)}\cos(0) + c_2 e^{-2t}\sin(0) = 0$$

$$\Rightarrow c_1 = 0$$

$$x'(0) = 4$$

$$\Rightarrow \frac{d}{dt}\left(c_2 e^{-2t}\sin(5t)\right)\Big|_{t=0} = 4$$

$$\Rightarrow c_2(-2)e^{-2t}\sin(5t) + 5c_2 e^{-2t}\cos(5t)\Big|_{t=0} = 4$$

$$\Rightarrow 5c_2 = 4$$

$$\Rightarrow c_2 = 4/5$$

$$x(t) = 4/5\, e^{-2t}\sin(5t)$$

46.

A.

$$\omega = \sqrt{\frac{k}{m}} \quad \text{where} \quad m\,x''(t) + k\,x(t) = 0$$

i. $\omega = 1/2$

ii. $\omega = 2$

iii. $\omega = 1/4$

iv. $\omega = 4$

Since $T = \frac{2\pi}{\omega}$, the highest ω will have the shortest period

Therefore iV has the shortest period

B.

we look for the largest $x(0)$

i. has the largest amplitude of 36 meters

C.

we know each equation is in the form

$$x(t) = C_1 \cos\left(\sqrt{\tfrac{k}{m}}\, t\right)$$

where C_1 is $x(0)$ from the table and $\omega = \sqrt{\frac{k}{m}}$

original functions			Derivatives
i.	$36\cos(\frac{1}{2}t)$		i. $-18\sin(\frac{1}{2}t)$
ii.	$25\cos(2t)$	$\Rightarrow$	ii. $-50\sin(2t)$
iii.	$4\cos(\frac{1}{4}t)$		iii. $-\frac{4}{4}\sin(\frac{1}{4}t)$
iv.	$\cos(4t)$		iV. $-4\sin(4t)$

Has the greatest magnitude for coeffecient

Therefore the highest possible velocity is eq ii.