Tutorial 1 Solutions

1.

A. $$\frac{d}{dt}\left(\sqrt{t^2-3t}\right) = \frac{d}{dt}\left(t^2-3t\right)^{1/2}$$

$$= \tfrac{1}{2}\left(t^2-3t\right)^{-1/2} \cdot \frac{d}{dt}\left(t^2-3t\right)$$

$$= \tfrac{1}{2}\left(t^2-3t\right)^{-1/2}\left(2t-3\right)$$

B. $$\frac{d}{dt}\left(\frac{1}{\sqrt{t^2-3t}}\right) = \left(t^2-3t\right)^{-1/2}$$

$$= -\tfrac{1}{2}\left(t^2-3t\right)^{-3/2}\frac{d}{dt}\left(t^2-3t\right)$$

$$= -\tfrac{1}{2}\left(t^2-3t\right)^{-3/2}\left(2t-3\right)$$

C. $$\frac{d}{dx}\left(\frac{1}{1+e^{-x}}\right) = \frac{d}{dx}\left(1+e^{-x}\right)^{-1}$$

$$= -1 \cdot \left(1+e^{-x}\right)^{-2}\frac{d}{dx}\left(1+e^{-x}\right)$$

$$= -1 \cdot \left(1+e^{-x}\right)^{-2} \cdot \left(-e^{-x}\right)$$

$$= \left(1+e^{-x}\right)^{-2} \cdot e^{-x}$$

D.

$$\frac{d}{dt}\left(\ln\left(8-t^3\right)\right) = \frac{d}{dt}\ln\left(8-t^3\right)$$

$$= \frac{1}{8-t^3}\cdot\frac{d}{dt}\left(8-t^3\right)$$

$$= \frac{1}{8-t^3}\cdot\left(-3t^2\right)$$

$$= \frac{-3t^2}{8-t^3}$$

E.

$$\frac{d}{dx}\left[\left(\ln\left(\cos^2(x)+1\right)\right)\cdot\left(e^x+\sin(x)\right)^3\right]$$

$$= \frac{d}{dx}\left(\ln\left(\cos^2(x)+1\right)\right)\left(e^x+\sin(x)\right)^3$$

$$+ \ln\left(\cos^2(x)+1\right)\frac{d}{dx}\left(e^x+\sin(x)\right)^3$$

$$= \frac{1}{\cos^2(x)+1}\cdot\frac{d}{dx}\left(\cos^2(x)+1\right)\left(e^x+\sin(x)\right)^3$$

$$+ \ln\left(\cos^2(x)+1\right)\cdot 3\left(e^x+\sin(x)\right)^2\frac{d}{dx}\left(e^x+\sin(x)\right)$$

$$= \frac{1}{\cos^2(x)+1} \cdot \left(-2\cos(x)\sin(x)\right)\left(e^x+\sin(x)\right)^3$$

$$+ \ln\left(\cos^2(x)+1\right) \cdot 3\left(e^x+\sin(x)\right)^2\left(e^x+\cos(x)\right)$$

2.

A.

$$\frac{d}{dt}(r(t)) = \frac{d}{dt}\left(\frac{5t^4 - 2t}{t^{1/2}}\right)$$

$$= \frac{d}{dt}\left(5t^{7/2} - 2t^{1/2}\right)$$

$$= \frac{35}{2}t^{5/2} - t^{-1/2}$$

B.

$$r'(t) = \frac{d}{dt}\left(\frac{\sin(7t)}{t^3}\right)$$

$$= \frac{\frac{d}{dt}(\sin(7t))\, t^3 - \sin(7t)\frac{d}{dt}(t^3)}{(t^3)^2}$$

$$= \frac{7\cdot\cos(7t)\, t^3 - \sin(7t)(3t^2)}{t^6}$$

$$= \frac{7\cdot\cos(7t)}{t^3} - \frac{3\sin(7t)}{t^4}$$

3.

A.

$$\vec{a}+\vec{b} = \langle 1, 2, 3 \rangle + \langle 0, -1, 1 \rangle$$
$$= \langle 1+0, 2-1, 3+1 \rangle$$
$$= \langle 1, 1, 4 \rangle$$

B.

$$\vec{b} - 3\vec{c} = \langle 0, -1, 1 \rangle - 3 \cdot \langle 2, 4, 6 \rangle$$
$$= \langle 0, -1, 1 \rangle - \langle 6, 12, 18 \rangle$$
$$= \langle -6, -13, -17 \rangle$$

C.

$$\| \vec{a} - \vec{b} \| = \| \langle 1, 2, 3 \rangle - \langle 0, -1, 1 \rangle \|$$
$$= \| \langle 1, 3, 2 \rangle \|$$
$$= \sqrt{1^2 + 3^2 + 2^2}$$
$$= \sqrt{14}$$

D.

$$\vec{b} \cdot \vec{c} = \langle 0, -1, 1 \rangle \cdot \langle 2, 4, 6 \rangle$$

$$= 0 \cdot 2 - 1 \cdot 4 + 1 \cdot 6$$

$$= 0 - 4 + 6$$

$$= 2$$

4.

A. Dot products of perpendicular vectors are $= 0$

$$\langle 1, k, 3 \rangle \cdot \langle 2, -3, 4 \rangle = 0$$

$$\Rightarrow 2 - 3k + 12 = 0$$

$$\Rightarrow k = \frac{14}{3}$$

B.

parallel $\vec{a} = n \cdot \vec{b}$ for any real number

$$\langle q, 2 \rangle = n \cdot \langle 2, 3 \rangle$$

$$\Rightarrow q = n \cdot 2, \quad 2 = n \cdot 3$$

$$\Rightarrow n = 2/3, \quad q = 4/3$$

5.

$$\langle a, b \rangle \cdot \langle -3, 4 \rangle = 0$$

$$-3a + 4b = 0$$

$$a = \frac{4}{3} b$$

Hence $\vec{v} = \langle \frac{4}{3}, 1 \rangle$ but this isn't a unit vector.

To change this we have

$$\frac{\vec{v}}{||\vec{v}||} = \frac{\langle \frac{4}{3}, 1 \rangle}{\sqrt{1 + \frac{16}{9}}} = \frac{3}{5} \langle \frac{4}{3}, 1 \rangle = \langle \frac{4}{5}, \frac{3}{5} \rangle$$

6.

$$\vec{w} = \langle 0, 0, 0 \rangle$$