Tutorial #2

7. A.

$$\langle x, y \rangle = \langle \sqrt{t-1}, \tfrac{t}{2} \rangle$$

From this we can get the following equations

① $x = \sqrt{t-1}$

② $y = \frac{t}{2}$

The question asks us to "Eliminate the parameter" which just means that we want to get rid of t and express y in terms of x.

There are multiple ways of eliminating t but should all lead to the same answer

① $x = \sqrt{t-1} \Rightarrow x^2 + 1 = t$

we substitute here

② $y = \frac{t}{2} \Rightarrow y = \frac{(x^2+1)}{2}$

Hence we get

$$y = \frac{(x^2+1)}{2}$$

B.

Although it may be tempting to graph $y = \frac{(x^2+1)}{2}$ we need take into account the following,

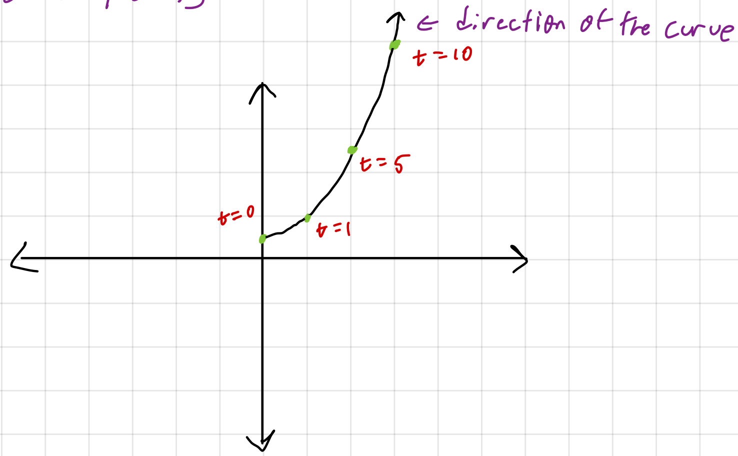
Things to take note of here:

- Domain of t
- How Domain of t affects values of x and y

$x = \sqrt{t-1}$ is only real when $t \geq 1$, for any $t < 1$ we get a negative number in the root.

Knowing $t \geq 1$ lets plot some points

t	$\langle x, y \rangle$
1	$\langle 0, \frac{1}{2} \rangle$
2	$\langle 1, 1 \rangle$
5	$\langle 2, \frac{5}{2} \rangle$
10	$\langle 3, 5 \rangle$



Hence from the graph we can see that the curve goes towards the right and passes over $y = \frac{(x+1)^2}{2}$ when $x \geq 0$

8. A.

When two particles collide we know that

$$r_1(t_0) = r_2(t_0)$$

for some value t_0.

$$r_1(t) = \langle t^2, 7t-12, t^2 \rangle$$
$$r_2(t) = \langle 4t-3, t^2, 5t-6 \rangle$$

$$r_1(t_0) = r_2(t_0) \Rightarrow \langle t_0^2, 7t_0-12, t_0^2 \rangle = \langle 4t_0-3, t_0^2, 5t_0-6 \rangle$$

$$\Rightarrow \begin{array}{l} t_0^2 = 4t_0 - 3 \\ 7t_0 - 12 = t_0^2 \\ t_0^2 = 5t_0 - 6 \end{array}$$

with these 3 equations, solve for t_0

$$\begin{array}{l} t_0^2 = 4t_0 - 3 \\ t_0^2 = 7t_0 - 12 \end{array} \Rightarrow \begin{array}{l} 4t_0 - 3 = 7t_0 - 12 \\ 3t_0 = 9 \\ t_0 = 3 \end{array}$$

plug $t_0 = 3$ back into vector to verify solution

$$r_1(3) = \langle (3)^2, 7(3)-12, (3)^2 \rangle = \langle 9, 9, 9 \rangle$$

$$r_2(3) = \langle 4(3)-3, (3)^2, 5(3)-6 \rangle = \langle 9, 9, 9 \rangle$$

So the paths indeed intersect at $\langle 3, 3, 3 \rangle$ at $t = 3$.

B.

When the paths intersect, the curves go through the same point but at different times.

$$r_1(t_1) = r_2(t_2)$$

for some point t_1 and t_2

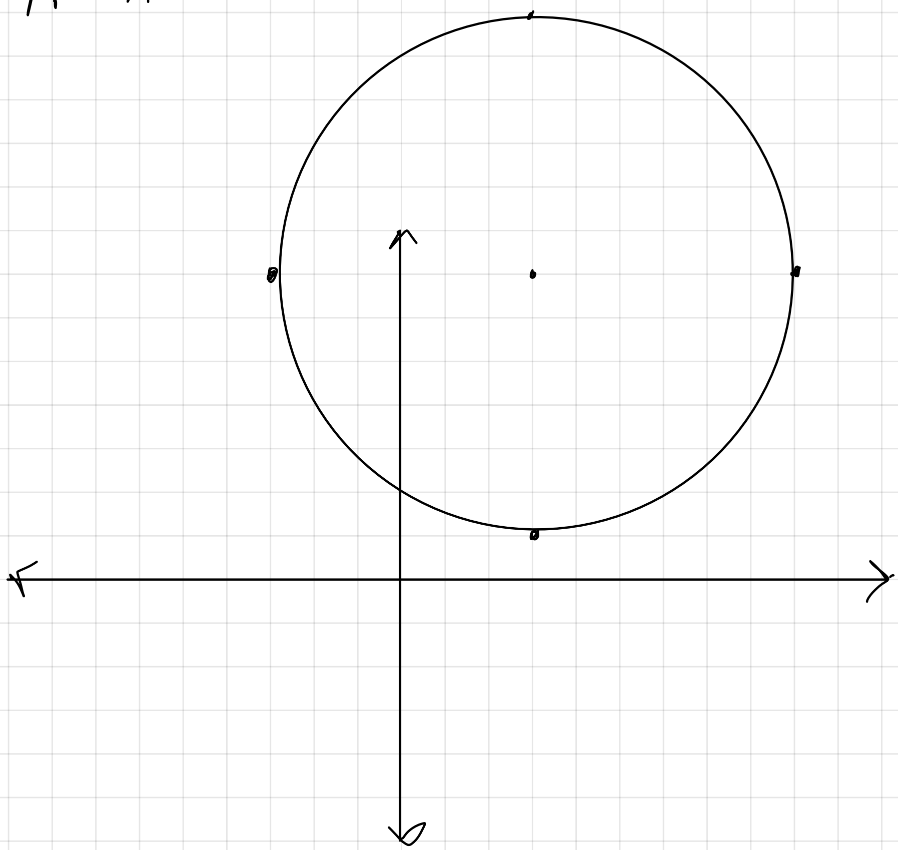
$$\langle t_1^2, 7t_1 - 12, t_1^2 \rangle = \langle 4t_2 - 3, t_2^2, 5t_2 - 6 \rangle$$

$$\Rightarrow \begin{aligned} t_1^2 &= 4t_2 - 3 \\ 7t_1 - 12 &= t_2^2 \\ t_1^2 &= 5t_2 - 6 \end{aligned}$$

$$\Rightarrow \quad t_1 = 3, \quad t_2 = 3$$

Hence the only values that satisfy these equalities is $t_1 = 3$ and $t_2 = 3$. Therefore the only time the paths intersect is when they collide at $t = 3$

9. A.



B.

$r(t) = \langle 6\cos(t)+3,\ 6\sin(t)+7 \rangle$

C.

$r(t) = \langle 6\cos(t)+3,\ 6\sin(t)+7 \rangle$

$\Rightarrow \quad x = 6\cos(t)+3$

$\quad\quad y = 6\sin(t)+7$

plug these values back into our original equation:

$$((6\cos(t)+3)-3)^2+((6\sin(t)+7)-7)^2=36$$

$$(6\cos(t))^2+(6\sin(t))^2=36$$

$$36(\cos^2(t)+\sin^2(t))=36$$

$$36=36$$

Hence r(t) is always on the curve regardless of the value of t.