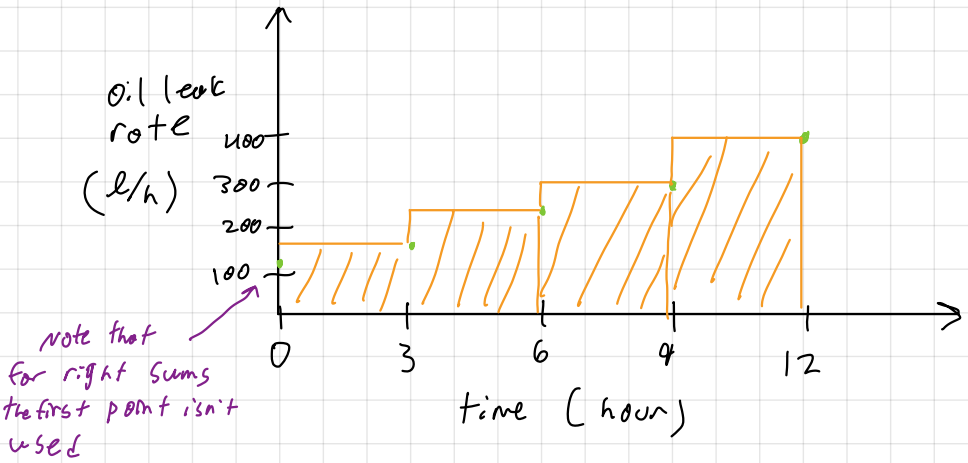
Question 23:

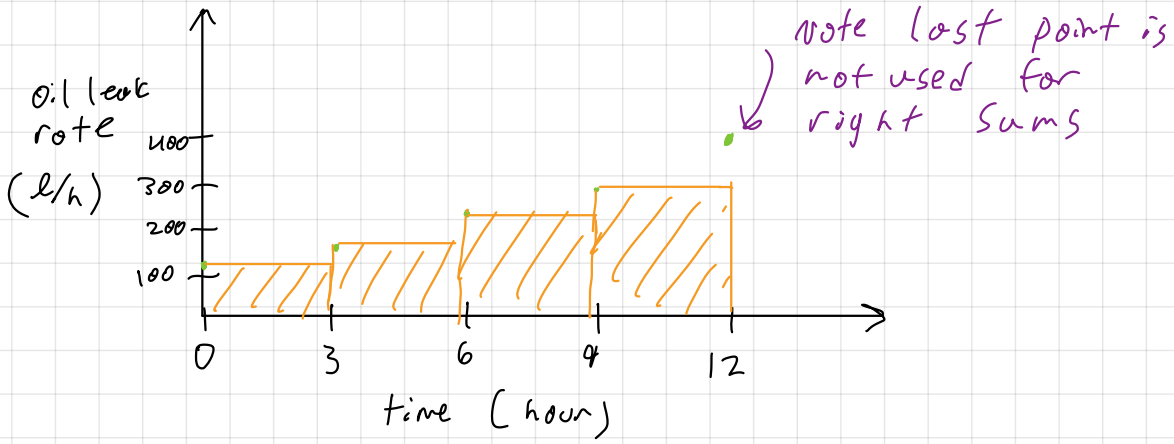
upper estimates (right riemann sums):



$$RIGHT(4) = (3-0)(154) + (6-3)(235) + (9-6)(295) + (12-9)(404)$$

$$= 3,279$$

Lower estimates:



$$LEFT(4) = (3-0)(114) + (6-3)(154) + (9-6)(235) + (12-9)(295)$$

$$= 2,409$$

Question 24:

$$\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right) dx$$

Key is to solve this geometrically

$$y = 1 + \sqrt{9 - x^2}$$

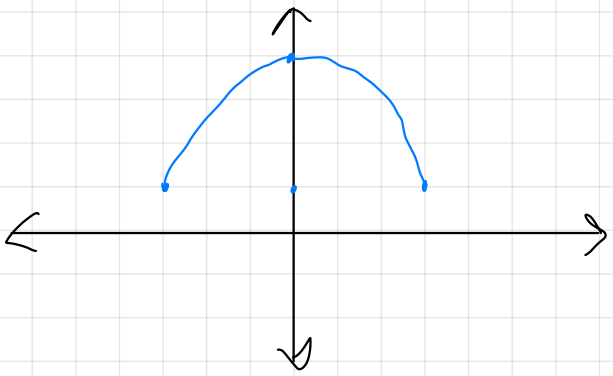
$$\Rightarrow y - 1 = \sqrt{9 - x^2}$$

$$\Rightarrow (y-1)^2 = 9 - x^2$$

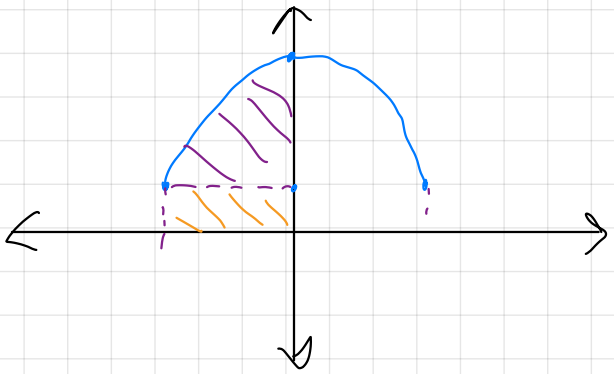
$$\Rightarrow (y-1)^2 + x^2 = 9$$

Note because $y = 1 + \sqrt{9-x^2}$ and not $y = 1 \pm \sqrt{9-x^2}$, y only describes the top half of the circle

Hence the function is a circle of radius 3 centered at $(0, 1)$



Hence, the area under the curve on the interval $[-3, 0]$ is



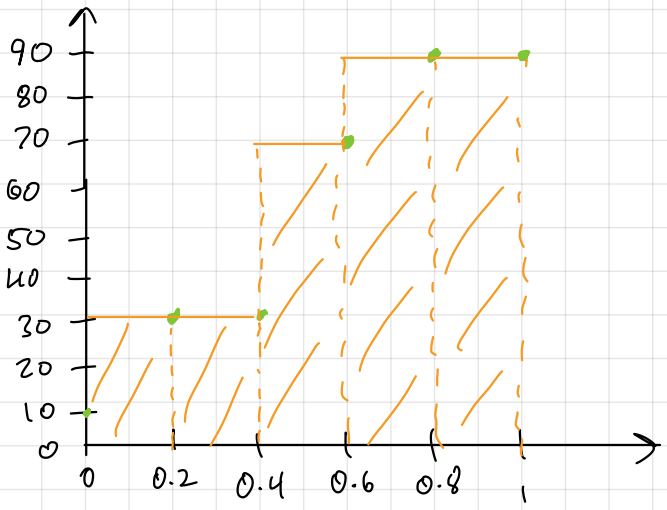
Area = $\frac{1}{4}$ circle + rectangle

Area = $\frac{1}{4}(\pi(3^2)) + (3 * 1)$

Area = $9/4\,\pi + 3$

Question 25:

A.



$$RIGHT(5) = 30(0.2) + 30(0.4-0.2) + 70(0.6-0.4) + 90(0.8-0.6) + 90(1-0.8)$$

$$RIGHT(5) = 62.0 \text{ km}$$

B.

$$\text{gas consumed} = \text{distance} \cdot \frac{\text{efficiency}}{100}$$

$$= \text{km} \cdot \frac{\text{liters}}{100 \text{ km}}$$

$$= \frac{\text{liters}}{100}$$

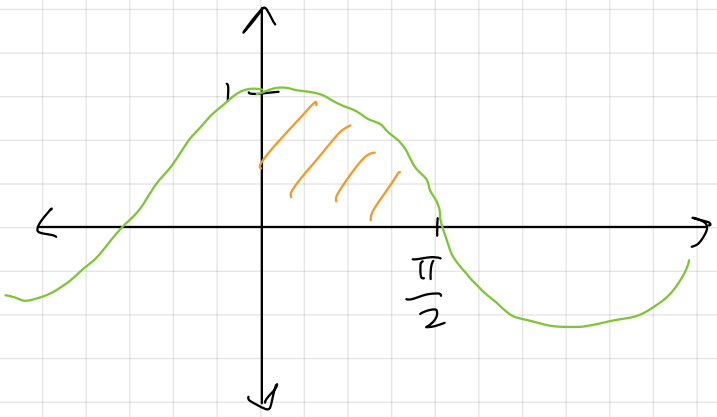
The units are liters as expected

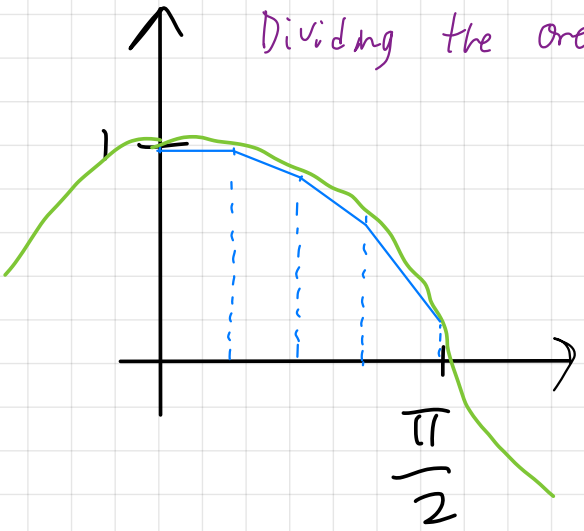
$$\text{overall gas consumed} = (30 \tfrac{\text{km}}{\text{h}} \cdot 0.2\text{h}) \cdot \frac{15 \text{ liters}}{100 \text{ km}} + \frac{(30)(0.2)(15)}{100} + \frac{(70)(0.2)(7)}{100} + \frac{(90)(0.2)(8)}{100} + \frac{(90)(0.2)(8)}{100}$$

$$= 5.66 \text{ liters}$$

Question 26:

A.





Dividing the area into trapezoids:

$$\text{TRAPEZOID} = \frac{b-a}{n}\left[\frac{f(a)}{2} + \sum_{k=1}^{n-1} f\left(a + k\frac{(b-a)}{n}\right) + \frac{f(b)}{2}\right]$$

$$\text{TRAPEZOID}(4) = \frac{\frac{\pi}{2}-0}{4}\left[\frac{\cos(0)}{2} + \cos(\pi/8) + \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{3\pi}{8}\right) + \frac{\cos(\pi/2)}{2}\right]$$

$$= 0.987$$