Question 18:

By implicit differentiation:

$$\frac{d}{dx}\left(y^2 + xy - y^3\right) = \frac{d}{dx}(7)$$

$$\Rightarrow \quad 2y\frac{dy}{dx} + \frac{dx}{dx}y + x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 2y\frac{dy}{dx} + y + x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}\left(2y + x - 3y^2\right) = -y$$

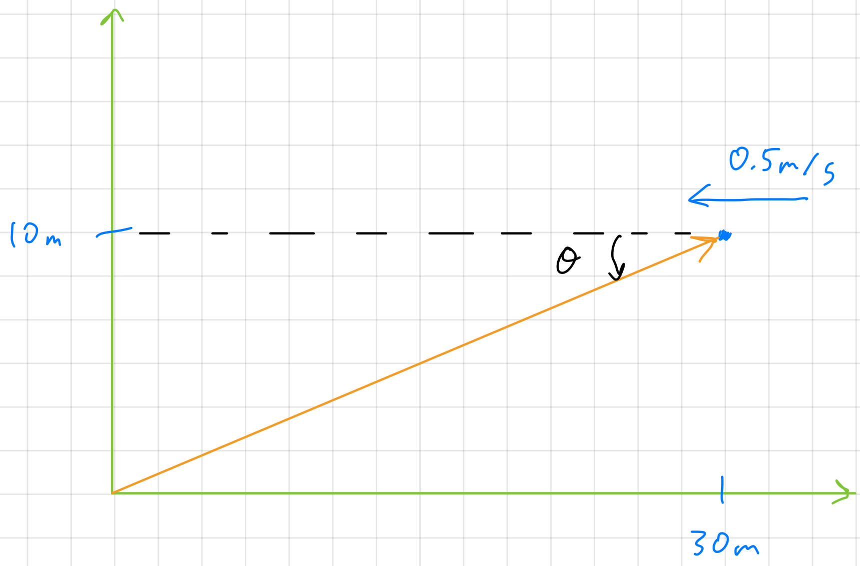
$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2y + x - 3y^2}$$

Plug in the point $(3, 2)$:

$$\frac{dy}{dx} = \frac{-2}{2(2) + 3 - 3(2)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-2}{-5} = 2/5$$

Question 14:



Goal is to find the change in θ caused by the velocity of the drone.

$$\tan(\theta) = \frac{10}{30 - 0.5t}$$

(we could use other trig functions, but we will use tan for now.)

$$\Rightarrow \frac{d}{dt}\tan(\theta) = \frac{d}{dt}\frac{10}{30-0.5t}$$

$$\Rightarrow \sec^2(\theta)\frac{d\theta}{dt} = \frac{10\cdot(0.5)}{(30-0.5t)^2}$$

$$\Rightarrow \frac{1}{\cos^2\theta}\frac{d\theta}{dt} = \frac{5}{(30-0.5t)^2}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{5\cos^2(\theta)}{(30-0.5t)^2}$$

Now we need to calculate the initial values of t and θ:

For t:

From the way the equation is structured, the drone is 30 m away at $t=0$. Hence the initial value of t is $t = 0$.

For θ:

Initial position is $(30, 10)$

$$\tan(\theta) = \frac{10}{30} = 1/3$$

$$\Rightarrow \quad \theta = \arctan(1/3) \approx 0.322 \text{ rad}$$

Plugging these values back into the original equation yields:

$$\frac{d\theta}{dt} = \frac{5\cos^2(\theta)}{(30 - 0.5t)^2}$$

$$\frac{d\theta}{dt} = \frac{5\cos^2(0.322)}{(30 - 0.5(0))^2} = 5 \cdot 10^{-3} \text{ rad/s}$$

Question 20:

Volume of a cone:

$$V = \frac{1}{3}\pi r^2 h$$

We know volume remains constant and the height decreases at 0.1 m/s. Hence,

$$\frac{dh}{dt} = 0.1 \text{ m/s} \qquad \frac{dV}{dt} = 0$$

By implicit differentiation:

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{1}{3}\pi r^2 h\right)$$

$$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi\left(\frac{d}{dt}(r^2)\,h + r^2\frac{dh}{dt}\right)$$

$$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$$

$$\Rightarrow 0 = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2(-0.1)\right)$$

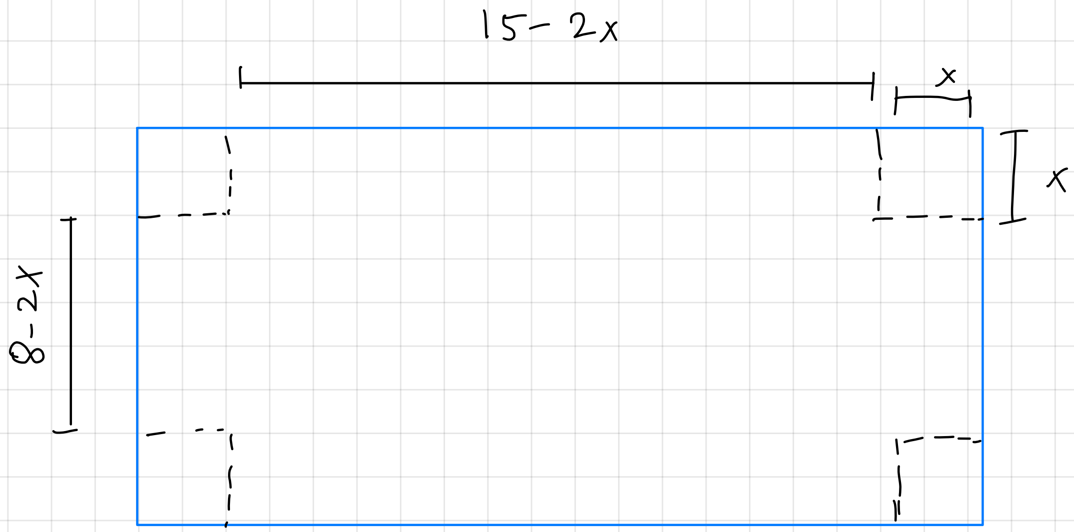
$$\Rightarrow 2r\frac{dr}{dt}h = r^2(0.1)$$

$$\Rightarrow \frac{dr}{dt} = \frac{r(0.1)}{2(h)}$$

From the problem we know $r = 3$ and $h = 2$, hence

$$\frac{dr}{dt} = \frac{(3)(0.1)}{2(2)} = 0.075 \text{ m/s}$$

Question 21:



A.

$$V = (8-2x)(15-2x)(x)$$

$$= 4x^3 - 46x^2 + 120x$$

B. we first find where the dervative is zero then see if this a local minimum or maximum

$$\frac{d}{dx}V = 12x^2 - 92x + 120 = 0$$

$$\Rightarrow (x-6)(3x-5)(4) = 0$$

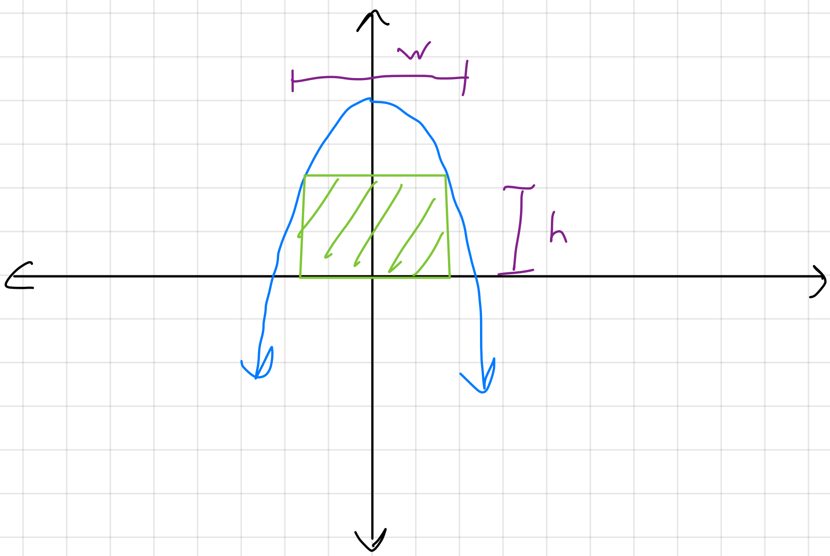
$$\Rightarrow x = 6 \text{ or } 5/3$$

$$\frac{d^2}{dx^2}V = 24x - 92$$

x	$V(x)$	$V''(x)$	Local min or max
6	−72	52	min
5/3	40.74	−52	max

Hence volume is optimized when squares of 5/3 cm are cut. The volume of this box would be 40.74 cm^3.

Question 22



equation for height in terms of width:

$$h = 20 - 4x^2$$

volume:

$$V = (20 - 4x^2)(2x) = -8w^3 + 40w$$

optimizing equation:

$$\frac{dV}{dx} = -24x^2 + 40 = 0$$

$\Rightarrow \quad x = 1.30$

$w = 2x = 2.6$

$h = 13.33$

$\Rightarrow v = 34.66$