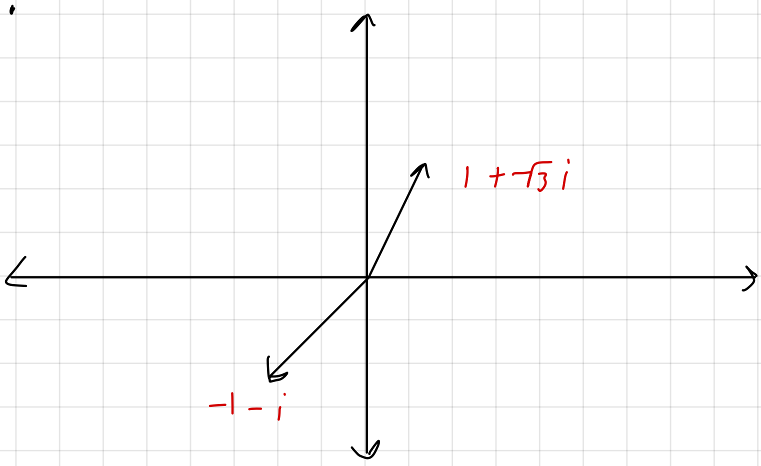
Question 40.

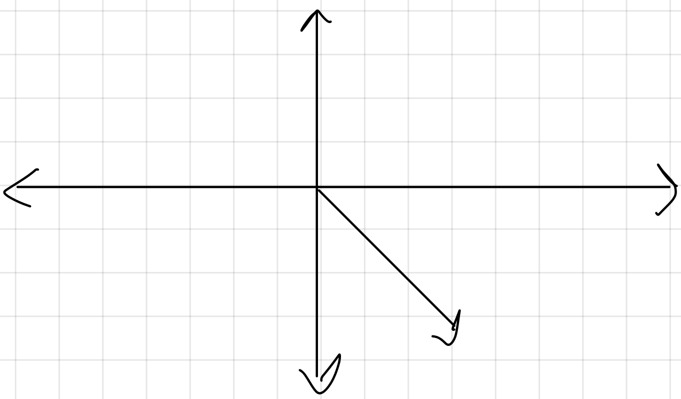
A.



B.

$$(1+\sqrt{3}i)(-1-i) = -1 - i - \sqrt{3}i - \sqrt{3}i^2$$
$$= -1+\sqrt{3} - (1+\sqrt{3})i$$

C.



D.

$$|z_1| = \sqrt{(\sqrt{3})^2+(1)^2} = 2$$

$$\angle \arg |z_1| = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \pi/3$$

$$|z_2| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$\arg \angle z_2 = \tan^{-1}\left(\frac{-1}{-1}\right) = \pi/4 + \pi = \frac{5\pi}{4}$$
(since in Q3)

E.

$$\angle \arg(z) = \tan^{-1}\left(\frac{-(1+\sqrt{3})}{-1+\sqrt{3}}\right)$$
$$= -\frac{5\pi}{12}$$

$$|z| = \sqrt{(1+\sqrt{3})^2 \; (-1+\sqrt{3})^2}$$
$$|z| = 2\sqrt{2}$$

F.

$$z_1 \cdot z_2 = 2\sqrt{2} \cdot \left(\cos\left(-\frac{5\pi}{12}\right) + i\sin\left(-\frac{5\pi}{2}\right)\right)$$
$$= 2\sqrt{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4} + i\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right)\right)$$
$$= -1+\sqrt{3} - (1+\sqrt{3})i$$

Question 41

A.

$$ma = F_{net} = -kx - mg - cv$$

$$m\,x''(t) = -k\,x(t) - mg - c\,x'(t)$$

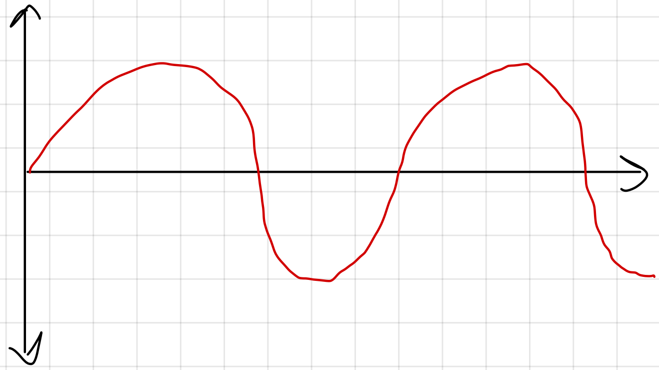
B. $kx = mg \Rightarrow x = \frac{mg}{k}$

$$y(t) = x(t) - \frac{mg}{k}$$

$$m\,y''(t) = -k\,y(t) - c\,y''(t)$$

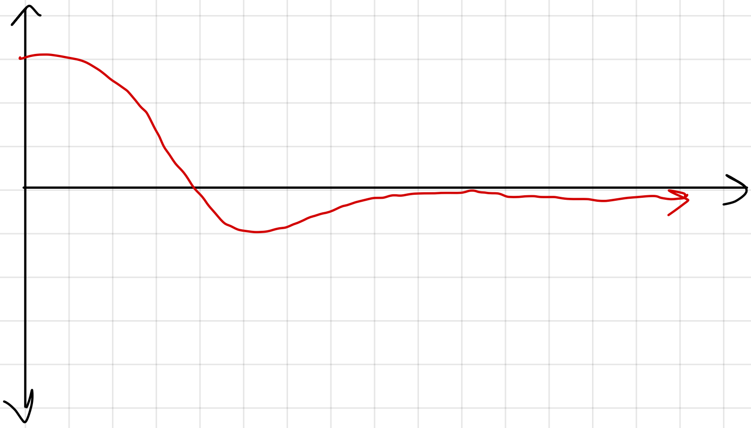
C.

$$m\,y''(t) = -k\,y(t)$$



D.

E.



43.

A. $i^{95} = (i^4)^{23} \cdot i^3$

$= 1 \cdot (-i)$

$= -i$

B.

$$(1+i)^{200} = \left(\sqrt{2}\cdot\left(\cos\left(\pi/4\right)+i\sin\left(\pi/4\right)\right)\right)^{200}$$

$$= 2^{100}\cdot\left(\cos\left(50\pi\right)+i\sin\left(50\pi\right)\right)$$

$$= 2^{100}\cdot 1$$

$$= 2^{100}$$

Edit:

where $arg(z) = \theta$ or the angle made in the complex plane

$$|z|, \angle arg(z) = |z|(\cos(\theta) + i\sin(\theta))$$

This is a way to convert polar coordinates into rectangular ones, or $|z|, \angle arg(z)$ to $a+bi$ form.

Example: $|2|, \angle \pi/4$

$$|2|, \angle \pi/4 = |2| \cdot (\cos(\pi/4) + i\sin(\pi/4))$$

$$= |2| \cdot \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$$

$$= \sqrt{2} + \sqrt{2} i \leftarrow \text{Rectangular coordinates}$$

polar form

Useful Identities:

Let, $z_1 = |z_1| \cdot (\cos(\theta_1) + i\sin\theta_1) = |z_1|, \angle arg(z_1)$

and $z_2 = |z_2| \cdot (\cos(\theta_2) + i\sin(\theta_2)) = |z_2|, \angle arg(z_2)$

Then we have,

$$z_1 \cdot z_2 = |z_1| \cdot |z_2| \cdot (\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)) = |z_1| \cdot |z_2| \angle arg(z_1) + arg(z_2)$$

$$z_1 / z_2 = |z_1| / |z_2| \cdot (\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)) = |z_1| / |z_2|, \angle arg(z_1) - arg(z_2)$$

$$(z_1)^n = |z_1|^n \cdot (\cos(n \cdot \theta_1) + i\sin(n \cdot \theta_1)) = |z_1|^n, \angle n \cdot arg(z_1)$$

for $z = a + bi$ $\quad |z| = \sqrt{a^2 + b^2}$