Question 27:

$$\int_0^5 \frac{1}{(x+1)^3}\,dx$$

By fundamental theorem of calculus:

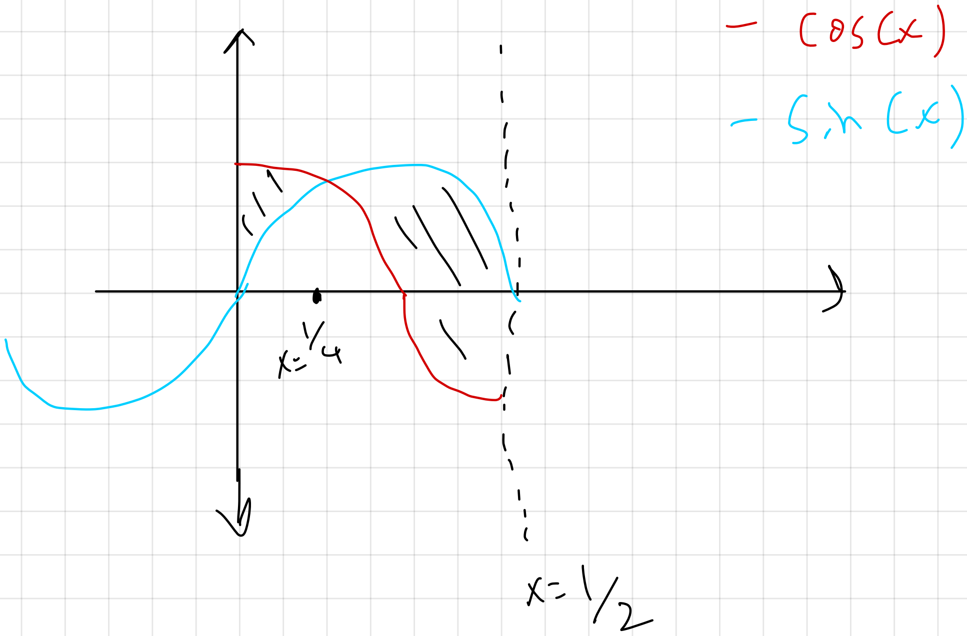
$$\int_0^5 (x+1)^{-3}\,dx = \left(-\frac{1}{2}\right)(x+1)^{-2}\Big]_0^5$$

$$= \left(-\frac{1}{2}\right)(5+1)^{-2} - \left(-\frac{1}{2}\right)(0+1)^{-2}$$

$$= \left(-\frac{1}{2}\right)\left(\frac{1}{36}\right) + \frac{1}{2}$$

$$= \frac{35}{72}$$

Question 28:



$$\text{Area} = \int (\text{top} - \text{bottom})\,dx$$

$$A = \int_0^{1/4} \left(\cos(\pi x) - \sin(\pi x)\right) dx + \int_{1/4}^{1/2} \left(\sin(\pi x) - \cos(\pi x)\right) dx$$

$$= \frac{1}{\pi}\sin(\pi x) + \frac{1}{\pi}\cos(\pi x) \Big]_0^{1/4} + \left(-\frac{1}{\pi}\cos(\pi x) - \frac{1}{\pi}\sin(\pi x)\right)\Big]_{1/4}^{1/2}$$

$$= \frac{1}{\pi}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \frac{1}{\pi}(1) \quad - \frac{1}{\pi}\left[1 - \sqrt{2}\right]$$

$$= \frac{1}{\pi}(\sqrt{2} + \sqrt{2}) - \frac{2}{\pi}$$

$$= \frac{1}{\pi}(2\sqrt{2} - 2)$$

Question 29:

$$r(t) = 20 + 0.1t^2$$

$$V_{\text{final}} - V_{\text{initial}} = \int r(t)\,dt$$

$$V_{\text{final}} - (200) = \int_0^{10} (20 + 0.1t^2)\,dt$$

$$V_{\text{final}} = 200 + \left(20t + \frac{0.1}{3}t^3\right)\Big]_0^{10}$$

$$= 200 + 200 + \frac{100}{3}$$

$$= 1300/3 \text{ liters}$$

30.

$$\int \frac{x^3 - 2}{\sqrt{x}}\, dx$$

$$= \int \frac{x^3 - 2}{x^{1/2}}\, dx$$

$$= \int x^{5/2} - 2x^{-1/2}\, dx$$

$$= \frac{2}{7} x^{7/2} - 4x^{1/2} + C$$

Question 31:

$$\int_0^9 [2f(x) + 3g(x)]\, dx$$

$$= 2\int_0^9 f(x)\, dx + 3\int_0^9 g(x)\, dx$$

$$= 2(37) + 3(16)$$

$$= 122$$

why $x = 1/4$

$\cos(\pi x) = \sin(\pi x)$ (since they intersect at this point)

$\Rightarrow \quad 1 = \dfrac{\sin(\pi x)}{\cos(\pi x)}$

$\Rightarrow \quad 1 = \tan(\pi x)$

$\Rightarrow \quad \arctan(1) = \arctan(\tan(\pi x))$

$\Rightarrow \quad \pi/4 = \pi x$

$\Rightarrow \quad x = 1/4$