

# MTHE 474 Notes

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# Information Measures for Discrete Systems

## Definitions

- **Definition 2.2:** Entropy of discrete random variable  $X$  with pmf  $P_X(*)$  is defined as

$$H(x) := - \sum_{x \in X} P_X(x) * \log_2 P_X(x)$$

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- **Definition 2.8 (Joint entropy):**

$$H(X, Y) := - \sum_{(x, y) \in \mathcal{X} \times \mathcal{Y}} P_{X, Y}(x, y) * \log_2 P_{(X, Y)}(x, y)$$

- **Definition 2.9 (Conditional entropy):**

$$H(Y|X) := \sum_{x \in \mathcal{X}} P_X(x) \left( - \sum_{y \in \mathcal{Y}} P_{Y|X}(y|x) * \log_2 P_{Y|X}(y|x) \right)$$

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## Lemmas/Theorems

- **Lemma 2.4 (Fundamental Inequality):**  $\forall x > 0$  and  $D > 1$  we have

$$\log_D(X) \leq \log_D e * (x - 1)$$

- **Lemma 2.5 (Non-negativity):**  $H(X) \geq 0$
- **Lemma 2.6 (Entropy Upper-Bound):**  $H(X) \leq \log_2 |\mathcal{X}|$  where random variable  $X$  takes values from finite set  $\mathcal{X}$
- **Lemma 2.7 (Log-Sum inequality)** Write this one out later
- **Theorem 2.10 (Chain rule for entropy):**  $H(X, Y) = H(X) + H(Y|X)$