NAME: ______________________________

Final Exam

Physics 375/475, Winter 2020

There is information attached at the end of the exam that you may find useful. If you use scratch paper that you wish me to grade, please make sure to put the problem number and your name on every sheet.

Good Luck !

Time out:____________ **Time in:** ____________

1.) (*20 points*) Answer the following questions. Make sure you fully explain your reasoning and provide detailed steps for any calculations.

(a) Hubble's law is $v = H_o d$. At what distance d will the recession velocity v start exceeding the speed of light? Comment on whether this answer might be a problem, since special relativity forbids the transfer of information at speeds greater than the speed of light. Assume $H_0 = 70$ km s^{-1} Mpc^{-1}.

(b) A friend challenges you by claiming that the Cosmic Microwave Background (CMB) radiation is just radiation that comes to us from stars and galaxies. Provide *at least two reasons, with adequate explanation*, why this could not be the case.

(c) Explain in detail the problems that *inflation* addresses and how it solves those problems

(d) Describe the mechanism(s) by which Baryon Acoustic Oscillations (BAO) are formed and what they tell us about basic cosmology.

2)(*20 points*) A negatively curved universe has,

$$\dot{a} = \pm\frac{c}{R_o}; \quad a(t) = \frac{t}{t_o} \text{ where } t_o = \frac{R_o}{c}$$

Consider such a universe that is expanding and has $\Omega_m \ll 1$.

(a) Derive the equation for how matter density fluctuations evolve in such a universe.

(b) Compare your results in (a) to the matter dominated universe. Discuss in which universe a matter density fluctuation would grow faster. Make sure you refer to your results and discuss specifically how you arrived at your conclusion(s) from these results.

3)(*20 points*) The Hubble length (in units of GeV) is defined as

$$R_H \equiv \frac{1}{H}$$

where H the Hubble parameter. In units of GeV the Hubble parameter is

$$H = \frac{8\pi\sqrt{\epsilon}}{3} \cdot 1.2 \times 10^{19} \text{ GeV}$$

where ϵ is the energy density of the universe.

(a) What was the value of the energy density (in units of GeV) of the universe at the time when mass contained inside one Hubble volume was equal to 100 kg (note 100 kg $= 5.6 \times 10^{28}$ GeV).

(b) Roughly, what was the age of the universe at the time when the mass contained inside one Hubble volume was equal to 100 kg. (Note to convert to years, multiply your answer by 2.1×10^{-32}.)

4.)(*20 points*) In all our discussions, we considered a cosmological constant for the dark energy with equation of state parameter $w = -1$. In general this need not be the case. Consider a universe whose dark energy component has an equation of state given by

$$P_{DE} = \left(\frac{m}{3} - 1\right)\epsilon_{DE} \quad \text{with } m \leq 2.$$

(a) Starting with the Friedmann equation, derive an expression for the Hubble parameter, $H(t)$.

(b) Find an expression for the age of this universe as a function of redshift. That is find an expression for $t(z)$.

5.)(*20 points*) Consider the decoupling era. Assume the universe is matter dominated at this epoch and that the benchmark model is correct.

(a) Derive the value of z for the decoupling era.

(b) Derive the value of T, the temperature for the decoupling era.

(c) Derive the time, t, at which the decoupling era occurs.

Useful and Useless Information

You may use the text, `Introduction to Cosmology` first or second editions by Barbara Ryden for this exam. This is the only resource you may use.