Physics 440, Autumn 2017 **Activity 15: Hamiltonians and Canonical Equations**

1. The Lagrangian of a one-dimensional harmonic oscillator is $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$
   a. Derive the equation of motion using the Lagrangian formalism and find the solution.
   b. Find the Hamiltonian.
   c. Using Hamilton's canonical equations, re-derive the equation of motion and show that it is identical to the equation of motion you found in a.
2. For the spherical pendulum the Lagrangian is

$$L = \frac{1}{2}ml^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right) - mgl\cos\theta$$

   The equations of motion derived with the Lagrangian formalism are (see p. 24)

$$ml^2\ddot{\theta} - ml^2\dot{\phi}^2\sin\theta\cos\theta - mgl\sin\theta = 0$$

$$\frac{d}{dt}\left(ml^2\sin^2\theta\,\dot{\phi}\right) = 0$$

   a. Find the Hamiltonian.
   b. Derive the equations of motion using the Hamiltonian formalism and show that they are identical to the ones given above.