Inflation and Introduction to structure formation

1. You will now begin exploring a possible physical reason that could lead to the kind of inflation behavior needed to address the flatness and horizon problem.

   (a) In the lecture you were given that

   $$\epsilon_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 + V(\phi)$$

   $$P_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi)$$

   Using these two expressions, rewrite the fluid equation,

   $$\dot{\epsilon}_\phi + 3H(t)\left(\epsilon_\phi + P_\phi\right) = 0$$

   to come up with a differential equation for ϕ. The solution will have three terms, give a physical meaning to each of the terms.

   (b) At your table discuss how the equation just derived would account for the behavior of inflation, i.e., the conditions in question (1 a).

   (c) Find the condition for a terminal velocity (when $\ddot{\phi} = 0$.)

   (d) Suppose the inflation field changes very slowly with time. For example suppose

   $$\dot{\phi}^2 \ll \hbar c^3 V(\phi).$$

   Substitute this value in for the condition found in part (d) and find a condition on $dV/d\phi$.

2. At your table, re-cap the physics we've introduced to explain inflation. Make sure you discuss the dynamical behavior of the scalar field.

3. Writing a scientific paper is one of the important aspects of becoming a scientist that is often not taught. Here we will go over some of the main aspects of a paper and have you tease out the important factors.

   (a) Consider the following abstract from the guide provided to you on writing a paper,

   > We explore the use of Euler's method for numerically solving differential equations. We use the well known Logistic equation to test the accuracy of the method by comparing the numerical results with the known analytic solution. We find that Euler's method reproduces the analytic result to within 1times tested and captures the important feature of the Logistic equation such as the carrying capacity...

   At your table, pick out the key elements of an abstract. Especially discuss how an abstract *is not* an introduction.

   (b) Now consider the following introduction taken from the same source,

   > Most ordinary differential equations (ODEs) have no analytic solution. Hence one is forced to use non-analytic techniques. An especially important class of techniques for solving ODEs are numerical techniques. Here we look at one type of numerical technique that is useful in solving autonomous, linear, first order ODEs ...

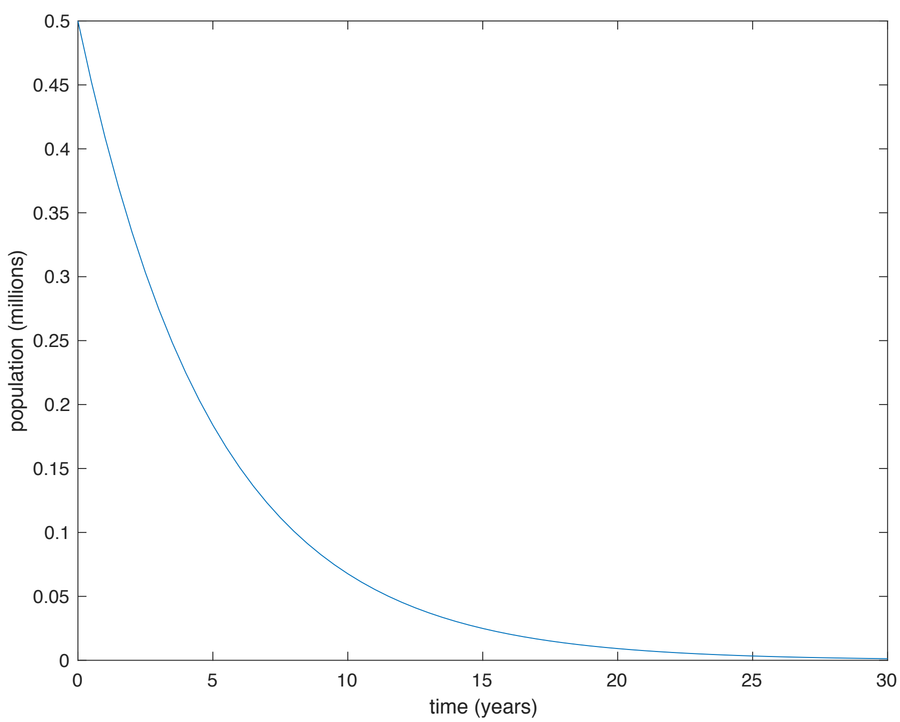
   At your table, pick out the key elements of an introduction. Especially discuss how an introduction *is not* an abstract.

(c) Suppose in your paper you are discussing Euler's method of numerically solving ordinary differential equations and you write the following expression

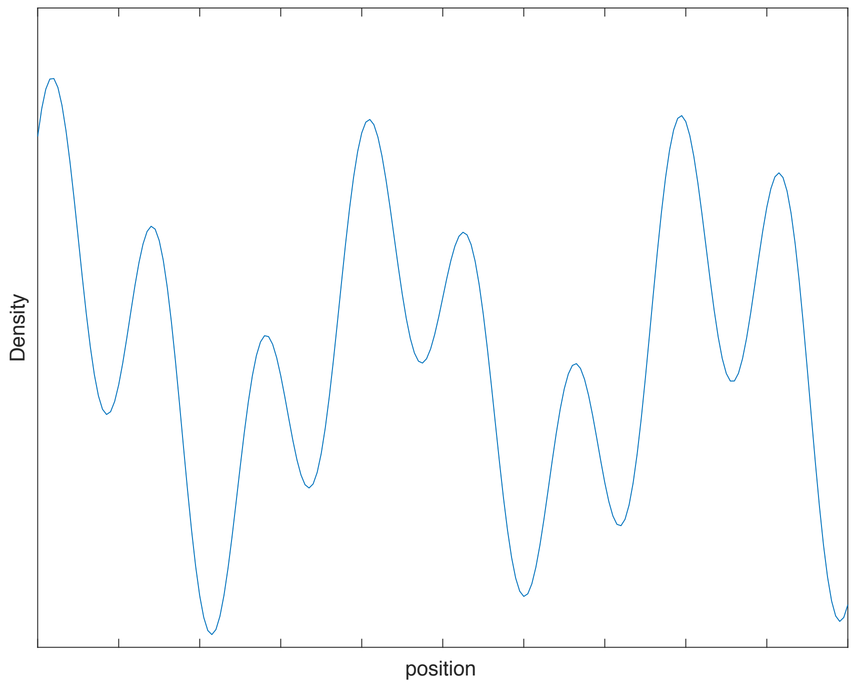
$$\frac{dy}{dt} \approx \frac{\Delta y}{\Delta t} = f(y,t).$$

At your table, come up with a narrative to attach to this equation.

(d) Suppose the figure below appeared in your paper. At your table, come up with a narrative that would be attached to this figure.



4. Large structures in the universe form via gravitational instability. To get a general idea of the theory behind how structure forms, consider a (highly unrealistic) 1-D universe, whose density as a function of position a time t_o is shown in the figure below.



The units are arbitrary, and assume that the fluctuations are small. On the figure, sketch how the density field might appear at a later time. Explain your reasoning.

5. In the lecture we introduced the density fluctuation defined as

$$\delta(\vec{r},t) = \frac{\epsilon(\vec{r},t) - \bar{\epsilon}(t)}{\bar{\epsilon}(t)},$$

where $\epsilon(\vec{r},t)$ is the energy density at time t and position, $\vec{r}$, and $\bar{\epsilon}(t)$ is the mean energy density at time t.

(a) Discuss in words what $\delta(\vec{r},t)$ is.

(b) What does $\delta(\vec{r},t) < 1$ tell you about the density fluctuation at $(\vec{r},t)$. Do the came for $\delta(\vec{r},t) > 1$.

(c) Assuming the energy density can only be positive, what is the minimum value that $\delta(\vec{r},t)$ can have.

(d) What is the maximum value that $\delta(\vec{r},t)$ can have.

6. In the lecture, we've seen how a sphere with a slight over density will have it's surface evolve in time as,

$$\frac{\ddot{R}}{R} = -\frac{4\pi\, G\, \bar{\rho}}{3}\delta(t) \tag{1}$$

(a) Suppose $\delta > 0$, what happens to the sphere?

(b) How many unknowns appear in Eq. (1). What principle might you use to find another relation involving the unknowns.

(c) In the lecture we've seen that an auxiliary condition we can use is the conservation of mass which leads to

$$R(t) = R_o \left[1 + \delta(t)\right]^{-1/3}$$

where

$$R_o \equiv \left(\frac{3M}{4\pi\bar{\rho}}\right)^{1/3} = \text{constant}$$

Using the fact that for $\delta \ll 1$, we can write

$$R(t) \approx R_o \left[1 - \frac{1}{3}\delta(t)\right]$$

take the second derivative of $R(t)$ with respect to time and substitute the result into Eq. (1) to form a new differential equation for $\delta(t)$. (Note make the substitution that $R_o \approx R$ after taking the second derivative.)

(d) Solve the differential equation and interpret the results.

Homework 04–Due Friday,March 6

1. Problem 7.3
2. Problem 7.4
3. Suppose $\Omega = 0.5$ in the early universe when the energy density is $\epsilon = 10^{16}$ GeV m^{-3}. At his time, suppose all the matter in the universe obeys $P = -\epsilon$ (i.e., single component universe).
   (a) After the scale factor increases by 60 e-foldings, what is the new value of Ω
   (b) Suppose at the end of the expansion described in part (a), all the energy density is instantly transformed into radiation (so the value of ϵ does not change, but the equation of state does). Assuming that the matter in the universe is composed *entirely* of radiation, what is the value of Ω when $T = 10^4 K$. The starting value of Ω you start here is the value you got in part **a.).**
4. Problem 11.4
5. **Grad Problem.** In this problem, you will carry out a very simple version of the parameter space process that cosmologists use to determine cosmological parameters in the BenchMark model. Please use a plotting software package to do this assignment, I do not want hand-drawn figures.
   (a) Draw a graph in which the x-axis is $\Omega_{m,o}$ and the y-axis is Ω_Λ. Each axis should go from 0 to 1. As you know the best shows that $\Omega_{m,o} + \Omega_\Lambda = 1$. Plot this line on the graph.
   (b) Observations of supernovae show that $\Omega_{m,o} - \Omega_\Lambda = -0.4$. Plot this line on the graph
   (c) If the CMB and supernovae results are both correct, what can you conclude about the values of $\Omega_{m,o}$ and Ω_Λ
   (d) What does the quantity $\Omega_{m,o} - \Omega_\Lambda$ tell you about the universe? For example, if the value $\Omega_{m,o} - \Omega_\Lambda = +0.4$ instead of what you plotted on the graph, what would be different about the universe? Explain in detail.