Introduction to structure formation

1. For the case of a static universe, we obtained the expression,

$$\ddot{\delta} = 4\pi G\bar{\rho}\delta.$$

When we do the same analysis for an expanding universe we get

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}\delta$$

as we've just seen in the lecture.

(a) At your table discuss the differences on density fluctuations between the two expression above.

(b) The expression we derived above was done in the context of Newtonian gravity. The full GR derivation gives,

$$\ddot{\delta} + 2H\dot{\delta} = \frac{4\pi G}{c^2}\bar{\epsilon}_m\delta$$

Rewrite this equation in terms of the density parameter for matter,

$$\Omega_m = \frac{\bar{\epsilon}_m}{\epsilon_c} = \frac{8\pi G\bar{\epsilon}_m}{3c^2H^2}$$

(c) Is the expression in (b) generally applicable. If not, when does it apply?

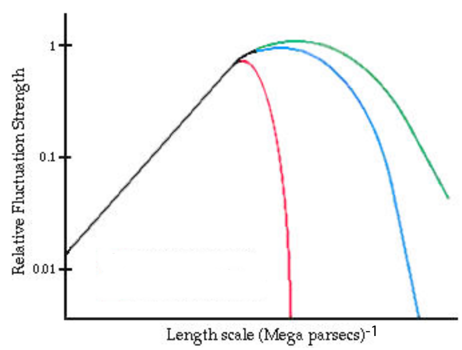
(d) Consider the epoch in which the universe was radiation dominated.

In this case $\Omega_m \ll 1, H = 1/(2t)$. Write your expression in (5b) for this era.

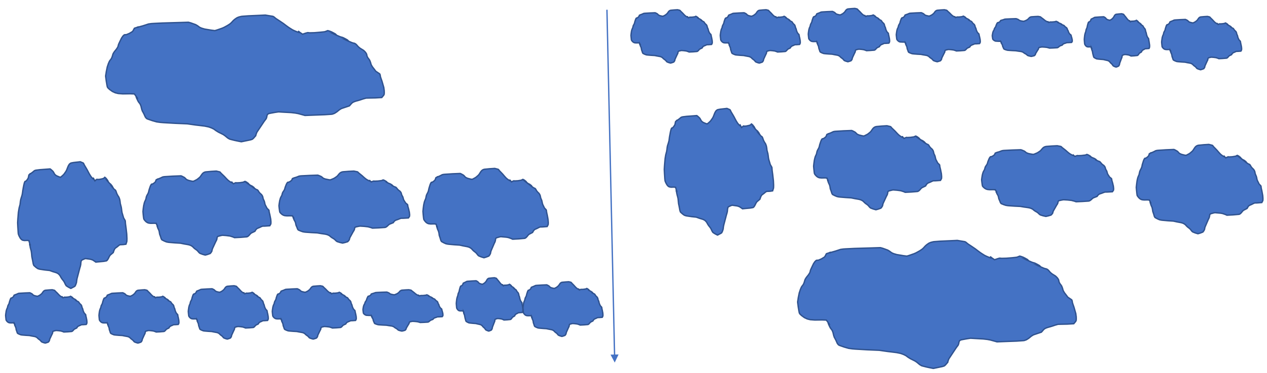
(e) Do the same for a universe that is dominated by a cosmological constant, in which case $H = H_\Lambda$.

2. In the lecture, we've discussed the power spectrum and how it will change depending on the nature of the dark matter.

(a) The figure below shows three different power spectra corresponding to cold, hot, and mix dark matter models. Label which is which and explain your reasoning.



(b) The figure below shows in a cartoon fashion, how structure may have evolved in the universe. On both the left and right sides time advances down. Which scenario corresponds to a hot dark matter universe and which to a cold dark matter universe. Explain your reasoning.



Scenario 1 Scenario 2

Homework 04–Due Monday,March 9

1. Problem 7.3
2. Problem 7.4
3. Suppose $\Omega = 0.5$ in the early universe when the energy density is $\epsilon = 10^{16}$ GeV m^{-3}. At his time, suppose all the matter in the universe obeys $P = -\epsilon$ (i.e., single component universe).
   (a) After the scale factor increases by 60 e-foldings, what is the new value of Ω
   (b) Suppose at the end of the expansion described in part (a), all the energy density is instantly transformed into radiation (so the value of ϵ does not change, but the equation of state does). Assuming that the matter in the universe is composed *entirely* of radiation, what is the value of Ω when $T = 10^4 K$. The starting value of Ω you start here is the value you got in part **a.).**
4. Problem 11.4
5. **Grad Problem.** In this problem, you will carry out a very simple version of the parameter space process that cosmologists use to determine cosmological parameters in the Benchmark model. Please use a plotting software package to do this assignment, I do not want hand-drawn figures.
   (a) Draw a graph in which the x-axis is $\Omega_{m,o}$ and the y-axis is Ω_Λ. Each axis should go from 0 to 1. As you know the best shows that $\Omega_{m,o} + \Omega_\Lambda = 1$. Plot this line on the graph.
   (b) Observations of supernovae show that $\Omega_{m,o} - \Omega_\Lambda = -0.4$. Plot this line on the graph
   (c) If the CMB and supernovae results are both correct, what can you conclude about the values of $\Omega_{m,o}$ and Ω_Λ
   (d) What does the quantity $\Omega_{m,o} - \Omega_\Lambda$ tell you about the universe? For example, if the value $\Omega_{m,o} - \Omega_\Lambda = +0.4$ instead of what you plotted on the graph, what would be different about the universe? Explain in detail.