(1) $H_0 = 70\ Km\,s^{-1}\,Mpc^{-1}$ $\qquad v = H_0 d$

(a) $C = 3\times10^5\ Km/s$ $\qquad C = v$

$$\frac{3\times10^5\ \cancel{Km}\cdot\cancel{s}\ mpc}{70\ \cancel{Km}\cdot\cancel{s}} = 4285\ mpc$$

Not a problem as long as it is (velocity) less than the speed of light relative to something else.

(B) - The whole sky has the same microwave radiation that we can tell comes from the Horizon

- It is a black body, temperature is unique

-

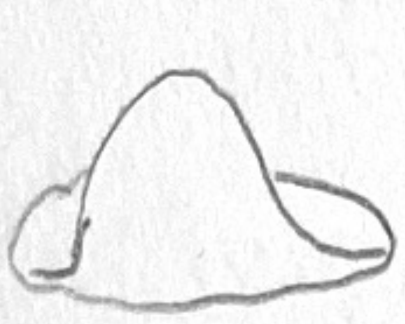
(C) Has the

- Flatness problem
  - ~~Everything is very close~~, Fabric of space was far from flat, expanded quick and flattened out
    - Blanket streached out
- monopole problem
  - In the early universe there were no monopoles

- Horizon problem
  - looking at the sky ~~Horizon~~ temp is the same but spread out meaning things had to be close at one point.
  - Explains isotopy and homogeneity
  - Coffee from the same pot

(D) Baryon Acoustic Oscillations (BAO)

- Sound wave at recombination, we get Density Rings with Dark matter in the middle and Baryonic matter on the rings

(2) Negatively curved universe

$$\ddot{a} = \pm \frac{c}{R_0}, \quad a(t) = \frac{t}{t_0} \qquad t_0 = \frac{R_0}{c}$$

$$a(t) = \frac{tc}{R_0}$$

$\Omega_m << 1$

Density fluctuation $\delta(\vec{r}, t) = \dfrac{\varepsilon(\vec{r}, t) - \bar{\varepsilon}(t)}{\bar{\varepsilon}(t)} \to \delta \cdot \dfrac{\varepsilon_m - \bar{\varepsilon}_m}{\bar{\varepsilon}_m}$

$$\Omega_m = \frac{\bar{\varepsilon}_m}{\varepsilon_c} = \frac{8\pi G \bar{\varepsilon}_m}{3c^2 H^2} \qquad t_0 = -1/c = -3.0 \times 10^{-8} \, m^{-1} s$$

$$\frac{\ddot{R}}{R} = -\frac{1}{3} \ddot{\delta} \Rightarrow \frac{\ddot{R}}{R} = +\frac{1}{3} 4\pi \bar{S} \cdot \delta$$

$$\delta(t) = A_1 e^{t/t_0} + A_2 e^{-t/t_0}$$

$$\delta(t) = A_1 e^{tc/R_0} + A_2 e^{-tc/R_0}$$

(B) Matter dominated is filled with more matter which ↑ pressure and ↑ Density flux.

(3) $R_H = \frac{1}{H}$ $H = \frac{8\pi\sqrt{\epsilon}}{3} \cdot 1.2\times10^{19}\,GeV$

$V_{sphere} = \frac{4}{3}\pi r^3$

(a) $\rho = \frac{m}{V}$ $m = 100\,kg \to 5.6\times10^{28}\,GeV$

$\epsilon = \frac{9}{64\pi^2 H^2} \cdot (1.2\times10^{-19}\,GeV)^2$

$\epsilon = \frac{9}{64\pi^2} \cdot R_H^3 \cdot (1.44\times10^{-38}\,GeV^{-2})$

Needs to be volume

$\epsilon = \frac{9}{64\pi}(4.3\times10^{39}\,GeV^3)(1.44\times10^{-38}\,GeV^{-2})$

$\epsilon = 2.7\,GeV$

$V = \frac{1}{H^3} \to R_H^3$

$\rho = \frac{m}{R_H^3} \to \frac{5.6\times10^{28}\,GeV}{1.3\times10^{-11}\,GeV}$

$H = \frac{c}{H_0} = \frac{3\times10^5\,\cancel{km}/s}{70\,\cancel{km\,s^{-1}}\,mpc^{-1}}$

$= 4285\,mpc$

$R_H = 1/(4285)^3$

$= 1.3\times10^{-11}\,GeV$

ρ 5.6 $4.3\times10^{39}\,GeV^3$

$R_H = \frac{1}{\sqrt[3]{5.6\times10^{28}\,GeV}} = 3.82\times10^{9}\,GeV$

$\epsilon = \frac{3^2}{8^2\pi^2} \cdot (1.2\times10^{-19}\,GeV)^2 \cdot R_H^2 = 3.0\times10^{-21}\,GeV$ wrong

(B)

$3.0\times10^{-21}\,GeV \cdot 2.1\times10^{-32}$

$= 6.3\times10^{-53}$

wrong

(4) $P_{DE} = \left(\frac{m}{3} - 1\right) \epsilon_{DE} \quad m \leq 2$

(2) $\epsilon_{DE} = P_{DE}\left(\frac{3}{m} - 1\right)$

$$H(t)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{kc^2}{R_0^2 a^2}$$

$$H(t)^2 = \frac{8\pi G}{3c^2}\left(P_{DE}\left(\frac{3}{m} - 1\right)\right) - \frac{kc^2}{R_0^2 a^2}$$

$$H(t) = \sqrt{\frac{8\pi G}{3c^2}\left(P_{DE}\left(\frac{3}{m} - 1\right)\right) - \frac{kc^2}{R_0^2 a^2}}$$

(B) $\left(\frac{\dot{a}}{a}\right)^2 \sqrt{\frac{8\pi G}{3c^2}\left(P_{DE}\left(\frac{3}{m} - 1\right)\right) - \frac{kc^2}{R_0^2 a^2}} \qquad a(t) = \frac{1}{(1+z)}$

$$H_0 t = \int_0^a \frac{da}{\sqrt{\frac{8\pi G}{3c^2}\left(P_{DE}\left(\frac{3}{m} - 1\right)\right) - \frac{kc^2}{R_0 a^2}}}$$

After integrating plug $a(t) = \frac{1}{(1+z)}$ to have in terms of z

(5) Decoupling Era - Matter Dominated

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \cancel{\frac{\Omega_{r,0}}{a^4}}^{0} + \cancel{\Omega_{\Lambda,0}}^{0} + \cancel{\frac{1-\Omega_0}{a^2}}^{0} \quad \Omega_0 = \Omega_{m,0} = 1$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} \rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{a^3} \Rightarrow \frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} = \Omega_{m,0}(1+z)^3$$

$$\Gamma = H$$

$$H^2 = (1+z)^3 \qquad z_{dec} = \frac{2\times10^{20}}{X(z_{dec})^{2/3}} - 1$$

(B)

$$T_{rec} \sim \frac{Q}{2.7K}$$

(c)

$$\Gamma = c/\lambda = 5.0\times10^{-21}\,s^{-1}$$

$$\tau(t) = \int_t^{t_0} \Gamma(t)\,dt$$

$$\int_0^1 \frac{dt}{5.0\times10^{+21}} = \left[\frac{t}{5.0\times10^{+21}}\right]_0^1$$

$$= \frac{1}{5.0\times10^{21}} = \boxed{2\times10^{-22}\,s}$$