**PHY 474/374** **Spring 2021**

**Week 6—Wednesday, May 5—Discussion Worksheet**

## The Main Sequence

Stars spend most of their lifetime fusing hydrogen to helium on the Main Sequence.

**1.** Let's begin by finding an approximate relation to predict how long a star will stay on the Main Sequence.

(a) Write $L = E/t$, where $E = fMc^2$ is the energy produced per unit time; f is the fraction of mass converted to energy, and $M$ is the mass of the star. Using the mass-luminosity relationship, $L \sim M^{3.5}$, for main sequence stars, show that the time spent on the Main Sequence is

$$t_{\rm MS} \sim M^{-2.5}$$

$L = E/t \rightarrow L = fc^2 \frac{M}{t}$ where $L \sim M^{3.5}$

$t = fc^2 \frac{M}{M^{3.5}}$

we drop $fc^2$ and $M^1 \cdot M^{3.5} = M^{-2.5}$

$t_{MS} \sim M^{-2.5}$

(b) In terms of the Sun's lifetime of $10^{10}$ yr, the relation above can be written as

$$\frac{t_{\rm MS}}{10^{10}\ \rm yr} = \left(\frac{M}{M_\odot}\right)^{-2.5}$$

Use this to calculate the lifetime of a 0.5 $M_\odot$ and a 15 $M_\odot$ star, respectively, on the Main Sequence.

$0.5\ M_\odot$: $t_{MS} = \left(\frac{0.5}{1}\right)^{-2.5} 10^{10}\ yr = 5.66 \times 10^{10}\ yr$

$15\ M_\odot$: $t_{ms} = \left(\frac{15}{1}\right)^{-2.5} 10^{10}\ yr = 1.14 \times 10^{7}\ yr$

**2.** Although we assumed $L \sim M^{3.5}$ in the previous question, this is not true over all mass ranges. Let's look at this in more details, and its implications for stars on the Main Sequence.

Starting from

$$L = \frac{\mu^4 M^3}{\kappa}$$

where $\kappa$ is the opacity, examine whether the $L$ *vs.* $M$ slope will be more or less steep for stars like our Sun than for high mass stars.

**Hint:** You should be able to show that $L \sim \mu^{7.5}\, M^{5.2}/Z(1+X)$ for lower mass stars like our Sun, provided you assume $R \sim M^{0.6}$ for such stars.

$$\kappa = \kappa_0 \rho \; T^{-3.5} \qquad \kappa_0 = Z(1+X)$$

$$T = \frac{GMm_p\mu}{R^3} \quad , \quad \rho = \frac{M}{R^3}$$

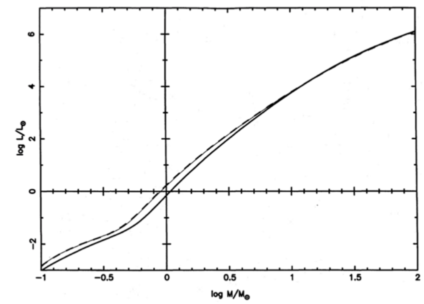
$$T = \cancel{\left(G\, m_p\right)^{-3.5}} \cdot \frac{M^{-3.5}}{R^{0.5}}\, \mu^{-3.5}$$

$$\kappa = Z(1+X)\, \frac{M^{-3.5}}{R^{0.5}}\, \frac{M}{R^3} \rightarrow Z(1+X)\, \frac{M^{-2.5}}{R^{3.5}}$$

$$\rightarrow Z(1+X)\, \frac{M^{-2.5}}{M^{2.1}} \rightarrow Z(1+X)\, M^{-4.6}\mu^{-3.5}$$

$$L = \frac{\mu^{7.5}\, M^{5.2}}{Z(1+X)}$$

**3.** Consider the plot below of $\log(L/L_\odot)$ *vs.* $\log(M/M_\odot)$ taken from Tout et al. (1996).




(a) Does the solid line in the plot support your conclusion in Question 2 regarding whether the slope is more or less steep for stars like our Sun than for high mass stars? At about what value of $M_\odot$ does this change in slope appear, according to the graph?

yes, for stars like the Sun, the slope is steeper

change in slope $\log\left(\frac{M}{M_\odot}\right) = 1$

Radiative transfer thus, around $10 M_\odot$

(b) Does the slope of the solid line change prominently in any other mass range, and if so, what could be the reason?

yes, for lower mass stars, slope is less steep than for stars like our sun $\log\left(\frac{M}{M_\odot}\right) = -0.2 \approx 0.6 M_\odot$

BC stars are convective

(c) The solid line in the plot is for a metallicity $Z = 0.02$, whereas the broken line is for a metallicity $Z = 0.0003$. Are the expressions you obtained in Question 2 consistent with the behavior exhibited by the metallicity in this graph?

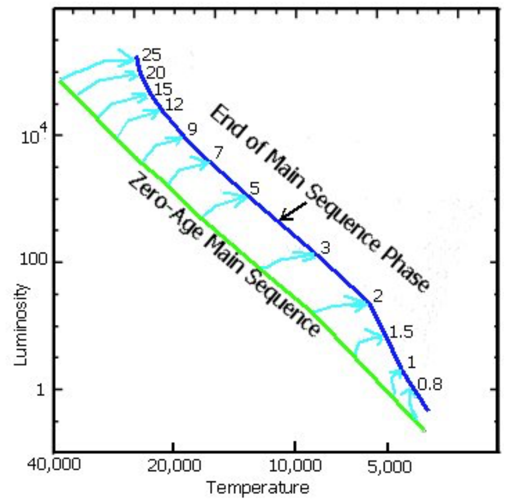
yes, for high mass stars $L \sim \frac{\mu^4 M^3}{1+X}$.

independent of Z

for stars like the Sun $L \sim \frac{\mu^{7.5} M^{5.2}}{Z(1+X)}$ $L \sim Z$

**4.** Although descriptions of the Main Sequence (MS) might make an impression of a static point in ($L$-$T_e$) space, this is certainly not the case. During the course of its time on the Main Sequence, a star moves from the point where it entered the MS (known as ZAMS for Zero Age Main Sequence) upward and to the right as shown in the figure below (source: sites.uni.edu):

(a) First, figure out what this graph is telling you. How are $L$ and $T$ changing as a star ages on the MS?




Higher Mass Stars see a higher luminocity and temperature, lower Mass Stars are Dimmer and Cooler

L ↑

T ↓

(b) Now, consider this quantitatively and explain why $L$ and $T$ change as shown in the plot above during the time that a star is on the Main Sequence.

**Hint:** Among others, relations that we derived earlier (or could obtain from what we derived earlier) and may be of use here include

$$L \propto \frac{\mu^4 M^3}{\kappa} \qquad P_c \sim \frac{M^2}{R^4} \qquad T_c \sim \frac{\mu M}{R} \qquad L \sim R^2 T_e^4$$

along with the information that core temperature, $T_c$, remains approximately constant during the Main Sequence phase.

μ ↑ as chemical comp increases

h ↑

Tc - Stays the Same

M - Stays the Same

Te ↓

h will have to change faster than L

**5.** We will now look at a numerical modeler online. Go to Lionel Siess' isochrones page at http://www.astro.ulb.ac.be/~siess/pmwiki/pmwiki.php/WWWTools/Isochrones

and leaving everything else at default, choose yes for additional information in output, then deselect all but 0.5 and 5.0 $M_\odot$ (i.e., keep only these two mass values).

(a) For the 0.5 and 5.0 $M_\odot$ stars that you selected, choose `Compute ZAMS`, and hit the submit button. Use the output to compare the ages of stars with 0.5 and 5.0 $M_\odot$ respectively when they reach the Main Sequence.

$0.5 M_\odot$: $1.02 \times 10^8$ yr

$5 M_\odot$: $7.92 \times 10^5$ yr

$5 M_\odot$ Stars reach MS much earlier than $0.5 M_\odot$ Stars.

(b) How much brighter is a 5.0 $M_\odot$ star when it reaches the Main Sequence compared to a 0.5 $M_\odot$ star?

$$\left.\begin{array}{l} 0.5\ M_\odot \longrightarrow 4.63 \times 10^{-2}\ L_\odot \\ 5\ M_\odot \longrightarrow 6.822 \times 10^2\ L_\odot \end{array}\right\} \quad \frac{L_{5M_\odot}}{L_{0.5M_\odot}} = \frac{6.82 \times 10^2}{4.63 \times 10^{-2}}$$

$= 14,724$

about 15,000 times Brighter.

(c) Write down and compare the core temperatures of a 0.5 and 5.0 $M_\odot$ star when it reaches the Main Sequence.

$0.5 M_\odot$: $8.74 \times 10^6$ K

$5 M_\odot$: $2.29 \times 10^7$ K

$5 M_\odot$ is larger but not by a whole lot, $T_c$ is about 2.6 times that of $0.5 M_\odot$