Physics 412—Harmonic Oscillator Dynamics Winter 2020

Suppose that the state of a quantum harmonic oscillator at time $t = 0$ is

$$|\Psi_0\rangle = c_m|m\rangle + c_n|n\rangle.$$

Assume that the state is normalized so that $|c_m|^2 + |c_n|^2 = 1$.

(1) Find $|\Psi(t)\rangle$.

(2) If you measure the energy at time t, what values can you measure and with what probabilities?

(3) Calculate $\langle X\rangle$ and $\langle P\rangle$ as functions of time for this state. How do your results depend on the relative values of m and n? For this part you'll probably want to use

$$X = \frac{d_0}{\sqrt{2}}\left(a + a^\dagger\right), \quad P = \frac{-i\hbar}{\sqrt{2}d_0}\left(a - a^\dagger\right).$$

(4) Discuss how the values of c_m and c_n affect $\langle X\rangle$ and $\langle P\rangle$ at time $t = 0$. How would you pick c_m and c_n to force $\langle P\rangle$ or $\langle X\rangle$ to be zero when $t = 0$? How would you pick c_m and c_n to force $\langle X\rangle$ or $\langle P\rangle$ to be nonzero at $t = 0$?