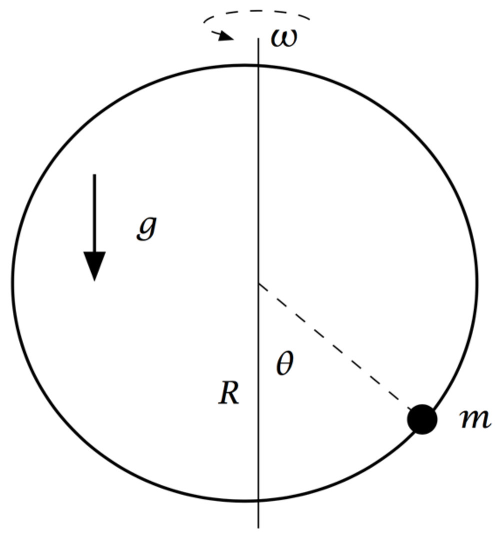
A massless, frictionless vertical hoop rotates with constant angular frequency ω about a diameter, as shown in the figure below. A particle of mass m is free to move along the hoop. The angle θ measures the angle of the particle relative to the axis of rotation.



a. Show that the Lagrangian for this system in spherical coordinates is.
$$L = \frac{1}{2} mR^2\left(\dot{\theta}^2 + \omega^2 \sin^2\theta\right) + mgR\cos\theta$$

b. Compare this Lagrangian to the one for the horizontal rotating hoop from activity 10, which had an explicit time dependence:
$$L = \frac{mR^2}{2}\left[\dot{\theta}^2 + 2\omega\dot{\theta}\cos(\theta - \omega t)\right]$$
(I eliminated the constant term ω^2, which doesn't affect the motion of the system.) What causes the explicit time dependence in the Lagrangian of the horizontal hoop? What's different for the vertical hoop?

c. Show that the equation of motion is $\ddot{\theta} = \left(\omega^2 \cos\theta - \frac{g}{r}\right)\sin\theta$.

d. Find the equilibrium solutions from the equation of motion. Carefully look at the equation. You should notice that the number of equilibrium solutions depends on how fast the hoop is spinning. At what ω does the number of solutions change, and what are the equilibrium solutions when the hoop spins more slowly and faster than this ω. Which of these equilibrium solutions are stable?