Physics 412—Quantum Mice Basics Winter 2020

Consider a quantum lab mouse and some of its properties. One property might be the size of the mouse, which we could measure by putting the mouse on a quantum scale. Since the mouse is quantized, its weight can take on only one of two values, either $w = 2$ (small mouse), or $w = 10$ (large mouse). If a quantum mouse is small, it's in the state $|s\rangle$ (labeled s for small), while if it's large, it's in the state $|l\rangle$ (labeled l for large). These are eigenstates of the weight operator W, with eigenvalue equations

$$W|s\rangle = 2|s\rangle \quad \text{and} \quad W|l\rangle = 10|l\rangle.$$

Being either large or small is normal for a quantum mouse, so the states $|s\rangle$ and $|l\rangle$ are assumed to be normalized. Because the operator W corresponds to a measurable quantity, it is Hermitian.

A second property of the quantum mice is their attitude, which can be measured by looking at a mouse's expression, yielding either a smile (attitude = +1), or a frown (attitude = −1). We'll call the corresponding quantum states $|h\rangle$ (for happy) and $|u\rangle$ (for unhappy). These are eigenstates of the attitude operator A, with eigenvalue equations

$$A|h\rangle = +|h\rangle \quad \text{and} \quad A|u\rangle = -|u\rangle.$$

As with the size states, the attitude states are normalized, and the operator A is Hermitian.

(1) What can you say about the inner products $\langle s|s\rangle$, $\langle l|l\rangle$, $\langle s|l\rangle$, $\langle h|h\rangle$, $\langle u|u\rangle$, and $\langle h|u\rangle$? If you think some of these inner products are zero, prove it! (See page 16 of the course notes, starting with Eq. (2.17).)

(2) There is a relationship between size and attitude for the quantum mice. Suppose that

$$|s\rangle = \frac{1}{\sqrt{5}}|h\rangle + \frac{2}{\sqrt{5}}|u\rangle.$$

I guess that means that small quantum mice are more than a little bit stressed! (Do you see that from the equation?)

(a) Expand the "large" size state in the "attitude basis": $|l\rangle = a|h\rangle + b|u\rangle$. Find the constants a and b. (Use your results from Question 1.)

(b) Represent all four states $|s\rangle$, $|l\rangle$, $|h\rangle$, and $|u\rangle$ as column vectors in the attitude basis.

(c) Find the representations of the operators W and A in the attitude basis $\{|h\rangle, |u\rangle\}$. Express your answers as 2×2 matrices. Verify that the representations of W and A satisfy the condition for a Hermitian operator.

(d) Invert the equations relating the size states to the attitude states to represent $|h\rangle$ and $|u\rangle$ as column vectors in the size basis.

(e) Find the representations of the operators W and A in the size basis $\{|s\rangle, |l\rangle\}$. Express your answers as 2×2 matrices. Verify that the representations of W and A satisfy the condition for a Hermitian operator.

(f) Are the size states $|s\rangle$ and $|l\rangle$ eigenstates of the attitude operator A? Explain.

(g) Are the attitude states $|h\rangle$ and $|u\rangle$ eigenstates of the size operator W? Explain.