Homework 8

(1a) Explain where the gate $\sqrt{X}$ gets its name

The X operator is also known as the NOT gate. The $\sqrt{NOT}$ or $\sqrt{X}$ is found by using a unitary matrix multiplied by itself results in $\sqrt{X}$.

$$\sqrt{X} = \sqrt{} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$$

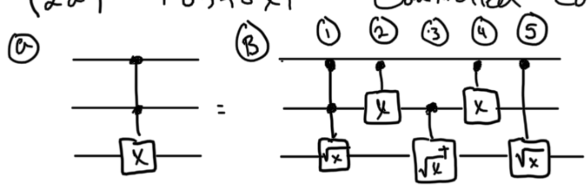
where

$$\sqrt{X}\sqrt{X} = X \qquad U^{\dagger} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \qquad U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$$

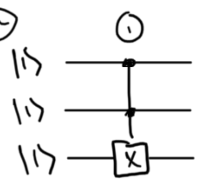
$$U^{\dagger}U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$UU^{\dagger} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(2a) Toffoli - Controlled-Controlled-NOT gate, three-qubit



a



a

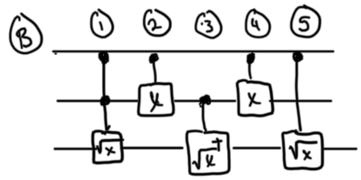
$\alpha\ |1,1,1\rangle \xrightarrow{1} \boxed{|1,1,0\rangle}$, $\epsilon\ |0,0,0\rangle \xrightarrow{1} \boxed{|0,0,1\rangle}$

$\beta\ |1,1,0\rangle \xrightarrow{1} \boxed{|1,1,1\rangle}$, $G\ |0,0,1\rangle \xrightarrow{1} \boxed{|0,0,0\rangle}$

$\gamma\ |1,0,0\rangle \xrightarrow{1} \boxed{|1,0,1\rangle}$, $\eta\ |0,1,1\rangle \xrightarrow{1} \boxed{|0,1,0\rangle}$

$c\ |1,0,1\rangle \xrightarrow{1} \boxed{|1,0,0\rangle}$, $\emptyset\ |0,1,0\rangle \xrightarrow{1} \boxed{|0,1,1\rangle}$

(B)



$\alpha \; |1,1,1\rangle \xrightarrow{1} |1,1,0\rangle \xrightarrow{2} |1,0,0\rangle \xrightarrow{3} |1,0,1\rangle \xrightarrow{4} |1,1,1\rangle \xrightarrow{5} \boxed{|1,1,0\rangle}$

$\beta \; |1,1,0\rangle \xrightarrow{1} |1,1,1\rangle \xrightarrow{2} |1,0,1\rangle \xrightarrow{3} |1,0,0\rangle \xrightarrow{4} |1,1,0\rangle \xrightarrow{5} \boxed{|1,1,1\rangle}$

$\gamma \; |1,0,0\rangle \xrightarrow{1} |1,0,1\rangle \xrightarrow{2} |1,1,1\rangle \xrightarrow{3} |1,1,0\rangle \xrightarrow{4} |1,0,0\rangle \xrightarrow{5} \boxed{|1,0,1\rangle}$

$\delta \; |1,0,1\rangle \xrightarrow{1} |1,0,0\rangle \xrightarrow{2} |1,1,0\rangle \xrightarrow{3} |1,1,1\rangle \xrightarrow{4} |1,0,1\rangle \xrightarrow{5} \boxed{|1,0,0\rangle}$

$\epsilon \; |0,0,0\rangle \xrightarrow{1} |0,0,1\rangle \xrightarrow{2} |0,1,1\rangle \xrightarrow{3} |0,1,0\rangle \xrightarrow{4} |0,0,0\rangle \xrightarrow{5} \boxed{|0,0,1\rangle}$

$\zeta \; |0,0,1\rangle \xrightarrow{1} |0,0,0\rangle \xrightarrow{2} |0,1,0\rangle \xrightarrow{3} |0,1,1\rangle \xrightarrow{4} |0,0,1\rangle \xrightarrow{5} \boxed{|0,0,0\rangle}$

$\eta \; |0,1,1\rangle \xrightarrow{1} |0,1,0\rangle \xrightarrow{2} |0,0,0\rangle \xrightarrow{3} |0,0,1\rangle \xrightarrow{4} |0,1,1\rangle \xrightarrow{5} \boxed{|0,1,0\rangle}$

$\theta \; |0,1,0\rangle \xrightarrow{1} |0,1,1\rangle \xrightarrow{2} |0,0,1\rangle \xrightarrow{3} |0,0,0\rangle \xrightarrow{4} |0,1,0\rangle \xrightarrow{5} \boxed{|0,1,1\rangle}$

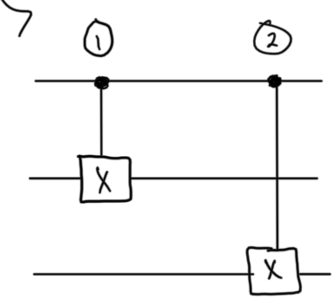
(3) input $|\psi,0,0\rangle = a|0,0,0\rangle + b|1,0,0\rangle$

output $a|0,0,0\rangle + b|1,1,1\rangle$

$|\psi\rangle \to a|0,0,0\rangle + b|1,0,0\rangle$

$|0\rangle \to |0\rangle$

$|0\rangle \to |0\rangle$



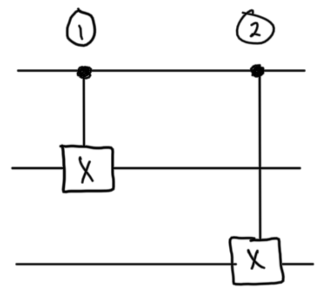
$|\psi\rangle \xrightarrow{1} a|0,0,0\rangle + b|1,1,0\rangle$

$\xrightarrow{2} \boxed{a|0,0,0\rangle + b|1,1,1\rangle}$

$= a|0,0,0\rangle + b|1,1,1\rangle$

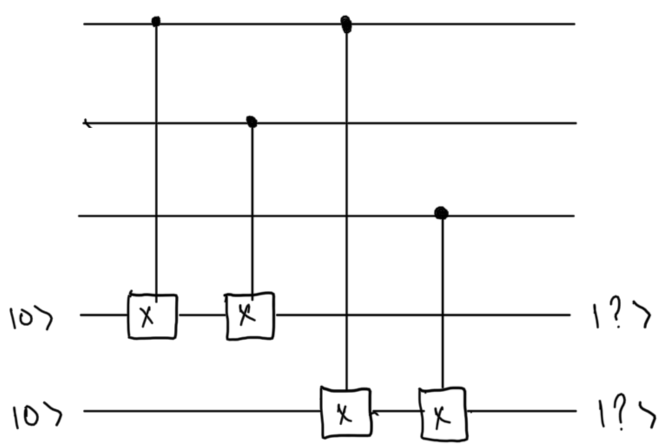
(B) input $|\Psi_L\rangle = a|0,0,0\rangle + b|1,1,1\rangle$
output $|\Psi,0,0\rangle = a|0,0,0\rangle + b|1,0,0\rangle$

would stay the same



(C)

$$|\Psi_L\rangle \to \begin{cases} a|0,0,0\rangle + b|1,1,1\rangle & \text{No error,} \\ a|1,0,0\rangle + b|0,1,1\rangle & \text{qubit 1 flipped,} \\ a|0,1,0\rangle + b|1,0,1\rangle & \text{qubit 2 flipped,} \\ a|0,0,1\rangle + b|1,1,0\rangle & \text{qubit 3 flipped,} \end{cases}$$



$|\Psi_L\rangle_1 = |0,0\rangle$

$|\Psi_L\rangle_2 = |1,0\rangle$

$$|\psi_L\rangle_3 = |0,1\rangle$$

$$|\psi_L\rangle_4 = |1,1\rangle$$