Physics 460—Particle in a Sphere: Superpositions **Spring 2020**

A particle in a sphere with angular momentum $l = 2$ can be in a superposition of energy eigenstates with $-2 \le m \le 2$ and any positive integer n

$$|\Psi\rangle = \sum_{n=1}^{\infty} \sum_{m=-2}^{2} c_{nm} |n, 2, m\rangle.$$

These states have energy

$$E_{2n} = \frac{{z_{2n}}^2 \hbar^2}{2ma^2},$$

where z_{2n} is any one of the values in the line with $l = 2$ in Table 2.1 on page 40 of the course notes.

(1) The system is in a state with

$$c_{30} = \frac{1}{2}, \quad c_{41} = \sqrt{\frac{3}{8}}, \quad c_{61} = \mathrm{i}\sqrt{\frac{3}{8}},$$

and all other coefficients equal to zero.

(a) If you measure the z-component of the angular momentum of the system, what values can you obtain and with what probability? What is the expectation value of L_z for this state?

(b) If you measure the energy of the system, what values can you obtain and with what probability? What is the expectation value of energy for this state?

(2) The system is in a state with

$$c_{11} = \frac{\mathrm{i}}{4}, \quad c_{12} = \sqrt{\frac{3}{4}}, \quad c_{31} = \mathrm{i}\sqrt{\frac{1}{8}}, \quad c_{50} = \frac{1}{4},$$

and all other coefficients equal to zero.

(a) If you measure the z-component of the angular momentum of the system, what values can you obtain and with what probability? What is the expectation value of L_z for this state?

(b) If you measure the energy of the system, what values can you obtain and with what probability? What is the expectation value of energy for this state?

(1) $|\psi\rangle = \frac{1}{2}|3,2,0\rangle + \sqrt{\frac{3}{8}}|4,2,1\rangle + i\sqrt{\frac{3}{8}}|6,2,1\rangle$

(a)

$$\left.\begin{array}{l} m = 0 : \; P = \frac{1}{4} \\ m = 1 : \; P = \frac{3}{8} + \frac{3}{8} = \frac{3}{4} \end{array}\right\} \langle L_z \rangle = \frac{1}{4}(0\hbar) + \frac{3}{4}(\hbar) = \frac{3\hbar}{4}$$

(b) $n = 3, \; l = 2 : \; E_{32} = \frac{{z_{32}}^2 \hbar^2}{2ma^2}$ $\quad z_{32} = 3.923$

$P = \frac{1}{4}$

$n=6, \ell=2 \qquad E_{62} = \dfrac{z_{62}^2 \hbar^2}{2ma^2}$

$P = 3/8$

$n=4, \ell=2 \qquad E_{42} = \dfrac{z_{42}^2 \hbar^2}{2ma^2}, \quad P = 3/8$

$$\langle E \rangle = \frac{E_{32}}{4} + \frac{3E_{62}}{8} + \frac{3E_{42}}{8}$$