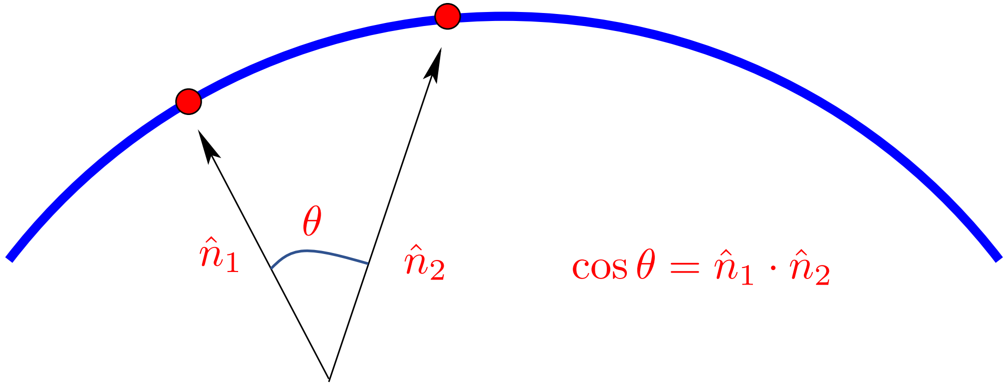
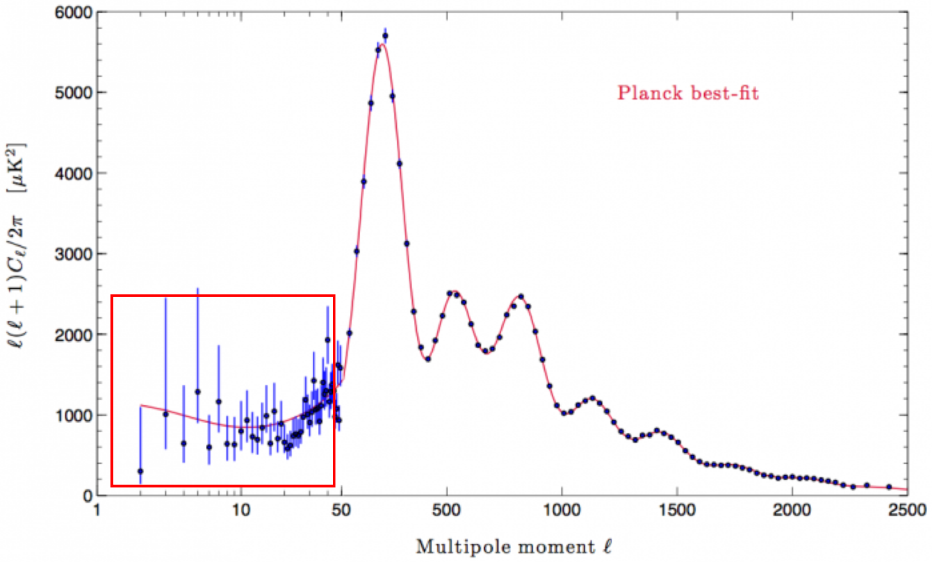




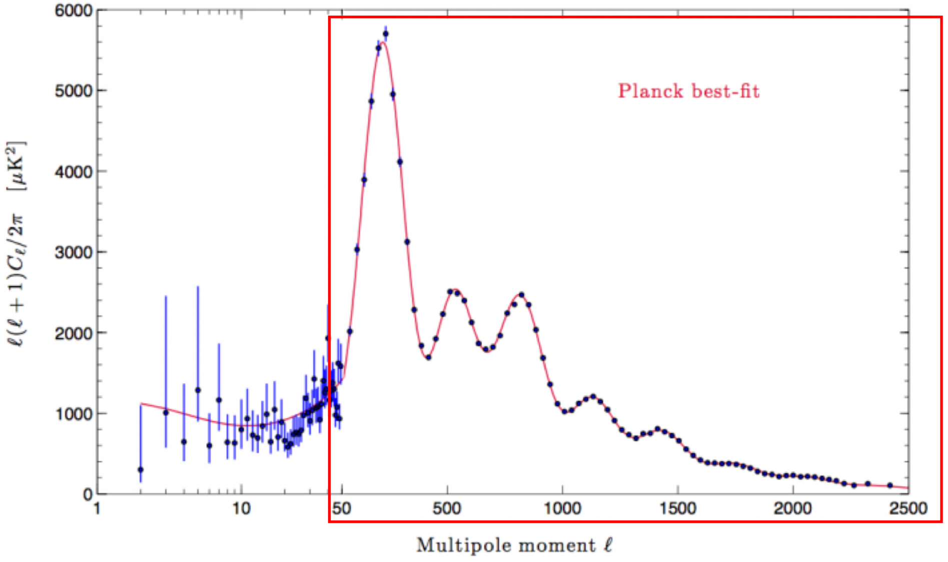
$$C(\theta) = \left\langle \frac{\delta T}{T}(\hat{n}_1), \frac{\delta T}{T}(\hat{n}_2) \right\rangle = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1)\, C_l\, P_l(\cos\theta)$$

This region of the CMB is *constant.* Detailed analysis shows that

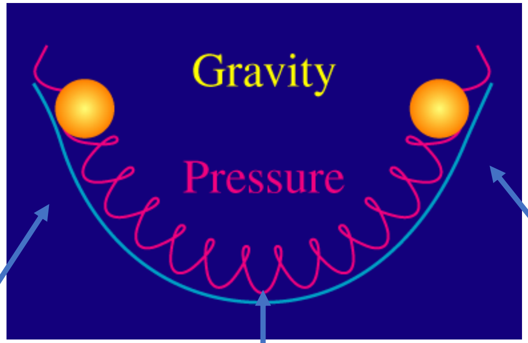
- Fluctuations on these scales occur because the distribution of dark matter is *not* perfectly homogeneous
- This is called the *Sach-Wolfe* effect and detailed analysis shows that

$$\epsilon_{\rm DM}(r) = \bar{\epsilon}_{\rm DM} + \delta\epsilon_{\rm DM}(r) \Rightarrow \frac{\delta T}{T} = \frac{1}{3}\frac{\delta\phi}{c^2}$$

where $\delta\phi$ is gravitational potential associated with $\delta\epsilon_{\rm DM}(r)$



Baryons and photons



Gravity potential well

baryon

baryon

photon

Do question (1) on the worksheet and S T O P

Dark Matter

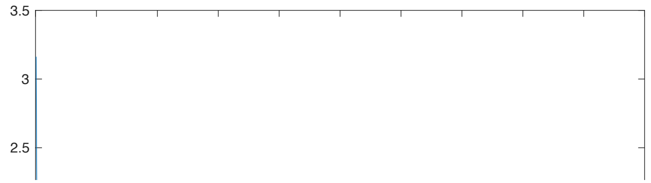
We begin, as most topics do, with first year physics. Do question (2) on the worksheet and S T O P

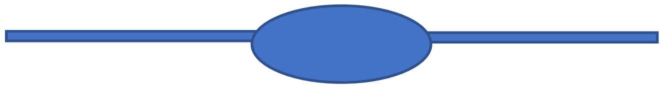
(2a) For a star to stay in circular orbit, its *speed* must be constant. For uniform circular motion,

$$a = \frac{v^2}{R}$$

(2b) $F = ma \Rightarrow \frac{GMm}{R^2} = ma \Rightarrow a = \frac{GM}{R^2}$

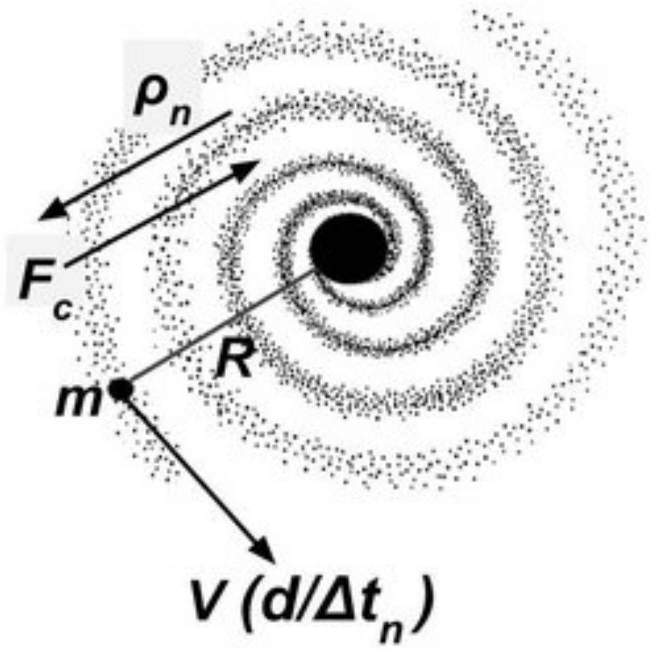
(2c) $a = \frac{GM}{R^2}$ and $a = \frac{v^2}{R}$; so $v = \sqrt{\frac{GM}{R}}$ Note that *M* here is the mass of the galaxy at distance *R*

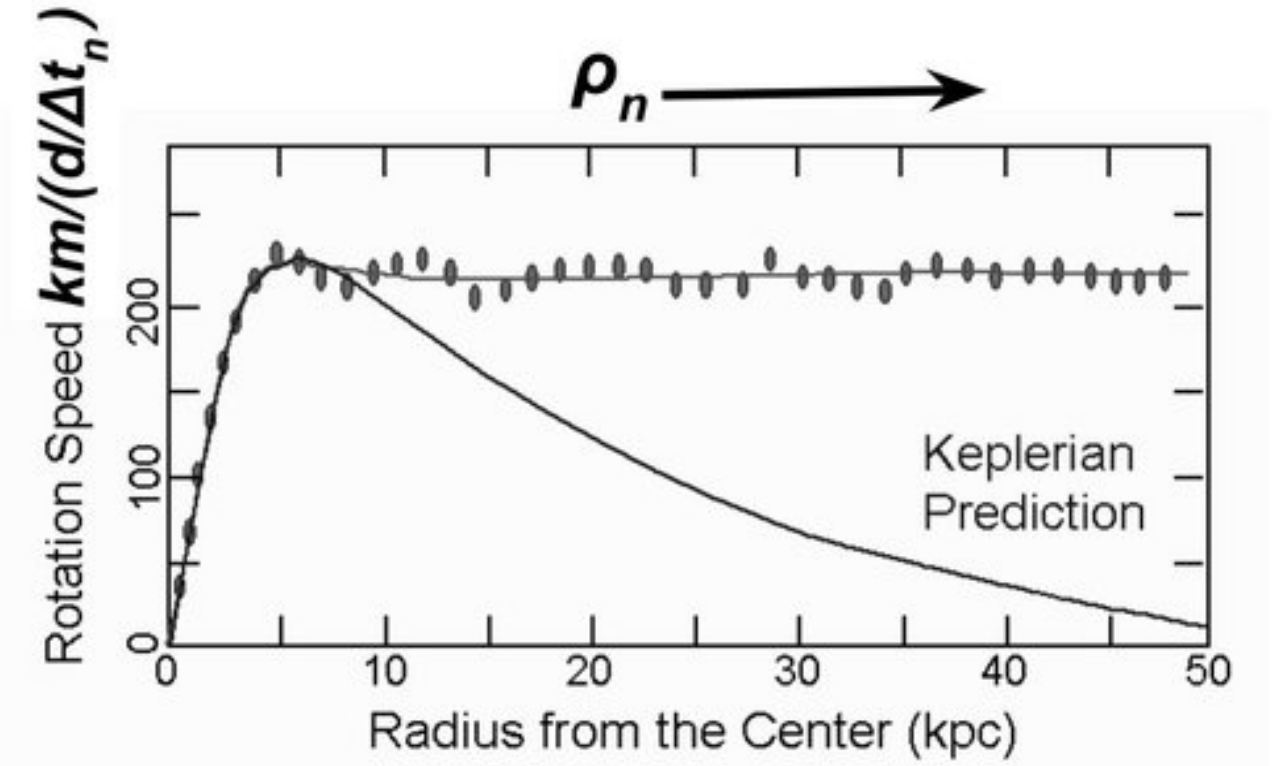




As R increases, centripetal force (F_c) is perfectly balanced:

v ($d/\Delta t_n$) and, subsequently, ρ_n proportionally increase





Clusters of galaxies—but first a digression

The virial theorem.

$$v_i = \frac{dr_i}{dt}$$

$$p_i = m_i v_i$$

$$\frac{dp_i}{dt} = F_i$$

Now define: $H = \sum_i p_i \cdot r_i$ Taking time derivative gives

$$\frac{dH}{dt} = \sum_i \frac{dp_i}{dt} \cdot r_i + \sum_i p_i \cdot \frac{dr_i}{dt}$$

$$\frac{dH}{dt} = \sum_i \underbrace{\frac{dp_i}{dt}}_{=\text{ Force}} \cdot r_i + \sum_i \underbrace{p_i \cdot \frac{dr_i}{dt}}_{=\text{ 2 K}}$$

$$\frac{dH}{dt} = \sum_i F_i \cdot r_i + 2K$$

Time average

$\bar{K} + \frac{1}{2}\sum_i \bar{F}_i \cdot \bar{r}_i = 0$ But for gravity $F = -\frac{dV}{dr} \Rightarrow F \cdot r = V$

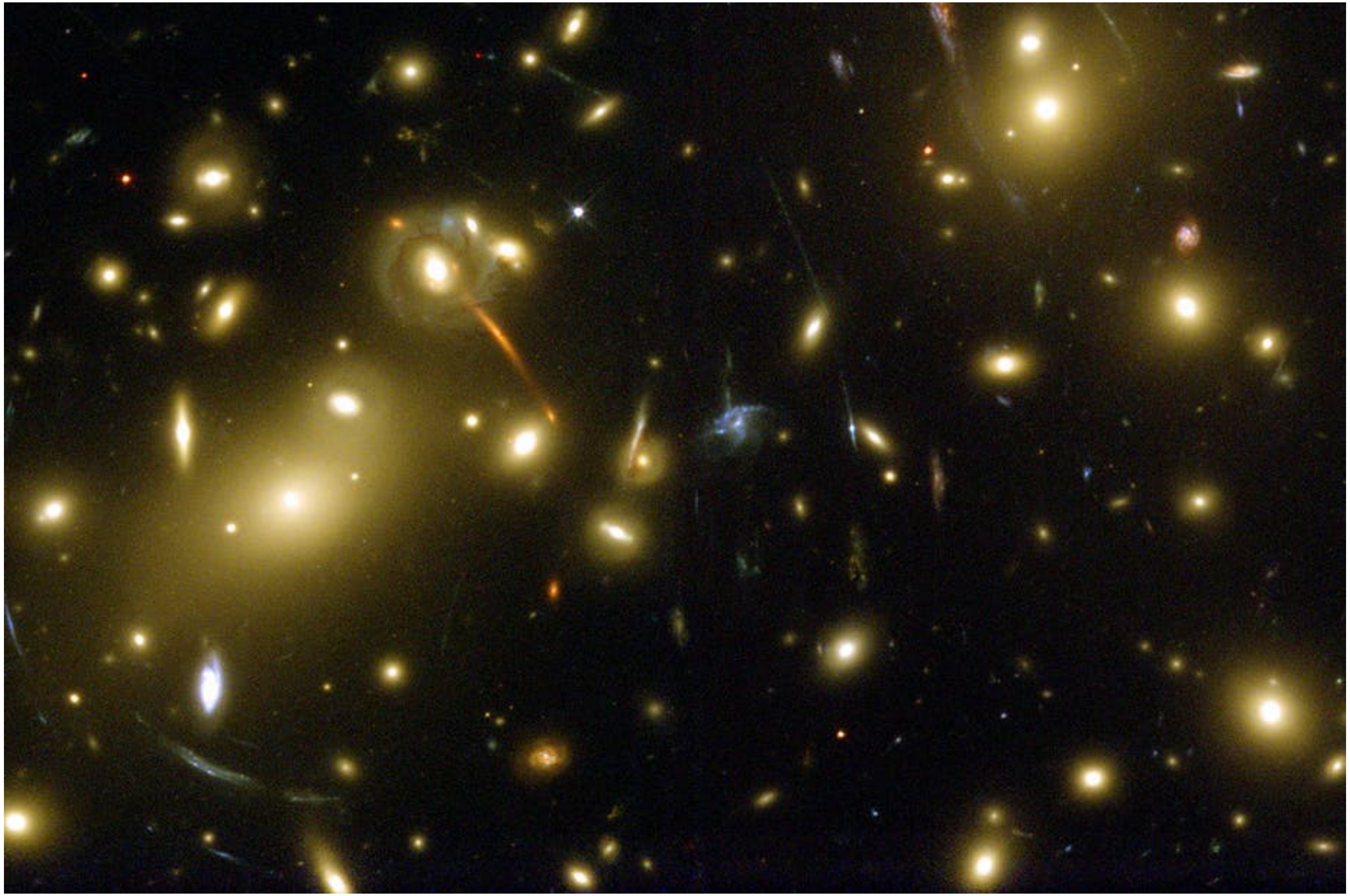
So

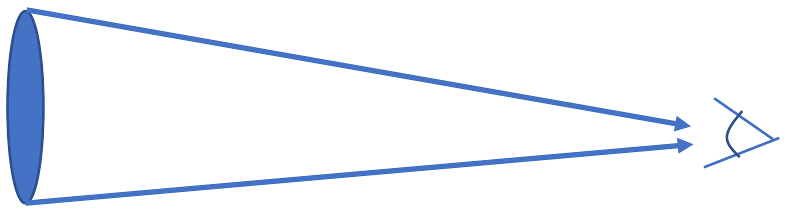
$$\boxed{\bar{K} = -\frac{1}{2}\bar{V}}$$ Virial theorem

Now back to clusters of galaxies

$$\boxed{\bar{K} = -\frac{1}{2}\bar{V}} \qquad \bar{K} = \frac{1}{2}m\langle v^2\rangle \quad \bar{V} \propto \frac{1}{r} = \frac{\alpha}{2}\frac{GM^2}{r_h} \qquad \text{So : } \langle v^2\rangle = \alpha MG\frac{1}{r_h}$$

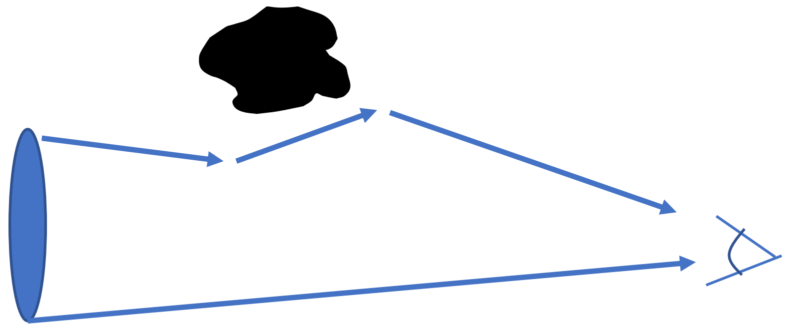
Gravitational lensing.





The presence of matter will distort the image.

This is gravitational lensing.



While there is now a lot of evidence of dark matter, we have no idea what it is. And most of the matter is in the form of dark matter. We have that

$$\Omega_{\mathrm{bary,o}} = 0.048; \quad \Omega_{\mathrm{DM,o}} = 0.262$$

Do question (3) on the worksheet.