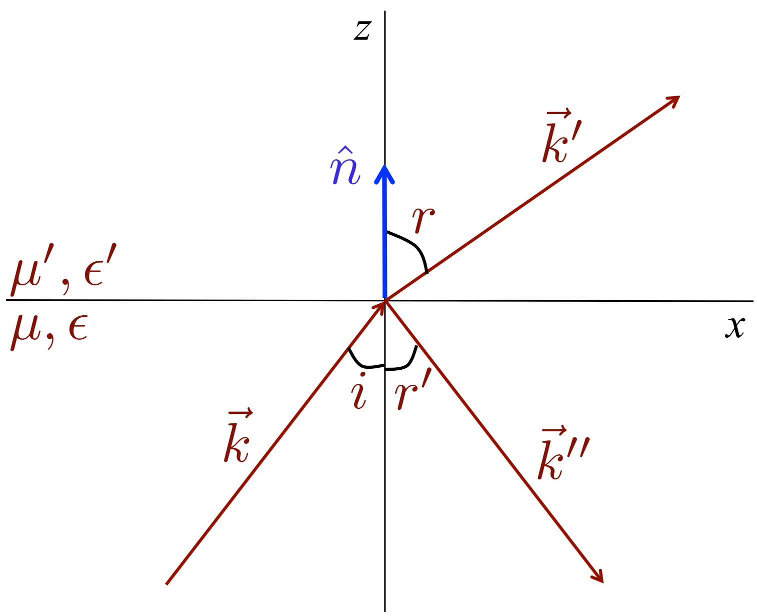
PHY 411 Winter 2021

Class Summary—Week 3, Day 1—Tuesday, Jan 19

Reflection and Refraction of Electromagnetic Waves

Having learned the fundamentals of propagating plane waves obtained from the Maxwell equations, we will now study some well known phenomena in optics by considering a plane wave that is incident on a plane surface between two media of different dielectric properties — one with permeability μ and permittivity ϵ, and the other with permeability μ' and permittivity ϵ' respectively.

The geometry of the problem is shown in Figure 7.5 in Jackson (page 303), and reproduced below.



- The plane interface separating the two media is at $z = 0$.
- The media below the plane $z = 0$ has permeability μ and permittivity ϵ.
- The media above the plane $z = 0$ has permeability μ' and permittivity ϵ'.
- A plane wave with wave vector $\vec{k}$ and frequency ω is incident from medium (μ, ϵ).
- The refracted wave in medium (μ', ϵ') has wave vector $\vec{k}'$.
- The wave reflected back into medium (μ, ϵ) has wave vector $\vec{k}''$.
- Jackson also defines a unit normal $\hat{n}$ from medium (μ, ϵ) toward medium (μ', ϵ'), but take care you don't confuse it with the $\hat{n}$ that he (and we) used in §7.1 (where it was used to indicate the direction of $\vec{k}$)!
- And, to add to the fun, we will also be using the symbol n for the index of refraction in the medium (μ, ϵ), and n' for the index of refraction in the medium (μ', ϵ').

  If you feel frustrated by this duplication of symbols, you aren't alone. In fact, research groups publish with a variety of symbols, so you might as well get used to it if you're considering an academic career.

With the geometry defined above, we can now write the incident, refracted, and reflected waves.

Incident Wave

$$\vec{E} = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$
$$\vec{B} = \sqrt{\mu\epsilon}\,\frac{\vec{k}\times\vec{E}}{k} \tag{7.30}$$

Refracted Wave

$$\vec{E}' = \vec{E}'_0 \, e^{i(\vec{k}'\cdot\vec{x}-\omega t)}$$
$$\vec{B}' = \sqrt{\mu'\epsilon'}\,\frac{\vec{k}'\times\vec{E}'}{k'} \tag{7.31}$$

Reflected Wave

$$\vec{E}'' = \vec{E}''_0 \, e^{i(\vec{k}''\cdot\vec{x}-\omega t)}$$
$$\vec{B}'' = \sqrt{\mu\epsilon}\,\frac{\vec{k}''\times\vec{E}''}{k} \tag{7.32}$$

Notice that $\vec{k}$ and $\vec{k}''$ have the same magnitude, k — *you showed this on the worksheet in class.*

Proof that $k = k''$ — Question 1 on today's worksheet:

- Recall that the phase velocity $v = 1/\sqrt{\mu\epsilon}$— this means that the incident and reflected waves have the same phase velocity because they are in the same medium, so $v = v''$.
- But phase velocity $v = \omega/k$ also. And the frequency ω must remain the same in all the three media; why? — because the frequency is determined by the source, it can't change at the interface — if you have N waves coming in per second, then N waves must reflect and refract per second.
- Therefore, we get that $k = \omega/v$ is equal to $k'' = \omega/v''$.

With $k = k''$, we get equation (7.33) in Jackson:

$$\left|\vec{k}\right| = \left|\vec{k}''\right| = k = \omega\sqrt{\mu\epsilon}$$
$$\left|\vec{k}'\right| = k' = \omega\sqrt{\mu'\epsilon'} \tag{7.33}$$

We will now study two aspects of such reflection and refraction phenomena.

The first are the so-called kinematic properties, which we obtain from the wave nature of the phenomena and the fact that there are boundary conditions to be satisfied, but do not depend on the detailed nature of the waves or the boundary conditions. By using these criteria, we obtain the law of reflection (angle of incidence equals angle of reflection) and Snell's law for refraction, both of which you've encountered in introductory physics classes.

The other aspect concerns the dynamic properties which depend on the specific nature of electromagnetic fields and their boundary conditions, as we'll see below. By doing this, we can figure the intensities of the reflected and refracted waves, and their phase changes and polarization.

Kinematic Properties

Kinematic properties are those that we obtain from the wave nature of the phenomena and the fact that there are boundary conditions to be satisfied, but do not depend on the detailed nature of the waves or the boundary conditions.

Because the boundary conditions must be satisfied at all points on the plane $z = 0$ at all times, the spatial (and time) variation of all the three fields must be the same at $z = 0$. This means that the phase factors of all the three waves must be equal at $z = 0$, and so

$$(\vec{k} \cdot \vec{x})_{z=0} = (\vec{k}' \cdot \vec{x})_{z=0} = (\vec{k}'' \cdot \vec{x})_{z=0} \tag{7.34}$$

Note that equation (7.34) has nothing to do with the boundary conditions themselves (i.e., it is "independent of the nature of the boundary conditions" according to Jackson). Instead, equation (7.34) is just a mathematical requirement stemming from the very existence of the boundary conditions.

Equation (7.34) contains the laws of reflection and refraction.

- All the three wave vectors $\vec{k}, \vec{k}'$, and $\vec{k}''$ lie on a plane.
- The angle of incidence (i) is equal to the angle of reflection (r').
- Snell's law for refraction: $n \sin i = n' \sin r$

The three results above can be obtained from equation (7.34) by showing that it yields

$$k \sin i = k' \sin r = k'' \sin r' \tag{7.35}$$

Let's do the first term here as an example:

- Begin by writing $\vec{k} = k\hat{k}$, using $\hat{k}$ to indicate the direction of $\vec{k}$ because $\hat{n}$ is being used to indicate a different direction in this section (see the first page of this summary). Then

$$(\vec{k} \cdot \vec{x})_{z=0} = (k\hat{k} \cdot \vec{x})_{z=0} = k\,(\hat{k} \cdot \vec{x})_{z=0}$$

- We need to find the dot product $\hat{k} \cdot \vec{x} = \hat{k} \cdot (\hat{x}\,x + \hat{y}\,y + \hat{z}\,z)$.
- To find the dot product written above, we note that the incident ray makes an angle i with the normal ray, so by extending the incident ray into the space $z > 0$ (use a dashed line), it is easy to see that it makes an angle $(90^\circ - i)$ with the positive direction of the x-axis, and an angle i with the z-axis. The y-axis is perpendicular to the xz-plane, so the incident ray makes an angle of 90° with the y-axis. See the posted video for a figure.
- So the dot product $\hat{k} \cdot \vec{x}$ evaluates to

$$\begin{aligned}
\hat{k} \cdot \vec{x} &= \hat{k} \cdot (\hat{x}\,x + \hat{y}\,y + \hat{z}\,z) \\
&= x\,\hat{k} \cdot \hat{x} + y\,\hat{k} \cdot \hat{y} + z\,\hat{k} \cdot \hat{z} \\
&= x \cos(90^\circ - i) + y\,\cos 90^\circ + z\,\cos i
\end{aligned}$$

- On the previous page, we showed that the dot product $\hat{k}\cdot\vec{x}$ evaluates to

$$\hat{k}\cdot\vec{x} = x\cos(90^\circ - i) + y\,\cos 90^\circ + z\,\cos\,i$$

- Inserting this into the first term in equation (7.34), we obtain

$$(\vec{k}\cdot\vec{x})_{z=0} = k\,(\hat{k}\cdot\vec{x})_{z=0}$$

$$\text{so that } (\vec{k}\cdot\vec{x})_{z=0} = k\left[x\,\cos(90^\circ - i) + y\,\cos 90^\circ + z\,\cos\,i\right]_{z=0}$$

- The second term on the right hand side vanishes because $\cos\,90^\circ = 0$, the third term vanishes because we're evaluating the expression at $z = 0$.
- Therefore, we are left with

$$(\vec{k}\cdot\vec{x})_{z=0} = k\left[x\,\cos(90^\circ - i)\right]$$

- Finally, therefore, we get

$$(\vec{k}\cdot\vec{x})_{z=0} = kx\,\sin\,i$$

Likewise for the other two terms (*as you showed in Question 2(a) of the worksheet for today*)

$$(\vec{k}'\cdot\vec{x})_{z=0} = k'x\,\sin\,r \qquad \text{and} \qquad (\vec{k}''\cdot\vec{x})_{z=0} = k''x\,\sin\,r'$$

Putting it all together, and canceling x, we get equation (7.35) written above.

In summary, starting from equation (7.34):

$$(\vec{k}\cdot\vec{x})_{z=0} = (\vec{k}'\cdot\vec{x})_{z=0} = (\vec{k}''\cdot\vec{x})_{z=0}$$

we have obtained that

$$kx\,\sin\,i = k'x\,\sin\,r = k''x\,\sin\,r'$$

Since this must be valid for all values of x, we obtain finally that

$$k\,\sin\,i = k'\,\sin\,r = k''\,\sin\,r'$$

which is equation (7.35).

- The derivation above shows why equation (7.34) implies that $\vec{k}, \vec{k}'$, and $\vec{k}''$ lie on a plane — if, for example, $\vec{k}$ and $\vec{k}'$ were not in the same plane, then they would have different components along the axes and equation (7.34) would not hold.
- Since $k = k''$, meaning that $k''\,\sin\,r' = k\,\sin\,r'$, we get from equation (7.35) that

$$k\,\sin\,i = k''\,\sin\,r' = k\,\sin\,r'$$

  This gives $i = r'$, which is the law of reflection, i.e., the angle of incidence (i) is equal to the angle of reflection (r'), *as you proved in Question 2(b) of today's worksheet.*

- Snell's law for refraction:

  Recall that in equation (7.5), we defined the index of refraction as

$$n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$$

  Then, from equation (7.35), we get

$$\frac{\sin\ i}{\sin\ r} = \frac{k'}{k} = \frac{\omega\sqrt{\mu'\epsilon'}}{\omega\sqrt{\mu\epsilon}}$$

  using equation (7.33). Then

$$\frac{\sin\ i}{\sin\ r} = \frac{\sqrt{\mu'\epsilon'}}{\sqrt{\mu\epsilon}} = \frac{\sqrt{\mu'\epsilon'/\mu_0\epsilon_0}}{\sqrt{\mu\epsilon/\mu_0\epsilon_0}} = \frac{n'}{n} \tag{7.36}$$

  Therefore, when a wave incident at angle i to the normal travels from a medium with index of refraction n into a medium with index of refraction n', it bends at angle r to the normal such that

$$n\ \sin\ i = n'\ \sin\ r$$

  which is *Snell's law. You proved this in Question 2(c) of today's worksheet.*

Dynamic Properties

Now, let's do the dynamics by looking at the boundary conditions. So, what are the boundary conditions?

There are four, as you no doubt learned in undergrad E&M. **If you've forgotten, or didn't get a chance to learn them during your undergrad, they are derived** in the *Introduction and Survey* chapter in Jackson: § I.5 (pages 16-18). I've demonstrated how to derive the first one in the video, the other three you should read by yourself if you're interested. *Note that deriving the conditions is not part of this course; our objective is to use them.* Essentially, you derive the conditions by writing the integral form of Maxwell's equations and choosing Gaussian pillboxes and Amperian current loops at the surface. As a reminder, if the fields in medium 1 are $\vec{E}_1, \vec{D}_1, \vec{B}_1, \vec{H}_1$ and those in medium 2 are $\vec{E}_2, \vec{D}_2, \vec{B}_2, \vec{H}_2$, then

$$(\vec{D}_2 - \vec{D}_1)\cdot\hat{n} = \sigma \tag{I.17}$$

$$(\vec{B}_2 - \vec{B}_1)\cdot\hat{n} = 0 \tag{I.18}$$

$$\hat{n}\times(\vec{E}_2 - \vec{E}_1) = 0 \tag{I.19}$$

$$\hat{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{K} \tag{I.20}$$

where σ is the surface charge density, $\vec{K}$ is the surface current density, and $\hat{n}$ is a unit vector pointing from medium 1 to medium 2. In studying reflection and refraction, of course, there is no surface charge or surface current at the interface.

Although deriving equations (I.17)-(I.20) is not part of this course, understanding what they tell us and then deriving from them the four equations in equation (7.37) certainly constitutes part of this course. We will now discuss in detail how to go from equations (I.17)-(I.20) to the four expressions in equation (7.37).

What do the four equations (I.17)-(I.20) tell us, and starting from them, how do we get the four equations in (7.37)?

- Equation (I.17) tells us that the discontinuity in the component of $\vec{D}$ perpendicular to the interface at any point on the interface between the two media is equal to the surface charge density at that point. As noted above, since there is no surface charge density at the interface when studying reflection and refraction, we will match the perpendicular components of $\vec{D}$ at the interface.

  So, set the component of the electric displacement $\vec{D}$ perpendicular to the interface between the two media to be continuous at the boundary $z = 0$. This gives

  $$\left[\vec{D}_0 + \vec{D}_0''\right] \cdot \hat{n} = \vec{D}_0' \cdot \hat{n}$$

  where, as shown in Figure 7.5 in Jackson and reproduced on the first page of this summary, $\hat{n}$ is a unit vector perpendicular to the plane $z = 0$ pointing from the surface (μ, ϵ) to the surface (μ', ϵ'). In the equation above, and all the equations to follow, we're going to be using $\vec{D}_0$, $\vec{B}_0$, etc., because we've already determined the phase factors like $e^{i(\vec{k}\cdot\vec{x}-\omega t)}$ to be equal at the interface $z = 0$ in our discussion on the kinematic properties.

  To get the first of the four equations written by Jackson in equation (7.37), move all fields to the same side:

  $$\left[\left(\vec{D}_0 + \vec{D}_0''\right) - \vec{D}_0'\right] \cdot \hat{n} = 0$$

  and write in terms of the electric field, since $\vec{D} = \epsilon\vec{E}$:

  $$\left[\epsilon\left(\vec{E}_0 + \vec{E}_0''\right) - \epsilon'\,\vec{E}_0'\right] \cdot \hat{n} = 0$$

  which is the first of the four equations written by Jackson in equation (7.37).

- Equation (I.18) tells us that the component of $\vec{B}$ perpendicular to the surface must be continuous at the boundary $z = 0$. So

  $$\left[\vec{B}_0 + \vec{B}_0''\right] \cdot \hat{n} = \vec{B}_0' \cdot \hat{n}$$

  or

  $$\left[\vec{B}_0 + \vec{B}_0'' - \vec{B}_0'\right] \cdot \hat{n} = 0$$

  Writing $\vec{B}$ in terms of $\vec{E}$ by using equations (7.30), (7.31), and (7.32), we get

  $$\left[\sqrt{\mu\epsilon}\,\frac{\vec{k}\times\vec{E}_0}{k} + \sqrt{\mu\epsilon}\,\frac{\vec{k}''\times\vec{E}_0''}{k} - \sqrt{\mu'\epsilon'}\,\frac{\vec{k}'\times\vec{E}_0'}{k'}\right] \cdot \hat{n} = 0$$

  Setting $k = \omega\sqrt{\mu\epsilon}$ and $k' = \omega\sqrt{\mu'\epsilon'}$ from equation (7.33), the equation above becomes

  $$\left[\sqrt{\mu\epsilon}\,\frac{\vec{k}\times\vec{E}_0}{\omega\sqrt{\mu\epsilon}} + \sqrt{\mu\epsilon}\,\frac{\vec{k}''\times\vec{E}_0''}{\omega\sqrt{\mu\epsilon}} - \sqrt{\mu'\epsilon'}\,\frac{\vec{k}'\times\vec{E}_0'}{\omega\sqrt{\mu'\epsilon'}}\right] \cdot \hat{n} = 0$$

  from which we obtain (*as you showed in Question 3 on the worksheet for today*)

  $$\left[\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0'' - \vec{k}'\times\vec{E}_0'\right] \cdot \hat{n} = 0$$

  which is the second of the four equations written by Jackson in equation (7.37).

- Equation (I.19) tells us that the electric field parallel to the surface (i.e., the tangential component of $\vec{E}$) must be continuous at the boundary $z = 0$, so

$$\left[\vec{E}_0 + \vec{E}_0''\right] \times \hat{n} = \vec{E}_0' \times \hat{n}$$

from which we obtain

$$\left[\vec{E}_0 + \vec{E}_0'' - \vec{E}_0'\right] \times \hat{n} = 0$$

which is the third of the four equations written by Jackson in equation (7.37).

- Equation (I.20) tells us that the tangential component of the magnetic field $\vec{H}$ is continuous at the boundary $z = 0$:

$$\left[H_0 + H_0''\right] \times \hat{n} = H_0' \times \hat{n}$$

or

$$\left[H_0 + H_0'' - H_0'\right] \times \hat{n} = 0$$

Since $\vec{B} = \mu\vec{H}$, this can be written as

$$\left[\frac{1}{\mu}\,B_0 + \frac{1}{\mu}\,B_0'' - \frac{1}{\mu'}\,B_0'\right] \times \hat{n} = 0$$

Writing $\vec{B}$ in terms of $\vec{E}$ by using equations (7.30), (7.31), and (7.32), we get

$$\left[\frac{1}{\mu}\left(\sqrt{\mu\epsilon}\,\frac{\vec{k}\times\vec{E}_0}{k}\right) + \frac{1}{\mu}\left(\sqrt{\mu\epsilon}\,\frac{\vec{k}''\times\vec{E}_0''}{k}\right) - \frac{1}{\mu}\left(\sqrt{\mu'\epsilon'}\,\frac{\vec{k}'\times\vec{E}_0'}{k'}\right)\right] \cdot \hat{n} = 0$$

and, again, setting $k = \omega\sqrt{\mu\epsilon}$ and $k' = \omega\sqrt{\mu'\epsilon'}$ from equation (7.33), we get

$$\left[\frac{1}{\mu}\left(\sqrt{\mu\epsilon}\,\frac{\vec{k}\times\vec{E}_0}{\omega\sqrt{\mu\epsilon}}\right) + \frac{1}{\mu}\left(\sqrt{\mu\epsilon}\,\frac{\vec{k}''\times\vec{E}_0''}{\omega\sqrt{\mu\epsilon}}\right) - \frac{1}{\mu}\left(\sqrt{\mu'\epsilon'}\,\frac{\vec{k}'\times\vec{E}_0'}{\omega\sqrt{\mu'\epsilon'}}\right)\right] \cdot \hat{n} = 0$$

so that finally

$$\left[\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0''\right) - \frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\right] \times \hat{n} = 0$$

which is the fourth of the four equations written by Jackson in equation (7.37).

So, now we have all the four equations in equation (7.37):

$$\left[\epsilon\left(\vec{E}_0 + \vec{E}_0''\right) - \epsilon'\,\vec{E}_0'\right] \cdot \hat{n} = 0 \tag{7.37.a}$$

$$\left[\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0'' - \vec{k}'\times\vec{E}_0'\right] \cdot \hat{n} = 0 \tag{7.37.b}$$

$$\left[\vec{E}_0 + \vec{E}_0'' - \vec{E}_0'\right] \times \hat{n} = 0 \tag{7.37.c}$$

$$\left[\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0''\right) - \frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\right] \times \hat{n} = 0 \tag{7.37.d}$$

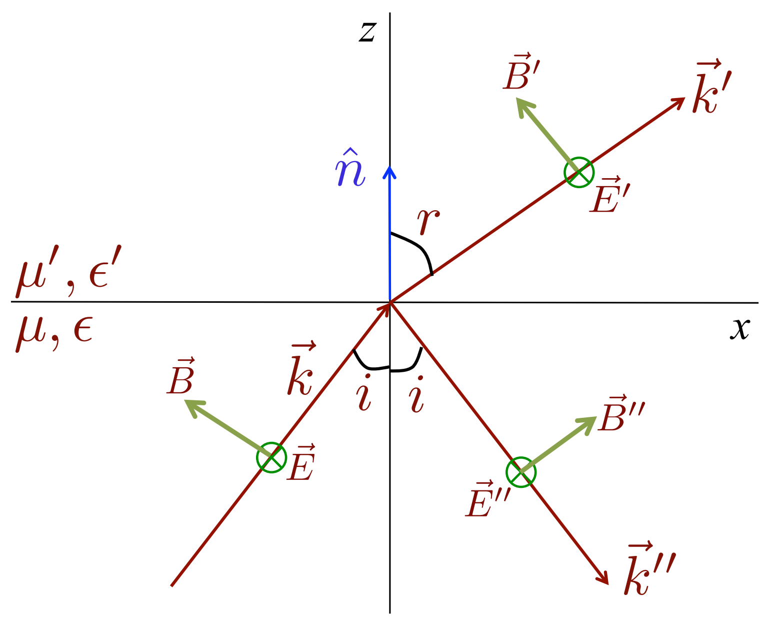
A glance at the boundary conditions in equation (7.37) indicates very messy algebra ahead!

So, is there anything we can do to simplify procedures without losing generality?

Yes, there is — we can consider the case in which the incident plane wave is linearly polarized with its polarization vector perpendicular to the plane of incidence separately from the case in which the polarization vector is parallel to the plane of incidence. The plane of incidence is the plane defined by $\vec{k}$ and $\hat{n}$; recall that $\hat{n}$ is now a unit normal from medium (μ, ϵ) toward medium (μ', ϵ'). We can consider these two cases separately because the general case of arbitrary elliptical polarization can be built by appropriate linear combination of the two cases, as we studied in § 7.2 earlier.

$\vec{E}$ perpendicular to the plane of incidence

We will first consider the case in which the incident plane wave has its polarization vector (i.e., $\vec{E}$-field) perpendicular to the plane of incidence. The geometry of the problem is shown in Figure 7.6(a) on page 305 in Jackson; the figure is reproduced below. The plane of the page is the plane of incidence, and the electric fields are into the page, directed away from us (hence represented by the tail end of an arrow: $\otimes$). The $\vec{B}$ are directed as shown, so that when the fingers of the right hand are curled from $\vec{E}$ to $\vec{B}$, the energy flow is in the direction of the $\vec{k}$-vectors.



Let us now investigate what we get from equation (7.37) for the case of the $\vec{E}$-fields perpendicular to the plane of incidence that we are considering here.

Since the plane of incidence is defined by $\vec{k}$ and $\hat{n}$, and so all the $\vec{E}$-fields are perpendicular to $\hat{n}$, the dot products of all the $\vec{E}$-fields with $\hat{n}$ will be zero. Another way to think about this is that the $\vec{E}$-fields are all parallel to the boundary surface (because they are directed into the plane of the page); therefore, $\vec{E} \cdot \hat{n} = 0$ for all the waves — incident, reflected and refracted. The net result is that equation (7.37.a) yields nothing.

On the other hand, if the $\vec{E}$-fields are perpendicular to the plane of incidence or, equivalently, parallel to the boundary surface ($z = 0$), then $\vec{E} \times \hat{n} \neq 0$ for the incident, reflected, and refracted waves. Therefore, equation (7.37.c) and equation (7.37.d) are the ones to pursue; yes, the latter is for the $\vec{B}$-fields, but the $\vec{B}$-fields are at an angle to the surface, so we'll definitely get something out of equation (7.37.d). You might wonder about equation (7.37.b), and the answer is yes, we could also use it, but it will duplicate the results from equation (7.37.c) — try it out for yourself and see, so we won't consider it here.

So, begin with the third equation (7.37.c):

$$\left[\vec{E}_0 + \vec{E}_0'' - \vec{E}_0'\right] \times \hat{n} = 0$$

- There are three terms on the left hand side. If we distribute and write them out, we have

$$\vec{E}_0 \times \hat{n} + \vec{E}_0'' \times \hat{n} - \vec{E}_0' \times \hat{n} = 0$$

- Consider the magnitude of the first of the three terms (*as you did on the worksheet for today*): $\vec{E}_0$ is perpendicular to the plane of incidence, so it makes an angle of 90° with the unit vector $\hat{n}$, and so the cross product

$$\left|\vec{E}_0 \times \hat{n}\right| = \left|\vec{E}_0\right| \left|\hat{n}\right| \sin 90^\circ = E_0 \left(1\right)\left(1\right) = E_0$$

- Likewise, for the second term, $\vec{E}_0''$ is perpendicular to the plane of incidence, so it makes an angle of 90° with the unit vector $\hat{n}$, and so

$$\left|\vec{E}_0'' \times \hat{n}\right| = E_0''$$

- Likewise for the third term, we get

$$\left|\vec{E}_0' \times \hat{n}\right| = E_0'$$

- Moreover, all three cross products are along the same direction, as you'll realize if you curl the fingers of your right hand and establish the direction of the cross product with your thumb. Therefore, equation (7.37.c) gives us the *scalar* relation

$$E_0 + E_0'' - E_0' = 0 \tag{7.38.a}$$

  which is the first of the two equations in (7.38).

- As mentioned above, the second equation (7.37.b) gives the same equation (7.38.a) as above (using Snell's law), so we won't consider it.

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Next, consider the fourth boundary condition in equation (7.37), given by

$$\left[\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0+\vec{k}''\times\vec{E}_0''\right)-\frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\right]\times\hat{n}=0 \tag{7.37.d}$$

On Question 5 of today's Discussion Worksheet, you were asked to show that equation (7.37.d) above reduces (in magnitude) to

$$\sqrt{\frac{\epsilon}{\mu}}\,E_0\cos\,i-\sqrt{\frac{\epsilon}{\mu}}\,E_0''\cos\,i-\sqrt{\frac{\epsilon'}{\mu'}}\,E_0'\cos\,r=0 \tag{7.38.b}$$

This has now been **moved to Homework 3**.

On Question 6 of today's Discussion Worksheet, you were asked to show that equation (7.38.a) on the previous page and equation (7.38.b) together yield

$$\begin{aligned}\frac{E_0'}{E_0}&=\frac{2n\,\cos\,i}{n\,\cos\,i+\dfrac{\mu}{\mu'}\sqrt{n'^{\,2}-n^2\,\sin^2\,i}}\\[1em] \frac{E_0''}{E_0}&=\frac{n\,\cos\,i-\dfrac{\mu}{\mu'}\sqrt{n'^{\,2}-n^2\,\sin^2\,i}}{n\,\cos\,i+\dfrac{\mu}{\mu'}\sqrt{n'^{\,2}-n^2\,\sin^2\,i}}\end{aligned} \tag{7.39}$$

This has now been **moved to Homework 3**.

Having obtained the ratios in equation (7.39), we now have a complete description of the problem (for $\vec{E}$-fields perpendicular to the plane of incidence). In the next class, we will write down how we can use the ratios in equation (7.39) to find the energy flow by using the time-averaged Poynting vector (note, however, that we will do it explicitly only for the specific case of $\vec{E}$-fields *parallel* to the direction of polarization).