PHY 411 Winter 2021

Homework 6—due by 9:00 PM, Monday, Feb 22

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Friday (Feb 26). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

1. In class, we learned how to solve Laplace's equation in different geometries. Here is a chance to work with its counterpart, Poisson's equation: do **Problem 1.5** in Jackson (*page 51*).

Note: Jackson is cryptic in his statement to "find the charge distribution (both continuous and discrete)" — what he means is that your solution for the charge density $\rho(r)$ will contain a piece that blows up at $r \to 0$. You'll need to take care of this by splitting out your solution at $r = 0$; that is, write your solution in two parts, one valid for $r > 0$, and the other for $r = 0$. The latter case has a well known form; Jackson tells you how to handle such cases at the bottom of page 35.

2. The two-dimensional Laplace equation, in which the potential can be assumed to be independent of one of the coordinates, has applications in, e.g., a long uniform transmission line. Consider such a case in which the potential is independent of z, so that the Laplace equation is

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

with boundary conditions $\Phi = 0$ at $x = 0$ and $x = a$, $\Phi = V$ at $y = 0$ for $0 \le x \le a$ and $\Phi \to 0$ for large y (e.g., see Figure 2.10 on page 73 in Jackson).

By the method of separation of variables, show that the potential in $0 \le x \le a, y \ge 0$ is given by

$$\Phi(x, y) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{a}\right) e^{-n\pi y/a}$$

where A_n is a constant.

3. Use the appropriate boundary condition(s) to evaluate A_n in the previous problem in the usual way, and show that

$$A_n = \begin{cases} \dfrac{4V}{\pi n} & \text{for } n \text{ odd} \\ 0 & \text{for } n \text{ even} \end{cases}$$

Homework 6 continues on the next page.

4. A compact representation of Legendre polynomials is given by *Rodrigues' formula*:

$$P_l(x) = \frac{1}{2^l\, l!}\,\frac{d^l}{dx^l}\,(x^2-1)^l$$

Using Rodrigues' formula, derive that

$$\frac{dP_{l+1}}{dx} - \frac{dP_{l-1}}{dx} - (2l+1)\,P_l = 0$$