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(a)

Convolution - will tell a singual based on another

$$p \otimes q$$

Correlation - tells how similar signals are

$$p \odot q$$

(B)

Fourier Series - Is an expansion of a function with a basis solved with summation

$$f(x) = \sum_{-\infty}^{\infty} c_n e^{-i 2\pi i}$$

Fourier Transform - Has Discrete frequencies and is solved with an integral $\int_{-\infty}^{\infty} c_n e^{-i\omega t}$

Discrete Fourier Transform - Solve with summation

$$\sum_{n=0}^{N-1} f(n) e^{-i 2\pi k n/N}$$

(C)

Cubic Spline - Takes derivatives at end points.

Cubic polynomial - passes through every point

2a)

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} + 4y = 0$$

$$\frac{dy}{dt} = v \,, \quad \frac{dv}{dt} = v - 4y$$

2b) $t = 0,\ y = 0,\ \dot{y} = v = 1,\ \Delta t = h = 0.5$

t	y	v
0	0	1
1	1.62	0.47
2	2.25	-0.11
3		

See Note why this is wrong

$$S = \begin{pmatrix} y \\ v \end{pmatrix} \quad F = \begin{pmatrix} v \\ v - 4y \end{pmatrix}$$

$$f_0 = f(t_0, y_i)$$
$$f_1 = f(t_0 + h/2,\ y_0 + \tfrac{h}{2} f_0)$$
$$f_2 = f(t_0 + h/2,\ y_0 + \tfrac{h}{2} f_1)$$
$$f_3 = f(t_0 + h,\ y_0 + f_2)$$
$$y_{i+1} = y_i + \tfrac{h}{6}(f_0 + 2f_1 + 2f_2 + f_3)$$

$$f_0 = \begin{pmatrix} 1 \\ 1 - 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$f_1 = f\begin{pmatrix} 1 + 1/4(1) \\ 0 + 1/4(1) \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.25 \end{pmatrix}$$

$$f_2 = f\begin{pmatrix} 1 + 1/4 \cdot 0.25 \\ 0 + 1/4 \cdot 0.25 \end{pmatrix} = \begin{pmatrix} 1.0625 \\ 0.0625 \end{pmatrix}$$

$$f_3 = f\begin{pmatrix} 1 + 1.0625 \\ 0 + 0.0625 \end{pmatrix} = \begin{pmatrix} 2.0625 \\ 0.0625 \end{pmatrix}$$

$$y_1 = \begin{pmatrix} 1 + 1/12(1 + 2(0.25) + 2(1.0625) + 2.0625) = 1.47 \\ 0 + 1/12(1 + 2(0.25) + 2(0.0625) + 0.0625) = 0.14 \end{pmatrix}$$

Should have Been Reverse... messed up early on calcs Fixed later ...

$$f_0 = \begin{pmatrix} v \\ v-4y \end{pmatrix} = \begin{pmatrix} 1 \\ 1-0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$f_1 = \begin{pmatrix} v = 1 + 1/4(1) \\ y = 0 + 1/4(1) \end{pmatrix} = \begin{pmatrix} 1.25 \\ 0.25 \end{pmatrix} \to \begin{pmatrix} 1.25 \\ 1.25-(4)1/4 \end{pmatrix} = \begin{pmatrix} 1.25 \\ 1.25-1 \end{pmatrix} = \begin{pmatrix} 1.25 \\ 0.25 \end{pmatrix}$$

$$f_2 = \begin{pmatrix} v = 1 + 1/4(1.25) \\ y = 0 + 1/4(0.25) \end{pmatrix} = \begin{pmatrix} 1.3125 \\ 0.0625 \end{pmatrix} \to \begin{pmatrix} 1.31 \\ 1.31-(4)0.0625 \end{pmatrix} = \begin{pmatrix} 1.31 \\ 1.06 \end{pmatrix}$$

$$f_3 = \begin{pmatrix} 0 + 1.31 \\ 1 + 1.06 \end{pmatrix} = \begin{pmatrix} 1.31 \\ 2.06 \end{pmatrix}$$

$$y_1 = \begin{pmatrix} 0 + 1/12(1 + 2(1.25) + 2(1.31) + 1.31) \\ 1 + 1/12(1 + 2(0.25) + 2(1.06) + 2.06) \end{pmatrix} = \begin{pmatrix} 1.62 \\ 0.47 \end{pmatrix}$$

$$f_0 = \begin{pmatrix} v \\ v-4y \end{pmatrix} = \begin{pmatrix} 0.47 \\ 0.47-4(1.62) \end{pmatrix} = \begin{pmatrix} 0.47 \\ -6 \end{pmatrix}$$

$$f_1 = \begin{pmatrix} 1.62 + 1/4(0.47) \\ 0.47 + 1/4(-6) \end{pmatrix} = \begin{pmatrix} 1.7375 \\ -1.03 \end{pmatrix}$$

$$f_2 = \begin{pmatrix} 1.62 + 1/4(1.7375) \\ 0.47 + 1/4(-1.03) \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.21 \end{pmatrix}$$

$$f_3 = \begin{pmatrix} 1.62 + 0.7 \\ 0.47 + 0.21 \end{pmatrix} = \begin{pmatrix} 2.32 \\ 0.68 \end{pmatrix}$$

$$y_1 = \begin{pmatrix} 1.62 + 1/12(0.47 + 2(1.7375) + 2(0.7) + 2.32) = 2.28 \\ 0.47 + 1/12(-6 + 2(-1.03) + 2(0.21) + 0.68) = -0.11 \end{pmatrix}$$

(3a) $A = (3\,7\,2\,1\,0\,8\,0\,1)$

$$g(n\Delta\omega) = \sum_{m=0}^{N-1} f(m\Delta t)\, e^{-i2\pi mn/N}$$

$$g(n\Delta\omega) = E + O$$

$E \downarrow EE + EO$, $O \downarrow OO + OE$

$$g(n\Delta\omega) = g_{even}(n\Delta\omega) + \underbrace{e^{-i2\pi n/N}}_{\omega} g_{odd}(n\Delta\omega)$$

$N = 8 \quad W_N^n$

$$f(0) + f(1)W_2^n + f(2)W_4^n + f(3)W_4^n W_2^n$$
$$+ f(5)W_2^n + f(6)W_4^n + f(7)W_4^n W_2^n$$

B/c

$A = (3\,1\,0\,0) \quad N = 4$

$$g(n\Delta\omega) = f(0) + f(1)W_2^n + f(2)W_4^n + f(3)W_4^n W_2^n$$

$n=0 \rightarrow 3 + 1 + 0 + 0 = \boxed{4}$ First is the sum of f

$n=1 \rightarrow 3 + 1e^{-i2\pi/4} + 0 + 0 \rightarrow \boxed{3+2i}$

$n=2 \rightarrow 3 + 1e^{-i2\pi} + 0 + 0 \rightarrow \boxed{3-i}$

$n=3 \rightarrow 3 + 1e^{-i3\pi/2} + 0 + 0 \rightarrow \boxed{3-2i}$

(4a)

$$u_t = u_x$$

$$u_x = \frac{u_{i+1}^{j} - u_{i-1}^{j}}{2\Delta x}$$ - grid space

$$u_t = \frac{u_i^{j+1} - u_i^{j}}{\Delta t}$$ - time space

Next - current / time step

(B)

$u_0^j = 0$; $u_4^j = 1$; $u_i^0 = 0$ for all $i \neq 4$

$\Delta t = 0.1$ $\Delta x = 0.2$ $2\Delta x =$ ~~0.04~~ 0.4

Since $u_0^j = 0$ and $u_j^0 = 0$

t	$u_{i=0}$	$u_{i=1}$	$u_{i=2}$	$u_{i=3}$	$u_{i=4}$
t=0.0	0	0	0	0	1
t=0.1	0	0	0	0.25	1
t=0.2	0	0	0	0.5	1

$$\frac{0 - 0}{0.04} = \frac{0 - 0}{0.1} = 0$$

$$\frac{1 - u_{i-1}^{j}}{2\Delta x} = \frac{u_i^{j+1} - u_i^{j}}{\Delta t}$$

$$\frac{0 - 0}{0.04} = \frac{u_i^{j+1} - 0}{0.1}$$

$$\cancel{= \frac{1-1}{0.1}}$$

$$\frac{u_3^1 - 0}{0.1} = \frac{1-0}{0.4} \quad u_3^1 = 0.25 \quad \frac{u_2^1 - 0}{0.1} = \frac{0-0}{0.4}$$

$$\frac{u_3^2 - 0.25}{0.1} = \frac{1-0}{0.4}$$

(5a)

$$f(t) = a_1 e^{a_2 t} + a_3 + a_4 t$$

$$S = \sum (y_i - (a_1 e^{a_2 t} + a_3 + a_4 t))^2$$

$$\frac{\partial S}{\partial a_2} = \sum 2(y_i - (a_1 e^{a_2 t} + a_3 + a_4 t))(a_1 t e^{a_2 t})$$

Does not stay linear

(B)

$$F(z) = \frac{\pi \mu_0}{4} k^2 \left[\frac{1}{z^2} + \frac{1}{(z+a_1)^2} - \frac{2}{(z+a_2)^2} \right]$$

$$S = \sum (y_i - (F(z)))^2 \qquad \frac{2}{(z+a_2)^2} \to 2(z+a_2)^{-2} \to \frac{-4}{z}$$

$$\frac{\partial S}{\partial a_2} = \sum 2(y_i - (F(z)))\left(-\frac{4}{z}\right)$$

Would be linear

(c)

$$f(t) = \frac{k}{1 + e^{-r(t-t_0)}}$$

$$S = \sum (y_i - (F(t)))^2 \to \frac{\partial S}{\partial k} = \sum 2(y_i - (f(t)))\left(\frac{1}{1+e^{-r(t-t_0)}}\right)$$

Would not remain linear