EXAM 1

- Monday, Oct. 12
  - Interpolation/Curve Fitting
  - Monte Carlo
  - ODEs

Today's Learning goals

1. 2-D and more.
2. Second order ordinary differential equations

- Most physics problems involve *multiple dependent* variables

$$\frac{d\boldsymbol{p}}{dt} = m\frac{d\boldsymbol{v}}{dt}$$

← Three equations which may be coupled

- Thus we need to study how *systems of ODEs* can be solve numerically.

- It turns out we can handle *systems of ODEs* pretty straight-forwardly

1. Write the dependent variables as a vector, $\vec{\boldsymbol{S}} = \begin{pmatrix} x \\ y \end{pmatrix}$
2. Do the same with RHS of the ODEs so that we have $\vec{\boldsymbol{F}} = \begin{pmatrix} f(x,y) \\ g(x,y) \end{pmatrix}$
3. The *system of ODEs* can now be written as $\dfrac{d\vec{\boldsymbol{S}}}{dt} = \vec{\boldsymbol{F}}(x,y).$

Rather then sending over the variable *y* to function **derivs**, one sends the vector ***S*** so that the function looks like

```
function [der] = derivs(t,S)
    x = S(1);
    y = S(2);
    der = [f(x,y); g(x,y)]
end
```

Second order ODEs occur often in physics. For example, Newton's second law is a second order ODE since the derivative of the position, *x*, with respect to time, *t*, appears at second order.

$$\frac{d^2x}{dt^2} = \frac{\boldsymbol{F}}{m}$$

At first glance, *Runge-Kutta methods* would **not** seem to be applicable to these kinds of ODEs. And indeed, they are not, or at least, not directly. But if one is clever, there are ways to apply *Runge-Kutta methods* to second order ODEs

So, let's get clever, do worksheet questions 1 and 2 then S T O P

(1)

$$\begin{aligned}\frac{d\boldsymbol{Y}}{dt} &= \boldsymbol{Z}\\ \frac{d\boldsymbol{Z}}{dt} &= f(t, \boldsymbol{Y})\end{aligned}$$

Let

(2)

$$\boldsymbol{S} = \begin{pmatrix}\boldsymbol{Z}\\ \boldsymbol{Y}\end{pmatrix}$$

where

$$\boldsymbol{F} = \begin{pmatrix}\boldsymbol{Z}\\ f(t, \boldsymbol{Y},)\end{pmatrix}$$

then the system of first order ODEs can be written as a *single* vector equation,

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{F}(t, \boldsymbol{S})$$

The last expression *looks* just like a first order ODE. This gives us the hint on how to solve second order ODES with *Runge-Kutta*. We replace the original scalar variable, *y*, with a vector variable ***S*** and use our *method twice*. Let's put this into practice, do question 3 on the Worksheet.

(3) The code is virtually unchanged from the code we used for multiple independent variables.

We are now ready to proceed to second order ODEs in multiple dimensions.
Do worksheet questions 4--6 and S T O P

(4) Let

$$\boldsymbol{S} = \begin{pmatrix} \boldsymbol{Z} \\ \boldsymbol{Y} \\ \boldsymbol{X} \\ \boldsymbol{W} \end{pmatrix}$$

where

$$\boldsymbol{Z} = \frac{d\boldsymbol{Y}}{dt}; \quad \boldsymbol{W} = \frac{d\boldsymbol{X}}{dt}$$

$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{Z} \\ f(t, \boldsymbol{Y}, \boldsymbol{X}) \\ \boldsymbol{X} \\ g(t, \boldsymbol{X}, \boldsymbol{Y}) \end{pmatrix}$$

then the system of first order ODEs can be written as a *single* vector equation,

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{F}(t, \boldsymbol{S})$$