(1) Consider the unit vector $\hat{\mathrm{n}} = \sin\theta\,\hat{\mathrm{i}} + \cos\theta\,\hat{\mathrm{k}}$ and its associated spin operator

$$S_{\hat{\mathrm{n}}} = \vec{S}\cdot\hat{\mathrm{n}} = \sin\theta\,S_x + \cos\theta\,S_z.$$

We'll work with spin-1/2 for the time being.

(a) What physical quantity does $S_{\hat{\mathrm{n}}}$ represent?

(b) Write down the representation of $S_{\hat{\mathrm{n}}}$ in the z-state basis, $\{|+_z\rangle,\ |-_z\rangle\}$.

(c) Find the eigenvalues and eigenvectors of $S_{\hat{\mathrm{n}}}$. (Call them $|+_{\hat{\mathrm{n}}}\rangle$ and $|-_{\hat{\mathrm{n}}}\rangle$.) Again, express your answers in the z-state basis.

(d) If the state of the system is $|+_x\rangle$ and you measured the physical quantity associated with $S_{\hat{\mathrm{n}}}$, what values could you measure and with what probabilities?