We developed the Friedmann equation and studied a series of possible universes:

- Empty universe
- Single component universe
- Multi-component universe
- Bench mark model

Do question (1a) on the worksheet and S T O P

(1a) $$\frac{\dot{a}^2}{a^2} = H_o^2\left[\frac{\Omega_{r,o}}{a^4} + \frac{\Omega_{m,o}}{a^3} + \Omega_\Lambda + \frac{1-\Omega_o}{a^2}\right]$$

We can measure $\Omega_{r,o},\ \Omega_{m,o},\ \Omega_\Lambda,\ H_o$ but we cannot directly measure a_o

Do question (1b) on the worksheet and S T O P

(1b) Distance. Much of observational cosmology has to do with getting accurate distance measurements.

$$\frac{\dot{a}^2}{a^2} = H_o^2\left[\frac{\Omega_{r,o}}{a^4} + \frac{\Omega_{m,o}}{a^3} + \Omega_\Lambda + \frac{1-\Omega_o}{a^2}\right]$$

Measuring these only gives *a(t)* if we assume the Friedmann equation is correct.

To confirm the Friedmann equation we need to measure *a(t)* independently and see if the L.H.S = R.H.S

We now do what any good (and bad) physicists does when confronted with the unknown… We Taylor expand.

$$a(t) = a(t_o) + (t - t_o)\left.\frac{da}{dt}\right|_{t=t_o} + \frac{(t-t_o)^2}{2}\left.\frac{d^2a}{dt^2}\right|_{t=t_o} + \cdots$$

Do question (2) on the worksheet and S T O P

(2a) $$\frac{a(t)}{a(t_o)} = 1 + H_o\,(t-t_o) + \frac{1}{2}\frac{\ddot{a}(t)}{a(t_o)}\,(t-t_o)^2 + \cdots$$

(2b) $$\frac{a(t)}{a(t_o)} = 1 + H_o\,(t-t_o) - \frac{q_o}{2}H_o^2\,(t-t_o)^2 + \cdots$$

(2c)
$$\begin{aligned}
\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3c^2}\sum_w \epsilon_w(1+3w) \\
-\frac{\ddot{a}}{aH^2} &= \frac{1}{2}\sum_2 \Omega_w(1+3w) \\
q_o &= \frac{1}{2}\sum_w \Omega_{w,o}(1+3w)
\end{aligned}$$

(2d) $$q_o = \frac{1}{2}\Omega_{m,o}$$

Notice, that if we could measure q_o, this gives us a way of finding Ω_m, without actually having to weigh all the stuff in the universe.

Do question 3

(3a) $w_r = 1/3, w_m = 0, w_\Lambda = -1 \Rightarrow q_o = \Omega_{r,o} + \frac{1}{2}\Omega_{m,o} - \Omega_\Lambda$

(3b) $q_o < 0$ and the universe is accelerating. Benchmark model has $q_o = -0.55$

Recall that $\frac{a(t)}{a(t_o)} = 1 + H_o\,(t - t_o) - \frac{q_o}{2} H_o^2\,(t - t_o)^2 + \cdots$ so to measure a, we just need to measure q_o and H_o

We've also just seen that to measure q_o, we need to measure the critical densities…this is hard !

Oh well, let's table that for now, and see what we can do with H_o

Recall that $c\,z = H_o\,d$. Ok, well z is easy to measure, so we're almost there? What about d?

$$d_p(t_o) = c \int_{t_e}^{t_o} \frac{dt}{a(t)}$$

But this is no good, it's $a(t)$ we need find!...crap.

We now do what any good (and bad) physicists does when confronted with the unknown… We Taylor expand.

$$\frac{1}{a(t)} \approx 1 - H_o(t - t_o) + \frac{1 + q_o}{2} H_o^2 (t - t_o) + \cdots$$

Do question (4) on the worksheet and S T O P

(4) The integral is straightforward to do since it's a polynomial. To second order with the limits inserted it yields,

$$d_p \approx c\,(t_o - t_e) + \frac{cH_o}{2}\,(t_o - t_e)^2$$

However this doesn't get us any closer because we don't know what $t_o - t_e$ is. Crap. What now?

We now do what any good (and bad) physicists does when confronted with the unknown... We Taylor expand.

$$\begin{aligned} z &= \frac{1}{a(t_e)} - 1 \quad \text{Taylor expanding} \\ z &\approx H_o\,(t_o - t_e) + \left(\frac{1+q_o}{2}\right) H_o^2\,(t_o - t_e))^2 \,\text{solving for the time} \\ t_o - t_e &\approx \frac{1}{H_o}\left[z - \left(\frac{1+q_o}{2}\right) z^2\right] \end{aligned}$$

Substituting into the proper distance we get that $d_p \approx \frac{c}{H_o} z \left[1 - \frac{1+q_o}{2} z\right]$ and crap, we need H_o and q_o to find d_p.

But we wanted d_p so we could find H_o !

Do question (5) on the worksheet and S T O P

Taylor expanding has not completely worked. Time to try something else. Recall that we seek a distance measurement we can use to determine, at the very minimum H_o.

Maybe we can use some other type of distance ?

- **Proper distance** roughly corresponds to where a distant object would be at a specific moment of cosmological time, which can change over time due to the expansion of the universe.
- **Comoving distance** between fundamental observers does not change with time, as comoving distance accounts for the expansion of the universe.
- **Luminosity distance** is the distance associated with the amount of flux one measures from a distant object
- **Transverse comoving distance**
- **Angular diameter distance**
- **Light Travel distance**

Well at least we have choices. Let’s first look at the *luminosity distance.*

Luminosity distance, components

- *L*, luminosity of a known *standard candle.*
  - *Standard candle* is a source whose luminosity, electromagnetic energy per time, is known.
- Flux, *f* , the luminosity per unit area
  - Bolometric flux is flux over all wavelengths. Instruments cannot detect all wavelengths so we do not typically measure the bolometric flux.
  - Unit Area is a geometric quantity that will change depending on the metric

$$f = \frac{L}{4\pi d_L^2}; \text{ or } d_L \equiv \left(\frac{L}{4\pi f}\right)^{1/2}$$

Surface area

$$A(t_o) = 4\pi S_k(r)^2$$

$$S_k(r) = \begin{cases} R_o \sin(r/R_o) & k = 1 \\ r & k = 0 \\ R_o \sinh(r/R_o) & k = -1 \end{cases}$$

We pick up $(1 + z)^2$ fall off in energy due to expansion of the universe (see chapter 3)

$$\boxed{d_L = S_k(r)(1+z)}$$

Currently we seem to have ***k = 0*** so $\boxed{d_L = r\,(1+z) = d_p(t_o)\,(1+z)}$ Hey, maybe some progress finally.

Do question (7) on the worksheet