Physics 460—Homework Report 3
Due Tuesday, Apr. 21, 1 pm **Name:** ____________________

Complete all the problems on the accompanying assignment.

List all the problems you worked on in the space below. Circle the ones you fully completed:

Please place the problems into the following categories:

- These problems helped me understand the concepts better: ____________________
- I found these problems fairly easy: ____________________
- I found these problems very challenging: ____________________

In the space below, show your work (even if not complete) for any problems you still have questions about. Indicate where in your work the question(s) arose, and ask specific questions that I can answer.

Use the back of this sheet or attach additional paper, if necessary.

If you have no remaining questions about this homework assignment, use this space for one of the following:

- Write one or two of your solutions here so that I can give you feedback on its clarity.
- Explain how you checked that your work is correct.

(1) Suppose that the initial wave function for a particle in a cube is

$$\psi_0(x,y,z) = -Axyz(x-L)(y-L)(z-L),$$

where A is a normalization constant.

(a) Find A.

(b) The wave function can be expressed in terms of the energy eigenstates ψ_{n_x,n_y,n_z} as

$$\psi_0 = c_{n_x,n_y,n_z}\psi_{n_x,n_y,n_z}.$$

Find the coefficients c_{n_x,n_y,n_z}.

(c) If you measure the energy of the particle, what is the probability of obtaining the result $E = 11\hbar^2\pi^2/(2mL^2)$? What is the probability of obtaining the result $E = 27\hbar^2\pi^2/(2mL^2)$? Your answers should be numbers!

(2) Consider a particle in a cube of sides L in initial state

$$\psi_0(x,y,z) = \begin{cases} A, & 0 \le x \le L,\ 0 \le y \le L/4,\ 0 \le z \le L/4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the normalization constant A.

(b) If you measure the energy of the particle, what are the three most likely results of this measurement, and what is the probability of each of these results?

(3) The Hamiltonian for the isotropic harmonic oscillator in three dimensions in Cartesian coordinates is

$$H = \frac{{P_x}^2 + {P_y}^2 + {P_z}^2}{2m} + \frac{k\,(X^2+Y^2+Z^2)}{2}.$$

This is an isotropic oscillator because there is a single value of the spring constant k for the springs in all three directions.

(a) Show that the wave functions for the energy eigenstates can be written in the form

$$\psi_n(x,y,z) = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z),$$

where $\psi_{n_x}(x)$ *etc.* are the energy eigenstates for the one-dimensional harmonic oscillator.

(b) Show that the energy eigenvalues for these states are

$$E_n = \left(n + \frac{3}{2}\right)\hbar\omega_0,$$

where $n = n_x + n_y + n_z$, and n_x, n_y, and n_z can take on the values 0, 1, 2, etc.

(c) Write out explicitly the energy eigenstates for the four states $(n_x,n_y,n_z) = (0,0,0)$, $(n_x,n_y,n_z) = (1,0,0)$, $(n_x,n_y,n_z) = (0,1,0)$, and $(n_x,n_y,n_z) = (0,0,1)$. The relevant states for the one-dimensional oscillator are

$$\psi_0(x) = \frac{e^{-x^2/2{d_0}^2}}{\sqrt{\sqrt{\pi}d_0}}, \quad \text{and} \quad \psi_1(x) = \sqrt{\frac{2}{\sqrt{\pi}{d_0}^3}}\,x e^{-x^2/2{d_0}^2}, \quad \text{where} \quad d_0 = \sqrt{\frac{\hbar}{m\omega_0}}.$$

(d) Explain how that you know that the angular momentum eigenstates, $|l,m\rangle$, are also eigenstates of the Hamiltonian.

(e) Express your four states from part (c) in spherical coordinates. Then express the angular dependence in terms of the spherical harmonics.

(f) Using your results from part (e), if you measured the z component of the angular momentum of each of the four states from part (c), what values could you obtain, and with what probabilities?

Homework 3

(1) $\Psi_0(x, y, z) = A\,xyz\,(x-L)(y-L)(z-L)$,

(a) Find A,

$$\int |\Psi_0(x)|^2 dx = 1 = A^2 \int_0^L x^2 y^2 z^2 (x-L)^2 (y-L)^2 (z-L)^2$$

$$= A^2 \left[\int_0^L x^2 (x-L)^2 dx\right]^3 = A^2 \left(\frac{L^5}{30}\right)^3 = A^2 \left(\frac{L^{15}}{27000}\right)$$

$$\sqrt{\frac{27000}{L^{15}}} = A \Rightarrow A = \frac{30\sqrt{30}}{\sqrt{L^{15}}}$$

$$\Psi_0 = \frac{30\sqrt{30}}{\sqrt{L^{15}}} xyz(x-L)(y-L)(z-L)$$

(B) $\Psi_0 = C_{n_x,n_y,n_z} \Psi_{n_x,n_y,n_z}$ Find C_{n_x,n_y,n_z}

$$C_{n_x,n_y,n_z} = \langle n_x, n_y, n_z | \Psi \rangle$$

$$\Psi(x,y,z) = \sum_{n_x,n_y,n_z} \underset{\uparrow \langle \Psi_n | \Psi \rangle}{C_{n_x,n_y,n_z}} \Psi_{n_x,n_y,n_z}(x,y,z)$$

$$C_{n_x,n_y,n_z} = \langle \Psi_n | \Psi \rangle$$

$$\frac{30\sqrt{30}}{\sqrt{L^{15}}} \int_0^L (x(x-L)\,n_x)^3 dx$$

(C)

(2)
(a) $\Psi_0(x,y,z) = \begin{cases} A, & 0 \le x \le L,\ 0 \le y, L/4,\ 0 \le z \le L/4 \\ 0, & \text{otherwise} \end{cases}$

$$\int |\Psi_0(x)|^2 = 1 \rightarrow A^2 \int_0^L dx \int_0^{L/4} dy \int_0^{L/4} dz$$

$$\rightarrow A^2 \Big[x\Big]_0^L \Big[y\Big]_0^{L/4} \Big[z\Big]_0^{L/4} = 1 \rightarrow A^2 [L] \left[\frac{L}{4}\right]\left[\frac{L}{4}\right]$$

$$\rightarrow A^2 \frac{L^3}{16} = 1 \rightarrow A^2 = \frac{16}{L^3} \Rightarrow A = \frac{4}{L^{3/2}}$$

$$\Psi_0(x,y,z) = \begin{cases} 4/L^{3/2}, & 0 \le x \le L,\ 0 \le y, L/4,\ 0 \le z \le L/4 \\ 0, & \text{otherwise} \end{cases}$$

(B) $\cancel{E_{111} = \frac{3\pi \hbar^2}{2mL^2}} \rightarrow \cancel{E_{211} = E_{121} = E_{112} = \frac{6\pi^2\hbar}{2mL^2}}$

$$\langle \Psi_n | \Psi \rangle = \frac{4}{L^{3/2}} \int_0^L n_x dx \int_0^{L/4} n_y dy \int_0^{L/4} n_z dz$$

$$= \frac{4}{L^{3/2}} \left[n_x L + \frac{n_y L}{4} + \frac{n_z L}{4} \right]$$

$$P = |\langle \Psi_n | \Psi \rangle|^2$$

$\vdots$

(3)

$$H = \frac{P_x^2 + P_y^2 + P_z^2}{2m} + \frac{k(x^2 + y^2 + z^2)}{2}$$

(a)

(B)

$$E_n = \left(n + \frac{3}{2}\right)\hbar\omega_0 = \hbar\omega_0\left(n_x + n_y + n_z + \frac{3}{2}\right)$$

(C) $\Psi_{000} = \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-m\omega r^2/2\hbar}$

$\Psi_{100} = \sqrt{\frac{2m\omega}{\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-m\omega r^2/2\hbar}$

$\Psi_{010} = \sqrt{\frac{2m\omega}{\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-m\omega r^2/2\hbar}$

$\Psi_{001} = \sqrt{\frac{2m\omega}{\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-m\omega r^2/2\hbar}$

(D)

$H\Psi_{100} = \frac{3}{2}\hbar\omega\Psi_{000}$

$H\Psi_{100} = \frac{5}{2}\hbar\omega\Psi_{100}$

$H\Psi_{010} = \frac{5}{2}\hbar\omega\Psi_{010}$

$H\Psi_{001} = \frac{5}{2}\hbar\omega\Psi_{001}$