(1) The Hamiltonian for negatively-charged particle in a magnetic field is $H = -\gamma \vec{S} \cdot \vec{B}$. Section 1.6 of the course notes discusses the precession of a negatively-charged spin-1/2 particle when the magnetic field is $\vec{B} = B_0\,\hat{\text{k}}$. In this activity, you'll explore the precession of a negatively-charged spin-1 particle under the same conditions.

(a) Consider a negatively-charged spin-1 particle in state

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\Big(|{+1_z}\rangle - |{-1_z}\rangle\Big).$$

Find the time evolution of this state if the magnetic field is $\vec{B} = B_0\,\hat{\text{k}}$. Express your answer in terms of the Larmor frequency $\Omega_0 = |\gamma B_0|$.

(b) Calculate $\mathcal{P}_{+1_x}(t)$ for this particle (the probability that a measurement of spin along the x axis have a result of $+\hbar$).

(c) Suppose instead that the initial state of the particle is

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\Big(|{+1_z}\rangle + |{-1_z}\rangle\Big).$$

Find the time evolution of this state given the same magnetic field, and calculate $\mathcal{P}_{+1_x}(t)$ for this state.

(d) Suppose instead that the initial state of the particle is

$$|\Psi_3\rangle = \frac{1}{\sqrt{3}}\Big(|{+1_z}\rangle + |0_z\rangle + |{-1_z}\rangle\Big).$$

Find the time evolution of this state given the same magnetic field, and calculate $\mathcal{P}_{+1_x}(t)$ for this state.

(e) Compare your answers for $\mathcal{P}_{+1_x}(t)$ for these three states. How are they similar? How do they differ?

$$H = -\gamma B_0 S_z \qquad U(t) = e^{-iHt/\hbar}$$

$$S_z|{+1_z}\rangle = \hbar|{+1_z}\rangle, \quad S_z|0_z\rangle = 0|0_z\rangle$$

$$S_z|{-1_z}\rangle = -\hbar|{-1_z}\rangle$$

(a)

$$|\psi(t)\rangle = \frac{e^{+i\Omega_0 t}}{\sqrt{2}}|{+1_z}\rangle - \frac{e^{-i\Omega_0 t}}{\sqrt{2}}|{-1_z}\rangle$$

$$|\psi(t)\rangle \leftrightarrow \frac{e^{+i\Omega_0 t}}{\sqrt{2}}\begin{bmatrix}1\\0\\0\end{bmatrix} - \frac{e^{-i\Omega_0 t}}{\sqrt{2}}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$|\psi(t)\rangle \leftrightarrow \frac{1}{\sqrt{2}}\begin{bmatrix}e^{+i\Omega_0 t}\\0\\-e^{-i\Omega_0 t}\end{bmatrix}$$

(b) $\mathcal{P}(+1_x) = |\langle +1_x | \psi(t)\rangle|^2$

$$= \left| \frac{1}{2}\begin{bmatrix} 1 & \sqrt{2} & 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} e^{+i\Omega_0 t} \\ 0 \\ -e^{-i\Omega_0 t} \end{bmatrix} \right|^2$$

$$= \frac{1}{8} \left| e^{+i\Omega_0 t} - e^{-i\Omega_0 t} \right|^2$$

$$= \frac{1}{8} \left| +2i\sin\Omega_0 t \right|^2 = \frac{1}{2}\sin^2\Omega_0 t$$

(c) Same but with a + in third row

$$|\psi(t)\rangle \leftrightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} e^{+i\Omega_0 t} \\ 0 \\ e^{-i\Omega_0 t} \end{bmatrix}$$

$$\Rightarrow \mathcal{P}(+1_x) = \frac{1}{8} \left| e^{+i\Omega_0 t} + e^{-i\Omega_0 t} \right|^2$$

$$= \frac{1}{8} \left| 2\cos\Omega_0 t \right|^2 = \frac{1}{2}\cos^2\Omega_0 t$$

(d)

$$|\psi(t)\rangle \leftrightarrow \frac{1}{\sqrt{3}} \begin{bmatrix} e^{i\Omega_0 t} \\ 1 \\ e^{-i\Omega_0 t} \end{bmatrix}$$

$$\mathcal{P}(+1_x) = \left| \frac{1}{2}\begin{bmatrix} 1 & \sqrt{2} & 1 \end{bmatrix} \frac{1}{\sqrt{3}} \begin{bmatrix} e^{i\Omega_0 t} \\ 1 \\ e^{-i\Omega_0 t} \end{bmatrix} \right|^2$$

$$= \frac{1}{12} \left| e^{i\Omega_0 t} + \sqrt{2} + e^{-i\Omega_0 t} \right|^2$$

$$P(+1_x) = \frac{1}{12}\left(2\cos\Omega_0 t + \sqrt{2}\right)^2$$

$$= \frac{1}{12}\left(2 + 2\sqrt{2}\cos\Omega_0 t + 4\cos^2\Omega_0 t\right)$$