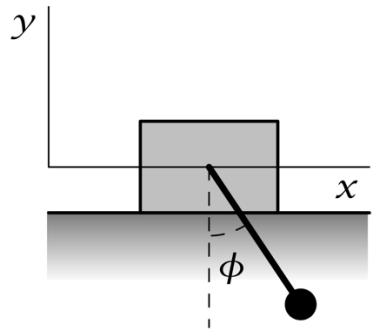
Activity 6: Horizontal Block and Pendulum

1. A block of mass *M* slides along a frictionless horizontal surface as shown below. Hanging from the block is a pendulum of length l and mass m.



a. Using the $x - y$ axes shown, let the position of the block be (x_1, y_1) and the position of the pendulum bob be (x_2, y_2). Write down the kinetic and potential energy for the block and the pendulum.
b. Expressing the position of the pendulum in terms of the angle shown, find the Lagrangian and the equations of motion for the system.
c. You should find that there is a cyclic coordinate. Find the corresponding constant of the motion. What does this constant represent in terms of the physics of the system?
d. Let the constant from part (3) be called P. Use P to eliminate $\dot{x}$ from the equation of motion and show that the angle of the pendulum motion is determined by the equation
$\ddot{\phi} = \left(\frac{g}{l}\sin\phi + \frac{\alpha}{2}\sin(2\phi)\,\dot{\phi}^2\right)/(1 - \alpha\cos^2\phi)$
where $\alpha = \frac{m}{m+M}$
e. What happens with the equation of motion for ϕ if $m \ll M$? Provide a physical explanation for this.