An object of mass m moving in two dimensions in a potential

$$V(r) = \frac{k}{n+1} r^{n+1}$$

has the following Lagrangian in polar coordinates:

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$$

If $n = -2$ this potential corresponds to the gravitational or Coulomb potential, while if $n = 1$ it corresponds to the two-dimensional simple harmonic oscillator.

a. Find the conjugate momenta and the Hamiltonian.
b. The angle θ is an ignorable coordinate in the Hamiltonian, so its conjugate momentum, p_θ, is a constant of the motion, and we can replace it with a constant, $\mathcal{L}$, in the Hamiltonian. This reduces the problem to one degree of freedom.
c. Plot phase space trajectories for $n = -2$ (a gravitational or Coulomb potential). Try both negative and positive values for the Hamiltonian. Also try different values for $\mathcal{L}$.
d. From the Hamiltonian, find the equations of motion for (r, p_r).
e. Find the equilibrium solutions for (r, p_r). What shape of trajectory do these equilibria correspond to?