Physics 412—Projection Operators Winter 2020

A certain quantum mechanical operator A has eigenvalues a_1, a_2, and a_3, with corresponding eigenstates

$$|a_1\rangle \leftrightarrow \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ \mathrm{i}\\ 0\end{bmatrix}, \quad |a_2\rangle \leftrightarrow \frac{1}{\sqrt{3}}\begin{bmatrix}\mathrm{i}\\ 1\\ 1\end{bmatrix}, \quad \text{and} \quad |a_3\rangle \leftrightarrow \frac{1}{\sqrt{6}}\begin{bmatrix}\mathrm{i}\\ 1\\ -2\end{bmatrix}.$$

(1) Find the representations of the projection operators that correspond to measurements of a_1, a_2, and a_3.

(2) Verify that your projection operators sum to the identity matrix.

(3) If the state of the system is

$$|\Psi\rangle \leftrightarrow \frac{1}{2}\begin{bmatrix} i \\ 1 \\ 1-i \end{bmatrix},$$

use the appropriate projection operator to find

(a) the probability of obtaining each of the three possible values a_1, a_2, or a_3 if you measure A.

(b) the state of the system after the measurement.