Suppose universe had the following behavior $a(t) = \begin{cases} \sqrt{t/t_i} & t < t_i \\ ae^{H_i(t-t_i)} & t_i < t < t_f \\ ae^{H_i(t_f-t_i)}\sqrt{t/t_f} & t > t_f \end{cases}$ (pay attention to subscripts on t)

It is usual in studying inflation to compare the how much the scale factor changed between t_i and t_f by forming the ratio:

$$\frac{a(t_f)}{a(t_i)} = e^N; \text{ where } N \equiv H_i(t_f - t_i); \; H_i \equiv \sqrt{\frac{\Lambda}{3}}$$

N is called the number of *e*-foldings

As a review of what where we stopped Monday, do question (1) on the worksheet and S T O P

- This scenario describes a universe that up until time t_i is growing `` normally”,
- then between t_i and t_f undergoes exponential expansion
- After t_f, it resumes ``normal growth”

(1 b) $\epsilon_\Lambda = \dfrac{3c^2}{8\pi G} H_i^2 \approx 10^{105}\ \mathrm{TeV\ m^{-3}}$ (1 c) $|1 - \Omega(t_f)| = \mathrm{e}^{-2N}\,|1 - \Omega(t_i)|$ (1 d) $|1 - \Omega(t_f)| = \mathrm{e}^{-2N}$

The horizon problem. Recall that the horizon distance is given by the relation

$$d_{\mathrm{hor}}(t) = a(t) c \int_0^t \frac{dt'}{a(t')}$$

Do question (2a) on the worksheet and S T O P Do question (2b) and (2c) and S T O P

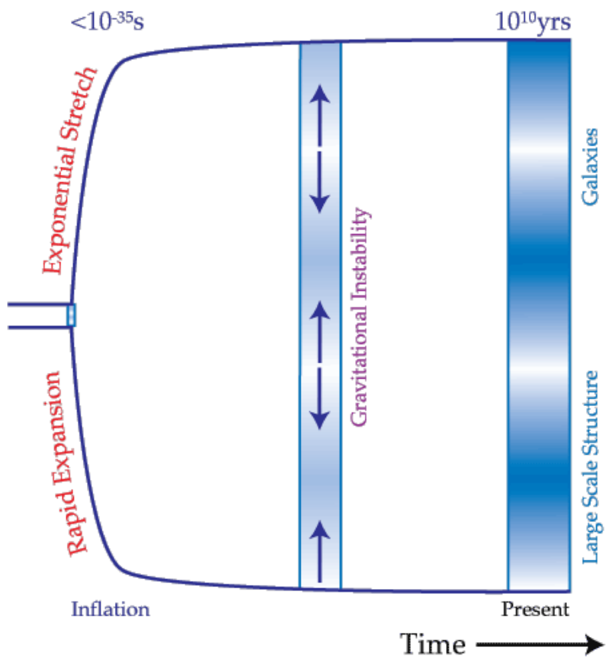
(2 b) $d_{\mathrm{hor}}(t_i) = a_i\, c \displaystyle\int_0^t \frac{dt'}{a_i \sqrt{t/t_i}} = 2ct_i$ (2c) $d_{\mathrm{hor}}(t_f) = a_i\, c\, e^N \displaystyle\int_0^{t_i} \frac{dt'}{a_i \sqrt{t/t_i}} + \int_{t_i}^{t_f} \frac{dt}{a_i \exp[H_i(t - t_i)]}$

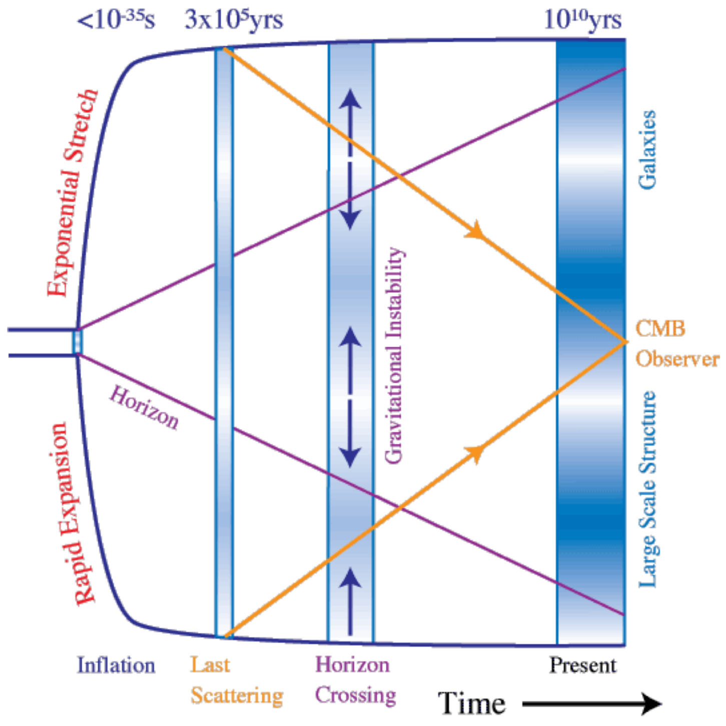
When *N* is large, then the integrals in (2c) yield $d_{\mathrm{hor}}(t_f) = e^N c \left(2t_i + H_i^{-1}\right)$

Finish question (2) on the worksheet and S T O P

$$d_{\mathrm{hor}}(t_i) = 6 \times 10^{-28}\,\mathrm{m} \qquad d_{\mathrm{hor}}(t_f) = 15\,\mathrm{m}$$

Do question (3) on the worksheet





The physics of inflation:

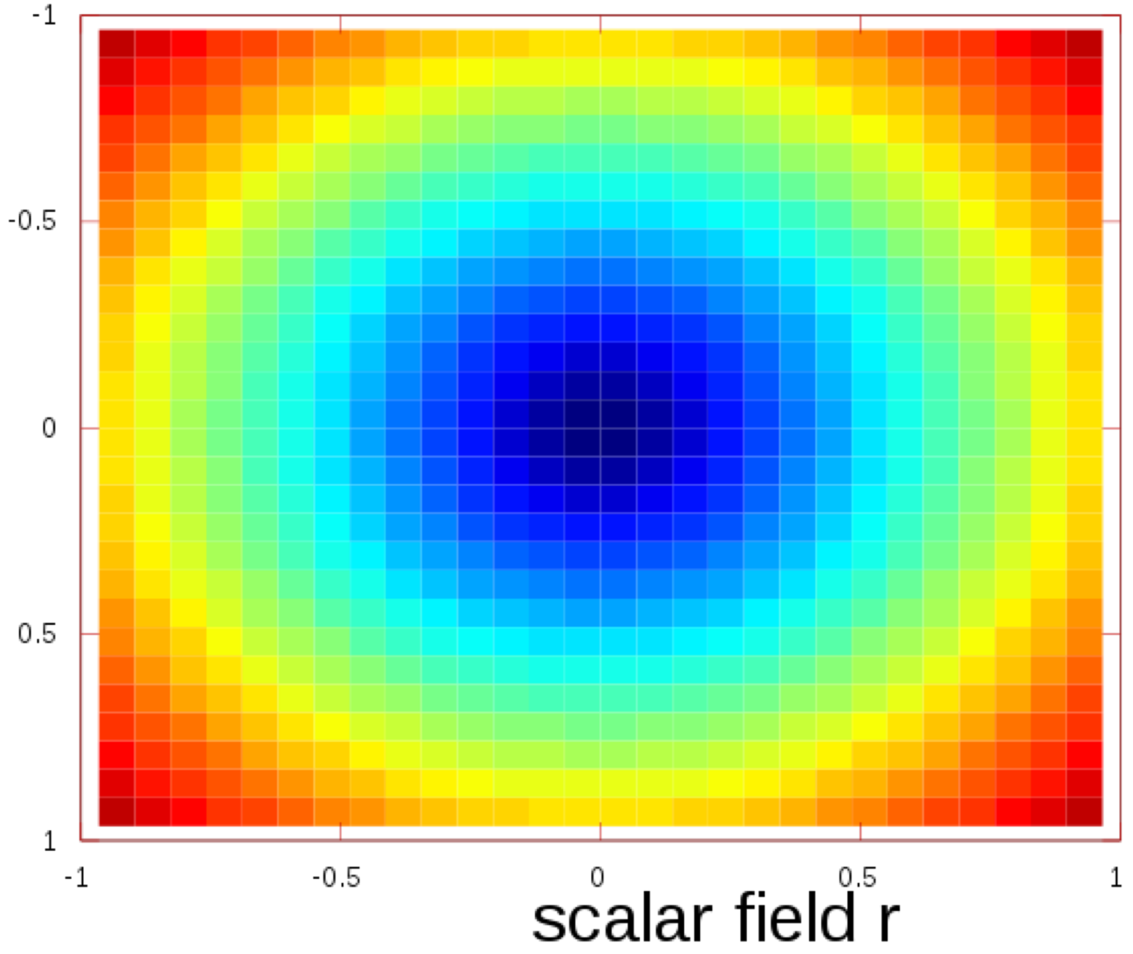
- There is no one satisfactory theory
- Many theories, at least in outline, will have the components we will explore
- An entire different class of theories exist typified by *vector fields.* We will not look into these at all.

We've seen that a universe that behaves like this: addresses the flatness and horizon problems

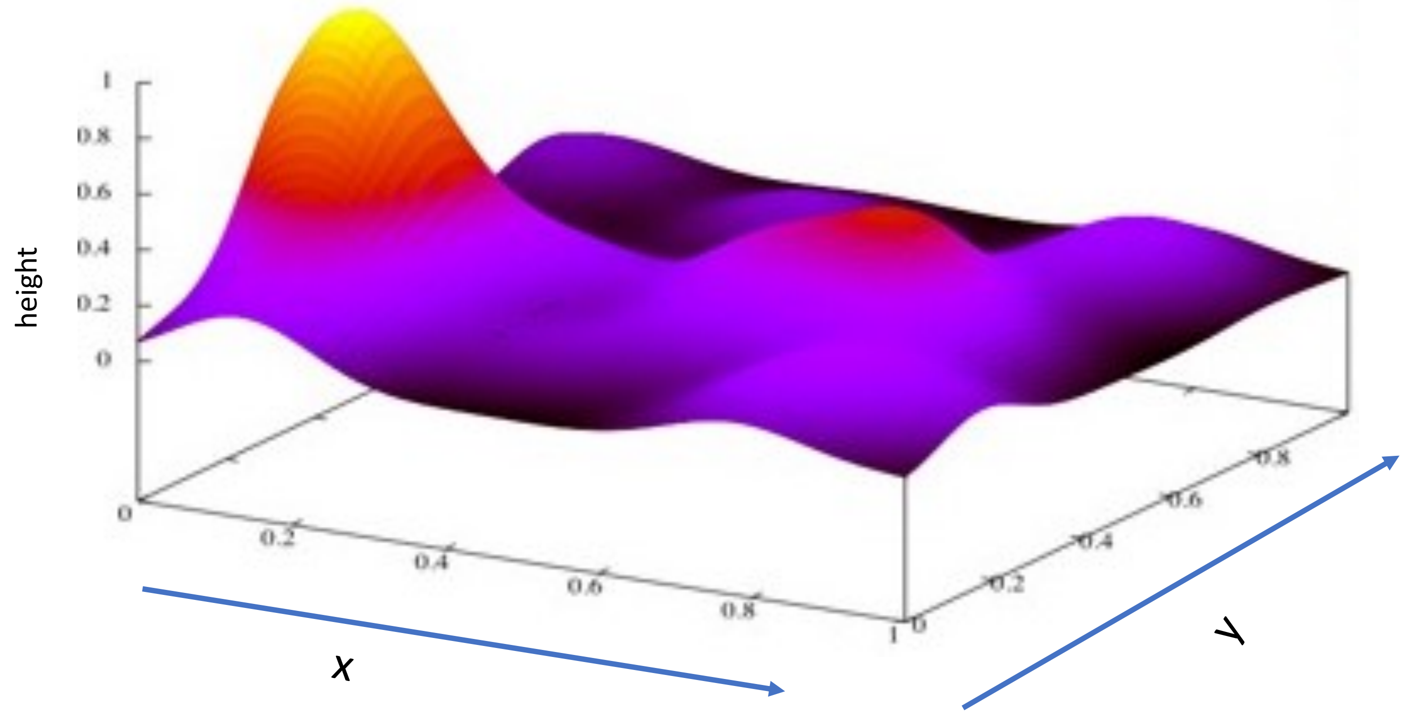
$$a(t) = \begin{cases} \sqrt{t/t_i} & t < t_i \\ a e^{H_i(t-t_i)} & t_i < t < t_f \\ a e^{H_i(t_f - t_i)} \sqrt{t/t_f} & t > t_f \end{cases}$$

The next question is, what possible physical process could cause the universe to behave like this?

Scalar fields.



Generally, scalar fields can have associated potential energies



The scalar field here is the height at position (*x,y*). Let's call this: $\phi(x, y, t)$

At each height, there is a gravitational potential given as $V(\phi) = g\phi$

Do question (4 a) on the worksheet and S T O P

Suppose there exists a scalar field and its associated potential labeled as $\phi;\ V(\phi)$ respectfully.

The energy density associated with this field is $\epsilon_\phi = \underbrace{\frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2}_{\text{kinetic}} + \underbrace{V(\phi)}_{\text{potential}}$ with $\underbrace{P_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi)}_{\text{From GR}}$

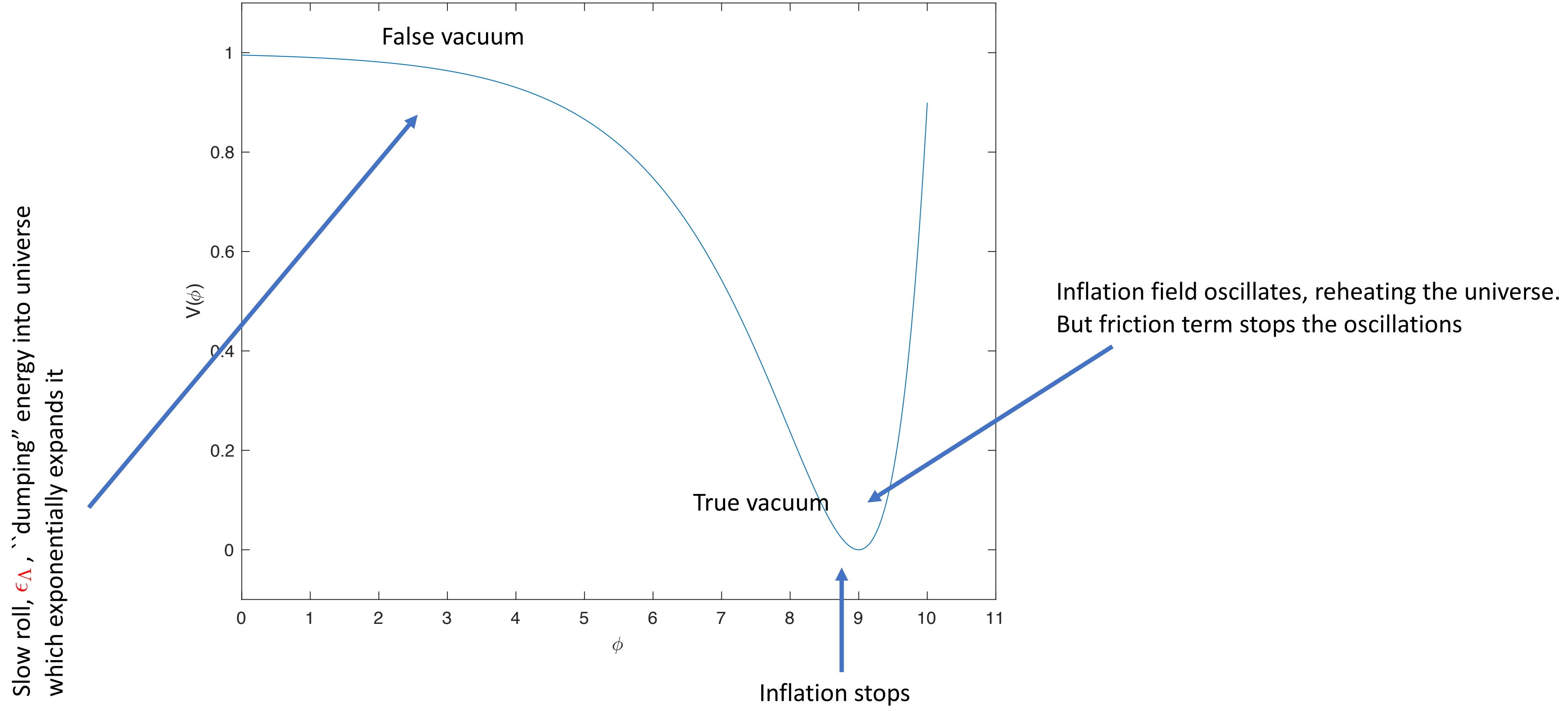
Do question (4 b and c) on the worksheet and S T O P.

(4b and c) $\underbrace{\ddot{\phi}}_{\text{acceleration}} + \underbrace{3H(t)\dot{\phi}}_{\text{friction}} = \underbrace{-\hbar c^3 \frac{dV}{d\phi}}_{\text{driving force}}$

Finish question (4) on the worksheet and S T O P

(4d) $3H\dot{\phi} = -\hbar c^3 \frac{dV}{d\phi}$

(4e) $\left(\frac{dV}{d\phi}\right)^2 \ll \frac{9H^2V}{\hbar c^3}$



Very much like a *phase transition* during which *latent heat* is released into the surroundings.

Do question (5) on the worksheet.