Homework 1

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Problem 1

Some calculation can be found in the appendix where some calculations in this problem were carried out in python. The Thomson scattering cross section is gave by

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^2$$

and the value we obtain is

$$\sigma_T = \frac{8\pi}{3}\left(\frac{(1.602\times10^{-19}C)^2}{4\cdot\pi\cdot 8.85\times10^{-12}Fm^{-1}\cdot 9.109\times10^{-31}kg\cdot 3.00\times10^{2}ms^{-1}}\right)^2 = 6.68\times10^{-29}m^2.$$

The photon scattering timescale is then given by

$$t_s = \frac{l}{c}, \text{ where } l = \frac{1}{n_e\sigma_T}\text{is the mean free path.}$$

In order to find the photon scattering timescale, the electron density of the sun needs to be found first. The electron density of the sun can be found by

$$n_e = \frac{\rho}{\mu_e m_u} = \frac{1.4\times10^3\ kg\ m^{-3}}{(1.5)\cdot 1.6\times10^{-27}kg} = 5.8\times10^{29}m^{-3}$$

Given the electron density, the mean free path is

$$l = \frac{1}{n_e \sigma_T} = \frac{1}{5.8 \times 10^{29}\ m^{-3} \cdot 6.68 \times 10^{-29}\ m^2} = 0.026\ m$$

Finally, the photon scattering timescale is

$$t_s = \frac{0.026\ m}{3.00 \times 10^8\ m\ s^{-1}} = 8.6 * 10^{-11} s$$

Problem 2

(a)

The equation to find the angular separation of two stars as seen from earth is given by

$$tan(p) = \frac{AU}{d} \rightarrow tan(p) = \frac{3 \times 10^{-4}\ pc}{6\ pc} \rightarrow p = tan^{-1}(5 \times 10^{-5}) = (3.2 \times 10^{-3})^{\circ}$$

converting this to arcseconds gives

$$(3.2 \times 10^{-3})^{\circ} \cdot \frac{60'}{1^{\circ}} \cdot \frac{60''}{1'} = 11.5''.$$

(b)

Given that the angle of separation is $6.2''$, this value in degrees is

$$6.2'' = \frac{1'}{60''} \cdot \frac{1^{\circ}}{60'} = (1.72 \times 10^{-3})^{\circ}.$$

The value of degrees can be converted into radians by

$$\frac{(1.72 \times 10^{-3})^{\circ}}{180^{\circ} \cdot \pi} = 3 \times 10^{-6}$$

Using the equation from part (a) we have

$$tan(3 \times 10^{-6}) = \frac{x}{6\ pc} \rightarrow x = tan(3 \times 10^{-6}) \cdot 6\ pc = 3.7AU$$

Problem 3

(a)

The absolute magnitude of star system β. The absolute magnitude M is gave by

$$m - M = 5\ log_{10}\ d - 5$$

where $m = 3.18$ and $d = 119.6\ pc$. Therefore, the absolute magnitude is

$$M = (3.18 - 5\ log_{10}\ 119.6 + 5)) = -2.21$$

(b)

The apparent magnitude m of a star is

$$m_1 - m_2 = -2.5 log\left(\frac{l_1}{l_2}\right)$$

where $m_1 = 3.18$ and $l_1 l_2 = 5.9$. Therefore,

$$m_2 = 2.5 log(5.9) + 3.18 = 5.1$$

Problem 4

The Lorentzian distribution is gave by

$$\phi(\nu) = \frac{\gamma_n/4\pi^2}{(\nu-\nu_0)^2 - (\gamma_n/4\pi)^2}.$$

To find if it normalized to unity, the integral has to be set up as

$$\int_0^\infty \phi(\nu)d\nu \to \int_0^\infty \frac{\gamma_n/4\pi^2}{(\nu-\nu_0)^2 - (\gamma_n/4\pi)^2}d\nu$$

For simplicity, let $\alpha = \gamma_n/4\pi$ so that the integral can be expressed as

$$\frac{\alpha}{\pi}\int_0^\infty \frac{1}{(\nu-\nu_0)^2 - \alpha^2}d\nu.$$

Evaluating the integral results in

$$\begin{aligned}
&= \frac{1}{\pi}\Bigg[-tan^{-1}\Big(\frac{\nu_0-\nu}{\alpha}\Big)\Bigg]_0^\infty \\
&= \frac{1}{\pi}\Bigg[-tan^{-1}\Big(\frac{4\pi(\nu_0-0)}{\gamma}\Big) + tan^{-1}\Big(\frac{4\pi(\nu_0+\infty)}{\gamma}\Big)\Bigg].
\end{aligned}$$

The first component must have a value that is very large. Therefore, we need an assumption where the numerator will be much larger than the denominator. We need values to approach infinity, moreover, values that will fix this problem are when the following is satisfied: $\nu_0 >> \gamma$ and this results in a large enough number. When these conditions are met, the following is true,

$$\begin{aligned}
&= \frac{1}{\pi}\Bigg[-tan^{-1}(-\infty) + tan^{-1}(\infty)\Bigg] \\
&= \frac{1}{\pi}\Bigg[\frac{\pi}{2} + \frac{\pi}{2}\Bigg] = 1.
\end{aligned}$$

Thus,

$$\int_0^\infty \phi(\nu)d\nu = 1$$

Appendix

Problem 1 Calculations

```
import numpy as np

G = 6.67 * 10 ** (-11) # gravitational constant (N m^2 kg^-2)
M_solar = 1.99 * 10 ** 30 # solar mass (kg)
R_solar = 6.96 * 10 ** 8 # solar radius (m)
m_p = 1.67 * 10 ** (-27) # (kg)
k_B = 1.38 * 10 ** (-23) # Boltsmanns (J/K)

e   = 1.602 * 10 ** (-19) # electron charge (C)
m_e = 9.109 * 10 ** (-31) # electron mass (kg)
c   = 3.00 * 10 ** 8 # speed of light (m/s)
epsilon = 8.85 * 10 ** (-12) # permittivity (F m^-1)

sigma_T = (8 * np.pi)/3 * ((e ** 2)/(4 * np.pi * epsilon * m_e * (c **
    2))) ** 2
print('Value of sigma_T: {}'.format(sigma_T))

n_e = 1

l = 1/(n_e * sigma_T)
t_s = l/c
print('Value of t_s: {}'.format(t_s))
```