Physics 460—Homework Report 1
Due Tuesday, Apr. 7, 1 pm **Name:** Timothy Holmes

Complete all the problems on the accompanying assignment.

List all the problems you worked on in the space below. Circle the ones you fully completed:

Please place the problems into the following categories:

- These problems helped me understand the concepts better: ________________
- I found these problems fairly easy: ________________
- I found these problems very challenging: ________________

In the space below, show your work (even if not complete) for any problems you still have questions about. Indicate where in your work the question(s) arose, and ask specific questions that I can answer.

Use the back of this sheet or attach additional paper, if necessary.

If you have no remaining questions about this homework assignment, use this space for one of the following:

- Write one or two of your solutions here so that I can give you feedback on its clarity.
- Explain how you checked that your work is correct.

Problem
1 and 2

(1) Using the properties of the angular momentum operators and their eigenstates, evaluate

$$J_x|j,m\rangle \quad \text{and} \quad J_y|j,m\rangle.$$

(2) Show that

$$\langle J_x\rangle = \langle J_y\rangle = 0$$

for the states $|\Psi\rangle = |j,m\rangle$.

(3) Show that

$$\langle {J_x}^2\rangle = \langle {J_y}^2\rangle = \hbar^2[j(j+1) - m^2]/2$$

for the states $|\Psi\rangle = |j,m\rangle$. Using a symmetry argument, explain why these values must be true, in light of the values for $\langle {J_z}^2\rangle$ and $\langle J^2\rangle$ for these states.

(4) Find the representation of the operators J_x, J_y, J_z, and J^2 using the states $|j,m\rangle$ as your basis for the case $j = 3/2$. Since there are four possible values for m, your answers should all be 4×4 matrices. Use the following representation for the basis states:

$$|3/2,3/2\rangle \leftrightarrow \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \quad |3/2,1/2\rangle \leftrightarrow \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \quad |3/2,-1/2\rangle \leftrightarrow \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \quad |3/2,-3/2\rangle \leftrightarrow \begin{bmatrix}0\\0\\0\\1\end{bmatrix}.$$

(5) Use your answers from Problem (4) to construct the representations of J_+ and J_- for the case $j = 3/2$. Explain why these matrices have the form they do—do they “look” like raising and lowering operators?

Homework 1

(1) Using the properties of the angular momentum operators and their eigenstates, evaluate

$$J_x|j,m\rangle \text{ and } J_y|j,m\rangle$$

$$J_+|j,m\rangle = \hbar\sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle$$
$$J_-|j,m\rangle = \hbar\sqrt{j(j+1)-m(m-1)}\,|j,m-1\rangle$$

$$J_x|j,m\rangle = \frac{1}{2}(J_+ + J_-)|j,m\rangle$$
$$= \frac{\hbar}{2}\left[\sqrt{(j-m)(j+m+1)}\,|j,m+1\rangle + \sqrt{(j+m)(j-m+1)}\,|j,m-1\rangle\right.$$

$$J_y|j,m\rangle = \frac{1}{2i}(J_+ + J_-)|j,m\rangle$$
$$= \frac{\hbar}{2i}\left[\sqrt{(j-m)(j+m+1)}\,|j,m+1\rangle + \sqrt{(j+m)(j-m+1)}\,|j,m-1\rangle\right.$$

(2) Show $\langle J_x\rangle = \langle J_y\rangle = 0$ where $|\psi\rangle = |j,m\rangle$

$$\langle J_x\rangle = \langle\psi|J_x|\psi\rangle = \langle m,j|J_x|j,m\rangle$$

$$\frac{1}{i\hbar}\langle m,j|[J_y, J_z]|j,m\rangle$$

$$= \frac{1}{i\hbar}\langle m,j|J_yJ_z - J_zJ_y|j,m\rangle$$

$$= \frac{1}{i\hbar}\left[\langle m,j|J_yJ_z|j,m\rangle - \langle m,j|J_zJ_y|j,m\rangle\right]$$

$$= \frac{1}{i\hbar}\left[\hbar m\langle m,j|J_y|j,m\rangle - \hbar m\langle m,j|J_y|j,m\rangle\right]$$

$$= 0$$

Same example for $\langle J_y\rangle$

(3) $\langle J_x^2 \rangle = \langle J_y^2 \rangle = \hbar^2 [j(j+1) - m^2] / 2$

$\langle J_x \rangle = \langle \psi | J_x^2 | \psi \rangle = \langle m, j | J_x^2 | j, m \rangle$

$$J_x^2 = \frac{1}{4}(J_+^2 + J_+ J_- + J_- J_+ + J_-^2) = J_y^2$$

$\langle J_x^2 \rangle = \langle J_y^2 \rangle$

$$= \frac{1}{2}\Big[\langle j, m | J^2 | j, m \rangle$$

$$- \langle j, m | J_z | j, m \rangle \Big] = \frac{\hbar^2}{2}\left[j(j+1) - m^2 \right]$$

(4) Spin: $|+3/2\rangle, |+1/2\rangle, |-1/2\rangle, |-3/2\rangle$

$$J_x = \begin{bmatrix} \langle +3/2 | J_x | +3/2 \rangle & \langle +1/2 | J_x | +3/2 \rangle & \langle +1/2 | J_x | +3/2 \rangle \\ \langle +3/2 | J_x | +1/2 \rangle & \langle +1/2 | J_x | +1/2 \rangle & \langle +1/2 | J_x | +1/2 \rangle \\ \langle +3/2 | J_x | -1/2 \rangle & \langle +1/2 | J_x | -1/2 \rangle & \langle +1/2 | J_x | -1/2 \rangle \\ \langle +3/2 | J_x | -3/2 \rangle & \langle +1/2 | J_x | -3/2 \rangle & \langle +1/2 | J_x | -3/2 \rangle \end{bmatrix}$$

$$\begin{bmatrix} \langle +1/2 | J_x | +3/2 \rangle \\ \langle +1/2 | J_x | +1/2 \rangle \\ \langle +1/2 | J_x | -1/2 \rangle \\ \langle +1/2 | J_x | -3/2 \rangle \end{bmatrix}$$

$\vdots$

$$\hbar \begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \end{bmatrix} = \hbar \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$J_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix} \qquad J_z = \frac{\hbar}{2}\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$J_y = \frac{i\hbar}{2}\begin{bmatrix} 0 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & -2 & 0 \\ 0 & 2 & 0 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix} \qquad J^2 = \frac{\hbar^2}{2}\begin{bmatrix} -3 & 0 & 2\sqrt{3} & 0 \\ 0 & -1 & 0 & 2\sqrt{3} \\ 2\sqrt{3} & 0 & 1 & 0 \\ 0 & 2\sqrt{3} & 0 & 3 \end{bmatrix}$$