Physics 412—Quantum Mice Measurements — Winter 2020

We will continue our studies of the properties of quantum mice, moving onto the implications of compatibility for the certainty or uncertainty of measurements. The four properties of our quantum mice are attitude (operator A, eigenstates $|h\rangle$ and $|u\rangle$), behavior (operator B, eigenstates $|p\rangle$ and $|a\rangle$), energy (operator H, eigenstates $|4\rangle$ and $|2\rangle$), and size (operator W, eigenstates $|s\rangle$ and $|l\rangle$). If we use the attitude states as our basis, we can represent the operators as matrices and the states as column vectors as follows:

$$A \leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad |h\rangle \leftrightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad |u\rangle \leftrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$B \leftrightarrow \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \qquad |p\rangle \leftrightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \mathrm{i} \end{bmatrix}, \qquad |a\rangle \leftrightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -\mathrm{i} \end{bmatrix},$$

$$H \leftrightarrow \frac{2}{5} \begin{bmatrix} 6 & 2 \\ 2 & 9 \end{bmatrix}, \qquad |4\rangle \leftrightarrow \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad |2\rangle \leftrightarrow \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 1 \end{bmatrix},$$

$$W \leftrightarrow \frac{2}{5} \begin{bmatrix} 21 & -8 \\ -8 & 9 \end{bmatrix}, \qquad |s\rangle \leftrightarrow \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad |l\rangle \leftrightarrow \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

Note that H and W share eigenstates because they commute. None of the other operators commute.

You can answer all of these working in the attitude basis, using the representation given above. It's probably easiest that way. Note: these questions are meant to be somewhat qualitative, though you can give quantitative answers to them as well.

(1) Suppose that you have a mouse that you know to be small (state of the mouse is $|s\rangle$). What can you say about measurements of other properties of the mouse (attitude, behavior, or energy)? Can you say with certainty what the attitude of a small mouse is? What about the behavior, or the energy?

(2) Suppose that you have a mouse that you know to be happy (state of the mouse is $|h\rangle$). What can you say about measurements of other properties of the mouse (energy, behavior, or size)? Can you say with certainty what the size of a happy mouse is? What about the behavior, or the energy?

(3) Suppose that you have a mouse that you know to be aggressive (state of the mouse is $|a\rangle$). What can you say about measurements of other properties of the mouse (energy, attitude, or size)? Can you say with certainty what the size of a happy mouse is? What about the attitude, or the energy?

(4) Suppose that you have a mouse that you know to be energetic (state of the mouse is $|4\rangle$). What can you say about measurements of other properties of the mouse (attitude, behavior, or size)? Can you say with certainty what the attitude of a happy mouse is? What about the behavior, or the size?

(1) W commutes with H, not A & B.

If state is $|s\rangle$, energy is 4 with 100% certainty.

Measurement of attitude could result in happy or unhappy.

Measurment of behavior could result in passive or aggressive.

(2) A does not commute with any other operators, so if the state is $|h\rangle$, measurements of the other quantities could result in either of the two possible values.

(3) Same as 2.

(4) Same as 1.