Physics 412—Assessment F–1 Name:

F–1: I can use the postulates of quantum mechanics to analyze physically interesting systems in one spatial dimension or described by a finite-dimensional Hilbert space.

Unsatisfactory Progressing Acceptable Polished

This exam is open book and open notes, of course. The only constraint is that you work on these problems individually. This means that you are *not* to discuss these problems, potential approaches to the problems, or compare answers with other students or anyone else. You are welcome (and encouraged) to ask me questions if you get stuck, but try not to get stuck too easily.

I've created a submission folder on D2L. The final exam is due Wednesday, March 18, at noon. Multiple submissions are allowed, and all the files you submit will be kept.

You should submit an electronic copy of your exam. It is your responsibility to make sure that your exam is legible. A single pdf is strongly preferred. Please let me know if you have any questions.

(1) Work in the z-state basis for this problem about spin-1/2 particles.

(a) Verify the following useful relationships:

$$S_x|{+z}\rangle = \frac{\hbar}{2}|{-z}\rangle, \quad S_x|{-z}\rangle = \frac{\hbar}{2}|{+z}\rangle, \quad S_y|{+z}\rangle = \frac{\mathrm{i}\hbar}{2}|{-z}\rangle, \quad S_y|{-z}\rangle = -\frac{\mathrm{i}\hbar}{2}|{+z}\rangle.$$

(b) Find a state for a spin-1/2 particle for which $\langle S_x\rangle = \langle S_y\rangle = \langle S_z\rangle$.

(2) A particle is in a one-dimensional box of length L, with sides at $x = \pm L/2$ as in the course notes. Suppose that at $t = 0$ you know that the state of the system is given by

$$\psi(x,0) = A\left[\sin\frac{4\pi x}{L} + \mathrm{i}\cos\frac{\pi x}{L}\right],$$

where A is a normalization constant. Find $\psi(x,t)$ and calculate the probability that the particle will be found in the right-most quarter of the box, $x > L/4$, as a function of time.

(3) Consider operator $S(p_0) = \mathrm{e}^{\mathrm{i}p_0 X/\hbar}$, where p_0 is a real number and X is the position operator. This problem is about applying this operator to the ground state of the simple harmonic oscillator.

(a) Find $\left[a,\, S(p_0)\right]$, where a is the lowering operator for the simple harmonic oscillator.

(b) Show that $|\Psi\rangle = S(p_0)|0\rangle$ is an eigenstate of the lowering operator a. Find the eigenvalue.

(c) Assuming the system is in state $|\Psi\rangle$, find $\langle X\rangle$ and $\langle P\rangle$. How do they compare to the expectation values of X and P for the ground state itself?