# Physics 412—Harmonic Oscillator Operators and States **Winter 2020**

Starting with the definition of $a$, $N$, and $H$ for the simple harmonic oscillator,

$$H = \frac{P^2}{2m} + \frac{m{\omega_0}^2 X^2}{2}, \quad a = \frac{1}{\sqrt{2}d_0}\left[X + \frac{\mathrm{i}P}{m\omega_0}\right], \quad d_0 = \sqrt{\frac{\hbar}{m\omega_0}}, \quad N = a^\dagger a,$$

verify all the following properties, using the commutation relation for $X$ and $P$, $\left[X, P\right] = \mathrm{i}\hbar$, as necessary. Note that $|E\rangle$ means an eigenstate of $H$ with eigenvalue $E$ and $|n\rangle$ means and eigenstate of $N$ with eigenvalue $n$. You should do this without referencing the course notes!!

**(1)** $H = \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right) = \hbar\omega_0\left(N + \frac{1}{2}\right)$.

**(2)** $\left[a, a^\dagger\right] = I$.

**(3)** $\left[N, a\right] = -a$.

**(4)** $\left[N, a^\dagger\right] = a^\dagger$.

**(5)** $N\left(a|n\rangle\right) = (n-1)\left(a|n\rangle\right)$.

**(6)** $N\left(a^\dagger|n\rangle\right) = (n+1)\left(a^\dagger|n\rangle\right)$.

**(7)** $a|n\rangle = \sqrt{n}\,|n-1\rangle$.

**(8)** $a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$.

**(9)** $X|n\rangle = \frac{d_0}{\sqrt{2}}\left[\sqrt{n+1}\,|n+1\rangle + \sqrt{n}\,|n-1\rangle\right]$.

**(10)** $P|n\rangle = \frac{\mathrm{i}\hbar}{\sqrt{2}d_0}\left[\sqrt{n+1}\,|n+1\rangle - \sqrt{n}\,|n-1\rangle\right]$.

**(11)** $\left[H, N\right] = 0$.

**(12)** $\left[H, a\right] = -\hbar\omega_0 a$.

**(13)** $\left[H, a^\dagger\right] = \hbar\omega_0 a^\dagger$.

**(14)** $H\left(a|E\rangle\right) = (E - \hbar\omega_0)\left(a|E\rangle\right)$.

**(15)** $H\left(a^\dagger|E\rangle\right) = (E + \hbar\omega_0)\left(a^\dagger|E\rangle\right)$.