The CMB

(1) In the lecture, you've been presented with three facts,

$$\sigma_e = 6.65 \times 10^{-29} \, \text{m}^2; \quad \lambda = \frac{1}{n_e \sigma_e}; \quad n_e = \frac{0.25 \, \text{m}^{-3}}{a^3}$$

(a) Given that photons travel with a speed c, find an expression for the rate at which photons scatter. Call this term Γ. (*Hint:* Think dimensionally).

(b) Use n_e and λ to find how often photons scatter when $a = 10^{-5}$.

(c) Write the Friedmann equation for this radiation dominated era. Given that $\Omega_{r,o} = 9.0 \times 10^{-5}$, find the Hubble parameter (in terms of a).

(d) Let $a = 10^{-5}$ and find the Hubble parameter. Compare this value to Γ obtained in part (b). Interpret your results.

(2) In studying the physic of the CMB, one of the key ideas is that the reactants in the reaction

$$H + \gamma \rightleftharpoons p + e$$
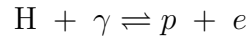
are in thermal equilibrium. At some point the equilibrium is *broken* and neutral hydrogen forms. Using the figure in the lecture as a guide, discuss at your table what might be the key factors that determine recombination.

(3) In the lecture, you've been given two number densities (which are derived from quantum statistics),

$$n_\gamma = 0.2436 \left(\frac{kT}{\hbar c} \right)^3$$

$$n_x = g_x \left(\frac{m_x kT}{2\pi\hbar^2} \right)^{3/2} \exp\left(\frac{-m_x c^2 + \mu_x}{kT} \right)$$

Describe every term in each of the equations, and discuss what the kind of species n_γ and n_x apply to,

(4) Recall that we defined the fractional ionization, X, as

$$X \equiv \frac{n_p}{n_p + n_H}.$$

(a) Solve for n_H from the fractional ionization.

(b) Substitute your result in (a) into the *Saha* equation,

$$\frac{n_{\text{H}}}{n_p\, n_e} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{Q}{kt}\right)$$

to arrive at an expression for X.

(5) At your table, discuss the process by which have arrived at the various recombination quantities $(T, z, \ldots)$

Homework 03–Due Friday, Feb. 21

1. Problem 6.3

2. Problem 6.8

3. Problem 8.2

4. Problem 8.4