Physics 412—Practice T–1 (Due Jan. 21, 11 am) Name:

T–1: I can use a set of basis vectors to represent both states and operators.
Unsatisfactory   Progressing   Acceptable   Polished

(1) Suppose that we have a three-dimensional vector space, with two operators, A and B on this space, with representations

$$A \leftrightarrow \begin{bmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -2a \end{bmatrix} \quad \text{and} \quad B \leftrightarrow \begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -2ib \\ 0 & 2ib & 0 \end{bmatrix}.$$

The above representations are using the eigenstates of A as basis vectors. Using the usual notation, we can write these eigenstates and their relationship to A as $A|a_1\rangle = a|a_1\rangle$, $A|a_2\rangle = -a|a_2\rangle$, and $A|a_3\rangle = -2a|a_3\rangle$. Note that A is Hermitian.

(a) Find its eigenvalues and eigenvectors of B. Normalize the eigenvectors and express them as linear combinations of the original basis kets, for example $|b_1\rangle = c_1|a_1\rangle + c_2|a_2\rangle + c_3|a_3\rangle$, where c_1, c_2, and c_3 are constants.

(b) Write down the eigenkets of B as column vectors in the A representation and show that they are orthogonal.

(c) You now have a second orthonormal basis, $|b_1\rangle$, $|b_2\rangle$, $|b_3\rangle$. Write down the representations of both sets of states in this basis, as column vectors. Show your work or explain your answer.

(d) Find the matrix representations of both operators in the basis $|b_1\rangle$, $|b_2\rangle$, $|b_3\rangle$.