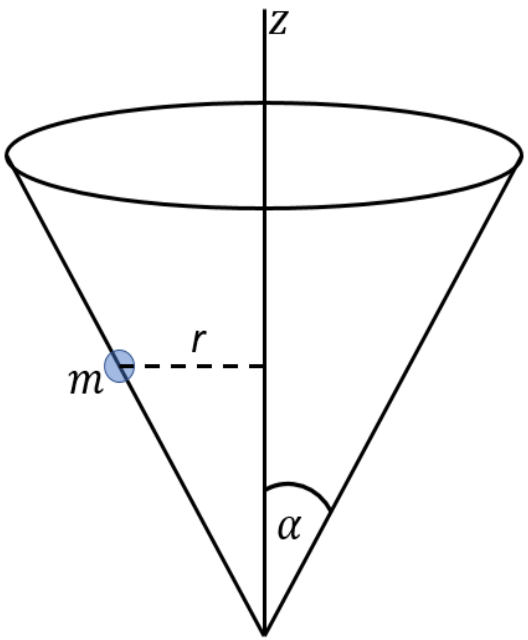
Upcoming deadlines

- Homework 2 due today
- Corrections for homework 2 on Tuesday
- Reading assignment (sections 2.1-2.4) and warm-up quiz 3 due on Tuesday.
- Review of weeks 1 and 2
- Today: Activity 7.

Lagrangian Dynamics

- Determine how many degrees of freedom there are
- Find the constrain equations
- Identify appropriate generalized coordinates and find the transformation equations
- Derive the Lagrangian in appropriate generalized coordinates.
- Derive the equation(s) of motion.
- Identify cyclic variables
- Derive conjugate momenta to identify constants of motion.

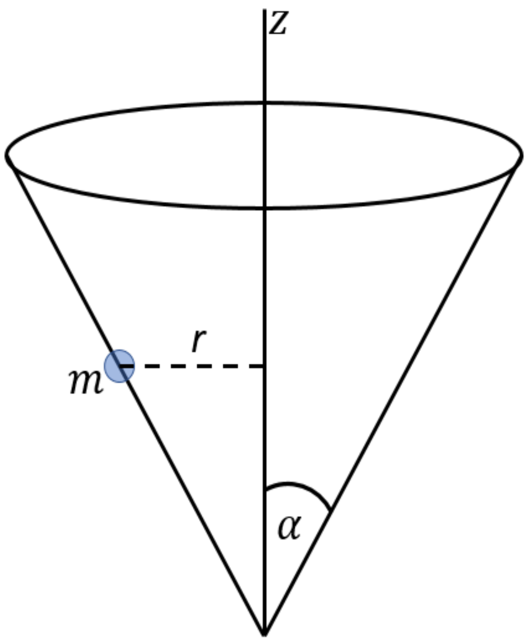
Example



Lagrangian in Cartesian coordinates

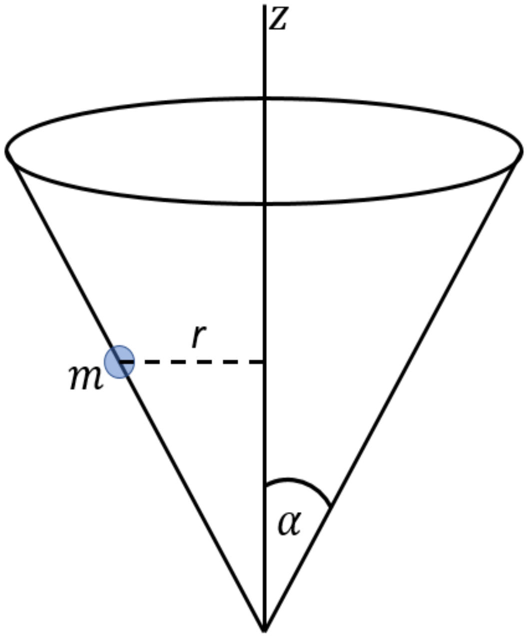
$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$$

Example



- What are the constraints and constraint equations?

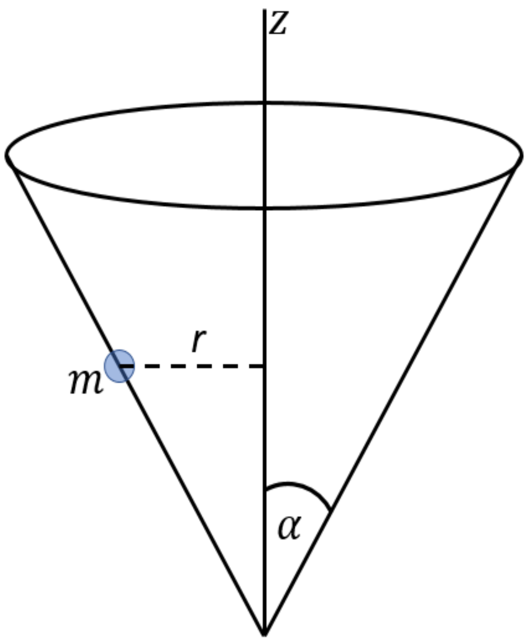
Example



- What are the constraints and constraint equations?
- Particle constraint to move on the cone:

$$\frac{r}{z} = \tan \alpha$$

Example

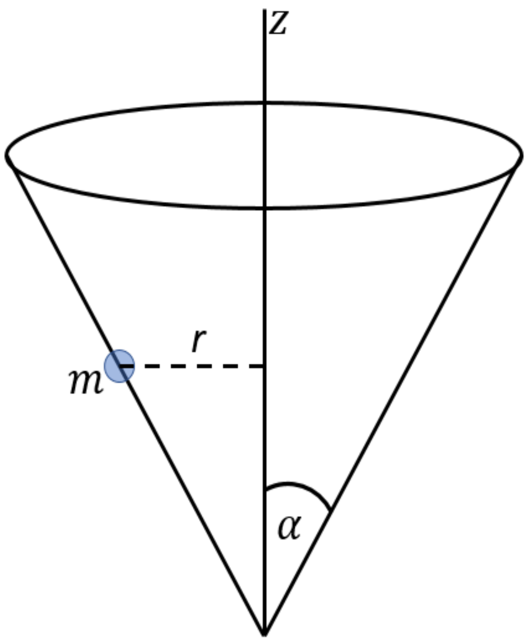


- What are the constraints and constraint equations?
- Particle constraint to move on the cone:

$$\frac{r}{z} = \tan \alpha$$

- How many degrees of freedom?
- One particle and one constraint: 3-1 = 2 degrees of freedom.

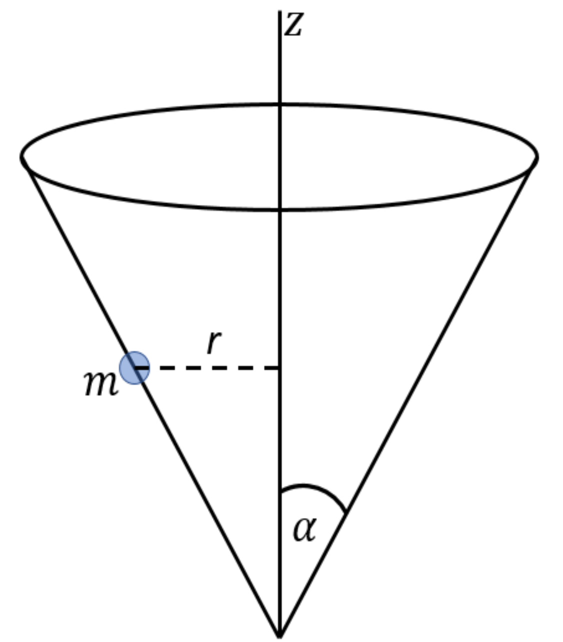
Example



How many generalized coordinates are needed to completely describe the configuration of this system? Which ones should we choose?
2 degrees of freedom means we need two coordinates
r and ϕ (azimuthal angle)

Coordinate transformations

- $x = r\cos\phi\,, \dot{x} = \dot{r}\cos\phi - r\dot{\phi}\sin\phi$
- $y = r\sin\phi\,, \dot{y} = \dot{r}\sin\phi + r\dot{\phi}\cos\phi$
- $z = r/\tan\alpha, \dot{z} = \dot{r}/\tan\alpha$



Lagrangian in the new coordinates

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$$

- Plug in transformations and simplify

$$L = \frac{1}{2}m\left[\dot{r}^2 \csc^2 \alpha + r^2\dot{\phi}^2\right] - mgr\cot\alpha$$

Identify cyclic coordinates

$$L = \frac{1}{2}m\left[\dot{r}^2 \csc^2\alpha + r^2\dot{\phi}^2\right] - mgr\cot\alpha$$

ϕ is not in the Lagrangian, so it is cyclic or ignorable. Changing the value of ϕ does not affect the motion of the system.

Conjugate Momenta and Constants of Motion

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} = \frac{\partial L}{\partial \phi} = 0$$

Because ϕ does not appear in the Lagragian. Therefore

$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} = 0$ and $\frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi} = const$

$$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}$$

Is a constant of motion. It is called the generalized momentum or the conjugate momentum of the cyclic generalized variable ϕ.