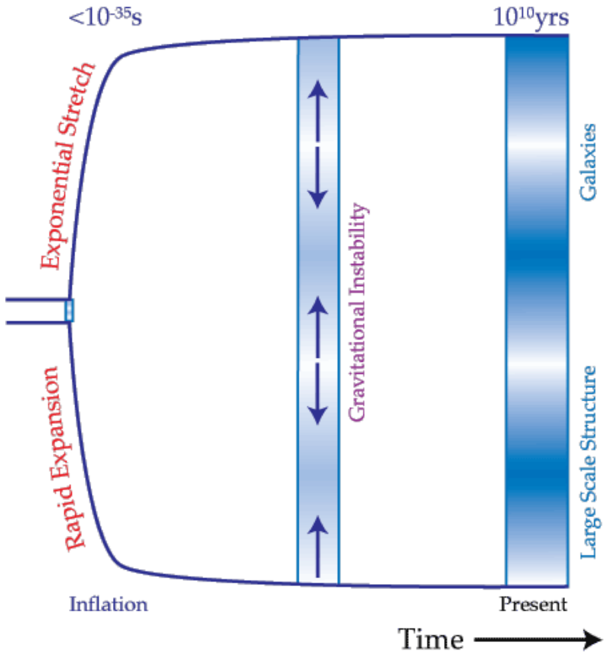

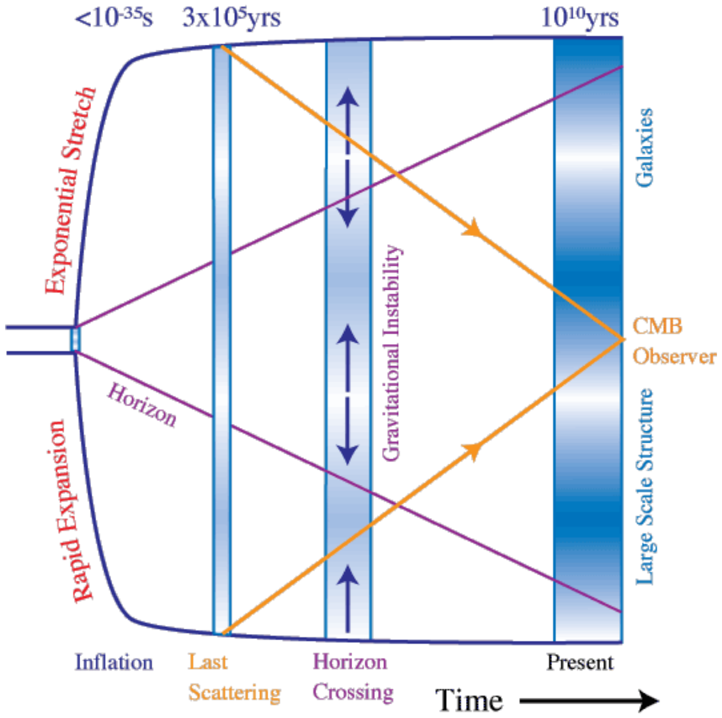

The physics of inflation:

Suppose there exists a scalar field and its associated potential labeled as $\phi;\ V(\phi)$ respectfully.

The energy density associated with this field is $\epsilon_\phi = \underbrace{\frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2}_{\text{kinetic}} + \underbrace{V(\phi)}_{\text{potential}}$ with $\underbrace{P_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi)}_{\text{From GR}}$

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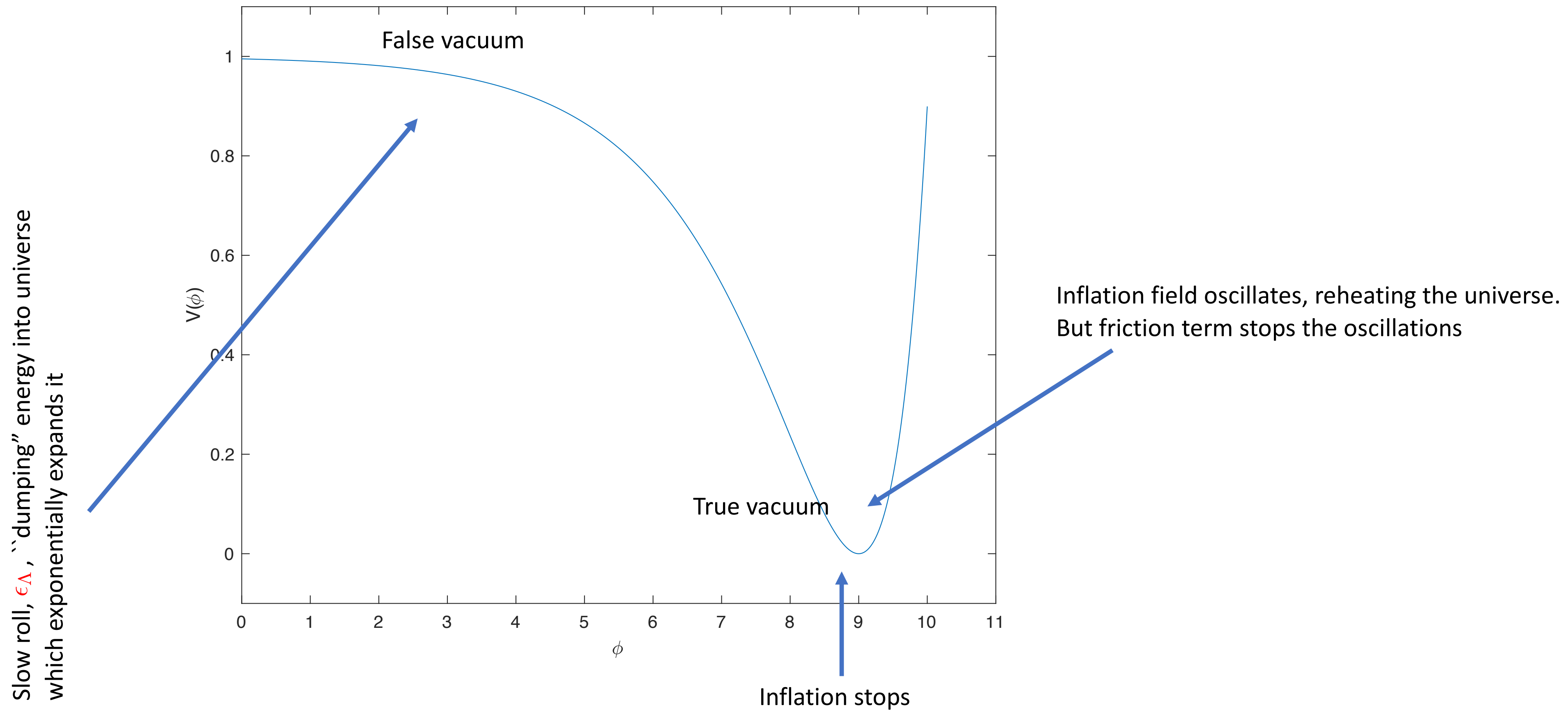
Do question (1)on the worksheet and S T O P.

(1 a and b) $\underbrace{\ddot{\phi}}_{\text{acceleration}} + \underbrace{3H(t)\dot{\phi}}_{\text{friction}} = \underbrace{-\hbar c^3 \frac{dV}{d\phi}}_{\text{driving force}}$

(1c) $3H\dot{\phi} = -\hbar c^3 \frac{dV}{d\phi}$

(1d) $\left(\frac{dV}{d\phi}\right)^2 \ll \frac{9H^2 V}{\hbar c^3}$



Very much like a *phase transition* during which *latent heat* is released into the surroundings.

Do question (2) on the worksheet.

Elements of the paper.

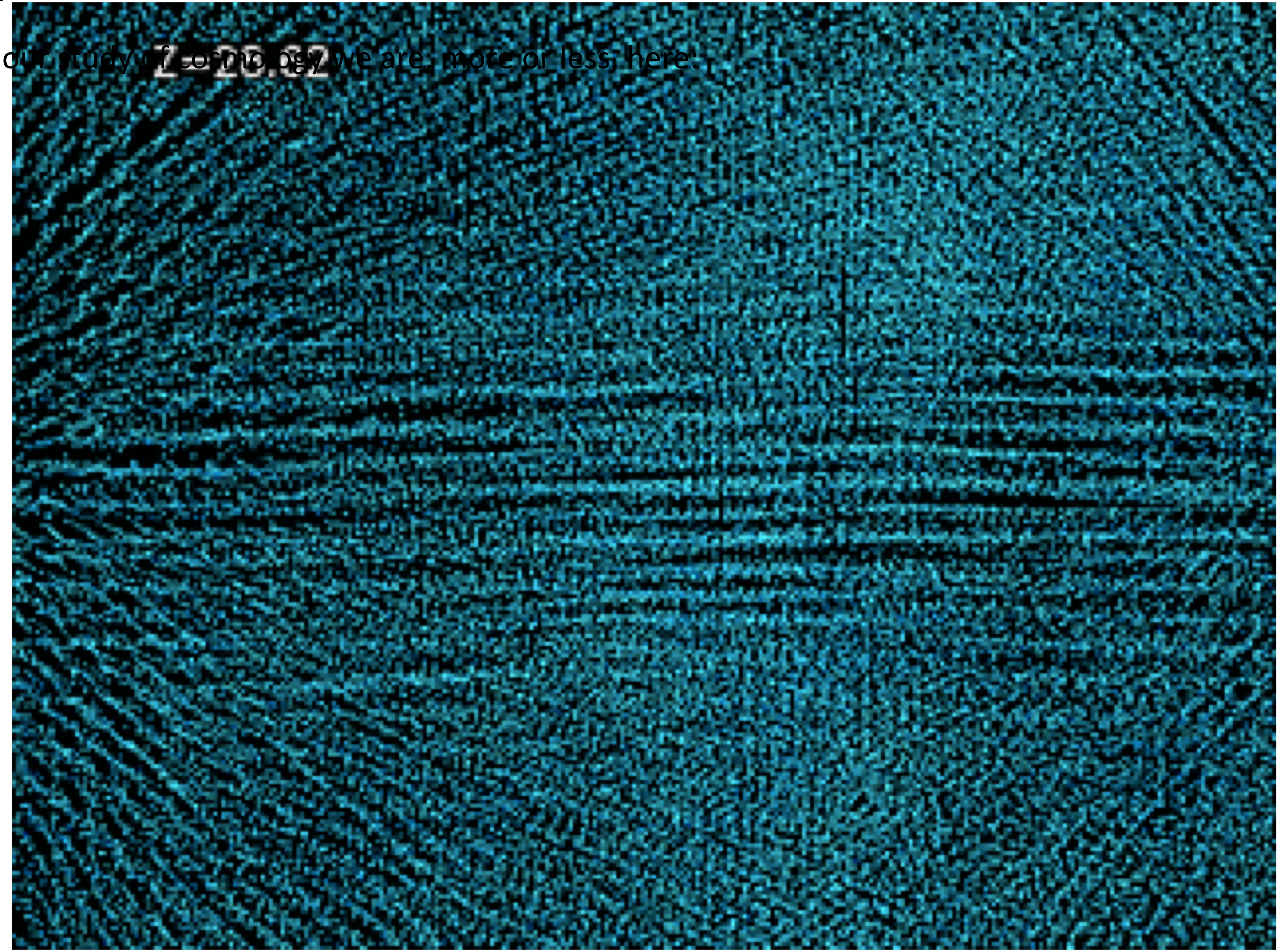
Do question (3a) on the worksheet and S T O P

Do question (3b) on the worksheet and S T O P

Do question (3c) on the worksheet and S T O P

Do question (3d) on the worksheet and S T O P

So far in our study of cosmology we are, more or less, here:



How does this happen?

Structure Formation:

- On scales > 100 Mpc, the universe is isotropic and homogeneous. But on scales less than this, the universe is *clumpy*.
- We now explore how the universe became clumpy from an initial very smooth density field.
- The *clumps* we investigate are on the order a few Mpc to 50 Mpc.
- The basic mechanism for growing these large structures is *gravitational instability.*

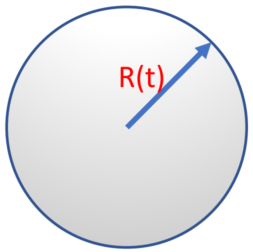
To get a general feel for the idea behind the theory of large scale structure formation, do question (4) on the worksheet and S T O P

Now let's get more quantitative. From question (4), we probably got the intuition that it is the density fluctuations, not the density themselves that lead to structure formation. Let's define the density fluctuation as

$$\delta(\vec{r},t) \equiv \frac{\epsilon(\vec{r},t) - \bar{\epsilon}(t)}{\bar{\epsilon}(t)}$$

Here, $\epsilon(\vec{r},t)$ is the energy density at a specific time and position, and $\bar{\epsilon}(t)$ is the mean energy density at this time.

Do question (5) on the worksheet and S T O P



Suppose we have a static, homogeneous, matter-only universe that has a small over-density suddenly added to it. The new density is described by

$$\rho = \bar{\rho}\,(1+\delta); \quad \delta \ll 1$$

Applying Newton's second law, the acceleration at the sphere's surface due to this extra mass is:

$$\ddot{R} = -\frac{G(\Delta M)}{R^2} = -\frac{G}{R^2}\left(\frac{4\pi}{3}R^3\bar{\rho}\,\delta\right)$$

or

$$\frac{\ddot{R}}{R} = -\frac{4\pi\,G\,\bar{\rho}}{3}\delta(t)$$

Let's explore some of the consequences of this result. Do question (6 a and b) on the worksheet and S T O P

(6 a and b) If there is an over density, the sphere will *collapse.* There are two unknowns, *R(t)* and $\delta(t)$.

To find another relationship, we use conservation of mass which tell us that

$$M = \frac{4\pi}{3}\bar{\rho}[1+\delta)t)]\;R(t)^3 \Rightarrow \boxed{R(t) = R_o\,[1+\delta(t)]^{-1/3}}$$

where

$$R_o \equiv \left(\frac{3M}{4\pi\bar{\rho}}\right)^{1/3} = \text{constant}$$

Finish question (6)

(6c)
$$\begin{aligned} R(t) &= R_o\,[1+\delta(t)]^{-1/3} \text{ now take } \ddot{R} \\ \ddot{R} &= -\frac{1}{3}R_o\ddot{\delta} \approx \frac{1}{3}R\ddot{\delta} \text{ substitute into eq (1)} \\ \ddot{\delta} &= 4\pi G\,\bar{\rho}\,\delta \end{aligned}$$

(6 d) Straight forward second order ODE. It yields, $\delta(t) = A_1 e^{t/t_{\mathrm{dyn}}} + A_2 e^{-t/t_{\mathrm{dyn}}}$ where $t_{\mathrm{dyn}} = \dfrac{1}{\sqrt{4\pi G\bar{\bar{\rho}}}}$

The positive term quickly dominates and the perturbation grows exponentially.