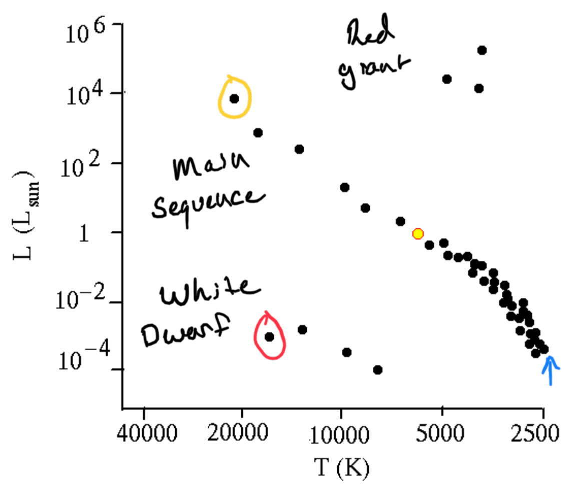
PHY 474/374 Spring 2021

Week 6—Monday, May 3—Discussion Worksheet

Evolution before the Main Sequence

So far, we have been learning about the properties of stars and physical processes. Today, we will begin learning the subject of Stellar Evolution. Stars form in sub-parsec scale clumps (sometimes called cores) deep inside Giant Molecular Clouds when these clumps become unstable to gravitational collapse. Although we won't go into the details here (since they belong to our course on Star Formation), it is worth noting that the net effect is to accrete enough material onto the central protostar (usually through a disk) so that the temperature at the core of the protostar increases until it is high enough to begin hydrogen fusion.

1. To learn about stellar evolution, we will need a (theoretical) Hertzsprung-Russell (HR) Diagram. First let us figure out what an observational HR diagram looks like. The HR diagram takes a little getting used to, because of its unusual axes (a historical artifact; the physics of the arrangement wasn't known when it was first constructed). Consider the HR diagram of the stars nearest to the Sun shown below (source: U. Oregon), with the Sun marked in yellow.



(a) Mark the hottest ***and*** brightest star shown on the plot.

In yellow

(b) Mark the coolest ***and*** dimmest star shown on the plot.

In Blue

(c) Mark the hottest ***but*** dimmest star shown on the plot.

In Red

(d) What can you conclude about the ***sizes*** of the three stars that are offset to the top right on the plot? Justify.

Very large stars

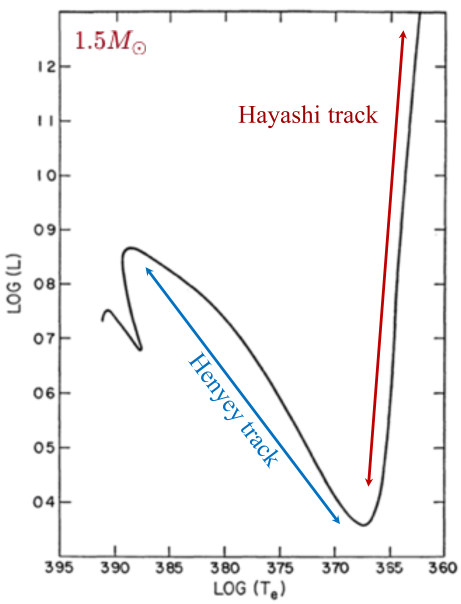
- Red giants

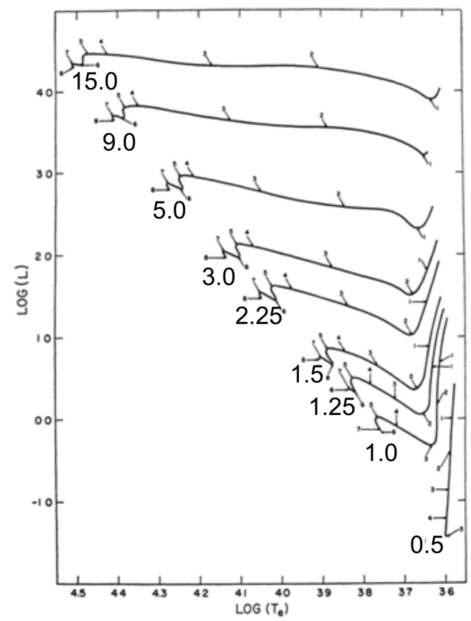
↑ luminosity

↓ surface temps

$L = 4\pi R^2 \sigma T_{eff}^4$, high L = large R, T is low

2. Now let's look at a theoretical HR diagram. Below on the left is one for a 1.5 $M_\odot$ star from Iben (1965), plots for additional stellar masses are shown on the right. Luminosity is in units of $L_\odot = 3.86 \times 10^{33}$ erg/sec and surface temperature T_e in units of K. Note that numbers on both axes have a decimal point after the first number, that is, $\log(T_e)$ values are 3.95, 3.90, and so on from left to right, whereas L values are 0.4, 0.5, and so on from bottom to top.





(a) Initially, when the star is convective, it follows an almost vertical track down the HR diagram (marked as **Hayashi track**). Since the surface temperature T_e is almost constant, discuss what is happening to the luminosity and radius of the 1.5 $M_\odot$ star in this segment.

This happens Because of a weak dependence on R

$T_c \propto \frac{M^{1/4}}{R^{1/8}}$ $\quad L \propto R^{5/2} M$

Radius is contracting

(b) Due to the change in radius that you described in part (a), what must be happening to the **internal temperature** (careful: not T_e, which is the surface temperature), and as a result, the **opacity** of the 1.5 $M_\odot$ star?

↑ internal temperature · Bc ↓ in R

↓ opacity - Higher temps tell us this is dropping

3. Consider again the figure from Iben (1965) on the previous page.

(a) As a result of the change you described in Question 2(b), the 1.5 $M_\odot$ star goes from being largely convective to having a radiative core. This makes it take a sharp turn to the left and move along the **Henyey track**. Given that the star must still be shrinking at this stage, describe why you see the changes in the luminosity and surface temperature in the Henyey segment.

↑ Luminocity

↑ surface temp

Star is collapsing

(b) In the plot on the right (previous page), the last point marked for each track indicates when the star reaches the Main Sequence and begins nuclear fusion. How is the track for a 0.5 $M_\odot$ different from that for a 1.5 $M_\odot$ star?

- lower mass has a smaller temperature range
- lower mass has a larger luminocity range

(c) Again based on the plot on the right (previous page), how is the track for a higher mass star (like the 5.0 $M_\odot$ or 15.0 $M_\odot$) different?

- The higher the mass the higher the luminocity
- the higher the mass, the larger the temperature range
- Smaller Dip as mass increases

4. Consider now the following figure from Iben (1965), which shows the variation with time t (in s) of several quantities for a star with mass $M = M_\odot$, the surface temperature T_e (in K), the luminosity L (in units of $L_\odot = 3.86 \times 10^{33}$ erg/sec), stellar radius R (in $R_\odot = 6.96 \times 10^{10}$ cm), the ratio of central to mean density $\rho_c/\overline{\rho}$, and mass fraction in the radiative core $Q_{\rm rc}$ (i.e., the ratio of the mass through which energy is transferred by radiation as a fraction of the mass of the star). The maximum and minimum scale limits for these quantities correspond to: $3.58 < \log T_e < 3.78$, $-0.4 < \log L < 0.6$, $-0.4 < \log R < 0.6$, $0.0 < \log(\rho_c/\rho) < 2.0$, and $0 < Q_{\rm rc} < 1$.

(a) Mark the portion in the plot that corresponds to the Hayashi track, and explain on what basis you chose it. Discuss the behavior of some of the quantities in this segment, and check whether they are consistent with your answers on the previous pages.

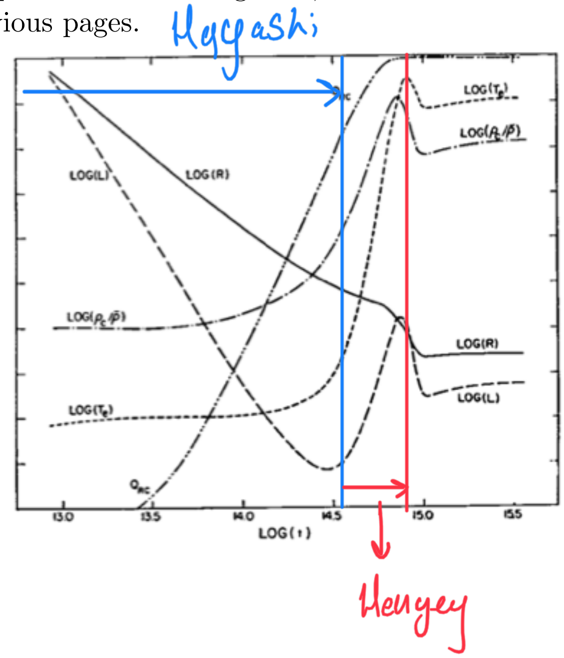
On Hayashi track,

T_e remains approx. constant

L falls

must be in region shown

R is decreasing in marked Region



(b) Mark the portion in the plot that corresponds to the Henyey track, and explain on what basis you chose it. Discuss the behavior of some of the quantities in this segment, and check whether they are consistent with your answers on the previous pages.

on Henyey track,

T_e ↑

L ↑

R still increases

ρ_c/ρ large increase

5. A contracting protostar will become a star only if the temperature in its core becomes high enough to initiate nuclear fusion. Core temperature increases with contraction as long as the gas stays ideal, but the gas also moves toward being degenerate as it contracts. Once degenerate, the temperature no longer increases with contraction. Thus, the temperature must become sufficiently high to initiate nuclear fusion ***before*** the gas becomes degenerate.

(a) Consider a star of mass M and radius R. Starting from equation (4.9): $T_c = \dfrac{G\mu_c m_p M}{kR}$, which assumes that the star is comprised of an ideal gas, show that

$\rho^{1/3} = (M/R^3)^{1/3}$

$$T_c = 2.06 \times 10^6\, \mu_c \left(\frac{M}{M_\odot}\right)^{2/3} \rho^{1/3}$$

$$T_c = \frac{(6.67 \times 10^{-11}\, m^3\, kg^{-1}\, s^{-2})(1.67 \times 10^{-27}\, kg)}{(1.38 \times 10^{-23}\, m^2\, kg\, s^{-2}\, K^{-1})} (1.99 \times 10^{30}\, kg)^{2/3} \ldots$$

$$\frac{4\pi}{3} \qquad \mu_c \left(\frac{M}{M_\odot}\right)^{2/3} \rho^{1/3}$$

$$T_c = 2.06 \times 10^6\, \mu_c \left(\frac{M}{M_\odot}\right)^{2/3} \rho^{1/3}$$

(b) If, during contraction, the gas becomes degenerate, then the temperature will no longer increase with contraction. By setting kT equal to the so-called Fermi energy which describes a degenerate electron gas, we get a critical density for the onset of degeneracy, and the temperature at which the critical density is reached in the protostar is given by

$$T \simeq 5.6 \times 10^7\, \mu\mu_e^{1/3} \left(\frac{M}{M_\odot}\right)^{2/3}$$

If the temperature for nuclear fusion is 10 million K, calculate the minimum mass required for a star to form. Use $\mu\mu_e^{1/3} \sim 1$.

$$\frac{M}{M_\odot} = \left[\frac{T}{5.6 \times 10^7\, \mu\mu_e^{1/3}}\right]^{3/2} = \left[\frac{10 \times 10^6\, K}{5.6 \times 10^7 (1)}\right]^{3/2} = 0.075$$

$$M \sim 0.08\, M_\odot$$

minimum mass for a star