Physics 412—The Dual Space Winter 2020

In quantum mechanics every ket $|\Psi\rangle$ has a corresponding bra $\langle\Psi|$ in the dual space. For R^2, the vectors in the dual space are represented by row vectors, not column vectors. So if a vector $\vec{A}$ has components A_x and A_y, the representations are

$$\vec{A} \leftrightarrow \begin{bmatrix} A_x \\ A_y \end{bmatrix}, \quad \text{dual of } \vec{A} \leftrightarrow \begin{bmatrix} A_x & A_y \end{bmatrix}.$$

As in quantum mechanics, when we operator on a dual vector with an operator, we do so from the left, and the answer is a new dual vector.

(1) Are the operators R_{30} and T_{45} Hermitian? If so, prove it, by calculations using the representations of vectors and operators. If not, find the representations of the adjoints (Hermitian conjugates) of the operators R_{30} and T_{45}.

(2) Using your matrix representations, find the eigenvalues and eigenvectors of R_{30} and T_{45}.