We've looked at a series of toy universes and we'll look at a few more today.

Recall that the starting point for all our analysis is the Friedmann equation in the form:

$$\frac{H^2}{H_o^2} = \frac{\Omega_{r,o}}{a^4} + \frac{\Omega_{m,o}}{a^3} + \Omega_\Lambda + \frac{1-\Omega_o}{a^2} \qquad (1)$$

Do question (1a) on the worksheet and S T O P

(1a) $\Omega_o = \Omega_{r,o} + \Omega_{m,o} + \Omega_\Lambda;$ here $\Omega_{r,o} \Rightarrow \Omega_o = \Omega_{m,o} + \Omega_\Lambda$

so

$$\boxed{\frac{H^2}{H_o} = \frac{\Omega_{m,o}}{a^3} + \frac{1-\Omega_{m,o}-\Omega_\Lambda}{a^2} + \Omega_\Lambda}$$

Finish question (1)

(1b) The universe starts out contracting. The last term is dominant. At a certain, *a*, the middle term becomes dominant. But this leads to unphysical $H^2 < 0$, so there are values of the scale factor that are prohibited!

(1c,d)

$$\begin{aligned}
\frac{H^2}{H_o^2} &= \frac{\Omega_{r,o}}{a^4} + \frac{\Omega_{m,o}}{a^3} \\
H_o dt &= \frac{a\,da}{\sqrt{\Omega_{r,o}}}\left[1 + \frac{a}{a_{rm}}\right]^{-1/2} \\
H_o t &= \frac{4a_{rm}^2}{3\sqrt{\Omega_{r,o}}}\left[1 - \left(1 - \frac{a}{2a_{rm}}\right)\left(1 + \frac{a}{a_{rm}}\right)^{1/2}\right]
\end{aligned}$$

Our best observations form the basis of the *benchmark model.* This model posits a universe that:

- Is spatially flat
- Contains radiation, matter, and a cosmological constant.
- Has a Hubble constant, H_o = 68 km s^{-1} Mpc^{-1}
- The radiation consists of photons and neutrinos with $\Omega_{r,o} = 5.35 \times 10^{-5}$
- The matter is in the form of both *baryonic matter* and *dark matter* with $\Omega_{bm,o} = 0.048, \quad \Omega_{dm,o} = 0.262$
- The cosmological constant is $\Omega_\Lambda = 0.69$
- Has a horizon distance of $d_{\mathrm{hor}}(t_o) \approx 14000\,\mathrm{Mpc}$

Do question (2) on the worksheet.