Physics 460—Practice S–7 (Due May 21, 1 pm) Name:

S–7: I can make predictions about systems described by entangled states and explain the EPR paradox and Bell's theorem.   Unsatisfactory Progressing Acceptable Polished

(1) A colleague claims that they have found an entangled state for a two-particle spin-1/2 system with the following property: they can find a measurement direction $\hat{n}$ along which the uncertainty in the measurement of the spin of one of the particles is zero: $\Delta S_{\hat{n}} = 0$.

Do you believe them? If so, provide an entangled state that has this property. If not, why not?

(2) Consider a hidden variable λ that has the range $-\pi \leq \lambda \leq \pi$, and let the measurement function $f(\hat{n}, \lambda)$ be

$$f(\hat{n}, \lambda) = \begin{cases} +1, & \lambda \geq \theta, \\ -1 & \lambda < \theta, \end{cases}$$

where θ is the angle between $\hat{n}$ and the z axis (so $0 \leq \theta \leq \pi$). This says that the result of the measurement depends on a comparison of the value of λ to the orientation of the measurement axis. We will apply Bell's result, so the measurement functions for the two particles obey the property $f_1(\hat{n}, \lambda) = -f_2(\hat{n}, \lambda)$.

(a) Let the probability density be $\rho(\lambda) = A\lambda^2$, where A is a constant. Find A.

(b) Using the three angles from Figure 4.2 of the course notes, with $\hat{n}_1$ along the z axis, $\hat{n}_2$ along the x axis, and $\hat{n}_3$ at a 45° angle in the x–z plane, calculate $\epsilon(\hat{n}_1, \hat{n}_2)$, $\epsilon(\hat{n}_1, \hat{n}_3)$, and $\epsilon(\hat{n}_2, \hat{n}_3)$ for this hidden variable theory.

(c) Show that (for these three angles at least) this hidden variable theory obeys Bell's inequality,

$$|\epsilon(\hat{n}_1, \hat{n}_2) - \epsilon(\hat{n}_1, \hat{n}_3)| \leq 1 + \epsilon(\hat{n}_2, \hat{n}_3).$$