PHY 411 Winter 2021

Homework 3—due by 9:00 PM, Monday, Jan 25

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Friday (Jan 29). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

1. Starting from the general boundary conditions on electric and magnetic fields at an interface between two media, we wrote four boundary conditions tailored to the incident, refracted, and reflected fields that we are discussing in this chapter; see equation (7.37) in Jackson, or the posted class summary. Of these, the fourth boundary condition is

$$\left[\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0+\vec{k}''\times\vec{E}_0''\right)-\frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\right]\times\hat{n}=0 \tag{7.37.d}$$

For the case of the $\vec{E}$-fields ***perpendicular*** to the plane of incidence, show that equation (7.37.d) above reduces (in magnitude) to

$$\sqrt{\frac{\epsilon}{\mu}}\,E_0\cos\,i-\sqrt{\frac{\epsilon}{\mu}}\,E_0''\cos\,i-\sqrt{\frac{\epsilon'}{\mu'}}\,E_0'\cos\,r=0 \tag{7.38.b}$$

Note: This was Question 5 on the worksheet for Tuesday (Jan 19), and it has been moved here because we didn't have time to get to it in class.

2. For $\vec{E}$-fields ***perpendicular*** to the plane of incidence, you obtained on the worksheet that

$$E_0+E_0''-E_0'=0 \tag{7.38.a}$$

and equation (7.38.b) written above.

Show that equation (7.38.a) and equation (7.38.b) together yield, for $\vec{E}$-fields ***perpendicular*** to the plane of incidence, that

$$\begin{aligned}\frac{E_0'}{E_0}&=\frac{2n\,\cos\,i}{n\,\cos\,i+\dfrac{\mu}{\mu'}\sqrt{n'^{\,2}-n^2\,\sin^2\,i}}\\[2ex]\frac{E_0''}{E_0}&=\frac{n\,\cos\,i-\dfrac{\mu}{\mu'}\sqrt{n'^{\,2}-n^2\,\sin^2\,i}}{n\,\cos\,i+\dfrac{\mu}{\mu'}\sqrt{n'^{\,2}-n^2\,\sin^2\,i}}\end{aligned} \tag{7.39}$$

Hint: It is easier to derive the second equation first!

Note: This was Question 6 on the worksheet for Tuesday (Jan 19).

3. For $\vec{E}$-fields ***parallel*** to the plane of incidence, the ratios of transmitted amplitude to incident amplitude (E_0'/E_0) and reflected amplitude to incident amplitude (E_0''/E_0) are given in equation (7.41) in Jackson:

$$\begin{aligned} \frac{E_0'}{E_0} &= \frac{2nn' \cos i}{\dfrac{\mu}{\mu'} n'^{\,2} \cos i + n\sqrt{n'^{\,2} - n^2 \sin^2 i}} \\ \frac{E_0''}{E_0} &= \frac{\dfrac{\mu}{\mu'} n'^{\,2} \cos i - n\sqrt{n'^{\,2} - n^2 \sin^2 i}}{\dfrac{\mu}{\mu'} n'^{\,2} \cos i + n\sqrt{n'^{\,2} - n^2 \sin^2 i}} \end{aligned} \tag{7.41}$$

(a) Use equation (7.41) above to find the transmission coefficient $T_{\|}$ for nonpermeable media where $\mu = \mu'$. Express all angles in your answer in terms of the angle of incidence i.

(b) Use equation (7.41) above to find the reflection coefficient $R_{\|}$ for nonpermeable media where $\mu = \mu'$. Again, express all angles in your answer in terms of the angle of incidence i.

4. For $\vec{E}$-fields ***parallel*** to the plane of incidence, you obtained expressions for the transmission coefficient $T_{\|}$ and the reflection coefficient $R_{\|}$ in Question 3 above (in nonpermeable media where $\mu = \mu'$).

(a) For $n = 1$ and $n' = 1.5$, plot $T_{\|}$ and $R_{\|}$ *on the same graph* as a function of the angle of incidence i, where i runs from 0 to $\pi/2$. *Graphs sketched by hand will be given a grade of zero.*

(b) Describe the graph, pointing out notable features in words. In particular, verify by direct calculation that Brewster's angle is showing up at the appropriate place in your graph.