PHY 474/374 **Spring 2021**

Week 8—Wednesday, May 19—Discussion Worksheet

Elements beyond Iron

We have learned that stars cannot produce elements beyond iron through the regular fusion processes, because high Coulomb barriers for heavier elements render their formation improbable at low temperatures, and photodissociation by high energy photons at high temperatures prevents formation of stable heavier nuclei.

1. Neutrons, however, are not subject to the Coulomb barrier. It is widely believed that neutron capture processes are responsible for formation of elements heavier than the iron group.

(a) Write down *in words* the effect, if any, on the atomic number and atomic weight of the absorption of a neutron in a nucleus.

Atomic mass A will ↑ by 1 when a neutron is added to the nucleus, the Atomic number Z will remain the same

(b) Use your answer above to complete the reaction below.

$$(\mathcal{Z}, \mathcal{A}) \;+\; n \;\rightarrow\; (\underline{Z}, \underline{A+1}) \;+\; \gamma \tag{1}$$

(c) Suppose the reaction in equation (1) renders the nucleus unstable to β-decay. Write down the subsequent reaction below.

$$(\underline{Z}, \underline{A+1}) \;\rightarrow\; (\underline{Z+1}, \underline{A+1}) \;+\; \mathrm{e}^- \;+ \bar{\nu}_\mathrm{e} \tag{2}$$

where $\bar{\nu}_\mathrm{e}$ is the electron anti-neutrino.

(d) In reality, the situation is more complicated because the β-decay in equation (2) is typically fairly slow. Thus, the nucleus produced in equation (1) may have time to capture another neutron before it decays according to equation (2).

Starting from the product in equation (1), write down the result of absorbing another neutron.

$$(\underline{Z}, \underline{A+1}) \;+\; n \;\rightarrow\; (\underline{Z}, \underline{A+2}) \;+\; \gamma \tag{3}$$

Thus, neutron capture may proceed in two ways:

- In the process of slow neutron capture (s-process), neutron capture is much slower than the β-decay.
- In the process of rapid neutron capture (r-process), neutron capture is much more rapid than the β-decay.

2. In order to understand the s-process and r-process, we must first consider the timescales.

(a) Let's derive an expression for the *lifetime of a nucleus against neutron capture*, τ_n. Assuming that the cross section for neutron capture is independent of energy, we get that $\tau_n = 1/n_n(\sigma v)$. For a typical neutron cross-section of $\sigma \sim 10^{-25}$ cm^2, and a temperature of 5×10^8 K, show that this gives

$$\tau_n \sim \frac{10^9 \text{ yr}}{n_n}$$

provided n_n is in cm^{-3}.

Note: Use velocity $v = \sqrt{2kT/\mu_n}$, and although the mass of a neutron is marginally greater than that of a proton, it will suffice for these calculations to use $m_p = 1.67 \times 10^{-24}$ grams; also, the Boltzmann constant is $k = 1.38 \times 10^{-16}$ erg/K.

$$\tau_n = \frac{1}{n_n\left(10^{-25}\,\text{cm}^2 \cdot \sqrt{\frac{2(1.38\times10^{-16}\,\text{erg}/\text{K})(5\times10^8\,\text{K})}{1.67\times10^{-24}\,\text{g}}}\right)} = \frac{3.5\times10^{16}\,\text{s}}{n_n} = \frac{10^9\,\text{yr}}{n_n}$$

(b) Now, consider an environment in which the neutron density is low, $n_n \sim 10^5$ cm^{-3}. Use the expression in part (a) to find the lifetime of the nucleus against neutron capture in this region.

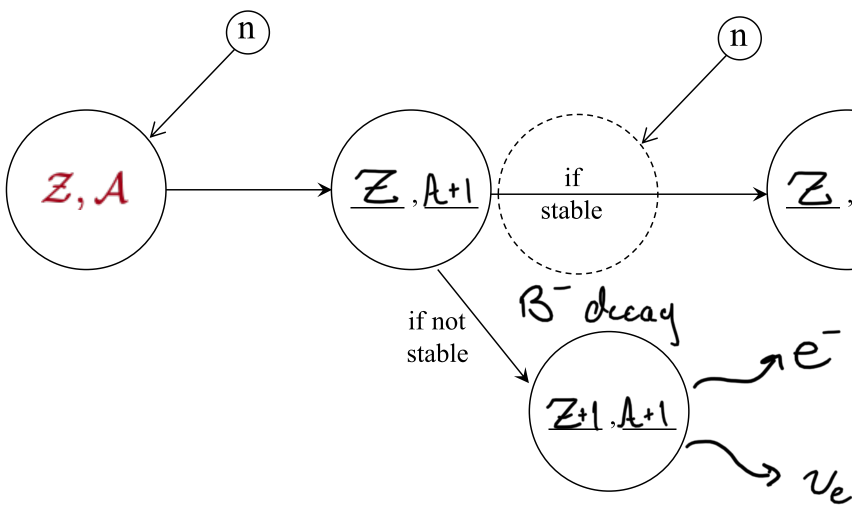
$$\tau_n \sim \frac{10^9\,\text{yr}}{n_n} = \frac{10^9\,\text{yr}}{10^5\,\text{cm}^{-3}} = 10^4\,\text{yr cm}^3$$

(c) Next, consider an environment in which the neutron density is high, $n_n \sim 10^{23}$ cm^{-3}. Find the lifetime of the nucleus against neutron capture in this region.

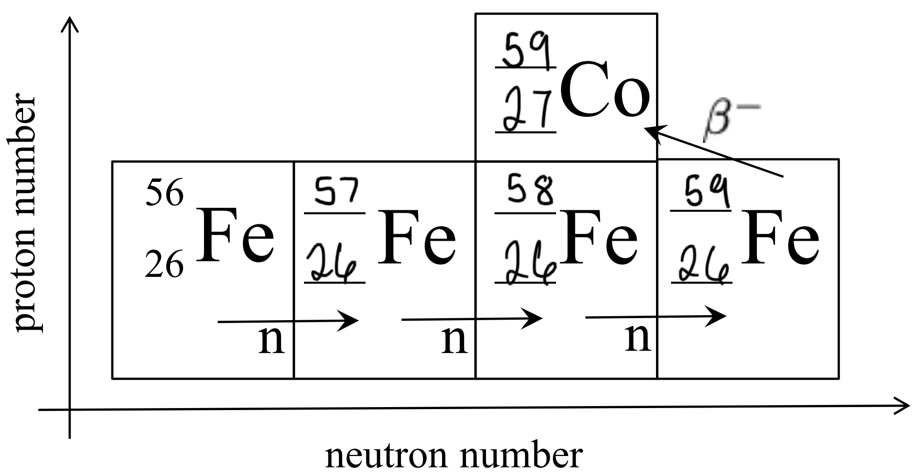
$$\tau_n \sim \frac{10^9\,\text{yr}}{n_n} = \frac{10^9\,\text{yr}}{10^{23}\,\text{cm}^{-3}} = 1\times10^{-14}\,\text{yr cm}^3$$

3. The s-process corresponds to your calculation in Question 2(b) for which the lifetime of the nucleus is long, compared to β-decay (which is typically on the order of hours). Thus, in time, the nucleus will capture a neutron undergoing the reaction in equation (1) in Question 1(b). If this nucleus is stable, the next higher nucleus will be created, as in equation (3). Instead, if this nucleus is unstable to β-decay, the decay will occur before the next neutron capture. Thus, the s-process proceeds along the valley of stability (in the $p-n$ graph) forming a series of stable nuclei, until an unstable nucleus is formed, which will then undergo β-decay to reach a stable isotope.

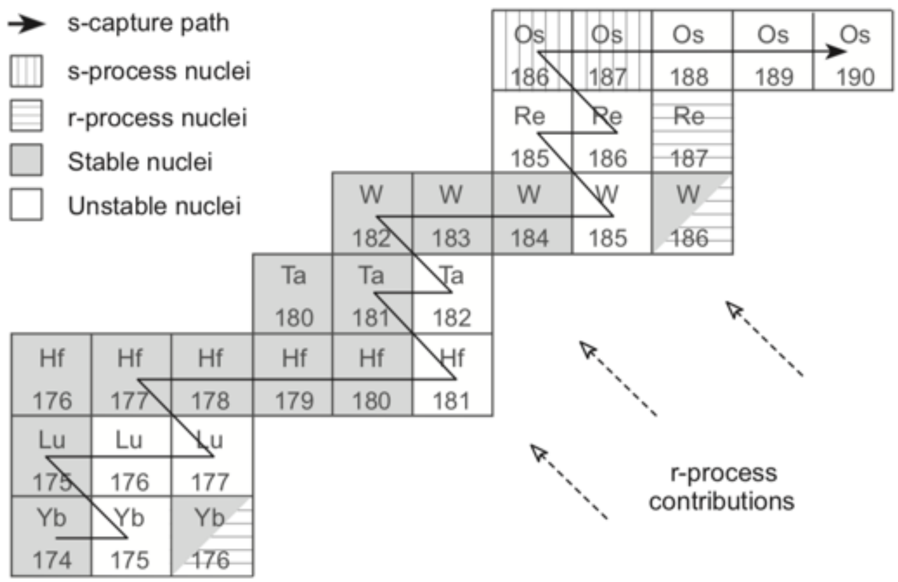
(a) Use the information above to fill in the schematic below for the s-process.



(b) Let's look at an actual example of the s-process that involves capture of neutrons to form a series of stable nuclei, until an unstable nucleus is formed that β-decays. Use what you learned from part (a) above to fill in the blank spaces below.



4. Together, the s-process and r-process allows for the heavier elements to be built. Some nuclei can be built through s-processes only, some through r-processes only, and some through both. Consider the figure below which shows the building of the elements in the Yb-Os region.



(a) For Yb (Ytterbium), $\mathcal{Z} = 70$. Follow the s-process starting from Yb ($\mathcal{Z} = 70$, $\mathcal{A} = 174$) and ending at Os ($\mathcal{Z} = 76$, $\mathcal{A} = 190$), and verify it all works as expected.

(b) Notice how the path stays very near the valley of stability. The r-process generally populates very neutron-rich isotopes that subsequently β-decay toward the valley of stability. Explain why some isotopes like ^{186}W can be populated ***only*** by the r-process.

Unstable isotope lies to the left blocking the s-process path.

5. It can be shown the the product $\sigma_{\mathcal{A}} N_{\mathcal{A}}$ is approximately constant along the *s*-process path (although more sophisticated analyses show that it is a slowly varying function of $\mathcal{A}$), where $\sigma_{\mathcal{A}}$ is the cross section for neutron capture and $N_{\mathcal{A}}$ is the abundance of the nuclei.

(a) Explain why the relative abundance of a given *s*-process element depends, according to this relation, on its neutron capture cross-section.

$\sigma_A N_A \approx$ constant

σ_A is low, nuclei from previous will build up

σ_A is high isotope will quickly move to next

(b) If you look at a graph of cross section *vs*. atomic weight, you'll see that the cross section has minimums at values of neutron number $n = 50, 82$, and 126. Explain why.

Note: In the graphs shown in *Dalsgaard*, the neutron number is written in uppercase; take care you don't confuse it with the abundance.

Nuclear magic number, If a neutron shell or Proton shell is filled in a nucleus, the nucleus is more stable. Nuclei in which neutron shells have filled ($n = 50, 82, 126$) won't absorb another neutron, so their cross section will be low