**PHY 420** **Spring 2021**

**Class Summary—Week 4, Day 2—Thursday, Apr 22**

## Radiation: The story so far . . .

In order to learn about how electromagnetic waves are generated, we began with a system of oscillating charges in otherwise empty space. We assumed a harmonic time dependence for a system of charges and currents that vary in time, e.g.,

$$\rho(\vec{x},t) = \rho(\vec{x})\, e^{-i\omega t}, \qquad \vec{J}(\vec{x},t) = \vec{J}(\vec{x})\, e^{-i\omega t}$$

and likewise for all potentials and fields, e.g., for $\Phi, \vec{A}, \vec{E}, \vec{B}, \vec{D}, \vec{H}$.

We then applied the Green function technique to obtain an expression for the **vector potential** $\vec{A}(\vec{x})$ of a localized system of charges and currents that vary sinusoidally in time:

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{x}\,') \frac{e^{ik|\vec{x}-\vec{x}\,'|}}{|\vec{x}-\vec{x}\,'|}\, d^3x' \tag{9.3}$$

Instead of directly evaluating equation (9.3), however, we defined three **zones of interest**:

- the **near (or static) zone**: $d \ll r \ll \lambda$;
- the **intermediate (or induction) zone**: $d \ll r \sim \lambda$; and
- the **far (or radiation) zone**: $d \ll \lambda \ll r$

where $r$ is the distance to the observation point: $r = |\vec{x}|$.

Henceforth, we will concern ourselves mostly with the far (radiation) zone, since it is where we are most of the time for the major sources of electromagnetic radiation — our detectors are always located a great number of wavelengths away from the sources.

In the far (radiation) zone, the observation point $r$ is very far from the source and much larger than the wavelength of the light. Thus, since $(r/\lambda) \gg 1$ and $k = 2\pi/\lambda$, we have $kr \gg 1$ in the far zone. To proceed, we used the relation

$$\left|\vec{x}-\vec{x}\,'\right| \simeq r - \hat{n}\cdot\vec{x}\,' \tag{9.7}$$

where $\hat{n}$ is a unit vector in the direction of $\vec{x}$. In fact, *from your derivation of this approximation on Tuesday's worksheet*, you know that equation (9.7) is valid for $r \gg d$ (independent of $kr$), so it is a reasonable approximation even in the near zone. With the approximation in equation (9.7), the **vector potential** can be written as

$$\lim_{kr\to\infty} \vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \vec{J}(\vec{x}\,')\, e^{-ik\hat{n}\cdot\vec{x}\,'}\, d^3x' \tag{9.8}$$

where I've used the limit to signify that we're in the far zone. Notice that $e^{ikr}/r$ is just an outgoing spherical wave, so equation (9.8) tells us that in the far zone the vector potential behaves as an outgoing spherical wave times a coefficient that depends on an integral over the source.

*On Tuesday's worksheet*, you showed by writing the exponential term as a series expansion and taking the summation and some terms outside the integral, that

$$\lim_{kr\to\infty} \vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\,\frac{e^{ikr}}{r}\sum_{n=0}^{\infty}\frac{(-ik)^n}{n!}\int \vec{J}(\vec{x}\,')\,(\hat{n}\cdot\vec{x}\,')^n\,d^3x' \tag{9.9}$$

The magnitude of the $n^{\text{th}}$ term in equation (9.9) above is given by

$$\frac{1}{n!}\int \vec{J}(\vec{x}\,')\,(k\hat{n}\cdot\vec{x}\,')^n\,d^3x' \tag{9.10}$$

Since the order of magnitude of $\vec{x}\,'$ is $d$ and $kd \ll 1$, the successive terms in the expansion of $\vec{A}(\vec{x})$ written in equation (9.9) fall off rapidly with $n$. Consequently, the radiation emitted from the source will come mainly from the first non vanishing term in the expansion of equation (9.9).

**There isn't much material for today because I decided to do a mini-lecture to go over important points from Tuesday's class, and then let you complete Questions 4-6 that we didn't get to on Tuesday's worksheet**.