Two numbers

(1) The general Friedmann equation can be written as

$$\frac{\dot{a}^2}{a^2} = H_o^2 \left[\frac{\Omega_{r,o}}{a^4} + \frac{\Omega_{m,o}}{a^3} + \Omega_\Lambda + \frac{1-\Omega_o}{a^2} \right] \tag{1}$$

(a) In order to confirm any particular model of the universe (e.g. single component, multicomponent, curvature, etc), what quantities in the Friedmann equation, Eq (1), do we need to *measure.* Which quantities can and cannot be directly measured?

(b) In order to obtain $a(t)$ and H_o, what basic physical quantity do we need to measure?

(2) In the lecture, we've Taylor expanded the scale factor to arrive at

$$a(t) = a(t_o) + (t - t_o) \left. \frac{da}{dt} \right|_{t=t_o} + \frac{(t-t_o)^2}{2} \left. \frac{d^2a}{dt^2} \right|_{t=t_o} + \cdots \tag{2}$$

You'll now do some manipulations of this expression.

(a) Rewrite Eq. (2) by dividing both sides by $a(t_o)$ and using the fact that $H_o = \dot{a}(t_o)/a(t_o)$.

(b) Define a new quantity called the *deceleration* parameter as

$$q_o = -\frac{\ddot{a}(t_o)}{a(t_o)} \frac{1}{H_o^2} = -\frac{a(t_o)\ddot{a}(t_o)}{\dot{a}(t_o)}$$

to rewrite your result in part(a) using q_o.

(c) Now recall that the acceleration equation is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\sum_w (\epsilon_w(1+3w)$$

Divide this expression by $-H_o^2$ and rewrite using the q_o parameter

(d) Using (c) find q_o for the case of non-relativistic matter, $w = 0$. Notice, that if we could measure q_o, this gives us a way of finding Ω_m, without actually having to weigh all the stuff in the universe.

(3) We've seen that we can write,

$$q_o = \frac{1}{2}\sum_w \Omega_{w,o}(1+3w).$$

(a) Consider the radiation + matter + Λ universe. Find q_o in term of the Ω_w.

(b) If $\Omega_\Lambda > \Omega_{r,o} + \Omega_{m,o}/2$, is q_o negative or positive? How does the acceleration of the universe relate to the sign of q_o.

(4) In the lecture we saw that

$$\frac{1}{a(t)} \approx 1 - H_o(t-t_o) + \left(\frac{1+q_o}{2}\right) H_o^2(t-t_o)^2 + \cdots$$

Use this expression to calculate the integral,

$$d_p = c\int_{t_e}^{t_o} \frac{dt}{a(t)}$$

to second order in $(t_o - t_e)$

(5) Has this solved our problem, that is, has this given us d_p so that we can solve H_o?

(6) Summarize what we've done thus far today. Make sure you note all the key ideas. What, if any, progress has been made. How we tried to overcome problems when ran into them. Be specific and make sure that the idea of measurement plays a key role in your answers.

(7) Repeat question 6 but now with the new information we've just developed in the lecture especially concentrating what quantities we need information on to get distance measurements..

Homework 03–Due Friday, Feb. 21

1. Problem 6.3
2. Problem 6.8
3. Problem 8.2
4. Problem 8.4