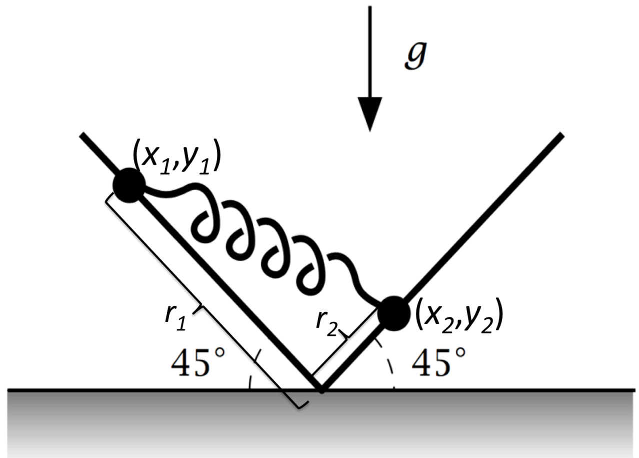
Use the principle of virtual work in the form

$$\frac{\partial V}{\partial q_\alpha} = 0$$

to find the equilibrium configuration of the two particles shown below. Both particles have mass *m*, the spring constant is *k,* and the rest length of the spring is *l.*



$$V = mgy_1 + mgy_2 + \frac{1}{2}k\left(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} - l\right)^2$$

In terms of generalized coordinates r_1, r_2:

$$V = mgr_1 \sin(45°) + mgr_2 \sin(45°) + \frac{1}{2}k\left(\sqrt{r_1^2 + r_2^2} - l\right)^2$$

$$\frac{\partial V}{\partial r_1} = \frac{mg}{\sqrt{2}} + k\left(\sqrt{r_1^2 + r_2^2} - l\right)\frac{r_1}{\sqrt{r_1^2 + r_2^2}} = 0$$

$$\frac{\partial V}{\partial r_2} = \frac{mg}{\sqrt{2}} + k\left(\sqrt{r_1^2 + r_2^2} - l\right)\frac{r_2}{\sqrt{r_1^2 + r_2^2}} = 0$$

If $r_1 \neq 0, r_2 \neq 0$: Subtract these two equations from each other:

$$2k\left(\sqrt{r_1^2 + r_2^2} - l\right)\frac{r_2 - r_1}{\sqrt{r_1^2 + r_2^2}} = 0 \;\Rightarrow r_1 = r_2$$

To find the value of r_1 and r_2 we can set $r_2 = r_1$ in the equation for $\frac{\partial V}{\partial r_1}$:

$$\frac{mg}{\sqrt{2}} + k\left(\sqrt{r_1^2 + r_1^2} - l\right)\frac{r_1}{\sqrt{r_1^2 + r_1^2}} = 0 \Rightarrow \frac{mg}{\sqrt{2}} + k\left(\sqrt{2}r_1 - l\right)\frac{1}{\sqrt{2}} = 0$$

$$\frac{1}{\sqrt{2}}\left(mg + \sqrt{2}kr_1 - kl\right) = 0 \Rightarrow r_1 = r_2 = \frac{1}{\sqrt{2}}(l - \frac{mg}{k})$$