**PHY 420** **Spring 2021**

## Homework 3—due by 9:00 PM, Monday, Apr 19

**You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.**

***Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Friday (Apr 23). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.***

**1.** A hollow right circular cylinder of radius $b$ has its axis coincident with the $z$ axis and its ends at $z = 0$ and $z = L$. The potential on the end faces of the cylinder is zero, while the potential on the cylindrical surface is $V(\phi, z)$.

Using the appropriate separation of variables in cylindrical coordinates, find a series solution for the potential anywhere inside the cylinder.

**2.** For the same arrangement as in Question 1 above, suppose the cylindrical surface is made up of two equal half-cylinders so that

$$V(\phi, z) = \begin{cases} V & \text{for } -\dfrac{\pi}{2} < \phi < \dfrac{\pi}{2} \\ -V & \text{for } \dfrac{\pi}{2} < \phi < \dfrac{3\pi}{2} \end{cases}$$

Find the potential inside the cylinder.

**Note:** You may use your results from Question 1; there is no need to derive them again.

**3.** The solutions $J_{\pm\nu}(x)$ are called *Bessel functions of the first kind* of order $\pm\nu$. If $\nu$ is an integer, the solutions are linearly dependent. For $\nu = m$, an integer, show that we get

$$J_{-m}(x) = (-1)^m J_m(x)$$

**4.** Explore plots of the modified Bessel functions $I_\nu(x)$ and $K_\nu(x)$ in Matlab (or Python) by plotting the following:

$$I_0(x), I_1(x), I_2(x), I_3(x) \qquad \text{and} \qquad K_0(x), K_1(x), K_2(x), K_3(x)$$

Submit your code. Plots generated using an online calculator like Desmos will get zero points.

**Note:** Both functions go to infinity on one end, and some may be zero on the other end, so you will need to work with the axes limits until you get a reasonably good visual representation.