PHY 420 Midterm Examination Help Document

Maxwell Equations:

$$\vec{\nabla}\cdot\vec{D}=\rho \qquad \vec{\nabla}\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0$$

Continuity equation:

$$\vec{\nabla}\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0$$

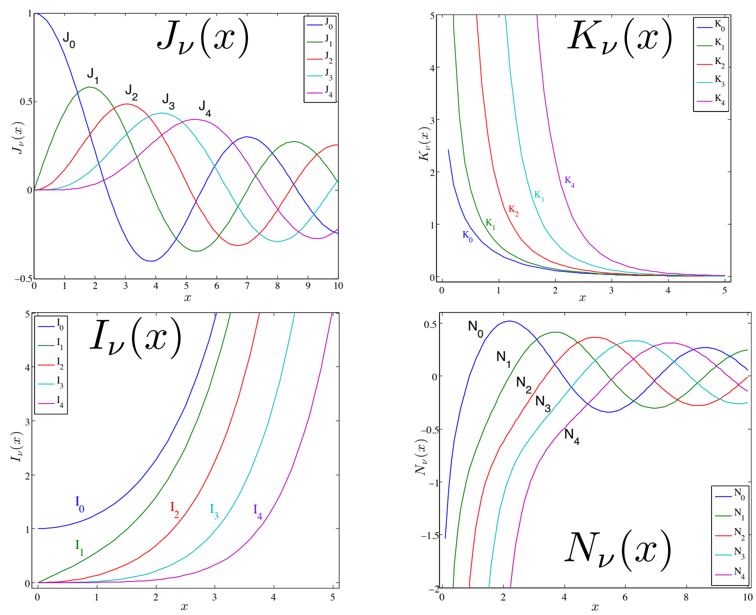
Laplace equation in cylindrical coordinates (ρ,ϕ,z):

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2}+\frac{\partial^2\Phi}{\partial z^2}=0$$

Useful Functions:

$$\cosh x=\frac{e^x+e^{-x}}{2}$$

$$\sinh x=\frac{e^x-e^{-x}}{2}$$



Macroscopic Definitions:

$$\vec{D}=\epsilon_0\vec{E}+\vec{P} \qquad \vec{H}=\frac{1}{\mu_0}\vec{B}-\vec{M}$$

Energy and Momentum:

$$W=\frac{1}{8\pi\epsilon_0}\sum_i\sum_j\frac{q_iq_j}{|\vec{x}_i-\vec{x}_j|} \qquad W=\frac{1}{2}\int\rho(\vec{x})\,\Phi(\vec{x})\,d^3x \qquad W=\frac{\epsilon_0}{2}\int\left|\vec{E}\right|^2d^3x$$

$$Q_i=\sum_{j=1}^{n}C_{ij}V_j \qquad W=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}C_{ij}V_iV_j \qquad W=-\frac{1}{2}\int_{V_1}\vec{P}\cdot\vec{E}_0\,d^3x \qquad W=\frac{1}{2}\int_{V_1}\vec{M}\cdot\vec{B}_0\,d^3x$$

Maxwell Stress Tensor: $T_{\alpha\beta}=\epsilon_0\left[E_\alpha E_\beta+c^2B_\alpha B_\beta-\frac{1}{2}\left(\vec{E}\cdot\vec{E}+c^2\vec{B}\cdot\vec{B}\right)\delta_{\alpha\beta}\right]$

$$\frac{d}{dt}\left[\left(\vec{P}_{\text{mech}}+\vec{P}_{\text{field}}\right)_\alpha\right]=\oint_S\sum_\beta T_{\alpha\beta}\,n_\beta\,da$$

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Radiation:

$$\vec{p} = \int \vec{x}\,' \,\rho(\vec{x}\,')\, d^3x'$$

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2}\, k^4 \left|\hat{n} \times \vec{p}\right|^2$$

$$q_{lm} = \int Y^*_{lm}(\theta', \phi')\, r'^{\,l}\, \rho(\vec{x}\,')\, d^3x'$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

Coordinate System Information:

Volume Element—Spherical (r, θ, ϕ): $r^2 \sin\theta\, dr\, d\theta\, d\phi$

Cylindrical (ρ, ϕ, z): $\rho\, d\rho\, d\phi\, dz$

Unit vectors—Spherical to Cartesian:

$$\hat{r} = \hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta$$

$$\hat{\theta} = \hat{x} \cos\theta \cos\phi + \hat{y} \cos\theta \sin\phi - \hat{z} \sin\theta$$

$$\hat{\phi} = -\hat{x} \sin\phi + \hat{y} \cos\phi$$

Unit vectors—Cartesian to Spherical:

$$\hat{x} = \hat{r} \sin\theta \cos\phi + \hat{\theta} \cos\theta \cos\phi - \hat{\phi} \sin\phi$$

$$\hat{y} = \hat{r} \sin\theta \sin\phi + \hat{\theta} \cos\theta \sin\phi + \hat{\phi} \cos\phi$$

$$\hat{z} = \hat{r} \cos\theta - \hat{\theta} \sin\theta$$

$\boldsymbol{\delta}$-function information:

$$\int \delta(x)\, dx = 1$$

$$\int f(x)\, \delta(x-a)\, dx = f(a)$$

Vector Formulas:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

$$\vec{\nabla} \cdot (\psi \vec{a}) = \vec{a} \cdot \vec{\nabla}\psi + \psi \vec{\nabla} \cdot \vec{a}$$

$$\vec{\nabla} \times (\psi \vec{a}) = \vec{\nabla}\psi \times \vec{a} + \psi \vec{\nabla} \times \vec{a}$$

$$\vec{\nabla}(\vec{a} \cdot \vec{b}) = (\vec{a} \cdot \vec{\nabla})\vec{b} + (\vec{b} \cdot \vec{\nabla})\vec{a} + \vec{a} \times (\vec{\nabla} \times \vec{b}) + \vec{b} \times (\vec{\nabla} \times \vec{a})$$

$$\vec{\nabla} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{\nabla} \times \vec{a}) - \vec{a} \cdot (\vec{\nabla} \times \vec{b})$$

$$\vec{\nabla} \times (\vec{a} \times \vec{b}) = \vec{a}(\vec{\nabla} \cdot \vec{b}) - \vec{b}(\vec{\nabla} \cdot \vec{a}) + (\vec{b} \cdot \vec{\nabla})\vec{a} - (\vec{a} \cdot \vec{\nabla})\vec{b}$$

$$\vec{\nabla} \times \vec{\nabla}\psi = 0$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{a}) = 0$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \nabla^2 \vec{a}$$

$$\vec{\nabla}' \left(\frac{1}{|\vec{x} - \vec{x}\,'|} \right) = \frac{\vec{x} - \vec{x}\,'}{|\vec{x} - \vec{x}\,'|^3}$$