Applications of d’Alembert’s Principle

$$\sum_{\alpha=1}^{N}(F_\alpha^{ext} - \dot{p}_\alpha)\delta r_\alpha = 0$$

Particle falling in a gravitational field:

Force of gravity: $F^{ext} = -mg$

$$\delta W_G = -mg\delta z$$

Inertial force: $F_I = m\ddot{z}$

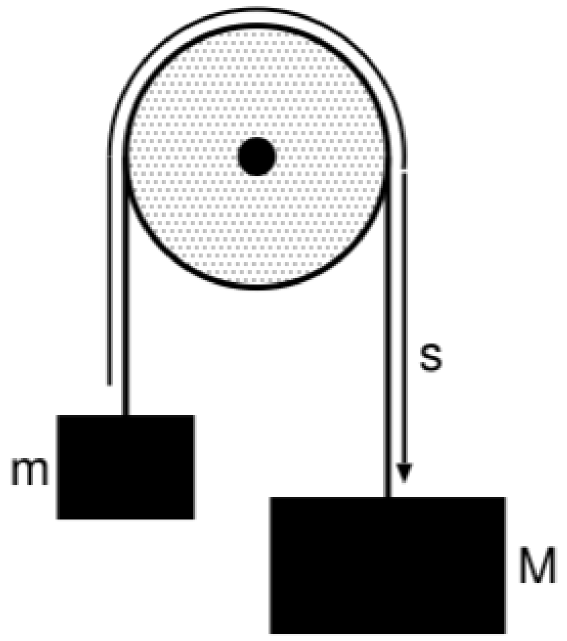
$$\delta W_I = m\ddot{z}\delta z$$

$$(-mg - m\ddot{z})\delta z = 0 \Leftrightarrow \ddot{z} = -g$$

Applications of d'Alembert's Principle

$$\sum_{\alpha=1}^{N} (F_\alpha^{ext} - \dot{p}_\alpha)\delta r_\alpha = 0$$

Atwood's Machine



Left side:

$$F_L^{ext} = -mg$$
$$\delta W_G^L = -mg\delta s$$

Right side:

$$F_R^{ext} = Mg$$
$$\delta W_G^R = Mg\delta s$$

Inertial force:

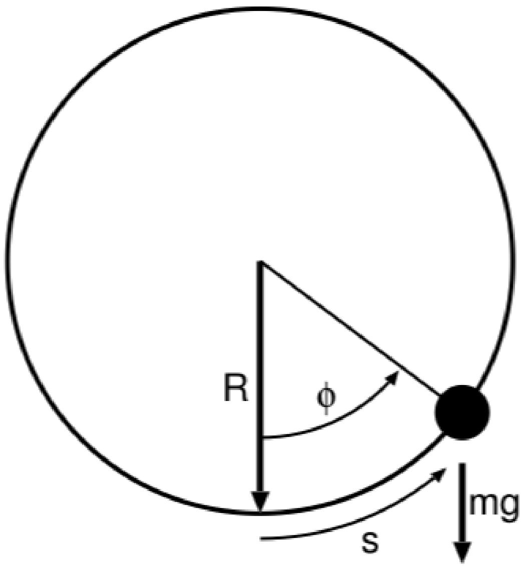
$$(M+m)\ddot{s}$$
$$\delta W_I = (M+m)\ddot{s}\delta s$$

$$[(Mg - mg) - (M+m)\ddot{s}]\delta s = 0$$
$$\ddot{s} = \frac{(M-m)}{M+m} g$$

Applications of d’Alembert’s Principle

$$\sum_{\alpha=1}^{N}(F_{\alpha}^{ext}-\dot{p}_{\alpha})\delta r_{\alpha}=0$$

Bead on a vertical hoop:



Radial component of gravity does no work

Tangential component of gravity: $F^{ext}=-mg\sin\phi$

$$\delta W_G=-mg\sin\phi\,\delta s=-mgRsin\phi\delta\phi$$

Radial component of the inertial force does no work

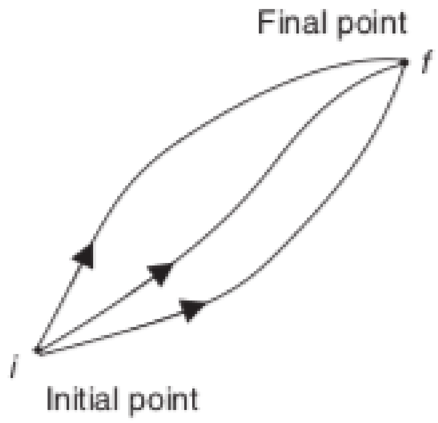
Tangential component of the inertial force: $\mathrm{m}\ddot{s}=mR\ddot{\phi}$

$$\delta W_I=mR\ddot{\phi}\delta s=mR^2\ddot{\phi}\delta\phi$$

$$\left(-mgRsin\phi-mR^2\ddot{\phi}\right)\delta\phi=0\Leftrightarrow\ddot{\phi}=-\frac{g}{R}\sin\phi$$

Hamilton’s Principle

Action integral: $I = \int_{t_1}^{t_2} L dt$ The integral of the Lagrangian over time from the initial time to the final time of the process



Which path will the particle take?

The path that will minimize the integral, so $\delta I = 0$!

$$I = \int_{t_1}^{t_2} (T - V) dt$$

“The average kinetic energy less the average potential energy is as little as possible for the path of an object going from one point to another.”

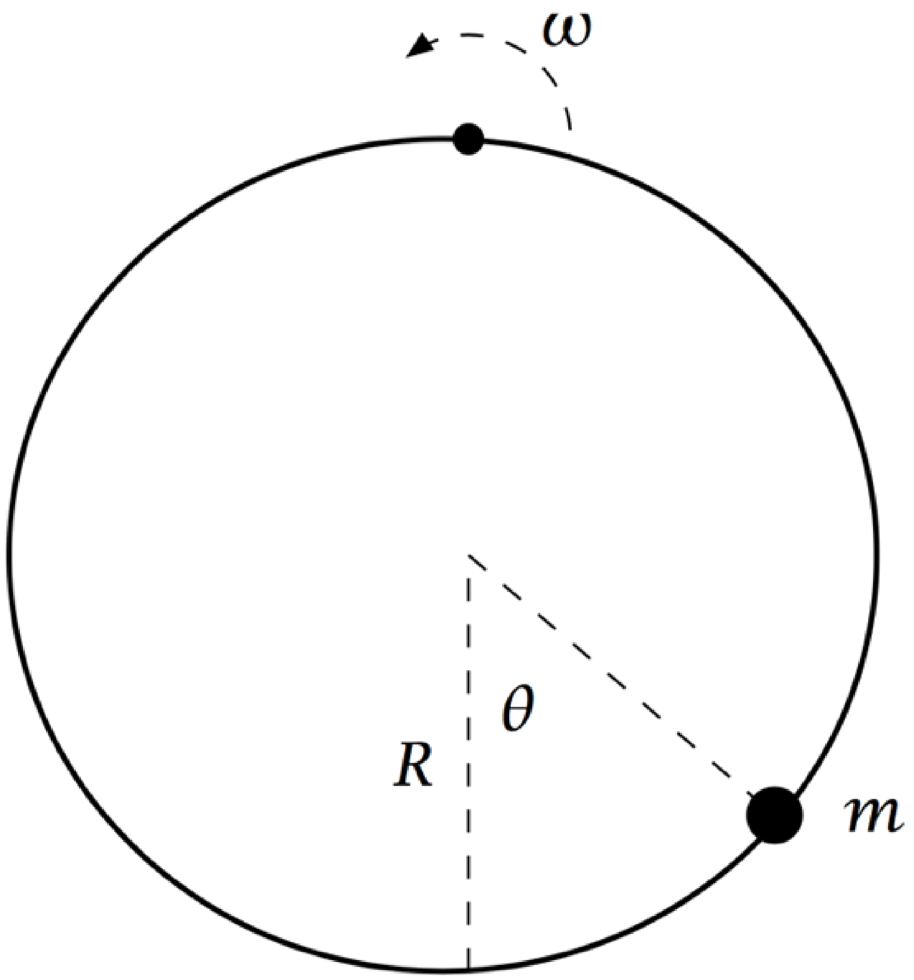
Hamilton’s Principle

$F = ma$ in Newtonian mechanics is easy to understand: Every moment the particle gets an acceleration from a force, so it knows what to do.

$\delta I = 0$ In Lagrangian mechanics is not so easy to understand (for me). The action is an integral over the entire path the particle will take. How does the particle know what path will have the smallest average kinetic energy less average potential energy? And why is it that a particle wants to follow a path where the average $T - V$ is minimized in the first place?

(This may be a good topic for a term project!)

Now something practical and and hopefully more intuitive:

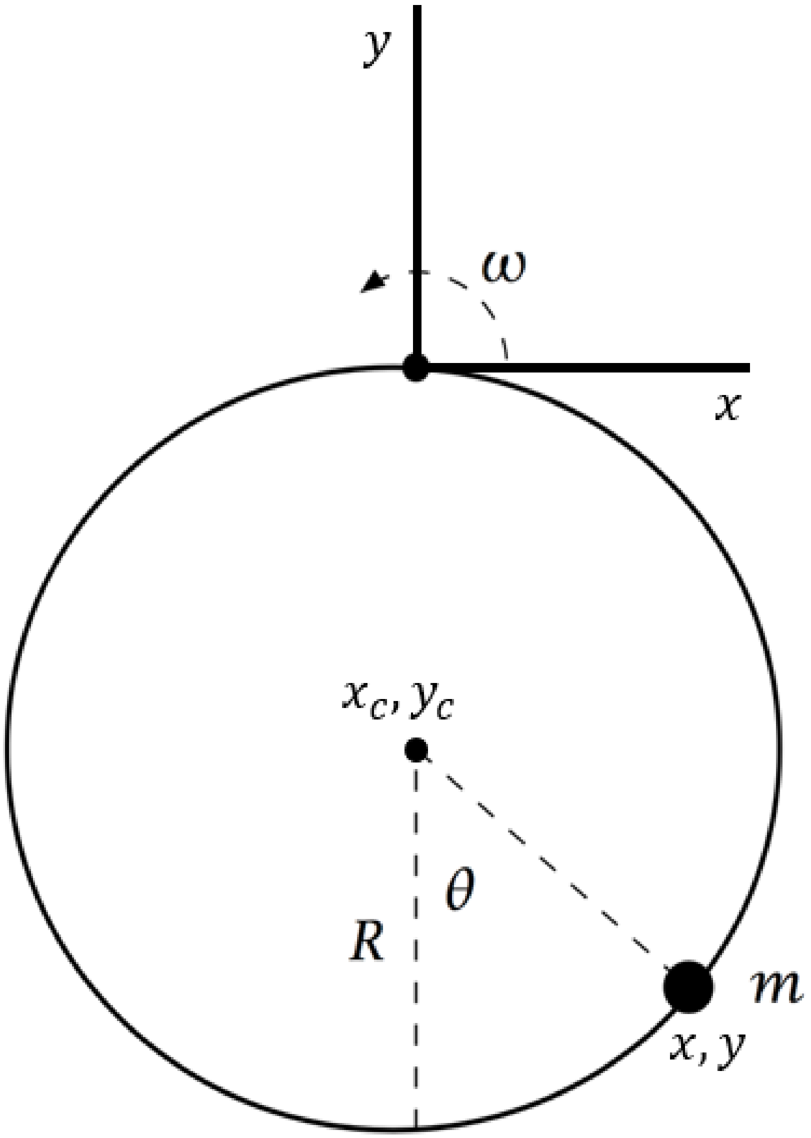


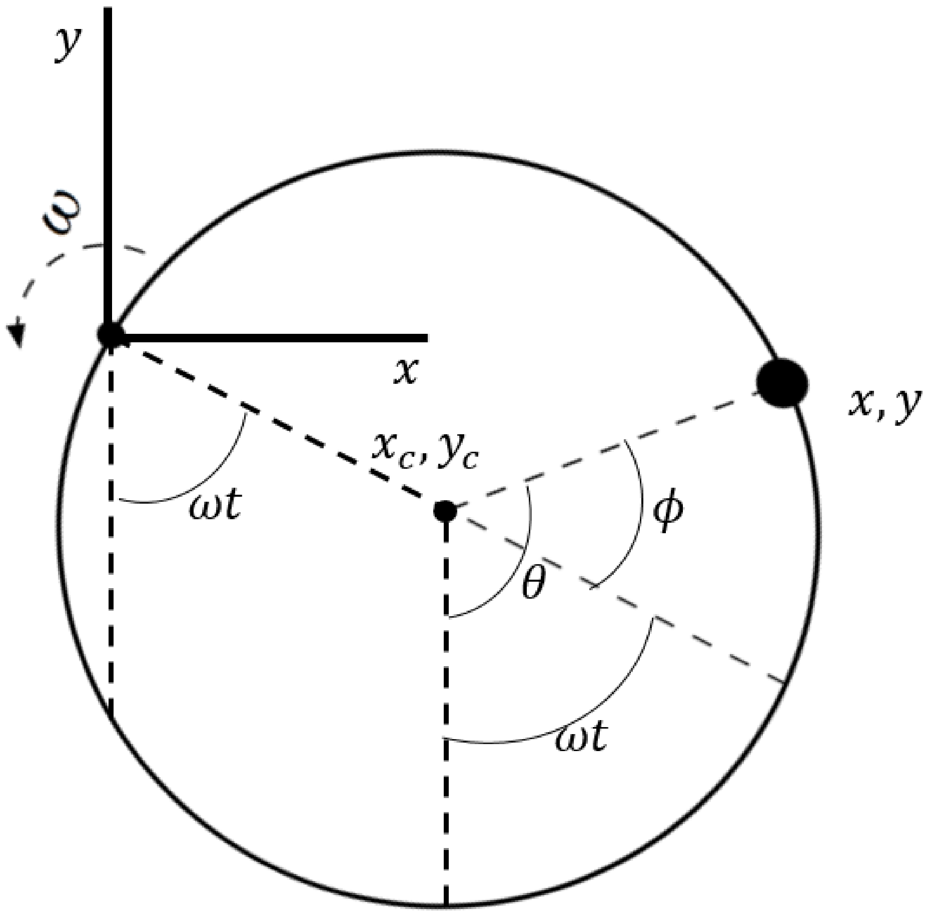
Horizontal hoop

Spinning horizontally with constant angular velocity

Bead with mass m, no friction.

How does the bead move?





Understanding $\phi = \theta - \omega t$