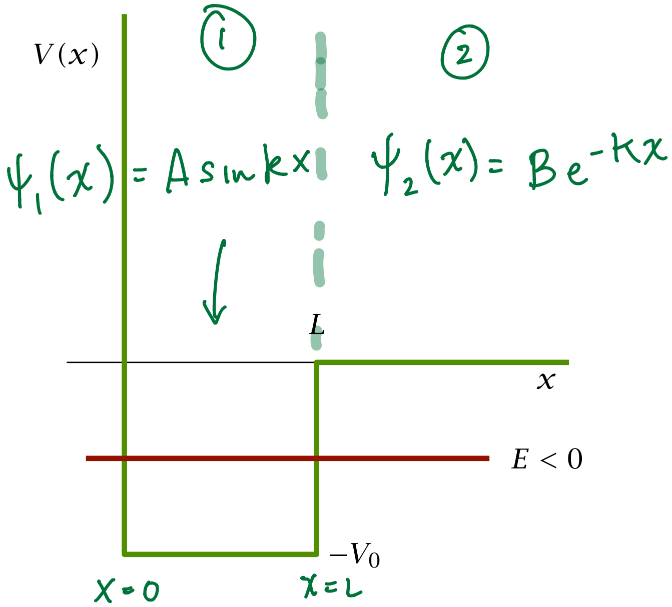
Physics 412—Bound States for Half of a Finite Square Well — Winter 2020

(1) Consider a particle in the finite square well, pictured below. Find the bound states of the particle (the energy eigenstates for which $E < 0$).



① $$-\frac{\hbar^2}{2m}\frac{d^2\psi_1}{dx^2} = (E+V_0)\psi_1$$

$$+\frac{\hbar^2 k^2}{2m} = E + V_0 \Rightarrow k = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

② $$-\frac{\hbar^2}{2m}\frac{d^2\psi_2}{dx^2} = E\psi_2$$

$$-\frac{\hbar^2\kappa^2}{2m} = E \qquad \kappa = \sqrt{\frac{-2mE}{\hbar^2}}$$

To do this, divide the system up into two regions: the region to the right of the well and the well itself. The energy eigenstates must satisfy the energy eigenvalue equation in both regions (Eq. (5.142) or Eq. (5.147) as appropriate), and they must be continuous and smooth (continuous first derivative) at the boundary where the two regions meet.

Boundary conditions:

$$\psi_1(L) = \psi_2(L), \qquad \psi_1'(L) = \psi_2'(L)$$

$$A\sin kL = Be^{-\kappa L} \qquad kA\cos kL = -\kappa Be^{-\kappa L}$$

Divide $$k\frac{\cos kL}{\sin kL} = -\kappa \Rightarrow \boxed{k\cot kL = -\kappa}$$

Given L & V_0 ... this fixes E.

This is the "odd" case for the finite square well.
These bound states are those functions & energies!