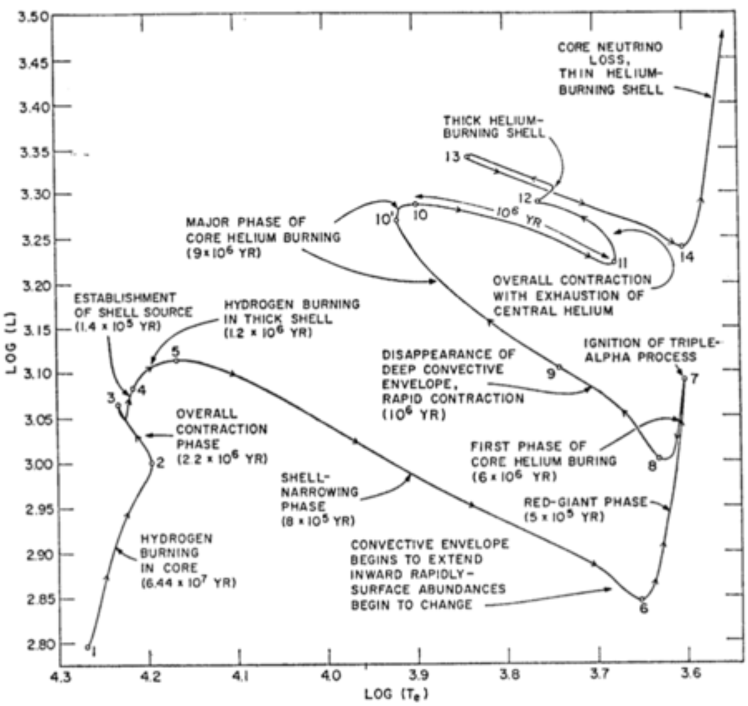
PHY 474/374 **Spring 2021**

Class Summary—Week 7, Day 2—Wednesday, May 12

Post Main Sequence Evolution

At the end of the Main Sequence stage, the star is left with a core of helium and a small amount of heavy elements. To understand what happens inside the star once it goes off the Main Sequence, it is best to follow the evolutionary track in an HR diagram. One can construct physical arguments for almost all of the track, although small loops here and there may not have simple physical explanations; they come out of the numerical simulations because of the details of how quantities are interacting and adjusting inside the star.



Consider the evolutionary track of a 5 $M_\odot$ star shown above (Iben 1967). It contains more detail than we want to go into, but will help in following the major events after the star goes off the Main Sequence.

We have already discussed the stage from point 1 to 2 in an earlier class; starting from ZAMS (at the point marked 1), the star moves up and to the right in the HR diagram during its lifetime on the Main Sequence.

At point 2, its H abundance has become very low (about 0.05 by mass; Iben 1967), so the star tries to continue energy production by raising its core temperature. It achieves this by overall contraction; this releases gravitational potential energy, half of which is converted to thermal energy that goes into heating the core. The rest escapes as radiation from the star, and is primarily responsible for the increase in luminosity (L) and surface (i.e., effective) temperature (T_e) from point 2 to point 3.

When it reaches point 3, the hydrogen in the core is almost completely exhausted (about a fraction of 1%; Iben 1967). We now have a helium core, but its temperature is not high enough yet to initiate He fusion (which requires about 100 million K).

Just outside the core, however, a shell of hydrogen has progressively built up the required density and temperature needed for hydrogen fusion. Between points 3 and 4, the region of major nuclear energy production shifts from the core to this thick shell concentric with the core, in which hydrogen is abundant. During this time, the core also becomes rapidly isothermal.

While the core is still in the process of contracting and cooling, the hydrogen shell just outside the core ignites. Iben (1967) states that this ignition is mildly explosive, in the sense that it pushes matter away in both directions from the shell. Matter in the envelope expands outward and the stellar radius increases. Since energy is required for this expansion, not all of the nuclear energy generated in the shell reaches the surface. This is why the luminosity decreases immediately following point 3 in the figure on the previous page. Another way to think about this is by using the **shell-burning law** in *Dalsgaard*, which some authors call the mirror principle. Very concisely, the shell-burning law maintains that shell sources produce mirrored motion of mass shells above and below them. Thus, if the matter interior to the shell contracts, then matter outside the shell will expand, and vice versa. This can be extended to multiple shells. Thus, as fusion ignites in the hydrogen shell concentric with and outside the core, core contraction is accompanied by expansion in the outer parts of the star.

Between points 3 and 4, the total rate of nuclear energy production in the H-shell increases with time. At point 4, conditions have again become reasonably stable. The release of gravitational energy in the core (which is now mostly helium) and the absorption of energy in the envelope of the star are minimal.

Between points 4 and 5, fusion of hydrogen occurs in a fairly thick shell containing about 5% of the star's mass (Iben 1967). Fusion in the shell adds more mass to the helium core, which continues to contract. Although I have found no direct reference to this, I assume that from points 4 to 5, the energy produced by fusion in the thick hydrogen shell is radiated away, thus the increase in luminosity in this segment. Moreover, since absorption of energy in the expanding envelope is minimal, the surface temperature must go down, and that is indeed what we are seeing in the HR diagram.

But why does the core collapse continue during this shell burning stage? To answer this question, we have to know about yet another interesting feature. With the sole energy source being in a concentric shell outside the core, the core cannot maintain a thermal gradient, and becomes isothermal. Fusion of hydrogen in the shell increases the core size (because hydrogen in the shell is becoming helium), but the helium core is still supported primarily by the pressure of the helium gas, which is still nondegenerate. There is a limit, though, to the mass of an isothermal helium core that can be supported by gas pressure. Called the Schönberg-Chandrasekhar limit, it is given by

$$M_c \simeq 0.37 \left(\frac{\mu_{\rm env}}{\mu_c}\right)^2 M$$

where M is the total mass of the star, M_c is the mass of the isothermal core, μ_c is the mean molecular weight in the core, and $\mu_{\rm env}$ is the mean molecular weight in the envelope. When the Schönberg-Chandrasekhar limit is reached, the core can no longer support itself, or the layers above it, against gravity and begins to collapse. As a 5 $M_\odot$ star leaves the Main Sequence, the mass of the helium core is already very close to this limit, and with the growth of the core during shell burning, this limit is quickly exceeded. This is why the core contracts; this collapse will continue until helium fusion ignites in the core, or the electron gas in the core becomes degenerate.

Past point 5 (see figure on page 1), with the Schönberg-Chandrasekhar limit exceeded, the core begins to contract more rapidly; applying the shell-burning law, the envelope also begins to expand more rapidly. But the hydrogen-fusing shell also narrows; thus, a decreasing rate of energy production in the shell and absorption of energy in the expanding radiative envelope together contribute to the decreasing luminosity from points 5 to 6. The star evolves so rapidly through the region between points 5 and 6 that the observational probability of seeing a star in this phase is very low; hence this part is called the **Hertzsprung gap**.

Within the expanding and cooling envelope while the star is moving toward point 6, the opacity is increasing. Shortly before point 6, the temperature gradient becomes steeper than the adiabatic gradient and convection becomes the dominant mode of energy transport in a growing region of the envelope extending inward from the surface. Iben (1967) then describes how, between the outer edge of the growing convective envelope and the surface where energy still flows by radiation, the dominant source of opacity is the H^- ion, in which electrons supplied by metals of low ionization potential are loosely attached to H. Opacity tends to act the opposite way for H^- ions; instead of increasing with decreasing temperature like it does conventionally, the opacity caused by H^- at the surface actually decreases with decreasing temperature. Since smaller opacities favor a larger energy flow, the star's luminosity begins to increase past point 6, even as its surface temperature decreases. This causes the star to climb the Hayashi track in the reverse direction to its pre-Main Sequence stage evolution, that is, the star rapidly climbs up the **Red Giant Branch (RGB)** from point 6 to 7. While on this segment, the star can exhibit significant mass loss; rates as large as $10^{-6}\ M_\odot\ \text{yr}^{-1}$ have been observed for RGB stars.

At point 7, the core temperature (and density) have become high enough to trigger the fusion of helium via the **triple-α process**. The first stage is

$$^4\text{He} + {}^4\text{He} \rightarrow {}^8\text{Be}$$

but ^{8}Be is very unstable, so the third ^{4}He must arrive within a short time to undergo the reaction

$$^8\text{Be} + {}^4\text{He} \rightarrow {}^{12}\text{C} + \gamma$$

Thus, this reaction is effectively a three-body process, and the energy generation rate is

$$\epsilon(3\alpha) \propto \rho^2 Y^3 T^{30}$$

Notice the strong temperature dependence, T^{30}, as opposed to T^4 for the PP-chain of hydrogen. Note also the dependence on $\rho^2 Y^3$, a consequence of the effectively three-body reaction, as opposed to the two-body PP interaction, which is proportional to ρX^2.

For stars above $\sim 2\ M_\odot$, this transition is quite smooth. For stars below about $2\ M_\odot$, however, contraction of the core up to point 7 has rendered it electron-degenerate. This is a state in which electrons are packed into their tightest possible configuration based on the Pauli Exclusion Principle, which tells us that no two electrons may occupy the same energy state. For non-degenerate matter, a rise in temperature due to the onset of fusion causes a corresponding rise in pressure. Not so for degenerate matter, for which the pressure is independent of the temperature. Thus, the rise in temperature leads to a rapid increase in reaction rates, the fusion reactions run faster, which raises temperature even further, and we get a thermonuclear runaway, called a **helium flash**. This continues until enough electrons are excited to lift the degeneracy. There are no observable consequences, however, since the enormous energy release is almost entirely absorbed in the envelope.

Once helium fusion begins, the release of energy in the core causes it to expand. The hydrogen shell outside the core is still present and undergoing fusion, so the outer parts of the star contract, as predicted by the shell-burning law. Thus, immediately after triple-α ignition, the luminosity of the star drops, reversing the direction of evolution, as the star moves down the Hayashi track (point 7 to 8).

Between points 7 and $10'$ shown in the figure (page 1), the envelope of the star continues to contract, and the surface temperature increases. Iben (1967) ascribes this behavior to the fact that the major source of energy production is still the shell undergoing hydrogen fusion. Averaged over the entire track between points 7 and 10, the shell contributes about 85% of the star's energy output. In order to maintain this energy production, matter in the shell must be kept at sufficiently high densities and temperatures. This is accomplished by the contraction of the envelope, which by the shell-burning law is in turn a result of the expansion of the core interior to the H-shell as a result of the triple-α reactions causing helium fusion. The contracting envelope compresses and heats the hydrogen-rich matter at the leading edge of the shell.

Iben (1967) points out that envelope contraction is particularly rapid between points 8 and 9 (figure, page 1). However, this phase of contraction is not associated with nuclear fusion processes. Instead, Iben (1967) associates it with a recession of the convective envelope which occurs as a result of decreasing opacity due to increasing temperature. As the dominant mode of energy flow switches from convection to radiation, matter in the envelope adjusts rapidly. The envelope of the star is again dominated by radiative energy transport, even as core helium fusion leads to the establishment of a convective central region, which grows with time during most of the core helium fusion phase from points 7 to 10.

As fusion depletes helium in the convective core, the fraction of ^{12}C increases, and it reacts with the still-available helium to form ^{16}O. Temperatures in the convective core continue to rise during the core-helium fusion phase. Between points 9 and 10, the rate of energy production in the core increases relative to the rate of energy production in the hydrogen-fusing shell.

As the helium fraction in the core decreases, a replay of earlier phases near the end of the Main Sequence phase begins to takes place. A little before point $10'$ (figure, page 1), the core begins to contract in order to heat itself. Between points $10'$ to 11, the star responds to core contraction in much the same manner as between points 5 to 6; core contraction is accompanied by envelope expansion, and the star begins to increase in size. Past point 10, the luminosity decreases as a result of decrease in the energy production rate in the hydrogen-fusing shell and radiative absorption in the expanding envelope.

Between points 11 and 12, an overall contraction takes place as all parts of the star move toward the center. This phase, associated with the exhaustion of helium in the core and the formation of a helium-fusing shell concentric with the core, is analogous to the phase from points 2 to 3 related to the exhausted of hydrogen in the core. Another way to think about this is with the shell-burning law; contraction in the core causes expansion between the helium and hydrogen shells, and a contraction of the envelope outside the hydrogen shell. The absorption of energy in the expanding matter between shells contributes to rapid weakening of the hydrogen-fusing shell.

Between points 12 and 13, helium fusion takes place in a thick shell outside the core, even as the helium-exhausted core continues to contract and heat steadily. The strength of the helium-fusing shell continues to increase relative to that of the narrow hydrogen-burning shell.

As point 13 is approached, the helium-fusing shell narrows rapidly. The matter outside the helium-fusing shell expands and cools. Such is the cooling that a little beyond point 13 the fusion in the hydrogen shell is halted. More recent calculations than Iben (1967) indicate that the hydrogen-fusing shell may be extinguished even earlier, immediately after the He-fusing shell ignites.

Evolution beyond point 13 is analogous to the earlier phase between points 5 and 7. The star now has only the helium shell source. The core, without any means to support itself, contracts rapidly and heats up. By the shell-burning law, the envelope outside the helium-fusing shell expands, decrease T_e. This expansion of the envelope absorbs energy from the helium-fusing shell, as a result of which the strength of helium fusion decreases in the shell. Convective transport eventually dominates over radiative in the cooling envelope, and as opacities drop near the surface, the luminosity rises and the star again goes up a Hayashi track, but this time at a higher luminosity than in the Red Giant Branch. This segment is called the **Asymptotic Giant Branch (AGB)**.

Evolution up the AGB is quite complex. A thermal instability develops in the helium shell source, and causes thermal pulses where the star alternates between having a hydrogen and a helium shell source. These lead to rapid changes in surface temperature and luminosity, and we will learn next week how heavier elements are also produced during this stage by the so-called s-process.

For reasons that are not fully understood, but likely due to the instabilities, the star begins to lose mass at a fairly rapid rate during its time on the Asymptotic Giant Branch. Stars with $M < 8M_\odot$will eventually lose essentially all the material outside the degenerate carbon-oxygen core, become Planetary Nebulae, and ultimately end up as White Dwarfs. More massive stars will initiate carbon fusion and continue on a different evolutionary track.

Lower Mass Stars

Stars with masses lower than 5 $M_\odot$ qualitatively display a similar progression in the post-Main Sequence stages, with some important differences. A major difference is that a 1 $M_\odot$ star does not have a convective core during the Main Sequence stage. Thus, hydrogen is used up initially only at the center, and there is a gradual transition from core hydrogen burning to hydrogen burning in a thick shell. The overall contraction phase accompanied by increasing surface temperature from point 2 to 3 (figure, page 1) becomes less pronounced with decreasing mass and disappears somewhere between 1.25 $M_\odot$ and 1 $M_\odot$ (see the figure for Question 3 on today's worksheet, except there the labeling is changed and 2 to 3 corresponds to 3 to 4!). This is because convection in the core of the larger mass stars ensures that hydrogen is reduced uniformly over a relatively large fraction of the nuclear burning region, whereas it reduces more gradually toward the center of a 1 $M_\odot$ star. Due to this quicker reduction over relatively large regions in the more massive stars, rapid contraction and heating must take place to maintain core burning at a sufficiently high level (Iben 1967). Also during the shell-narrowing phase from points 5 to 6 (figure, page 1) corresponding to points 7 to 10 in the figure for Question 3 on today's worksheet, the drop in luminosity decreases with decreasing mass, whereas the increase in luminosity between points 1 and 5 (figure, page 1) corresponding to points 1 to 7 in the figure for Question 3 on today's worksheet diminishes with increasing mass. Also, as noted before, unlike the gradual onset of the triple-α process in a 5 $M_\odot$ star, lower mass stars like 1 $M_\odot$ see an explosive event known as the helium flash, although this has no observable consequences since the explosion is confined deep within the star. Finally, the end points also differ; a 0.5 $M_\odot$ star will not undergo helium fusion and end up as a helium white dwarf, whereas a 1 $M_\odot$ star ends up as a carbon-oxygen white dwarf.