7.15

PHY 474/374 **Spring 2021**

Week 5—Wednesday, April 28—Discussion Worksheet

Energy Generation in Stars (continued)

Stars generate energy by nuclear fusion in their cores.

1. During most of their lifetime, stars generate energy from the fusion of hydrogen into helium. Schematically, the reaction is

$$4\,{}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2\mathrm{e}^+ + 2\nu_e \tag{8.45}$$

(a) Nuclear reactions must conserve *baryon number*, *lepton number*, and *charge*. Verify this is the case for the reaction in equation (8.45).

Note: *Quarks* and *leptons* are the basic building blocks of matter. Baryons (e.g., protons and neutrons) are made up of three quarks. There are six leptons in the Standard Model of Particle Physics: the electron, muon, and tau particles, and their associated neutrinos.

Baryon #: $4\,{}^1H \Rightarrow$ protons $\rightarrow$ 4 baryons on left hand side

$4\,He \Rightarrow \left.\begin{matrix}\text{2 protons}\\ \text{2 neutrons}\end{matrix}\right\} \rightarrow$ 4 baryons on right hand side

Baryon # conserved

Lepton #: Nothing on left hand side, 2 leptons (ν_e) and 2 anti-leptons (e^+, positrons) on right hand side

Lepton # conserved.

Charge: 4+ on left hand side, $\left.\begin{matrix}\text{2 protons}\\ \text{2 positrons}\end{matrix}\right\}$ 4+ on right hand side

charge conserved

(b) Compute the energy liberated in the reaction in equation (8.45).

Hint: Use the method from the previous class, subtracting the mass excess of the reactants (4^1H) and products (^{4}He). The mass excesses are tabulated in Appendix B in *Dalsgaard*.

$$Q = c^2\left[4\,\Delta m({}^1H) - \Delta m({}^4He)\right]$$

$$= 4\,(7.28899\ \mathrm{MeV}) - 2.42\ \mathrm{MeV}$$

$$= 26.73\ \mathrm{MeV}$$

2. The reaction shown in equation (8.45) does not take place as indicated in the formula since the probability that four protons can come together at a point and react is negligible; instead, it proceeds through one of two paths, each of which has some variations.

(a) One pathway for the fusion of H to He is the proton-proton (PP) chain. Using compact notation to represent the equation

$$A + a \to Y + y \qquad \text{as} \qquad A(a, y)Y$$

Dalsgaard writes the sequence of reactions in the PP-I chain (one of three main pathways in the PP chain) as

$$^{1}\mathrm{H}\,(^{1}\mathrm{H}, e^{+}\nu_e)\,^{2}\mathrm{H}\,(^{1}\mathrm{H}, \gamma)\,^{3}\mathrm{He}\,(^{3}\mathrm{He}, 2^{1}\mathrm{H})\,^{4}\mathrm{He}$$

Write out all the three equations in expanded form, **then verify** that the net reaction is indeed equation (8.45). Note that I've chosen to write ^{2}H for deuterium, instead of ^{2}D in *Dalsgaard*.

$^{1}H + {^{1}H} \rightarrow {^{2}H} + e^{+} + \nu_e \Rightarrow 2\,{^{1}H} \rightarrow {^{2}H} + e^{+} + \nu_e$

$^{2}H + {^{1}H} \rightarrow {^{3}He} + \gamma$

$^{3}He + {^{3}He} \rightarrow {^{4}He} + 2\,{^{1}H} \Rightarrow 2\,{^{3}He} \rightarrow {^{4}He} + 2\,{^{1}H}$

(b) The energy liberated is less than what you found in Question 1(b), because the neutrino carries away 0.263 MeV of energy on average in the $^{1}\mathrm{H}\,(^{1}\mathrm{H}, e^{+}\nu_e)\,^{2}\mathrm{H}$ reaction. Compute the effective energy liberated in the PP-I chain.

2 neutrinos

$Q_{eff} = 26.73\ \text{MeV} - (0.263\ \text{MeV})\,2$

$= 26.2\ \text{MeV}$

3. Following the production of ^{3}He, there are two additional pathways in the PP chain to produce ^{4}He.

(a) The PP-II chain (after the first two reactions in the PP-I) chain that produces ^{3}He is

$$^3\mathrm{He}\,(^4\mathrm{He},\gamma)\,^7\mathrm{Be}\,(\mathrm{e}^-,\nu_e)\,^7\mathrm{Li}\,(^1\mathrm{H},{}^4\mathrm{He})\,^4\mathrm{He}$$

whereas the PP-III chain (after the first two reactions in the PP-I) chain that produces ^{3}He is

$$^3\mathrm{He}\,(^4\mathrm{He},\gamma)\,^7\mathrm{Be}\,(^1\mathrm{H},\gamma)\,^8\mathrm{B}\,(,\mathrm{e}^+\nu_e)\,^8\mathrm{Be}\,(,{}^4\mathrm{He})\,^4\mathrm{He}$$

Write the first two reactions that are common to all three branches of the PP-chain on the two lines in the middle below, then fill in the remaining reactions in the spaces indicated.

$2\,^1H \rightarrow {}^2H + e^+ + \nu_e$

$^2H + {}^1H \rightarrow {}^3He + \gamma$

PP-I

$2\,^3He \rightarrow {}^4He + 2\,^1H$

$^3He + {}^4He \rightarrow {}^7Be + \gamma$

PP-II

$^7Be + e^- \rightarrow {}^7Li + \nu_e$

$^7Li + {}^1H \rightarrow {}^4He + {}^4He$

PP-III

$^7Be + {}^1H \rightarrow {}^8B + \gamma$

$^8B \rightarrow {}^8Be + e^+ + \nu_e$

$^8Be \rightarrow {}^4He + {}^4He$

(b) Identify the reactions that compete to decide the relative importance of the PP-I *vs*. the PP-II and PP-III branches, and the reactions that compete to decide the relative importance of the PP-II *vs*. the PP-III branches.

The PP-II has 1 less reaction than the PP-III branch.

4. Another pathway is possible for conversion of H to He in stars that contain C, N, and O. Summing net reactants and products around this so-called CNO cycle, we get

$$^{12}\text{C} + 4p \rightarrow {}^{12}\text{C} + {}^{4}\text{He} + 2e^{+} + 2\nu_e$$

(a) Based on the net reaction equation above, explain why ^{12}C is considered a catalyst in this cycle.

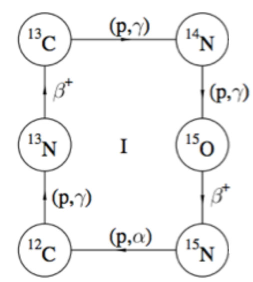
^{12}C is at the end of the sequence

$^{12}C\,(4p,\ ^4He\ 2e^+\ 2\nu_e)\ ^{12}C$

Stays unchanged (left and right side).

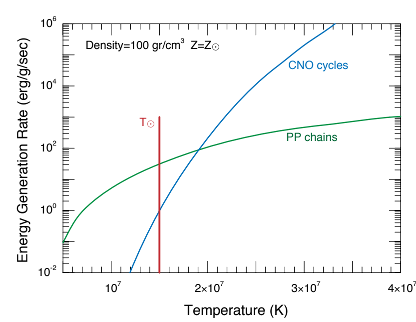
(b) Now, consider the full set of reactions shown in the graphic below (source: Li, Stanford). Explain why this is called a *cycle*, as opposed to *process*, and why C, N, O are called catalysts for this cycle.

It is a cycle because the reaction eventually repeats itself



(c) Consider the graph below (source: asu.edu). Which process, PP chain or CNO cycle, would you expect to dominate in our Sun? Justify.

PP chain → at Sun's temp (as marked), energy generation rate for pp chain >> that for CNO cycle



(d) Which process, PP chain or CNO cycle, was likely responsible for the generation of energy in the earliest generation of stars? Why?

High mass stars generate by CNO cycle, earliest stars (Despite being high mass) would not have C, N, O in their natal clouds, hence likely generated energy by P-P chains.

5. Begin doing Question 1(a) on Homework 3. The setup is a little tricky, so I'm hoping you will get into as much detail as possible during class.

In homework solutions.