Toy universes, the Benchmark Model, mid-term review

(1) The general Friedmann equation is

$$\frac{H^2}{H_o^2} = \frac{\Omega_{r,o}}{a^4} + \frac{\Omega_{m,o}}{a^3} + \Omega_\Lambda + \frac{1-\Omega_o}{a^2} \tag{1}$$

(a) Consider a universe consisting of matter, Lambda, and curvature. Write the Friedmann equation for this universe in a form that does not contain Ω_o.

(b) Qualitatively describe how the scale factor evolves in the case in which the following conditions hold:

- $\Omega_{m,o} > 0; \quad \Omega_\Lambda > 0; \quad \Omega_{m,o} + \Omega_\Lambda > 1$ and
- $a(t=0) >> 1; H(t=0) < 0.$

(c) Now consider a matter + radiation flat universe. Write the Friedmann equation for this universe.

(d) Consider the time of radiation-matter equality. The resulting equation in (c) is separable. Separate the equation and try to do the integral.

(2) The Benchmark model has been presented in the lecture. Using the given parameters, write the Friedmann equation for this model. Write the integral that solves for $H_o t$ for this model. And finally discuss the benchmark model by comparing it to the other toy universes we discussed.

Mid Term Review

1. Discuss the importance and relevance to, cosmology of the following observational facts:

   - The night sky is dark
   - the universe contains stuff
   - galaxies are moving away
   - The universe if filled with a background radiation
   - The universe is isotropic and homogeneous

2. What are the meanings of and the mathematical expressions for:

   - Redshift
   - Scale factor
   - Hubble parameter and Hubble law

3. From general relativity, give the meaning of, and where appropriate, the mathematical expression for:

   - Euclidean space
   - Curvature
   - Metric
   - Robertson-Walker metric and when its the appropriate metric to use

- Null geodesic
- Proper distance
- The relationship between Redshift and scale factor.

4. For items related to the Friedmann equation

   - Write the general Friedmann equation in all its different forms and discuss the advantage of each form
   - What is the fluid equation and how does the w parameter come into play.
   - What is the acceleration equation.
   - Write the Friedmann equation in terms of the density parameters. What is the meaning of the critial density and what is it's mathematical expression. How is Ω_o related to the curvature of the universe.
   - Write down the Friedmann equation for single component universes and discuss some of the behavior for each component.
   - Write down the Friedmann equation for examples of multi-component toy universes and discuss the physical consequences of these universes. Compare the toy universes to the one we seem to live in.