

S-8: I can explain the relationships between the energy eigenstates of the simple harmonic oscillator and use the eigenstates to make predictions about measurements.

Unsatisfactory

Progressing

Acceptable

Polished

(1) The harmonic oscillator Hamiltonian can be written in terms of the raising and lowering operators as

$$H = \hbar\omega_0 \left(a^\dagger a + \frac{I}{2} \right) = \hbar\omega_0 \left(N + \frac{I}{2} \right), \quad \text{where } N = a^\dagger a.$$

The raising and lowering operators have the following properties:

$$[a, a^\dagger] = I, \quad a|n\rangle = \sqrt{n}|n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \quad N|n\rangle = n|n\rangle.$$

Explain, as best you can, how we know the coefficients above are \sqrt{n} and $\sqrt{n+1}$. Where do these come from? Hint: Use the relationship between a , a^\dagger , and N , and the fact that the states $|n\rangle$ are eigenstates of N with eigenvalue n ...

(2) The momentum operator can be written in terms of the raising and lowering operators as

$$P = \frac{i\hbar}{\sqrt{2}d_0} (a^\dagger - a).$$

Let the state of a simple harmonic oscillator be

$$|\Psi\rangle = \frac{1}{5} (3|1\rangle + 4i|3\rangle).$$

Calculate the expectation value of the kinetic energy for this state, $\langle T \rangle = \langle P^2 \rangle / 2m$.