Physics 460—Homework Report 5
Due Tuesday, May 5, 1 pm

Name: Timothy Holmes

Complete all the problems on the accompanying assignment.

List all the problems you worked on in the space below. Circle the ones you fully completed:

1ab 2abcde

Please place the problems into the following categories:

- These problems helped me understand the concepts better: 2cde
- I found these problems fairly easy: 1ab 2ab
- I found these problems very challenging: _____

In the space below, show your work (even if not complete) for any problems you still have questions about. Indicate where in your work the question(s) arose, and ask specific questions that I can answer.

Use the back of this sheet or attach additional paper, if necessary.

If you have no remaining questions about this homework assignment, use this space for one of the following:

- Write one or two of your solutions here so that I can give you feedback on its clarity.
- Explain how you checked that your work is correct.

I found these problems to be much more understandable / doable than last weeks.

(1) An ensemble of pairs of spin-1/2 particles is in the state

$$|\Psi\rangle = \frac{1}{\sqrt{5}}|+;+\rangle + \frac{2\mathrm{i}}{\sqrt{5}}|-;+\rangle.$$

(a) You measure the z-component of the spin of both particles. What is the correlation coefficient for this measurement?

(b) Can you find a pair of axes that would lead to a correlation coefficient of $C = 1$ for measurements of the spin of these particles? If so, what are they? If not, why not?

(2) Consider the most general state for a system of two spin-1/2 particles:

$$|\Psi\rangle = a|+;+\rangle + b|+;-\rangle + c|-;+\rangle + d|-;-\rangle.$$

Here a, b, c, and d are complex constants subject to the normalization condition $|a|^2+|b|^2+|c|^2+|d|^2 = 1$.

(a) Suppose you want to find the most general state that has a correlation coefficient of $C = 1$ for measurements of the spin of both particles along the z axis. What does that tell you about the coefficients a, b, c, and d?

(b) Is your state from part (a) necessarily entangled? Explain.

(c) What is the relationship between the average values of the spins for the two particles, $\langle S_{z1}\rangle$ and $\langle S_{z2}\rangle$, for your state from part (a)? Explain or interpret your answer.

(d) What is the relationship between the uncertainties of the spins for the two particles, ΔS_{z1} and ΔS_{z2}, for your state from part (a)? Explain or interpret your answer.

(e) Suppose you want to find the most general state that has a correlation coefficient of $C = 1$ for measurements of the spin of both particles along the z axis and has the largest uncertainty for the spin of the first particle, ΔS_{z1}. What does that tell you about the coefficients a, b, c, and d?

(f) How do your answers to parts (a) through (e) change if you're looking for the state with $C = -1$? Explain.

Homework 5

(1) $|\psi\rangle = \frac{1}{\sqrt{5}}|+;+\rangle + \frac{2i}{\sqrt{5}}|-;+\rangle$

(a)

$|\psi\rangle = \frac{1}{\sqrt{5}}\left[|+;+\rangle + 2i|-;+\rangle\right] \quad |\psi^*\rangle = \frac{1}{\sqrt{5}}\left[|+;+\rangle - 2i|-;+\rangle\right]$

$C = P(+;+) + P(-;-) - P(+;-) - P(-;+)$

$P_{++} = |\langle +;+|\psi\rangle|^2 = 1/5 \qquad P_{--} = |\langle -;-|\psi\rangle|^2 = 0$

$P_{+-} = |\langle +;-|\psi\rangle|^2 = 0 \qquad P_{-+} = |\langle -;+|\psi\rangle|^2 = 4/5$

$C = 1/5 + 0 - 4/5 + 0 = -3/5$ Somewhat anti correlated?

(B) $C = 0 \rightarrow$ No relationship
Between two measurements $\rightarrow$ uncorrelated

No, there is no possible way to make this state correlated!

(2) $|\psi\rangle = a|+;+\rangle + b|+;-\rangle + c|-;+\rangle + d|-;-\rangle$

(a) A correlation of $C = 1$ would mean that the measured values are always aligned. If we are not able to change the state and

$C = P(+;+) + P(-;-) - P(+;-) - P(-;+)$

Then we would need $P(+;+)$ and $P(-;-)$ to sum to 1. Therefore, c and d would be 0, $a+b=1$.

(B) yes, they are correlated