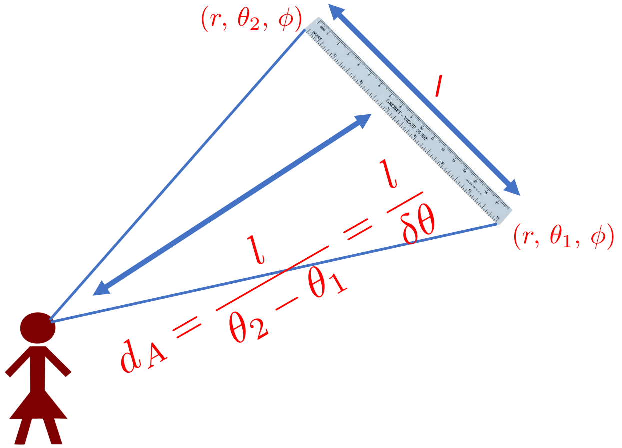
- **Proper distance** roughly corresponds to where a distant object would be at a specific moment of cosmological time, which can change over time due to the expansion of the universe.
- **Comoving distance** between fundamental observers does not change with time, as comoving distance accounts for the expansion of the universe.
- **Luminosity distance** is the distance associated with the amount of flux one measures from a distant object
- **Transverse comoving distance**
- **Angular diameter distance**
- **Light Travel distance**

$$d_p \approx \frac{c}{H_o} z \left[1 - \frac{1+q_o}{2} z\right] \qquad q_o = -\frac{\ddot{a}(t_o)}{a(t_o)} \frac{1}{H_o^2} = -\frac{a(t_o)\ddot{a}(t_o)}{\dot{a}(t_o)} \qquad f = \frac{L}{4\pi d_L^2}; \text{ or } d_L \equiv \left(\frac{L}{4\pi f}\right)^{1/2} \qquad \boxed{d_L = S_k(r)(1+z)}$$




But remember that the universe is expanding.

This means that the light you see coming from the ends of the ruler has undergone expansion. So

$$ds = l = a(t_e)\, S_k(r)\, \delta\theta = \frac{S_k(r)\delta\theta}{1+z}$$

And so

$$d_A = \frac{l}{\delta\theta} = \frac{S_k(r)}{1+z}$$

Galaxies and clusters of galaxies are both large enough to serve as standard yardsticks at large distances.

Do question (1) on the worksheet and S T O P

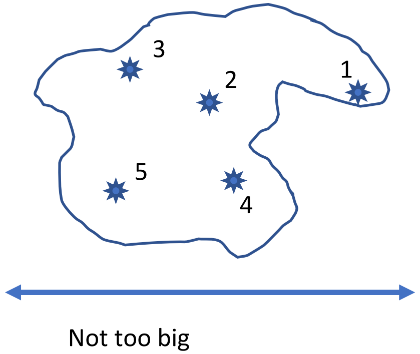
(1) Galaxies are not solid objects, cannot find the edges to find *l*.

We are back to this $f = \frac{L}{4\pi d_L^2}$; or $d_L \equiv \left(\frac{L}{4\pi f}\right)^{1/2}$. But we need to find a standard candle that we can use to find *L*

Recall we want a way to at least measure $H_o$, and to do so, we need a distance.

- Find a population of standard candles. This gives us *L.* They need to be bright enough that we can see them from far away.
- Measure the flux, *f*, and redshift, *z,* for each standard candle
- Compute, $d_L = \sqrt{L/4\pi f}$ for each standard candle
- Plot *cz* versus $d_L$
- Measure the slope of *cz* versus $d_L$ when *z<<1,* this will give $H_o$

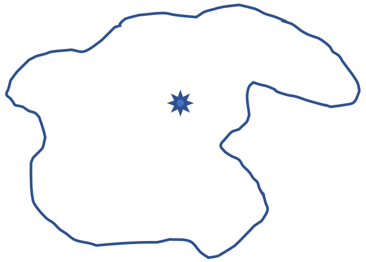
Cepheids, the work of Henrietta Leavitt




| Star | Period (days) | Mean flux* |
|---|---|---|
| 1 | 10 | 20 |
| 2 | 25 | 45 |
| 3 | 15 | 30 |
| 4 | 12 | 21 |
| 5 | 27 | 47 |

Difference in mean flux is due to their mean luminosity, not their distance.

So there is a period—luminosity relationship



Galaxy far away, star has a period 10 days, but a flux 100 times less than above.

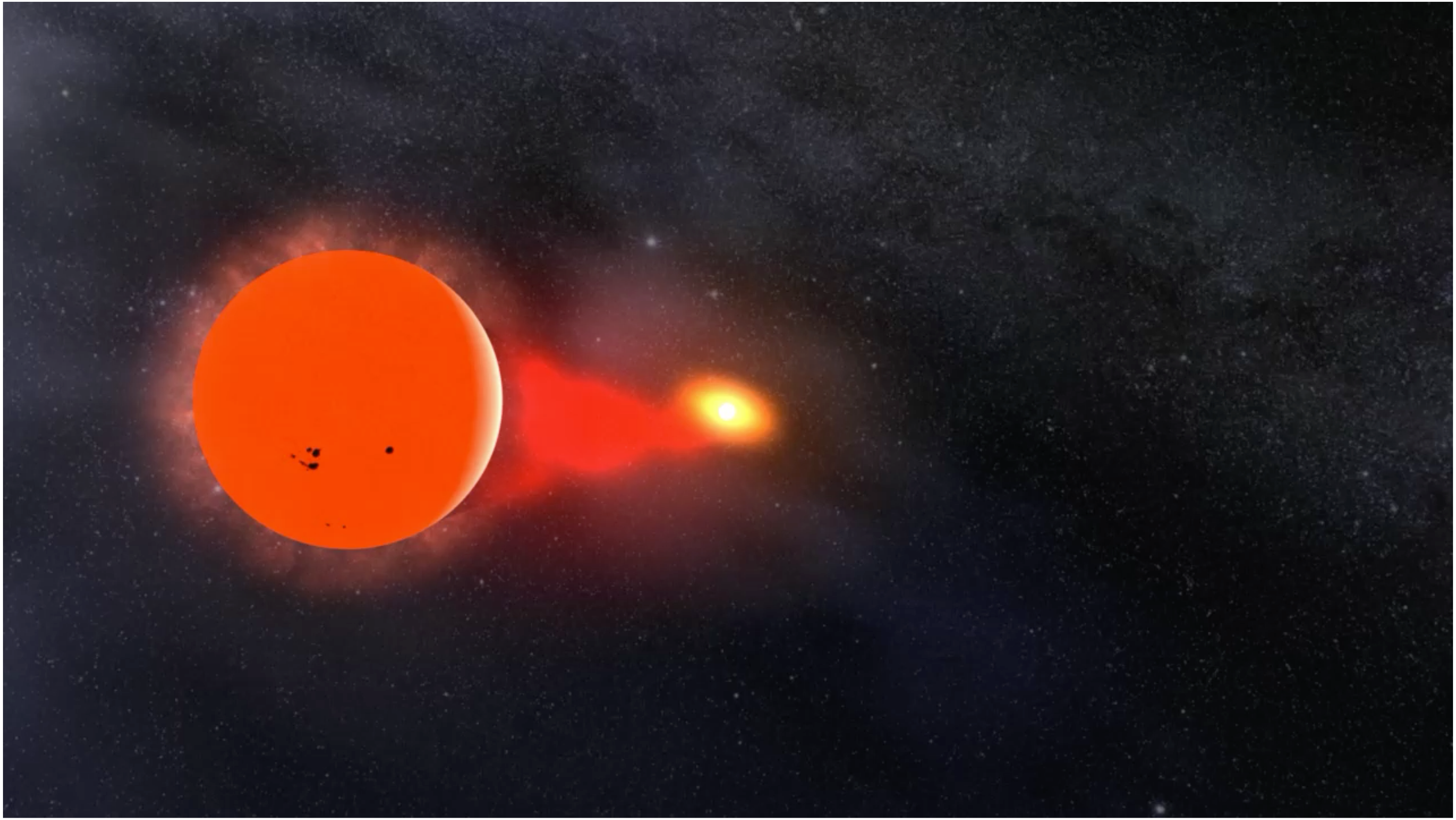
$$d_L = \left(\frac{L}{4\pi f}\right)^{1/2}$$

But from period, we know luminosity is the same as above, so

$$\frac{d_{L,1}}{d_{L,2}} = \left(\frac{\frac{L}{4\pi f_1}}{\frac{L}{4\pi f_2}}\right)^{1/2} = \left(\frac{f_2}{f_1}\right)^{1/2} = \sqrt{100} = 10$$ times further

Unfortunately, this only gets us to about 30 Mpc, and the universe is definitely not isotropic/homogeneous on these scales. We can't use Cepheids as standard candles for cosmology

Do question (2) on the worksheet and STOP



Type IA supernova

- Extremely luminous. A type IA supernova will outshine all the stars in the host galaxy for a few days
- We have a very good idea of their inherent luminosity, so we have a good standard candle.

Do question (3) on the worksheet and S T O P

(3)

- Find nearby Cepheid's whose distance can be found by accurate methods (parallax, etc) and determine their luminosity
- Find Cepheid's in nearby galaxies and measure the distances to those galaxies.
- Hope to find a Type 1A supernova in those galaxies and get their luminosity since we know the distance to the galaxy
- Use Type IA supernova to measure distances to many farther away galaxies, and determine $H_o$

# Cosmic Microwave Background (CMB )Radiation

- We've pushed the *distance* base tests of cosmology about as far as we can. Time to change tactics
- We observe the universe on cosmological scales is
  - Isotropic and homogeneous
  - Obeys the Robertson-Walker Metric and Friedmann equations
  - Is close flat with $\Omega_m$ = 0.3-ish and $\Omega_\Lambda$ = 0.7-ish
  - Filled with a background radiation

The universe is extremely bright in the microwave range. Some characteristics of the radiation include

- $T_o = 2.725\,\mathrm{K}$
- $\epsilon_{\gamma,o} = 0.261\,\frac{\mathrm{MeV}}{\mathrm{m}^3}$
- $n_{\gamma,o} = 4.11 \times 10^8\,\mathrm{m}^{-3}$
- $n_{\mathrm{baryons},o}/n_{\gamma,o} \approx 5 \times 10^{-10}$

The CMB is observed across the entire sky and there are 2 key important results of these observations that will need explaining

1. The CMB is very close to a blackbody spectrum at *T = 2.73*. Recall that a blackbody spectrum is

$$I_\nu = \frac{2h\nu^3}{c^2}\left[e^{h\nu/T} - 1\right]^{-1}$$

2. After subtracting the effects of the motion of our galaxy (due to the local environment) we have that

$$\langle T \rangle = \frac{1}{4\pi}\int T(\theta,\phi)\,\sin\theta\,d\theta\,d\phi \approx 2.73$$

   but there are small fluctuations in the temperature distribution,

$$\frac{\delta T}{T}(\theta,\phi) \equiv \frac{T(\theta,\phi) - \langle T \rangle}{\langle T \rangle} \approx 1.1 \times 10^{-5}$$

Do question (4) on the worksheet and S T O P

Wednesday—Week06

The physics. Three terms you need to be aware of: *recombination, decoupling, last scattering surface*.

- Recombination—the epoch during which previously ionized baryonic matter becomes neutral
- Decoupling—the time at which photons stop interacting with baryonic matter
- Last scattering– the time at which a CMB photon undergoes its last scattering from an electron

Some preliminaries

- H is neutral hydrogen
- *p* is ionized hydrogen (i.e. a proton)
- *e* is electrons

Is all baryonic matter ionized. No. The fraction that is given by $X \equiv \frac{n_p}{n_p + n_{\mathrm{H}}} = \frac{n_p}{n_{\mathrm{bary}}} = \frac{n_e}{n_{\mathrm{bary}}}$

Hydrogen is ionized when *E = 13.6* eV so a photon with much energy can cause the reaction

$$\begin{aligned} \mathrm{H} + \gamma &\rightarrow p + e \\ p + e &\rightarrow \mathrm{H} + \gamma \end{aligned}$$

An *X* depends on the details of these two reactions

Now let's play with the physics a bit. We'll go back to a moment, $a = 10^{-5}$ , $z = 10^{5}$ and the universe is about 70 years old (using the benchmark model).

At that moment, $T = 3 \times 10^5\,\mathrm{K};\ E_\gamma = 60\,\mathrm{eV}$ and since there are so many photons than baryons, very little chance that neutral hydrogen is present, and $X = 1$.

With the universe fully ionized, the dominant reaction is $\gamma + e \rightarrow \gamma + e$

Three facts: $\sigma_e = 6.65 \times 10^{-29}\,\mathrm{m}^2$ ; $\lambda = \dfrac{1}{n_e \sigma_e}$ ; $n_e = n_{\mathrm{baryons}} = \dfrac{n_{\mathrm{baryons,o}}}{a^3} = \dfrac{0.25\,\mathrm{m}^{-3}}{a^3}$

Do question (5) on the worksheet

(5a) $\Gamma = \frac{c}{\lambda} = n_e \, \sigma_e \, c$

(5b) $\Gamma = n_e \, \sigma_e \, c = 5.0 \times 10^{-6}\,\mathrm{s}^{-1}$

(5c) $\frac{H^2}{H_o^2} = \frac{\Omega_{r,o}}{a^4} \Rightarrow H = \frac{2.1 \times 10^{-20}\mathrm{s}^{-1}}{a^2}$

(5d) $H = 2.1 \times 10^{-10}\,\mathrm{s}^{-1}$

The expansion is much slower than the collision rate, so photons are well-coupled at this time.