PHY 474 Spring 2021

Homework 4—due by 5:00 PM, Friday, May 14

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Monday (May 17). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

1. Consider the PP-I chain that we learned in class. Calculate the energy released in ***each of the three steps*** of the PP-I chain. Express your answer in MeV if you want full credit.

Note: The mass of a proton (p) is 1.0073 u, a deuteron (^{2}H or ^{2}D) is 2.0141 u, the light helium nucleus (^{3}He) is 3.0160 u, and that of the ^{4}He nucleus is 4.0015 u, where the unified atomic mass unit is 1 u $= 1.660538782 \times 10^{-27}$ kg (nist.gov).

2. Nuclear fusion converts H to He, so the mass fractions X and Y are going to change from the beginning of the Main Sequence (ZAMS) to the end of the Main Sequence (also known as TAMS). In this problem, you will calculate X and Y at TAMS.

(a) Begin by calculating the mass reduction rate dM/dt associated with the Sun's current luminosity, $L_\odot = 3.828 \times 10^{26}$ watts. State your answer in both kg/s and $M_\odot$/yr (you'll see why!). **Hint:** Start from $E = mc^2$.

(b) For H fusion to He, the efficiency of energy released is only $\epsilon \approx 0.007$. Use this to calculate the rate of reduction in the mass of hydrogen in the Sun, dM_H/dt. Again, state your answer in both kg/s and $M_\odot$/yr.

(c) Next, calculate the (total) decrease in the mass of H over the Main Sequence lifetime of the Sun. Express your answer in $M_\odot$.

(d) At ZAMS, the Sun had a mass fraction of H given by $X = 0.72$, and the mass fraction of He was $Y \approx 0.26$. Use your calculations above to compute the average mass fractions X and Y at TAMS, the end of the Sun's Main Sequence lifetime.

3. The energy generation rate in the PP chain and CNO cycle are given by, respectively

$$\epsilon_\mathrm{pp} = (1.08 \times 10^{-12})\, \rho X^2 T_6^4 \qquad \text{and} \qquad \epsilon_\mathrm{pp} = (8.24 \times 10^{-31})\, \rho\, X\, X_\mathrm{CNO}\, T_6^{20}$$

where T_6 is the temperature in units of 10^6 K, and the constants within parentheses are both in units of W m^3 kg^{-2}. Use $\rho = 10^3$ kg m^{-3}, $X = 0.7$, and $X_\mathrm{CNO} = 0.02$.

(a) Compute the temperature T at which the PP chain and CNO cycle will generate the same energy per unit time.

(b) Draw a graph of $\log_{10} \epsilon$ *vs.* $\log_{10} T_6$ for the PP chain and the CNO cycle, putting both on the same plot.

Question 4 is on the next page.

4. Recall the *Hayashi track* for pre-main sequence stars.

(a) Discuss what is meant by the *Hayashi forbidden region*.

(b) The mass-luminosity relation for convective stars is

$$\frac{L_s}{L_\odot} \simeq 0.03 \left(\frac{M}{M_\odot}\right)^{-7} \left(\frac{T_s}{2400\text{ K}}\right)^{-40}$$

where L_s and T_s are the luminosity and temperature on the surface of the star respectively, and M is the mass of the star. Plot log L_s *vs.* log T_s for $T = 2400$ to 2800 K.

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.