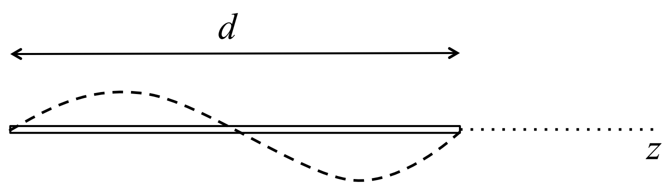
PHY 420 Spring 2021

Homework 5—due by 9:00 PM, Tuesday, May 11

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Friday (May 14). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

A thin linear antenna of length d lies along the z-axis with its center at the origin, as shown in the figure below. The antenna is excited in such a way that the sinusoidal current makes a full wavelength of oscillation (as shown by the dashed line in the figure).



1. By inspection, one can write the current density as

$$\vec{J}(\vec{x})\,e^{-i\omega t} = I\,\sin(kz)\,\delta(x)\delta(y)\,e^{-i\omega t}\,\hat{z}, \qquad \text{if } -\frac{d}{2} < z < \frac{d}{2}$$

$$= 0, \qquad \text{if } |z| > \frac{d}{2}$$

Use this to show that the vector potential $\vec{A}(\vec{x})$ in the radiation zone ($kr \gg 1$) is given by

$$\vec{A}(\vec{x}) = \hat{z}\,\frac{\mu_0 I}{2\pi}\,\frac{e^{ikr}}{ikr}\left[\frac{\sin(\pi\cos\theta)}{\sin^2\theta}\right]$$

2. Use your derived expression for $\vec{A}(\vec{x})$ to find $\vec{B}$ and $\vec{E}$ in the radiation zone.

Hint: It helps to change to spherical coordinates at this stage. Also, instead of trying to differentiate $\vec{A}$ explicitly, it helps to use $\vec{B} = ik\hat{n}\times\vec{A}$, as Jackson says to do in equation (9.39).

3. Calculate $\dfrac{dP}{d\Omega}$, the power radiated per unit solid angle.

4. Please present three choices (only) of topic from the list below, ranked in order of preference (1, 2, 3).

I'll try and award your highest choice if there is a sufficient diversity of picks, but if not, I'll make the choice for you. Please make sure you identify your choices clearly.

- Energy in capacitors: Jackson problems 1.6 & 1.8 (p. 51, 52): ____________________
- Green Function for the sphere; conducting sphere with ±V (p. 64-67): ____________________
- Fields and Charge Densities in 2-D corners and along edges (p. 75-79): ____________________
- Behavior of fields near conical hole or sharp point (p. 104-107): ____________________
- Boundary value problems with dielectrics: sphere (p. 157-159): ____________________
- $\vec{A}$ and $\vec{B}$ for a circular current loop (p. 181-184, up to eq. 5.41): ______________
- Boundary value problems: Uniformly magnetized sphere (p. 198-200): ____________________
- Magnetized sphere in external field & magnetic shielding (p. 200-203): ____________________
- Thomas Precession (p. 548-553): ____________________
- Superconductivity (based on p. 603-605): ____________________
- Synchrotron Radiation: Bridging Particle Accelerators and Astrophysics: ____________________ (based on Chapter 14):
- Bremsstrahlung Radiation in Astronomy (based on p. 714-721): ____________________