NAME: ______________________________

Exam 1.

Physics 342/442, Fall 2020

There is information attached at the end of the exam that you may find useful. No books or notes allowed. Good Luck !

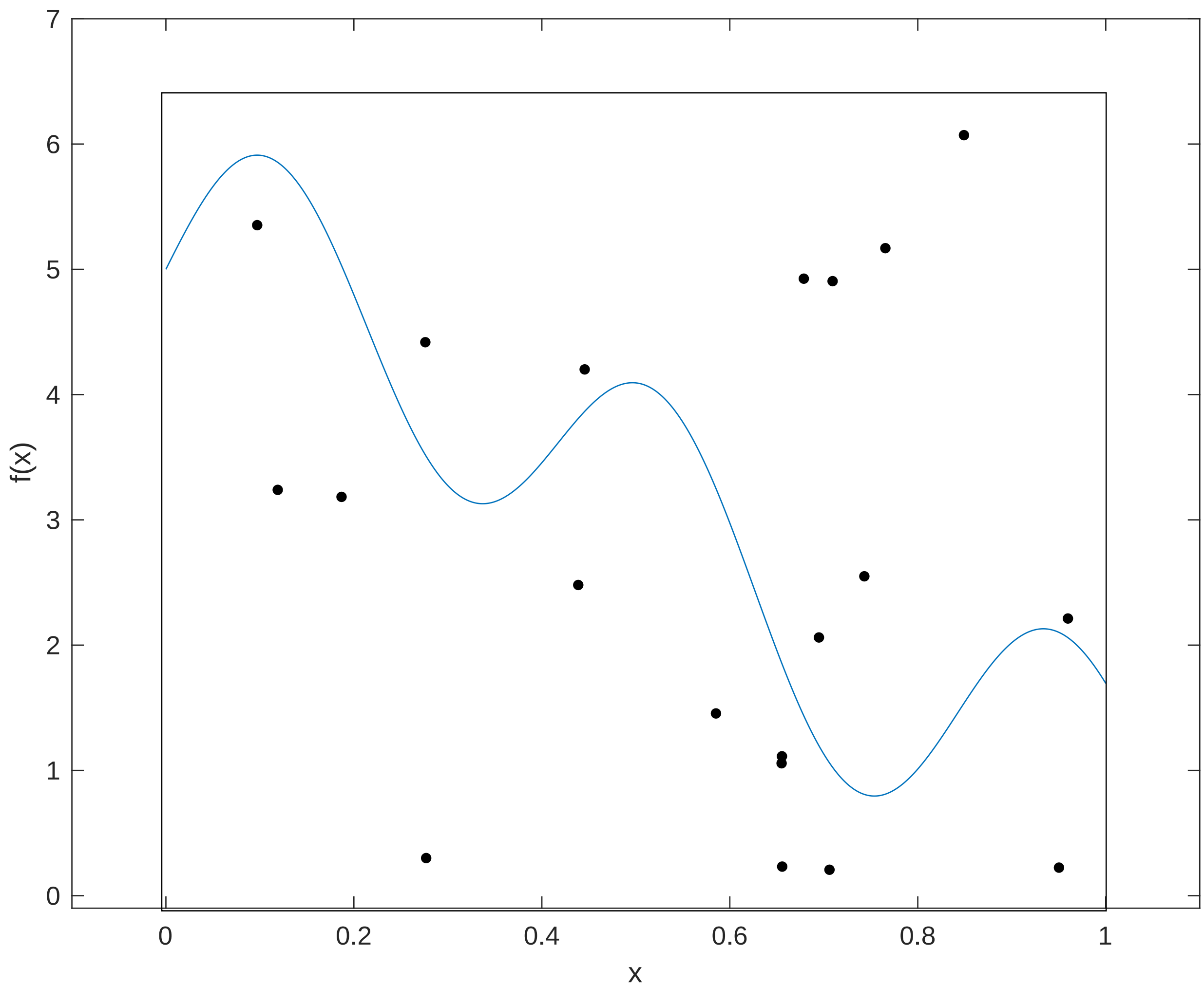
1.) Answer the following in a clear and concise way. Usually more words means less points.

(a) (*5 points*) **Briefly** describe how adaptive time-step Runge-Kutta methods work to solve ODEs. Please note the emphasis on briefly.

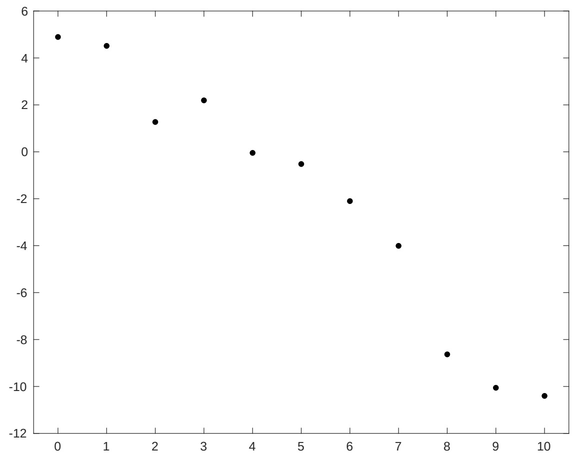
(b) (*5 points*) **Briefly** describe why a very small determinant, say $\sim 10^{-12}$, when using **LU** might cause one to doubt the results. Please note the emphasis on briefly.

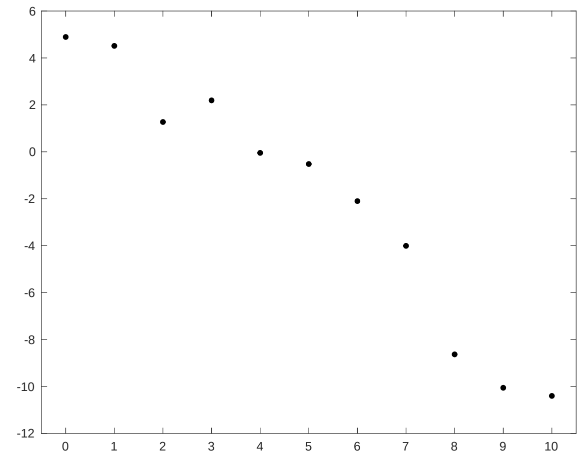
(c) (*5 points*) **Briefly** describe two methods for fitting a functions whose normal equations are non-linear. Please note the emphasis on briefly.

2. (*10 points*) Use the figure below to estimate the integral of the function. The dots have been randomly generated, and the height of the rectangle is 6.5



3.) (*10 points*) The figures below show the same experimentally obtained data. On the left panel, sketch a cubic spline appropriate for this data, and on the right panel sketch your estimate of a best fit line for this data. Below each figure describe the conditions that that must be met to obtain a cubic spline and a least-squares best fit line.





4.)(*15 points*) By explicitly computing the appropriate partial derivatives, determine if the functions require linear or non-linear fits.

(a)

$$f(t) = a_o + a_1 \sin(2\pi t) + a_2 \sin(4\pi t); \quad a_o, a_1, a_2 \text{ parameters}$$

(b)

$$f(x) = a_1 \exp(-a_2 x) + a_3 x; \quad a_1, a_2, a_3 \text{ parameters}$$

(c)

$$f(x) = \exp(-a_1 x) + a_2 x; \quad a_1, a_2 \text{ parameters}$$

5.a) (*5 points)* Convert the following 2nd order ODE to a system of first order ODEs. This requires that a new variable be introduced, call that variable, v.

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 1 = 0.$$

5.b) (*15 points*) Use Euler's and Runge-Kutta fourth order methods on the first order system of ODEs found in part (a) to fill in the following table. The initial conditions are that at $t = 0$,$y = 0$, $v = 2$. Use a time step of $\Delta t = h = 1$.

	Euler		Runge-Kutta	
t	y	v	y	v
0	0	1	0	1
1				
2				
3				

Useful and Useless Information

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$	$x_{i+1} = x_i - f(x_i)\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$	$x_{N-1} = \frac{\rho_{N-j} - c_{N-j}x_{N-j+1}}{\beta_{N-j}}$
$S = \sum \left(y_i - Y(x_i, a_1, \ldots, a_m)\right)^2$	$\boldsymbol{A} = \boldsymbol{LU}$	$S_{ij} = \frac{\partial^2 S}{\partial a_i \partial a_j}$
$y_{i+1} = y_i + hf(t_o, y_o)$	$f(t_{mid}, y_{mid}) = \frac{y(t_o+h) - y(t_o)}{h}$	$y_{i+1} = y_i + hf(t_{mid}, y_{mid})$
$f_o = f(t_o, y_o)$ $f_1 = f(t_o + \frac{h}{2}, y_o + \frac{h}{2} f_o)$ $f_2 = f(t_o + \frac{h}{2}, y_o + \frac{h}{2} f_1)$ $f_3 = f(t_o + h, y_o + f_2)$ $y_{i+1} = y_i + \frac{h}{6}\left(f_o + 2f_1 + 2f_2 + f_3\right)$		$y_i' = \frac{y_{i+1} - y_{i-1}}{2h}$ $y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$