F-1 problem 1

(1) $t=0$

$$H = \frac{4\omega}{\hbar} S_z^{(n)} \otimes S_x^{(a)}.$$

Spin operator neutron $\rightarrow S_z^{(n)}$
Spin operator atom $\rightarrow S_x^{(a)}$

$$|\Psi(0)\rangle = \tfrac{1}{2\sqrt{2}}\left(|+z^{(n)}\rangle + |-z^{(n)}\rangle\right) \otimes \left((1+i)|+x^{(a)}\rangle + (1+i)|-x^{(a)}\rangle\right)$$
$$= \tfrac{1}{2\sqrt{2}}\left((1+i)|+z,+x\rangle + (1+i)|-z,+x\rangle + (1-i)|+z,-x\rangle + (1-i)|-z,-x\rangle\right)$$

$$|\Psi(t)\rangle = \frac{(1+i)}{2\sqrt{2}} e^{-i\omega t}|+z,+x\rangle + \frac{(1-i)}{2\sqrt{2}} e^{i\omega t}|+z,-x\rangle + \frac{(1+i)}{2\sqrt{2}} e^{i\omega t}|-z,+x\rangle + \frac{(1-i)}{2\sqrt{2}} e^{-i\omega t}|-z,-x\rangle$$

(a) At $t=0$ the state becomes

$$|\Psi(0)\rangle = \frac{(1+i)}{2\sqrt{2}}|+z,+x\rangle + \frac{(1-i)}{2\sqrt{2}}|+z,-x\rangle + \frac{(1+i)}{2\sqrt{2}}|-z,+x\rangle + \frac{(1-i)}{2\sqrt{2}}|-z,-x\rangle$$

An entangled state means that it cannot be factored into the product of states for each of the particles. In this state we can see that we are unable to factor out $+z, -z, +x, -x$. If we look at the first particle, there is two instances of a $+z$ and two instances of a $-z$. If we look at the second particle, there are two instances of $+x$ and two instances of $-x$. Therefore, none of this is able to be factored. This means the state $|\Psi(0)\rangle$ is an entangled state.

(B)

$$P_{+z+x} = |\langle +z,+x|\psi(0)\rangle|^2 = \left|\frac{(1+i)}{2\sqrt{2}}\right|^2 = \frac{1}{4}$$
$$P_{+z-x} = |\langle +z,-x|\psi(0)\rangle|^2 = \left|\frac{(1-i)}{2\sqrt{2}}\right|^2 = \frac{1}{4}$$
$$P_{-z+x} = |\langle -z,+x|\psi(0)\rangle|^2 = \left|\frac{(1+i)}{2\sqrt{2}}\right|^2 = \frac{1}{4}$$
$$P_{-z-x} = |\langle -z,-x|\psi(0)\rangle|^2 = |(1-i)|^2 = 1$$

$|\langle -z,-x|\psi(0)\rangle|^2 \quad \left|\frac{\quad}{2\sqrt{2}}\right|^2 \quad 4$

(C)

	Particle 1: $+z$	$-z$
Particle 2: $+x$	$1/4$	$1/4$
$-x$	$1/4$	$1/4$

$C = 1/4 + 1/4 - 1/4 - 1/4 = 0$

$\boxed{\text{Uncorrelated}}$

(D) $t = \pi/4\omega \qquad e^{-i\omega t} \to e^{-i\pi/4} \qquad e^{i\omega t} \to e^{i\pi/4}$

$$|\psi(\tfrac{\pi}{4\omega})\rangle = \frac{(1+i)}{2\sqrt{2}} e^{-i\pi/4} |+z,+x\rangle + \frac{(1-i)}{2\sqrt{2}} e^{i\pi/4} |+z,-x\rangle + \frac{(1+i)}{2\sqrt{2}} e^{i\pi/4} |-z,+x\rangle + \frac{(1-i)}{2\sqrt{2}} e^{-i\pi/4} |-z,-x\rangle$$

F-1 problem 2

(2)

$$|\psi\rangle = 1/3\,(2|2,1,1\rangle - (2-i)|2,1,-1\rangle)$$
$$= 1/3\,(2\psi_{2,1,1}(r,\theta,\varphi) - (2-i)\,\psi_{2,1,-1}(r,\theta,\varphi))$$

(a) $P(r = 4a_0 \to \infty)$

$$P(r,\theta,\varphi) = \psi^*\psi = |\psi(r,\theta,\varphi)|^2$$
$$= 1/3\left[2\psi^*_{2,1,1}(r,\theta,\varphi) - (2+i)\psi^*_{2,1,-1}(r,\theta,\varphi)\right]$$
$$\cdot\, 1/3\left[2\psi_{2,1,1}(r,\theta,\varphi) - (2-i)\psi_{2,1,-1}(r,\theta,\varphi)\right]$$

$$= 1/9\left[\left[2R^*_{21}(r)\,Y_1^{1*}(\theta,\varphi) - (2+i)R^*_{21}(r)\,Y_1^{-1*}(\theta,\varphi)\right]\right.$$
$$\left.\cdot\left[2R_{21}(r)\,Y_1^1(\theta,\varphi) - (2-i)R_{21}(r)\,Y_1^{-1}(\theta,\varphi)\right]\right]$$

$$= 1/9\left[4|R_{21}|^2\cdot|Y_1^1|^2 - (4-2i)|R_{21}|^2 Y_1^1 Y_1^{-1} - (4+2i)|R_{21}|^2 Y_1^1 Y_1^{-1}\right.$$
$$\left. + 5|R_{21}|^2|Y_1^{-1}|^2\right]$$

$$= 1/9\left[4|R_{21}|^2\cdot|Y_1^1|^2 - 8|R_{21}|^2 Y_1^1 Y_1^{-1} + 5|R_{21}|^2|Y_1^{-1}|^2\right]$$

Using

$$\langle l_1, m_1 | l_2, m_2\rangle = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta\left[Y_{l_1}^{m_1*} Y_{l_2}^{m_2}\right] = \delta_{l_1,l_2}\delta_{m_1,m_2}$$

Will reduce the expression we simplified above. Our new state will have all the same Y terms go to $\to 1$ and unlike Y terms go to $\to 0$.

$$= 1/9\left[4|R_{21}|^2\cdot\underbrace{|Y_1^1|^2}_{\to 1}\ \cancel{-\,8|R_{21}|^2\,\underbrace{Y_1^1 Y_1^{-1}}_{\to 0}}\ + 5|R_{21}|^2\underbrace{|Y_1^{-1}|^2}_{\to 1}\right]$$

Therefore we are only left with the r terms where

$$P(4a_0 \le r \le \infty) = \int_{4a_0}^{\infty}\frac{r^2}{9}\left[4|R_{21}|^2 + 5|R_{21}|^2\right]dr = \int_{4a_0}^{\infty}\frac{r^2}{\cancel{9}}\left[\cancel{9}|R_{21}|^2\right]dr$$

$$P(4a_0 \le r \le \infty) = \int^{\infty} \; e^{-r/a} \; r^4 \, dr \;\; 103 \approx 0.609 \approx \boxed{60.9\%}$$

$$P(4a_0 \le r \le \infty) = \int_{4a_0} \frac{\quad}{3(2a_0)^3} \frac{\quad}{a_0^2}\, dr = \frac{\quad}{3e^4} \approx 0.4 \approx \boxed{42.\%}$$

(B) Bounds with range φ and θ $\quad 0 \le \varphi \le \pi/2$ and $0 \le \theta \le \pi/2$
With no bound restriction on r.

Using

$$\langle l_1, m_1 | l_2, m_2 \rangle = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta \left[Y_{l_1}^{m_1 *} Y_{l_2}^{m_2} \right] = \delta_{l_1, l_2} \delta_{m_1, m_2}$$

will no longer work. This is because for this to work we need the bounds of $0 \le \varphi \le 2\pi$ and $0 \le \theta \le \pi$. But this is no longer true. The new Bounds are $0 \le \varphi \le \pi/2$ and $0 \le \theta \le \pi/2$. Therefore, going back to the State

$$= 1/9 \left[4|R_{21}|^2 \cdot |Y_1^1|^2 - 8|R_{21}|^2 Y_1^1 Y_1^{-1} + 5|R_{21}|^2 |Y_1^{-1}|^2 \right]$$

we can find the probability by integrating over

$$P = \left[\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\infty} 1/9 \left[4|R_{21}|^2 \cdot |Y_1^1|^2 - 8|R_{21}|^2 Y_1^1 Y_1^{-1} + 5|R_{21}|^2 |Y_1^{-1}|^2 \right] \sin\theta\, r^2\, d\theta\, d\varphi\, dr \right]$$

$$P = \left[\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\infty} \frac{1}{9} \left[4|R_{21}|^2 \cdot |Y_1^1|^2 - 8|R_{21}|^2 Y_1^1 Y_1^{-1} + 5|R_{21}|^2 |Y_1^{-1}|^2 \right] \sin\theta\, r^2\, d\theta\, d\varphi\, dr \right]$$

$$= \frac{1}{9} \left[\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\infty} \frac{4e^{-r/a}}{3(2a_0)^3} \frac{r^4}{a_0^2} \cdot \frac{3}{8\pi} \sin^3\theta\, \overbrace{\cancel{e^{-i\varphi} e^{i\varphi}}}^{\text{Complex conj}}\, d\varphi\, d\theta\, dr \right.$$

$$- \int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\infty} \frac{8e^{-r/a}}{3(2a_0)^3} \frac{r^4}{a_0^2} \cdot \left(-\frac{3}{8\pi} \sin^3\theta \right) d\theta\, d\varphi\, dr$$

$$+ \int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\infty} \frac{5e^{-r/a}}{3(2a_0)^3} \frac{r^4}{a_0^2} \cdot \frac{3}{8\pi} \sin^3\theta\, \overbrace{\cancel{e^{-i\varphi} e^{i\varphi}}}^{\text{Complex conj}}\, d\theta\, d\varphi\, dr$$

1) $\int^{\pi/2} \sin^3(\theta)\, d\theta = 2/3$

Note: $\int_0 \sin(x)dx = /3$

$$= \frac{1}{9}\left[\int_0^\infty \frac{4e^{-r/a}}{3(2a_0)^3}\frac{r^4}{a_0^2}\cdot\frac{3}{8\pi}\cdot\frac{2}{3}\frac{\pi}{2}\right.$$

$$- \int_0^\infty \frac{8e^{-r/a}}{3(2a_0)^3}\frac{r^4}{a_0^2}\cdot\frac{-3}{8\pi}\cdot\frac{2}{3}\cdot\frac{\pi}{2}\,dr$$

$$\left. + \int_0^\infty \frac{5e^{-r/a}}{3(2a_0)^3}\frac{r^4}{a_0^2}\cdot\frac{3}{8\pi}\cdot\frac{2}{3}\frac{\pi}{2}\,dr\right]$$

$$= \frac{1}{9}\left[\int_0^\infty \frac{e^{-r/a_0}r^4}{48a_0^5}dr - \int_0^\infty \frac{e^{-r/a_0}r^4}{24a_0^5}dr + \int_0^\infty \frac{5e^{-r/a_0}r^4}{192a_0^5}dr\right.$$

$$= \frac{1}{9}\left[\frac{1}{8} + \frac{1}{4} + \frac{5}{32}\right] \boxed{= \frac{17}{288} \approx 0.059 \approx 5.9\%}$$

(C) $\psi_{n\ell m}$

Magnitude

$|L| = \sqrt{\ell(\ell+1)}\,\hbar$ one ℓ value $\ell = 1$

$|L| = \sqrt{1(1+1)} = \boxed{\sqrt{2}\,\hbar}$ $P = 1$

Z-Component

$L_z = m\hbar$

$L_z = 1\cdot\hbar = \boxed{\hbar}$ $P_\hbar = |\langle 1|\psi\rangle|^2 = \boxed{4/9}$

$L_z = -1\cdot\hbar = \boxed{-\hbar}$ $P_{-\hbar} = |\langle -1|\psi\rangle|^2 = \boxed{5/9}$

$m_s = 1/2$
$S = 1/2$

(D) $|\psi\rangle = 1/3\left(2|2,1,1\rangle - (2-i)|2,1,-1\rangle\right)\otimes|+\rangle$

$n=2,\ \ell=1,\ m_\ell=1$ $n=2,\ \ell=1,\ m_\ell=-1$

$m_\ell + m_s = 3/2$ $m_\ell + m_s = -1/2$