

$$ds^2 = dx^2 + dy^2 + dz^2$$

Cylindrical: $x = r \cos \theta$, $y = r \sin \theta$, $z = z$

$$dx = -dr \sin \theta - r \cos \theta d\theta \quad dy = dr \cos \theta + r \sin \theta d\theta$$

$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2$$

Spherical: $x = r \cos \phi \sin \theta$ $y = r \sin \phi \sin \theta$ $z = r \cos \theta$

$$dx = dr \cos \phi \sin \theta - r \sin \phi \sin \theta d\phi + r \cos \phi \cos \theta d\theta$$

$$dy = dr \sin \phi \sin \theta + r \cos \phi \sin \theta d\phi + r \sin \phi \cos \theta d\theta$$

$$dz = -dr \cos \theta - r \sin \theta d\theta$$

$$ds^2 = dr^2 \sin^2 \theta + r^2 \sin^2 \theta d\phi^2 + r^2 \cos^2 \theta d\theta^2$$

$$+ dr^2 \cos^2 \theta + r^2 \sin^2 \theta d\theta^2$$

$$+ 2r \cos \theta \sin \theta dr d\theta - 2r \cos \theta \sin \theta dr d\theta$$

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

On the surface of a cylinder, $r = \text{constant} \Rightarrow dr = 0$

On the surface of a sphere, $r = \text{constant} \Rightarrow dr = 0$