PHY 474/374 Spring 2021

Class Summary—Week 5, Day 2—Wednesday, April 28

Energy Generation in Stars (continued)

Stars generate energy by **nuclear fusion** in their cores. We will now learn about the generation of energy in stars in more detail.

Hydrogen fusion

During most of their lifetime, stars derive their energy from the fusion of hydrogen into helium. Schematically

$$4\,{}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + 2\mathrm{e}^{+} + 2\nu_e \tag{8.45}$$

where the release of the two positrons (e^{+}) is required to maintain charge balance (because two protons have been effectively converted into neutrons), and the two electron neutrinos (ν_e), which are leptons[1] are released to conserve lepton number, since the two positrons are anti-leptons. Nuclear reactions not only conserve the lepton number and the charge, but also the baryon number. This is indeed the case here, *as you determined in Question 1(a) on today's worksheet.* You also found *in Question 1(b) on today's worksheet* that the energy liberated due to the process in equation (8.45) is 26.73 MeV.

However, the reaction does not take place quite as indicated in the formula in equation (8.45); the probability that four protons can come together at a point and react is negligible. Instead, the reaction proceeds through a number of different paths, which we will learn now in more detail. Basically, there are two different ways, each with some variations, for the overall reaction:

- the **proton-proton chain (PP-chain)**, which involves direct fusion of protons, produces most of the energy in the Sun, and is dominant in stars of a solar mass or less.
- the **CNO cycle**, in which fusion occurs through a sequence of reactions involving C, N, and O, which effectively act as catalysts, quickly surpasses the PP-chain in energy production as soon as the mass exceeds about 1 $M_{\odot}$.

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[1] Quarks and leptons are the basic building blocks of matter. The present structure of Particle Physics has six leptons: electron, muon, and tau particles, and their associated neutrinos.

The proton-proton (PP) chain

One pathway for the fusion of H to He is the **proton-proton (PP) chain**. Using compact notation to represent the equation

$$A + a \to Y + y \qquad \text{as} \qquad A(a, y)Y$$

Dalsgaard writes the sequence of reactions in the **PP-I chain** (one of three main pathways in the PP chain) as

$$^{1}\mathrm{H}\,(^{1}\mathrm{H}, e^{+}\nu_{e})\,^{2}\mathrm{H}\,(^{1}\mathrm{H}, \gamma)\,^{3}\mathrm{He}\,(^{3}\mathrm{He}, 2^{1}\mathrm{H})\,^{4}\mathrm{He} \tag{8.50}$$

In Question 2(a) of today's worksheet, you wrote out these three equations in expanded form, then verified that the net reaction is indeed given by equation (8.45). Note that I've chosen to write $^{2}\mathrm{H}$ for deuterium in the equation above, instead of $^{2}\mathrm{D}$ in *Dalsgaard*.

In Question 2(b) of today's worksheet, you worked out the energy liberated in the PP-I chain. This is less than what you found in Question 1(b), because the neutrino carries away 0.263 MeV of energy on average in the $^{1}\mathrm{H}\,(^{1}\mathrm{H}, e^{+}\nu_{e})\,^{2}\mathrm{H}$ reaction. Thus the energy liberated is

$$26.73\ \mathrm{MeV} - 2\,(0.263\ \mathrm{MeV}) = \mathbf{26.2\ MeV}$$

There are two additional pathways in the PP chain to produce $^{4}\mathrm{He}$. On the worksheet, I wrote the pathways after the production of $^{3}\mathrm{He}$, but I'll include the whole pathway here.

- The **PP-II chain** is

$$^{1}\mathrm{H}\,(^{1}\mathrm{H}, e^{+}\nu_{e})\,^{2}\mathrm{H}\,(^{1}\mathrm{H}, \gamma)\,^{3}\mathrm{He}\,(^{4}\mathrm{He}, \gamma)\,^{7}\mathrm{Be}\,(\mathrm{e}^{-}, \nu_{e})\,^{7}\mathrm{Li}\,(^{1}\mathrm{H}, ^{4}\mathrm{He})\,^{4}\mathrm{He}$$

  where I've shown the entire pathway, including the first two steps that are also in the PP-I chain.

- whereas the entire pathway for the **PP-III chain** is

$$^{1}\mathrm{H}\,(^{1}\mathrm{H}, e^{+}\nu_{e})\,^{2}\mathrm{H}\,(^{1}\mathrm{H}, \gamma)\,^{3}\mathrm{He}\,(^{4}\mathrm{He}, \gamma)\,^{7}\mathrm{Be}\,(^{1}\mathrm{H}, \gamma)\,^{8}\mathrm{B}\,(, \mathrm{e}^{+}\nu_{e})\,^{8}\mathrm{Be}\,(, ^{4}\mathrm{He})\,^{4}\mathrm{He}$$

You wrote out these reactions *in Question 3(a) of today's worksheet*; please see the posted video to check your answers.

In Question 3(b) of today's worksheet, you identified the reactions that compete to decide the relative importance of the PP-I *vs.* the PP-II and PP-III branches, and the reactions that compete to decide the relative importance of the PP-II *vs.* the PP-III branches.

- Competition between the reactions

$$^{3}\mathrm{He}\,(^{3}\mathrm{He}, 2^{1}\mathrm{H})\,^{4}\mathrm{He} \qquad \text{and} \qquad ^{3}\mathrm{He}\,(^{4}\mathrm{He}, \gamma)\,^{7}\mathrm{Be}$$

  decides the relative importance of the PP-I *vs.* the PP-II and PP-III branches. PP-I dominates because the $^{3}\mathrm{He}\,(^{3}\mathrm{He}, 2^{1}\mathrm{H})\,^{4}\mathrm{He}$ is faster than $^{3}\mathrm{He}\,(^{4}\mathrm{He}, \gamma)\,^{7}\mathrm{Be}$ by about four orders of magnitude.

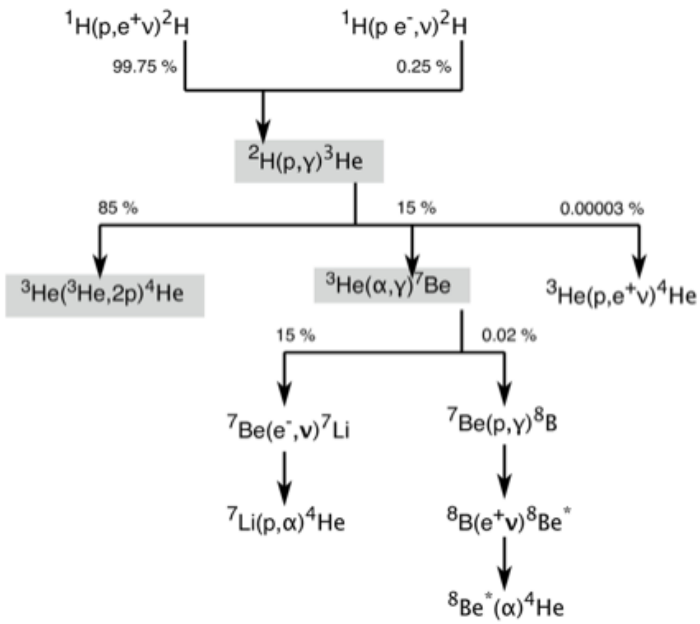
- Competion between electron or proton capture in $^{7}\mathrm{Be}$, that is, between the reactions

$$^{7}\mathrm{Be}\,(\mathrm{e}^{-}, \nu_{e})\,^{7}\mathrm{Li} \qquad \text{and} \qquad ^{7}\mathrm{Be}\,(^{1}\mathrm{H}, \gamma)\,^{8}\mathrm{B}$$

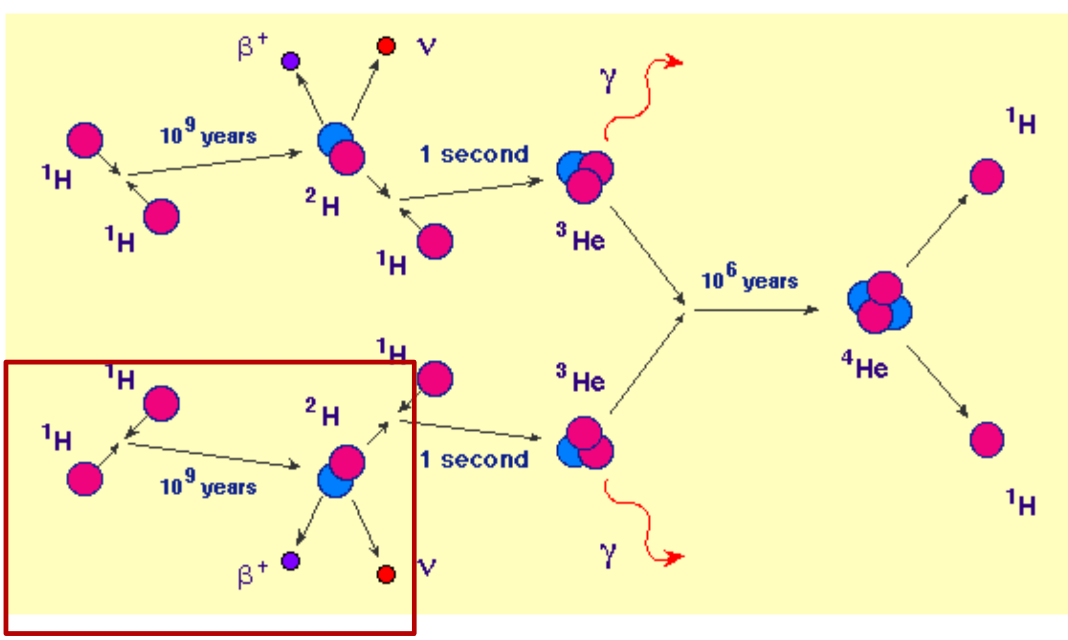
  decides the relative importance of the PP-II *vs.* the PP-III branches. At the temperature in the Sun's core, the electron capture dominates and so the PP-II reaction is dominant.

Let's look at more details of the reactions.

Staying on the theme of competition between reactions, let's find which paths are likelier than others. The figure below (source: *Broggini 2006*) shows the details of the reactions. Note that this figure also shows additional (low probability) pathways that we have not included in our discussion. Thus, the very first reaction to build ^{2}H has another pathway, but with a very low probability of 0.25%, so we have not considered it in our discussion on the worksheet. We see from this figure that the **PP-I pathway is the dominant one (85%)**.



Consider the figure below (source: *pas.rochester.edu*). It shows all the reactions of the PP-I chain, with the time for each reaction written. Notice that the first reaction takes a long time, $\sim 10^9$ yr. Thus, the first reaction is the bottleneck, and it sets the time scale for the main sequence lifetime of the star.



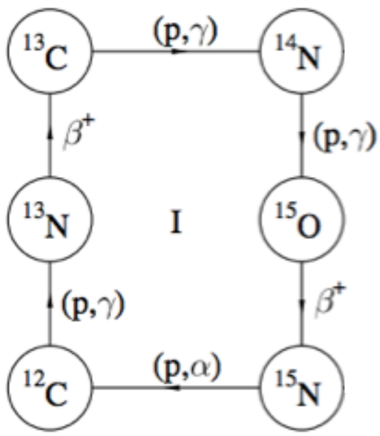
The CNO Cycle

Another pathway is possible for conversion of H to He in stars that contain C, N, and O. Summing net reactants and products around this so-called **CNO cycle**, we get

$$^{12}\mathrm{C} + 4p \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} + 2e^{+} + 2\nu_e$$

We see that C is a catalyst in this reaction; it enables the reactions to take place but remains unchanged (notice that ^{12}C is on both the left and right hand sides).

Now, consider the full set of reactions shown in the graphic below (source: *Li, Stanford*). We can see now why this is called a *cycle*, as opposed to *chain*; in principle, you could start anywhere in the loop and end up back where you started. Thus, it is also clear from the graphic why C, N, O are called catalysts.



It is also instructive to write out the **full set of equations for the CNO cycle**. Let's start from ^{12}C, as in the net reaction written at the top of this page, and follow the path along the direction marked in the graphic above. Note that the p marked in the graphic is ^{1}H in our notation, whereas β^+ is e^+, and α is ^{4}He. Note also that that graphic does not display the neutrinos, so we will need to work those out wherever appropriate (occurrences of positrons, for example, will need to have neutrinos present to conserve the lepton number).

Starting from ^{12}C at the bottom left corner of the graphic above, we have

$$^{12}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{13}\mathrm{N} + \gamma$$

$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu_e$$

$$^{13}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + \gamma$$

$$^{14}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{15}\mathrm{O} + \gamma$$

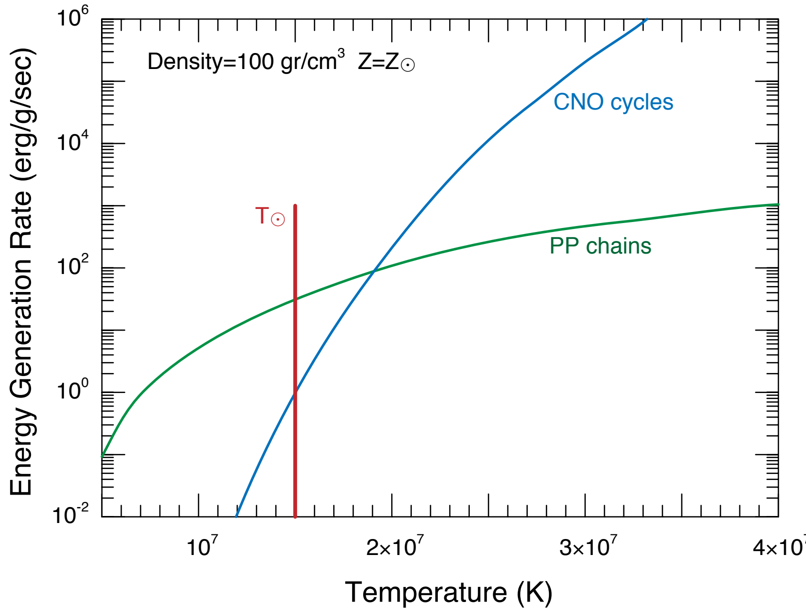
$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu_e$$

$$^{15}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$$

and we end again at ^{12}C at the bottom left corner of the graphic above. The key point is that starting from ^{1}H, we have ended up with ^{4}He.

Finally, we need to know which branch (PP or CNO) occurs where.

Consider the plot below (source: *asu.edu*). The vertical red line marks the temperature at the core of our Sun. From this plot, we see that the PP chain will dominate in our Sun. Indeed, in stars of a solar mass or less, the PP chain will dominate.



In high mass stars, on the other hand, where the core temperatures are higher, we would expect the CNO cycle to dominate. The only exception would be the earliest generation of stars, discussed below.

The raw material for the **earliest generation of stars** came from the hydrogen and helium left over from the Big Bang. Thus, even though the earliest generation of stars were likely among the highest mass stars ever formed, they would've had to generate energy by the PP chain, since C, N, and O had not yet been formed in the hydrogen and helium gas left over from the Big Bang from which this earliest generation of stars drew its raw material.