Today we'll finish our look at the overall picture of cosmology. We'll start our look in detail next week.

Modern cosmology can be built by addressing the following observations:

1. Night is *dark*
2. On large scales, the universe is *isotropic and homogeneous*
3. Galaxies are *moving away* from us and *the further they are, the faster* they are moving
4. The universe is made up of *different stuff*
5. The universe is *filled with a background radiation* whose character is almost a *perfect black body*

The night is dark...

Suppose we have an ∞ universe in size and stars. Let's explore the consequences:

$$\begin{aligned} \bar{n} &\equiv \text{Density of stars} \\ L &\equiv \text{Luminosity of stars} \end{aligned}$$

Do question (1 a, b) on the worksheet and S T O P

(1 a) Luminosity is the rate at which *energy* is radiated away from a star

(1 b) Flux is stuff per time per area. Using luminosity (*energy/time*) we have

$$f(r) = \frac{L}{4\pi r^2}$$

Finish question 1 and S T O P

(1 c) Integrating flux over all space gives

$$\begin{aligned} dJ(r) &= \frac{L}{4\pi r^2} \cdot n \cdot \overbrace{r^2 dr}^{\text{volume element}} \\ J = \int dJ &= \frac{nL}{4\pi} \int_0^\infty dr = \infty \end{aligned}$$

Night sky should be infinitely bright !!

The night is dark…

Olber's paradox

$$dJ(r) = \frac{L}{4\pi r^2} \cdot n \cdot \overbrace{r^2 dr}^{\text{volume element}}$$

$$J = \int dJ = \frac{nL}{4\pi} \int_0^\infty dr = \infty$$

Night sky should be infinitely bright !!

Assumptions made to get to this point

- *nL* = constant
- *f ~ 1/r²*
- Universe is infinite in size and age ✔

Resolution:

- Even if universe is infinite is size, it is not infinite in age
- Stars are not infinitely longed lived
- Do question (2) on the worksheet and S T O P

On large scales, the universe is *isotropic and homogeneous*

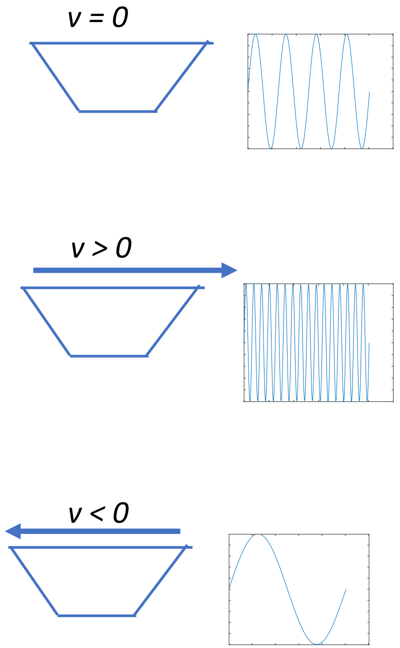
- *Isotropy:* The universe is *isotropic* if there it does ***not*** have a preferred direction
- *Homogeneous:* The universe is *homogeneous* if it does ***not*** have a preferred location
- Note that the universe is *neither* homogeneous nor isotropic at small scales, only on scales > 150 Mpc
- Do question (3) on the worksheet and S T O P

3a

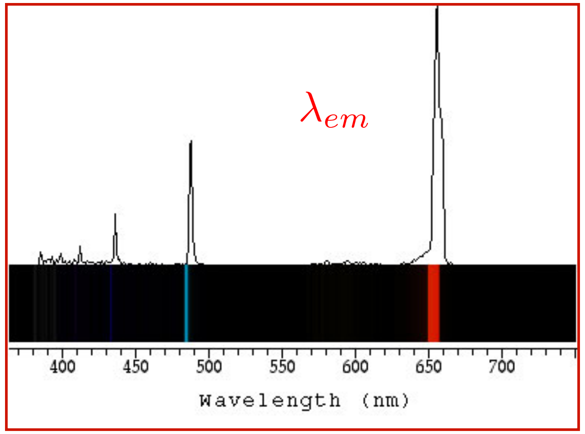
3b

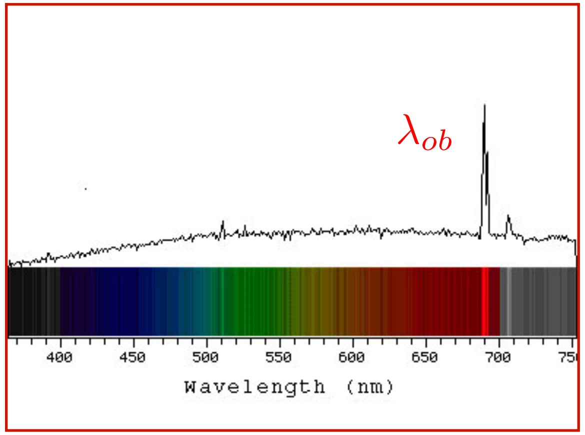
3c One doesn't need to study the whole universe to understand the universe as a whole

Galaxies are *moving away* from us and *the further they are, the faster* they are moving



$$z \equiv \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}}$$

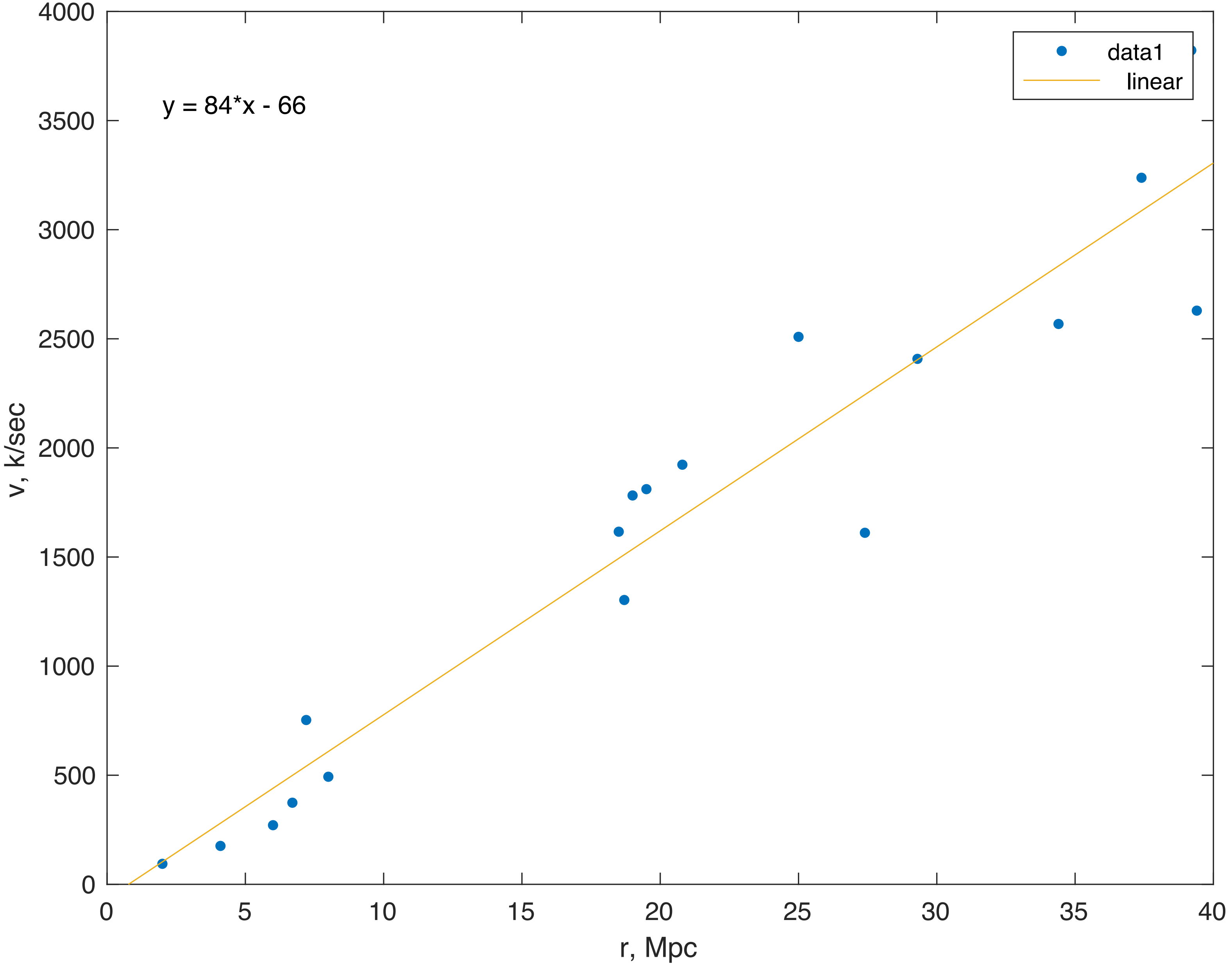




For small *z*, we have that $v = z \times c$

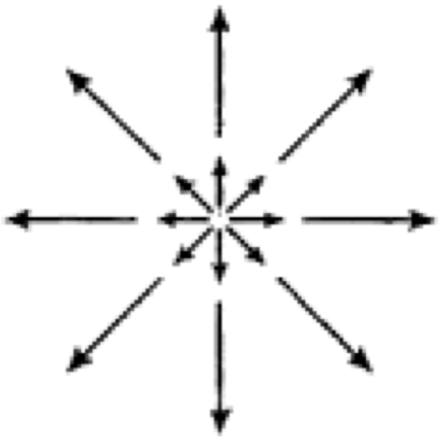
Do question (4) on the worksheet and S T O P

(4 c) $v = H_o r$

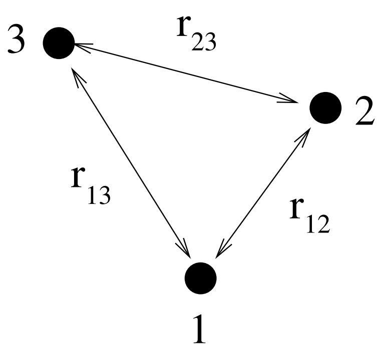

y = 84*x - 66
data1
linear
v, k/sec
r, Mpc
0
500
1000
1500
2000
2500
3000
3500
4000
0
5
10
15
20
25
30
35
40

Galaxies are *moving away* from us and *the further they are, the faster* they are moving

What we just found, looks something like this →



So is the universe not homogeneous?



$$
\begin{aligned}
r_{12} &\equiv |\vec{r_1} - \vec{r_2}| \\
r_{23} &\equiv |\vec{r_2} - \vec{r_3}| \\
r_{13} &\equiv |\vec{r_1} - \vec{r_3}|
\end{aligned}
$$

Homogeneity means shape of triangle is preserved as galaxies move away

$$
\begin{aligned}
r_{12}(t) &= a(t)\, r_{12}(t_o) \\
r_{23}(t) &= a(t)\, r_{23}(t_o) \\
r_{13}(t) &= a(t)\, r_{13}(t_o)
\end{aligned}
$$

a is called the *scale factor*

Now as the universe expands, the distances change as a function of time and the galaxies pick up a *velocity*. That is,

$$v_{12}(t) = \frac{dr_{12}}{dt} = \frac{da(t)}{dt} r_{12}(t_o) = \frac{\dot{a}}{a}\, r_{12}(t)$$

Do question (5) on the worksheet and S T O P

$$
\begin{aligned}
v = \frac{r}{t} &= H_o r \\
t &= \frac{1}{H_o} \\
t &\approx 14\ \mathrm{Gy}
\end{aligned}
$$

Hubble time

But recall that (4 c) $v = H_o r$ so $v = Hr \Rightarrow H = \frac{\dot{a}}{a}$

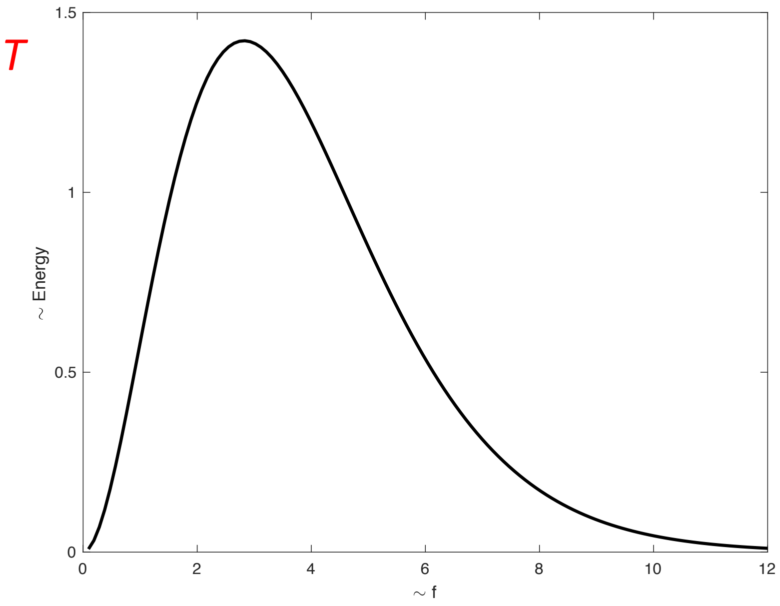
If we let $v = c$, we get the *Hubble distance* $r = c/H_o \cong 4300$ *Mpc*

The universe is made up of *different stuff*

- Baryons, the stuff you and I are made up of
- Radiation (light)
- Dark Matter
- Dark Energy
- These components are dominant at different times and affect the evolution of the universe

The universe is *filled with a background radiation* whose character is almost a *perfect black body*

At a given *T*



The energy of photons between frequency *f* and *f + df* is

$$\mathcal{E}(f)df = \frac{8\pi\hbar}{c^3}\frac{f^3 df}{\exp(\hbar f/kT) - 1}$$

And integrating over all frequencies gives

$$\mathcal{E}_\gamma = \alpha T^4$$

$$\alpha = \frac{\pi^2 k^4}{15\hbar^3 c^3} = 7.56 \times 10^{-16}\ \mathrm{J\ m^{-3} K^{-4}}$$

The universe is *filled with a background radiation* whose character is almost a *perfect black body*

$\mathcal{E}_\gamma = \alpha T^4;\ P_\gamma = \mathcal{E}_\gamma/3$ (we'll see why later)

Now let's look at 1st law of thermodynamics $dU = dQ - PdV$ In a homogeneous universe there is no heat flow. Why?

$$\begin{aligned} dU = dE &= \mathcal{E}V \\ dE &= \alpha T^4 V \end{aligned} \qquad \begin{aligned} -PdV &= -\frac{\mathcal{E}_\gamma}{3}dV \\ &= \frac{\alpha T^4}{3}dV \end{aligned}$$

In an expanding universe, these quantities are changing as a function of time, so we have

$$\begin{aligned} \frac{dE}{dt} &= \alpha\left(4T^3\frac{dT}{dt}V + T^3\frac{dV}{dt}\right) \\ -P\frac{dV}{dt} &= -\frac{1}{3}\alpha T^4\frac{dV}{dt} \end{aligned}$$

So

$$\begin{aligned} \alpha\left(4T^3\frac{dT}{dt}V + T^3\frac{dV}{dt}\right) &= -\frac{1}{3}\alpha T^4\frac{dV}{dt} \\ \frac{1}{T}\frac{dT}{dt} &= -\frac{1}{3V}\frac{dV}{dt} \\ \frac{d}{dt}(\ln T) &= \frac{d}{dt}(\ln V^{1/3}) \end{aligned}$$

But for a homogeneous expanding universe, $V \propto a(t)^3$ so

$$\frac{d}{dt}(\ln T) = \frac{d}{dt}\ln(a(t)) \Rightarrow \boxed{T \propto \frac{1}{a(t)}}$$

Do question (6) on the worksheet