Physics 412—Vectors and Operators Winter 2020

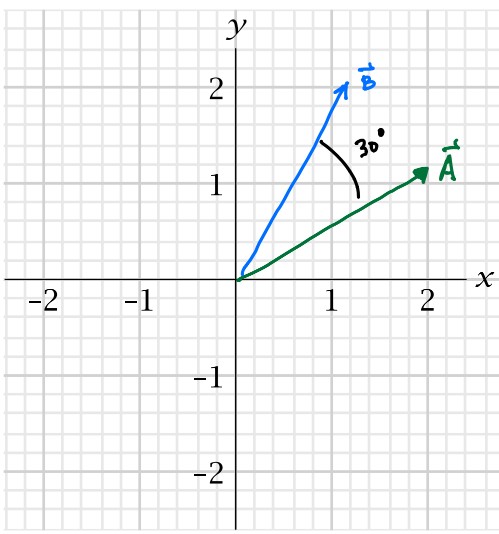
Since we don't quite know enough yet to work with vectors and operators in Hilbert space, we'll begin by working with vectors and operators in R^2, the Euclidean plane. We'll use standard notation like $\vec{A}$ and $\vec{B}$ for the vectors. Consider the two operators, R_{30} and T_{45}, defined as follows:

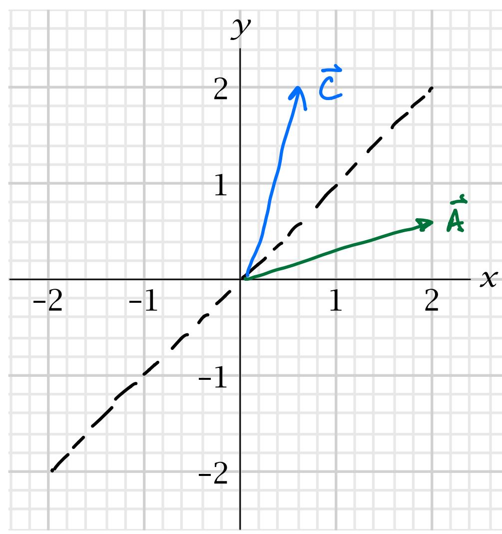
- The operator R_{30} rotates any vector by 30° counter-clockwise.
- The operator T_{45} reflects any vector through the line that makes a 45° angle with the x axis, *i.e.* the line with slope 1.

In other words, if $\vec{A}$ is an arbitrary vector, then the vector $\vec{B} = R_{30}\vec{A}$ is the vector obtained by rotating $\vec{A}$ 30° counter-clockwise. The vector $\vec{C} = T_{45}\vec{A}$ is the vector obtained by reflecting $\vec{A}$ through the line with slope 1.

(1) On the axes below, implement these operators for two or three example vectors, by explicitly constructing $\vec{B} = R_{30}\vec{A}$ and $\vec{C} = T_{45}\vec{A}$.

$\vec{B} = R_{30}\vec{A}$ $\vec{C} = T_{45}\vec{A}$





(2) In quantum mechanics, operators are supposed to be linear, meaning that

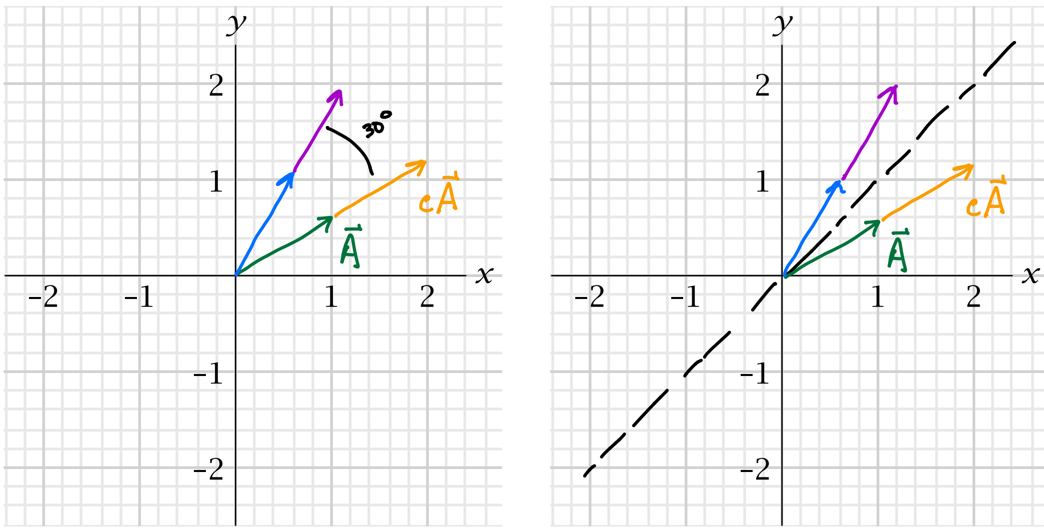
1. $\hat{A}\left(c|\Psi\rangle\right) = c\left(\hat{A}|\Psi\rangle\right)$, where c is a scalar (a complex number).
2. $\hat{A}\left(|\Psi\rangle + |\Phi\rangle\right) = \hat{A}|\Psi\rangle + \hat{A}|\Phi\rangle$.

Show by explicit construction, using the axes below, that R_{30} and T_{45} are linear. (Of course in this context, the scalar c is a real number.)

When adding vectors together, use the tip-to-tail rule to add them graphically.

Property 1

pick $c=2$



Property 2