(1) A particle is confined to one corner of a cubic box of sides L in quantum state

$$|\psi\rangle \leftrightarrow \begin{cases} A\sin\frac{2\pi x}{L}\sin\frac{2\pi y}{L}\sin\frac{2\pi z}{L}, & |x,y,z| < L/2, \\ 0, & |x,y,z| \geq L/2. \end{cases}$$

(a) Find the normalization constant A.

(b) If you measured the energy of the particle, what values could you measure, and with what probabilities?

(c) Describe as best you can the time evolution of this state.

(a) $$\langle\psi|\psi\rangle = 1 = A^2\int_0^{L/2}dx\int_0^{L/2}dy\int_0^{L/2}dz\,\sin^2\frac{2\pi x}{L}\sin^2\frac{2\pi y}{L}\sin^2\frac{2\pi z}{L}$$

$$= A^2\left[\int_0^{L/2}dx\,\sin^2\frac{2\pi x}{L}\right]^3 = A^2\left(\frac{L}{4}\right)^3 = \frac{A^2L^3}{64}$$

$$\Rightarrow \boxed{A = \frac{8}{L^{3/2}}}$$

(b)

$$\langle\psi_n|\psi\rangle = \frac{8\sqrt{8}}{L^3}\int_0^{L/2}\sin\frac{2\pi x}{L}\sin\frac{2\pi y}{L}\sin\frac{2\pi z}{L}\sin\frac{n_x\pi x}{L}\sin\frac{n_y\pi y}{L}\sin\frac{n_z\pi z}{L}\,dx\,dy\,dz$$

$$= \frac{8\sqrt{8}}{L^3}\left[\int_0^{L/2}\sin\frac{2\pi x}{L}\sin\frac{n\pi x}{L}\right]^{\text{"}3\text{"}}$$

$$[\ ] = \frac{2L\sin n\pi/2}{4\pi-\pi^2n^2}$$ $n \neq 2!$

$$= \frac{8\sqrt{8}}{L^3}\cdot 8L^3\left(\frac{\sin\frac{n_x\pi}{2}}{4\pi-\pi^2n_x^2}\cdot\frac{\sin\frac{n_y\pi}{2}}{4\pi-\pi^2n_y^2}\cdot\frac{\sin\frac{n_z\pi}{2}}{4\pi-\pi^2n_z^2}\right)$$

$[\ ] = \frac{L}{4}$ if $n=2$

$$P = |\langle\psi_n|\psi\rangle|^2$$

1st if any of n_x, n_y, n_z are even (except 2!) then $P = 0$

2nd all odd n's appear.

3rd $n = 2$ is possible for any or all n's.

(c)

$$\psi(x,y,z) = \sum_{n_x,n_y,n_z}' \underbrace{c_{n_x,n_y,n_z}}_{\langle \psi_n | \psi \rangle} \psi_{n_x,n_y n_z}(x,y,z)$$

$$\psi(x,y,z,t) = \sum_{n_x,n_y,n_z}' c_{n_x,n_y,n_z} e^{-iE_{n_x,n_y,n_z}t/\hbar} \psi_{n_x,n_y n_z}(x,y,z)$$