1 Maxwell Equations

In free spaceIn free space

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \nabla\cdot\vec{B}=0$$

$$\vec{\nabla}\cdot\vec{D}=0 \qquad \nabla\cdot\vec{D}=\rho$$

$$\vec{\nabla}\times\vec{E}-i\omega\vec{B}=0 \qquad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B}+i\omega\mu\varepsilon\vec{E}=0 \qquad \vec{\nabla}\times H=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

1.1 Constructive Relations

$$\vec{D}=\varepsilon\vec{E} \qquad \vec{B}=\mu\vec{H}$$

2 Electromagnetic Waves and Propagation

2.1 Helmholtz wave equations

$$(\nabla^2+\mu\varepsilon\omega^2)\vec{E}=0 \qquad (\nabla^2+\mu\varepsilon\omega^2)\vec{B}=0$$

2.2 Constructive Relations

1. **Wave number:** $k=\omega\sqrt{\mu\varepsilon}$
2. **Phase velocity:** $v=\frac{\omega}{k}=\frac{1}{\sqrt{\mu\varepsilon}}=\frac{c}{n}$
3. **Index of refraction of the medium:** $n=\frac{\mu\varepsilon}{\mu_0\varepsilon_0}$

2.3 Plane Electromagnetic Waves

$$\vec{E}=\vec{E}_0e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \vec{B}=\sqrt{\mu\varepsilon}\frac{\vec{k}\times\vec{E}}{k}$$

2.4 Polarization of Waves

$$\vec{E}_1=\hat{\varepsilon}_1E_1e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \vec{E}_2=\hat{\varepsilon}_2E_2e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\vec{E}(\vec{x},t)=(\hat{\varepsilon}_1E_1+\hat{\varepsilon}_2E_2)e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

2.5 Stokes Parameters

Linear polarization basis:

$$\vec{E}(\vec{x},t)=(\hat{\varepsilon}_1E_1+\hat{\varepsilon}_2E_2)e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$E_1=a_1e^{i\delta_1} \qquad E_2=a_2e^{i\delta_2}$$

Circular polarization basis:

$$\vec{E}(\vec{x},t)=(\hat{\varepsilon_+}E_++\hat{\varepsilon_-}E_-)e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$E_+=a_+e^{i\delta_+} \qquad E_-=a_-e^{i\delta_-}$$

2.6 Reflection and Refraction: Kinematic Properties

Incident wave:

$$\vec{E}=\vec{E}_0e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \vec{B}=\sqrt{\mu\varepsilon}\frac{\vec{k}\times\vec{E}}{k}$$

Refracted wave:

$$\vec{E}'=\vec{E}_0'e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \vec{B}'=\sqrt{\mu'\varepsilon'}\frac{\vec{k}'\times\vec{E}'}{k}$$

Reflected wave:

$$\vec{E}''=\vec{E}_0''e^{i(\vec{k}''\cdot\vec{x}-\omega t)} \qquad \vec{B}''=\sqrt{\mu\varepsilon}\frac{\vec{k}''\times\vec{E}''}{k}$$

2.7 Reflection and Refraction: Boundary condition

Normal components:

$$[\varepsilon(\vec{E}_0+\vec{E}_0'')-\varepsilon'\vec{E}_0']\cdot\hat{n}=0$$

$$[\vec{k}\times E_0+\vec{k}''\times\vec{E}_0''-\vec{k}'\times\vec{E}_0']\cdot\hat{n}=0$$

Tangential components:

$$[\vec{E}_0+\vec{E}_0''-\vec{E}_0']\times\hat{n}=0$$

$$\left[\frac{1}{\mu}(\vec{k}\times\vec{E}_0+\vec{k}''\times\vec{E}_0'')-\frac{1}{\mu'}(\vec{k}'\times\vec{E}_0'\right]\times\hat{n}=0$$

2.8 Brewster's Angle

2.9 Snell's Law

2.10 Total Internal Reflection

2.11 Reflection and Transmission Coefficients

$$\vec{s}\cdot\hat{n}=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|E_0|^2cos(i)$$

$$\vec{s'}\cdot\hat{n}=\frac{1}{2}\sqrt{\frac{\varepsilon'}{\mu'}}|E_0'|^2\cos(r)$$

$$\vec{s''}\cdot\hat{n}=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|E_0''|^2\cos(r)'$$

$$T=\frac{\vec{s'}\cdot\hat{n}}{\vec{s}\cdot\hat{n}} \qquad R=\frac{\vec{s''}\cdot\hat{n}}{\vec{s}\cdot\hat{n}}$$

2.12 Dispersion Model for time-varying field

$$m[\ddot{\vec{x}}+\gamma\dot{\vec{x}}+\omega_0^2\vec{x}]=-e\vec{E}(\vec{x},t)$$

2.13 Dispersion

$$\frac{\varepsilon(\omega)}{\varepsilon_0}=1+\frac{Ne^2}{\varepsilon_0 m}\sum_j\frac{f_j}{\omega_j^2-\omega^2-i\omega\gamma_j}$$

2.14 Attenuation of a plane wave