Physics 412—Quantum Mice Care and Feeding Winter 2020

You're considering changing suppliers for quantum mouse feed, switching to Energy Boost Mouse Chow™ from Acme Conglomerate. All that you know about this new feed is that it will change the Hamiltonian for the mice to be

$$H' \leftrightarrow \frac{2}{5}\begin{bmatrix} 6 & 3 \\ 3 & 14 \end{bmatrix},$$

which has eigenvalues 2 and 6. (This is represented in the attitude basis.) How will this affect your mice? Use what you know about quantum mechanics to discuss the repercussions of switching to this new feed on the attitude, behavior and/or size of the mice. As a reminder, the states and operators for the mice are

$$A \leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad |h\rangle \leftrightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad |u\rangle \leftrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$B \leftrightarrow \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad |p\rangle \leftrightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}, \qquad |a\rangle \leftrightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix},$$

$$H \leftrightarrow \frac{2}{5}\begin{bmatrix} 6 & 2 \\ 2 & 9 \end{bmatrix}, \qquad |4\rangle \leftrightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad |2\rangle \leftrightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \end{bmatrix},$$

$$W \leftrightarrow \frac{2}{5}\begin{bmatrix} 21 & -8 \\ -8 & 9 \end{bmatrix}, \qquad |s\rangle \leftrightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad |l\rangle \leftrightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

As before you can assume that the energy eigenvalues are expressed in units where $\hbar = 1$, so you can drop the factor of $\hbar$ from the exponents involving time. (Alternatively, you can assume that the energy eigenvalues are actually proportional to $\hbar$.)

Under the old feed, the evolution of a happy mouse was given by

$$|h(t)\rangle \leftrightarrow \frac{1}{5}\begin{bmatrix} e^{-4it} + 4e^{-2it} \\ 2e^{-4it} - 2e^{-2it} \end{bmatrix}$$

The probability of the mouse remaining happy was

$$P(h) = \frac{1}{25}\left(17 + 8\cos 2t\right) \qquad \text{Minimum: } \frac{9}{25} = 0.36$$

Let's see what will happen with the new feed.

The eigenstates of H' are

$$|H',6\rangle \leftrightarrow \frac{1}{\sqrt{10}}\begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \text{and} \quad |H',2\rangle \leftrightarrow \frac{1}{\sqrt{10}}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

The propagator is then

$$U(t) = e^{-6it}|H',6\rangle\langle H',6| + e^{-2it}|H',2\rangle\langle H',2|$$

$$\leftrightarrow \frac{e^{-6it}}{10}\begin{bmatrix}1\\3\end{bmatrix}\begin{bmatrix}1 & 3\end{bmatrix} + \frac{e^{-2it}}{10}\begin{bmatrix}-3\\1\end{bmatrix}\begin{bmatrix}-3 & 1\end{bmatrix}$$

$$\leftrightarrow \frac{e^{-6it}}{10}\begin{bmatrix}1 & 3\\3 & 9\end{bmatrix} + \frac{e^{-2it}}{10}\begin{bmatrix}9 & -3\\-3 & 1\end{bmatrix}$$

$$\leftrightarrow \frac{1}{10}\begin{bmatrix}e^{-6it}+9e^{-2it} & 3e^{-6it}-3e^{-2it}\\3e^{-6it}-3e^{-2it} & 9e^{-6it}+e^{-2it}\end{bmatrix}$$

Now, apply to $|h\rangle$:

$$|h(t)\rangle \leftrightarrow \frac{1}{10}\begin{bmatrix}e^{-6it}+9e^{-2it}\\3e^{-6it}-3e^{-2it}\end{bmatrix}$$

Then $$P(h) = \frac{1}{100}\left|e^{-6it}+9e^{-2it}\right|^2$$

$$= \frac{1}{100}\left(1+81+9e^{-4it}+9e^{4it}\right)$$

$$= \frac{1}{100}\left(82+18\cos 4t\right)$$

minimum: $0.64 > 0.36$

Frequency: $\omega = 4$ compared to $\omega = 2$

Now apply to $|s\rangle \longleftrightarrow \frac{1}{\sqrt{5}}\begin{bmatrix}1\\2\end{bmatrix}$

$$|s(t)\rangle \rightarrow \frac{1}{10}\begin{bmatrix} e^{-6it}+9e^{-2it} & 3e^{-6it}-3e^{-2it} \\ 3e^{-6it}-3e^{-2it} & 9e^{-6it}+e^{-2it}\end{bmatrix}\left(\frac{1}{\sqrt{5}}\begin{bmatrix}1\\2\end{bmatrix}\right)$$

$$\rightarrow \frac{1}{10\sqrt{5}}\begin{bmatrix} e^{-6it}+9e^{-2it}+6e^{-6it}-6e^{-2it} \\ 3e^{-6it}-3e^{-2it}+18e^{-6it}+2e^{-2it}\end{bmatrix}$$

$$|s(t)\rangle \leftrightarrow \frac{1}{10\sqrt{5}}\begin{bmatrix} 7e^{-6it}+3e^{-2it} \\ 21e^{-6it}-e^{-2it}\end{bmatrix} \leftarrow$$

to find $P(s,t)$, calculate $|\langle s|s(t)\rangle|^2$

$$P(s,t) = \left|\frac{1}{50}\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix} 7e^{-6it}+3e^{-2it} \\ 21e^{-6it}-e^{-2it}\end{bmatrix}\right|^2$$

$$= \frac{1}{2500}\left|7e^{-6it}+3e^{-2it}+42e^{-6it}-2e^{-2it}\right|^2$$

$$= \frac{1}{2500}\left|49e^{-6it}+e^{-2it}\right|^2$$

$$= \frac{1}{2500}\left(2401+1+49e^{-4it}+49e^{4it}\right)$$

$$= \frac{1}{2500}\left(2402+98\cos 4t\right)$$

mostly small, but some large mixed in!