PHY 474 — Spring 2021

Homework 5—due by 5:00 PM, Friday, May 21

Late submissions will be accepted until 8 AM on Monday (May 24); also see page 2.

1. In class, we calculated the Main Sequence lifetime using a general relation between luminosity and mass. Let's try a different approach. The Main Sequence star α Centauri A has luminosity $L = 1.5\ L_\odot$, and mass $M = 1.1\ M_\odot$. Assume α Centauri A was initially made of pure hydrogen, and that about 10% of its mass is converted to helium during its Main Sequence lifetime.

Useful Information: $M_\odot = 1.99 \times 10^{30}$ kg, $L_\odot = 3.828 \times 10^{26}$ W.

(a) How many helium nuclei are produced during the Main Sequence lifetime of α Centauri A?

(b) How much energy does α Centauri A produce during its Main Sequence lifetime? Recall that the energy released when four hydrogen nuclei are fused into one helium nucleus in the pp-chain is 26.7 MeV. Express your answer in J.

(c) Compute the Main Sequence lifetime of α Centauri A in Gyr, where 1 Gyr $= 10^9$ yr.

2. The kinetic energy of protons at which quantum mechanical tunneling (through the Coulomb barrier) has a significant probability is

$$E \approx \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{2m_p}{h^2}$$

(a) Use this equation to show that for helium to fuse into carbon on the horizontal branch, the kinetic energy per particle at the star's core (and hence T_c) has to be 64 times higher than is required for hydrogen fusion on the Main Sequence.

(b) Use the equation for E above, together with $E = (3/2)\,kT$, to compute the temperature for hydrogen fusion and helium fusion.

3. In class, we learned about the Standard Solar Model. Plots of this model are very useful for understanding what is going on in the Sun. The file provided was downloaded from `sns.ias.edu`; the first row is self-explanatory. Read the data into your software of choice and plot the following.

(a) Plot X *vs.* $R/R_\odot$ and Y *vs.* $R/R_\odot$, where X is the hydrogen mass fraction and Y is the helium mass fraction. Put both on the same plot. Submit *only* this program.

Note: Be aware that $R/R_\odot$ is in the second column (not the first, which is $M/M_\odot$).

(b) Comment on your plot in part (a), especially any interesting features that stand out.

(c) Plot T *vs.* $R/R_\odot$.

(d) Plot $L/L_\odot$ *vs.* $R/R_\odot$ and also $M/M_\odot$ *vs.* $R/R_\odot$, putting both on the same.

4. In class we learned that all stars lose mass. There are various empirical relationships for such mass loss. One of them is Reimer's law for the mass loss rate on the Asymptotic Giant Branch, and it is given by

$$\frac{dM}{dt} = -\frac{c\eta LR}{M}$$

where L is the luminosity, R is the radius, and M is the mass of the AGB star in solar units, and dM/dt is the mass loss rate of the AGB star in $M_\odot$ yr^{-1}. Meanwhile, $c = 4 \times 10^{-13}$ units is a constant, and η is a free parameter $\sim$1; for this problem, assume $\eta = 1$.

Integrate Reimer's law to derive an expression for the mass $M(t)$ of the AGB star as a function of time. Use boundary conditions M_0 at time $t = 0$, and mass equal to M at time t. You can make the simplifying assumption that L and R of the AGB star do not change as it loses mass.

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.