Exam 1 Corrections

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PHY 440 Classical Mechanics

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Problem 1

On the exam I completely misread/misinterpreted this problem. I read this problem as find the constrains for whatever reason. Looking back at this problem I can now clearly solve these problems. To find these we just take the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{q} = 0.$$

1a.

Given,

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

After the derivatives there are 3 const. motions.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \rightarrow \ddot{x}m = const.$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = 0 \rightarrow \ddot{y}m = const.$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{z}} - \frac{\partial L}{\partial z} = 0 \rightarrow \ddot{z}m = const.$$

1c.

Given,

$$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 - mg(l-x)sin(\alpha)$$

x will becomes $x = \theta R$ for this transformation.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \rightarrow \ddot{x}m + mgsin(\alpha)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \rightarrow m\ddot{\theta}R^2 + I\ddot{\theta} = mgRsin(\alpha)$$

θ is not a constant of motion for this Lagrangian since the momentum in not conserved.

1d.

Given,

$$L = \frac{1}{2}ml^2(\dot{\theta}^2 + \dot{\phi}^2 sin^2\theta) + mglcos(\theta)$$

There will not be a const. of motion for θ since there is a $\dot{\theta}$ and θ.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} - \frac{\partial L}{\partial \phi} = 0 \rightarrow \ddot{\phi}ml^2 sin^2\theta = const.$$

1e.

Given,

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GMm}{r}$$

since there is a $\dot{r}$ and r there will not be an const. of motion.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} - \frac{\partial L}{\partial \phi} = 0 \rightarrow \ddot{\phi}ml^2 sin^2\theta = const.$$

Problem 2

2a.

I was initial confused on this problem because I was unsure if the block could move or not. For the corrections I will assume that the wedge does move. For this problem we also need to know the distance the block is from the top of the wedge. I had erased this even though it was correct. The wedge of mass m_1 can only move in the x direction so it is

$$T = \frac{1}{2}m_1\dot{x}_1^2$$

and the smaller mass of m_2 is

$$T = \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2)$$

Transforming the coordinates brings

$$x_2 = lcos(\alpha) \qquad \dot{x}^2 = \dot{x}_1 + \dot{l}cos\alpha \tag{1}$$
$$y_2 = h - lsin(\alpha) \qquad \dot{y}_2 = -\dot{l}sin(\alpha) \tag{2}$$
$$y_1 = 0 \tag{3}$$

where l is the distance from the block to the top of the wedge and h is the height of the wedge.

$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(2\dot{x}\dot{l}cos(\alpha) + \dot{l}^2 + \dot{x}^2)$$

and the potential is

$$V = m_2 g(h - lsin(\alpha)).$$

The Lagrangian becomes

$$L = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(2\dot{x}\dot{l}cos(\alpha) + \dot{l}^2 + \dot{x}^2) - m_2 g(h - lsin(\alpha))$$

2b.

The potential energy of the system is actually

$$V = kx^2 + 2kL(L - \sqrt{x^2 + y^2})$$

since there is a need for 2 potential terms that may have different lengths at different times. The final Lagrangian becomes

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - kx^2 + 2kL(L - \sqrt{x^2 + y^2})$$

2c.

Let $\theta = \omega t$ and let

$$x = Rcos(\theta) + lsin(\phi) \quad (4)$$
$$y = Rsin(\theta) - lsin(\phi) \quad (5)$$

We have that

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$
$$= \frac{1}{2}m(-R\omega sin(\theta) + l\dot{\phi}cos(\phi))^2 + \frac{1}{2}m(R\omega cos(\theta) + l\dot{\phi}sin(\phi))^2.$$

Which will reduce down to

$$\frac{1}{2}m(R^2\omega^2 + 2R\omega l\dot{\phi}sin(\phi - \theta) + l^2\dot{\phi}^2).$$

The potential energy for the system is

$$V = mg(Rsin(\theta) - lcos(\phi)).$$

Thus, the Lagrangian is

$$L = \frac{1}{2}m(R^2\omega^2 + 2R\omega l\dot{\phi}sin(\phi - \theta) + l^2\dot{\phi}^2) - mg(Rsin(\theta) - lcos(\phi)).$$

2d.

If $x = x - l$ then l would go to zero since the time derivative of l with respect to x is 0. I need to watch my derivatives closer and make better sense of them to avoid this problem.

Problem 3

The system constrains are based on the length of the rod and the angle the masses depend on. Therefore we have that the virtual work is

$$x = lcos(\theta) \qquad x\delta = lcos(\theta)\delta\theta \tag{6}$$
$$y = lsin(\theta) \qquad y\delta = lsin(\theta)\delta\theta \tag{7}$$

For virtual work we have that

$$F_i \frac{\partial x_i}{\partial q_\alpha}.$$

Which will become

$$mg\delta y = -F\delta x \rightarrow mg\delta y + F\delta x = 0.$$

Finally, let's solve for F,

$$F = mg\frac{lcos(\theta)\delta\theta}{lsin\theta\delta\theta} \rightarrow F = mgcot(\theta).$$

Problem 4

I tend to have problems with my signs when writing out the Lagrangian. When I right it in the form of

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}.$$

I do tend to have more success when writing it in the form of

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0.$$

This allows me to control most of the problems I may have with the signs. In this case on the exam I must have skipped over it. Making sense of the final equation of motion would also help me determine if it were correct or not. A $-m_2 g$ makes more sense than a $+m_2 g$ for this system, since $m_2 > m_1$ the mass m_2 will be moving downwards.