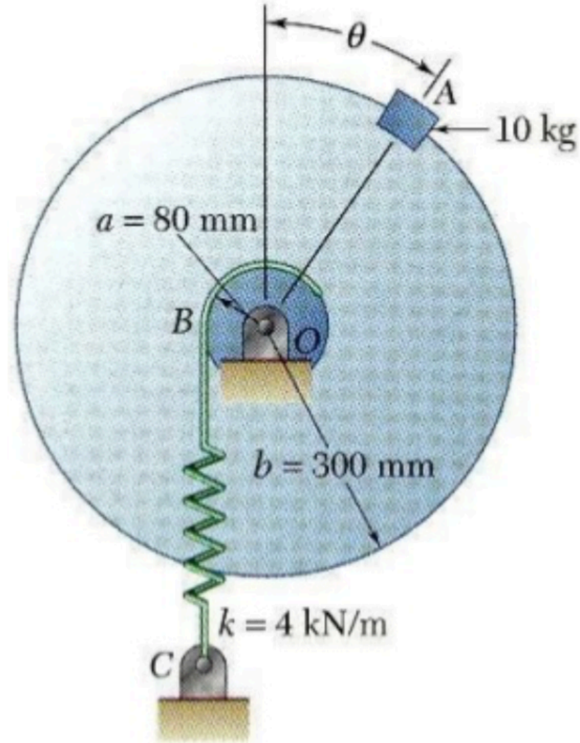
Virtual Work in the form $\partial V / \partial q_\alpha$ example problem:

The spring is unstretched for $\theta = 0$. The wheel can rotate clockwise. Derive an expression for the total potential energy of the system. Determine θ when the system is in equilibrium by using the principle of virtual work in the form $\frac{\partial V}{\partial q_\alpha} = 0$.



$$V_s = \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}k(a\theta)^2$$
$$V_g = mgy = mgb\cos\theta$$
$$V = \frac{1}{2}k(a\theta)^2 + mgb\cos\theta$$

Determine the equilibrium angle by setting

$$\frac{\partial V}{\partial \theta} = 0$$

$$\frac{\partial V}{\partial \theta} = ka^2\theta - mgb\sin\theta = 0$$
$$\sin\theta = \frac{ka^2\theta}{mgb}$$
$$\sin\theta = \frac{(4000)(0.08^2)}{(10)(9.81)(0.31)}\theta = 0.8699\theta$$

There are two solutions for $\theta = 0$ and $\theta = 0.902\ rad$

Take the second derivative to determine if these equilibrium positions are stable of unstable.

$$\frac{\partial^2 V}{\partial \theta^2} = ka^2 - mgb\cos\theta$$

At $\theta = 0$: $\frac{\partial^2 V}{\partial \theta^2} = -3.82 < 0$ (unstable)

At $\theta = 0.902$: $\frac{\partial^2 V}{\partial \theta^2} = 7.36 > 0$ (stable)