F-1 problem 1

(1) $t=0$ $\quad H = \frac{4\omega}{\hbar} S_z^{(n)} \otimes S_x^{(a)}$.

Spin operator neutron $\to S_z^{(n)}$
Spin operator atom $\to S_x^{(a)}$

$$|\Psi(0)\rangle = 1/2\sqrt{2}\left(|+z^{(n)}\rangle + |-z^{(n)}\rangle\right) \otimes \left((1+i)|+x^{(a)}\rangle + (1+i)|-x^{(a)}\rangle\right)$$
$$= 1/2\sqrt{2}\left((1+i)|+z,+x\rangle + (1+i)|-z,+x\rangle + (1-i)|+z,-x\rangle + (1-i)|-z,-x\rangle\right)$$

$$|\Psi(t)\rangle = \frac{(1+i)}{2\sqrt{2}} e^{-i\omega t}|+z,+x\rangle + \frac{(1-i)}{2\sqrt{2}} e^{i\omega t}|+z,-x\rangle + \frac{(1+i)}{2\sqrt{2}} e^{i\omega t}|-z,+x\rangle + \frac{(1-i)}{2\sqrt{2}} e^{-i\omega t}|-z,-x\rangle$$

(a) At $t=0$ the state becomes

$$|\Psi(0)\rangle = \frac{(1+i)}{2\sqrt{2}}|+z,+x\rangle + \frac{(1-i)}{2\sqrt{2}}|+z,-x\rangle + \frac{(1+i)}{2\sqrt{2}}|-z,+x\rangle + \frac{(1-i)}{2\sqrt{2}}|-z,-x\rangle$$

An entangled state means that it cannot be factored into the product of states for each of the particles. In this state we can see that we are unable to factor out $+z, -z, +x, -x$. If we look at the first particle, there is two instances of a $+z$ and two instances of a $-z$. If we look at the second particle, there are two instances of $+x$ and two instances of $-x$. Therefore, none of this is able to be factored. This means the state $|\Psi(0)\rangle$ is an entangled state.

(B)

$$P_{+z+x} = |\langle +z,+x|\Psi(0)\rangle|^2 = \left|\frac{(1+i)}{2\sqrt{2}}\right|^2 = \frac{1}{4}$$
$$P_{+z-x} = |\langle +z,-x|\Psi(0)\rangle|^2 = \left|\frac{(1-i)}{2\sqrt{2}}\right|^2 = \frac{1}{4}$$
$$P_{-z+x} = |\langle -z,+x|\Psi(0)\rangle|^2 = \left|\frac{(1+i)}{2\sqrt{2}}\right|^2 = \frac{1}{4}$$
$$P_{-z-x} = |\langle -z,-x|\Psi(0)\rangle|^2 = \left|\frac{(1-i)}{2\sqrt{2}}\right|^2 = \frac{1}{4}$$

$|\langle -z,-x|\psi(0)\rangle|^2 \quad \left|\frac{\quad}{2\sqrt{2}}\right|^2 \quad 4$

(C)

Particle 2 \ Particle 1	+z	-z
+x	1/4	1/4
-x	1/4	1/4

$C = 1/4 + 1/4 - 1/4 - 1/4 = 0$

$\boxed{\text{Uncorrelated}}$

(D) $t = \pi/4\omega \qquad e^{-i\omega t} \rightarrow e^{-i\pi/4} \qquad e^{i\omega t} \rightarrow e^{i\pi/4}$

$$|\psi(\tfrac{\pi}{4\omega})\rangle = \frac{(1+i)}{2\sqrt{2}} e^{-i\pi/4} |{+z},{+x}\rangle + \frac{(1-i)}{2\sqrt{2}} e^{i\pi/4} |{+z},{-x}\rangle + \frac{(1+i)}{2\sqrt{2}} e^{i\pi/4} |{-z},{+x}\rangle + \frac{(1-i)}{2\sqrt{2}} e^{-i\pi/4} |{-z},{-x}\rangle$$