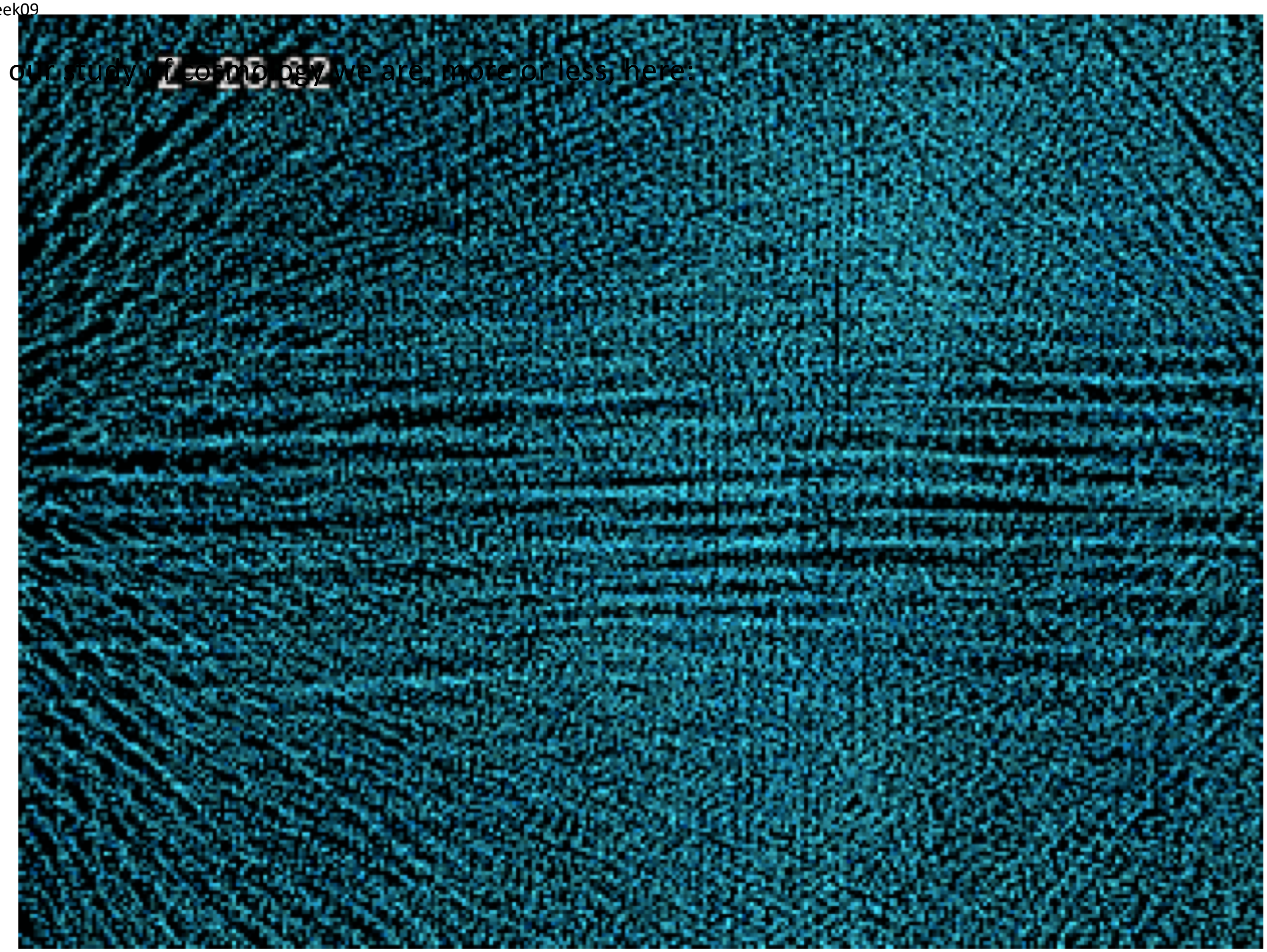
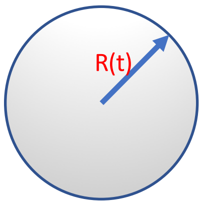
Some logistics

- Next homework now due Monday, March 9th
- Unexpectedly I have to travel to DC again next week, so no class on March 11th
  - I would like to do presentations on March 16th, does this work for everyone?
  - Review for final would also be on the 16th.

So far in our study of cosmology we are, more or less, here:



How does this happen?



Suppose we have a static, homogeneous, matter-only universe that has a small over-density suddenly added to it. The new density is described by

$$\rho = \bar{\rho}\,(1+\delta); \quad \delta \ll 1$$

Applying Newton's second law, the acceleration at the sphere's surface due to this extra mass is:

$$\ddot{R} = -\frac{G(\Delta M)}{R^2} = -\frac{G}{R^2}\left(\frac{4\pi}{3}R^3\bar{\rho}\,\delta\right)$$

or

$$\frac{\ddot{R}}{R} = -\frac{4\pi\,G\,\bar{\rho}}{3}\delta(t)$$

Let's explore some of the consequences of this result. Do question (1 a and b) on the worksheet and S T O P

(1 a and b) If there is an over density, the sphere will *collapse.* There are two unknowns, *R(t)* and $\delta(t)$.

To find another relationship, we use conservation of mass which tell us that

$$M = \frac{4\pi}{3}\bar{\rho}[1+\delta)t)]\ R(t)^3 \Rightarrow \boxed{R(t) = R_o\,[1+\delta(t)]^{-1/3}}$$

where

$$R_o \equiv \left(\frac{3M}{4\pi\bar{\rho}}\right)^{1/3} = \text{constant}$$

Finish question (1)

(1c)
$$\begin{aligned} R(t) &= R_o\,[1+\delta(t)]^{-1/3} \text{ now take } \ddot{R} \\ \ddot{R} &= -\frac{1}{3}R_o\ddot{\delta} \approx \frac{1}{3}R\ddot{\delta} \text{ substitute into eq (1)} \\ \ddot{\delta} &= 4\pi G\,\bar{\rho}\,\delta \end{aligned}$$

(1 d) Straight forward second order ODE. It yields, $\delta(t) = A_1 e^{t/t_{\rm dyn}} + A_2 e^{-t/t_{\rm dyn}}$ where $t_{\rm dyn} = \dfrac{1}{\sqrt{4\pi G\bar{\rho}}}$

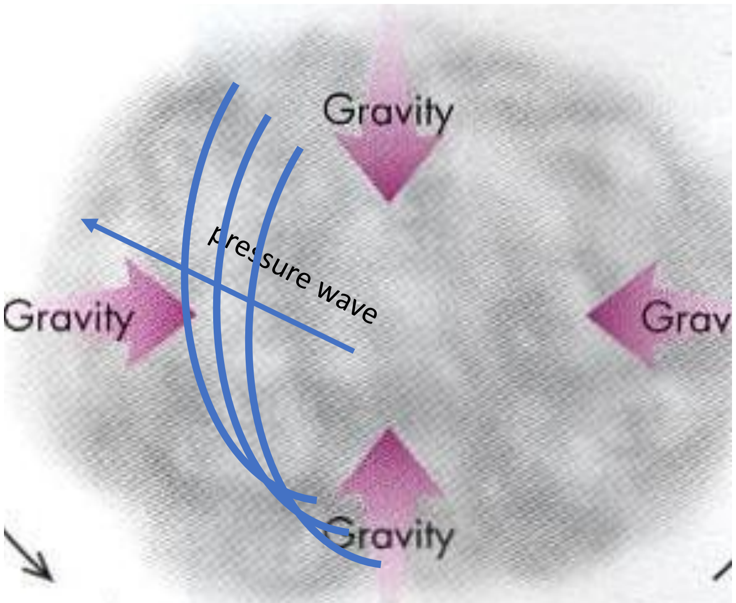
The positive term quickly dominates and the perturbation grows exponentially. Yikes! Even with $\delta \ll 1$ a density fluctuation grows very large very fast.

The term $\dfrac{1}{\sqrt{4\pi\,G}}$ has a value of 9.6 hours, so $t_{\rm dyn} = \dfrac{9.6\,\text{hours}}{\sqrt{\bar{\rho}}}$

Notice the result depends on $\bar{\rho}$ only and not on *R*. This result is problematic for the universe.

To see why, and to speculate about a solution, do question 2 on the worksheet and S T O P

The Jean's Length



Okay, now we have the opposite problem, how does collapse ever happen?

The resolution. Speed of pressure wave in some region of radius ***R***, is

$$c_s \sim \frac{R}{t_{\rm P}} \text{ or } t_{\rm P} \sim \frac{R}{c_s}$$

Do question (3 a) on the worksheet and S T O P (3 a) $t_{\rm P} < t_{\rm dyn}$

Do question (3 b) on the worksheet and S T O P

(3 b) $t_{\rm P} < t_{\rm dyn}$ but $t_{\rm P} = R/c_s \;\Rightarrow R = c_s t_{\rm dyn}$ Finish question (3) $\lambda_j = c_s \left(\frac{\pi c^2}{G\bar{\rho}}\right)^{1/2} = 2\pi\, c_s\, t_{\rm dyn}$

Gravitational collapse on cosmic scales

For a spatially flat universe, we have $t = \frac{1}{H} = \left(\frac{3c^2}{8\pi G\bar{\epsilon}}\right)^{1/2}$ and note that this is just $\frac{1}{H} = \left(\frac{3}{2}\right)^{1/2} t_{\rm dyn}$

The Jeans length for this universe will then be $\lambda_j = 2\pi\, c_s\, t_{\rm dyn} = 2\pi \left(\frac{2}{3}\right)^{1/2} \frac{c_s}{H}$

Previously we had seen that $c_s = c\,\sqrt{w}$ so $\lambda_j = 2\pi\, c_s\, t_{\rm dyn} = \boxed{2\pi \left(\frac{2}{3}\right)^{1/2} \sqrt{w}\,\frac{c}{H}}$

Do question (4) on the worksheet

$$c_s(\text{photons}) = c/\sqrt{3} \approx 0.58c$$

(4) $$c_s(\text{baryon}) = \left(\frac{0.26\,\text{eV}}{1140 \times 10^6\ \text{eV}}\right) c = 1.5 \times 10^{-5}\ c$$

$$\frac{c_s(\text{baryon})}{c_s(\text{photon})} \approx \frac{1.5 \times 10^{-5}}{0.58} \approx 2.6 \times 10^{-5}$$

Some additional terminology. The ***Jeans mass*** is the mass contained within a sphere of radius λ_J and for baryons is given by:

$$M_J \equiv \rho_{\rm baryon} \left(\frac{4\pi}{3}\lambda_J^3\right)$$