PHY 411 **Winter 2021**

Homework 4—due by 9:00 PM, Friday, Feb 5

There is no late deadline due to the Midterm next week.

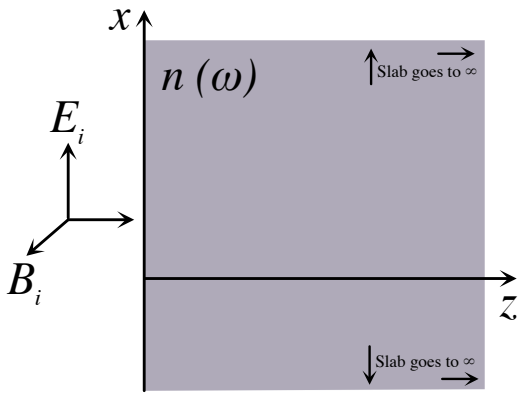
1. A plane wave of frequency ω is incident normally from vacuum (see figure below) on a semi-infinite slab of material with a complex index of refraction $n(\omega)$, where $n^2(\omega) = \epsilon(\omega)/\epsilon_0$.

Show that the reflection coefficient is given by

$$R = \left|\frac{1 - n(\omega)}{1 + n(\omega)}\right|^2$$

whereas the transmission coefficient is given by

$$T = \frac{4\,\mathrm{Re}\,n(\omega)}{|1 + n(\omega)|^2}$$



2. In class we discussed a static model for a substance in the presence of an electric field. The polarization of neighboring molecules gives rise to an internal field $\vec{E}_i$ in addition to the average macroscopic field $\vec{E}$, so that the dipole moment is modified to

$$\left\langle \vec{p}_{\mathrm{mol}} \right\rangle = \epsilon_0 \gamma_{\mathrm{mol}} \left(\vec{E} + \vec{E}_i \right)$$

where γ_{mol} is the molecular polarizability. Jackson finds that $E_i = \vec{P}/3\epsilon_0$.

Starting from the definition that $\vec{P} = N\langle \vec{p}_{\mathrm{mol}} \rangle$, where N is the number of molecules per unit volume, derive the Clausius-Mossotti equation

$$\gamma_{\mathrm{mol}} = \frac{3}{N} \frac{(\epsilon/\epsilon_0 - 1)}{(\epsilon/\epsilon_0 + 2)}$$

3. Consider the following experimental data for nitrogen.

Temperature (K)	296.9	296.9	296.9	296.9
Pressure (Pa)	1.0200×10^5	57.50×10^5	221.6×10^5	1011.6×10^5
Density (kg m^{-3})	1.180	66.04	236.1	578.0
Dielectric constant	1.00052	1.03109	1.11413	1.29633

(a) Calculate γ_{mol} for each of the four sets of data given above.

(b) In principle, γ_{mol} is a function of the electric field, but for a wide range of field strengths, it is a constant that characterizes the response of the molecules to an applied field. Did you find that γ_{mol} is constant in all the four instances that you calculated above? Comment.

Question 4 is on the next page . . .

4. Answer the following. *You must support your answer with appropriate calculations and explanations if you want full credit. In both parts (a) and (b) below, a yes/no answer that is not supported by calculations and explanations will be awarded zero credit.*

The electron density in the ionosphere reaches a maximum value of 1.5×10^{12} m^{-3}.

(a) Can a 2 MHz wave be used to communicate with a satellite?

(b) Can a 2 GHz wave be used to communicate with a satellite?