Physics 460—Practice S–3 (Due Apr 28, 1 pm) Name: Timothy Holmes

S–3: I can apply and interpret wave functions in three dimensions in Cartesian coordinates.

Unsatisfactory Progressing Acceptable Polished

The normalized energy eigenstates for the particle in a cube are

$$\psi_{n_x,n_y,n_z}(x,y,z) = \sqrt{\frac{8}{L^3}}\sin\frac{n_x\pi x}{L}\,\sin\frac{n_y\pi y}{L}\,\sin\frac{n_z\pi z}{L},$$

with energies

$$E_{n_x,n_y,n_z} = \frac{\hbar^2\pi^2}{2mL^2}\left(n_x{}^2 + n_y{}^2 + n_z{}^2\right).$$

(1) A particle is prepared in the state

$$\psi(x,y,z) = A\Big[\psi_{111}(x,y,z) - 2\mathrm{i}\psi_{122}(x,y,z)\Big].$$

(a) Find A and explain why you don't have to evaluate any integrals to do so.

(b) If you measured the energy of the particle, what values could you obtain and with what probabilities?

(c) If you measured the position of the particle, what is the probability that you would find it in the lower corner of the box, in the region $0 \le x, y, z \le L/2$? Your answer should be a number!

Practice Assessment 3

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$$\psi_{n_x,n_y,n_z}(x,y,z) = \sqrt{\frac{8}{L^3}} \sin\frac{n_x \pi L x}{L} \sin\frac{n_y \pi L}{L} \sin\frac{n_z \pi z}{L}$$

With energies

$$E_{n_x,n_y,n_z} = \frac{\hbar^2 \pi^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

(1) A particle is prepared in the state

$$\psi(x,y,z) = A\left[\psi_{111}(x,y,z) - 2i\,\psi_{122}(x,y,z)\right]$$

(a) Find A and explain why you don't have to evaluate any integrals to do so.

We can rewrite the state as

$$|\psi\rangle = \left[\psi_{111}(x,y,z) - 2i\,\psi_{122}(x,y,z)\right]$$

Since we write this in bracket notation we can then solve is such that $A = 1/\sqrt{\langle\psi|\psi\rangle}$. Therefor, we have

$$1 = \langle\psi|\psi\rangle = A^2\left[\left(\langle 1,1,1| + 2i\langle 1,2,2|\right)\left(|1,1,1\rangle - 2i|1,2,2\rangle\right)\right]$$

$$1 = A^2\left[\cancel{\langle 1,1,1|1,1,1\rangle}^{\,1} + 4\cancel{\langle 1,2,2|1,2,2\rangle}^{\,1}\right] \Rightarrow 1 = A^2\left[5\right]$$

$A^2 = 1/_{5}$ $\Rightarrow$ $\boxed{A = 1/\sqrt{5}}$

$$A^2 = 1/5 \rightarrow \boxed{A = 1/\sqrt{5}}$$

$$|\psi\rangle = \frac{1}{\sqrt{5}}\Big[\,|1,1,1\rangle - 2i\,|1,2,2\rangle\Big]$$

(B)

$$E_{111} = \frac{\hbar^2\pi^2}{2mL^2}\left(1^2+1^2+1^2\right) = \boxed{\frac{3\hbar^2\pi^2}{2mL^2}}$$

$$E_{221} = \frac{\hbar^2\pi^2}{2mL^2}\left(2^2+2^2+1^2\right) = \boxed{\frac{9\hbar^2\pi^2}{2mL^2}}$$

$$P = |\langle\psi_{n_x,n_y,n_z}|\psi\rangle|$$

$$\left.\begin{aligned} P_{111} &= \frac{1}{5}\,|\langle 111|111\rangle| = \boxed{\frac{1}{5}} \\ P_{221} &= \frac{1}{5}\,|4\,\langle 221|221\rangle| = \boxed{\frac{4}{5}} \end{aligned}\right\} 1\ \checkmark$$

Not Required (For Practice)

$$\langle H\rangle = \sum E_n P_n = \left(\frac{1}{5}\right)\frac{3\hbar^2\pi^2}{2mL^2} + \left(\frac{4}{5}\right)\frac{9\hbar^2\pi^2}{2mL^2}$$

$$= \frac{3\hbar^2\pi^2}{10mL^2} + \frac{36\hbar^2\pi^2}{10mL^2} = \boxed{\frac{39\hbar^2\pi^2}{10mL^2}}$$

(C) $\displaystyle\int_0^{L/2} \psi^*(x,y,z)\,\psi(x,y,z)$

$$= \int_0^{L/2}\int_0^{L/2}\int_0^{L/2} \frac{1}{5}\Bigg[\Bigg[\sqrt{\frac{8}{L^3}}\sin\frac{\pi x}{L}\sin\frac{\pi y}{L}\sin\frac{\pi z}{L} +$$

$$2i\sqrt{\frac{8}{L^3}}\sin\frac{\pi x}{L}\sin\frac{2\pi y}{L}\sin\frac{2\pi z}{L}\Bigg]\Bigg[\sqrt{\frac{8}{L^3}}\sin\frac{\pi x}{L}\sin\frac{\pi y}{L}\sin\frac{\pi z}{L}$$

$$+ 2i \left| \frac{8}{L^3} \sin \frac{\pi x}{L} \sin \frac{2\pi y}{L} \sin \frac{2\pi z}{L} \right| \Big] dx\,dy\,dz$$

Cross terms will cancel

$$\int_0^{L/2}\int_0^{L/2}\int_0^{L/2} \frac{1}{5}\left[\frac{8}{L^3} \sin^2 \frac{\pi x}{L} \sin^2 \frac{\pi y}{L} \sin^2 \frac{\pi z}{L}\right.$$

$$\left. + 4\,\frac{8}{L^3} \sin^2 \frac{\pi x}{L} \sin^2 \frac{2\pi y}{L} \sin^2 \frac{2\pi z}{L}\right]$$

$$\rightarrow \frac{8}{5L^3}\left[\left(\int_0^{L/2} \sin^2 \frac{\pi x}{L}\, dx\right)^3 + 4\int_0^{L/2} \sin^2 \frac{\pi x}{L} \sin^2 \frac{2\pi y}{L} \sin^2 \frac{2\pi z}{L}\right]$$

$$\rightarrow \frac{8}{5L^3}\left[\frac{L^3}{64} + 4\left[\frac{L^3}{64}\right]\right] \rightarrow \frac{8}{\cancel{5L^3}}\left[\frac{\cancel{5L^3}}{64}\right] = \frac{8}{64}$$

$$\boxed{= \frac{1}{8}}$$