PHY 474/374 Spring 2021

Class Summary—Week 8, Day 2—Wednesday, May 19

Elements beyond Iron

The production of elements heavier than iron calls for processes different from the fusion reactions that occur in the cores of stars. We have already learned that even the cores of high mass stars cannot produce elements beyond iron. It is worth taking a moment to remind ourselves why this is the case, that during the normal course of events, stars cannot produce elements heavier than iron by fusion. Recall that the Coulomb barrier for heavier elements will be much higher than for hydrogen, and so at lower temperatures there is very little probability of tunneling through these higher barriers. On the other hand, at high temperatures there are lots of photons present that would lead to the photodissociation any heavier nuclei that were formed. In fact, we have already learned that photodissociation of nuclei by high energy photons leads to quasi-equilibrium processes which predominantly generate nuclei around the maximum in the binding energy, i.e., around the iron group.

So, how can elements heavier than iron be formed? The answer comes from neutrons. Neutrons are electrically neutral, so they have no Coulomb barrier to overcome. Thus, if matter is exposed to a flux of neutrons, then heavier elements can be built up via neutron capture. The capture of a neutron in a nucleus increases its atomic mass without changing its atomic number, *as you discussed in Question 1 on today's worksheet.* Specifically, a nucleus with atomic number $\mathcal{Z}$ and atomic mass $\mathcal{A}$ transforms as

$$(\mathcal{Z}, \mathcal{A}) \quad + \quad n \quad \rightarrow \quad (\mathcal{Z}, \mathcal{A}+1) \quad + \quad \gamma \tag{15.1}$$

If the resulting nucleus is unstable to β-decay, then it decays according to

$$(\mathcal{Z}, \mathcal{A}+1) \quad \rightarrow \quad (\mathcal{Z}+1, \mathcal{A}+1) \quad + \quad \mathrm{e}^- \quad + \bar{\nu}_\mathrm{e} \tag{15.2}$$

where $\bar{\nu}_\mathrm{e}$ is the electron anti-neutrino. If you're familiar with this version of the beta decay, you will recall that a neutron inside the nucleus changes to a proton (which is why the atomic mass remains the same but the atomic number changes meaning that the nucleus has changed to that of a new element), and this is accompanied by the release of an electron and an electron anti-neutrino.

In reality, the situation is more complicated because the β-decay in equation (15.2) is typically fairly slow. Thus, the nucleus produced in equation (15.1) may have time to capture another neutron before it decays according to equation (15.2). The result of absorbing another neutron is then

$$(\mathcal{Z}, \mathcal{A}+1) \quad + \quad n \quad \rightarrow \quad (\mathcal{Z}, \mathcal{A}+2) \quad + \quad \gamma \tag{15.3}$$

Thus, neutron capture may proceed in two ways:

- In the process of slow neutron capture (***s*-process**), neutron capture is much slower than the β-decay.
- In the process of rapid neutron capture (***r*-process**), neutron capture is much more rapid than the β-decay.

We will now discuss these in more detail.

In order to understand the s-process and r-process, we must first consider the timescales. Let's begin by deriving an expression for the **lifetime of a nucleus against neutron capture**, τ_n.

Assuming that the cross section for neutron capture is independent of energy, we get that

$$\tau_n = \frac{1}{n_n \langle \sigma v \rangle}$$

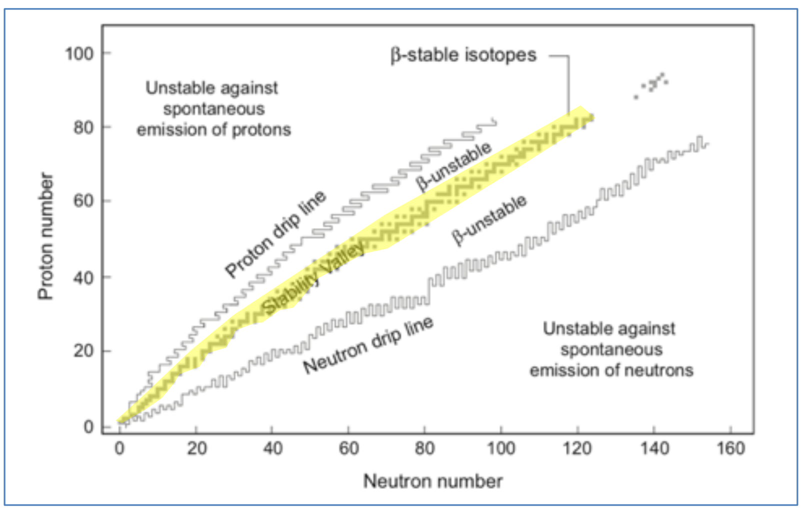
For a typical neutron cross-section of $\sigma \sim 10^{-25}$ cm^2, and a temperature of 5×10^8 K, *you showed in Question 2(a) of today's worksheet* that this gives

$$\tau_n \sim \frac{10^9 \text{ yr}}{(n_n/\text{cm}^{-3})}$$

Now, consider an environment in which the neutron density is low, $n_n \sim 10^5$ cm^{-3}. Using the expression above, we get that the lifetime of the nucleus against neutron capture in this region is

$$\tau_n \sim \frac{10^9 \text{ yr}}{10^5} = 10^4 \text{ yr}$$

as you obtained in Question 2(b) of today's worksheet. The s-process corresponds to this situation, for which the lifetime of the nucleus is long, compared to β-decay (which is typically on the order of hours). Why? Since the lifetime is long, the nucleus is stable. Thus, the nucleus can wait until another neutron comes by, at which point the next higher nucleus will be created, as in equation (15.3). Thus, the s-process proceeds along the **valley of stability** (see the $p - n$ graph below, where this is highlighted in yellow) forming a series of stable nuclei, until an unstable nucleus is formed, which will then undergo β-decay.

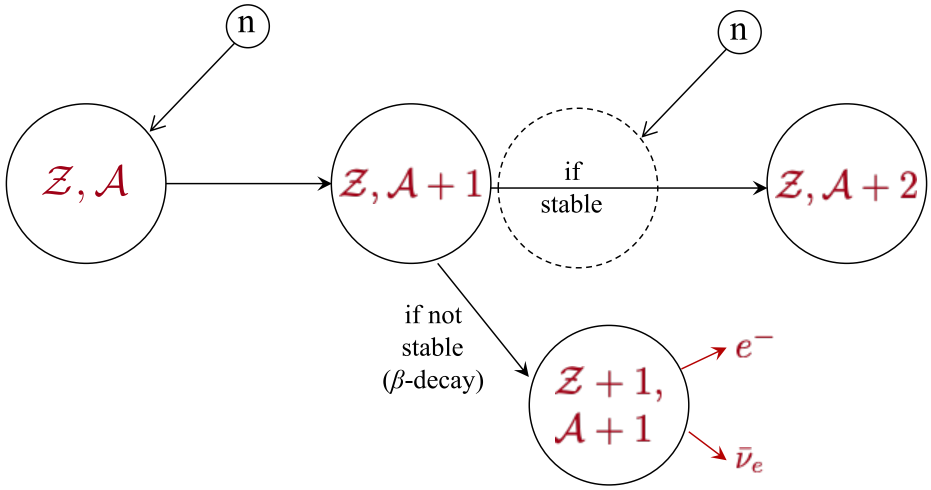


Next, consider an environment in which the neutron density is high, $n_n \sim 10^{23}$ cm^{-3}. Again, using the expression for τ_n above *as you did in Question 2(c) of today's worksheet*, we get that

$$\tau_n \sim \frac{10^9 \text{ yr}}{10^{23}} = 3 \times 10^{-7} \text{ s} \sim 10^{-7} \text{ s}$$

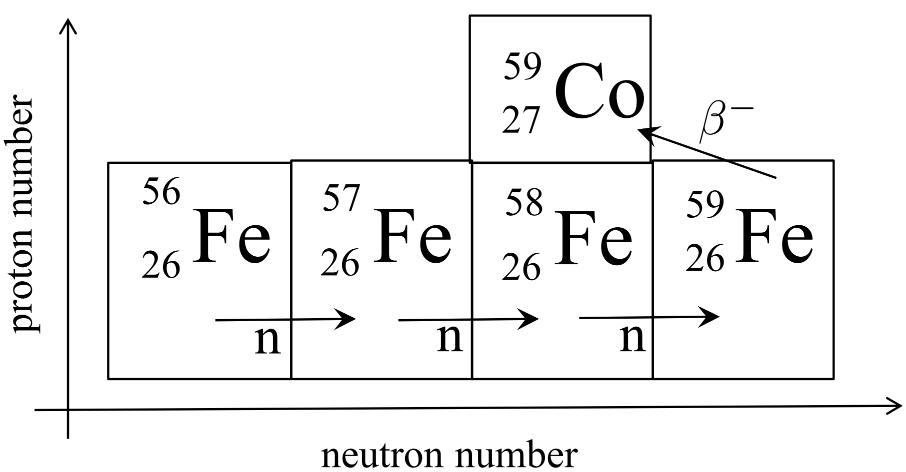
It is in this environment that the r-process takes place. Since the lifetime of the nucleus against neutron capture is a fraction of a second, and thus the neutron capture is much more rapid than the β-decay, the r-process produces predominantly neutron-rich nuclei.

Now, let's see how we can build up nuclei using the s-process. To begin, It is worth drawing the following flow chart, *as you did in Question 3(a) on today's worksheet.*



The flow chart shown above is based on equation (15.1) through equation (15.3). In the first step, the addition of a neutron to a nucleus with atomic number $\mathcal{Z}$ and atomic mass $\mathcal{A}$ produces an isotope with atomic mass $(\mathcal{A}+1)$, as we wrote in equation (15.1). If the resulting nucleus is unstable to β-decay, it will decay according to equation (15.2) and produce a new element with atomic number $\mathcal{Z}+1$, and atomic mass $(\mathcal{A}+1)$, as we wrote in equation (15.2). However, if the nucleus has an opportunity to capture a neutron before it β-decays, then it will produce another isotope with atomic mass $(\mathcal{A}+2)$. In this isotope has an opportunity to capture a neutron before it β-decays, it will create another isotope with atomic mass $(\mathcal{A}+2)$, and so on.

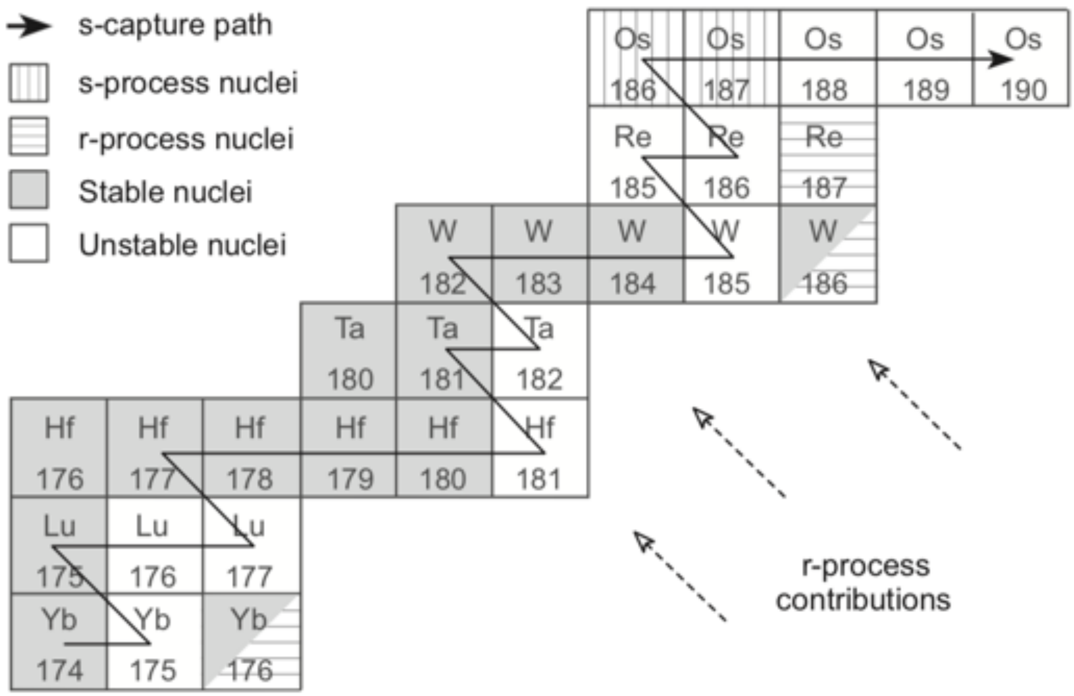
An actual example of how the s-process can build up a sequence of isotopes until it reaches an unstable isotope that β-decays is shown in the figure below, *which you completed in Question 3(b) on today's worksheet.*



In the figure above, we see that ^{56}Fe changes to ^{57}Fe by capturing a neutron, which then captures another neutron to become ^{58}Fe. The process continues until the unstable isotope ^{59}Fe is reached, which then β-decays to ^{59}Co. Note that, by convention, the neutron captures are drawn to proceed toward the right, whereas the β-decay is drawn diagonally backward toward the upper right. In this manner, the process accommodates increase in the neutron number toward the right of the graph, and increases in the proton number toward the top, while also making the β-decay process more visible.

Together, the *s*-process and *r*-process allows for the heavier elements to be built. Thus, nuclei that cannot be produced by regular fusion processes in the cores of stars can be built in this way. Some nuclei can be built through *s*-processes only, some through *r*-processes only, and some through both.

Consider the figure below which shows the building of the elements in the Yb-Os region.

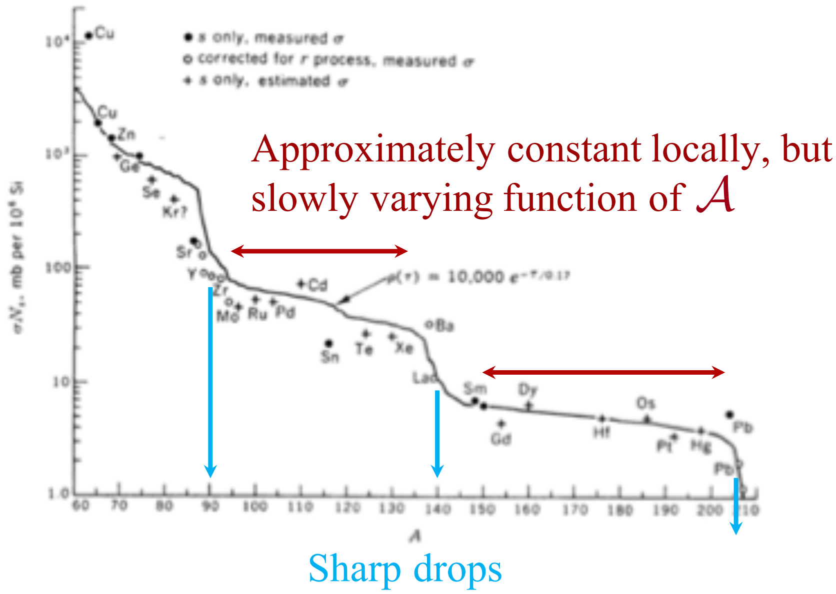


In Question 4(a) on today's worksheet, you followed the *s*-process starting from the nucleus of Ytterbium, Yb ($\mathcal{Z} = 70$, $\mathcal{A} = 174$). This captures a neutron to become ^{175}Yb, which then β-decays to Lutetium, Lu ($\mathcal{Z} = 71$, $\mathcal{A} = 175$). By capturing a neutron, ^{175}Lu then becomes ^{176}Lu. Now, notice that ^{175}Lu is marked as a stable nucleus, but ^{176}Lu is marked as unstable. Yet, it captures a neutron to become ^{177}Lu. This is where the issues of timescales comes in; the timescale for β-decay of ^{176}Lu is such that it can capture a neutron and form ^{177}Lu before it β-decays. It is the ^{177}Lu that β-decays to Hafnium, Hf ($\mathcal{Z} = 72$, $\mathcal{A} = 176$). Hafnium then goes through the *s*-process to form a series of isotopes of increasing atomic mass until ^{181}Hf β-decays to Tantalum, Ta ($\mathcal{Z} = 73$, $\mathcal{A} = 181$). After capturing a neutron, the next isotope ^{182}Ta β-decays to Tungsten, W ($\mathcal{Z} = 74$, $\mathcal{A} = 182$). After a series of *s*-process neutron captures, ^{185}W decays to Rhenium, Re ($\mathcal{Z} = 75$, $\mathcal{A} = 185$). The final step shown is Osmium Os ($\mathcal{Z} = 76$, $\mathcal{A} = 190$). Thus, we have verified that the *s*-process works as expected. Notice how the path stays very near the valley of stability.

Meanwhile, the *r*-process generally populates very neutron-rich isotopes. Some of these like ^{176}Yb are stable. Others, like ^{187}Re will subsequently β-decay to the valley of stability. Thus, many isotopes can be populated by both the *s*-process and the *r*-process.

Notice also that some isotopes like ^{186}W can **only** be populated by the *r*-process, because an unstable isotope lies to their left, blocking the *s*-process path.

The product $\sigma_{\mathcal{A}} N_{\mathcal{A}}$ is a slowly varying function of $\mathcal{A}$), where $\sigma_{\mathcal{A}}$ is the cross section for neutron capture and $N_{\mathcal{A}}$ is the abundance of the nuclei. This is shown in the figure below.



Given its slow variation, $\sigma_{\mathcal{A}} N_{\mathcal{A}}$ can be considered approximately constant locally along the s-process path, that is, in the segments marked in the figure above. *In Question 5(a) on today's worksheet*, you discussed why the relative abundance of a given s-process element depends, according to this relation, on its neutron capture cross-section.

In Question 5(b) on today's worksheet, you discussed the reason for the sharp drops marked in the figure above at atomic masses $\mathcal{A}$ equal to $\sim$90, 140, and 209, corresponding to neutron numbers neutron number $n = 50, 82$, and 126. Note that in the graphs shown in *Dalsgaard*, the neutron number is written in uppercase; take care you don't confuse it with the abundance. This can also be seen in a graph of cross section *vs.* atomic weight, which has minimums at values of n written above.