(1) A particle is in the ground state ($n_x = n_y = n_z = 1$) of a cubical box of sides $L$ as described in Section 2.1 of the course notes. The box is suddenly doubled in size in the $z$ direction, so its length along the $z$ axis is now $2L$.

(a) What are the energies and the energy eigenstates of this new, larger box? In other words, how would you modify Eqs. (2.20) and (2.21) to account for the new box size?

(b) Take the state of particle to be its state before the box expanded: the ground state of the original box (Eq. (2.20) with $n_x = n_y = n_z = 1$) for $z < L$ and zero for $z > L$. If you measure the energy of the particle in the new box, what is the most probable value you would measure and what is the probability of measuring that value? What is the second most probable value, and what is its probability?

(a) Let $L_z = 2L$. Replace one $L$ with $L_z$:

$$\psi_{n_x,n_y,n_z}(x,y,z) = \sqrt{\frac{8}{L^2 L_z}} \sin\frac{n_x \pi x}{L} \sin\frac{n_y \pi y}{L} \sin\frac{n_z \pi z}{L_z}$$

$$E = \frac{\hbar^2 \pi^2}{2m^2}\left(\frac{n_x^2}{L^2} + \frac{n_y^2}{L^2} + \frac{n_z^2}{L_z^2}\right)$$

Now replace $L_z$ with $2L$

$$\psi_n = \sqrt{\frac{4}{L^3}} \sin\frac{n_x \pi x}{L} \sin\frac{n_y \pi y}{L} \sin\frac{n_z \pi z}{2L}$$

$$E = \frac{\hbar^2 \pi^2}{2m L^2}\left(n_x^2 + n_y^2 + \frac{n_z^2}{4}\right)$$

(b) This part is very similar to the previous activity

Take

$$\psi_0(x,y,z) = \begin{cases} \sqrt{\frac{8}{L^3}} \sin\frac{\pi x}{L} \sin\frac{\pi y}{L} \sin\frac{\pi z}{L} & z \le L \\ 0 & L < z \le 2L \end{cases}$$

Expand in terms of the new energy eigenstates.