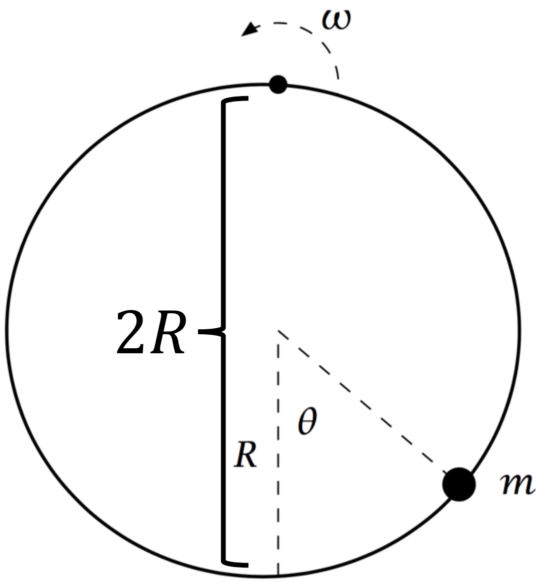
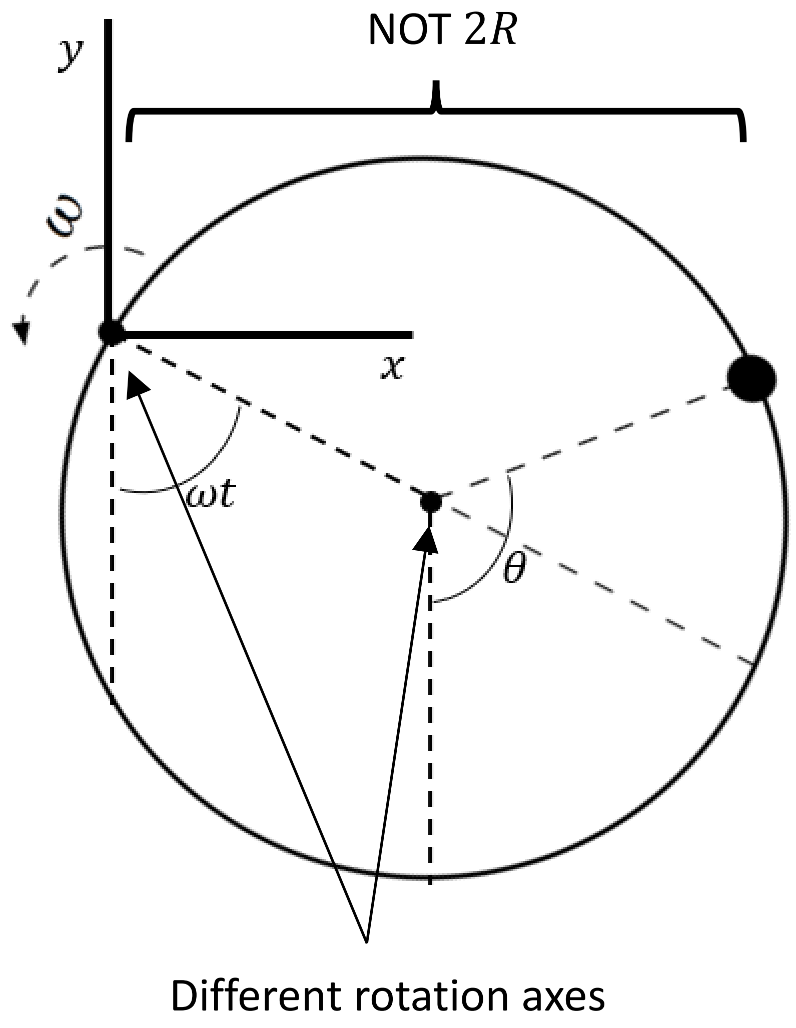
Activity 10 from last Thursday:

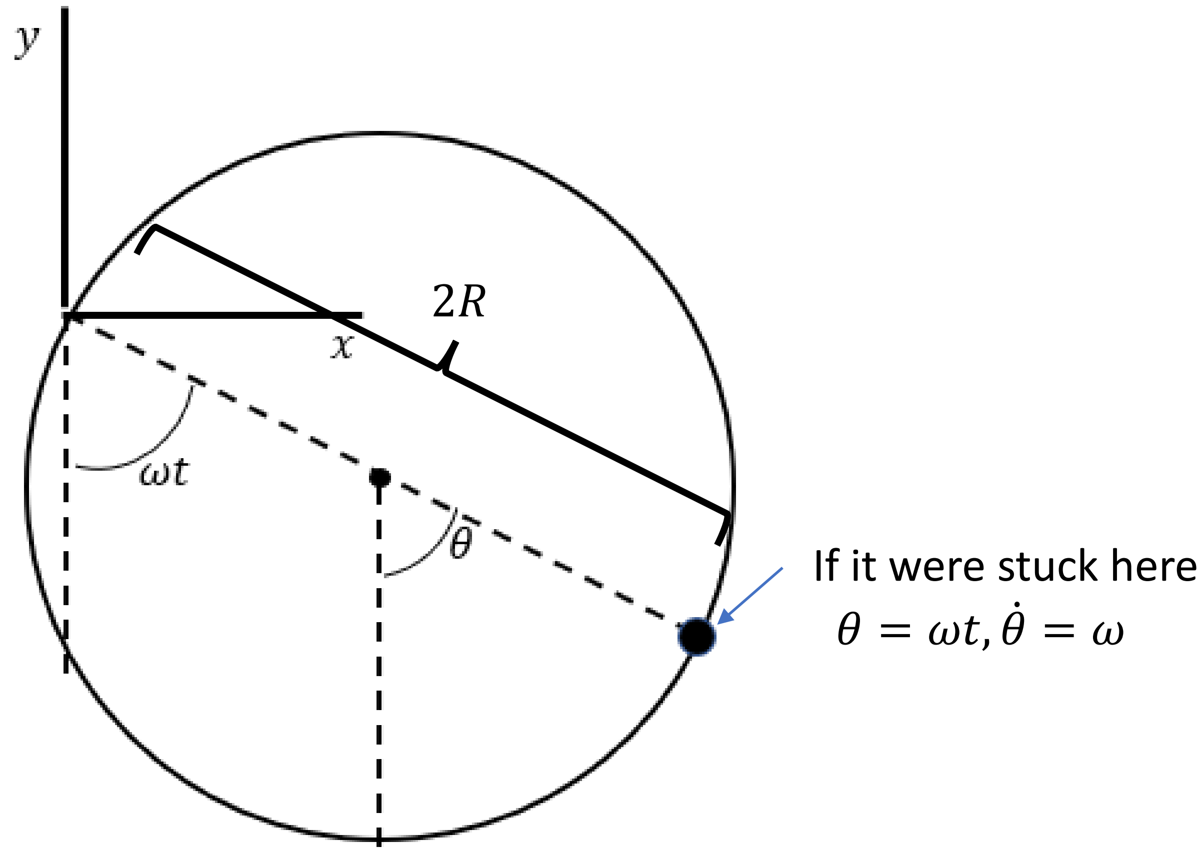
$$L = \frac{mR^2}{2}\left[\omega^2 + \dot{\theta}^2 + 2\omega\dot{\theta}\cos(\theta - \omega t)\right]$$

Why isn't the for this system Lagrangian simply

$$L = \frac{m(2R)^2\dot{\theta}^2}{2}$$





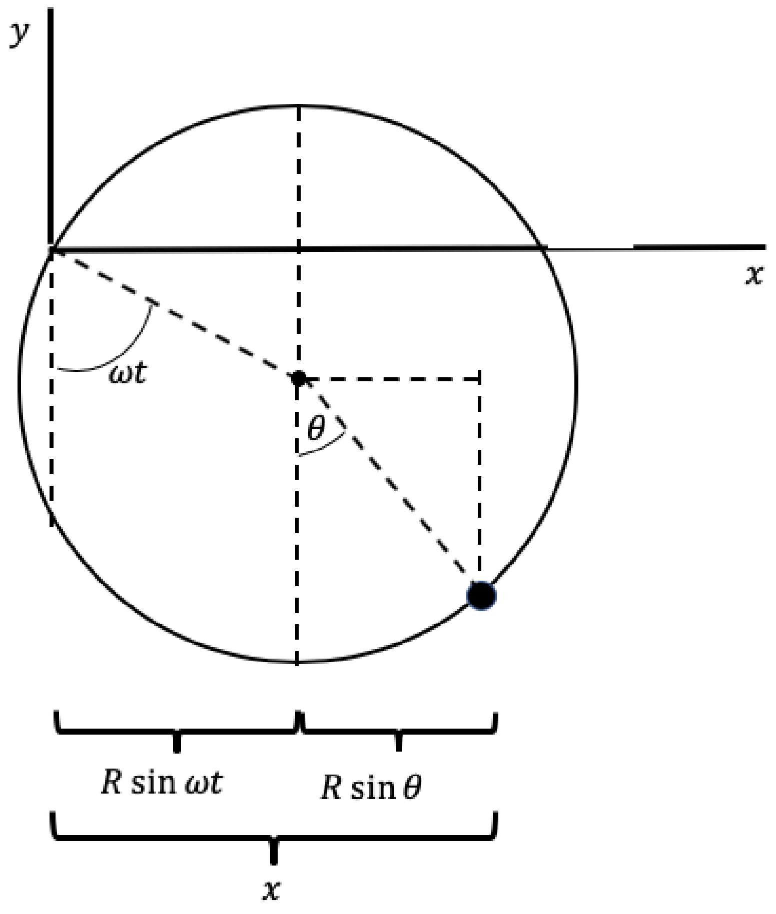


$$L = T = \frac{mR^2}{2}\left[\omega^2 + \dot{\theta}^2 + 2\omega\dot{\theta}\cos(\theta - \omega t)\right]$$

$$= \frac{mR^2}{2}\ [\omega^2 + \omega^2 + 2\omega^2\cos(\omega t - \omega t)]$$

$$\frac{mR^2}{2}\ [4\omega^2] = \frac{m(2R)^2\dot{\theta}^2}{2}$$

- $x = x_c + R\sin\theta = R\sin\omega t + R\sin\theta$
- $y = y_c - R\cos\theta = -R\cos\omega t - R\cos\theta$
- $\dot{x} = R\omega\cos\omega t + R\dot{\theta}\cos\theta$
- $\dot{y} = R\omega\sin\omega t + R\dot{\theta}\sin\theta$



- $\dot{x}^2 + \dot{y}^2 = \left(R\omega\cos\omega t + R\dot{\theta}\cos\theta\right)^2 + \left(R\omega\sin\omega t + R\dot{\theta}\sin\theta\right)^2$
- $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{mR^2}{2}[\omega^2 + \dot{\theta}^2 + 2\omega\dot{\theta}\cos(\theta - \omega t)]$

Explicit time dependence

- $\phi = \theta - \omega t \Rightarrow \theta = \phi + \omega t, \dot{\theta} = \dot{\phi} + \omega$
- $L = \frac{mR^2}{2}\left[\omega^2 + \dot{\theta}^2 + 2\omega\dot{\theta}\cos(\theta - \omega t)\right]$
- $= \frac{mR^2}{2}\left[\omega^2 + (\dot{\phi} + \omega)^2 + 2\omega(\dot{\phi} + \omega)\cos(\phi)\right]$

We eliminated the explicit time dependence!

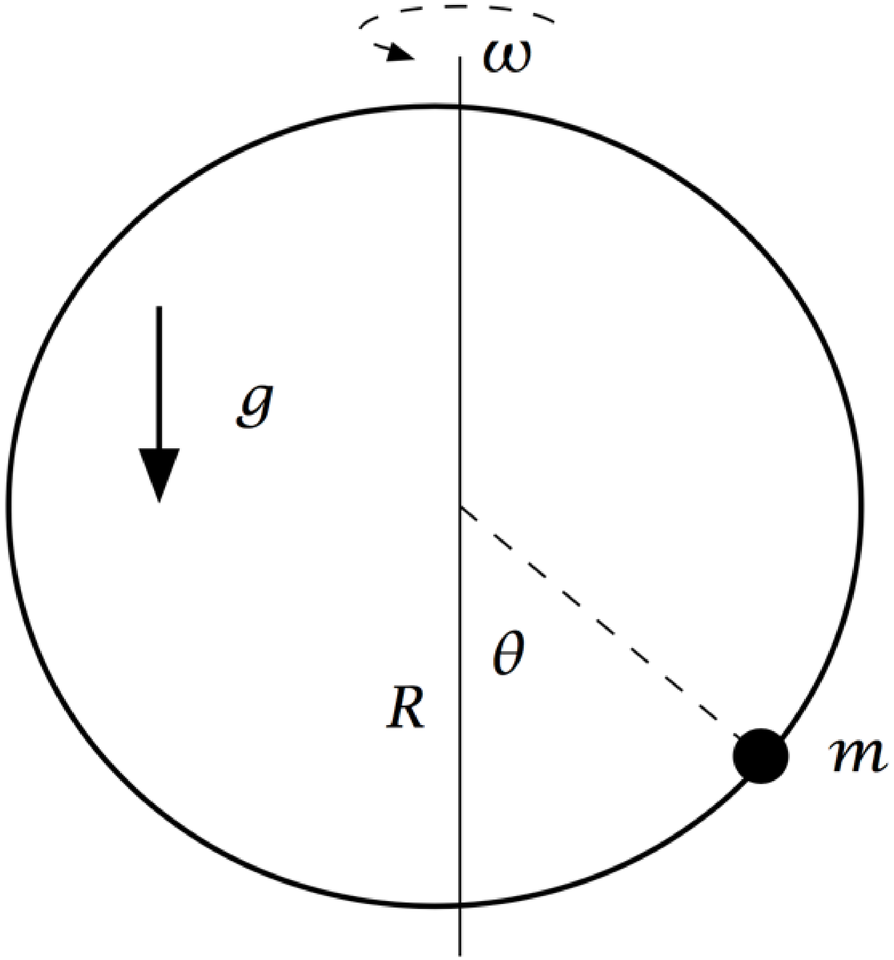
$V = \frac{mR^2}{2}\ 2\omega^2 \cos\phi = m(\omega^2 R)R\cos\phi$
behaves like a potential energy term

Plug into Lagrange's equation:

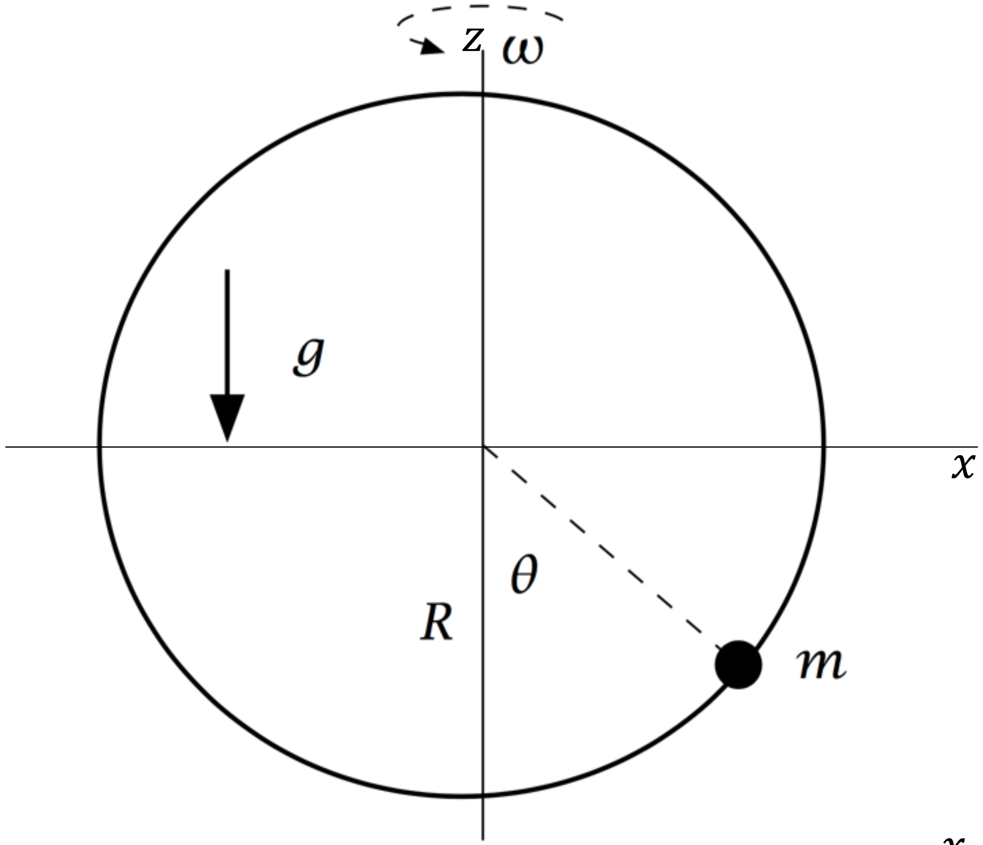
$$\ddot{\phi} = -\omega^2 \sin\phi$$

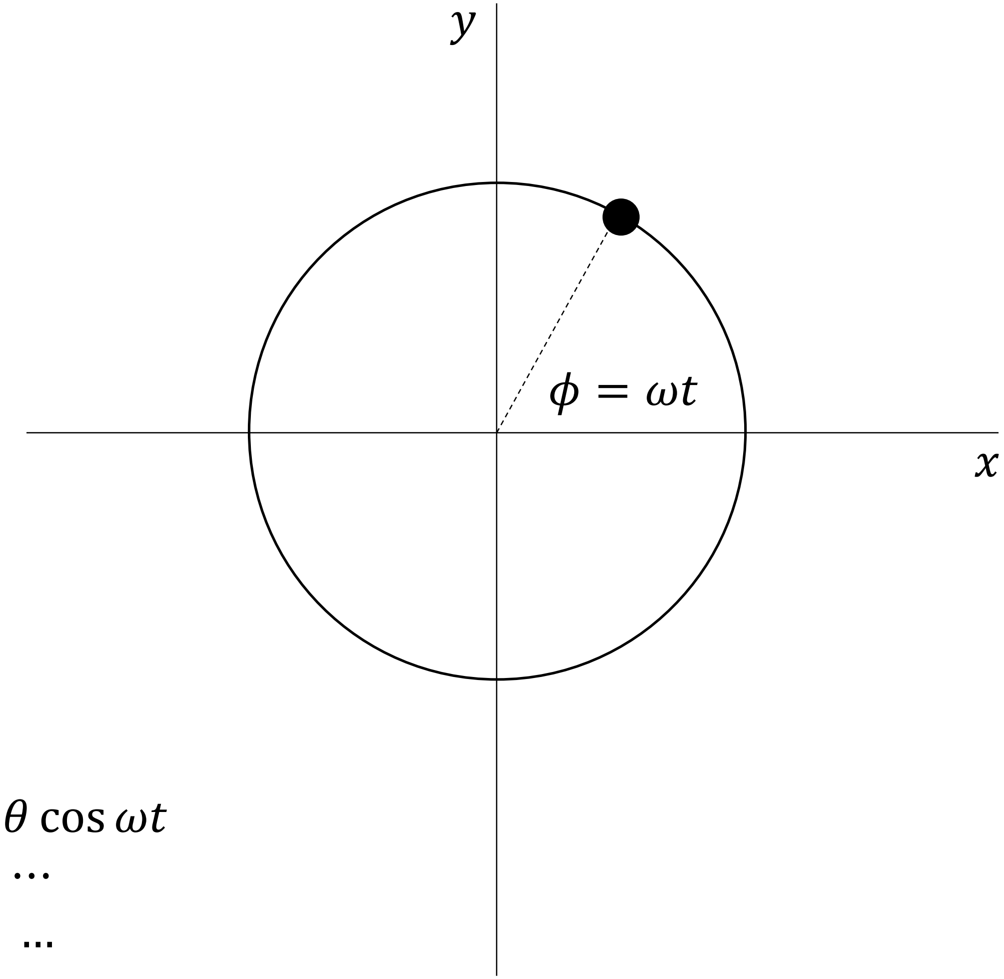
Behaves like a simple pendulum with $g = \omega^2 R$

Activity 11



Coordinate transformation





$$x = R \sin\theta \cos\omega t$$
$$y = \cdots$$
$$\dot{x} = \ \ldots$$
$$\dot{y} = \ \ldots$$

- $L = \frac{1}{2} mR^2 \left(\dot{\theta}^2 + \omega^2 \sin^2 \theta \right) + mgR \cos \theta$

- $\ddot{\theta} = \left(\omega^2 \cos \theta - \frac{g}{R} \right) \sin \theta$

What are the equilibrium solutions?

https://www.youtube.com/watch?v=kS9WoYj2AaY
https://www.youtube.com/watch?v=jpKn0F5idXY