Homework 4

(1) The wave functions for energy eigenstates of

$$\psi_{nlm}(r,\theta,\varphi) = A j_0(k_{n0} r) Y_0^0(\theta,\varphi)$$

(a) $$k_{n0} = \frac{z_{0n}}{a}$$

$$A^2 \int_0^a r^2 |j_0(k_{n0} r)|^2 dr = 1 \quad \text{where } k_{nl} = \frac{z_{nl}}{a}$$

let $x = \frac{r}{a} \Rightarrow dr = a\,dx \quad r^2 = a^2x^2 \quad k_{n0} r = z_{n0} x$

$$A^2 a^3 \int_0^1 x^2 |j_0(n\pi x)|^2 dx = 1$$

$$1 = \frac{1}{2\pi^2 n^2 a^2} a^3 A^2 \rightarrow \boxed{A = \frac{\sqrt{2}\pi n}{\sqrt{a}}}$$

$$\psi_{n00}(r,\theta,\varphi) = \frac{\sqrt{2}\pi n}{\sqrt{a}} \frac{\sin(n\pi/a\, r)}{n\pi/a\, r} Y_0^0(\theta,\varphi)$$

(B) $$A^2 \int_0^{r/a} r^2 |j_0(k_{n0} r)|^2 dr \quad r_{00} = x_{00} a =$$

$$A^2 \int_0^{r/a} r^2 |j_0(n\pi r)|^2 dr$$

$$\rightarrow A^2 a^3 \int_{z_{0n}}^{z_{0n}} x^2 |j_0(z_{0n} x)|^2 dx \quad \boxed{= 1/2}$$

Not dependent on

(C) $$\langle r \rangle = \int_0^\infty r^2 dr \left(r |\psi(r)|^2 \right)$$

$$\int_0^{\infty} \frac{\sqrt{2\pi n}}{\sqrt{a}} \left(\frac{\sin(n\pi/a\, r)}{n\pi/a\, r} \sqrt{\frac{1}{4\pi}} \right)^2 r^2\, dr$$

$$\frac{2\pi n^2}{a} \frac{1}{4\pi} \int_0^{\infty} \frac{\sin(n\pi/a\, r)}{n\pi/a} r^2\, dr$$

$$= \boxed{1/2\, a}$$

(D) Spherical Bessel Function is not $\emptyset$ at $r = 0$