An Introduction to Cosmological Kinematics

(1) Gravity is the only force that plays a key role on cosmological scales. Newton's law of gravitaion is

$$F_G = -\frac{GM_g m_g}{r^2}$$

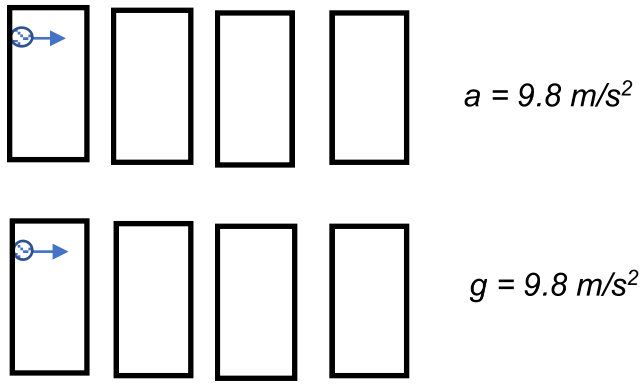
(a) What are the units of G?

(b) What does the negative sign indicate?

(c) For point particles, it is easy to determine from where r should be measured. Where should r be measured from for extended objects?

(d) Apply Newton's gravitational law to Newton's second law ($F = ma$). What do you obtain for the acceleration?

(2) Consider the figure below. Each rectangle represents an enclose box an instant of time later, with time progressing left to right. In the top panel, an object is launched with a horizontal velocity while the box accelerates upward with $a = 9.8\text{m/s}^2$, while in bottom panel, the object is launched with the same horizontal velocity while the box sits on the surface of the earth.



(a) On the top row of rectangles, sketch the position of the object as time advances.

(b) Do the same for the bottom row of rectangles.

(c) Should the sketches be the same or not? What does this tell you about objects accelerating and objects in a gravitational field?

(3) We are going to work our way through some of the notational aspects associated with metrics.

(a) Consider the following two vectors,

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \;;\;\; \vec{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

Take the dot product $\vec{A} \cdot \vec{B}$ using matrix mulitplication. In order to take the dot product using matrix mulitplication, what did you have to do to one of the vectos? Does the answer change if you did $\vec{B} \cdot \vec{A}$ instead?

(b) Now write vectors, $\vec{A}, \vec{B}$ as a linear combination of the standard Cartesian basis vectors,

$$\hat{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad \hat{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad \hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};$$

(c) Now repeat part (a), but using the vectors expressed as in part (b)

(d) Show explicitly using the vectors defined in part (a) that the dot product is found by

$$A_\nu \left(e^\nu \cdot e_\mu\right) B^\mu$$

(e) To make things a little easier, let's go to the 2-D Euclidean case so that the basis vectors are now

$$\hat{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \hat{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Find all the terms for $\hat{e}_\mu \cdot \hat{e}_\nu$.

(f) Show that $\vec{A} \cdot \vec{B} = g_{\mu\nu} A^\mu B^\nu$.

(4) Recall that we previously defined the Hubble distance as $d_H(t_o) \equiv c/H_o$.

(a) In lecture we derived, $v_p(t_o) = H_o d_p(t_o)$. Substitute the Hubble distance, $d_H(t_o)$ for $d_p(t_o)$. What results when you do this.

(b) The Hubble distance is currently about $d_H = 4380 \pm 130\text{Mpc}$ (using $H_o = 68 \pm 2$ km s/(km Mpc). Suppose two objects are separated by 6000 Mpc. What is their velocity? Is it possible for objects to be separated greater than d_H? Explain your answer.

(c) We previously saw that $v = H_o d$, what is the difference between what we just derived and what we found previously, especially given that they have the exact same functional form.

(5) Recap the important topics covered today. Compare and constrast your results with others at your table

Homework 01–Due Friday Jan 17

1. Problem 2.2
2. Problem 3.3
3. Problem 3.5

Additional Grad Student Problem(s)

4. Problem 2.4
5. The critical energy density at the present time is,

$$\epsilon_{c,o} = \frac{3c^2}{8\pi G} H_o^2.$$

   (a) Calculate $\epsilon_{c,o}$ in SI units using $H_o = 70$ km s^{-1} Mpc^{-1}. Since H_o is known to within 10%, express your answer for $\epsilon_{c,o}$ to with in 20% limits.

   (b) Convert your answer to units more often used in cosmology, GeV m^{-3}.

   (c) Find the equivalent mass density, $\rho_{c,o} = \epsilon_{c,o}/c^2$ in SI units.

   (d) Convert your answer to solar masses per Mpc^{-3}. A solar mass, $M_\odot = 1.99 \times 10^{30}$ kg