PHY 411 Winter 2021

Homework 2—due by 9:00 PM, Tuesday, Jan 19

Note: The submission has been moved one day later from our usual due to the MLK holiday.

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Friday (Jan 22). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

1. The plane wave solutions to the Helmholtz wave equation are

$$\vec{E}(\vec{x},t) = \vec{\mathcal{E}}\, e^{i(k\hat{n}\cdot\vec{x}-\omega t)}$$
$$\vec{B}(\vec{x},t) = \vec{\mathcal{B}}\, e^{i(k\hat{n}\cdot\vec{x}-\omega t)}$$

where $\vec{\mathcal{E}}$ and $\vec{\mathcal{B}}$ are constant vectors.

By substituting the plane wave solutions written above into Faraday's law, $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$, derive the expression

$$\vec{B} = \sqrt{\mu\epsilon}\, \left(\frac{\vec{k}\times\vec{E}}{k}\right)$$

Note: You may assume without proof that

$$\vec{\nabla}\times\vec{\mathcal{E}}\, e^{i(k\hat{n}\cdot\vec{x}-\omega t)} = -\vec{\mathcal{E}}\times\vec{\nabla}\, e^{i(k\hat{n}\cdot\vec{x}-\omega t)}$$

2. In class, we derived the Helmholtz equation for $\vec{E}$. Starting from the Maxwell equations, and following similar procedures, derive the Helmholtz equation for $\vec{B}$:

$$\left(\nabla^2 + \mu\epsilon\omega^2\right)\vec{B} = 0$$

3. Consider the electric field given by

$$\vec{E} = E_0\,\frac{\hat{x}-i\hat{y}}{\sqrt{2}}\, e^{i(kz-\omega t)}$$

(a) By explicit computation, verify Gauss' law $\vec{\nabla}\cdot\vec{E} = 0$ for this field.

(b) Use Faraday's law to find $\vec{B}$.

(c) What is the state of polarization of this wave? Explain your answer clearly.

4. In class, we represented the electric and magnetic fields along two orthogonal directions by using unit vectors $\hat{\epsilon}_1$ and $\hat{\epsilon_2}$; this can be termed the *linear polarization basis.*

Another general representation of the polarization may be done in terms of the unit vectors:

$$\hat{\epsilon}_+ = \frac{1}{\sqrt{2}}\left(\hat{\epsilon}_1 + i\hat{\epsilon}_2\right) \qquad \text{and} \qquad \hat{\epsilon}_- = \frac{1}{\sqrt{2}}\left(\hat{\epsilon}_1 - i\hat{\epsilon}_2\right)$$

and this can be termed the *circular polarization basis.*

A useful way to express the state of polarization is via the four Stokes parameters, $s_0, s_1, s_2,$ and s_3. Suppose these have the values

$$s_0 = 3, \qquad s_1 = -1, \qquad s_2 = 2, \qquad s_3 = -2$$

(a) For the values of $s_0, s_1, s_2,$ and s_3 given above, determine the amplitude of the electric field up to an overall phase in the linear polarization basis.

(b) For the values of $s_0, s_1, s_2,$ and s_3 given above, determine the amplitude of the electric field up to an overall phase in the circular polarization basis.