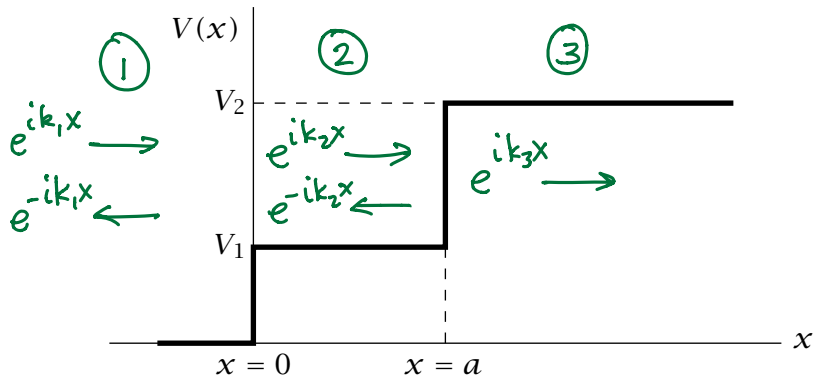
(1) Consider the "double step" potential shown below. Find the reflection and transmission coefficients for a beam of particles incident from the left when $E > V_2$.

$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$ $k_2 = \sqrt{\frac{2m(E-V_1)}{\hbar^2}}$ $k_3 = \sqrt{\frac{2m(E-V_2)}{\hbar^2}}$



① $\psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$

② $\psi_2(x) = Ce^{ik_2x} + De^{-ik_2x}$

③ $\psi_3(x) = Fe^{ik_3x}$

We want to find $R = \left|\frac{B}{A}\right|^2$

Boundary Conditions:

$x=0$: $\psi_1(0) = \psi_2(0)$ $\quad A + B = C + D$ ⓐ

$\psi_1'(0) = \psi_2'(0)$ $\quad ik_1(A - B) = ik_2(C - D)$ ⓑ

$x=a$: $\psi_2(a) = \psi_3(a)$ $\quad Ce^{ik_2a} + De^{-ik_2a} = Fe^{ik_3a}$ ⓒ

$\psi_2'(a) = \psi_3'(a)$ $\quad ik_2\left(Ce^{ik_2a} - De^{-ik_2a}\right) = ik_3Fe^{ik_3a}$ ⓓ

Note: all the i's cancel from the coefficient in the derivative B.C.s!

Multiply ⓒ by $-k_3$ and add to ⓓ

$-k_3Ce^{ik_2a} - k_3De^{-ik_2a} + k_2Ce^{ik_2a} - k_2De^{-ik_2a} = 0$

$$(k_2 - k_3)Ce^{ik_2a} - (k_2 + k_3)De^{-ik_2a} = 0$$

Multiply ⓐ by k_2 and add to ⓑ

$k_2A + k_2B + k_1A - k_1B = 2k_2C$

$$(k_2+k_1)A + (k_2-k_1)B = 2k_2 C$$

Multiply ⓐ by k_2 and subtract ⓑ

$$k_2 A + k_2 B - k_1 A + k_1 B = 2k_2 D$$

$$(k_2-k_1)A + (k_2+k_1)B = 2k_2 D$$

← solve for C & D and substitute ↓

From above,

$$(k_2 - k_3)e^{ik_2 a} C = (k_2+k_3)e^{-ik_2 a} D$$

$$\frac{(k_2-k_3)e^{ik_2 a}}{2k_2}\left((k_2+k_1)A + (k_2-k_1)B\right)$$

$$= \frac{(k_2+k_3)e^{-ik_2 a}}{2k_2}\left((k_2-k_1)A + (k_2+k_1)B\right)$$

Gather B terms on left, A terms on right ...

$$\left[(k_2-k_3)(k_2-k_1)e^{ik_2 a} - (k_2+k_3)(k_2+k_1)e^{-ik_2 a}\right]B$$

$$= \left[-(k_2-k_3)(k_2+k_1)e^{ik_2 a} + (k_2+k_3)(k_2-k_1)e^{-ik_2 a}\right]A$$

$$\therefore \quad \frac{B}{A} = \frac{\left[-(k_2-k_3)(k_2+k_1)e^{ik_2 a} + (k_2+k_3)(k_2-k_1)e^{-ik_2 a}\right]}{\left[(k_2-k_3)(k_2-k_1)e^{ik_2 a} - (k_2+k_3)(k_2+k_1)e^{-ik_2 a}\right]}$$

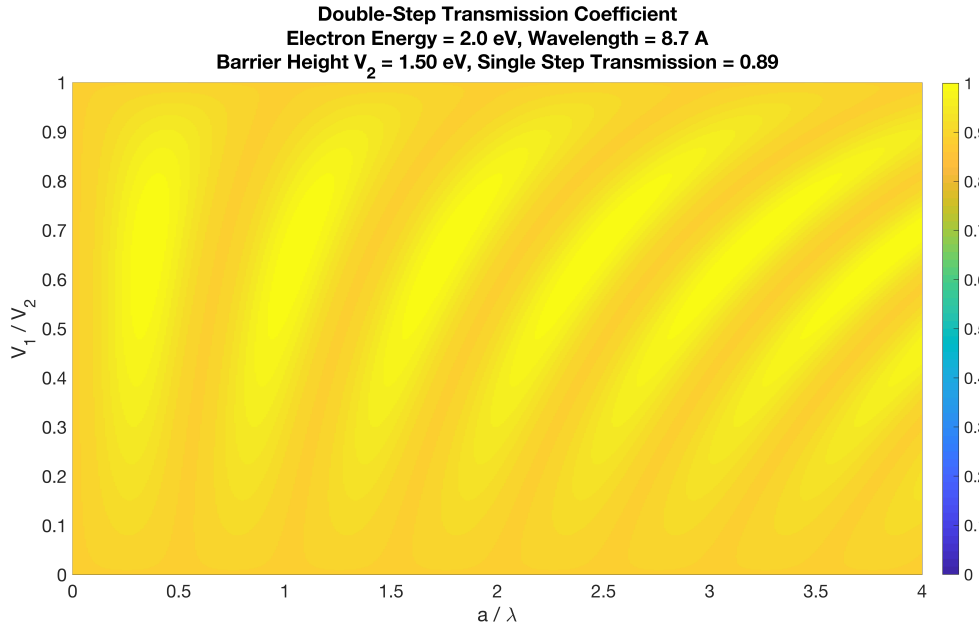
Not 100% sure this is correct ...

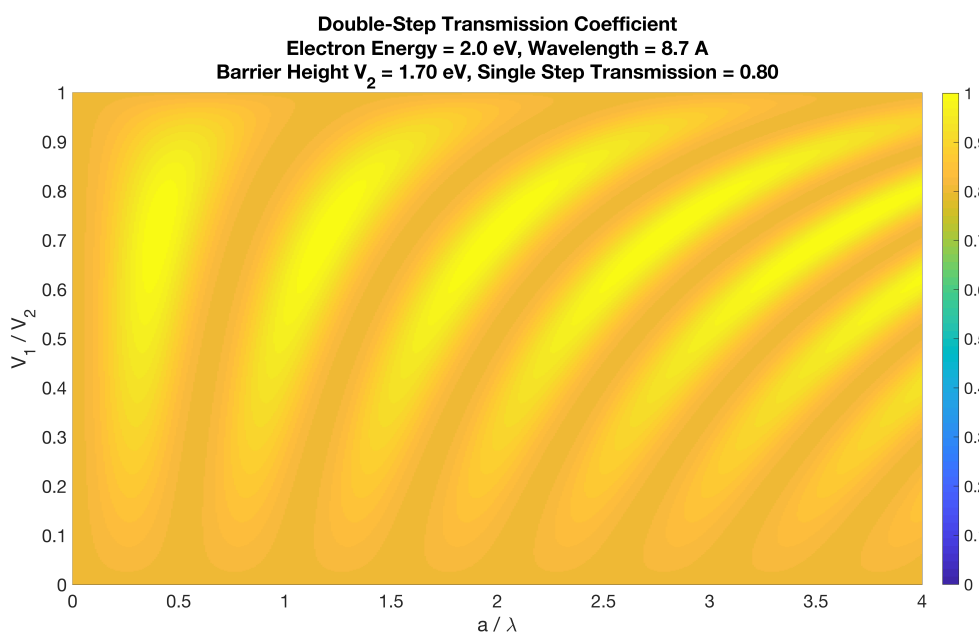
of course what we really want is

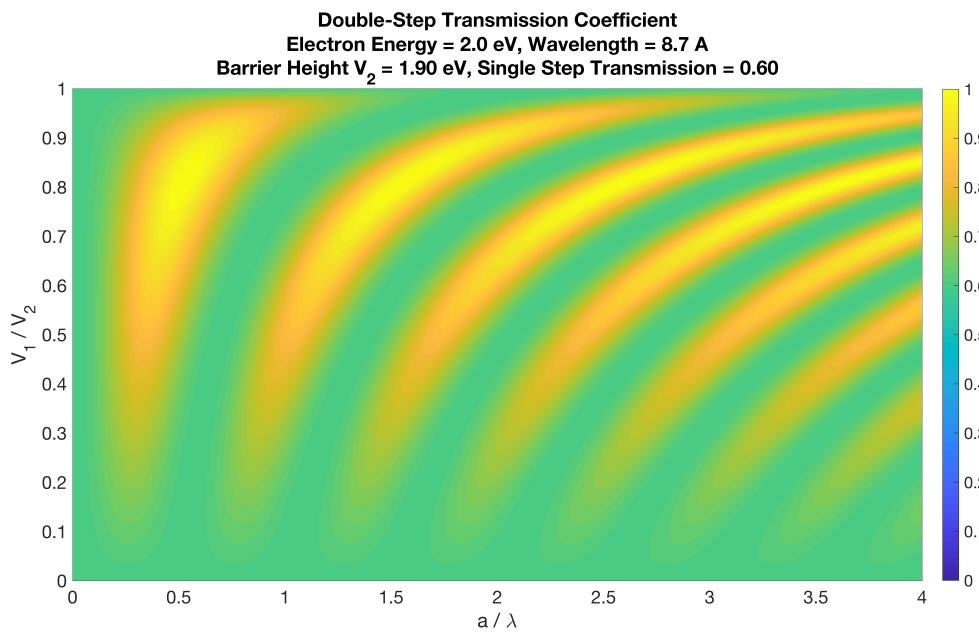
$$R = \frac{|B|^2}{|A|^2}$$

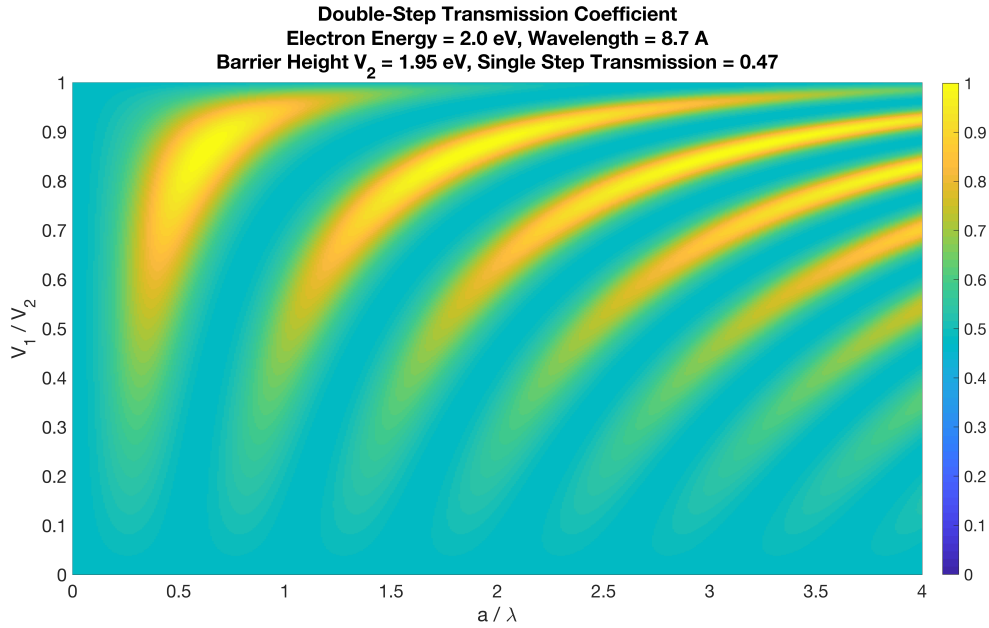
I will let Matlab calculate that for me.
I don't see the point in writing it out...

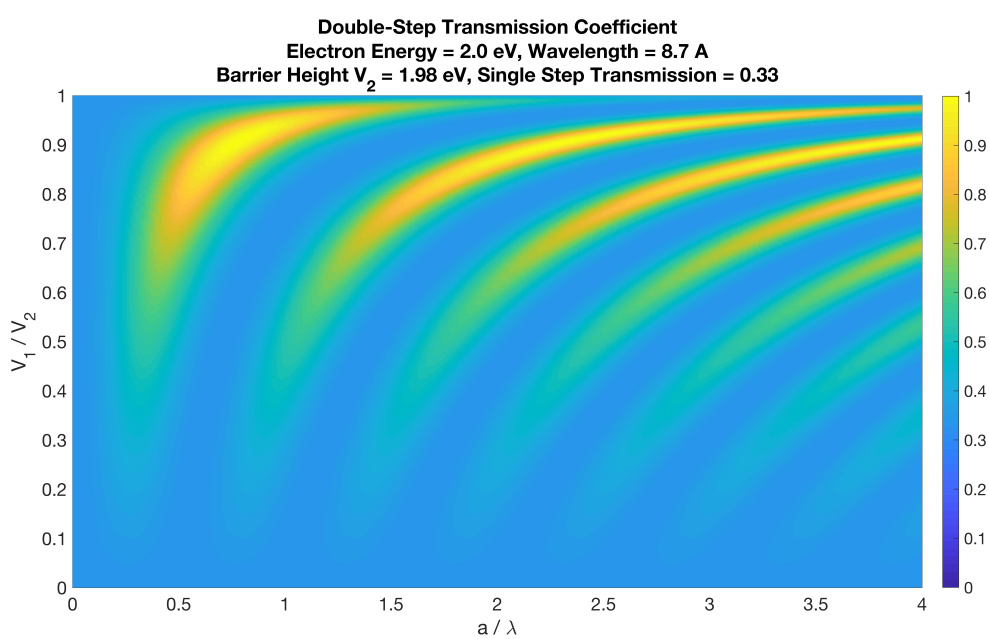
Everything on the right-hand side is known
from the values of E, V_1, V_2 and a.

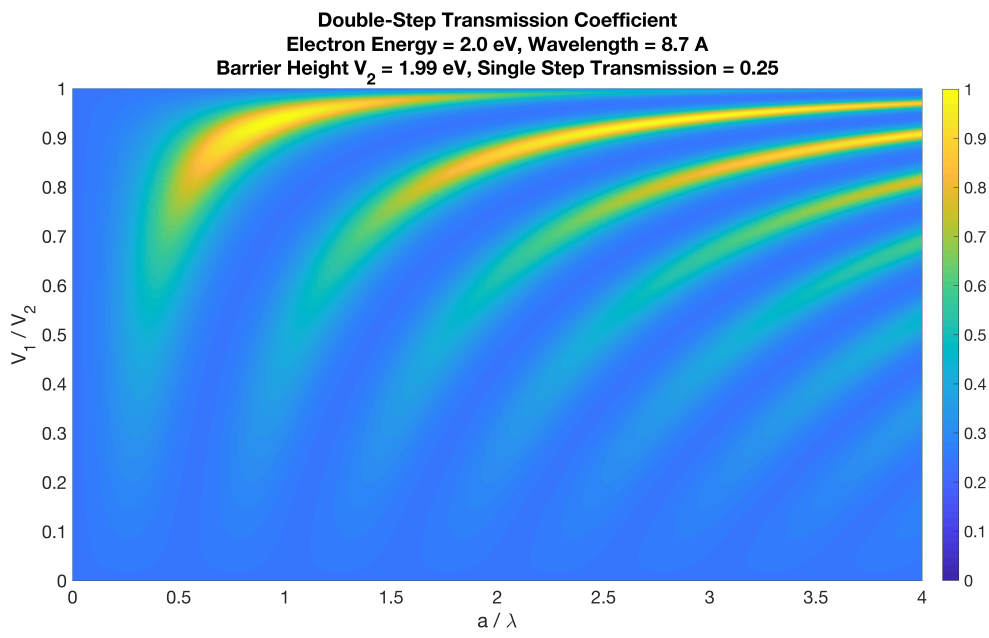

Double-Step Transmission Coefficient
Electron Energy = 2.0 eV, Wavelength = 8.7 A
Barrier Height V2 = 1.50 eV, Single Step Transmission = 0.89
V1 / V2
a / λ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4


Double-Step Transmission Coefficient
Electron Energy = 2.0 eV, Wavelength = 8.7 A
Barrier Height V2 = 1.70 eV, Single Step Transmission = 0.80
V1 / V2
a / λ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4


Double-Step Transmission Coefficient
Electron Energy = 2.0 eV, Wavelength = 8.7 A
Barrier Height V2 = 1.90 eV, Single Step Transmission = 0.60
V1 / V2
a / λ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4


Double-Step Transmission Coefficient
Electron Energy = 2.0 eV, Wavelength = 8.7 A
Barrier Height V2 = 1.95 eV, Single Step Transmission = 0.47
V1 / V2
a / λ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4


Double-Step Transmission Coefficient
Electron Energy = 2.0 eV, Wavelength = 8.7 A
Barrier Height V2 = 1.98 eV, Single Step Transmission = 0.33
V1 / V2
a / λ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4


Double-Step Transmission Coefficient
Electron Energy = 2.0 eV, Wavelength = 8.7 A
Barrier Height V2 = 1.99 eV, Single Step Transmission = 0.25
V1 / V2
a / λ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4