Homework 3

(1) $\Psi_0(x,y,z) = A\,xyz\,(x-L)(y-L)(z-L)$,

(a) Find A,

$$\int |\Psi_0(x)|^2 dx = 1 = A^2 \int_0^L x^2 y^2 z^2 (x-L)^2 (y-L)^2 (z-L)^2$$

$$= A^2 \left[\int_0^L x^2 (x-L)^2 dx\right]^3 = A^2 \left(\frac{L^5}{30}\right)^3 = A^2 \left(\frac{L^{15}}{27000}\right)$$

$$\sqrt{\frac{27000}{L^{15}}} = A \Rightarrow A = \frac{30\sqrt{30}}{\sqrt{L^{15}}}$$

$$\Psi_0 = \frac{30\sqrt{30}}{\sqrt{L^{15}}} xyz(x-L)(y-L)(z-L)$$

(B) $\Psi_0 = C_{n_x,n_y,n_z} \Psi_{n_x,n_y,n_z}$ Find C_{n_x,n_y,n_z}

$$C_{n_x,n_y,n_z} = \langle n_x, n_y, n_z | \Psi \rangle$$

$$\Psi(x,y,z) = \sum_{n_x,n_y,n_z} \underbrace{C_{n_x,n_y,n_z}}_{\langle \Psi_n | \Psi \rangle} \Psi_{n_x,n_y,n_z}(x,y,z)$$

$$C_{n_x,n_y,n_z} = \langle \Psi_n | \Psi \rangle$$

$$\frac{30\sqrt{30}}{\sqrt{L^{15}}} \int_0^L \left(x(x-L)\,n_x\right)^3 dx$$

(C)

(2)

(a)

$$\Psi_0(x,y,z) = \begin{cases} A, & 0 \le x \le L,\ 0 \le y,\ L/4,\ 0 \le z \le L/4 \\ 0, & \text{otherwise} \end{cases}$$

$$\int |\Psi_0(x)|^2 = 1 \rightarrow A^2 \int_0^L dx \int_0^{L/4} dy \int_0^{L/4} dz$$

$$\rightarrow A^2 \left[x \right]_0^L \left[y \right]_0^{L/4} \left[z \right]_0^{L/4} = 1 \rightarrow A^2 [L] \left[\frac{L}{4}\right]\left[\frac{L}{4}\right]$$

$$\rightarrow A^2 \frac{L^3}{16} = 1 \rightarrow A^2 = \frac{16}{L^3} \rightarrow A = \frac{4}{L^{3/2}}$$

$$\Psi_0(x,y,z) = \begin{cases} 4/L^{3/2}, & 0 \le x \le L,\ 0 \le y,\ L/4,\ 0 \le z \le L/4 \\ 0, & \text{otherwise} \end{cases}$$

(B)

$$\cancel{E_{111} = \frac{3\pi \hbar^2}{2mL^2}} \rightarrow \cancel{E_{211}} = \cancel{E_{121}} = \cancel{E_{112}} = \cancel{\frac{6\pi^2\hbar}{2mL^2}}$$

$$\langle \Psi_n | \Psi \rangle = \frac{4}{L^{3/2}} \int_0^L n_x dx \int_0^{L/4} n_y dy \int_0^{L/4} n_z dz$$

$$= \frac{4}{L^{3/2}} \left[n_x L + \frac{n_y L}{4} + \frac{n_z L}{4} \right]$$

$$P = |\langle \Psi_n | \Psi \rangle|^2$$

⋮

(3)

$$H = \frac{P_x^2 + P_y^2 + P_z^2}{2m} + \frac{k(x^2 + y^2 + z^2)}{2}$$

(a)

(B)

$$E_n = \left(n + \frac{3}{2}\right)\hbar\omega_0 = \hbar\omega_0\left(n_x + n_y + n_z + \frac{3}{2}\right)$$

(C) $$\Psi_{000} = \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-m\omega r^2/2\hbar}$$

$$\Psi_{100} = \sqrt{\frac{2m\omega}{\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-m\omega r^2/2\hbar}$$

$$\Psi_{010} = \sqrt{\frac{2m\omega}{\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-m\omega r^2/2\hbar}$$

$$\Psi_{001} = \sqrt{\frac{2m\omega}{\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-m\omega r^2/2\hbar}$$

(D)

$$H\Psi_{100} = \frac{3}{2}\hbar\omega\Psi_{000}$$

$$H\Psi_{100} = \frac{5}{2}\hbar\omega\Psi_{100}$$

$$H\Psi_{010} = \frac{5}{2}\hbar\omega\Psi_{010}$$

$$H\Psi_{001} = \frac{5}{2}\hbar\omega\Psi_{001}$$