Let's recap where we are.

- Isotropic/homogenous universe leads to Robertson-Walker metric
- Using this metric in Einstein's General Relativity leads to the Friedmann Equations.
- The Friedmann equations determine the cosmic scale evolution of the universe.
- So are we done with this part of cosmology?
- Do question (1) on the worksheet and S T O P

(1b) $1-\Omega(t) = \dfrac{H_o^2\ (1-\Omega_o)}{H(t)^2\ a(t)^2}$

(1c) $1-\Omega(t) = \dfrac{(1-\Omega_o)\ a^2}{\Omega_{r,o} + a\,\Omega_{m,o}}$

(1d) $|1-\Omega(t)| \leq 2\times 10^{-6}$

(1e) $|1-\Omega(t)| \leq 7\times 10^{-16}$

(1f) $|1-\Omega(t)| \leq 2\times 10^{-62}$

This result is referred to as the *Flatness problem*. The problem simply stated is that to see the amount of *flatness* we see today requires Ω to be within 2×10^{-62} of unity. That's an incredible number.

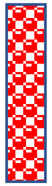
We keep saying that the universe is isotropic and homogeneous, and that's a good thing we've said. But consider

- At the time of the last scattering, the distance to the horizon is 0.251 Mpc
- Today that distance translates to an angular separation of 1.1^{o}
- The largest temperatures fluctuations in the CMB are on the order of 30 μK
- There are about *40,000 patches* in the sky of angular separation 1.1^{o}
- Do question (2) on the worksheet and S T O P

This problem is referred to as the *horizon* problem.

The monopole problem

n



s

But Maxwell's equation allow for  n (and/or s)

These *monopoles* are not seen even though (at least in most modern theories) they should be as prevalent as *dipoles*.

Inflation to the rescue. Do question (3 a) on the worksheet and S T O P. Finish question (3) on the worksheet and S T O P

(3b) $P < -\frac{\epsilon}{3}$

(3c) $P = w \cdot \epsilon \Rightarrow w < -\frac{1}{3}$

(3d)
$$
\begin{aligned}
\frac{\ddot{a}}{a} &= \frac{\Lambda}{3} \\
\left(\frac{\dot{a}}{a}\right)^2 &= \frac{\Lambda}{3} \\
&\text{solving for } a \\
a &\propto e^{H_i t} \quad \left(H_i = \sqrt{\frac{\Lambda}{3}}\right)
\end{aligned}
$$

How does this work to solve problems?

Suppose universe had the following behavior $a(t) = \begin{cases} \sqrt{t/t_i} & t < t_i \\ a e^{H_i(t-t_i)} & t_i < t < t_f \\ a e^{H_i(t_f - t_i)} \sqrt{t/t_f} & t > t_f \end{cases}$

(pay attention to subscripts on t)

Do question (4 a) and S T O P

- This scenario describes a universe that up until time t_i is growing `` normally",
- then between t_i and t_f undergoes exponential expansion
- After t_f, it resumes ``normal growth"

It is usual in studying inflation to compare the how much the scale factor changed between t_i and t_f by forming the ratio:

$$\frac{a(t_f)}{a(t_i)} = e^N; \text{ where } N \equiv H_i(t_f - t_i); \; H_i \equiv \sqrt{\frac{\Lambda}{3}}$$

N is called the number of *e*-foldings

Finish problem 4

(4 b) $\epsilon_\Lambda = \frac{3c^2}{8\pi G} H_i^2 \approx 10^{105}\ \mathrm{TeV\ m^{-3}}$ (4 c) $|1-\Omega(t_f)| = 2^{-2N}\,|1-\Omega(t_i)|$ (4 d) $|1-\Omega(t_f)| = 2^{-2N}$

The horizon problem. Recall that the horizon distance is given by the relation

$$d_{\rm hor}(t) = a(t)c\int_0^t \frac{dt'}{a(t')}$$

Do question (5a) on the worksheet and S T O P Do question (5b) and (5c) and S T O P

(5 b) $d_{\rm hor}(t_i) = a_i\, c\int_0^t \frac{dt'}{a_i\sqrt{t/t_i}} = 2ct_i$ (5c) $d_{\rm hor}(t_f) = a_i\, c\, e^N \int_0^{t_i} \frac{dt'}{a_i\sqrt{t/t_i}} + \int_{t_i}^{t_f} \frac{dt}{a_i \exp[H_i(t-t_i)]}$

When *N* is large, then the integrals in (5c) yield $d_{\rm hor}(t_f) = e^N c\left(2t_i + H_i^{-1}\right)$

Finish question (5) on the worksheet and S T O P

$$d_{\rm hor}(t_i) = 6\times 10^{-28}\,\mathrm{m} \qquad d_{\rm hor}(t_f) = 15\,\mathrm{m}$$