Physics 412—Practice S–2 (Due Jan. 27, 4 pm) Name:

S–2: Given a state vector $|\Psi\rangle$ and a Hermitian operator A, I can calculate the results of measurements of the observable A for a quantum ensemble in terms of the probabilities of possible results, the expectation value, and the uncertainty.

Unsatisfactory Progressing Acceptable Polished

(1) Suppose that we have a three-dimensional vector space and an operator A with representation

$$A \leftrightarrow \begin{bmatrix} a & -2ia & a \\ 2ia & 3a & 4ia \\ a & -4ia & -3a \end{bmatrix}.$$

(a) If the system is in the state below, find $\langle A\rangle$ and $\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$.

$$|\Psi\rangle \leftrightarrow \frac{1}{\sqrt{3}} \begin{bmatrix} 1-i \\ 0 \\ -i \end{bmatrix}.$$

(2) The eigenvalues of A are $-5a$, 0, and $6a$. Their corresponding eigenstates are

$$|{-5a}\rangle \leftrightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} 0 \\ -\mathrm{i} \\ 2 \end{bmatrix}, \quad |0\rangle \leftrightarrow \frac{1}{\sqrt{30}}\begin{bmatrix} -5\mathrm{i} \\ 2 \\ \mathrm{i} \end{bmatrix}, \quad |6a\rangle \leftrightarrow \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 2\mathrm{i} \\ 1 \end{bmatrix}.$$

Assuming the system is in the same state $|\Psi\rangle$ as part (a), find the probability of obtaining each of the the possible values when A is measured, and verify that your expectation value from part (a) is correct.