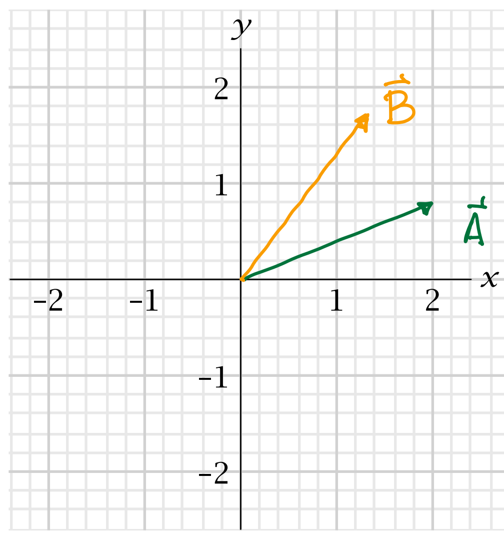
Physics 412—Representations of Vectors and Operators   Winter 2020

In quantum mechanics we represent every state vector as a set of number (components) relative to a set of basis vectors. We do the same thing in R^2, and generally use the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ as our basis vectors. We then represent an arbitrary vector $\vec{A}$ as a column vector with two elements:

$$\vec{A} \leftrightarrow \begin{bmatrix} \vec{A} \cdot \hat{\imath} \\ \vec{A} \cdot \hat{\jmath} \end{bmatrix}.$$

(1) On the axes below, draw in a vector $\vec{A}$, find its components, and write down its representation with respect to the unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (This will probably involve counting boxes.) Then draw in $\vec{B} = R_{30}\vec{A}$ and $\vec{C} = T_{45}\vec{A}$ and find *their* components and representations. (More counting boxes!)

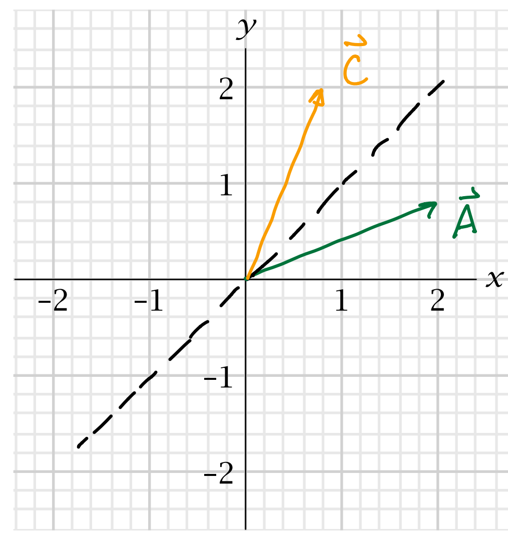
$\vec{B} = R_{30}\vec{A}$



$$\vec{A} \leftrightarrow \begin{bmatrix} 2.0 \\ 0.8 \end{bmatrix}$$

$$\vec{B} \leftrightarrow \begin{bmatrix} 1.25 \\ 1.65 \end{bmatrix}$$

$\vec{C} = T_{45}\vec{A}$



$$\vec{A} \leftrightarrow \begin{bmatrix} 2.0 \\ 0.8 \end{bmatrix}$$

$$\vec{C} \leftrightarrow \begin{bmatrix} 0.8 \\ 2.0 \end{bmatrix}$$

In quantum mechanics, we represent operators based on their effect on the basis vectors. The result is a set of *matrix elements* that we arrange in a square matrix. In the context of R^2, we will represent operators as 2×2 matrices as follows.

Let H be an operator. We first calculate the results of H acting on the unit vectors $\hat{\imath}$ and $\hat{\jmath}$. Call the results $\vec{h}_{\mathrm{i}}$ and $\vec{h}_{\mathrm{j}}$. The representation of the operator H is then the matrix

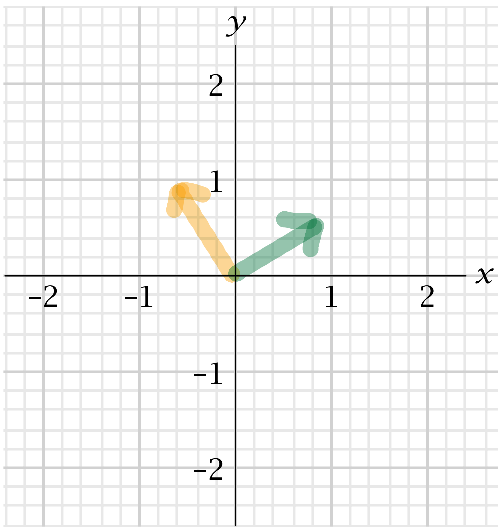
$$H \leftrightarrow \begin{bmatrix} \vec{h}_{\mathrm{i}} \cdot \hat{\imath} & \vec{h}_{\mathrm{j}} \cdot \hat{\imath} \\ \vec{h}_{\mathrm{i}} \cdot \hat{\jmath} & \vec{h}_{\mathrm{j}} \cdot \hat{\jmath} \end{bmatrix}.$$

The first column of this matrix is the representation of the vector $\vec{h}_{\mathrm{i}}$, while the second is the representation of the vector $\vec{h}_{\mathrm{j}}$.

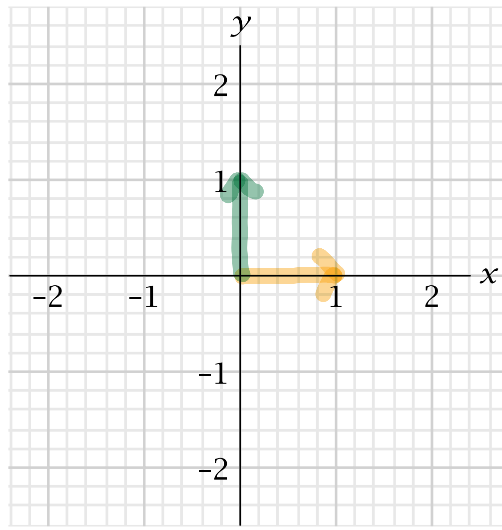
(1) One the axis below, draw in the results of acting with R_{30} and T_{45} on the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ and use the results to construct the representation of these operators.

When you're done with that use your representation of these operators to calculate the results of acting with them on your vector $\vec{A}$ from the previous side, and compare the result with what you found there.

$R_{30}\hat{\imath}$ and $R_{30}\hat{\jmath}$



$T_{45}\hat{\imath}$ and $T_{45}\hat{\jmath}$



$$R_{30}\hat{\imath} \leftrightarrow \begin{bmatrix} 0.87 \\ 0.5 \end{bmatrix} \qquad T_{45}\hat{\imath} \leftrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$R_{30}\hat{\jmath} \leftrightarrow \begin{bmatrix} -0.5 \\ 0.87 \end{bmatrix} \qquad T_{45}\hat{\jmath} \leftrightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$R_{30} \leftrightarrow \begin{bmatrix} 0.87 & -0.5 \\ 0.5 & 0.87 \end{bmatrix} \qquad T_{45} \leftrightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$