Physics 460—More Spin Correlations Spring 2020

The operator that represents the spin of a spin-1/2 particle in an arbitrary direction $\hat{n}$ is

$$S_{\hat{n}} \leftrightarrow \frac{\hbar}{2}\begin{bmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{bmatrix}.$$

Here θ and ϕ are the usual angles for spherical coordinates. This operator of course has eigenvalues $\pm\hbar/2$, and its eigenstates are

$$|+_{\hat{n}}\rangle = \cos\frac{\theta}{2}\,|+\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|-\rangle, \quad \text{and} \quad |-_{\hat{n}}\rangle = \sin\frac{\theta}{2}\,|+\rangle - e^{i\phi}\cos\frac{\theta}{2}\,|-\rangle,$$

where $|\pm\rangle$ are the usual z-state basis vectors. This transformation can be reversed, and the z-state basis vectors can be written as

$$|+\rangle = \cos\frac{\theta}{2}\,|+_{\hat{n}}\rangle + \sin\frac{\theta}{2}\,|-_{\hat{n}}\rangle, \quad \text{and} \quad |-\rangle = e^{-i\phi}\sin\frac{\theta}{2}\,|+_{\hat{n}}\rangle - e^{-i\phi}\cos\frac{\theta}{2}\,|-_{\hat{n}}\rangle.$$

A two-particle spin-1/2 system is prepared in the state (expressed in the z-state basis)

$$|\Psi_A\rangle = \frac{1}{\sqrt{2}}\Big(|+;-\rangle - |-;+\rangle\Big).$$

This state is known as the singlet state, and because it cannot be factored into the product of states for each of the two particles, it is an entangled state.

(1) The spin of the first particle is measured along the z axis, and the spin of the second particle is measured along the direction $\hat{n}$. Show that the correlation coefficient between these two measurements,

$$C = \mathcal{P}(+;+) + \mathcal{P}(-;-) - \mathcal{P}(+;-) - \mathcal{P}(-;+),$$

is $C = -\cos\theta$. Interpret this result. Does it make sense based on what you know about quantum mechanics, spin, and the meaning of the correlation coefficient?

(2) You decide you want to see if your correlation from question (1) can be reproduced by a two-particle state that is not entangled. Since measurements along the z axis for state $|\Psi_A\rangle$ have equal probability of resulting in spin up and spin down, you decide to try the state

$$|\Psi_B\rangle = \frac{1}{\sqrt{2}}\Big(|+\rangle + |-\rangle\Big) \otimes \Big(a|+\rangle + b|-\rangle\Big),$$

where a and b are undetermined (complex!) coefficients.

(a) What constraint does the normalization of $|\Psi_B\rangle$ place on the coefficients a and b?

(b) Calculate the correlation coefficient for this state for measurements of the spin of particle 1 along the z axis and the spin of particle 2 along the direction $\hat{n}$.

(c) Interpret your result from part (b). Does it make sense based on what you know about quantum mechanics, spin, and the meaning of the correlation coefficient? Does this state reproduce the correlation of the singlet state?

(3) What if we looked at the statistics of each measurement rather than the correlations? Suppose we calculated the average of the measurements of the spin of particle 1 and compared that value to the average of the measurements of the spin of particle 2. Do you think these averages would be the same for the two states, or different? You can calculate them if you're curious!