Inflation

(1) Recall that the Friedmann equation can be written as

$$1 - \Omega(t) = -k\left(\frac{c/H(t)}{a(t)R_o}\right)^2$$

or in the present moment

$$1 - \Omega_o = -k\left(\frac{c/H_o}{R_o}\right)^2$$

Current observations indicate that

$$|1 - \Omega_o| \leq 0.005$$

(a) What do you deduce from the fact that $|1 - \Omega_o| \leq 0.005$

(b) Combine the first two equations above to find the density parameter as a function of time (note your answer will contain the Hubble parameter and constant as well as the scale factor).

(c) Recall that in the matter + radiation era, the Friedmann equation is

$$\frac{H(t)^2}{H_o^2} = \frac{\Omega_{r,o}}{a^4} + \frac{\Omega_{m,o}}{a^3}.$$

Using this expression, substitute for the Hubble parameter and constant in your answer for (b) to obtain an expression for $\Omega(t)$ in terms of the $\Omega_o, \Omega_{r,o}, \Omega_{m,o}$, and, $a(t)$

(d) The benchmark model has $\Omega_{m,o} = 0.31, \Omega_{r,o} = 9.0 \times 10^{-5}, |1 - \Omega_o| \leq 0.005$. Using these figures in your expression obtained in part (c), find $|1 - \Omega(t)|$ at the time of radiation-matter equality when $a = 2.9 \times 10^{-4}$.

(e) Let's continue going back time, find $|1 - \Omega(t)|$ at the time of Big Bang nucleosynthesis (when protons and neutrons first fused) which occurs when $a = 3.6 \times 10^{-9}$.

(f) Now lets go back even further, to the Planck time when $a = 2 \times 10^{-32}$ and again find $|1 - \Omega(t)|$.

(g) Now note, that in order for $|1 - \Omega_o|$ (today), the universe had to have $1 - \Omega(t)$ to within 2 parts in 10^{62}. At your table discuss this result.

(2) In the lecture we've noted the following facts

- At the time of the last scattering, the distance to the horizon is 0.251 Mpc
- Today that distance translates to an angular separation of 1.1^o

- The largest temperatures fluctuations in the CMB are on the order of $30\mu K$
- There are about 40,000 patches in the sky of angular separation 1.1^o

(a) Are the different *patches* in the sky casually connected?

(b) Do you find anything unusual in having 40,000 patches in the sky that are not casually connected and yet have the same temperature? Discuss this with your table mates and come up with an analogy that demonstrates how unusual this fact is.

(3) The most commonly accepted idea to resolve the three problems discussed above is *inflation.* We now introduce you to its basic ideas

(a) At your table, discuss how a short period of rapid acceleration early in the existence of the universe might provide a solution to the flatness, horizon, and monopole problem.

(b) Consider the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P).$$

In order for the acceleration of the scale factor, a, to be positive, what condition(s) must be true of the pressure, P.

(c) Using (b), what is the condition on the equation of state parameter, w.

(d) Consider the case in which $w = -1$ (the cosmological constant case). Define the $\Lambda \equiv 8\pi G/(3c^2)\epsilon$. Write the acceleration for this case and determine a condition on $\Lambda/3$. Then write the Friedmann equation for a flat universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon$$

in terms of Λ and solve for $a(t)$

(4) In question (3) you derived some results for having $w = -1$. You will now probe some of the consequences.

(a) In the lecture you were given the following scenario,

$$a(t) = \begin{cases} \sqrt{t/t_i} & t < t_i \\ a e^{H_i(t-t_i)} & t_i < t < t_f \\ a e^{H_i(t_f-t_i)}\sqrt{t/t_f} & t > t_f \end{cases}$$

Describe in words what is happening in the universe in this scenario.

(b) In the lecture we've introduced the ratio,

$$\frac{a(t_f)}{a(t_i)} = e^N; \text{ where } N \equiv H_i(t_f - t_i); \ H_i \equiv \sqrt{\frac{\Lambda}{3}}$$

One possible model for inflation has the exponential growth occurring at the Grand Unified Theory (GUT) time of $t_i \approx 10^{-36}$s, so that $H_i = 10^{36}$s. Find the energy density,

$$\epsilon_\Lambda = \frac{3c^2}{8\pi G} H_i^2.$$

Note that the current value of $\epsilon_\Lambda = 0.0034$ TeV m^{-3}. Compare the results and think about the consequences of this energy density at the GUT time scale.

(c) Now we'll see how this scenario might solve the flatness problem. Recall that

$$|1 - \Omega(t)| = \frac{c^2}{R_o^2 a(t)^2 H(t)^2}$$

During the inflation time, all terms but $a(t)$ are constants. In this case,

$$|1 - \Omega(t)| \propto e^{-2H_i t}.$$

Compare the density parameter at the beginning of inflation ($t = t_i$) with the density parameter at the end of inflation ($t = t_f = [N+1]t_i$).

(d) Now consider a universe that was initially very strongly curved, for example, a universe such that

$$|1 - \Omega(t)| \approx 1.$$

Find the value of the density parameter after it has undergone N e-foldings of inflation.

(e) At your table, discuss how this scenario addresses the flatness problem.

(5) We now address the horizon problem.

(a) At your table, speculate how inflation might address the horizon problem.

(b) The horizon distance is given by

$$d_{\rm hor}(t) = a(t)\, c \int_0^t \frac{dt'}{a(t')}$$

Recall that in the model we are discussing that before inflation occurs, $a(t) = a_i\sqrt{t/t_i}$. Find the horizon distance at the beginning of inflation.

(c) Set up the integral to find the horizon distance at the end of inflation.

(d) Suppose inflation began at $t_i \approx 10^{-36}$, use (5b) to find the distance to the horizon. Then suppose inflation lasts for $N = 65$ e-foldings, using (5c) find the horizon distance after inflation ends.

(e) At your table discuss how this addresses the horizon problem.

Homework 04–Due Friday,March 6

1. Problem 7.3
2. Problem 7.4