PHY 474 Spring 2021

Homework 3—due by 5:00 PM, Friday, Apr 30

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Monday (May 3). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

1. Let h be the height in the atmosphere above the photosphere. Then, the mean free path $\lambda = 1/\kappa\rho$ must satisfy the relation

$$1 \simeq \int_0^\infty \kappa\rho\, dh \tag{1}$$

Approximate the opacity κ by a power law

$$\kappa = \kappa_0^{(\mathrm{ph})}\, \rho^a T^b \tag{2}$$

where (ph) stands for the photosphere, the surface of the star that is at the base of its atmosphere. Take care that you don't confuse the use of a in the exponent for the density with the use of a for the radiation density constant (e.g., in Question 4); they are different.

Assuming an *isothermal atmosphere*, the variation of density with height is given by

$$\rho = \rho_{\mathrm{ph}} \exp\left(-\frac{h}{H}\right) \tag{3}$$

where the density scale height H is the same as the pressure scale height we wrote in class, that is

$$H = \frac{k_B T}{\mu m_p g} \tag{4}$$

where $g = GM/R^2$ is the *gravitational acceleration* in the atmosphere.

(a) Replacing κ in equation (1) with the expression in equation (2), then using equation (3) to substitute for ρ, show that equation (1) integrates to

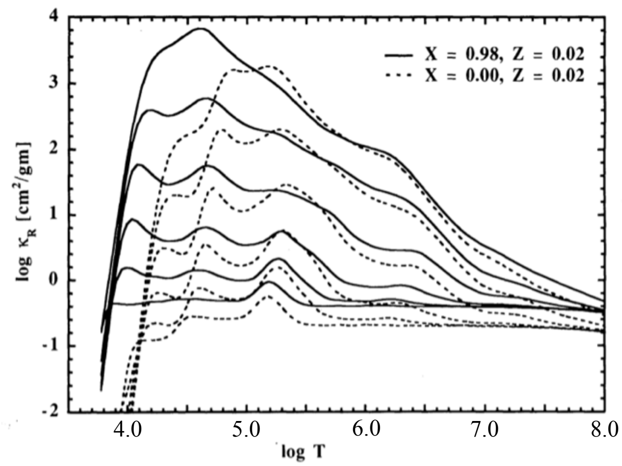
$$1 \simeq \frac{H}{a+1}\, \kappa_{\mathrm{ph}}\, \rho_{\mathrm{ph}}$$

where $\kappa_{\mathrm{ph}} = \kappa_0^{(\mathrm{ph})}\, \rho_{\mathrm{ph}}^a\, T_{\mathrm{eff}}^b$ is the opacity of the photosphere.

(b) Using the ideal gas law in the form $P = \rho k_B T/\mu m_p$ that we wrote in class, show that

$$P_{\mathrm{ph}} = \left[\frac{GM(a+1)}{R^2 \kappa_0^{(\mathrm{ph})}}\right]^{1/(a+1)} \left[\frac{k_B}{\mu m_p}\right]^{a/(a+1)} T_{\mathrm{eff}}^{(a-b)/(a+1)}$$

2. Consider the graph of opacity *vs.* temperature shown below.



Recall that although we wrote the opacity in terms of physical characteristics of the star, the opacity is usually determined from computed tables. The plot above shows the opacity as a function of temperature for different values of the density (actually a quantity R which has a relation to the density, the details of which we won't get into here) taken from such a table. You may ***ignore*** the dotted lines for answering this question (if you're interested, the solid line is for a star with a hydrogen mass fraction $X = 0.98$, thus containing no helium, whereas the dotted line is for a star with $X = 0$).

(a) Discuss why the opacity dips sharply at low temperatures, as seen on the left of the graph above.

(b) Discuss why the opacity levels off at high temperatures, as seen on the right of the graph above.

3. Consider a star that has values for relevant physical quantities near its center as given below.

r	$m(r)$	$L(r)$	$T(r)$	$\rho(r)$	κ
$0.1\ R_\odot$	$0.028\ M_\odot$	$24.4\ L_\odot$	2.2×10^7 K	3.1×10^4 kg m^{-3}	0.04 m^2 kg^{-1}

Assuming a mean molecular weight, $\mu = 0.7$, determine by appropriate calculations whether the energy transport at this location is radiative or convective. *Choosing an answer without mathematical justification may result in negative points.*

4. In class, you showed that the luminosity $L(r)$ is given by

$$L = -\frac{4\pi r^2 ac}{3\kappa\rho}\frac{d}{dr}\left(T^4\right)$$

Replacing r by R, $-d(T^4)/dr$ by T^4/R, the opacity by a power law $\kappa \simeq \kappa_0\, \rho^\lambda T^{-\nu}$, and other appropriate steps, show that the surface luminosity of a star is given by

$$L_s \simeq \frac{ac}{\kappa_0}\left(\frac{G\mu m_p}{k_B}\right)^{4+\nu} R^{3\lambda-\nu}\, M^{3+\nu-\lambda}$$