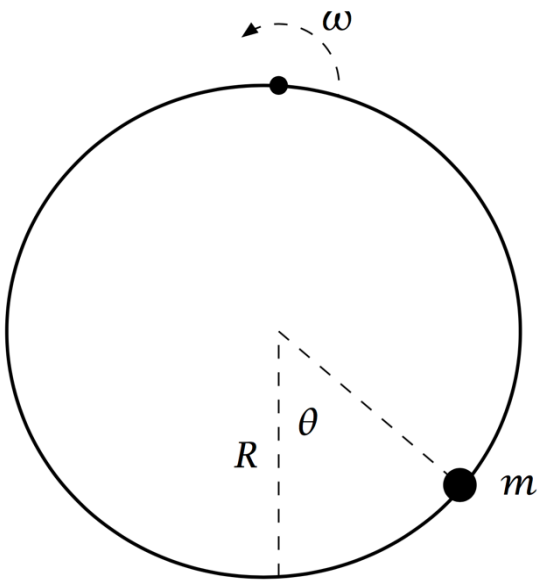
Activity 10: Bead on a Horizontal Rotating Hoop

A massless, frictionless hoop lying flat on a table rotates with constant angular frequency ω about one point on the rim, as shown in the figure below. A particle of mass *m* is free to move along the hoop. The angle θ measures the angle of the particle relative to a fixed reference (the vertical direction in the figure).



a. Show that the Lagrangian for this system can be written as
$$L = \frac{mR^2}{2}\left[\omega^2 + \dot{\theta}^2 + 2\omega\dot{\theta}\cos(\theta - \omega t)\right]$$
Note that the first term in brackets, ω, is a constant, so it won't affect the equations of motion.

b. Show that the change of variables $\phi = \theta - \omega t$ results in a Lagrangian that does not have an explicit time dependence. What is the physical meaning of the variable ϕ?

c. Working now with this new Lagrangian, find the equations of motion.

d. Find the equilibrium solutions and their stability. *[think about whether to include the stability analysis]*