Physics 460—Homework Report 2
Due Tuesday, Apr. 14, 1 pm **Name:** ____________________

Complete all the problems on the accompanying assignment.

List all the problems you worked on in the space below. Circle the ones you fully completed:

Please place the problems into the following categories:

- These problems helped me understand the concepts better: ____________________
- I found these problems fairly easy: ____________________
- I found these problems very challenging: ____________________

In the space below, show your work (even if not complete) for any problems you still have questions about. Indicate where in your work the question(s) arose, and ask specific questions that I can answer.

Use the back of this sheet or attach additional paper, if necessary.

If you have no remaining questions about this homework assignment, use this space for one of the following:

- Write one or two of your solutions here so that I can give you feedback on its clarity.
- Explain how you checked that your work is correct.

(1) A particle with positive electric charge and spin11/2 is placed in a uniform magnetic field $\vec{B} = B_0(\hat{\imath}+\hat{k})/\sqrt{2}$. The Hamiltonian is then $H = -\gamma\vec{S}\cdot\vec{B}$, where γ is a positive constant called the gyromagnetic ratio.

(a) Working in the $\{|+_z\rangle, |-_z\rangle\}$ basis, find the matrix representation of the Hamiltonian H. [Continue working explicitly in this representation for parts (b)–(d).]

(b) Find the eigenvalues and eigenstates of H.

(c) At time $t = 0$, the particle is in state $|\Psi_0\rangle = |-_z\rangle$. If you measure the energy of the particle, what values can you measure, and what is the probability of measuring each value?

(d) Find the state vector at time t, and calculate $\langle S_x\rangle$ at time t.

(2) Consider a spin-1/2 particle in a rotating magnetic field

$$\vec{B} = B_1 \cos\omega t\,\hat{\imath} - B_1 \sin\omega t\,\hat{\jmath} + B_0\,\hat{k}.$$

This magnetic field has a constant z-component and a component that rotates in the x–y plane with frequency ω. If the initial state of the particle is $|+_z\rangle$, it can be shown that the state of the system at a later time t is

$$|\Psi(t)\rangle = \left(\cos\frac{\Omega t}{2} + \mathrm{i}\frac{\Omega_0-\omega}{\Omega}\sin\frac{\Omega t}{2}\right)\mathrm{e}^{\mathrm{i}\omega t/2}|+_z\rangle + \frac{\mathrm{i}\Omega_1}{\Omega}\sin\frac{\Omega t}{2}\,\mathrm{e}^{-\mathrm{i}\omega t/2}|-_z\rangle,$$

where

$$\Omega_0 = \gamma B_0, \qquad \Omega_1 = \gamma B_1, \qquad \text{and} \qquad \Omega = \sqrt{{\Omega_1}^2 + (\Omega_0-\omega)^2}.$$

(a) Explain how you would verify that $|\Psi(t)\rangle$ given above is the state of the system at a later time t. You don't have to carry out all the algebra (unless you want to!), but explain what you would have to do to verify that this $|\Psi(t)\rangle$ is correct.

(b) The situation of *paramagnetic resonance* occurs when $\omega = \Omega_0$. How does the state $|\Psi(t)\rangle$ simplify in this case? Interpret your result in terms of the spin of the particle.

(c) Suppose that in the paramagnetic case the rotating field is applied to the particle for a finite time τ such that

$$\Omega_1\tau = \frac{\pi}{2}.$$

This field is called a 90° pulse. Why do you think it has this name?

(d) Suppose that in the paramagnetic case the rotating field is applied to the particle for a finite time τ such that

$$\Omega_1\tau = \pi.$$

This field is called a 180° pulse. Why do you think it has this name?