1 Random Numbers

(1) PDF

(a) In the definitions of discrete and continuous PDFs what is meant by *countable* or *not countable*.

Countable - one - to - one integers

(b) Come up with an example of a discrete PDF that does **not** generate equally likely random numbers.

Lots of different ones

(c) Suppose there was a process that generated discrete random numbers and after many trials a *histogram* of the results looked like this:

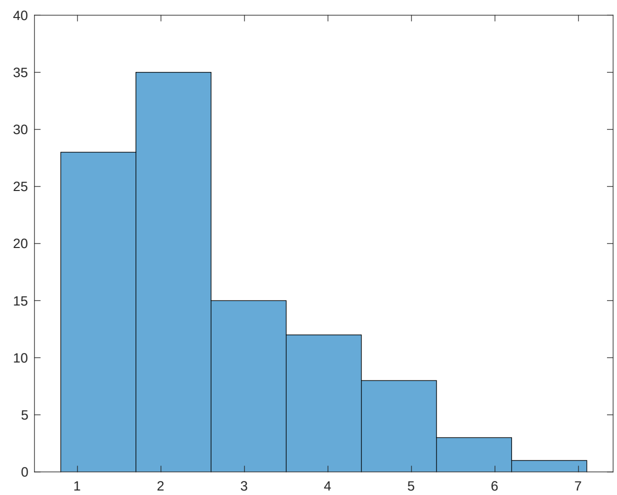

Figure 1: Discrete PDF. The vertical axis is the number of times the number on the x axis has occurred

What is the most likely number to occur? Compared to the other numbers, would this number be much more likely to be chosen or not?

2

(d) Consider now a continuous PDF that generates a pool of random numbers between 0 and 1. Assume further that, like the die, every number is equally likely. What is the probability of choosing a specific number, like say 0.5.

0

(2) Consider the *PDF*

$$p(x) = \int_0^1 3x^2 dx.$$

$x^2 - 3/4\,x - 9/16$

$3x^4 - 9x^3/4 - 27x^2/16$

Find the mean and variance of this *PDF*.

$\mu_x = \int_0^1 3x^3 dx \rightarrow \left[3/4\, x^4\right]_0^1 = 3/4$

$\sigma_x^2 = \int_0^1 (x - 3/4)^2\, 3x^2 \rightarrow -0.525$

(3) From the Teams page, download the file `x-square-pdf.dat`. Find the sample mean and then estimate the true mean from this data. The data was generated using the *PDF* from question (2). Compare the sample mean to the theoretical mean obtained in question (2).

(4) **The rejection method.** The rejection method is an ingenious approach to generating random numbers for a given *PDF*. It can be used for generating sample values for any random variable that assumes values only within a finite range and for which the *PDF* is bounded.

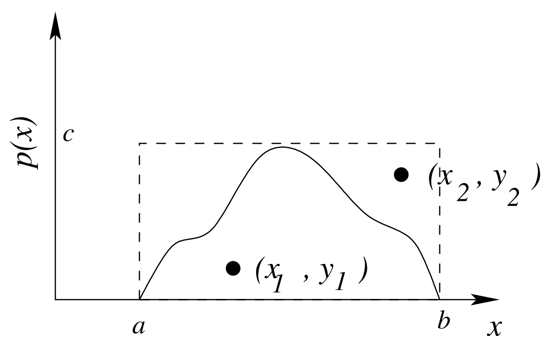

Figure 2: Rejection method

The idea of this method is as follows. Consider figure 2. The *PDF* is the solid curved line. It's range is $a \leq x \leq b$ and it's maximum value is c. The dotted line shows a rectangle that fully contains the *PDF*. The algorithm is as follows

(i) Enclose the *PDF* in the smallest rectangle that fully contains it. Thus, in the figure the rectangle is $(b-a) \times c$. Note that in real applications a small quantity, ϵ may need to be added to a, b, c so that the boundaries are properly captured.

(ii) Generate two uniform random numbers, (x_r, y_r). Scale these by multiplying x_r by $(b-c)$ and y_r by c.

(iii) If y_r is "below" the *PDF*, accepts the $x-$coordinate. This is your random number drawn from the appropriate distribution. If it "above" the *PDF* reject the pair and do step (ii) again.

So in figure 2, the point x_1 would be accepted, while the point x_2 would be rejected.

Write a `MatLab` function that will generate 100 random numbers drawn from the *PDF*, $p(x) = 3x^2, \quad , 0 \leq x \leq 1$. Find the sample mean from your data and compare it to the theoretical mean found in problem 2. Some handy `MatLab` commands you might investigate are `rand`, `mean`, and `var`.

(5) Write a `MatLab` function to find

$$g(x) = \int_0^1 x \exp(x) dx$$

by enclosing the function $g(x)$ in a rectangle of length 1 and width $\exp(1)$.

(6) Construct a step-by-step procedure for using **Method II** to compute integrals numerically. Then write the `MatLab` code to implement this procedure and calculate the same integral you did in question (5) using **Method II**. Use 100 points first, then use 1000 points. Find the error in your calculations.