Homework 7

(1) $|+z\rangle = \cos\frac{\theta}{2}|+\hat{n}\rangle + \sin\frac{\theta}{2}|-\hat{n}\rangle$

$|-z\rangle = e^{i\varphi}\sin\frac{\theta}{2}|+\hat{n}\rangle - e^{i\varphi}\cos\frac{\theta}{2}|-\hat{n}\rangle$

Correlation for the singlet state

$E(\hat{k},\hat{u}) = -\cos\theta$

(a) $|\psi\rangle = |+,-\rangle$

$P_{++} = |\langle++|\psi\rangle|^2 = \cos^2\theta/2$
$P_{+-} = |\langle+-|\psi\rangle|^2 = \sin^2\theta/2$
$P_{-+} = |\langle-+|\psi\rangle|^2 = (e^{-i\varphi})^2\sin^2\theta/2 \to e^{-2i\varphi}\sin^2\theta/2$
$P_{--} = |\langle--|\psi\rangle|^2 = (e^{i\varphi})^2\cos^2\theta/2 \to e^{-2i\varphi}\cos^2\theta/2$

$C = P_{++} + P_{+-} - P_{-+} + P_{--}$

$C = \cos^2\theta/2 + \sin^2\theta/2 - (e^{2i\varphi}\sin^2\theta/2) + (e^{-2i\varphi}\cos^2\theta/2)$

(B) $|E(\hat{n}_1,\hat{n}_2) - E(\hat{n}_1,\hat{n}_3)| \leq 1 + E(n_2,n_3)$

(c) $|\psi\rangle = |+,+\rangle$

$P_{++} = |\langle++|\psi\rangle|^2 = \cos^2\theta/2$
$P_{+-} = |\langle+-|\psi\rangle|^2 = \sin^2\theta/2$
$P_{-+} = |\langle-+|\psi\rangle|^2 = 0$
$P_{--} = |\langle--|\psi\rangle|^2 = 0$

$C = P_{++} + P_{+-} - P_{-+} + P_{--}$

$C = \cos^2\theta/2 + \sin^2\theta/2$

(D)

(e)

(2) $f(\hat{n}, \lambda) = \begin{cases} +1 & , \lambda \geq \cos\theta \\ -1 & , \lambda < \cos\theta \end{cases}$

(A) $\rho(\lambda) = A \qquad 1 = \int_{-1}^{1} A \, d\lambda \Rightarrow 1 = A\left[\lambda\right]_{-1}^{1}$

$\rightarrow 1 = A\left[1 - (-1)\right] \rightarrow 1 = 2A \quad A = 1/2 \quad \rho(\lambda) = \frac{1}{2}$

(B)

$$E(\hat{n}_1, \hat{n}_2) = -\int \frac{1}{2}(1)(1) = -1$$

$$E(\hat{n}_1, \hat{n}_3) = -\int \frac{1}{2}(1)(1) = -1$$

$$E(\hat{n}_2, \hat{n}_3) = -\int \frac{1}{2}(+1)(1) = -1$$

(C) $|E(\hat{n}_1, \hat{n}_2) - E(\hat{n}_1, \hat{n}_3)| \leq 1 + E(n_2, n_3$

$\cdot\ 1 - 1 - (-1)| \leq 1 + (-1)$

$0 \leq 0$