PHY 474/374 Spring 2021

Week 9—Monday, May 24—Discussion Worksheet

Stellar Evolution

We will now take a look at the advanced stages of high mass stellar evolution that we didn't get to do in detail during earlier classes.

1. The masses of high mass star cores during the various fusion phases are given in the table below (Hirschi et al. 2004). The first column shows the initial mass (at ZAMS). For each of these initial masses, the 3rd through 7th columns show masses at various stages corresponding to zero initial rotation ($v_{\rm ini} = 0$) and $v_{\rm ini} = 300$ km/s; $M_{\rm final}$ is the final mass before the star explodes, so that $(M_{\rm ini} - M_{\rm final})$ is the mass lost during the evolution, M_α is the mass of the He core after the Main Sequence phase, $M_{\rm CO}$ is the mass of the CO core after the He-fusion phase, $M_{\rm Fe}$ is the mass of the iron core after the Si-fusion phase, and $M_{\rm remn}$ is the mass of the stellar remnant after the supernova.

$M_{\rm ini}/M_\odot$	$v_{\rm ini}$ [km s^{-1}]	$M_{\rm final}$	M_α	$M_{\rm CO}$	$M_{\rm Fe}$	$M_{\rm remn}$
9	0	8.663	2.185	0.920	–	0.920
9	300	8.375	2.547	1.413	–	1.239
12	0	11.524	3.141	1.803	–	1.342
12	300	10.199	3.877	2.258	–	1.462
15	0	13.232	4.211	2.441	1.561	1.510
15	300	10.316	5.677	3.756	2.036	1.849
20	0	15.694	6.265	4.134	1.622	1.945
20	300	8.763	8.654	6.590	2.245	2.566
25	0	16.002	8.498	6.272	1.986	2.486
25	300	10.042	10.042	8.630	2.345	3.058
40	0	13.967	13.967	12.699	2.594	4.021
40	300A	12.646	12.646	11.989	2.212	3.853
60	0	14.524	14.524	13.891	2.580	4.303
60	300A	14.574	14.574	13.955	2.448	4.323
85	0	17.236	17.236	16.564	–	5.115
85	300A	12.314	12.314	11.666	–	3.776
120	0	16.254	16.254	15.591	–	4.819
120	300A	11.270	11.270	10.663	–	3.539

(a) Which stars will produce supernovae rich in H-lines, $M_{\rm ini} < 25M_\odot$ or $M_{\rm ini} > 25M_\odot$? Why?

$M_{ini} > 25 M_\odot$

M_{final} and M_α have the same value

(b) In 2018, Luciano Rezzolla set an upper limit of 2.16 $M_\odot$ on the maximum mass of non-rotating neutron stars. Based on the data in the table above, stars above what $M_{\rm ini}$ are likely to become Black Holes if Rezzolla's limit is correct?

2. Let's look at the energetics of core-collapse supernovae.

(a) Assuming that the core collapses from a white dwarf ($M_c \sim 1.4M_\odot, R \sim 10^4$ km) to a neutron star ($R \sim 20$ km), compute the energy released during the core collapse. Use $E \simeq -GM^2/R$. Express your answer in erg.

$$E \simeq \frac{-GM^2}{R}$$

$$= \frac{-6.67\times10^{-8}\,\text{cm}^3\,\text{g}^{-1}\,\text{s}^{-2}\,(2.79\times10^{33}\,\text{g})^2}{1\times10^9\,\text{cm}} = 519\times10^{48}\,\text{erg}$$

$$= \frac{-6.67\times10^{-8}\,\text{cm}^3\,\text{g}^{-1}\,\text{s}^{-2}\,(2.79\times10^{33}\,\text{g})^2}{2\times10^6\,\text{cm}} = 259\times10^{51}\,\text{erg}$$

(b) The potential energy needed to expel the envelope is calculated from models to be $\sim 10^{50}$ erg. Using an envelope mass, $M_{\text{env}} \sim 6M_\odot$ and velocity 10^4 km/s, find the kinetic energy of the envelope in erg.

$$KE = \tfrac{1}{2}MV^2$$

$$= \tfrac{1}{2}(6M_\odot)(10^4\,\text{km/s})^2$$

$$= \tfrac{1}{2}(1.99\times10^{30}\,\text{kg})(10^4\,\text{km/s})^2$$

$$= 6\times10^{51}\,\text{erg}$$

(c) The supernova has a peak luminosity of 10^8-10^9 $L_\odot$, lasting about 60 days. Find the energy radiated by the supernova in erg. **Note:** $L_\odot = 3.846 \times 10^{33}$ erg/s.

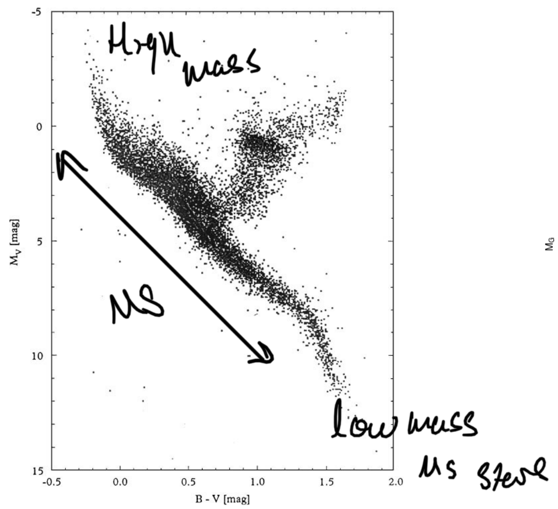
$$E_{rad} = (10^{8-9})(3.846\times10^{33}\,\text{erg/s})(60\,\text{day})(3600)(24)\,\text{s/day}$$

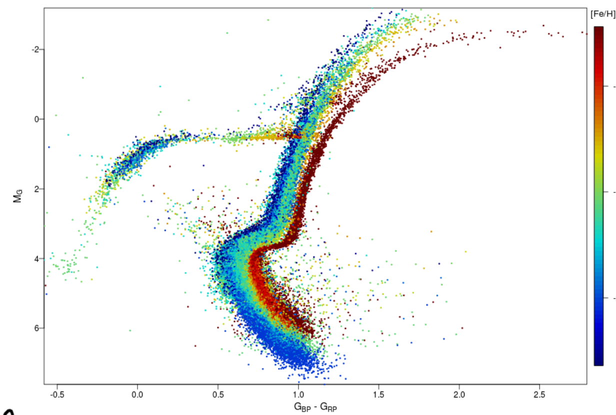
$$= 1.99\times10^{48-49}$$

$$\sim 2\times10^{48} - 2\times10^{49}\,\text{erg}$$

You should find that only a small fraction of the energy released in the core collapse is used for ejecting the envelope and emitting light. Evidently, most of the energy comes out in the form of neutrinos.

3. HR diagrams provide useful information on stars.





(a) The figure above ***on the left*** shows an HR diagram for 16,631 stars in the Hipparcos catalog. Mark the Main Sequence in the figure, and point out the location of the most massive stars and the location of low mass stars on the Main Sequence.

(b) Stars in a cluster formed at about the same time from material with the same chemical composition. Therefore, analysis of star clusters can provide a useful test of stellar evolution models. The figure above ***on the right*** shows the HR diagram for 14 clusters of stars. Discuss why this looks different from the figure above on the left.

High mass stars in the cluster have evolved
off the Main Sequence

4. Black Holes are the end points of high mass stars that leave behind remnants above a certain mass limit.

(a) The Schwarzschild radius (R_s) is the radius of the Event Horizon surrounding a non-rotating Black Hole. Any object with a physical radius smaller than its Schwarzschild radius will be a Black Hole. It is given by

$$R_s = \frac{2GM}{c^2}$$

Compute R_s for a Black Hole with 5 $M_\odot$.

$$R_s = \frac{2(6.67\times10^{-11}\,Nm^2/kg^2)[5(1.99\times10^{30}\,kg)]}{(3\times10^8\,m/s)^2}$$

$$= 14.7\,km$$

(b) If a ray of light passes a Black Hole, it can be captured into orbit if it gets closer than $1.5R_s$. This is known as the photon capture radius. Calculate the photon capture radius for a Black Hole with 5 $M_\odot$.

$R_{photon\ capture} = 1.5\,R_s = 1.5(14.7\,km) = 22.1\,km$

(c) If a particle of matter gets closer than $R = 3.0R_s$ from a black hole, it will not be able to remain in a stable orbit, no matter how it moves. It will eventually fall into the Black Hole. What is the radius of the Last Stable Particle Orbit for a Black Hole with 5 $M_\odot$?

$R_{last} = 3.0\,R_s = 3(14.7\,km) = 44.1\,km$

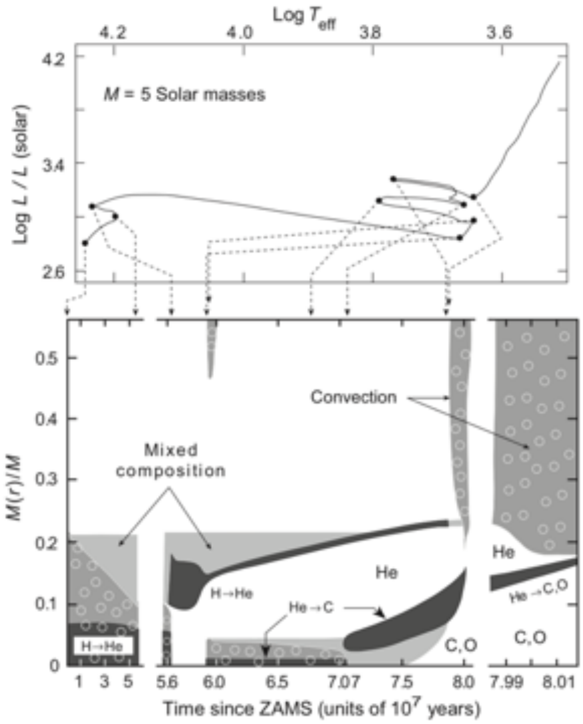
(d) An astronomer detects an asteroid orbiting a Black Hole at a distance of 150 km. If the black hole mass is 8 $M_\odot$, is the asteroid in a stable orbit, or will it be dragged into the Black Hole?

$R_s = 8(2949.6\,m) = 23.6\,km$

$3(23.6\,km) = 71\,km$

$150\,km > 71\,km$, yes, stable in orbit

5. We will use this question as an opportunity for review. Consider the path of a 5 $M_\odot$ star in the HR diagram after it evolves off the Main Sequence. The figure in the bottom panel is known as a Kippenhahn diagram, and provides a useful way to chart the evolutionary path of a star.



Discuss the stages after the Main Sequence. Use the Kippenhahn diagram to discuss the details of what is going on in the star.