PHY 420 **Spring 2021**

Homework 2—due by 9:00 PM, Monday, Apr 12

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Friday (Apr 16). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

1. An infinitely long straight wire along the z-axis carries a uniform current I (moving upward toward the positive z-direction). A spherical shell of radius R with a total charge Q uniformly distributed over its surface is centered at the origin (through which the wire also passes, since the wire is along the z-axis).

(a) Write down an expression for $\vec{E}$ (i.e., *magnitude and direction*) produced by this configuration.
Note: This is a simple Gauss' law problem, since you may assume that the electric field is produced only by the charge distributed over the surface of the spherical shell.

(b) Write down an expression for $\vec{B}$ (*magnitude and direction*) produced by this configuration.
Note: This is a simple Ampere's law problem, since you may assume that the magnetic field is produced only by the current in the infinitely long wire.

(c) Use your results above to write down the *magnitude and direction* of the Poynting vector $\vec{S}$.

(d) Use $\vec{J} \cdot \vec{E}$ to determine the sign of the work done, and verify that this is consistent with the sign of the energy flow due to this configuration.

2. A transverse plane wave is incident normally in vacuum on a perfectly absorbing flat screen. From the law of conservation of linear momentum that we discussed in class, we know that $\sum_\beta T_{\alpha\beta}\, n_\beta$ is the αth component of the force per unit area on the surface. Use this to show that the pressure (called radiation pressure) exerted on the screen is equal to the field energy per unit volume in the wave.

Hint: Whether you can do this problem on one page, or five, depends on the initial setup, so spend some time thinking about it. I would recommend putting the screen on the xy-plane, for example, and having the wave travel along the $\hat{z}$ direction (remember, then, that the outward normal will be $\hat{n} = -\hat{z}$); you could then also choose $\vec{E} = E_x\hat{x}, \vec{B} = B_y\hat{y}$.

3. In the neighborhood of the Earth, the flux of electromagnetic energy from the Sun is approximately 1.4 kW/m^2. If an interplanetary sailplane had a sail of mass 1 g/m^2 of area and negligible other weight, what would be its maximum acceleration in m/s^2 due to the solar radiation pressure? **Hint:** The previous problem might be of use.

4. Consider a circular toroidal coil of mean radius a and N turns, with a *small* uniform cross section of area A, that is, both the height and width of the toroid are small compared to a. The toroid has a current I flowing in it. There is also a point charge Q located at the center of the toroid. Assume that the toroid is in the xy-plane, so that its axis is along the z-direction.

Calculate all the components of the electromagnetic field momentum of the system. You should find that in the plane of the toroid

$$\left(\vec{P}_{\text{field}}\right)_x = 0 \qquad \text{and} \qquad \left(\vec{P}_{\text{field}}\right)_y = 0$$

whereas the component of the field momentum along the axis of the toroid is

$$\left(\vec{P}_{\text{field}}\right)_z \approx \pm\frac{\mu_0 QINA}{4\pi a^2}$$

where the sign depends on the sense of the current flow in the coil. Assume that the electric field of the charge penetrates unimpeded into the region of nonvanishing magnetic field, as would happen for a toroid that is actually a set of N small nonconducting tubes inside which ionized gas moves to provide the energy flow.

Hint: Since the height and width of the toroid are small compared to a, we can use an Amperian loop with radius equal to a. This means there is only a ϕ-component of $\vec{B}$. Applying Ampere's Law, you should find that the magnetic field is

$$\vec{B} = \pm\frac{\mu_0 NI}{2\pi a}\,\hat{\phi}$$

where the $\pm$ is present to incorporate the direction of current flow in the coil (and will be present in the final expression, too). Meanwhile, the electric field is just that due to a point charge.