

Physics 460—Homework Report 9

Due Tuesday, Jun 2, 1 pm

Name: Timothy Holmes

Complete all the problems on the accompanying assignment.

List all the problems you worked on in the space below. Circle the ones you fully completed:

(1a)

Please place the problems into the following categories:

- These problems helped me understand the concepts better: _____
- I found these problems fairly easy: _____
- I found these problems very challenging: 1 2

In the space below, show your work (even if not complete) for any problems you still have questions about. Indicate where in your work the question(s) arose, and ask specific questions that I can answer.

Use the back of this sheet or attach additional paper, if necessary.

If you have no remaining questions about this homework assignment, use this space for one of the following:

- Write one or two of your solutions here so that I can give you feedback on its clarity.
- Explain how you checked that your work is correct.

Course Notes
MIT OpenCourseWare

- (1) In the course notes, we consider quantum teleportation using the state $|\Psi^-\rangle$ as the entangled state. In class we discussed using $|\Psi^+\rangle$ as the entangled state. Could we equally well use either $|\Phi^-\rangle$ or $|\Phi^+\rangle$ as the entangled state for quantum teleportation.

In other words, suppose that the initial state of the three particle system is one of

$$|\Psi_{123}\rangle = (a|0\rangle + b|1\rangle) \otimes |\Phi_{23}^-\rangle \quad \text{or} \quad |\Psi_{123}\rangle = (a|0\rangle + b|1\rangle) \otimes |\Phi_{23}^+\rangle.$$

Is it possible in either of these cases to transfer the state of particle 1 to particle 3? If so, how? If not, what goes wrong?

- (2) Suppose that we have two pairs of entangled particles with each pair being in the singlet state:

$$|\Psi_{12}\rangle = \frac{1}{\sqrt{2}}[|0;1\rangle - |1;0\rangle] = |\Psi_{12}^-\rangle \quad \text{and} \quad |\Psi_{34}\rangle = \frac{1}{\sqrt{2}}[|0;1\rangle - |1;0\rangle] = |\Psi_{34}^-\rangle.$$

The state of the four-particle system is then

$$|\Psi_{1234}\rangle = |\Psi_{12}^-\rangle \otimes |\Psi_{34}^-\rangle.$$

- Expand this state in terms of the unentangled basis states for the four particles.
- If you immediately measured the states of particles 2 and 3, what results could you obtain and with what probabilities?
- What would the state of the remaining system (composed of particles 1 and 4) be for each of the results from part (b)? Are any of these states entangled?
- Rewrite the original state (from part (a)) in terms of the Bell-basis states for particles 2 and 3.
- If you put particles 2 and 3 through a Bell-state analyzer (converting each of the Bell-states from part (d) to the corresponding unentangled state), and then measured the states of particles 2 and 3, what results could you obtain and with what probabilities?
- What would the state of the remaining system (composed of particles 1 and 4) be for each of the results from part (e)? Are any of these states entangled?
- Explain why the process of parts (d) and (e) is known as “entanglement swapping”. Have particles 1 and 4 interacted with each other during this process?

Homework 9

$$(1) \quad |\psi_{123}\rangle = (a|0\rangle + b|1\rangle) \otimes |\Phi_{23}^-\rangle$$

$$|\psi_{123}\rangle = (a|0\rangle + b|1\rangle) \otimes |\Phi_{23}^+\rangle$$

$$|\Phi_{23}^-\rangle = \frac{1}{\sqrt{2}} (|0\rangle_2 \otimes |0\rangle_3 - |1\rangle_2 \otimes |1\rangle_3)$$

$$\rightarrow \frac{1}{\sqrt{2}} (|0,0\rangle - |1,1\rangle)$$

$$|\psi_{123}\rangle = (a|0,0,0\rangle + b|1,0,0\rangle - a|0,1,1\rangle - b|1,1,1\rangle)$$

$$|\Phi_{23}^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle_2 \otimes |0\rangle_3 + |1\rangle_2 \otimes |1\rangle_3)$$

$$\rightarrow \frac{1}{\sqrt{2}} (|0,0\rangle + |1,1\rangle)$$

$$|\psi_{123}\rangle = (a|0,0,0\rangle + b|1,0,0\rangle + a|0,1,1\rangle + b|1,1,1\rangle)$$

$$a|0,0,0\rangle = \frac{a}{\sqrt{2}} (|\psi^+,0\rangle + |\psi^-,0\rangle) \quad b|1,0,0\rangle = \frac{b}{\sqrt{2}} (|\psi^+,0\rangle - |\psi^-,0\rangle)$$

$$a|0,1,1\rangle = \frac{a}{\sqrt{2}} (|\psi^+,1\rangle + |\psi^-,1\rangle) \quad b|1,1,1\rangle = \frac{b}{\sqrt{2}} (|\psi^+,1\rangle - |\psi^-,1\rangle)$$

$$|\psi\rangle = \frac{1}{2} (a(|\psi^+,0\rangle + |\psi^-,0\rangle) + b(|\psi^+,0\rangle - |\psi^-,0\rangle) + a(|\psi^+,1\rangle + |\psi^-,1\rangle) + b(|\psi^+,1\rangle - |\psi^-,1\rangle))$$

$$|\psi\rangle = \frac{1}{2} (|\psi^+\rangle \otimes (a|1\rangle + b|0\rangle) + |\psi^-\rangle \otimes (a|1\rangle + b|0\rangle) + |\psi^+\rangle \otimes (a|0\rangle + b|1\rangle) + |\psi^-\rangle \otimes (a|0\rangle + b|1\rangle))$$