Physics 412—Quantum Mice Observations   Winter 2020

We will continue our studies of the size and attitude of quantum mice, moving onto observations. Quantum mice come in two sizes, small and large. The size property is represented mathematically by the operator W, with two eigenvalues, 2 and 10, and corresponding eigenstates $|s\rangle$ and $|l\rangle$. The attitude property is represented by the operator A, with eigenvalues $+1$ (happy) and -1 (unhappy), and corresponding eigenstates $|h\rangle$ and $|u\rangle$.

The relationship between size and happiness can be found from the relationship between the eigenstates of W and A:

$$|s\rangle = \frac{1}{\sqrt{5}}\Big[|h\rangle + 2|u\rangle\Big], \quad |l\rangle = \frac{1}{\sqrt{5}}\Big[-2|h\rangle + |u\rangle\Big].$$

If we use the attitude states as our basis, we can represent the operators as matrices and the states as column vectors:

$$A \leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad |h\rangle \leftrightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad |u\rangle \leftrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$W \leftrightarrow \frac{1}{5}\begin{bmatrix} 42 & -16 \\ -16 & 18 \end{bmatrix}, \qquad |s\rangle \leftrightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad |l\rangle \leftrightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

Note: You can answer all of these working in the attitude basis, using the representation given above. It's probably easiest that way.

(1) If you measure the attitude of a small mouse (that is, a mouse in state $|s\rangle$), what results can you obtain, and with what probabilities? Same question for a large mouse.

(2) If you measure the size of a happy mouse (that is, a mouse in state $|h\rangle$), what results can you obtain, and with what probabilities? Same question for an unhappy mouse.

(3) If you wanted to create a mouse that has a 90% probability of being happy when you measure its attitude, what state should (could?) it be in?

(4) If you measured the size of your mouse from Question 3, what results could you obtain, and with what probabilities?

(5) Is your answer to Question 3 unique, or are there multiple states that will predict the same probabilities for measurements of the mouse's attitude? Explain.

(6) If the answer to Question 3 is not unique, do different states give different probabilities if you measure the size of the mouse? Explain.

① We can obtain happy $(A=+1)$ or unhappy $(A=-1)$.

If the state is $|\psi\rangle = |s\rangle$, then the probabilities are

$$P(h) = |\langle h|s\rangle|^2 = \left|\begin{bmatrix} 1 & 0 \end{bmatrix}\frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right|^2 = \left|\frac{1}{\sqrt{5}}(1+0)\right|^2 = \frac{1}{5}$$

$$P(u) = |\langle u|s\rangle|^2 = \left|\begin{bmatrix} 0 & 1 \end{bmatrix}\frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right|^2 = \left|\frac{1}{\sqrt{5}}(0+2)\right|^2 = \frac{4}{5}$$

If the state is $|\psi\rangle = |\ell\rangle$, then the probabilities are

$$P(h) = |\langle h|\ell\rangle|^2 = \left|\begin{bmatrix}1 & 0\end{bmatrix}\frac{1}{\sqrt{5}}\begin{bmatrix}-2\\1\end{bmatrix}\right|^2 = \left|\frac{1}{\sqrt{5}}(-2+0)\right|^2 = \frac{4}{5}$$

$$P(u) = |\langle u|\ell\rangle|^2 = \left|\begin{bmatrix}0 & 1\end{bmatrix}\frac{1}{\sqrt{5}}\begin{bmatrix}-2\\1\end{bmatrix}\right|^2 = \left|\frac{1}{\sqrt{5}}(0+1)\right|^2 = \frac{1}{5}$$

2) If the state is $|\Psi\rangle = |h\rangle$, then

$$P(s) = |\langle s|h\rangle|^2 = \frac{1}{5}$$

$$P(\ell) = |\langle \ell|h\rangle|^2 = 4/5$$

If the state is $|\Psi\rangle = |u\rangle$, then

$$P(s) = |\langle s|u\rangle|^2 = \frac{4}{5}$$

$$P(\ell) = |\langle \ell|u\rangle| = 1/5$$

3) We want $|\Psi\rangle = a|h\rangle + b|u\rangle$ where

$$|a|^2 = 0.9 \quad \text{and}$$

$$|a|^2 + |b|^2 = 1$$

Pick $a = \sqrt{0.9}$ and $b = \sqrt{0.1}$

Then $|\Psi\rangle = \sqrt{0.9}\,|h\rangle + \sqrt{0.1}\,|u\rangle$

and $|\Psi\rangle \longleftrightarrow \frac{1}{\sqrt{10}}\begin{bmatrix}3\\1\end{bmatrix}$

4 $$P(s) = |\langle s|\Psi\rangle|^2 = \left|\left(\frac{1}{\sqrt5}\begin{bmatrix}1 & 2\end{bmatrix}\right)\left(\frac{1}{\sqrt{10}}\begin{bmatrix}3\\1\end{bmatrix}\right)\right|^2$$

$$= \left|\frac{1}{\sqrt{50}}(3+2)\right|^2 = \frac{25}{50} = \frac{1}{2}$$

$$P(\ell) = |\langle \ell|\Psi\rangle|^2 = \left|\left(\frac{1}{\sqrt5}\begin{bmatrix}-2 & 1\end{bmatrix}\right)\left(\frac{1}{\sqrt{10}}\begin{bmatrix}3\\1\end{bmatrix}\right)\right|^2$$

$$= \left|\frac{1}{\sqrt{50}}(-6+1)\right|^2 = \frac{25}{50} = \frac{1}{2}$$

5 a & b don't have to be real!!

$a = \sqrt{0.9}$ $\quad b = \sqrt{0.1}\,i$

6 $$P(s) = |\langle s|\Psi\rangle|^2 = \left|\left(\frac{1}{\sqrt5}\begin{bmatrix}1 & 2\end{bmatrix}\right)\left(\frac{1}{\sqrt{10}}\begin{bmatrix}3\\i\end{bmatrix}\right)\right|^2$$

$$= \left|\frac{1}{\sqrt{50}}(3+2i)\right|^2 = \frac{13}{50}$$

$$P(\ell) = |\langle \ell | \Psi \rangle|^2 = \left| \left(\frac{1}{\sqrt{5}} \begin{bmatrix} -2 & i \end{bmatrix} \right) \left(\frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ i \end{bmatrix} \right) \right|^2$$

$$= \left| \frac{1}{\sqrt{50}} (-6 + i) \right|^2 = \frac{37}{50}$$