# Homework 6

(a) $l = 2$ $s = 1/2$

From Clebsh-Gordan table $2 \times 1/2$

$j = 5/2$ and $m = 5/2$

$\rightarrow \sqrt{j(j+1)}\,\hbar = \sqrt{35}\,\hbar/6$ $\quad P = 1$

(B) angular momentum of electron is $\sqrt{35}\,\hbar/2$, $j = 5/2$
$m = [-2, 2]$

| $J_1$ | $J_2$ | $M_1$ | $M_2$ | |
|---|---|---|---|---|
| 5/2 | +5/2 | +2 | +1/2 | $P = 1$ |
| 5/2 | +3/2 | +2 | -1/2 | $P = 4/5$ |
| 5/2 | +1/2 | +1 | -1/2 | $P = 3/5$ |
| 5/2 | -1/2 | 0 | -1/2 | $P = 2/5$ |
| 5/2 | -3/2 | -1 | -1/2 | $P = 1/5$ |
| 5/2 | -5/2 | -2 | -1/2 | $P = 1$ |

(C)

| $J_1$ | $J_2$ | $M_1$ | $M_2$ | | |
|---|---|---|---|---|---|
| 5/2 | 5/2 | 2 | 1/2 | $J = \sqrt{55}\,\hbar/6$ | $P = 1$ |
| 5/2 | 3/2 | 1 | 1/2 | $J = \sqrt{55}\,\hbar/6$ | $P = 4/5$ |
| 3/2 | 3/2 | 1 | 1/2 | $J = \sqrt{15}\,\hbar/2$ | $P = 4/5$ |
| 5/2 | 1/2 | 0 | 1/2 | $J = \sqrt{55}\,\hbar/6$ | $P = 3/5$ |
| 3/2 | 1/2 | 0 | 1/2 | $J = \sqrt{15}\,\hbar/2$ | $P = 4/5$ |
| 5/2 | -1/2 | -1 | 1/2 | $J = \sqrt{55}\,\hbar/6$ | $P = 2/5$ |
| 3/2 | -1/2 | -1 | 1/2 | $J = \sqrt{15}\,\hbar/2$ | $P = 4/5$ |
| 5/2 | -3/2 | -2 | 1/2 | $J = \sqrt{55}\,\hbar/6$ | $P = 1/5$ |
| 3/2 | -3/2 | -2 | 1/2 | $J = \sqrt{15}\,\hbar/2$ | $P = 4/5$ |

(D)

(2) $|\psi\rangle = \frac{1}{\sqrt{3}}\left\{|311\rangle \otimes |-\rangle + \left[|210\rangle - |211\rangle\right] \otimes |+\rangle\right\}$

$j = 3/2 \; ; \; m = +1/2$

$$|3/2 ; +1/2\rangle = \sqrt{\frac{1}{3}}\,|+1 ; -1/2\rangle + \sqrt{\frac{2}{3}}\,|-1 ; +1/2\rangle$$

(B)

$$|\langle +1 ; +1/2 \,|\, +1 ; -1/2\rangle|^2 = |\sqrt{1/3}|^2 = 1/3$$

$$|\langle -1 ; -1/2 \,|\, -1 ; -1/2\rangle|^2 = |\sqrt{2/3}|^2 = 2/3$$

$$|\langle \pm 1 ; \mp 1/2 \,|\, \pm 1 ; \pm 1/2\rangle|^2 = |\sqrt{1/3}\cdot 0 + \sqrt{2/3}\cdot 0|^2 = 0$$

(C)

| $J_1$ | $J_2$ | $m_1$ | $m_2$ | |
|---|---|---|---|---|
| 3/2 | 3/2 | 1 | 1/2 | $P = 1$ |
| 3/2 | 1/2 | 1 | -1/2 | $P = 1/3$ |
| 3/2 | -1/2 | 0 | -1/2 | $P = 2/3$ |
| 3/2 | -3/2 | -1 | -1/2 | $P = 1$ |

↑

[1 1]

$[1, -1]$

(3a) $$H_{hf} = \frac{A}{\hbar^2}(\vec{S}_1 \cdot \vec{S}_2) + H_A$$