Inflation

(1) We have derived some results for having $w = -1$ as applied to the inflation scenario. You will now probe some of the consequences.

(a) In the lecture you were given the following scenario,

$$a(t) = \begin{cases} \sqrt{t/t_i} & t < t_i \\ a e^{H_i(t-t_i)} & t_i < t < t_f \\ a e^{H_i(t_f-t_i)}\sqrt{t/t_f} & t > t_f \end{cases}$$

Describe in words what is happening in the universe in this scenario.

(b) In the lecture we've introduced the ratio,

$$\frac{a(t_f)}{a(t_i)} = e^N; \text{ where } N \equiv H_i(t_f - t_i);\ H_i \equiv \sqrt{\frac{\Lambda}{3}}$$

One possible model for inflation has the exponential growth occurring at the Grand Unified Theory (GUT) time of $t_i \approx 10^{-36}$s, so that $H_i = 10^{36}$s. Find the energy density,

$$\epsilon_\Lambda = \frac{3c^2}{8\pi G} H_i^2.$$

Note that the current value of $\epsilon_\Lambda = 0.0034$ TeV m^{-3}. Compare the results and think about the consequences of this energy density at the GUT time scale.

(c) Now we'll see how this scenario might solve the flatness problem.

Recall that

$$|1 - \Omega(t)| = \frac{c^2}{R_o^2 a(t)^2 H(t)^2}$$

During the inflation time, all terms but $a(t)$ are constants. In this case,

$$|1 - \Omega(t)| \propto e^{-2H_i t}.$$

Compare the density parameter at the beginning of inflation ($t = t_i$) with the density parameter at the end of inflation ($t = t_f = [N+1]t_i$).

(d) Now consider a universe that was initially very strongly curved, for example, a universe such that

$$|1 - \Omega(t)| \approx 1.$$

Find the value of the density parameter after it has undergone N e-foldings of inflation.

(e) At your table, discuss how this scenario addresses the flatness problem.

(2) We now address the horizon problem.

(a) At your table, speculate how inflation might address the horizon problem.

(b) The horizon distance is given by

$$d_{\rm hor}(t) = a(t)\, c \int_0^t \frac{dt'}{a(t')}$$

Recall that in the model we are discussing that before inflation occurs, $a(t) = a_i \sqrt{t/t_i}$. Find the horizon distance at the beginning of inflation.

(c) Set up the integral to find the horizon distance at the end of inflation.

(d) Suppose inflation began at $t_i \approx 10^{-36}$, use (5b) to find the distance to the horizon. Then suppose inflation lasts for $N = 65$ e-foldings, using (5c) find the horizon distance after inflation ends.

(e) At your table discuss how this addresses the horizon problem.

(3) At you table, recap the issues that gave rise to the idea of inflation. Then recap exactly how inflation overcomes the flatness and horizon problems.

(4) You will now begin exploring a possible physical reason that could lead to the kind of inflation behavior needed to address the flatness and horizon problem.

(a) In the lecture we gave the example of a scalar field associated with height as function of position, $\phi(x, y, t)$. What are the units of this scalar field? Now speculate what kind of units a scalar field associated with inflation might have. Discuss at your table.

(b) In the lecture you were given that

$$\begin{aligned}\epsilon_\phi &= \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 + V(\phi) \\ P_\phi &= \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi)\end{aligned}$$

Using these two expressions, rewrite the fluid equation,

$$\dot{\epsilon}_\phi + 3H(t)\left(\epsilon_\phi + P_\phi\right) = 0$$

to come up with a differential equation for ϕ. The solution will have three terms, give a physical meaning to each of the terms.

(c) At your table discuss how the equation just derived would account for the behavior of inflation, i.e., the conditions in quesion (1 a).

(d) Find the condition for a terminal velocity (when $\ddot{\phi} = 0$.)

(e) Suppose the inflation field changes very slolwly with time. For example suppose

$$\dot{\phi}^2 \ll \hbar c^3 V(\phi).$$

Substitute this value in for the condition found in part (d) and find a condition on $dV/d\phi$.

(5) At your table, recap the physics we've introduced to explain inflation. Make sure you discuss the dynamical behavior of the scalar field. Then begin working on the homework,

Homework 04–Due Friday,March 6

1. Problem 7.3
2. Problem 7.4
3. Suppose $\Omega = 0.5$ in the early universe when the energy density is $\epsilon = 10^{16}$ GeV m^{-3}. At his time, suppose all the matter in the universe obeys $P = -\epsilon$ (i.e., single component universe).
   (a) After the scale factor increases by 60 e-foldings, what is the new value of Ω
   (b) Suppose at the end of the expansion described in part (a), all the energy density is instantly transformed into radiation (so the value of ϵ does not change, but the equation of state does). Assuming that the matter in the universe is composed *entirely* of radiation, what is the value of Ω when $T = 10^4 K$. The starting value of Ω you start here is the value you got in part **a.).**
4. Problem 11.4
5. **Grad Problem.** In this problem, you will carry out a very simple version of the parameter space process that cosmologists use to determine cosmological parameters in the BenchMark model. Please use a plotting software package to do this assignment, I do not want hand-drawn figures.
   (a) Draw a graph in which the x-axis is $\Omega_{m,o}$ and the y-axis is Ω_Λ. Each axis should go from 0 to 1. As you know the best shows that $\Omega_{m,o} + \Omega_\Lambda = 1$. Plot this line on the graph.
   (b) Observations of supernovae show that $\Omega_{m,o} - \Omega_\Lambda = -0.4$. Plot this line on the graph
   (c) If the CMB and supernovae results are both correct, what can you conclude about the values of $\Omega_{m,o}$ and Ω_Λ
   (d) What does the quantity $\Omega_{m,o} - \Omega_\Lambda$ tell you about the universe? For example, if the value $\Omega_{m,o} - \Omega_\Lambda = +0.4$ instead of what you plotted on the graph, what would be different about the universe? Explain in detail.