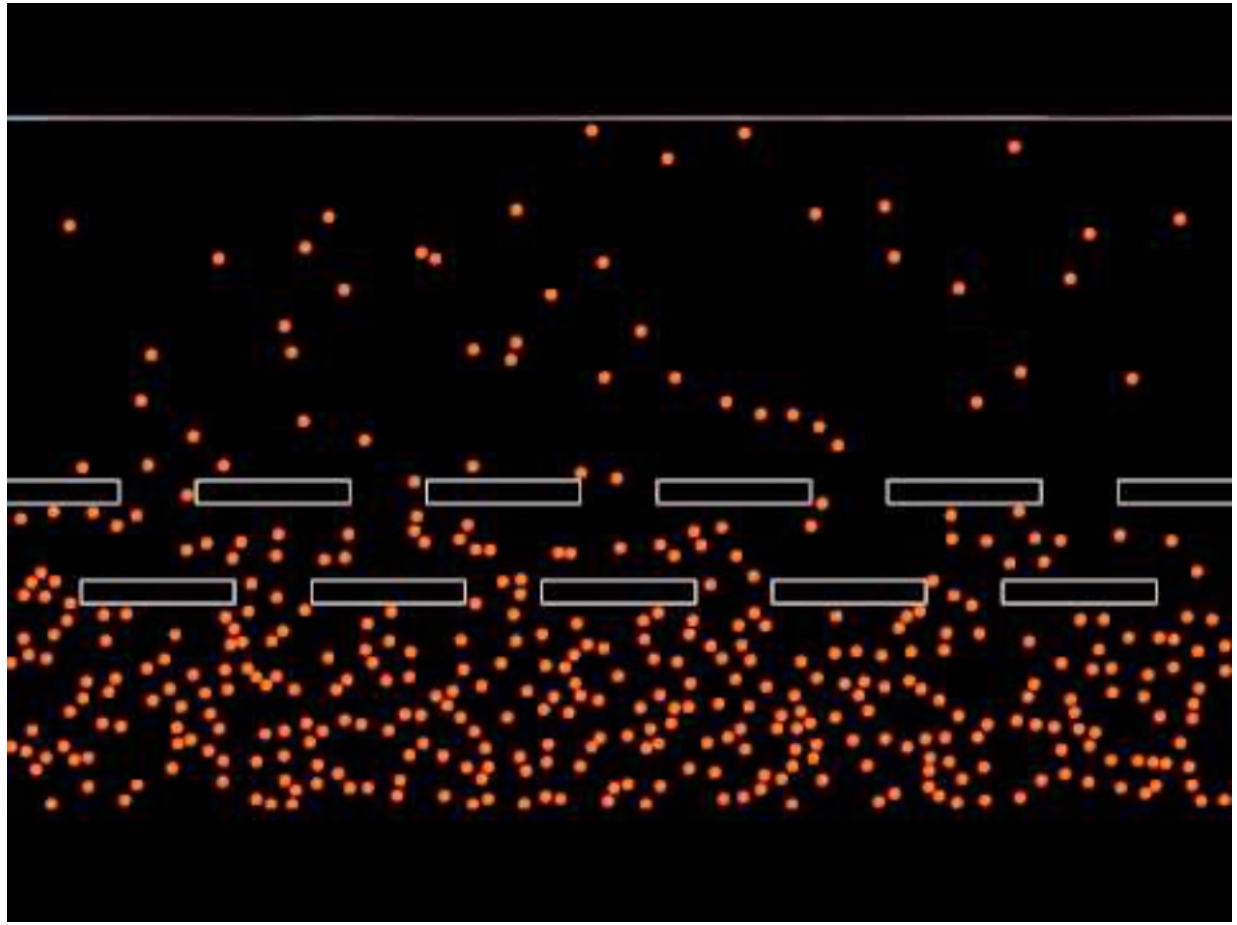
Monte Carlo techniques. We will apply these techniques to two different cases:

1. Integration
2. Monte Carlo simulation

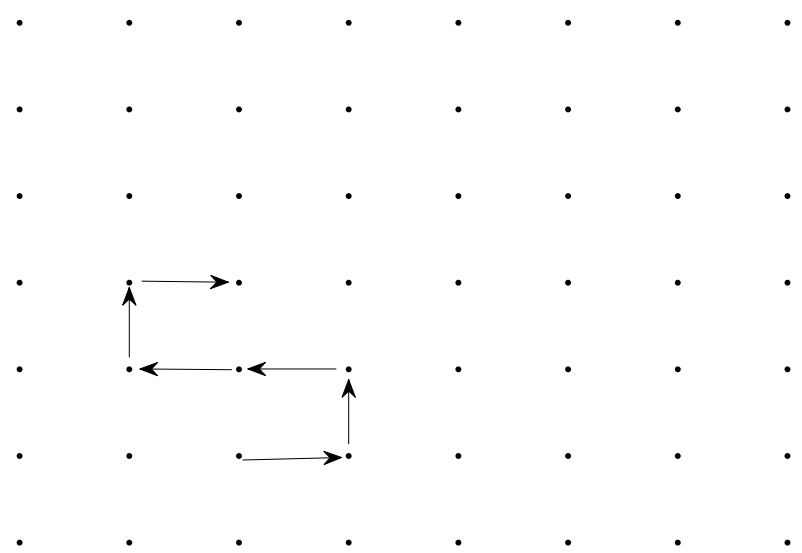


$$m_i \vec{v}_{i\,\text{before}} = m_i \vec{v}_{i\,\text{after}}$$

$$\tfrac{1}{2} m_i \left|\vec{v}_i\right|^2_{\text{before}} = \tfrac{1}{2} m_i \left|\vec{v}_i\right|^2_{\text{before}}$$

We'll explore how *Monte Carlo techniques* can add insight to complicated situations like the one just introduced.

Let's start with a simple 2-dimensional example.



- Assume particle can only move to nearest-neighbor and can only take one step each iteration
- Generate a random number, x_r between 0 and 1
  - If $0 \le x_r \le 0.25$, move one step left
  - If $0.25 \le x_r \le 0.50$, move one step up
  - If $0.50 \le x_r \le 0.75$, move one step right
  - If $0.75 \le x_r \le 1.00$, move one step down

Do question (1) on the worksheet and S T O P

```
function montesim01(n,r)
%Drunken Sailor simulation

% first some stuff to plot grid points

xy = 1:n;
[X, Y] = meshgrid(xy);
% xyo = xy([4 11]);
figure(1)
plot(X, Y, '.k')
hold on

% starting pint
x = n/2;
y = n/2;
% starting simulation
for j = 1:r
    stp = rand;
    if stp<.25 % step left
        %x = x -1;
        u = -1;
        v = 0;
    elseif stp>0.25 && stp<0.50 % move up
        %y = y + 1;
        u = 0;
        v = 1;
    elseif stp>0.5 && stp<0.750 % move right
        %x = x + 1;
        u = 1;
        v = 0;
    elseif stp>0.75 && stp<1 % move down
        %y = y - 1;
        u = 0;
        v = -1;
    end
    quiver(x,y,u,v)
    x = x +u;
    y = y + v;
    hold on
    pause
end
end
```

This carries out the algorithm

Do question (2) on the worksheet

Let's now move to the continuous 3-D case using diffusion as our example. Think of perfume diffusing through a room.

- A single molecule that changes directions randomly
- The molecule can be scattered in direction in the solid angle centered at the collision.

$$\mathrm{d}\,\Omega = \sin\theta\,\mathrm{d}\,\theta\,\mathrm{d}\,\phi$$

$$\mathrm{d}\,\Omega = \mathrm{d}\,q\,\mathrm{d}\,\phi$$

$$\mathrm{d}\,q = \sin\theta\,\mathrm{d}\theta \Rightarrow q = \cos\theta$$

- Select $\phi, [0, 2\pi]$, select q [-1, 1]
- Allow molecule to move some distance before colliding
- Repeat.

From this very simple simulation, very important physics emerges as you will soon see. Do question (3) on the worksheet.

```
for prt = 1:400  %prt is number of particles
    x1 = 5;
    y1=5;
    z1=5;
for n = 1:collision  % collison is number of collisoions each particle has
    theta = acos(-1 +2*rand);
    phi = 2*pi*rand;
    [x,y,z] = sph2cart(theta,phi,r);
    x1 = x +x1;
    y1 = y + y1;
    z1 = z + z1;
    dis(prt,n) = sqrt((x1 - 5)^2 + (y1 - 5)^2 + (z1 - 5)^2);
    if prt ==400 %show path of last particle
        scatter3(x1,y1,z1,'MarkerFaceColor','k')
        hold on
    end
end

end
```

Finish the worksheet.