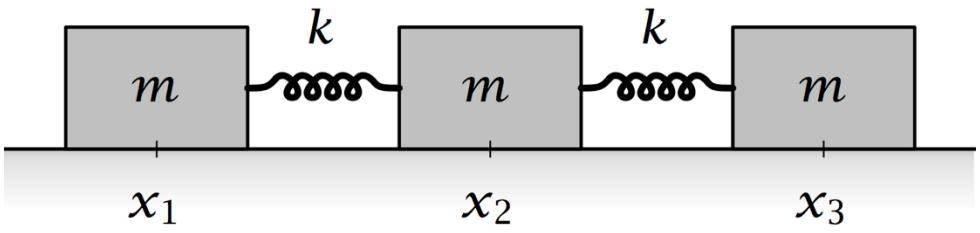
Three identical blocks connected by springs slide along a frictionless horizontal surface as shown below. The mass of each block is m, the spring constant of both springs is k, and their rest length is l.



a. How many degrees of freedom does this system have? How many coordinates do you need to completely describe its configuration?
b. Find the Lagrangian in cartesian coordinates $(x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3)$. Does it have any cyclic coordinates? If not, does that mean there aren't any constants of motion?
c. Thinking about this system sliding along the surface as it oscillates, do you think that there should be a conserved quantity (a constant of the motion)? If so, what is it? Without doing any calculations, can you write down an expression for this quantity?
d. Even though using the x coordinates of the three blocks allowed us to write the Lagrangian with the minimum required number of coordinates (three degrees of freedom, three coordinates), using x_1, x_2 and x_3 didn't allow us to find the constant of motion we know should be there. Can you find a new set of three variables (call them X_1, X_2 and X_3) so that if you express the Lagrangian in these variables there is a cyclic variable?
e. Once you have found the cyclic variable, find the corresponding conserved quantity by calculating its conjugate momentum.