PHY 411 **Winter 2021**

Formula Sheet for Midterm Examination

Maxwell Equations: $\vec{\nabla} \cdot \vec{D} = \rho$ $\qquad$ $\vec{\nabla} \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

$\vec{\nabla} \cdot \vec{B} = 0$ $\qquad$ $\vec{\nabla} \times \vec{E} + \dfrac{\partial \vec{B}}{\partial t} = 0$

Helmholtz Wave Equation: $\left(\nabla^2 + k^2\right) \vec{u} = 0$, where $\vec{u} = \vec{E}$ or $\vec{B}$

Plane Waves: $\vec{B} = \sqrt{\mu\epsilon}\, \dfrac{\vec{k} \times \vec{E}}{k}$

Poynting vector: Re $\vec{S} = \frac{1}{2}\, \vec{E} \times \vec{H}^*$; (time-averaged) $\qquad$ For incident wave: Re $\vec{S} \cdot \hat{n} = \dfrac{1}{2} \sqrt{\dfrac{\epsilon}{\mu}} \left|E_0\right|^2 \cos i$

Constants:

Electron charge $= 1.602 \times 10^{-19}$ C $\qquad$ $\epsilon_0 = 8.854 \times 10^{-12}$ in SI units (farad/m)

Electron mass $= 9.11 \times 10^{-31}$ kg $\qquad$ $\mu_0 = 4\pi \times 10^{-7}$ in SI units (N/A^2)

Vector Formulas:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

$$\vec{\nabla} \times \vec{\nabla}\psi = 0$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{a}) = 0$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \nabla^2 \vec{a}$$

$$\vec{\nabla} \cdot (\psi \vec{a}) = \vec{a} \cdot \vec{\nabla}\psi + \psi \vec{\nabla} \cdot \vec{a}$$

$$\vec{\nabla} \times (\psi \vec{a}) = \vec{\nabla}\psi \times \vec{a} + \psi \vec{\nabla} \times \vec{a}$$

$$\vec{\nabla}(\vec{a} \cdot \vec{b}) = (\vec{a} \cdot \vec{\nabla})\vec{b} + (\vec{b} \cdot \vec{\nabla})\vec{a} + \vec{a} \times (\vec{\nabla} \times \vec{b}) + \vec{b} \times (\vec{\nabla} \times \vec{a})$$

$$\vec{\nabla} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{\nabla} \times \vec{a}) - \vec{a} \cdot (\vec{\nabla} \times \vec{b})$$

$$\vec{\nabla} \times (\vec{a} \times \vec{b}) = \vec{a}(\vec{\nabla} \cdot \vec{b}) - \vec{b}(\vec{\nabla} \cdot \vec{a}) + (\vec{b} \cdot \vec{\nabla})\vec{a} - (\vec{a} \cdot \vec{\nabla})\vec{b}$$
