Physics 460—Practice S–1 (Due Apr 9, 1 pm) **Name:** Timothy Holmes

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S–1: I can apply the fundamental properties of the angular momentum operators and use and interpret their eigenstates.

Unsatisfactory Progressing Acceptable Polished

Section 1.3 of the course notes is dense with derivations of the properties of the angular momentum eigenstates. Explain the following in detail in your own words.

(1) How do we know that the eigenvalues of J^2 must be positive?

The eigenvalues of J^2 are given by

$$\lambda = \frac{j(j+1)\hbar^2}{2}$$

where j is a non negative number making $\lambda \geq 0$. The only possible values for j are $(0, 1/2, 1, 3/2, 2, \ldots)$.

(2) How do we know that m has a maximum value?

given the equation

$$J_\pm |\lambda, m\rangle = C_\pm(\lambda, m)|\lambda, m \pm 1\rangle \quad (1.40)$$

These ladder operators step the value of m up or down with the eigenkets of J^2. The limitation is given by $m^2 \leq \lambda(m, l)$, that bounds it with a min, m_{min}, and max, m_{max}. Thus, there can be no state $m > m_{max}$ or $m < m_{min}$.

(3) How do we know that $\lambda = m_{\max}(m_{\max} + 1)$?

If we take the lowering and raising operator we can prove this. Given, we want to raise for m_{max} we have

$J_+ |l, m_{max}\rangle = \emptyset$. multiply the lowering operator so

$J_- J_+ |l, m_{max}\rangle = \emptyset \Rightarrow$ Substitute $\Rightarrow (J^2 - J_z^2 - \hbar J_z)|l, m_{max}\rangle = 0$

Rearrange so $J^2|l, m_{max}\rangle = (J_z^2 + \hbar J_z)|l, m_{max}\rangle$

$$= \lambda = m_{max}(m_{max} + 1).$$