Physics 440, Autumn 2019 **In-Class Exam 1**

Tuesday, October 8 **Score: _____/17 points**

The following equations might be useful:

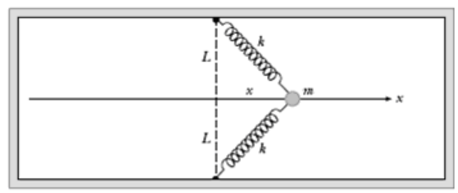
$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}$ $\quad L = T - V \quad \delta W = \sum_{\alpha=1}^{n} Q_\alpha \, \delta q_\alpha \quad Q_\alpha = \sum_{i=1}^{3N} F_i \frac{\partial x_i}{\partial q_\alpha}$

$\sin^2\alpha + \cos^2\alpha = 1 \quad \sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha - \beta)$

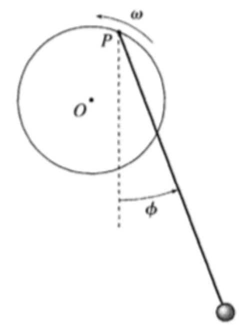
1. Find the constant(s) of motion for the following five Lagrangians. If there are no constants of motion please explain how you know this.
   a. (1 point) $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$
   b. (1 point)$L = \frac{1}{2}m\dot{z}^2 - mgz$
   c. (1 point)$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 - mg(l - x)\sin\alpha$
   (Disk with radius R and moment of inertia I rolling down an inclined plane.)
   d. (1 point)$L = \frac{ml^2}{2}\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) + mgl\cos\theta$
   e. (1 point)$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{GMm}{r}$

2. Find the Lagrangians in appropriate generalized coordinates for the following four systems.
    a. (1 point) A box sliding down a frictionless inclined ramp with inclination angle α.

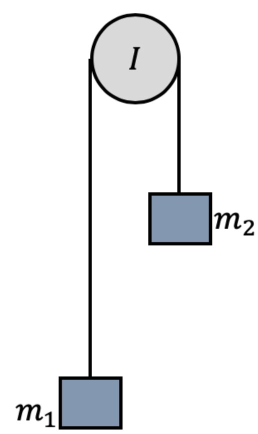
    b. (1 point) A mass attached between two identical springs with spring constant k and rest length L on a horizontal frictionless tabletop. (The figure on the right shows the view from above the table.)



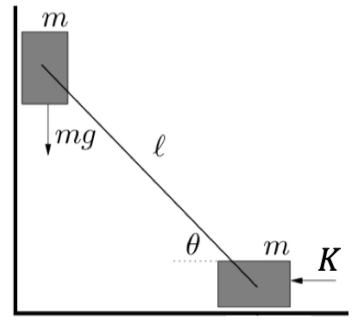
c. (2 points) A pendulum of mass m and length l attached to the edge of a wheel with radius R that rotates at angular velocity ω.



d. (2 points) Atwood's machine with a pulley with mass M, radius R and moment of inertia $I = \frac{1}{2}MR^2$. The rope length is l.



3. (3 points) Two blocks on frictionless surfaces are constrained by a rod of length l to move together. Use the principle of virtual work to determine the force K needed to keep the system in static equilibrium.



4. (3 points) The figure on the right shows a disk of mass m_1 tethered to a hanging mass m_2 by a massless string of length l. The disk is free to move on a frictionless, horizontal table. The string in threaded through a small hole in the table. m_2 can move vertically but cannot not swing back and forth. The Lagrangian of this system is

$$L = \frac{1}{2} m_1 (\dot{r}^2 + r^2 \dot{\theta}^2) + \frac{1}{2} m_2 \dot{r}^2 + m_2 g (l - r)$$

Derive the equation(s) of motion using the Lagrangian formalism.