PHY 420 Final Examination Help Document

Maxwell Equations (SI form):

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

Maxwell Equations (Gaussian form):

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\,\vec{J} + \frac{1}{c}\,\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{E} + \frac{1}{c}\,\frac{\partial \vec{B}}{\partial t} = 0$$

Laplace equation, cylindrical (ρ, ϕ, z):

$$\frac{1}{\rho}\,\frac{\partial}{\partial \rho}\left(\rho\,\frac{\partial \Phi}{\partial \rho}\right) + \frac{1}{\rho^2}\,\frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

Maxwell Stress Tensor:

$$T_{\alpha\beta} = \epsilon_0 \left[E_\alpha E_\beta + c^2 B_\alpha B_\beta - \frac{1}{2}\Big(\vec{E}\cdot\vec{E} + c^2 \vec{B}\cdot\vec{B}\Big)\,\delta_{\alpha\beta}\right]$$

Continuity equation:

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Radiation:

Dipole Moment: $\vec{p} = \int \vec{x}\,'\,\rho(\vec{x}\,')\,d^3x'$

For Quadrupole: $\dfrac{dP}{d\Omega} = \dfrac{c^2 Z_0}{1152\pi^2}\,k^6 \left|\Big\{\hat{n} \times \vec{Q}(\hat{n})\Big\} \times \hat{n}\right|^2$

$dP/d\Omega$ for dipole radiation will be on the questions sheet, if needed.

Lorentz transformations:
(for boost along x_1 axis)

$$ct' = \gamma\,(ct - \beta x_1) \qquad ct = \gamma\,(ct' + \beta x_1')$$

$$x_1' = \gamma\,(x_1 - \beta x_0) \qquad x_1 = \gamma\,(x_1' + \beta x_0')$$

$$x_2' = x_2 \qquad x_2 = x_2'$$

$$x_3' = x_3 \qquad x_3 = x_3'$$

Transformations of Fields:
(for boost along x_1 axis)

$$E_1' = E_1 \qquad B_1' = B_1$$

$$E_2' = \gamma\Big(E_2 - \beta B_3\Big) \qquad B_2' = \gamma\Big(B_2 + \beta E_3\Big)$$

$$E_3' = \gamma\Big(E_3 + \beta B_2\Big) \qquad B_3' = \gamma\Big(B_3 - \beta E_2\Big)$$

Field-strength tensor:

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

Note: $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

where $\vec{\beta} = \vec{v}/c$

The dual field-strength tensor $\mathcal{F}^{\alpha\beta}$ can be obtained from $F^{\alpha\beta}$ by putting $\vec{E} \to \vec{B}$ and $\vec{B} \to -\vec{E}$, whereas the covariant form $F_{\alpha\beta}$ can be obtained from $F^{\alpha\beta}$ by putting $\vec{E} \to -\vec{E}$.

PHY 420 Final Examination Help Document (page 2)

Spherical to Cartesian:

$\hat{r} = \hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta$

$\hat{\theta} = \hat{x} \cos\theta \cos\phi + \hat{y} \cos\theta \sin\phi - \hat{z} \sin\theta$

$\hat{\phi} = -\hat{x} \sin\phi + \hat{y} \cos\phi$

Cartesian to Spherical:

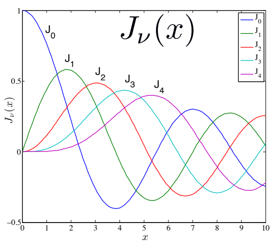
$\hat{x} = \hat{r} \sin\theta \cos\phi + \hat{\theta} \cos\theta \cos\phi - \hat{\phi} \sin\phi$

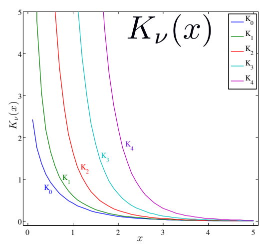
$\hat{y} = \hat{r} \sin\theta \sin\phi + \hat{\theta} \cos\theta \sin\phi + \hat{\phi} \cos\phi$

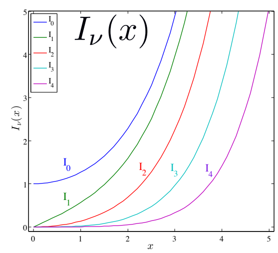
$\hat{z} = \hat{r} \cos\theta - \hat{\theta} \sin\theta$

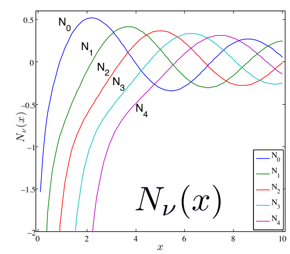
Useful Functions and Plots:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\sinh x = \frac{e^x - e^{-x}}{2}$$









Vector Formulas:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

$$\vec{\nabla} \times \vec{\nabla}\psi = 0$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{a}) = 0$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \nabla^2 \vec{a}$$

$$\vec{\nabla} \cdot (\psi\vec{a}) = \vec{a} \cdot \vec{\nabla}\psi + \psi\vec{\nabla} \cdot \vec{a}$$

$$\vec{\nabla} \times (\psi\vec{a}) = \vec{\nabla}\psi \times \vec{a} + \psi\vec{\nabla} \times \vec{a}$$

$$\vec{\nabla} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{\nabla} \times \vec{a}) - \vec{a} \cdot (\vec{\nabla} \times \vec{b})$$

$$\vec{\nabla} \times (\vec{a} \times \vec{b}) = \vec{a}(\vec{\nabla} \cdot \vec{b}) - \vec{b}(\vec{\nabla} \cdot \vec{a}) + (\vec{b} \cdot \vec{\nabla})\vec{a} - (\vec{a} \cdot \vec{\nabla})\vec{b}$$

Divergence theorem: $\displaystyle\int_V \vec{\nabla} \cdot \vec{A}\, d^3x = \int_S \vec{A} \cdot \hat{n}\, da$