1 Definitions and Ideas

1.1 Stellar Timescales

1. dynamical timescale: $t_{dyn} = \left(\frac{R^3}{GM}\right)^{1/2}$
2. Kelvin-Helmholtz timescale: $t_{KH} = \frac{GM^2}{RL_s}$
3. nuclear timescale: $t_{nuc} = 10^{10} years \left(\frac{M}{M_\odot}\right)\left(\frac{L_s}{L_\odot}\right)^{-1}$

1.2 Distance Measures

1. Astronomical Unit (AU)
2. parsec (pc)
3. Light Year (Ly)

1.3 Stellar Brightnesses

1. magnitude scale

1.4 Stellar Spectra

interstellar absorption: - The interstellar space contains matter, in the form of gas and dust, which affects the light on the way from a star to the observer. Thus, except for the nearest stars, we cannot immediately measure the intrinsic properties of the stars.

Natural Broadening: - Since we cannot know how long the atom or electron will remain in its upper state, but only assign a probability, we will have a spread in the energy of each level. This leads to a broadening of the line.

Lorentzian profile: : $\phi(\nu)d\nu = \frac{\gamma_n/4\pi}{(\nu-\nu_0)^2+(\gamma_n/4\pi)^2}\frac{d\nu}{\pi}$

Collisional (or pressure) Broadening: - results from the fact that atoms are not isolated. Instead, they interact with their neighbors. They will collide directly with some (neutral) neighbors, and positive ions will also feel the electric fields of nearby (charged) particles (e.g., electrons)

Doppler Broadening: - arises because the atoms in a gas move around randomly with a distribution of speeds that is described by the Maxwell-Boltzmann distribution.

2 The Physics of Stars

2.1 Thermodynamic Equilibrium

Maxwellian distribution: :

$f(v) = 4\pi\left(\frac{m}{2\pi kT_{kin}}\right)^{3/2} v^2 exp\left(-\frac{mv^2}{2kT_{kin}}\right)$

Boltzmann distribution: :

$\frac{n_2}{n_1} = \frac{g_2}{g_1} exp\left(-\frac{E_2-E_1}{kT_{ex}}\right)$

Planck distribution: :

$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{exp(h\nu/kT_{rad})-1}$

2.1.1 Ideal Gas

1. mass fraction of H and He by X and Y respectively, heavy elements by Z

mean molecular weight: : $\mu = \frac{4}{3+5X-Z}$

2.2 Hydrostatic Equilibrium

Balance of forces inside a star

Mass in shell: : $\frac{dm}{dr} = 4\pi r^2\rho$

Central pressure: : $P_c = \frac{GM^2}{R^4}$

Central temperature: : $T_c = \frac{\mu_c m_p}{k}\frac{GM}{R}$

Hydrostatic equilibrium: : $\mathrm{dP}\frac{}{dr=-\rho\frac{Gm}{r^2}}$

2.2.1 The Virial Theorem

Describes a system in equilibrium

Viral theorem: : $\Omega + 2U = 0$ where Ω is the gravitational potential energy and U is the (total) internal energy

2.2.2 Polytropic models, Lane-Emden equation

Polytropic relation: : $P(r) = K[\rho(r)]^\gamma$

Lane-Emden equation: : $\frac{1}{\zeta^2}\frac{d}{d\zeta}\left(\zeta^2\frac{d\theta}{d\zeta}\right) = -\theta^n$

2.3 Energy Transport in Stellar Interiors

radiation: - energy carried by photons

convection: - energy carried by bulk motions of gas

2.4 Stellar Atmospheres

The atmosphere of a star is where the opaque interior becomes semi-transparent.

Scale height: : $H = \frac{kT}{\mu m_p g}$ where $g = \frac{Gm}{r^2}$

Photosphere: - the bottom of the atmosphere, corresponds to the visible surface of the star.

2.5 Energy Production

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2.6 Energy Transport by Convection

Adiabatic gradient: $\nabla_{ad} = \frac{\Gamma_2-1}{\Gamma_2} = \left(\frac{\partial\, ln\, T}{\partial\, ln\, P}\right)_{ad}$

1. For a fully ionized ideal gas, $\nabla_{ad} = 2/5$

Radiative temperature gradient: :

$\nabla_R = \frac{d\, ln\, T}{d\, ln\, P} = \frac{3k_B}{16\pi acGm_p}\frac{\kappa}{\mu}\frac{L(r)}{m(r)}\frac{\rho}{T^3}$

Condition for instability: : $\nabla_R > \nabla_{ad}$

Convection may occur under the following circumstances:

1. The ratio $L(r)/m(r)$ is large (massive stars)
2. The opacity (κ) is large (low mass stars)
3. The quantity ρ/T^3 is large (low mass stars)
4. The adiabatic temperature gradient (∇_{ad}) is small (low mass stars)

2.7 Mass-Luminosity Relations

1. mean free path (covey the same as opacity):

$\lambda = \frac{1}{n\sigma}$

particle density: : $n = \rho/(\mu m_p)$

opacity: - parametrizes the microscopic interaction between radiation and matter: $\kappa = \frac{\sigma n}{\rho}$

2.7.1 Contributions to Opacity

2. electron scattering

Bound-free (bf) absorption: - an electron that is initially bound to an atom is ejected by the interaction with a photon of sufficient energy; the freed electron then has a kinetic energy equal to the difference between the energy of the photon and the ionization potential:

$\kappa = \kappa\rho T^{-3.5}$

Free-free (ff) absorption: - a free electron in the vicinity of an ion absorbs energy from a photon and becomes a free electron with greater energy.

Bound-bound (bb) absorption: - electrons bound to a neutral or partially ionized atom absorb energy from a photon and are excited to a higher energy state, but one which is still bound.

Thomson cross-section of the electron (cgs): :

$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2$

power law: : $\kappa = \kappa_0\rho^\lambda T^{-\nu}$

Surface luminosity: :

$L_s = \frac{ac}{\kappa_0}\left(\frac{G_p}{k_B}\right)^{4+\nu} R^{3\lambda-\nu} M^{3+\nu-\lambda}$

where $a = 7.565\times 10^{-15}\ erg\ cm^{-3}\ K^{-4}$

3 Energy Generation in Stars

3.1 Nuclear Energy Generation

Coulomb barrier: : $E_coul = \frac{Z_1 Z_2 e^2}{r_0} = Z_1 Z_2 MeV$

cross sections: : $\sigma(E) = \frac{S(E)}{E} exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar v}\right)$

binding energy: - the mass difference is released as energy: $Q(Z,N) = c^2[zm_p + Nm_n - m(Z,N)]$ where Z is protons and N is neutrons.

mass excess: : $= m - m_p(Z+N)$

3.2 Hydrogen Fusion

PP chain: - involves direct fusion of protons, produces most of the energy in the Sun, and is dominant in stars of a solar mass or less.

CNO cycle: - fusion occurs through a sequence of reactions involving C, N, and O, which effectively act as catalysts, quickly surpasses the PP-chain in energy production as soon as the mass exceeds about $1\ M_\odot$

4 Stellar Evolution

4.1 Evolution before the Main Sequence

Hayashi track:

Henyey track:

4.2 The Main Sequence

Stars spend most of their lifetime fusing hydrogen to helium on the Main Sequence.

4.2.1 The Zero Age Main Sequence (ZAMS)

Time spent on the Main Sequence: : $t_{MS} \propto M^{-2.5}$

4.2.2 Evolution during core hydrogen fusion

4.3 The Sun

1. Standard Solar Model - 1 $M_\odot$ ZAMS has evolved to the present-day Sun subject to the following assumptions:

- The Sun was formed from a homogeneous mixture of gases.
- It is powered by nuclear reactions in its core.
- It is approximately in hydrostatic equilibrium, with gravitational forces exactly compensated by gradients arising from gas and radiation pressure.
- Some deviations from equilibrium are permitted as the Sun evolves, but these are small and slow.
- Energy is transported from the core to the surface by photons (radiative) and by large-scale vertical motion of packets of gas (convection).

2. solar neutrinos - The production of each 4^He nucleus during nuclear fusion in the Sun is accompanied by two neutrinos

4.4 Post Main Sequence Evolution

At the end of the Main Sequence stage, the star is left with a core of helium and a small amount of heavy elements.

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4.5 High Mass Stars

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4.6 Stellar Remnants: White Dwarfs and Neutron Stars

% mass loss: : $\left(\frac{\text{original mass} - \text{remnant mass}}{\text{original mass}}\right) \times 100$

4.7 Elements beyond Iron

s-process: - (slow neutron capture) neutron capture is much slower than the β-decay

r-process: - (rapid neutron capture) neutron capture is much more rapid than theβ-decay