**Physics 412—Practice S-6 (Due Feb. 24, 4 pm) Name:**

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| **S-6:** I can use the wave function in position or momentum space to make predictions about measurements for a free particle in one dimension.<br>Unsatisfactory Progressing Acceptable Polished |

**(1)** A particle is in a quantum state with momentum-space wave function

$$\phi_0(p) = Ae^{-p^2/2\hbar^2 a^2}(1 + e^{-ipx_0/\hbar}).$$

Here $p$ has the range $-\infty < p < \infty$. For this question, I want you to explicitly write out *and simplify* any integrals necessary to answer the questions, but you don't try to evaluate them.

(a) Explain how to find the normalization constant $A$.

(b) Does the $e^{-ipx_0/\hbar}$ term affect the probability density for the momentum? If so, how? Explain.

(c) If this is the momentum-space wave function of the particle at time $t = 0$, what is the momentum-space wave function at a later time $t$?

(d) Explain how you would calculate the probability that the particle will be found in the range $x = [x_1, x_2]$ if its position were measured at time $t = 0$.

(a) $\langle \varphi_0 | \varphi_0 \rangle = 1 = A^2 \int_{-\infty}^{\infty} |\varphi_0(p)|^2 \, dp \rightarrow A = \sqrt{1/|\varphi_0(p)|^2}$

(B) yes, we are in momentum space and there is a p term in $e^{-ipx_0/\hbar}$

(C) Propigator $U(t)|\varphi(p,0)\rangle$

$= \int_{-\infty}^{\infty} e^{-iHt/\hbar} |x\rangle |\varphi_0(p)| \rightarrow \varphi(x,t)$

(D) Convert to position space $\rightarrow \psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int e^{ixp/\hbar} \cdot \varphi(p) \, dp$

$P[x_1, x_2] = \int_{x_1}^{x_2} |\psi(x,0)|^2 \, dx$