Physics 460—Quantum Rigid Rotor Spring 2020

The quantum rigid rotor is described by the Hamiltonian

$$H = \frac{L^2}{2I},$$

where I is the moment of inertia of the rotor and L^2 is the total angular momentum operator. Clearly, the energy eigenstates of the rotor are the eigenstates of L^2, the states $|l, m\rangle$, represented in position space by the spherical harmonics, $Y_l^m(\theta, \phi)$.

(1) At time $t = 0$ the wave function of a particular rigid rotor is $\psi(\theta, \phi, 0) = A \sin^2\theta(1 - \cos 2\phi)$.

(a) Write this state as a superposition of spherical harmonics.

(b) Find the normalization constant A.

(c) Find the state and its position-space wave function at a later time t.

(d) If you measure the total angular momentum at time t, what results could you obtain and with what probabilities?

(e) If you measure the z-component of the angular momentum at time t, what results could you obtain and with what probabilities?

(f) At time $t = 0$ you measure the orientation of the rotor. What is the probability that you will find it in the range $\theta = [0, \pi/2]$ and $\phi = [0, \pi]$?