**PHY 474/374** **Spring 2021**

**Week 1—Monday, March 29—Discussion Worksheet**

## Stellar Timescales

**1.** Timescales are very important in stellar astrophysics because they allow us to learn about the impact of different physical processes. Today we will discuss three relevant timescales.

(a) Starting from the gravitational acceleration at the surface of a star of mass $M$ and radius $R$ given by

$$g_s = \frac{GM}{R^2}$$

and by writing the time required for a particle to fall through a distance $l$ in the gravitational field of the star, show that the **dynamical timescale**, $t_{dyn}$, obtained by putting $l = R/2$ is

$$t_{dyn} = \left(\frac{R^3}{GM}\right)^{1/2}$$

$$y = \cancel{V_0}^{\,0} t + \tfrac{1}{2} g_s t^2 \rightarrow t = \sqrt{\frac{2y}{g_s}} \quad y = l \rightarrow t = \sqrt{\frac{2l}{g_s}}$$

let $l = R/2$, $g_s = GM/R^2$

$$t_{dym} = \left[\frac{\cancel{2}(R/\cancel{2})}{GM/R^2}\right]^{1/2} \rightarrow \left(\frac{R^3}{GM}\right)^{1/2}$$

(b) Find $t_{dyn}$ for our Sun. The mass of the Sun is $1.99 \times 10^{30}$ kg, its radius is $6.96 \times 10^8$ m, and the gravitational constant is $G = 6.67 \times 10^{-11}$ N m$^2$ kg$^{-2}$.

$$t_{dyn} = \left(\frac{(6.96 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(1.99 \times 10^{30} \text{ kg})}\right)^{(1/2)}$$

$$= 1593.77 \text{ s} \approx 1600 \text{ s}$$

Stellar masses vary roughly from $0.1 M_\odot$ to $100 M_\odot$, and radii from 0.01-1000 $R_\odot$, so $t_{dyn}$ ranges from seconds to years. We find no evidence for motion on these timescales in stars, indicating that forces on the star are roughly in balance. This is known as hydrostatic equilibrium, and we will learn about it in great detail later this quarter.

**2.** The Kelvin-Helmholtz timescale, $t_{\mathrm{KH}}$, is the time required by a star to radiate away its gravitational potential energy at its current luminosity.

(a) Given that the gravitational potential energy of a star of mass $M$ and radius $R$ is $\Omega = -GM^2/R$, show that

$$t_{\mathrm{KH}} = \frac{GM^2}{RL_s}$$

where $L_s$ is the (surface) luminosity of the star.

$$\rightarrow \Omega = -\frac{Gm^2}{R}$$

$$\rightarrow L_s = \frac{GM^2}{R\,t_{KH}} = \frac{\text{energy}}{\text{time}}$$

$$\boxed{t_{KH} = \frac{GM^2}{RL_s}}$$

(b) The luminosity of the Sun is $4 \times 10^{26}$ watts. Find $t_{\mathrm{KH}}$ for the Sun (in yr).

$$t_{KH} = \frac{GM^2}{RL_s} = \left(\frac{(6.67\times10^{-11}\,\mathrm{N\,m^2\,kg^{-2}})(1.99\times10^{30}\,\mathrm{kg})^2}{(6.96\times10^{8}\,\mathrm{m})(4\times10^{26}\,\mathrm{W})}\right)$$

$$\boxed{= 30\times10^{6}\ \text{years}}$$

**3.** For most of its lifetime, the energy produced by a star comes from the fusion of hydrogen into helium. The length of time that a star can generate energy by fusing H into He depends on how much fuel it has and how fast it is using that fuel.

(a) If the nuclear timescale, $t_{\rm nuc}$, is equal to $f\epsilon Mc^2/L_s$, where $f$ is the fraction of the star's mass available for fusion, and $\epsilon$ is the matter-energy conversion efficiency, show that

$$t_{\rm nuc} = 7 \times 10^{-4}\,\frac{Mc^2}{L_s}$$

given that fusion occurs deep in the star's interior and only about 10% of the star's mass is available for fusion, and in the fusion of H to He, $\epsilon$ is only about 0.7%.

$f = 0.1 \qquad \epsilon = 0.007$

$$t_{nuc} = (0.1)(0.007)\frac{MC^2}{L_s}$$

$$t_{nuc} = (7 \times 10^{-4})\frac{Mc^2}{L_s}$$

(b) Calculate $t_{\rm nuc}$ for the Sun. Recall from the previous page that the luminosity of the Sun is $4 \times 10^{26}$ watts.

$$t_{nuc} = 7 \times 10^{-4}\,\frac{(1.99 \times 10^{30}\,kg)(3 \times 10^{8}\,m/s)}{4 \times 10^{26}\,J/s}$$

$$= \frac{3.134 \times 10^{17}\,s}{(60)(60)(24)\,s/yr} = 9.9 \times 10^{9}\,years$$

## Distances

You should be familiar with the usual distance scales used in astronomy: AU, pc, Ly. Here is an opportunity to re-familiarize yourself with them, in case you've forgotten (or never learned about them).

**4.** The nearest star to the Sun, Proxima Centauri, has a parallax angle of 0.768″ (where ″ stands for arcseconds).

(a) Calculate the distance to Proxima Centauri in pc.

$$d \text{ in pc} = \frac{1}{p \text{ in arcsec}}$$

$$D = \frac{1}{0.768''} = 1.30 \text{ pc}$$

(b) Calculate the distance to Proxima Centauri in Ly. **Note:** 1 pc = 3.26 Ly.

$$\frac{1.30 \text{ pc}}{1 \text{ pc}} \cdot 3.26 \text{ ly} = 4.24 \text{ ly}$$

(c) Calculate the distance to Proxima Centauri in AU. **Note:** 1 AU = 150 million km

$$1 \text{ pc} = 206265 \text{ AU}$$

$$\frac{1.30 \text{ pc}}{1 \text{ pc}} \cdot 206265 \text{ AU} = 268{,}144.5 \text{ AU}$$

(d) How many stars do you think have a parallax angle of 1″, or larger? Discuss.

Zero, Nearest star has a parallax angle 0.768″
Stars further than this will have smaller parallax angles

## Stellar Brightness

The measurement of stellar brightnesses has a cherished history, so much so that we still adhere to the historically used system of magnitudes. You should be familiar with apparent magnitude, absolute magnitude, measurement of magnitudes using filters (most commonly $U, B, V$ at optical wavelengths), the color index, and Planck's black body radiation law (since stars are assumed to emit black body radiation).

**5.** Consider a star of apparent magnitude $m_1$ and another with $m_2$. The difference $(m_1 - m_2)$ is related to the brightness ratios of the two stars via

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{B_1}{B_2}\right)$$

where the minus sign is present because brighter stars have lower $m$ values.

(a) In the constellation of Cassiopeia, there is a binary star system $\eta$ Cas. The brighter component of the binary ($\eta$ Cas A) has an apparent magnitude of 3.45, whereas its companion ($\eta$ Cas B) has an apparent magnitude of 7.51 (both of these are in the visual $V$ band, although this information is not required in this problem). By what factor is $\eta$ Cas A brighter than $\eta$ Cas B?

$$\frac{B_1}{B_2} = 10^{-\left(\frac{m_1 - m_2}{2.5}\right)}$$

$$\rightarrow \quad \frac{B_1}{B_2} = 10^{-\left[\frac{(3.45 - 7.51)}{2.5}\right]} = 42.1$$

(b) The absolute magnitude ($M$) is defined to be the apparent magnitude an object would have if it were located at distance 10 pc. Thus, $m$ and $M$ are related via

$$m - M = 5 \log_{10} d - 5$$

where $d$ is the distance to the object in pc. The $\eta$ Cas binary star system is about 6 pc away from us. What is the absolute magnitude of $\eta$ Cas A?

$$M = m - 5\log_{10} d + 5$$

$$= 3.45 - 5\log_{10} 6 + 5$$

$$= 4.56$$