PHY 411 Winter 2021

Homework 3 solutions

1. Starting from the general boundary conditions on electric and magnetic fields at an interface between two media, we wrote four boundary conditions tailored to the incident, refracted, and reflected fields that we are discussing in this chapter; see equation (7.37) in Jackson, or the posted class summary. Of these, the fourth boundary condition is

$$\left[\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0+\vec{k}''\times\vec{E}_0''\right)-\frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\right]\times\hat{n}=0 \tag{7.37.d}$$

For the case of the $\vec{E}$-fields ***perpendicular*** to the plane of incidence, show that equation (7.37.d) above reduces (in magnitude) to

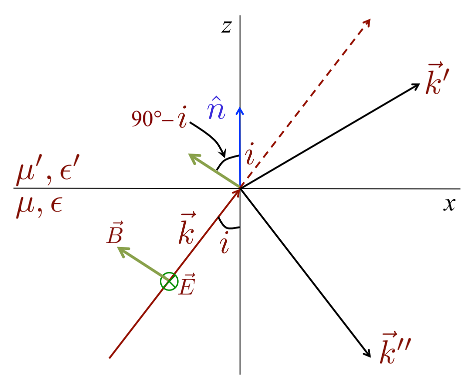
$$\sqrt{\frac{\epsilon}{\mu}}\,E_0\cos\,i-\sqrt{\frac{\epsilon}{\mu}}\,E_0''\cos\,i-\sqrt{\frac{\epsilon'}{\mu'}}\,E_0'\cos\,r=0 \tag{7.38.b}$$

Solution: Begin with equation (7.37.d) written above. There are three terms on the left hand side, which can be written separately as

$$\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0\right)\times\hat{n}+\frac{1}{\mu}\left(\vec{k}''\times\vec{E}_0''\right)\times\hat{n}-\frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\times\hat{n}=0$$

Consider the first of the three terms (which I've written in purple font above).

Consider first the cross product $\vec{k}\times\vec{E}_0$. I've drawn a figure to make it easy to follow along (where I've suppressed information that is not necessary for this part, and added labels that will be of use). Curling the fingers of your right hand from the direction of $\vec{k}$ to $\vec{E}_0$ (which is into the page) puts your thumb along the direction of $\vec{B}$, so the direction of $\vec{k}\times\vec{E}_0$ is along the direction of $\vec{B}$.



So, we have: $\vec{k}\times\vec{E}_0 = kE_0$ in magnitude, and direction is along that of $\vec{B}$ marked in the figure.

Next we need $(\vec{k}\times\vec{E}_0)\times\hat{n}$. I've redrawn the green-colored $\vec{B}$ vector at the origin where x and z axes meet for better perspective on the angles. The direction of the $\vec{B}$ vector makes an angle $(90°-i)$ with the $\hat{n}$ direction (see figure above), so the cross product $(\vec{k}\times\vec{E}_0)\times\hat{n}$ contributes a $\sin(90°-i)=\cos i$ term. Curling the fingers of the right hand from the direction of $\vec{B}$ toward $\hat{n}$ puts the thumb pointing into the page, so the direction of $(\vec{k}\times\vec{E}_0)\times\hat{n}$ is into the page (i.e., along $\hat{y}$).

Putting it all together, we get that the first of the three terms in the fourth equation (7.37.d) gives

$$\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0\right)\times\hat{n}=\left(\frac{1}{\mu}\,kE_0\cos\,i\right)\hat{y}$$

Using $k = \omega\sqrt{\mu\epsilon}$ from equation (7.4), the equation at the bottom of the previous page becomes

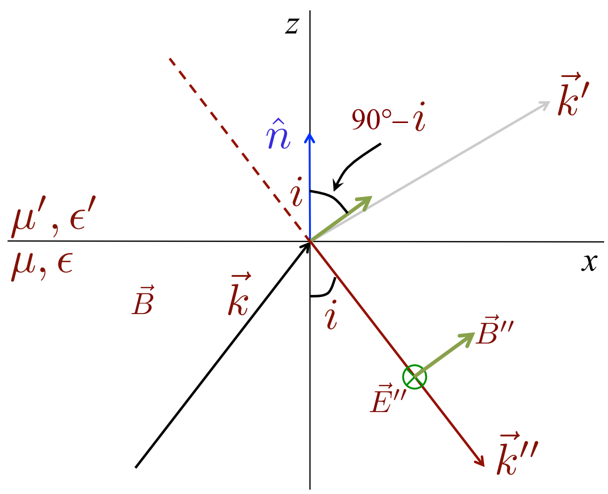
$$\frac{1}{\mu}\left(\vec{k} \times \vec{E}_0\right) \times \hat{n} = \left(\frac{1}{\mu}\,\omega\sqrt{\mu\epsilon}\,E_0 \cos\,i\right)\hat{y} = \left(\omega\sqrt{\frac{\epsilon}{\mu}}\,E_0 \cos\,i\right)\hat{y} \tag{7.37.d.1}$$

Next, consider the second of the three terms in equation (7.37.d), which is

$$\frac{1}{\mu}\left(\vec{k}'' \times \vec{E}_0''\right) \times \hat{n}$$

Again, I've drawn a figure with the relevant details highlighted, and other information suppressed. You can see from this figure that curling the fingers of your right hand from the direction of $\vec{k}''$ to $\vec{E}_0''$ (which, again, is into the page) puts your thumb along the direction of $\vec{B}''$, so the direction of $\vec{k}'' \times \vec{E}_0''$ is along the direction of $\vec{B}''$.

So, we have: $\vec{k}'' \times \vec{E}_0'' = k'' E_0''$ in magnitude, and direction is along that of $\vec{B}''$ marked in the figure.



Next we need $(\vec{k}'' \times \vec{E}_0'') \times \hat{n}$. I've redrawn the green-colored $\vec{B}''$ vector at the origin where x and z axes meet for better perspective on the angles. The direction of the $\vec{B}''$ vector makes an angle $(90° - i)$ with the $\hat{n}$ direction (as marked in the figure above), so the cross product $(\vec{k}'' \times \vec{E}_0'') \times \hat{n}$ contributes a $\sin(90° - i) = \cos i$ term.

This time, though, curling the fingers of the right hand from the direction of $\vec{B}''$ toward $\hat{n}$ puts the thumb pointing out of the page, so the direction of $(\vec{k}'' \times \vec{E}_0'') \times \hat{n}$ is out of the page (i.e., along $-\hat{y}$).

With $k'' = \omega\sqrt{\mu\epsilon}$, we get finally that the second of the three terms in the fourth equation (7.37.d) gives

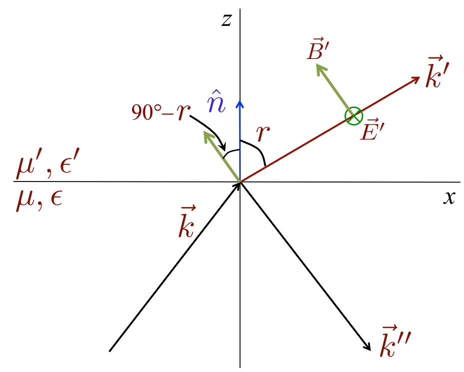
$$\frac{1}{\mu}\left(\vec{k}'' \times \vec{E}_0''\right) \times \hat{n} = \left(\omega\sqrt{\frac{\epsilon}{\mu}}\,E_0'' \cos\,i\right)\Big[-\hat{y}\Big] \tag{7.37.d.2}$$

Note again that this term gives a vector pointing out of the plane of the page (i.e., along $-\hat{y}$).

Finally, consider the third of the three terms in equation (7.37.d), which is

$$\frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\times\hat{n}$$

Yet again, I've drawn a figure with the relevant details highlighted, and other information suppressed).



You can see from this figure that curling the fingers of your right hand from the direction of $\vec{k}'$ to $\vec{E}_0'$ (which, again, is into the page) puts your thumb along the direction of $\vec{B}'$, so the direction of $\vec{k}'\times\vec{E}_0'$ is along the direction of $\vec{B}'$.

So, we have: $\vec{k}'\times\vec{E}_0' = k'E_0'$ in magnitude, and direction is along that of $\vec{B}'$ marked in the figure.

Next we need $(\vec{k}'\times\vec{E}_0')\times\hat{n}$. I've redrawn the green-colored $\vec{B}'$ vector at the origin where x and z axes meet for better perspective on the angles. The direction of the $\vec{B}'$ vector makes an angle $(90°-r)$ with the $\hat{n}$ direction (as marked in the figure above), so the cross product $(\vec{k}'\times\vec{E}_0')\times\hat{n}$ contributes a $\sin(90°-r)=\cos r$ term. Just as for the first term, curling the fingers of the right hand from the direction of $\vec{B}'$ toward $\hat{n}$ puts the thumb pointing *into the page*, so the direction of $(\vec{k}'\times\vec{E}_0')\times\hat{n}$ is into the page (i.e., along $\hat{y}$).

With $k'=\omega\sqrt{\mu'\epsilon'}$, we get finally that the third of the three terms in the fourth equation (7.37.d) gives

$$\frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\times\hat{n} = \left(\omega\sqrt{\frac{\epsilon'}{\mu'}}\,E_0'\cos r\right)\,\hat{y} \tag{7.37.d.3}$$

So equation (7.37.d):

$$\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0\right)\times\hat{n}+\frac{1}{\mu}\left(\vec{k}''\times\vec{E}_0''\right)\times\hat{n}-\frac{1}{\mu'}\left(\vec{k}'\times\vec{E}_0'\right)\times\hat{n}=0$$

can now be replaced by the results from equation (7.37.d.1), equation (7.37.d.2), and equation (7.37.d.3) , so that we obtain

$$\left(\omega\sqrt{\frac{\epsilon}{\mu}}\,E_0\cos\,i\right)\hat{y}+\left(\omega\sqrt{\frac{\epsilon}{\mu}}\,E_0''\cos\,i\right)\Big[-\hat{y}\Big]-\left(\omega\sqrt{\frac{\epsilon'}{\mu'}}\,E_0'\cos\,r\right)\,\hat{y}=0$$

After canceling common factors and writing only the magnitudes (i.e., without the unit vectors), we get

$$\sqrt{\frac{\epsilon}{\mu}}\,E_0\cos\,i-\sqrt{\frac{\epsilon}{\mu}}\,E_0''\cos\,i-\sqrt{\frac{\epsilon'}{\mu'}}\,E_0'\cos\,r=0 \tag{7.38.b}$$

which is the desired solution.

2. For $\vec{E}$-fields ***perpendicular*** to the plane of incidence, you obtained on the worksheet that

$$E_0 + E_0'' - E_0' = 0 \tag{7.38.a}$$

and equation (7.38.b) written above.

Show that equation (7.38.a) and equation (7.38.b) together yield, for $\vec{E}$-fields ***perpendicular*** to the plane of incidence, that

$$\begin{aligned} \frac{E_0'}{E_0} &= \frac{2n \cos i}{n \cos i + \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2 \sin^2 i}} \\ \frac{E_0''}{E_0} &= \frac{n \cos i - \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2 \sin^2 i}}{n \cos i + \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2 \sin^2 i}} \end{aligned} \tag{7.39}$$

Solution: Let us write equation (7.38.a) and equation (7.38.b) together below:

$$\begin{aligned} E_0 + E_0'' - E_0' &= 0 \\ \sqrt{\frac{\epsilon}{\mu}}\left(E_0 - E_0''\right)\cos i - \sqrt{\frac{\epsilon'}{\mu'}}\,E_0' \cos r &= 0 \end{aligned} \tag{7.38}$$

From equation (7.38), we can find the relative amplitudes of the refracted and reflected waves written in equation (7.39).

I'm going to derive the second relation E_0''/E_0 first. To do so, eliminate E_0' from the second equation in (7.38) above using the first. That is, from the first equation (7.38.a), we get

$$E_0 + E_0'' = E_0'$$

and substituting this into the second equation (7.38.b), we get

$$\sqrt{\frac{\epsilon}{\mu}}\left(E_0 - E_0''\right)\cos i - \sqrt{\frac{\epsilon'}{\mu'}}\left(E_0 + E_0''\right)\cos r = 0$$

Moving terms with E_0 and E_0'' to opposite sides of the equals sign, we get

$$\left[\sqrt{\frac{\epsilon}{\mu}}\cos i - \sqrt{\frac{\epsilon'}{\mu'}}\cos r\right] E_0 = \left[\sqrt{\frac{\epsilon}{\mu}}\cos i + \sqrt{\frac{\epsilon'}{\mu'}}\cos r\right] E_0''$$

so that

$$\frac{E_0''}{E_0} = \frac{\sqrt{\dfrac{\epsilon}{\mu}}\cos i - \sqrt{\dfrac{\epsilon'}{\mu'}}\cos r}{\sqrt{\dfrac{\epsilon}{\mu}}\cos i + \sqrt{\dfrac{\epsilon'}{\mu'}}\cos r}$$

We're done as far as the expression for ratio of amplitudes is concerned, but we still need to get it in the form of equation (7.39) in Jackson. I'll do this starting on the next page.

So far, we have obtained the ratio of amplitudes as

$$\frac{E_0''}{E_0} = \frac{\sqrt{\dfrac{\epsilon}{\mu}}\,\cos i - \sqrt{\dfrac{\epsilon'}{\mu'}}\,\cos r}{\sqrt{\dfrac{\epsilon}{\mu}}\,\cos i + \sqrt{\dfrac{\epsilon'}{\mu'}}\,\cos r}$$

To get this expression into equation (7.39) in Jackson, I'm going to multiply numerator and denominator by μ, so that we get

$$\frac{E_0''}{E_0} = \frac{\mu\sqrt{\dfrac{\epsilon}{\mu}}\,\cos i - \mu\sqrt{\dfrac{\epsilon'}{\mu'}}\,\cos r}{\mu\sqrt{\dfrac{\epsilon}{\mu}}\,\cos i + \mu\sqrt{\dfrac{\epsilon'}{\mu'}}\,\cos r} = \frac{\sqrt{\dfrac{\mu^2\,\epsilon}{\mu}}\,\cos i - \mu\sqrt{\dfrac{\epsilon'}{\mu'}}\,\cos r}{\sqrt{\dfrac{\mu^2\,\epsilon}{\mu}}\,\cos i + \mu\sqrt{\dfrac{\epsilon'}{\mu'}}\,\cos r}$$

and then I'm going to multiply the numerator and denominator of the fraction in the second term by μ' to get

$$\frac{E_0''}{E_0} = \frac{\sqrt{\dfrac{\mu^2\,\epsilon}{\mu}}\,\cos i - \mu\sqrt{\dfrac{\mu'\epsilon'}{\mu'\mu'}}\,\cos r}{\sqrt{\dfrac{\mu^2\,\epsilon}{\mu}}\,\cos i + \mu\sqrt{\dfrac{\mu'\epsilon'}{\mu'\mu'}}\,\cos r} = \frac{\sqrt{\mu\,\epsilon}\,\cos i - \dfrac{\mu}{\mu'}\sqrt{\mu'\epsilon'}\,\cos r}{\sqrt{\mu\,\epsilon}\,\cos i + \dfrac{\mu}{\mu'}\sqrt{\mu'\epsilon'}\,\cos r}$$

Multiply numerator and denominator by $1/\sqrt{\mu_0\epsilon_0}$ so this becomes

$$\frac{E_0''}{E_0} = \frac{\sqrt{\dfrac{\mu\,\epsilon}{\mu_0\,\epsilon_0}}\,\cos i - \dfrac{\mu}{\mu'}\sqrt{\dfrac{\mu'\epsilon'}{\mu_0\,\epsilon_0}}\,\cos r}{\sqrt{\dfrac{\mu\,\epsilon}{\mu_0\,\epsilon_0}}\,\cos i + \dfrac{\mu}{\mu'}\sqrt{\dfrac{\mu'\epsilon'}{\mu_0\,\epsilon_0}}\,\cos r} = \frac{n\,\cos i - \dfrac{\mu}{\mu'}\,n'\,\cos r}{n\,\cos i + \dfrac{\mu}{\mu'}\,n'\,\cos r}$$

since $n = \sqrt{\mu\,\epsilon/\mu_0\,\epsilon_0}$, etc.

The final step is to change i to r in the second term. To do so, we note that

$$n'\,\cos r = \sqrt{n'^2\cos^2 r} = \sqrt{n'^2\left(1 - \sin^2 r\right)} = \sqrt{n'^2 - n'^2\,\sin^2 r}$$

and apply Snell's law $n'\,\sin r = n\,\sin i$, so that the above expression becomes

$$n'\,\cos r = \sqrt{n'^2 - n^2\,\sin^2 i}$$

Substituting this into the ratio of amplitudes obtained above, we get

$$\frac{E_0''}{E_0} = \frac{n\,\cos i - \dfrac{\mu}{\mu'}\,\sqrt{n'^{\,2} - n^2\,\sin^2 i}}{n\,\cos i + \dfrac{\mu}{\mu'}\,\sqrt{n'^{\,2} - n^2\,\sin^2 i}}$$

which is the second relation in equation (7.39).

Finding the first relation in equation (7.39) is now easy. Start with the first relation in equation (7.38):

$$E_0 + E_0'' - E_0' = 0$$

and divide it by E_0 to get

$$\frac{E_0}{E_0} + \frac{E_0''}{E_0} - \frac{E_0'}{E_0} = 0$$

from which we obtain

$$\frac{E_0'}{E_0} = 1 + \frac{E_0''}{E_0}$$

Substituting the second relation in equation (7.39) which we've already proved to be true above, this becomes

$$\frac{E_0'}{E_0} = 1 + \frac{n\,\cos\,i - \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i}}{n\,\cos\,i + \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i}}$$

Note that you can proceed in this manner because you've already proven the relation for E_0''/E_0. In other words, if you are trying to prove two equations A and B, you can only use equation A to prove B if, and only if, you've *already proved equation* A *to be true*.

So, we get

$$\frac{E_0'}{E_0} = \frac{n\,\cos\,i + \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i} + n\,\cos\,i - \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i}}{n\,\cos\,i + \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i}}$$

The second and fourth terms in the numerator are identical except for opposite signs, so they cancel, and we obtain

$$\frac{E_0'}{E_0} = \frac{2n\,\cos\,i}{n\,\cos\,i + \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i}}$$

which is the first of the two relations in equation (7.39).

Therefore, we have now proved both equations below.

$$\begin{aligned} \frac{E_0'}{E_0} &= \frac{2n\,\cos\,i}{n\,\cos\,i + \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i}} \\ \frac{E_0''}{E_0} &= \frac{n\,\cos\,i - \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i}}{n\,\cos\,i + \dfrac{\mu}{\mu'}\sqrt{n'^{\,2} - n^2\,\sin^2\,i}} \end{aligned} \tag{7.39}$$

3. For $\vec{E}$-fields ***parallel*** to the plane of incidence, the ratios of transmitted amplitude to incident amplitude (E_0'/E_0) and reflected amplitude to incident amplitude (E_0''/E_0) are given in equation (7.41) in Jackson:

$$\begin{aligned} \frac{E_0'}{E_0} &= \frac{2nn' \cos i}{\dfrac{\mu}{\mu'} n'^{\,2} \cos i + n\sqrt{n'^{\,2} - n^2 \sin^2 i}} \\ \frac{E_0''}{E_0} &= \frac{\dfrac{\mu}{\mu'} n'^{\,2} \cos i - n\sqrt{n'^{\,2} - n^2 \sin^2 i}}{\dfrac{\mu}{\mu'} n'^{\,2} \cos i + n\sqrt{n'^{\,2} - n^2 \sin^2 i}} \end{aligned} \tag{7.41}$$

(a) Use equation (7.41) above to find the transmission coefficient $T_{\|}$ for nonpermeable media where $\mu = \mu'$. Express all angles in your answer in terms of the angle of incidence i.

Solution: The transmission coefficient is given by

$$T = \frac{\vec{S}' \cdot \hat{n}}{\vec{S} \cdot \hat{n}} = \frac{\frac{1}{2}\sqrt{\dfrac{\epsilon'}{\mu'}}\left|E_0'\right|^2 \cos r}{\frac{1}{2}\sqrt{\dfrac{\epsilon}{\mu}}\left|E_0\right|^2 \cos i} \tag{H3.1}$$

To save time, I won't work out the steps to derive the expressions for $\vec{S} \cdot \hat{n}$ and $\vec{S}' \cdot \hat{n}$ here, because I derived $\vec{S}' \cdot \hat{n}$ in the Class Summary for Week 3—Day 2.

Since $\mu = \mu'$ in this problem, we can write equation (H3.1) as

$$T = \frac{\sqrt{\dfrac{\mu'\epsilon'}{\mu'\mu'}}\left|E_0'\right|^2 \cos r}{\sqrt{\dfrac{\mu\epsilon}{\mu\mu}}\left|E_0\right|^2 \cos i} = \frac{\sqrt{\mu'\epsilon'}}{\sqrt{\mu\epsilon}}\left|\frac{E_0'}{E_0}\right|^2 \frac{\cos r}{\cos i} = \frac{\sqrt{\mu'\epsilon'/\mu_0\epsilon_0}}{\sqrt{\mu\epsilon/\mu_0\epsilon_0}}\left|\frac{E_0'}{E_0}\right|^2 \frac{\cos r}{\cos i} \tag{H3.2}$$

Since $n = \sqrt{\mu\epsilon/\mu_0\epsilon_0}$ and $n' = \sqrt{\mu'\epsilon'/\mu_0\epsilon_0}$, we get from equation (H3.2) above that

$$T = \frac{n' \cos r}{n \cos i}\left|\frac{E_0'}{E_0}\right|^2 = \frac{\sqrt{n'^{\,2}(1 - \sin^2 r)}}{n \cos i}\left|\frac{E_0'}{E_0}\right|^2 = \frac{\sqrt{n'^{\,2} - n^2 \sin^2 i}}{n \cos i}\left|\frac{E_0'}{E_0}\right|^2$$

where we have used Snell's law: $n \sin i = n' \sin r$.

Upon substituting for $|E_0'/E_0|$ from equation (7.41) written in the question above (with $\mu = \mu'$), we get

$$T = \frac{\sqrt{n'^2 - n^2 \sin^2 i}}{n \cos i}\left[\frac{2nn' \cos i}{n'^2 \cos i + n\sqrt{n'^2 - n^2 \sin^2 i}}\right]^2 \tag{H3.3}$$

which is the expression for the transmission coefficient that we were asked to find.

(b) Use equation (7.41) above to find the reflection coefficient $R_{\parallel}$ for nonpermeable media where $\mu = \mu'$. Again, express all angles in your answer in terms of the angle of incidence i.

Solution: Again, to save time, I won't work out the steps to derive the expressions for $\vec{S} \cdot \hat{n}$ and $\vec{S}'' \cdot \hat{n}$ here; the expressions are written in the Class Summary for Week 3—Day 2, and the steps are similar to the derivation for $\vec{S}' \cdot \hat{n}$ derived in that Class Summary.

The reflection coefficient is given by

$$R = \frac{\vec{S}'' \cdot \hat{n}}{\vec{S} \cdot \hat{n}} = \frac{\frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\left|E_0''\right|^2 \cos r'}{\frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\left|E_0\right|^2 \cos i} = \left|\frac{E_0''}{E_0}\right|^2 \tag{H3.4}$$

because $i = r'$.

Therefore, the desired reflection coefficient is just $|E_0''/E_0|^2$, with E_0''/E_0 taken from equation (7.41), after setting $\mu = \mu'$:

$$R = \left[\frac{n'^2 \cos i - n\sqrt{n'^2 - n^2 \sin^2 i}}{n'^2 \cos i + n\sqrt{n'^2 - n^2 \sin^2 i}}\right]^2 \tag{H3.5}$$

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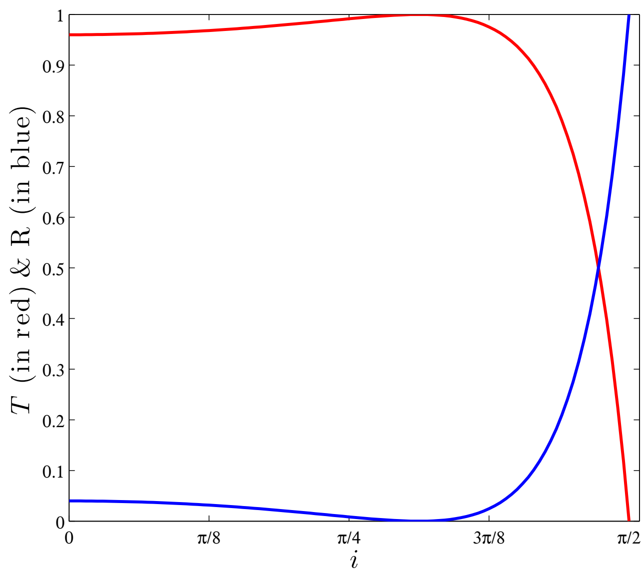
4. For $\vec{E}$-fields ***parallel*** to the plane of incidence, you obtained expressions for the transmission coefficient $T_{\parallel}$ and the reflection coefficient $R_{\parallel}$ in Question 3 above (in nonpermeable media where $\mu = \mu'$).

(a) For $n = 1$ and $n' = 1.5$, plot $T_{\parallel}$ and $R_{\parallel}$ *on the same graph* as a function of the angle of incidence i, where i runs from 0 to $\pi/2$. *Graphs sketched by hand will be given a grade of zero.*

Solution: I wrote a Matlab program to plot equation (H3.3) and equation (H3.5), with the appropriate values of n and n' written into the program. The graphs for the transmission coefficient (in red) and reflection coefficient (in blue) are shown below as a function of the angle of incidence i, where i runs from 0 to $\pi/2$. Notice that $T + R = 1$ at every value of i, as expected.



(b) Describe the graph, pointing out notable features in words. In particular, verify by direct calculation that Brewster's angle is showing up at the appropriate place in your graph.

Solution: In the graph drawn above, we see that at zero angle of incidence ($i = 0$ is also called normal incidence since the normal is a ray perpendicular to the reflecting surface in geometric optics), most of the light (about 95%) is transmitted, whereas at grazing incidence ($i = \pi/2$) when the light is traveling almost "parallel" to the surface, all the light is reflected.

Starting at normal incidence for $\vec{E}$ parallel to the plane of incidence, therefore, we find that the transmission starts off high (about 95% at $i = 0$) gradually increases up to about $i \approx 2.5\pi/8$, where $T = 1$, after which it sharply decreases, reaching zero transmitted intensity at grazing incidence of $i = \pi/2$. The reflected intensity is in agreement with $T + R = 1$ everywhere.

Of interest also is that there is no reflection at $i \approx 2.5\pi/8$ for $\vec{E}$ parallel to the plane of incidence. This is a known phenomenon that we studied in class — the angle in question is known as Brewster's angle. From equation (7.43) in Jackson, we get

$$i_B = \arctan\left(\frac{n'}{n}\right) = \arctan\left(\frac{1.5}{1}\right) = 56.3^\circ \approx \frac{2.5\pi}{8}$$

Therefore, our graph is consistent with the predicted value for Brewster's angle.