1 Vector Formulas

1.1 Triple Products

$$A\cdot(B\times C)=B\cdot(C\times A)=C\cdot(A\times B)$$
$$A\times(B\times C)=B(A\cdot C)-C(A\cdot B)$$

1.2 Product Rules

$$\nabla(fg)=f(\nabla g)+g(\nabla f)$$
$$\nabla((A\cdot B)=A\times(\nabla\times B)+B\times(\nabla\times A)+(A\cdot)B+(B\cdot\nabla)A$$
$$\nabla\cdot(fA)=f(\nabla\cdot A)+A\cdot(\nabla f)$$
$$\nabla\cdot(A\times B)=B\cdot(\nabla\times A)-A\cdot(\nabla\times B)$$
$$\nabla\times(fA)=f(\nabla\times A)-A\times(\nabla f)$$
$$\nabla\times(A\times B)=(B\cdot\nabla)-(A\cdot\nabla)B+A(\nabla\cdot B)-B(\nabla\cdot A)$$

1.3 Second Derivatives

$$\nabla\cdot(\nabla\times A)=0$$
$$\nabla\times(\nabla f)=0$$
$$\nabla\times(\nabla\times A)=\nabla(\nabla\cdot A)-\nabla^2 A$$

2 Maxwell Equations

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \nabla\cdot\vec{B}=0$$
$$\vec{\nabla}\cdot\vec{D}=0 \qquad \nabla\cdot\vec{D}=\rho$$
$$\vec{\nabla}\times\vec{E}-i\omega\vec{B}=0 \qquad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{B}+i\omega\mu\varepsilon\vec{E}=0 \qquad \vec{\nabla}\times H=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

2.1 Constructive Relations

$$\vec{D}=\varepsilon\vec{E} \qquad \vec{B}=\mu\vec{H}$$

3 Electromagnetic Waves and Propagation

3.1 Helmholtz wave equations

$$(\nabla^2+\mu\varepsilon\omega^2)\vec{E}=0 \qquad (\nabla^2+\mu\varepsilon\omega^2)\vec{B}=0$$

3.2 Constructive Relations

1. **Wave number:** $k=\omega\sqrt{\mu\varepsilon}$
2. **Phase velocity:** $v=\frac{\omega}{k}=\frac{1}{\sqrt{\mu\varepsilon}}=\frac{c}{n}$
3. **Index of refraction of the medium:** $n=\frac{\mu\varepsilon}{\mu_0\varepsilon_0}$

3.3 Plane Electromagnetic Waves

$$\vec{E}=\vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \vec{B}=\sqrt{\mu\varepsilon}\frac{\vec{k}\times\vec{E}}{k}$$

3.4 Polarization of Waves

$$\vec{E}_1=\hat{\varepsilon}_1 E_1 e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \vec{E}_2=\hat{\varepsilon}_2 E_2 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$
$$\vec{E}(\vec{x},t)=(\hat{\varepsilon}_1 E_1+\hat{\varepsilon}_2 E_2)e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

3.5 Stokes Parameters

Linear polarization basis:

$$\vec{E}(\vec{x},t)=(\hat{\varepsilon}_1 E_1+\hat{\varepsilon}_2 E_2)e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$
$$E_1=a_1 e^{i\delta_1} \qquad E_2=a_2 e^{i\delta_2}$$

Circular polarization basis:

$$\vec{E}(\vec{x},t)=(\hat{\varepsilon_+}E_+ +\hat{\varepsilon_-}E_-)e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$
$$E_+=a_+ e^{i\delta_+} \qquad E_-=a_- e^{i\delta_-}$$

3.6 Reflection and Refraction: Kinematic Properties

Incident wave:

$$\vec{E}=\vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \vec{B}=\sqrt{\mu\varepsilon}\frac{\vec{k}\times\vec{E}}{k}$$

Refracted wave:

$$\vec{E}'=\vec{E}'_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \vec{B}'=\sqrt{\mu'\varepsilon'}\frac{\vec{k}'\times\vec{E}'}{k}$$

Reflected wave:

$$\vec{E}''=\vec{E}''_0 e^{i(\vec{k}''\cdot\vec{x}-\omega t)} \qquad \vec{B}''=\sqrt{\mu\varepsilon}\frac{\vec{k}''\times\vec{E}''}{k}$$

3.7 Reflection and Refraction: Boundary condition

Normal components:

$$[\varepsilon(\vec{E}_0+\vec{E}''_0)-\varepsilon'\vec{E}'_0]\cdot\hat{n}=0$$
$$[\vec{k}\times E_0+\vec{k}''\times\vec{\vec{E}}''_0-\vec{k}'\times\vec{E}'_0]\cdot\hat{n}=0$$

Tangential components:

$$[\vec{E}_0+\vec{E}''_0-\vec{E}'_0]\times\hat{n}=0$$
$$\left[\frac{1}{\mu}(\vec{k}\times\vec{E}_0+\vec{k}''\times\vec{E}''_0)-\frac{1}{\mu'}(\vec{k}'\times\vec{E}'_0\right]\times\hat{n}=0$$

3.8 Brewster's Angle

3.9 Snell's Law

3.10 Total Internal Reflection

3.11 Reflection and Transmission Coefficients

$$\vec{s}\cdot\hat{n}=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|E_0|^2 cos(i) \qquad T=\frac{\vec{s}'\cdot\hat{n}}{\vec{s}\cdot\hat{n}}$$
$$\vec{s}'\cdot\hat{n}=\frac{1}{2}\sqrt{\frac{\varepsilon'}{\mu'}}|E'_0|^2\cos(r) \qquad R=\frac{\vec{s}''\cdot\hat{n}}{\vec{s}\cdot\hat{n}}$$
$$\vec{s}''\cdot\hat{n}=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|E''_0|^2\cos(r)' \qquad T+R=1$$

3.12 Dispersion Model for time-varying field

$$m[\ddot{\vec{x}}+\gamma\dot{\vec{x}}+\omega_0^2\vec{x}]=-e\vec{E}(\vec{x},t)$$

3.13 Dispersion

$$\frac{\varepsilon(\omega)}{\varepsilon_0}=1+\frac{Ne^2}{\varepsilon_0 m}\sum_j\frac{f_j}{\omega_j^2-\omega^2-i\omega\gamma_j}$$

3.14 Attenuation of a plane wave