

(1) A free particle has the following position-space wave function at time  $t = 0$ :

$$\psi(x, 0) = \begin{cases} Ae^{ip_0 x/\hbar} (b^2 - x^2), & |x| < b, \\ 0, & |x| \geq b. \end{cases}$$

- (a) Sketch the wave function and find the normalization constant,  $A$ .
- (b) Find the expectation value of  $x$  at time  $t = 0$ . (Maybe you don't need to integrate ...)
- (c) Find the momentum-space wave function at time  $t = 0$ ,  $\phi(p, 0)$ .
- (d) Find the expectation value of  $p$  at time  $t = 0$ . (Do you have to integrate ...?)
- (e) Use the propagator and your answer to part (c) to find the momentum-space wave function at a later time  $t$ .
- (f) Find the position-space wave function at a later time  $t$ .
- (g) What is the probability that the particle will be found with  $x > b$  at time  $t$ ? What is the probability that the particle will be found with  $x < -b$  at time  $t$ ?

Note: I encourage you to use WolframAlpha or equivalent to evaluate integrals. You can also leave integrals "as is" if you like.