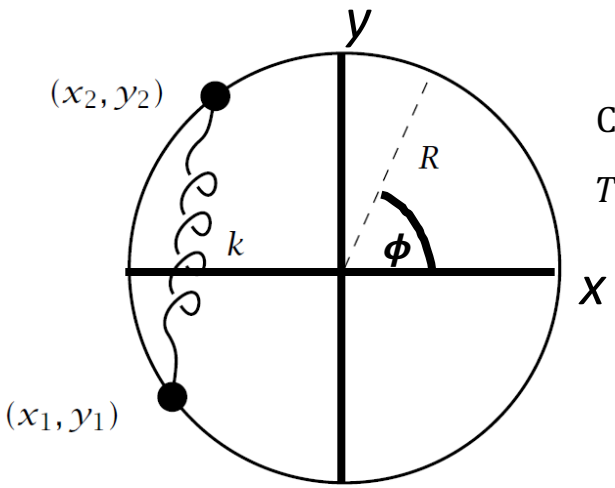
Activity 2: Beads on a hoop



Cartesian coordinates

$$T + V = \frac{1}{2}m\left(\dot{x_1}^2 + \dot{y_1}^2 + \dot{x_2}^2 + \dot{y_2}^2\right) + \frac{1}{2}k[(x_2 - x_1)^2 + (y_2 - y_1)^2] + mg(y_1 + y_2)$$

only for vertical hoop

$$x_1 = R\cos\phi_1$$
$$y_1 = R\sin\phi_1$$
$$x_2 = R\cos\phi_2$$
$$y_2 = R\sin\phi_2$$
$$\dot{x}_1^2 = R^2\dot{\phi}_1^2\sin^2\phi_1$$
$$\dot{y}_1^2 = R^2\dot{\phi}_1^2\cos^2\phi_1$$
$$\dot{x}_2^2 = R^2\dot{\phi}_2^2\sin^2\phi_2$$
$$\dot{y}_2^2 = R^2\dot{\phi}_2^2\cos^2\phi_2$$

Plug in transformations, apply Pythagorean identity

$$\sin^2\alpha + \cos^2\alpha = 1$$

And half-angle identity

$$\sin^2\alpha = \frac{1}{2}(1 - \cos 2\alpha)$$

$\Rightarrow$ Polar coordinates

$$T + V = \frac{1}{2}mR^2(\dot{\phi}_1^2 + \dot{\phi}_2^2) + 2kR^2\sin^2\left(\frac{\phi_2 - \phi_1}{2}\right) + mgR(\sin\phi_1 + \sin\phi_2)$$