PHY 420 Spring 2021

Homework 1—due by 9:00 PM, Tuesday, Apr 6

Please read carefully first. The following will be strictly enforced.

- You have **several choices** for writing up your homework. You may:

  → **write by hand and scan as a single PDF**, *or*

  → **write in latex (using the template file provided) and generate PDF**, *or*

  → **write in Word, and save as PDF.**

  No matter which method you use, **please submit one PDF file** only.

- Only questions and sub-parts that are **numbered clearly**, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

- Any graphs you may be asked to draw *must* be generated on the computer. **Hand-drawn graphs will be given a grade of zero**.

- You are **welcome and strongly encouraged to work in groups** on the homework; however, ***the actual homework submission must be your own work***. In particular, **do not** sit with the homework of someone in your group and copy down the solution.

- For homework, you are **only permitted** to look at your text (and Griffiths), everything posted for this course in D2L, and a math handbook. **Do not** look at any other text, **do not** look on the internet, **do not** discuss solutions with anyone outside of class, **do not** try to get solutions from someone not taking the class with you.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Friday (Apr 9). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

Homework problems begin on the next page

1. The alpha particle model of the nucleus postulates that, whenever possible, the neutrons and protons in a nucleus are arranged in alpha particles. It has been around for almost a hundred years, never verified, but never disproved either! According to this model, the nucleus of ^{12}C would be comprised of three alpha particles arranged in an equilateral triangle. If each side of this triangle has length 3×10^{-15} m, calculate the potential energy W of this arrangement. You may assume that the alpha particles are point charges.

Note: No tricks here! This is that easy, because it is the first homework problem of the quarter.

2. In class, we showed that the potential energy can be expressed as an integral of the square of the electric field over all space:

$$W = \frac{\epsilon_0}{2} \int \left|\vec{E}\right|^2 d^3x$$

Suppose we have a charge Q that is ***uniformly distributed within a sphere*** of radius R. Show that

$$W = \frac{3}{5}\left[\frac{Q^2}{4\pi\epsilon_0 R}\right]$$

Note: You may assume (without having to derive it using Gauss' Law) that the electric field is radial, and given by

$$E_r(r) = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3} \qquad \text{for} \quad r < R$$

whereas for $r > R$, $E_r(r)$ is simply the electric field of a point charge Q.

3. The magnetic analog of equation (4.86) for electrostatics is

$$\delta W = \int \vec{H} \cdot \delta\vec{B}\, d^3x$$

If a linear relation exists between $\vec{B}$ and $\vec{H}$, then show that the total magnetic energy will be

$$W = \frac{1}{2}\int \vec{H} \cdot \vec{B}\, d^3x$$

4. In class, we discussed how a problem of considerable interest is the change in energy when a dielectric object is placed in an electric field whose sources are fixed. Suppose initially that there exists an electric field $\vec{E}_0$ due to a distribution of charges $\rho_0(\vec{x})$ in a medium with ϵ_0, which may be a function of position. Then, with the sources fixed in position, a dielectric object of volume V_1 is introduced into the field, changing the field from $\vec{E}_0$ to $\vec{E}$. Then $\epsilon(\vec{x})$ has the value ϵ_1 inside V_1 and ϵ_0 outside V_1. The change in the energy is then given by

$$W = \frac{1}{2}\int \left(\vec{E} \cdot \vec{D} - \vec{E}_0 \cdot \vec{D}_0\right) d^3x$$

Show that this can be written as

$$W = \frac{1}{2}\int \left(\vec{E} \cdot \vec{D}_0 - \vec{D} \cdot \vec{E}_0\right) d^3x$$