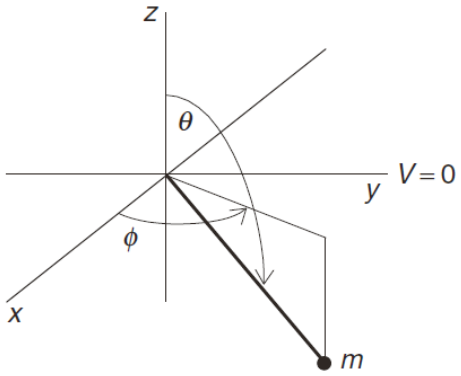
Activity 1: Energy of a spherical pendulum



Total energy in Cartesian coordinates

$$T + V = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + mgz$$

Coordinate transformation from Cartesian to spherical:

$x = l \sin\theta \cos\phi$

$y = l \sin\theta \sin\phi$

$z = l \cos\theta$

Apply chain and product rules:

$\dot{x}^2 = l^2[(\dot{\theta}\cos\theta\cos\phi)^2 + (\dot{\phi}\sin\theta\sin\phi)^2 - 2\dot{\theta}\dot{\phi}\sin\theta\cos\theta\sin\phi\cos\phi]$

$\dot{y}^2 = l^2[(\dot{\theta}\cos\theta\sin\phi)^2 + (\dot{\phi}\sin\theta\cos\phi)^2 + 2\dot{\theta}\dot{\phi}\sin\theta\cos\theta\sin\phi\cos\phi]$

$\dot{z}^2 = l^2(\dot{\theta}\sin\theta)^2$

plug into equation for $T + V$, remember Pythagorean trigonometric identity $\sin^2\alpha + \cos^2\alpha = 1$

Total energy in spherical coordinates:

$$T + V = \frac{1}{2}ml^2\left(\dot{\theta}^2 + sin^2\theta\,\dot{\phi}^2\right) + mgl\cos\theta$$