PHY 474/374 Spring 2021

Class Summary—Week 7, Day 1—Monday, May 10

The Sun

The Sun allows not only for a test of stellar models, but also the opportunity to verify them with exquisite precision not afforded by any other star. Solar astrophysicists have developed a Standard Solar Model, using knowledge from fields like nuclear and atomic physics, measured quantities, and key assumptions to describe all solar observations.

The essence of the **Standard Solar Model** is that a 1 $M_\odot$ ZAMS has evolved to the present-day Sun subject to the following assumptions (source: Guidry, *Stars and Stellar Processes*):

- The Sun was formed from a homogeneous mixture of gases.
- It is powered by nuclear reactions in its core.
- It is approximately in hydrostatic equilibrium, with gravitational forces exactly compensated by gradients arising from gas and radiation pressure.
- Some deviations from equilibrium are permitted as the Sun evolves, but these are small and slow.
- Energy is transported from the core to the surface by photons (radiative) and by large-scale vertical motion of packets of gas (convection).

Why do we assume that the Sun formed from a homogeneous mixture of gases? This is motivated by the strong convection expected in protostar during contraction to Main Sequence. We can then assume that the surface abundances are undisturbed in the subsequent evolution. Thus, the present-day surface abundances indicate the composition of the original solar core. The abundance of most elements on the surface can be inferred from spectroscopy (except He, Ne, Ar, which are not excited significantly by the blackbody spectrum of photosphere).

Two key points of interest are:

- How the various parameters, ρ, T, and so on, change from the core of the Sun to the surface; you'll work these out on the next homework.
- How the various parameters, L, T_c, and so on, change from ZAMS to the present-day Sun.

Let's look at the latter in more detail.

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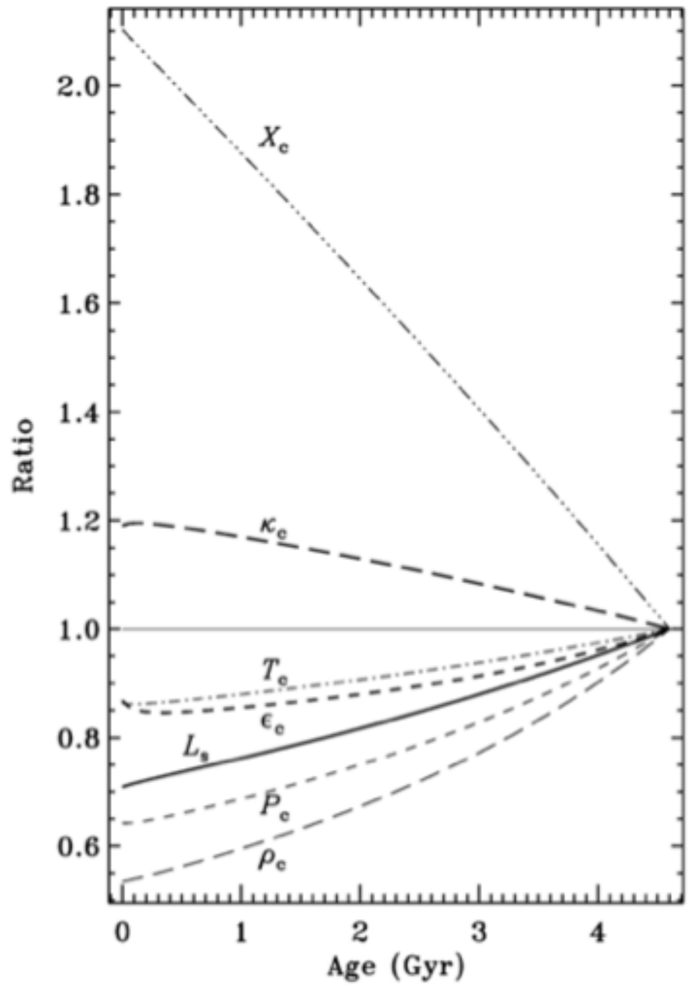
ZAMS to Present-day Sun

Consider the graph below, taken from Figure 11.8 in *Dalsgaard*. All variables have been normalized with respect to their value in the present-day Sun.



Seven key parameters are shown in this plot, from ZAMS until the present day; the x-axis is marked in Gyr, where 1 Gyr = 10^9 yr. All seven parameters are plotted as ratios, normalized to 1 in the present-day Sun.

Let's begin with T_c, the temperature in the core of the Sun. The core of the Sun is hotter today than it was at ZAMS, as is evident from the dot-dashed line for T_c. Intuitively, this makes sense; the Sun has been undergoing fusion of hydrogen to helium in its core for about 5 billion yr, and it stands to reason that this energy production would increase the temperature of the core over time. Since the ratio at ZAMS was 0.85 compared to today, we see that T_c has increased by about 15%, *as you calculated in Question 1(a) on today's worksheet.*

Next, let's look at X, the mass fraction of hydrogen in the core. Over time, we see that X has gone down significantly as hydrogen in the core has fused into helium. Using $X = 0.709$ at ZAMS, the value today is 0.338 *as you calculated in Question 1(b) on today's worksheet* (the generally accepted value is 0.335).

On the next page, we will discuss the implications of these changes.

In Question 2(a) on today's worksheet, you discussed how the change in X will impact μ, hence ρ_c, and in turn, T_c.

Recall that

$$\mu = \frac{4}{3 + 5X - Z} \qquad n = \frac{\rho}{\mu m_p} \qquad T_c = \frac{G \mu_c m_p M}{k_B R}$$

Since X has gone down, as we discussed on the previous page, μ has increased, according to the equation on the extreme left above. This means that the core temperature T_c has also increased, according to the equation on the extreme right above, and this is in agreement with our discussion on the previous page.

Now, changes in T_c should impact ϵ_c, the rate of energy generation at the center, since $\epsilon_c \propto \rho X^2 T^4$. However, the change in ϵ_c is modest than what this T^4 dependence would suggest, *as you found in Question 2(b) on today's worksheet.* This is because $\epsilon_c \propto X^2$ also, and the decrease in X likely offsets the increase in T_c enough to make the increase in ϵ_c be modest.

Even though the change in ϵ_c may be modest, it has resulted in a significant change to the surface luminosity of the Sun from ZAMS until today. *In Question 3(a) on today's worksheet*, you calculated that the percentage change in the luminosity of the Sun is 30% from ZAMS to the present day. Despite the modest change in ϵ_c, the fall in the opacity in the interior has enabled radiation resulting from the increased ϵ_c to be communicated more effectively to the surface, increasing the surface luminosity, *as you found in Question 3(b) on today's worksheet.*

One of the tests of the Standard Solar Model comes from our knowledge of the interior of the Sun as inferred from helioseismology, which involves the propagation of waves in the Sun's interior, much like a geologist learns about the Earth's interior from seismic waves.

There are three categories of waves: p-modes, f-modes, and g-modes.

- The p-modes are pressure waves trapped between the surface and the lower boundary of the convection zone (hence standing sound waves).

  In first-year lab, you learned about standing waves on a string; the idea is the same here except this is in 3-D, so instead of sin/cos functions to describe the modes, we need spherical harmonics; we won't go into them here.

- The g-modes correspond to oscillations in which the restoring force is gravity. If g-modes can be observed, they carry information on much deeper regions of the Sun than carried by p-modes.

- Finally, f-modes are essentially surface gravity waves, and they provide a diagnostic of flows and magnetic fields present in the surface regions.

For an example of how these are so useful in learning about the Sun, consider that since the p-modes are essentially sound waves, their frequencies are largely determined by the speed of sound in the stellar interior, which is given by

$$c^2 = \frac{\gamma P}{\rho} \simeq \frac{\gamma k_B T}{\mu m_p}$$

Thus, frequencies of p-modes can, in principle, give information about the temperature in stellar interiors, although in practice, this requires additional information on the chemical composition and hence the mean molecular weight.

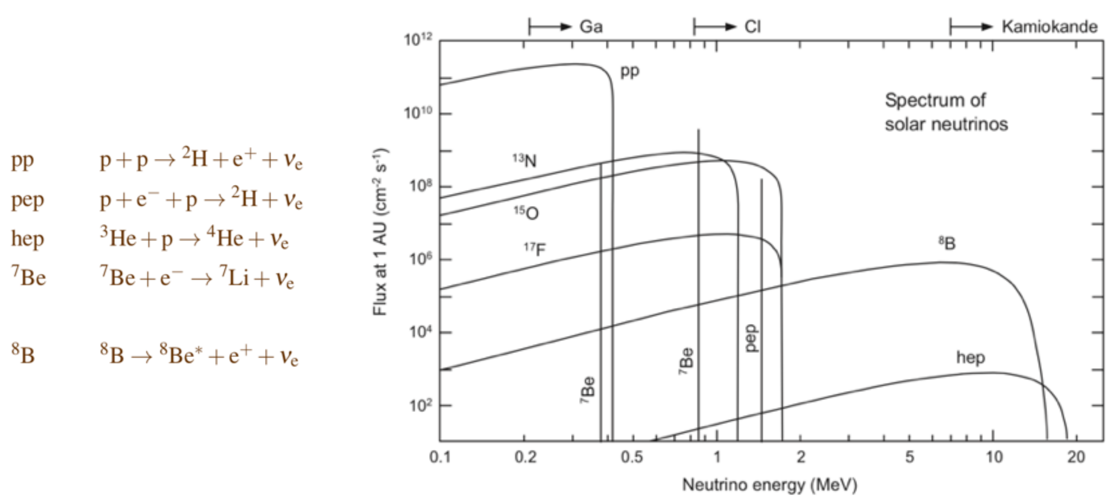
Solar Neutrinos

Neutrinos are massless, electrically neutral particles that interact only via the Weak force. They were first proposed by Pauli in 1930 to maintain energy and momentum conservation in beta-decay.

The production of each ^{4}He nucleus during nuclear fusion in the Sun is accompanied by two neutrinos. Since neutrinos interact very weakly, they are guaranteed to escape the Sun and in principle, therefore, measurement of the fluxes of neutrinos should give information on the rate of nuclear reactions in the core of the Sun. Remember, the energy responsible for surface luminosity must make its way from core to surface on a 100,000 yr timescale. On the other hand, neutrinos produced in the core move unimpeded through the Sun, and reach the Earth about 8.5 minutes after they were produced. Thus, neutrinos carry immediate and direct information about conditions in the solar core than the photons.

In practice, however, neutrinos have a very small cross section for reactions, and therefore are very difficult to detect. Given their low cross sections, detectors must be huge in order to be able to register a detectable signal. Moreover, the neutrinos are emitted in a number of different nuclear reactions in the Sun, each with different energies. The distribution of neutrinos depends on the branching ratios between the PP-I, PP-II, and PP-III chains that we learned about in an earlier class, and also on the relative important of the CNO cycle.

Consider the energy spectrum of neutrinos predicted by a model of the present-day Sun shown below, where the y-axis gives the flux of neutrinos at 1 AU. The reactions shown in the figure are written out in full in the left panel; note that we did not discuss pep and hep last week. Ignore the reactions marked ^{13}N, ^{15}O, and ^{17}F; they belong to the CNO cycle which makes only modest contributions to the solar neutrino flux.



The pioneering experiment to detect solar neutrinos was started by Raymond Davis in the early 1960s. It uses the reaction

$$\nu \ + \ ^{37}\mathrm{Cl} \ \ \rightarrow \ \ ^{37}\mathrm{Ar} \ + \ e^-$$

in a detector comprised of 600 tons of cleaning fluid (C_2Cl_4), placed 1500 m below ground in the abandoned Homestake gold mine in South Dakota. As marked in the figure above, the threshold minimum for this reaction to occur is 0.8 MeV, which means that this method is sensitive, in principle, to the reactions ^{8}B, ^{7}Be, hep, and pep, *as you found in Question 4(a) on today's worksheet.* In practice, however, this experiment detected largely the ^{8}B, and some ^{7}Be.

Another detector uses gallium to detect solar neutrinos via the reaction

$$\nu \ + \ ^{71}\text{Ga} \ \rightarrow \ ^{71}\text{Ge} \ + \ e^-$$

and has a threshold of only 0.23 MeV (see figure on the previous page where the sensitivity limit is marked at the top). Thus, this method is sensitive to PP *as you answered in Question 4(b) on today's worksheet*, which is important, since 90% of the Sun's energy is produced by the PP chain.

The table below shows solar neutrino fluxes from various experiments compared with a Standard Solar Model (SSM). All fluxes are in Solar Neutrino Units (SNU), where SNU is defined as 10^{-36} captures per target atom in the detector per second. Note that I've suppressed the systematic and statistical errors for brevity. The second and third entries (SAGE and GALLEX) refer to the Ga-based detection.

Experiment	Observed flux (SNU)	SSM (SNU)	Observed/SSM
Homestake	2.54	9.3	0.27
SAGE	72	137	0.53
GALLEX	69.7	137	0.51

From the ratio of observed neutrino flux to that predicted by a Standard Solar Model (SSM) written in the last column of the table above *that you calculated in Question 5(a) on today's worksheet*, we see that all three, and indeed all experiments to date, have detected fewer neutrinos than predicted, *as you noted in Question 5(b) on today's worksheet.*

There are two possible reasons for this discrepancy, *as you discussed in Question 5(c) on today's worksheet.* One could be that the Standard Solar Model is wrong. However, results from helioseismology agree very closely with observations, so it is difficult to believe that the Standard Solar Model could be so wrong! The other possibility is that our knowledge of neutrinos is wrong, or at least, incomplete.

Indeed, scientists have discovered that neutrinos have three flavors, the electron neutrino, muon neutrino, and the tau neutrino. They can oscillate between these three flavors as they travel, so detectors sensitive to one type of neutrino will undercount the neutrino flux.