PHY 411 Winter 2021

Homework 5—due by 9:00 PM, Tuesday, Feb 16

You may write by hand and scan as a single PDF, or write in latex (using the template file provided) or Word, and generate PDF. Please submit one PDF file only. Only questions and sub-parts that are numbered clearly, with numbers corresponding to those in this document, will be graded. See the syllabus for more detailed rules.

Submit late homework into the late D2L dropbox for reduced credit (see syllabus); late submissions will be accepted until 8 AM on Saturday (Feb 20). Emailed or paper copies of homework are never accepted; in particular, do not attach homework to email to make an end-run around the D2L deadline or late deadline; such emails are automatically deleted and do not count as submissions.

1. In the syllabus, it is noted that 5% of the course grade comes from a **Formal Write-up**, an enhanced homework activity in which you will write up problems in the formal manner of a publishable paper. The time has come to make your choice of problems. The formal write-up is due during Week 10 (*instructions and exact schedule will be provided in D2L once I assign the topics*).

Please list the following topics in **order of preference**, and **justify your first preference**.

(a) Discuss a **model of the ionosphere** as given in **Section 7.6** (*Jackson*, pages 316-319); in particular, you must decouple from equation (7.63) the transverse components to demonstrate explicitly the progression to equation (7.65) and equation (7.67), then discuss Figure 7.10 and Figure 7.11.

(b) Write fields and match boundary conditions to solve **Problem 7.2** (*Jackson*, pages 340-341).

(c) Work with me to solve **Problem 7.15** (*Jackson*, pages 345-346), and then use it to discuss dispersion and rotation measures of pulsars.

(d) By working through the equations of **Section 8.2** (*Jackson*, pages 356-359) on **waveguides**, be able to explain TM and TE waves, along with a TEM mode.

(e) **Toy MHD** (*Jackson*, pages 319-322): Starting from equation (7.68) and equation (7.69), work to linearize and derive equation (7.71) and equation (7.72), and hence discuss longitudinal and transverse wave modes, especially the Alfvén velocity.

I will likely assign two students per problem so you can discuss your approaches, but likely no more than two students per problem. I've tried to distribute topics so they appeal to different interests: part (a) is Plasma Physics, part (b) is Classical Optics, part (c) is Observational Astronomy, part (d) is Engineering Applications, and part (e) is Fluid Dynamics (with applications also in Astrophysics).

Questions 2, 3, and 4 are on the next page.

2. With the Midterm this week, here is some lighter activity with a couple of Griffiths-level problems (that should, hopefully, still help to enhance your understanding of vector and scalar potentials). Suppose the scalar potential Φ and vector potential $\vec{A}$ are given by

$$\Phi = 0 \qquad \text{and} \qquad \vec{A} = \hat{y}\, A_0 \, \sin(kx - \omega t)$$

where A_0 is a constant.

(a) Find the fields $\vec{E}$ and $\vec{B}$.

(b) Discuss whether these fields can represent an electromagnetic wave, and if so, provide a quantitative answer for the direction in which the wave would be traveling.

3. Consider the gauge transformation

$$\vec{A}' = \vec{A} + \vec{\nabla}\Lambda \qquad \text{and} \qquad \Phi' = \Phi - \frac{\partial \Lambda}{\partial t}$$

Suppose you are given the potentials:

$$\Phi(\vec{r}, t) = 0 \qquad \text{and} \qquad \vec{A}(\vec{r}, t) = -\frac{qt}{4\pi\epsilon_0 r^2}\,\hat{r}$$

and suppose the gauge function Λ is given by

$$\Lambda = -\frac{qt}{4\pi\epsilon_0 r}$$

(a) Determine explicitly the fields $\vec{E}$ and $\vec{B}$ corresponding to $\Phi(\vec{r}, t)$ and $\vec{A}(\vec{r}, t)$. **Note:** The cross product in spherical coordinates (r, θ, ϕ) is given on the inside back cover in Jackson, or see the Class Summary for Week 1—Day 1.

(b) Find the transformed potentials $\vec{A}'$ and Φ' for the gauge function Λ given above.

(c) Determine explicitly the fields $\vec{E}'$ and $\vec{B}'$ corresponding to Φ' and $\vec{A}'$.

(d) Comment on your results, e.g., how is $\vec{E}'$ related to $\vec{E}$, and is this expected? Likewise for $\vec{B}'$ and $\vec{B}$. Physically, what kind of charge distribution do we have here? Could you have figured that by looking at Φ and $\vec{A}$, or did you need to do the gauge transformation to figure this out?

4. In order to work with wave packets, we built up a superposition of solutions to the wave equation by writing

$$u(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k)\, e^{ikx - i\omega(k)t}\, dk \qquad \text{where} \qquad A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x, 0)\, e^{-ikx}\, dx$$

where the amplitude $A(k)$ describes the properties of the linear superposition of the different waves. Consider now an approximately monochromatic plane wave packet in one dimension that has the instantaneous form

$$u(x, 0) = \begin{cases} N\, e^{ik_o x} & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

(a) Calculate the wave-number spectrum $|A(k)|^2$ of this packet.

(b) Draw graphs of $|u(x, 0)|^2$ and $|A(k)|^2$. Attach your program and your graph to your submission. *Hand-drawn sketches will not be accepted.*