Longitudinal Homework 5

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1. Slope of Age

a. Age as a Class Variable

Check that matrix multiplication matches a model fit with R:

```
# First level is the reference group
X1 <- X[,c(1,3,4,5)]
beta <- ginv((t(X1)%*%ginv(V)%*%X1)) %*% (t(X1) %*% ginv(V) %*% ramus$height)
mod <- gls(height ~ factor(age),data = ramus,correlation=corAR1(form=~1|boy))
beta
```

```
##         [,1]
## [1,] 48.655
## [2,]  0.970
## [3,]  1.915
## [4,]  2.795
```

```
# Compare to R model
kable(summary(mod)$tTable)
```

	Value	Std.Error	t-value	p-value
(Intercept)	48.655	0.5864847	82.960397	0e+00
factor(age)8.5	0.970	0.1803995	5.376955	8e-07
factor(age)9	1.915	0.2520881	7.596552	0e+00
factor(age)9.5	2.795	0.3051001	9.160928	0e+00

Get the test statistic and its SE for linear contrast:

```
# Re-fit with full X
beta <- ginv((t(X)%*%ginv(V)%*%X)) %*% (t(X) %*% ginv(V) %*% ramus$height)
L <- c(0,-3,-1,1,3)
# Estimate and SE
lbeta <- L %*% beta
lbeta
```

```
##      [,1]
## [1,] 9.33
```

```
selbeta <- sqrt(L%*%(ginv(t(X)%*%ginv(V)%*%X))%*%matrix(L))
selbeta
```

```
##          [,1]
## [1,] 1.027713
```

```
lbeta/selbeta
```

```
##         [,1]
## [1,] 9.07841
```

Compare:

```r
# Check with R
emm <- emmeans(mod,specs = ~age)
contrast(emm,method = list("linear" = c(-3,-1,1,3)))
```

```
##  contrast estimate   SE   df t.ratio p.value
##  linear       9.33 1.03 61.9 9.078   <.0001
##
## Degrees-of-freedom method: satterthwaite
```

```r
# Check statistic using DF from R
2*pt(lbeta/selbeta,df = 57,lower.tail = FALSE)
```

```
##              [,1]
## [1,] 1.149952e-12
```

They match! $L\beta$ is proportional to the slope between the parameter estimates (without an intercept):

```r
x <- c(1:4)
y2 <- as.data.frame(emm)[,2]
kable(summary(lm(y2 ~ x))$coefficients)
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	47.7425	0.0397335	1201.56834	0.0000007
x	0.9330	0.0145086	64.30661	0.0002417

And because the estimate is statistically significant, we can say there is a linear trend for time.

b. Age as a Continuous Variable

Check that matrix multiplication matches a model fit with R:

```
# New V matrix
sigma_e_sq=6.8783;  phi=0.9542
V_i=sigma_e_sq*matrix(c(1,phi,phi^2,phi^3,phi,1,phi,phi^2,phi^2,phi,1,phi,
                        phi^3,phi^2,phi,1),nrow=4,ncol=4)
V=kronecker(diag(20),V_i)
# New X matrix
X_i=cbind(1,rep(c(8.0,8.5,9.0,9.5)))
X=NULL
for(i in 1:20){X=rbind(X,X_i)}
# Manually
beta <- ginv((t(X)%*%ginv(V)%*%X)) %*% (t(X) %*% ginv(V) %*% ramus$height)
beta
```

```
##            [,1]
## [1,] 33.750224
## [2,]  1.863342
```

```
# R
mod <- gls(height ~ age,data = ramus,correlation=corCAR1(form=~age|boy))
kable(summary(mod)$tTable)
```

	Value	Std.Error	t-value	p-value
(Intercept)	33.750224	1.8414735	18.327836	0
age	1.863342	0.2002349	9.305783	0

Get the estimate and SE:

```
L <- c(0,1)
lbeta <- L %*% beta
lbeta
```

```
##           [,1]
## [1,] 1.863342
```

```
selbeta <- sqrt(L%*%(ginv(t(X)%*%ginv(V)%*%X))%*%matrix(L))
selbeta
```

```
##            [,1]
## [1,] 0.2002725
```

```
lbeta/selbeta
```

```
##           [,1]
## [1,] 9.304034
```

Hypothesis test check using DF from R:

```
2* pt(lbeta/selbeta,df = 59,lower.tail = FALSE)
```

```
##               [,1]
## [1,] 3.570689e-13
```

These match too! So there is a statistically significant trend for time, and bone height increases by 1.86 units (95% CI: 1.46 - 2.26, $p < 0.0001$) on average with each year of life.

2. Publishing the Results

I don't think it particularly matters which approach you report in a journal. Both methods test for the same sort of linear trend and the conclusion doesn't change depending on the method (both suggest that the linear trend is highly significant). I suppose that the continuous approach might be slightly more intuitive for many people, since they're probably used to the concept of testing whether regression coefficients are equal to 0 and the estimate is easier to understand, but this really depends on the audience and question of interest.