

Masters Theory I

Homework #11

Note: Homework has 2 pages

Out: Thursday
Due / Quiz following Thursday

1. Prove Thm 5.3.1 (assume results of problem 5.15).
2. Prove Lemma 5.3.3 part (a), assuming $n=2$ (C&B page 221).
3. Example 5.2.12
4. Example 5.4.5
5. Example 5.4.7
6. C&B 5.3 (deja vu)
7. C&B 5.16
8. C&B 5.17
9. C&B 5.22
10. C&B 5.24
11. Use Theorem 5.2.7 to find the MGF of \bar{X} when X has the distributions below. For each, do you recognize the MGF of \bar{X} ?
 - (a) Normal
 - (b) Poisson
 - (c) exponential
12. Discrete order statistics: Assume that X_1, X_2, \dots, X_{20} are iid $f(x)$ such that

$$\begin{aligned}P(X = 1) &= .1 \\P(X = 3) &= .3 \\P(X = 5) &= .4 \\P(X = 7) &= .2\end{aligned}$$

Write out equations and solve (R) for the following

- (a) $P(X_{(7)} \leq 3)$
- (b) $P(X_{(8)} \geq 5)$
- (c) Find $P(X_{(20)} \leq 7)$
- (d) Find $P(X_{(4)} = 3)$

Hint the following is 1.0:

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sum(dbinom(size=20,prob=.5,x=c(0:20)))
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13. Let $Y_1 < Y_2 < \dots < Y_n$ be the order statistics of a random sample of size n from the exponential distribution with pdf $f(x) = e^{-x}$, $0 < x < \infty$ zero elsewhere.
 Show that $Z_1 = nY_1, Z_2 = (n-1)(Y_2 - Y_1), Z_3 = (n-2)(Y_3 - Y_2), \dots, Z_n = Y_n - Y_{n-1}$ are independent and that each Z_i has the exponential distribution.
 Hint: The joint pdf of all order statistics is given by (C&B page 230):

$$f_{X_{(1)}, \dots, X_{(n)}}(x_1, \dots, x_n) = n! f_X(x_1) * \dots * f_X(x_n) \\ -\infty < x_1 < \dots < x_n < \infty$$