Homework 10
BIOS 7731
Due 12/8 10:30am through Canvas.
Students may work together on homework assignments, but the assignment handed in must represent your own work. Problems based on a later lecture are labeled with *.

1. Let $Y_i = \beta_0 + \beta_1(X_i - \overline{X}) + \epsilon_i$, with $i = 1...n$, X_i known, and $\epsilon_i \sim_{iid} (0, \sigma^2)$. Let $\mathbf{X}$ be the design matrix for this regression problem, and show that the limiting distribution of $(\mathbf{X}'\mathbf{X})^{1/2}(\hat{\beta} - \beta)$ is $\mathcal{N}_2(\mathbf{0}, \sigma^2\mathbf{I})$, where $\hat{\beta}$ are the least squares estimates for $\beta = (\beta_0, \beta_1)$.

2. BD problem 6.1.4 page 422

3. BD problem 6.2.9 page 427

4. *Let $X_{i1}, ..., X_{in},\ i = 1, 2$, be independently distributed as beta distributions with p.d.f.'s:

$$\theta_i x^{\theta_i - 1} I_{(0,1)}(x) \text{ with } \theta_i > 0,$$

   $i = 1, 2$, respectively.

   (a) Find the form of the Score (Rao) test for testing $H_0 : \theta_1 = \theta_2$ versus $H_1 : \theta_1 \neq \theta_2$.

   (b) Find the form of the LR test for testing $H_0 : \theta_1 = \theta_2$ versus $H_1 : \theta_1 \neq \theta_2$

   (c) Find the form of the Wald test for testing $H_0 : \theta_1 = \theta_2$ versus $H_1 : \theta_1 \neq \theta_2$

5. *BD problem 6.3.4 page 429