Homework 6
BIOS 7731
Due 10/22 10:30am through Canvas. Students may work together on homework assignments, but the assignment handed in must represent your own work. Problems based on a later lecture are labeled with *.

1. Suppose $X_1, \ldots, X_n$ are IID $Poisson(\theta)$ and $\theta \sim Gamma(1, \lambda)$.

   (a) According to the loss function $l(\theta, a) = \theta^p(\theta - a)^2$, where p is a fixed positive constant, show that the Bayes rule is $\frac{T(X)+p+1}{n+\lambda}$, where $T(X) = \sum_{i=1}^n X_i$.

   (b) Is the Bayes rule also minimax? If so, for what values of λ and p is it minimax?

2. Empirical Bayes

   Consider estimation of regression slopes $\theta_1, \ldots, \theta_p$ for p pairs of observations, $(X_1, Y_1), \ldots, (X_p, Y_p)$, modeled as independent with $X_i \sim N(0, 1)$ and $Y_i | X_i = x \sim N(\theta_i x, 1)$.

   (a) Following a Bayesian approach, let the unknown parameters θ_i be iid random variables from $N(0, \tau^2)$. Find the Bayes estimate of θ_i in this Bayesian model with squared error loss.

   (b) Determine $E[Y_i^2]$ in the Bayesian model. Using this, suggest a simple method of moments estimator for τ^2.

   (c) Given an empricial Bayes estimator for θ_i combining the simple "empirical" estimate for τ in part b) with the Bayes estimate for θ_i when τ is known in part a).

3. BD 3.4.2 (pg 203)

4. BD 3.4.3 (pg 203)

5. BD 3.5.1 (pg 206)*

6. BD 3.5.11 (pg 208)*