Homework 1
BIOS 7731
Due 9/10 10:30 am through Canvas
Students may work together on homework assignments, but the assignment handed in must represent the students own work.

1. Using the properties of a probability measure, show (BD pg. 443)
   (a) A.2.2
   (b) A.2.3
   (c) A.2.5
   (d) A.2.7

2. Consider $f_X(x) = \frac{x^2 e^{-x}}{2}$ for $x \in (0, \infty)$, and zero otherwise.
   (a) Show this is a density function by verifying (BD pg. 449) A.7.6.
   (b) Find the distribution function, $F(x)$.
   (c) Find F(2).

3. The Gamma Distribution
   (a) The gamma function is defined for $\alpha > 0$ by
   $$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx.$$
   Use integration by parts to show that $\Gamma(x+1) = x\Gamma(x)$. Show that $\Gamma(x+1) = x!$ for $x = 0, 1, \ldots$.
   (b) Show that the function
   $$p(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad x > 0$$
   is a probability density function when $\alpha > 0$ and $\beta > 0$. This density is called the gamma density with parameters α and β. The corresponding probablity distribution function is denoted $\Gamma(\alpha, \beta)$.
   (c) Show that if $X \sim \Gamma(\alpha, \beta)$, then $E[X^r] = \beta^r \Gamma(\alpha + r)/\Gamma(\alpha)$. Use this formula to derive the mean and variance of X.

4. Suppose $X \sim N(0, 1)$, find the mean and covariance of the random vector $(X, I\{X > c\})$.

5. Let T be an exponential random variable and conditional on T, let U be uniform on $[0, T]$. Find the unconditional mean and variance of U.

6. For any two random variables X and Y with finite variances, prove that
   (a) $Cov(X, Y) = Cov(X, E[Y|X])$.
   (b) X and $Y - E[Y|X]$ are uncorrelated.