

Longitudinal Homework 4

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1. Cereal data

a. Poisson regression

Model

$$Y_i \sim \text{Poisson}(e^{\beta_0 + \beta_1 * \text{condition} + \beta_2 * \text{sex} + \beta_3 * \text{weight}})$$

Assuming that $\text{Var}(Y) = \mu$

Where Y_i is the number of servings of breakfast cereal eaten in week one. The outcome is the same for all of the following models.

Results

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.9979487	0.1987601	5.020869	0.0000005
Cond	0.8645418	0.1349809	6.404921	0.0000000
Sex	-0.1988221	0.0822303	-2.417869	0.0156117
Wt1	0.1973105	0.1323242	1.491114	0.1359317

The results of Poisson models are interpreted as changes in rate ratio and are given above on the log scale. The interpretation of these results is that on average the experimental program group ate cereal at a rate 2.73 times higher (95% CI: 1.84, 3.12, $p < 0.0001$) than the control group when adjusting for sex and weight, and women ate cereal at a rate of 0.82 times (95% CI: 0.70 0.96, $p = 0.016$) the men's rate after adjusting for experimental condition and weight.

b. Overdispersion (quasilikelihood)

Model

$$Y_i \sim \text{Poisson}(e^{\beta_0 + \beta_1 * \text{condition} + \beta_2 * \text{sex} + \beta_3 * \text{weight}})$$

Assuming that $\text{Var}(Y) = \phi\mu$

Results

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.9979487	0.3410728	2.9259112	0.0042961
Cond	0.8645418	0.2316274	3.7324672	0.0003229
Sex	-0.1988221	0.1411073	-1.4090131	0.1620961
Wt1	0.1973105	0.2270686	0.8689464	0.3870665

On average, the experimental program group ate cereal at a rate 2.73 times higher (95% CI: 1.55, 3.85, $p = 0.0003$) than the control group when adjusting for sex and weight.

The estimates are the same as the Poisson model above, but the confidence intervals and p values have changed, because of the introduction of the scale parameter. The standard errors in this model are simply the standard errors from the Poisson model with $\phi = 1$ multiplied by the scale factor in the quasiliikelihood model:

```
scale <- 1.716
pois_ses <- summary(pois_mod)$coefficients[,2]
pois_ses

## (Intercept)      Cond      Sex      Wt1
## 0.1987601 0.1349809 0.0822303 0.1323242

pois_ses * scale

## (Intercept)      Cond      Sex      Wt1
## 0.3410724 0.2316272 0.1411072 0.2270684

summary(quasi_pois_mod)$coefficients[,2]

## (Intercept)      Cond      Sex      Wt1
## 0.3410728 0.2316274 0.1411073 0.2270686
```

The scale parameter above comes from SAS, but to determine it in R you would just divide the SEs from the quasiliikelihood model by the SEs from the simple Poisson model. The point here is that the SEs have a linear relationship based on the scale parameter ϕ .

c. Normal error in linear predictor

Model

$$Y_i | \epsilon_i \sim \text{Poisson}(e^{\beta_0 + \beta_1 * \text{condition} + \beta_2 * \text{sex} + \beta_3 * \text{weight} + \epsilon_i})$$

with $\epsilon_i \sim N(0, \sigma^2)$

Results

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.8817447	0.3242098	2.7196732	0.0065346
Cond	0.8551232	0.1970427	4.3397869	0.0000143
Sex	-0.1534800	0.1433302	-1.0708139	0.2842531
Wt1	0.1637159	0.2361203	0.6933583	0.4880847

On average the experimental program group ate cereal at a rate 2.35 times higher (95% CI: 1.60, 3.50, $p < 0.0001$) than the control group when adjusting for sex and weight.

d. Negative binomial

Model

$$Y_i \sim \text{Negative Binomial}(e^{\beta_0 + \beta_1 * \text{condition} + \beta_2 * \text{sex} + \beta_3 * \text{weight}}, k)$$

Results

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.1055072	0.3228816	3.4238780	0.0006173
Cond	0.8573437	0.1954837	4.3857545	0.0000116

	Estimate	Std. Error	z value	Pr(> z)
Sex	-0.1710841	0.1436796	-1.1907338	0.2337581
Wt1	0.0994404	0.2339898	0.4249773	0.6708532

On average, the experimental program group ate cereal at a rate 2.35 times higher (95% CI: 1.60, 3.47, $p < 0.0001$) than the control group when adjusting for sex and weight.

2. Model comparison

	Poisson Regression	Poisson QL	Poisson + Normal Error	Negative Binomial
Intercept	0.998	0.998	0.882	1.106
Condition	2.374	2.374	2.352	2.357
Sex	0.820	0.820	0.858	0.843
Weight	1.218	1.218	1.178	1.105
Other Parameter	NA	NA	NA	NA