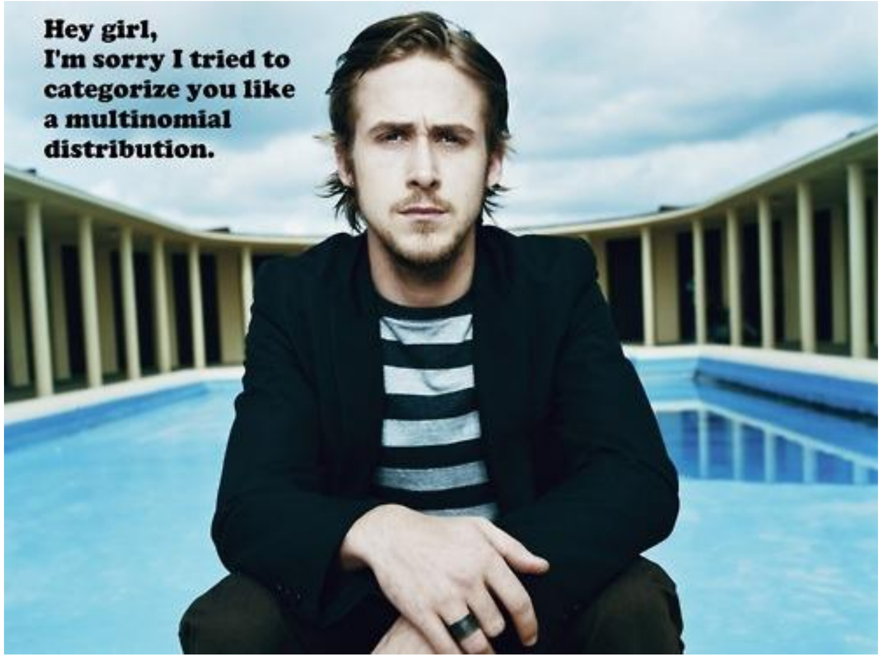


BIOS 6612
Lecture 2

Introduction to
Logistic Regression

Agretsi (2002) Catgorical Data Analysis, 2nd Edition. Section 4.2, Chapter 5 up to 5.1.3
Hosmer DW, Lemeshow S. *Applied Logistic Regression*. Wiley, 2000.
Allison PD. Logistic Regression Using SAS: Theory and Practice. SAS Publishing 1999.

Review (Lectures 1) / Current (Lecture 2)/ Preview (Lecture 3)

- Lecture 1: Model Selection
    - Adjusted R squared
    - Partial F-test
    - AIC
    - Mallow's Cp
    - Forward, Backwards and Stepwise selection

- Lecture 2: Introduction of Logistic Regression
    - Odds ratio is appropriate for case-control studies
    - Introduction to logistic regression

$$\ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = \beta_0 + \beta_1 X_1 + ... + \beta_k X_k \Rightarrow 0 < \hat{p} < 1$$

- Lecture 3:
- Logistic Regression
    - Maximum Likelihood Estimation

Logistic Regression

- Logistic regression can be used to model the association between
  - A binary outcome ($Y = 1$ or $Y = 0$) and
  - One or more explanatory variables
    - dichotomous, categorical, and/or continuous

- Binary or dichotomous variables have two levels:
  - Yes / No
  - Disease / No Disease
  - Case / Control
  - Dead / Alive
  - Cured / Not Cured
  - In Remission / Not in Remission
  - Low birthweight / Normal birthweight

- We can model the probability/odds of "success" or the probability of "failure."
  - It is important to keep in mind which probability you are modeling
  - By default SAS models the probability of the category with the lowest value, which is often '0'

Measures of Effect for 2 × 2 Categorical Data

Exposure/ Risk Factor	*Outcome/Response* Yes	No	
Yes	a	b	$a+b=n_1$
No	c	d	$c+d=n_2$

Let $\hat{p}_1 = a/n_1$ be the probability of disease among the exposed.

Let $\hat{p}_2 = c/n_2$ be the probability of disease among the unexposed.

Odds Ratio

- The odds is defined as $p/(1-p)$ where p=Pr(Y=1)
- The odds ratio is defined as $[p_1/(1-p_1)]/[p_2/(1-p_2)]$.
- The odds ratio is appropriate for cohort, cross-sectional, and case-control studies
    - Relative Risk is NOT appropriate for case-control studies

Odds interpretation

X is the outcome when rolling a fair 6-sided die such that:

$Pr(X=1) = 1/6$
$Pr(X\neq1) = 5/6$

Odds of X=1: $(1/6) / (5/6) = 1 / 5$ or "one-to-five"
Odds of X≠1: $(5/6) / (1/6) = 5 / 1$ or "five-to-one"

Bet on the outcome of X=1 then you have 1 chance to win and 5 chances to lose
Bet on the outcome of X≠1 then you have 5 chances to win and 1 chance to lose

Y is the outcome when rolling a "loaded" (not-fair) 6-sided die such that:

$Pr(Y=1) = 1/2$
$Pr(Y\neq1) = 1/2$

Odds of Y=1: $(1/2) / (1/2) = 1 / 1$ or "one-to-one"
Odds of Y≠1: $(1/2) / (1/2) = 1 / 1$ or "one-to-one"

Bet on the outcome of Y=1 then you have an equal number of chances to win or lose
Bet on the outcome of Y≠1 then you have an equal number of chances to win or lose

Odds Ratios:

$Odds_{X=1} / Odds_{Y=1} = (1/5) / (1/1) = 0.2$

$Odds_{X=1} = 0.2 * Odds_{Y=1}$

$Odds_{X\neq1} / Odds_{Y\neq1} = (5/1) / (1/1) = 5$

$Odds_{X\neq1} = 5 * Odds_{Y\neq1}$

Odds Ratio Example

Drinking Status	*Lung Cancer* Yes	No	
Heavy	33 a	1667 b	1700
Non	27 c	2273 d	2300
	60	3940	4000

$$\hat{O}R = \frac{p_1 / (1-p_1)}{p_2 / (1-p_2)}$$

$$= \frac{\left(\frac{a}{a+b}\right) \Big/ \left(\frac{b}{a+b}\right)}{\left(\frac{c}{c+d}\right) \Big/ \left(\frac{d}{c+d}\right)} = \frac{ad}{bc} = 1.6665$$

$$H_0 : OR = 1$$

$$\ln(\hat{O}R) = \ln(1.667) = 0.51075$$

Another Example: Exposure to passive smoking and cancer

Passive Smoking	*Cancer* Yes	No	
Yes	281 (a)	210 (b)	491
No	228 (c)	279 (d)	507
	509	489	998

$\hat{p}_1$ = 281/491 = 0.5723 (proportion passive smoke exposed with cancer)

$\hat{p}_2$ = 228/507 = 0.4497 (proportion non-exposed with cancer)

$\widehat{Odds}_{ex} = (p_1/(1-p_1)) = 281/210 = 1.3381$ (cancer odds given passive smoke exposure)

$\widehat{Odds}_{un} = (p_2/(1-p_2)) = 228/279 = 0.8172$ (cancer odds given not exposed)

$\widehat{OR} = 1.6374$

Odds Ratio $= (p_1/(1-p_1)) \ / \ (p_2/(1-p_2)) = Odds_{ex} / Odds_{un}$
$= (p_1*(1-p_2))/(p_2*(1-p_1))$
$=>$ ad/bc =(281*279)/(210*228)=1.6374

Modeling Binary Outcomes

- What would happen if we used linear regression to model a binary outcome (model the absolute risk)?
  - That is, what is wrong with using the binary outcome (coded 0,1) as the dependent variable in a linear regression analysis?

$$p = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k \Rightarrow -\infty < p < \infty$$

  - No constraint on values of p

- We could model relative risk by modeling the log of the probabilities (multiplicative model):

$$\ln(p) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k \Rightarrow 0 < p < \infty$$

- We could model the log of the odds.

$$\ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k \Rightarrow 0 < p < 1$$

$$\lim_{p\to 0} \frac{p}{1-p} = 0 \Rightarrow \lim_{p\to 0} \log\left(\frac{p}{1-p}\right) = -\infty \ \textit{and} \ \lim_{p\to 1} \frac{p}{1-p} = \infty \Rightarrow \lim_{p\to 1} \log\left(\frac{p}{1-p}\right) = \infty$$

Transforming the probability to an odds removes the upper bound of 1. If we then take the logarithm of the odds, we also remove the lower bound of 0, so our outcome can range between –∞ and ∞. When we backtransform, this contstains $\hat{p}$ to be between 0 and 1.

Linear Regression With a Binary Outcome

```
DATA smoke;
 INPUT passive cancer n;
 DATALINES;
 1 1 281
 1 0 210
 0 1 228
 0 0 279
;
```

```
PROC REG;
 MODEL cancer=passive;
 FREQ n;
RUN;
```

The FREQ options tells SAS that each line of data is observed *n* time. (e.g., there are 281 observations with passive=1 and cancer=1).

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	3.74904	3.74904	15.20	0.0001
Error	996	245.65076	0.24664		
Corrected Total	997	249.39980			

Parameter Estimates

Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	Intercept	1	0.44970	0.02206	20.39	<.0001
passive	Passive Smoke	1	0.12260	0.03144	3.90	0.0001

Linear Regression Equation: $\hat{y} = \hat{p} = 0.44970 + 0.12260 \times passive$

Unexposed to Passive Smoke: $\hat{p} = 0.44970 + 0.12260 \times 0 = 0.44970$

Exposed to Passive Smoke: $\hat{p} = 0.44970 + 0.12260 \times 1 = 0.57230$

Linear Regression Assumptions

1. Existence
2. Independence
3. Linearity
4. Homoskedasticity
5. Normality of the Errors

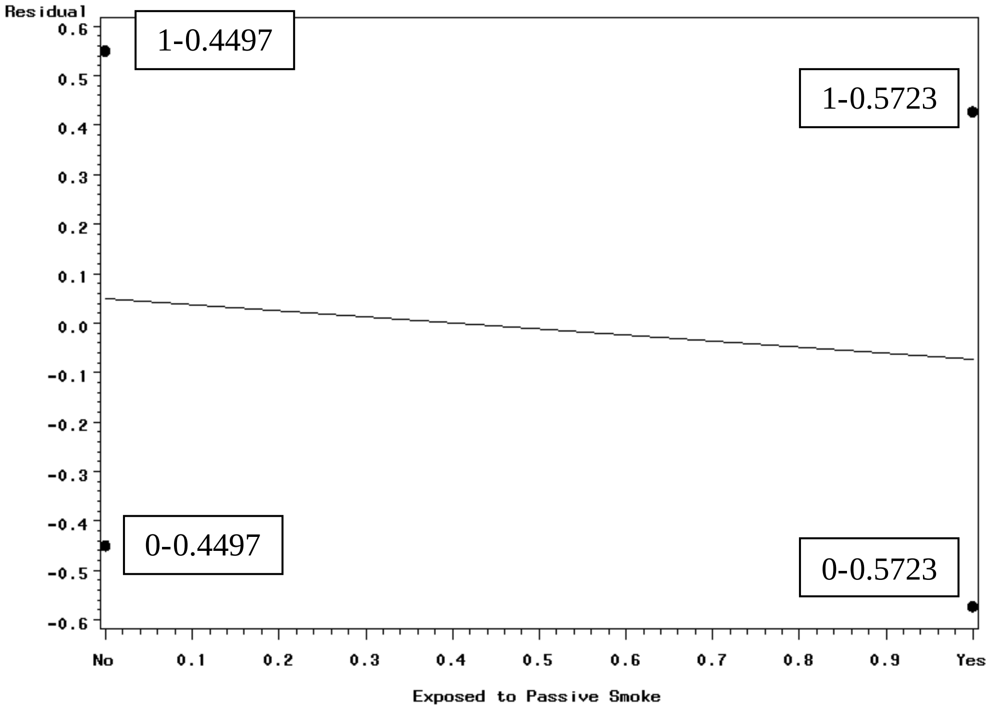
Assumptions 4 and 5 are violated with dichotomous outcomes.

For an individual *i*, $Y|X_i$ follows a Bernoulli distribution with

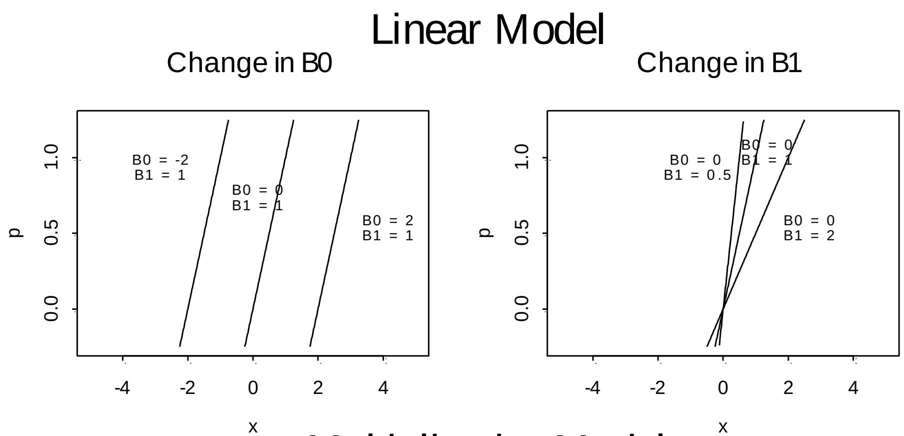
$Pr(success) = \pi(x_i)$

$Variance = \pi(x_i)*(1-\pi(x_i))$

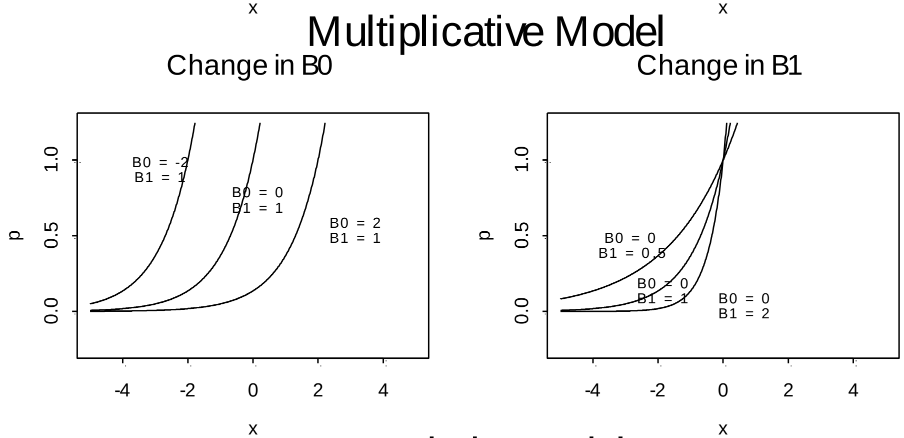
Residual Plot:



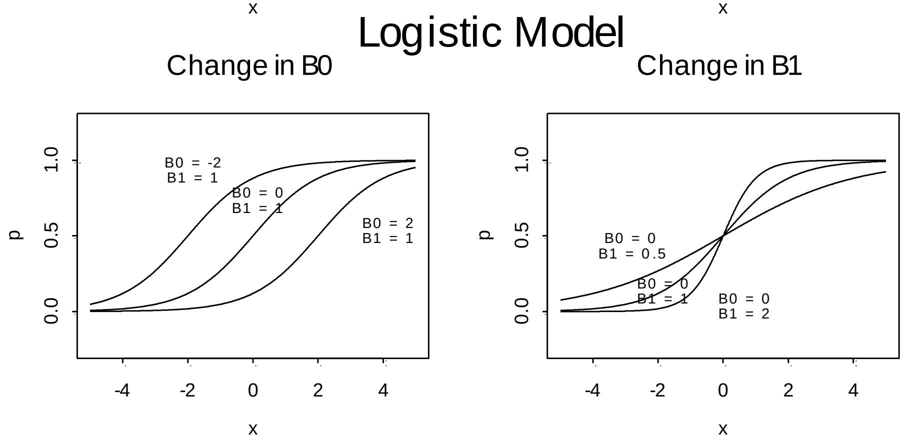
- Despite clear-cut violations of OLS assumptions, most applications of OLS regression to dichotomous variables give results that are qualitatively quite similar to results obtained using logistic regression.



$$p = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k \Rightarrow -\infty < p < \infty$$



$$\ln(p) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k \Rightarrow 0 < p < \infty$$



$$\ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k \Rightarrow 0 < p < 1$$

The Logistic Model

- The logistic model takes the following form: $\ln\left(\dfrac{p}{1-p}\right) = \overbrace{\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k}^{z}$

  - The log of the odds is known as the logit (logistic) transform: $\text{logit}(p) = \ln\left(\dfrac{p}{1-p}\right)$

  - The logit transform can take on any value from $-\infty$ to ∞, and thus p is constrained to lie between 0 and 1.

- If we solve the logistic model for p,

$$\ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k \Rightarrow \frac{p}{1-p} = \exp\left(\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k\right)$$

$$p = (1-\text{p})\exp\left(\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k\right) \Rightarrow p(1+\exp\left(\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k\right)) = \exp\left(\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k\right)$$

$$p = \frac{e^{\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k}}{1 + e^{\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k}} = \frac{e^z}{1+e^z} = \frac{1}{1+e^{-z}}$$

where z = $\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k$ and $\lim\limits_{z \to +\infty} = 1$ *and* $\lim\limits_{z \to -\infty} = 0$

Interpretation of Parameters in the Logistic Model

- In general, β_j is the change in the outcome/response for a 1-unit change in X_j.

- In logistic regression, this is interpretable as the expected change in the natural logarithm of the odds
  - i.e. change in the log(odds)) for a 1-unit change in X_j

- Therefore $e^{\beta j}$ is the odds ratio associated with a 1-unit change in X_j
  - For every one-unit change in X_j the odds of success changes by $e^{\beta j}$ times.

- In logistic regression, we choose the parameters (β_j's) that maximize the likelihood of the observed data.
  - Where the likelihood is defined as:

$$L = \prod_{i=1}^{n} \frac{e^{y_i(\beta_0+\beta_1 X_{1i}+\ldots+\beta_k X_{ki})}}{1+e^{(\beta_0+\beta_1 X_{1i}+\ldots+\beta_k X_{ki})}}$$

  - Maximum likelihood estimation (more on this next time).

Assumptions for Logistic Regression

- Independence of observations.
- The logit is a linear (additive) function of X
    - The odds and odds ratios are multiplicative on the ratio scale

- Even if the true relationship is nonlinear, we can still assess first order (linear) trends in the log odds of response across groups defined by the predictors.

- In this situation, the odds ratio describes a "general trend" in the ratio over the distribution of the X.

- That is, "on average, the odds is exp(β) times larger for every unit increase in X".

- For the i^{th} subject, $Y|X_i$ is Bernoulli with Pr(success) = $\pi(x_i)$ and variance $\pi(x_i)(1- \pi(x_i))$
    - NOTE: The mean $\pi(x_i)$ is a probability that is a function of the covariates, X_i
    - NOTE: The variance $\pi(x_i)(1- \pi(x_i))$ is a function of the covariates, X_i (not homoscedastic)
- The errors, e, are binomial with mean 0 and variance $\pi(x)(1- \pi(x))$.

Example (2x2 Table Link to Logistic Regression)

Crude estimate of the association between exposure to passive smoke and cancer

Total Sample:

Passive Smoking	*Cancer* Yes (1)	No (0)	
Yes (1)	281	210	491
No (0)	228	279	507
	509	489	998

Odds Ratio = $(p_1/(1-p_1)) / (p_2/(1-p_2))$ = $Odds_{ex} / Odds_{un} = (p_1*(1-p_2))/(p_2*(1-p_1))$
For the estimate
=> ad/bc =(281*279)/(210*228)=1.6374

$$\ln\left(\frac{p}{1-p}\right) = \ln(odds) = \beta_0 + \beta_1 passive$$

What are the estimates of β_0 and β_1?

Estimated odds of cancer for unexposed = (228/507)/(279/507) = 228/279 = 0.81720

$$\ln(odds_{un}) = \ln(228/279) = -0.20187 = \hat{\beta}_0$$

Estimated odds of cancer for exposed = (281/491)/(210/491) = 281/210 = 1.33810

$$\ln(odds_{ex}) = \ln(281/210) = 0.29125 = \hat{\beta}_0 + \hat{\beta}_1$$

$$0.29125 = -0.20187 + \hat{\beta}_1$$

$$\hat{\beta}_1 = 0.49312$$

```
DATA smoke;
 INPUT smoke passive cancer n;
 DATALINES;
 0 1 1 120
 0 1 0  80
 0 0 1 111
 0 0 0 155
 1 1 1 161
 1 1 0 130
 1 0 1 117
 1 0 0 124
;
```

```
PROC LOGISTIC DESCENDING;
 MODEL cancer = passive
       /COVB;
 FREQ n;
RUN;
```

The DESCENDING option models the probability of response, where a positive response is indicated by the largest value of the outcome variable (e.g., '1' for a '0','1' coded variable; 'Yes' for a variable with format 'No', 'Yes').

```
PROC LOGISTIC;
 MODEL cancer (EVENT='1') = passive
       /COVB;
 FREQ n;
RUN;
```

In recent versions of SAS, you can directly specify the value of the outcome with the EVENT option.

Requests covariance matrix for the betas

SAS Output:

Probability modeled is cancer=1.

Always verify which value of the outcome is being modeled!

Analysis of Maximum Likelihood Estimates

	Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
$\hat{\beta}_0$	Intercept	1	-0.2019	0.0893	5.1128	0.0238
$\hat{\beta}_1$	passive	1	0.4931	0.1276	14.9262	0.0001

Odds Ratio Estimates

Effect	Point Estimate	95% Wald Confidence Limits	
passive	1.637	1.275	2.103

95% Wald CI is first calculated for the LN(OR) and then exponentiated:

95% CI for LOG(OR): $0.4931 \pm 1.96\ (0.1276)$

(0.2430, 0.7432)

95% CI for (OR): (EXP(0.2430), EXP(0.7432))

(1.2751, 2.1026)

Estimated Covariance Matrix

Parameter	Intercept	passive
Intercept	0.00797	-0.00797
passive	-0.00797	0.016291

Interpretation odds, ln(odds) and OR

- $\ln\left(\dfrac{p}{1-p}\right) = \ln(odds) = \beta_0 + \beta_1 passive$ => (-.2019) + .4931*passive

- The intercept estimates the ln(odds) of disease when all covariates are set to zero.

  - The ln(odds) of cancer for individuals not exposed to passive smoke (when *passive*=0) are –0.2019.

  - The odds of cancer for those not exposed to passive smoke are thus $e^{-0.2019} = 0.8099$.

Interpretation odds, ln(odds) and OR

- β_j is interpretable as a change in the ln(odds) associated with a 1-unit change in X_j.

  - The ln(odds) of cancer increase by 0.4931 for individuals exposed to passive smoke compared to individuals not exposed to passive smoke (i.e., when *passive* increases by one unit from 0 to 1).

  - The ln(odds) of cancer for those exposed to passive smoke (when *passive*=1) are $-0.2019 + 0.4931 = 0.2912$.

  - The odds of cancer for those exposed to passive smoke are thus $e^{-0.2019+0.4931} = e^{0.2912} = 1.338$.

- $e^{\beta j}$ is the odds ratio associated with a 1-unit change in X_j.

$$OR = \frac{e^{-0.2019+0.4931}}{e^{-0.2019}} = e^{0.4931} = 1.637$$

Interpretation: We estimate that the population of people exposed to passive smoke have 1.64 times greater odds of cancer compared to a population not exposed to passive smoke.

Example: Logistic Regression with a continuous predictor

Example: Post-Surgery Hypothermia (temperature ≤ 96.8º). Modeling Odds of Hypothermia by Length of Surgery (hours).

```
PROC LOGISTIC;
 MODEL hypotherm (EVENT = 'Yes') = surgtime;
RUN;
```

```
PROC FORMAT;
 VALUE hypotherm 0 = 'No'
                 1 = 'Yes';
RUN;
```

```
                         The LOGISTIC Procedure

                           Model Information

      Data Set                     WORK.BODYCOVERS
      Response Variable            hypotherm           Hypothermia
      Number of Response Levels    2
      Number of Observations       60
      Model                        binary logit
      Optimization Technique       Fisher's scoring


                            Response Profile
                   Ordered                       Total
                     Value     hypotherm     Frequency

                         1     No                   46
                         2     Yes                  14
```

MODEL 1:
Surgery Time

Probability modeled is hypotherm='Yes'. ←

Always verify which value of the outcome is being modeled!

```
                        Model Convergence Status
                Convergence criterion (GCONV=1E-8) satisfied.
```

Model Fit Statistics

Criterion	Intercept Only	Intercept and Covariates
AIC	67.193	64.982
SC	69.287	69.171
-2 Log L	65.193	60.982

The LOGISTIC Procedure

H_0: $\beta_1 = \beta_2 = \ldots = \beta_p = 0$

H_0: $\beta_{surgtime} = 0$

Testing Global Null Hypothesis: BETA=0

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	4.2105	1	0.0402
Score	4.5522	1	0.0329
Wald	3.9427	1	0.0471

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	-2.6943	0.8480	10.0948	0.0015
surgtime	1	0.8470	0.4266	3.9427	0.0471

$(0.8470/0.4266)^2$

$e^{0.847} = 2.33$

Odds Ratio Estimates

Effect	Point Estimate	95% Wald Confidence Limits	
surgtime	2.333	1.011	5.382

<u>Decision and Uncertainty:</u>

There is a significant association between length of surgery and post-surgery hypothermia ($p = 0.0471$).
[Length of surgery is a significant predictor of post-surgery hypothermia ($p = 0.0471$)].

<u>Point and Interval Estimate (Log-Odds):</u>

$$\boxed{0.847 \pm 1.96 \times 0.4266}$$

On average, the ln(odds) of hypothermia increase by 0.847 (95% CI: 0.0109, 1.6831) for every additional hour of surgery time.

<u>Point and Interval Estimate (Odds Ratio):</u>

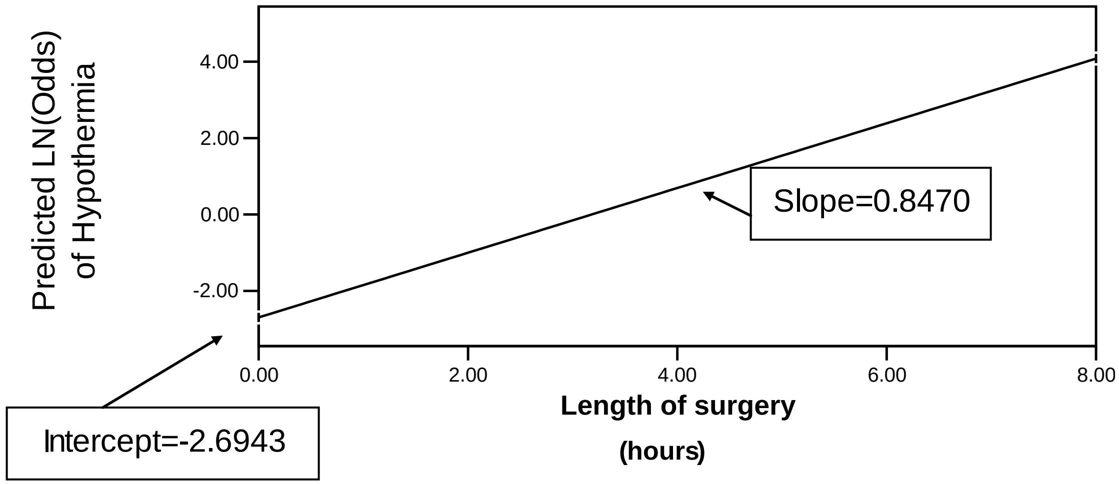
$$\boxed{(e^{0.0109},\ e^{1.6831})}$$

On average, the odds of hypothermia increase 2.33 times (95% CI: 1.011 to 5.382 times) for every additional hour of surgery time.

Fitted Model for Continuous Predictor

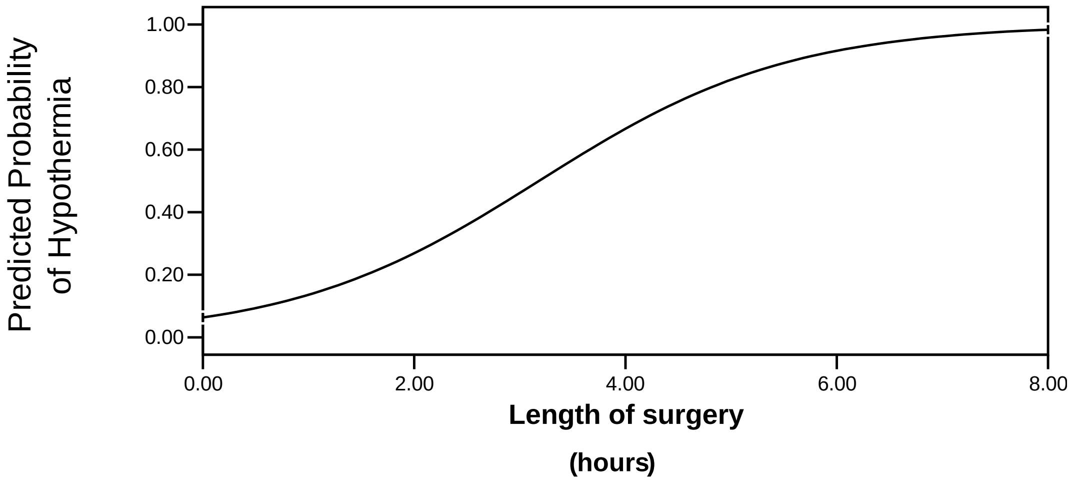
Predicted LN(Odds):

$$\text{LN(Odds)} = -2.6943 + 0.8470 \times \textit{surgtime}$$



Predicted Probability:

$$\hat{p} = \frac{e^{\left(-2.6943 + 0.8470 \times surgtime\right)}}{1+e^{\left(-2.6943 + 0.8470 \times surgtime\right)}}$$



Example: Logistic regression model examining factors associated with post-surgery hypothermia at PACU entry (tympanic temperature ≤ 96.8 degrees).

```
PROC LOGISTIC;
 MODEL hypotherm (EVENT= 'Yes') = age surgtime bmi blanket ;
RUN;
```

MODEL 2:
- Age
- Surgery Time
- BMI
- Use of Warmed Blanket

```
                    The LOGISTIC Procedure

                      Model Information
Data Set                      WORK.BODYCOVERS
Response Variable             hypotherm Hypothermia
Number of Response Levels     2
Number of Observations        60
Model                         binary logit
Optimization Technique        Fisher's scoring


                       Response Profile

              Ordered                        Total
                Value      hypotherm     Frequency
                    1      Yes                  14
                    2      No                   46

           Probability modeled is hypotherm='Yes'.

                    Model Fit Statistics
                                          Intercept
                             Intercept       and
           Criterion           Only       Covariates
           AIC                67.193         59.199
           SC                 69.287         69.670
           -2 Log L           65.193         49.199
```

Prediction

- We can use a multiple logistic-regression model to predict the probability of disease for an individual subject with covariate values $x_1,\ldots,x_k$.

- Logistic model: $\ln\left(\dfrac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k$

- If we solve for p, we get $p = \dfrac{e^{\beta_0+\beta_1 X_1+\ldots+\beta_k X_k}}{1+e^{\beta_0+\beta_1 X_1+\ldots+\beta_k X_k}}$

- For prospective/cohort studies, $\hat{p}$ is an estimate of the incidence of disease.

- For cross-sectional studies, $\hat{p}$ is an estimate of the prevalence of disease.

- For case-control studies, $\hat{p}$ is not interpretable unless the sampling fraction of cases and controls from the reference population is known, which is almost always *not* the case.

Example

What is the predicted probability of hypothermia at PACU entry for an individual 50 years old, with a BMI of 30, who underwent a 1.5-hour surgery with an unwarmed blanket? [Individual 1]

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	2.8881	2.6575	1.1810	0.2771
age	1	0.0143	0.0242	0.3465	0.5561
surgtime	1	1.0930	0.5332	4.2013	0.0404
bmi	1	-0.2413	0.0963	6.2738	0.0123
blanket	1	-0.9906	0.8068	1.5075	0.2195

$$\hat{p} = \frac{e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}}{1+e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}} = 0.1195$$

What is the predicted probability of hypothermia at PACU entry for an individual 50 years old, with a BMI of 30, who underwent a 3-hour surgery with an unwarmed blanket? [Individual 2]

$$\hat{p} = \frac{e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}{1+e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}} = 0.4114$$

Example

What are the relative odds of being hypothermic at PACU entry for individual 2 compared to individual 1?

$$\hat{OR} = \frac{\dfrac{\hat{p}_2}{1-\hat{p}_2}}{\dfrac{\hat{p}_1}{1-\hat{p}_1}} = \frac{\dfrac{0.4114}{1-0.4114}}{\dfrac{0.1195}{1-0.1195}} = 5.15$$

$$\hat{OR} = \frac{\left(\dfrac{\dfrac{e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}{1+e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}}{1-\dfrac{e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}{1+e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}}\right)}{\left(\dfrac{\dfrac{e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}}{1+e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}}}{1-\dfrac{e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}}{1+e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}}}\right)} = \frac{\left(\dfrac{\dfrac{e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}{1+e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}}{\dfrac{1}{1+e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}}\right)}{\left(\dfrac{\dfrac{e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}}{1+e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}}}{\dfrac{1}{1+e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}}}\right)}$$

$$= \frac{e^{2.8881+0.0143\times 50+1.0930\times 3.0-0.2413\times 30-0.9906\times 0}}{e^{2.8881+0.0143\times 50+1.0930\times 1.5-0.2413\times 30-0.9906\times 0}} = \frac{e^{2.8881}e^{0.0143\times 50}e^{1.0930\times 3.0}e^{-0.2413\times 30}e^{-0.9906\times 0}}{e^{2.8881}e^{0.0143\times 50}e^{1.0930\times 1.5}e^{-0.2413\times 30}e^{-0.9906\times 0}} = e^{1.0930\times(3-1.5)} = e^{1.0930\times(1.5)}$$

Example

What are the relative odds of being hypothermic at PACU entry for individual 2 compared to individual 1?

$$\text{OR}=e^{1.0930 \text{ x } 1.5}=5.15$$

Interpretation: For every 1.5-hour increase in surgery time, the odds of being hypothermic at PACU entry increase 5.2-fold after adjusting for age, BMI, and use of a warmed blanket.

- What is the odds ratio and 95% CI relating the additional risk of hypothermia per each 10-year increase in age after adjusting for other risk factors?

$$e^{0.0143\times 10}=1.1537 \Rightarrow e^{0.0143\times 10\pm 1.96(0.0242*10)}=(0.718,1.853)$$