Homework 8

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1. Model covariance matrix for subject i

$$G_i = \begin{pmatrix} \sigma_I^2 & \sigma_{IS}^2 \\ \sigma_{IS}^2 & \sigma_S^2 \end{pmatrix}$$

$$Z_i = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$Z_i^t = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$R_i = \begin{pmatrix} \sigma_e^2 & 0 & 0 \\ 0 & \sigma_e^2 & 0 \\ 0 & 0 & \sigma_e^2 \end{pmatrix}$$

$$V_i = Var(Y_i) = Z_i G_i Z_i^t + R_i = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \sigma_I^2 & \sigma_{IS}^2 \\ \sigma_{IS}^2 & \sigma_S^2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} + \begin{pmatrix} \sigma_e^2 & 0 & 0 \\ 0 & \sigma_e^2 & 0 \\ 0 & 0 & \sigma_e^2 \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_I^2 + \sigma_e^2 & \sigma_{IS}^2 + \sigma_I^2 & 2\sigma_{IS}^2 + \sigma_I^2 \\ \sigma_{IS}^2 + \sigma_I^2 & 2\sigma_{IS}^2 + \sigma_I^2 + \sigma_S^2 + \sigma_e^2 & 3\sigma_{IS}^2 + \sigma_I^2 + 2\sigma_S^2 \\ 2\sigma_{IS}^2 + \sigma_I^2 & 3\sigma_{IS}^2 + \sigma_I^2 + 2\sigma_S^2 & 4\sigma_{IS}^2 + \sigma_I^2 + 4\sigma_S^2 + \sigma_e^2 \end{pmatrix}$$

2. R code

```
mod1 <- lme(y ~ time, random=~time|id,data=dat)
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3. Is it possible for this structure to have covariance that decays as time between responses increases?

In order to show this, you compare the covariance for times 0 and 1, and for times 0 and 2. If there's more correlation between time 0 and 1 than there is between time 0 and 2, then cov(0,1) > cov(0,2). This turns out to be fairly easy to rearrange, and shows that there can be decay as time between measurements increases, as long as $\sigma_{IS}^2 < 0$

$$\sigma_{IS}^2 + \sigma_I^2 > 2\sigma_{IS}^2 + \sigma_I^2$$
$$\sigma_{IS}^2 > 2\sigma_{IS}^2$$
$$0 > \sigma_{IS}^2$$