Masters Theory III
Homework #5

Out: Thursday
Due / Quiz following Thursday

1. Casella and Berger problems 7.1, 7.2*, 7.6, 7.9

   * C&B 7.2 will require a numeric maximization since there is no closed form solution if both α and β are both unknown. I would like you to use R for this problem (code posted on Canvas). I will be happy to discuss this problem in detail during lab.

2. Lecture 11: Proof to theorem $\star$ (pages 8-9)

3. Additional R questions:

   (a) Run the binomial examples from Lecture 10 (Canvas). We should talk about how these functions were maximized (numerically) in lab. Which is better a maximum obtained by numeric methods (here a grid search) or those obtained using calculus? When might they differ?

   (b) Run the binomial examples where the likelihood is multiplied by a constant (Canvas). Does multiplying a likelihood by a constant (with respect to the parameter) change the maximum?

REVIEW PROBLEMS NOT ON THE QUIZ, BUT MAY BE ON EXAM

1. (C&B 7.10) The independent random variables $X_1, \ldots, X_n$ have the common distribution (cdf) for

$$\begin{aligned} P(X_i \le x|\alpha,\beta) &= 0 \text{ if } x < 0 \\ P(X_i \le x|\alpha,\beta) &= \left(\frac{x}{\beta}\right)^{\alpha} \text{ if } 0 \le x \le \beta \\ P(X_i \le x|\alpha,\beta) &= 1 \text{ if } x > \beta \end{aligned}$$

   where $\alpha > 0$ and $\beta > 0$.

   (a) Show that $\left(\prod X_i, X_{(n)}\right)$ is the two-dimensional sufficient statistic.

   (b) Show the MLEs of α and β are $\left[\frac{1}{n}\sum_{i=1}^{n}(logX_{(n)} - logX_i)\right]^{-1}$ and $X_{(n)}$. Remember to show that it is indeed a maximum.

   (c) The length in millimeters of cuckoo's eggs found in hedge sparrow nests can be modeled with this distribution. For the data below, find the MLE of α and β.
   22.0, 23.9, 20.9, 23.8, 25.0, 24.0, 21.7, 23.8, 22.8, 28.1, 23.1, 23.5, 23.0, 23.0

2. Suppose that the random variables $Y_1, \ldots, Y_n$ satisfy: $Y_i = \beta x_i + \epsilon_i, \quad i = 1, \ldots, n$ where $x_1, \ldots, x_n$ are known fixed constants (predictor covariates) and $\epsilon_1, \ldots, \epsilon_n$ are iid $N(0, \sigma^2)$.

   (a) Show that $Y_i \approx N(\beta x_i, \sigma^2)$.

   (b) Show that $(\sum Y_i^2, \sum x_i Y_i)$ is a complete sufficient statistic.

   (c) For a fixed σ^2, find the MLE of β and show that it is an unbiased estimator of β.

   (d) Find the dist'n of $\hat{\beta}$ (Hint: corollary 4.6.10).