ANOVA models and hypotheses
Lecture 10 (Supplement)

Effects and hypotheses: myostatin example

- Table for population mean leucine protein levels for `group*time` combinations

	time			
group	24h	48h	72h	
C	μ_{11}	μ_{12}	μ_{13}	$\mu_{1\cdot}$
M	μ_{21}	μ_{22}	μ_{23}	$\mu_{2\cdot}$
	$\mu_{\cdot 1}$	$\mu_{\cdot 2}$	$\mu_{\cdot 3}$	$\mu_{\cdot\cdot}$

- Write null hypotheses for the following tests:
  1. some difference in means
  2. main effect of Time
  3. main effect of Myostatin
  4. Time$\times$ Myostatin interaction

Cell means model

- Mean for the combination of group i and time j is μ_{ij}
- Mean for group i (ignoring time) is $\mu_{i\cdot}$
- Mean for time j (ignoring group) is $\mu_{\cdot j}$
- Overall (grand) mean is $\mu = \mu_{\cdot\cdot}$

Factor effects model

Can write the mean for each group and time combination as

$$\mu_{ij} = \mu + \alpha_i + \tau_j + \gamma_{ij}$$

- $\alpha_i = \mu_{i\cdot} - \mu$ is main effect for group at ith level
- $\tau_j = \mu_{\cdot j} - \mu$ is main effect for time at jth level
- $\gamma_{ij} = \mu_{ij} - \mu_{i\cdot} - \mu_{\cdot j} + \mu$ is interaction effect when group is at ith level and time is at jth level
- Interaction effect can also be written as $\gamma_{ij} = \mu_{ij} - (\mu + \alpha_i + \tau_j)$

Null hypotheses of interest

1. some difference in means
   - $H_0 : \mu_{ij} = \mu$ for all i, j (cell means model)
   - $H_0 : \alpha_i = \tau_j = 0$ for all i, j (factor effects model)
2. main effect of Time
   - $H_0 : \mu_{\cdot 1} = \mu_{\cdot 2} = \mu_{\cdot 3}$ (cell means model)
   - $H_0 : \tau_j = 0$ for all j (factor effects model)
3. main effect of Myostatin
   - $H_0 : \mu_{1\cdot} = \mu_{2\cdot}$ (cell means model)
   - $H_0 : \alpha_i = 0$ for all i (factor effects model)
4. Time $\times$ Myostatin interaction
   - $H_0 : \mu_{ij} - \mu_{i\cdot} - \mu_{\cdot j} + \mu = 0$ (cell means model)
   - Alternatively, null could be stated as the differences between group means being the same at each time,
     $H_0 : \mu_{11} - \mu_{21} = \mu_{12} - \mu_{22} = \mu_{13} - \mu_{23}$
   - $H_0 : \gamma_{ij} = 0$ for all i, j (factor effects model)

Degrees of freedom: two-factor effects model I

- In the cell means model, there are 6 means being estimated
  - $J = 3$ time levels
  - $I = 2$ myostatin levels
  - $IJ = 6$ cell means
- In the factor effects model
  - The total degrees of freedom is $n - 1 = 23$: one degree of freedom is lost for estimating the intercept
  - For the Time factor, we have $J = 3$ levels, so there are $J - 1 = 2$ degrees of freedom associated with this factor
  - For the Myostatin factor, we have $I = 2$ levels, so there are $I - 1 = 1$ degrees of freedom associated with this factor
  - The degrees of freedom for the interaction is equal to the product of the degrees of freedom for each factor, or $(I - 1)(J - 1) = 1 \times 2 = 2$
  - The "model" degrees of freedom is equal to $(I - 1) + (J - 1) + (I - 1)(J - 1) = 5$ (number of non-intercept parameters being estimated, so the sum of degrees of freedom in the first factor, second factor, and interaction)

Degrees of freedom: two-factor effects model II

- The error term has degrees of freedom is equal to the sample size minus the number of parameters being estimated
  - $n - IJ = 24 - 6 = 18$ for cell means model
  - $n - 1 - (I-1) - (J-1) - (I-1)(J-1) = 24 - 6 = 18$ for factor effects model

$\Rightarrow$ Using either model, there are 18 error degrees of freedom: whether you think about each cell of the table having its own mean (cell means model) or about an intercept and then an effect for each factor and their interaction (factor effects model), there are only

$$IJ = 1 + (I-1) + (J-1) + (I-1)(J-1)$$

uniquely estimable parameters.