Homework 2

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1 BD 1.1.1

1. Example (a)

   (a) Here let X be a R.V. indicating the diameter of a pebble and $Y = log(X)$. The logarithm of the diameter is normally distributed, so:
   $$P_Y(Y) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}(\frac{y-\mu}{\sigma})^2}$$
   To find the distribution of X, we can do a simple transformation using $\frac{d}{dx}Y = \frac{1}{X}$ and see that
   $$P_X(X) = \frac{1}{x\sqrt{2\pi}\sigma}e^{-\frac{1}{2}(\frac{log(x)-\mu}{\sigma})^2}$$

   (b) Pebble diameters must be $X \in (0, \infty)$, so $-\infty < log(X) < \infty$. Because we are assuming $log(X) \sim \mathcal{N}(\mu, \sigma^2)$, $-\infty < \mu < \infty$ and $\sigma > 0$.

   (c) This is a parametric model because we are assuming a distribution for the pebble diameters.

2. Example (b)

   (a) For this example we have the model $X_i = \mu + \epsilon_i$, for $1 \leq i \leq n$ and $\epsilon \sim \mathcal{N}(0.1, \sigma^2)$. Therefore
   $$P_X(X) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}(\frac{x-\mu+0.1}{\sigma})^2}$$

   (b) In this case the variance of the errors is known, so the parameter space is $\mu \in R$.

(c) This is also a parametric model because we are assuming a distribution for the errors.

3. Example (c)

    (a)

    (b)

    (c)

1.1 1.1.2