Homework 8
BIOS 7731
Due 11/10 10:30am through Canvas.
Students may work together on homework assignments, but the assignment handed in must represent your own work. Problems based on a later lecture are labeled with *.

Update: 3 Nov 2020: Everyone has decided to complete problem 0 as part of this assignment so as a result this assignment is quite long. I have decided to shorten it as follows:

- *Question 1:* You do not need to answer part (c).
- *Question 6:* This problem is now optional. I have left it on the assignment for extra credit. I have also added a hint.

0. **BD 5.3.28 part (a), pg 354** *This is problem number 5 from assignment 7. You do not have to resubmit it if you already included your solution in your submission of assignment 7.*

1. **BD 5.3.8 parts (a),(b) ~~,(c)~~ , pg 349**

2. **BD 5.3.18, pg 352***

3. **BD 5.3.20, pg 352***
   Hint: Use the multivariate version of the delta method.

4. **BD 5.4.1 part (c), pg ~~352*~~ 356***

5. Suppose that $k_n[\delta_n - g(\theta)]$ converges in distribution to a continuous limit distribution H. Prove that:
   (a) If $k_n'/k_n \to d \neq 0$ then $k_n'[\delta_n - g(\theta)]$ also converges to a continuous limit distribution.
   (b) If $k_n'/k_n \to 0$ or ∞, then $k_n'[\delta_n - g(\theta)]$ converges in probability to zero or infinity, respectively.
   (c) If $k_n \to \infty$, then $\delta_n \to_p g(\theta)$.

6. **Extra Credit.** Suppose $\rho(x)$ is an even function, nondecreasing and non-negative for $x \geq 0$ and positive for $x > 0$. Then, $E\{\rho[\delta_n - g(\theta)]\} \to 0$ for all θ implies that δ_n is consistent for estimating $g(\theta)$.

Hint: Prove the following proposition and then use this lemma to show consistency of δ_n.

Prop: Let $g \geq 0$ be an even function (i.e., $g(-x) = g(x)$) which is increasing on $[0, \infty)$. Then for all random variables X and all $\epsilon > 0$,

$$P(|X| \geq \epsilon) \leq \frac{Eg(X)}{g(\epsilon)}$$