Homework 7
BIOS 7731

Due 10/29 10:30am through Canvas. Students may work together on homework assignments, but the assignment handed in must represent your own work. Problems based on a later lecture are labeled with *.

1. (a) Suppose we toss a fair ($p = 0.5$) coin n times. Use the Chebychev inequality to find how many times a coin must be tossed in order that the probability will be at least 0.90 that the observed frequency of heads will lie between 0.4 and 0.6.
   (b) Alternatively, use the normal approximation to determine the number of of tosses required in part a). How do the results in part a) and b) compare?

2. Let sample space $S = [0, 1]$ with the uniform density function and define $Z(s) = s$ and $Z_1, Z_2 \ldots$ as

$$Z_1(s) = s + I_{[0,1]}(s), \quad Z_2(s) = s + I_{[0,1/2]}(s), \quad Z_3(s) = s + I_{[1/2,1]}(s)$$
$$Z_4(s) = s + I_{[0,1/3]}(s), \quad Z_5(s) = s + I_{[1/3,2/3]}(s), \quad Z_6(s) = s + I_{[2/3,1]}(s)$$

   (a) Does $Z_n \to^{\mathcal{P}} Z$?
   (b) Does $Z_n \to^{a.s.} Z$?

3. Let F be a cdf and let F^{-1} denote its left-continuous inverse.
   (a) Show that $F \circ F^{-1}(t) \geq t$ for all $0 < t < 1$ with equality iff t is in the range of F. Hence $P(F(X) \leq t) \leq t$.
   (b) Show that $F^{-1} \circ F(x) \leq x$ for all $x \in (-\infty, \infty)$ with strick inequality iff $F(x - \epsilon) = F(x)$ for some $\epsilon > 0$. Hence $P(F^{-1} \circ F(X) \neq X) = 0$ for $X \sim F$.

4. BD 5.3.13, pg 351*

5. BD 5.3.28, pg 354*