Homework 5
BIOS 7731
Due 10/15 10:30am through Canvas. Students may work together on homework assignments, but the assignment handed in must represent your own work.

1. BD 3.2.3 (See lecture notes from 10/8 for solution to BD 3.2.2)
2. BD 3.2.5 a) b)
3. BD 3.2.8 a) c)

   Hint: For part a), you need to find the minimum conditional Bayes risk (also known as the posterior risk from class notes). You still need to solve this part but as a hint it should simplify to:
   $$r(\delta^*|X) = \sum_{j=1}^{r} c_j^2 \frac{\beta_j(\beta_0 - \beta_j)}{\beta_0^2(\beta_0+1)} - 2\sum_{j<k} c_j c_k \frac{\beta_j \beta_k}{\beta_0^2(\beta_0+1)},$$
   where $\beta_j = n_j + \alpha_j$ and $\beta_0 = \sum_{j=1}^{r} \beta_j = \alpha_0 + n$.
4. (Old Exam Question) Consider a Bayesian model in which the random parameter Θ has a Bernoulli prior distribution with success probability $\frac{1}{2}$. That is,
   $$\pi(\theta) = \begin{cases} \frac{1}{2}, & \theta = 0; \\ \frac{1}{2}, & \theta = 1. \end{cases}$$
   Given $\theta = 0$, the random variable X has density f_0 and given $\theta = 1$, X has density f_1.
   (a) Find the Bayes estimate (aka Bayes rule) of θ under squared loss.
   (b) Find the Bayes estimate (aka Bayes rule) of θ if $L(\theta, d) = I\{\theta \neq d\}$ (zero-one loss).
5. Let $X_1, \ldots X_n$ be iid Uniform$(0, \theta)$, with $\theta > 0$.
   (a) Show that the density of the largest order statistic $X_{(n)}$ is $p(x, \theta) = n\theta^{-n}x^{n-1}I_{(0,\theta)}(x)$, where $I_{(0,\theta)}(x)$ is an indicator for $0 < x < \theta$.
   (b) Find an unbiased estimator for θ based on $X_{(n)}$ and determine its variance. Note that $X_{(n)}$ is complete and sufficient for θ, so any function of $X_{(n)}$ that is an unbiased estimator of θ is UMVU. (Read Definition 6.2.21, Example 6.2.23 and Theorem 7.3.23 in Casella & Berger)
   (c) Find the Fisher information matrix $I(\theta)$. Show that the Fisher information inequality does NOT hold for the UMVU estimator in b).
6. Let $X_1, \ldots, X_n$ be iid $N(\theta, 1)$
   (a) Show that an unbiased estimator of θ^2 is $\bar{X}^2 - \frac{1}{n}$.
   (b) Calculate its variance and show that it is greater than the Information Inequality Bound.

   Hint: Use Stein's Lemma

   Let $X \sim N(\theta, \sigma^2)$, and let g be a differentiable function satisfying $E|g'(X)| < \infty$. Then
   $$E[g(X)(X - \theta)] = \sigma^2 E[g'(X)].$$