HW 2 (continued)

1.3.19. a) Consider a state of nature with 2 possible values, θ_1 and θ_2. Next consider the loss function for two actions a_1 and a_2:

$\ell(\theta, a)$:

	a_1	a_2
θ_1	0	2
θ_2	3	1

and X a RV with $p(x|\theta) =$

	$X=0$	$X=1$
θ_1	0.2	0.8
θ_2	0.4	0.6

The non-randomized decision rules $\delta_i(x)$ can be shown in a table:

$i=$	1	2	3	4
$X=0$	a_1	a_1	a_2	a_2
$X=1$	a_1	a_2	a_1	a_2

So, risk can be calculated (see BD pg 25) as:

$$R(\theta, \delta) = E[\ell(\theta, \delta(x))] = \ell(\theta, a_1)\, P[\delta(x) = a_1] + \ell(\theta, a_2)\, P[\delta(x) = a_2]$$

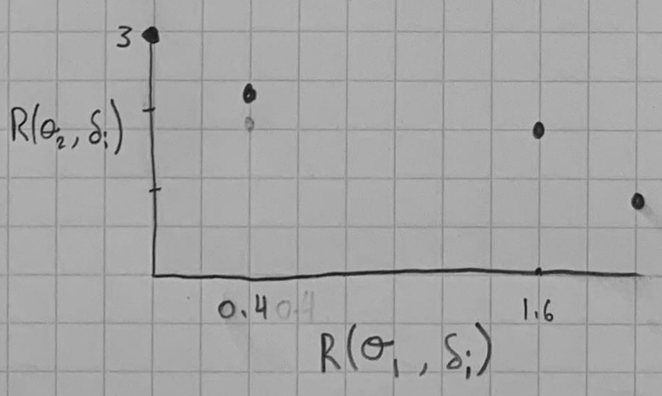
And we can construct a risk table:

$i=$	1	2	3	4
$R(\theta_1, \delta_i)$	$0(0.2) + 0(0.8)$	$0(0.2) + 2(0.8)$	$2(0.2) + 0(0.8)$	$2(0.2) + 2(0.8)$
$R(\theta_2, \delta_i)$	$3(0.4) + 3(0.6)$	$3(0.4) + 1(0.6)$	$1(0.4) + 3(0.6)$	$1(0.4) + 1(0.6)$

which equals:

$i=$	1	2	3	4
$R(\theta_1, \delta_i)$	0	1.6	0.4	2
$R(\theta_2, \delta_i)$	3	1.8	2.2	1

A plot:



The minimax rule is the one in which

$$\sup_{\theta} R(\theta, \delta^*) = \inf_{\delta} \sup_{\theta} R(\theta, \delta)$$

$$\sup_{\theta} R(\theta, \delta_{1-4}) = 3,\ 1.8,\ 2.2,\ 2$$

$\therefore$ δ_2 is minimax rule.

Now assume a prior distribution for θ s.t. $\pi(\theta_1) = 0.1$ and $\pi(\theta_2) = 0.9$. By BD 1.3.10, the Bayes risk is:

$$r(\delta_i) = 0.1\, R(\theta_1, \delta_i) + 0.9\, R(\theta_2, \delta_i)$$

Therefore our Bayes Risk table is:

$i =$	1	2	3	4
$r(\delta_i)$	$0.1(0)+0.9(3)$	$0.1(1.6)+0.9(1.8)$	$0.1(0.4)+0.9(2.2)$	$0.1(2)+0.9(1)$
$r(\delta_i) =$	2.7	1.78	2.02	1.1

So, δ_4 is the Bayes rule for this prior.

1.1.2. b) For estimation of λ set:

$$P_X(x \mid \lambda_1) = \frac{e^{-\lambda_1} \lambda_1^x}{x!} = \frac{e^{-\lambda_2} \lambda_2^x}{x!} = P_X(x \mid \lambda_2)$$

It's obvious that for any value of $x = 0, 1, \dots$ the densities are the same if $\lambda_1 = \lambda_2$

$$\frac{e^{-\lambda_1} \lambda_1^x}{x!} \Big/ \frac{e^{-\lambda_2} \lambda_2^x}{x!} = \left(\frac{e^{-\lambda_1} \lambda_1}{e^{-\lambda_2} \lambda_2} \right)^x$$

The same can be said for the second part of the model:

$$P_Y(Y \mid n) = \binom{n}{y} p_1^y (1-p_1)^{n-y} = \binom{n}{y} p_2^y (1-p_2)^{n-y} = P(Y \mid n, p_2)$$

when n is a known integer, and therefore not a parameter, $p_1 = p_2$ implies $P(Y \mid n, p_1) = P(Y \mid n, p_2)$. However, if n is unknown and therefore a parameter, there are combinations of (n, p) that will make $P_Y(Y \mid n_1, p_1) \neq P_Y(Y \mid n_2, p_2)$.