# BIOS 7731 HW 6

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## BD 3.5.11

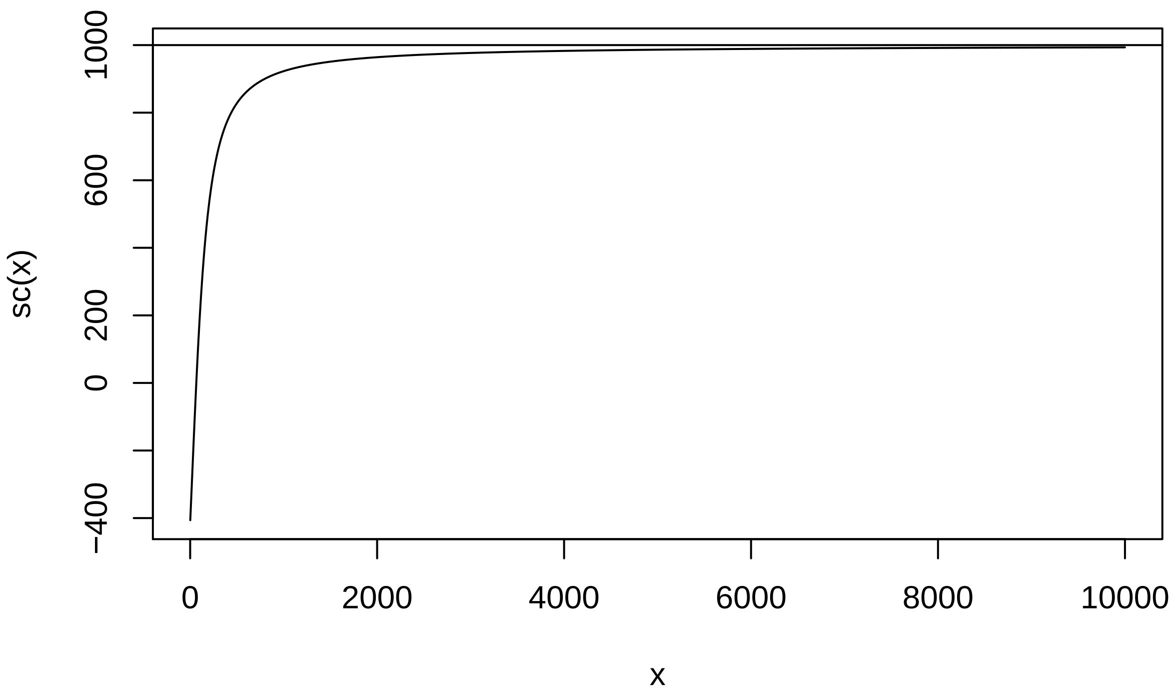
If we set $\mu_0 = 0$ and the ideal sample mean of $x_1, ..., x_{n-1}$, $\bar{X}_{n-1} = 0$, then the sensitivity curve of $t = \frac{\sqrt{n}(\bar{x}-\mu_0)}{\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}}$ simplifies to:

$$sc(x) = n(\frac{\sqrt{n}(\bar{X}-\mu_0)}{\sqrt{\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}}} - 0) = n[\frac{\sqrt{n}(\bar{X})}{\sqrt{\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}}}]$$

### a)

From this we can see that the limit of $sc(x)$ as $|x| \to \infty$ is 1, assuming $n$ is fixed. When the observation $x$ is added to the ideal sample with sample mean 0, the new sample mean is pushed away from 0 (with the direction depending on the sign of $x$). As $x$ gets extremely large, the function approaches $n\frac{\bar{X}}{\sqrt{\bar{X}^2}} = n$ due to the Law of Large Numbers. In order to check this, I wrote some quick R code:

```r
set.seed(1017)
# Make n-1 sample with mean 0 (or close enough)
xn_1 <- rnorm(999,0,5)
# N
n <- length(xn_1)+1
# Values of x going toward infinity
xs <- 1:10000
# SC function
sc <- lapply(xs, function(x){
  xn <- c(xn_1,x)
  stat <- n*sqrt(n)*mean(xn)/sd(xn)
  stat
})
# Plot
plot(xs,unlist(sc),type = "l",xlab = "x",ylab = "sc(x)")
abline(n,0)
```




**b)**

It's a little more obvious to see the limit of $sc(x)$ as $n \to \infty$ with $x$ fixed. The function can be rearranged to $[\frac{n\sqrt{n}\sqrt{n-1}(\bar{X})}{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2}}]$. With $x$ fixed this is increasing in $n$, so the limit as $n$ approaches $\infty$ does not exist.

So, the t-ratio is robust as a function of $x$, but not $n$.