January 20 2015

[37%] 1. For $k = 1, 2, \ldots,$ let $A_k = \{ a^n \mid n \geq 0,\ n \text{ is divisible by } k \}$.

(a) Show that A_2 and A_3 are regular, by drawing a DFA that accepts each of these.

(b) Show that $A_2 \cap A_3$ is regular by constructing a DFA that accepts it, using the product construction. Give its transition table.

(c) Show that $A_2 \cup A_3$ is regular similarly, using the product construction.

(d) Show that $A_2 \cup A_3$ is regular by another method, using an NFA.

[31%] 2. Is each of the following languages regular?
If it is, give a regular expression; if not, give a proof.
(**Note:** You may assume that the intersection of two regular sets is regular.)

(a) $A = \{ a^m b^n \mid m \leq 2n \}$.

number of a in x

(b) $B = \{ x \in \{a,b\}^* \mid \#a(x) \leq 2, \}$.

(c) $C = \{ x \in \{a,b\}^* \mid \#a(x) \leq 2 \times \#b(x) \}$.

[27%] 3. Give CFGs for the following sets of strings:

(a) the set of palindromes over $\{a, b\}$

(b) $\{ a^n b^m \mid n \leq m \leq 2n \}$ $1 \leq m \leq 2$

(c) $\{ a^n b^m c^n \mid n, m \geq 0 \}$

[10%] 4. Let Σ be a finite alphabet, and $A \subseteq \Sigma^*$.
Prove or disprove: If A is finite, then A is regular.

THE END

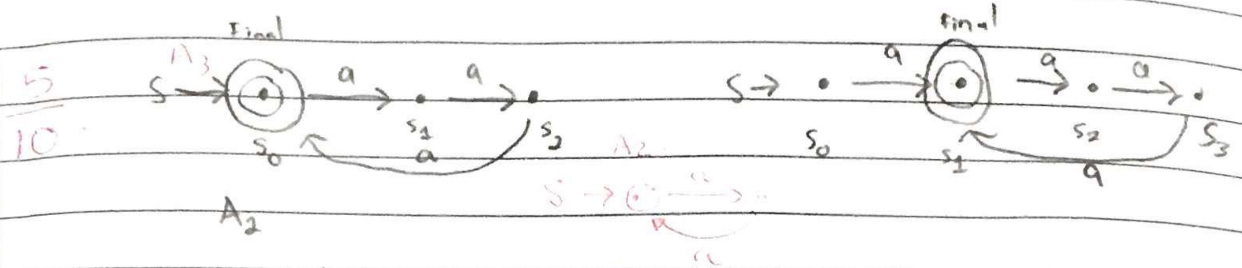
Yi Wang
1317934
T01

Sasan

CS2FA3

1. a) $A_2 = \{a^n \mid n \geq 0,\ n \text{ is divisible by } 2\}$
$A_3 = \{$ " " $3\}$

5/10



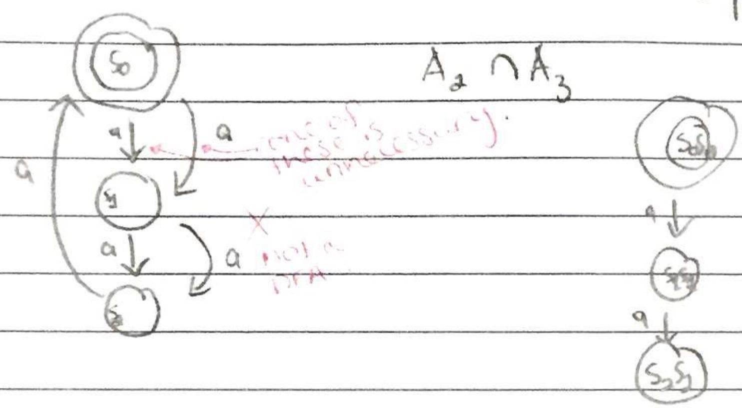
b)

	a
F → S → S_0	S_1
S_1	S_2
S_2	S_0

	a
S → S_0	S_1
F → S_1	S_2
S_2	S_3
S_3	S_0

$A_2 \cap A_3$
$= \{(S_0, S_1)\}$

$A_2 \cap A_3$



0/10

c) ~~$A_2 \cup A_3 = \{(S_0,S_0), (S_1,S_1), (S_2,S_2), (S_1,S_3), (S_1,S_2)\}$~~

4/8

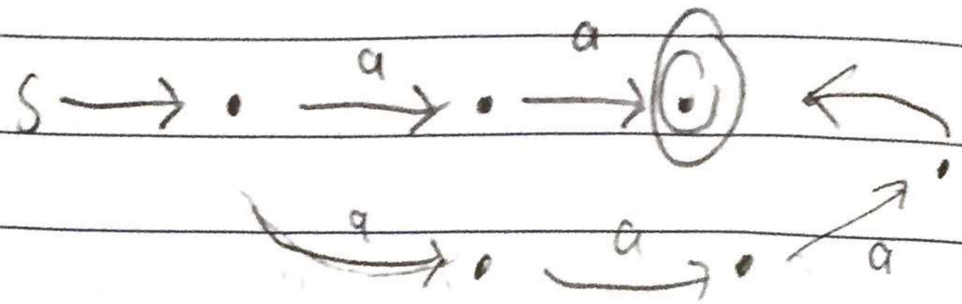
	a
(S_0, S_0)	(S_1, S_1)
(S_1, S_1)	(S_2, S_2)
(S_2, S_2)	(S_0, S_3)
(S_0, S_3)	(S_1, S_0)
(S_1, S_0)	(S_2, S_1)
(S_2, S_1)	(S_0, S_2)
(S_0, S_2)	(S_1, S_3)
(S_1, S_2)	(S_2, S_0)

→

	a
(S_2, S_0)	(S_0, S_1)
(S_0, S_1)	(S_1, S_2)
(S_1, S_2)	(S_2, S_3)
(S_2, S_3)	(S_0, S_0)

NFA

d)



$m \le 2k$

cng $\qquad k \qquad \frac{1}{2}k$

2. a) irregular

$A = \{a^m b^n \mid m \le 2n\}$

Assume A is regular

let $k >$ number of states of DFA M where $L(M) = A$

$S = a^{k} b^{k}$ $\quad$ ($2k$, $2k$)

$x = \varepsilon \qquad y = a^{2k} \qquad z = b^{2k}$

By P.L

$\exists\, u, v, w$ where $v \neq \varepsilon$ such that

$y = uvw$

and $\forall i \ge 0$

$y = uv^iw$

let $|v| = l \qquad l > 0 \quad (v \neq \varepsilon)$

$S_i = x\,uv^iw\,z$

$S_0 = x\,uv^0w\,z$

$= x\,uw\,z$

$= a^{k-l}\, b^{2k}$ $\qquad$ 8/13

$S_1 = x\,uv^1w\,z$

$= x\,uvw\,z$

$= a^{k+l}\, b^{2k}$

$S_2 = x\,uv^2w\,z \qquad k + 2l > 2k$

$= a^{k+2l}\, b^{2k}$

$\therefore$ contradiction

A is not regular

b) $B = \{x \in \{a,b\}^* \mid \#a(x) \leq 2\}$

Regular given by the following regular expression

10/8 $b^* + ab^* + aab^* + b^*a + b^*aa$

~~$C = \{x \in \{a,b\}^* \mid \#a(x) \leq 2 \times \#b(x)\}$~~

c) Irregular

~~Assume C is regular~~

let $k >$ the number of stats of M where

$L(M) = C$

~~$S = a^{2k} b^k$~~

~~$x = \varepsilon \quad y = a^{2k} \quad z = b^k$~~

~~By P.L~~

$\exists u, v, w$ where $v \neq \varepsilon$

$y = uvw$

and $\forall i \geq 0$

$y = uv^i w$

let $|v| = l \quad l > 0 \quad (v \neq \varepsilon)$

$S_i = x u v^i w z$ 7/10

~~$S_0 = xuv^0wz$~~

$= a^{k-l} b^k$

$S_2 \neq xuv^2wz$

$= a^{k+2l} b^k \qquad k + 2l > 2k$

$\therefore$ Contradiction

~~C is not a regular~~

language

b) $B = \{x \in \{a,b\}^* \mid \#a(x) \leq 2\}$

Regular given by the following regular expression

4/8

$b^* + ab^* + aab^* + b^*a + b^*aa$

~~$C = \{x \in \{a,b\}^* \mid \#a(x) \leq 2 \times \#b(x)\}$~~

c) Irregular

~~Assume C is regular~~

let $k >$ the number of states of M where $L(M) = C$

~~$S = a^{2k} b^k$~~

$x = \varepsilon \quad y = a^{2k} \quad z = b^k$

~~By P.L~~

~~$\exists u, v, w$~~ where $v \neq \varepsilon$

~~$y = uvw$~~

~~and $\forall i \geq 0$~~

~~$y = uv^i w$~~

let $|v| = l \quad l > 0 \quad (v \neq \varepsilon)$

$S_i = x u v^i w z$

7/10

~~$S_0 = x u v^0 w z$~~

~~$= a^{k-l} b^k$~~

~~$S_2 = x u v^2 w z$~~

~~$= a^{k+2l} b^k$~~

$k + 2l > 2k$

$\therefore$ ~~Contradiction~~

~~C is not a regular~~

~~language~~

3. a) 4/9 $S \to aSa \mid bSb \mid \varepsilon \mid a \mid b \mid \cancel{ab} \mid \cancel{ba}$

b) $S \to \varepsilon \mid aSbb \mid aSb$

c) $S \to B \mid aSc$ 9/9

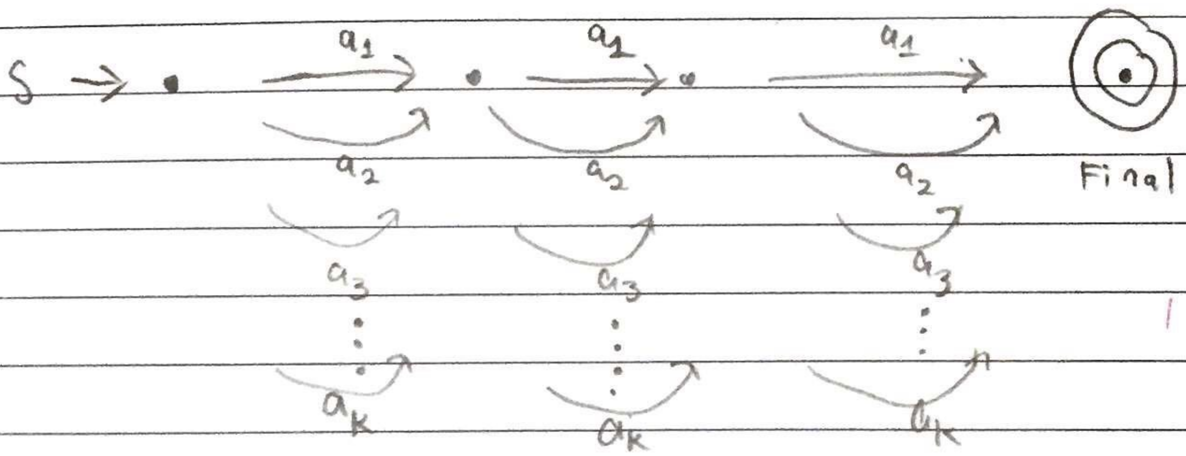
$B \to \varepsilon \mid b \mid Bb$

4. Σ be a finite alphabet

$A \subseteq \Sigma^*$

~~If A is finite~~ then A is regular

Suppose for an arbitary size of alphabet, you can construct the following DFA



10/10

~~let k represent the number of elements~~ in Σ

$\therefore$ you can construct a DFA over a finite alphabet

$\therefore$ $A \subseteq \Sigma^*$ is regular

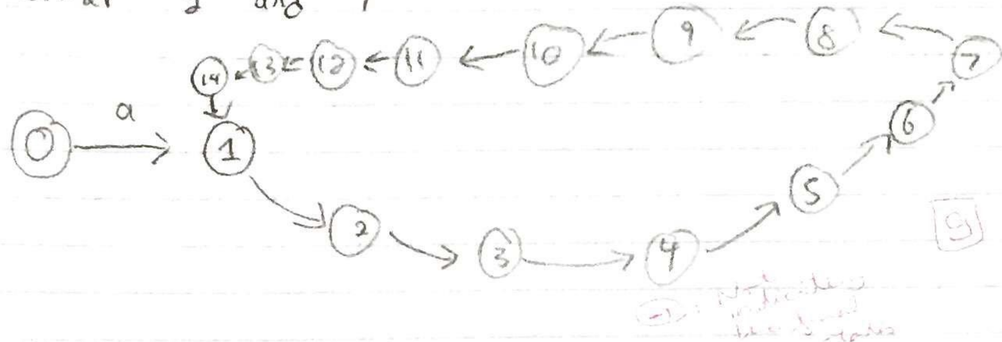
Yi wang
1317934

Jan 22 2015

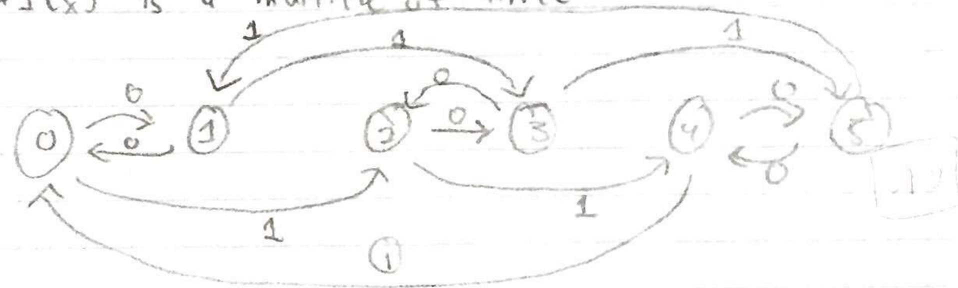
22/30

CS 2FA3 Assignment 1

1. b) DFA - set of strings in $\{a\}^*$ whose length is divisible by either 2 and 7



c) DFA - the set of strings $x \in \{0,1\}$ such that $\#0(x)$ is even and $\#1(x)$ is a multiple of three



2. Given an alphabet Sigma, are the following subsets of Σ^* regular or not

a) The empty set

The empty set is regular, since it is a subset of all sets and will always be closed under $\cup$ (union), $\cdot$ (concatenation)

b) The whole Σ^*

Σ^* is composed of the finite alphabet Σ. You can think of the composed languge with each letter as a state. knowing this you can draw a DFA and it will closed. Therefore it is also a regular language

Yi wang
1317934

January 28 2015

17/35

Comp sci 2FA3 Assignment 2

pg 302 2. For any $A \subseteq \Sigma^*$ if A is regular so is rev A

If A is regular, assume the following NFA (5)

$S \rightarrow \underset{s_1}{\cdot} \xrightarrow{a} \underset{s_2}{\cdot} \xrightarrow{b} \odot$ $\{ab\}$

The NFA of rev $\{ab\}$ = $\{ba\}$ is

$S \rightarrow \underset{s_1}{\cdot} \xrightarrow{b} \underset{s_2}{\cdot} \xrightarrow{a} \odot$

✗ This is an example, not a proof

∵ A Finite automata can be constructed for rev A for any $A \subseteq \Sigma^*$ where A is regular

∴ rev A is also regular

pg 315 2a)

	a	b
→ 1	2	2
2F	1	1

	a	b
→ 1	1	2
2F	2	1

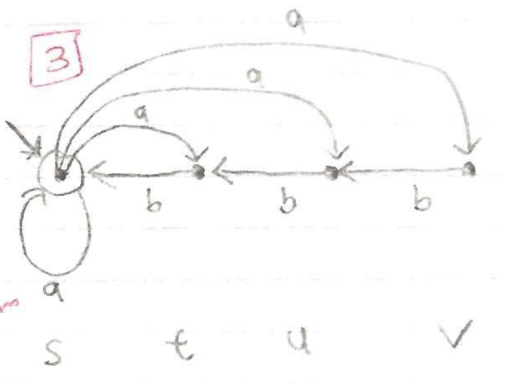
14

	a	b
(1,1)	(2,1)	(2,2)
(2,1)	(1,1)	(1,2)
(2,2)	(1,2)	(1,1)
(1,2)	(2,2)	(2,1)

i) $F_n = \{(2,2)\}$

ii) $F_n = \{(1,1), (2,1), (2,2), \cancel{(1,2)}\}$ (−1)

NFA 3



pg316 3. a) "abbb" 1 (−4) This string will be accepted

b)

	a	b
S→ F→ {s}	{s,t}	~~{∅}~~
{s,t}	{s,t,u}	{s,t}
{s,t,u}	{s,t,u,v}	{s,t,~~u~~}
{s,t,u,v}		
∅	∅	∅

2 (−1): wrong transitions

(−2): missing rows

DFA

•

Yi wang
1317934

10/35

February 5 2015

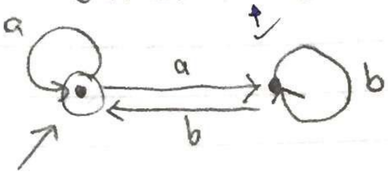
Comp SCI 2FA3 Hw 3

p303 1. a) $(a.a)^*$ X what about 'aabaa'?

$\frac{2}{15}$ b) $b.(b.b)^*$ X what about 'bbbaa'?

c) $(a.a)^* + (b.(b.b)^*)$ X

p316 4b)



$a^* + (ab)^* + (a.b^*.b)^*$ X what about 'aababbbbaaa'?

$\frac{2}{10}$

$(a + ab^*b)^*$

subset of $\{a, b\}^*$

p319 11. a) b^* ✓ $\{x \mid x$ does not contain the substring $a\}$

$\frac{6}{10}$ b) $(a+b)^* a^* + ba$ X $\{x \mid x$ does not contain the substring $ab\}$

this would accept 'aba' it contains the substring 'ab'!

Yi wang

1317934

February 12 2015

$\frac{16}{30}$

CS 2FA3 Hw 4

p 304 1. a) $A = \{a^n b^m \mid n = 2m\}$

Assume A is regular $m = 2m$

$\exists$ DFA M such that $L(M) = A$

choose $k >$ num of states of M ✓

$s = a^k b^{2k}$ ✓

$= xyz$ ✓

$x = \varepsilon$ $\underbrace{y = a^k}$ $z = b^{2k}$ ✓

$y = uvw$

$|y| = k$ ✓

$|uvw| = k$

By pumping lemma ✓ $i = 0, 1, 2, \ldots$

$s_i = xuv^iwz \in A$ ✓

$|v| = l$ $1 < l \leq k$

$\frac{9}{10}$ $v \neq \varepsilon$

$s_0 = xuv^0wz$ $k - l \neq k$

$= a^{k-l} b^{2k}$ ✓ so $xy^0z \notin L$

$\therefore$ A is not regular

1. b). $A = \{ x \in (a,b,c)^* \mid x$ is a palindrome, i.e $x = rev(x) \}$
Assume A is regular

$n \geq$ # of states in...

$S = a^n b a^n$ follows the form of a palindrome
an b is $\in$ of our set

$x = \varepsilon$ $y = a^n$ $z = ba^n$
$y = uvw$ ✓

By Pumping lemma ✓

$S_i = xuv^i w z$ ✓ $\notin$.

7/10

$S_0 = xuv^0 wz$
$= a^0 ba^{n-0}$ ✓

The string uw must consist of exactly n a's to form a palindrome

The string uw has less than n a's
From the pumping lemma we know that v contains at least one a

∵ there is a contradiction
∴ the set of all palindromes is not regular

d) the set of balanced strings of parentheses ( )
$\{ (^n)^n \mid n > 0 \}$

a = (
b =)

0/10

This set is not regular as it is just $\{ a^n b^n \mid n > 0 \}$ in which a^i where v^i can be pumped to show that the language is not regular
$y = uvw$

if you're going to use this then you need to prove it's not regular.

January 20 2015

CS 2FA3 Tut 1

Yasmine Sharoda
sharodym@mcmaster.ca
Office: ITB-207

{a, b}
ab, ba, ...

$\Sigma^* = \{ \varepsilon, "a", "b", "ab", "ba", "aa", "bb" \}$

{#, +} ε = "" empty string

group $(\mathbb{Z}, +, 0, -)$ — 0: identity

$x + x^{-1} = 0$
$x + y = 0$
$(-x)$

$(2 + 3) + 5 = 2 + (3 + 5)$ associtivity

$x + e = x$ $e + x = x$

defining group
closure
associativity
commutivety
identity

{0, 1} "Is $x \in L$?" is x a string of the language

$L \subseteq \Sigma^*$

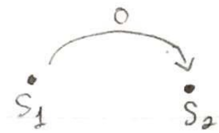
regular language
simplest language to answer the language no memory

0 1 1 0 0 traversal, no memory
does not remember the past

i

language - set of strings
set regular or not?

automata



states encode some knowledge

start state

transition - add knowledge

name of DFA $\rightarrow M = (Q, \Sigma, \delta, S, F \}$

Q ↑ set of states

F final

DFA : deterministic finite automaton

set of knowledges you could have

if there is a state you don't need to remember how you got to S_3

$S \rightarrow \bullet$

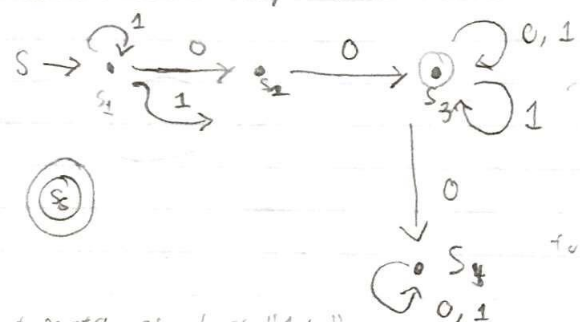
if a string belongs to the language of "00"

read string go through two 0s

transition on every state in 0

even final state needs a transition on every alphabet symbol

M_1



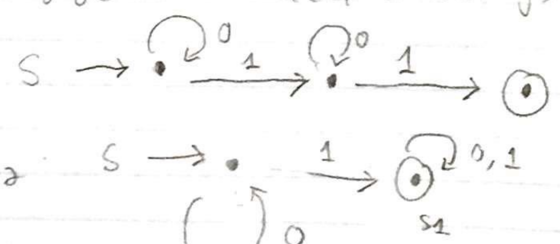
1 0 1 1 0 1 1 1 0

to not a final state

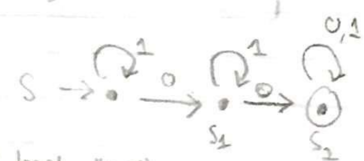
automata at least "11"

languge that accepts strings of at least "11"

★ you can have more than 1 final state



Fixed M_1



at least "00"

NFA

$(\{S, S_1, S_2\}, \{0, 1\}, M_1, S, \{S_2\})$

CS 2FA3 Tut 1

DFA ∃ intersection of two languages

"01"

$A = L(M_1)$

$B = L(M_2)$

$A \cap B = L(M_1 \times M_2)$

complement $\sim A = L(\sim M_1)$

$A \cup B = L(\sim(\sim M_1 \times \sim M_2))$

$= L(M_1 \sqcup M_2)$

DML

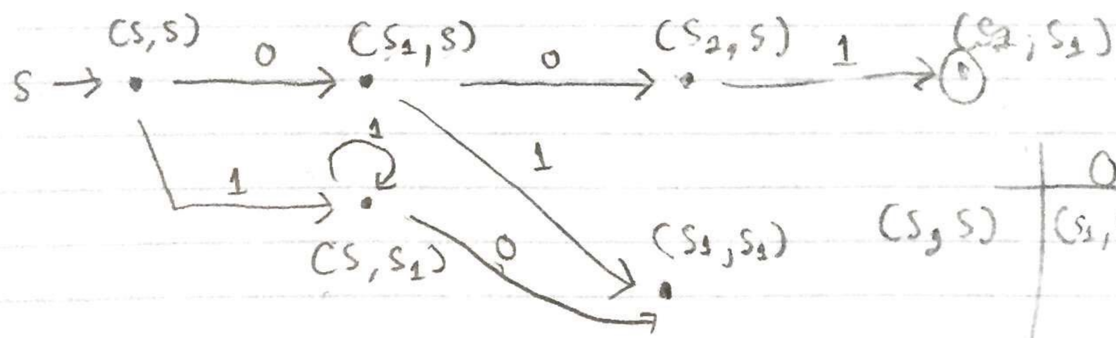
$S_1 \cup S_2 = \sim(\sim S_1 \cap \sim S_2)$

Not union of language $A \cup B$ since it would accept "00", "11"

(S,S) $A \cap B$ $M_1 \times M_2$

$M = (Q, \Sigma, \delta, s, F\}$

$\{(\underset{\text{start}}{\underline{S,S}}), (S,S_1), (S_1,S), (S_1,S_1), (S_2,S), \underset{\text{final}}{(S_2,S_1)}\}$



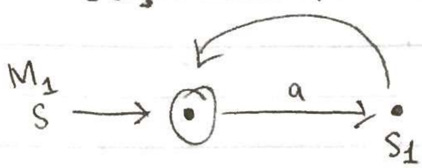
	0	1
(S,S)	(S1,S)	

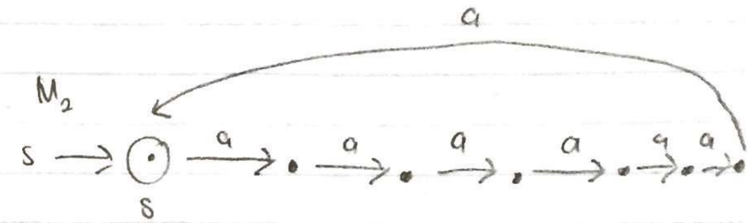
Jan 27 2015

CS 2FA3 Tut

Hw 1 solutions

1. b) $L \subseteq \{a\}^*$





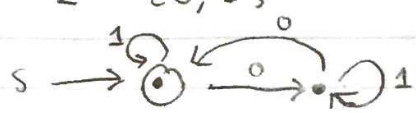
$$L(M) = L(M_1) \cup L(M_2)$$

$\sqcup$ square union

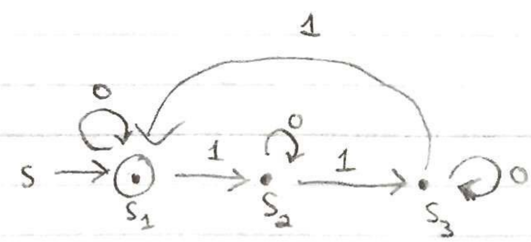
$$M = M_1 \sqcup M_2$$
$$= \sim(\sim M_1 \times \sim M_2)$$

c) $\Sigma = \{0, 1\}$

M_1



multiple of 3 for 0



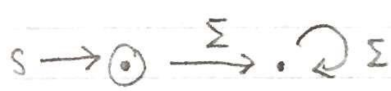
multiple of 3 for 1

$$L(M) = L(M_1) \cap L(M_2)$$
$$M = M_1 \times M_2$$

2. Draw DFA

a) $\{\} = \emptyset$ $\{" "\}$ empty language no final state

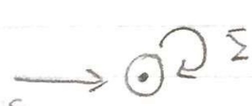
accepts nothing valid



0 a a a

when is a language not regular?

b) Σ^*



palidromes abba you need to remember

111 0 111

1111 0 111

Tutorial

L_1 $\quad L_2 = \sim L_1$ complement regular

$\Downarrow$

non deterministic

NFA $\quad \Sigma = \{a, b\}$

only things you accept are part of your language

a b a b a a a

accepting in an NFA is any path that can make you accept

any string ending in 3 as

Convert to DFA

	a	b
s → {s}	{s, t}	{s}
{s, t}	{s, t, u}	{s}
{s, t, u}	{s, t, u, v}	{s}
F → {s, t, u, v}	{s, t, u, v}	{s}

we don't know {S} ∪ {} = {S}

M_1

	a	b
s → 1	2	3
F → 2	3	1
F → 3	1	2

	a	b
F, s → 1	3	2
2	1	3
F → 3	2	1

	a	b
(1, 1)	(2, 3)	(3, 2)
(2, 3)	(3, 2)	(1, 1)
(3, 2)	(1, 1)	(2, 3)

intersect $\quad F_\cap = \{(2, 3),\}$

union $\quad F_\cup = \{(1, 1), (2, 3), (3, 2)\}$

Feb 3 2015

Comp SCI 2FA3 Tut

Hw 2

* Reverse of a string

$rev\ \varepsilon = \varepsilon$

$rev\ xa = a\ rev\ x$

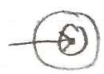
For $A \subseteq \Sigma^*$

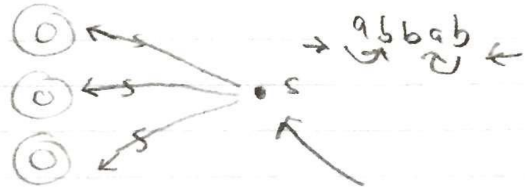
$rev\ A \stackrel{def}{=} \{ rev\ x \mid x \in A \}$

Prove that

If A is regular then $rev\ A$ is regular

M_A





E_A

$\varepsilon \longrightarrow \varepsilon$

$\emptyset \longrightarrow \emptyset$

$a \in \Sigma_A \longrightarrow a$

$+ \quad \cdot \quad *$

Assume α and β are 2 regular expression

$\alpha + \beta \longrightarrow L_1 \cup L_2$

$\alpha\beta \longrightarrow L_1 L_2$

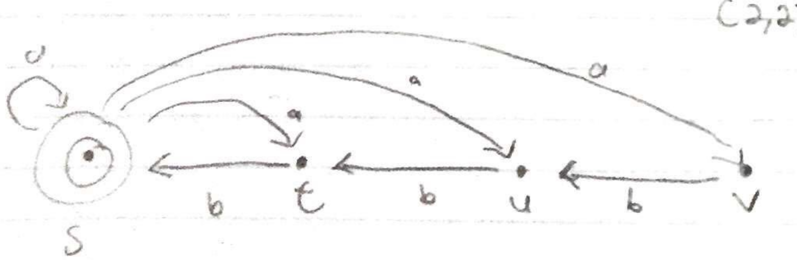
$\alpha^* \longrightarrow L_1^*$

when asked to a specific NFA does not prove a language is regular. subsets of a language might not be regular

	a	b
1	2	2
2F	1	1

	a	b
1	1	2
2F	2	1

	a	b
→ (1,1)	(2,1)	(2,2)
(2,1)	(1,1)	(1,2)
(1,2)	(2,2)	(2,1)
(2,2)	(1,2)	(1,1)



	a	b
{s}	{s, t, u, v}	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

w.c.s — regular expressions

- $a \in \Sigma$
- $\emptyset$
- Σ

$L = (a+b)^* = \{\varepsilon, a, b, aa,$

$\alpha = (a+b)^* \cdot (a+bb)$

Operations

$+$, $\bullet$, $*$

$\bullet$ concat, $*$ Kleene star: 0 or more occurrence

$a+b \rightarrow$ "a" or "b"

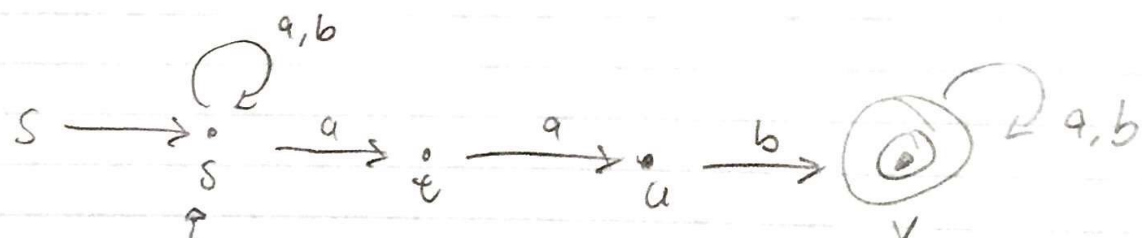
$a.b \rightarrow$ "ab"

$a^* \rightarrow \{\varepsilon, a, aa, \ldots\}$

$\Sigma = \{0, 1\}$

a) The language of strings having at least 2 1s

$0^* 1 0^* 1 (0+1)^*$



$(a+b)^* a a b (a+b)^*$

Feb 10 2015

CS 2FA3 Tutorial

HW #3 answers

1. a) $\Sigma = \{a,b\}$

$\alpha = (b^* a b^* a b^*)^*$ $\quad (aa)^* (bb)^*$

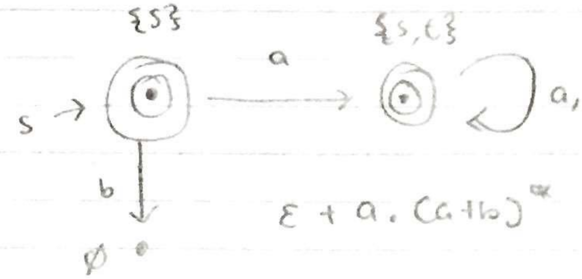
$b^* (ab^* ab^*)^*$ $\quad$ ba bbbbbb

$(ab^* a + b)^*$

b) $\beta = a^* b (a^* b a^* b a^*)^*$

c) $\alpha + \beta$

DFA



$\varepsilon + a . (a+b)^*$

pg 316 4b) $a^* + (ab^* ba^*)^*$

$(a + abb^* b)^*$

pg 319 11. a) b^*

b) $b^* a^*$

March 6 Midterm

Pumping Lemma

prove language is not regular

For any regular language A

$\exists k > 0$, such that

For all strings x, y, z

If $xyz \in A$ and $|y| \geq k$

Then $\exists$ strings u, v, w such that

1) $y = uvw$

2) $v \neq \varepsilon$

3) $\forall i \geq 0 \; xuv^iwz \in A$

$xyz \in A$ $\quad$ "a"

$xuvwz \in A$ $\quad$ $a^0 = \varepsilon$

$S_i = xuv^i wz \in A$

$S_0 = xuwz \in A$

$m \le 2n$

$a^m b^n$

$\underbrace{aa}\ bbb$

$aaaaa\ bbb$

$\Sigma = \{a, b\}$

$A = \{a^m b^n \mid m \le 2n\}$

- Assume A is regular
  $\exists$ DFA M such that $L(M) = A$
  choose $k >$ num of states of M
- $S = \underbrace{a^k}\, b^{2k}$.
  $= x\,y\,z$

$x = \varepsilon \qquad \underbrace{y = a^k} \qquad z = b^{2k}$

$y = uvw$
$|y| = k$
$|uvw| = k$

By pumping lemma

$S_i = x u v^i w z \in A$

$|v| = l, \quad 0 < l \le k$

$|uvvvw|$
$= k + 2l$

$S_3 = x \underbrace{uv^3w}_{k+2l} \underbrace{z}_{2k}$

Contradict the condition of the language

$S_{k+1} = x \underbrace{u v^{(k+1)} w}_{k + k\cdot l} z$

$|uv^{(k+1)}w| \le 2k$

$S_{k+2} = x u v^{(k+2)} w z$

$k + (k+1)\cdot l \ge 2kl + l \ge 2k$

we reached the contradiction

Y has ε so doing nothing
other languages do stuff

Feb 04 2015

CS 2FA3 Tutorial

1. a) $A = \{a^n b^m \mid n = 2m\}$

Hw #4 answers

Assume A is regular

let $k >$ number of states of DFA M where $L(M) = A$ — 2 points

let $S = a^{2k} b^k \in A$
$= xyz$

where — not important, you can pump

$x = \varepsilon \quad y = a^{2k} \quad z = b^k$

epsilon empty state

also valid:
$y = a^k \quad y = a^k \quad z = b^k$
$x = a^{2k} \quad y = b^k \quad z = \varepsilon$

By P.L (1 point), $\exists\, u, v, w$ and $v \neq \varepsilon$ (1 point) s.t

$y = u\,v\,w$

and $\forall i \geq 0$

$S_i = x u v^i w z \in A$

But let $|v| = l$ and $l > 0$ $(v \neq \varepsilon)$

$S_0 = x u v^0 w z$
$= x\, u w\, z$
$= a^{2k-l} b^k \notin A$

$|v| = l$

a^l

$y - v$
$a^{2k} - a^l$
a^{2k-l}

This contradicts with P.L
$\therefore$ A is not regular

$S_2 = x u v^2 w z$
$= x u v v w z$
$= a^{2k+l} b^k$

$2k - l < k$

$n \neq 2m$

a) $A = \{x \in \{a,b,c\}^* \mid x \text{ is a palindrome}\}$

Assume A is regular

let $k >$ number of states of DFA M

where $L(M) = A$

let $S = a^k b a^k \in A$

$= xyz$

where

$x =$	ε	$a^k b$
$y =$	a^k	a^k
$z =$	ba^k	ε

By P.L $\exists\ u, v, w$ and $v \neq \varepsilon$ s.t

$y = uvw$

and $\forall i \geq 0$ $\quad S_i = xuv^i wz \in A$

But let $|v| = l$ $\quad l > 0 \ (v \neq \varepsilon)$

$S_0 = xuv^0wz$

$= a^{k-l} b a^k \notin A$

This contradicts P.L

$\therefore$ A is not regular

$a^{k-l} \neq a^k$ not palindrome

Context free language (not regular or regular)

describe CFL using context free grammar

130

$$\begin{cases} S \to D \mid DS \\ D \to 0 \mid 1 \mid 2 \mid 3 \mid \ldots \mid 9 \end{cases}$$

$S \to DS \to 1S \to 1DS \to 13S \to 13D \to 130$

$$\begin{cases} N \to D \mid DN \\ D \to 0 \mid 1 \mid 2 \mid 3 \mid \ldots \mid 9 \end{cases}$$

$S \to N \mid S+S \mid S-S \mid S*S \mid S/S \mid (S) \mid$

130 + 120 − 4

$S \to S+S \to N+S \to 130+S \to 130+S-S \to 130+120-4$

March 3 2015

CS 2FA3 Tutorial

Sa San Va kili ITB 102
vakils@mcmaster.ca

Hw 5 Q1: Reg or not?

① DFA, NFA, Reg Exp
② ① pumping lemma
② closure property

c) $A = \{a^n b^{2m} \mid n \neq m\}$

Suppose A is Reg $\Rightarrow \sim A$ is reg
$\sim A = \Sigma^* - A = \{a^n b^n \mid n \geq 0\}$ is well known as context free

$\because$ contradiction.
$\therefore$ A is not Reg

Note P. 4-8
Note P. 5-1

$\left.\begin{matrix} L_1 \\ L_2 \end{matrix}\right\}$ $L_1 \cup L_2$ $L_1 \cap L_2$ $L_1 - L_2$

If you can't define as a regexe then not regular

CFG

$\{a^n b^{2n} \mid n \geq 0\}$

$S \rightarrow aSbb \mid \varepsilon$
string or empty

empty
ε $\emptyset$
epsilon

$\{a^n b^m \mid n \geq 0, m \geq 2n\}$

$S \rightarrow aSbb \mid \varepsilon \mid Sb$

a term has context free

$\{a^n b^n c^m \mid n, m \geq 0\}$

$S \rightarrow Sc \mid T$

$T \rightarrow aTb \mid \varepsilon$

March 17 2015

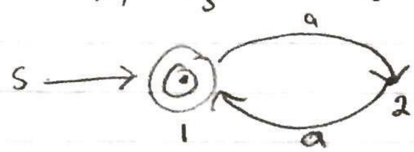
CS 2FA3 Tutorial

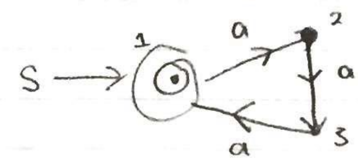
Q1 (37) $A_K = \{a^n \mid n \geq 0, n \text{ is divisble by } k\}$

$k = 1, 2, \ldots$

a) A_2, A_3 : Reg By DFA $A_3 = \varepsilon / a^3$

production construction or transition table





didn't need to show DFA

(10) b) $A_2 \cap A_3$: Product Const

	a
(2,3)	(1,1)

$f = \{(1,1)\}$

$M_2 \times M_3$:

	a
(1,1)	(2,2)
(1,2)	(2,3)
(1,3)	(2,1)
(2,1)	(1,2)
(2,2)	(1,3)

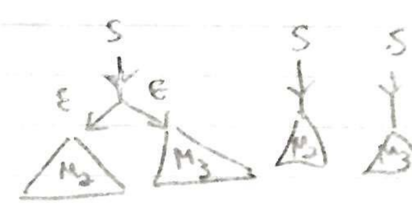
(5) c) $A_2 \cup A_3$: Product Construction

Transition table is the as part b

$F = \{(1,1), (1,2), (1,3), (2,1)\}$

$M_2 \cup M_3 = \sim(\sim M_2 \times \sim M_3)$

(9) d) $A_2 \cup A_3$ is Reg By NFA



Reg or Not

don't use lin's version pumping lemma

(31) Q2 a) $A = \{a^m b^n \mid m \leq 2n\}$

(8) b) $b^* + b^* a b^* + b^* a b^* a b^*$

3. a) $S \to aSa \mid bSb \mid a \mid b \mid \varepsilon$

b) $S \to aSbb \mid aSb \mid \varepsilon$

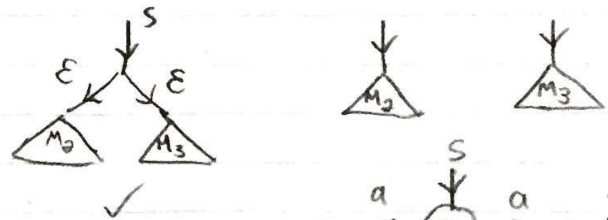
March 24 2015

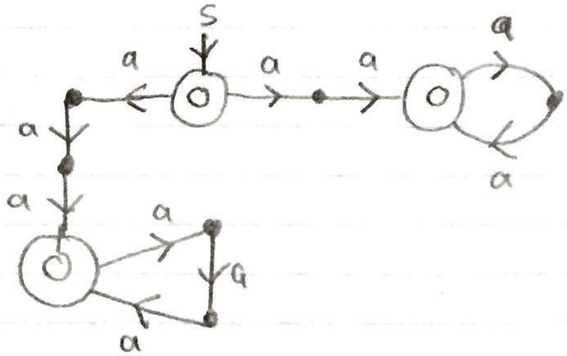
CS 2FA3 Tut

$A_k = \{a^n \mid n \geq 0, n \text{ is div by } k\}$

$A_2 \cup A_3$

NFA





Hw6 Q1. Given lang A, A is reg $\Leftrightarrow$ A is strong left linear

(Cf 9-5, Note 1): A is reg $\Leftrightarrow$ rev(A) is reg

(P 9-4, Thm): rev(A) is reg $\Leftrightarrow$ rev(A) has $\boxed{\text{SRL}}$ Gramm

(P 9-5, Notes 2 (3)): rev(A) has SRL $\Leftrightarrow$ A = rev(rev(A)) has SLL gramm

practice

CFG $\{a^n b^n c^m \mid n, m \geq 0\}$

$\{a^n b^m c^n \mid n, m \geq 0\}$

transform to chomsky grammar | palindrome

CFG

$S \to aSa \mid bSb \mid a \mid b \mid \varepsilon$

20

step 1: G: for every pair $A \to \varepsilon$ and $B \to \alpha A \gamma$ add $B \to \alpha\gamma$

alpha, gamma

$S \to aa$ $S \to bb$

step 2: Remove ε

G': $S \to aSa \mid bSb \mid a \mid b \mid aa \mid bb \mid$

step 3: Add non terminals A, B

$S \to ASA \mid BSB \mid AA \mid BB \mid A \mid B$

$S \to a$ $S \to b$ $A \to a$ $B \to b$

$S \to ASA$

it doesn't which pair you choose

AS or SA

Finally adding non terminal C, D

$C \to SA \qquad D \to SB$

$S \to AC \mid BD$

chomsky grammar normal form in order to make it easer to understand reduce complexity

chomsky grammar / what we have

$$\boxed{S \to AC \mid BD \mid AA \mid BB \mid a \mid b}$$

$C \to SA \qquad A \to a$

$D \to SB \qquad B \to b$

$S \to a$

$S \to b$

what we have broken up

---

chomsky let G be $S \to ABa \mid BA \mid a$

$A \to aab$

$B \to Ac \mid \varepsilon$

Step 1 : $\hat{G}$: $V \to \varepsilon$ and $V' \to \alpha V \gamma$

$V' \to \alpha\gamma$

$S \to Aa$

$S \to A$

$S \to A$

$A \to aab$

$S \to aab$

unit production : $V \to V'$ and $V' \to \alpha$ $\quad V \to \alpha$

step 1 $\hat{G}$: $S \to ABa \mid Aa \mid BA \mid aab \mid a$

$A \to aab$

$B \to Ac \mid \varepsilon$

last 2 classes problems

step 2 $\boxed{\text{remove } \varepsilon}$ $S \to ABa \mid Aa \mid BA \mid aab \mid a$

$A \to aab$

$B \to Ac$

step 3 Add production : $V_a \to a$ $\quad S \to ABa \quad S \to ABV_a$

can't choose

$V_b \to b \quad S \to Aa \quad S \to AV_a$

$V_c \to c \quad S \to aab \quad S \to V_a V_a V_b$

$A \to aab \quad A \to V_a V_a V_b$

$B \to AV_c$

$B \to A_c$

Final $S \to ABV_a$

$C \to AB \quad S \to CV_a$

Final step $S \to V_a V_b V_c$

$D \to V_a V_b \quad S \to DV_c$

$S \to CV_a \mid AV_a$

$BA \mid DV_b \mid a$

$C \to AB$

$D \to V_a V_b$

$S \to CV_a$

$S \to DV_c$

March 31 2015

CS2FA3 Tut

$A = \{a^m b^n a^n \mid m, n \geq 0\}$

Hw7

$S \to aS \mid T$

$T \to bTa \mid \varepsilon$

X $\boxed{T \to Tba \mid \varepsilon}$ $\to \underbrace{bababa\ldots}_{n}$

CFG

$m > 0$, $n = 0$

aaa

$m = 0$, $n > 0$

$\underbrace{bbb\ldots}_{n}\underbrace{aaa}_{n}$

$m = 0$
$n = 0$

ε

---

office hour ITB 102

PDA for $\{x \in \{a,b\}^* \mid \#a(n) = \#b(x)\}$

$Q = \{q\}$ all states

$\Sigma = \{a, b\}$ input alphabet

$\Gamma = \{\bot, A, B\}$ stack alphabet

A: number of a's over b's

B: number of b's over a's

δ: trans

only 2 care about case stack

$(q, a, \bot) \to (q, A\bot)$

$(q, a, A) \to (q, AA)$

$(q, a, B) \to (q, \varepsilon)$ cancel each other stack goes to empty epsilon

---

$(q, b, \bot) \to (q, B\bot)$

$(q, b, A) \to (q, \varepsilon)$

$(q, b, B) \to (q, BB)$

you only define top behaviour

$(q, \varepsilon, \bot) \to (q, \varepsilon)$

$(q, \varepsilon, A) \to (q, A)$

$(q, \varepsilon, B) \to (q, B)$

is b?

only need one variable
since $a^n b^n$ n

number of A

PDA for $\{ a^n b^n \mid n \geq 0 \}$

$\Sigma = \{ a, b \}$ $\quad Q\,(q, p)$ $\quad \Gamma = \{ \bot, A \}$

$\delta: (q, a, \bot) \rightarrow (q, A\bot)$

$(q, a, A) \rightarrow (q, AA)$

s $\quad$ # $\quad (q, b, \bot) \rightarrow$

$(q, b, A) \rightarrow (p, \varepsilon)$

$\cancel{(q, b, AA) \rightarrow \varepsilon}$

?

$(p, b, A) \rightarrow (p, \varepsilon)$

PDA for reg lang.? $\quad$ aa|bb

(number of A)