Homework 2 - Transformations
Solution

1. **Using the labels Translate, Rotation, Scale, or Reflect, label the transformation of point p when multiplied by matrix M (p' = Mp):**

   The main confusing thing is that these matrices are not all the same size. If its a 2x2 matrix, it must be meant to apply to a 2D point $[x, y]^T$, if its a 4x4 then this must be for a 3D homogeneous point $[xyzw]^T$. If we have a 3x3 matrix, we wouldn't know if it was 2D homogeneous $[xyw]^T$ or 3D $[xyz]^T$. Normally the problem should give this information. In this case it looks like a 2D translate, and if it was a 3D then it would be doing something really weird, so we guess the one that makes sense, since the problem didnt tell us which way to interpret.

   Now on to the question of how we know. The scale is identifiable by only having values on the diagonal. The translate by having values in the top right positions. The rotates I usually have to check and see if they are really rotates by picking some example points with real numbers and multiplying to see what happens.

   (a) Rotate

   $$M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

   (b) Rotate

   $$M = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

   (c) Scale

   $$M = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (d) Reflect

   $$M = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

   (e) Translate

   $$M = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

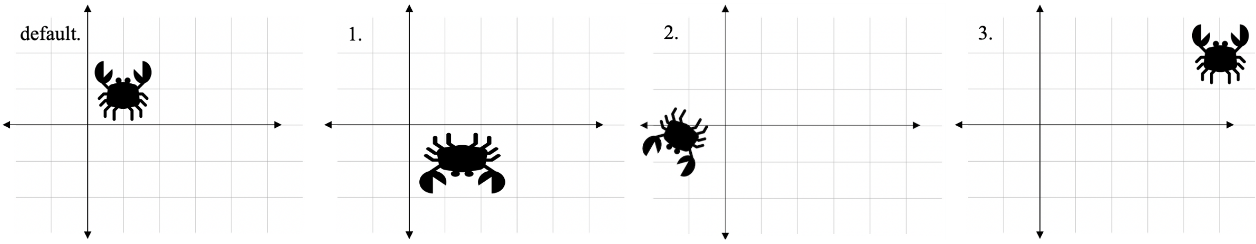
2. **Find the matrix for a $120°$ rotation about the axis defined by the vector r = (1,1,0).**

   The key here is to rotate our (1,1,0) axis-of-rotation to lie on the x-axis. This can be accomplished by rotating the whole world around the z-axis by -45 degrees. Then rotate around our axis-of-rotation (which is now the same as the x-axis). Finally we put the axis-of-rotation back in its original position.

   M=$R_z(45)R_x(120)R_z(-45)$

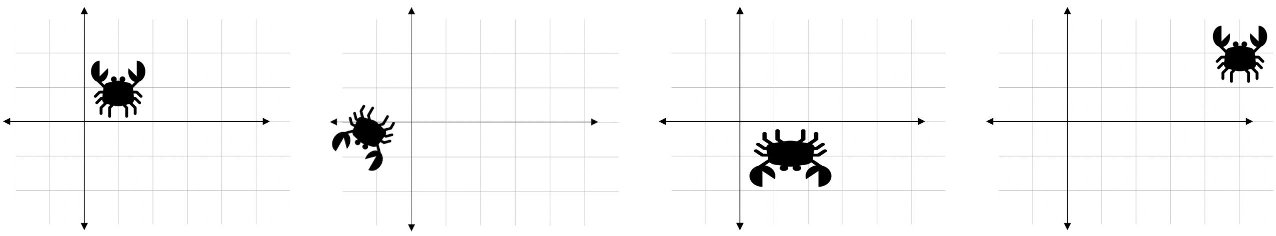
3. **Match the following 2D homogeneous matrices to the transformations in the image:**

a. $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{\sqrt{1}}{2} & 0 \\ \frac{\sqrt{1}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

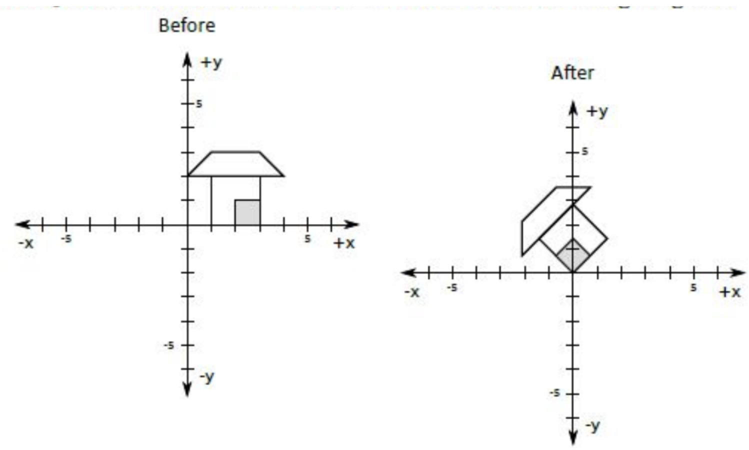


default

a. $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{\sqrt{1}}{2} & 0 \\ \frac{\sqrt{1}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

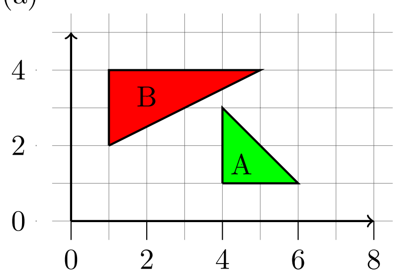


4. **Describe a sequence of Translate(x,y), Rotate(degrees), and/or Scale (by a value of) that when multiplied describe the below transformation to go from the 'Before' to 'After'.**



First Scale(-1,1), Translate(3,0), and then Rotate (not exact, but estimated around 45°)

5. (a) **Describe (using a sequence of (3 x 3) matrix multiplications) the transformations needed to transform the triangle from A to B in the figure below:**
(a)



$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} * \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} * \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} * \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Translate(1,4) * Scale(2,1) * Rotate(90 degrees clockwise) * Translate(-4,-1) * Point

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 0 & 2 & -1 \\ -1 & 0 & 8 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$$

Transformations can take multiple routes to reach the same resulting points. Another solution using reflecting, scaling, and then translating:

Translate(-7,5) * Scale(2,1) * Reflect(across the x-axis) * Point

$$= \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 2 & 0 & -7 \\ 0 & -1 & 5 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$$

Note that normally the same transformation should lead to exactly the same matrix, regardless of which operations you applied to get there. In this case the matrices don't match. So what is up?! The vertices aren't labeled, so the two matrices lead to different A vertices moving to different positions in the B triangle. If the same vertex went to the same point, the two matrices would have to be the same.

(b) **Using the transformations found in part (a), multiply the following points with the matrix.**
P_1.**(4, 1)**P_2.**(4, 3)**

As usual for 2D operations, w is set to a default of 1 in the vector during calculation.
Since this is an operation that takes a point on triangle A, to a point on triangle B, the answer better be a vertex on triangle B. We are having you multiply to see that this works!

P_1 (4, 1)

$$\begin{bmatrix}1 & 0 & 1\\0 & 1 & 4\\0 & 0 & 1\end{bmatrix} * \begin{bmatrix}2 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} * \begin{bmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 1\end{bmatrix} * \begin{bmatrix}1 & 0 & -4\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix} * \begin{bmatrix}4\\1\\1\end{bmatrix} = \begin{bmatrix}1\\4\\1\end{bmatrix}$$

(1,4)

P_2 (4, 3)

$$\begin{bmatrix}1 & 0 & -7\\0 & 1 & 5\\0 & 0 & 1\end{bmatrix} * \begin{bmatrix}2 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} * \begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix} * \begin{bmatrix}4\\3\\1\end{bmatrix} = \begin{bmatrix}5\\4\\1\end{bmatrix}$$

(5,4)