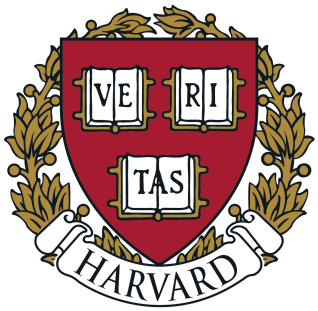

MATHEMATICAL PORTIONS
OF THE
1869 HARVARD
ENTRANCE EXAMINATION

A Recreation Typeset in LaTeX
Prepared by Tim Ricchuiti

Original Examination Administered
July 1869

ARITHMETIC.

☞ Give the work in full;—reduce the answers to their simplest form;—and write and arrange your exercise in a legible and orderly manner.
*Applicants for ADVANCED STANDING may omit Nos. 1, 2, 3, and 6.

*1. Reduce $\frac{184800}{1180410}$ to its lowest terms.
What is a *prime number?* When are two numbers said to be prime to *each other?* Reduce the numerator and denominator of the above fraction to their *prime factors.*

*2. From $5\frac{1}{3}$ subtract $\frac{3\frac{7}{16}}{3\frac{1}{9}} \div \left(\frac{3}{10} \text{ of } \frac{4\frac{5}{7}}{2\frac{2}{3}} \text{ of } 4\frac{1}{6}\right)$. Simplify by cancelling.

*3. Divide 33368949.63 by 0.007253. What is the quotient of 3336.894963 by 72530? What is the third power of 0.1? of 100? Write these answers in *words.*

4. Find the cube root of 0.0093 to five places of decimals.
Find the square root of 531.5 to three places of decimals.

5. Reduce to their lowest terms as vulgar fractions the infinite or circulating decimals $0.\dot{2}2\dot{5}$, $0.00\dot{2}2\dot{5}$, $0.25\dot{2}2\dot{5}$. Reduce $\frac{3}{7}$ to a circulating decimal.[1]

*6. From 1 sq. rod 5 sq. ft. subtract 7 sq. yd. 139 sq. in.

7. Find the *amount* of £50 12s. 5d. at simple interest at 8 per cent., at the end of 5 years 2 months and 3 days.

8. One metre = 39.37 inches. Compute from this datum the value of 4 miles in kilometres.

LOGARITHMS AND TRIGONOMETRY.

9. What is the logarithm of 1 in any system? of any number in a system of which that number is the *base?* In a system of which the base is 4, what is the logarithm of 64? of 2? of 8? of $\frac{1}{2}$?

10. Find by logarithms, using arithmetical complements, the value of the fraction[2]

$$\frac{(0.02183)^2 \times (7)^{\frac{2}{3}}}{\sqrt{(0.0046)} \times 23.309}$$

11. Prove the formula for the *cosine of the sum of two angles;* and deduce the formulas for the cosine of the *double* of an angle and the cosine of the *half* of an angle.

12. In what quadrants is the cosine *positive*, and in what quadrants *negative?* Prove the values of the cosine of $0°, 90°, 180°, 270°$.

13. Given in an oblique triangle $b = 0.254$, $c = 0.317$, $B = 46°$. Solve completely.

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ALGEBRA.

☞ Give the *work in full;* reduce the answers to their *simplest form*; and write and arrange your exercise in a *legible* and *orderly* manner.

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1. Reduce the following expression to its simplest form:

$$(9a^2b^2 - 4b^4)(a^2 - b^2) - (3ab - 2b^2)(3a[a^2 + b^2] - 2b[b^2 + 3ab - a^2])b.$$

2. Divide $36x^2 + 1 - 64x^4 - 12x$ by $6x - 1 - 8x^2$.

3. What is the *reason* that when different powers of the same quantity are multiplied together their exponents are *added?*

4. Reduce to one fraction, with the lowest possible denominator:

$$\frac{3a + 2b}{a + b} - \frac{25a^2 - b^2}{a^2 - b^2} - \frac{a}{2b}.$$

5. Divide $\dfrac{x + y}{x^2 - 2xy + y^2}$ by $\dfrac{x^2 + xy}{x - y}$; and reduce the answer to its lowest terms.

6. Find x, in terms of a, b, and c, from the equation $\dfrac{a - 2x}{b} = \dfrac{cx - bc}{a}$. What is the value of x when $a = 2$, $b = -1$, $c = 3$?

7. A man bought a watch, a chain, and a locket for \$216. The watch and locket together cost three times as much as the chain, and the chain and locket together cost half as much as the watch. What was the price of each?

8. Solve the equation

$$\frac{5x}{x + 12} - \frac{8 - 3x}{3x - 1} = 1.$$

9. Find $(a - b)^9$ and $\left(xy - \dfrac{x^3}{2y}\right)^6$ by the Binomial Theorem.

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PLANE GEOMETRY.

1. Prove that the perpendicular from the centre of a circle upon a chord bisects the chord and the arc subtended by the chord.

2. To circumscribe a circle about a given triangle.

3. Prove that two angles are to each other in the ratio of two arcs described from their vertices as centres with equal radii.

4. Prove that a line drawn through two sides of a triangle parallel to the third side divides those two sides into proportional parts.

5. State and prove the proportion which exists between the parts of two chords which cut each other in a circle. State what proportion exists when two secants are drawn from a point without the circle.

6. Prove that two regular polygons of the same number of sides are similar.

7. Prove that similar triangles are to each other as the squares of their homologous sides.

8. Show how the area of a polygon circumscribed about a circle may be found; then how the area of a circle may be found; then prove that circles are to each other as the squares of their radii.

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Notes

[1]The dot notation is archaic. Modern notation would use an overbar. The decimals in the document would now be written as $0.\overline{225}, 0.00\overline{225}$, and $0.25\overline{225}$.

[2]It's unclear whether the square root in the denominator was intended for the whole product or only the 0.0046. However, the use of parentheses around 0.0046 suggests the square root was only meant to be computed on that portion. In modern type-setting, that would be $\sqrt{0.0046} \times 23.309$