Distributional aspects of time to death in human populations

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Abstract

All lifetable summary indices are functions of age, but these are usually only calculated for age zero, or sometimes the age at maturity, retirement or some other particular age. We propose to let lifetable summary measures be calculated at any age. In this manuscript, we provide some elementary definitions of functions describing the conditional lifespan distribution, and apply these to human populations. We suggest a selection of applications for late life decisions, such as the decision to bequest or move into a care residence, based on these distributional measures.

Background

Typically demographers are satisfied to summarize the distribution of remaining lifetimes for age groups using only its mean, $e(a)$. However, remaining life expectancy is not an omnibus descriptor of time to death. There are other useful measures of longevity, such as the modal or median ages at death, and demographers also have a battery of indicators for lifespan variability or entropy. One aspect in common for many such indicators is that they refer to the age distribution of mortality in a snapshot of a stationary population or else the age at death distribution of a newborn cohort under constant mortality conditions. These measures are not typically made conditional on survival to later ages, i.e., demographers too seldom consider the properties of the distribution of remaining lifetimes.

It is our impression that mortality transitions have most often been described in terms of changes in the mortality hazard curve, except when framed in terms

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of compression (e.g., Fries 1980), which is a distributional observation. Hazards are attractive because rates in a given age group can be imagined as independent from other ages. Research on the deaths distribution (or else its translation as a survival curve) has either been based on the entire age range (e.g., Wilmoth and Horiuchi 1999) or else on left-truncated distributions on a few selected ages, in order to focus on senescent processes. For instance, Edwards and Tuljapurkar (2005) and Gillespie et al. (2014) left-truncate analyses at ages 10 and 15, respectively, in order to observe the deaths distribution without a preponderance of infant mortality. Another family of studies left-truncates at variable ages, depending on the age-specific value of some function. For example, Canudas-Romo and Engelman (2009) document historical trends in total maximum lifespan expectancy using in each instance the age that maximizes conditional lifespan expectancy, $a + e(a)$ (this was not always $e(0)$). Left-truncation is also common practice in studies of old-age mortality. Kannisto (2001) suggested measuring old-age mortality dispersion by left-truncating at the modal age of death, and several researchers have followed suit (e.g., Thatcher et al. 2010, Ouellette and Bourbeau 2011, among others). One can in practice repeat such analyses after left-truncating on each age in succession, thereby revealing an age pattern to the subject of interest (variation, inequality, and between-population divergence) in order to form a more complete picture over the lifecourse. We refer to this technique as age-conditioning. Engelman et al. (2010), for instance, condition on age to derive an age pattern to the standard deviation of remaining lifespan.

All lifetable global summary measures can be reworked as functions of conditional remaining lifetime (e.g., the modal remaining lifespan) to become functions of age. In this paper, we provide some elementary definitions of age-conditioned distribution functions, including variance, skewness and kurtosis, and we apply these to populations from the Human Mortality Database (HMD). As an example of the utility of this perspective we suggest a selection of simple heuristics for late life decisions based on these distributional measures.

Definitions

Remaining life expectancy conditional on survival to age a is defined as

$$e(a) = \frac{1}{l(a)} \int_0^\infty l(a+y)\, \mathrm{d}y \qquad . \tag{1}$$

Let lifespans for a given birth cohort be measured with the random variable, X, with distribution $d(X)$, which is identical to the d_x column of the lifetable if a radix of 1 is used. In other words, the distribution of lifespans for newborns is identical to the distribution of age at death in the stationary population. We are interested in the conditional density function, $f(X - a \mid X \geq a)$, which we denote using $f(y|a)$, where a is age attained and y is remaining years of life,

and which is defined as:

$$f(y|a) = \frac{1}{l(a)}\mu(a+y)l(a+y) \qquad , \tag{2}$$

i.e., the probability of surviving to and dying at age $a+y$ given survival to age a. (2) can also be used to calculate remaining life expectancy:

$$e(a) = \int_{y=0}^{\infty} yf(y|a)\ \mathrm{d}y \qquad . \tag{3}$$

Demographers make less frequent reference to $f(y|a)$, which is however useful for decomposing demographic counts into remaining lifetime classes. The conditional distribution of remaining lifetimes can be described empirically using quantiles, or other central measures such as the median or the mode, or perhaps more parsimoniously using its moments. The n^{th} central moment about the conditional mean of $f(y|a)$, $\eta_n(y|a)$ is defined as:

$$\eta_n(y|a) = \frac{1}{l(a)} \int_{y=0}^{\infty} (y - e(a))^n \mu(a+y)l(a+y)\ \mathrm{d}y \tag{4}$$

or just

$$\eta_n(y|a) = \int_{y=0}^{\infty} (y - e(a))^n f(y|a)\ \mathrm{d}y \qquad , \tag{5}$$

where $\eta_2(y|a)$ gives the variance of remaining lifespan about $e(a)$, $\sigma^2(y|a)$. Survival-conditioned variance is useful information, but it can be deceptive to display graphically, since lifespan variation is not usually symmetric around $e(a)$. The conditional skewness function, $Skew(y|a)$ is not a perfect measure of symmetry in $f(y|a)$, but it captures most such variation and can be roughly interpreted in this way. It is defined as

$$Skew(y|a) = \frac{\eta_3(y|a)}{\sigma(y|a)^3} \qquad , \tag{6}$$

the third standardized moment. The conditional excess kurtosis of $f(y|a)$, $Kurt(y|a)$, can be defined as

$$Kurt(y|a) = \frac{\eta_4(y|a)}{\sigma(y|a)^4} - 3 \qquad . \tag{7}$$

The age pattern of kurtosis describes how the peakedness, or the fatness of the tails of the remaining distribution change over age. The coefficient of variation

of remaining lifespan, $CV(y|a)$ is then simply

$$CV(y|a) = \frac{\sigma(y|a)}{e(a)} \qquad . \tag{8}$$

$CV(y|a)$ is dimensionless and comparable over age, and its reciprocal can be thought of as a signal to noise ratio of one's likely remaining lifespan, assuming a constant mortality pattern in ages higher than a.

Observed patterns

The above definitions are exact and amenable to calculation from the standard lifetable. We illustrate using the long series of Swedish data available from the HMD, and provide results for other HMD populations in a forthcoming online appendix. We begin by outlining the basic age pattern of some interesting distribution descriptors, and continue by plotting age-period trends as Lexis surfaces. Results show distinct transitions according to the index viewed. The mean, standard deviation, skewness and kurtosis occasionally suggest different onset dates.

To begin, let us compare some quantiles of the distribution of $f(y|a)$, translated to $f(a + y|a)$. Figure 1a provides shows the median and interquartile range of conditional age attained implied by period rates for Swedish females in 1900 and 2000. In a general sense, we can conclude from this Figure that the lower quantiles of conditional age attained change much more over age than do upper quantiles. We also see that lower mortality regimes year (2000) have more compact interquartile ranges than to do high mortality regimes (1900), meaning that one can wager an age at death with greater certainty, and even earlier in life. Further, in contemporary low mortality regimes, with low early life mortality, the conditional IQR holds nearly constant (in fact it always rises, albeit imperceptibly) until after typical midlife ages.

Figure 1b is similar to Figure 1a, but shows the mean of conditional lifespan, $a + e(a)$, as a function of age, plus and minus a single standard deviation about $e(a)$. This might be a less useful indicator of spread than is IQR due to assymetry, until later in life, when it yields similar results to the IQR.

Variance can be usefully supplemented after infant mortality has passed by referring to the skewness and kurtosis patterns over age, displayed in Figure 2, which also includes intermediate year, 1950. Skewness and kurtosis also display strong age patterns. Skewness, Figure 2a, essentially increases until old-age mortality deceleration, crossing zero somewhere the age where mean and median remaining life expectancy are equal, and also in the neighborhood of the late-life minimum in kurtosis, Figure 2b. In contemporary low mortality conditions, kurtosis follows a roller-coaster pattern over age, with positive excess kurtosis falling from birth to become negative around age 50 or 60, then positive again

Figure 1: Sweden, females in 1900 & 2000. Period mortality (HMD)

(a) 25th, 50th, and 75th percentiles of conditional lifespan.

(b) Variation around mean, $e(a) + / - \sigma(y|a)$ (a single standard deviation).

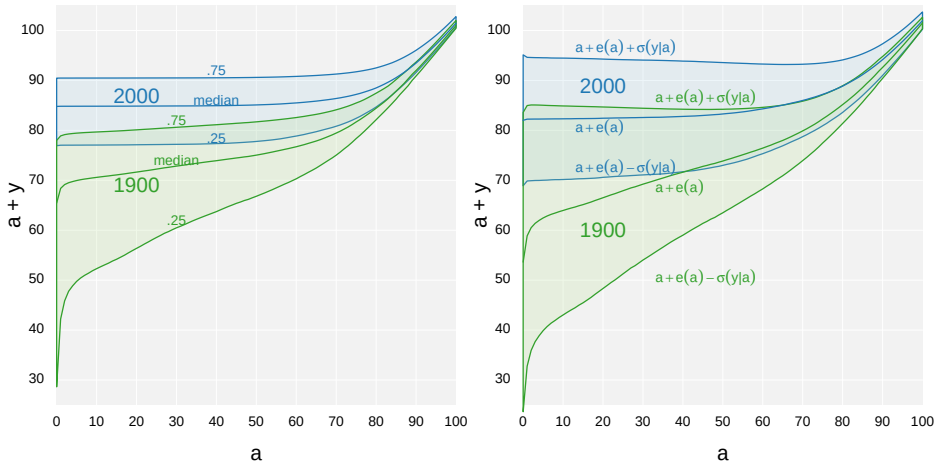
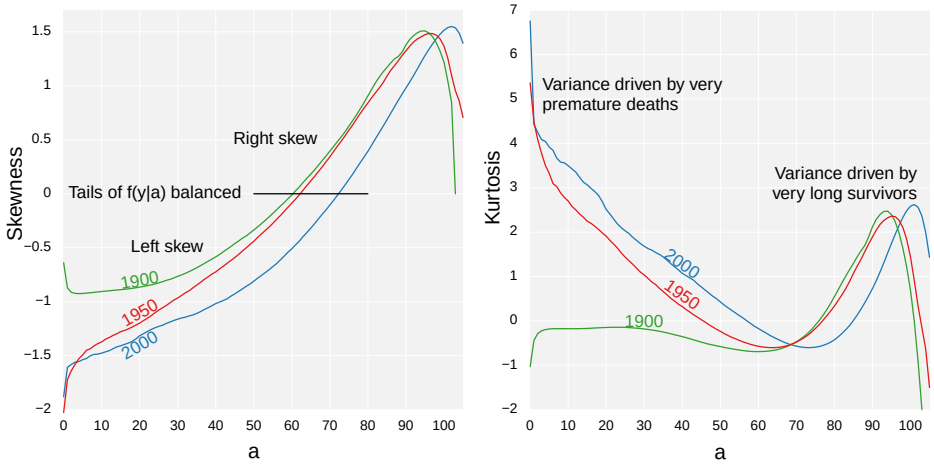


Figure 2: Sweden, females in 1900, 1950 & 2000. Period mortality (HMD).

(a) Skewness, $\gamma_1(y|a)$ from equation (6).

(b) Kurtosis, $\gamma_2(y|a)$ from equation (7).

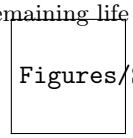


around age 80, and perhaps again negative sometime after age 100.[1] In high-mortality populations, such as 1900 Sweden, the kurtosis of remaining lifespan

[1]The observation of a platykurtic remaining lifespan distribution after age 100 is sensitive to data quality and adjustments/ smoothing used, and we do not analyse this in depth.

Figure 3: Sweden, females in 1751-2011, ages 0-110+. Period mortality (HMD).

(a) $e(a)$, remaining life expectancy.



(b) $\sigma(y|a)$, standard deviation of remaining lifespan.

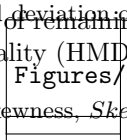
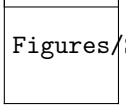
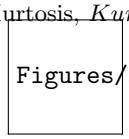


Figure 4: Skewness, Kurtosis and CV of remaining lifespan. Sweden, females in 1751-2011, ages 0-110+. Period mortality (HMD).

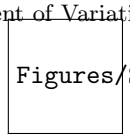
(a) Skewness, $Skew(y|a)$



(b) Kurtosis, $Kurt(y|a)$



(c) Coefficient of Variation, $CV(y|a)$



remained slightly platykurtic until between age 70 and 80.

Each distribution moment has undergone major shifts in the recorded past, which we plot on Lexis surfaces for Swedish females for the 260 years of data available from the HMD. Figure 3a shows the well-known mean remaining lifespan, $e(a)$, the isopeleths of which have maintained a steady linear increasing pattern since at least the 1950s. Figure 3b shows the standard deviation of remaining lifespan, the age pattern of which held roughly constant for the first 150 years of data, and appears to have started an abrupt shift, still underway, starting at the 1918 influenza pandemic.

Skewness, Figure 4a has undergone a much longer transition, starting before the mid 19th Century and accelerating after the 1918 pandemic. The trend has been one of decrease in all ages. Kurtosis, Figure 4b, began its transition around 1900, decreasing in ages above $e(a)$ and increasing in ages below $e(a)$, but always obtaining a local minimum near $e(a)$. The coefficient of variation, Figure 4c, began its decreasing trend in the 19th Century, driven initially by its denominator, $e(a)$, and after 1918 by both changes in the mean and standard deviation.

Data and code used to produce these results will be made available, and an appendix of results for other HMD populations and both sexes will also be made available.

Applications

(section in progress) Distributional aspects of age-conditioned mortality may play a decisive role in designing lifespan-related policies, interventions, hedges, and investments. By lifespan-related, we refer both to policies that affect and are affected by lifespan. For instance, Edwards (2013) derives an abstract approach to estimate or conceive of a cost to variance in human lifespan. This work was based on the lifetable deaths distribution as a whole, but we pose the question that perhaps a) there are costs and opportunities implicit in other moments and b) these costs may vary depending on the age of the beholder, and may not be implicit and homogenous over all ages. Choices, such as the decision to preemptively move into a care residence, are age dependant, and the age pattern of optimal behavior may hinge on mortality projected in ages higher than a.

The empirical distribution of $f(y|a)$ bears heavily on the consequences of many late-life decisions (in the aggregate), and so should help shape individual planning. Application domains include purchaser evaluations of life insurace, the decision to bequest, or the decision to pre-emptively move into a care residence. While the sustainability of pension and retirement plans is mostly a function of mean remaining lifetime, their equality with respect to post-labor lifespan depends on $f(y|a)$ for the general population and for subpopulations. To equalize the duration of retirement between individuals is at first glance a step toward equality, but there is no necessary or best way to determine this duration, and there is also no necessary way to adjust this duration as a function of lifespan itself. In other words, if the desired duration of retirement is 20 years, perhaps the meaning of 20 years is greater for an individual with a shorter lifespan and lesser for a longer total lifespan. Whether the goal is to equalize the duration of retirement, the proportion of life in retirement, or some other way of partitioning lifelines into lifecourse stages, distributional aspects of lifespans hold promise as an element in formulating analytic guidelines. Distributional differences between population subgroups vary greatly, as we demonstrate for sex differences. Sex differences and differences between other population subgroups may form the basis for research on group differences in the lifespan distribution.

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