# Variants of decomposition

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# Two classes

- bespoke decompositions
- generalized decomposition

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Partition the difference in a **crude** rate into additive **rate** and **structure** components.

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The total difference is the difference in rates times the mean structure (rate component) plus the difference in structure times the mean rates (structure component).

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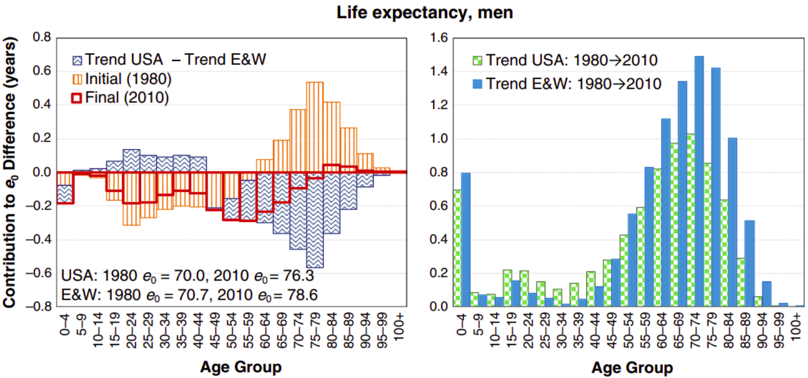
# Contour decomposition



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- Generalized counterfactual approach (Sudharsanan & Bijlsma 2020)

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