

KOSTAT-UNFPA Summer Seminar on Population

Workshop 1: Introduction to Demography

# Projection

27 June, 2025

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**ikerbasque**  
Basque Foundation for Science



Statistics  
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# Workshop plan, July 1-5, 2024

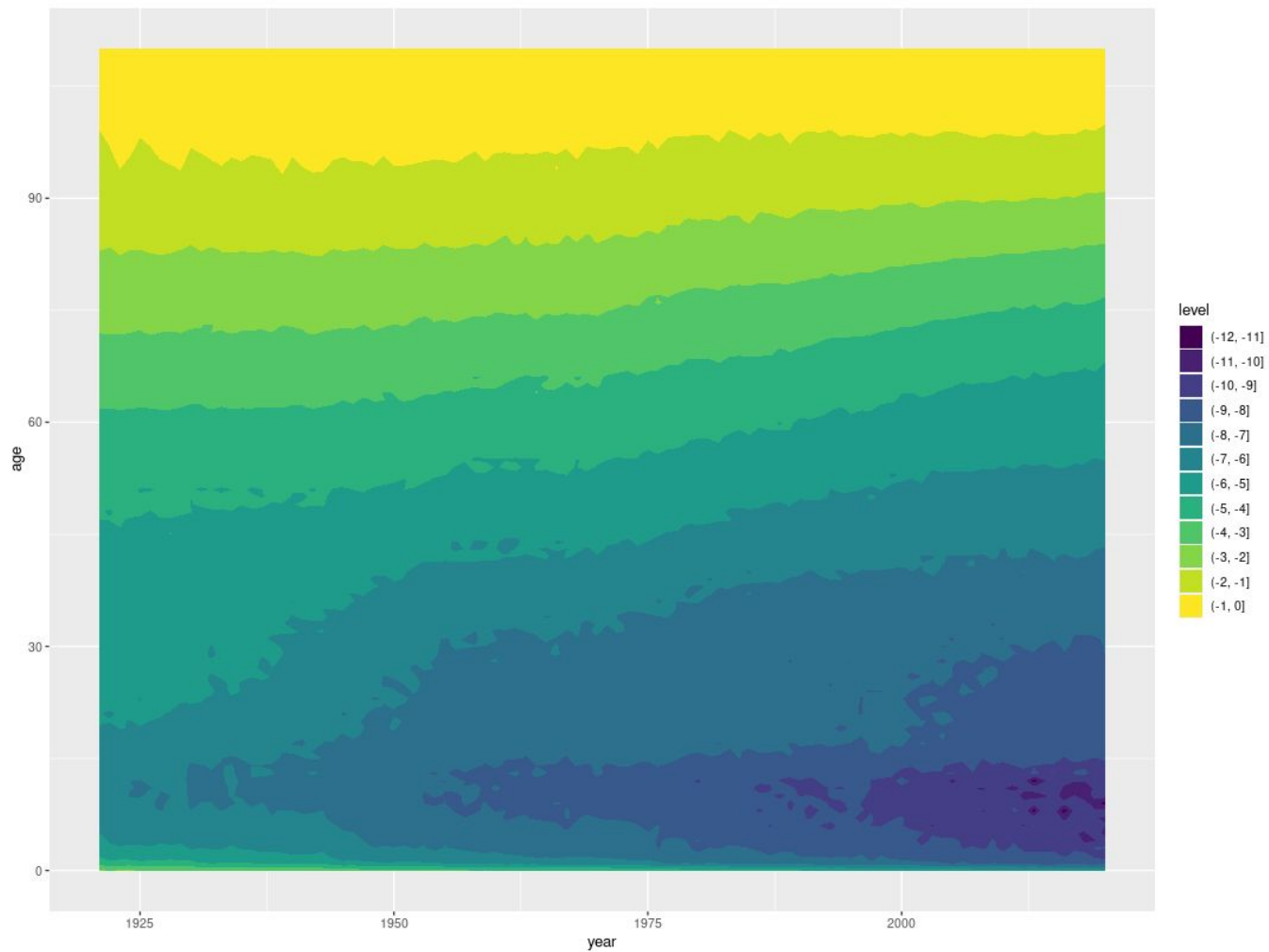
- |                  |                             |
|------------------|-----------------------------|
| 1: Monday        | Intro concepts, and R setup |
| 2: Tuesday       | Mortality and fertility     |
| 3: Wednesday     | Structure                   |
| 4: Thursday      | Growth                      |
| <b>5: Friday</b> | <b>Projection</b>           |

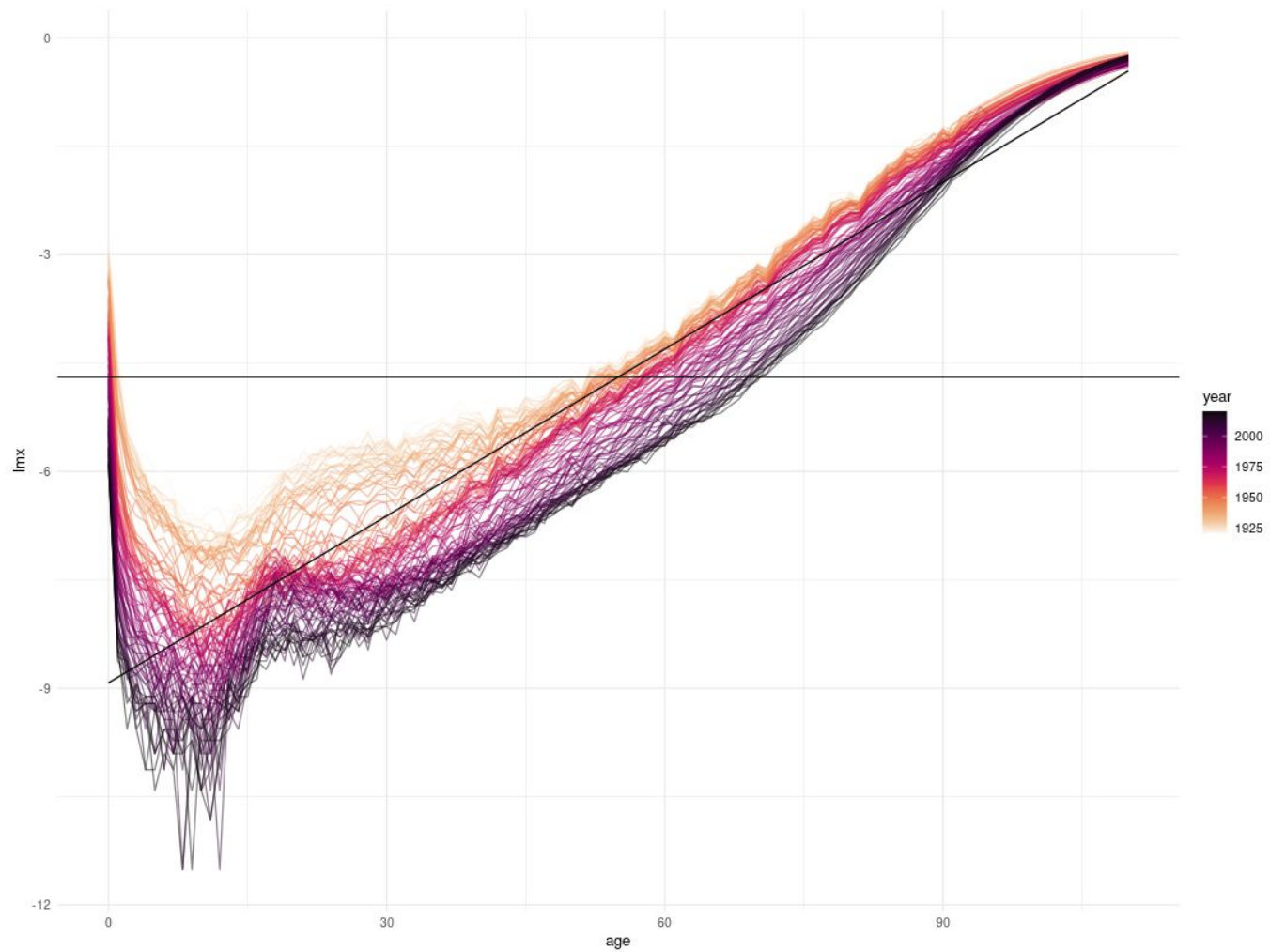
## Review of session 4

- Geometric and exponential growth
- Stationary and stable structure
- Stable structure in periods; stationary structure in cohorts
- Stable populations don't happen; change is constant
- `colorspace` for good palettes.

# Projection

- Predict the past/present to predict the future
- Simplify
- Extrapolate





## % Variance (of log rates) explained

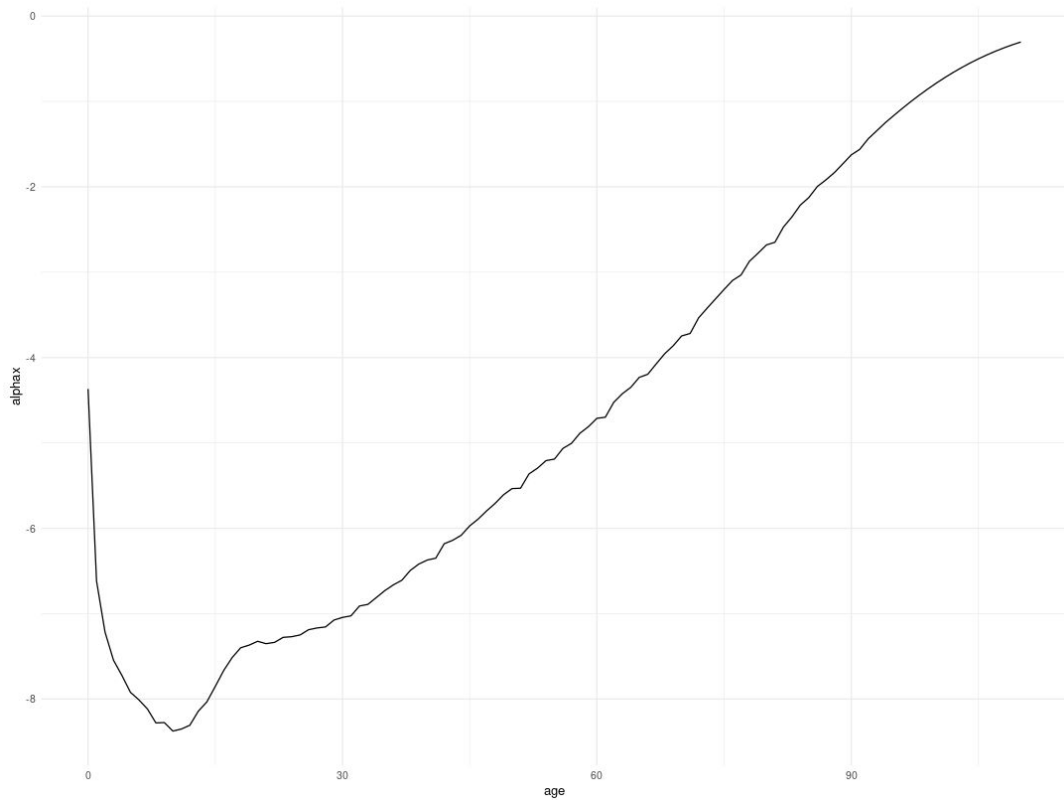
Parameters	
1 (simple intercept)	Total var = 276
2 (line)	66.4 %
3 (simple plane)	72.9 %
4 (plane with tilt)	75.0 %
200 (each year has own line)	75.2 %
111 (age intercepts)	76.5 %
222 (each age has own line)	93.9 %

## Lee Carter method

$$\ln(m_x(t)) = \alpha_x + \beta_x \kappa(t) + \epsilon_x(t)$$



# Alpha



# Calculating beta and kappa

SVD - singular value decomposition

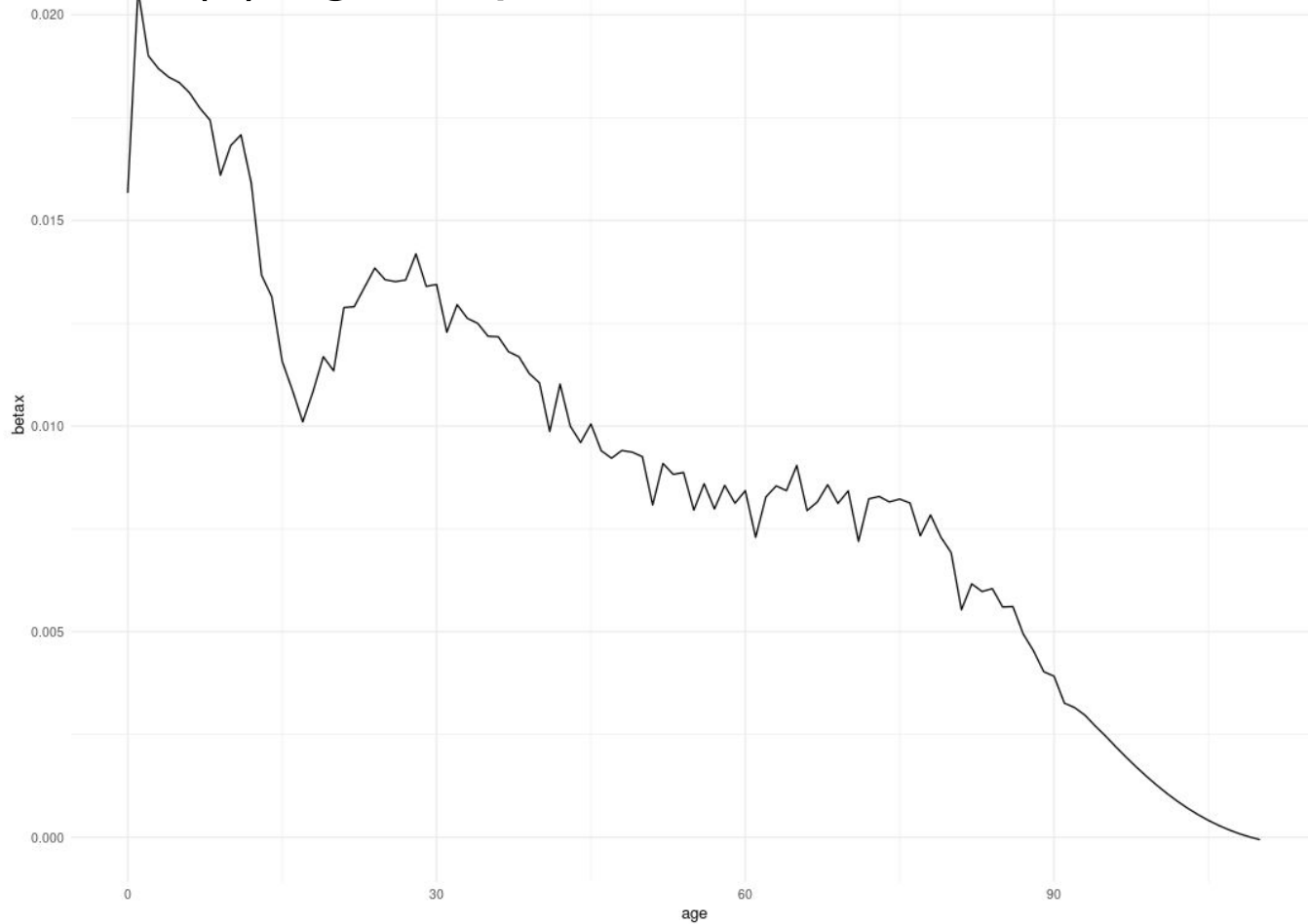
Factorize a matrix **M** into pieces **d**, **U**, **V**, such that:

$$\mathbf{M} = \mathbf{U} \text{diag}(\mathbf{d}) \mathbf{V}^T$$

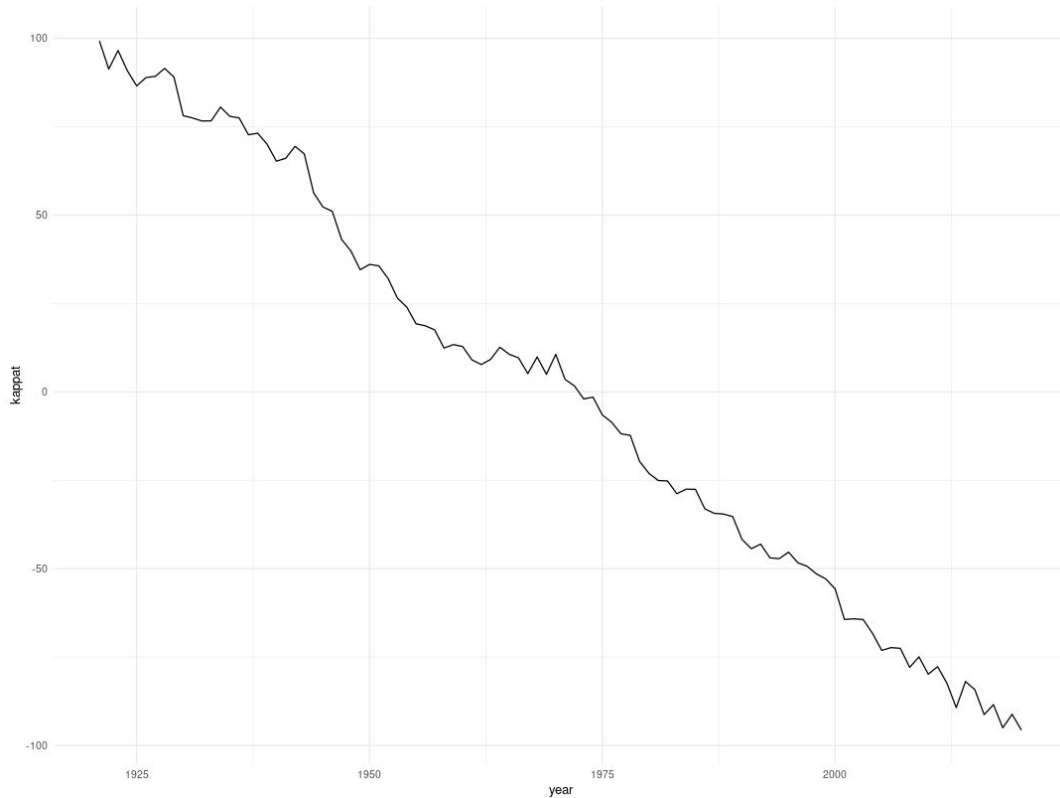
$\beta(x)$  (age slopes) derived from **U** (scale first column to 1)

$\kappa(t)$  (secular change) derived from **V** (scale first column of **V** to sum to first element of **d**)

# Beta(x) age slopes



# Kappa(t) the trend to extrapolate



## % Variance (of log rates) explained

Parameters	
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200 (each year has own line)	75.2 %
111 (age intercepts)	76.5 %
222 (each age has own line)	93.9 %
224 (LC, simple)	94.2 %



## Modeling and Forecasting U. S. Mortality

Ronald D. Lee; Lawrence R. Carter

*Journal of the American Statistical Association*, Vol. 87, No. 419 (Sep., 1992), 659-671.

Stable URL:

<http://links.jstor.org/sici?sici=0162-1459%28199209%2987%3A419%3C659%3AMAFUSM%3E2.0.CO%3B2-T>

*Journal of the American Statistical Association* is currently published by American Statistical Association.

$$\ln(m_x(t)) = \alpha_x + \beta_x \kappa(t) + \epsilon_x(t)$$