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FÜR DEMOGRAFISCHE  
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RESEARCH





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# **Birth, Death, & Thermodynamics**

**Tim Riffe**



## About that title

### On the one hand

Birth and death are events experienced by individuals. Both are conditioned by randomness, circumstance, and agency.

### On the other hand

Birth and death have very strong and steady age patterns. It's easy to think there might be *laws*.



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## Laws? two kinds:

### On the one hand

You can *only* enter a population via birth or in-migration. You can *only* leave a population via death or out-migration. (conservation)

### On the other hand

The forces of demographic change (mostly fertility and mortality) are bounded and very empirically regular, ergo law-abiding.



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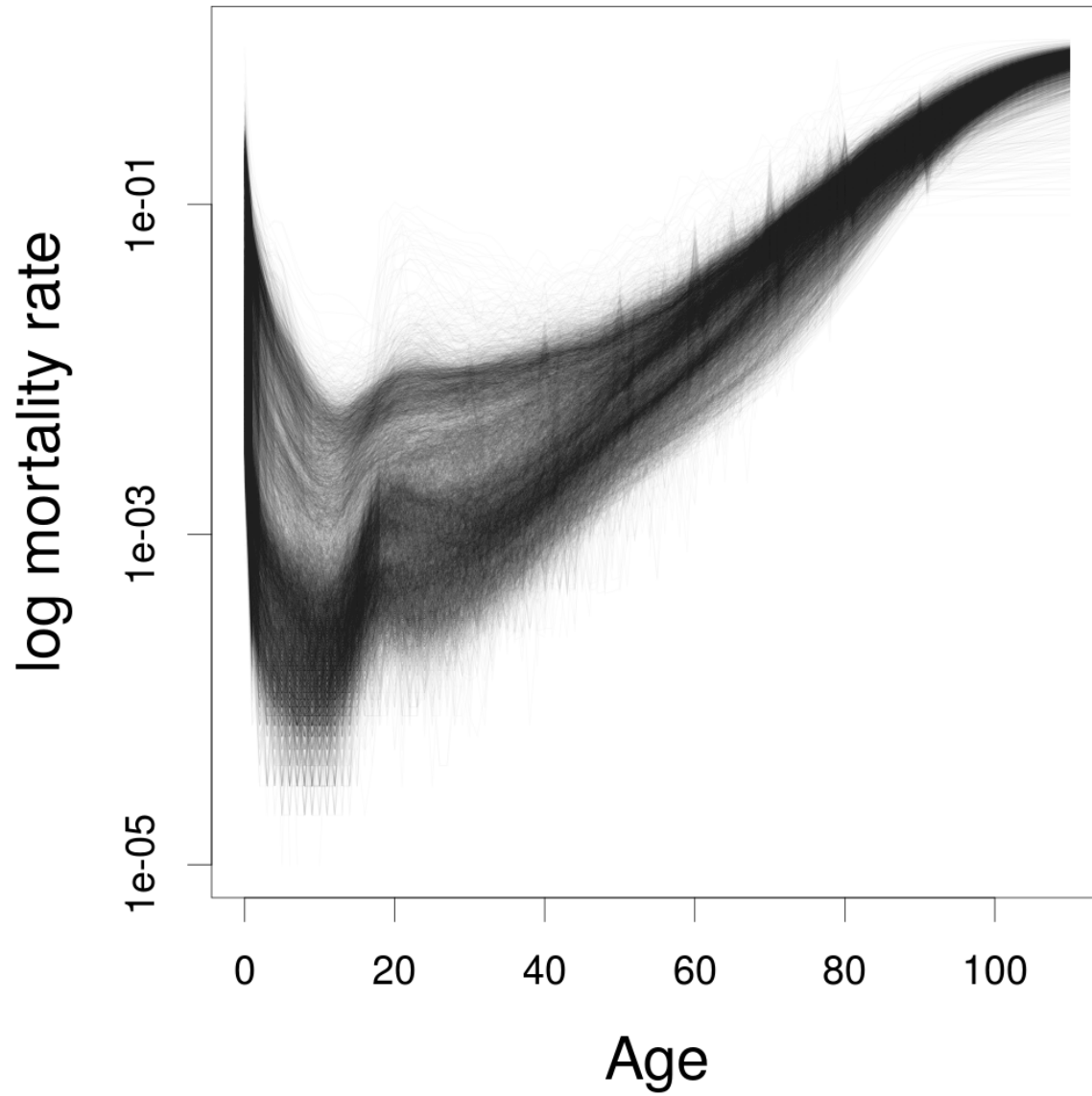
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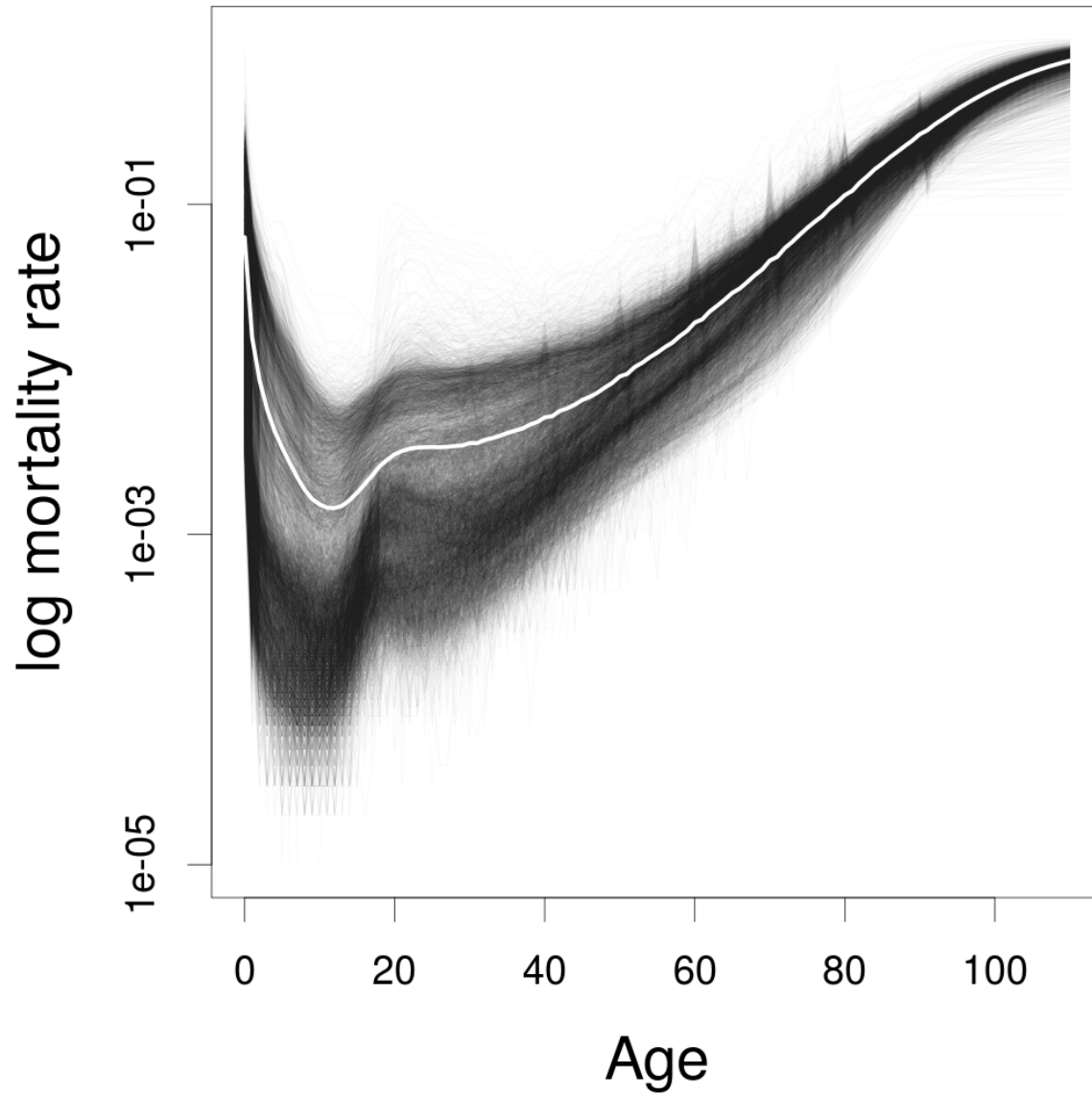


# laws for mortality





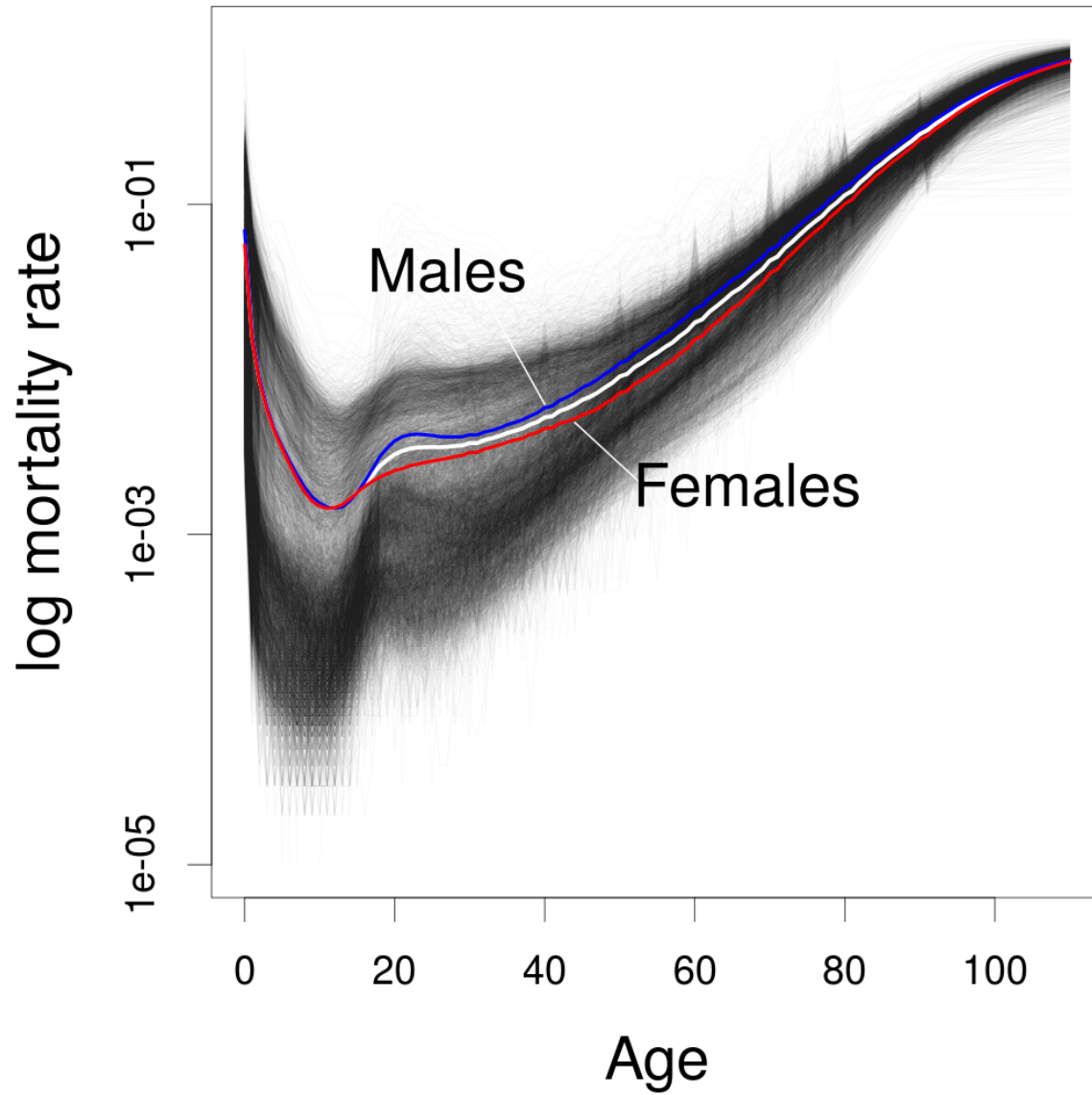
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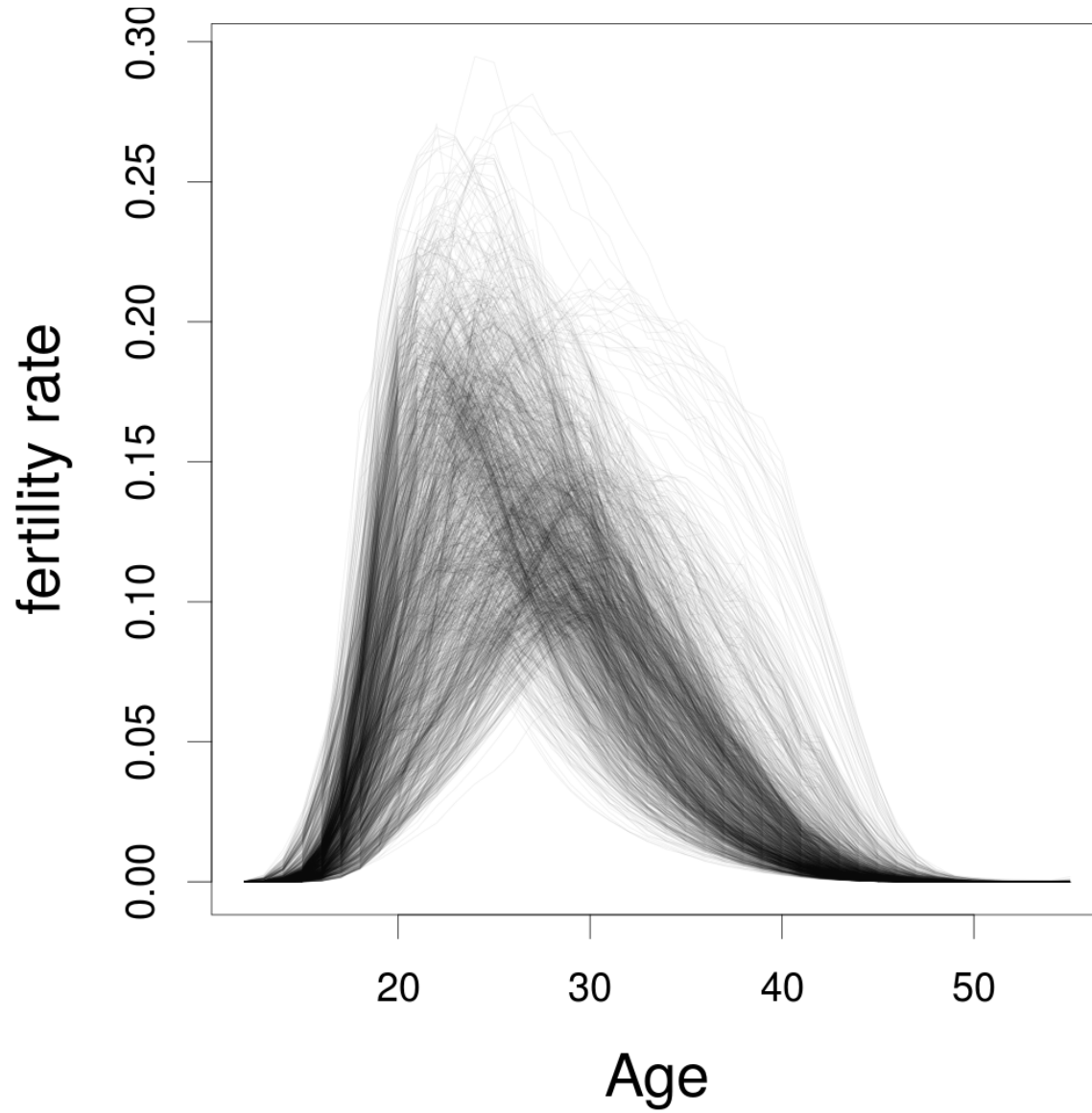


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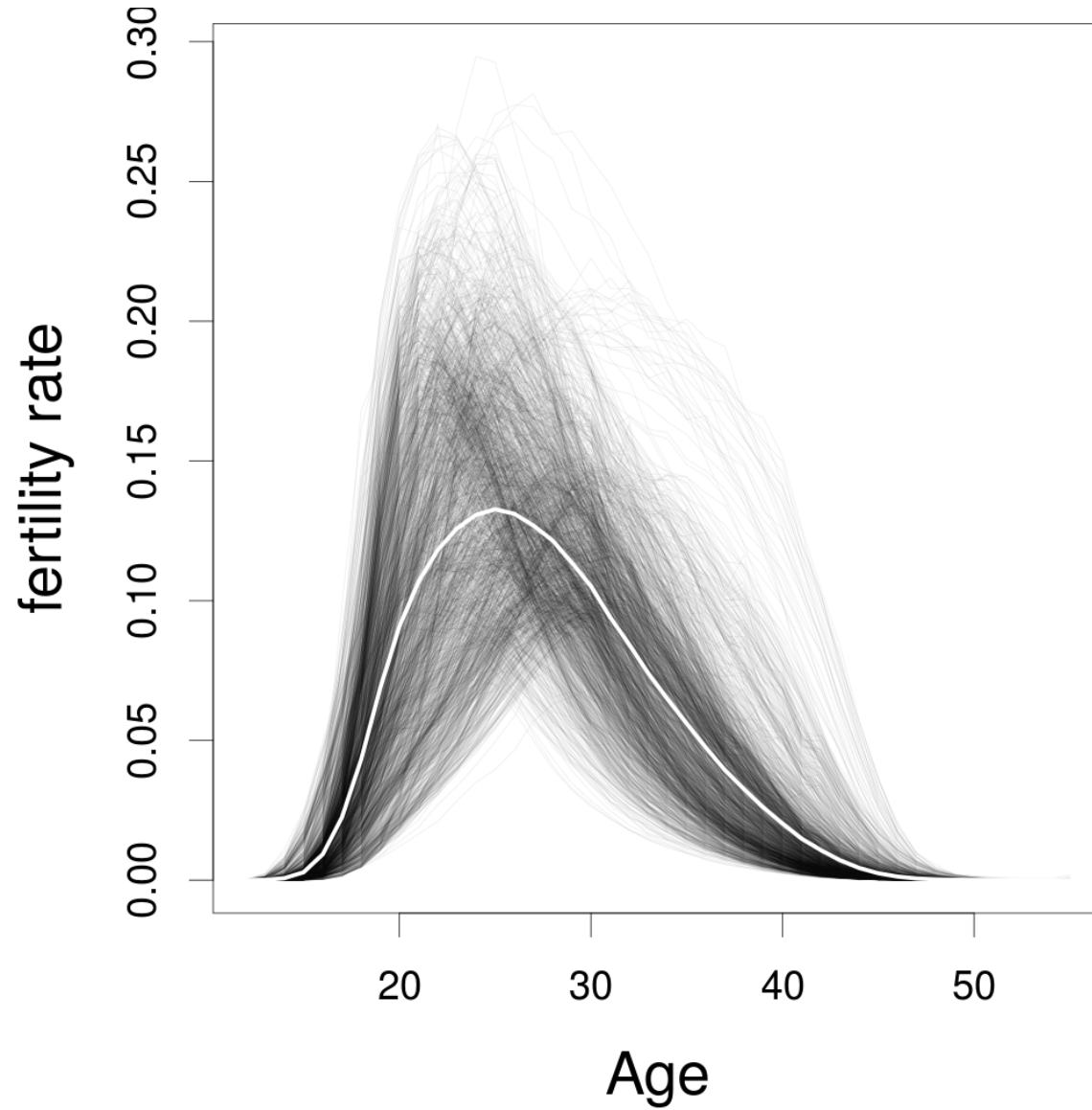


## laws for fertility



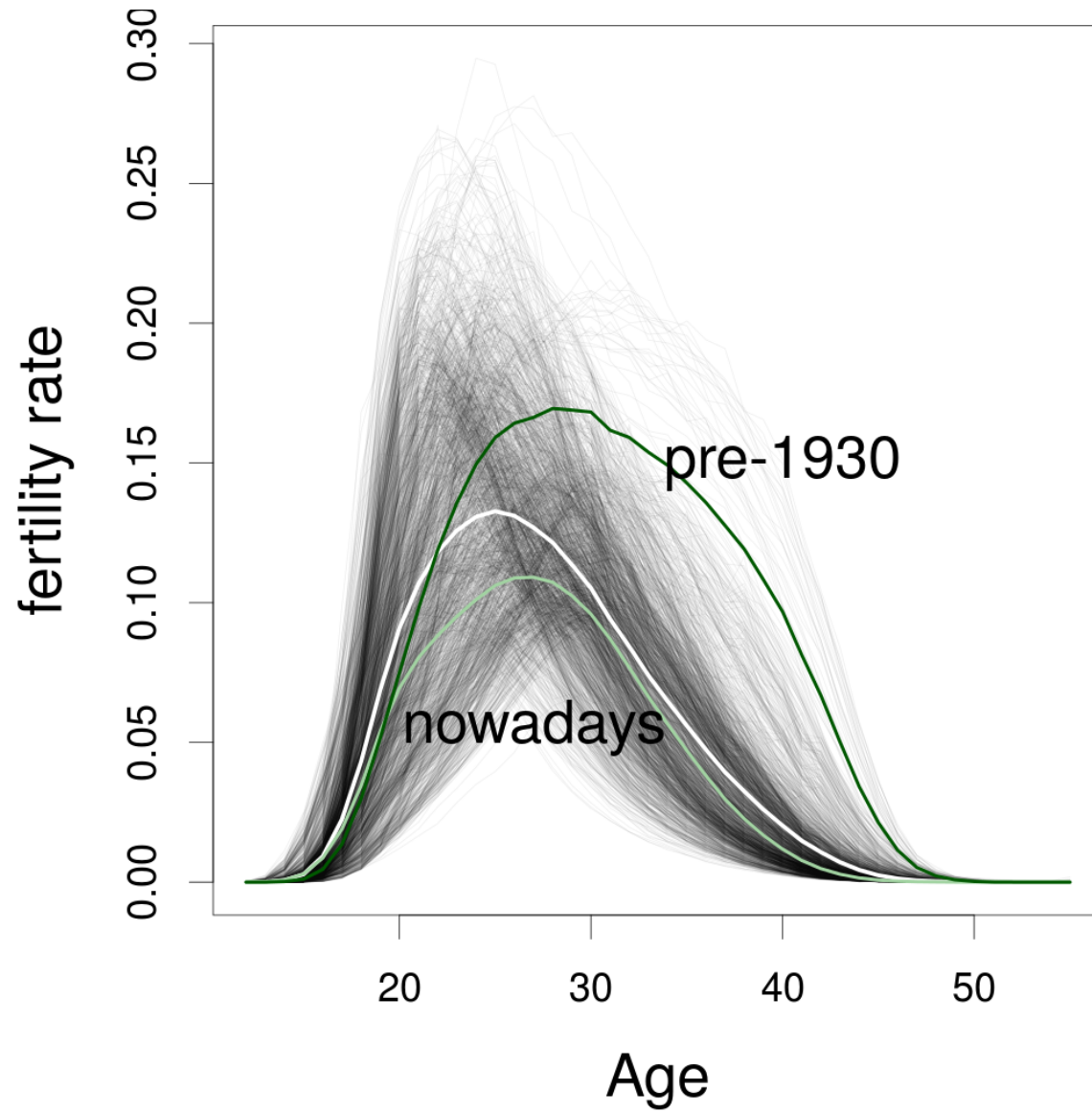


## laws for fertility



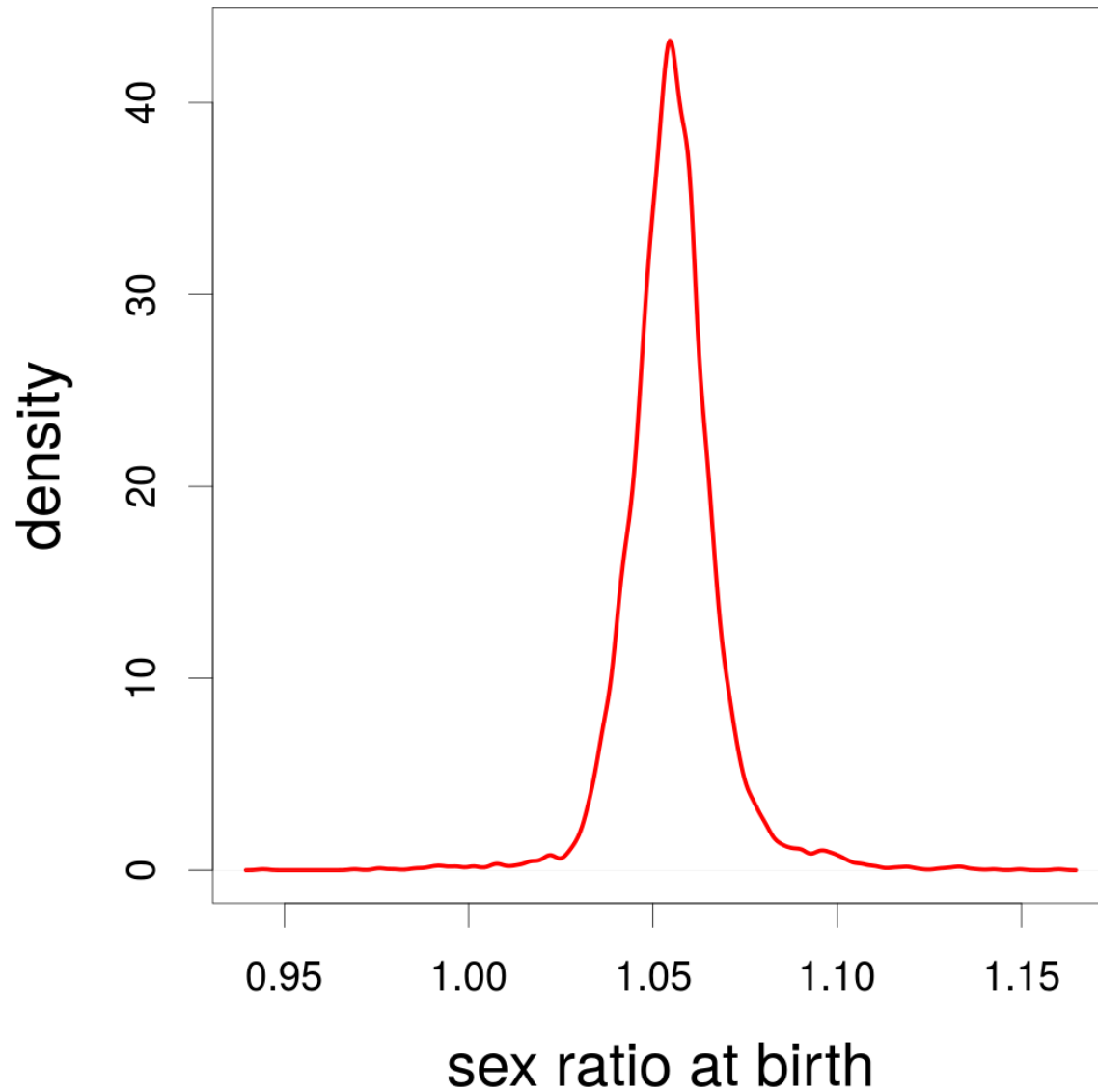


## laws for fertility





## laws for fertility: Sex ratio at birth





# continuous and discrete time

## On the one hand

Populations are composed of a finite number of individuals. Events (birth, death) are usually observed in discrete intervals. Problem for differentiable equations for things?

## On the other hand

We can think of underlying risk as a continuous and smooth function. And we can think of population processes in the limit as continuous functions. Plus continuous math is easier on the eyes.



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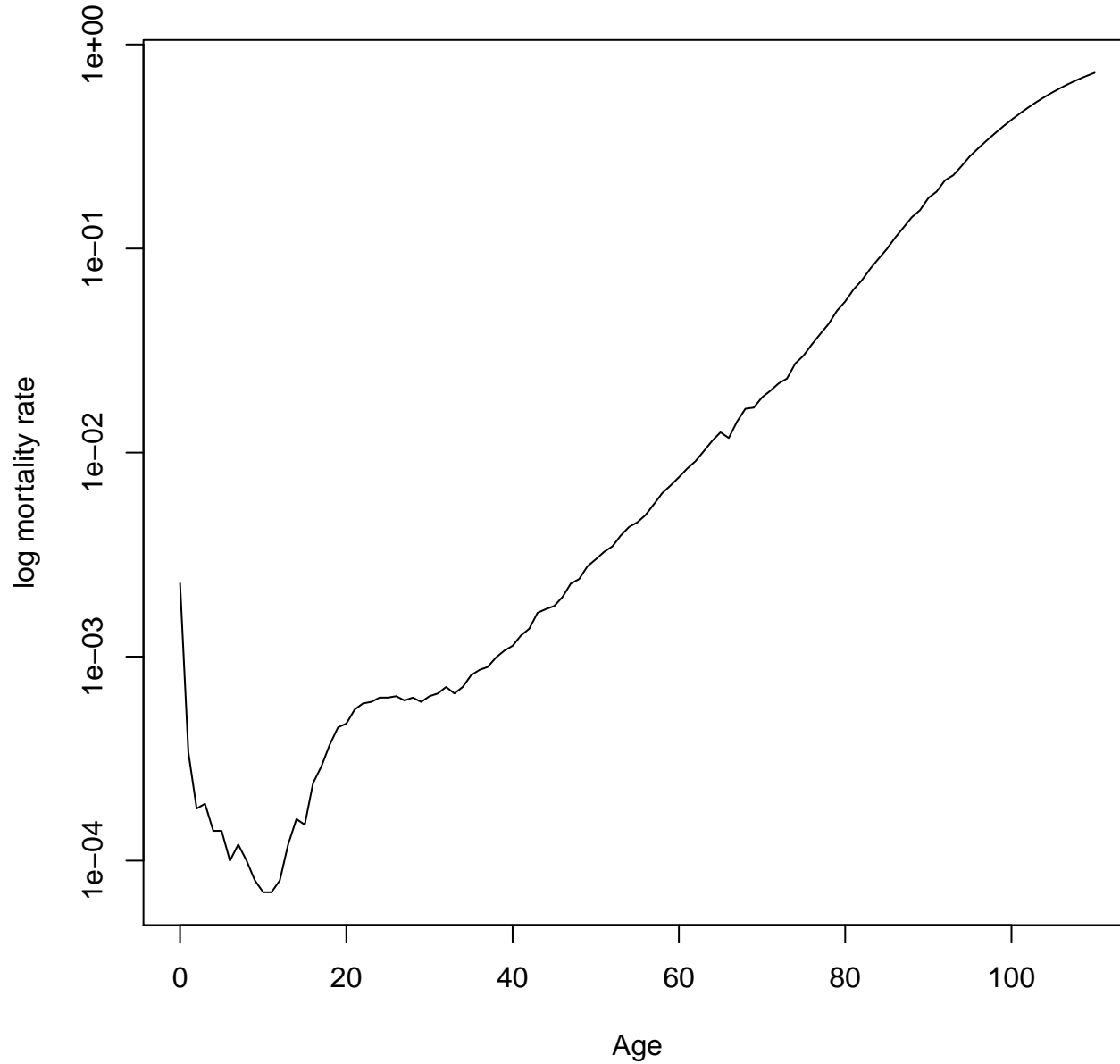
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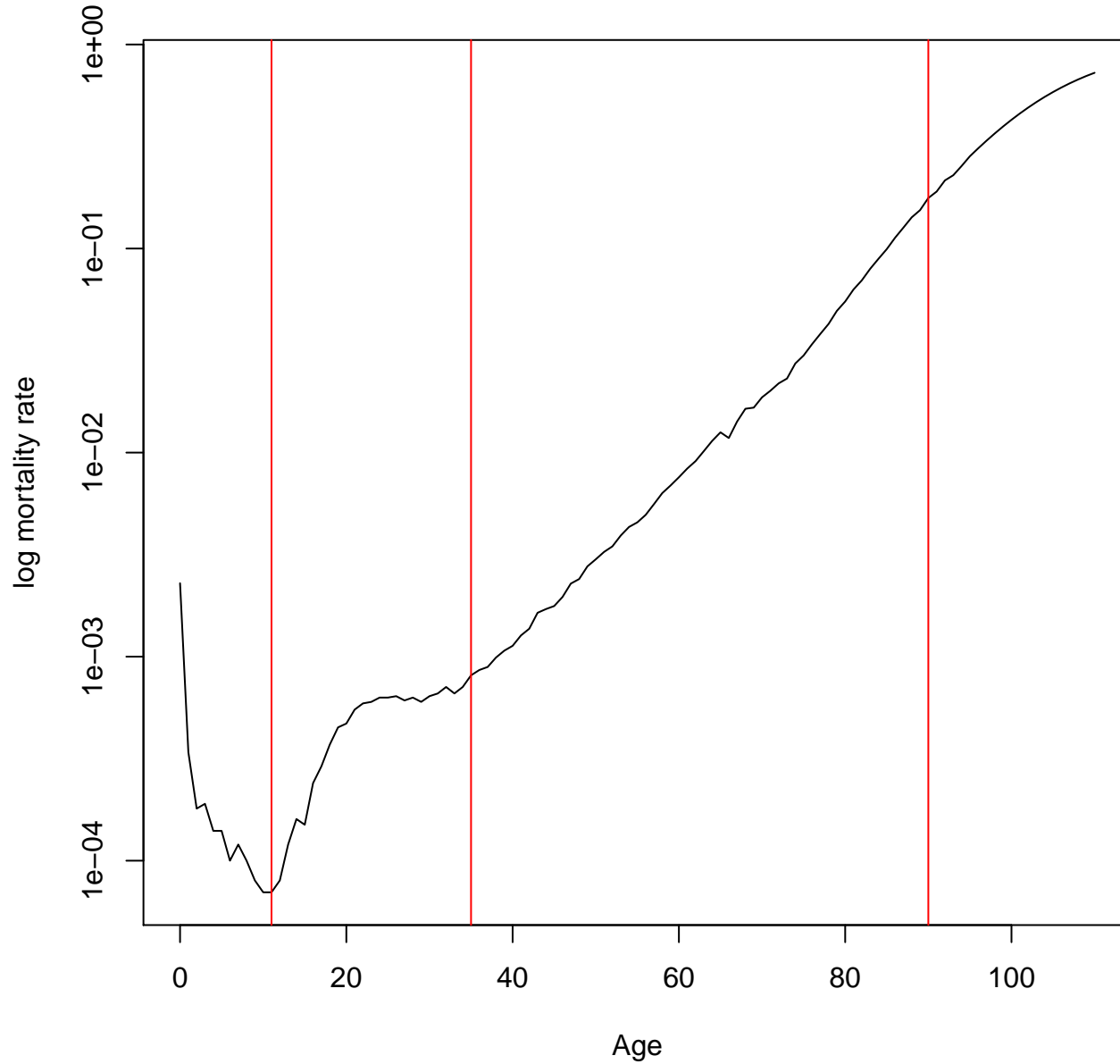
# Models of understanding





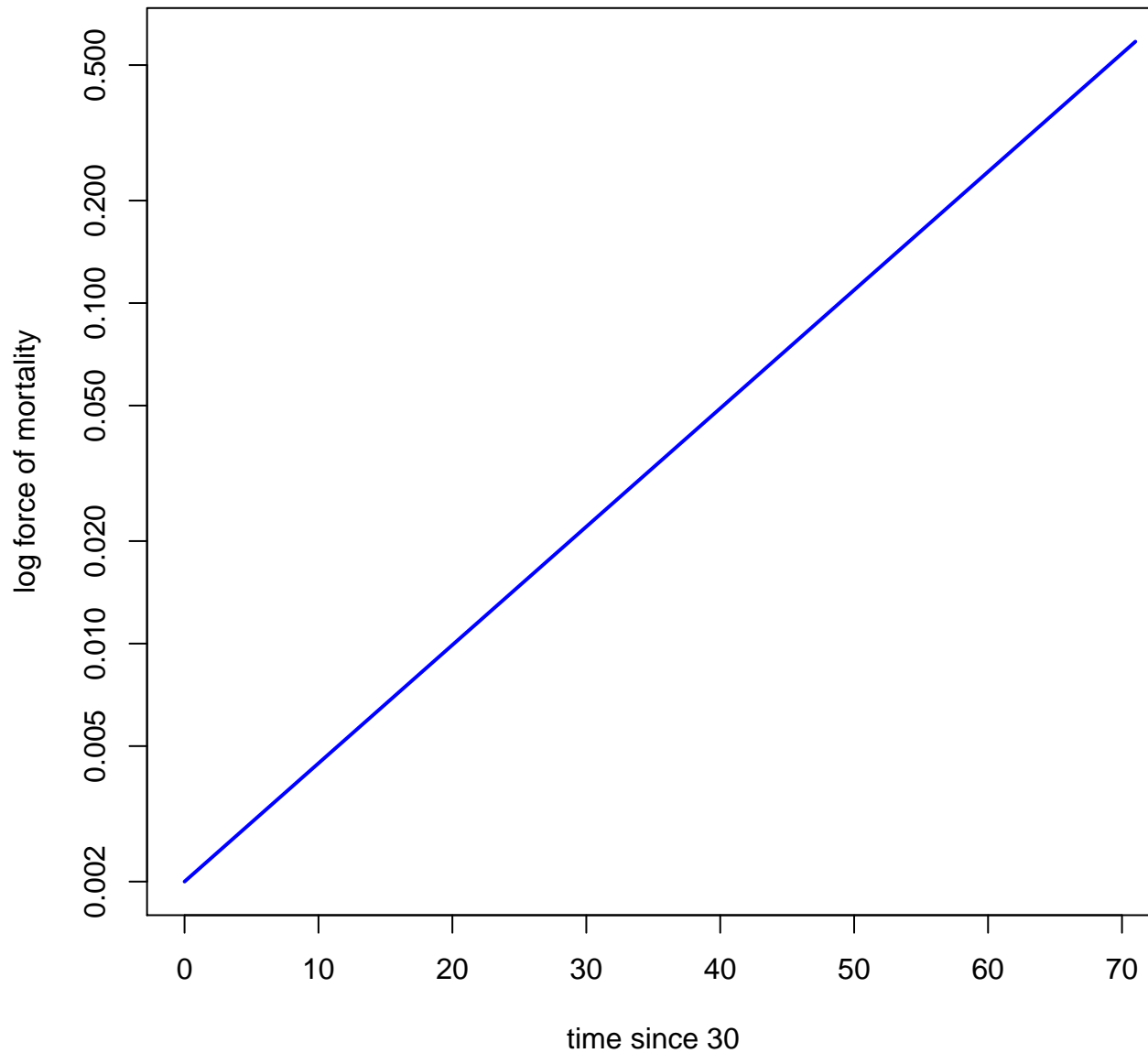


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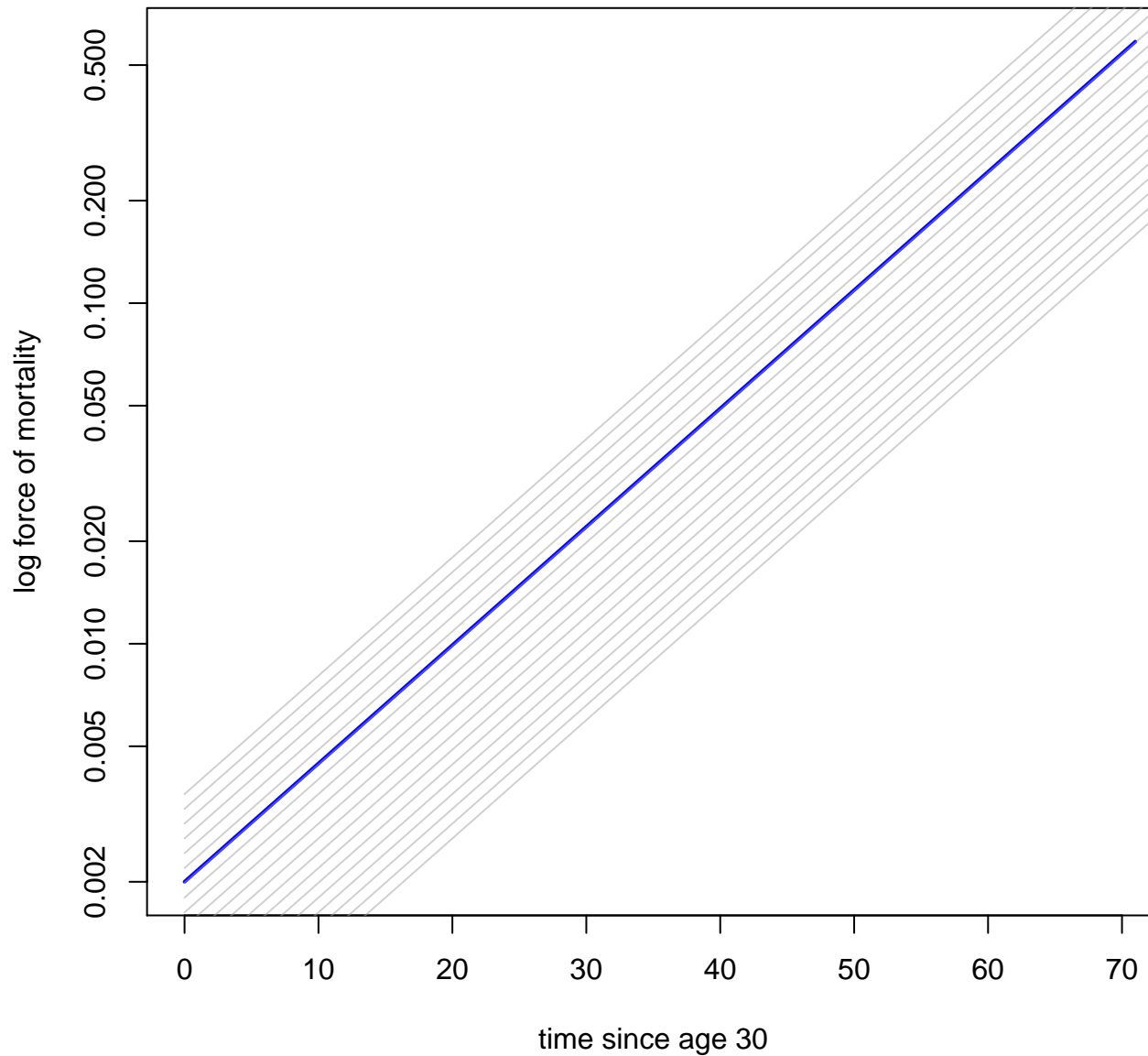


# Models of understanding: frailty



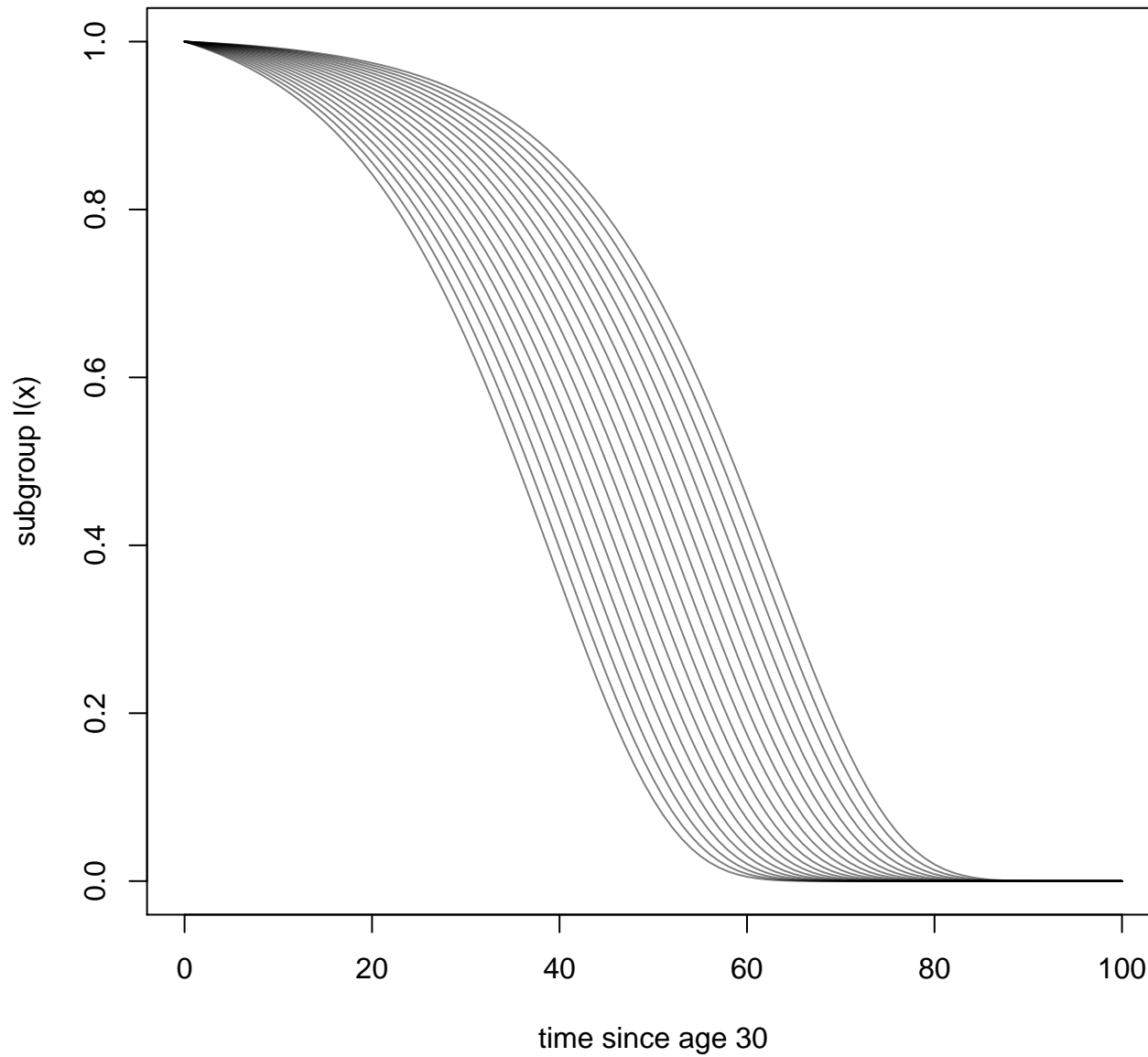


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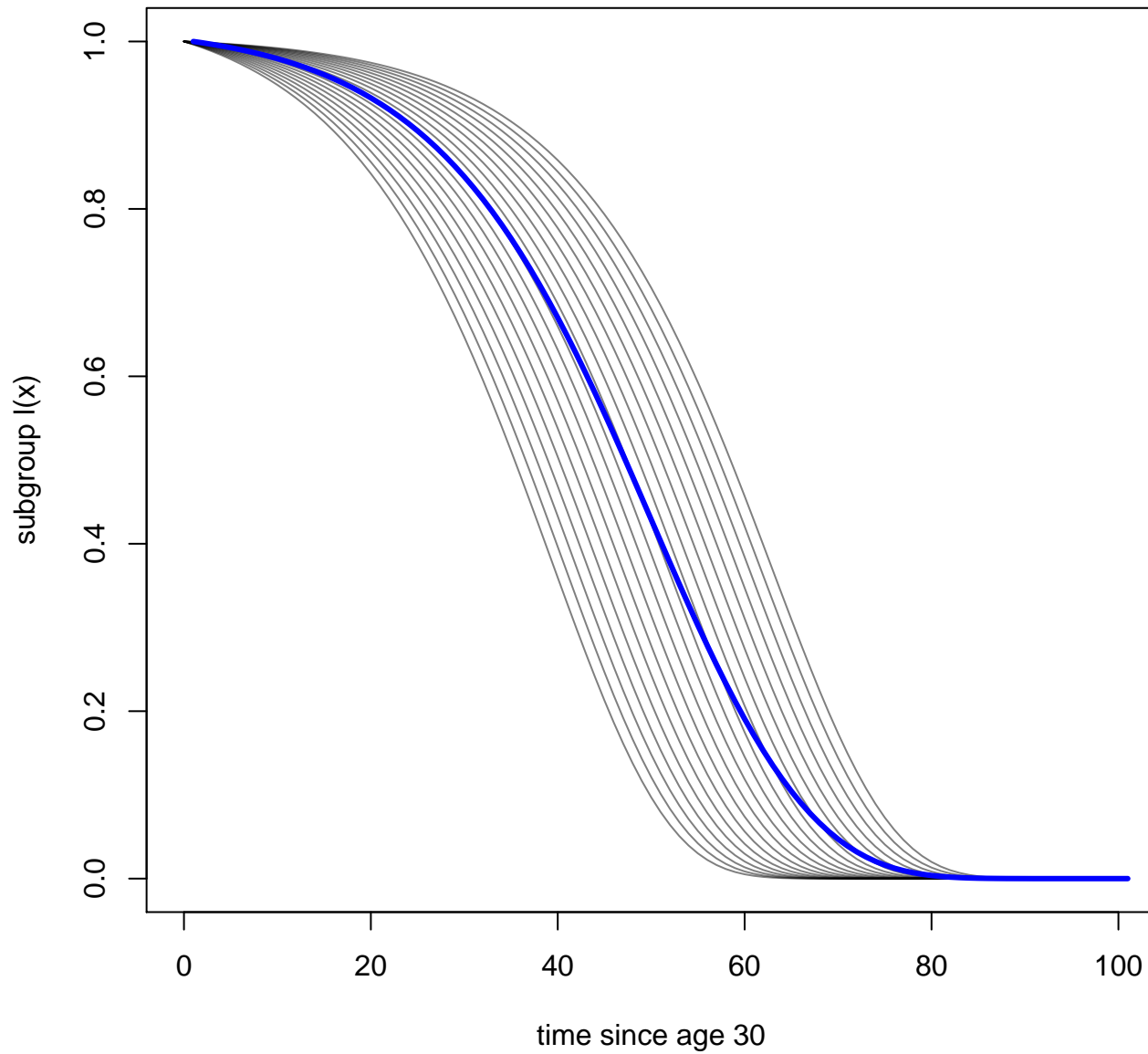


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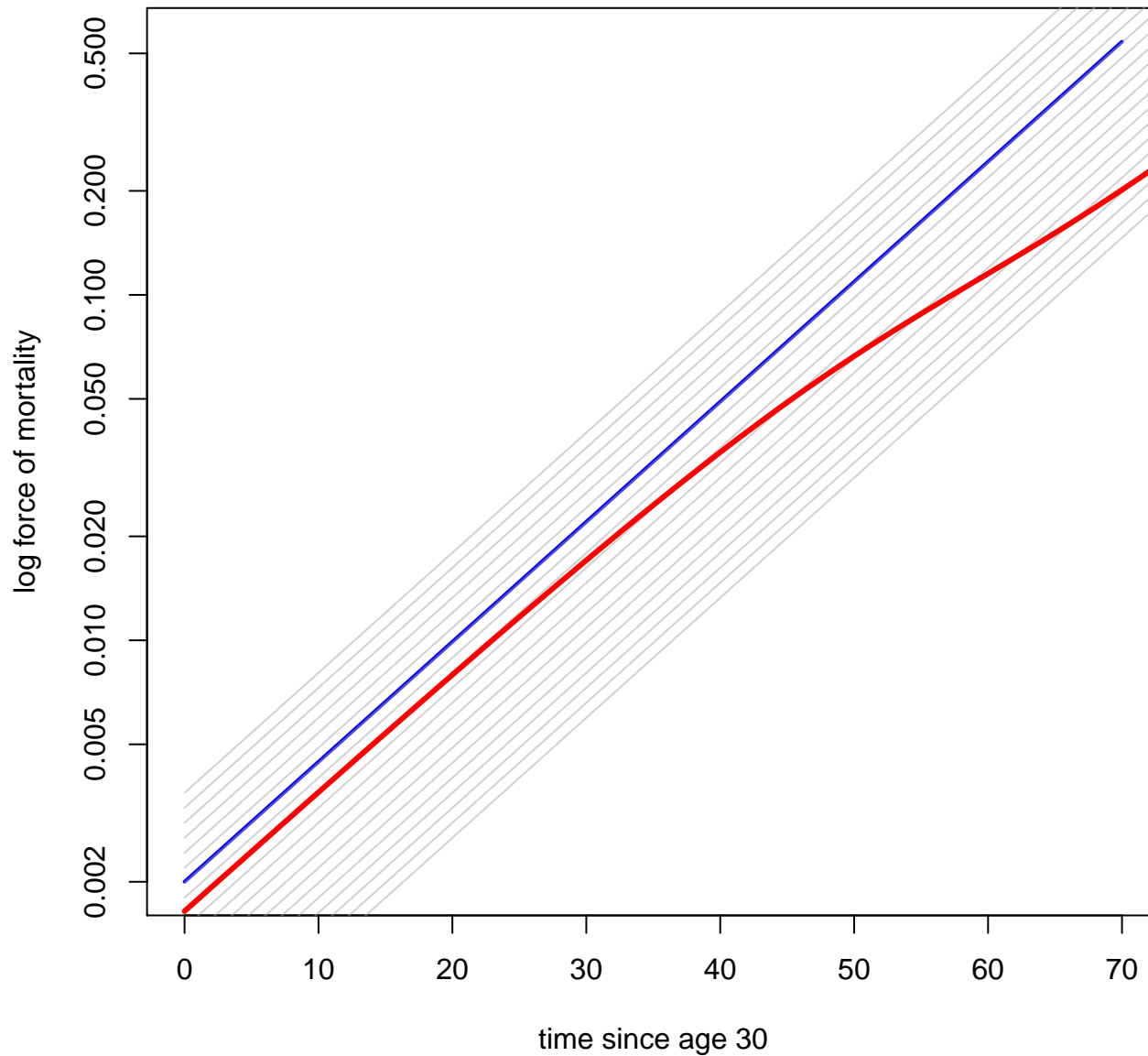


# Models of understanding: frailty





# Models of understanding: frailty





# Models of understanding: and yes, there's math

$$P_B(y, t) = \frac{1}{1 + h \exp(-k(t)(1 - \lambda_B)\phi_s(y))} \quad (8.16)$$

$$\begin{aligned} \frac{\partial P_B(y, t)}{\partial t} &= -\frac{\partial \ln(k(t))}{\partial t} [h k(t)\phi_s(y)(\lambda_B - 1)P_B(y, t)(1 - P_B(y, t))] \quad (8.17) \\ \frac{\partial \ln(P_B(y, t))}{\partial t} &= \frac{\partial \ln(k(t))}{\partial t} [k(t)\phi_s(y)(1 - \lambda_B)(1 - P_B(y, t))] \end{aligned}$$

where  $\phi_s(y) = \int_{Y_1}^y \mu_s(x)dx$  and  $h = (1 - g)/g$ .

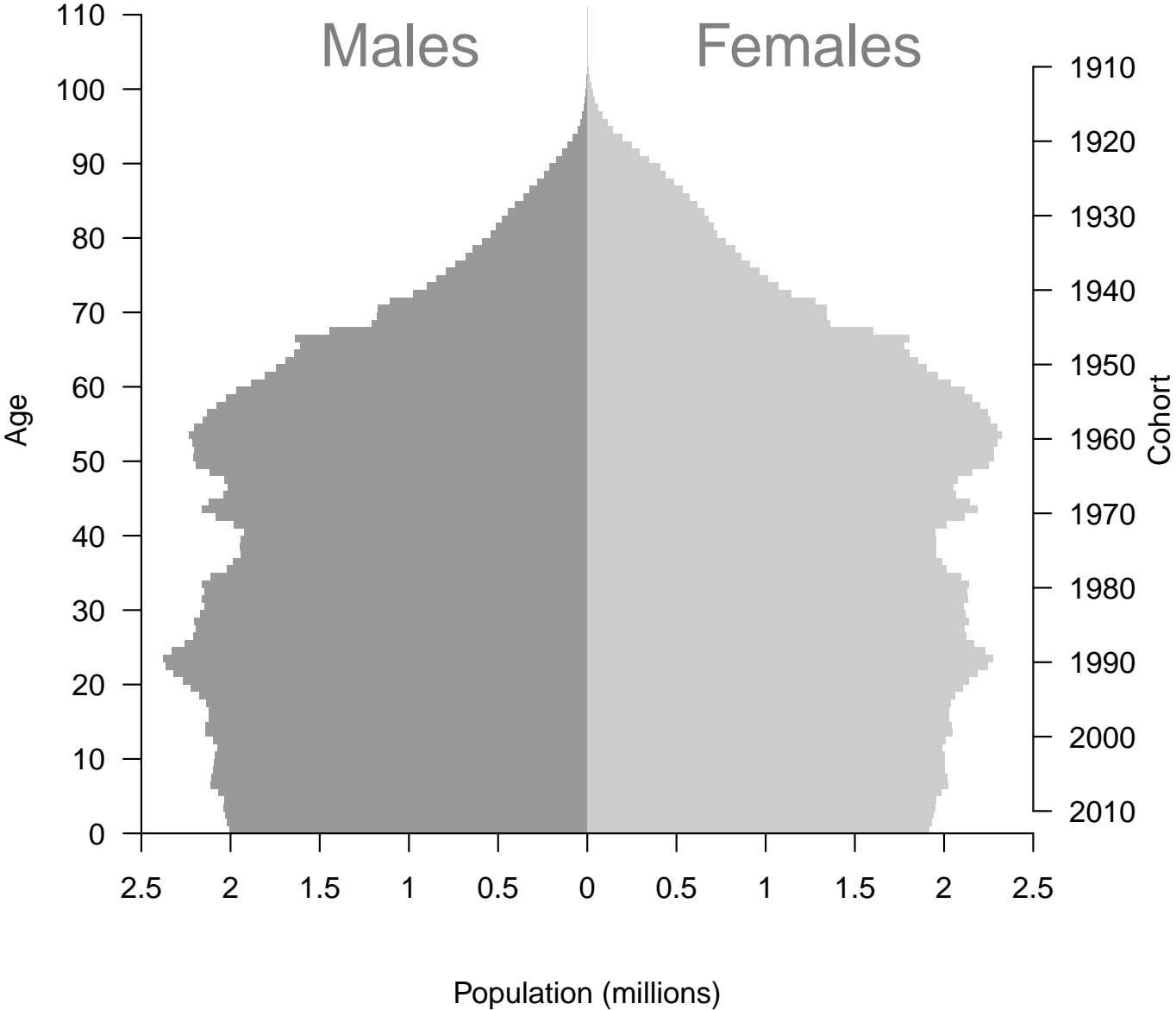
The mean value of frailty at age  $y$  and time  $t$  is:

$$E_{yt}(\lambda_B) = P_B(y, t)(\lambda_B - 1).$$

Taking logs in (8.15) and then derivatives with respect to time we get

$$\frac{\partial \ln(\bar{\mu}(y, t))}{\partial t} = \frac{\partial \ln(k(t))}{\partial t} - \left\{ \frac{(\lambda_B - 1)(\partial P_B(y, t)/\partial t)}{P_B(\lambda_B - 1) + 1} \right\}. \quad (8.18)$$

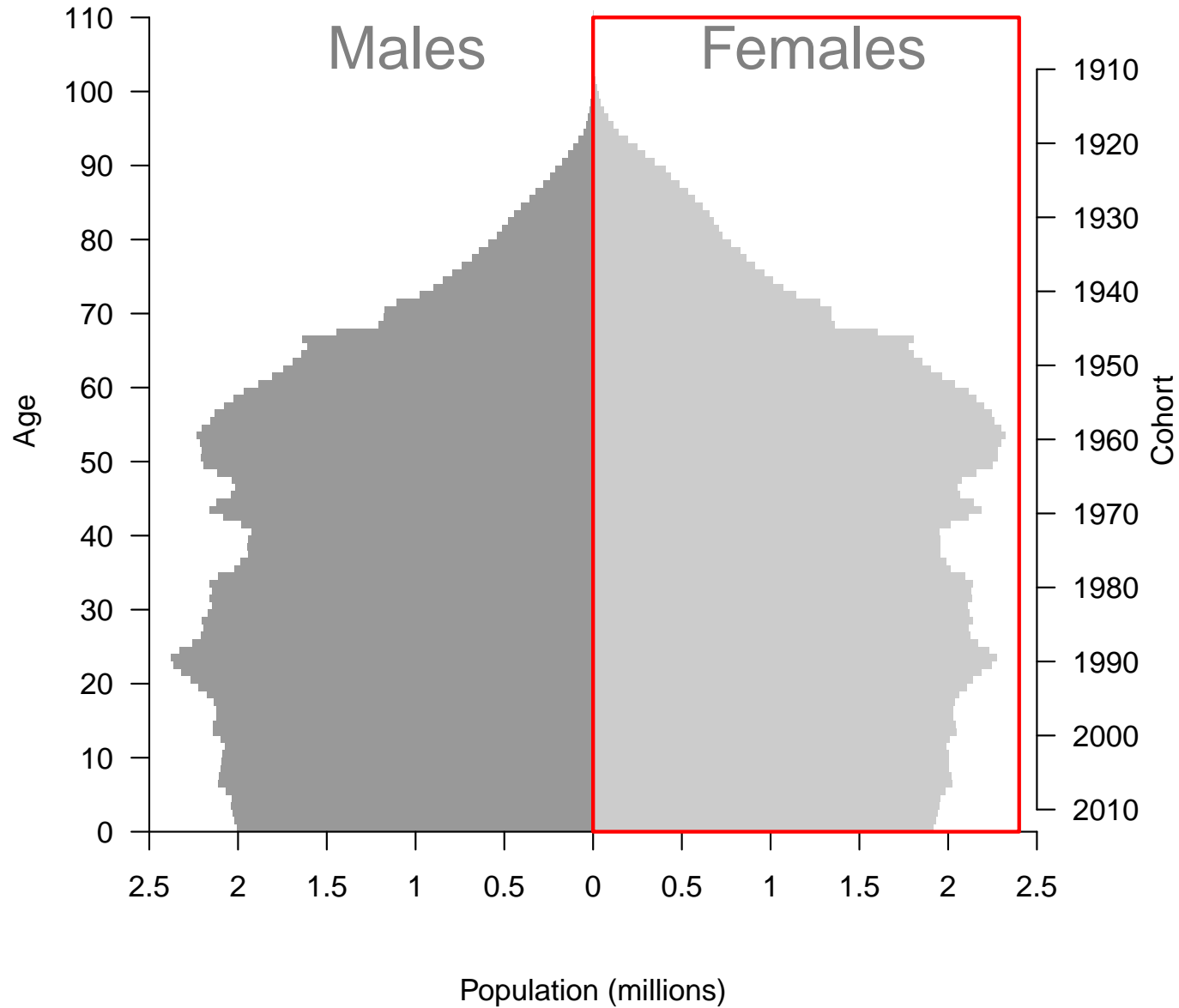
screenshot from Palloni & Beltran-Sanchez (2016)







# Population renewal





# Population renewal



## more math in demography

- indirect methods of estimation: requires models
- data quality control: requires models
- models of interactions, contagion, mixing: populations are heterogeneous
- models of health and disease transitions
- parametric models of mortality and fertility
- Math is at the core of demography



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- the Census Bureau, CDC, NIH, State govts
- academia: Berkeley, Princeton, Penn, IHME
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